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Foreword

VOLUME 61 of the Transactions of The American Society of Mechanical Engineers contains the individual papers published during 1939 under the sponsorship of the Society's professional divisions and technical committees, including the contributions of the Applied Mechanics Division issued originally in a quarterly known as *Journal of Applied Mechanics*, and the 1939 Society Records and Index. The technical papers and reports that make up this volume represent the Society's annual contribution to the permanent record of mechanical-engineering achievement. Most of these papers and reports were presented at meetings of the Society and of its professional divisions and local sections and were published in monthly issues, eight being distributed as the Transactions of The American Society of Mechanical Engineers and four as the *Journal of Applied Mechanics*. Other papers published by the Society but not included in this volume may be located by means of the Index on page RI-49 at the end of this volume.

Because of the fact that material of which this volume is composed was originally issued periodically as Transactions, *Journal of Applied Mechanics*, and Society Records, three sets of page numbers will be found. Numbers without letter symbols are those of the eight issues of the Transactions, those with letter symbol A preceding the number refer to the pages of the *Journal of Applied Mechanics*, which follow, and those with letter symbol RI to the Society Records section, which concludes the volume.

All sections of the Transactions are bound together at the end of the year for the convenience of libraries and of engineers who wish all of the papers in permanent form. Copies of the bound Transactions will be found in depositories located in selected engineering, university, and public libraries throughout the world. A complete list of these depositories will be found on pages RI-45 to RI-48 of the Society Records and Index. Copies of the Transactions have also been set aside for sale.

The Society Records of the A.S.M.E. for 1939, which form a part of these Transactions, are the permanent records of the Society's activities for the year, including lists of committees. The Index provides a means of locating special A.S.M.E. publications and articles in *Mechanical Engineering* as well as those in the Transactions.

THE COMMITTEE ON PUBLICATIONS

Some Problems Related to the Preparation of Illinois Coals[1]

By L. C. McCABE,[2] URBANA, ILL.

The author discusses the mixtures of coal and mineral matter from Illinois mines and relates the problems encountered in cleaning them. He points out the need for and the method of separating the coal and refuse, and discusses the losses encountered in coal separation. Attention is called to the reduction of ash and sulphur and to the enhanced heating value of washed coal, the value of which is difficult to overestimate because of the resultant reduction of fly ash and sulphur fumes during burning. The author states that any effort directed toward the reduction of atmospheric pollution by fly ash and sulphur fumes through the use of better prepared fuel should be encouraged.

COAL preparation in Illinois has long been characterized by good hand picking and screening. In recent years it has come to embrace mechanical cleaning, dedusting, drying, surface treatment by chemicals and oils, and crushing.

The second coal washer in the United States was erected in Illinois in 1870.[3] Development for a number of years was fairly rapid, and in 1908 Illinois ranked first as compared with other coal-producing states in the total amount of washed coal. The washed-coal tonnage increased until 1917 and then rapidly declined until in 1929 only 527,000 tons out of a production of 61,000,000 tons were washed. In 1930 there was a revival of interest in coal cleaning which reached such proportions that in 1936 more than 6,000,000 tons were mechanically cleaned out of a production of 51,000,000 tons. In the period of six years (1930–1936) the amount of washed coal has increased from less than 1 per cent to approximately 12 per cent of the total production. Today there are 21 cleaning plants in operation in the state and three or four awaiting construction. It is probable that when these plants are in full operation, they will produce twelve or fifteen million tons of washed coal annually.

Important factors involved in the trend toward mechanical cleaning are the increased production from mechanized underground workings and from strip mines which do not lend themselves to careful separation of coal and refuse during loading, increased competition from oil and gas, the perfection of mechanical coal-burning equipment with exacting fuel requirements, and the adoption of antismoke ordinances by municipalities.

Selected coals chosen from the producing beds of the State will best illustrate the problems encountered in modern preparation plants in Illinois and at the same time demonstrate the benefits accruing to the coal consumer.

The modern washery is operated to eliminate noncoal material from the feed entering the plant. Carbonaceous and clay shales from the roof and underclay from the floor of the seam are loaded with the coal to some extent in all mines, but their removal in the cleaning process is relatively simple because their specific gravity is considerably greater than that of the coal. Occasionally, however, clays or shales disintegrate and remain in suspension in the wash water. As a result, the coal in the wash box becomes coated or "whitewashed" and clean-water sprays must be used to remove the coating and give the coal a clean appearance. Where the water supply is limited, as it is in some parts of Illinois, elaborate settling and clarifying systems must be utilized for settling out the suspended solids before the water can again be introduced into the cleaning system.

The bands, lenses, or veins of shale, pyrite, and "bone" which are commonly associated with coal beds may be broken free in mining or may remain attached to the coal. In the latter case the coal adhering to the impurity is frequently wasted unless the washery reject is crushed and recleaned.

Intimate mixtures of coal and mineral matter present the most difficult cleaning problem. This may be illustrated by a comparison of the washability characteristics[4] of two Illinois screenings from mines *A* and *B*, as given in Table 1; such a comparison can also be made from Figs. 1 and 2.

The ash content of the face sample from mine *A* is higher than the average for Illinois and the ash in the sample from mine *B* is considerably lower, but both mines produce raw screenings of about the same ash content. The workings in mine *A* are under a black-slate or a limestone roof and on a bone or a hard clay floor, conditions which permit little introduction of impurities foreign to the coal bed. Mine *B* has a friable shale roof and a soft clay floor, and a considerable quantity of these materials is loaded with the coal.

When cleaned at 1.50 specific gravity, the curves of coal recovery versus ash in Fig. 1 show a recovery of 80.6 per cent for coal *A*, while from Fig. 2, it is 81.6 for coal *B*. The ash content of the former is 9.5 per cent and of the latter 5.0 per cent. The refuse-reject curve shows the refuse from coal *A* to contain only 53.5 per cent ash while the refuse from coal *B* is 73.8 per cent ash. This would indicate a relatively large quantity of combustible matter in the refuse from coal *A*.

The ash-distribution curves best show the extent of the admixture of mineral matter and coal. The gentle warping of the coal-*A* ash-distribution curve, in Fig. 1, is indicative of the dissemination of ash through the coal. The sharp change in direction to the right in the coal-*B* ash-distribution curve in Fig. 2, reflects the presence of the free shale and clay which found its way into the coal during mining.

Separation of coal and refuse is difficult when large quantities of the feed have a specific gravity approaching the effective specific gravity of the separating medium. The ±0.10 specific gravity curves of Figs. 1 and 2 show that 15 per cent of coal *A*

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[1] Published by permission of the Chief, Illinois Geological Survey, Urbana, Ill.

[2] Associate Geologist, Coal Division, Illinois Geological Survey.

[3] "Summary Facts on the Coal Resources and Coal Industry of Illinois," in collaboration with the Engineering Experiment Station, University of Illinois. State of Illinois Geological Survey Division, Urbana, Ill., 1935.

Contributed by the Fuel Division and presented at the Semi-Annual Meeting of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS, held in St. Louis, Mo., June 20–24, 1938.

Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th St., New York, N. Y., and will be accepted until February 10, 1939, for publication at a later date. Discussions received after the closing date will be returned.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[4] "Washability Characteristics of Illinois Coal Screenings," by D. R. Mitchell and L. C. McCabe, Illinois Geological Survey, Report of Investigations No. 48, 1937, p. 18.

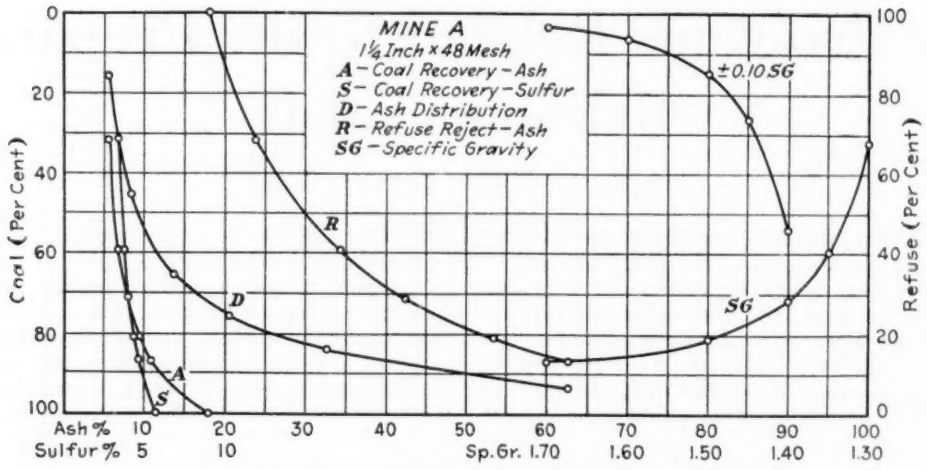


FIG. 1 WASHABILITY CHARACTERISTICS OF COAL FROM MINE *A*

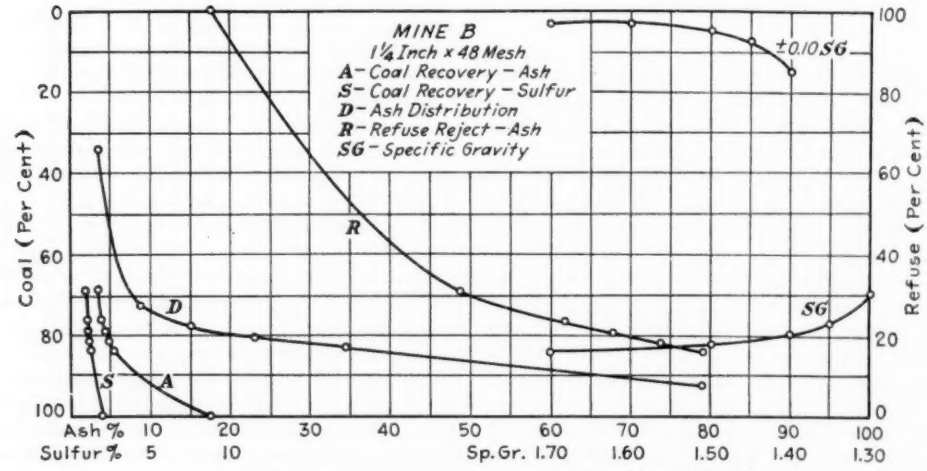


FIG. 2 WASHABILITY CHARACTERISTICS OF COAL FROM MINE *B*

and only 4.3 per cent of coal *B* is between 1.40 and 1.60 specific gravity. Because of the large quantity of material having a specific gravity near the separating gravity, coal *A* is a difficult coal to clean while coal *B*, with a relatively small amount of material of about 1.50 specific gravity, presents a simple cleaning problem. The use of specific-gravity–distribution curves makes it possible to select a specific gravity at which washing can be done most efficiently. Furthermore, they show that the possibility of efficient operation at one specific gravity does not assure similar efficiency at another.

Occasionally, in spite of a knowledge of the washability characteristics of the coal, considerations other than efficiency may direct the operation of a cleaning plant. The consumer, frequently without discrimination, objects to dull or slabby coal and to meet this objection a plant will operate to reject such coal, although it has a high calorific value. Sales departments may find that a washed coal having 7.1 per cent ash would be more readily marketed if the ash is reduced to 6.8 per cent. To obtain this reduction of 0.3 per cent in ash 1 or 2 per cent of coal may find its way into the washery reject.

For numerous reasons the tonnage losses in cleaning are relatively high. At some mines the feed may shrink 20 per cent or more in passing through the preparation plant. While the reject may contain little or no coal, it represents tonnage that formerly found its way into the market and must therefore be compensated for by increased price or expanded sales of washed coal.

TABLE 1 COMPARISON[a] OF RECOVERY, ASH AND SULPHUR PERCENTAGES IN COALS FROM TWO ILLINOIS MINES

	Ash						
Mine	Face sample	Unwashed screenings	Coal recovery	Float ash	Float sulphur	Sink ash	Sink sulphur
A	10.6	18.0	80.6	9.5	4.4	53.5	11.4
B	6.5	17.6	81.6	5.0	1.2	73.8	5.4

[a] Based on 1¼-in. to 48-mesh coal, dry basis, with 1.50 specific gravity

Wet-washing plants cannot effectively clean coal smaller than 48 mesh. In some instances it is removed from the larger coal by dedusting or by screening, but the most common practice in wet-washing plants is to remove it from the system with a thickener or centrifuge, or to permit it to escape to the settling pond.

The ash content of dusts from ten Illinois coals[4] recently studied ranged between 15 and 37 per cent ash. There is no ready market for such material and with few exceptions it is a waste product. It constitutes as much as 5 per cent of the tonnage in some mines.

In addition to tonnage losses, washing entails the loss of potential heat in the washery refuse. That the percentage of heat loss is not as great as the percentage of tonnage loss is apparent from Table 2, which gives a comparison of two coals in the unwashed and in the washed condition. The washability characteristics of coal from mines *C* and *D* are shown in Figs. 3 and 4,

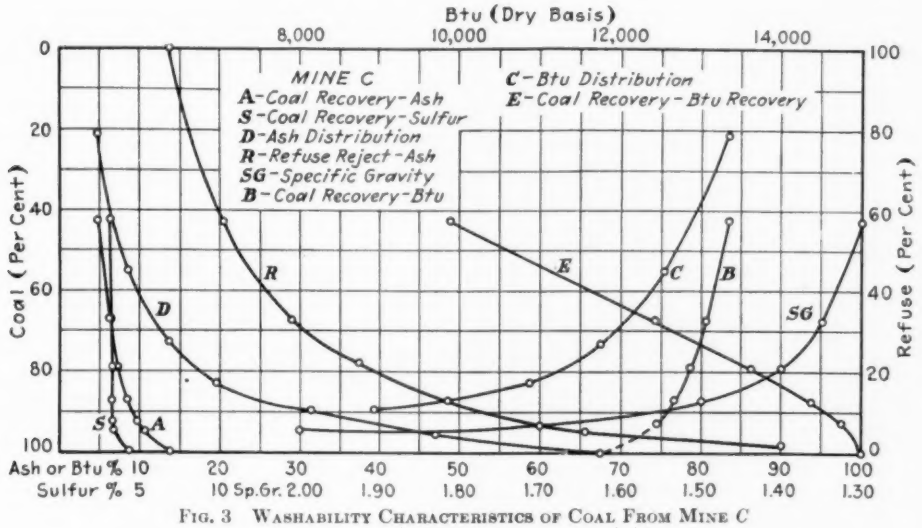


FIG. 3 WASHABILITY CHARACTERISTICS OF COAL FROM MINE *C*

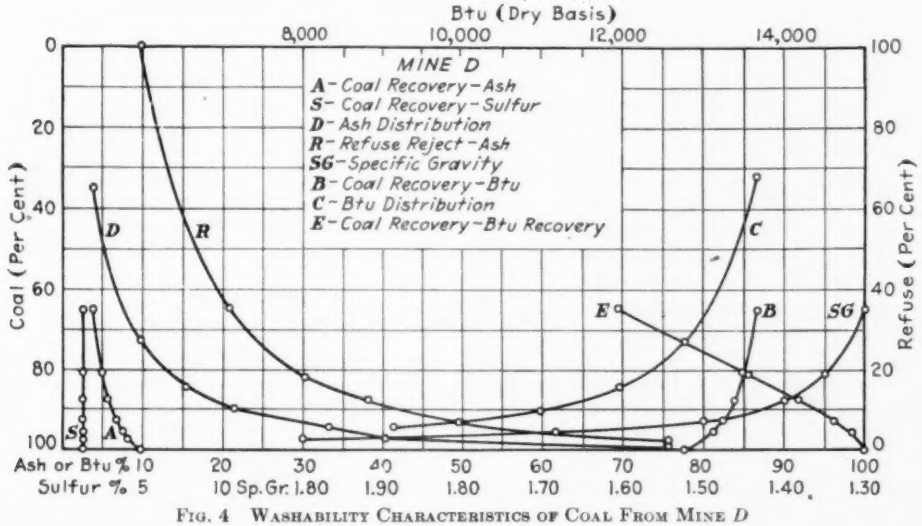


FIG. 4 WASHABILITY CHARACTERISTICS OF COAL FROM MINE *D*

TABLE 2 COMPARISON ON A DRY BASIS OF WASHED AND UNWASHED COALS FROM TWO ILLINOIS COAL MINES

Mine	Coal recovery, per cent	Ash, per cent	Sulphur, per cent	Heat value, Btu per lb	Btu recovered, per cent	Btu increase, per cent
C	100	13.62	4.5	11,756	100	...
	90	9.00	3.1	12,580	96	7.0
D	100	9.85	1.2	12,743	100	...
	90	6.50	1.0	13,300	94	4.4

respectively. When washed for a tonnage recovery of 90 per cent, 96 per cent of the Btu value is recovered in coal *C* and 94 per cent in coal *D*. The greater part of these losses of potential energy may be accounted for by the low calorific value of "bony" coal and pyrite which, in part, constitute the refuse.

Because of the reduction of its ash and sulphur content, a coal is enhanced in its calorific value by washing. Coal *C*, containing 13.6 per cent ash, 4.5 per cent sulphur, and 11,756 Btu per lb, as it comes from the mine, may be washed at 90 per cent recovery to produce a fuel with 9 per cent ash, 3.1 per cent sulphur, and 12,580 Btu per lb. Changes in the character of coal as a result of washing which are of interest to the consumer are 31 per cent reduction of sulphur, 34 per cent reduction of the ash percentage, and 7 per cent increase in calorific value. Coal *D* does not offer comparisons of the same magnitude because it has relatively low ash and sulphur content as it is mined. However, the heat value is raised from 12,743 to 13,300 Btu per lb by washing.

A discussion of the problems related to the preparation of Illinois coals would not be complete without some consideration of the distribution and varieties of sulphur[5] and their reaction to the cleaning process.

Sulphate, pyritic, and organic sulphur are the three varieties generally recognized. The sulphate form is thought to represent mainly the gypsum in the coal, but as the amount is usually less than 0.25 per cent it will be ignored in this discussion. Because of its yellow color, metallic appearance, and high specific gravity (4.95–5.1), pyrite is the most easily identified of the forms of sulphur. Organic sulphur is that part of the total sulphur which is neither chemically determined sulphate sulphur nor chemically determined pyritic sulphur.

The total sulphur content of most Illinois coals is high, averaging 4.37 per cent on an ash-and-moisture-free basis (3.4 per cent on the as-received basis). Fig. 5 shows the distribution of total sulphur, and Fig. 6 shows the minimum and maximum values for

[5] "Distribution of Sulfur in Illinois Coals and Its Geological Implications," by G. H. Cady, Illinois Geological Survey, Report of Investigations No. 35, 1935.

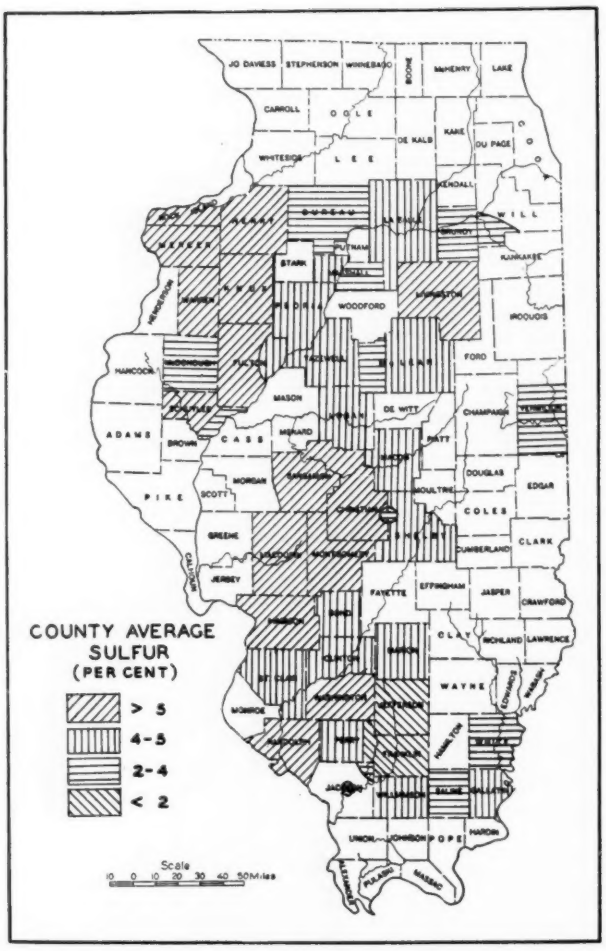


FIG. 5 COUNTY AVERAGE OF SULPHUR CONTAINED IN FACE SAMPLES OF ILLINOIS COALS ON AN ASH-AND-MOISTURE-FREE BASIS

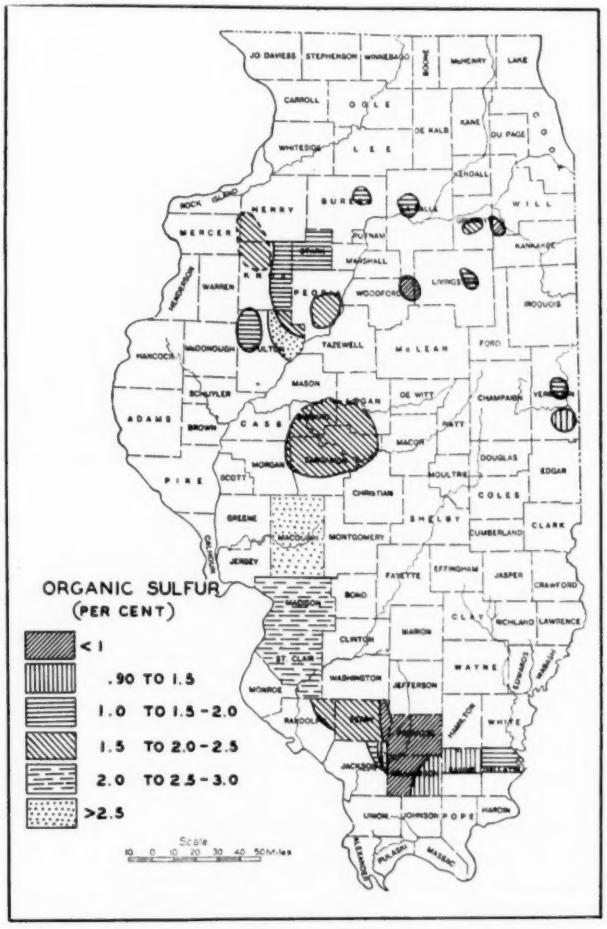


FIG. 6 MINIMUM AND MAXIMUM PERCENTAGES OF ORGANIC SULPHUR IN COAL FROM VARIOUS COUNTIES IN ILLINOIS

organic sulphur in the coal in various counties in the state,[6] information that is particularly useful in evaluating the possibility of sulphur removal by washing. The data for mapping sulphur distribution are from face samples and Cady[7] has pointed out that they do not tell the whole story in regard to the amount of pyritic sulphur present in the coal as more or less pyrite is discarded in collecting face samples. He concludes that the amount of pyrite discarded, if included in the face sample, would rarely increase the sulphur more than 2 per cent and commonly not more than 1 per cent. For practical consideration in pyrite reduction the amount of discarded pyrite need not be of concern as its specific gravity is so much greater than the coal that it is not involved in the economic limits of sulphur reduction.

There appears to be no reduction in the amount of organic sulphur in the washing process and in theory, should the pyritic sulphur be so concentrated as to be broken free in mining, and discarded in toto with the reject, the organic sulphur would represent the irreducible minimum of sulphur content. Actually it is not possible to remove all the pyritic sulphur, for much of it is disseminated through the coal in particles of microscopic size. In the high-sulphur coals, 15 to 35 per cent of the total sulphur may be removed by washing, but the low-sulphur coals are little affected, as shown for coal *D* in Fig. 4.

It was entirely relevant in discussing the problems of preparation to call attention to the magnitude of sulphur and ash reduction and to the enhanced heat value of washed coal. Greater fuel uniformity, less ashes, dust, and sulphur vapors are important corollaries. It would be difficult to overestimate the value to a municipality of the effective reduction of fly ash and sulphur fumes which can be brought about by the use of clean coal. Any effort directed toward reduction of atmospheric pollution by fly ash and sulphur fumes through the use of better prepared fuel should be encouraged because it is based on sound scientific premises.

Illinois coals can and are being burned smokelessly in equipment designed to raise the volatile matter to its ignition temperature before it passes into the stack. For the great number of domestic furnaces not particularly designed for the burning of high-volatile coals, installation of a conversion burner or stoker, or change to a processed coal appears to offer the only solution of the smoke problem without substitution of more expensive oil or gas.

[6] "Distribution of Sulfur in Illinois Coals and Its Geological Implications," by G. H. Cady, Illinois Geological Survey, Report of Investigations No. 35, 1935, pp. 24 and 28.

[7] Ibid., p. 26.

Methods Employed to Remedy Water-Hammer Shock in Pumping Systems

By E. BRUCE BALL,[1] KILMARNOCK, SCOTLAND

This paper gives certain typical problems which have been encountered in English practice in pumping-plant installations, where surge conditions have been encountered, and the changes made to remedy them. The severity of the conditions and the effectiveness of the remedies are shown by pressure-recorder charts showing the line surges caused by the operation of automatic control apparatus.

The paper is the contribution of The Institution of Mechanical Engineers of Great Britain to the Second Symposium on Water Hammer.

THE problem of surge and water hammer, accentuated by the introduction of automatic control on complex centrifugal-pumping installations, has become increasingly serious. Many cases have been reported where severe pressure changes have occurred and, after investigation, almost complete elimination of the surges was effected.

Examples of wide divergency have been selected to illustrate the method of analysis and solutions applied. The first two cases deal with check-valve closure on short pumping systems and the third with gate-valve closures on a long column. Finally, an example is given where an air vessel overcame the objectionable pressure surge.

Case 1 Hydraulic Pumping Plant for Operating Elevators

The plant consists of two 10-stage 5-in-diam turbine pumps each delivering 350 gpm at 1155 ft pumping head running at a speed of 1460 rpm. The pumps are independent but their suction and delivery pipes are connected, and only one pump is operated at a time. The pumps are supplied through an 8-in-diam suction main from a tank approximately 55 ft above them, and deliver through a 6-in-diam pipe to two hydraulic cylinders controlling two elevators. Between the pumps and the elevators there are three accumulators which are equally loaded and independent; all three of them have choke valves which come into operation when the accumulators are nearly at the top of their stroke. One of the accumulators carries a tripping gear which stops and starts the motors driving the pumps. Connected immediately to the vertical delivery line of each pump there is a 5-in-diam check valve, a 5-in. to 6-in. taper pipe, and then a 6-in-diam gate valve. Fig. 1 shows a diagrammatic layout of the plant.

Operating Conditions. When the pumps were cut out with the elevators using water the check valves slammed violently. The whole pumping system shook and the noise gave rise to alarm. Furthermore, the gun-metal seat ring on one of the check valves had loosened due to this slamming.

To find the magnitude and nature of the pressures set up in the delivery pipe, pressure connections were made in the covers of the check valves to carry recording indicators which registered variations of pressure on a time basis.

Fig. 2 shows reproductions of cards from the two valves. That for valve no. 1 shows that the pumping pressure was 405 lb per sq in., and the maximum pressure recorded due to the valve slam 785 lb per sq in., a rise of 380 lb per sq in. or 94 per cent. The violence of the shock is apparent from the jagged nature of the diagrams. Valve no. 2 had the same pumping pressure and a maximum pressure rise of 190 lb per sq in. or 47 per cent.

Both cards show similar characteristics. On a primary surge pressure with a measured period of about 0.08 sec (calculated period 0.075 sec) is superimposed a secondary wave of high frequency the time history of which is complicated by the layout of the pipe system. A notable point is that on valve no. 2 the maximum pressure rise was only half that on no. 1. Although the valves were identical and the pumping conditions the same, there was a slight difference due to the cross connecting of the pipes.

Remedy. Fundamental experiments on, and critical analysis of, ordinary check valves had led to the conclusion that these were unsuitable for dealing with rapid reversals of flow and a type of check valve had been evolved which incorporated all the mechanical and hydraulic properties essential for dealing with difficult conditions. Mechanical improvement was given by a combination of inertia reduction, mass concentration, ideal suspension, and localized movement of the valve disk within a certain region of the stream, and hydraulic improvement was given by avoidance of cavitation and provision for forward and backward water streams within the body and around the valve disk.

In the system under discussion these specially evolved "recoil" valves were substituted for the original check valves and the effect may be seen in Fig. 3. These diagrams were taken under the same conditions as those in Fig. 2.

The rise in pressure over the pumping pressure is 12 per cent on no. 1 valve, an improvement of 85 per cent. No. 2 valve shows an improvement of 100 per cent as the rise in pressure does not exceed the pumping pressure. The noise and vibration due to slam had been entirely eliminated, so that a stethoscope was necessary to tell exactly when the valve closed.

Case 2 Automatic Pumping Plant for Intermittent Supply

A 6-in-diam single-stage centrifugal pump delivers 1000 gpm at 25 lb per sq in. into a closed vessel half full of air. A 6-in-diam pipe from this vessel supplies an intermittent demand used for process work. Due to variation in draw off the pressure in the tank varies and a relay gage on the tank operates the motor switch, cutting out the motor at the maximum allowable pressure and cutting it in again at the minimum limit. A check valve was placed on the discharge side of the pump in a horizontal position. The pipe between the pump and the tank is about 20 ft and the suction lift 3 ft. Fig. 4 shows a diagrammatic layout.

Operating Conditions. As in the first case, a violent slam occurred at the check valve when the pump was cut out. Pressure-time diagrams were taken on the outlet and inlet sides of the valve and these are reproduced in Fig. 5. The shocks were extremely

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[1] Managing Director, Glenfield & Kennedy, Kilmarnock, Scotland.

Contributed by the Hydraulic Division and presented at the Annual Meeting of The American Society of Mechanical Engineers, in New York, N. Y., December 6–10, 1937.

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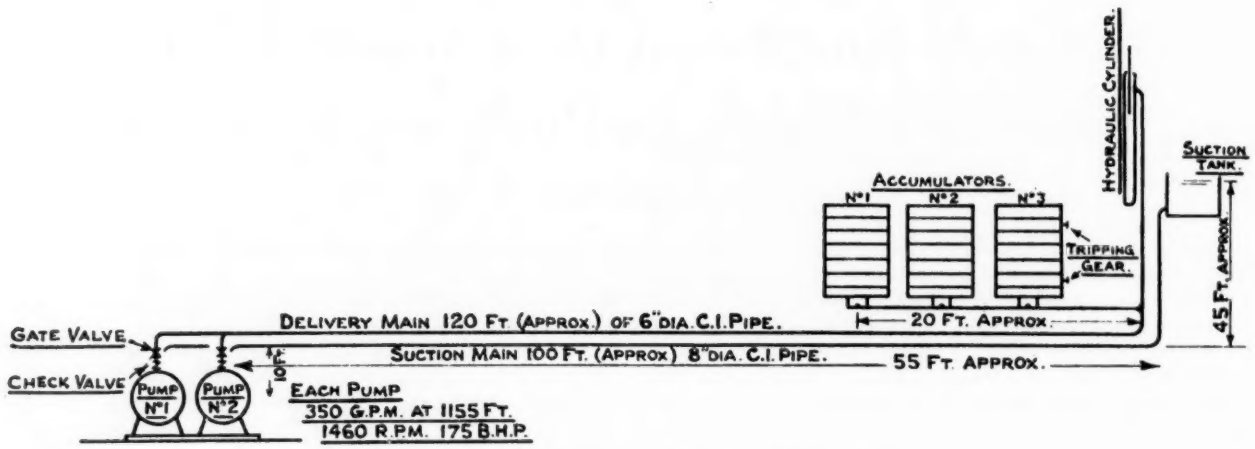


FIG. 1 DIAGRAMMATIC LAYOUT OF HYDRAULIC PUMPING PLANT

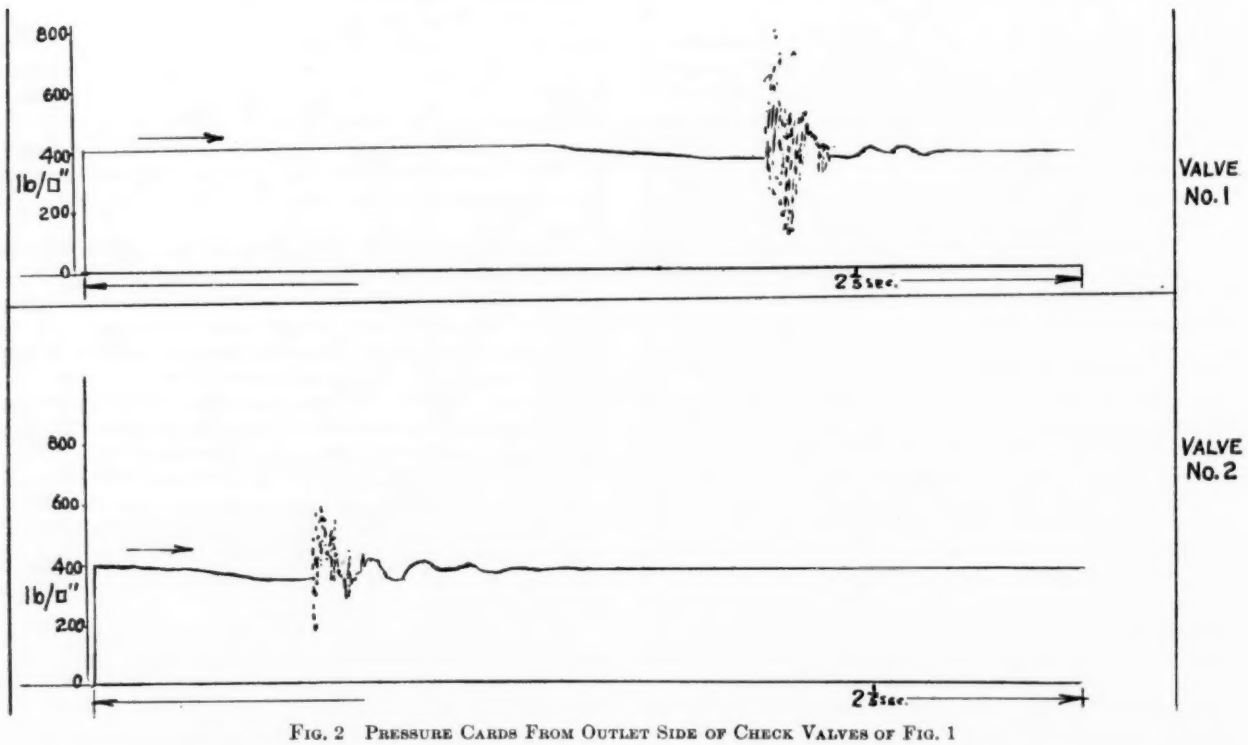


FIG. 2 PRESSURE CARDS FROM OUTLET SIDE OF CHECK VALVES OF FIG. 1

violent as shown by the jagged lines caused by the pencil jumping from the paper due to the vibration of the whole piping system and recording instrument. The recorded shock pressure on the outlet side of the valve was about 350 lb per sq in. and about 530 lb per sq in. on the inlet side.

An interesting point is that the shock pressures were about 50 per cent greater on the inlet side of the valve than on the outlet side. Confirmation of this apparently remarkable condition is provided by recorded cases of pipe fractures close to the inlet side of ordinary nonreturn valves.

Remedy. As in the previous case the check valves were replaced by recoil valves and the results of the change-over are seen in Fig. 6 which were taken under identical working conditions of pressure and pumping velocity.

The recorded surge pressure was 13 lb per sq in. on the outlet side of the valve and zero pressure on the inlet side. Thus 97½ per cent of the arresting shock was eliminated. The valve closed silently.

CASE 3 BOOSTING SYSTEM ON WATERWORKS SUPPLY MAIN

A general layout of the plant is shown in Fig. 7. Water from the reservoir is led to the suction side of an 8-in-diam pump which is set on a loop on the 15-in-diam main. Flow to the elevated tank is controlled by a 12-in-diam electrically operated gate valve on the downstream side of the pump. The 12-in. gate valve is closed first and when the "closed" limit switch is tripped the pumps are shut down.

Operating Conditions. During flow the pumping pressure was 390 ft on the downstream side of the gate valve. The valve is geared to travel 12 in. in 60 sec and, during closure, the valve traveled for 55 sec, or to within 1 in. of the shut position, before any changes were recorded by the pressure gages. Then the

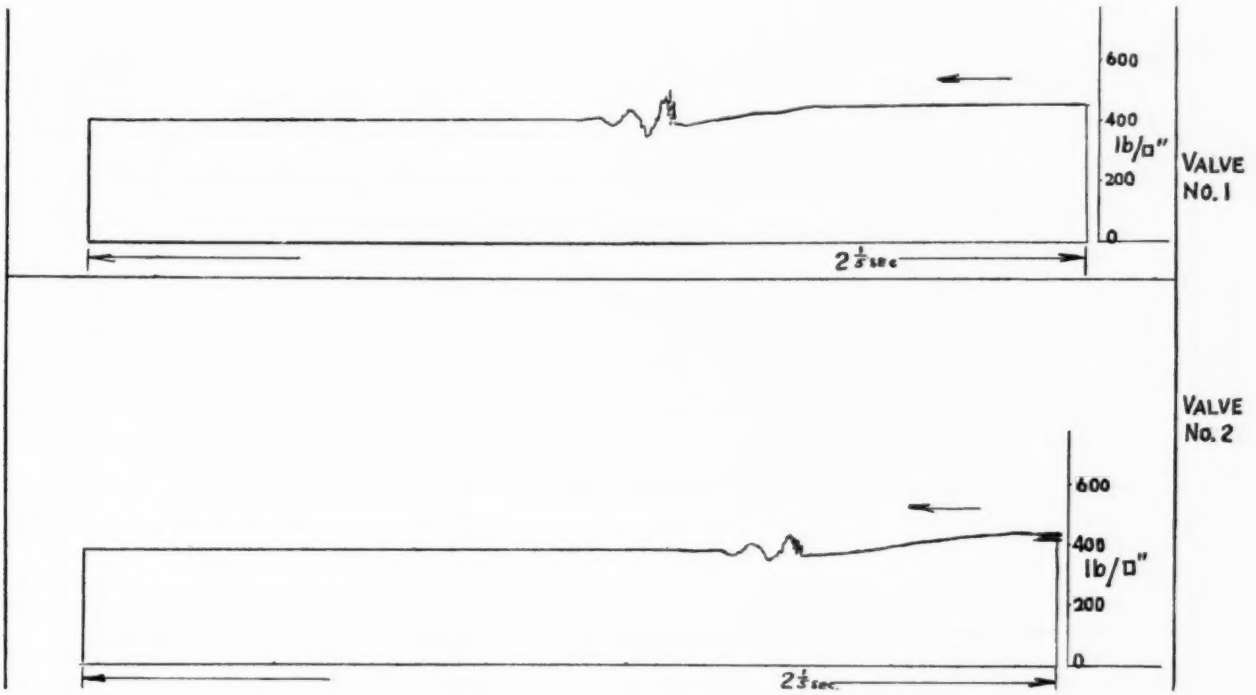


FIG. 3 PRESSURE CARDS WITH CHECK VALVES OF FIG. 1 REPLACED BY RECOIL VALVES

delivery pressure fell to 150 ft, rose to about 540 ft, and oscillated so rapidly that observations of any kind were impossible. The recording pressure gages were so agitated that the diagram was all jags, dots, and blots of ink.

On the delivery side of the valve, when closing down, the surge was a complicated series of pressure waves and reflections. Attempts to measure the period gave an average figure between 3 and 4 sec (which agrees well with the calculated value of 3.35 sec) for the fundamental surge, on which a number of erratic secondary waves were superimposed.

On the inlet side, when closing down, the surge exhibited a much less severe though similarly complicated series of pressure waves with a fundamental period of about 6½ sec. The estimated period was 6.9 sec.

The continued running of the pump aggravates the intensity of the surge downstream of the 12-in. gate valve during its closing operation. As the pressure is both maintained and built up, so that the difference of pressure between the two sides of the gate is increased, the result is that the jet velocity through the partially opened gate is increased. This has a driving effect, increasing the tendency of the column to travel away from the closing gate and still further increasing the fall of pressure. As the action is cumulative an abnormal depression is caused and, when the column recovers by reason of its elasticity, a pressure wave opposite in sign and of high intensity occurs.

A recoil valve had been installed originally in the plant between the pump and the electrically operated gate valve and when the pump was cut out, keeping the gate valve open, no shock or noise was experienced on either the delivery or suction mains and only a slight undulation, soon damped out, was noticed in the pressure gages. While this was entirely satisfactory, the previously outlined method of operating the plant was still desired.

Remedy. As pressure rise depends on time of valve closure, obviously the surge could be reduced by increasing the time of operation of the valve. The critical period, however, is the final part of the stroke and even doubling the time of closure would only approximately halve the surge pressure which would thus still be too severe. Also with this insufficiently extended closing time about 3000 gal of water would be put into the tank after the float gear had transmitted the signal to stop.

A valve was required that would occupy the same total time in closing, but would reduce the velocity in the main in direct proportion to the valve travel. While the spectacle-eye or follower-ring type of gate valve is an improvement in this respect over the disk type, to get the direct relationship required a ported follower-ring type was necessary, with the passage orifice so formed that as the valve closed the valve resistance and the pipe-line friction combined would retard the water column at a practically uniform rate.

Such a valve was made and installed and was successful in overcoming the undue surges. Although no actual test figures are available, the observed maximum peak-surge pressure does not exceed the pumping pressure.

REMARKS

The three cases described have been selected from a large number for their diversity of pumping conditions, pressure, and pipe layout and all are illustrative of the effect of valve behavior on a water column.

In general, a water system consisting mainly of long horizontal water columns at low pressure is sluggish in reversal, and a system, or portion of a system, consisting of short mobile water columns, or a system possessing some measure of elasticity, is rapid in reversal.

The fundamental relation determining the acceleration of reversal of a rigid water column is

$$\text{Acceleration} = \frac{\text{head}}{\text{column length}} \times g$$

where g is the acceleration due to gravity. In practice this relation becomes modified by elasticity and many other factors affecting the column behavior.

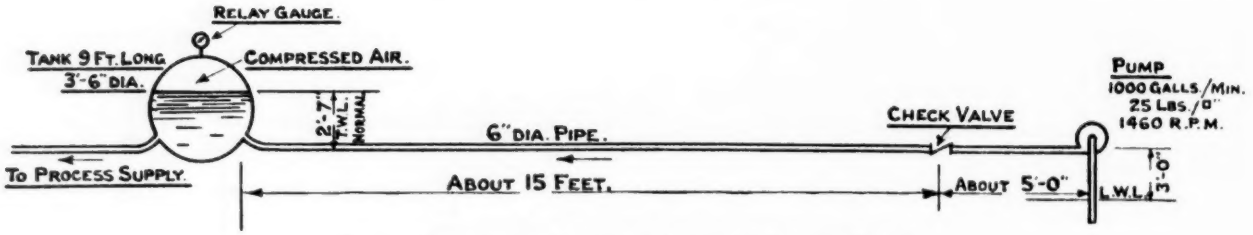


FIG. 4 DIAGRAMMATIC LAYOUT OF AUTOMATIC PUMPING PLANT

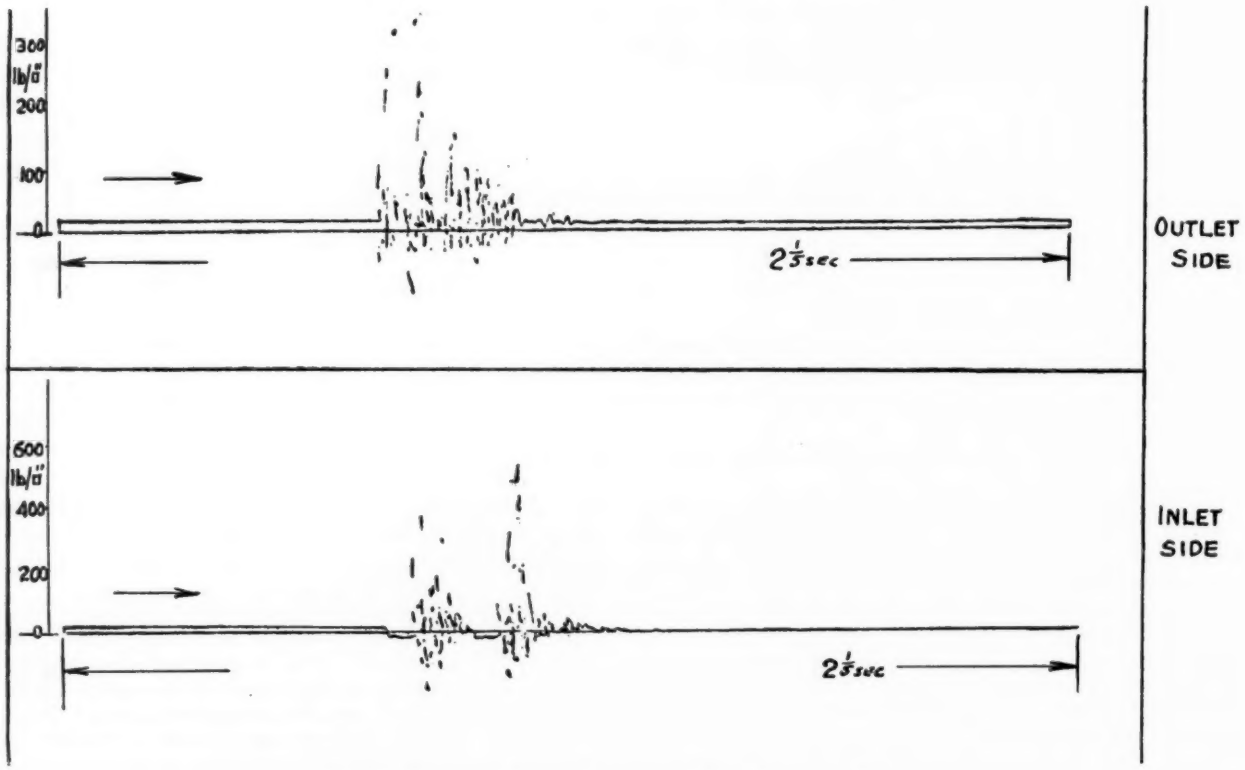


FIG. 5 PRESSURE CARDS TAKEN AT CHECK VALVE OF FIG. 4

A self-acting check valve depends for its operation only on the force due to gravity and, as the inertia of the parts must be overcome, the acceleration of the valve disk must be less than g. The water column is in contact with the disk, hence, if the column acceleration is greater than g and the disk acceleration less than g, it is obviously impossible for the valve to close more quickly than the column can reverse.

The criterion of perfect operation of a check valve is that the valve should reach its seat at the moment the water column is at rest. If the valve does this, any subsequent surge in the main is inherent in the system, cannot be cured by any valve, and must be remedied by other means. This is illustrated by the following case.

CASE 4

In a 6-in-diam pumping main, 18,500 ft in length conveying water at the rate of 83 gpm to a height of 396 ft, a recoil valve, placed on the delivery side of the pump, closed silently without any sign of slam when the pump was cut out. There was, however, a sudden drop of pressure equivalent to the friction head and then a surge varying from 525 ft maximum to 300 ft minimum. The normal pumping pressure is 450 ft.

When the pumps cut out the head conditions suddenly change and this change, not any arresting of backflow, causes the surge.

This type of problem may be solved satisfactorily by incorporating an air vessel in the pipe and, while absolute elimination of the surge cannot be expected, reduction of excess pressure to reasonable limits is possible by the use of a correctly designed vessel. In the case under discussion an air vessel 3 ft 0 in. in diam × 5 ft 9 in. high so reduced the surge that the maximum pressure never rose above 400 ft and the minimum never fell below 370 ft. Thus the original variation of 225 ft was reduced to 30 ft.

A properly designed recoil valve is a necessity in such an installation, as one effect of the air vessel is to increase the severity of the working conditions for the valve. The introduction of a free surface, in the air vessel and near the check valve, divides the water column into two lengths, viz., the main column and a short column from the air vessel to the pump. The short column then becomes exceedingly mobile and subject to rapid reversal of flow when the pump is shut down.

CHECK VALVES

Ordinary standard check valves meet all cases of sluggish water columns and frequently can deal sufficiently satisfactorily with cases where moderately rapid reversals of flow occur. More severe conditions can sometimes be met by fitting loaded disks to

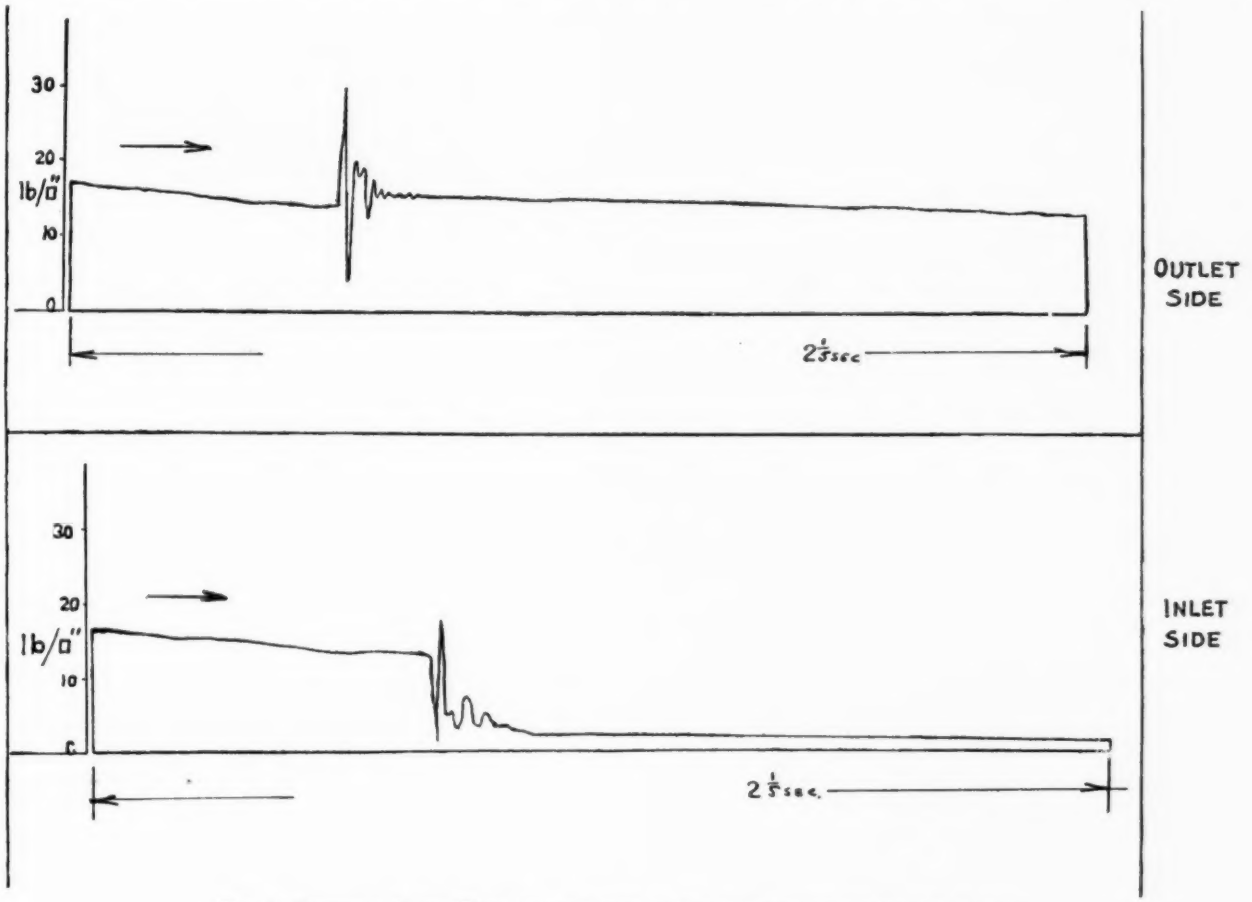


FIG. 6 PRESSURE CARDS WITH CHECK VALVE OF FIG. 4 REPLACED BY RECOIL VALVE

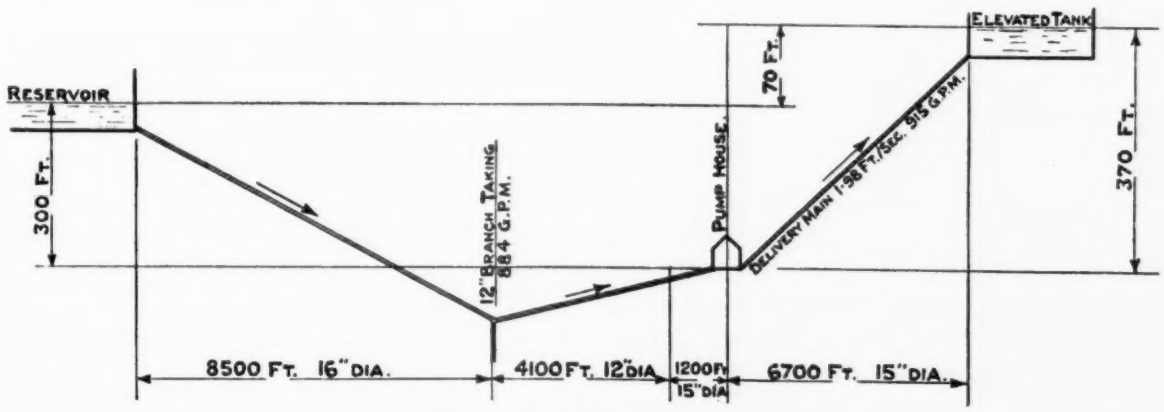


FIG. 7 DIAGRAMMATIC ARRANGEMENT OF PIPE LINE

otherwise standard types of check valves and occasionally so-called slow-closing or cushioned valves are installed. Apart from the heavy extra cost, experience with slow-closing valves proves that, in many cases when the conditions are really severe, the valve has been known to break down mechanically or become responsible for a fractured pipe. In other cases, the quantity of water lost during slow closing is either prohibitive or it is inconvenient to design the plant to accommodate the return water.

The check valve must, therefore, itself be capable of dealing with the rebounding water column.

A special elaborate plant on which exceedingly severe flow and return-flow conditions could be produced was installed in the hydraulic laboratory of Glenfield & Kennedy, Ltd., Kilmarnock, to investigate and develop the design of a recoil valve suitable for dealing with rapidly reversing water columns. Experiments were extended over a number of years and periodically the results were critically analyzed during the progress of the work.

It was found that the conventional type of check valve could be so improved by the application of scientific principles to every feature of the design that it could be completely transformed from a crude device to a highly responsive piece of apparatus eminently suitable for dealing with particularly mobile water columns.

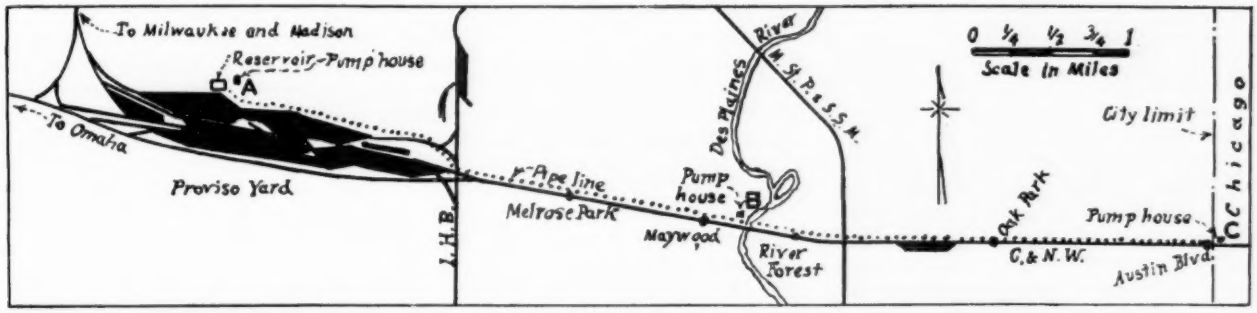


FIG. 1 MAP SHOWING LOCATION OF TRANSITE PIPE LINE FROM CITY MAIN TO RESERVOIR

(*A* Motor-driven gate valve and 5-in. relief valve at Proviso.) (*B* Vale booster station with mercoid pressure-controlled Worthington pump and 14-in. check valve.) (*C* Austin pumping station with mercoid pressure-controlled Worthington pump and 14-in. check valve.)

Speed of Water-Hammer Pressure Wave in Transite Pipe

BY LEWIS H. KESSLER,[1] MADISON, WIS.

THE PURPOSE of this paper is to show a determination of the speed of travel of the pressure wave due to valve closure in a class-150, 14-in. cement-asbestos (Transite) pipe 34,237 ft long, serving the Proviso Freight Terminal of the Chicago and Northwestern Railway Company, and to extend these data to other sizes of Transite pipe by means of determinations of the modulus of elasticity.

At the request of the Railroad Water & Coal Handling Company of Chicago, Illinois (contractors), tests were made on February 9, 1937, on this pipe line, a plan of which is shown in Fig. 1. Water is metered and discharged from Chicago city mains through a float-control valve into an open 3000-gal tank in the pumping station at Austin Boulevard, Oak Park, Illinois. The pump suction pipe connected to the tank is at elevation 47.0 ft. This is considerably above the discharge at the city mains and the layout eliminates a possible cross connection. From the pump suction and discharge piping the line rises to a maximum elevation of 66.0 ft and then drops to a minimum of 44.0 ft at the Des Plaines River crossing near Vale, the booster pumping station (*B*) of Fig. 1. Twenty summits and sags occur between these pumping stations because the line on the right of way has its profile broken by passing above subways.

From elevation 44.0 at the Vale booster station, the line rises to elevation 66.0 at the motor-operated gate valve in the control house adjacent to the Proviso reservoir (see Fig. 2). The valve is normally operated by float control in the reservoir but in the tests described herein it was manually operated.

The pipe line was placed in service on January 6, 1937. However, presumably due to water-hammer pressure caused by closure of the motor-driven gate, leaks developed at several joints in the line. A 5-in. Snow Company flat-disk spring-loaded relief valve was installed on an 8-in. saddle and riser directly upstream from the valve. This was the situation when the writer and assistants were engaged to conduct tests to determine the adequacy of the relief valve in reducing water hammer.

The February tests indicated that the relief valve was lowering the pressure to safe limits, and a report was rendered on this basis to the contractors. The writer suggested that since little reported information was available on the characteristics and behavior of the pressure wave in Transite pressure pipe, this pipe line might be used to study the speed of advance of the wave front. The management of the railroad would not permit tests to be made without the relief valve because of the necessity for an uninterrupted water service. Service had been disrupted previous to the installation of the relief valve, and the railroad did not want to go through this experience again. However, the writer was permitted to make tests with the relief valve operating and to

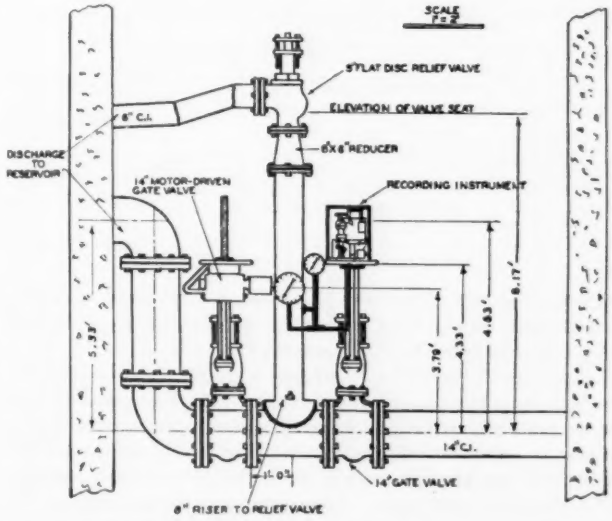


FIG. 2 INTERIOR OF GATE HOUSE AT PROVISO

[1] Associate Professor of Hydraulic and Sanitary Engineering, University of Wisconsin.

Contributed by the Hydraulic Division, and presented at the Second Water-Hammer Symposium in cooperation with the American Society of Civil Engineers and the American Water Works Association, at the Annual Meeting of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS, New York, N. Y., December 6–10, 1937.

Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y., and will be accepted until February 10, 1939, for publication at a later date. Discussion received after the closing date will be returned.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors and not those of the Society.

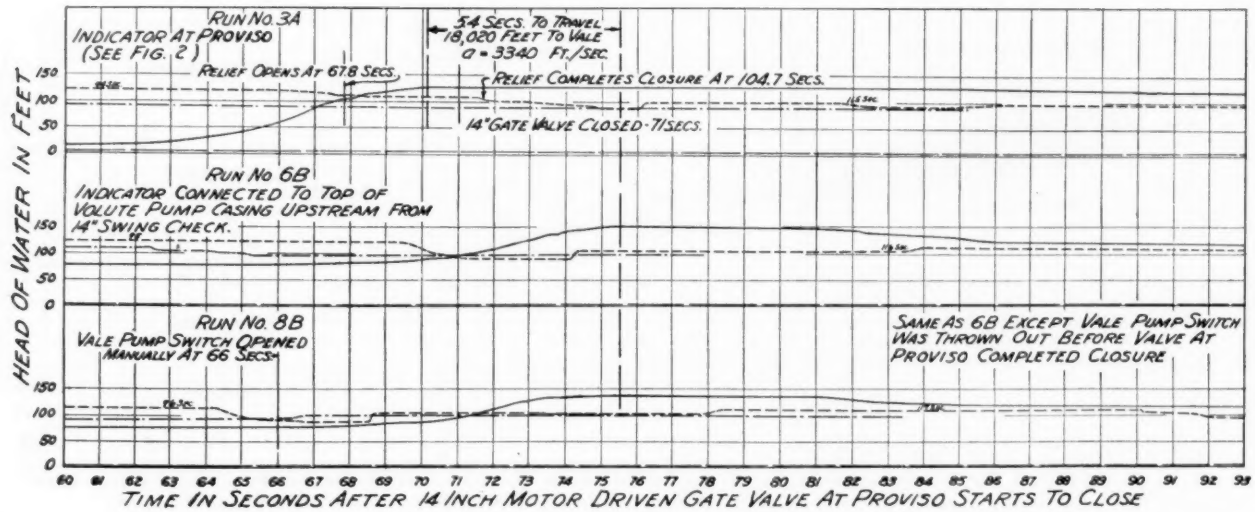


FIG. 3 TESTS OF AUGUST 27, 1937. PRESSURE-TIME DIAGRAMS ON 14-IN. TRANSITE CLASS 150 PRESSURE PIPE USED AS A PUMP-DISCHARGE MAIN

(Cards indicate advance of water-hammer pressure wave front from Proviso to booster pump at Vale, with 5-in. flat-disk-type spring-loaded relief valve operating at Proviso. Instant gate closure, V_0 = 3.38 fps, V_r = 2.16 fps.)

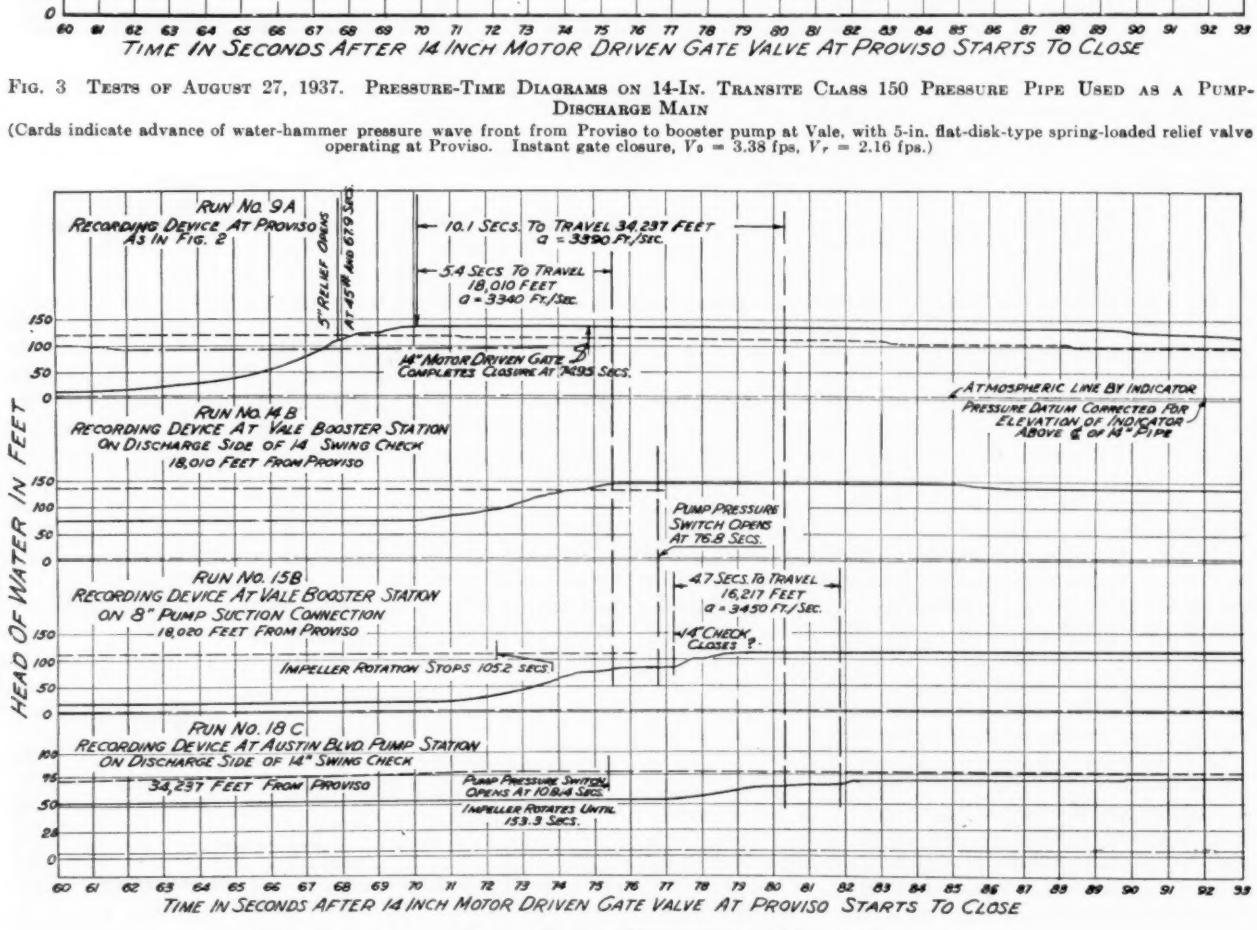


FIG. 4 TESTS OF NOVEMBER 14, 1937

measure the rate of travel of the pressure wave under operating conditions. These tests were made on August 27, 1937, and check tests were made on November 13 and 14.

Eight measurements of discharge were made by several observers using two 8-in. Neptune meters. The float control at the surge tank is sensitive and pump operation in the Austin Station causes no appreciable fluctuation in water level in the surge tank. Long-time observations showed the discharge to be fairly constant at 3.570 cfs or 1600 gpm. This result was checked against the head-discharge curves of the 6-L-1 Worthington centrifugal pumps. Incidentally a hydraulic-grade-line study using a coefficient of friction in the Hazen-Williams formula of C = 140, as suggested by the Johns-Manville Company, again checked the measured rate of discharge of 3.570 cfs. This discharge was reasonably constant for all tests.

There were available three possible methods of investigating the speed of the pressure wave: 1 By pressure-time diagrams showing the advance of the wave front as it progressed 18,020 ft from Proviso to the Vale booster station and 16,217 ft from Vale to Austin Boulevard Station.

2 By computation based on the hydraulic behavior of the relief valve with discharge characteristics as shown by tests in the hydraulic laboratory at the University of Wisconsin.

3 By comparing with field data the computed theoretical maximum pressure rise without the relief valve in operation.

The results obtained by these methods could be verified by tests to determine the modulus of elasticity of the cement-asbestos pipe. These tests were conducted in the materials-testing laboratory of the University of Wisconsin.

Advance of the Wave Front

Pressure-time diagrams were obtained by two methods. A Maihak high-speed engine indicator with chronomatic drum was used in the February tests. However, for this particular problem, it appeared that another type as used in the hydraulic laboratory at the University was more suitable. In the August and November tests a Crosby Diesel-engine indicator recorded the pressure history on a 24 × 4-in. rectangular card clamped to an aluminum drum which rotated at a uniform speed. The time history was recorded on the same card above the pressure trace by use of an electrically operated timer. In addition to the pressure-time cards, stop-watch and Bourdon test-gage readings were taken by five assistant observers.

In the tests of August 27, pressure-time cards were taken on the riser of the relief valve located 1 ft upstream from the motor-driven gate valve at Proviso. See Fig. 2. The recording device was taken to Vale booster station and four cards were taken on the top of the volute casing of the pump. In the tests of November 13 and 14 the location at Proviso, *A*, was used for four runs. Previous to these tests the railroad officials had the pipe line tapped on both sides of the check valve at Vale, *B*, and on the pump suction. A tap was also placed at Austin Boulevard Station, *C*, on the discharge side of the check valve. These additional connections to the pipe for pressure transmission permitted studies to be made of the advance of the wave front. All time was synchronized so that zero time was that time at which the motor-driven valve at Proviso began to close. The movement of the valve stem is uniform throughout the closure.

Figs. 3 and 4 are tracings of a few of the cards arranged to show the progress of the pressure wave caused by valve closure.

Table 1 gives the average values of measurements as determined in the August and November tests. Only one run, 13B, is not included since the card was damaged during the test. All time values shown in the table are from start of valve movement at Proviso to the maximum initial pressure rise at the respective stations.

Speed of Wave by Behavior of Relief Valve

The T. W. Snow Construction Company of Chicago loaned to the hydraulic laboratory a 5-in. relief valve and spring made to the same specification as the valve in use at Proviso. The relief valve was erected and tested on a setup practically identical with the Proviso layout and a similar spring setting and water pressures were used in order to obtain the coefficient of discharge of the valve for various ratios of lift to diameter. In the November tests at Proviso, the spring from the relief valve tested in the hydraulic laboratory was installed on the Proviso relief valve so as to eliminate a possible variable in spring behavior. Fig. 5 indicates the results obtained on the 5-in. relief valve, used in computations to determine speed of the pressure wave. The vertical lift of the relief valve was measured by means of a needle clamped to the valve stem. This needle recorded its vertical travel as a trace on a smoked glass clamped to the valve bonnet.

TABLE 1 AVERAGE VALUES OF MEASUREMENTS

	Time from start of valve movement to maximum initial rise of wave front, sec			
	At Proviso	At Vale 18,020 ft from Proviso	Difference, sec	Speed of wave, (*a*) ft per sec
August 27, 1937				
Runs 1A, 2A, 3A, 4A	70.16	75.55	5.39	3345
Runs 6B and 7B				
November 14, 1937				
Runs 9A, 10A, 11A, 12A	70.13	75.5	5.37	3355
14B, 15B				
	At Vale	At Austin, 16,227 ft from Vale		
Runs 14B and 15B	75.5	80.3	4.8	3380
16C, 17C, 18C, 15B				
Vale suction	77.2[a]	81.9[a]	4.7	3450

[a] Beginning of secondary wave.

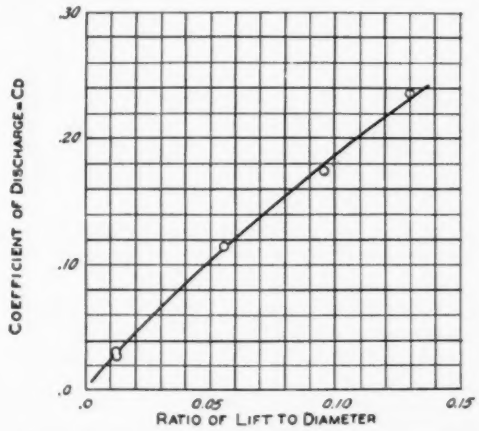


Fig. 5 Test Results on 5-In. Relief Valve at Hydraulic Laboratory of the University of Wisconsin

Indications were from data shown in Table 1 that the speed of propagation of the pressure wave was between 3340 and 3450 fps. Assuming the lower value of a, the time for one complete traverse of the acoustic wave $2L/a$ is 20.5 sec with L = 34,237 ft. From Table 2 it can be seen that the effective time of valve closure is between 15.25 and 16.4 sec. Instant closure occurs. The reason for the difference between the effective times of closure of the August and November tests is that the electrical control had been repaired during the interim between tests because some adjustments were necessary. In both cases the effective time of valve movement is less than 20.5 sec.

In this installation the fact that the relief valve opens and discharges water from the conduit means that the speed of the wave

TABLE 2 TEST DATA

	August tests						November tests					
H_0 = 8.41 ft												
Lift of relief valve, in.	0.496						0.480					
Diam relief-valve orifice, in.	5.09						5.090					
Ratio, lift/diam, in.	0.0975						0.0943					
c_d from Fig. 5	0.183						0.178					
Q_0 (= 1600 gpm), cfs	3.570						3.570					
V_0, fps	3.380						3.380					
Run no.	t,[a] sec	T,[b] sec	Difference, $T - t$,[c] sec	H, ft	h, ft	H_r, ft	Q_r, cfs	a, fps				
August												
1A	55.5	71.0	15.5	129.6	121.2	121.4						
2A	55.5	71.0	15.5	127.3	118.9	119.1						
3A	56.0	71.0	15.0	128.3	119.9	120.1						
4A	56.0	71.0	15.0	129.6	121.2	121.4						
Average			15.25	128.7	120.3	120.5	2.285	3210				
November												
9A	59.0	74.0	15.0	135.6	127.2	127.4						
10A	56.4	74.8	18.4	135.6	127.2	127.4						
11A	58.0	74.8	16.8	135.6	127.2	127.4						
12A	59.5	74.8	15.3	134.0	125.6	126.8						
Average			16.4	135.2	126.8	127.3	2.279	3360				

[a] t taken from pressure-time diagrams.
[b] T by stop-watch timing.
[c] $T - t$ = effective time of valve movement

can be computed, providing the pressure rise and relief-valve characteristics and behavior are known.

The following nomenclature and the data in Table 2 are used in computing the speed of the wave.

NOMENCLATURE

- t = time from start of valve movement to beginning of pressure rise above H_0, sec
- T = total time of valve movement, sec
- H_0 = flow head or pressure head under flow conditions, 8.41 ft
- H = pressure head for surge conditions, ft
- h = increase in pressure head above H_0 or $H - H_0$, ft
- H_r = pressure head on orifice of relief valve, when discharging = H — elevation head of orifice = H — 8.17, ft. See Fig. 2
- Q_0 = discharge in 14-in. pipe for flow conditions, 3.57 cfs
- V_0 = velocity in 14-in. pipe for flow conditions, 3.380 fps
- Q_r = discharge through relief valve at head of H_r, cfs
- V_r = velocity in 14-in. pipe due to discharge through relief, fps
- A = actual area of 14-in. Transite pipe at Proviso = 1.059 sq ft
- A_r = area of 5.09-in-diam relief-valve orifice = 0.1415 sq ft
- c_d = coefficient of discharge of relief valve for respective ratio of lift to diameter. See Fig. 5
- a = speed of water-hammer pressure wave, fps
- g = acceleration due to gravity, 32.2 ft per sec²
- E = modulus of elasticity of Transite pipe, lb per sq in.

The discharge through the relief orifice and the velocity V_r may be expressed as

$$Q_r = c_d A_r \sqrt{(2gH_r)} \quad [1]$$

$$V_r = Q_r/A \quad [2]$$

Therefore the velocity which has been destroyed by the valve closure is equal to the difference between the initial velocity under steady conditions and V_r or

$$V_0 - V_r = \Delta V$$

According to the Joukowsky formula the pressure rise under instantaneous conditions is directly proportional to the decrement in velocity.

$$h = \frac{a}{g}\Delta V \text{ or } a = \frac{h}{\Delta V}g \quad [3]$$

The values of a computed by the Equation [3] are indicated in Table 2.

At this point it will be observed that the values of a shown in Tables 1 and 2 vary from 3210 to 3450 fps with a median of about 3335 fps. Using the equation shown in Fig. 6, values of E are obtained for all the values of a determined by hydraulic measurement, as shown in Table 3.

TABLE 3

	a, fps	E, lb per sq in.
From Table 1	3345	3425000
	3355	3500000
	3380	3600000
	3450	3900000
From Table 2	3210	3000000
	3360	3500000
Median	3335	3375000

With the assistance of Professor Wendt of the department of mechanics, University of Wisconsin, tests were conducted on 4-in. 150-lb standard Transite pipe. Compression tests were made on both wet and dry specimens and a cross-bending test was made. Later upon receipt of data from the Johns-Manville Company which indicated a reduction in the modulus of elasticity with increase in diameter, it was decided to test a section of the 14-in. 150-lb pipe installed at Proviso. In addition the Johns-Manville Company kindly sent to the laboratory a length of new standard 14-in. 150-lb pipe with Simplex couplings. Compression, tension, and cross-bending tests were run on these specimens and final results of tests are shown in Table 4. It will be observed that one test was made to determine the circumferential modulus of elasticity. This modulus agreed for all practical purposes with the moduli obtained by standard methods of testing.

Having presented the values of E as obtained by both hydraulics and mechanics of materials, it appears that a value of E of 3,400,000 lb per sq in. should be used for the class-150 lb Transite pipe of the Proviso tests. The value of a computed with this modulus is 3340 fps and this value is used in computing the theoretical maximum pressure rise that occurred before installation of the relief valve.

SPEED OF WAVE BY COMPUTED THEORETICAL PRESSURE RISE WITHOUT RELIEF VALVE

As mentioned in connection with Table 2, the effective time of the valve movement is less than one interval of $2L/a$ because only the last 22 per cent of the valve movement causes a pressure rise in the pipe line. Hence, without the relief valve, the entire initial velocity V_0 must have been destroyed by valve closure. The excess pressure head, due to water hammer, that must have occurred at Proviso was, therefore, $h = V_0 a/g$. With a taken as 3340 fps and V_0 as 3.380 fps, h = 350.5 ft. When gage elevation of 4.33 ft (see Fig. 2) is considered then H = 354.6 ft or 153.6 lb per sq in.

This figure is substantiated by Henry Westenfeldt, operator of the control-gate house and the water-treating plant at Proviso, who states that the pressure rise previous to the installation of the relief valve, as measured by him on the Bourdon gage, was more than 150 lb per sq in., which was the limit of the gage. The author calibrated this gage and found it accurate. This point is mentioned as another confirmation of the fact that the speed of the wave was more than 3260 fps which is the value corresponding to a gage reading of 150 lb per sq in.

TABLE 4 TESTS ON TRANSITE PRESSURE PIPE TO DETERMINE MODULUS OF ELASTICITY

(Conducted in materials-testing laboratory, University of Wisconsin)

Date of test, 1937	Inside diam, in.	Outside diam, in.	Wall thickness, in.	Method of loading	Specimen	Modulus of elasticity, lb per sq in.
Oct. 8	3.965	5.071 End	0.553 End	Compression	4-in. standard 150-lb Transite end specimen	3430000
Oct. 12	3.978	5.22 Center	0.622 Center	Compression	4-in. center specimen immersed in water 95 hr	3475000
Oct. 14	4.048	5.060 End 5.270 Center	0.506 End	Cross-bending. Center-point load 6.0-ft span	4-in. standard 150-lb. Tested at standard 6.50-ft length	3460000
Nov. 16	13.89	16.26 End	1.185 End	Compression	14-in. pipe section from Proviso line	3250000
Nov. 16	Two tension specimens, 1.117 × 1.500 in., cut from 14-in. Proviso pipe line					3300000
Nov. 20	13.93	16.30 End 16.60 Center	1.185	Cross-bending. Center-point load. 12-ft span	14-in. standard 150-lb section shipped by Johns-Manville Co.	3250000
Nov. 27	16.99	19.95 End	1.480	Compression on ring perpendicular to pipe axis	Transite 14-in. Simplex coupling turned in lathe. 7.355-in. width	Circumferential modulus 3110000[a]

[a] Does not include additive effect due to shear.
Data by K. F. Wendt, L. H. Kessler, P. S. Davy, and B. R. Blackwell.

Conclusions

Until further information is available, the author suggests that the value of the modulus of elasticity for computing the speed of propagation of the water-hammer pressure wave in Transite pipe be taken as 3,400,000 lb per sq in. A value of 3,400,000 can definitely be taken for the 4-in. and 14-in. sizes and it is believed there is no question but that the same value holds for all sizes of class-150 Transite pipe likely to be used in water service. As has been mentioned before, this value of 3,400,000 corresponds to a speed of the pressure wave of 3340 ft per sec in a 14-in. pipe. Fig. 6 shows the speed of wave computed for class-150 lb Transite pipe of from 4 to 24 in. in diameter, inclusive, using this value of E.

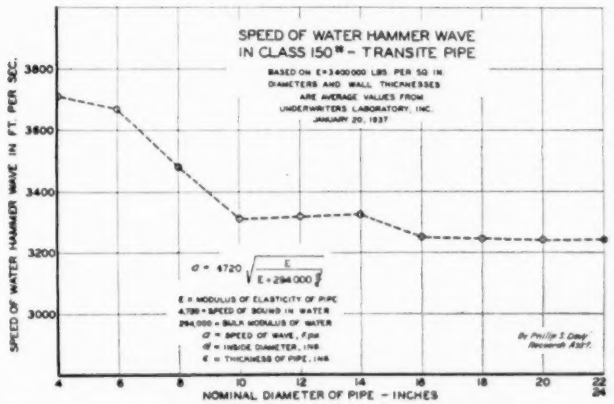


Fig. 6 Speed of Water-Hammer Wave in Class-150 lb Transite Pipe

Acknowledgment

The writer wishes to acknowledge the cooperation and assistance, in performing the test, of R. E. Coughlan, supervisor of water supply, H. W. Jensen, assistant division engineer, and H. Westenfeldt, water-treatment foreman, all of the C.&N.W. Ry. Co.; C. O. Congdon, vice-president, Railroad Water & Coal Handling Co., Chicago; Barton S. Snow, president, T. W. Snow Construction Co., Chicago; Merrill B. Gamet, instructor in civil engineering, Northwestern University; Gerard A. Rohlich, instructor in hydraulics, Carnegie Institute of Technology; E. R. Dodge, instructor in hydraulic engineering, University of Wisconsin; R. F. Zwettler, thesis student. Arrangements for tests were made through F. C. Huffman, assistant chief engineer of C.&N.W.Ry. Co., who also provided a profile of the pipe line and pump-discharge characteristic curves.

Prof. K. F. Wendt, department of mechanics, University of Wisconsin postponed some of his research work in order to supervise all tests on the cement-asbestos pipes. Prof. J. G. Woodburn, chief of department of hydraulic and sanitary engineering, University of Wisconsin, made the arrangements for financing this research project and gave valuable suggestions for the preparation of the paper. The men on W.P.A. research project no. 6605 in the hydraulic laboratory on water-hammer investigations assisted in the preparations for the field tests.

The Johns-Manville Company through their engineers, A. J. Maahs, E. F. Ries, and R. T. Walker provided all pipe required for test purposes and cooperated in every way possible.

The writer is especially indebted to Philip S. Davy graduate research assistant in hydraulics and sanitary engineering, University of Wisconsin, for his assistance in the analysis of test results and preparation of this paper.

Cavitation Characteristics of Centrifugal Pumps Described by Similarity Considerations

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Similarity considerations of hydraulic pumps and turbines can be obtained by comparing the known hydraulic performance of a given machine with the performance of other machines having geometrically similar waterways. The paper describes the application of this theory to the cavitation performance of centrifugal pumps and related machinery, and also introduces a characteristic parameter which, concerning cavitation performance, has properties similar to the specific speed with respect to other performance characteristics of such machines.

Introduction

THE WORK described in this paper had its origin at the Hydraulic Laboratory of the Metropolitan Water District of Southern California and the Pasadena Laboratory of the California Institute of Technology. The initial problem, concerning the evaluation of cavitation-test results on centrifugal pumps, was to work out a method of coordinating the results obtained at different speeds on the same pump. It was solved by applying the Thoma-Moody[4] similarity law for the cavitation performance of hydraulic pumps and turbines. This law essentially states that for changes in speed and proportional changes in the rate of flow through the machine, the absolute suction head above the vapor pressure of the fluid must be changed proportionally to the square of the speed of rotation, or to the first power of the total head of the machine, in order to maintain similarity of the cavitation phenomena in the machine. It can be expressed by means of the Thoma parameter

$$\sigma = \frac{H_{sv}}{H} = \frac{\text{Suction head above the vapor pressure of the fluid}}{\text{Total head of the machine}} \quad \ldots\ldots [1]$$

In other words, the cavitation behavior of a pump or turbine, relative to its normal behavior, will be the same if the Thoma parameter σ is held constant while speed and capacity are changed proportionally to each other. The law, as applied to centrifugal pumps, was verified at the Pasadena Laboratory with an unusual degree of accuracy. Fig. 1 shows the results of one experiment in which the test points obtained at different speeds of rotation practically all fall on one curve. Using the same results would form three widely separated curves if the drop in efficiency had been plotted against the suction head instead of the Thoma parameter. But the Thoma parameter cannot give any indications as to the influence of changes in the specific speed on the cavitation performance.

The concepts presented in this paper are but extensions of the fundamental ideas which have already been introduced by Thoma and Moody and which, concerning centrifugal pumps, were checked experimentally at the Pasadena Laboratory. Upon the suggestion of Professor Moody, the authors are also including in their derivations those of the Thoma parameter

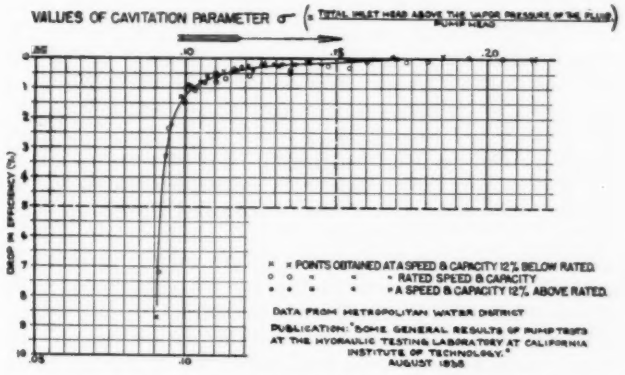


FIG. 1 TYPICAL RELATION BETWEEN INLET HEAD AND EFFICIENCY

and of the physical concepts underlying this whole field of investigation, thereby giving a more complete presentation of all questions involved.

Definitions

Our present knowledge of centrifugal pumps and related machines makes it necessary to base most considerations about their hydraulic behavior directly on test results. This means that in this field of engineering the most common and reliable method of evaluating the performance of any machine is to use the results obtained from tests on another machine of similar design and operating characteristics. This extension of test results, meaning application of conclusions to pumps and to operating conditions which differ in some way or another from the particular pumps and/or operating conditions for which test results are available, is generally made for some performance characteristics, such as capacity and head, by means of concepts like "unit speed," "unit capacity," and "specific speed." The purpose in the use of these concepts is to make test results independent of the particular size and speed of the test machine

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"Experimental Research in the Field of Water Power," by D. Thoma, First World Power Conference, London, June 30, to July 12, 1924.

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so that they may be applied to similar machines of different sizes and/or different speeds.

Such extensions, however, are not yet in general use for cavitation tests, particularly in connection with centrifugal or propeller-type pumps. Therefore, the purpose of this paper is to introduce concepts similar to specific speed but applicable to cavitation characteristics only. These concepts will permit reliable extensions of cavitation-test results from one pump to similar pumps of different sizes and/or different operating conditions. It should be understood that, on the other hand, no attempt is being made in this paper to predetermine by theoretical means the cavitation behavior of pumps from their design features such as the cross sections of waterways and the like.

In order to avoid misunderstanding, definitions are given of the most important terms used in this paper. These definitions, however, are not intended to be complete, but merely sufficient for the present purpose.

1 *Cavitation.* Cavitation in this paper refers to that well-known behavior of centrifugal pumps which is caused when somewhere inside the pump the pressure drops to the vapor pressure of the fluid so that vapor pockets are formed in the stream, as a result of which the performance of the pump is materially impaired. The erosion which is often found in connection with prolonged cavitation is not considered.

2 *Suction or Inlet Head.* For investigations on cavitation phenomena it is most convenient to deal with the so-called "Net Suction Head," namely, the total suction head above the vapor pressure of the fluid pumped (including the velocity head in the suction pipe). The suction head thus defined will be designated by H_{sv}. The reason for this definition is that cavitation must be expected to commence when somewhere in the pump the static pressure has dropped to the vapor pressure of the fluid so that vaporization begins. This definition of the suction head automatically takes care of the very pronounced influence of temperature and vapor-pressure variations. Hence for most practical cases we need not include the temperature, vapor pressure, or nature of the fluid in cavitation formulas or charts, except in so far as these factors have an effect on the type of pump used.

Furthermore it is assumed that the net suction head H_{sv} is measured vertically from the shaft center line of horizontal-shaft impellers and from a suitably chosen central plane of vertical-shaft impellers. This definition of course is not rigorous because it is actually the suction head at the point of lowest pressure in the pump which should be used. Since the precise location of this point is not known, it would, strictly speaking, be necessary to adhere to Froude's similarity relation (fluid heads proportional to linear dimensions of the machine) whenever the vertical dimensions of the inlet passages become appreciable in comparison with the net suction head. This refinement, however, will not be considered in this paper.

3 *Cavitation Limit.* The cavitation limit of a pump may be defined satisfactorily in several ways, but the most important two are based on:

(*a*) The upper limit of capacity for a given net suction head and speed of rotation.

(*b*) The lower limit of the net suction head at a given capacity and speed of rotation.

The first is illustrated by Fig. 2 showing the conventional characteristic curves for two different inlet heads, while Fig. 3 illustrates the second by showing the pump head, or efficiency, plotted against the net inlet head for constant capacity and speed. It is clear that the curve shown in Fig. 3 corresponds to the σ curve shown in Fig. 1.

Both figures show examples by which the respective limits may be defined, but the following investigations will be valid regardless of the particular definition chosen, as long as the chosen definition is strictly complied with in comparing cavitation characteristics of various machines. The following investigations may be expected to maintain their validity to a reasonable degree, even where the pitting of the runner blades and not the influence of cavitation on the momentary performance of the machine is used as a criterion for the cavitation limit.

4 *Similarity or Similitude of Flow Conditions.* The flow conditions in geometrically similar pumps shall be called "similar" only if the machine and fluid velocities at similarly located points change from one machine to another by a factor which is constant throughout any two pumps compared. The velocity diagrams computed from the rate of flow, cross section of waterways, and from the peripheral velocities of the rotating parts are, therefore, similar at similarly located points. This definition, which follows standard engineering practice, does not take into account the changes which arise from the influence of viscosity and turbulence, i.e., from the changes in Reynolds' number, nor does it consider the fact that the roughness of the waterways generally does not remain geometrically similar with respect to the other dimensions of the waterways, so that the "relative

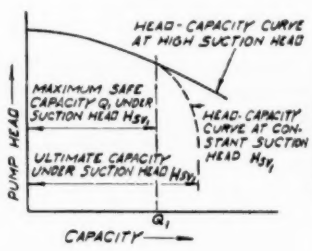


FIG. 2

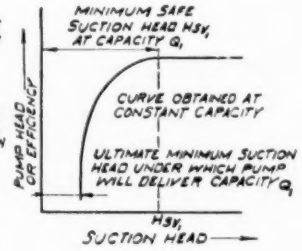


FIG. 3

roughness" is changed materially. The combined influence, however, of the changes in Reynolds' number and in the relative roughness is reasonably well known through test results with geometrically similar pumps of different sizes and/or different speeds, and it is to be expected that the influence of these factors on the cavitation behavior of a pump will be of the same order of magnitude as their influence on capacity, head or efficiency. This means that within the usual degree of approximation there is justification in neglecting the influence of changes in Reynolds' number and in the relative roughness on the cavitation performance of centrifugal pumps. While the validity of this approximation seems to be confirmed by practical experience, one should keep in mind that cavitation involves phenomena such as vaporization not present during normal pump operation, so that practical experience must be confirmed by more rigorous investigations.

Derivations

We are now ready to attack our initial problem, namely, that of extending given cavitation-test results to other pumps and/or other operating conditions.

The simplest form of such an extension is the well-known step from the performance of a model to the performance of the full-size pump, with the latter and the model operating under the same pump head. In this case the model and the full-size pump both will have to operate under the same net suction head, H_{sv}, in order to obtain the same conditions with respect to cavitation, i.e., either the same safety against cavitation or, if the pumps are cavitating, the same extent or influence of this phenomenon.

The characteristic of this simple step from the model to the full-size pump, both operating at absolutely the same head conditions, is that all machine and fluid velocities are equal to

each other at similarly located points in the two pumps, while all dimensions of the waterways are changed according to one fixed ratio.

The natural generalization of this method of extending given test results is to consider cases where, in addition to changing the waterway dimensions according to one ratio, all velocities at similarly located points are changed according to another fixed ratio. Flow conditions thus related have been defined previously as "similar," and it is in fact only under this condition of similarity of the flow conditions that simple and reliable conclusions can be drawn from the performance of one pump regarding the performance of geometrically similar pumps, or of the same pump under different operating conditions.

In order to include cavitation characteristics in such similarity considerations, it is expected that cavitation will commence when, somewhere in the pump, the static pressure has dropped to its natural lower limit, namely, the vapor pressure of the fluid in question. Under this condition the difference in total head between the suction pipe of a pump (corrected for elevation) and the point or region of the lowest pressure inside of the pump must be equal to the net suction head H_{sv}, as previously defined. However, in accordance with the given definition for similarity of flow conditions, all pressure or head differences inside of geometrically similar pumps will change proportionally to the square of the velocities as long as the flow conditions remain "similar." Since it is apparent that in any state of cavitation the net suction head H_{sv} actually exists as a head difference inside of the pump, the conclusion is that for similar flow conditions in similarly shaped waterways of centrifugal pumps the suction head above the vapor pressure (net suction head) is proportional to the square of the fluid velocities in these passages, i.e.

$$H_{sv} \text{ is proportional to } V^2, \text{ or } V \text{ is proportional to } \sqrt{H_{sv}} \quad [2]$$

where V may be any suitably chosen fluid velocity in the waterways of the pump.

This relation essentially expresses the Thoma-Moody law of similarity regarding cavitation conditions to which reference was made in the introduction. From this point on Thoma's method of extending given cavitation-test results is simply the following:

It is known that under similar flow conditions, as previously defined, the total head H of the machine is also proportional to the square of the fluid velocities. This means that according to relation [2] the total head H and the net suction head H_{sv} follow exactly the same law with respect to the velocities in the machine, i.e., H and H_{sv} are proportional to each other as long as the flow conditions are similar. For geometrically similar pumps, however, the flow conditions will be similar if the specific speeds are the same. Hence, if the Thoma parameter, $\sigma = H_{sv}/H$, and the specific speed of the pump are held constant, similarity is maintained with respect to the cavitation conditions, so that the influence of cavitation on the performance of the machine will remain the same. The relations

$$\sigma = \text{const} \quad \text{and} \quad N_s = \text{const} \quad [3]$$

therefore, express the desired condition under which cavitation-test results can be extended from one pump to a similar pump and/or similar operating conditions.

The last step in extending cavitation-test results is that from a given pump to another pump of a different specific speed. This problem is essentially the same as that of finding some theoretical background for calculating changes of σ as a function of the specific speed.

It should be understood that a rigorous solution of this problem does not seem to be within the reach of present-day fluid mechanics. However, useful approximations for such a solution are possible, and it is with such an approximation that the authors deal in this paper.

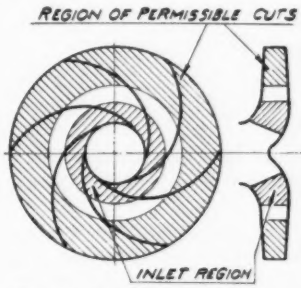


FIG. 4 LOW-SPECIFIC-SPEED IMPELLER

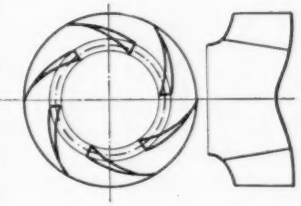


FIG. 5 HIGH-SPECIFIC-SPEED IMPELLER

As indicated in the introduction, the required relation between σ and the specific speed will be found by assuming that specific speed is changed by changing only the outside portions of the impeller, for instance, by cutting its outer diameter as shown in Fig. 4, leaving the suction passages of casing and impeller as well as capacity and speed of rotation unaltered. While it is true that the designer generally will not observe such a restriction on the design of the suction passages, the answer derived under this assumption will form the desired background for a theoretical relation between σ and N_s from which the effects of changes in the eye design can be measured and compared.

For the present the additional assumption is that changes in the outer waterways of the impeller, leaving speed and capacity unaltered, do not materially affect the flow conditions in the suction passages, i.e., in the region of lowest pressure where cavitation must be expected to commence. This assumption is quite accurate for impellers of the low-specific-speed type shown in Fig. 4, but is less accurate for high-specific-speed runners, such as shown in Fig. 5 because in this case it is no longer possible to consider suction and discharge passages separately from each other. For runners of specific speeds below 2000 (capacity through one eye in gpm) the assumption was confirmed experimentally at the Pasadena Laboratory.

It is clear that changes in the outer portions of the impeller, at the same capacity and speed of rotation, will primarily affect the pump head H (head produced by the pump) while, according to the authors' assumptions, the absolutely lowest pressure in the pump is not affected. Therefore, the net suction head H_{sv} which corresponds to a definite state of cavitation, will also remain the same. This means that such changes in the outer portions of the runner will influence the values of σ as well as of the specific speed N_s only by their influence on H. Hence, any relation between σ and N_s describing this particular form of change in the impeller must be such that resulting changes in the pump head H do not affect H_{sv}, while the speed of rotation N, and the capacity Q are held constant according to the definition of the change being considered. However, H_{sv}, N, and Q are, outside of H, the only remaining variables in σ and N_s, so that the above condition can be satisfied only if the pump head H cancels out of the desired relation between σ and N_s. This relation will therefore have to be of the form

$$\sigma = \text{const} \times N_s^{4/3} \quad [4]$$

or some power thereof. By substituting for σ and N_s their original expressions there is found

$$H_{sv} = \text{const} \times N^{4/3}Q^{2/3} \quad [5]$$

so that H no longer appears in the resulting equation

From the original assumptions and to the degree of their accuracy it follows that for this form of change in the specific speed, the constant in the last expressions cannot be a function of N_s or of H, but depends only on the suction qualities of the pump, particularly on the design of its inlet passages. For reasons of analogy with N_s this constant can be written in the form

$$\text{const}^{3/4} = S = \frac{N \times \sqrt{Q}}{H^{3/4}_{sv}} \quad [6]$$

which may be considered as a new form of cavitation parameter similar to σ but applicable in particular to the suction qualities of the inlet passages.

Naturally the assumption that changes in the discharge passages, and thereby in N_s, are without any influence on the flow conditions in the suction passages, cannot be strictly correct. Hence it is found that S actually will be a function of the specific speed so that, strictly speaking, the desired condition for the extension of cavitation-test results has now the form

$$S = \text{const} \quad \text{and} \quad N_s = \text{const} \quad [7]$$

which is essentially the same as the previously derived relation [3] where σ took the place of S. Therefore, from this theoretical point of view there exists no difference in physical significance between S and σ. From a practical point of view, however, such a difference does exist, because for moderate and low specific speeds, S will be very nearly independent of N_s. Hence for such specific speeds

$$S = \text{const} \quad [8]$$

is alone a sufficiently accurate condition for the desired extension of cavitation-test results from one pump to other pumps with similar inlet passages but of different specific speeds.

Returning later to the comparison between the parameter σ and the new parameter S, the authors now derive the latter by a straight similarity consideration quite similar to those leading to concepts, such as unit capacity, unit speed, and specific speed. This derivation, while offering nothing fundamentally new regarding the theoretical background of such quantities, will show that the validity of the new parameter S actually does not depend on the previous method of deriving the relation between σ and N_s, but has, in regard to cavitation characteristics, a significance very similar to the significance of the specific speed with respect to general operating conditions of centrifugal pumps.

This derivation starts again from the Thoma-Moody law on similarity of cavitation conditions in centrifugal pumps or turbines (see relation [2]).

For similarity considerations on cavitation or inlet characteristics it is natural to choose the eye diameter D_e of the impeller as the characteristic dimension of the waterways. The condition of similarity of the flow conditions demands that the fluid velocities V be proportional to the peripheral velocity of the eye of the impeller, which in turn is proportional to $D_e \times N$, where N is rpm.

Considering now a change in velocities and size from V_1, N_1, Q_1 (capacity), H_{sv1} and D_{e1} to a new combination V_2, N_2, Q_2, H_{sv2}, and D_{e2}, the following question is proposed: Which relation must be maintained between these operating conditions and dimensions in order to permit similar flow conditions in similarly shaped waterways of centrifugal pumps?

The condition of similarity of flow conditions demands that, according to relation [2]

$$\frac{V_1}{V_2} = \frac{D_{e1}N_1}{D_{e2}N_2} = \frac{\sqrt{H_{sv1}}}{\sqrt{H_{sv2}}} \quad [9]$$

and according to the usual condition of continuity in the waterways

$$\frac{Q_1}{Q_2} = \frac{D_e^2{}_1 V_1}{D_e^2{}_2 V_2} = \frac{D_e^2{}_1 \times D_{e1}N_1}{D_e^2{}_2 \times D_{e2}N_2} = \frac{D_e^2{}_1\sqrt{H_{sv1}}}{D_e^2{}_2\sqrt{H_{sv2}}} \quad [10]$$

From Equations [9] and [10] the condition for similar flow conditions may be written in the form

$$\frac{D_{e1}N_1}{\sqrt{H_{sv1}}} = \frac{D_{e2}N_2}{\sqrt{H_{sv2}}} = \text{const} \quad \text{and} \quad \frac{Q_1}{D_e^2{}_1\sqrt{H_{sv1}}} = \frac{Q_2}{D_e^2{}_2\sqrt{H_{sv2}}} = \text{const} \quad [11]$$

These two expressions, which may be designated by

$$S_n = \frac{ND_e}{\sqrt{H_{sv}}} \quad \text{and} \quad S_q = \frac{Q}{D_e^2\sqrt{H_{sv}}} \quad [11a]$$

correspond to the unit speed and unit capacity of centrifugal pumps, and are to be used in the same way for determining the scale ratio of similar inlet passages of two centrifugal pumps.

However, if it is desired to find the condition for similar flow with respect to the operation conditions only, i.e., without reference to the dimensions of the waterways, D_e must be eliminated from the Equations [11] or [11a]. From Equation [11a] is obtained by multiplication

$$S_n\sqrt{S_q} = \frac{N\sqrt{Q}}{H^{3/4}_{sv}} = S = \text{const} \quad [12]$$

This equation expresses the desired condition for similar flow conditions in similarly shaped inlet passages of centrifugal pumps under cavitation conditions. It is clear that the term "similar flow conditions" implies a similar extension of the field of cavitation (vaporization) in the pump. Hence the influence of cavitation on the performance characteristics, such as head or efficiency, will be the same for pumps with similar suction passages as long as Equation [12] is satisfied.

So far it has evidently been sufficient to extend the requirement of similarity of shape and flow conditions over the suction passages only, namely, from the suction pipe line to the point of lowest pressure in the pump. Similarity of shape and flow in the rest of the pump is required only to the extent in which this is necessary in order to maintain similarity in the suction passages. In other words, only those conditions in the pump, which can be expected to have a noticeable influence on the suction flow conditions, must be kept similar.

However, if the requirement of similarity over the whole pump is extended, then it is possible to eliminate the dimensions of the waterways from the expressions for S_n and S_q differently. In this case D_e is proportional to the impeller diameter D and therefore, ND_e is proportional to ND which is proportional to $\sqrt{H}$ (H = total pump head). Hence the condition S_n shown in Equation [11a] may be written in the form

$$\frac{\sqrt{H}}{\sqrt{H_{sv}}} = \text{const} \quad \text{or} \quad \frac{H_{sv}}{H} = \sigma = \text{const} \quad [13]$$

arriving thereby again at the well-known form of the Thoma parameter which was discussed before. The essential facts brought out by this second method of deriving the two cavitation parameters may be summed up as follows: Changes in the new parameter S necessitate corresponding changes in the suction flow conditions, while changes in the Thoma parameter σ do not necessarily require such changes but may be brought about by changing the discharge flow conditions only, e.g., by a change in the impeller diameter. For this reason the parameter σ will change as a function of the specific speed to the extent to which

the pump head changes with N_s, considering the other operating conditions as constant. The new parameter S on the other hand is influenced by changes in the specific speed but indirectly, i.e., only to the extent to which the inlet dimensions and flow conditions are influenced by changes in the discharge dimensions and flow conditions.

This leads to what is probably the most important practical difference between σ and S: For most comparisons of the cavitation behavior of centrifugal pumps of different specific speeds, it will be found that changes of S as a function of N_s only, are of a lower order of magnitude than its changes as a function of the design of the inlet passages. Hence it is generally sufficient to disregard the changes of S with respect to changes in N_s in comparison with its changes resulting from other variations in the design of the pump.

This is not true with respect to the σ parameter, where even for moderate changes in N_s the influence of the changes becomes equal to, or larger than, the influence of other changes in the design of the inlet passages. In other words changes of σ with respect to N_s must be considered generally as being at least of the same order of magnitude as changes of σ with respect to other design features and hence cannot be neglected.

Naturally, these two statements are true only if moderate changes in N_s are considered. For very large changes in N_s, which will affect the type of pump in question, the changes in the new parameter S with respect to N_s may be no longer negligible.

It should not be thought from the foregoing remarks that the authors wish to recommend replacing the Thoma parameter σ by the new parameter S in all cases. While it is true that for the comparison and prediction of cavitation characteristics of centrifugal pumps the parameter S seems to offer certain advantages over the parameter σ, the same thing is probably not true with respect to hydraulic turbines, where the total turbine head and the net suction head are more intimately related, so that a separate consideration of suction and discharge flow conditions may not offer a useful approximation. In connection with centrifugal pumps, the Thoma parameter σ will certainly be used in all cases where the enumerated advantages of the new parameter S do not come into play, as for instance in changing the speed of the same pump, or for comparisons between pumps of the same specific speed. In such cases the use of S would bring unnecessary numerical complications which are not present in connection with the numerically simpler parameter σ.

Since for these reasons the two parameters, σ and S, are likely to have parallel use in the same field of application, it will be necessary to give an analytic expression for the relation between the two parameters. This relation, which follows immediately from Equations [4] and [6], is

$$S = N_s \frac{1}{\sigma^{3/4}} \quad \text{or} \quad \sigma = \left(\frac{N_s}{S}\right)^{4/3} \qquad [14]$$

Finally it must be mentioned that for comparisons between single-suction and double-suction pumps, regarding their cavitation or inlet characteristics, the capacity of the double-suction pump should be divided by two when used in the specific speed and in the new parameter S. This merely means that only the rate of flow entering through one eye of the impeller is to be considered, which is natural for the desired comparison with single-suction pumps.

Examples and Applications

Using a numerical example it is assumed that a single-suction, single-stage pump, is tested at 1800 rpm, delivering satisfactorily 2100 gpm of cold water against a 215-ft pump head with a suction lift of 7 ft. The specific speed of this pump is $N_s = 1470$. For cold water, and at a moderate elevation above sea level the atmospheric pressure is equivalent to a column of water about 32 ft above its vapor pressure. The net suction head above the vapor pressure of the water, therefore, becomes

$$H_{sv} = 32 - 7 = 25 \text{ ft}$$

Now suppose it is desired to design a pump of the same specific speed, but running at 900 rpm and delivering 15,000 gpm against a 316-ft head.

Considering the first pump as a model of the second pump, it is possible to design the new pump from the first pump by "factoring," i.e., by changing all dimensions of the latter according to a constant factor. It is known that the new speed, capacity, and head conditions can be met by this procedure because the specific speeds are the same, and hence permit the use of geometrically similar pumps.

In this case, there is found the suction head necessary for operating the new pump against 316 ft from the suction head of the model while the latter was pumping against 215 ft by means of the previously discussed Thoma relation, Equations [3] and [13], so that the new suction head above the vapor pressure H_{sv2} is found as follows

$$\frac{H_{sv2}}{316} = \frac{25}{215}, \quad \text{or} \quad H_{sv2} = 36.7 \text{ ft}$$

Therefore, pumping cold water, the model pump as well as the full-size pump will have to work with a submergence of 36.7 — 32 = 4.7 ft in order to pump against 316 ft just as satisfactorily as the model has been pumping against 215 ft at 1800 rpm with a suction lift of 7 ft.

Of course, the scale factor to be used in "factoring" the full-size pump from its model can be found in a conventional way by means of a constant value of the "unit capacity" defined as

$$q_1 = \frac{Q_1}{D_1^2\sqrt{H_1}} = \frac{Q_2}{D_2^2\sqrt{H_2}} \qquad [15]$$

where Q is the capacity of the pump, D any suitably chosen linear dimension, usually the impeller diameter, H the total pump head and the subscripts 1 and 2 applying to the model and to the full-size pump, respectively. This definition is used in preference to the more common definition of the unit capacity $Q/\sqrt{H}$ because it is dimensionally more consistent. For this case, there is obtained from Equation [15]

$$\frac{D_2}{D_1} = 2.43$$

i.e., all waterway dimensions of the existing model have to be multiplied by 2.43 in order to obtain the corresponding dimensions of the new pump.

The next step would be to find what suction head would be required if the new pump were to be designed for 700 rpm instead of 900 rpm, still delivering 15,000 gpm against a head of 316 ft, i.e., if the specific speed were to be changed from 1470 to about 1142. As already stated, since a rigorous solution of this problem is not possible, a good approximation can be obtained by assuming that the inlet waterways of the pump are kept geometrically similar to those of the model, so that the change in the specific speed is brought about by a relative change of the outside diameter of the impeller (and of the discharge passages of the casing) with respect to the inlet dimensions. For this case it has been found that the new pump will work approximately under the same cavitation conditions as the model only if the value of S in Equation [12] is the same in both cases. In other words it is possible to "factor" all inlet waterway dimensions of the new pump from an existing pump with known cavitation

performance if S has for the new pump the same value as for the pump for which the cavitation behavior is empirically given.

In the present case it is found that for satisfactory operation of the "model"

$$S = \frac{1800\sqrt{2100}}{25^{3/4}} = 7360$$

but it is necessary to remember that the term "model" applies in this case to the inlet passages only, i.e., the previous model pump is no longer a model of the new pump as a whole.

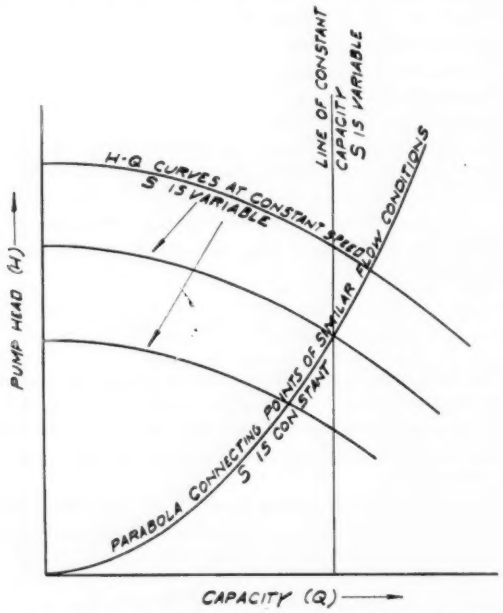


FIG. 6 BEHAVIOR OF S ON THE HEAD–CAPACITY DIAGRAM

If all suction passages of the new pump are designed by factoring from the previously considered model, the suction head necessary for an equally satisfactory operation of the new pump under the new operating conditions can be calculated from Equation [12] as follows

$$S = 7360 = \frac{700 \times \sqrt{15000}}{H_{sv2}^{3/4}} \quad \ldots\ldots\ldots [16]$$

where H_{sv2} is the required net suction head for the new pump. From Equation [16] it is found that $H_{sv2} = 26.5$ ft. This pump, pumping cold water will, therefore, operate satisfactorily with a suction lift of $32 - 26.5 = 5.5$ ft, compared with the submergence of 4.7 ft required for the pump of the same capacity and head running at 900 rpm.

The factor by which the inlet dimensions of the "model" have to be multiplied in order to obtain the corresponding inlet dimensions of the new pump may be calculated from Equation [11a].

Considering for instance the second part of this Equation it is found

$$\frac{D_{e2}}{D_{e1}} = \sqrt{\frac{Q_2}{Q_1} \times \frac{\sqrt{H_{sv1}}}{\sqrt{H_{sv2}}}} = \sqrt{\frac{15000\sqrt{25}}{2100\sqrt{26.5}}} = 2.64$$

which is the desired factor for the inlet dimensions. D_{e2} and D_{e1} are the eye diameters of the new pump and of the model, respectively. Evidently this procedure is exactly the same as that previously used in determining the scale factor for a new but completely similar design from the equality of the "unit capacities" of the two pumps in question, but here the suction head H_{sv} is used in place of the total pump head H. However, the factor thus determined does not apply at all to the discharge waterways of impeller or casing. Such a factor for the discharge waterways cannot be determined from the old model due to the change in specific speed. The outer portions of the new pump either have to be newly designed or factored from another pump of the proper specific speed. In this case it is left to the experience and general knowledge of the designer to decide how far the similarity with the model for the inlet dimensions has to be maintained and from which point on the discharge conditions become the controlling factor.

The approximations which were used in the previous examples (see Equation [12]), also permits the derivation of the following simple relations for pumps with geometrically similar inlet passages and flow conditions:

For N = const: H_{sv} is proportional to $Q^{3/2}$

For Q = const: H_{sv} is proportional to $N^{4/3}$

It is important to note that the changes of Q or N, considered here, must be accompanied by corresponding changes in the size

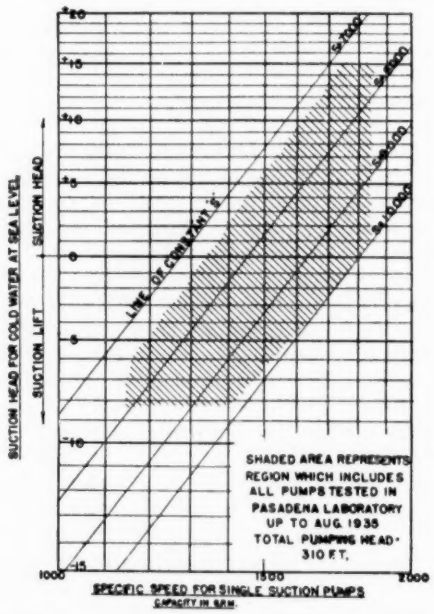


FIG. 7 DIAGRAM SHOWING RELATION BETWEEN SPECIFIC SPEED AND MINIMUM INLET HEAD FOR SATISFACTORY OPERATION
(Limit of satisfactory operation defined by 0.3 per cent drop in efficiency. See Figs. 1 and 3.)

of the pump in order to satisfy approximately the previous relations. These relations, therefore, generally cannot be true for changes of Q or N of one pump of definite size, because if either the capacity or the speed of a pump is changed without a corresponding change in the other, similarity cannot be maintained in the flow conditions in the pump. Therefore, changes in Q or N alone will cause (in the same pump) also changes in S, because the constant value of S can be obtained only if the flow conditions in the inlet passages are similar. This means that in general the value of S will vary along the conventional head-capacity curve of a pump (see Fig. 6). It will also vary if the capacity is held constant while the speed is changed. On the other hand, S will remain constant along any of the familiar parabolas in the head-capacity diagram along which the capacity is changed proportionally to the speed, thereby connecting points of similar flow conditions in the pump.

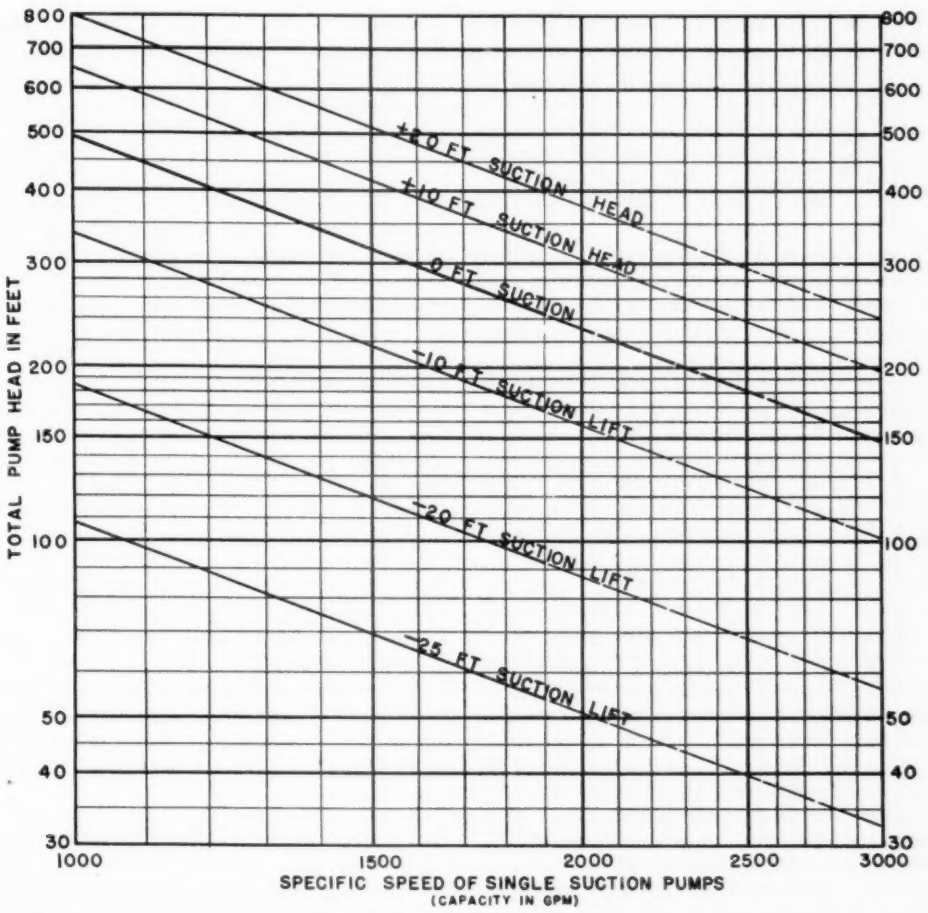


FIG. 8 FIELD OF EMPIRICAL POINTS IN THE S TO N_s DIAGRAM AND LINES REPRESENTING EXAMPLES OF CAVITATION LIMITS
(Empirical data taken from Pasadena Laboratory results and replotted from Fig. 7.)

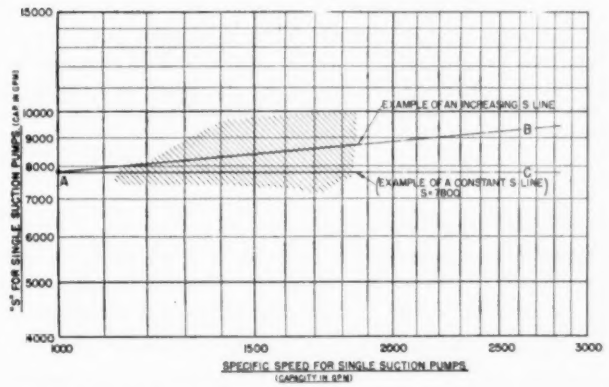


FIG. 9 EXAMPLE OF A CAVITATION LIMIT CHART FOR SINGLE-SUCTION, SINGLE-STAGE PUMPS PUMPING COLD WATER AT SEA LEVEL
(Computed on the basis of increasing S, line A-B in Fig. 8.)

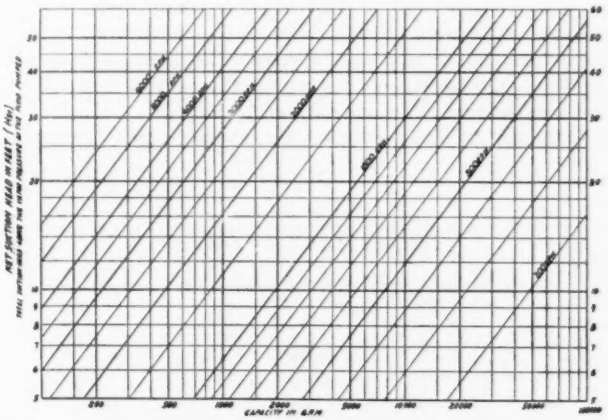


FIG. 10 SAMPLE CHART OF A CAVITATION LIMIT CHART FOR SINGLE-SUCTION, SINGLE-STAGE PUMPS
(Calculated on the basis of constant S = 7800.)

The question of the variations of S along the head-capacity curves as well as along the vertical lines of constant capacity is a most interesting and instructive one, but definitely exceeds the general scope of the present paper.

A rather startling verification of the practical usefulness of the new cavitation parameter S is shown in Fig. 7. In this figure the shaded area shows the range of net suction heads which is required for satisfactory operation for a rather wide range of specific speeds as well as pump designs.[5]

There has been plotted into this diagram lines of constant S values. The comparison shows an excellent agreement between the general slope of this area and the lines of constant S, showing

[5] "Some General Results of Pump Tests at the Hydraulic Testing Laboratory at the California Institute of Technology," compiled by the Metropolitan Water District of Southern California, August, 1935.

that the variation of the inlet heads with specific speed, as given by the law S = constant is generally verified by the pumps tested. It shows on the other hand that variations in design are reflected in corresponding variations in S regardless of the specific speed. In other words, according to these tests and within the range covered, S is practically independent of the specific speed, and dependent on other design features only. The upper limit of this comparison is at a specific speed of 1900 for single-suction pumps.

These test data also give an opportunity to illustrate how the new factor S can serve as a basis for comparing cavitation behaviors of a wide range of pumps, and for building up corresponding cavitation charts in a logically consistent manner.

Fig. 8 shows the region which is covered by the Pasadena Laboratory tests (see Fig. 7), but replotted so as to show S as a function of specific speed. Within this area lines can be chosen which would define cavitation limits for the pumps in question. The line A-B may be considered as an example of a limit chosen in this way. Using this line as a basis it then becomes possible to derive the cavitation limit chart shown in Fig. 9, which has been brought into the form of the familiar chart for double-suction pumps compiled by the Hydraulic Institute. The method of deriving the chart in Fig. 9 from the line A-B in Fig. 8 is based on Equations [13] and [14], which, in connection with line A-B, yield one definite value of H_{sv}/H for each value of N_s. The values of H_{sv}/H thus obtained, determine the points of intersection between the inclined lines of constant suction head and the vertical lines of constant N_s in Fig. 9.

Since in this example the chosen relation between S and N_s forms a straight line on the logarithmic diagram in Fig. 8, the resulting lines of constant suction head on the logarithmic chart in Fig. 9 also will be straight lines, requiring but two values of the ratio N_s/S for their complete determination. If, on the other hand, the S to N_s relation in Fig. 8 forms a general curve, then the constant-suction-head lines in Fig. 9 will also be curved and have to be determined more or less point by point.

In the special case, S = constant (line A-C in Fig. 8), a still more convenient form of the cavitation limit chart becomes possible. This form which is shown in Fig. 10, is really nothing but a nomographic solution of Equation [12], giving a value of S = 7800. The method of constructing this chart is conventional and does not require any special explanations.

Inasmuch as line A-C, on the average, falls below line A-B of Fig. 8, the resulting diagram in Fig. 10 yields more conservative values for the permissible inlet heads than those obtained from Fig. 9.

It should be noted that this paper, with all examples and charts, is concerned with specific speed and head limitations which are due only to the danger of cavitation (as heretofore defined), and does not deal with the additional restrictions imposed by mechanical and geometric limitations in the design of such machines. These restrictions will rule out the practical use of certain regions in the diagrams of Figs. 9 and 10, but intentionally have been disregarded in order not to confuse the principal issue of this paper.

Acknowledgments

The authors wish to thank A. H. Borchardt and M. Spillmann of the Worthington Pump and Machinery Corporation for encouraging these studies and for advocating their presentation before the Hydraulic Institute in December, 1937. Furthermore they are greatly indebted to Professor L. F. Moody of Princeton University whose repeated advice and criticism was an essential factor in arriving at the present form of presentation. They also wish to thank A. Hollander of the Byron-Jackson Company, R. M. Peabody of the Metropolitan Water District of Southern California, and others who have given their valuable comments. Finally they wish to emphasize the fact that this problem could not have been attacked with such confidence without the foundation formed by the previous work of Moody and Thoma, nor without the help of the experimental studies carried out at the laboratories of the Metropolitan Water District of Southern California and the California Institute of Technology.

Addendum

After the presentation of the preceding studies before the Hydraulic Institute, the authors learned from Professors L. Bergeron and P. Danel that the new cavitation parameter S had been used for some time in France, and was contained in a technical memorandum from Professor P. Bergeron to the Union d'Électricité, dated November 27, 1934. In this memorandum the new parameter has been given the very appropriate name "suction specific speed." The independent derivation and use of this parameter by the French engineers only confirms the authors' belief in its fundamental and useful nature.

Cooling-Tower Fundamentals and Their Application to Cooling Diesel and Gas Engines

By GUSTAVE J. BISCHOF,[1] NEW YORK, N. Y.

The size and cost of a cooling tower varies with certain conditions, some of which are inherent in the design, others are influenced by location and type of service, and still others may be fixed by the purchaser. In the last group are proper relation of water circulated and range of cooling for a given heat dissipation, the location of the cooling range in the temperature scale, and the relative size of the cooling tower compared to heat exchanger when both are operated as a unit. The purchaser is often unaware that reasonable modification of these factors may considerably reduce the size and cost of the cooling system.

Narrow cooling ranges with relatively high water circulations are conducive to optimum heat transfer, both from the standpoint of high temperature differences and high heat transfer coefficients in the heat exchanger.

The choice of the kind of engine-cooling system depends on many factors besides first cost, such as space requirements, operating cost, and trouble-free operation. The use of closed systems, while relatively high in first cost is often justified on the basis of yearly operating charges and low maintenance cost.

VARIOUS FORMS of equipment for evaporatively cooling water are so common that detailed descriptions are not needed in this paper. A brief outline, classified by nature of water surface and means of air movement, as shown in Table 1, is sufficient as a review. Some fundamental characteristics of evaporative water-cooling equipment are of importance to those responsible for its selection and use.

TABLE 1 CLASSIFICATION OF EVAPORATIVE COOLING EQUIPMENT

Name of equipment	Water distribution	Kind of water surface	Air movement
Spray pond	High-pressure spray nozzles	Drop	Lateral by prevailing wind
Spray-type atmospheric cooling tower	High-pressure spray nozzles	Drop	Lateral by wind and vertical by spray effect
Spray-type mechanical-draft cooling tower	High-pressure spray nozzles	Drop	Vertical counterflow by fans
Atmospheric deck cooling tower	Low-pressure distributors or troughs	Drop and wetted surface	Lateral by prevailing wind
Filled mechanical-draft cooling tower	Low-pressure distributors or troughs	Drop and wetted surface	Vertical counterflow by fans and baffled by filling
Combined spray-type tower and atmospheric-tube heat exchangers	High-pressure spray nozzles	Drop and wetted exchanger surface	Lateral by wind and vertical by spray effect
Evaporative surface	High-pressure spray nozzles	Drop and wetted exchanger surface	Vertical by fans

Evaporative cooling of water depends on these factors:

1 Large extent of water surface, either in drops or wetted surface.

2 The wet-bulb temperature, which is a function of the dry-bulb temperature and the relative humidity. The wet-bulb temperature is the absolute lowest limit to which water can be cooled evaporatively. In practice this temperature is only approached but never reached.

3 The quantity of air coming in contact with the water surface, its time of contact, and its relation to the quantity of water cooled, known as the air-water ratio.

4 The velocity with which the air scours the water surface.

In the spray types of equipment, nozzles operating at from 5 to 10 lb pressure finely break up the water to produce the desired drop surface, and the water remains in the air a comparatively short time in its unbroken fall to the bottom. Towers with filling or decks present a considerable wetted surface. In addition, the filling interrupts the fall of the water, thus increasing the time of contact of water and air. Table 2 shows the length of time for a drop of water to fall 30 ft with equispaced decks for interruptions. It appears that more than nine decks are not warranted by the gain in time of air contact and, remembering that decks offer resistance to lateral air flow induced by wind, it is not surprising to find that 30-ft-high atmospheric deck towers are standardized with about nine or ten decks. Filled towers are more expensive than spray types, but are more uniformly efficient over a large range of water-circulation rates. The water need not be finely broken up by pressure nozzles when filling is employed but only uniformly distributed over the top of the filling by means of low-pressure distributors or troughs. Spray nozzles dribble water without proper breakup when operating below designed capacity and hence, cooling falls off.

TABLE 2 VARIATION IN TIME FOR WATER TO FALL 30 FT WITH NUMBER OF DECKS

No. of decks	No. of falls	Total time of drop in air, sec.
0	1	1.36
1	2	1.92
4	5	3.05
9	10	4.30
14	15	5.25

In mechanical-draft towers, the filling is spaced more closely than in the atmospheric deck types, the height of filling being only from 12 to 15 ft out of a stack height of from 25 to 35 ft. The positive movement of air by forced- or induced-draft fans insures upward flow of air against filling and eliminator resistance quite independent of wind conditions. Spray types of mechanical-draft towers eliminate the considerable weight of filling and reduce the resistance against which the fans operate. On the other hand, they are subject to the limitations mentioned previously. Their proper field of application is where the ratio of

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Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y., and will be accepted until Feb. 10, 1939, for publication at a later date. Discussion received after the closing date will be returned.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

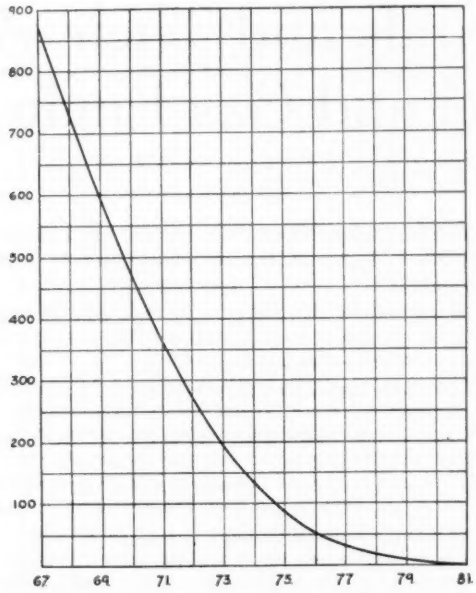


FIG. 1 HOURS DURING JUNE, JULY, AUGUST, AND SEPTEMBER THAT WET-BULB TEMPERATURES ARE EXCEEDED
(New York City observations.)

wet-bulb approach[1] to cooling range is rather high. Spray towers require from 20 to 40 per cent more stack area than filled mechanical types and also require higher air-water ratios. As the approach-to-range ratio decreases, indicating more severe cooling conditions, the size of spray-type towers compared with filled towers increases rapidly. This disparity in size and air-water ratio is accounted for by the short time the descending water is in contact with the counterflowing air, which prevents the air from taking up its full quota of heat and moisture. The discharged air has a considerably lower wet-bulb temperature than that of filled towers. Filling with its wetted surface acts very much like additional surface and baffled flow in a heat exchanger.

FACTORS INFLUENCING SIZE OF COOLING TOWERS

The cooling conditions in a specific situation which influence the size of a tower are: The design of the tower; the wet-bulb temperature of the location; the heat to be dissipated, which depends on the quantity of water circulated together with the cooling range; and the position of the cooling range on the temperature scale, which is fixed by the wet-bulb temperature and the approach of the off-tower water temperature to that wet-bulb temperature.

The wet-bulb temperature cannot be controlled. It suffices to say that the wet-bulb temperature to be chosen as the basis of tower selection should not be so high as to occur but a few hours per year, nor so low as to result in inadequate cooling over much of the hot season. The mean of the daily high wet-bulb temperatures of the hottest month of the year usually results in a suitable figure. A closer study of the wet-bulb conditions may be made from local weather-bureau data, which will indicate the probable frequency of each wet-bulb temperature during a year or a season. Fig. 1 shows the hours for June, July, August, and September in New York City that the wet-bulb temperatures shown are exceeded. It will be noted that 70 F wet bulb is exceeded about 475 hr out of a total of 2928 hr, over 16 per cent of the 4 months or $5^1/_2$ per cent of the year. A 75 F wet-bulb temperature is only exceeded about 88 hr in the 4 months, 3 per cent of the time or 1 per cent of the year, while 78 F wet bulb is exceeded only 15 hr in the 4 months, 0.5 per cent of the time or 0.17 per cent of the year. For most purposes 75 F wet bulb in this locality would be amply high, as it would be more economical to use city water to supplement the cooling tower for the remaining few hours or to operate at slightly higher temperatures, rather than to invest in excess tower size.

The design of the tower rests largely with the manufacturer and is, except as discussed before, beyond the scope of this paper. The choice of water quantities to be cooled and the cooling range to meet required heat dissipation, together with its position in the temperature scale is, however, subject to considerable choice on the part of the purchaser. Extensive experience in the cooling-tower field indicates to the writer that the purchaser is often unaware that modification of these factors may reduce the size of cooling tower considerably. These modifications depend to some extent on the requirements of the equipment which the tower serves, especially so in the case of condensers and heat exchangers. In general a larger tower calls for a smaller heat exchanger and vice versa, so that the selection of both tower and heat exchanger should be studied as a combined unit to obtain greatest economy. It therefore seems desirable to attempt an evaluation of the factors, aside from design, which influence cooling-tower size, namely: (1) Water circulation, (2) wet-bulb temperature, (3) approach of off-tower water temperature to wet-bulb temperature, (4) cooling range or difference between on- and off-tower water temperatures.

Fig. 2 shows the effect on mechanical-draft-tower size, i.e., the horizontal projected area inside of the stack, of variations in the amount of water circulated, wet-bulb temperature, approach to wet-bulb temperature, and cooling range. Each curve shows the influence of but one variant when others are kept constant. The horizontal scale gives factors for tower sizes and unity factor has been assigned to a 20 F cooling range, a 10 F approach to the wet-bulb temperature, and a wet-bulb temperature of 70 F. By assigning a suitable constant to convert area factors into square feet and developing a quantity-of-water-circulation scale in gallons per minute, all to conform with a specific tower design, this set of curves could be used to obtain the correct size of a mechanical-draft tower of such a design to meet any

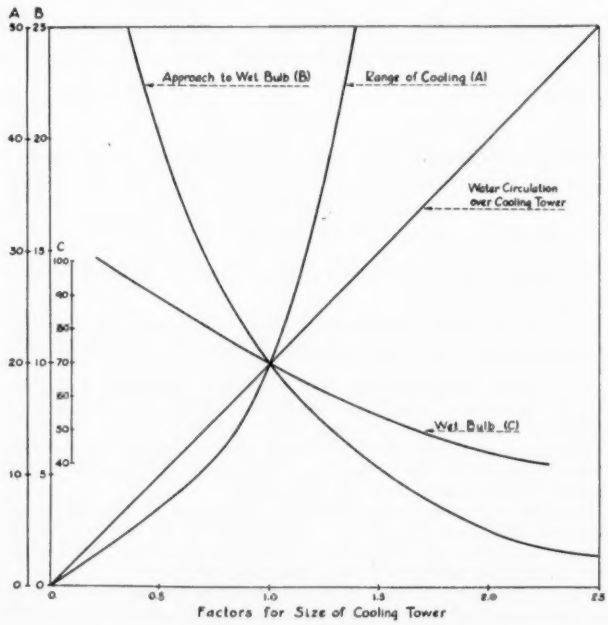


FIG. 2 EFFECT OF VARIANTS ON SIZE OF COOLING TOWER

[1] The wet-bulb approach is the difference between the temperature of the cooled water leaving the tower and the wet-bulb temperature.

conditions. The size of the tower is equal to the product of the four size factors times a constant.

For the present purpose, the curves are useful however, in showing how the size of the tower varies with the cooling conditions to be met. It strikingly illustrates how the purchaser can, by a wise choice of such cooling conditions as lie within his power of modification, obtain a reasonably sized cooling tower to give complete satisfaction at a reasonable price.

Analysis of each curve separately shows that:

1 The size of a cooling tower will be in direct proportion to the quantity of water circulated and cooled.

2 For equally close approaches to wet-bulb temperature, a low wet-bulb temperature requires a larger tower. The percentage of increase in tower size increases as the wet-bulb temperature falls.

3 For equal cooling ranges, the closer the approach of the temperature of the off-tower water to the wet-bulb temperature, the larger is the necessary tower size. Close approaches call for disproportionately large tower sizes.

4 Greater ranges of cooling, i.e., greater difference between on- and off-tower water temperatures, do not require proportionately larger towers. Hence it is more economical, for a given heat dissipation, to raise the on-tower water temperature and decrease the quantity of water circulated.

The curves of Fig. 2 are based for convenience on mechanical-draft towers. A similar set with but slight modification could be drawn up for an atmospheric tower which would have the same general characteristics but with the additional variant of wind velocity. In general all that has been said of variants in a forced-draft tower are equally true of an atmospheric tower.

Where water is scarce, or of such quality that treatment becomes imperative, the quantity of make-up water required is important. Water lost by evaporation is a direct function of the heat dissipated and is roughly equal to one pound for every 1000 Btu. To this amount must be added a constant driftage loss which depends upon the water circulation, the kind of equipment, and its design. In general terms and under average wind conditions a spray pond loses from 5 to 5 per cent of its circulation, an atmospheric tower from 1 to 1.5 per cent, and a mechanical-draft tower with proper eliminator system from 0.2 to 0.5 per cent.

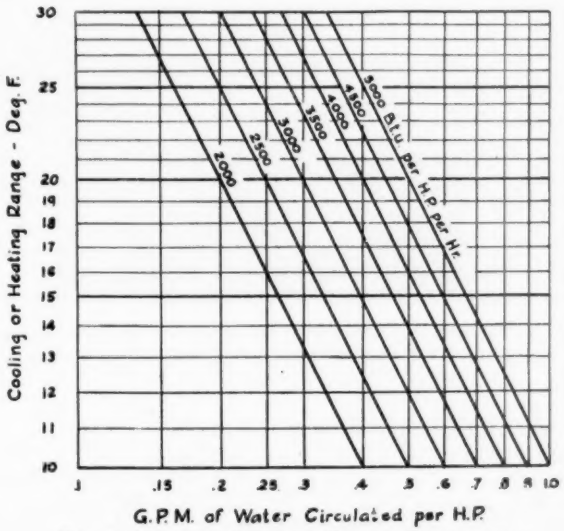


FIG. 3 RELATION BETWEEN COOLING RANGE AND CIRCULATION

(Problem: A Diesel engine of 1500 hp is to be cooled by removal of 3000 Btu per hp per hr. If the water is to be cooled through a 20-F range, how much should be circulated?

Solution: Trace horizontal line from 20 F to intersection with diagonal line of 3000 Btu per hp per hr. Drop vertically from intersection and read 0.3 gpm per hp. Then for 1500 hp, 1500 × 0.3 = 450 gpm.)

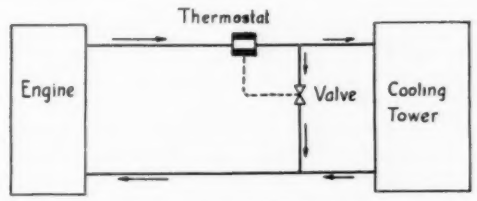


FIG. 4 THERMOSTATIC BY-PASS CONTROL

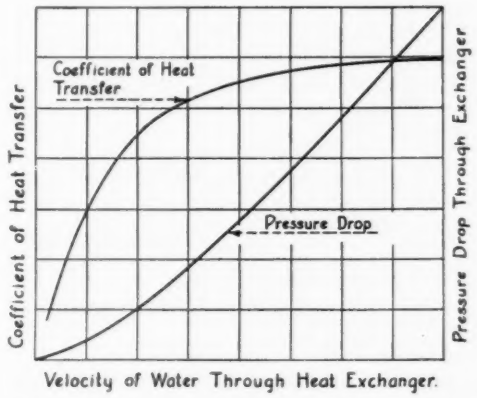


FIG. 5 VARIATION OF COEFFICIENT OF HEAT TRANSFER AND PRESSURE DROP WITH VELOCITY IN HEAT EXCHANGERS

APPLICATIONS TO DIESEL- AND GAS-ENGINE COOLING

Satisfactory oil- and gas-engine cooling is recognized as necessary for efficient operating and low maintenance costs. Determination of the proper amount of heat dissipation is the first consideration. Recent advances in engine design have enabled more of the heat input to be converted to energy at the crankshaft, thereby diminishing the heat lost to cooling water. Engine efficiencies have steadily risen from 28 to 39 per cent. A recent questionnaire sent to leading American Diesel- and gas-engine manufacturers disclosed that the heat to be dissipated by the cooling water varied from 1300 to 3500 Btu for Diesel engines and for gas engines from 3800 to 5400 Btu per bhp per hr. The figures depend upon design and whether or not exhaust manifolds and pistons are cooled. With such wide variations generalizations are dangerous but in the great majority of cases values for Diesel engines were between 2500 and 3000 Btu, while for gas engines they ranged from 4000 to 5000 Btu per bhp per hr.

A convenient chart for establishing circulation in accordance with cooling range and vice versa, when once the heat to be dissipated by the cooling water per horsepower per hour is determined is shown in Fig. 3. In general, ranges should be kept low, 12 to 20 F, and circulation of jacket water high so as to reduce thermal stress and secure turbulent circulation without dead pockets for steam formation and scale deposit. Partial recirculation is often advisable with thermostatic control, keeping off-engine water temperatures uniform without changing jacket-water circulation. This is done by means of by-passing the cooling equipment as illustrated diagrammatically in Fig. 4.

In recent years there has been a decided trend to the closed system of engine cooling. While the cost of the closed system is roughly three times the cost of the open system, the total yearly charges are more nearly the same.

The over-all rate of heat transfer through the heat-exchanger surface in closed systems depends upon the velocity of water in the tubes and around the tubes, the cleanness and nature of the

surfaces, and the mean temperature difference existing between raw water and jacket water. Maintenance of clean surfaces is an operating problem, while the kind of surface should largely be left to the manufacturer. Velocities, on the other hand, depend both upon the quantity of water circulated through the jackets and the arrangement of heat-exchanger tubes. Parallel tubes decrease the velocity and a series arrangement increases it. Fig. 5 shows the gain in over-all coefficient of heat transfer with increasing velocities and it also shows the increase in pressure drop especially at higher velocities. The nature of these curves indicates that due to relatively small gain in coefficient at high velocities and relatively large pressure-drop increases, there is a definite point beyond which velocities should not be increased. In practice, velocities may vary from 1.5 to 6 fps within the exchanger tubes.

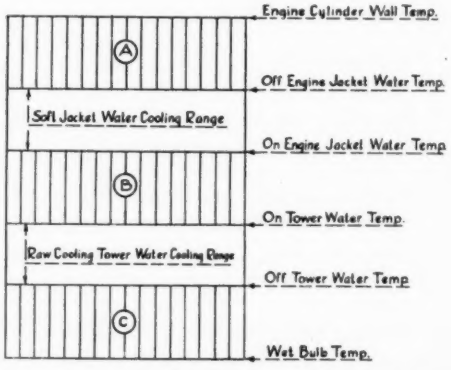


FIG. 6 STEPS IN HEAT DISSIPATION FROM ENGINE CYLINDERS TO ATMOSPHERE

(Temperature differences: (*A*) Engine cylinder wall and soft jacket water; (*B*) soft jacket water and raw tower water; (*C*) raw tower water and wet-bulb temperature.)

By increasing the quantities of water circulated the coefficients of heat transfer are not only increased but the cooling and heating range is kept narrow; an important consideration in obtaining the necessary mean temperature difference for heat transfer with reasonably sized exchanger surfaces. Heat dissipation from the engine cylinder to atmosphere takes three distinct steps as illustrated diagrammatically in Fig. 6: (*A*) from cylinder wall to jacket water, (*B*) from jacket water to raw water, and (*C*) from raw water to atmosphere.

The lower the wet-bulb temperature and the higher the permissible cylinder temperature, the greater is the total temperature difference available for these steps and hence the smaller the equipment may be. These top and bottom temperatures usually cannot be controlled, but by circulating larger quantities of both raw and soft water, which can be controlled, the ranges are narrowed leaving a greater temperature difference between hot jacket water and cool raw water (*B*).

EQUIPMENT FOR COOLING SYSTEMS

Scarcity or high cost of water and frequently problems of disposal necessitate recooling and reuse of water rather than running it to waste. A cooling tower saves about 95 per cent or more of the water. When soft water is available for make-up, open systems can be used. In such cases the jacket water itself is pumped through the engine and over some form of tower, after which it is recirculated through the engines. The choice of the particular type of tower depends largely on location and cost of ground space. In order of ascending investment costs are spray ponds, spray atmospheric towers, deck atmospheric towers and mechanical-draft towers. This also is the order of decreasing ground space required. Unless a pond of considerable extent already exists with cheap ground rent, spray-pond equipment, despite its low first cost, will prove expensive. At the other extreme, forced-draft towers, with their higher initial and operating costs, occupy small ground space (especially the filled induced-draft types), need no exposure, and are even adaptable for indoor use.

Heat exchangers used with cooling towers are of two kinds, the shell-and-tube type shown in Fig. 7 and the atmospheric tube type, placed directly in the base of a cooling tower as shown in Fig. 8. Table 3 compares the salient features of each type.

Radiators have found a wide application for cooling engines of small and medium size. They are particularly suitable for

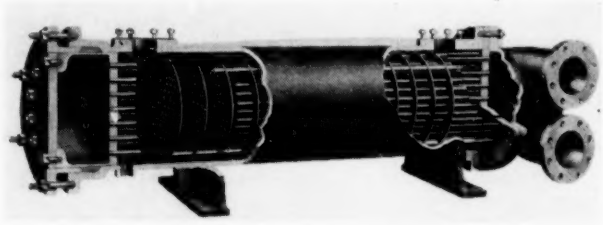


FIG. 7 SHELL-AND-TUBE TYPE OF HEAT EXCHANGER

(Courtesy of Griscom Russell Company.)

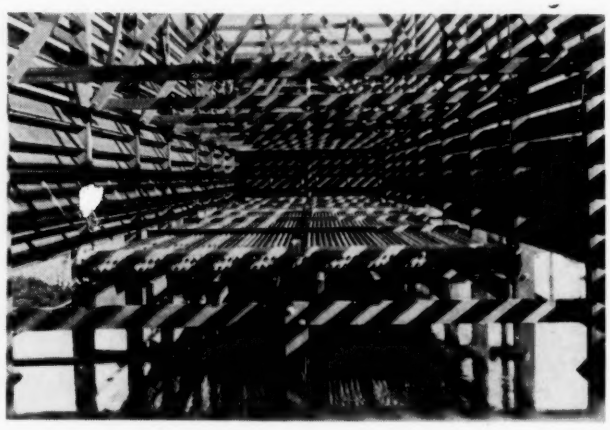


FIG. 8 INTERIOR OF SPRAY-TYPE COOLING TOWER WITH ATMOSPHERIC-TUBE HEAT EXCHANGERS IN BASE

(Courtesy of The Cooling Tower Company, Inc.)

TABLE 3 ATMOSPHERIC-TUBE AND SHELL-AND-TUBE HEAT EXCHANGERS COMPARED

	Atmospheric-tube exchanger	Shell-and-tube exchanger
Water flow	Raw water showered on outside of tubes, soft water inside. Raw-water side easily cleaned by simple descaling without dismantling	Raw water through tubes and soft water baffled around tubes, causing all water flow to have high positive velocities
Over-all coefficient of heat transfer	From 125 to 170 Btu per hr per deg F mean temperature difference per sq ft	From 125 to 325 Btu per hr per deg F mean temperature difference per sq ft
Ratio, raw to soft water circulated	From 1/1 to 2/1	From 1/1 to 3/1
Location	Directly in base of cooling tower and showered by descending raw water	May be placed indoors to prevent water freezing during periods of shutdown in cold weather

portable field engines and in some cases for indoor installations. As they do not employ the principle of evaporation, they are free from scale deposit and need no make-up water. On the other hand, they require the movement of a large amount of air and consequently high consumption of fan power. If off-engine water temperatures are to be kept below 160 F, under dry-bulb conditions exceeding 100 F, their size and horsepower requirements make them uneconomical.

The evaporative condenser was designed primarily as a combined cooling tower and condenser for refrigerant condensing, but it is also finding a field in Diesel-engine cooling. A photograph of one of the larger ones is shown in Fig. 9. It consists of a sheet-

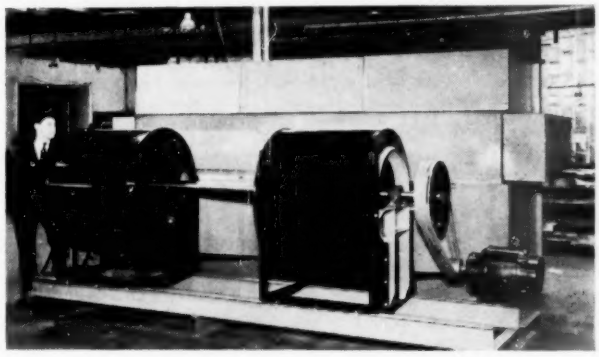


FIG. 9 LARGE EVAPORATIVE-CONDENSER-TYPE WATER COOLER (Courtesy of the Trane Company.)

metal shell through which air travels vertically driven by induced- or forced-draft fans. In the path of the ascending air are located a series of horizontal tubes carrying the soft water. These tubes have finned surfaces, and a spray-nozzle system discharges the raw water over the fins where evaporation occurs. Excess water collects in a pan from which it is pumped back to the nozzles together with the make-up water. The equipment is essentially an evaporatively cooled radiator requiring about half the amount of air necessary for a dry radiator. Jacket water is cooled through a 20 F range with off-engine water temperature between 140 and 160 F. The raw-water circulation is low compared with other types of heat-exchanger systems, being only about one third of the soft-water circulation.

The evaporative-radiator system seems well-adapted to engines of small and medium size, and forms a compact unit for indoor closed systems from which the moist air is discharged to the exterior through ducts. A serious disadvantage is the accumulation of insulating scale on the fins, resulting from salt deposits left by the evaporation of the raw water. Cleaning in place is out of the question and ordinary descaling methods cannot be used on fins. The finned sections must therefore be periodically replaced by spares while chemical descaling is applied to the fouled surfaces. Tubes without fins may be used where only hard untreated water is available for make-up, but this reduces the surface to such an extent that about 50 per cent more tubes are required.

ECONOMY OF CLOSED VERSUS OPEN SYSTEMS

The trend to closed cooling systems in preference to open systems has been so marked in recent years that the writer undertook to establish total yearly costs and analyze the components of these costs for both systems over a wide range of engine sizes. For the open system the investment covered a spray-type atmospheric tower mounted over a concrete foundation basin, all erected in place, together with a suitable pump and motor for water circulation. Cooling was from 115 F to 95 F at 75 F wet bulb. This resulted in a circulation of 0.3 gpm per hp. The closed-system investment included a much larger spray-type atmospheric tower, with atmospheric-tube heat exchangers mounted in the base, erected over a concrete basin foundation. Included also in the investment cost were a raw- and a soft-water circulating pump with motors. The jacket water cooling was much higher in the temperature scale (a permissible and desirable condition with closed systems), being from 140 F to 125 F, with both raw- and soft-water circulation at 0.4 gpm per

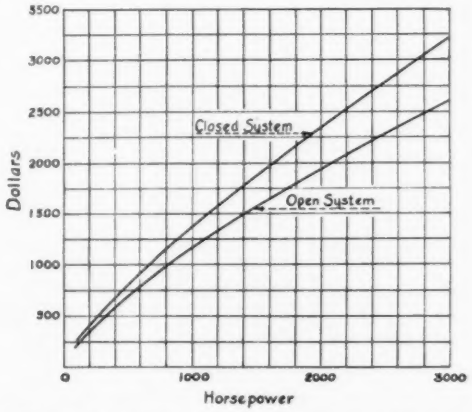


FIG. 10 TOTAL YEARLY CHARGES FOR OPEN AND CLOSED COOLING SYSTEMS FOR ENGINES

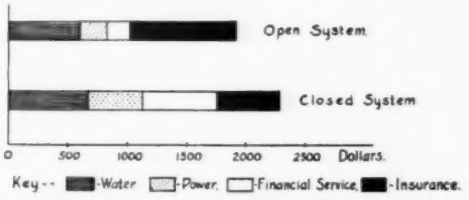


FIG. 11 COMPONENTS OF TOTAL YEARLY CHARGES

hp. No piping was included for either type of system. The investment costs, depending on size of installation serviced, varied from \$4 to \$1 per hp for the open system and from \$11 to \$3 for the closed system.

Fig. 10 graphically illustrates the variation of total yearly charges for both systems, servicing installations from 100 to 3000 hp.

Total yearly charges are composed of investment charges, operating charges (pumping cost and make-up water), and insurance. Investment charges as taken were based on 5 per cent interest on installation cost and yearly equalized depreciation payments for 10 years into a sinking fund compounded annually at 5 per cent interest. The operating charges assume operation for 3000 hr per year, make-up water costing \$1 per 1000 cu ft, and electric current at \$0.01 per kwhr. The insurance rates, were based on engines built since 1925, are for 3-yr premiums, and are the so-called "objective charges" to which is added a small insurance charge, not included.

Fig. 11 analyzes these components for a cooling system servicing a 2000-hp installation. It is interesting to note that while power charges, because of double pumping, and financial charges, because of larger investment, are materially higher for the closed system, the low insurance rates, due to its use, make total yearly charges only about 20 per cent higher than for the open system. The insurance component is a measure, in some degree, of the engine-repair costs but when other items associated with trouble-free and uninterrupted service are considered, it is readily seen that closed systems are economically justified unless very soft water or elaborate treatment is used.

Theory of Elastic Engine Supports

By S. ROSENZWEIG,[1] NEW YORK, N. Y.

It is obvious that the application of spring suspension to a foundation represents a great departure from the methods previously used. Instead of rigidly bolting down a machine, it is now installed so as to operate on a yielding structure. Instead of comparatively simple static laws, complicated dynamic conditions have to be considered. This paper presents the theory of the dynamic principles underlying machine operations. It is shown that not only capacity, speed, and weight of a machine must be considered, but also acceleration and free forces of reciprocating and rotating engines, irregularity of motion, inertia forces of individual parts, forced frequencies of operation, and natural frequencies of the subsoil.

Frequencies of Elastic Supports

THE SIMPLEST form of a vibratory motion can be demonstrated by the action of a spring, Fig. 1. A spring is loaded by a weight L and suffers a static deflection of d. An external force is applied to the loaded spring so as to displace the weight from its rest position C to some other position A. This external force is now removed and the weight oscillates between points A and B, due to the elasticity of the spring. These oscillations gradually die out, principally due to air resistance, till the weight stops at rest position C. The spring, elastically speaking, functions in identically the same manner as a swinging pendulum whose length 1 equals the deflection d of the spring. This analogous function is of great importance in the design of elastic suspensions.

The movement of the spring from C to A and C to B, designated x, is called the amplitude of the vibratory motion. By traversing the distance of $4x$ (from C to A, from A to B, and back to C) a cycle of the vibrating body is completed. The time required to complete the cycle is called the period, designated by T, and is usually measured in seconds. The reciprocal of T is termed the frequency of vibration, f, which indicates the number of periods that occur in a given unit of time, usually in cycles per second. For instance, if the frequency of vibration is ten cycles per second ($f = 10$), ten cycles are completed in one second with the duration of each cycle T being $^1/_{10}$ of a second.

The rate at which the spring oscillates has one particular value which is called its *natural frequency of vibration*. It is that frequency at which a system will vibrate after the external force which displaced it is removed. This natural frequency always remains the same, no matter how great or small the initial amplitude x of the spring or in the case of a pendulum the distance the pendulum moves from its rest position. Only the physical characteristics (wire diameter, outside diameter, etc.) of the spring or the load will affect its frequency and in the last analysis the static deflection d which is determined by both these factors. For an undamped simple linear system this natural frequency can be figured by the formula[2]

$$f = 3.13 \sqrt{\frac{1}{d}} \quad \ldots\ldots\ldots\ldots\ldots\ldots [1]$$

in which f is the frequency in cycles per second and d the deflection in inches.

The action of the spring under a momentary blow, as occurs, for instance, in a hammer installation, is of sudden and short duration. It may occur once or repeatedly, depending upon the number of blows that are struck; but from a vibratory standpoint, each blow produces an individual motion which gradually dies down.

In the case of rotary machines, such as motors and turbines, or reciprocating machines, such as steam and oil engines, there is

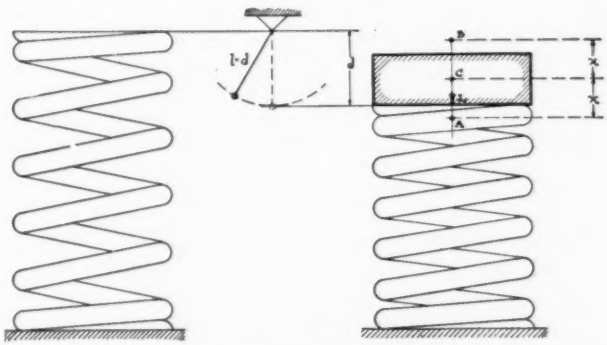


Fig. 1 Action of a Spring After Application of a Static Force

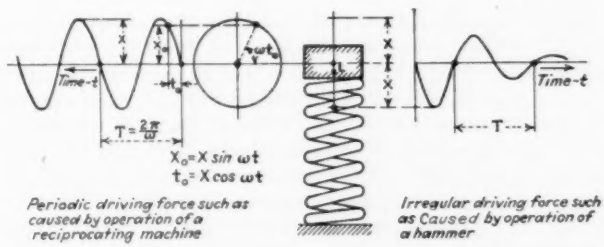


Fig. 2 Action of a Spring From Periodic and Irregular Forces

a periodically varying motion which reverses itself twice during each cycle and continuously repeats itself in all its particulars after a certain interval of time, called the *period of vibration*. To produce either motion, an external driving force, having a definite frequency called *forced frequency* is required. In the case of the hammer, it is provided by the blows struck on the anvil. In the case of engines, it is provided by the continuously rotating and reciprocating mechanism. The difference in the action of the irregular and periodic driving forces is apparent from Fig. 2 which shows the respective vibratory motions. Different physical laws apply to each. This paper will concern itself with the laws governing periodic forces produced by reciprocating engines.

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[1] President, The Korfund Co., Inc.

Presented at the Eleventh National Oil and Gas Power Meeting of the Oil and Gas Power Division, The American Society of Mechanical Engineers, at Dallas, Texas, June 6–9, 1938. Abridged.

Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y., and will be accepted until February 10, 1939, for publication at a later date. Discussion received after the closing date will be returned.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[2] "Vibration Prevention in Engineering," by Arthur L. Kimball, John Wiley and Sons, Inc., New York, N. Y., 1932, p. 7.

Damping

Vibratory movement is always impeded by some sort of resistance. Generally speaking, this resistance is produced by what is technically termed either *absolute damping* or *relative damping*. Absolute damping, the more objectionable type, is usually caused by the sides of a foundation rubbing against other surfaces, such as earth, or soundproofing material against which the foundation is poured. This friction is usually not excessive since the foundation has a tendency to shrink away from the surrounding pit walls. An air space around the foundation is always preferable and, where it is provided, reduction in any vibratory transmission present will be noticeable. Relative damping is due to the quality of the substructure on which the foundation rests. All soils and, naturally, all organic shock-absorbing materials, such as rubber, felt, or cork, have a considerable damping factor under compressing action. Contrary to the still prevailing belief of many engineers, damping is decidedly harmful because it increases the transmitting effect of vibrations.

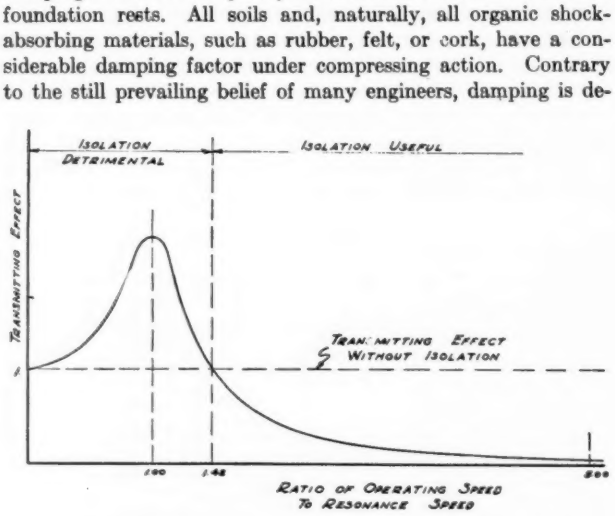


Fig. 3 Simple Form of Vibration-Isolation Analysis

Fig. 3 proves mathematically the correctness of this statement, which is equally well demonstrated by practical experience. Steel springs have a damping coefficient of less than one half of one per cent, a fact which contributes greatly to the excellent results obtainable with them as shock absorbers. Absorbing vibrations by means of damping implies dissipation of energy in the form of heat. It is energy irretrievably lost to the machine. This leads to a most important conclusion. When the operation of a machine sets up a vibratory motion in an adjoining structure, the energy used is spent for an entirely different purpose from that for which the machine is built. This energy is subtracted from the productive output of the machine. It is, therefore, profitable to regain as much as possible of this energy. A correctly designed spring suspension with a minimum of damping will save a considerable amount of this otherwise dissipated energy. This effect can be demonstrated most strikingly on large motors, where it appears as an appreciable increase in speed when motors are correctly isolated. No definite figures about actual savings are available; some prominent European engineers claim increase in power up to ten per cent. Damping, to prevent excessive movements, is of advantage only when the machine is operated near resonance speed, but better means than damping are available for this purpose to the designer of a spring-suspended foundation.

Power and Operating Amplitudes

The first complete analysis of vibration isolation was worked out by Soderberg.[3] A diagrammatical representation in its simplest form is shown in Fig. 3. The ordinates show the amount of the vibratory force which is transmitted to the substructure of an isolated foundation in comparison with a rigidly mounted foundation represented by the horizontal line 1. The abscissa is the ratio of the forced frequency (due to the operation of the machine and, often, corresponding with it's revolutions) to the natural frequency of the vibration absorber. For instance, if this frequency ratio is 4:1 the transmitted force is only about one tenth that of a rigidly mounted machine. When this ratio sinks below 1:42 there is a most astonishing result. The use of a so-called vibration absorber actually makes conditions worse. This is not just theory but has often been demonstrated in actual practice, particularly when the frequency ratio was in the region between zero and one. Many an engineer has assumed that, when a certain thickness of felt, cork, or rubber placed under a machine still permitted transmission of vibration, doubling the thickness of the isolating material would certainly improve conditions. To his astonishment, he found just the contrary to be true. The diagram clearly shows the reason for the failure.

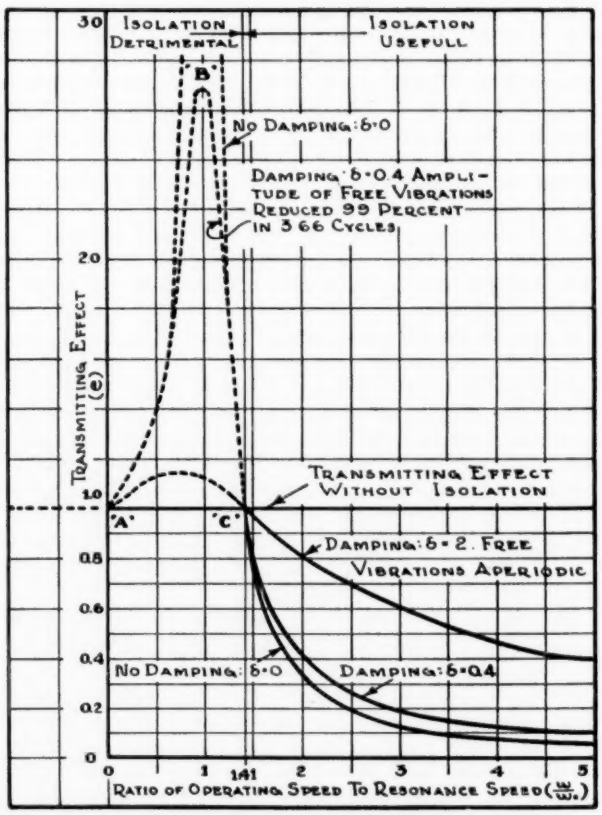


Fig. 4 Effect of Isolators With Different Degrees of Damping

This effect can also be clearly observed when standing on the floor adjoining a correctly isolated foundation. When starting and accelerating the engine, the frequency ratio is changed due to the varying speeds of the engine. A quickly increasing vibration transmission, which may be noticeable during the low-frequency region, will gradually diminish and suddenly disappear as a suitable frequency ratio is reached.

[3] "Vibration Absorbers," by C. R. Soderberg, *The Electric Journal*, vol. 21, April, 1924, Fig. 26, p. 162.

Another point in this diagram is of great importance. When the frequency ratio is one, i.e., when the two frequencies are alike, the transmitting effect is at a maximum, B. The system is now operating at, or passing through, what is called the *resonance speed of the system.* This is a dangerous condition if maintained, because large amplitudes of motion and excessive transmitted vibrations may be built up by comparatively small forces. This is the reason why soldiers are ordered to break step when marching over a bridge. To avoid disaster, the frequency of the combined rhythm of marching feet must not correspond with the natural frequency of the bridge.

In the next illustration, Fig. 4, the representation of Fig. 3 is repeated by curves showing the transmitting effect of vibration isolators having different degrees of damping. The curve $D = 0$ (zero damping), represents the result of isolation by means of steel springs, and the other curves illustrate the gradual decrease in efficiency as damping increases. This vibration-absorbing efficiency is more clearly represented by the curves

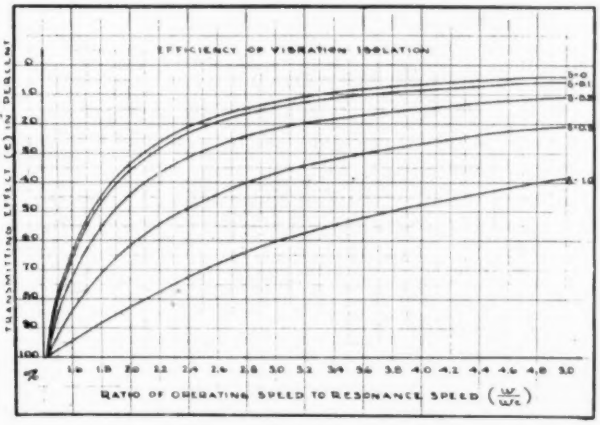


FIG. 5 EFFICIENCY OF VIBRATION ISOLATION

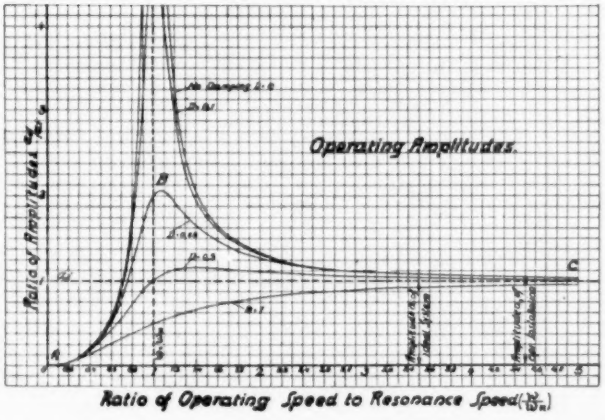


FIG. 6 RELATION OF ACTUAL OPERATING AMPLITUDES TO IDEAL CONDITIONS

in Fig. 5 where, for each degree of damping, it is plotted over the entire range of frequency ratios, varying between 1:42 and 5:1.

Therefore to make the force transmitted to the substructure as small as possible, the forced frequency of the operating machines must be large, compared to the natural frequency of the shock absorber, with a minimum of damping. This ratio should be at least 3:1, and in many cases, considerably more, depending upon local conditions and the magnitude of unbalanced force. It is clear that if this unbalanced force is small, a frequency ratio of 2.5:1 or 3:1 may be sufficient to make the transmitted vibratory force so small that it will not be noticeable. If, however, this force is great, an efficiency of the isolating medium of 80 to 90 per cent may not be sufficient. In such cases frequency ratios of from 5:1 to 8:1 are required.

With these facts established, it now becomes necessary to examine the movement which the running engine possesses when placed on a spring-suspended foundation. The operating amplitudes shown in Fig. 6 are all related and compared with the *ideal system.* The term, ideal system, refers to an imaginary system suspended in space and provided with an infinitely elastic suspension, i.e., no vibrations are transmitted. These conditions may be approached but can never be realized in actual practice.

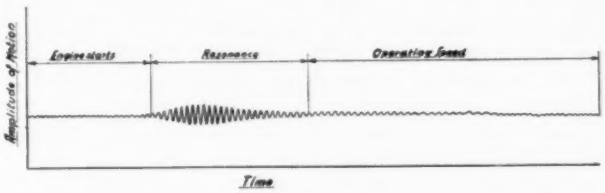


FIG. 7 AMPLITUDE OF MOTION OF FOUNDATION SYSTEM

The horizontal line 1 represents the operating amplitude of the ideal engine, obviously constant because the frequency ratio is infinitely large. This amplitude is, therefore, properly termed the *limit amplitude.* For the actual engine, in comparison, amplitudes are small for the low frequency ratios but gradually increase until they become large at resonance speed. From point B they decrease asymptotically until at point C they approach the amplitude of the ideal system.

Some very interesting conclusions can be drawn from this diagram. It is possible that somewhat steadier engine operation is obtainable with an unisolated or incorrectly isolated foundation, as the range of curves to the left of the vertical 1.0 line indicates, though ordinarily on account of the resiliency of the subsoil, operation will take place along the 0.4 to 0.6 abscissa. But this causes excessive wear in the engine, reduced efficiency, and the annoyance which always accompanies transmitted vibrations. The benefits obtainable with a spring-suspended foundation are so great, physically and economically, that slight oscillation, hardly exceeding a few thousandths of an inch, is immaterial. That these operating amplitudes are negligible, is apparent from Fig. 7, representing the record taken from an actual engine. As shown, the amplitudes vary from the time the engine starts, passes through resonance, and attains operating speed. By making the springs in the isolator very soft, resonance speed will occur at a very low engine speed and therefore be passed through so quickly that the small increase in movement due to resonance is hardly noticeable.

It is evident that a vibration isolation of 100 per cent with no oscillation of the operating system is impossible. However, for a frequency ratio of 5:1 and with zero damping, the amplitude of motion will be within 4 per cent of the ideal engine and vibration-absorbing efficiency approximately 97 per cent. This means that the motion of the mass would be virtually imperceptible and the slight vibration transmission would be well beyond the human sensitivity range, hardly detectable even by the finest instruments.

FOUNDATION AS A STABILIZER

Unbalance and other inertia forces have the tendency to upset the equilibrium of the machine system, free forces tending to move it in a translatory direction and free moments in a rotational direction. Even in machines in which an attempt

is made to provide perfect balance, as for instance for the primary and secondary forces and moments on six- and eight-cylinder Diesel engines of the four-cycle type, slight unavoidable changes in the manufacturing process will upset the theoretical calculations. These unbalanced forces in Diesel engines can be accurately measured. Their presence or absence depends upon

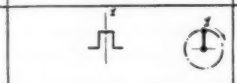
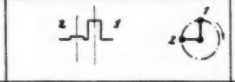
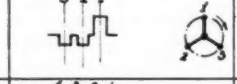
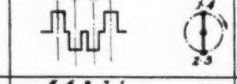
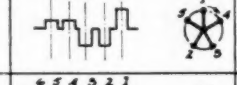
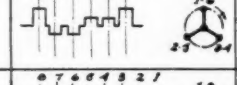
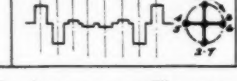
No. of Cylinders	Crank Sequence Diagrams	Unbalance	
		Free Forces	Free Couples
1		Primary and secondary forces unbalanced	Nil.
2		Secondary vertical force unbalanced	Primary couples unbalanced
2		Primary force unbalanced	Primary and secondary couples unbalanced
3		Nil.	Primary and secondary couples unbalanced
4		Secondary vertical force unbalanced	Nil.
5		Nil.	Primary and secondary couples unbalanced
6		Complete primary and secondary balance	
0		Complete primary and secondary balance	

FIG. 8 ANALYSIS OF UNBALANCE FOR VARIOUS CRANK ARRANGEMENTS

the number of cylinders and the crank arrangement, and their magnitude depends upon the design of the particular machine.

Fig. 8 shows the type of unbalance for Diesel engines of any given number of cylinders with specific crank arrangement. Operation of the engine will provide additional changes in the balance, such as variations in piston clearances due to unequal expansion, difference in the size of combustion chambers when bearings and connecting rods are adjusted, etc. It is, therefore, necessary to provide a stabilizing mass in the weight of the engine and foundation which will counteract the disturbing effect of free forces and moments. The calculation of the stabilizing mass is essentially the work of the mathematician backed by the experience of actual installations to check his calculations. The curves in Fig. 9 are the results of such calculations in which Q represents the stabilizing mass, a_0 the operating amplitude, and K a dynamic-balance coefficient which depends upon the perfection of balance in the engine. The coefficient K which can be calculated from the data furnished by engine manufacturers, governs the foundation weight for a given amplitude and, therefore, becomes also a concrete measure of the dynamic-balance quality of an engine's design and construction.

It should be noted that the results obtained from the curves given in Fig. 9 apply to the ideal engine and give, therefore, optimum conditions. Fig. 6 indicates that a close approach to the ideal engine, within 4 per cent, is possible by means of correct isolation by employing only minimum foundation weights.

The mathematical calculations which are the basis of Fig. 7 have been covered by Hartz.[4] An outline of these calculations illustrated by Fig. 10 is given here.

Using a single-cylinder engine which has a primary unbalanced force and a two-cylinder engine which, with cranks at 180 deg, has a primary unbalanced moment, the respective translatory and rotational movements caused by such unbalance are shown and can be mathematically calculated. In a mass system as represented by the ideal engine and its foundation, the moving masses, consisting of piston, piston rod, connecting rod, crank, etc., move relatively to the engine frame and the foundation to which it is rigidly bolted. Dynamics teaches that in such a system every movement of the revolving and reciprocating masses is counteracted by movement of the engine frame and foundation, which tends to maintain equilibrium by keeping the combined center of gravity S stationary. Let

m_1 = the combined revolving and reciprocating mass
r_1 = crank radius = displacement of m_1
m_2 = mass of foundation and stationary engine parts
r_2 = displacement of m_2 = a_0 in Fig. 9
m = $m_1 + m_2$ = total stabilizing mass
g = 32.2 ft per sec per sec

Therefore, if m_1 and m_2 move in counterphase to each other in such a way that S remains stationary, the law of gravitation is complied with if

[4] Vorschau auf die 75. V.D.I. Hauptversammlung in Kiel, H. Hartz, *Zeitschrift des Vereines deutscher Ingenieure*, vol. 81, June, 1937, p. 638.

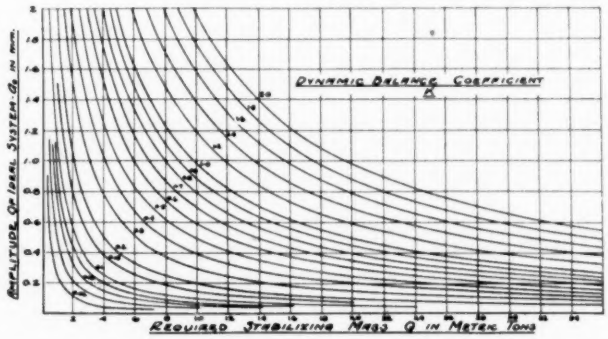


FIG. 9 RELATION OF STABILIZING MASS TO OPERATING AMPLITUDE

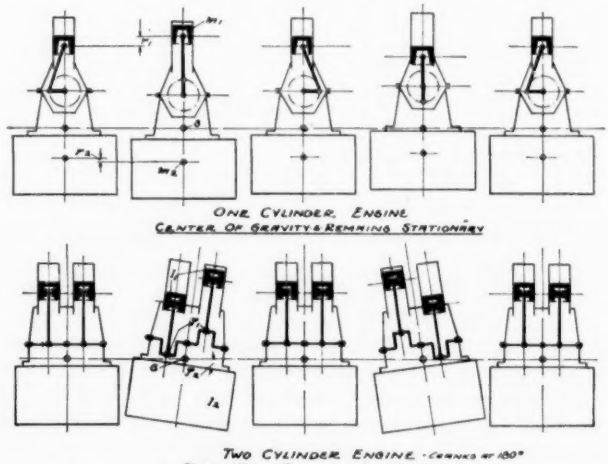


FIG. 10 ACTION OF RECIPROCATING AND STABILIZING MASSES IN ONE- AND TWO-CYLINDER ENGINES

$$m_1 \times r_1 = m_2 \times r_2 \ldots\ldots\ldots\ldots\ldots\ldots [2]$$

(for the translatory movement). Since the free force of the crank mechanism is

$$P_{max} = m_1 r_1 \omega^2 = m_2 r_2 \omega^2 \ldots\ldots\ldots\ldots\ldots [3]$$

the dynamic-balance coefficient is

$$K = \frac{P_{max}}{\omega^2} = m_2 r_2 = m a_0 \ldots\ldots\ldots\ldots\ldots [4]$$

and, hence, the stabilizing mass becomes

$$Q = g \times K \times a_0^{-1} \ldots\ldots\ldots\ldots\ldots [5]$$

The formulas indicate that the stabilizing mass is independent of the revolutions or frequency of the engine. It is, therefore, a fallacy to assume that high-speed machines require heavier foundations than low-speed machines, or that the same machine can have a lighter foundation when it is being operated at a reduced speed. This may be illustrated by the following example:

P_{max} = 3000 lb (secondary free force)
n = 500 rpm
ω = 104.4

therefore

$$K = \frac{P_{max}}{\omega^2} = \frac{3000}{10900} = 0.275 \text{ and}$$

for

$$A_0 = 0.002 \text{ in.}; \quad Q = 32.2 \times \frac{0.275}{0.002} = 4430 \text{ lb}$$

for

$$A_0 = 0.004 \text{ in.}; \quad Q = 2215 \text{ lb}$$

An amplitude of 0.004 in. to 0.006 in. is unquestionably permissible with the flexible pipe connections generally provided

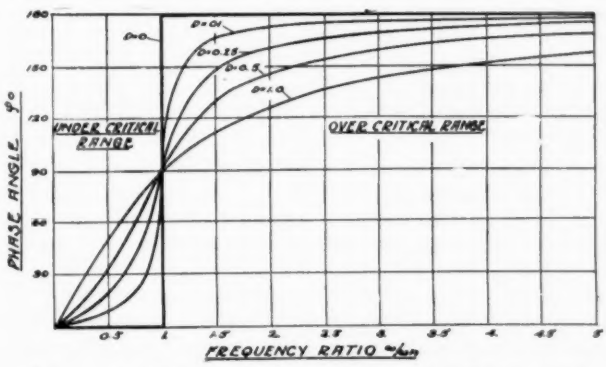


FIG. 11 VARIATION OF FREQUENCY RATIO WITH PHASE ANGLE

for Diesel engines. If the necessary foundation weight cannot be accommodated and greater amplitudes are desirable, the free force must be reduced by additional balancing.

The calculation of rotational movements provides greater difficulties because it is not simply a matter of weight in any shape for the stabilizing mass but also the inertia moment which requires consideration, and, therefore, the shape as well as the size of the mass is, in the final analysis, of principal importance. Hence, it can be concluded that by giving the foundation the proper shape, it will be possible to provide a relatively lesser stabilizing mass for free moments than for free forces. This will make it sometimes preferable in crowded localities to use an engine having an unbalanced moment, rather than an engine having a free force. The mathematical calculations are similar to those given for free forces, the only change being a substitution of inertia mass I for weight mass m, angular displacement φ_0 for amplitude a_0, and moment M for force P.

As the calculations apply to the ideal engine, which by definition is a system suspended in space, the amplitudes obtained for a given foundation weight will be slightly greater in accordance with Fig. 6.

It was previously shown that the disturbing reciprocating and revolving masses should swing in counterphase to the stabilizing mass to provide equilibrium and to satisfy Equation [2]. This is correct for the ideal engine where the phase angle between the two opposing masses is always 180 deg.

The conditions in the actual engine are somewhat different as Fig. 11 shows. Here again is a confirmation of all the previous lessons, that at low frequency ratios and with increased damping an increase of vibration transmission takes place; and for the low frequency ratio the phase angle is not 180 deg but 0 deg. In other words, the stabilizing mass is moving in the same direction as the disturbing mass instead of against it and there is increased vibration transmission. The phase angle is less than 90 deg below resonance (the vertical 1 line). It increases rapidly, a practically instantaneous change to 180 deg with zero damping. That is why with a foundation suspended on springs, the amplitude of motion is reduced so quickly (see Fig. 6). The mass becomes an effective stabilizing mass as the phase angle rather suddenly changes from zero to 180 deg. The greater the damping factor, however, the slower does the change in phase angle take place, an added indication of the disadvantage of damping.

CHOICE OF THE ISOLATING MATERIAL

It is a fundamental law that to retard the progress of vibratory waves, there must be interposed between the source of the disturbance and the structure on which it is mounted, a material which has a different mechanical resistance to sound conductivity. The mechanical resistance to sound conductivity (hereafter designated *R.S.* conductivity) is the product of the density of the material and the velocity at which the sound progresses through it.

TABLE 1 SOUND CONDUCTIVITY OF VARIOUS SUBSTANCES

Material	Sound velocity ft per sec v	Density, lb per in.^{3} d	Mechanical resistance to sound conductivity $v \times d$
Steel	17200	0.283	4860
Wrought iron	15500	0.278	4320
Copper	11700	0.320	3750
Cast iron	12400	0.260	3230
Lead	4150	0.411	1710
Glass	18000	0.094	1690
Concrete	16500	0.072	1190
Ash (with fiber)	15300	0.0272	416
Water	4700	0.036	169
Pine (with fiber)	11000	0.0145	159
Cork (with fiber)	1600	0.0086	13.9
Rubber	200	0.0442	8.84
Air	1080	0.000047	0.05

A machine is usually made of iron and steel, while the structure is composed of concrete and steel, all having a high *R.S.* conductivity as shown in Table 1. It is obvious, therefore, that for the purpose cited, a material has to be chosen from the bottom of the table. Air is unsurpassed as an isolating medium, but for obvious reasons can be employed only against lateral impact vibrations. Steel springs act admirably as isolators because of their shape, form, and heat-treatment possibilities.

In comparing the different materials shown in Table 1, it will be found that the density decreases fairly proportionally with the sound conductivity and, therefore, it can be stated that the more resilient a material the more suitable it is as a vibration absorber.

All materials deflect under a static load and follow a general law

$$c = \frac{dP}{dx} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots [6]$$

which indicates that any increase in load P results in an increase in deflection x. In Fig. 12 (upper right-hand corner) are shown the deflection curves for three materials which are generally used as vibration absorbers—cork, rubber, and steel springs. Cork and rubber show a variable c curve. Rubber becomes harder and cork becomes softer as the load increases. The c curve of the steel spring is a straight line because it follows Hooke's law providing proportionality between load and deflection. K is then called the *spring constant* which denotes the number of pounds necessary to deflect the spring one inch. The formula may be written as

$$K = \frac{L}{d} = \text{const}$$

As is shown in Equation [1] for vertical vibratory motions, the deflection d controls the natural frequency of the isolator, and therefore, must have a definite value regardless of the weight it has to support. Hence it is important that the calculation

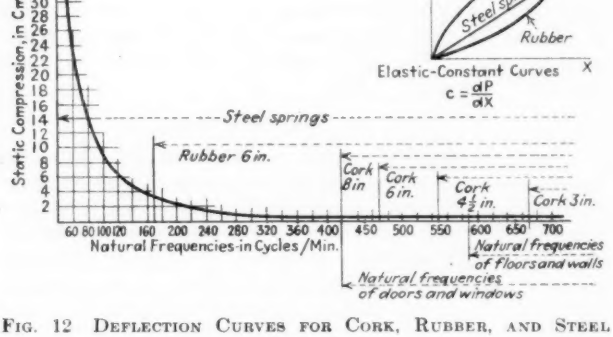


FIG. 12 DEFLECTION CURVES FOR CORK, RUBBER, AND STEEL SPRINGS

of such deflection in the isolator be made on a simple and reliable basis. The calculation of steel springs, based on modern practice employing the Wahl coefficient for the determination of deflection and stress, is very simple indeed. The large deflections necessary for maximum isolating efficiency are easily obtainable through the choice of the proper physical characteristics, especially with a coil spring which has a higher loading capacity per pound of spring material than any other type of spring usually applied for this purpose. Therefore the author adopted this type of spring for the most important and difficult isolation problems.

Organic materials do not show the simplicity and advantages of the steel spring which does not depend upon its material, but rather, upon its wire diameter, outside diameter, etc., which may be chosen at will, while the former are restricted in their use as their elasticity depends upon the material itself and only to a minor degree upon shape. This defect explains why organic materials cannot provide the necessary large deflections in the isolator for any reasonable thickness. The restrictions imposed upon them are clearly shown in Fig. 12, indicating the lowest natural frequencies that are obtainable for different conditions.

The curve shows the relation between the natural frequency of an undamped, single-degree system and the static deflection of the spring in accordance with the formula of $f = 188\sqrt{(1/d)}$ cycles per minute. Therefore, for a deflection of one inch, the frequency is 188 cycles per minute.

As stated, all the calculations given so far refer to static stresses. Isolators placed under operating machinery, also deal with dynamic stresses. The behavior of all materials, with the exception of steel springs, is different when stressed dynamically. On the basis of many experiments, it was found that the stiffness of cork and rubber is two to three times greater under a dynamic than under a static load and, therefore, their natural frequencies should be correspondingly increased over the values given in Fig. 12. Thus their useful employment range becomes still more limited.

Kimball[5] states, "Cork is not a perfectly elastic material, and tests show that it should be compressed three to four times as much as an ideal spring to produce the required frequency.... Considering its poor elastic qualities from the test standpoint, it is surprising how well it works in some applications....High-grade rubber is elastically much superior to cork, nor does it lose its resilience appreciably with time, as is commonly believed, unless cheap grades are used." Fig. 11 fully substantiates these statements in the light of most recent developments.

The considerations in this section so far are based upon a vertically acting force and, therefore, upon the use of a vibration isolator having what is technically termed *one degree of freedom.*

Since practically all foundation systems show six degrees of freedom, three in translatory and three in a rotational direction for the three space axes, the elastic suspension should be designed accordingly. Reference has been made to the analogous action of the spring and pendulum. By correct arrangement and construction of the isolators, this pendulum action can be used to provide the necessary frequencies, not only in a vertical direction, but also in a horizontal direction.

CONCLUSION

In conclusion it may be stated that from the standpoint of vibration control and for dynamic reasons, the following principal points must be observed in the design and arrangement of spring suspensions:

1 Large frequency ratio—at least 3:1
2 Minimum damping
3 Carefully calculated stabilizing mass
4 Correctly selected isolating materials.

ACKNOWLEDGMENT

The author wishes to give credit to the many engineers who have done such valuable work in developing and applying vibration isolation theories. It is impossible to mention all the names, but particular recognition is given to C. R. Soderberg, Westinghouse Electric Company; J. P. DenHartog, Harvard University; J. Ormondroyd, University of Michigan; A. L. Kimball, General Electric Company; E. H. Hull, General Electric Company; V. O. Knudsen, University of California; J. Geiger, Augsburg, Germany; and Ernst Lehr, Darmstädt, Germany.

[5] "Vibration Prevention in Engineering," by Arthur L. Kimball, John Wiley and Sons, Inc., New York, N. Y., 1932, pp. 105 and 106.

Discussion

Some Data on Diesel-Electric Switching Locomotives[1]

J. P. Laux.[2] The percentage of availability mentioned by the author parallels quite closely the results which we have obtained with Diesel-electric switchers on the Lehigh Valley Railroad. We have found also that the locomotives of more modern design are giving better performance results than the older types. However, we do not find it necessary, as the author mentions, to hold locomotives out of service for 16 hr for monthly inspection—this work is done in 8 hr, thereby increasing the service availability of our Diesel-electric locomotive.

We have experienced some trouble with our Diesel-electric locomotives, due to design defects, which have now been corrected. The writer would estimate that the Diesel engine is responsible for about 75 per cent of Diesel-electric-locomotive failures, while the electrical equipment is responsible for the remaining 25 per cent.

The operating costs given by the author compare very closely with the cost of operating this type of power on the Lehigh Valley Railroad.

H. M. Rugg.[3] An official of one of the companies manufacturing Diesel-electric switching locomotives gave the writer some interesting figures regarding their operation. Speaking of the life of the locomotives he said, even considering the length of time they have been manufacturing and using these engines, they have yet to find any rule or any data which would allow them to estimate the length of life. Some of the locomotives have been in service constantly from 10 to 12 years with no indication of their passing out of service except for obsolescence.

In regard to economy, there were several items of exceptional interest. For instance, in 1936, class-1 railroads spent about 35 million dollars for fuel in switching service exclusive of the cost of moving the fuel to the point of usage and return of empties; Diesel-electric switchers will normally reduce this expense 75 per cent. In ordinary switching service each tank car of fuel oil covers as many service hours as 12 cars of coal.

Again, the railroads spent about 45 million dollars repairing locomotives used in switching service; Diesels will reduce this item approximately 50 per cent. They spent 16 million dollars for enginehouse expense caring for locomotives used in switching service. This includes cleaning fires, handling ashes, cleaning grates, watching, boiler washing, and government inspections of firebox and boiler. This service, he stated, on Diesels is practically negligible.

That same year they spent $3\frac{1}{2}$ million dollars for water to be used in switching locomotives. The Diesel practically eliminates this expense. In substituting Diesels for steam switching locomotives the capital investment will not be as high, as three Diesels will normally cover as many hours of service as four steam locomotives. Diesel switchers are normally available for not less than 8000 hours, or 330 twenty-four-hour days per year while steam switching locomotives are rarely available for more than 6000 hours per year.

From quite complete studies and this company's experience with Diesel power it is estimated that if class-1 railroads were to put 3000 Diesel locomotives into switching service, this number of Diesels—about 25 per cent of the present steam-switching-locomotive inventory—would cover approximately 50 per cent of the present switching hours and effect a saving in operating expense of 52 million dollars yearly. This saving would be sufficient to liquidate fully the investment in Diesel locomotives within five years and would permit the retirement, without replacement, of not less than 5000 old and obsolete steam locomotives with their current excessive operating cost, which should more than meet the cost of acquiring the Diesels. The full economy of Diesel operation will not become effective until Diesels completely supplant the steam locomotives in a given engine district, thereby eliminating the supporting services of steam that are not needed for Diesels, such as coal docks, water stations, track pans, and water-treating facilities.

A tremendous improvement has taken place during the last 10 or 12 years as the use of this type of equipment has advanced. Prior to 1924, single cars utilizing gas-electric power were the most common type of motor vehicle in railroad service. The engines used in these cars ranged from 150 to 175 bhp and most of them were gasoline engines. In the next six or seven years the size of the power units increased to as much as 900 bhp.

While use of the Diesel-electric locomotive in switching service began in 1924 with a 300-bhp engine, it was not until 1932 and 1933 that its application to both streamlined trains and switching service brought the special designs of high power. Now we have 1200- and 1600-bhp engines and, by combining various units, the most powerful locomotives ever built are being placed in service, the latest and largest being the two 5400-bhp locomotives for hauling the specially designed fast trains of the Union and Southern Pacific Railroads to California. Some of the locomotives already in service have covered more than half a million miles without a major overhaul.

R. Tom Sawyer.[4] Table 5 of this paper[1] gives comparative operating costs which are being substantiated every day by Diesel-electric locomotives built by the American Locomotive Company. Toward the end of the paper the author states: "In comparing the figures in Table 5 it is obvious that the three important items are fuel, enginehouse expense, and repairs; and in the case of all three items the steam-switcher costs are much higher than those for the Diesel switchers." The writer would like to discuss these three items.

Fuel. Railroads are asking why one locomotive burns more fuel than another locomotive of the same size; the locomotive that burns the most fuel is doing the most work—is using more of its brake horsepower. For example, an Alco 600-bhp locomotive in switching service will use from 5 to 6 gal of fuel per hr, while the same locomotive will burn 8 to 10 gal per hr in transfer service; in road service this same engine is using up to 32 gal per hr.

Enginehouse Expense. For steam locomotives this expense is justified, but for Diesels it is often a matter of bookkeeping or policy of management. The entire policy of the railroad may revolve around this one idea. If the steam and Diesels are mixed,

[1] Published as paper OGP-60-5, by John W. Anderson, in the April, 1938, issue of the A.S.M.E. Transactions, vol. 60, p. 221.

[2] Superintendent of Motive Power, Lehigh Valley Railroad, Bethlehem, Pa. Mem. A.S.M.E.

[3] Automotive Engineer, Birmingham, Mich. Mem. A.S.M.E.

[4] Sales Engineer, Diesel Locomotive Division, American Locomotive Company, New York, N. Y. Mem. A.S.M.E.

both using the same enginehouse, the enginehouse expense on the Diesels will be higher than if the same Diesels are placed in their own house, with steam locomotives eliminated from that section or yard. The road steam locomotives should be maintained at a main central enginehouse if possible, and the outlying yard engine-houses should preferably be closed when Diesel switchers are used. This allows a reasonable but low enginehouse expense for Diesels.

Repairs. The author pointed out that repairs deal with the entire locomotive, not just the Diesel engine. The entire locomotive must be built throughout so that it will have a large margin of safety on all parts. Alco Diesel locomotives today are coming well within the repair figures which are given in Table 5 of the paper.

The cost of repairs depends primarily upon two things: (1) The complete locomotive properly built for the service, and (2) railroad personnel and equipment.

Regardless of how good the personnel is, the locomotive will be costly to maintain if not properly built. We have certainly passed through an experimental era as pointed out by the author, but today we have learned how to build the heavy-duty Diesel-electric switcher; also, we have excellent cooperation with the railroad personnel and know how to hold maintenance costs to a minimum.

For example, we have learned that the mean effective pressure of a Diesel engine means nothing in so far as maintenance is concerned, while maximum pressures mean everything. The life of a Diesel engine also depends upon its temperature. We have learned by seven years of experience on the road that if an Alco engine has an exhaust temperature below 900 F, maintenance will be low. We have also found out that it is possible to obtain 900 bhp out of a 600-bhp engine simply by supercharging without increasing maximum bearing pressures and exhaust temperatures, thereby holding maintenance to a minimum and engine life to a maximum.

RALPH MILLER.[5] The writer would like to explain the manner in which figures on availability are obtained. Take as an example a certain railroad which now operates a number of 600-hp Diesel switches. These units work seven 24-hour days per week with the exception of eight hours out of service for I.C.C. inspection each week, making 6000 hours for 37½ weeks. At the end of that time the locomotive is taken into the shop for engine overhauling; heads, pistons, and rods are removed, main bearings inspected, the injection system is gone over, and all parts that show wear or defects which render them unfit for another 6000 hours of operation are replaced.

Now due to the deplorable condition under which nearly all the railroads conduct Diesel-engine maintenance work, both as to the actual work and supervision, the work I have just outlined will take from six to eight weeks, whereas if it were done by the engine-manufacturer's repair crew, or men of similar experience, it could easily be done in four or five days. Thus, in this case, if we have 37½ weeks of service and eight weeks out for work that should be done in five or six days that forces the availability factor down to nearly 80 per cent when it ought to be up close to 98 per cent, which latter availability is regularly obtained by the few railroads who have capable men in charge of maintenance.

We who were the pioneers in the development of Diesel engines for railroad service can truthfully say that it is one of the most difficult applications ever undertaken, primarily due to lack of proper maintenance personnel and supervision on the part of the railroads.

W. L. H. DOYLE.[6] The writer would like to ask the author this question dealing with one point discussed in his paper: Is there an indicated tendency on the part of some engine operators toward use of lubricating oils having viscosity characteristics other than those recommended by the manufacturer? The writer believes that it would be more logical to hold to the use of oils complying with manufacturer's recommendations, which are based on operating experience with the engines on the test floor and in a broad field of application.

F. H. DUTCHER.[7] The author does not mention benefits accruing from supercharging. If Diesel-electric switchers have been supercharged, have there been any beneficial or adverse effects on costs and on the general availability of these locomotives? The writer would also like to know if the percentage of Diesel-electric switchers, relative to steam locomotives, increasing.

AUTHOR'S CLOSURE

In the particular case referred to by Mr. Doyle, the engine manufacturer desired to use what the author would consider as an extra-heavy oil, but the railroad's experience with the oil was such that the railroad considered the lubricating-oil consumption too high and the general engine performance none too satisfactory. Therefore, contrary to the manufacturers recommendation, the railroad experimented with a lighter grade of lubricating oil; however, the oil used was only slightly lighter than the one recommended. After several months, the railroad returned to the use of the recommended oil, and gave the engine manufacturer comparative engine-performance data obtained with the two grades of oil. In this regard, the railroad used the same grade of oil in all of its engines, regardless of type or make.

With reference to Mr. Dutcher's first question, one manufacturer has built a number of Diesel-electric switchers with supercharged engines rated at 900 bhp; the rating of the engine without the supercharger being 600 bhp. These locomotives are mentioned in Mr. Sawyer's discussion of the paper.

The performance of the 900-bhp supercharged engines is, the author understands, better than that of the same engine when it was rated at 600 bhp with normal aspiration. Upon examination after service, the mechanical condition of the engine rated at 900 bhp was better than that when the engine was rated at 600 bhp and examined after a similar period of service; less evidence of heat was found, and the appearance of all the engine parts was better.

Table 5 of the paper can be referred to for a comparison of the cost of fuel for the 900-bhp and 600-bhp engines, which of course increases for the higher rated engine. Since it is the same size of engine, the lubricating oil, of course, doesn't change. Enginehouse expense is the same, supplies cost the same, and wages are the same. The figure for repairs has been increased slightly; however, the amount of the increase is, of course, purely arbitrary. The superchargers themselves require very little attention in service. The figures in the right-hand column of Table 5 of the paper for the 900-bhp Diesel can be taken as representative of supercharged performance. The first cost of the locomotive is very little more than for the normal 600-bhp rating; hence, the saving over steam is even greater.

Fig. 1 of the paper shows that the rated power of the locomotives in use is rising. Probably it is a fair statement to say that

[5] Chief Engineer, National Supply Company, Springfield, Ohio. Mem. A.S.M.E.

[6] Chief Engineer, Diesel Division, Baldwin Southwark Corp., Philadelphia, Pa. Jun. A.S.M.E.

[7] Associate, Mechanical Engineering, Columbia University, New York, N. Y.

most of the locomotives going into service now and in the near future for switching purposes will be rated 900 bhp instead of 600 bhp, that is, for the normal switcher. The transfer locomotives carry a much higher rating.

Regarding Mr. Dutcher's second question, the author believes that Diesel-electric switchers are being ordered in preference to steam locomotives. A list of Diesel-electric locomotives placed in service in the last two years is to be found on page 223 of the June, 1938, issue of the *Railway Mechanical Engineer;* a chart accompanying this table indicates the rapid growth of Diesel-engine applications in railroad service, which chart can be used to augment Fig. 1 of the paper. A comparison of the number of steam and Diesel-electric switching locomotives ordered during 1937 can be made by referring to the table of locomotive orders to be found on page 77 of the January 1, 1938, issue of *Railway Age.*

Slag and Deslagging of Steam-Generating Equipment[1]

P. Nicholls.[2] Mr. Bailey does not state how many kinds of powders he has tried, but presumably he has tried at least the seven, which he describes. He makes a nearly neutral statement on the results of the tests, for as he says, it is not possible to duplicate results in such tests even if every effort is made to maintain conditions the same. Thus, it is doubtful whether the differences are caused by the chemicals or by other factors, but the tendency is to hope that beneficial changes are due to the treatment, and to give it the benefit of the doubt.

By what means might accumulations of slag on boiler tubes be prevented? First, and preferably, the particles of slag could be prevented from leaving the bed. Although it is known that with stokers, the weight of the slag carried out of the bed increases with an increase in the rate of combustion, yet it is also known that it decreases as the burning over the area of the bed becomes more uniform, and that holes or uneven burning are responsible for much of the trouble. Thus, there is a possibility at least, of reducing the slagging by insuring a better grade of fuel bed. In a pulverized-fuel furnace, the percentage of slag retained in the furnace can be, and has been, increased by causing the flame to impinge upon the slag bed, and there is the possibility that the ash that does pass to the tubes may have a lower fusion temperature than the average ash.

Second, the temperature of the particles of slag can be reduced so that they will not stick to the tubes. The method that has been employed has been to increase the height of the furnace and use slag-screen tubes, widely spaced, to cool the droplets before they reach the more closely spaced tubes.

Third, the ability of the droplets to adhere to the bare metal might be decreased. For a liquid to stick to a surface it must wet it. There are no data on the adherence property of slags, which property involves equally the nature of the slag and surface. There is of course a greater tendency for the slag to stick to a rougher surface, such as one that has been oxidized. As the surface of contact of the slag particle with the tube is cooled directly the particle impinges on the tube, there would appear to be some hope that this fact, together with the properties of the slag and the surface, could be utilized to prevent adherence; studies may disclose a type of tube surface to which the slag will not stick. It is claimed for some proprietary products that they coat the surface of the tubes, but no conclusive proof has been published that this is a fact.

The alkalies vaporize at relatively low temperatures and can condense on the tubes. That this occurs has been shown by analyses of slag taken from tubes. The inner layer was found to be rich in alkalies but the alkali content of the outer layers was less. The reason for this is that as the slag thickens the temperature of its outer surface becomes too high to permit the alkalies to condense. Whether a thin deposit of alkali would increase or decrease the tendency of slag to stick is problematical, but if alkali or anything else deposits, the nonsticking property of a special surface might be destroyed.

If the chemical does not volatilize, any action it might have on the slagging of the tubes would depend on its being carried out of the bed on the slag droplet. If the chemical combines with the slag, the properties of the slag would be changed. The change might be favorable with some slags and reduce the sticking ability of the droplet, but because the composition of slags varies, one chemical could not be expected to have the same effect on all slags.

It has been reported, but no experimental data can be offered in proof, that common salt thrown on a hot fuel bed causes considerable slag to fall from the tubes. This can be explained by the fact that the sudden fluxing of portions of the slag sets up mechanical strains in the deposit, or that the tentacles of slag around the tube soften so that they weaken and cannot hold the weight of the block. It is less likely that such actions would result if the chemical left the bed at a constant rate, as it would with a treated coal.

Additions of salt to a fuel bed, repeated at intervals, will loosen scale on boiler surfaces. This refers to thin scale and not to blocks of slag. The writer would ascribe this action to mechanical strain but, although he has seen it occur, he has not conducted controlled experiments. He would expect such action to be more definite with occasional heavy doses than with the application of a uniform treatment to the coal.

The writer's conclusion is that the only manner in which the treatment of coals with small quantities of chemicals could materially reduce the slagging of boiler tubes is that a coating of chemical might reduce the tendency of the slag particles to stick.

Maurice Ratel.[3] To eliminate slag it is better to prevent its formation rather than to try to get rid of it after its formation. It is the writer's opinion that slagging is due to a poor and nonuniform porosity of the fuel bed. It is well known that, in any fuel bed, some parts burn more quickly than others, and this is due to a deficiency of air supply in places caused by a varying porosity of the fuel bed. This condition occurs particularly with multiple-retort underfeed stokers where because of the feeding of the fuel by means of rams, a fuel bed is obtained that is more compact in sections in line with the rams, and which therefore produces stratification of gases. Parts of the fuel bed, specially along the center line of the retorts do not receive sufficient air so that coal is volatilized but not burned entirely. The tarry products driven off are drawn through the bank of tubes of the boiler and condense on the tubes to form a slimy product to which fly ash and unburned particles of fuel stick. Due to the effect of radiation from the burning fuel bed this coating hardens and often becomes a vitrified mass adhering solidly to the tubes.

To prevent slagging it is therefore necessary to eliminate this stratification of gases and this can only be done by obtaining a fuel bed of uniform porosity which makes possible proper air distribution.

The writer has accomplished this result with his improved

[1] Published as paper FSP-60-7, by Alex D. Bailey, in the April, 1938, issue of the A.S.M.E. Transactions, p. 209.

[2] Supervising Fuel Engineer, Bureau of Mines, Pittsburgh Experiment Station, Pittsburgh, Pa. Mem. A.S.M.E. Published by permission of the Director, Bureau of Mines.

[3] Ingenieur E.C.P., Bievres, France.

multiple-retort underfeed stokers and the results obtained have proved the correctness of the theory. To obtain good porosity of the fuel bed several series of stepped secondary pushers are used in each retort, with adjacent series working in opposite directions.[4] By these means a good porosity is maintained in every point of the fuel bed, and hence a better distribution of air and elimination of the stratification of gases.

The results are plainly proved by comparative tests of stokers using this principle with well-known multiple-retort stokers of American design.

Table 1 shows some test results, taken at random, in which boilers, coal, and other conditions are identical.

TABLE 1

	Ratel multiple-retort stoker		American multiple-retort stoker of modern design	
Dates, 1934	May 5	May 12	May 5	May 12
Evaporation rate, tons per hr	97	100	102	95
CO_2 at the time of readings, per cent	12	12	11	14
Temperature superheated steam	817 F	842 F	767 F	640 F
Temperature gas, boiler outlet	662 F	717 F	817 F	914 F
Temperature gas, economizer outlet	275 F	292 F	329 F	345 F

The differences in temperatures are most convincing. With the competitive stoker, temperatures of gases at the boiler outlet as high as 977 F have often been observed after operation for three weeks. Table 2 shows the results of another test with the Ratel stoker.

TABLE 2

Stoker lighted April 6, 1934		Stopped May 2, 1934
Evaporation rate, tons per hr	75	85
Temperature superheated steam	797 F	795 F
Temperature gas, boiler outlet	574 F	597 F
Temperature gas, economizer outlet	221 F	239 F

With the competitive stoker, after operation for 3 weeks, in addition to the high gas temperature at the boiler outlet, there was frequent clogging of the passages through the tubes which compelled shutdown at the boiler. All these troubles have been eliminated since the Ratel type of pushers were installed.

In regard to the use of sanding or water lancing to remove slag, it is the writer's opinion that neither of these methods should be used.

When lancing tubes which are at a relatively high temperature in comparison to that of the water used, sudden contractions occur which cause leaks and act on the structure of the metal rendering it brittle because of this mechanical action. Secondly, water combines chemically with the SO_2 contained in the gases of combustion, forming H_2SO_4 which attacks the steel of the economizer tubes. This action causes pitting and leaks if not the bursting of the tubes.

The sanding of tubes is always dangerous. When practiced, even very carefully, sanding causes a wear of the tubes which weakens them rapidly, and sooner or later, causes them to burst.

To summarize, the best way is to fight slagging and fouling of boiler tubes by proper regulation of the fuel bed and the combustion process.

Author's Closure

The author is fortunate indeed to have Dr. Nicholls' discussion of this paper as he knows more about the subject of treatment for the prevention of slag than anyone I know. Both he and Mr. Ratel have sounded a fundamental truth in saying that the best way to eliminate slag is to prevent its formation, but unfortunately with some coals this is impossible. It is hoped that ways will be devised for improving these fuels and removing some of their objectionable characteristics; washing these coals, in many cases, is particularly beneficial as it seems to remove those ingredients of the ash which are most conducive to slagging trouble.

Operating men will without doubt agree with Mr. Ratel that water lancing should be avoided if possible, and when it is necessary it should be carefully done. Unfortunately this treatment is necessary in many installations using slagging coals, even when firing conditions are the best. This applies to equipment fired with pulverized coal as well as stoker-fired.

The author is glad to take this opportunity to thank these gentlemen for their discussion.

[4] U. S. Patent No. 1,539,208.

Some Results From Research on Flow Nozzles[1]

Ronald B. Smith.[2] This progress report marks the first concerted effort in the United States toward the establishment of coefficients on standardized nozzles. In view of the experimental background established by research over the last ten years in Germany, Italy, Sweden, France, and other European countries on a standardized profile, now known as the I.S.A. shape, these efforts seem timely.

On the basis of the tests on various sizes of I.S.A. nozzles in smooth and rough pipe reported in Fig. 3, it is interesting to note that while the authors are justified in concluding that the coefficients of the twelve tests do not all fall within the tolerance band prescribed by the V.D.I., there are only four that exceed it and in none of these cases is the difference greater than 0.1 per cent. A comparison of the tests and the standardized coefficients is shown in Table 1. For each test the minimum and maximum coefficient above the limiting Reynolds number is recorded and then converted to include the approach velocity, so that a comparison can be made directly with the standardized data. The standardized tolerances are those of the V.D.I. corrected where necessary for roughness in the pipe. In this country, in fact in all the countries where the I.S.A. nozzle has been adopted, except Germany, the close tolerances of the V.D.I. are not used with the I.S.A. profile, but instead, the I.S.A. tolerances are employed. These tolerances allow ± 1 per cent for small-ratio nozzles increasing to ± 1.8 per cent for the larger ratios, with additions for pipe roughness. All the tests made on I.S.A. nozzles by the authors fall well within the I.S.A. tolerance zone; so, too, do the most recent investigations in Italy.[3] It is interesting to observe that the Italian studies, which were carried to Reynolds' numbers as high as the present work, indicated that, except for a few inconsistent tests at the University of Padova, the coefficients fell within the V.D.I. tolerance band, but the recommendation of the Italian standardizing body was for the adoption of the I.S.A. tolerance zone for reasons of manufacture. There is no evidence from the Italian tests of a continually rising coefficient above the limiting Reynolds number, and it seems to the writer rather difficult to generalize this conclusion on the basis of the limited data of Fig. 3.

The effect of upstream pipe roughness has been recognized for some time and generally treated by an adjustment of the coefficient and of the tolerance on the coefficient. The roughness effect is more severe on orifices than on nozzles and if the comparison between Figs. 3 and 5 is admitted it is more on the long-radius nozzle than on the I.S.A. The effect of pipe roughness has its influence mainly on the upstream velocity profile. A relatively rough pipe will have a more peaked distribution than a smooth approach with the result that the integral of the kinetic energy in

[1] Published as paper RP-60-3, by H. S. Bean and S. R. Beitler, in the April, 1938, issue of the A.S.M.E. Transactions.

[2] Development Engineer, Elliott Company, Jeannette, Pa. Jun. A.S.M.E.

[3] "Le Esperienze Italiane sui Boccagli E sui Diaframmi Unificati," Uni 160, Uni, Foro Bonaparte 16, Milan.

TABLE 1 I. S. A. NOZZLE TESTS

β	Tested coefficient, Fig. 3, Rough	Tested coefficient, Fig. 3, Smooth	Tested coefficient, K, Rough	Tested coefficient, K, Smooth	Standard I.S.A.[b] coefficient, K, Rough	Standard I.S.A.[b] coefficient, K, Smooth	Standard V.D.I.[c] tolerance, per cent, Rough	Standard V.D.I.[c] tolerance, per cent, Smooth	Deviation of test from standard, per cent, Rough	Deviation of test from standard, per cent, Smooth
0.3	0.984	0.987	0.988	0.991	0.9885	0.9885	±0.5	±0.5	−0.05	+0.25
	0.983	0.985	0.987	0.989					−0.15	+0.05
0.4	0.986	0.987	0.9985	0.9995	0.9940	0.9940	±0.5	±0.5	+0.45	+0.55
	0.986	0.984	0.9985	0.9965					+0.45	+0.25
0.5	0.981	0.981	1.013	1.013	1.007	1.007	±0.5	±0.5	+0.6	+0.6
	0.977	0.975	1.009	1.007					+0.2	0
0.6	0.972	0.963	1.042	1.032	1.033	1.031	±0.8	±0.6	+0.9	+0.1
	0.971	0.963	1.041	1.032					+0.8	+0.1
0.7	0.953	0.944	1.094	1.083	1.082	1.076	±1.6	±0.9	+1.1	+0.65
	0.953	0.945	1.094	1.084					+1.1	+0.75
0.8	0.918	0.907	1.195	1.181	1.190	1.176	±2.6	±1.3	+0.4	+0.4
	0.917	0.907	1.194	1.181					+0.3	+0.4

[a] Coefficient K is the tested coefficient corrected for approach velocity.
[b] "Rules for Measuring Flow of Fluids," I.S.A. Bulletin No. 12.
[c] "Standards for Discharge Measurements" V.D.I. Regeln, 4th edition, 1937.

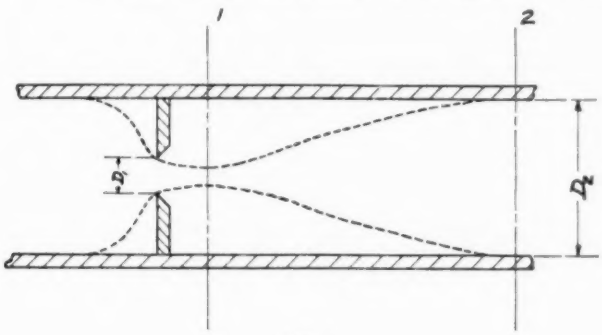


FIG. 1

TABLE 2

OVER-ALL PRESSURE LOSS IN A NOZZLE, PER CENT (φ = 0.98; μ = 1.0)

β^2	Pressure loss calculated from Carnot relation	Pressure loss from authors' Fig. 16	Pressure loss from V.D.I. tests[a]
0.2	68	70	68
0.3	56	58	56
0.4	45	48	45
0.5	36	39	32
0.6	28	30	

OVER-ALL PRESSURE LOSS IN AN ORIFICE, PER CENT (φ = 1.00; μ = variable)

β^2	Contraction coefficient μ	Pressure loss calculated from Carnot relation	Pressure loss from fluid-meters tests, Fig. 9	Pressure loss from V.D.I. tests[a]
0.09	0.598	89.8	90.0	89.0
0.16	0.606	82.3	82.0	82.0
0.25	0.618	73.2	73.0	73.0
0.36	0.636	62.7	63.0	62.0
0.49	0.664	51.0	51.0	50.0
0.64	0.709	37.5	38.0	37.0

[a] "Standards for Discharge Measurements," V.D.I. Regeln, 4th edition, 1937.

relation to the square of the mean velocity will be greater for the rough approach. As Pardoe[4] has shown, this brings about an increase in the discharge coefficient. This conclusion is opposite to that drawn by the authors but it is verified by Fig. 5 and in part by Fig. 4. Possibly for very high area ratios the effect of the initial kinetic energy on the coefficient is distorted, but on the basis of the results of Figs. 4 and 5 it is difficult to rationalize.

The investigation of the expansion factors of a nozzle is much to be desired. Will the authors please explain on what style of nozzle the present investigation was made and at what point the pressures were measured?

In regard to the pressure distribution upstream from the nozzle face, there does not appear to be uniformity between the results of the Bailey Meter Company and those of the Bureau of Standards with the long-radius nozzle. Inasmuch as the size of the static-pressure hole can influence the result it would be of interest to know the sizes that were employed. The upstream measurements by the Bailey Meter Company on the I.S.A. nozzles agree when plotted against diameter ratio with the results of Witte.[5] The indications from the tests of Table 3 are that the build-up in pressure at the upstream corner is from 40 to 45 per cent of the velocity head in the pipe for area ratios up to about 0.45. For area ratios higher than 0.45 the build-up is less. The explanation for this may rest with the fact that for area ratios above 0.45 the inlet curvature of the nozzle reaches to the pipe wall with a consequent influence on the direction of the approach velocity. A build-up of from 40 to 45 per cent is approximately the same as that found with orifices of equivalent area ratio.

On the downstream side of the long-radius nozzle the authors suggest standardizing the pipe-wall tap at 0.5 pipe diameter. When the tests of Tables 2, 4, 5, and 6 are plotted against area ratio for this location, the results are fairly regular up to diameter ratios of about 0.65, but beyond this point there is considerable inconsistency. This condition, as the authors suggest, is undoubtedly associated with the close clearance between the pipe and the nozzle wall. In view of the fact that the clearance is not fixed by the nozzle proportions, and because of its magnitude will undoubtedly be affected by roughness and other minor influences, would it not be a good idea to limit the use of downstream pipe taps to nozzles with a diameter ratio less than 0.65?

On the basis of the downstream-pressure measurements the authors have been able to ascertain the over-all pressure loss of a nozzle, concluding that this loss is proportioned to $1 - \beta^2$. It may be of interest to know that this result could have been predicted for both orifices and nozzles once the discharge coefficients were established. Consider the orifice installation shown in Fig. 1 of this discussion, and apply the momentum equation between plane 1 at the vena contracta and plane 2 on the downstream side of the diaphragm where the pressure recovery is complete. With the notation:

p_1, c_1 = pressure and velocity at plane 1
p_2, c_2 = pressure and velocity at plane 2
ρ = mass density
β = diameter ratio = D_1/D_2
μ = coefficient of contraction
φ = a discharge coefficient
G = mass flow

the conservation of momentum prescribes that

$$\frac{\pi D_2^2}{4}(p_2 - p_1) = G(c_1 - c_2)$$

[4] "The Effect of Installation on the Coefficient of Venturi Meters," by W. S. Pardoe, Trans. A.S.M.E., vol. 58, 1936, paper HYD-58-6, pp. 677–684.

[5] "Die Strömung durch Düsen und Blenden," by R. Witte, *Forschung auf dem Gebiete des Ingenieurwesens*, vol. 2, July, 1931, p. 245.

or upon simplification

$$\delta_2 = p_2 - p_1 = \mu\beta^2\rho(1 - \mu\beta^2)c_1^2 \quad [1]$$

Now the mass flow through the system may be represented by

$$G = \frac{\varphi \frac{\pi D_1^2}{4} \mu}{\sqrt{(1 - \beta^4\mu^2)}} \sqrt{(2\rho\Delta)} = \frac{\pi D_1^2}{4} \mu\rho c_1 \quad [2]$$

with the result that

$$\frac{\delta_2}{\Delta} = 2\varphi^2 \frac{\mu\beta^2}{1 + \mu\beta^2}$$

and the over-all pressure loss for the installation becomes

$$1 - \frac{\delta_2}{\Delta} = 1 - 2\varphi^2 \frac{\mu\beta^2}{1 + \mu\beta^2} \quad [3]$$

Equation [3] is the classical Carnot relation.

In Table 2 herewith, is shown a comparison between the calculated pressure loss from the Carnot equation and the losses measured by the authors as well as by other investigators. For nozzles a coefficient of $\varphi = 0.98$ and $\mu = 1.0$ was employed, whereas with orifices a friction coefficient $\varphi = 1.0$ and a contraction coefficient based on the A.S.M.E.-A.G.A. data and calculated on the basis of Equation [2] were used.

Authors' Closure

The authors are grateful to Mr. Smith for his extensive discussion of their paper, which, as stated in the introduction, is essentially a progress report. It is altogether probable that their final conclusions may differ from those given in these progress reports because of the inclusion of test data still being received, and from such discussions as Mr. Smith has given.

While it is interesting to have it shown that the tests on I.S.A. nozzles are within the I.S.A. tolerance zone, the authors are not sure whether to regard this as a compliment to the quality of the test work done, or as a critical comment on the uncertainty of fluid-flow measurements with nozzles in general.

In regard to whether the points in Fig. 3 do or do not tend to show a continually rising coefficient, it seems to the authors that there are only 3 which appear definitely to have reached a constant value. These are the two for $\beta = 0.70$ and that in brass pipe for $\beta = 0.80$. All of the others are either very definitely still increasing, or at least not definitely constant. While it is possibly unfair to the readers, it is nevertheless natural for the authors to be more or less influenced by other test data which are still in process of being prepared for publication. In many cases these tests yet to be reported have not indicated that a constant coefficient was reached, irrespective of the nozzle shape or pressure-tap locations, even when with water the differential pressure was carried up to over 50 ft.

The tests with steam reported in the paper were made on "long-radius" nozzles. The pressure-tap locations were approximately one pipe diameter upstream and one-half pipe diameter downstream. These distances were measured from the faces of the pipe flanges.

The authors do not have at hand a record of the pressure-hole sizes used by the Bailey Meter Company in their pressure-gradient tests. At the National Bureau of Standards the diameters of the pressure holes in the 4-in. pipe varied from 0.04 in. for the holes very close to the faces of the pipe flanges to 0.25 in. for the holes one pipe diameter and more from the flange faces.

It does not seem to the authors that Mr. Smith's suggestions of limiting the use of a downstream pipe wall tap to nozzles of a diameter ratio of 0.65 and under is desirable or practical. If this were to be followed it would most certainly have to apply to the use of corner taps because that tap is even farther under the shadow of the nozzle throat. There are three other possibilities: (1) To place a limit on the width of the annular space between the inner surface of the pipe and the outer surface of the nozzle. This would be possible because it is seldom if ever necessary for the wall of the nozzle throat to be very thick. (2) To make use of the tolerance of ± 0.25 pipe diameter in locating the downstream tap so as to have it nearer 0.75 than 0.5 pipe diam. (3) To use a somewhat shorter nozzle. In justification of this alternative, attention is called to the fact that as stated in other reports, the committee is conducting tests with other shapes of nozzles than the I.S.A. and the long-radius as defined in the first paper.[6] These tests are still going on, but a report on them will be made at the earliest possible time.

The Friction of Reciprocating Engines[1]

E. S. Dennison.[2] Near the end of his paper the author states: "It is of interest that the friction-mean-pressure values obtained in the present test do not differ widely, either in value or trend, from those obtained by Taylor[3] with a somewhat similar engine." In both cases, friction expressed as mean effective pressure was found to vary in almost linear relation to the mean (gage) pressure exerted on the piston crown. The similarity in results is of special interest because of the difference in the method of applying pressure. Taylor maintained an almost uniform pressure during the cycle, while the author refers his results to the mean during a compression cycle, maximum gas pressure being about 10 times the initial. The results, taken together, form a useful check. The direct proportion between gas pressure and piston and ring friction, found by Taylor, evidently is not much upset by a change over to fluctuating conditions such as are found in an operating engine. Moreover, this proportionality seems to extend to the region of actual combustion pressures, since the author's maximum was in excess of 500 lb per sq in.

Attempts are sometimes made to identify the friction of pistons and rings either as fluid (i.e., due to an oil film), or as solid or Coulomb friction. The present results show the friction force to be proportional roughly to (1) gas pressure and (2) speed. The first result is inconsistent with film lubrication; the second is contrary to the law of solid friction. Evidently no simple classification of this type of friction is possible at the present time.

Conclusions regarding piston and ring friction, based upon the author's data, have to be made on the assumption that journal-bearing friction at a given speed is constant irrespective of load. This is true only as an approximation. It is unfortunate that the author did not attempt to segregate journal friction from the total. Division of "mechanical" loss into the portions contributed by bearings, pistons, and rings, respectively, is essential in any thorough investigation of the subject which the author undertook.

C. W. Good.[4] About eight years ago the writer tested an

[6] "Researches on Flow Nozzles," by H. S. Bean, *Mechanical Engineering*, vol. 59, July, 1937, p. 500.

[1] Published as paper OGP-60-6, by F. H. Dutcher, in the April, 1938, issue of the A.S.M.E. Transactions, p. 225.

[2] Engineer in charge of Diesel Engine Development, Electric Boat Company, Groton, Conn. Mem. A.S.M.E.

[3] "The Effect of Gas Pressure on Piston Friction," by M. P. Taylor, *S.A.E. Journal*, vol. 38, May, 1936, pp. 200–205.

[4] Assistant Professor of Mechanical Engineering, and Assistant Director of Engineering Research, University of Michigan, Ann Arbor, Mich. Mem. A.S.M.E.

engine to determine the probable variation in friction losses due to the change in direction of heat flow through the cylinder walls. The following procedure was used to determine the effect of the change of heat conditions in the engine operating under its own power as compared to that with the engine being motored by the dynamometer: With the engine operating under stable conditions, the ignition and fuel were shut off, the dynamometer was changed over from a generator to a motor as rapidly as possible, and a succession of load readings were taken for a few minutes. The supply of cooling water was also shut off at the time the ignition was cut off, the purpose being to retain the heat in the engine during the test. It was found that the friction increased at a decreasing rate with time, indicating, as the writer believes, that the cylinder walls on the inside were cooling down as the heat flowed to the water jacket, with the result that the oil film on the walls became more viscous. This fact is pointed out in order to indicate the effect of the temperature gradient in comparing test results where the engine is operating under its own power.

As you increase the combustion pressure, you are putting more heat into the cylinder walls. That means a higher temperature gradient through the walls to the water, which in the author's tests was maintained at 180 F. Therefore, it is quite possible to conceive that the oil would be getting thinner and thinner for successive points along the author's curves and the writer is inclined to believe that this is a fairly good reason for the slope of the author's curves, although the writer has no definite research results other than the previously mentioned observations with which to prove it. Further, there is probably a higher inside wall temperature as you go up along the curve. That would mean high oil temperatures on the walls, which would mean less friction due to the decrease in oil viscosity.

G. B. Karelitz.[5] There are two main sources of mechanical friction in engines, that is, the journal bearings and the piston rings. One might regret that no attempt to estimate the friction in bearings has been made, because that is one part of the friction which can be estimated, probably even tested separately—Dennison's paper[6] gives the necessary data. Does the author know what proportion of the total friction is attributed to the pistons and what proportion to the bearings?

Curves drawn originally by the author but not included in the published paper have an upward bend at the lower speeds, characteristic of friction in bearings.

When it comes to the friction in pistons, we encounter difficulties in spite of the number of papers written and reports made on the physics of piston rings. We do not know what happens under a piston ring during engine operation. The pressure behind the piston ring could be measured, but this does not mean that the metal-to-metal pressure on the cylinder would be known. There may be pools of oil or gas between the rubbing surfaces, in which case part of the load would be carried by the very thin liquid film and part by the metal-to-metal contact. Under the circumstances there is no way to guess what would be the coefficient of friction between the piston ring and the cylinder wall.

This is similar to what happens in a journal bearing on a railway car. In a railway motor axle bearing, which carries about 40 lb per sq in. on its projected area, the coefficient of friction is usually rather low, only about 0.75 per cent. However, if a speck of dirt were to come into the clearance, the friction would increase immediately. The part of the load carried on the liquid film is lost, with the result that solid friction takes place and the friction increases to 2 or 3 per cent, or even higher.

The same thing may happen under a piston ring in an internal-combustion engine. The total pressure behind the piston ring must be balanced by the wall. However, we do not know how much of this pressure is carried by an oil film, gas film (which may act very well as a lubricant), metal-to-metal contact, or a speck of carbon. The amount of pressure carried by these various factors will influence the coefficient of friction to a marked degree.

Harte Cooke.[7] A method used some years ago by the writer for determining friction losses in steam engines might prove of interest in discussing this paper,[1] inasmuch as the same method could be used to investigate friction losses in internal-combustion engines. The writer used the method while testing a 2500-bhp engine which had heavy flat gridiron valves. In determining the indicated horsepower at no load, indicator cards were taken as rapidly as possible for 30 min, the average mean effective pressure from which permitted the calculation of a good average no-load indicated horsepower. It then became the problem to determine how to allocate the various friction losses.

It looked as though the heavy flat gridiron valves would require a lot of power. An assumption was made to determine the distribution of the friction losses. The coefficient of friction of all rubbing surfaces outside the valve chamber and cylinder was taken as X, while the coefficient of friction of the piston rings, piston, and valves was taken as $2X$. The loads on all the bearings were obtained, and, with the coefficient of friction assumed, the friction losses of the engine were determined. If the calculated horsepower was 250 and the result of the writer's estimate gave 322 horsepower, the ratio of these two horsepowers was the ratio between the assumed and actual coefficients of friction. Then, when the writer knew the coefficient, he calculated the horsepower for the various parts of the engine.

The writer found that the valves, which were heavily loaded and unbalanced, took very little power because the stroke was short, whereas the piston rings, which had natural tension pressure of 3.5 lb per sq in., accounted for one of the principal losses, because while the pressure was low the distance traveled per minute by the pistons was very high.

A similar method could be applied to allocate the friction losses in an internal-combustion engine. The engine could be motored to determine the total friction and pumping losses, after which the head could be taken off and the test repeated to eliminate the pumping losses. The test could be again repeated with the pistons out of the engine to determine the shaft friction. The trouble will then be the determination of the friction loss of the piston rings, which will vary because of differences in their natural tension and differences in pressures under the various rings at all load conditions. Tests at full load on internal-combustion engines could be conducted and the pressures behind the piston rings determined, from which the friction losses and friction horsepower could be determined. However, these pressures, the coefficients of friction, and the friction losses could be calculated in turn, and the latter checked with the actual losses.

There is another method available for determining the friction losses in a Diesel engine: Using quarter, half, and three-quarter load as abscissas and fuel consumption as ordinates, draw a line for the total fuel consumption for the various loads and project it until it cuts the base line. From this intersection a fair estimate of friction losses can be made.

Referring to the balancing of pressures behind and in front of piston rings, the writer on one occasion took some steam-engine

[5] Professor of Mechanical Engineering, Columbia University, New York, N. Y. Mem. A.S.M.E.

[6] "Film Lubrication Theory and Engine-Bearing Design," by E. S. Dennison, Trans. A.S.M.E., vol. 58, 1936, paper OGP-58-1, pp. 25–36.

[7] Mechanical Engineer, Diesel Engine Division, American Locomotive Works, Auburn, N. Y. Mem. A.S.M.E.

rings, which were 1 1/2 in. thick and had a 2-in. face, and cut a groove in the face near each edge, after which holes were drilled from the bottom of the grooves to the space behind the rings. This was done to balance the rings by cutting down excessive pressures which might occur behind them. These rings stayed in place which gave the writer reason to believe the pressure behind and in front of rings was balanced.

There are many uncertainties to consider when trying to determine piston friction losses, such as clearances, improper adjustment of rings, face conditions at the rings, quality of lubricants, and pressures behind the rings. It is essential that operating data be obtained relative to these factors, all of which bear on piston friction losses and the answer to which would be of inestimable value to engineers interested in the subject.

LOUIS ILLMER.[8] In the next to the last paragraph of this paper, reference is made to a paper[9] published by the writer, and states that in the author's tests no sudden building-up of ring-pocket pressures was found conforming with the writer's deductions.

The paper under discussion does not break down the frictional losses sufficiently to permit a direct check of a phenomenon of this kind. In fact, the results are masked by failure to separate bearing, gear, and other losses from the piston-ring losses, as such.

The writer's hypothesis did not deduce any sudden building up of pocket pressure, but rather an accumulative or gradual self-loading pressure that progressively increases at higher piston speeds. It is only at maximum speeds that the ratio of piston-ring drag to total frictional losses becomes a significant factor. The ring width and its side fit may exert a considerable influence on the intensity and distribution of pocket pressure, but neither is specified in the author's paper.[1]

When replotting the reported test points (after deducting for pump work), the graph lines do not converge as stated; instead they may justifiably be drawn in substantial parallelism. When taken thus, the total drag including bearing and piston losses, may be evaluated as

$$\text{FMP} = \left(\frac{P_{cyc}}{8.2} + 2\right) + \frac{\text{RPM}}{275} \quad \ldots\ldots\ldots\ldots [1]$$

where P_{cyc} = mean cyclic pressure acting behind the piston, lb per sq in. gage.

The accompanying bearing and valve-gear losses usually constitute the major portion of this FMP value. While the paper under discussion affords no appraisal of such bearing losses, a rough separation thereof may be arrived at by assuming that the piston-ring loss becomes substantially equal to the bearing losses when running normally without supercharging under full-speed conditions at $P_{cyc} = 50$. Accordingly, the following expression may be taken as an approximation for such bearing losses on a power-stroke basis in terms of pounds per square inch of piston area

$$= \text{abt } 5 + 1/3\ \sqrt[3]{(\text{RPM})}\sqrt[3]{(P_{cyc})} \quad \ldots\ldots\ldots\ldots [2]$$

When resorting to such interpolation, the piston and crosshead drag may be determined by subtracting Equation [2] of this discussion from Equation [1].

As applied to the test conditions prevailing in the paper[1] under discussion, the corresponding first pocket pressures have been worked out in accordance with the equations laid down in the writer's paper.[9] Fig. 1 of this discussion plots the resulting values, assuming the rings to be 1/8 in. wide and their side faces snugly fitted. It will be observed that such estimated pocket pressures start to build up at different speeds in a proportionate relation to the supercharging pressure used, that is to say, the bore-contacting pressure of the first ring is not directly responsive to the initial manifold pressure alone but to piston speed also.

Going a step further, by dividing the remaining unit piston-ring drag by the pocket pressures plotted in Fig. 1 of this discussion, some idea may be gained regarding the prevailing friction

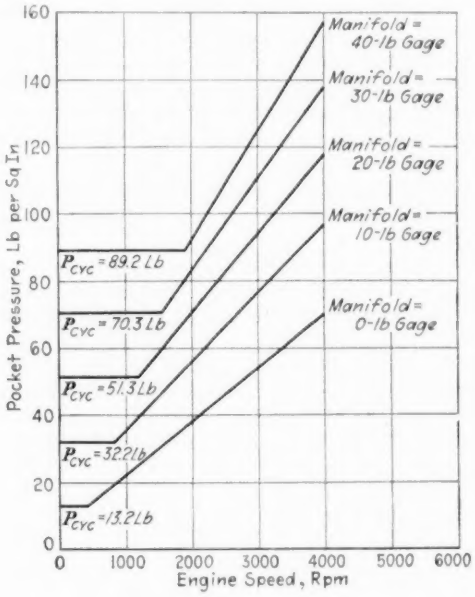


FIG. 1 ESTIMATED POCKET PRESSURES P_r

coefficient f_{exc} that applies to the first ring. Without elaborating on this aspect, it is found such coefficient takes on the following characteristic

$$f_{exc} = 0.08\ [5\sqrt[3]{\text{RPM}} - (80 - P_{cyc})] \quad \ldots\ldots\ldots [3]$$

This finding applies after the oil film has partially broken down and the frictional ring drag starts to increase directly with augmented pocket loading. The numerical value of Equation [3] of this discussion is dependent upon both the P_{cyc} and rpm factors, and when plotted reaches a common abrasive maximum value of $f_{exc} = 0.08$ at around 4000 rpm. The rpm factor evidently bears a relationship to the rate of accumulative pocket-pressure rise. Actually f_{exc} never drops to zero, but at a moderate P_{cyc} value goes over into an oil-borne-film condition for which the coefficient is relatively low. Although the rated engine speed was 3600 rpm, all the reported tests were for some undisclosed reason restricted to 3000 rpm.

According to Fig. 1 of this discussion, the estimated ring-pocket pressure rises from 13 to 70 lb per sq in. between 500 to 4000 rpm for nonsupercharged conditions. If the alleged accumulative action did not actually occur, the maximum ring-friction coefficient, because of its lowered bore-contacting pressure, would have to be correspondingly increased to about $5.4 \times 0.08 = 0.43$. Such an excessive value would be entirely unreasonable under well-lubricated high-speed conditions.

Therefore, the writer believes that the author's tests, as far as they go, tend to prove rather than disprove the existence of an accumulative pocket pressure. Furthermore, it is questioned whether his supercharging method does constitute a rational basis for fixing upon ring-pocket behavior, particularly where the bearing and incidental fixed-gear losses cannot be sufficiently broken

[8] Research Engineer, Cortland, N. Y. Mem. A.S.M.E.

[9] "Piston Ring Friction in High-Speed Engines," by L. Illmer, Trans. A.S.M.E., vol. 59, January, 1937, paper AER-59-1, pp. 1–6.

down to allow of definitely segregating piston-ring drag. In any event, no accurate deduction can be drawn from combined piston and bearing measurements where the latter factor remains inherently large.

B. J. ROBERTSON.[10] There are many inconsistencies arising from the conventional method of obtaining friction horsepower by motoring an engine with an electric dynamometer. A compression-ignition engine of the precombustion-chamber type in the laboratories at the University of Minnesota gave the following results during dynamometer tests at full load and 850 rpm: Brake horsepower = 42; friction horsepower = 24; fuel consumption = 0.445 lb per bhp-hr; fuel consumption = 0.283 lb per ihp-hr; brake mean effective pressure = 74.3 lb per sq in.; friction mean effective pressure = 42.3 lb per sq in.; and indicated mean effective pressure = 116.6 lb per sq in. It is quite evident that the fuel consumption per indicated horsepower hour is impossibly low, so that it may be presumed that the friction horsepower as determined by motoring is too high.

Moore and Collins[11] report a friction mean effective pressure of 5 lb per sq in. due to the friction produced by the air passing from the main chamber to the antechamber of a compression-ignition single-cylinder engine. The nozzle between the precombustion chamber and the main clearance chamber of the engine was $^1/_4$ in. in diameter so that the friction losses due to air movement in and out of the precombustion chamber would undoubtedly be much greater than in the N.A.C.A. engine.[11] It seems probable that the density of the gases passing through the precombustion-chamber nozzle would be somewhat greater when the engine is motored than when it is operated under power, and that the power loss when motoring the engine is excessive because of greater density.

Another source of discrepancy between the friction losses while motoring and under power is found by reason of the heat rejected to the water jacket while motoring the engine. The expansion line falls below the compression line, producing a compression loop. The mean effective pressure of this loop in the engine with a five-to-one compression ratio tested by Moss[12] was given as slightly over 1 lb. A high compression ratio with correspondingly high air temperatures during the friction run should increase the area of the compression loop.

Moss[12] also reports that the friction losses in the single-cylinder Benz engine were twice as great when operating under full power and full spark advance as when the engine was motored. The discrepancy between the friction losses of the two previously mentioned engines when tested by the motoring method, indicates that there is a great need for further investigation along this line. The method employed by the author would yield valuable additional information if applied to engines of different types.

The author emphasizes the effect of the high pressure of the piston rings against the cylinder walls. The excessive friction and consequent wear produced by the high pressure of the top piston ring against the cylinder wall is indicated by the familiar ridge found in any used engine immediately above the highest point to which the ring travels. The depth of the wear at the extreme top is greatest with high-compression engines. It is undoubtedly true that the lubrication between the ring and the cylinder wall is poorest at the top position, but the hypothesis that excessive pressures against the cylinder wall are responsible for wear is borne out by the excessive clearance found at the mid-stroke position on some aircraft engines. Side thrust of a rather short piston itself is very likely responsible for this condition. Excessive side-wall pressures which are caused either by rings or by piston skirts may be expected to result in excessive wear.

Just what pressures exist behind the top ring at high speeds is not known. The work done by Professor Ford and the writer on a slow-speed engine,[13] indicates that there is, without a doubt, a time lag in transmitting the pressure to the second ring groove. The two curves in Fig. 2 of this discussion represent pressure behind the top ring and the pressure behind the second ring. It will be noticed that the pressure behind the second ring is very nearly constant for the compression and expansion strokes, while the pressure behind the first ring shows a wide fluctuation. At high speeds there may be a lag in transmitting the cylinder pressures to the top ring groove also. Until information is available on the ring-groove pressures at high speeds it seems necessary that friction tests be made with pressure variation in the cylinder approximating those occurring when the engine is operating under load. The author's method employs cylinder pressures that fluctuate in a normal manner.

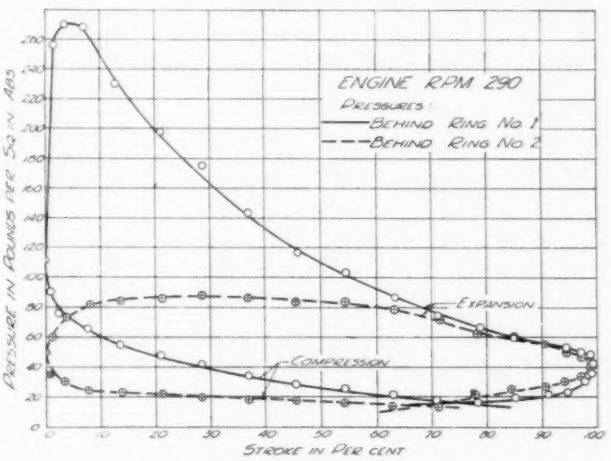


FIG. 2 COMPARISON OF PRESSURES BEHIND THE FIRST AND SECOND RINGS OF AN INTERNAL-COMBUSTION ENGINE

Fig. 8 of the paper[1] indicates that there is lag in the transmission of the pressures to the rear of the rings, because the curves of mechanical friction versus speed become flatter with increased mean pressures on the pistons.

It is reasonable to suppose that the friction of the individual rings will be proportional to the pressure in the respective ring grooves. Tests made by removing the first ring to evaluate the friction on the second ring, etc., do not give the true picture because the gas pressures in the combustion chamber are then transmitted directly to the second ring and the pressures produced by it are very nearly the same as those that existed behind the first ring when it was in position.

Mechanical friction is a large fuel consumer in many types of internal-combustion engines. The difference between the losses in different types is so great that a more complete understanding of the contributing causes is greatly needed.

[10] Associate Professor of Internal Combustion Engines, Institute of Technology, University of Minnesota, Minneapolis, Minn. Mem. A.S.M.E.

[11] "Friction of Compression-Ignition Engines," by C. S. Moore and J. H. Collins, Jr., N.A.C.A. Technical Notes No. 577, August, 1936.

[12] "Motoring Losses in Internal Combustion Engines," by H. Moss, Technical Report of the Aeronautical Research Committee (British Air Ministry) for 1927–1928, vol. 2, Reports and Memoranda 1128, pp. 835–844.

[13] "An Investigation of Piston-Ring Pressures," by B. J. Robertson and A. R. Ford, *Diesel Power*, vol. 12, October, 1934, pp. 548–551.

Author's Closure

Messrs. Dennison, Karelitz, and Illmer have mentioned the failure to separate piston and journal-bearing friction. It is regrettable that this was not done. It was felt that it would be a mistake to make any structural changes in the engine until the work was completed, and for this reason it is impossible at present to furnish data on bearing clearances, ring pressures, and the separation of mechanical losses. It is hoped that these figures may be presented in a later paper.

Professor Good noted the effect on friction of change in piston and ring temperatures, and suggests that this is one of the reasons for the decrease in slope of the curves of friction mean pressure, both in relation to speed and also to indicated mean effective pressure. This is very likely the case. In fact, this is one of the reasons that the compression-expansion cycle, as described, was used in this investigation.

Mr. Cooke's suggested method of determining Diesel-engine friction losses by plotting fuel consumptions over a load range might well be used in the case of a constant-speed injection engine, particularly where the fluid-pumping losses are known or can be determined. For carburetor engines, however, the effects of change of mixture proportionality and fluid-pumping losses with load variation are believed to be so serious as to make this method unsatisfactory as a means of determining mechanical friction losses.

Mr. Illmer's discussion states his belief, and gives his reasons therefore, that this paper tends to prove his hypothesis of a building-up of piston-ring-pocket pressures at high speeds. He also notes that the maximum speed reported by the author was 3000 rpm. As was stated in the paper, it is regretted that the equipment available for this work was not capable of higher speeds.

When the results as given in the paper are examined in connection with the work of Taylor, which has already been mentioned, and with the discussion of this paper by Professor Robertson, particularly with respect to his suggestion of increased pocket-pressure lag with increased speed, Mr. Illmer's conclusion would seem, in the author's opinion, to be unwarranted.

In Mr. Taylor's investigation a constant air pressure was maintained above the pistons, which would tend to result in constant pressure, subject to blowby, back of the several piston rings. If his figures are replotted to give the relation of friction mean pressure to rpm for any constant air pressure, the resulting curve will be seen to have substantially the same shape as those presented in the paper, except that the Taylor curve has a somewhat higher and more constant slope.

The author believes that this bears out Professor Robertson's idea of increased lag, and a consequently lower mean ring-pocket pressure as speed is increased.

Income Versus Production[1]

Comfort A. Adams.[2] The writer has for some years been well aware of the trend pointed out so clearly in the diagrams, although never before has he seen such complete statistics over a long period. The logic of the situation, as the writer sees it, is briefly as follows:

1 Every dollar that goes into the savings or investment pocket beyond the need for capital, means that much less demand for consumer products, which are the only ultimate products of industry.

2 Since the receivers of interest and dividends, as a whole, spend a much smaller percentage of their income on consumer products than do the recipients of wages and small salaries, it is obvious that the larger the interest-and-dividend end of the nation's income, the smaller is the demand for consumer products, and the more unstable is the whole economic structure.

However, it was not intended to tell the author what he already knows, but merely to indicate a vital interest in the article, which to the writer's mind touches the sore spot of the whole economic order.

John B. Coleman.[3] From the data given in Table 1 of the paper the writer has prepared the chart shown in Fig. 1, herewith, in which the incomes of the various groups are plotted as percentages of the group incomes in 1919. The assumption is made that the 1919 income was reasonably fair in amount and in distribution and, according to the writer's recollection, this was so since almost everybody that wanted work was working, incomes were high, people were spending money, and profits were good. What really happened was that from 1919 to 1929 a much larger share of the goods produced went into profit and overhead. The additional profit must have been on the order of twenty-five billion dollars in that ten years, and this provided the surplus money which was invested in foreign bonds and certain "monuments to imbecility," that served no useful purpose.

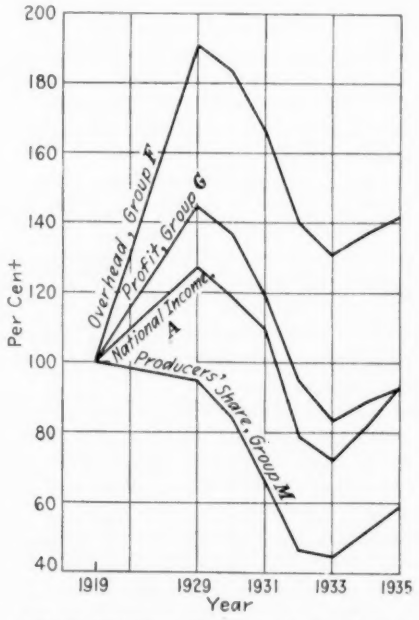


Fig. 1 Chart Showing Percentage of National and Group Incomes Based on the Year 1919
(Data taken from Table 1 of the paper.)

Now it is perfectly obvious that the additional profit spent in this way used up goods that the producers were not able to take, and therefore, there was a fictitious sense of prosperity and good times, accompanied by a side effect of rising prices on the New York Stock Exchange. When the debacle came in 1929, there was no place in which these surplus goods could be sold, and unemployment commenced.

It would seem that it is perfectly obvious that the three lines—profit, national income, and producers' share of the national income—should follow practically the same course. Overhead might increase, although perhaps a great deal of the increase

[1] Published as paper MAN-60-1, by Walter Rautenstrauch, in the April, 1938, issue of the A.S.M.E. Transactions.

[2] Consulting Engineer, E. G. Budd Mfg. Co., Philadelphia, Pa. Mem. A.S.M.E.

[3] Chief Engineer in charge of fire-protection engineering, Grinnell Co., Inc., Providence, R. I. Mem. A.S.M.E.

shown in Fig. 1 was unjustified. In other words, those engaged in trade probably got more than their share in the years 1919 to 1929. Certainly the other three lines must be held close to one another if any reasonable degree of security and employment is to be attained for the great mass of the people.

W. D. ENNIS.[4] With much of the author's paper it is possible to agree. The paper does real service in stressing the fact that national income equals national production. Disagreements will start with two questions of fact:

1 *Is the National Overhead Increasing?* The paper treats as productive, only the salaries and wages of workers in six of the fourteen groups contributing to national income, plus total incomes of farm owners. All else is overhead—for example, public-health service, education, merchandising, policing, and government road building. The paper shows, as between 1919 and the period from 1929 to 1935, a startling increase in the ratio of overhead (so called) to production.

Whatever this ratio is, it is not overhead. The implication is that the purchase of services is something to be limited, while the purchase of labor for producing commodities is something to be promoted.

Fig. 2 of this discussion reflects the results of a similar study, continuously from 1919 to 1935, according to the uniform technique of the National Bureau of Economic Research. The author's figures indicated are from two different sources, and in both cases are based on unrevised data. Here groups 1 to 4 correspond to the author's commodity-producing groups; not merely the labor included therein, but the whole of the national income credited thereto.

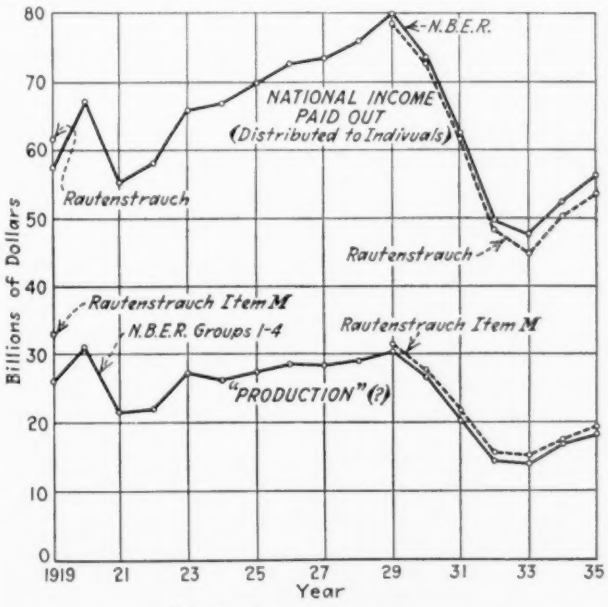


FIG. 2 COMPARISON OF DATA

(Solid lines represent data from National Bureau of Economic Research Publication 32, p. 18.)

The author's groups 5 to 7 are not included therein and not shown on the diagram. They, with group 8 (trade), bear a fairly constant ratio (0.219 to 0.249, lowest in 1935) to total income distributed to all groups, and may therefore be eliminated from the discussion.

[4] Humphreys Professor, Stevens Institute of Technology. Mem. A.S.M.E.

These curves, like the figures of the paper, spell a falling ratio of production to total income paid out, and hence a rising proportion of "overhead" (groups 9–14) to "production" (groups 1–8). The ratio of total overhead to commodity production (groups 1–4) rose from 120 per cent to 206 per cent. But, the reference[5] tentatively concludes, this increase was due "exclusively to the decline in the prices of commodities, which was greater than (that) in the prices of services." This point is ignored in the paper. The reference quoted in Fig. 2 makes the ratio referred to, when price fluctuations are eliminated, 148 per cent in 1919 and 138 per cent in 1933. The average for the first 8 years of the period was 158 per cent, and for the second 8 years it was 154 per cent. Here is no increase whatever.

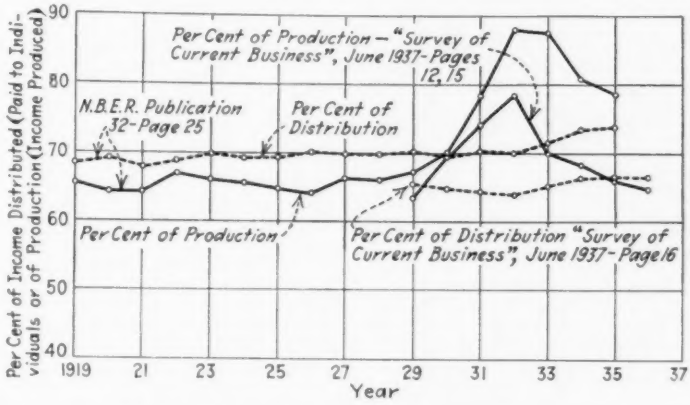


FIG. 3 LABOR'S SHARE IN THE NATIONAL ECONOMY

2 *Is Labor's Share Decreasing?* The paper stresses a decline in the percentage of total income paid as compensation to employees, although admitting that the amount paid employees has increased. Global figures were available, and are presented in Fig. 3. Labor's proportion in the total economy, on either basis referenced, has not decreased, it has increased. The share of labor in production, as here contrasted with the share in total distribution, is particularly noteworthy.

The paper shows declining shares to labor in manufacturing, mining, contract construction, and other activities. These are activities of increasing mechanization. Now mechanization has to be paid for, in interest and dividends. Thus, in such industries, labor's share tends to decline (not necessarily labor's amount). Relief is afforded by employment in service activities—which the paper labels unfavorably as overhead.

To sum up, the paper proposes a purely arbitrary definition of overhead. It exaggerates the increase in what it defines as overhead. There has, however, been some increase in the proportion of total income (actual dollars) paid out to the service activities. That this is disadvantageous has not been demonstrated. It is chiefly and perhaps wholly due to irregular price variations.

Dollar payments for labor have increased slightly faster than total national income. In hard times, they hold up better than national production. The percentage of income paid directly for labor may decline in particular industries, especially where labor is costly, and must do so where mechanization is involved. But in total, labor's share has not decreased. On the contrary, it is one of the most stable figures in our economy.

RALPH E. FLANDERS.[6] It is an original idea to analyze the operations of the national economy as a whole in much the same

[5] National Bureau of Economic Research, Publication 32, p. 22.

[6] President, Jones and Lamson Machine Company, Springfield, Vt. Past-President A.S.M.E.

way as would be done in analyzing the operations of an individual business. The paper is very much worth-while on this account. The writer's criticisms for the most part relate to factors which would be taken into account in the individual business, but whose counterparts in the national economy do not seem to have been recognized by the author.

If the analogy of private business is to be followed, it is not surprising to find the national overhead and, in general, the percentage of indirect expenses growing. That is the case as more and more efficient methods of production are applied to the individual business. Less of the expenditure goes to direct labor, and more to indirect labor and to capital expenditure, particularly in the recovery of capital costs through depreciation. It may be said roughly that the greater the efficiency of a business, the higher are indirect costs in relation to direct. The net cost, however, is lower, and all the indirect expenses serve as a means of distributing income for the purchase of the output.

These points are of particular importance in the treatment on page 215, in which it seems not to have been realized that capital costs are in a large measure distributed and become available as purchasing power of the product produced; and similarly on page 218, the process of distribution seems to have been assumed to be in some way separate from production and not a part of the process of putting finished goods in the consumers' hands. This seems to be fallacious. Distribution is part of the overhead in the total process and serves as one of the means for the distribution of the purchasing power for obtaining possession of the goods as they reach the market. In general, the author seems to have looked at the economic process too narrowly, as though some few workers most directly concerned with production were the only ones whose work contributed to the total result and whose wages constituted the necessary purchasing power.

In making comparisons with private business operations, it is not at all surprising to find that social overhead increases in a depression; it does in private business as well.

Another point that seems to need clearing up is whether the rendering of all services belongs in the overhead. It would seem that this is not the case. The great mass of people engaged in rendering personal services, whether in beauty parlors or kitchens, and those of higher grade, furnishing concert music and operating libraries, are as much a part of the process of making goods and services available as are the men in an automobile factory, for instance. Sometimes one wants to take an automobile ride, and sometimes one wants to read a book from the library. The whole nexus of organization which permits the one or the other to be done, stands very nearly on the same footing, except that the one is related to goods and the other to services.

A useful line of inquiry, it seems to the writer, would be to make some estimate of the proper proportion of the returns from industry that should go into capital expenditure and the proportion that should go into consumption goods. There is, of course, an optimum division of activity between capital and consumer goods which could theoretically be determined, although practically it would be difficult to do.

Capital-goods production must be great enough to replace worn-out equipment and finance an expanding population, as well as finance an increasing standard of living. It must also be great enough to replace uneconomic equipment with new and better equipment where economic conditions justify it. It must be great enough, furthermore, to finance the shorter working week and also single-shift operation in noncontinuous industries, unless two- and three-shift operation is considered to be a part of the "more abundant life." In addition, the proportion which goes into capital expenditures must be great enough to finance experimentation, research, and development work, much of which will be thrown away, as well as leaving a share for making good on the other types of losses inherent in a free economy.

To the writer the diagrams are the most interesting part of the paper. To appraise their significance, however, it would be necessary to have them carried over a greater span of years so that it could be seen whether the tendencies noted are cyclical, secular, or catastrophic.

RUSSELL SPRAGUE.[7] In connection with the matter of the overhead on the total national production, the writer has recently completed an analysis corresponding to that of the author for the whole period from 1910 to 1929. The picture presented in the paper seems somewhat distorted. The author compares the abnormal period from 1929 to 1935 with the single abnormal year, 1919. A truer picture is obtained when the periods immediately before and after 1919 are also studied.

Table 1 of the paper shows a "par" ratio $(F + G)/M$ of 85.4 per cent in 1919 and a large increase thereafter. But this ratio varied from 99 per cent in 1918 to 133 per cent in 1910, averaging 128 per cent for the whole period from 1910 to 1916.[8]

The author deplores the fact that while in 1919 the overhead group received only 46 per cent of the national income, this had increased to 68 per cent in 1932. But his Fig. 1 also shows that in 1909 the overhead group received 54 or 55 per cent of the national income.

Data exist for the period following 1919,[9] and also revised figures from King for 1919. The ratio varies from a minimum of 94.4 per cent in 1920 to a maximum 149 per cent in 1929. The maximum of 149 per cent compares with 148 per cent in the paper, but the figure for 1919 is 103 per cent, not 85.4 per cent as stated in the paper. For the whole period from 1921 to 1929, the ratio is 137 per cent, which is not significantly above the figure of 128 per cent for the period from 1910 to 1916.

G. L. TUVE.[10] The paper is an excellent example of engineering analysis applied to economics, and the author is to be commended for such a compact and unbiased presentation.

When he suggests that the facts are obscured "to those who think in terms of money instead of in terms of goods," he has certainly pointed out one of our basic troubles. Engineers should do their share to bring the attention back from the "counters" to the goods themselves, or better still, to the human effort necessary to produce the goods.

The author states (p. 218) that "the issuance of claims to goods (money) not associated with the processes of production always constitutes a hazard to the national economy when the issuance of such claims tends to increase at a rate greater than the rate of production." As an example he cites "government expenditures for relief which are not balanced by the production of goods." Is it not possible that if economic difficulties are being produced by an insufficient number of claims for consumer goods (low consumer purchasing power), the increasing of such claims may improve the condition of the national economy? According to Figs. 3 to 7, inclusive, the "claims to consumption of goods in excess of current personal needs," in other words savings, have been accumulating faster than they could be used in the capital-goods industries. If some of these claims to consumer goods are transferred by taxation to immediate consumers (i.e., families on relief), who will at once exchange them for consumers goods, will not the stoppage and unemployment in the consumers-goods industries be partially relieved at the same time that human need

[7] Instructor, Stevens Institute of Technology. Jun. A.S.M.E.

[8] "Income in the United States—Its Amount and Distribution—1909–1919," National Bureau of Economic Research.

[9] "National Income and Capital Formation, 1919–1935," by Simon Kuznets, National Bureau of Economic Research.

[10] Professor, Heat-Power Engineering, Case School of Applied Science, Cleveland, O.

is being met? Granted that better ways should be found to relieve the stoppage, nevertheless the taxation method should improve matters in an emergency, because it is actually a reversing of the trends shown in Figs. 3 to 7.

Readers of this paper should not miss the analysis made by another A.S.M.E. member, Dr. H. C. Dickinson, in his book "The Mechanics of Prosperity."

Author's Closure

Professor Ennis attempts to refute the argument that the overhead on the national plant has increased materially during the last 17 years, by using an entirely different grouping of income disbursement from that used in the paper. It is the author's belief that his own groupings are both rational and important and that the ratios shown in the paper have important significance in our economy.

However, their meaning should be more fully explained. This Professor Ennis fails to do. What the author had hoped would ensue from this discussion was more light on the economic and social significance of the ratios shown.

The Fig. 2 drawn by Professor Ennis does not "reflect the results of a similar study," but of a wholly different study. The study he refers to is found in chapter 3 of "National Income and Capital Formation," entitled "Distribution According to Industrial Origin;" the author's study related to "Income Payments to Individuals." In the study to which Professor Ennis refers, incomes by industries, a comparison is made between commodity-producing industries, not individuals, and "other industries." The complete reference, of which Professor Ennis quotes only a part, is, "If the estimates underlying Table 5 are to be trusted, the apparent decline over the period in the relative share of the commodity-producing industries in the economic system *as measured by the industrial distribution of the national product*, was due exclusively to the decline in the prices of commodities, which was greater than the price of services."

This reference also continues: "But in view of the slender basis upon which the difference and adjustment of price changes rest, it is perhaps best to leave this conclusion as a tentative suggestion deserving further explanation." In view of these considerations, Professor Ennis' criticism cannot be taken seriously.

Under the heading, "Is labor's share decreasing?" Professor Ennis cites the percentage which employees' compensation, both productive and overhead, is of the total national income. He does not address himself to the issues raised in the paper, but uses ratios which, as stated in the paper, do not set forth the important relationships. Under the same heading, Professor Ennis states, "New mechanization has to be paid for in interest and dividends." Many of us are not so sure of the "has to be," particularly as to the amounts involved. The paper raises the questions of the workableness of the relative shares of capital and labor to the goods produced. That is the engineering question involved. The concluding remarks of Professor Ennis are so obviously outside the scope of the author's thesis that a reply is not called for.

The comments by Professor Adams and Mr. Coleman throw additional light on the important problems of the relationships with which the paper deals.

Mr. Flanders suggests that because private business is increasing its ratio of indirect to direct expenses, it is not surprising to find the overhead on the national plant increasing. Quite true, but both the national plant and private businesses are now in a rather bad way. Mr. Flanders seems to have failed to grasp the main points in the paper, i.e., that it is the relationships between labor's claims and capital's claims to the goods produced which cause the first disturbance; and the relationships between the trend in employment and wages in creating form utility, and the trend in employment, wages, and income in creating other utilities, which cause the second disturbance. This is apparent in his statements, "All the indirect expenses serve as a means of distributing income for the purchases of the output" and "capital costs. . . . . become available as purchasing power of the product produced." Such statements are too general to have any value in a critical analysis. Furthermore, capital costs have not been expressing themselves in purchasing power, as Mr. Flanders well knows, and that is one of the causes of unemployment.

Mr. Sprague presents a more critical analysis and quite properly calls attention, as did the writer in his oral presentation, to the fact that 1919 was a "low" year for the producers. But to state that the period from 1929 to 1935 and the year 1919 were all abnormal, implies that Mr. Sprague knows what the normal is. The author does not. Certainly the data for the period including the war years cannot be regarded as relating to a normal period. The author does not understand how Mr. Sprague arrives at a ratio of overhead to production of 103 per cent in 1919. The data of Table 2, which are believed to be correct, show a total national income of 61,628 millions of dollars, of which the producers received 33,224 millions and the overhead was 28,404 millions. This gives a ratio of 85.4 per cent of overhead to production.

Professor Tuve suggests that taxation of savings and the use of the resulting funds in purchasing power through unemployment relief may improve the conditions of the national economy. It will, temporarily, but as a permanent measure it is, in the author's opinion, unsound. The very fact that this measure must be resorted to demonstrates that our present methods of distributing claims to goods are not workable.

Effect of Temperature Variation on the Creep Strength of Steels[1]

J. J. Kanter.[2] A much needed clarification of the effect of temperature variations on the creep strength of steels is afforded by the author's skillful and instructive treatment of the subject. Both the metallurgist who seeks to smooth his creep curves and the engineer who must incorporate creep data in the design of machines and structure, will find these correction formulas and methods helpful.

In discussing this paper it is the aim to suggest a somewhat different basis for the functional relationships between secondary creep rate, stress, and temperature, which in no way reflects upon the practical aspect of the author's analysis in so far as the end results are concerned. It is the writer's thought that a choice of such functions as lead to physical interpretations of the creep phenomena itself might ultimately provide a sounder basis for analysis than the purely empirical choice of functions and constants which are made in the paper. The advantages of generalizing the creep-strength equations in a manner that leads to fundamental constants which bear relations to other properties of the material should be obvious.

In a recent paper[3] the writer attempted a representation of the relationship between secondary creep rate, stress, and temperature, based upon certain concepts borrowed from thermodynamics and statistical mechanics. Consider a series of secondary creep rates obtained at a single stress over a range of temperature containing no critical points. If these are plotted with the logarithm of rate versus the reciprocal of the absolute temperature, a

[1] Published as paper RP-60-5, by Ernest L. Robinson, in the April, 1938, issue of the A.S.M.E. Transactions, p. 253.

[2] Research Metallurgist, Crane Co., Chicago, Ill.

[3] "The Problem of the Temperature Coefficient of Tensile Creep Rate," by J. J. Kanter, A.I.M.E. Metals Technology Division, Technical Publication No. 863, December, 1937.

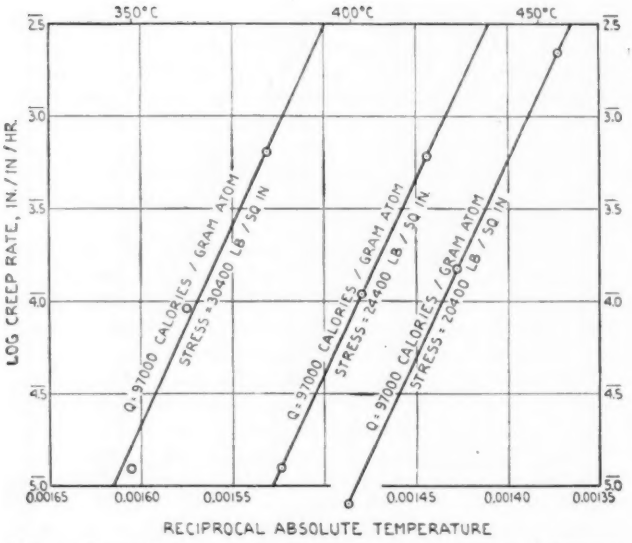


FIG. 1 CREEP RATE OF ARMCO IRON AS A FUNCTION OF ABSOLUTE TEMPERATURE[4]

straight line is determined as illustrated in Figs. 1 and 2. The slopes of lines so determined can be interpreted in terms of the energy involved in the process leading to creep strain over that range of temperature. It is noteworthy that the slope upon such diagrams, regardless of the stress involved is a constant value within the temperature range. If the logarithm of the creep rate divided by stress be plotted versus a suitable stress function the isotherms upon this diagram are linear and parallel, and the ratio of values also leads to a definite slope when plotted against the reciprocal of the absolute temperature, as illustrated in Fig. 3. These findings are mathematically expressed in the following equation, applying within the specified range of temperature

$$\ln\left(\frac{v}{\varphi_0 \sigma}\right) = f(\sigma) - \frac{Q}{RT} \quad \ldots\ldots\ldots\ldots [1]$$

where ln = natural logarithm; v = constant tensile creep rate, in. per in. per hr; σ = tensile stress, lb per sq in.; φ_0 = flowability constant; $f(\sigma)$ = stress function relating creep rate; Q = quantity of energy, cal per g mol; R = gas constant, 1.987 cal per deg mol; and T = absolute temperature, k. This analysis gives a kinetic picture of creep in which metal atoms, having an average energy proportional to the absolute temperature, must chance to

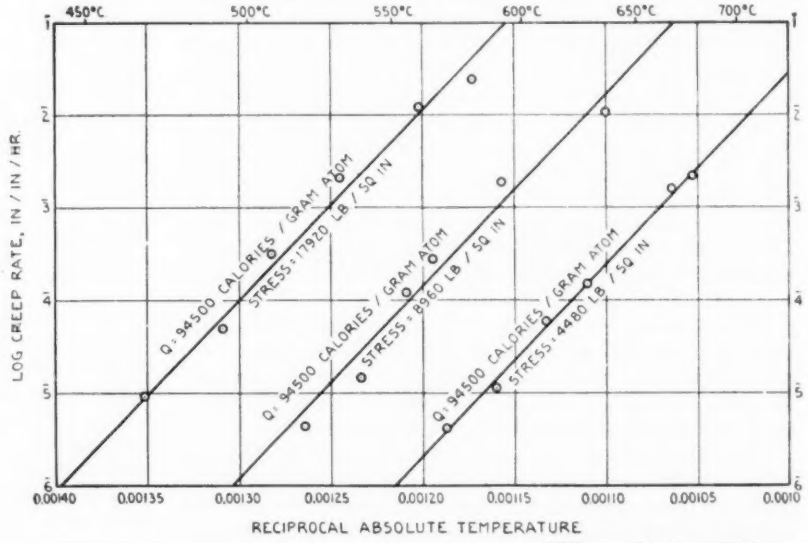


FIG. 2 CREEP RATE OF CAST CARBON STEEL AS A FUNCTION OF ABSOLUTE TEMPERATURE[5]

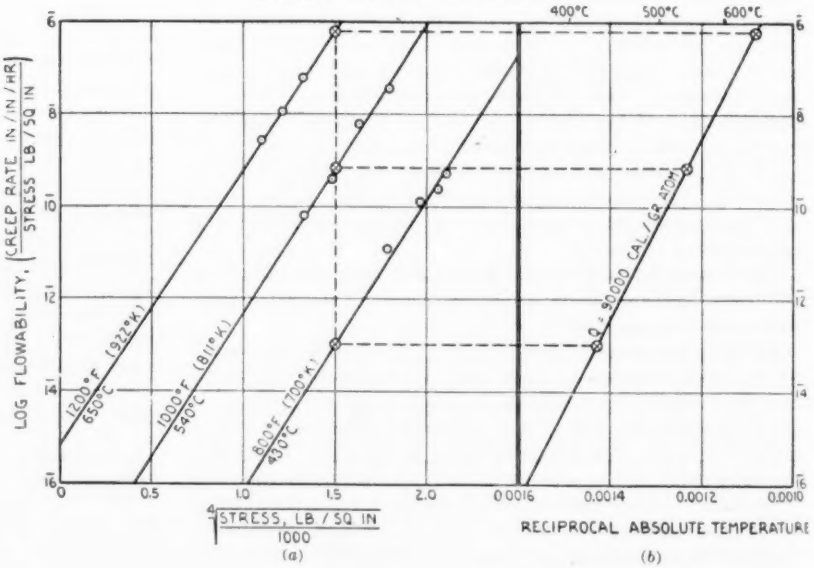


FIG. 3 "FLOWABILITY" OF OPEN-HEARTH WROUGHT STEEL, 0.17 PER CENT CARBON[6]
[(a) As a function of stress. (b) As a function of absolute temperature.]

[4] "Properties of Materials at High Temperatures," by H. J. Tapsell and W. J. Clenshaw, Engg. Research Report No. 1, Department of Science and Industrial Research, of London, 1927.

[5] "The Strength at High Temperature of a Cast and a Forged Steel as Used for Turbine Construction," by H. J. Tapsell and A. E. Johnson, Engg. Research Report No. 17, Department of Science and Industrial Research. London, 1931.

[6] "Some Long-Time Tension Tests of Steel at Elevated Temperatures," by J. J. Kanter and L. W. Spring, Proceedings A.S.T.M., vol. 30, 1930, part 1, pp. 110–132.

attain great enough activation to surmount a barrier energy Q, before participation in creep strain. The fact that similar values of Q are found over large ranges of stress suggests that strain energy has but little effect upon the necessary activation, but rather influences the probability that metal atoms will attain the necessary activation.

Suppose it is desired to apply Equation [1] to the problem of effect of temperature variation on the creep strength of steels with which the author is dealing—the variable is then defined as a constant, v_0, and $v_0/\varphi_0 = \sigma_0$. Furthermore, since a range of stress above zero is to be considered, a stress function compatible with Equation [1] of the paper can be chosen by letting

$$f(\sigma) = (n - 1) \ln (\sigma/\sigma_0) \quad [2]$$

where n is the same slope of a log-log plot as used by the author and σ_0 is the stress constant.

Substituting these values, Equation [1] becomes

$$\ln (\sigma/\sigma_0)^n = \frac{Q}{RT} \quad [3]$$

or, written as an explicit function of stress

$$\sigma = \sigma_0 e^{\frac{Q}{nRT}} \quad [4]$$

where e is the base of natural logarithms. Defining σ_T as the nominal creep strength at absolute temperature T, and σ_{T_0} as

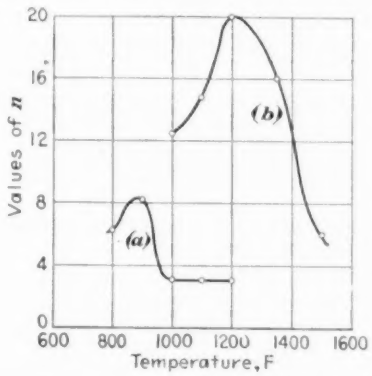


FIG. 4 VALUES OF n VERSUS TEMPERATURE

(n = slope of plot of log-log creep rate versus stress. Curve (a), 0.15 per cent carbon steel, Fig. 3 of the paper. Curve (b), austenitic steel, Fig. 6 of the paper.)

the nominal strength at any other absolute temperature, T_0, within the range where the data are represented by the log-log type of diagram

$$\frac{\sigma_T}{\sigma_{T_0}} = e^{\left(\frac{Q}{nR}\right)\left(\frac{\Delta T}{TT_0}\right)} \quad [5]$$

in which ΔT is the spread in temperature ($T - T_0$).

Equation [5] is then similar to case 1 of the paper. The value Q serves as a strength index as does the author's k. When considering steels and iron, Q is usually found to have values in the vicinity of 90,000 cal per g mol and merits being considered a physical constant of iron and steel. The author's k can be shown to be a function of Q and the temperature range by comparing his Equation [2] with the writer's Equation [5,] from which it is readily deduced that

$$nk = \frac{Q}{RTT_0} \quad [6]$$

Thus the various cases of temperature variation for which the

TABLE 1

Temp, F	Creep stress, lb per sq in. (1% in 10,000 hr)	Log-log slope
800	26800	6.33
900	16900	8.27
1000	5750	3.04
1100	1800	3.08
1200	620	3.06

TABLE 2

Temp, F	Creep stress, lb per sq in. (1% in 10,000 hr)	Log-log slope
1000	23000	12.5
1100	11000	14.8
1200	6100	20.0
1350	3200	16.0
1500	1200	6.1

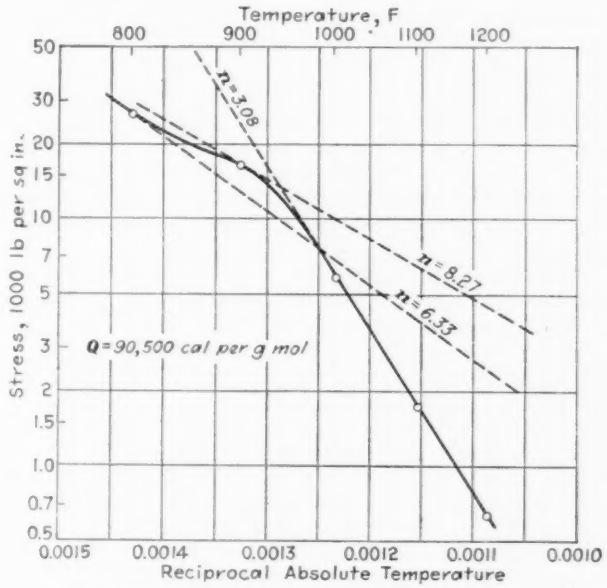


FIG. 5 GRAPHICAL TEST UPON DATA OF TABLE 1

(0.15 per cent carbon steel, Fig. 3 of the paper.)

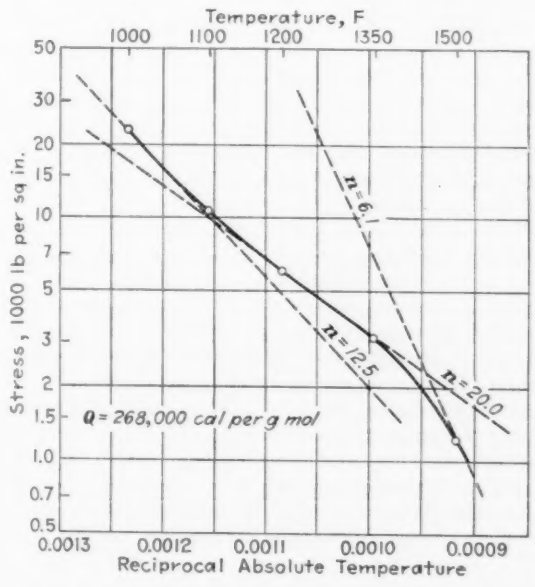


FIG. 6 GRAPHICAL TEST UPON DATA OF TABLE 2

(Austenitic steel, Fig. 6 of the paper.)

author has derived formulas may be conveniently related to the creep of any metal for which Q and n can be evaluated, merely by making use of Equation [6].

The power n from the log-log plot assumes an interesting significance in connection with Equation [4]. As pointed out in the paper, experimental values of n vary from unity to as high as 20, depending upon the material, the temperature, and even the portion of stress range considered. Data upon the creep of lead at small stresses show the value approaching unity, in which case the rate of creep is directly proportional to the applied stress. Creep under this circumstance obeys the law of pure viscous flow, for which in general there is but little partition of energy, and motions or displacements of molecules are free to take place essentially as do the molecules of liquids or gases. The average energy of such relatively unrestrained molecules, as for instance those composed of grain-boundary atoms, is approximated by RT. The greater the restraints upon the atoms, however, the greater becomes their partition of energy, with the result that the effective average energy appears to become some value nRT. In general, as temperature increases and the molecular or lattice restraints of metals grow weaker, it is found from creep data that the values of n diminish and tend to approach unity as a limit. The decrease, as will be shown, however, does not always proceed uniformly, presumably due to phase changes which may occur.

Examining the references to the original data for Figs. 5 and 6 of the paper, one finds considerable variation in the slopes of the plots of log-log creep rate versus stress between various temperatures. Thus, the value of n cannot be considered a constant independent of the temperature, except over limited ranges. For the 0.15 per cent carbon steel tested by Clark and White, for which the author uses $n = 3$ at temperatures between 900 and 1200 F, the writer deduces the values shown in Table 1.

While $n = 3$ may be a fair value to use between 1000 and 1200 F, too large a change takes place between 900 and 1000 F to warrant its use as far down the scale as 900 F. Again, in the case of Norton's data for austenitic steel from Fig. 6 of the paper for which the author uses $n = 15$ for temperatures between 1000 and 1400 F, the following array of n values has been deduced by the writer from the published log-log plots.

While $n = 15$ represents a fair average of these values between 1000 and 1400 F, it is the writer's belief that the spread between $n = 12.5$ and $n = 20$ is too large to be discharged as error incident to experiment, but rather that it bears a definite physical significance. It is interesting to note that for both ferritic and austenitic steels the n values reach maxima at temperatures corresponding to the point of most intense temper embrittlement for the respective steels. See Fig. 4, herewith. These are also the temperatures usually associated with strain-hardening relief and equicohesion. All these facts are suggestive of the thought that transitions are in progress at these temperatures, which may well lead to anomalies in the observed values of n. If, as suggested, n is an index of energy partition, it is not surprising to find it rising to a maximum in the vicinity of a critical temperature.

Equation [4] suggests a diagram where the logarithm of the creep stress is plotted versus the reciprocal of the absolute temperature as illustrated by Figs. 5 and 6. As shown in Fig. 4, the power n is a function of temperature reaching a maximum value over the temperature ranges considered. It follows then that the creep-stress curve has an inflection point at the temperature corresponding to the maximum of the curve representing the index n. It then follows that the slopes of the tangents to the creep-stress curves in Figs. 5 and 6 determine the partition energies Q/n, involved in creep strain at the particular rate represented. The energy Q, then, is proportional to the product of the slope and the value of n for the corresponding temperature. Expressed analytically, this statement is equivalent to the equation

$$n \frac{\Delta\sigma}{\Delta T} = -\left(\frac{Q}{RT}\right)\left(\frac{\sigma}{T}\right) \quad \ldots\ldots\ldots\ldots [7]$$

This gives a method for testing the self-consistency of several creep stresses for a metal determined over a range of temperature. According to Equation [7] a temperature curve passed through the creep-stress points should have tangent slopes which, when multiplied by n for the corresponding temperature, are equivalent to the right-hand expression. Graphical tests upon the data of Tables 1 and 2 are illustrated in Figs. 5 and 6. If a Q value of 90,500 cal per g mol is taken for the 0.15 per cent carbon steel, tangents are determined which pass a smooth curve through the stress points, with an inflection point at about 870 F. In the case of the austenitic steel, a value for Q of 268,000 cal per g mol is deduced by this method, giving the curve an inflection point at about 1200 F.

While the writer holds no fixed brief for the validity of the physical conceptions herein expressed, he does believe that the mathematical methods used give a more precise representation of the actual data than is afforded by the corresponding devices proposed in the paper. The formulas for various cases of temperature variation, which the author has so ingeniously developed, will reflect actual data more accurately, if the index nk is computed according to the requirements of the writer's Equation [6].

Author's Closure

The author is greatly indebted to Mr. Kanter for his discussion of fundamental metallurgical questions. The paper as published was intended to represent the application or utilitarian viewpoint. The author's formulas are based on two experimentally determined coefficients or indexes which are usually subject to rather limited variation over the range of conditions required for the analysis of high-temperature machines and equipment. Thus they may be considered as constants except for great temperature differences. These limitations were discussed in the paper.

It is of interest to note in Mr. Kanter's Equations [6] and [7] that as T approaches T_0 the expression

$$-\frac{1}{\sigma}\frac{\Delta\sigma}{\Delta T} = \frac{-\frac{\Delta\sigma}{\sigma}}{\Delta T} \text{ approaches } k$$

thus establishing the essential identity of Equations [6] and [7]. (Mr. Kanter expresses k as a ratio instead of a percentage as used by the author.)

These expressions lead to the conclusion that the product kn, the increase of rate per degree, varies inversely as the square of the absolute temperature so that materials are likely to increase in rate somewhat less rapidly at higher temperatures than according to the author's formulas. Certainly this is a welcome conclusion.

Equations [6] and [7] enable the computation of the product, kn, from a thermodynamic constant of the material Q or vice versa. It is, of course, highly desirable to be able to do this when the constant Q is known for the material in question, especially if extensive creep data are not available. However, when good creep data are available, it may well be that they will serve to determine the value of Q. In any event the results computed from both sources should agree, and there is no reason why they should not. This whole process is a step toward rational interpretation of test results.

Mr. Kanter gives values of n for various temperatures in Tables 1 and 2. Tables 3 and 4 give, in addition, values of k, the product kn, and the values of Mr. Kanter's constant Q computed from the existing creep data by the use of his Equations [6] and

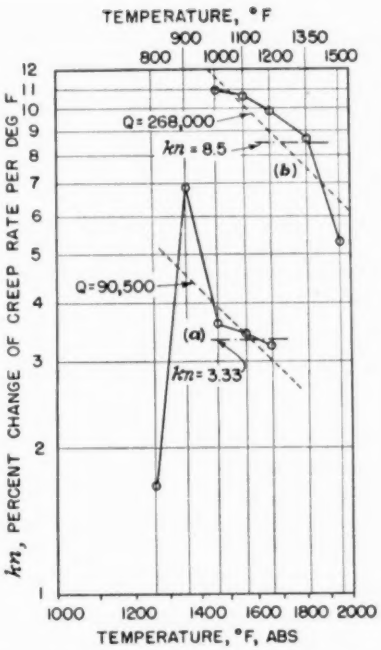


FIG. 7 VALUES OF kn, THE PER CENT CHANGE OF CREEP RATE PER DEGREE F, BASED UPON THE TEST RECORDS

(*a*) 0.15 per cent carbon steel, Fig. 5 of paper; (*b*) austenitic steel, Fig. 6 of paper.]

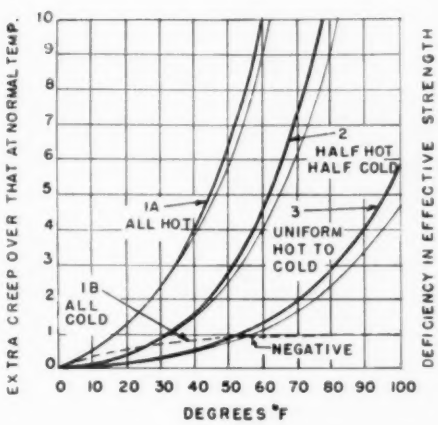


Fig. 8A

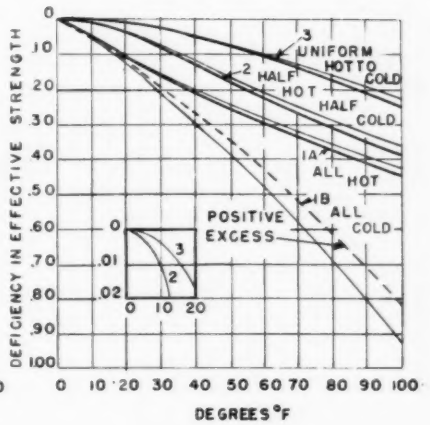


Fig. 8B

FIG. 8 CURVES FOR EXTRA CREEP AND DEFICIENCY IN EFFECTIVE STRENGTH DUE TO TEMPERATURE VARIATION 0.2 to 0.3 PER CENT MEDIUM-CARBON STEEL

(n = 6.67 k = 0.6 kn = 4.0)

[Heavy lines duplicate Figs. 4*A* and 4*B* of paper. Fine lines are based on linear relation between logarithm of creep and reciprocal of absolute temperature.]

[7]. It is not the intention of the author to discuss the significance of Q. It is recorded here simply because the data are available to determine it at the various temperatures of the tests.

TABLE 3

(0.15 per cent carbon steel, Figs. 3 and 5 of paper)

Temp, F	n	k	kn	Q
800	6.33	0.263	1.67	29,100
900	8.27	0.835	6.90	140,000
1000	3.04	1.175	3.58	84,000
1100	3.08	1.11	3.42	91,500
1200	3.06	1.05	3.22	97,500

TABLE 4

(Austenitic steel, Fig. 6 of paper)

Temp, F	n	k	kn	Q
1000	12.5	0.87	10.9	256,000
1100	14.8	0.715	10.6	284,000
1200	20.0	0.495	9.9	300,000
1350	16.0	0.541	8.7	314,000
1500	6.1	0.870	5.3	224,000

Fig. 7 has been drawn to illustrate how the original data correspond with the rules proposed by Mr. Kanter and those used by the author. The scales are logarithmic and the temperature scale has been doubled so that a 45-deg line has the required two-to-one slope for a line of constant Q; whereas horizontal lines represent constant kn.

The test points are connected by straight lines. The broken lines represent the values of Q suggested by Mr. Kanter while the shorter dot-and-dash lines represent the values chosen by the author for use over a limited range of temperatures. The carbon steel between 1000 and 1200 F appears to follow the author's rule more closely than Mr. Kanter's while the austenitic material, especially in the middle range of the data, appears to follow Mr. Kanter's equation.

The question arises as to how the results computed in the paper as published would be affected if the original creep-test data had been plotted against the reciprocal of the absolute temperature (as required by Mr. Kanter's theory), instead of the temperature itself, with a straight line on semilog paper being used to represent the results in each case. The difference is easily computed for a bar all too hot or cold, or for a bar hot half the time and cold half the time, but for the important case of uniform variation from cold to hot, the algebra does not seem easily integrated. However, graphic methods may be used. Figs. 8*A* and 8*B* duplicate Figs. 4*A* and 4*B* of the paper and show the altered results in fine lines. Looking at the left, the relations proposed by Mr. Kanter indicate just slightly less excess creep than the formulas published by the author. Similarly at the right, the proposed relations show slightly less deficiency of strength. In other words, the author's formulas are just slightly "on the safe side," as the engineer says, although for variations of ±30 F the difference is less than the width of the lines in the diagram.

Decomposition of Sodium-Sulphite Solutions at Elevated Temperatures[1]

R. M. HITCHENS[2] AND J. W. PURSSELL.[3] While the authors have been studying the behavior of sodium-sulphite solutions at high temperatures in laboratory bombs, the writers have been making a similar study on high-pressure boilers in which the temperatures are comparable but the concentration of sulphite is far less than that used in the bombs.

Tests have included examinations of boiler water and steam in boilers of 675 and 1400 lb pressure for traces of sulphide and sulphite. The purpose of the work was to investigate the possibility of hydrolysis of the sulphite to liberate sulphur dioxide, or autoxidation and reduction to sulphide and sulphite.

It was found possible to detect colorimetrically with lead plumbite as little as 0.02 ppm sulphide in a colorless boiler water, and 0.4 ppm in a colored water by distillation of the hydrogen sulphide into ammoniacal cadmium chloride.

By collecting condensed-steam samples in an inert atmosphere, reducing with aluminum and alkali, acidifying, and collecting the sulphide produced on a lead-acetate strip, it was found

[1] Published as paper RP-60-6, by W. O. Taff, H. E. Johnstone, and F. G. Straub, in the April, 1938, issue of the A.S.M.E. Transactions

[2] Monsanto Chemical Co.

[3] Boston Edison Co.

possible to detect 0.01 ppm sulphur and to estimate the amounts. Sulphide could be detected with the same sensitivity and technique by omitting the reduction step, or could be detected to 0.02 ppm colorimetrically.

These tests were applied to the boilers of the Monsanto Chemical Co. of St. Louis, Mo., which operate at 675 lb pressure, saturated-steam temperature of 497 F, and superheater outlet temperature of 730 F. The boiler water was dosed with 130 ppm of sodium sulphite. No sulphide whatever could be detected in the water. Likewise no sulphide or sulphite (less than 0.01 ppm S) could be found in steam condensed from the superheater inlet or outlet. The boiler-water sample was taken from the continuous blowdown line and cooled under pressure in a small steel-tube condenser. The steam samples from the superheater inlet and outlet were taken from connections welded into the tops of the two horizontal headers. From these connections, the steam passed through a short length of $^1/_2$-in. steel tubing to a stainless-steel-tube condenser in which it was condensed under pressure.

The absence of sulphite in the steam is to be expected since the boilers are equipped with steam washers which reduce the carry-over to a minimum. Failure to find sulphite likewise demonstrates the absence of hydrolysis of sodium sulphite.

The experiments at the Boston Edison Co. were carried out on boiler no. 7, Edgar Station, at 1400 lb pressure, saturated-steam temperature of 577 F, and superheated-steam temperature of 725 F.

With 100 ppm of sodium sulphite in the boiler water no sulphide whatever (less than 0.02 ppm S) could be found in the water by the sodium-plumbite method. This indicates the absence of autodecomposition to soluble sodium sulphide and sulphate. As mentioned before, this does not preclude the possibility of the reaction proceeding directly to a solid ferrous sulphide and sodium sulphate. The possible extent of this reaction will be discussed later.

Extensive tests were made of the sulphite and sulphide content of the saturated and superheated steam. The results of tests with the boiler on full load are tabulated in Table 1, which shows the sodium-sulphite content of the boiler water, the sulphite-plus-sulphide content of the steam, and the sulphide content of the steam.

TABLE 1 STEAM ANALYSES, BOSTON EDISON CO.

	Saturated steam, 577 F		Superheated steam, 725 F	
Sulphite in, boiler water, ppm Na_2SO_3	Sulphite + sulphide, ppm S	Sulphide only, ppm S	Sulphite + sulphide, ppm S	Sulphide only, ppm S
0.05[a]	0.01–0.02	Neg	0.01–0.02	Neg
9	0.03	0.01	0.03	0.01
20[a] (60% load)	0.02	Neg	0.04	Neg
35	0.03	0.03	0.03	0.01
64	0.07	0.04	0.06	0.02
175[a]	0.2	Neg	0.2	Neg

[a] These tests run at a later time. See text

Even with no sodium sulphite in the boiler water, i.e., after the boiler had been drained, refilled, and run three days without sulphite addition, there was a trace of sulphite in the steam. This amount rose to 0.03 ppm S with 9 ppm sodium sulphite in the water and to 0.2 ppm with 175 ppm sulphite in the water. The tests for sulphide are conflicting. In the first series of tests sulphide was found in traces, increasing in amount with the sulphite content of the water. In a second series of tests run several months later, no sulphide whatever was found, even with 175 ppm sulphite in the water. See[a] Table 1. The latter tests were run on a boiler which had been drained, refilled with fresh water, and operated for three days without sulphite treatment. It is possible that any sulphide formed reacted with oxides produced on the metal surfaces during operation without sulphite.

There is in these tests evidence that if water containing sulphite goes over into the steam it can undergo autodecomposition, both while passing to and while passing through the superheater.

That the sulphite found in the steam is the result of carry-over is indicated by the reduction in the sulphite content of the steam which occurs with decreasing the load, as shown in Table 2.

TABLE 2

		Saturated steam		Superheated steam	
Load, lb steam per hr	Sodium sulphite in boiler water, ppm Na_2SO_3	Sulphite + sulphide, ppm S	Sulphide only, ppm S	Sulphite + sulphide, ppm S	Sulphide only, ppm S
120000	20	0.02	Neg	0.04	Neg
50000	20	..	..	Neg	Neg
30000	20	..	..	Neg	Neg
20000	20	Neg	Neg	Neg	Neg
200000	130–175	0.23	Neg	0.20	Neg
50000	130–175	0.05	Neg	0.07	Neg

If hydrolysis were occurring, its extent would be dependent only upon the concentration of sulphite in the water, the pH value, and the temperature, and would be independent of the load. Thus, the partial pressure of sulphur dioxide in the steam should be constant and the amount of sulphite would not vary.

Now a comparison of the reactions in the boiler water will again be considered. At 579 F, essentially the 577 F in the Edgar Station boiler water, the authors observed that a sodium-sulphite solution underwent autodecomposition to sodium sulphide and sulphate, the reaction rate conformed to a differential equation of the first order, and progressed half way in 90 min. If the same thing happened in the boiler, it would be necessary to feed an enormous amount of sodium sulphite to the water to maintain any appreciable residual. Likewise, dose feeding would show a high initial concentration which would drop to one half in 90 min, to one quarter in 3 hr, and so on. This, however, was not the case. Addition of sulphite was often omitted for a week, and the water still contained sulphite at the end of the period.

During a certain 44-hr period the sulphite concentration fell from 130 to 30 ppm sodium sulphite. Of this, a loss of 72 ppm was accounted for by carry-over. The remaining 28 ppm not accounted for, 35 per cent of the average amount in the boiler during the period, amounted to only 0.20 lb per million lb of steam, sufficient to react with only 0.025 ppm oxygen. Assuming no oxygen whatever, 35 per cent in 44 hr is the maximum loss of sulphite by autoxidation and reduction. The authors found this much in 40 min. Further, there was no evidence of any such decomposition.

A similar condition existed in a 1790-lb boiler recently visited. The sodium-sulphite concentration of this boiler was maintained continuously at 30 ppm, which required the addition of 2.5 lb per day.

Extrapolation of the reaction rate found in the bombs to 613 F the temperature of this boiler water, would indicate a consumption of 50 lb per day, which is far in excess of the total amount fed.

Conclusions

1 Hydrolysis of sodium sulphite does not occur in 675-lb and 1400-lb boilers with as much as 170 ppm sodium sulphite in the water and with pH values of about 11.5. The steam is free from sulphur dioxide (less than 0.01 ppm S).

2 Similarly the waters in these boilers are free from soluble sulphides (less than 0.02 ppm S).

3 Steam from the 675-lb plant is free from sulphite and sulphide (less than 0.01 ppm S).

4 Steam from the 1400-lb plant contains traces of sulphite as carry-over. It likewise contains traces of sulphide in some

tests, but not in others, showing that decomposition of the sulphite carried over can occur under some conditions.

5 There cannot be accounted for more than 35 per cent autodecomposition of sulphite to sulphide and sulphate in the boiler water at 1400 lb pressure and 577 F in 44 hr; further, there is no evidence that any occurs. The authors found 35 per cent autodecomposition in 40 min, 50 per cent in 90 min, and 75 per cent in about 3 hr at this temperature. In view of the variance between these findings, it is evident that there is a wide difference in the behavior of concentrated and dilute solutions and that the reaction is really only pseudo first order.

6 Operating conditions in a 1790-lb plant indicate the same situation as described in the preceding paragraph.

7 In view of these results, there seems to be no objection to the use of sodium sulphite for oxygen removal in high-pressure boilers. Continuous feeding to maintain 10 to 20 ppm of sodium sulphite in the boiler water is indicated.

This is not offered as a final report, because the results were obtained on a limited number of boilers. The details of the methods of test are available and will be furnished gladly to those interested upon request. Suggestions and criticisms are solicited

Authors' Closure

It is important to note that there is no essential disagreement between the results reported by Hitchens and Purssell and those of the authors. The tests made by the latter were run at concentrations of about 4000 ppm of sodium sulphite. Under these conditions decomposition was found in every case at temperatures of 545 F and higher. These tests have been extended to lower concentrations in some experiments made by Professor Straub in bombs equipped with a steel reflux condenser. His results are shown in Table 3 herewith.

TABLE 3 DECOMPOSITION OF SODIUM-SULPHITE SOLUTIONS IN STEAMING BOMBS

Test	Temp, F	Time, hr	Sodium sulphite, ppm At start of test[a]	Sodium sulphite, ppm At end of test	Per cent original sulphur present in solution
1	420	48	500	220	92
2	420	28	500	175	...
3	580	44	500	30	78
4	580	45	500	30	68

[a] Before bomb was closed; oxidation by dissolved oxygen is expected.

They show that there is an abnormal decrease in the sulphite and in the total sulphur in the water at 580 F when the initial concentration of sulphite is from 200 to 500 ppm. While soluble sulphide was not found in the residual solution, evidence that it was present in the steam was indicated by the copper-sulphide scale identified from the condenser.

There were two reasons for choosing the high initial concentration for the first tests: (1) To explain the disappearance of sodium sulphite from a high-pressure boiler in which localized concentration was believed to exist; and (2) to permit following the reaction without using complicated semimicroanalytical methods. At the high concentrations the rate of decomposition is proportional to the concentration of residual sulphite, a relationship which is generally said to indicate a first-order reaction. In all of the tests made over a 36-hr period, however, the percentage of sulphite remaining was between 2.4 and 4.1 per cent of the original amount, corresponding to from 96 to 160 ppm. Since this concentration would have been reached in a much shorter time (from 6 to 23 hr at from 600 to 550 F), if the initial rate of decomposition had prevailed, it is evident that the reaction either reached equilibrium, or slowed down appreciably at low concentrations. The small concentrations of sulphide found in high-pressure boilers by Hitchens and Purssell using sensitive methods of analysis, show that the latter alternative is probably true. The results of the two investigations, therefore, are not at variance and may be considered as actually confirmatory.

The fact that the decomposition is "pseudo first order" rather than of true first order is only of academic interest. The practical engineer is primarily interested in knowing the answer to the question, "Does sodium sulphite decompose in actual operation?" On the evidence obtained so far it appears that at concentrations not exceeding 30 ppm in a boiler in which no abnormal concentrations are likely to occur, the salt does not decompose at a rate of practical importance. At concentrations between 30 and 200 ppm and above 545 F, some decomposition does occur and this becomes rapid in still more concentrated solutions.

Even when decomposition does occur little is known about the nature of the products and whether their presence would be considered detrimental to boiler operation. It would be expected that the hydrolysis of alkali sulphides to give hydrogen sulphide in the steam would take place as readily as that of carbonates to give carbon dioxide. There is indeed some evidence that this hydrolysis does take place. First, in the laboratory tests under steaming conditions the total sulphur content decreases when the vapors come in contact with copper or clean iron surfaces. Second, in the tests reported by Hitchens and Purssell on the high-pressure boilers, in Table 1 of their discussion, no sulphide was found in the boiler water but in several cases corresponding analyses by their sensitive methods showed sulphide in the saturated steam entering the superheater, before concentration of carry-over could have taken place. Thirdly, in the production of hydrogen sulphide by the reaction of sulphur with caustic, it is reported that the sulphide is completely removed from the solution when boiled at 90 lb steam pressure.

It is obvious that the chemical investigation of this subject is not complete. In view of the complexity of the reactions of sulphur compounds as shown by the extensive work of Foerster on decomposition of sulphurous acid and its salts, further investigation would seem to be justified as a precautionary measure.

Combustion of Pulverized Coal[1]

E. Kilburn Scott.[2] Naturally, there is much said in this paper about oxygen. The writer notes especially the remark on page 294: ". . . .the large particles must complete their combustion in the end of the furnace where the concentration of oxygen is low and where there is only limited turbulence," and again on page 296: "In a boiler furnace the concentration varies from 21 to about 2 per cent, and the average concentration in which the large particles burn probably does not exceed 6 per cent."

May the writer suggest the installation of an oxygen-making plant, as an adjunct to superpower plants, which could furnish an intermittent or continuous supply of oxygen at points where there is still some fuel and the oxygen has fallen below, say, 5 per cent.

As a means of stopping smoke nuisance, this extra oxygen supply could be used to good advantage, especially since almost any expenditure would be justified to stop excessive smoke in populated areas. Modern power stations are not supposed to create smoke, but there are times, as when bringing a new boiler onto load, when it is almost impossible to prevent smoke entirely; therefore, the additional apparatus the writer suggests seems justified. The oxygen supply could be made automatic through a connection with the invisible-ray apparatus used for smoke-density detection.

[1] Published as paper FSP-60-8, by Henry Kreisinger, in the May, 1938, issue of the A.S.M.E. Transactions, p.289.

[2] London, N. 1, England.

At places where there would be danger of burning the oxygen-jet nozzles, and where there is impinging due to change in direction of hot gases, powdered magnesite might be tried. That which the writer has in mind would be shrunk entirely in an electric furnace, then ground to a powder, mixed with a binder and brushed or sprayed on nozzles, etc. As long ago as 1902 the writer suggested such a use for some Indian magnesite which he had shrunk electrically in a furnace at Meraker, Norway. It was used for the protection of nozzles of water-gas burners at a plant in Hunslet, Leeds, where corrugated flues were being made for marine boilers; it was rather too expensive a proposition for those days, but that consideration would not apply today.

F. K. Ovitz.[3] The combustion reactions in a boiler furnace are practically completed within the flame boundary, and it appears that temperature in the flame rather than the temperature of the surrounding furnace gases would exert the major influence on the reactions.

The burning of fuel oil is more complex than the burning particle of coal. It is generally considered that the atomized oil entering the furnace is vaporized into gaseous form and raised to the ignition temperature before burning. The hydrocarbons from the vaporized oil combine with oxygen to produce primary oxidation products which react with a further supply of oxygen and finally are burned to carbon dioxide and water. Some of the hydrocarbons decompose thermally at the temperature of the flame to produce carbon, hydrogen, and lighter hydrocarbons which react also. Oxidation and thermal decomposition (cracking) occur simultaneously. The vaporization and cracking absorb heat which must be provided by the flame and furnace walls. Rapid vaporization and early ignition aid combustion as shown by experiments in a small furnace burning oil. These showed that higher CO_2 with the same degree of cleanness of combustion was obtained in a combustion chamber constructed of insulating firebrick than in one constructed of standard firebrick. The same burner and oil rate were used in each case. The difference is attributed to the higher level of temperature.

The writer's experience confirms the statement of the author that mixing is the controlling factor in combustion. Many examples have been encountered where the mixing characteristics of oil burners change significantly when the capacity at which they are operated is changed. Better control of mixing will solve many combustion difficulties.

W. C. Rudd.[4] Although this paper, with its many ramifications, is of utmost interest to those in charge of the design, construction, and operation of coal-burning power plants, there are many phases of pulverized-coal burning which merit further consideration. With this in mind, the writer submits the following questions and discussion:

How does pulverized-coal burning compare with stoker-fired plants? How does the investment in equipment of the two types of plants compare? What are the relative efficiencies of the two types? What are the costs of steam production in both types of plants over a long period of time? What grade and cost of coal can be used in a pulverized-coal plant as compared with a stoker-fired plant? Then too, there are other related operating problems, such as the explosion hazards in a pulverized-coal plant and the need for minimizing them. Probably one of the most important problems in a plant burning pulverized coal is the disposal of ash refuse, especially the elimination of the fly-ash nuisance.

[3] Standard Oil Company of Indiana, Whiting, Ind.

[4] Engineer of Water System Construction, Water Department, Detroit, Mich. Mem. A.S.M.E.

There is another field for the burning of pulverized coal, that is, its use in conjunction with burning sewage sludge for the development of power in sewage-treatment plants. Incinerators for burning sewage sludge can be constructed so as to have steam boilers, thereby generating steam from by-products; a plant of this type is now being built for the Sanitary District of Chicago. Sewage sludge, depending upon the process of treatment, has a heating value of from 5400 to 6500 Btu per lb on a dry basis, and normally contains from 60 to 80 per cent moisture. As long as the moisture is below 75 per cent, the sludge has a definite heating value and can be used for steam generation.

When the new Detroit sewage-treatment plant was designed, nonheat-recovery and heat-recovery sewage incinerators were considered, but the former was chosen because the engineers in charge believed that the heat-recovery incinerator for steam generation was still in the experimental stage.

The new Detroit sewage treatment plant, now under construction, is expected to have a load of raw sludge of 280 tons daily (dry). This includes screenings and grit and the net heating value is about 1.7 billion Btu per day; the heat for evaporating moisture in the sludge has been deducted in arriving at this value. This heat used in a steam-generating plant burning both pulverized coal and sewage sludge would produce about 90,410 kwhr of electric power daily (33,000,000 kwhr per yr), or 80 per cent of the total requirement. The writer believes that there are great possibilities for the development of the sewage-treatment plant for steam generation, wherein the sewage-sludge incinerator for heat recovery and the burning of pulverized coal can be combined.

John Blizard.[5] The author has given an interesting description of the combustion of pulverized fuel and has dealt at some length with the rate of combustion in furnaces the walls of which are cooled. He was one of the first to advocate cooling the furnace and his early predictions that this cooling would not increase the combustible loss have been proved correct by tests on many boilers equipped with waterwalls. If, today, the combustible loss is unduly high, very few endeavor to reduce it by reducing the cooling surface in the furnace, but instead one looks to the fineness of the coal and the manner of providing it with air as a means of reducing this loss.

The commonly accepted idea that raising the temperature of reacting substances increases the velocity of the reaction very rapidly arose from experience with gaseous reactions. This increase of velocity is explained by assuming that it is necessary for a molecule to possess a minimum of energy E before it can react. The kinetic theory of gases tells us that the proportion of all the molecules at the absolute temperature T deg F abs, which possess kinetic energy in excess of E, is given by $e^{-E/RT}$, where R is the gas constant. If we assume E to have the value, for example, of 50,000 Btu per lb mol, we find that the number of molecules possessing energy in excess of E is twelve times as great at 2500 F abs as it is at 2000 F abs, and it is 47 per cent greater at 2600 F abs than at 2500 F abs.

However, when burning pulverized fuel we have a heterogeneous reaction, and the author suggests that the temperature of the oxygen is of little importance provided the temperature of the fuel is above the ignition temperature. He further suggests that when a particle of fuel at a suitably high temperature is in the presence of air, the rate at which the particle burns is governed largely by the rate at which oxygen molecules will impinge upon a surface, regarding presumably the diffusion of the oxygen and products of combustion to and from the particle as a negligible part of the process. He uses these suggestions as a means of ex-

[5] Consulting Engineer, Foster Wheeler Corp., New York, N. Y. Mem. A.S.M.E.

plaining the reduction of the time of combustion of a particle of fuel in an atmosphere of air when the temperature of the furnace is decreased. This does not appear to be a valid explanation of the U. S. Bureau of Mines experiments as may be seen from the following considerations.

Let us assume that a cube of fuel, each side of which has a length of d inches, is subjected to the impingement of oxygen at the rate of M lb per sq ft sec and that each molecule impinging on the surface combines with the fuel. Then, if S is the pounds of oxygen required to burn one pound of fuel, the density of which is w lb per cu ft, the total amount of oxygen required to burn the cube of fuel is $Sw(d/12)^3$. The average surface on which the oxygen impinges during the course of the complete combustion of the cube is equal to one third of the original surface or $(6/3)(d/12)^2$, and the total amount of oxygen impinging on the particle during the time t sec required for the completion of combustion is given by

$$2Mt(d/12)^2 = Sw(d/12)^3$$

from which the time for complete combustion is given by

$$t = wdS/(24M)$$

and for charcoal with $w = 30$ and $S = 2.7$, the time of combustion is given by $3.4\ d/M$.

The rate of impingement of oxygen under a pressure of 0.21 atm is given by

$$M = 145(1/T)^{1/2}$$

where T is the temperature of the oxygen, F abs; which, for a temperature of 1651 F ($T = 2111$ F abs) gives a value of 3.1 lb per sq ft sec for the rate of impingement. Using this value of M for the combustion of charcoal would mean that a cube of charcoal of 1 cu in. volume would burn in a little over 1 sec with air at this temperature, and a particle which is a cube 0.01 in. on a side would burn in something over 0.01 sec. These times of combustion are only a small fraction of the actual times and the impingement of molecules on a surface calculated from the kinetic theory of gases appears to have little bearing on the rate of combustion unless used in combination with principles of diffusion and some theory of molecular activation.

If the writer interprets the Bureau of Mines experiments correctly, they show the time during which a particle glows when dropped into a furnace which is held at various temperatures. While these times should represent fairly well the time taken to burn the particles, it must be remembered that they contain no record of the time taken to heat the particle up to a temperature at which it glows, and during this period the particle has oxidized to some extent. It seems reasonable to suppose that the particle begins to glow at the same temperature regardless of the temperature of the furnace, and since more heat will penetrate into the particle, when it is heated slowly in the cooler furnace than it will when heated more rapidly in the hotter furnace, the particle in the cooler furnace will actually be hotter as a whole on reaching the temperature at which it glows than will the particle in the hotter furnace. It appears to the writer that this higher internal temperature at the moment when glowing commences, coupled with the greater intensity of the convection currents of air caused by the greater difference between the temperature of the products of combustion and surrounding air in the cooler furnace, is a more likely explanation of the phenomenon, though it can only partially explain the large differences in the times of combustion with the temperature.

Messrs. Tu, Davis, and Hottel[6] give a good account of the theory of the rate of combustion of carbon and apply it to their experiments on the rate of combustion under forced convection. They found that at the highest surface temperatures and lower velocities there was little difference in the rate of combustion, so that it appears probable that, when as in a powdered-fuel furnace the relative velocity of the air and size of the particle are both smaller than that obtaining in these experiments, there will be little difference in the rate of combustion with a change of temperature of the particle of coal once the particle has reached a high temperature.

While it seems probable that a particle of coal will burn somewhat more rapidly in a furnace which is not cooled than in one which is cooled, it must be remembered that this does not mean necessarily that the particle will be burned more nearly completely in the hotter furnace. In fact, since the density of the gases in the hotter furnace is less than the density of the gases in the cooler furnace it will take the particle longer to pass through the cooler furnace, give it more time to burn, and so may render the combustible loss less in the cooler furnace than it is in the hotter furnace.

The theory of combustion of coal in suspension involves so many complicated aerodynamic, thermodynamic, and chemical processes that it is unlikely that any satisfactory equation will be established which will give a true picture of the rate of combustion. Certainly one would make a rash and ill-considered statement, were one to state that theory shows that combustion will be less complete in a furnace that is cooled than in one that is not cooled.

RALPH BRISCOE.[7] The author has apparently taken into consideration the basic principles of combustion of pulverized coal in connection with the most modern design of boiler and furnace. He stated that water-cooled furnaces absorbed a large quantity of heat, thereby lowering the temperature of the furnace gas and of the fly ash carried by the gas. As a result these entrained ash particles do not stick to the boiler tubes and the superheater elements.

The writer believes that the application of pulverized coal antedated the development of the water-cooled furnace as we now know it by a good many years. There were, though, very few installations made burning pulverized fuel that did not have some water cooling of the furnace even if it were only a hearth screen. The operation of these furnaces was very interesting from the standpoint of attempting to maintain low and constant furnace temperature over a wide range of rating and maintain these conditions over long periods of operation of the boiler.

In this connection furnaces were designed of relatively large size to keep down the heat liberation per unit of volume. Also, knowledge of the time element necessary for complete combustion of pulverized coal in suspension was perhaps very limited. The writer wonders if the latter were not more the reason than the former. Pulverized-coal furnaces were designed for a heat liberation not much in excess of the stoker, and the stoker installations had been tolerated without water-cooled furnaces because we did not know any different. With the stoker, apart from the high furnace-maintenance cost, most of the trouble due to clinker and slag was in the stoker itself. With pulverized coal burning in suspension, the ash would be in suspension and might stick or adhere to the walls with results only too well known. And stick it did in the early installations, not only on the walls, but also on the hearth where in some cases it was removed with chisel bars and sledges.

The use of pulverized coal was here to stay, because the expected increase in efficiency, greater flexibility, and adaptability

[6] "Combustion Rate of Carbon," by C. M. Tu, H. Davis, and H. C. Hottel, *Industrial and Engineering Chemistry*, vol. 26, July, 1934, pp. 749–757, and August, 1934, pp. 889–892.

[7] Plant Engineer, Trenton Channel Power House, The Detroit Edison Company, Detroit, Mich. Mem. A.S.M.E.

to larger installations looked very desirable to the power-plant designer in comparison with the then stoker. However, something had to be done with the furnace. The early installations demonstrated that the solid refractory wall was far from being suitable, and, in fact, would be the retarding factor in the application of pulverized coal. But at this time, the air-cooled wall was developed. This generally consisted of a 9-in. refractory wall held back to another outer wall by rows of headers with an air space between the two. A part of the air for combustion was admitted with the coal and the balance circulated between the two walls.

The thought that prompted this was that the inner wall would be kept sufficiently cool with the circulating air so as to help keep the furnace temperature down somewhat and keep the refractory from melting and the ash from sticking to it, but, more particularly, the writer believes it was expected to reduce the thermal stresses that would be set up in a thick heavy wall.

Space will not permit of a review of all the interesting experiences obtained in the operation of large furnaces 25,000 cu ft in volume and over 60 ft high, built with the so-called air-cooled wall. As in most cases we condemn the materials of which the whole is made, this case was no exception to the rule. But experience gained in the operation of these furnaces over the last ten years has demonstrated that there are a large number of details which go to make up the design, and which had just as much effect on the life of the wall as did the kind of brick used.

On these refractory walls, there was a tendency to accumulate, even if it were fragile, a layer of slag over parts of the refractory surface which shaded some of the heat-absorbing surfaces, and, which, in turn, resulted in softening this slag, perhaps forming a ledge and then a concentration of flame on this part of the furnace with detrimental results. There would be some coating of the front bank of tubes and what little extra cooling medium was employed, with slag or ash in different forms, necessitating periodic cleaning and scraping to bring the furnace temperature back somewhere near that desirable and tolerable.

The writer has always thought of the change in furnace temperature with this change in furnace conditions as "furnace sensitivity," all brought about by the ash sticking to the side walls, the front bank of tubes, and the hearth screen. The writer wonders how much or what would be the furnace sensitivity of a completely water-cooled furnace burning pulverized coal of low ash-fusing temperature, operating at a comparatively high rating. Can the furnace temperature be maintained sufficiently low so that the ash when projected at high velocity against the tubes will not be so molten as to stick to the tubes, not necessarily in the form of heavy slag but as a thin layer of hard material sufficiently thick to reduce heat absorption and driven with such force against the tubes as to need exerted mechanical effort to remove?

The author has pointed out that almost any kind of fuel that can be pulverized can be burned. He also pointed out that temperatures above that necessary for combustion are not necessary, and, in the writer's opinion, are very undesirable, for, if we are to burn coals with low-fusing-temperature ash, the furnace temperature should be kept as low as possible. This is desirable because at high ratings we are apt to have high velocities of the gases entering the first bank of tubes and the superheater. If these gases carry the very fine ash in a molten condition at high velocity, it will be projected against the tubes, and, although the tubes might be staggered to prevent bridging of the ash, there will be an accumulation on the outside of the tube that is extremely hard to remove which results in a change in furnace temperature and an aggravated condition, completely undesirable.

Can a water-cooled furnace be designed with a very low sensitivity rating? Can the designer, within reason, predict in advance what the furnace sensitivity will be when burning a certain grade of coal? With the boiler proper reaching a greater perfection in design from a mechanical standpoint and with a furnace of low sensitivity and refractory maintenance costs practically eliminated, is the cleaning of the furnace and cleaning off of the tubes and the water-cooling surface to be the cause of the major outage factor as is now the case in some refractory installations?

It is not expected that combustion efficiency will increase much with the most modern design over the best designs of the last ten years. But the great output per unit now possible, and with an increase in the reliability factor, it would tend to reduce the necessity for so much reserve.

There is another very important factor not to be lost sight of in connection with furnace sensitivity. That is the superheater design. It has been recognized for some time that a nearly straight line of temperature for the superheated steam is desirable. It seems that outside methods—that is, methods used outside the boiler setting—to obtain straight line superheat have given way to the control of the distribution of the gases over the water and steam heat-absorbing surfaces. With low furnace sensitivity, the designer's problem is much easier. Although the design of superheaters and the control and distribution of the gases through the boiler passes and over the superheater might be outside the scope of this paper, it is unfortunate that the author was not asked to review this subject in connection with this paper. His knowledge of the subject in connection with large installations would have made it very interesting.

Naturally, it was desirable, and the writer presumes it is with the latest designs today, to admit the air so that the whole of the combustion space would be used, rather than have large heat liberation in local spots with resultant high temperature. It was surprising the effects on flame length, the furnace temperature, and the heat absorbed by what little furnace cooling was employed by the change in the air distribution into the furnace.

The author states that the impingement of flame against the side of the furnace does not damage the water-cooled wall. It is the writer's belief that the author had in mind the wall would not be eroded away or melted down or slagged over, as would be the case with refractory furnaces. Impingement of flame against water tubes is undesirable unless circulation is exceptionally good.

The author has also stated that the stability of fire is largely a matter of burner design. In the early designs of water-cooled furnaces, the question of fire stability and excess carbon in the ash, at low ratings was a topic. Has not the use of preheated air played a great part in the stability of fire, reduction of carbon in the ash, and high rates of combustion? Is there an upper limit in air temperature from the combustion standpoint above which it is not desirable to go? A discussion of this question would be of interest.

With the older style of furnace, when only a small portion of the air for combustion was admitted with the coal, it was an interesting experiment, mostly cut and try, to determine where and in what portions should the balance of the air be admitted at each place. Contrary to some opinions combustion should not be completed close to the burner, but should envelop the whole of the furnace and be complete before entering the boiler tubes.

The changes which have taken place in the last 15 years in the application of pulverized coal, the development of the equipment to produce it in proper form, and the development of the water-cooled furnace have been so great that only those who have had experience with an installation of recent design are in a position to discuss it with authority. From the bin-and-feeder system of 15 years ago to the present unit system with its improved burner, improved mill and classifier, and the all-water-cooled furnace is a very great step. Again it is unfortunate that the author was not asked to discuss, if only in a limited way, the part the mill

with its complement of auxiliaries plays in connection with success of any pulverized-coal installation. Coal in all its different forms is a very complex mixture and to reduce a larger share of its different forms in one piece of apparatus to usable material in pulverized form is an achievement

E. A. SITTER.[8] The author has stated many advantages of using water-cooled furnaces and has covered them very thoroughly with the exception of the one pertaining to feedwater treatment. Before heat-absorbing surfaces surrounded our furnaces the question of feedwater treatment was not of such vital importance as it is today. The installation of the water-cooled furnace has made it necessary to treat the feedwater to the extent of keeping the heating surfaces entirely free from scale and dirt and to maintain them so continuously. Not only has this practice effected a saving in maintenance costs but it has also contributed largely to higher boiler and furnace efficiencies.

Pulverized fuel fired in water-cooled furnaces makes a very flexible arrangement. Changes in steaming rate of the boiler can easily be followed; however, it also sets up a problem of water-level control in the boiler. A sudden change in steaming rate will tend to change the level of the water because of the difference in density of water in circulation through the wall tubes. Has the author had any trouble maintaining water level in pulverized-fuel-fired boilers where there were wide fluctuations in steaming rate?

Pulverized fuel is not difficult to ignite in the case of starting a fire because the furnace is water-cooled. The difficulty lies in the fact that the air which carries the fuel from the pulverizer to the furnace and that supplied for combustion is at a temperature below that at which pulverized fuel will readily ignite. The temperature of the air and the coal must be raised to and maintained above the igniting temperature by lighting torches of ample capacity until such time as preheated air at reasonably high temperatures is available. A question arises regarding the fuel that should be used for the torches: What are the advantages and disadvantages of fuel oil versus natural gas?

Pulverized-fuel-fired furnaces lend themselves very easily to maintenance of fuel-air ratios, and thus maintain a low excess-air value in combustion at all times; however, the design must be such that an interruption in coal feed from one pulverizer will not seriously reduce the steaming rate of the boiler, and the combustion control regulators must be easily and quickly adjusted to establish a new correct fuel-air ratio, otherwise the furnace efficiency will be considerably lower and there is danger of losing ignition. What precautions should be taken when taking one pulverizer out of service where two are supplying fuel to one boiler? Where in the conventionally designed tangential burner will the greatest heat release occur?

The writer would like to ask a few questions regarding the burning of various types of coal: Is it possible to obtain bituminous coal so low in volatile matter that ignition cannot be maintained? What changes in distribution of air entering the furnace should be made for coals of high volatile content as compared with coals of low volatile matter? Where a plant has a load which varies from high rating to low rating on boilers, is it advisable to select a fuel with high volatile content rather than low volatile content to support ignition at low rating loads? In our plant we have always admitted the major volume of air around the coal nozzles, believing this method will stimulate turbulence. Is this method preferred to the one that admits secondary air through the regular air passages and retards the flow of air around the coal nozzles? Is it not a fact that considerable savings can be made with a pulverized-fuel plant versus a stoker-fired plant where operation is on a five-day-week basis because of the fact that no banking is required?

We have had some interesting experiences in burning wet coal. Recently we pulverized $^3/_8$-in. Pittsburgh no. 8 screenings containing 10.6 per cent moisture. No difficulty was experienced anywhere along the coal path from the bunker to the Richardson scales, to the feeders located 15 ft above the pulverizers, in the pulverizers, or in the coal pipes to the burners with one exception, that was in the coal pipes between the feeders and the pulverizers. The design of these pipes is such that the coal fell about 12 ft and then turned an angle of 25 deg to the pulverizer. It was at this turn that stoppage occurred. The design of the pipe was altered to eliminate this angle as much as possible by dividing it between two angles of approximately 11 deg and 14 deg, respectively. While this alteration improved conditions materially it did not produce a satisfactory solution, some stoppage still occurred. It was then decided to admit sufficient preheated air to the feeder pipe to maintain the air temperature within the pipe at 140 F; this was done with satisfactory results, no stoppage has occurred since these changes were made. It appears as though preheated air could be used in the mill feeder pipes advantageously as standard practice. What has been the experiences of other operators of pulverized-fuel plants in this respect?

C. SHOWERS.[9] While it is true that the most important source of heat for ignition lies in the propagation of flame along the fuel stream, or along eddy currents of that stream, and that a well-designed burner will maintain ignition in the open air, it is also true that for boilers generating up to 1500 boiler hp, where one direct-firing mill and one burner is used and continuity of operation is important, it is safer to have some uncooled refractory in the furnace wall in which the burner is located so that in case of a slight interruption in the fuel supply to the burner due to mill feeder stoppage, caused by wet coal or foreign material, the heat from the refractory will ignite the coal stream immediately when it again issues from the burner.

It might be well to point out that regardless of the fact that combustion of fuel progresses more rapidly at lower temperatures, it is also true that the rate of heat absorption by waterwalls drops much faster than the furnace temperature. For instance, 1 sq ft of projected tube area will develop 2.4 boiler hp at a furnace temperature of 2350 F, whereas the same area will only develop 1.2 boiler hp or one half as much at 1900 F, so that on a basis of first cost it is possibly more economical to design for the highest furnace temperature possible with respect to the fusion temperature of the ash and try to design the burner so that the coal particles pass through the furnace by such a tortuous path that they have sufficient time to burn before entering the tube bank.

J. R. JAMES.[10] Although pulverized-coal firing has its advantages, it also has the disadvantage of collection and disposal of fly ash in those localities where public policy demands it.

In speaking only of dry-bottom furnaces there are two methods which are successful in the cleaning of the flue gases. One is the scrubbing of the gas by water spray and the other the use of electrostatic devices. However, as more uses for fly ash develop it is reasonable to believe that the collection of the ash in a dry state will predominate, for in that condition it is in a more salable state. Large sums of money have been spent in research by various agencies endeavoring to discover uses for fly ash. The

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[10] Senior Engineer, Engineering Division, The Detroit Edison Company, Detroit, Mich. Mem. A.S.M.E.

present situation, however, is that there is no well-known generally accepted use for this by-product.

Fly ash as collected by electroprecipitators has been used as follows: (1) Due to its puzzolanic properties it has been used as an admixture to Portland cement; (2) it has displaced clay as a raw product in the manufacture of Portland cement; (3) it has been used as a filler in the asphalt and rubber industry; and (4) it has been used in the manufacture of lightweight concrete, building block, and insulation.

The Detroit Edison Company has developed a lightweight concrete (100 lb per cu ft) composed of fly ash, prepared cinders, and cement. A mixture of five parts cinders, two parts fly ash, and one part of cement by volume gave a compressive strength of 3000 lb per sq in. at the end of 28 days. This same proportion by weight, using damp cinders and fly ash, is 1500 lb of cinders, 650 lb of fly ash, and 500 lb of cement.

This company has also developed a building block similar in shape to ordinary cement or cinder block. This block, which is 18 in. long instead of the usual 16 in., is of a slate color due in part to carbon in the ash and for an 8-in. block weighs about 28 lb in contrast to 40 lb for a concrete or cinder product. Due to its being formed in the steel mold box of a tamping machine it is always of an exact size with sharp edges. Its most valuable characteristic is its smooth surface, which lends itself to painting or enameling, and with proper construction can be mistaken for a plastered surface. It has ample strength in compression but has to be handled carefully during construction as its sharp edges are readily chipped. A bare 8-in. wall as tested at the University of Minnesota gave a heat-conductivity test of 0.318 Btu per sq ft per hr per deg of temperature difference. After its hollow cells were filled with rock wool it gave a coefficient of 0.16. The rock wool was removed and the spaces refilled with ordinary dry fly ash. The coefficient was 0.18.

The process for making this building block is similar to that of a sand-lime products plant. A mixture of 100 per cent ash, 10 per cent hydrated lime, 0.6 per cent rosin with about 20 per cent water is mixed in a wet pan for about 8 min. A tamping machine receives this material and forms about 360 blocks (8 in.) per hour. Loaded on cars the blocks are placed in a steam kiln at about 120-lb pressure where they remain for about 7 hr and are then taken to stock piles.

To date, about ten homes, ranging in value from $3000 up to $10,000, two churches, and several breweries have been constructed. At the present time, The Detroit Edison Company is constructing a six-story service building, all the floors of which are formed of fly-ash cinder concrete, about 3500 cu yd. In this building more than 100,000 blocks will be used.

The process used in the manufacture of the building block is controlled by the Rostone Company of Lafayette, Ind. The three uses of fly ash as developed commercially by this company, namely, fly-ash cinder concrete, building block, and insulation in connection with the block, appear to have an encouraging future, especially so in that private capital has recently shown its willingness to take over the manufacture and sale of these products.

T. W. Jeffords.[11] The author has called attention to certain rumors regarding the supposed difficulties experienced in maintaining combustion in water-cooled furnaces. An interesting experiment was run at our Mistersky Power Station, which throws a further sidelight on this question. In 1932 the City of Detroit faced a situation of having no place to dispose of its garbage due to certain financial circumstances encountered by the local reduction company. The Common Council instructed the Public Lighting Commission to experiment with the disposal of the city's garbage by dehydrating and burning it in the powdered-coal furnaces at the Mistersky Power Station. These furnaces are of the usual vertical firing design, being approximately 19 ft square at the base and 30 ft high. In carrying out these experiments an inclined hearth was constructed on top of the water screen at the bottom and rear of the furnace. This hearth was composed of cast-iron blocks, attached to the water screen, extending the full width of the furnace end to within about 5 ft of the front wall.

The garbage was fed in on top of this hearth through openings at the rear of the furnace. The powdered coal was fed into the furnace vertically through the arch at the top of the front wall. The powdered-coal flames were allowed to follow the usual U-shaped path encountered in vertically fired storage systems, except that the flame was shortened so that it turned just above the garbage on the hearth. The radiant heat from the powdered-coal flame drove off the moisture from the garbage as it was pushed down the hearth. Actual burning of the garbage took place, after it had become sufficiently dried, at the lower end of the hearth.

This bed of garbage at the bottom of the furnace formed a black heat-absorbing surface producing an effect similar to that of a waterwall. In addition, however, volumes of water vapor were driven off from this bed of garbage, thus producing a still further cooling effect upon the powdered-coal flames. It was found possible to burn approximately 9 tons of dehydrated garbage per hour. The moisture content of this garbage averaged between 50 and 60 per cent, which means that it was necessary for the powdered-coal flames to evaporate between 4 and 5 tons of water per hour, which passed off with the gases. At no time was any difficulty experienced in keeping the powdered-coal flames burning or in carrying low ratings on the boiler. The only change which was found necessary in the operation of the furnace was to carry a higher percentage of excess air. Normally the CO_2 at the boiler outlet on coal alone averages approximately 15 per cent. It was necessary to reduce this to approximately 13 per cent while burning garbage.

We are pleased to submit these comments on our garbage-burning experiments as further evidence of proof of Dr. Kreisinger's contention that water-cooled surfaces have no detrimental effects upon the stability of powdered-coal firing. We believe that these garbage-burning experiments have again proved that the powdered method of firing coal is quite flexible in that it can not only be burned in conjunction with blast-furnace gas and oil, but also with waste products such as rubbish and garbage.

W. D. Drysdale.[12] The author brings out the fact that a water-cooled furnace absorbs a large quantity of heat, thereby lowering the temperature of the furnace gas and of the fly ash carried by the gas. What provision has been made when burning coal of low-fusion-temperature ash, to maintain the temperature of the superheated steam at a steam load output between 50 to 75 per cent of boiler rating? As higher steam pressures and superheater temperatures are coming into use, the cooling of the gases to a point of preventing the fly ash sticking on boiler and superheater tubes becomes a major problem in maintaining the desired superheater outlet temperature.

Also, as a boiler operator and thinking of "safety-first" operations when lighting a burner using pulverized coal, oil, or gas, the safe thing to do is to open the boiler outlet damper so as to purge the furnace before applying the torch and feeding pulverized coal, oil, or gas to the torch. This reduces the hazard of a puff blowing back on the operator when applying the torch to the burner.

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[12] Assistant Engineer of Power Plants, The Detroit Edison Company, Detroit, Mich. Mem. A.S.M.E.

H. W. STRONG.[13] The author states: "Almost any kind of coal can be reduced to powdered form and burned like gas." It is generally acknowledged that the pulverized, fuel furnace has proved to be a very flexible, dependable, and economical means of producing steam. It has lived up to expectations. We are convinced from experience that this furnace will burn, more or less successfully, many grades of rather poor-quality coals which are abundant.

A rather new problem in coal utilization is now receiving the attention of the coal industry, that is, the advent of the small domestic stoker. This appliance is rapidly becoming recognized as a very useful labor-saving device and will, if given proper combustion-engineering guidance, become a user of a substantial tonnage of low-quality coals.

There are now on the market several small stokers, designed, engineered, and built by reliable companies, which, when provided with suitable grades of coal, do an excellent job of providing safe, dependable, automatic heat with a minimum of attention at a cost which is substantially below that of natural gas or fuel oil.

The present small-stoker design favors low-quality, small-size ($^3/_8$- to $1^1/_4$-in.), low-fusion, medium-ash, and highly volatile coals, without excessive fines. The fines are very objectionable in home basements. The pulverized-fuel furnace, on the other hand, also favors such low-quality coals but the fines are not objectionable. Therefore, a proper correlation between these two outlets would be of mutual benefit to both consumers and coal producers.

A. C. PASINI.[14] The writer agrees with the author regarding the use of pulverized fuel and the use of water-cooled walls. However, the advent of pulverized fuel and the use of water-cooled walls also stimulated the development of stoker-fired furnaces. The use of waterwalls in the Conners Creek Plant stoker-fired furnaces has produced a tremendous change in the design. In the old stoker-fired furnaces the refractory walls had always been a source of trouble because of their inherent weakness. The use of refractories also required carrying higher total air values in the furnace. With continued high ratings, the replacement costs for refractory walls were much too high. Water-cooled walls have completely changed the picture in so far as stoker-fired furnaces are concerned.

In the fall of 1934 the first new high-pressure boiler went into service at the Conners Creek Plant. This boiler is stoker-fired with automatic metered-air control and 100 per cent waterwalls. It has a furnace volume of 11,000 cu ft as compared to a furnace volume of 25,000 cu ft in the pulverized-fuel furnaces at Trenton Channel. The maximum heat release in the Conners Creek furnace of 50,000 Btu per cu ft per hr is comparable to some pulverized-fuel installations and greater than most pulverized-fuel-fired furnaces. This heat release has been maintained for as long as 12 hr without any trouble. At this rate the dust loading in the stack was approximately 1.3 lb of cinders per 1000 lb dry gas. The carbon content at this loading was approximately 55 per cent. The sieve analysis at normal rating showed 30 per cent passing through a 325 mesh, 13 per cent through a 200 mesh, 28 per cent through a 100 mesh, 28 per cent through a 50 mesh, and only 1.2 per cent retained on a 16 mesh.

Waterwalls, while an aid in cooling the gases and fly ash in large furnaces, do not tend to prevent fly-ash adherence on the boiler tubes where the furnace volume is small and heat release high. In the Conners Creek furnaces the tubes must be sprayed with water approximately every two days, despite the fact the flame travel is very small on the stokers.

No trouble has been experienced in lighting fires or in steaming at small ratings with stokers and waterwalls.

With metered-air-control equipment on the stokers, no secondary combustion has been noticed. We are able to maintain 16.5 per cent CO_2 with no CO at the superheater outlet. The semiradiant superheaters are located in the first pass and the convection superheater in the second pass.

As to stoker maintenance from November, 1934, to January, 1937, the cost of all iron replacement was 50 per cent less in the new stokers using metered-air-control equipment as compared to the old stoker-fired furnaces. This low cost was maintained even though the new stokers operated with 360 to 400 F preheated-air temperatures.

The author did not mention ash disposal. We are able to dispose of all our ash at a profit—no small item when you consider that the Conners Creek plant consumes approximately 1500 tons of coal per day.

T. C. TAYLER.[15] The best proof of a theory is to carry it to extremes. A boiler furnace with its demands for a short flame constitutes one extreme, while a rotary lime-kiln furnace demanding a long flame is the opposite extreme. Ideal conditions in the company with which the writer is associated call for a flame 50 ft long in a horizontal furnace 4 ft 6 in. inside diameter by 100 ft long. The cooling surfaces consist of the insulated fire-brick walls and partly and completely calcined lime varying in temperature from about 2000 F at 50 ft from the burner up to 2300 F about 25 ft from the burner and continuing at this temperature as it moves to the burner. Our conditions demand that we maintain flame temperature at 2500 F from the burner tip to a distance of 50 ft. To hold the flame temperature down to 2500 F at all points, the walls and lime must absorb heat as generated. There will be little absorption in the first 10 ft from burner and almost constant absorption in each foot from 10 to 50 ft from the burner tip. We require quick ignition at the burner tip, slow rate of combustion in first 10 ft, then a faster but almost constant rate for 40 feet.

Our problem of combustion is complicated by the facts that first we burn petroleum coke which is about 85 per cent fixed carbon and 14 per cent volatile, and second because our entire volume of combustion air is preheated to 1000 F. Moisture in fuel and cold air admitted to fuel pulverizer reduce primary air only enough to eliminate preignition.

The author names three factors as necessary for quick ignition: (1) Small amount of primary air, (2) fine pulverization, and (3) turbulence. It is not difficult to obtain all three. We need quick ignition, but having obtained it find ourselves facing the opposite problem of preventing almost complete combustion close to the burner. If the three factors give quick ignition and short flame, conversely, the opposites should give long flame. Hence, we might expect to find it necessary in our case to supply simultaneously both small and large amounts of primary air, both fine and coarse pulverizing, and have both great and little turbulence. These combined conditions may not be quite as impossible as might at first appear. We can have a burner that supplies a rich mixture at low velocity around the rim while it supplies a lean mixture at high velocity in the center; we can have both fine and coarse fuel simultaneously; and we can have great turbulence at one point and less at another.

We burn with an exceptionally small amount of excess air and, with high preheat, have no difficulty in getting quick ignition; our difficulty is to get sufficient heat into the space 35 to 50 ft

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[15] Vice-President, Belle Isle Lime Company, Detroit, Mich.

from the burner without having too much in the space 20 to 30 ft from the burner.

The author seems to eliminate fuel sizing as a factor in control of our flame temperatures. We have evidence that the larger particles failed to burn.

On the strength of the facts presented by the author, it seems that we will have to go further with burner design to get rich mixture at low velocity around the rim and lean mixture at high velocity in the center, and also go further in our attempts to increase turbulence without effect upon the streams from the burner. Design may take the form of means to create a slight whirl to the secondary air, the burner being in the vortex but having no whirl of its own unless perhaps in the opposite direction.

It is interesting to note that instead of water-cooled walls, we use lime-cooled walls, the rotating furnace carrying the heated walls under the bed of lime to give up some heat during a part of each revolution. We seek controlled rates of combustion rather than high rates. We seek retarded mixing, which is difficult in a rotary kiln where all fuel and air must enter through the end wall within 2 ft of the burner. We seek to avoid cooling lime close to burner by putting a blanket of 1000 F air between the flame and lime. It would be ideal if we could rotate that blanket of air between the part of the flame having excessive temperature and lime subject to overburning without actually creating at that point such turbulent mixing as to actually make matters worse. The author states that excessive furnace temperatures are not desirable He tries with water-cooled walls to extract heat as it is produced; we also try to extract heat as produced but for a different reason, namely, temperatures above 2500 F cause overburning of lime, tending to flux acid brick with basic lime.

It is interesting that our experience under conditions which represent the opposite extreme from boiler-furnace combustion are in harmony with the author's experiences, and as a result of his paper we are forced to rule out fuel sizing as a factor in our plans to control flame. We hope through concentrating our efforts on the burner velocities and turbulence in combustion space to approximate eventually ideal combustion conditions.

A. W. Thorson.[16] The data on furnace temperature versus time for combustion are very interesting, and seem to agree with subsequent work at Carnegie Institute of Technology on coal carbonization. It has been found that when coal is heated, the amount and composition of distilled products depend upon the rate of heating. As this rate is increased, due to higher furnace temperature, more products of heavier molecular weight are distilled off which require more oxygen for combustion. In addition, a less reactive residue coke remains as the carbonizing temperature is increased. Both tend to increase the time required for combustion.

The statement that temperature in excess of that for ignition retards combustion is further borne out in a more practical way. Recent furnace designs for traveling-grate-stoker firing show a replacement of the conventional long ignition arch by a very short one, supplemented by a long rear arch. Such a change prevents excessive temperature in the ignition zone and assists in burning out fixed carbon at the rear of the stoker. Results with such installations show reduced solid-carbon loss and increased burning rates.

There are three questions the writer would like to ask:

(1) Replacement of mill fan blades is one of the larger items of maintenance in a pulverizer plant. There has been a recent tendency to locate the mill fan on the clean-air side of the pulverizer. How much does this reduce fan maintenance, as compared to that of a fan handling both air and coal?

[16] Junior Engineer, Production Department, The Detroit Edison Company, Detroit, Mich. Jun. A.S.M.E.

(2) There has been some discussion on the relative ability of underfeed stokers and pulverized fuel to pick up load. Some engineers find it faster with the bin-and-feeder system than with the stokers, whereas some with direct pulverizer firing find that slower than with stokers operating on the same coal, presumably due to lack of storage in mills, and to boiler lag. The latter is reported to be a factor in the case of slag-tap furnaces where slag gives up heat when dropping load and absorbs it when increasing load, thereby contributing to boiler lag. The writer would like to have the author discuss this point.

(3) A recent compilation showed that of all new capacity under construction, or recently placed in service, over 80 per cent was pulverized coal. Is this the result of a tendency toward a wider latitude in fuel selection? Will the author please explain this trend?

F. J. Linsenmeyer.[17] In this short discussion of the paper the writer's views may be considered in the nature of support for pulverized-fuel applications. The writer believes that the use of pulverized fuel has brought a great many benefits and improvements to industrial and central-station plants, and that it opened up opportunities to the designer for some real ingenuity in the selection of water and steam circulation. The older form of Stirling boiler with vertical or horizontal tubes, gives way to circuits which permit steaming in any or all banks of tubes. The direct result is that the stoker furnaces have also adopted new circuits. The writer has always wondered why boiler designers resisted any attempt at steaming in the waterwalls. Certainly the pulverizer designers led the way here. These same water and steam circuits may have been the incentive for high-pressure plants such as the Loeffler, Schmidt, and Benson Boilers. While all Americans may not agree with such unconventional designs, they are adopting some of the fringes of these designs.

It may be true that pulverized fuel uses larger furnace volumes per unit output, but that is the very nature of the burning, stokers being a concentrated-heat application. As stated, pulverized fuel is more like liquid or gaseous combustion. A heat release of 33,000 Btu per cu ft of furnace is not as high, perhaps, as may be obtained with stokers, but the ease in control with slow-speed burning is evident. It seems then that the very nature of the pulverized fuel approaching the suspended-particle condition is in its favor as having greater potentiality for a complete combustion with minimum excess air.

Pulverized fuel is so finely divided that it may be compared with a Diesel spray wherein the small charge is broken up into particles of even smaller size than the coal and can be projected to any desired position in the combustion chamber to suit maximum control of the flame propagation. This analogy is a rather typical one for an instructor to use and has its limitations, but our knowledge of fluid flow is increasing so rapidly that the application seems an excellent subject for further speculation.

Fly ash is an ever-present nuisance and the closer the preparation of the fuel is to a real suspended solid, the more serious this fly ash becomes. The writer has been thinking of the possibility of still better preparation of the fuel until the fly ash will be so fine as to be suspended further in the atmosphere and carried away by the wind.

The draft characteristics of the pulverized system, where all the pressure drop is in fluid friction and not through a fuel-bed restriction, is commendable. This same effect is attained by vertical jet furnaces.

The burning of pulverized coal in a boiler furnace is a slower but more continuous process of combustion than the burning of pulverized coal in an internal-combustion engine. By proper

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mixing at ignition temperatures, one can get an explosion of pulverized solids. The problem may then very well be one of holding down the flame propagation to reasonable limits until combustion is complete. The writer believes that the author has made an excellent point in discussing the control of ignition temperatures; in liquid fuels at high flame rates this is called thermo-equilibrium and while still in a formative state, seems to follow the same idea proposed by the author.

The writer believes that the particle size of pulverized coal may be a little small; 1/1,000,000 in. is 1/25,000 micron. This comes well below the scale of "smokes" as indicated in the A.S.H.andV.E. scale of air-borne solids. If we say none would go through a 1000-mesh screen (if such a screen were available), we may use this figure for comparison. Assume the particle is 10 microns. With a surface area of 0.0003 sq mm, a volume of particle of 0.0000005 cu mm, a volume per pound of 30,000 cu mm, a total number of 600,000,000,000 particles per lb, and an area of 180,000,000 sq mm area per lb, then the pulverized coal, compared with a single piece of 1 lb, has 600,000,000,000 times as many particles with 8000 times the surface. It would seem that if the particles could be kept from fusing together, fly ash would be one tenth as small or at least one tenth the density (if as large).

The writer does not quite understand the time allowed for combustion. It would seem that the maximum would be 1 sec instead of 2 sec or more. Would the author give more detailed information on the final stage of combustion, that is, that of the burning of fixed carbon. Assuming 10,000 lb of coal produces 5,000,000,000 cu ft of combustion gases per hr in a 900-sq-ft furnace, then the velocity would be 150 fps, and the time therefore less than 1 sec.

W. D. Canan.[18] The author's paper deals largely with theory and design, but there are several matters in connection with the operation of pulverized-fuel equipment which are worthy of some consideration, and the writer would therefore ask that he give more information in connection therewith.

(1) What is the power consumption of the equipment for coal preparing and firing, i.e., of auxiliaries such as the pulverizers, primary-air fans, and coal feeders? Is the power used by this equipment a considerable proportion of the total energy output of the boiler? When it is taken into account is the resulting over-all boiler efficiency equal to or better than a stoker-fired job?

(2) Is the emission of fly ash from the stack a serious problem, and what if anything is being done to solve it? The writer has been quoted figures ranging from 20 to 80 per cent of the total ash produced as being the amount which would pass up the stack. It would therefore appear that, especially in locations where there are a large number of manufacturing plants and residences, considerable objection would be raised to the emission of any such quantity of ash. Some users object to using pulverized coal on account of contamination of the product which they manufacture.

In addition to the fly-ash nuisance there is also the matter of the disposal of the ash taken from the boiler. This ash is not desirable for fill, nor can it be dumped into rivers. Some other method must, therefore, be suggested for its disposal if neither of the above methods of disposal are available. Some years ago several wet gas-washing systems were given prominence as a means of collecting and disposing of the fly ash, but experience with this type of system has led the writer to believe that it will not satisfactorily solve the problem.

(3) Can a pulverized-fuel installation be made entirely safe from an operating standpoint? Are explosions in the boiler furnace itself due to incorrect design, or is it entirely a matter of operation? If it is a matter of operation, what, if any, safeguard can be incorporated in the design of the equipment to guard against explosions?

(4) The author has stressed the advantages of water-cooled furnaces, and their desirable features have not been overstated. Almost every boiler installation that goes in today has more or less of this type of construction. There is a tendency to cut down boiler heating surface and add waterwalls, and the writer is wondering how long it will be until the boiler heating surface as such disappears entirely. In this connection it might be interesting to know what proportion of the total heat absorption of a boiler of modern design takes place in the waterwalls.

[18] Rust Engineering Company, Pittsburgh, Pa. Mem. A.S.M.E.

Author's Closure

Mr. Briscoe refers to the change in furnace temperatures caused by a slag or ash deposit on the water-cooled furnace walls and the first row of boiler tubes as "furnace sensitivity." Such slag or ash deposit reduces the heat absorption by these surfaces, thereby raising the temperature of the products of combustion when they enter the boiler and superheater. This higher gas temperature raises the superheat and may also result in slag or ash deposit in the superheater and boiler.

With most designs of water-cooled furnaces the slag or ash accumulates to a certain degree and then begins to drop off as fast as it is deposited. The temperature then remains nearly constant. This nearly constant condition of slag deposit and temperature depends a great deal on the adjustment of burners, air supply, and the fineness of coal. The slag deposit is less when the coal is fine and when the burners and the air supply are so adjusted that the combustion is nearly complete when the gases reach the boiler. Delayed combustion is the greatest factor in the deposition of slag on heating surfaces. The air supply should be adjusted according to excess air or CO_2 content in the gases leaving the furnace, and not at the exit from the boiler, economizer, or air heater, where the gases may be greatly diluted by air leakage. Careful operation and good maintenance of the mills, furnace, and setting are as important for good results as good design.

Low temperature of gases entering the boiler or superheater eliminates much of the slag trouble, but there is a limit to the amount which this temperature can be reduced, particularly in an installation with high superheat temperature. The gases must be hot enough when they enter the superheater to contain enough heat at sufficiently high temperature to produce the desired superheat. If the temperature difference between the hot gases and the steam is too small, a large and expensive superheater is required.

Large cross section of the gas passage at the entrance into the boiler results in lower gas velocity and greatly reduces the slagging of the boiler and superheater. Careful operation contributes much toward the elimination of slagging. However, with coals having low-fusion-temperature ash, some slag deposits must be periodically removed. Therefore, it is desirable that facilities for cleaning during operation should be provided when the steam-generating equipment is designed. There are many installations which are kept in operation for many months at a time. If any cleaning of slag is necessary it is done without any interference with operation.

The temperature of superheated steam rises with boiler rating. The following methods are used to keep the superheat nearly constant over a desired range in steaming rate.

(*a*) The superheat is kept nearly constant by a damper-controlled gas by-pass which permits a varying volume of the hot gases to by-pass the superheater. The superheater is designed for the lowest steaming rate at which the full superheat is desired. As the steaming rate rises above this point, part of the

volume of hot gases is made to by-pass the superheater. Thus, the rising temperature of the hot gases entering the superheater is compensated by their decreasing volume. This method of control requires a larger and more costly superheater, an idle gas pass around the superheater, and a damper to regulate the volume of gases flowing through the by-pass. The regulation of the by-pass damper can be done manually or by an automatic control.

A modification of this method is used in a double boiler with a large superheater on one side and a smaller one on the other side. Varying quantities of gases are passed through the two sides. Superheat is controlled by regulating the flow of gases through the two sides of the unit.

(*b*) The superheat can be controlled by the manipulation of the fires. In this method burners are placed at different heights in the furnace. At low rating the burners located in the upper part of the furnace are put in operation and the fires are brought closer to the superheater. At high rates of steaming the lower burners are used. For superheat control over a wide range of rating this method can be used in combination with the by-pass-damper method.

(*c*) Superheat is maintained nearly constant by the combination of radiant with convection superheater. The radiant superheater is placed in the furnace as a part of the wall cooling surface, or, part of the superheating surface is placed between widely spaced boiler tubes in the first bank of the boiler. The surface in such locations is exposed to radiation from the furnace and absorbs a greater proportion of heat at low rating. The main part of the superheater is located in the usual place back of the first bank of boiler tubes and absorbs heat by convection. The steam flows first through the radiant section and then through the convection superheater. This method of superheat control is largely a matter of proportioning the amount of surface in the two parts of the superheater. It may be combined with the methods outlined under (*a*) and (*b*).

(*d*) The superheat can be controlled by a separately fired superheater. In this method the superheater is set over a separate furnace and the superheat is controlled by the amount of fuel burned. This method has the widest steaming range with constant superheat. It is also the most expensive and is now seldom used.

(*e*) The superheat can be controlled by partial desuperheating at the high rates of steaming. In this method the superheater is designed to give full superheat at the desired low rating. At higher ratings the steam is partly desuperheated after it has gone about half way through the superheating process, and the extent of the desuperheating is controlled by the final temperature. The superheater is installed in two parts with the desuperheating apparatus between them. This method of superheater control is rather complicated and is seldom used in this country.

Direct impingement of concentrated flame against water-cooled walls is not desirable, but flame sweeping or brushing over the walls is not objectionable. By direct impingement is understood the condition in which the flame strikes the wall nearly at right angles a short distance from the burner. A flame is sweeping over a water-cooled wall when the flame moves nearly parallel to the wall a considerable distance from the burner. In tangential firing the flame may sweep over the walls after it has traveled nearly diagonally across the furnace.

Preheated air helps the ignition because less heat is required to bring the incoming mixture of coal and air to ignition temperature. The ignition is quicker and closer to the burner. There are practical limits to the temperature to which air can be preheated. This limit is usually set by the efficiency of the steam-generating unit. If high efficiency is desired the temperature of gases entering the air heater should not exceed about 650 F. It is not economical to bring the temperature of preheated air closer than about 100 F to the temperature of gases entering the heater because the air heater would become too large and expensive. The weight of gases is always greater than the weight of air to be heated. The specific heat of the gases is also higher than that of the air. Therefore, the temperature rise of the air is greater than the temperature drop of the gases. The weight of gases is equal to the weight of air going through the air heater plus the air leaking into the setting plus the weight of the ash-free fuel. The specific heat of the products of combustion, particularly with coals of high hydrogen and high moisture content, is from 10 to 15 per cent higher than the specific heat of air. Usually for every 300 F of the temperature rise of the air there is only about 200 F drop of the temperature of the gases. Thus, if the temperature of the gases is to be reduced from 650 to 350 F the temperature of air would rise from 100 to 550 F. If the temperature of gases entering the air heater were 900 F, it would require a large air heater to bring the temperature of air to 800 F. The air temperature rise would be 700 F and the gas-temperature drop would be about two thirds of 700, that is, about 466 F. The resulting temperature of gases would then be 434 F. The over-all efficiency of the steam-generating unit would then be about 2 per cent lower than with the gases leaving the air heater at 350 F.

A further disadvantage of the highly heated air would be the oxidation of the air heater and air ducts, more expensive expansion joints, and insulation covering. To reduce the oxidation of the air heater and the ducts they would have to be made of heat-resisting steel alloy. There is no limitation to the temperature of preheated air from the standpoint of combustion.

During the last few years efforts have been made to develop a mill which would pulverize coal to more uniform fineness with lower power consumption and lower maintenance. By more uniform fineness is meant the reduction of the percentage of oversize and the superfine coal.

Mr. Sitter brought up the question of the necessity of feedwater treatment with water-cooled furnaces. The water-cooled furnace walls absorb heat at a high rate and must be kept clean internally to prevent their overheating. With internally clean tubes the temperature of the metal seldom rises over 50 F above the temperature of the water inside of the tube. To keep the surfaces internally clean the feedwater should be properly treated to prevent scale formation. All make-up water should be treated in a separate feed water-treatment plant. This primary treatment should be followed by a secondary treatment with phosphate which is supplied directly to the boiler through a separate line. The methods of feedwater-treatment have been sufficiently developed so that there is no excuse for the formation of scale inside of the boiler.

Widely fluctuating loads require large steam drums to take care of the variation in water level and at the same time leave sufficient space in the drum to separate the water from the steam. It is much easier to obtain clean steam with a steady rating than with widely fluctuating load.

Oil torches are more stable and are preferable to gas torches for lighting pulverized-coal fires. The oil torch has a hot and luminous flame and if the torch is blown out the furnace is not filled with explosive gas as may be the case with gas lighting torches. Oil leaks are more easily detected than gas leaks particularly leaks due to incompletely closed valves.

To take one mill out when two mills are supplying the fuel to one boiler the output of the mill to be taken out should be reduced in about three equal steps to the minimum output at which it will operate satisfactorily. At the same time the output of the mill to remain in operation sould be increased by similar steps. The air supply should also be shifted to maintain the stability

of flame. When the operator has assured himself that the flame remains stable, the mill to be taken out is shut down and the load fully taken by the other mill.

There are coals which are classed as bituminous coal with the percentage of volatile matter as low as 14 per cent. When the volatile matter drops below 20 per cent a delay in the ignition becomes quite noticeable. For burning coals so low in volatile matter, provision must be made for the reduction of primary air. This is at times difficult to do in direct firing because the mill must have a definite minimum quantity of air to operate satisfactorily. This minimum air for the mill may be too much air with the coal to produce good ignition. When coal low in volatile matter is burned in a tangentially fired furnace the air supply around the coal nozzle should be cut off and the secondary air should be supplied through the air port above the upper burner. With highly volatile coal the air around the coal nozzle should be reduced to bring the ignition to the burner without allowing the flame to back into it. The air ports above and below the burner should be kept fully open, and the air supply should be controlled by the air pressure at the burner, by regulating the speed of the pressure fan or the damper in the air supply line. Such air regulation supplies the greatest amount of energy to the furnace for mixing. The greatest heat release will occur in the center of the furnace in the plane of the burners where the streams of coal and air meet and where the mixing is most intensive. Any mixing that is done at the burners should be done to quicken the ignition. Mixing for rapid combustion is done in the center of the furnace away from the burner where the secondary air does not interfere with the ignition.

Highly volatile coal is better suited for a widely fluctuating load particularly with direct firing. The ignition is more stable because the volatile matter makes a rich mixture which ignites readily even with wide fluctuation in the air supply.

Irregular flow of coal to the pulverizer due to moisture and an excessive proportion of fines is one of the drawbacks of direct firing. If the hanging of the coal is between the feeder and the pulverizer, hot air supplied to the pipe connecting the two may prevent the hanging, but this increases the flow of air through the mill which at times may not be desirable for good ignition. If the hanging occurs before the feeder it is difficult to remedy.

Mr. Showers points out the advantage of some refractory surface in a pulverized-coal furnace. Such surface when hot reignites the fires after a short interruption in the fuel supply to the furnace. In many of the pulverized-coal installations with horizontal firing such surface is on the front, or the burner wall. This wall is made of refractory for structural and economic reasons; it is easier to make the front wall of refractory than of water-cooled surface, because the water-cooled tubes have to be bent around the burners and cannot be made to cover the burner wall completely. Slag may stick to such refractory surface and accumulate and may have to be removed by hand tools. In other cases the refractory may wash out and the wall require frequent patching. The reignition of the fuel after a short interruption may not occur promptly. It may be delayed until the furnace is filled with coal mixture and then ignite with a violent puff which causes leakage into the setting. If feeding of coal to the furnace is unreliable a small oil burner is a better security against a loss of fire.

The temperature of a pulverized-coal flame even in a completely water-cooled furnace is considerably above 2000 F. At lower rating the flame may be reduced to a smaller volume but the temperature of the flame is practically as high as it is at high ratings. A match can be burned in a completely water-cooled furnace with an average rate of heat liberation of a fraction of a Btu per cubic foot of the furnace volume. The flame will occupy a volume less than a cubic foot and have a temperature of about 2000 F. Furnaces are made completely water-cooled to avoid refractories in the furnace, the sticking of slag to them, and also to get away from the maintenance of such refractories.

It is largely the extent of the surface of the flame that determines the amount of heat radiated to the water-cooled walls. At low rating the surface is small because the volume of the flame is small, and the amount of heat radiated to the walls is small because of the small flame surface. At high ratings the flame fills the furnace almost completely. The surface is large and a large amount of heat is radiated to the furnace walls.

Mr. James presents interesting and complete information on the use of pulverized-coal ash. His discussion is a valuable addition to our knowledge about the use of pulverized coal.

Mr. Drysdale brings up an interesting point in connection with maintaining nearly constant superheat with varying steam output. This problem is more difficult to deal with in the design of units for high steam pressure and temperature. The gases entering the superheater must be hot enough to give the desired superheat and they should not be hot enough to cause slagging of the boiler and superheater. The designer has to work between two serious limitations. The methods for obtaining nearly constant superheat have been described in connection with Mr. Briscoe's discussion. The method outlined under the manipulation of fires promises a satisfactory solution. This method strives to maintain the temperature of gases entering the superheater nearly constant regardless of their volume.

Mr. Thorson asked three questions which can be answered as follows:

The maintenance of mill fans averages about 0.5 cent per ton of coal pulverized. If other maintenance remains the same, placing the mill fan ahead of the mill would reduce the over-all mill maintenance by 0.5 cent. There are some objections to such a mill-fan location. The mill is put under pressure and there may be leakage of coal dust from the mill and particularly from the coal feeder. Coal dust may be forced into bearings and increase their maintenance.

It does not seem likely that a stoker installation would have a quicker pickup than a direct-fired pulverized-coal installation. It takes time to build up a good stoker fire that will carry heavy load over extended periods. Banked stoker fires or low-rating fires are not usually in good condition for heavy load.

There is a decided trend toward pulverized-coal firing which is probably due to wider latitude in fuel selection. "Change of coals upsets the operation of a pulverized-coal plant to a much smaller degree than a stoker-fired plant."

Explosions in pulverized-coal furnaces are usually caused by faulty operation. The explosions are generally less disastrous than explosions in oil- or gas-fired furnaces. Coal is pulverized to make it more inflammable or combustible. The greater inflammability makes it more explosive. The greatest asset of safety is a good operator who does not take risks.

Originally the boiler was made to evaporate water. In modern high-pressure steam-generating units the boiler performs mostly a structural service. The boiler supplies rigid water-cooled walls, water-cooled supports for the superheater, water-cooled frame for partitions for gas passages through or around the superheater, and a water-cooled chamber for the economizer. The water-evaporating duties have taken secondary place. However, a boiler will always be necessary to perform the structural service.

A water-cooled furnace makes from 25 to 50 per cent of the steam made by the steam-generating unit.

Professor Linsenmeyer asks about the time required for combustion. This time depends on the size of the particles; the finest particles burn instantaneously, the large ones require more than two seconds because they contain many times more com-

bustible than the small ones, and because their combustion must be completed after the concentration of free oxygen in the furnace gases has been greatly reduced. In the last part of the path of gases in the furnace the oxygen concentration is 2 or 3 per cent. In fact some of the large particles never burn completely.

Mr. Canan's questions may be answered as follows:

The power consumption of the equipment for coal preparation and burning in a pulverized-coal plant is greater than it is in a stoker-fired plant. In general the difference probably does not exceed 2 per cent of the energy represented by the boiler output, and is offset by a higher over-all operating efficiency. The application of pulverized coal is about fifteen years old and has ceased to be a novelty. There must be a tangible reason for the preference for pulverized coal. There are undoubtedly some cases where stokers have an advantage over pulverized coal.

The emission of fly ash is greatly reduced by various dust-collecting devices. Some of the mechanical devices can be installed at low cost. They collect most of the coarser dust which would settle near the plant if allowed to go out of the stack. The emission of stoker cinders has also become objectionable and in some cases provision must be made for collecting them. If dust collectors must be used with either firing, engineers apparently prefer to use pulverized coal.

The disposal of the collected dust is a problem that seems to lack a good general solution. There are apparently many special solutions: Mr. James presented one of them.

Mr. Scott proposes supplying pure oxygen to the furnace near the end of the combustion process to raise the concentration of free oxygen and thereby obtain more complete combustion of the large coal particles. Pure oxygen for the combustion of fuel in boiler furnaces has been proposed before but the cost of supplying all the oxygen seemed prohibitive. Besides, the extremely high temperatures at the beginning of the combustion process, due to all pure oxygen, might damage the furnace. Supplying only a small quantity of oxygen near the end of the combustion process looks more practical because it would increase the oxygen concentration without high losses due to a great excess of air. In the Velox boiler air for combustion is compressed to obtain more oxygen per cubic foot of combustion space. Supplying pure oxygen near the end of the combustion process may give the same amount of free oxygen per cubic foot without the necessity of compressing a large quantity of inert nitrogen.

Mr. Rudd submits several questions referring to comparisons of pulverized-coal firing with stoker-fired plants. To answer such questions completely would require a thorough study of a number of representative plants using the two methods of firing. Some of the questions have been partly answered in connection with the discussions by Mr. Thorson and Mr. Canan. Although a majority of the new boilers now being installed are designed for pulverized-coal firing there are many new installations with stoker firing. These later installations undoubtedly have good economic reasons for the use of stokers.

Part of Mr. Blizard's discussion is devoted to mathematical analysis of the author's attempt, given on page 296 under (*a*), (*b*), and (*c*), to account for the slower combustion at higher furnace temperatures as shown by the results of the laboratory experiments made by the Bureau of Mines. The first two factors considered under (*a*) and (*b*) when combined agree with Mr. Blizard's equation for the rate of molecular impingement $M = 145(1/T)^{1/2}$.

However, the diffusion or rather the factor which removes the products of combustion from the surfaces of the coal particle and brings free oxygen within the range of impingement, must be given full consideration. The effect of temperature on this factor has been considered under (*c*). In the case of the laboratory experiments, the factor was the combination of natural diffusion and the movement, caused by the force of gravity, of the coal particles through the furnace atmosphere. The author is of the opinion that if the particle of coal were moved through the furnace atmosphere at sufficient speed to maintain the partial pressure of free oxygen within the range of impingement at 0.21 atmosphere, the time of combustion of the coal particle would be close to the value indicated by Mr. Blizard's calculation. It is certain that under the conditions of the experiments the partial pressure of free oxygen within the range of impingement was much smaller than 0.21 atm, because the products of combustion could not be moved out of the range of impingement and free oxygen brought into the range fast enough to maintain the oxygen pressure. With the time of combustion indicated by the experiments the same mathematical method gives the partial oxygen pressure to be of the order of 0.005 atm. This partial oxygen pressure within the impingement zone depends largely on the speed with which the burning particle of coal is moved out of one place where all oxygen has been consumed to another place where free oxygen is available. In all cases where rapid combustion is obtained it is produced by intensive mixing.

Mr. Blizard's explanation, that the particle as a whole is hotter when it begins to glow in a lower furnace temperature, may hold with particles of charcoal or coke. In the case of coal the first part of the heat absorbed by the particle is used to drive off the volatile matter, and this volatile matter bursts into flame long before the particle is brought to glowing temperature. Fig. 9 shows that combustion begins abruptly and ends gradually. During the first part of the process of combustion the coal particle moves downward out of the rising envelope of flame, and in doing so continually moves into new locations with high concentration of free oxygen. On the other hand, during the last part of combustion the burning particle rises with the rising products of combustion, and remains in an atmosphere with low free-oxygen concentration or low partial pressure.

Locomotive Axle Testing[1]

HUBER O. CROFT.[2] The writer would like to have the authors answer the following three questions: (1) What method is used to discover fatigue cracks when they form initially? (2) What stress effects are developed at the hub by the temperature increase due to the braking action of brake shoes? (3) What is the effect on fatigue stresses of the impact of the wheel when passing over crossings at high speed?

P. W. KIEFER.[3] It should be pointed out that the application of roller bearings has changed considerably the functional characteristics of the different axles and has made necessary the development of additional information on axle designs and materials in order to obtain a reasonable expectation of service life. In addition, it has become necessary to increase the power output of new locomotives to meet the requirements of improved schedules, but at the same time to avoid the higher unit costs for the more complicated driving-wheel and running-gear arrangements when the number of driving wheels is increased. Similarly, the weights of existing passenger equipment cars operating on faster

[1] Published as paper RR-60-2, by T. V. Buckwalter, O. J. Horger, and W. C. Sanders, in the May, 1938, issue of the A.S.M.E. Transactions, p. 335.

[2] Head of the Mechanical Engineering Department, College of Engineering, State University of Iowa, Iowa City, Iowa. Mem. A.S.M.E.

[3] Chief Engineer of Motive Power and Rolling Stock, New York Central System, New York, N. Y.

schedules are being materially increased because of air-conditioning installations and other improvements. All of these factors impose more exacting demands on the equipment axles.

Although the paper under discussion is confined to the subject of locomotive-axle testing, it is of interest to state that a series of tests on passenger-car axles now in progress on one of the Timken machines is being conducted under the direction of W. I. Cantley, mechanical engineer of the mechanical division of the A.A.R., through the cooperative activities of a joint subcommittee consisting of representatives from the directly interested main committees, including the A.A.R. mechanical division committee on car construction. Encouraging progress is being currently reported on these tests. To the best of the writer's knowledge, this method of testing represents a pioneer effort, but it is too early to attempt reaching conclusions from the data thus far collected.

In a few months from now much additional information will be available to the railroads and related industries from which it is believed many of the problems involved in the successful operation of axles under modern railroad equipment, either with or without roller bearings, may be answered, and the performance of such axles greatly improved.

Authors' Closure

Professor Croft raises three questions which may be answered as follows:

1 The only certain method known by which fatigue cracks can be detected in the wheel fit of the axle requires pressing the wheel off the axle. Magnaflux examination of the axle is then made to detect any minute cracks which are not evident to the eye. In some tests the wheel center has been cut into halves so that there would be no question about the influence of pressing the wheel off the axle as a contributing factor in causing the fatigue cracks to develop. It is believed, however, that the pressing-off operation does not develop such fatigue cracks.

2 Little knowledge is available concerning the stress effects at the hub due to temperature increase resulting from braking action. The feeling exists that, with the rim of the wheel in a heated condition, it would tend to expand and relieve the press-fit stresses at the hub. Available information from tests indicates that the press-fit pressure required to prevent a wheel from loosening on an axle is of such magnitude that any considerable increase or decrease in such press-fit pressure would cause little, if any, change in the fatigue strength of the axle. For this reason it is considered that brake-shoe action has small, if any, influence on the axle, fatigue-failure problem, although this is a subject which may deserve further consideration.

3 The case of stresses set up in an axle due to impact conditions which may occur when a wheel passes over crossings at high speed is a subject which is not investigated in this paper. Such stresses have actually been measured in service and it is known that peak stresses occur under such operating conditions. If such peak stresses are of sufficient magnitude and quantity, early axle failure will develop.

Mr. Kiefer suggests that the axle life may be considerably extended by the use of roller bearings except as it is limited by the development of fatigue cracks as explained in this paper. It is the ultimate object of the reported tests as well as those mentioned by Mr. Kiefer now being conducted for A.A.R. in this same laboratory to obtain an axle design specification which will result in a life much longer than is now being obtained. Based on information available now, the influence of roller bearings on the fatigue failure of axles has little or no effect on the fatigue strength of the axle at the wheel seat.

Quantitative Analysis of Process Lags[1]

J. C. Peters.[2] While the terms "lag" and "time lag" have been widely used for a number of years in describing the characteristics of industrial processes, the meaning of these terms has, in general, been somewhat vague and indefinite. The author suggested[3] in 1933 that the general effects be referred to as process lags and that these be classified as capacity lags, transfer lags, and velocity-distance lags. Such a breakdown is valuable in that it concentrates attention on certain fundamental factors involved. A process designed with these factors in mind is likely to be easier to control than would otherwise be the case, and the operation of existing processes can sometimes be improved by following similar considerations.

When temperature is the controlled variable, few processes lend themselves to exact mathematical analysis. In general, the equations contain variable parameters not too definitely known and are of such complication as to defy solution. In these cases, simplifying assumptions are required which often make the results useful only on a qualitative basis. Nevertheless, the writer feels that such solutions may be very valuable and should be encouraged.

The solution for the author's single-capacity process represented by Figs. 1 and 2 of the paper is one that is directly useful. When set up on percentage basis, the equations for it can be readily applied without the use of curves. An equivalent of the author's Equation [8] is

$$(T_a - T_{a0}) = (T_p - T_{a0})[1 - \epsilon^{-(ft/B)}] \quad \ldots\ldots [1]$$

where T_a = temperature of the liquid in the tank at time t; T_{a0} = initial value of T_a; T_p = final value of $T_a = (Q_s/cf) + T_0$; T_0 = temperature of entering liquid; Q_s = rate of electric heat supply, expressed in heat units; f = rate of liquid flow, mass per unit time; B = mass of liquid in the tank; c = specific heat of the liquid; and $\epsilon = 2.718$. When written in this way, the equation is equally useful whether a sudden change is made in the rate of electric heating, the rate of liquid flow, or the temperature of the incoming fluid.

On the left-hand side of Equation [1] of this discussion we have the amount by which the temperature in the tank has increased in time t. The ultimate change is $(T_p - T_{a0})$. Dividing both sides of the equation by this quantity and multiplying by 100, we obtain that percentage of the total change P that takes place in time t, or

$$P = 100[1 - \epsilon^{-(ft/B)}] \quad \ldots\ldots [2]$$

It is evident immediately that when $t = B/f$, $P = 100[1 - (1/\epsilon)] = 63$, approximately. The change is therefore 63 per cent complete in the time equal to that quantity referred to in physics and electrical engineering as the time constant.

Solving Equation [2] of this discussion for t gives

$$t = \frac{B}{f}\left[\log_\epsilon\left(\frac{100}{100 - P}\right)\right] \quad \ldots\ldots [3]$$

This is a very convenient equation for practical use. With the aid of a curve or table giving the value of the factor in brackets as a function of P, solutions are obtainable by simple slide-rule

[1] Published as paper PRO-60-1, by C. E. Mason, in the May, 1938, issue of the A.S.M.E. Transactions, p. 327.

[2] Research Department, Leeds & Northrup Company, Philadelphia, Pa.

[3] "Science of Automatic Control in Refining," by C. E. Mason, *World Petroleum*, June, 1933, p. 189.

calculation. The curve of Fig. 1 of this discussion gives the required relation for values of P between zero and 99 per cent.

When a knowledge of the actual magnitude of the ultimate change is desired, it may be obtained from

$$(T_p - T_{a0}) = \frac{Q_s}{cf} + T_0 - T_{a0} \quad [4]$$

Other equations of considerable interest and value are those giving the per cent change per unit time, obtained by differentiating Equation [2] of this discussion. These are

$$\frac{dP}{dt} = 100\,\frac{f}{B}\,\epsilon^{-ft/B} \quad [5]$$

and for the initial rate when $t = 0$

$$\frac{dP}{dt} = 100\,\frac{f}{B} \quad [6]$$

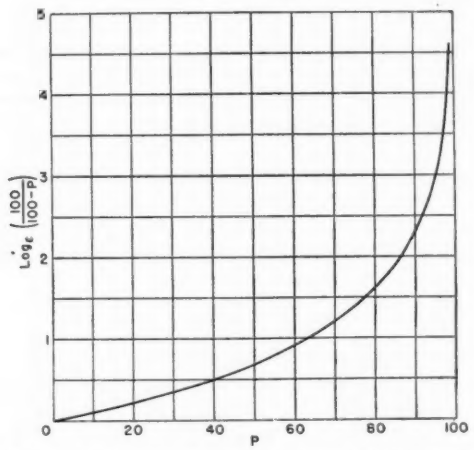


FIG. 1 VALUES OF P BETWEEN 0 AND 99 PER CENT

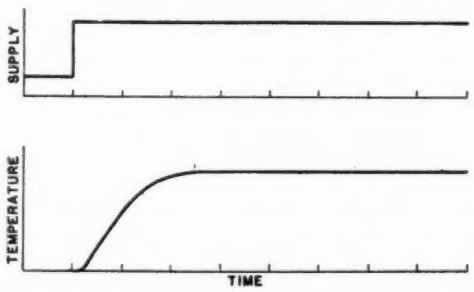


FIG. 2 TYPICAL RESPONSE CURVES AS OBTAINED WHEN HEAT TRANSFER TAKES PLACE THROUGH A MEDIUM INVOLVING BOTH THERMAL RESISTANCE AND THERMAL CAPACITY

With regard to hydraulic analogies, it appears that these can be of only limited service in connection with temperature-control problems, even when rate of fluid flow is assumed to vary directly with head. Considering the author's Fig. 4, capacities A_a and A_b are assumed to be separated by a simple thermal resistance. Most of the more difficult temperature-control problems involve resistance associated with capacity. With such a partition separating simple capacities A_a and A_b, response of the temperature in A_b to a sudden change in Q_0 would be of the type shown in Fig. 2 of this discussion. This curve is an experimental one taken from Fig. 6 of the paper by Bristol and Peters.[4] The temperature change is insignificant for a more or less definite time following a change in supply, after which it becomes similar to that for a single-capacity system. To give a practical example, we observe this effect when we heat a long iron rod at one end and continually observe the temperature at a distance from the point of application of heat. The theoretical solution of a corresponding ideal case is found in books on the theory of heat, where an infinitely long rod, perfectly insulated except at the ends and initially at uniform temperature throughout, is suddenly subjected to some higher temperature at one end. The temperature at any distance x from the heated end, expressed as that percentage P of the total change complete in time t, is

$$P = \frac{2}{\sqrt{\pi}} \int_0^{x/2\sqrt{(\kappa t)}} \epsilon^{-y^2}\,dy \quad [7]$$

For wrought iron, the diffusivity κ, expressed in cgs units, is 0.173. Curves a and b of Fig. 3 of this discussion apply to this material and are for distances from the heated end of 5 and 10 cm, respectively. At 10-cm distance, the change is 1 per cent complete in 0.72 min, and 63 per cent complete in 20.7 min; the ratio of the latter to the former time is 28.6. This ratio for the experimental curve of Fig. 2 of this discussion is approximately 5.

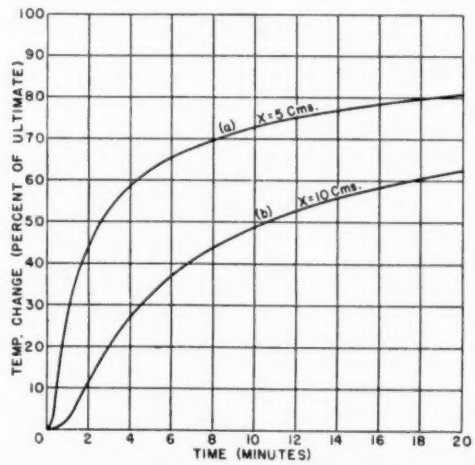


FIG. 3 RISE OF TEMPERATURE AT POINTS ON AN INFINITELY LONG IRON ROD FOLLOWING A SUDDEN CHANGE IN TEMPERATURE AT THE END

Experimentally obtained response curves of the type shown in Fig. 2 are of considerable interest to those working on the theory of automatic control. It would be very helpful to them if those in a position to make such tests on industrial processes would do so, submitting their results to the secretary of the A.S.M.E. Committee on Industrial Instruments and Regulators.

ED S. SMITH, JR.[5] This paper[1] is a creditable example of the type which the A.S.M.E. Committee on Industrial Instruments and Regulators was formed to consider and use. It is restricted to a single phase of the large subject of automatic regulation or control, and represents a worth-while attempt to provide a generalized and graphical treatment, following methods used occasionally on individual problems by others in this field. The generalized treatment promises to become a widely used tool as facility in the technique is attained by those working in this field, especially since the mathematics involved is only undergraduate calculus with simple solutions. The paper creates confidence in the use of the hydraulic analogy and technique; the value of the

[4] "Some Fundamental Considerations in the Application of Automatic Control to Continuous Processes," by E. S. Bristol and J. C. Peters, presented at the Annual Meeting of the A.S.M.E., held in New York, N. Y., December 6–10, 1937.

[5] Mechanical Engineer in charge of Patents, C. J. Tagliabue Manufacturing Company, Brooklyn, N. Y. Mem. A.S.M.E.

paper is unquestionable and not lessened by the fact that the methods per se (e.g., the use of the hydraulic analogy for temperature relations) are not original as presented by the author, but old.[6]

The hydraulic analogy breaks down into one more use of model testing, which method is of value in so many different fields. It is not necessary, of course, to stop with the tanks and resistances, since an automatic regulator may be added and the performance of the whole determined. It is old, especially in the hydraulic field, to test regulators on models of the proper sort to simulate working conditions fairly. In other words, the technique of the paper may be used with far more confidence than the unnecessarily conservative approach of the paper would indicate.

As to terms,"transportation lag" seems preferable to the hyphened "distance-velocity" lag. Also, the use of "interval" seems better than the use of "lag" for capacity and transfer lags, where the retardation, rather than seconds lag, for the transportation lag is meant.

However, there is another and valid approach. The lag may be expressed for the case of a gradual instead of an instantaneous change, in the former case the lag is expressed in seconds. There is a corresponding number of seconds for the interval required, following the sudden change, for the effect to approach its final value within 10 per cent of the total change. Further, where the process is complex and multiple time constants are involved, these may have corresponding multiple lags and multiple intervals in seconds. In most practical recorder-controller applications, it appears that the initial rate of change of the effect per unit change of the variable is more important than either the lag or interval, but such rate of change may be readily determined from the interval.[7] The writer favors the decimal interval because, in his experience, its use with semilog paper has proved to be most convenient.

The unnecessary difficulty in following the present paper would be largely obviated if the closure were to include a table which would present at a glance the corresponding types of lags, figures, cases, plots, formulas, and whether the example is thermal or hydraulic.

The low price of usual industrial controllers, or regulators, leaves an inadequate margin for the research which is required to put the subject of automatic regulation on the most useful basis. Consequently, cooperation among all those concerned is needed to distribute the common burden so that the best results will be obtained. Mr. Mason and the Foxboro Company, which made possible such use of his time, are thanked for their generous cooperation.

Model testing and dimensional analysis are inseparable. Instead of a formidable mathematical structure, the writer hopes to see in the future a simple graphical analysis with the performance of the several types of regulators and processes falling within generally recognized zones when plotted against the appropriate operator, a dimensionless ratio derived by dimensional analysis from the pertinent variables.

For example, the zones of hunting and nonhunting performance of a simple throttling regulator for various sensitivities and lags should be correlatable with a dimensionless ratio involving the product of terms involving the sensitivity (which is taken by Fairchild as a dimensionless ratio, i.e., effect/cause in the same units) and the ratio of metering to process lags. From the standpoint of common sense or common experience, an increase in sensitivity tends to cause increased hunting, as does an increase of metering lag, while capacity lag generally acts as a stabilizing balance wheel. Thus, without resorting to a blind use of the formal π theorem of Buckingham, the operator would reasonably appear simply as s/i, where s is the sensitivity and i is ratio of the capacity lag to the metering lag, expressed in the same units. Considering limits, unless metering lag were present, hunting would be impossible. Similarly, with zero capacity lag, hunting would result from the least actual metering lag. These considerations check the direction but not the powers of the terms of the operator. Using s/i as the abscissa, the exponent factor k would be used as the ordinate; the expected critical value of s/i would be of the order of unity. A single line would appear on the graph for the simple throttling regulator under these simple conditions; however, a surface would appear if an additional dimensionless ratio for another contour line, for more complex conditions, were used. For example, a surface would be obtained in the case of adding reset to such a regulator, or for a fixed-speed motor-operated valve and a fixed-speed reset; a logical operator for the contours would be the ratio of the reset speed to the valve speed. The writer regrets that he is unable under present conditions to take the time to confirm or expand these tentative operators but submits them as of related interest and of possible use in helping others attain the expressed aim of the paper of better attacking the general problem.

[6] See "The Hydrocal," by A. D. Moore, *Industrial and Engineering Chemistry*, vol. 28, June, 1936, pp. 704–710; also, U. S. Patent No. 2,082,211, June, 1937, and file history thereof.

[7] For a further treatment of these interrelated terms see comments made by author on Behar's discussion in the closure of the paper "Industrial Instruments, Their Theory and Application," by Ed S. Smith, Jr., and C. O. Fairchild, Trans. A.S.M.E., vol. 60, October, 1938, p. 595.

Author's Closure

It is of course true, as mentioned in the discussion submitted by Mr. Peters, that exact solutions for complex thermal problems are often so intricate as to be prohibitive. It was precisely this intricacy which originally suggested investigation of the possibilities of the many schemes of hydraulic analogy in order to determine a procedure which would be reasonably practical and which would reduce to relatively simple analysis.

A careful study of fundamentals is essential for the development of a practical scheme of analogous representation, in order to avoid faulty premises in the construction. The scope of the paper permitted only the introduction of some of the simpler basic principles. The author's experience has shown that, when properly constructed, such analogs can be made to characterize the behavior of complex systems far more faithfully than might be expected. Most every discouraging result in the earlier work has been traceable to insufficient care in the representations.

Equations [1] to [6] of Mr. Peters' discussion, paraphrasing the author's development, are exceedingly welcome if they add to the clarity of the paper.

In these forms the equations are not general but apply only to single-capacity thermal processes in which heat is conducted by a medium. The author's presentation for this type of thermal process followed a general scheme which would be equally applicable for other types of single-capacity processes as well as for the more complex multiple-capacity processes.

Whether the integrated equations are written in terms of $T_a - T_{ao}$ or in terms of $T_a - T_p$ seems most a matter of choice. Mr. Peters' alternative forms are further welcome since they may better serve to correlate the author's development with that of other phases of engineering. Solutions in terms of $T_a - T_{ao}$ are probably most easily obtained by adding $T_p - T_{ao}$ to both sides of the integrated equations which are given in the paper.

Expressing the results in terms of $T_a - T_p$ gives a convenient form for plotting directly in semilogarithmic coordinates. However, the exponential function

$$C(1 - e^{kt})$$

does not give a straight line in semilogarithmic coordinates. This function could be represented by two straight lines in these

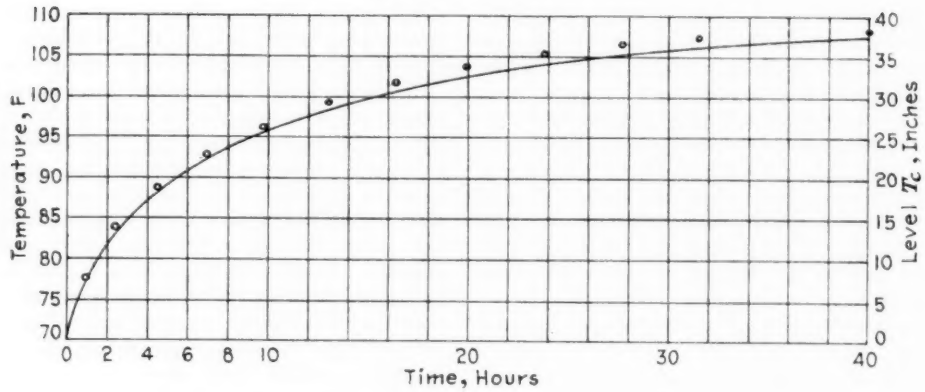


FIG. 4

coordinates. The possibility of confusion in using this function is more apparent when dealing with multiple-capacity processes. When the values of the fraction

$$\frac{T_a - T_{ao}}{T_p - T_{ao}}$$

are desired it seems more convenient to obtain them after the construction of the semilogarithmic plot.

In process analyses, and particularly in those which include an automatic controller, it seems we are more concerned with the deviation at any time from the eventual value for the present condition than with the deviation from some previous value which was the result of a previous condition.

When a process has been in static balance the value of T_{ao} for a new condition is the value of T_p for the previous condition. When changes are imposed on processes not in static balance the value of T_{ao} for a new condition does not have the same importance as a reference value.

Every actual thermal process will have capacity associated with resistance. It is possible, by proper construction of hydraulic analogs, to represent this condition with sufficient accuracy by means of a system of concentrated capacities and resistances more complex than those given in the paper. The inclusion of additional capacities results in differential equations of increased order. Many times the solutions of these more complex systems are not materially different from those of simpler systems involving fewer segregated capacities and resistances. It seems to the author that it would be a worthy purpose for the combined groups of the Committee on Industrial Instruments and Regulators to extend this field of investigation to determine the degree of complexity necessary for practical industrial use of the hydraulic analogy.

As a concrete example of the representation of a thermal process having distributed resistance and capacity by means of a hydraulic system, we have considered a problem given by Doherty and Keller in a recent text.[8] This problem concerns an insulated box which is constructed of cork and lined inside and out. The inside dimensions of the box are 5 × 5 × 4 ft, while the thickness is 4 in. The linings are of $^1/_{16}$-in. sheet steel. It is required to find the difference in temperature between the inner and outer walls as a function of time following the energizing of a 100-watt bulb within the box, the whole system being initially in equilibrium at an ambient temperature of 70 F. A careful solution is given by Doherty and Keller for this problem, using the methods of function theory. Their results are shown by the encircled points of Fig. 4

Fig. 5 shows an extremely simple hydraulic system to be considered as representing the above thermal process. The fundamental correspondences between the two systems are as follows:

Thermal process	Hydraulic analog
Btu of heat	Lb of liquid
Degrees F above 70 F	Inches of liquid level
Hours	Hours

The inflow of liquid corresponds to the heat generated by the bulb, while the outflow of liquid corresponds to the heat lost by the box to its surroundings. The liquid level T_b represents the temperature difference between the inner and outer walls of the box.

The constants of the hydraulic system, as shown in Fig. 5, have been evaluated as follows:

A_a = Total heat capacity of cork walls
= Volume × density × sp ht = 226 (Btu / deg F)

R_a = R_b = Half the thermal resistance of cork walls

$$= \frac{\text{thickness}}{2 \times (\text{mean area}) \times (\text{conductivity})} = 0.05665 \left(\text{deg F} \Big/ \frac{\text{Btu}}{\text{hour}} \right)$$

A_b = Total heat capacity of inner lining plus enclosed air
= 40 (Btu/deg F)

A solution by the usual methods as outlined in the paper, of the corresponding problem involving the hydraulic system, yields the following result

$$T_b = 39.1 - 13.026e^{-0.5328t} - 26.074e^{-0.06469t}$$

in which T_b is the level in inches (corresponding to the temperature of the inner wall in deg F above 70 F) while t is the time in hours following the application of heat. The solid curve of Fig. 4 is a plot of this equation.

Considering that the analog has been simplified by being restricted to a two-capacity system, the representation may be thought to be quite close. It is evident that a closer representation could have been obtained with a more complicated analog.

The theoretical solution for the ideal infinitely long rod cited by Mr. Peters, if considered as representative of a practical process, assumes all capacities and resistances of the process to be uniformly distributed, and permits the adjustment of only one parameter. It is difficult to imagine an industrial process which exhibits a resemblance to such a hypothetical case. It is the author's opinion that, by using hydraulic analogs, practicability may be attained with fewer simplifying assumptions due to the availability of any desired number of parameters.

The probability-integral, or error function

$$\frac{2}{\sqrt{\pi}} \int_0^z e^{-y^2}\, dy$$

[8] "Mathematics of Modern Engineering," by Doherty and Keller, John Wiley and Sons, 1936.

has found wide use for the solution of linear-heat-flow equations. Equation [7] of Mr. Peters' discussion, as written, would indicate a decrease in P as time increased. For the case of the semi-infinite rod, heated suddenly at the finite end at $t = 0$, the theoretical solution in per cent of ultimate change for any point at any time is given by

$$P = 100\left[1 - \frac{2}{\sqrt{\pi}}\int_0^{\left(\frac{x}{2}\sqrt{\frac{RC}{t}}\right)} e^{-y^2}\,dy\right]$$

where R and C are the distributed thermal resistance and thermal capacity, respectively, of the rod. The diffusivity κ, used by Mr. Peters, is the reciprocal of the product of the quantities R and C, and is a constant parameter.

The author wishes to acknowledge the new viewpoints contained in Mr. Smith's discussion but hastens to explain that there was no intention to imply any element of originality in the use of hydraulic analogies, but rather to extend their field of usefulness in connection with automatic-control analysis.

One of the most promising uses of the simplified mathematical representation of process characteristics lies in the analytical

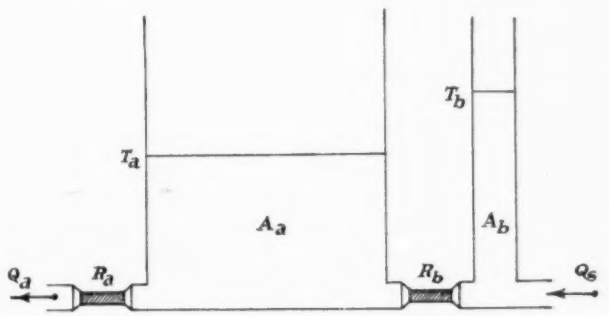


FIG. 5

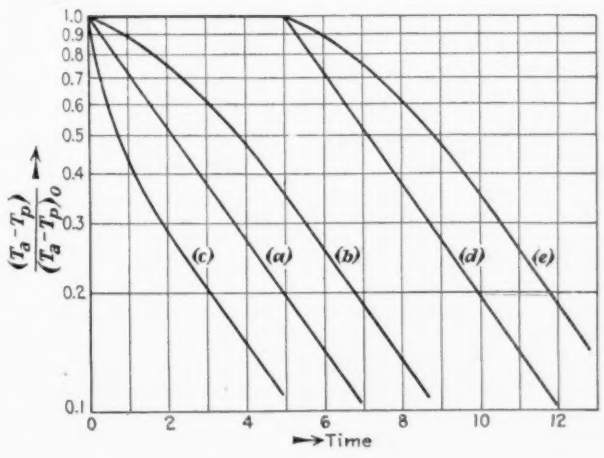


FIG. 6

study of the reactions of processes to the application of automatic control.

There has always been a great deal of confusion in the terminology with respect to process lags and the author would welcome any carefully considered standardization. The term "lag" has become almost useless due to a variety of ill-defined usages.

Many useful conclusions can be drawn from an analysis of single-capacity processes but only a limited number of these have any counterpart in the analysis of a two-capacity process. The author's experiences in the analysis of multiple-capacity processes has led him to despair of any interpretation of these processes in terms of the properties of simpler processes. The opposite development, in which simpler processes are considered as special cases of more complex processes, has permitted a greater degree of generality. The author hopes in future publications to extend his general development along the lines here indicated.

In view of the limited scope of the present paper, the author does not feel that the development has progressed sufficiently to substantiate any practical classification of processes. It is, however, within the scope of the present development to make a qualitative characterization of the three types of process lag as defined and discussed in the paper. Fig. 6 shows curves on semilogarithmic coordinates representing, in a purely qualitative way, the characteristics of these various process lags and of their possible combinations. These several curves represent: (*a*) Capacity lag alone; (*b*) transfer lag for which the initial rate of change is zero; (*c*) transfer lag for which the initial rate of change is finite; (*d*) distance-velocity lag associated with capacity lag alone; and (*e*) distance-velocity lag associated with type-(*b*) transfer lag.

There is no question but that dimensionless ratios and dimensional analysis will play an important part in the future development of theories governing automatic control; but dimensional analysis cannot be intelligently applied in any field until the nature of the phenomena involved is fully understood.

The portion of Mr. Smith's discussion relative to automatic controllers applied to processes has definitely gone beyond the development of process analysis as covered by the paper under discussion. It seems, however, that some of the conclusions concerning hunting and nonhunting charcteristics must be limited to these controls applied to single-capacity processes.

The order of the combined differential equation representing both the instrument and process can be increased either by the characteristics of the instrument or those of the process; thus any second-order process, combined with a simple proportional controller, may produce damped oscillations, if the sensitivity of the instrument affects the coefficients so as to give a pair of imaginary roots. Proportional control applied to third-order processes may produce continuous or increasing oscillations when the sensitivity is sufficiently high.

Many mathematical truths may be used to formulate theorems relating to automatic control but the author feels that a much more substantial progress can be had by following a gradual development and by the application of general theorems developed for the more complex processes and instruments, to the simpler processes considered as special cases of the more complex.

Design of Flanged Joints for Valve Bonnets[1]

E. C. PETRIE.[2] The subject of valve-bonnet-joint design is one which is of considerable interest to valve manufacturers and users. The high pressures which are now being used in many industries coupled with high temperatures have made flange-bonnet-joint design a problem which must be considered seriously. The authors have presented in the paper under discussion one of the first practical applications of the design

[1] Published as paper FSP-60-9, by J. D. Mattimore, N. O. Smith-Petersen, and H. C. Bell, in the May, 1938, issue of the A.S.M.E. Transactions, p. 297.

[2] Design and Development Department, Crane Company, Chicago, Ill.

TABLE 1 APPROXIMATE INSTALLATION STRESSES FOR ALLOY-STEEL BOLT STUDS DUE TO MANUALLY APPLIED TORQUE LOADS[a]

Size of bolt stud, in.	Installation stress, lb per sq in.[b]
3/4	52,000
7/8	48,000
1	45,000
1 1/8	42,500
1 1/4	40,000
1 3/8	38,000
1 1/2	36,500
1 5/8	35,000
1 3/4	34,000
1 7/8	33,000
2	32,000
2 1/4	30,000
2 1/2	28,500
2 3/4	27,000
3	26,000

[a] "The Ring Joint—Its Relative Merit and Application," by E. C. Petrie, *Heating, Piping, and Air Conditioning*, vol. 9, April, 1937, p. 213–220.

[b] Based on the formula: $S = 45{,}000/\sqrt{d}$, where S = installation stress, lb per sq in.; and d = nominal diameter of bolt, in.

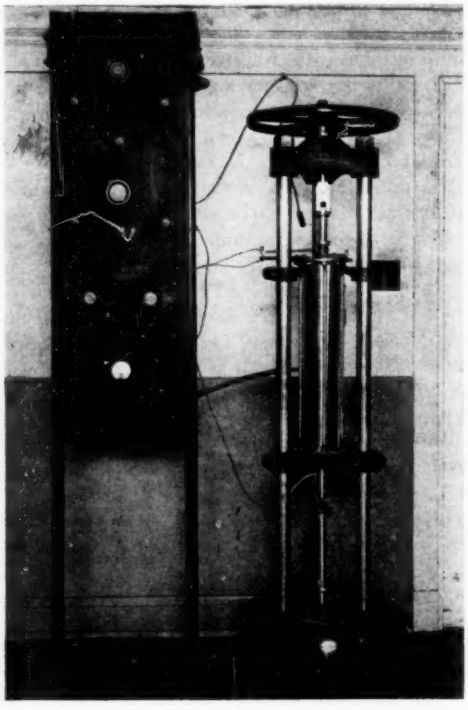


FIG. 1 RELAXATION-TEST UNIT

formula for flanges as developed by Waters, Wesstrom, Rossheim, and Williams.[3]

The authors have pointed out that for flanges smaller than 24 in. the calculation of the distributed moments should be based upon the mean diameter of the hub at the junction of the hub and flange. The writer believes that this same change from the original formula should be incorporated in the formula for X where the value of B should be changed to $(B + g_0)$. Similarly, the value of $h/\sqrt{(Bg_0)}$ should be changed to $h/\sqrt{[(B + g_0)g_0]}$.

In designing a flanged bonnet joint the authors correctly state that the starting point is to select a proper ratio of residual gasket contact pressure to line pressure. This ratio is one which must be based upon actual operating experiences and tests combined. The authors have eliminated the gasket material as a criterion by proportioning the width of the gasket face on the flange so that the compressive yield strength of the flange material will not be exceeded. This method for determining gasket dimensions may be suitable when the gasket material is strong enough to withstand the compressive loads applied.

There is another point which must be considered, that is, in most cases the bolt load applied to a flange joint is in excess of any load used in the theoretical calculations. For example, in the determination of the flange dimensions for a 6-in. 1500-lb bonnet joint the authors obtain a gasket width of 5/16 in. and a total gasket area of 9.94 sq in. The bolting used in this joint is sixteen 1 3/8-in. bolts. In sledging or pulling up these bolts in the shop or field, stresses of approximately 40,000 lb per sq in. are usually applied. A stress of this magnitude produces a total bolt load of 18.48 × 40,000 or 739,200 lb. This in turn develops a compressive stress of 739,200/9.94 or 74,400 lb per sq in. in the gasket or flange material. The writer believes that probable maximum stresses which may be applied when bolting a bonnet joint should be used in determining the proper gasket area.

In order to illustrate the probable magnitude of stresses which are applied in service, the writer has included Table 1 of this discussion which gives stresses based on a check up of bolt stresses in joints made up in the field and in the shop.

The American Standard steel end flange male and female facings give, as a general rule, low residual gasket pressures due to the wide gasket face. Experience has indicated that the gasket width should be decreased to approximately the width of the wide tongue-and-groove facing to insure a tight joint. Under this condition the average compression load applied by the bolts, when pulled up to stresses given in Table 1 of this discussion, is about 30,000 lb per sq in. as this compressive stress has produced satisfactory joints in the past, it seems to the writer that proper gasket proportions can be obtained by considering 30,000 lb per sq in. as the design gasket compression using bolt stresses as given in Table 1 of this discussion.

DESIGN OF HIGH-TEMPERATURE FLANGE JOINTS

When flange joints are designed for temperatures above 750 F, factors enter into the problem which do not have to be considered at the lower temperatures. At these high temperatures, the materials used for the bolts and flange are subjected to a plastic deformation which has been termed "creep." This plastic deformation causes residual gasket compression to decrease as increases occur in the time the joint is subjected to the temperature. If the decrease in gasket compression becomes great enough, the joint will leak.

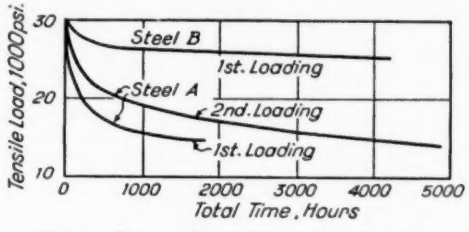


FIG. 2 TYPICAL RELAXATION-TEST CURVES

Tests conducted by the company with which the writer is affiliated, on a 10-in. 1500-lb flanged tee equipped with Sarlun-type joints, indicated that joint leakage occurred when the bolt stress dropped to approximately twice the total load exerted by the pressure on the area bounded by the outside diameter of the contact face.

In the last few years an attempt has been made by several laboratories to determine the amount of creep which occurs in steels used for manufacturing bolts, in order to develop design stresses for bolting materials. The first tests which were made

[3] "Formulas for Stresses in Bolted Flanged Connections," by E. O. Waters, D. B. Wesstrom, D. B. Rossheim, and F. S. G. Williams, Trans. A.S.M.E., vol. 59, April, 1937, paper FSP-59-4, pp. 161–169.

measured the creep at various temperatures under constant loading. This method of testing has now been supplanted by attempts to reproduce the same conditions which prevail in flange joints by testing under loads which become less as creep increases. This method of testing has been referred to as relaxation testing of bolting materials. A special machine for obtaining relaxation curves of steels under conditions similar to those occurring in a flange joint is shown in Fig. 1 of this discussion.

The test specimen under stress is held at a constant temperature in a special heater. The stress is measured by the strain in a calibrated external bar which is subjected to the same load as the test specimen. As creep progresses, the relaxation of load is measured on this external "weigh bar" and the values obtained are plotted against time in hours. Fig. 2 of this discussion shows the relaxation curves for several grades of alloy-steel bolting material.

It will be noted by referring to Fig. 2 of this discussion that the load-retention characteristics of steel *B* is superior to that of steel *A* due to its lower primary creep. It should be noted that the second loading of steel *A* indicates a greater load retention than the first loading. This leads to the conclusion that if bolts were retightened in a flange joint after the joint has been subjected to the operating temperature for a short period of time the joint will remain tight for a longer period of time than if the bolts were not retightened.

Another conclusion not apparent from Fig. 2 of this discussion is that if higher initial stresses be applied to the test specimen, the initial relaxation curve will be much steeper and the final stress after a period of time will be approximately the same or only slightly higher than that shown at, say, 4000 to 5000 hr.

Since the load-retention curve for bolting steels indicates the amount of stress which remains in a bolt after being subjected to temperature for a certain period of time, it is the writer's opinion that design bolt stresses can be obtained in this manner. If we again refer to Fig. 2 of this discussion it can be seen that the design stress for steel *B* at 850 F can be assumed as approximately 25,000 lb per sq in.

A.S.A. Flanges

The authors state in their conclusion: "In fairness to the A.S.A. flanges, it should be stated that they were purposely designed with excessive bolting." It must be remembered that these flanges are generally used with the raised face or male-and-female type of joint which comprises a gasket width and outside diameter of gasket much greater than the narrow tongue and groove illustrated by the authors. Under these conditions the residual gasket compression using A.S.A. bolting is much lower than that which has been used by the authors. With reference to the dimensions of the flanges, the writer does not believe that the differences shown are due to excessive bolting, but to the fact that loose flanges, such as the lap-and-screwed type, were used in determining the flange dimensions.

E. O. Waters.[4] Slightly over a year ago, the writer presented, along with three coworkers, the results of a very extensive study of flange stresses and deflections.[3] At that time, it was evident that there was some misapprehension on the part of designers and users that the authors were claiming a particular degree of potency for the three main formulas of their paper[3] (Fig. 1 of the present paper). Nothing could have been further from the truth; the formulas were no more nor less than the results of a formal stress analysis applied to specific locations of the flange structure at which it could reasonably be assumed that the stresses would have maximum values. Having accomplished this work, there still remained the problem of interpreting these results, determining suitable factors of safety which might, or might not, permit exceeding the elastic limit at certain points, and finding out how best to use the formulas, not only for checking but for assisting in the actual design of new sizes. The question of bolt stress and gasket pressure was also involved, and here again the paper[3] made no attempt to lay down any rules or even make suggestions.

It is extremely gratifying to find that others have been eager to carry on the work which the writer helped to initiate. Full-scale tests have been made,[5] while others it is hoped will be made public in the near future. The Taylor Forge and Pipe Works have developed one very practical design system, based on an orderly arrangement of calculations outlined in special work-sheets which they have constructed; however, in the present paper[1] we have a detailed account of another design procedure, independently arrived at, which shows clearly the difficulties that beset the flange designer, and gives an extremely interesting and effective means of overcoming them.

In the writer's opinion, the chief merit of the method outlined in this paper[1] lies in its elimination of many troublesome variables at the outset by the simple assumption that $f = 1$, $g_1/g_0 = 1.25$, and K is determined by the bolt circle that will carry the necessary bolting with the minimum practical clearance for the nuts. Nothing then remains but to find the flange thickness t in relation to one of the stresses, by means of the cubic or fifth-degree equation as indicated. Designers may differ when it comes to the auxiliary computations, such as that for gasket load and dimensions, bolt load and dimensions, and flange moments, but these are really nonessential to the method, and should be welcomed as adding to the body of information available on points that are still subject to differences of opinion. In particular, attention should be called to the choice of the value $f = 1$, since it not only contributes to the concise nature of the method, but eliminates at one stroke any argument as to the possibility of high stress at the small end of the hub or the critical nature of such a stress, if high. The analysis of the effect of hydrostatic pressure is worth noting, since it gives a good qualitative picture of the resultant variation in S_H, S_R, and S_T. The arbitrary 10 per cent reduction is of course a judgment factor and must be accepted as such in lieu of any more precise data on the effect of internal pressure.

In their concluding paragraphs, the authors make a comparison with A.S.A. flanges which, to say the least, is remarkable. There is no doubt that in many cases, flanges are subjected to heavy loads caused by dead weight and thermal expansion which are largely indeterminate, and here the A.S.A. standards offer the designer a set of proportions which experience has shown to be well within the limits of safety. When, however, the actual stress conditions are predictable with some degree of accuracy, it is interesting to note that the use of rational formulas, based upon an analysis of these conditions, makes possible a large saving in space and weight over what would result from the use of empirical data.

In their fourth paragraph, the authors mention an approximation that was made in the original development of the formulas for S_H, S_R, and S_T which has some bearing on their application to small-size flanges. It may be well to throw some further light on this matter by the following two considerations:

1 In deriving the equations, three simultaneous equations were set up in these three unknowns: Bending moment between flange and hub, shear between flange and hub, and rotation of

[4] Associate Professor in Mechanical Engineering, Yale University, New Haven, Conn. Mem. A.S.M.E.

[5] "Tests of Heat Exchanger Flanges," by D. B. Rossheim, E. H. Gebhardt, and H. G. Oliver, Trans. A.S.M.E., vol. 60, May, 1938, paper FSP-60–10, pp. 305–314.

flange and hub, all at the point of juncture. Now, as far as the flange is concerned, this point was taken at the inside diameter and middle surface, whereas for the hub it was taken at the large end and the mean diameter, for convenience in computation. These points are indicated approximately in Fig. 1 of the paper,[1] in the sectional view of the flange and hub, where the arrows representing S_R and S_H have zero length, and it is obvious that they are far from coincident.

The writer believes that the rotations at the two points are practically identical in the vast majority of cases, since, to a first approximation, the flange section acts as a rigid rectangle and rotates without curvature of the middle line. However, there may be an appreciable difference between the respective moments and shears, the importance of which cannot be determined without much additional computation. It is suggested that in any doubtful case, calculations be made on the basis of $K = A/B$, and then compared with those made on a basis of $K = A/(B+g_1)$; if the first assumption is possibly on the unsafe side, the second one certainly errs on the side of safety, since it neglects the material of the flange between the diameters $B + g_1$ and B.

2 In the analysis of the hub, which is based on the theory of a beam on an elastic foundation, it was assumed that the effective diameter of the hub is equivalent to the inside diameter, and two items of considerable importance for small-size flanges were completely ignored, namely: (*a*) The shear P on a plane normal to the axis of the hub is not uniformly distributed, since the hoop stress (proportional to the hoop strain) is greater at the inside diameter than at the outside diameter; (*b*) the unit slice of hub, which is considered equivalent to an elastically supported beam, is not rectangular in section, but trapezoidal, with a minimum width of 1 and a maximum width of $1 + (2g/B)$, so that the neutral axis and moment of inertia are subject to correction. For a hub of constant thickness, correction of these two items is equivalent to using an effective hub diameter of

$$B\left[\frac{2 + 4\frac{g}{B} + \frac{4}{3}\left(\frac{g}{B}\right)^2}{\left(\frac{B}{g} + 1\right)\log_e\left(1 + \frac{2g}{B}\right)}\right]^{1/2}$$

or approximately

$$B\left[1 + \left(2\frac{g}{B}\right) + \frac{1}{3}\left(\frac{g}{B}\right)^2\right]^{1/2}$$

for hubs of constant thickness.

This corrected diameter should be used in all the formulas and charts involving the ratio or product of B and g_0. For tapered hubs, the correction can be closely approximated by using the mean of g_0 and g_1 in the correction factor.

The calculation in Table 2 of this discussion shows the effect of these corrections in a deliberately chosen extreme case, and indicates the danger of a too carefree application of the formulas. As already stated, the change of K from 5 to 2.5 is overconservative; nevertheless, the true stresses probably lie somewhere between those in columns 2 and 3 of Table 2 of this discussion, which would make them quite different from the results of column 1

TABLE 2

Let A = 5 in., B = 1 in., $g_0 = g_1$ = 1 in., t = 1 in., $h = \infty$, and ν = 0.3. Then:

	Uncorrected: B = 1, K = 5	Corrected for B only: B' = 1.8269	Corrected for B: $K = A/$mean hub diam.
K	5.0000	5.0000	2.5000
B/g_0	1.0000	1.8269	1.8269
F	0.9089	0.9089	0.9089
V	0.5501	0.5501	0.5501
f	1.0000	1.0000	1.0000
T	0.4412	0.4412	1.3391
U	1.3272	1.3272	2.4736
Y	1.2078	1.2078	2.2510
Z	1.0833	1.0833	1.3810
X	0.2109	0.2441	0.7075
S_H	0.2109 M	0.2441 M	0.7075 M
S_R	0.4665 M	0.4060 M	1.3418 M
S_T	0.7024 M	0.7680 M	0.3980 M

ARTHUR MCCUTCHAN.[6] This paper is a valuable complement to the paper by Waters, Rossheim, Wesstrom, and Williams.[3] The transition from purely theoretical flange analyses to specific application often is a difficult process and this detailed account[1] should serve a useful purpose. As pointed out by the authors, the requirements of a formula for checking existing designs do not necessarily coincide with the requirements of a formula for developing new designs. The authors' device of using a similarity of proportions to reduce some of the formula ratios to constants appears to have merit.

The comparison of the new series of bonnet flanges with fitting flanges of the American Steel Flange Standards having approximately the same bore helps tie in these recent analyses with the original Waters-Taylor formulas. The reduction in weight of the flange ring is noteworthy, but should be attributed in part to the greater number of smaller bolts and the type of flange facing employed. Incidentally, it would seem that even lighter flanges would have resulted if the tongue-and-groove facing had been located close to the bolt holes rather than near the bore of the flange.

The writer is inclined to question the statement that the bolting in the Steel Flange Standards is heavier than necessary for the raised-face type of joint for which the standard was designed. Considerable improvement in joint life was obtained in an experimental high-temperature installation with which the writer is familiar by reaming out the bolt holes of 600-lb flanges and installing $^1/_8$-in. oversize bolts. Had the flanges been the weaker part of the assembly, the use of larger and stronger bolting would have been less effective in increasing joint life.

In view of the urgent need for practical rules for the design of bolted flange connections which can be incorporated in the A.S.M.E. Boiler Construction Code, A.S.A. Code for Pressure Piping, and other codes, this paper giving a demonstration of the practicability of designing flanges by the recent formulas developed by Waters and his associates,[3] is particularly timely.

AUTHORS' CLOSURE

The discussion by Professor Waters discloses that certain corrections are required in the stress formulas given in "Stresses in Bolted Flanged Connections"[7] where such formulas are used for the design or analysis of flange connections having high K values ($K = A/B$; A = outside diameter, B = inside diameter of flange), such as occur with flange connections for valve bonnets.

In addition to the corrections, the discussion also suggests that an alternative set of calculations of flange stresses be made for such types of flanges by using a smaller value of K, namely, $K = A/(B + g_1)$, in the corrected formulas, instead of the true value, A/B.

The object in making this alternative set of calculations is to obtain another set of flange-stress values above which the true stresses will not go. "The true stresses," it is stated, "probably lie somewhere between" stresses figured for $K = A/B$ and for

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[7] By E. O. Waters, D. B. Wesstrom, D. B. Rossheim, and F. S. G. Williams, Trans. A.S.M.E., vol. 59, April, 1937, paper FSP-59-4, pp. 161–169.

$K = A/(B + g_1)$ in the corrected formulas. The reason stated for considering the use of the smaller values of K as giving, so to speak, an upper limit of stress values is, that the addition of the flange-hub thickness g_1 to B in the denominator is equivalent to neglecting the material of the flange between the diameters B and $(B + g_1)$ and consequently such calculation "errs on the side of safety."

A flange weakened by a cutout of this amount of material at this particular place, most certainly will show an upper limit of stress values, providing the formulas which are used for calculating the stresses apply to this weakened and odd form of flange. This proviso must be made, otherwise the stresses calculated lose their meaning.

To the authors, however, it seems more natural to interpret the smaller value of K as simply creating another flange with a smaller outside diameter. Such smaller diameter is then to be considered as equal to $[B/(B + g_1)]A$, for this value of it will give the suggested smaller value of K of $A/(B + g_1)$. However, if this interpretation is accepted as correct, then the authors do not any more see clearly why a reduction in the outside diameter should cause the flange stresses calculated accordingly always to become a logical upper limit for flange stresses in the original flange. The authors realize that such stresses will be higher than the original stresses but cannot agree always to consider them as limiting stresses. Further, there does not seem to be any need for the upper limit of the stresses. Tests have shown the formulas to give reliable and safe stress results and therefore the authors prefer to accept the originally calculated stresses as final.

The correction of the diameter B given in the second paragraph of the discussion is most helpful and the authors are grateful for this information. Of the two formulas therein given, the last or approximate formula appears to be the most practical one to use in calculating flanges, and this formula when converted to tapered-hub flanges will be

$$B_c = B\left[1 + \frac{g_1 + g_0}{B} + \frac{1}{12}\frac{(g_1 + g_0)^2}{B^2}\right]^{1/2}$$

This B_c, therefore, or the theoretically correct one, is to be used instead of B in all the formulas and charts involving the ratio or product of B and g_0. That means the charts for f, F, and V and the formulas for X and h.

In Mr. McCutchan's discussion it is suggested "that even lighter flanges would have resulted if the tongue-and-groove facing had been located close to the bolt holes." It is quite possible to believe that this would be so for the moment arms of the forces concerned appear to be shortened by moving the gasket toward the bolt holes. However, as the gasket is moved outward the forces acting on the gasket and on the annular surface between the gasket and the inside diameter will increase in amount. This increase in the forces requires more bolt area, therefore, larger bolts, and larger bolts in turn will increase the bolt-circle diameter, thereby causing an increase in the moment arms. To show this, a practical example has been worked out for a flange of 10 in. inside diameter with a constant hub thickness of 3 1/8 in. and for a pressure of 1500 lb per sq in. On this flange the gasket has been moved out progressively in four equal steps 1 in. apart, keeping the gasket width constant. The results are tabulated in Table 3.

TABLE 3

Gasket dimensions, ID, in.	Gasket dimensions, OD, in.	Moment, in-lb	Bolt load, lb	Bolts, No.	Bolts, Size, in.	Bolt stress lb per sq in.	Bolt-circle diam, in.	Flange OD, in.
11 1/8	11 13/16	1005000	275800	16	1 1/2	12300	19 3/8	22 1/2
13 1/8	13 13/16	1229000	355600	14	1 3/4	12800	19 13/16	23 3/8
15 1/8	15 13/16	1423000	445000	14	2	12000	20 1/4	24 1/4
17 1/8[a]	17 13/16	1660000	543600	14	2 1/8	12800	21	25 1/4

[a] Diameter at inside of bolt hole is 18 13/16 in., or gasket OD is 1/2 in. from bolt hole.

In the table the number of bolts has been kept practically the same in order to have a fair comparison. It is, however, perfectly possible to start out with a few large bolts and to increase the number of them as required by the increase in force, thereby keeping the bolt-circle diameter almost constant, but that would hardly constitute proper design practice and accordingly would not give a reasonable or fair comparison.

Attempts have been made by the authors to put the gasket location in a purely mathematical form with one variable, namely, the distance of the gasket from the inside diameter of the flange, but bolt sizes cannot be reduced to a continuous function, therefore, approximations and assumptions are necessary. These approximations complicate the formulas to an extent that makes it difficult, if not impossible, to interpret the result and obtain a universal answer. The actual example given is typical of the flanges which the authors have investigated, and for such flanges it is well to locate the gasket as near to the inside diameter as it is practical and possible.

The bolting for standard flanges referred to in the discussion is a matter of considerable scope. The authors meant with the term "excessive bolting," to express the facts that the bolting requirements for a line flange are higher than for a bonnet flange and that due to the several different flange facings for which the standard bolting is designed, such bolting for some facings, of necessity, will be excessive, while for others, not at all excessive. Overbolting and excessive bolting are terms needing a definition or qualification before they can be fully discussed or understood.

In the discussion by Mr. Petrie there are certain calculations pertaining to a gasket unit pressure calculated by giving the bonnet bolts an assumed unit stress of 40,000 lb per sq in., whereby the gasket unit pressure becomes excessive. The assumption of 40,000 lb per sq in. is rather unwarranted in view of the authors' expressed statement in the paper that the gasket was designed to prevent an overloading of it. From their statements it naturally follows that the bonnet bolts would not be stressed to 40,000 lb per sq in. or any other arbitrarily selected stress, but to a stress in full conformity with the design of the gasket.

A bonnet flange is quite different in one respect from a line flange in that a bonnet flange is made up in the shop where full control of the bolt stresses can be exercised, while a line flange is made up in the field where such control is not often present, if ever. This control is exercised by the company with which the authors are connected, and the bonnet bolts are all uniformly stressed to a predetermined stress, and so marked by tag. That stress is not over 40,000 lb per sq in., but depends upon the flange-material yield point and the hydrostatic test pressure, as explained in the paper.

The authors wish to call attention to an error which appears on page 299 of the paper in Eq [4]. This formula should read

$$T = \frac{3K^2\left(1 + 2\frac{1 + \nu}{1 - \nu} \times 2.3026 \log_{10} K\right) - 3}{\pi(K - 1)\left(1 + \frac{1 + \nu}{1 - \nu}K^2\right)} \quad \ldots [4]$$

Operation of Emergency Shutoff Valves in Pipe Lines[1]

ROBERT W. ANGUS.[2] In his paper the author called attention to the very important case of the bursting of a pipe line above

[1] Published as paper HYD-59-10, by F. Knapp, in the November, 1937, issue of the A.S.M.E. Transactions, vol. 59, p. 679.

[2] Professor of Mechanical Engineering, University of Toronto, Toronto, Canada. Mem. A.S.M.E.

TABLE 1 COMPARISON OF THE EFFECTS OF A PIPE BURSTING JUST BELOW THE VALVE, AS OBTAINED BY THREE DIFFERENT METHODS[a]

Time after pipe failure occurs, sec...	0	0.20	0.40	0.60	0.80	1.00	1.2
Pressure head at gate A, ft:							
Loss concentrated at two points, Fig. 1	340	232.00	129.00	25.00	161.00	323.00	625.0
Loss concentrated at pipe entry, Fig. 2, solid lines	340	236.00	132.00	28.00	178.00	320.00	582.0
Friction loss neglected, Fig. 2, dashed lines	400	277.00	155.00	32.00	248.00	523.00	1010.0
Velocities at gate A, fps:							
Loss concentrated at two points, Fig. 1	4	5.08	5.91	6.67	6.42	5.98	4.2
Loss concentrated at pipe entry, Fig. 2, solid lines	4	5.04	6.08	7.15	6.62	5.92	4.0
Friction loss neglected, Fig. 2, dashed lines	4	5.23	6.46	7.70	7.88	7.60	5.3

[a] The methods referred to are shown in Figs. 1 and 2 of this discussion.

NOTE: This table shows the type of error that may result from neglecting friction head in this case. The results by the two methods of allowing for friction agree fairly well, but neglecting friction makes a great difference both in pressure and velocity.

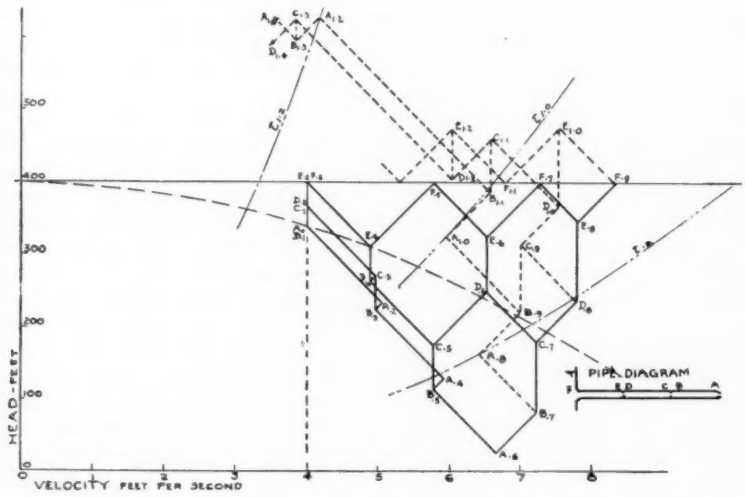


FIG. 1 EFFECT OF PIPE BURSTING JUST BELOW THE VALVE, WITH VALVE CLOSURE AS GIVEN BY τ AND ALLOWING FOR FRICTION

(H_0 = 400 ft, V_0 = 4 fps, a = 3216 fps, and L = 965 ft.)

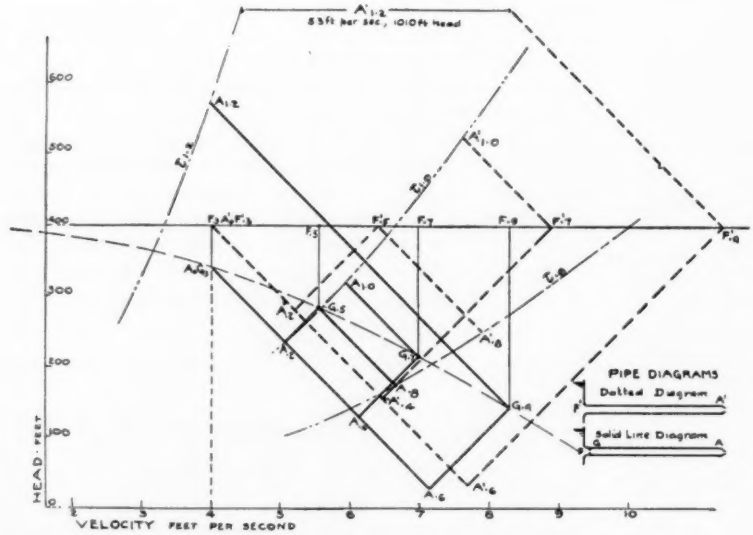


FIG. 2 EFFECT OF PIPE BURSTING JUST BELOW THE VALVE WITH FRICTION NEGLECTED (DASHED LINES) AND WITH FRICTION ALLOWED FOR BY SCHNYDER'S METHOD (SOLID LINES)

(H_0 = 400 ft, V_0 = 4 fps, a = 3216 fps, and L = 965 ft.)

or below a valve. Such a condition tests the value of the valve and its controls to the limit, particularly if the break were to occur just below the valve, because the pressure just above the valve would then become nearly atmospheric, exceeding the latter only by the pressure necessary to overcome the loss in the valve and jagged opening in the pipe. Accompanying this drop in pressure is a corresponding increase in discharge, and closure of the valve must be effected with great care to avoid further ruptures in the line, however, the operation must usually be rapid or serious damage may be done by the discharging water.

The author shows a shutoff valve with a self-acting by-pass valve, an arrangement of much value in larger pipes, because when the large valve becomes nearly closed it is difficult to control the areas in the way necessary to prevent water hammer, therefore, the smaller by-pass is most helpful. The writer discussed elsewhere[3] one such case, but if the by-pass connections are short and free from losses, the writer believes that the action of the by-pass differs in no way from a properly controlled large valve, assuming such control practicable.

In the case of the burst pipe, with large increase in velocity and great decrease in pressure, it does not appear to be possible to neglect friction loss and velocity head. In some experiments made by the writer on a $1^1/_4$-in. pipe about 160 ft long and discharging freely into the atmosphere, it was found that the calculation without considering friction gave answers far from those determined by measurements. This is to be expected, because while in normal operation on power plants and distribution piping the losses are usually much under 10 per cent, yet with a burst pipe the entire head available is lost in friction or converted into velocity head.

In order to observe the effect of friction, the writer solved the case shown in Figs. 1 and 2 of this discussion, allowing for losses by two different methods, and then he solved the same problem neglecting the losses. The data are not the same as those used by the author of the paper[1] under discussion, but the cases compared are all on exactly the same basis.

The problem solved by the writer is for a pipe with H_0 = 400 ft, V_0 = 4 fps, and a/g = 100 sec. The friction loss has been assumed as shown on the dashed line through A_0 and the point H_0 = 400 ft, V = O; in this case, in order to make a clear figure, the friction loss has been taken as 60 ft at a steady velocity of V_0 = 4 fps. Further, the selected value of L/a = 0.3 sec, and, when the pipe bursts, the pressure drop at the valve is assumed to vary uniformly with time, a statement which is not greatly in error, however, the exact relation is easily allowed for. It is further assumed that the minimum pressure at the gate is

[3] "Air Chambers and Valves in Relation to Water Hammer," by R. W. Angus, Trans. A.S.M.E., vol. 59, November, 1937, p. 667.

slightly different for all three cases, in view of the fact that the calculated velocities at this minimum pressure are not the same.

In Fig. 1 of this discussion, all of the losses are assumed concentrated at two points, as shown in the sketch of the piping. The broken lines have no significance beyond helping the reader to keep different parts of the diagram distinct.

In Fig. 2 of this discussion are two diagrams, one in solid lines and the other in broken lines. The one in solid lines shows one of Schnyder's constructions, which assumed the entire head loss concentrated near the reservoir end of the pipe. The diagram in broken lines shows the construction if friction were neglected. The principles used in constructing these diagrams are explained in the writer's papers.[3,4,5]

The same law of valve closure has been used throughout, and corresponds to the values of τ shown; it is assumed that there is no by-pass. The valve is assumed to have no effect until after 0.6 sec. Table 1 of this discussion compares the results obtained with the three constructions, that is, Fig. 1 of this discussion and the two constructions in Fig. 2 of this discussion.

Tests of Heat-Exchanger Flanges[1]

S. C. Hollister.[2] The writer believes that in the test vessel used by the authors the length of the upper section between the tapered hub and the dished-head knuckle radius is so short that the hub would not be free from effects of distortion arising from the performance of the head. The writer believes the minimum dimension should be 18 in. in order to prevent the possibility of affecting the flange region against the restraints offered by the head; and the writer is of the opinion that there is enough restraint to account for at least part of the discrepancies shown in Figs. 10 and 11 of the paper.

N. O. Smith-Petersen[3] and J. D. Mattimore.[4] A study of the authors' flange tests leads to the conclusion that the formulas developed by E. O. Waters and his associates, presented in another paper,[5] produce designs which are on the safe side by an appreciable margin. The test results are more conservative than the calculated results except in the case of the flange rotation due to bolt load for flange *C*-2 and for the hub stresses in flanges *C*-1*B* and *C*-1*C* in the higher ranges of bolt load where it would appear some yielding was taking place. In the case of flange *C*-2, the test points in Fig. 9 of the paper for flange rotation due to bolt load practically coincide with the calculated curve for the worst moment condition. In view of the complexity of the stresses involved and the difficulties of obtaining exact test data, the writers believe the correspondence of the theoretical figures with the test results is very good. The test is also of interest in several of its details.

Of interest is the banjo attachment of the Huggenberger tensometers to the hub. It is a clever solution of this problem and should prove very helpful to others contemplating similar tests.

The arrangement of ball bearings at both ends of flange studs is a refinement which assures a greater accuracy in stud-length measurement than obtained by flat surfaces on the studs.

The arrangement of two ball bearings on a few studs for measuring stud bending does not appear to have given conclusive results, for no data or comments concerning the usefulness of it are given. However, stud bending appears to be present in results recorded on the many charts. Upon them are shown calculated curves for the stud load acting through the center line of the studs and acting on the inside shank line of the studs. The actual measured results seem to change from one of these lines of action toward the other as the bolt load increases. The direction of the change is from the shank line toward the center line of the the stud. This direction is opposite to that expected to take place were the change due to flange deflection only, and therefore appears to be due to bending of the studs. A simple calculation will confirm this and will show that high stresses are produced in the studs when eccentrically loaded. In fact, when the stud load acts on the shank line of the stud the bending stress is exactly four times as great as the direct tensile stress caused by the load. Accordingly, for high stud stresses it seems likely that the stud reaction is more nearly through the center line of the stud than elsewhere.

Fig. 7 of the paper, showing the influence of the internal pressure upon the bolt load, is of considerable importance. Again it is shown by tests that an appreciable loss of bolt load is caused by the internal pressure, but the extent to which it is due to radial and to axial internal pressure is not stated. The gasket location may have much to do with the axial effect.

In the design of flanges for heat exchangers and similar vessels for comparatively low pressures and temperatures, it may be permissible, from an economic standpoint, to allow relatively high hub stresses. On the other hand, it is the writers' opinion that a flange designed for both high-pressure and high-temperature operation should have a much lower permissible hub stress. A high initial hub stress, under high-temperature conditions, might conceivably result in a readjustment of stress conditions within the structure which would defeat the designer's purpose. Actually there should be little, if any, economic hardship involved in this since conditions involving high pressure and high temperature usually bring about proportions producing low hub stresses without much effort on the designer's part.

The authors made an excellent selection of the types of flanges to be tested, since they constitute two extreme designs between which almost all other designs can find a classification.

T. McLean Jasper.[6] The authors have tested two sets of flanges made of ductile steel which have tapered hubs. In the use of ductile-steel flanges, the ductility acts as an extra insurance with reference to the operation of the flanges in service because at the point of maximum stress there is a certain amount of relief long before much deformation occurs. However, a test on a formula for design could best be carried out with something like cast iron, or some other brittle material, which does not adjust itself and for this reason will not tend to obscure the problem.

The tapered-hub flange does not in general represent a large quantity of flanges used in pressure vessels or in industrial work of all kinds. One important fact is brought out by Fig. 7 of the paper, which reaffirms the idea that the greatest bolt loads, and incidentally the greatest stresses, occur with the type of flange and gasket considered when the bolts are tightened and not when the pressure is exerted.

The question of the value of the tapered-hub flange is one which

[4] "Simple Graphical Solution for Pressure Rise in Pipes and Pump Discharge Lines," by R. W. Angus, *The Engineering Journal* of the Engineering Institute of Canada, February, 1935, pp. 72–81, and May, 1935, pp. 264–273.

[5] "Water Hammer in Pipes, Including Those Supplied by Centrifugal Pumps, Graphical Treatment," by R. W. Angus, Proceedings of The Institution of Mechanical Engineers, vol. 136, 1937, pp. 245–331.

[1] Published as paper FSP-60-10, by D. B. Rossheim, E. H. Gebhardt, and H. G. Oliver, in the May, 1938, issue of the A.S.M.E. Transactions, p. 305.

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[3] Design Engineer, Walworth Company, New York, N. Y.

[4] Works Engineer, Walworth Company, New York, N. Y.

[5] "Formulas for Stresses in Bolted Flanged Connections," by E. O. Waters, D. B. Wesstrom, D. B. Rossheim, and F. S. G. Williams, Trans. A.S.M.E., vol. 59, April, 1937, paper FSP-59-4, pp. 161–169.

[6] Director of Research, A. O. Smith Corporation, Milwaukee, Wis. Mem. A.S.M.E.

can lead to very important discussions. The method of putting the gasket near the bolt hole in the test flanges increases the inside pressure area, and, incidentally, an increased requirement is experienced in the design for the pressure times the area on which the pressure acts. The taper of the hub increases the diameter of the bolt circle, and this increases the moment with a fixed gasket circle.

Has it been established that a tapered-hub flange is superior to a flange without a taper and with a reasonable fillet? May it not be asked that the Special Committee establish also values for hubbed flanges without taper. This is important where outlet necks are relatively thick for design reasons.

The writer would like to know whether the stresses shown in Figs. 12 to 15, inclusive, are actual stresses or strains multiplied by a factor. If the latter were true, then the real stress values would be something very different from those shown. In Figs. 14 and 15 are the hub stresses, due to pressure, negative? They are shown as increases, but Fig. 7 shows a decrease in bolt length and therefore in load, as the pressure is increased. How were these values obtained?

E. O. WATERS.[7] The writer is interested in the results secured by the authors of this paper[1] and regrets that lack of time has prevented him from undertaking an independent analysis of the data. As far as the effect of flange loading is concerned, whether by bolt pull or by hydrostatic end force, the theoretical formulas for flange rotation used as a standard of comparison by the authors show reasonable agreement with these tests. The same is true for hub stress. The fact that this agreement is improved by taking the effective bolt-circle diameter as *B.C.-d* instead of *B.C.* is beside the point; for after all, when the two major loads (gasket reaction and bolt pull) are applied at points which are only one third of the full flange width apart, and the moment which they produce is equated to a fictitious moment caused by forces at the extreme inside diameter and outside diameter of the flange, there is bound to be some discrepancy between calculated and test results, and a small change in the bolt-pull location makes a big change in the value of the equivalent moment. To state the matter from another viewpoint, tests such as those reported by the authors should be regarded not so much as a check on proposed design formulas, as a determination of empirical constants by which such formulas should be multiplied in order to fit designs of the type tested. In the present case, where B is large and K is close to unity, it appears that the empirical correction is obtained by using *B.C.-d* for the effective bolt-circle diameter. This does not mean that the bolt reaction is actually moved in to this circle; it is merely a way of correcting for the approximations involved in the highly artificial calculation of effective flange moment, the arbitrary location of bolt reaction, and other simplifying assumptions in the design formulas.

The foregoing statement is by no means intended as criticism, but simply as a warning not to extend the authors' conclusions to miscellaneous flanges of widely different proportions. On the other hand, the authors deserve the thanks of all individuals and organizations connected with the manufacture and use of flanges for their very carefully planned and thorough-going research. It is to be hoped that the work may be extended along the lines suggested, with a view to determining how much yielding it is safe to permit in the hub, especially at the small end. Possibly further light could be shed on other questions of a controversial nature, such as the relation of allowable bolt stress to allowable flange stress, and the actual location of the bolt reaction which may shift toward the flange center as the flange rotation increases.

In regard to the superimposed effect of pressure on flange rotation and hub stress, shown in Figs. 10*a*, 10*b*, 11, and 14*a* to 15*c*, inclusive, it is obvious that the calculated and test results are not in as good harmony as could be desired. However, these flange rotations are small in magnitude as compared with the basic rotations caused by flange loading, and it is frequently found that elastic measurements made under such conditions fail to check closely with theory. In the case of hub-stress increase, most of the disparity appears to be confined to two locations on the long-hub unit, which may be due to undiscovered causes of a local nature. At any rate, the agreement in general appears to be good enough to justify an effort to extend the design formulas for S_H, S_R, and S_T to include the effect of hydrostatic pressure, for certain specified values of the latter.

In conclusion, a brief comment would seem to be in order regarding the authors' doubts as to the accuracy of appendixes B and W of reference 6. As far as flat plate theory is concerned, there is no reason to expect a more or less fixed relationship between the flange rotations at the inside and outside radii. For all practical purposes, the two are equal for rings of the proportions tested, and any theoretical difference would probably be less than the range of experimental error to be expected in measuring the rotation at several points, either at the outside or inside radius. The theory itself indicates that the ratio of the two rotations may be either greater or less than unity. For a simple ring having K, B, and t as given in the authors' Appendix 3, sheet no. 1, and a loading equivalent to M according to the conventional method of flange load distribution, but no moment concentrated at the inner edge, calculation shows that the inner slope exceeds the outer slope by $0.9197\ M/E$. If the loading consists only of a moment $-0.147269\ M$ at the inner edge, the inner slope (now in the opposite direction) exceeds the outer slope by only $0.5808\ M/E$, which is not enough to compensate for the slope difference caused by the first loading. In other words, for the designs tested by the authors, the hubs were not stiff enough to make the flanges dish in the manner which the authors expected.

ARTHUR MCCUTCHAN.[8] The authors of this paper and the Heat Exchange Institute are to be commended for the realistic view they have taken of the problem of flange design. Testing such large heavy structures under conditions so nearly simulating actual use is reassuring as an indication that too great significance will not be attached to localized high hub stresses which have little if any relation to the problem of securing tight joints.

The choice by the authors of flange rotation as a measure of flange rigidity rather than flange deflection eliminated uncertainty as to the effect of crushing the gasket and contact faces which has attended many previous flange loading tests. The writer has particular reference to the tests described by Bailey[9] in which this direct compression had to be subtracted in attempting to correlate the results with derived formulas.

While the one-quarter-scale models of a flange with an 8-in. bore tested by Bailey are rather a far cry from the flanges of $30^3/_4$ in. outside diameter described in the paper under discussion the greater deflection found for drilled flanges over undrilled flanges in Bailey's tests suggests that this factor may need to be considered in analyzing the authors' test data. Apparently Bailey's findings on drilled and undrilled tensile specimens would indicate that the flange rotation for a given bolt loading might be expected to be about 15 per cent greater for a drilled flange than for an undrilled flange of the proportions given in Fig. 1 of the paper.[1] Tests reported by Solsnoes[10] on light flange rings ap-

[7] Associate Professor of Mechanical Engineering, Yale University, New Haven, Conn. Mem. A.S.M.E.

[8] Engineer, The Detroit Edison Co., Detroit, Mich. Jun. A.S.M.E.

[9] "Flanged Pipe Joints for High Pressures and Temperatures," by R. W. Bailey, *Engineering*, vol. 144, October 15, 1937, p. 419.

[10] Discussion by Leif Solsnoes of the paper "Flanged Pipe Joints for High Pressures and Temperatures," by R. W. Bailey, *Engineering*, vol. 145, April 22, 1938, p. 443.

proximately $13^3/_8$ in. outside diameter, $^5/_8$ in. thick, and $^{23}/_{32}$ in. wide with 0.351-in. holes showed the drilled ring to rotate approximately 30 per cent more than the undrilled ring. While this effect of bolt holes was stated to be less than deduced by Bailey[9] from tests on drilled and undrilled tensile specimens of similar proportions, it appears to be an important factor.

The shifting of the point of application of the bolt forces and gasket reaction as the bolt load is applied with its resulting reduction in moment arm has plagued the steps of all who have tried to apply flange formulas to actual flange design. The authors' tests throw some light on this question but in the opinion of the writer additional tests are needed to establish the effect of this reduction in moment arm on the relation between bolt loading and flange rotation.

The reason for providing the small nubbin at the center of the tongue facing as shown in Fig. 1 of the paper, is understandable since it tended to define the location of the gasket reaction. But, this simplification destroyed part of the value of the test since the customary shortening of the moment arm due to shifting of the

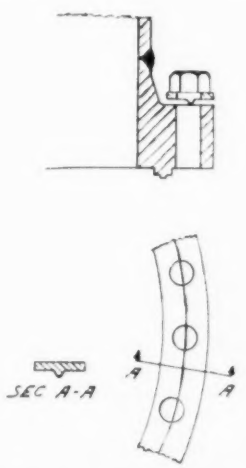


FIG. 1 CENTERING RING FOR PROPOSED CONSTANT-LEVER-ARM TEST

gasket reaction was prevented. If two additional tests were made, definite information could be obtained as to the effect of this reduction in moment arm due to shifting of the points of application of the bolt forces and of the gasket reaction. The following two tests are suggested:

1 Determine the bolt load versus the angular flange rotation for constant lever arm by providing an annular ring with centering rib, as shown in Fig. 1 of this discussion, to fit under the nuts and definitely locate the bolt forces on the bolt circle. The height of the nubbins on the gasket faces should be increased to give more positive location of the gasket reaction at the center of the tongue facing.

2 Determine the bolt load versus the angular flange rotation for variable lever arm by removing the nubbins from the gasket faces and applying bolting as in the present tests.[1]

The ratio of the bolt loads required to cause a given flange rotation determined from these tests should give a direct measure of the reduction in moment arm resulting from shifting of the points of application of the bolt forces and gasket reaction in an actual flange assembly.

The reduction in moment arm determined from such tests should give quantitative answers to one of the most intangible factors entering into the application of derived formulas to actual flange design.

It is hoped that these suggestions will be helpful and that the authors and the Heat Exchange Institute may find it possible to make these further tests.

AUTHORS' CLOSURE

It would be extremely helpful in a further study of the test results if Professor Hollister could make available any calculations which he has made on the effect of the hub length and dished head on deflections and stresses. It is admitted that a longer cylindrical section would more closely satisfy the assumption of an infinite hub. However, in line with the usual reasoning, hub lengths beyond (shell diameter $\times$ shell thickness) are considered to have little effect on the moment distribution and are usually neglected.

The authors are in agreement with Messrs. Smith-Petersen and Mattimore in their interpretations of the shift in the bolt reaction. The permanent loss in bolt load is attributed to relaxation of the gasket while the change in bolt load with internal pressure is probably a function of flange rotation. The opinion expressed that high hub stresses are not compatible with high-temperature conditions deserves further amplification, since it is not apparent why a readjustment of stresses or moments should prove harmful, although the proportions of such flanges are usually such that hub stresses do not control.

The authors agree with Mr. Jasper that the use of a brittle material would have afforded a better check on the initial hub stresses. However, it was not the object of this research to determine conditions at fracture, but rather to explore the readjustment of stresses between the flange and the hub due to yielding, and accordingly the use of a ductile material was prescribed. Tapered flanges are used on the shells of a large number of heat exchanger and pressure vessels as well as piping flanges, and are the rule rather than the exception, as stated by Mr. Jasper. While the taper does increase the bolt circle, it would appear obvious that the stress distribution is more favorable in a design with a reasonable taper than in a straight hub, which attracts a large part of the moment into the hub while offering no increase in resistance over that of a tapered hub of the same thickness at the root. It is generally found that a hub the thick end of which is twice the thickness of the small end gives the best conditions, combined with a taper length of approximately one half (shell diameter $\times$ shell thickness). While a location of the gasket close to the bolts produces an increase in the bolt load, the bolt moment which controls the flange design is reduced, in general, by such an arrangement.

Professor Waters' comments on the practical significance of referring the bolt moment to the bolt circle or the circle tangent to the inside shank of the bolts, are agreed with fully. This adjustment of the lever arm can be considered as an empirical correction of the formula which would take account of the stiffness of the flange.

Mr. McCutchan refers to Bailey's and Solsnoes' tests on the influence of the bolt holes. While some effect no doubt must be ascribed to holes in the flange, the authors know of no simple way of compensating for them, and believe that this effect together with the effect of gasket grooves and bolt spacing should be merged into the safety factor for the present. The additional tests proposed would no doubt be of great value in establishing the location of the bolt and gasket reaction, and when further tests are contemplated, will receive consideration. However, due to the wide variety of gaskets and facings in use, the results obtained would be valid only for the ideal conditions of loading used in such tests.

Trends in Fuel-Burning Equipment and Furnace Design as Shown by Recent Eastern-Seaboard Applications

By RALPH D. BOOTH,[1] BOSTON, MASS.

By reviewing the steam-generator installations made during the last five years in eastern-seaboard industrial plants, it is the purpose of the author in this paper to determine trends in the design of furnaces and fuel-burning equipment. The subject matter is divided into three general parts: (*a*) A brief summary of the apparent trends in kinds of fuel, fuel-burning equipment, and furnaces; (*b*) a more detailed examination of the characteristics of representative installations; and (*c*) a summary of comments and suggestions with respect to future design.

Résumé of General Characteristics

A CASUAL review of the characteristics of steam generators utilized for industrial purposes along the eastern seaboard during the last five years shows little that is unique to the industrial boiler in contrast with the boiler of central stations of comparable size and age, nor any marked difference in the trends in the East from those of the country at large. It is, of course, well known that practically all of the larger units are being equipped with waterwalls and that pulverized-coal firing has made tremendous advances in its general application and popularity. We have, during this period, increased markedly the use of bent-tube boilers in contrast with straight-tube types, in the sizes above approximately 30,000 lb per hr. Also during this period there has been developed as one of the bent-tube types the two-drum integral-furnace steam generator for general application in sizes of 20,000 to 30,000 lb per hr up to 150,000 lb per hr and more. These boilers have met with considerable popular acclaim because of the lower prices possible on the more or less standardized designs in which they are available. One manufacturer reports that during the last $2^1/_2$ years the integral-furnace type of boiler has constituted some 25 per cent of his total boiler sales and approximately 40 per cent of the sales of the waterwall type, and that currently it constitutes approximately 50 per cent of his sales of waterwall-type boilers.

From a review of the industrial plants in this area, it would be fair to say that the use of stokers on new boilers in excess of 100,000 lb per hr in contrast with the use of pulverized coal or oil, is the exception to the general rule.

Data given in Table 1, submitted by one manufacturer on 50 representative sales of larger boilers for industrial application, show the effects of these predominant trends.

It is impossible to generalize with respect to the fuel or firing equipment on boilers in the range of size from 40,000 to 100,000 lb. This size finds application in plants serving loads of markedly different load factors and often loads subject to appreciable variation. Also this is a size frequently used in replacement units which must be installed within the limitations and under the conditions surrounding the existing boilerhouse. These conditions create an ideal "no-man's land" where each individual application needs a careful study to determine upon the appropriate firing equipment.

We do find that on the eastern seaboard in general the fight is largely between pulverized-coal installations firing eastern bituminous coal, and multiple-retort stokers firing the same fuel. Only in the states of Pennsylvania, New Jersey, and New York do we find any appreciable number of the traveling-grate stokers firing anthracite coals. In the last five years there has been to the author's knowledge only one new anthracite-burning plant constructed in New England and there have been several anthracite-burning plants replaced with equipment for burning powdered coal and oil. In passing, it should be noted that within the last three years there have been two applications of pulverized-coal units burning anthracite culm with apparent success.

The most that can be said of boilers in this range of sizes is that a greater proportion of pulverized-coal or oil-fired boilers is finding recent application than those of the stoker-fired types. Also the industrial pulverized-coal furnaces in these sizes on the eastern seaboard are almost entirely of the dry-bottom type. Very probably the integral-furnace type boilers, which are particularly adapted for the use of pulverized coal and/or oil, and all of which with one exception are of the dry-bottom type, are playing an appreciable part in these trends.

In sizes below 40,000 lb per hr, stokers are still finding an appreciable field of application and, in this size of boiler, a relatively small percentage of waterwall furnaces is found.

One does not have to search long to find the cause of these trends. The development of waterwalls has so increased the availability factor of boilers and so decreased the furnace maintenance cost, particularly in conjunction with the application of pulverized coal that their wide acceptance in boilers to be operated on high load factor is readily understandable.

The development of the unit type of direct-firing pulverizer to its present degree of reliability, which is comparable to that of the boiler unit itself, has contributed largely to the present wide acceptance of pulverized-coal-fired boilers. This development has materially reduced the cost of pulverizing equipment. It has eliminated the coal-preparation building and storage bins for the pulverized coal and the space requirement of the equipment itself

TABLE 1 USE OF STOKERS COMPARED WITH PULVERIZED COAL OR OIL FIRING

Type	Capacity, M lb per hr 100–250	250–499	750 and over	Total
Bent-tube	39	5	2	46
Straight-tube	3	1	0	4
				50
Firing:				
Pulverized coal	27	5	2	34
Stokers	5	0	0	5
Oil or gas	10	1	0	11
				50

NOTE: Approximately 30 per cent of pulverized-coal burners are tangentially fired and the majority of the balance, horizontally fired.

[1] Partner, Jackson & Moreland, Engineers.

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NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors and not those of the Society.

TABLE 2 CHARACTERISTICS OF MODERN EASTERN-SEABOARD COAL-BURNING INDUSTRIAL STEAM GENERATORS, PULVERIZED-COAL FIRING

Plant no.	Location	Installation date	Type firing	Maximum capacity, lb steam per hr	Boiler: Type	Boiler: Heating surface, sq ft	Boiler: Evap lb per hr per ft of furnace width	Superheater: Pressure at outlet lb per sq in.	Superheater: Temp F	Economizer: Type	Economizer: Water temp F In	Economizer: Water temp F Out	Air Heater: Type	Air Heater: Temp Gas	Air Heater: F out Air	Fuel: Btu per lb as fired	Fuel: Fusion temp F ash	Fuel: Analysis (as fired), per cent: Fixed carbon	Volatile	Ash	Moisture	Sulphur, by separate determination, per cent
1	Md.	4/38	P.C.	312000	B.T.	6400	16650	850	830	Conv.	Int.	350	480	Regen.	380	578	13854	2700	68.77	20.50	7.00	3.73	1–1.32
2	N. C.	3/38	P.C.	275000	B.T.	17100	11700	675	825	Conv.	None	...	...	Regen.	371	594	14080	2300	74.41	15.25	5.10	5.00	0.65
3	Pa.	...	P.C.	275000	B.T.	5360	16300	1265	900	Conv.	Int.	212	407	Regen.	344	547	13839	2714	71.22	17.43	8.65	2.70	1.05
4	Va.	8/38	P.C.	225000	B.T.	12775	11250	415	700	Conv.	None	...	...	Tubular	413	472	14940	2800	76.6	18.2	3.2	2.0	0.74
5	Ont.	1/37	P.C.	200000	B.T.	9010	11100	802	820	Conv.	Int.	325	419	Plate	385	371	13750	2350	58.0	38.5	3.5	3.1	0.50
6	Mass.	7/32	P.C.	155000	S.T.	11400	8740	450	750	Conv.	None	...	...	Regen.	299	572	14700	2600	73.0	21.0	6.0	...	Less than 1 per cent
7	N. Y.	3/31	P.C.	150000	S.G.	15671	8100	750	750	Conv.	None	...	...	Plate	346	535	13330	2350	61.0	25.0	10.0	4.0	1.5
8	Conn.	8/37	P.C.	130000	B.T.	9140	7650	650	825	Conv.	None	...	...	Regen.	331	575	14200	2482	71.6	17.8	4.6	6.0	...
9	Pa.	12/36	P.C.	125000	B.T.	9351	7580	415	753	Conv.	None	...	...	Tubular	414	495	13815	2450	54.98	36.03	6.26	2.75	1.39
10	Md.	5/35	P.C.	125000	B.T.	10070	7580	630	700	Conv.	None	...	...	Plate	419	535	14000	2700	66.5	24.0	7.5	2.0	1.0
11	W. Va.	5/32	P.C.	120000	S.T.	10164	7250	260	585	Conv.	None	...	...	Plate	357	392	13326	2000	48.87	39.68	6.57	4.88	1.91
12	Va.	4/35	P.C.	120000	B.T.	8560	8730	435	716	Conv.	None	...	...	Tubular	416	467	14100	2600	70.7	17.8	8.8	3.0	0.62
13	N. H.	11/36	P.C.	120000	S.G.	10470	7460	250P 425F	550P 715F	Conv.	None	...	...	Regen.	350P 365F	426P 436F	13800	2300	67.3	19.0	5.7	8.0	...
14	N. J.	9/37	P.C.	120000	B.T.	9340	8000	850	762	Conv.	None	...	...	Plate	412	523	14300	...	68.87	22.21	8.02	0.90	1.35
15	N. Y.	1/34	P.C.	75000	B.T.	5082	7500	180	...	None	None	...	...	Plate	497	373	14450	2100	73.08	22.78	4.14	1.33	1.15
16	S. C.	7/35	P.C.	70000	S.T.	7210	4910	785	182.5 (supht.)	Conv.	None	...	...	Regen.	360	...	14200	2240	59.48	36.32	2.08	2.12	0.05
17	S. C.	1/37	P.C.	60000	S.G.	6523	4030	250	526	Conv.	None	...	...	None	...	...	14000	2400	56.0	32.75	6.25	5.0	...
18	N. C.	9/35	P.C.	60000	S.G.	6606	4610	425	554	Conv.	None	...	...	Plate	365	460	14200	2760	61.8	28.9	5.3	4.0	...
19	N. J.	4/37	P.C.	55000	S.G.	4330	5360	625	650	Conv.	None	...	...	Plate	429	...	13500	2650	73.9	16.1	8.5	1.5	1.2
20	N. Y.	1/37	P.C.	50000	S.G.	5621	4370	160	...	None	None	...	...	Plate	359	333	13149	2198	47.3	33.0	9.8	9.9	2.08
21	N. C.	9/37	P.C.	45000	S.G.	4780	4650	150	...	None	None	...	...	Aero-fin	...	...	15006	2200	71.92	23.09	3.84	1.15	...
22	Ga.	8/36	P.C.	45000	S.G.	3960	4900	430	745	Conv.	None	...	...	Plate	357	420	13750	2400	55.98	33.62	6.0	4.4	...
23	N. Y.	5/38	P.C.	43200	S.G.	4760	4470	175P 400F	477 561	Conv.	None	...	...	Plate	385	370	13500	2100	56.4	31.8	8.4	3.4	...
24	N. Y.	1/38	P.C.	40000	S.G.	3248	4710	175	...	None	None	...	...	Plate	429	341	14000	2300	57.0	32.0	9.0	2.0	1.9
25	N. Y.	2/37	P.C.	20000	S.G.	2660	2730	335	700	Conv.	None	...	...	Plate	383	357	13500	...	56.4	31.8	8.4	3.4	...

Plant no.	Furnace and firing equipment: Burners: Capacity: No.	Burners: Capacity: ER	Burners: Type	Furnace dimensions: Width ft-in.	Furnace dimensions: Depth ft-in.	Furnace dimensions: Height ft-in.	Waterwall surface, sq ft	Waterwall surface per cu ft of furnace volume	10^6 Btu fired	Effective furnace volume	Btu per cu ft per hr, max	Pulverizers: No. mills per unit	Pulverizers: Capacity each, maximum: Rated	Pulverizers: Capacity each, maximum: Required	Pulverizers: Per cent fine through 200 mesh	Pulverizers: Grindability	Draft, natural or induced	Efficiencies, over-all: Normal load	Efficiencies, over-all: Normal load: Per cent	Efficiencies, over-all: Maximum load	Efficiencies, over-all: Maximum load: Per cent
1	4	7125	Hor.	18–2	20–0	48–9	4140	0.254	4190	16350	25600	2	14500	14250	75	90	Ind.	250000	87.01	312000	85.95
2	9	2910	Vert.	23–0	22–0	35–3	5985	0.335	3690	17850	20700	2	14000	26200	70	80	Ind.	225000	86.9	275000	86.0
3	4	7200	Hor.	16–3	22–0	44–0	5465	0.348	3990	15700	25400	2	17300	14400	80	80	Ind.	205000	88.60	275000	87.50
4	4	4450	Hor.	19–4	18–6	33–6	3624	0.305	2660	11900	22300	2	12500	8900	65	58	Ind.	200000	85.3	225000	No. guar.
													16200		75	100					
5	4	4212	Cor.	17–11	17–10	26–0	4345	0.587	2320	7400	31400			Bin system				180000	86.0	200000	85.0
6	2	7050	Hor.	18–9	18–8	22–0	2065	0.268	2075	7700	26900	2	8000	7050	77	...	Ind.	149500	82.7	155000	82.3
7	8	1880	Cor.	18–7	16–6	25–9	2859	0.366	2080	7820	26600	2	8000	7530	...	...	Ind.	120000	...	150000	85.5
8	2	5785	Hor.	17–0	16–0	23–3	3033	0.473	1660	6400	25900	2	7250	5785	...	...	Ind.	115000	87.7	130000	87.2
9	4	3125	Hor.	16–3	15–6	28–3	2280	0.322	1730	7090	24400	2	8000	6250	...	...	Ind.	100000	85.5	125000	84.2
10	2	6175	Hor.	16–6	16–2½	26–9	2296	0.320	1730	7160	24100	2	8800	6175	65–70	...	Ind.	75000	86.0	125000	84.7
11	3	3933	Hor.	16–7	18–0	25–0	1730	0.232	1570	7460	21000			Bin system			Ind.	90500	86.25	120000	85.0
12	4	2850	Cor.	13–6	15–0	29–6	4540	0.712	1610	6230	26800	1	16000	11400	65	...	Ind.	100000	...	120000	85.3
13	2	5600P 6100F	Hor.	16–1	14–1⅛	20–9	2370	0.518	1685	4570	32900P 36800F	2	8500	5600P 6100F	...	...	Ind.	100000F	86.3	120000 120000	85.4 84.7
14	4	2820	Hor.	15–0	18–6	24–0	2951	0.442	1618	6680	24200	2	8000	5650	...	...	Ind.	100000	85.3	120000	84.9
15	1	6440	Hor.	10–0	16–0	25–0	1636	0.410	930	3990	23300	1	8000	6440			Ind.	50000	85.3	75000	82.9
16	2	3380	Hor.	14–0	16–6	20–9	1780	0.365	960	4875	19700	1	...	6760	...	...	Ind.	....	...	70000	84.0
17	1	5540	Hor.	14–11	14–1⅛	14–3	1727	0.575	775	3000	25800	1 1	4800 1800	5540	70	...	Natural	40000	82.2	60000	80.3
18	1	5500	Hor.	13–0	14–2	15–9	1797	0.561	780	3200	24400	2	3000	2750	...	...	Ind.	....	...	60000	85.5
19	1	5740	Hor.	10–3	16–1⅞	14–6	1415	0.585	775	2410	32200	1	6500	5740	65	70	Ind.	40000	86.0	55000	80.9
20	1	4920	Hor.	11–5	13–11⅛	15–0	1679	0.678	647	2400	27000	1	6000	4920	...	...	Ind.	35000	85.5	50000	84.1
21	2	1965	Hor.	9–8	14–1⅛	15–6	1100	0.458	590	2400	24600	2	2300	1965	75	47½ to 110	Natural	30000	80.3	45000	77.3
22	1	4100	Hor.	9–2	13–4	14–0	900	0.540	564	1670	33700	1	4300	4100	...	...	Ind.	35000	86.0	45000	84.0
23	2	1910	Hor.	9–8	14–1⅛	15–6	1090	0.500	516	2180	23700	1	4500	3820	65	55	Ind.	....	...	43200	85.3
24	2	1785	Hor.	8–6	14–1⅛	11–9	1254	0.883	500	1420	35200	1	4500	3570	65	...	Ind.	30000	84.5	40000	82.2
25	2	1035	Hor.	7–4	11–6⅞	11–9	1266	1.260	280	1000	27900	1	2900	2070	...	...	Ind.	15000	85.4	20000	84.9

P.C. = pulverized coal.
B.T. = bent-tube boiler.
S.G. = 2-drum integral-furnace boiler.
S.T. = straight-tube boiler.
P. = present
F. = future

TABLE 3 CHARACTERISTICS OF MODERN EASTERN SEABOARD COAL-BURNING INDUSTRIAL STEAM GENERATORS, STOKER FIRED

Plant no.	Location	Installation date	Type firing	Boiler: Maximum capacity, lb steam per hr	Boiler: Type	Boiler: Heating surface, sq ft	Boiler: Evap lb per hr per ft of furnace width	Superheater: Pressure at outlet, lb per sq in.	Superheater: Temp F	Superheater: Type	Economizer: Type	Economizer: Water temp F In	Economizer: Water temp F Out
51	Va.	8/30	Stoker	270000	S.T.	12840	10900	450	720	Conv.	Ext.	190	230
52	D. C.	1/34	Stoker	237000[1]	S.T.	25243	8950[1]	200	...	None	None	...	...
53	Md.	12/36	Stoker	200000	S.T.	16400	8050	360	700	Conv.	Ext.	220	310
54	Ga.	11/37	Stoker	90000	B.T.	8411	7200	160	...	None	None	...	...
55	Md.	3/35	Stoker	45000	B.T.	5083	3750	175	...	None	None	...	...
56	N. C.	9/37	Stoker	40000	B.T.	4958	4000	125	...	None	None	...	...
57	Va.	10/37	Stoker	25000	S.T.	4017	2780	250	...	None	None	...	...
58	N. J.	3/36	Stoker	82000	B.T.	8771	5125	400	...	Conv.	None	...	...
59	N. J.	3/36	Stoker	80000	B.T.	7700	5330	430	600	Conv.	Ext.	223	339
60	Pa.	11/35	Stoker	70000	B.T.	6700	6360	160	...	None	Integ. with lower drum	210	310
61	Pa.	12/33	Stoker	52700	B.T.	7026	4580	440	588	Conv.	None	...	...
62	Pa.	4/37	Stoker	51000	B.T.	7300	4250	215	525	Conv.	None	...	...
63	Pa.	...	Stoker	41250	B.T.	6658	3440	149	480	Conv.	None	...	...
64	N. J.	11/35	Stoker	40140	B.T.	4460	4460	165	...	...	None	...	...
65	N. J.	11/34	Stoker	20400	B.T.	2950	3400	165	...	None	None	...	...

Plant no.	Air heater: Type	Air heater: Temp F out Gas	Air heater: Temp F out Air	Fuel: Btu per lb as fired	Fuel: Fusion temp F ash	Fuel: Analysis (as fired), per cent Fixed carbon	Volatile	Ash	Moisture	Sulphur by separate determination, per cent
51	None	...	...	14100	2600–2700	56.4	35.8	5.35	2.45	0.95
52	None	...	...	13890	2540	64.6	23.7	7.90	3.80	0.8
53	None	...	...	14000	...	73.0	16.0	8.00	3.00	...
54	None	...	...	13400	2300–2700	53.92	37.40	8.68	2.64	1.2
55	None	...	...	14361	...	73.76	15.73	7.52	2.99	0.94
56	None	...	...	13500	2400	50.0	35.0	7.00	2.00	1.0
57	None	...	...	14750	27000	72.5	19.8	5.00	5.00	0.65
58	Plate	475	305	11150	...	72.9	3.6	13.50	10.00	0.5
59	None	...	...	11900	...	...	...	13.00	6.00	...
60	None	...	...	11230	...	72.3	6.25	11.10	10.35	0.85
61	None	...	...	10800	...	65.0	8.0	22.00	5.00	...
62	Plate	421	...	10800	...	68.0	7.0	20.00	5.00	...
63	None	...	...	10500	...	66.0	4.0	20.00	10.00	...
64	None	...	...	11500	...	75.0	4.5	14.00	6.50	...
65	None	...	...	11500	...	75.0	4.5	14.00	6.50	...

Plant no.	Type stoker	Net area, sq ft	Stoker dimensions: No. retorts	Stoker dimensions: Width, ft-in.	Stoker dimensions: Length, ft-in.	Waterwall surface, sq ft	Lb coal per sq ft grate	Draft, natural or induced	Efficiencies, overall: Normal load	Efficiencies, overall: Per cent	Efficiencies, overall: Maximum load	Efficiencies, overall: Per cent
51	MR	420	14	24–6	17–1½	664	57	Nat.	....	....	....	...
52	MR	485.6	15	26–4	18–6	1032	41.5	Ind.	129000 215000	82.0 80.0	237000[1]	...
53	MR	441.0	14	24–6	18–0	1050	49.7	Nat.	200000	85.0	225000	...
54	MR	202.1	7	12–4	16–6	721	46	Nat.	40000 70000	...	90000	
55	MR	105.4	6	10–6	10–0	387	40.8	Ind.	30000 37500	78.6 77.9	45000	75.8
56	MR	105.3	5	8–9	12–0	145	39.0	Nat.	15000 30000	77.25 76.75	40000	75.25
57	MR	75.25	4	7–0	10–9	30	29.3	Nat.	10000 18000	79.9 80.2	25000	79.3
58	T.G.	320	..	16–0	20–0	None	29.4	Ind.	65000	78.2	82000	...
59	T.G.	266	..	14–0	19–0	88	28.0	Nat.	60000	75.0	80000	...
60	T.G.	196.5	..	10–8⅜	18–4	290	43.3	Nat.	52000	79.3	70000	74.8
61	T.G.	204	..	10–8⅜	19–2	370	42.4	Nat.	31800 42150	70.0 67.5	52700	63.0
62	T.G.	223	..	11–8⅜	19–0	212	33.8	Nat.	40000	75.2	51000	68.7
63	T.G.	199.22	..	11–8⅜	17–0	None	31.0	Nat.	....	....	41250	69.0
64	T.G.	130.8	..	8–8⅜	15–0	None	43.8	Nat.	...	...	40140	68.0
65	T.G.	74.3	..	5–8⅜	13–0	None	33	Nat.	15300	72.0	20400	70.0

[1] F and A.
S.T. = straight-tube boiler. B.T. = bent-tube boiler.

is little more than that required by stokers. To a large extent, it has also reduced the hazards formerly associated with the storage of large quantities of finely pulverized coal.

The use of pulverizing equipment in general permits the use of a wider range of coals than is permissible with the stoker-fired installation which usually requires the use of premium-quality fuels. Today in New England this advantage will vary from 20 to 30 cents per ton of coal. Also, the boilers designed for pulverized coal are suitable for oil or gas firing and can, in fact, fire either or both fuels at the same time.

In the light of the equipment development, the trends found appear logical and it is therefore to be presumed that they will continue. The two largest items entering into the cost of steam or power are the fuel costs and the investment charges. In the large plant operating at high load factor the fuel costs usually outweigh the fixed charges on the investment. The development of high-availability pulverizers at a lower investment cost, making the low-priced fuels available, would naturally throw these larger plants into the pulverized-fuel class.

In the smallest units of 40,000 lb per hr or less, where we usually find low load factors and a wide variation in loads to be carried, the investment costs constitute a much larger portion of the annual cost. The additional investment burden to apply pulverizers in this range is often disproportionately high, because of the limitations imposed by the range of loads to be carried, and often the use of waterwalls would be unjustified except with pulverizers. It is therefore not surprising, in those plants that have not gone to oil firing, to find stokers still holding the field.

Under today's price situation, intermediate-sized units, applied under widely different loads, load factors, and plant conditions should logically be unclassifiable as to type of firing equipment

Comparison and Discussion of Typical Furnaces

In Tables 2 and 3, are presented data on two groups of furnaces and firing equipment found in representative installations on the eastern seaboard. Table 2 presents the data on pulverized-coal-fired boiler installations ranging in size from 20,000 lb per hr to

approximately 300,000 lb per hr, which are equipped with water-wall furnaces of the dry-bottom type. As such, this group is representative of the greater portion of the furnaces which have been installed on the eastern seaboard in the last five years. All of these units are in service and, according to the author's understanding, meet the ratings. It should be noted in passing that the first three units in Table 2 are utility units, included to permit a comparison of furnace characteristics over a slightly wider range of installation.

Table 3 is a presentation of the characteristics of representative stoker-fired installations which in the main are of partial water-wall design. The comparative number of units on which data are presented was chosen to cover with some degree of completeness the range of unit sizes found, and is not intended to reflect the relative proportion of these two types of installation. Because of the space and time limitations, the detailed discussion will be confined to the pulverized-coal-fired units, which are representative of the majority of industrial installations made within the last five years on the eastern seaboard.

These data purposely include boilers of one manufacturer only in order to permit comparisons which might be invalid if the variable of different furnace-wall design, as introduced by the different manufacturers, were not eliminated.

Before comparing the furnaces, it might be well to consider for a moment the primary requisites of a furnace. The earning power of any modern steam generator depends on its availability and its ability to stay on the line under sustained high-load operation. The availability depends to a very large extent on the provisions for preventing the fouling of the furnace, boiler, and superheater with ash or slag and to permit the removal of ash with the type of ash-removal system provided. It must, of course, simultaneously burn the fuel efficiently. The modern water-cooled furnace has therefore a double function. It must be so proportioned that the fuel will be burned economically and it must provide radiant heat-absorbing surface so that the gases will be reduced sufficiently in temperature to avoid excessive slagging of the boiler convection surface.

These requirements are to a large degree conflicting and obviously the responsibility for the final solution lies with the boiler manufacturers rather than with the users or the consulting engineer. The latter, however, must find some mechanics for producing specifications which first, will procure equipment to meet the requirements satisfactorily and second, will insure true comparability of the equipment offered by the various competitive manufacturers. It is, of course, a matter of primary importance to the manufacturers also that these two objectives should be realized.

It is therefore interesting to analyze the data available in Table 2 to see if any general relationship with respect to furnace design can be observed in terms of those characteristics which the engineer must, of necessity, transmit to the boiler manufacturer, namely, the characteristics of fuel, the type of ash-removal systems, etc. Since all of the furnaces listed in Table 2 are of the waterwall, dry-bottom type, such effect as the characteristics of ash-removal equipment can properly be considered as a constant in these comparisons.

In considering furnaces, it has long been customary, although now properly declaimed by the boiler manufacturers, for engineers and users to evidence some interest in heat release per cubic foot. It will be noted, in looking over the tabulated data, that there is a range of heat release from 19,700 to 36,800 Btu per cu ft, although this does not cover the extreme range of furnace designs found in representative installations in this area. It is apparent that there is no readily observable relation of heat release to boiler size.

Certainly, if the first three units which are utility installations are excepted, the steam release per foot of drum is so low it could not have been a controlling factor on furnace dimensions. A comparison of the furnace widths as actually installed with the permissible minimum widths, recommended by the manufacturers for burner spacing (assuming the number of burners, size, and type tabulated on this comparison) indicates that these furnaces as a whole, with one or two exceptions, are designed for minimum widths. This is logical, since, in general, the over-all cost of a boiler unit of a given capacity is appreciably reduced as the width of the furnace is decreased. The furnace depths as tabulated are also substantially the minimum depth permissible for the type and size of burner equipment installed.

It is when we come to the determination of the furnace height, dependent as it is upon the type and disposition of the waterwall surface and the characteristics of the fuel, that it becomes difficult to rationalize from the data available to the consulting engineer or the user (as contrasted with the boiler manufacturer) the characteristics found in the various units.

One manufacturer uses, as a preliminary criterion of the probable desirable furnace volume, the limitation of heat release, expressed as a per cent of the volatile in the fuel. It should be

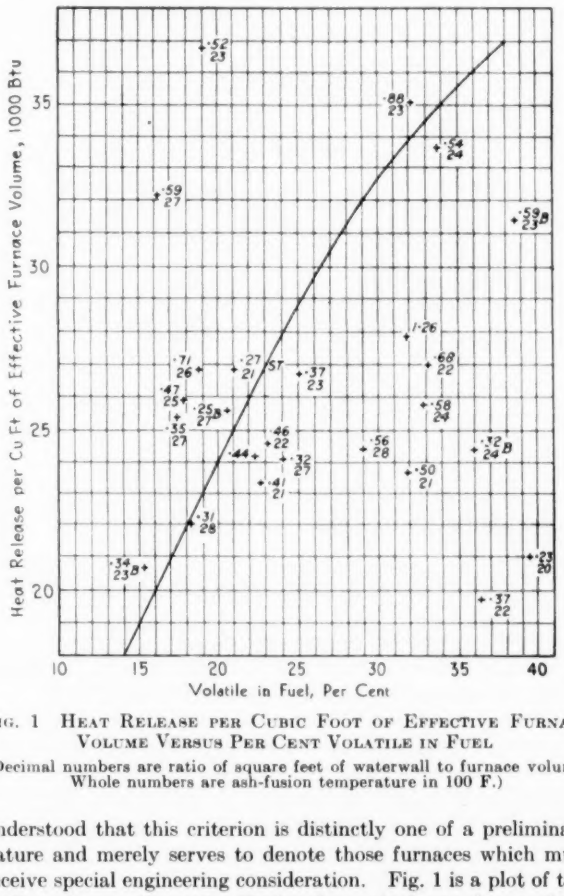


FIG. 1 HEAT RELEASE PER CUBIC FOOT OF EFFECTIVE FURNACE VOLUME VERSUS PER CENT VOLATILE IN FUEL

(Decimal numbers are ratio of square feet of waterwall to furnace volume. Whole numbers are ash-fusion temperature in 100 F.)

understood that this criterion is distinctly one of a preliminary nature and merely serves to denote those furnaces which must receive special engineering consideration. Fig. 1 is a plot of this preliminary requirement. It also shows the heat releases of the typical installations. On each point is noted, as a decimal number, the waterwall surface per cubic foot and, as a whole number, the fusion temperature of the ash expressed in hundreds of degrees F. While proportionately more of these furnaces have heat releases less than that indicated as permissible by this curve, an appreciable percentage of the installations exceed this preliminary maximum heat release. The tabulated installations do not appear to show any general trends such as might be

expected as a result of such a preliminary determinant. One might expect to find those installations, which had more water-cooled surface per cubic foot of effective furnace volume or those using fuel with the higher ash-fusion temperatures, operated at the higher heat releases, but no such general relationship is apparent.

Fig. 2 shows these same characteristics in terms of ash-fusion

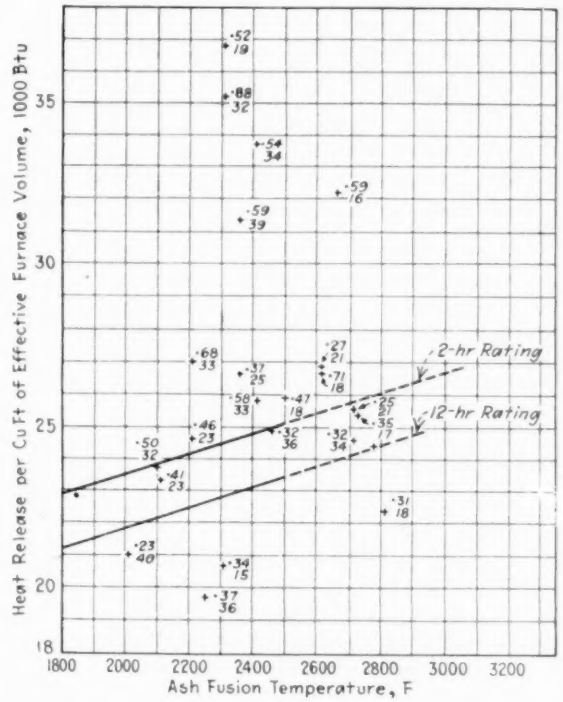


Fig. 2 Heat Release per Cubic Foot of Effective Furnace Volume Versus Ash-Fusion Temperature

(Decimal numbers are ratio of square feet of waterwall to furnace volume. Whole numbers are per cent volatile in fuel.)

temperature, together with a curve which another manufacturer uses as a preliminary criterion in determining the permissible heat releases per cubic foot. Again each point plotted on the chart has noted as a decimal number the waterwall surface per cubic foot of furnace volume, and in this instance, the whole number gives the per cent volatile in the fuel. It will be observed that those furnaces employing more waterwall surface per cubic foot, which might logically be expected to operate at higher heat releases for any given ash-fusion temperature, do not do so.

It could be argued rationally that, for any given ignition characteristics of the coal, it should be possible to operate at a higher heat-release value in a furnace which had more waterwall surface per cubic foot; and also, for the same ignition and water-wall characteristics, to operate at higher heat releases with fuels having higher ash-fusion temperatures. With this in mind, it is interesting to compare the furnaces against the product of the ash-fusion temperature and the waterwall surface ratio. This comparison is presented in Fig. 3. Each point is designated by the per cent volatile in the fuel, and here again, it will be noted that there is no readily observable relationship either to the points as a whole or to those points indicating fuels of approximately the same per cent volatile.

The only relationship generally observable in any of these three attempts at comparison is that most of the installations are working at heat releases of from 23,000 to 27,000 Btu per cu ft. It is certainly farthest from the author's mind to imply that operation should properly be limited to heat releases in that general range. It is rather the purpose of these three charts to point to successful installations at varying heat releases, and to the inability to express, in terms of any logical or empirical relationship to conditions which the engineer should specify to the boiler manufacturer, those values of heat release which have proved practicable. However, it may well be that owner's specifications or expressed desires have induced the manufacturers to keep their designs within this general range of heat release.

It should not be the function of the consulting engineer or the owner to assume responsibility for the proportioning of the boiler and its furnace. The successful combustion of fuel is obviously one of temperature control of the gases and, as such, the appropriate installation must be gaged to the type of surface and the particular arrangement of the surface employed by the manufacturer. It is however, necessary for the consulting engineer and the owner to devise some way of transmitting properly to the manufacturer the requirements of the installation and the characteristics of the fuels and other equipment associated with the furnace, which are not to be provided by the boiler manufacturer. The definition of the latter is simple, but the definition of the fuels is as yet far from simple. The difference in tendency of a

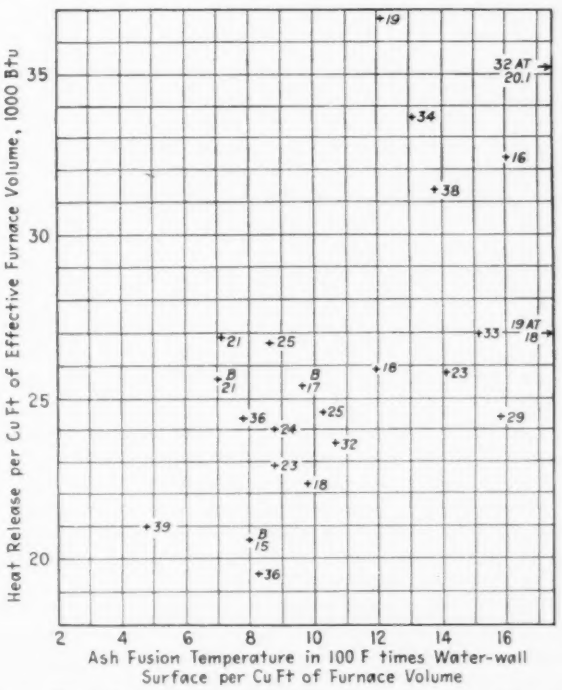


Fig. 3 Heat Release per Cubic Foot of Effective Furnace Volume Versus Ash-Fusion Temperature Times Waterwall Ratio

(Numbers in chart are per cent volatile in coal.)

noncoking coal and a coking coal to agglomerate in the furnace, and the effect of such agglomeration on the tendency to slag can be introduced only by experience. To date we have found no adequate mechanics for expressing these tendencies.

The industrial plants utilizing lower steam temperatures, as in general they do, are not suffering from this lack of information to the same degree as do the utilities operating at higher temperatures. The use of higher steam temperatures, involving as it does the necessity for higher gas temperatures on the desuperheaters and on the slag-screen tubes ahead of the superheater, is driving the boiler designers ever higher in the required gas temperatures

entering the slag screen. It is generally recognized as practicable to so proportion a furnace by reducing gas velocities that the heavier particles, which constitute the bulk of the slag nuisance, moving more slowly than the gas, can be cooled by the radiant action of the black surface to a lower temperature than the gas itself. In the past it has been quite general to specify that the gas temperature entering the slag screen shall be not higher than the ash-fusion temperature of the fuel. However, as we go higher in steam temperatures, and by necessity higher in the gas temperature entering the slag screen, the difficulty of keeping the gas temperatures to such low values, particularly where low ash-fusion-temperature coals are encountered, is forcing us to adopt gas temperatures in excess of the ash-fusion temperature.

Today there is relatively little known as to the differential in temperature of the slag particle and the gas except as it can be theoretically computed for an assumed size of particle. Also, there is relatively little knowledge as to the tendency of particles to agglomerate or the size of agglomerated particles, and hence estimates of particle temperatures are largely guesses. We, as engineers, charged with the duty of obtaining suitable equipment and procuring truly comparable competitive bids, find ourselves unable to define completely to the manufacturer those characteristics whose definition is our primary concern.

Comparison and Discussion of Firing Equipment and Pulverizer Applications

As a natural consequence of the type of furnaces (dry-bottom) compared in Table 2; practically all of the burner installations are of the horizontal type. It will be noted that the maximum sized burner has a capacity of approximately 7000 lb per hr, of coal (or approximately 75,000 lb per hr of steam). The choice of the number of burners was undoubtedly dictated by the characteristics of the load, and without complete knowledge of that it is impossible to comment.

The pulverizer tabulation does not disclose it, but all mills are of the direct-firing ball, or bowl, or impact type, with the exception of units for plants nos. 5 and 11 (which are of the storage type). Most of the pulverizer applications on the eastern seaboard are of these types, since relatively little of the low grindability fuels is used.

In general the pulverizers have a rated capacity at least 10 per cent in excess of the required capacity, corresponding to the recommendations of the pulverizer manufacturers. The limit to the amount of excess capacity which should be provided, either for wear allowances, protection of loads, or to permit the use of coals having a wide range of grindability, is dictated by the range of load swings to be handled.

Naturally the fineness of pulverization of the fuel to be provided is of primary importance even from the furnace viewpoint alone. Fig. 4 is a plot of the fineness of pulverization and the recommendation of one of the boiler manufacturers.

It is the author's belief that the trend in the future will be toward higher fineness and even higher values than those recommended by the manufacturers.

The fineness of pulverization is adequate if, under a given furnace design and operating conditions and for the range of fuels economically available, the incremental cost of further increasing the fineness exceeds the corresponding value of:

(*a*) The reduction in carbon loss due to the increase in fineness.

(*b*) The reduction in the cost of boiler cleaning and outage which can be effected by a reduced tube fouling and slagging, as increased fineness reduces the number and size of the particles impinging on the tubes; and either of the following:

(*c*) The reduction in the duty on dust-collecting equipment of the electrical type, which is adversely affected by increased particle size and increased carbon loss; or

(*d*) The reduction in stack nuisance, where no dust collectors or mechanical collectors are used, by increasing the whiteness of the stack discharge through reduction of the carbon content in the dust.

(*e*) The reduction in maintenance cost of slag-tap bottoms.

It is the author's opinion that manufacturers and users in the past have given consideration largely to (*a*), namely, the effect on carbon loss, and have considered this largely from a "coal-pile" point of view. At 30 per cent volatile, for example, an increase of 5 per cent in fineness over 70 per cent will reduce the carbon loss by approximately 20 per cent. This carbon loss, being of the order of 1 per cent, a change of 20 per cent in such a small quantity is relatively unimportant when viewed from the coal pile. However, for coking coals, whose particles tend to soften and agglomerate in the furnace, the increase in fineness may well be justified through the reduction in slagging tendency of the furnace.

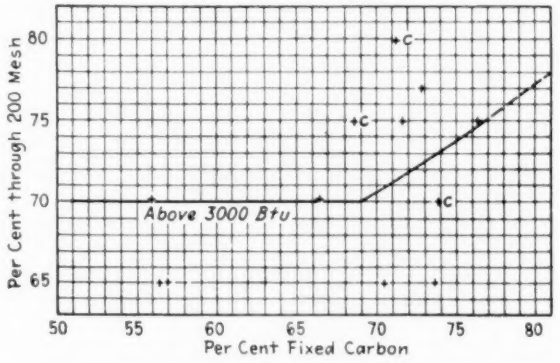


Fig. 4 Fineness of Pulverized Coal, Per Cent Above 200 Mesh Versus Fixed Carbon

A change of 20 per cent in the amount of carbon in the fly ash going through an electrostatic precipitator would markedly affect its efficiency. If, for example, the carbon formed 25 per cent in the dust, a reduction of $^1/_8$ would change the efficiency of the precipitator by approximately 2.5 per cent. Such a change in efficiency would decrease the emission from a stack, equipped with a precipitator to recover normally 90 per cent of the flue dust, by 25 per cent. The cost of providing a precipitator which would effect the same percentage recovery with the higher carbon content in the flue dust would materially exceed in the normal case the excess cost of the pulverizers and the increased power consumption.

These effects on the precipitator may not be as important with slag-tap or two-stage furnaces, since the carbon loss is somewhat lower than with dry-bottom furnaces. However, there is some evidence that increased fineness will help the slag-tap furnace under some conditions. It has been noted that an increase in fineness tends to reduce the deposition of iron sulphide in the slag pool, thus materially minimizing the hazards of punctured floor tubes and slag leaks.

Summary

If, for the purpose of one about to construct an industrial boiler plant on the eastern seaboard, a summary of the most pertinent suggestions were required, the author would abstract them as follows:

1 Ascertain with the help of a local fuel dealer, the cost of the available coals and oil, considering the entire range of fuels which could be reasonably applied to the type of plant in question.

2 If the plant is of low load factor and of a capacity less than

30,000 lb per hr, the probabilities are that an oil-fired or stoker-fired installation will prove most attractive, and that complete waterwalls will not be justified.

If the plant is of the range of 30,000 lb per hr to 150,000 lb per hr, unless it is in an area of high coal cost and unless the load factor is extremely high, the cost must be carefully ascertained for both pulverized-coal equipment and stoker-fired installations. For locations adjacent to tidewater, careful consideration should be given to the advantages accruing through the additional provision of oil-firing equipment in combination with pulverized-coal equipment. The choice between these two types of firing depends on many factors, such as the available coal, the load factor, the fixed charges on the initial investment, the cost of power, the required flexibility, the cost of ash disposal, and the necessity for providing a means for fly-ash removal. The arrangement of the plant may well have a controlling influence, particularly if the boilers are to be installed in an existing building with limited headroom.

If the plant is in excess of 150,000 lb per hr and of high load factor, in all probability a pulverized-coal-fired job (alone or in combination with oil) will prove to be economic.

3 In all probability bent-tube boilers will be found to be economic in the size of 30,000 to 40,000 lb per hr upward, except in those applications where existing conditions definitely control the type of boiler application. On the pulverized-coal or oil-fired installations, in the sizes of 30,000 to 150,000 lb per hr, the two-drum bent-tube type boiler with integral furnace will usually, except under unusual feedwater conditions, prove to be attractive.

4 If a pulverized-coal installation is indicated, it is advisable in the light of the recent trends in fuel prices to be as generous in fixing the capacity for range of fuels and degree of fineness as the variations in loads will permit.

5 While for ordinary moderate-capacity installations on the eastern seaboard, and particularly in New England, where high-grade coals only will prove to be in the economic range, it is most probable that a dry-bottom furnace will prove desirable, the final determination between the dry-bottom and the wet-bottom system should be made only after careful analysis of actual operating experience with similar fuels and similar boiler designs.

6 In determining upon the furnace and its proportions do not invade the field of the boiler manufacturer but give him that degree of freedom in his design which is necessary to meet his responsibility of providing a satisfactory installation. Supply the boiler manufacturers with as complete information on the load characteristics and the characteristics of the fuel as are practicable, and base your specifications on data from plants operating with comparable conditions.

In comparing the furnaces offered by the various manufacturers seek a boiler which has:

(*a*) A large furnace with as moderate a heat release as is consistent, under the load variations and fuel characteristics, with ignition requirements and completeness of combustion.

(*b*) Radiant-heat-absorbing surface to cool the ash particles in suspension to a chilled or solidified condition before they reach the furnace outlet and, where practicable within the limitations of the required superheat temperature, to cool the gas temperature also to the ash-fusion temperature.

(*c*) A furnace cross section such that the gas velocity in the clear space will be sufficiently low to permit the larger particles of fuel to hover or move more slowly than the gas, and a furnace height such that, in the time available after a particle leaves the flame and falls through the clear atmosphere to the bottom, it may radiate sufficient heat to the surrounding heat-absorbing surface to freeze at least to a sufficient crust thickness so that it will not stick to the furnace bottom.

(*d*) A relatively large furnace outlet so that the entrance velocity at the boiler aperture will be low to minimize the squashing of those ash globules or particles which are only surface-frozen.

(*e*) A furnace bottom so arranged with respect to the flame that: (1) With the slag-tap design the entire furnace atmosphere between the flame and the floor will be opaque in order that the particles may not be affected by an appreciable part of the black surface and thus remain in a molten condition; and (2) with the dry-bottom furnace, whether flat or of the hopper design, the entire furnace atmosphere between the flame and the bottom should be clear and entirely surrounded by radiant-heat-absorbing surface.

(*f*) Burner equipment that will produce a short flame without impingement, and which will distribute the heat evenly across the furnace.

(*g*) Where practicable, it should be so arranged that the particles are initially moving away from the boiler entrance aperture to minimize carbon loss, increase ash deposition in the furnace, and reduce impact of ash particles on the boiler tubes.

(*h*) A tube spacing in the boiler entrance aperture ahead of the superheater sufficiently wide so that no bridging of such slag as might stick can occur. Fortunately, we find under ordinary circumstances that the more economic installations for these particular sizes of boiler are of the bent-tube type in which the tubes in the boiler aperture and the superheater surface are arranged vertically or at a relatively sharp angle, thus providing a tendency toward self-cleaning.

Acknowledgment

The author wishes to express his appreciation to H. S. Colby of the Combustion Engineering Company, Inc., P. C. Idell and E. J. Billings of the Babcock & Wilcox Company; Martin Frisch of the Foster-Wheeler Corporation; and E. H. Smitz of C. H. Sprague & Son Company, for data which they supplied and for their material assistance otherwise in the preparation of this paper.

Operating Methods and Problems of a Combined Hydro- and Steam-Electric System

By H. L. HARRINGTON[1] and E. B. STROWGER,[2] BUFFALO, N. Y.

Today a combined hydro- and steam-electric system requires the use of operating methods based upon the principles of economic loading together with satisfactory and reliable service to all customers, and it is essential to arrange for some plan of centralized supervision of the power-supply facilities in order to put these methods into effect.

Certain system changes, such as the proper interconnections of generating facilities and load areas will in general be required to make such a plan effective and, if input-output data regarding the hydro- and steam-generating facilities are not available, tests should be carried out to furnish this information for the load supervisors and plant operating personnel.

Further, maximum use should be obtained from all storage reservoirs, reserve-capacity requirements should be established on a system basis with the highest degree of service reliability reasonably possible, and satisfactory standards and methods for the maintenance of system frequency and voltage must be followed.

THE PURPOSE of this paper is to illustrate the operating problems and methods involved in the application of the principles of economic loading to a large interconnected electric power system, the power-supply facilities of which consist of hydro- and steam-electric generating stations, as well as interconnections with other electric systems. There is considered a state-wide integrated electric system which has been formed over a period of years by successive consolidation of local units, which formerly operated in more or less isolated areas. Physical interconnection has resulted in a system containing hydroelectric stations diversified as to location due to the inclusion of various drainage areas in the operating territory and diversified as to characteristics depending on the character of the water supply and the design adopted for a particular project. Some of these stations are located on streams with well-regulated flow, some on streams with practically no regulation, and others on streams where the regulation is a function of joint use for flood control and for power development. Steam-electric generating stations include plants of relatively high economy suitable for base-load operation, as well as older and less efficient stations which may be used for peak load or reserved for emergency use. In spite of any such diversified character of the various sources of supply, however, all types of stations, whether steam or hydro, have a definite function in the power-supply facilities of the main system, and their operations must be coordinated with the objective of securing their full inherent economy.

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Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y., and will be accepted until March 10, 1939, for publication at a later date. Discussion received after the closing date will be returned.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

With this general picture in mind it is apparent that a territorial grouping of the generating facilities would be a desirable step for setting up an operating organization to control the station output; that is, to dispatch loads to the various stations to effect a maximum of economy in operation. Such a territorial grouping should be made in accordance with more or less well-defined major load areas, with main trunk transmission lines interconnecting these areas and permitting a free interchange of power between them. To effect this interchange with a minimum of limitations due to transformer capacity between one voltage and another, transmission voltages should be standardized as far as reasonably possible.

If the system is further complicated by the fact that electric service is rendered at more than one frequency, adequate frequency-changer capacity should be provided to allow a transfer of energy from the system of one frequency to that of another.

These general statements regarding the main features of a large electrical system are given in order to present a general picture to which the principles of economical and reliable operation of combined hydroelectric and steam-electric generating facilities are to be applied.

Fundamental Objectives

The fundamental objectives for the operation of a system as described, or one of similar characteristics, may be briefly summarized as follows:

1 An adequate and reliable supply of electric power must be assured at all times to all load areas.

2 The maximum possible use should be made of all available hydro energy which, when utilized in conjunction with steam-generated energy, will meet the varying requirements of the system load at a minimum over-all cost to the system as a whole.

Specific Requirements

For the most successful application of the foregoing principles, the following specific requirements must be met:

1 All load and generation areas must be interconnected with tie lines of sufficient capacity (*a*) to transmit the required energy into a particular area from other sources when a deficiency of energy exists in that area, and (*b*) to allow the surplus energy of a particular area to be made available to other areas where it can be economically utilized.

2 In order that the maximum use may be made of all hydro energy available, with the resulting conservation of fuel-generated energy, it is essential that the operation of all generating facilities be coordinated through some form of centralized supervision, where complete and up-to-the-minute information is available as to load requirements, availability of water supply and generating facilities, and the cost of power production from all sources.

3 All generating stations, both steam and hydro, must be provided with input-output data which will assist in determining how each station should carry its assigned load with the minimum use of water or fuel as the case may be and to assist the system load supervisors in assigning loads to all stations for maximum over-all economy.

4 All storage reservoirs should be operated so as to secure full advantage of the storage available with the least possible limitations due to local requirements.

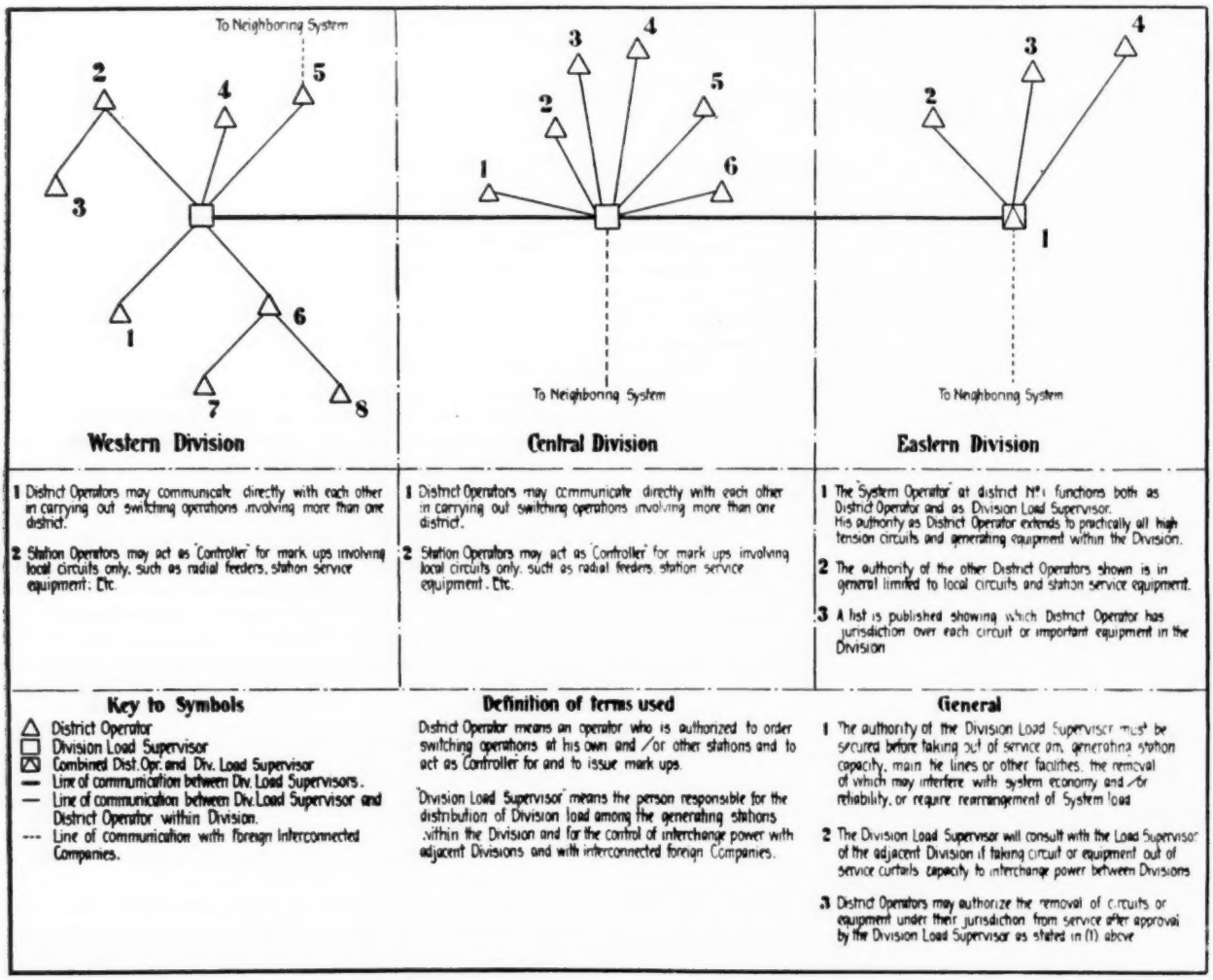


FIG. 1 OPERATING DIVISIONS OF A LARGE ELECTRIC UTILITY SYSTEM

5 Reserve capacity for protection against equipment failures should be limited to the smallest amount consistent with the highest degree of service reliability reasonably possible.

6 All the foregoing requirements must be related to maintenance of satisfactory system frequency and voltage. Scheduled tie-line loads between divisions and with other systems must be maintained in order that generating-station outputs may be produced as scheduled for maximum economy.

METHODS OF MEETING SPECIFIC REQUIREMENTS

1 *Interconnection of Generating Facilities and Load Areas.*

When the system under consideration has resulted from the consolidation of several formerly separate and more or less isolated units, additions and improvements to the existing transmission facilities made to effect a completely integrated system require careful planning and design to obtain the maximum advantage of the transmission facilities already provided. This requires a general survey and study of the system as a whole with special attention to

(*a*) Adequacy of transmission-line capacity to each load area.

(*b*) Standardization of voltages of the main transmission system to remove limitation due to capacity of step-up and step-down transformers.

(*c*) Interconnection with utilities serving adjoining load areas: (1) For possible marketing of surplus hydroelectric energy generated during high-flow periods through reduction of an equivalent amount of steam-generated energy, (2) for obtaining for each utility, additional capacity and energy sources during times of emergency, (3) for effecting a possible reduction in the maximum demand on steam plants by storing water in the ponds of hydrostations during light-load periods through the use of energy from other available sources so that this water can be used at times of peak demand, and (4) for reducing joint requirements for reserve capacity.

(*d*) Relay protection and circuit-breaker characteristics directed toward satisfactory operation of the system as a whole, both under normal and fault conditions.

2 *Centralized Supervision of Generating Facilities.*

Even with adequate interconnections, it is quite apparent that the power-supply facilities of the various areas of a system consisting of a multiplicity of steam- and hydro-generating stations, widely diversified as to location and character, together with interconnections with other systems, could not be operated independently as separate units and still develop their full inherent possibilities in meeting the requirements of the system load as a whole in the most economical manner possible. Some form of centralized control and supervision is, therefore, necessary. In the case of a compact system this may take the form of one central

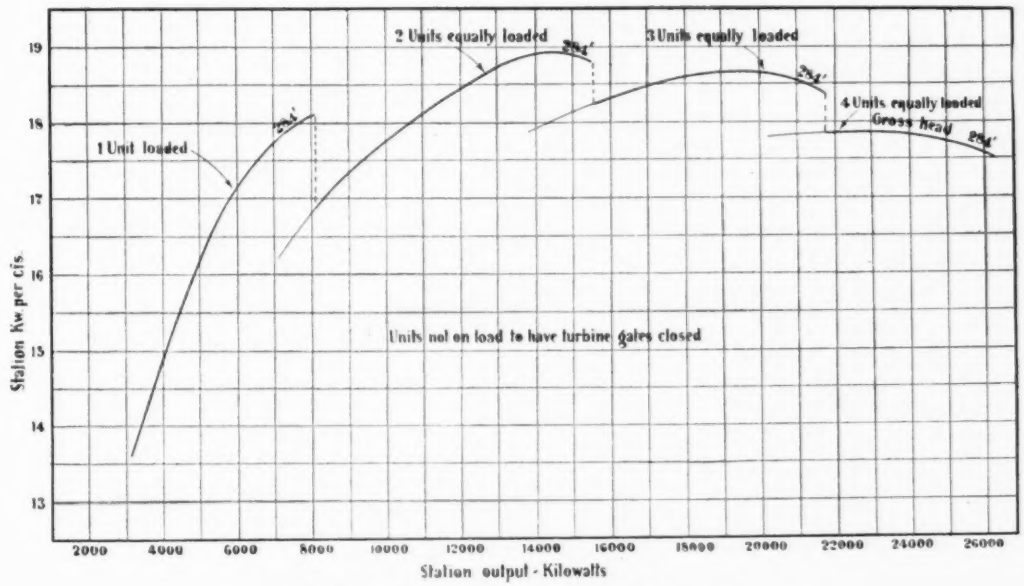


FIG. 2 LOAD DISPATCHING CURVE FOR STATION A

office from which control is exercised over all the plants in the territory. On the other hand, a large number of plants, an extensive operating territory, and complex local limitations may indicate that a completely centralized control would be cumbersome and would defeat the very objectives which it was set up to meet. In order to coordinate the operation of all the power-supply facilities and at the same time provide flexibility of control and close contact with local operating problems and limitations, it will prove to be desirable, in a widespread system, to set up a load-dispatching organization with offices in each of several major operating divisions, as indicated in Fig. 1. In the case shown, three division offices are indicated and are called the Western, Central, and Eastern. The location of these dispatching offices should correspond with the territorial grouping of principal load centers and power-supply facilities referred to previously. The dispatching offices should be connected with each other and with the generating stations in their respective divisions by direct telephone lines which will permit the prompt and frequent communication necessary in order to secure the most economical operation of the system as a whole. These lines of communication are indicated in Fig. 1. Competent and trained supervisors are required for duty at all times in each of these offices and each office should be under the direction of a chief supervisor responsible to the engineering and operating executives in his division. Coordination between divisions should be secured by constant contact, both by telephone and by frequent meetings of the chief supervisors and the engineering and operating personnel. At these meetings, current problems may be discussed and methods of procedure agreed upon.

It is the duty of the dispatching offices to keep informed as to system load conditions and power supply available at all times and to direct the load to be carried by the generating stations in the respective divisions in order that the varying requirements of the system load as a whole may be met in the most economical manner possible.

In order that maximum use may be made of all hydro power available, the load dispatchers must carry out the well-known rules of operation of a combined hydro and steam system, under which (*a*) during periods of low stream flow the steam stations operate on base load and the limited water supply at the hydro stations is used for carrying peak loads and (*b*) during periods of high stream flow the hydro stations on rivers with limited regulation of flow operate on base load with the steam plants and hydro plants on rivers which are highly regulated carrying the peak load.

A system completely balanced with respect to hydro and steam capacity will be able to carry its load throughout the year with little or no loss of hydro energy. However, such a balance may be difficult to obtain, particularly in a system not designed from the start with that objective in mind. There will probably be times during each year when all the available hydro energy cannot be fitted into the system load curve; hence, the desirability of interconnection with neighboring systems capable of absorbing surplus hydro energy during periods of high stream flow and light load.

In order that the dispatching offices may attain the foregoing objectives for the system, they must have complete and up-to-date information on hydro power available, the cost of power production from all sources, and on the efficiency characteristics of all stations.

3 *Data on Operating Characteristics of Generating Stations.*

In order that each hydro station may be operated so as to produce the maximum amount of energy from the water supply available, it is essential to know its operating characteristics. A knowledge of these will enable the station operators to divide any assigned load among the various units in such a way as to require the minimum use of water for that load and will likewise aid the load supervisors in assigning such a load to each station as may be most efficiently carried by it. The same principles, of course, are applied in the case of the steam stations.

Accordingly, if not already made, input-output tests should be carried out on most, if not all the generating stations, and the detail and refinement of the test for a given station should be commensurate with its importance and the possible gain to be made by ideal operation. In the case of hydroelectric plants the tests may be made by any accepted method such as those given by the A.S.M.E. Power Test Code for hydraulic prime movers. The measurements taken should include all those necessary to segregate the losses between the various component parts of the project such as the water conduit above the forebay, the penstock, the water wheel, the tailrace, and the generator.

The information derived from tests on hydro plants should be

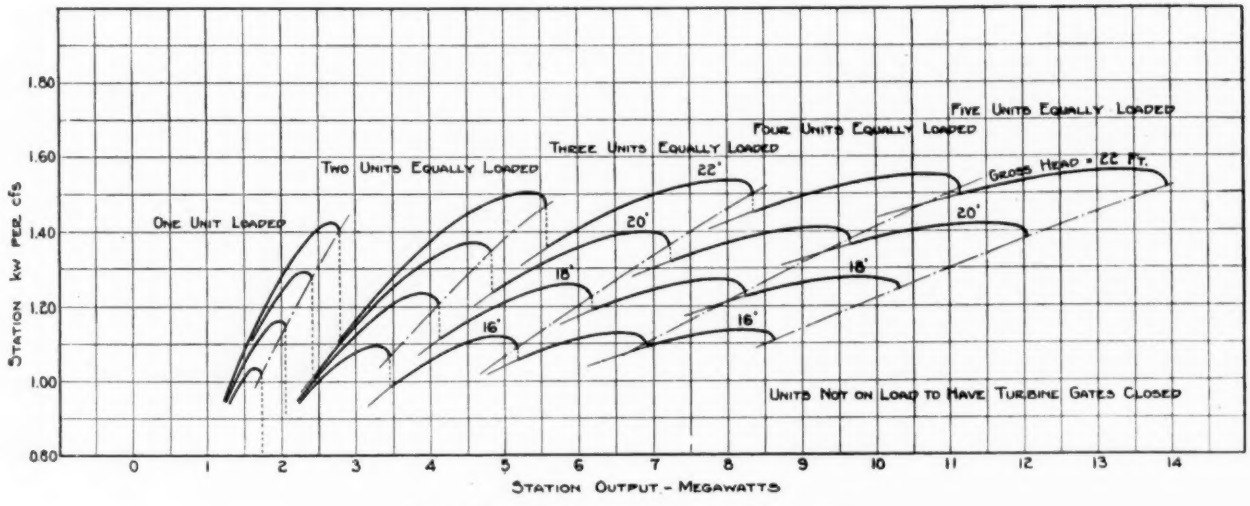


Fig. 3 Load Dispatching Curve for Station B

presented in the form of preferred-loading curves and tables, and in the case of at least one station on each river a manual of instructions should be prepared to enable the operators to keep a record of basic hydraulic information in the form of the "hydraulic log" as described hereinafter. Fig. 2 presents a preferred-loading curve for a hydroelectric station having a long pipe line which supplies four 10,000 hp units. For the sake of simplicity, the curve is given for only one gross head but actually the curve sheet would cover a family of gross heads ranging from 257 to 284 ft, which correspond to minimum and maximum reservoir elevations, respectively. It will be noted that for single-unit operation, the leakage of the other three units causes a relatively low over-all plant efficiency. As additional units are loaded up, the relative effect of leakage is reduced and plant efficiency is increased to the point where the effect of increased pipe-line loss overbalances the reduced leakage. Beyond this point the over-all efficiency begins to drop. In this case the point of best operation is reached with two units loaded to 14,500 kw and with the turbine wicket gates of the other two closed.

Fig. 3 shows curves for several heads for a station having individual penstocks. With this type of station the operating efficiency is progressively greater as units are put in operation, on account of eliminating the leakage loss in the stand-by units. From this curve it is apparent that, with a gross head of 20 ft and with one unit loaded, a maximum of 1.29 kw is obtained per cfs, but with five units operating under this head a maximum of 1.42 kw is obtained per cfs. Also for greatest economy of operation when the head is 20 ft five units should be operated with a total load of 11,500 kw. Information derived from these curves is put in the form of tables for the use of the station operators and the system load supervisors.

In assigning load to this and other similar stations, consideration must be given to the system requirement for peak capacity and to the water supply available. In all cases, however, the assigned load must be divided among the units according to the preferred-loading schedule.

For a station containing several units with unlike characteristics a loading "diagram" should supplement the preferred-load-

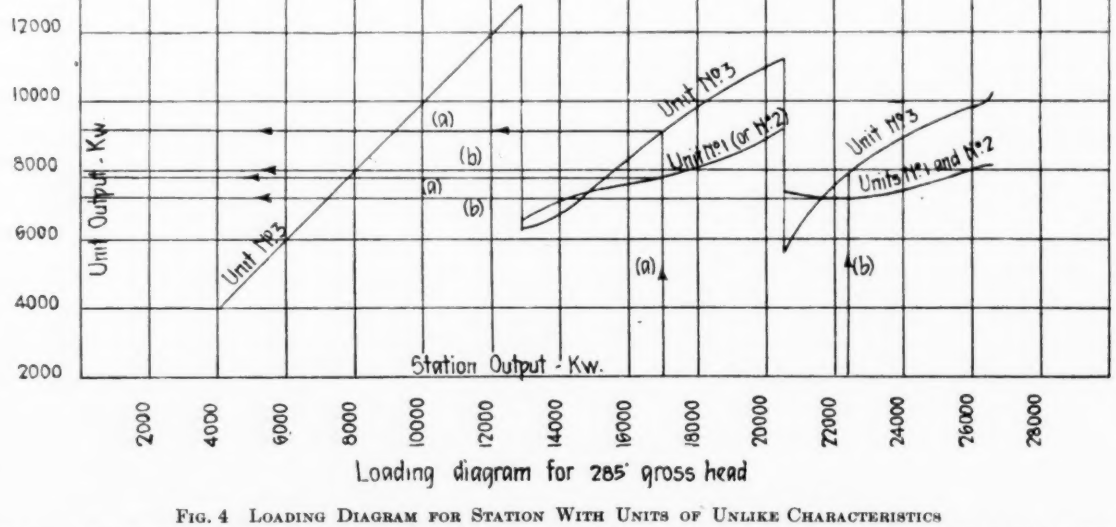


Fig. 4 Loading Diagram for Station With Units of Unlike Characteristics

(Examples: (*a*) For station load of 17,000 kw, follow vertical line from this point to intersections on curves. From these points follow horizontal lines to unit-output scale to determine load on each unit, viz.: No. 3 unit, 9150 kw; No. 1 (or No. 2) unit, 7850 kw. (*b*) For station load of 22,600 kw, unit-output figures are: No. 3 unit, 8000 kw; No. 1 and No. 2 units, 7200 kw each.)

STATION A DATE Sun. Feb 20, 1938

TIME	POND ELEV	FORE-BAY ELEV	TAIL-RACE ELEV	① OPER HEAD	②TOTAL STATION KWH	UNIT NO.1 KWH GEN.	UNIT NO.1 C.F.S.	UNIT NO.2 KWH GEN.	UNIT NO.2 C.F.S.	UNIT NO.3 KWH GEN.	UNIT NO.3 C.F.S.	UNIT NO.4 KWH GEN.	UNIT NO.4 C.F.S.	UNIT NO.5 KWH GEN.	UNIT NO.5 C.F.S.	UNIT NO.6 KWH GEN.	UNIT NO.6 C.F.S.	UNIT NO.7 KWH GEN.	UNIT NO.7 C.F.S.	③ TOTAL STA. C.F.S.	GATE C.F.S.	SPILL-WAY C.F.S.	LOG ICE TRASH-CHUTE C.F.S.	④ TOTAL C.F.S.	⑤ MAX STA. CAP C.F.S.	⑥ UN-USED C.F.S.	⑦ EXCESS C.F.S. ④-⑤
1 AM	931.8		892.1	279.7	71.000	9000	331	9100	336	4400	256	4500	261							1154				1154	1484		
2	931.8		892.1	279.7	7900	3800	219	3800	219	0	15	0	15							568				568	1484		
3	931.8		892.1	279.7	15000	7400	389	7900	397	0	15	0	15							813				813	1484		
4	931.8		890.4	281.4	7700	3200	192	4500	248	0	15	0	15							470				470	1484		
5	931.8		858.4	281.4		0	15	0	15	0	15	0	15							60				60	1484		
6	931.8		890.4	281.4		0	15	0	15	0	15	0	15							60				60	1484		
7	931.8		890.4	281.4	1500	0	15	1500	107	0	15	0	15							152				152	1484		
8	932.0		890.4	281.6		0	15	0	15	0	15	0	15							60				60	1484		
9	932.0		890.4	281.6	2400	0	15	2400	156	0	15	0	15							201				201	1484		
10	932.0		890.4	281.6		0	15		15	0	15	0	15							90				90	1484		
11	932.0		890.4	281.6		0	15		15	0	15	0	15							60				60	1484		
NOON	932.0		890.4	281.6		0	15		15	0	15	0	15							90				90	1484		
1 PM	932.0		890.4	281.6		0	15		15	0	15	0	15							90				90	1484		
2	932.0		890.4	281.6		0	15		15	0	15	0	15							90				90	1484		
3	932.0		890.4	281.6		0	15		15	0	15	0	15							90				90	1484		
4	932.0		890.4	281.6		0	15		15	0	15	0	15							90				90	1484		
5	932.3		890.4	281.9		0	15		15	0	15	0	15							60				60	1486		
6	932.3		890.4	281.9		0	15		15	0	15	0	15							60				60	1486		
7	932.3		892.5	279.7	17000	7000	372	7200	383	2800	179	0	15							949				949	1486		
8	932.3		892.1	280.2	17000	7400	393	7980	398	2100	147	0	19							950				950	1486		
9	932.3		892.1	280.2	14700	7300	378	7400	383	0	15	0	15							791				791	1486		
10	932.3		892.1	280.2	15000	7400	383	7900	394	0	15	0	19							807				807	1486		
11	932.3		890.4	281.9	3400	1900	117	1900	121	0	15	0	15							268				268	1486		
MID	932.3		890.4	281.9		0	15	0	19		15	0	15							60				90	1486		
TOT	932.0			97470	171.800	91000		97000		9300		4900								7933				7933			

* WHEN ④ IS GREATER THAN ⑤ THEN ⑥ = ④ - ⑤ - ⑧ WHEN ④ IS LESS THAN ⑤ THEN ⑨ = ④ - ⑤

TABLE OF LOAD CHANGES (CHANGED TO FIGURE OPPOSITE TIME) (USE INDICATED KW METER-READINGS)

TIME	UNIT NO 1	UNIT NO 2	UNIT NO 3	UNIT NO 4	UNIT NO 5	UNIT NO 6	UNIT NO 7
12 06 AM	318,376	516,943	780,644	780,858	Meter Readings		
12 06 AM	9900	5905	6000	60096			
12 43 A M	7800	7068	0(M)	0(M)			
1 06 A M	0(M)	0(M)	0(M)	0(M)			
1 39 A M	7000	7069	0(M)	0(M)			
3 26 A M	0(M)	7800	0(M)	0(M)			
5 36 A M	0(M)	0(M)	0(M)	0(M)			
6 26 A M	0(M)	7800	0(M)	0(M)			
6 36 A M	0(M)	0(M)	0(M)	0(M)			
8 22 A M	0(M)	7800	0(M)	0(M)			
8 44 A M	0(M)	0(M)	0(M)	0(M)			
6 00 P M	7300	7300	0(M)	0(M)	Wide open		
6 36 P M	9700	9700	9750	0(M)	-	-	
7 18 P M	7300	7300	0(M)	80(M)	-	-	
10 12 P M	0(M)	0(M)	0(M)	0(M)			

DAILY SUMMARY

1	ELEVATION OF PERMANENT CREST	935.0
2	ELEVATION OF SPILLING CREST	937.0
3	MAXIMUM POND ELEVATION	952.5
4	MAXIMUM POND DRAFT	0
5	AVERAGE OPERATING HEAD	281.1
6	TOTAL KWH GENERATED	171,800
7	TOTAL KWH MOTORING	23,750
8	STATION FLOW C.F.S. (AVERAGE)	325
9	TOTAL FLOW C.F.S. (AVERAGE)	325
10	KW.H. LOSSES DUE TO OPERATION	9700
11	KW.H. LOSSES DUE TO LOW POND	7036.9
12	KW.H. LOSSES DUE TO UNUSED C.F.S.	
13	KW.H. LOSSES DUE TO EXCESS C.F.S.	
14	RAINFALL FOR DAY	0.0"
15	RAINFALL FOR MONTH	4.27"
16	TEMPERATURE - MAXIMUM	37°F
17	TEMPERATURE - MINIMUM	17°F

REMARKS — Gate openings

½ 25 ∞ Units wide open

HEADWATER GAGE CHECKED AT M ERROR FT

MAKE NOTE HERE OF ALL FLASHBOARD AND SLUICE GATE CHANGES (FROM- TO-)

MACHINE HOURS	TOTALS				
GENERATING	17	7	8	1	1
MOTORING	79	17	18	23	23
SPEED NO LOAD	0	0	0	0	0
SHUT DOWN	0	0	0	0	0
TOTALS	96	24	24	24	24

OPERATOR	FIRST SHIFT 7 32 A M N M R	SECOND SHIFT 3 30 P M F H P	THIRD SHIFT 11 49 P M J B.
	7 49 A M E H H	3 30 P M W T P	11 49 P M S C E

FIG. 5 HYDRAULIC LOG

ing schedule in order to provide the allocation of loads between the units operating. An example of such a diagram is shown in Fig. 4.

Many companies have found that substantial improvements in hydro output have been obtained by the use of such curves and tables. It will prove to be of great advantage to maintain a constant check of operating results by means of the hydraulic log previously mentioned or by some similar routine. See Fig. 5. This sheet is designed to supplement the electrical log which is ordinarily kept by all hydroelectric stations. The data recorded can be arranged in such a way that the losses caused by departures from ideal loading can be readily determined and corrective measures taken.[3]

The manual of instructions for the use of the hydraulic log at a particular station should contain a complete explanation of all terms used and should give sample computations where they are considered necessary. Among the data tabulated in the manual there should appear the station capacity, preferred-loading tables, values of turbine-gate leakage, various constants, and discharge tables for spillway, log chute, and trash-and-ice chute.

It is highly desirable that complete input-output tests be made on all steam stations and that the results of such tests be continuously checked from operating records. The results of the test should show among other things (1) the no-load intercept for each combination of units, i.e., the fuel required to keep that number of units on the line without load (a theoretical condition only, as steam units must carry a certain definite minimum load in order that sufficient steam may pass through the turbine to dissipate the heat generated by friction at the high peripheral speeds of the low-pressure wheels), (2) the slope of the input-output curve over the whole capacity range for each combination of units when such units are loaded for maximum over-all economy, (3) a preferred-loading curve or table for each combination of units which will show the division of any given load among the units for maximum economy, (4) fuel consumption of boilers banked and

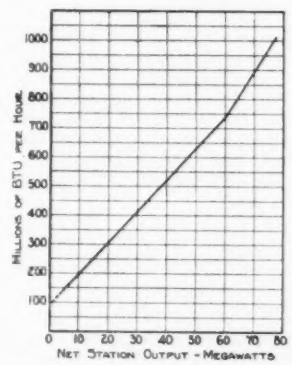


FIG. 6 INPUT-OUTPUT CURVE FOR TYPICAL STEAM UNIT

ready to pick up load, (5) the fuel required to start up each unit from a cold condition. Fig. 6 is a typical example of the input-output data obtained from the test of a large steam unit and its associated boilers and auxiliary equipment.

All these quantities will be required by the load dispatchers in planning the operation of the system for maximum over-all economy and in deciding which plants and units shall be in operation to provide the required running reserve generating capacity referred to later.

4 *Operation of Storage Reservoirs.*

Beside the economical operation of the hydro stations them-

[3] For a detailed presentation of the methods used and results obtained from the hydraulic log, reference may be made to an article, "How We Raise Hydro Efficiencies," by E. B. Strowger, *Electrical World*, vol. 103, April 14, 1934, p. 535.

selves, there is the operation of storage reservoirs to be considered. Instead of basing their operations upon the requirements of a relatively small or local section of the operating territory, as was done of necessity when certain parts of a system were formerly operated as more or less isolated units, they can now be made to function to the best advantage of the system as a whole. As a part of the larger system, full advantage can be taken of reservoir drawdown in anticipation of flood periods, because deficiencies occurring as a result of unexpected intervening periods of low flow can be made up from any available source on the combined system through the interconnected transmission facilities. In this way waste of water during high flow periods can be substantially reduced, with the resulting increase in annual hydro output. As a typical example, one company reports that since complete interconnection, it has increased the drawdown of one of the main reservoirs on its system by an average of 15 ft in anticipation of periods of high runoff. It has been estimated that in this case this type of operation has allowed an annual increase in system hydroelectric output of 75,000,000 kwhr which would otherwise have been wasted. Interconnection and coordinated operation has provided the outlet for the energy during the drawdown period as well as a backup source of supply to the area in case the reservoirs are overdrawn for an unexpected period of low flow.

5 *Reserve Capacity.*

As an example in setting up reserve generating-capacity requirements for protection against equipment failures, the following has been suggested by a company with a system similar to that described in the opening paragraphs, and is cited to show the influence of coordinated operation on the reserve generating capacity required.

1 There shall be installed on each system (25- and 60-cycle) sufficient reserve capacity to allow for the loss of the largest generating unit on that system at time of peak demand without reduction in load.

2 There shall be running on the 60-cycle system sufficient reserve generating capacity to allow for the loss of the largest unit on that system at all times. Under certain specified conditions, this 60-cycle running reserve may be reduced provided there is sufficient running reserve on the 25-cycle system available to the 60-cycle through the frequency changers or on neighboring interconnected systems to make up a total equivalent to the capacity of the largest unit. Running reserve is provided for the 25-cycle system either from the 60-cycle system through the frequency changers or on the 25-cycle system itself as may be required from time to time by operating conditions. The installed reserve allows the reestablishment of the running reserve in case of the loss or unavailability of the largest unit on either system at time of peak load.

The foregoing constitutes what is considered to be the minimum reserve generating capacity consistent with a high degree of service reliability. Operating conditions and the provision of generating capacity for growth of load will in general result in reserve capacities somewhat higher than the minimum required.

Here again, interconnection and coordinated operation have made substantial operating savings, for it is obvious that were the various units comprising the system operated separately while maintaining the same degree of service reliability, the amount of reserve capacity required, both installed and running, would be greatly increased.

6 *Regulation of System Frequency, Voltage, and Tie-Line Load.*

The regulation of system frequency, voltage, and tie-line loads is intimately connected with all the preceding requirements for economical and reliable operation. Satisfactory service to the customer requires maintenance of substantially constant voltage and frequency, and all other requirements of system operation must be coordinated to meet this condition. Within the last few years, the demand for accurate electric time service has introduced the further requirement of maintaining integrated frequency within what might be termed precision limits.

Maintenance of scheduled tie-line loads between divisions of the system and with neighboring systems is necessary in order that station outputs which have been scheduled to meet the total load with maximum over-all economy can be realized.

Again referring to the system outlined in the opening paragraphs, let it be assumed that electric service is rendered at both 25 and 60 cycles and that the two systems are connected together through a substantial capacity in variable-ratio frequency changers with positive and automatic control of load transfer within certain limits of variation of the two frequencies. The operating procedure as reported for such a setup may be described as follows:

In the preparation of programs for the operation of power-supply facilities, commonly called schedules, the 25- and 60-cycle systems may be considered separately in the initial steps. Referring to Fig. 1 the Western Division supervisor in whose territory the bulk of the 25-cycle system lies, prepares an estimate of the 25-cycle load for, say, the following day and schedules the operation of his power-supply sources to meet this load in accordance with the requirements stated previously. Any surplus or excess energy may then be scheduled for transfer through the frequency changers under positive load control. He also prepares a similar estimate and schedule for the Western division 60-cycle load and can then arrive at a combined excess or deficiency for the division as a whole. Similar schedules prepared by the Central and Eastern division supervisors, when combined with that for the Western division, will then show the over-all excess or deficiency of hydroelectric and other energy of low increment cost. Deficiency of energy of low increment cost must be made up by the steam plants which are then scheduled for operation in the order of their increment production costs with due allowance for losses from point of generation to point of use. Excess energy of low increment cost, if found to exist, is scheduled for sale to neighboring systems.

A complete operating schedule, prepared for the whole system in accordance with the requirements of maximum over-all economy, will call for definitely varying rates of transfer of energy between divisions and other systems throughout the whole period, the maintenance of which becomes the duty of the division load supervisors in addition to the fundamental requirements of maintaining system voltage and frequency. Transfer between the 25- and 60-cycle systems is under positive control and involves no serious problem. The transfer between divisions and with neighboring systems on the 60-cycle system, however, may present some complications in the matter of regulation. To illustrate: Suppose the economical operating schedule calls for a delivery of 100,000 kw to a neighboring system for a period of several hours. This represents possibly 5 per cent of the total generating capacity running in synchronism on the two systems combined. The transfer must be held within reasonably close limits if the greatest over-all economy is to be obtained, while at the same time the system frequency, both instantaneous and average, must be held within narrow limits.

One company reports that, in operating in parallel with another system, maintenance of instantaneous and average frequency was first handled by a large steam plant on the neighboring system by hand control and later by means of an automatic frequency controller of the frequency-bridge type. The company was charged with the responsibility of holding scheduled tie-line loads. This procedure resulted in certain difficulties, principally in rapid load changes on the regulating steam station during load-change periods with the attendant smoke nuisance and inefficient operation. This resulted from the fact that a load change anywhere on the combined systems resulted first in a corresponding change in sys-

tem frequency which was corrected by the regulating station regardless of the location of the load change. After the frequency returned to normal that part of the load change occurring on the company's system was reflected in a departure of the tie-line load from schedule, which departure would then be corrected by the company's load dispatchers, which then resulted in a further load change on the regulating station.

This situation was corrected by installing a blocking device, actuated by tie-line load and operating on the frequency controller of the regulating steam plant, together with the use of automatic frequency-control apparatus of the integrated-time-error type on the company's system. The blocking device on the tie line prevents the operation of the frequency controller at the steam station, if its operation as called for by system frequency would tend to throw the tie-line load off schedule. Thus a correction for deviation of frequency due to a load change is forced on the system upon which the load change occurs and the regulating duty is divided between the two systems in proportion to the load changes occurring on each.

Accurate integrated frequency for electric time service has been secured by use of a standard 60-cycle wave supplied to the regulating steam station by means of a crystal-controlled oscillator and accurate within precision limits. This frequency is applied to one circuit of a synchronoscope and system frequency to the other; thus the system time error in cycles (sixtieths of a second) is indicated by the revolutions of the synchronoscope. The rotation of the synchronoscope may be used to unbalance the bridge circuit of the frequency controller, thus causing it to regulate for a higher or lower frequency until the time error is corrected. Or the time error thus measured may be communicated to a large plant on the company's system having an integrated time-error type of frequency controller, used as a check on the master clock at that station, and as a guide in setting the time-error corrector. The time-error corrector is a device on the integrated time-error frequency controller which has a somewhat similar effect to unbalancing the frequency bridge of the controller at the regulating steam plant.

The results from this procedure have been satisfactory in that scheduled tie-line loads have been held within close limits, while at the same time instantaneous frequency is normally held within a band of ± 0.1 cycle per sec in 60 cycles per sec and maximum time error within $\pm$ 1 sec.



FIG. 1 GENERAL VIEW OF BONNEVILLE PROJECT ON THE COLUMBIA RIVER
(Spillway dam in main channel. Powerhouse and navigation locks in Bradeford Slough, foreground. The long concrete structures in center of picture are fish ladders. The pool is partially filled.)

The Kaplan Turbines at Bonneville

BY PAUL L. HESLOP[1] AND GEORGE A. JESSOP[2]

The Bonneville project has an initial installation of two 66,000-hp Kaplan turbines designed to operate under heads varying from 30 to 69 ft. Noteworthy features are the physical size of the units, the extensive model tests, pioneering in the use of welded construction, new ideas in auxiliaries, and the care exercised in setting embedded parts.

GENERAL DESCRIPTION OF PROJECT

THE BONNEVILLE project is at tidewater on the Columbia River 42 miles east of Portland, Oregon. It is the lowest of the ten dam sites selected by the U. S. Army Engineers as part of a projected scheme of river development.[3] Of these ten, Rock Island was constructed as a private power project some years ago, Grand Coulee (the uppermost) is under construction, and Bonneville is approaching completion. Navigation and power are provided by the Bonneville Dam, but navigation requirements at the dam or in the river will not affect operations of the power plant. River conditions are given in Table 1.

There is no seasonal storage available in the Bonneville pool, but daily load fluctuations can readily be accommodated with 3 ft or less of drawdown. The record flood of 1894, and more, has been provided for in spillway design but so large a flow is not expected again due to increased and increasing upriver storage. Consideration of these facts and other data led to the decision that the turbines should operate under heads of from 30 to 69 ft. With the authorization of Bonneville there was projected into the northwest territory a power development of large proportions with character of load, shape of load curve, immediate demand, probable rate of growth, and interconnections—all matters for conjecture. For such conditions with low variable head and variable output the Kaplan adjustable-blade turbine is well adapted to give the highest efficiency over the widest range.

The size of the turbines was given careful consideration. The units are about as large as is practicable under existing shop conditions. There are also certain practical mechanical limitations, of which might be mentioned the enormous vertical thrust and the high pressure on the bearings of the overhung turbine blades. Reduction of form work for fewer units is partially offset by the expense of longer structural spans, heavier cranes, and larger gates. As finally designed, the turbines have a rating of 66,000 hp under 50 ft head. They are the largest Kaplans in this country, the highest powered Kaplans ever built, and the generators they drive are the largest in physical dimensions so far constructed. Various characteristics are given in Table 2.

TABLE 1 RIVER HYDROLOGY

	Cfs	Cfs per sq mile[a]	Corresponding head, ft
Mean annual flow	211000	0.88	54
Minimum flow	40000	0.166	65[b]
Normal high flow	800000	3.23	40
1894 flood, maximum	1170000	4.65	25.7[c]

[a] Drainage area, 240,000 sq miles.
[b] The head can be 69 ft under certain conditions.
[c] Estimated.

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[3] See "Improvement of the Columbia River," by Col. Thomas M. Robins, *Civil Engineering*, vol. 2, September, 1932, pp. 563–567.

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NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

TABLE 2 TURBINE DESIGN DATA

	Main turbines	Station-service turbine
Type	Kaplan	Kaplan
Blade adjustment	Automatic	Manual
Number, present	2	1
Number, ultimate	10	1 or 2
Diameter of runner (throat), in.	280	81.0
Number of blades	5	5
Rated horsepower	66000	5000
Rated head, ft	50	50
Rated discharge, cfs	13000	1025
Rated speed, rpm	75	257
Runaway speed, 69-ft head, rpm	216	750
Range of head, ft	30 to 69	30 to 69
Usual minimum head, ft	40	40
Usual minimum tail water, el, ft	7	7
Possible minimum tail water, estimated, el, ft	3	3
Usual maximum tail water, el, ft	40	40
Maximum tail water for design, el, ft	53	53
Specific speed at rating	145	137
Phi at rating	1.61	1.59
Peripheral speed, turbine, normal, fpm	5498	5450
Peripheral speed, turbine, runaway, fpm	15800	15800
Peripheral speed, generator, normal, fpm	7975	8070
Peripheral speed, generator, runaway, fpm	22950	23400
Coef of gates as orifice, C in $CA\sqrt{(2gh)}$, at rating	0.455	
Center line of speed ring, el, ft	12.5	10
Center line of runner, el, ft	3.9167	7.516
Center to center of units, ft	82	
Weight of rotating element, lb	1500000	120000
Hydraulic thrust at 69-ft head, lb	1500000	
Total load on thrust bearing, lb	3000000	
WR^2 of turbine and generator, ft²-lb	158000000	920,000
Type of scroll	Concrete	Plate steel
Velocity at entrance to turbine scroll, fps	6.5	10.04
Type of draft tube	Elbow, horizontal splitter	Elbow, horizontal splitter
Number of intermediate piers in draft tube (see text)	1	1
Velocity, draft-tube exit, fps	6.3	7.1
Velocity through trash racks, gross area, fps	2.89	
Velocity through trash racks, net area, fps	3.75	
Manufacturer	S. Morgan Smith	S. Morgan Smith

The initial installation will consist of two main units and a station-service unit of 5000 hp capacity. Foundations are built for four more main units. At the site of the project the Columbia River divides into two parts around Bradford Island, as shown in Fig. 1. The power plant is located in the smaller of the two channels and the space required for the present installation plus four future units nicely forms a dam across the channel. The installation of the last four of the ultimate ten units will require further major operations of cofferdamming and excavating.

The conditions under which the turbines must operate are graphically illustrated by the power-duration curves shown in Fig. 2. The curve of flow Q is a 55-year hydrograph of the Columbia River plotted in duration form. Any vertical line contains simultaneous values of flow Q, headwater elevation, tail-water elevation, head H, and power. The curves are for the run of the river without any regulation. At low and moderate river stages, or about three fourths of the time, the pool will be held at elevation 72, which is the approximate level of the top of the spillway gates. As the river flow approaches 250,000 cfs the rise in tail water threatens to decrease the head sufficiently to limit output, but for flows up to around 500,000 cfs the effect of the rise in tail water can be counteracted by raising the headgates and the pool level to maintain the rated power output of the units. This can be carried on until headwater reaches elevation 82.5 when upriver culture prevents further adjustment. For flows above 800,000 cfs the spillway gates must be opened wide and the turbines left to do the best they can. For such conditions maximum output under the head then existing is the desired quality.

Based on past records, the head may get down to 40 ft almost any year, but would not have gone below that in the last 30 years. There are enough years of record to indicate the necessity for specifications of a 30-ft head, but the extreme minimum of 1894 should not occur again.

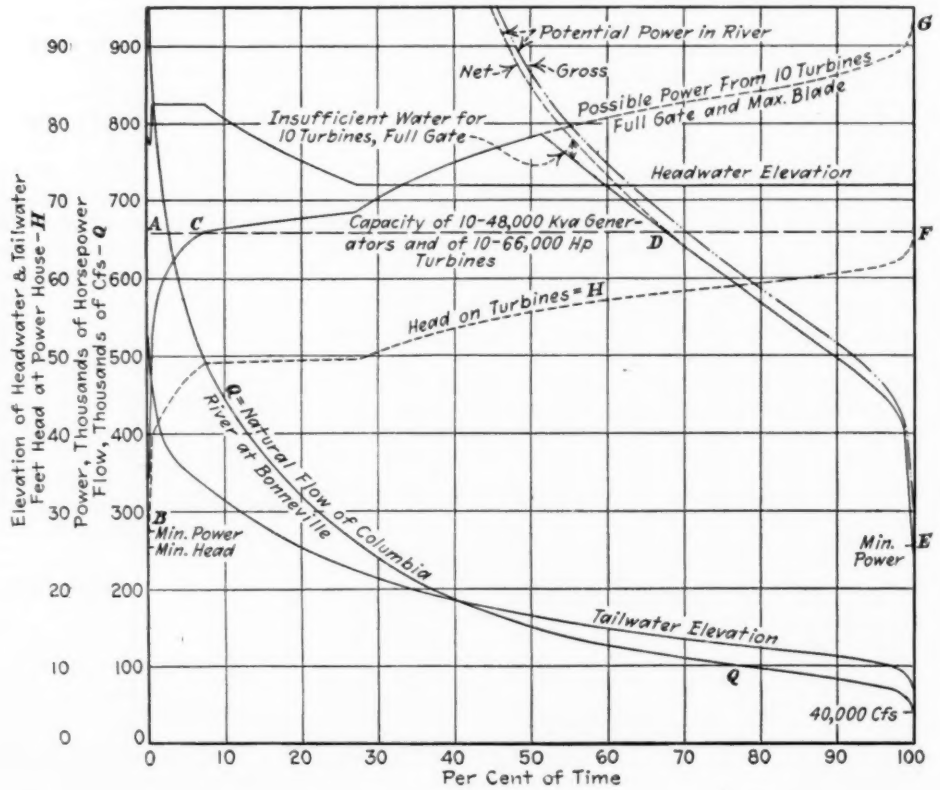


FIG. 2 HEAD AND POWER AT BONNEVILLE HYDROELECTRIC PLANT

(Curves showing hydraulic conditions at Bonneville, expressed in duration form; 55 years are covered, including the maximum flood of record in 1894. The initial installation consists of two 66,000-hp turbines designed for 50-ft head. Ultimately 10 units are to be installed.)

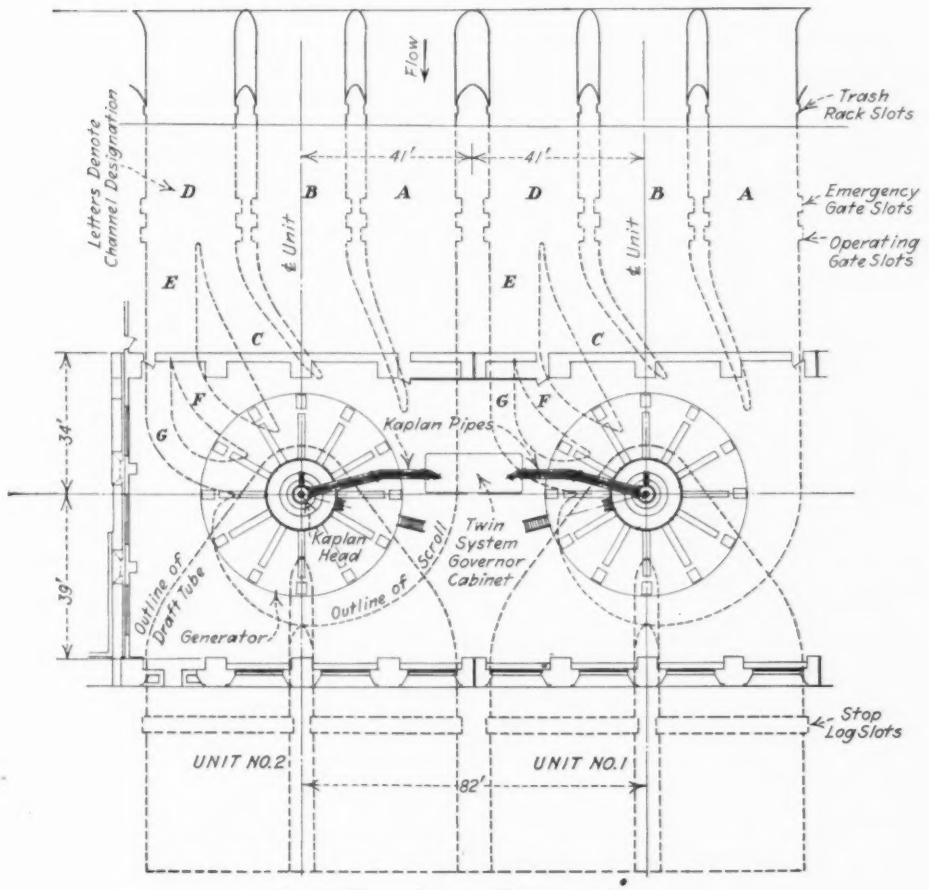


FIG. 3 FLOOR PLAN OF GENERATOR ROOM
(Units and governor shown superimposed on outlines of intakes, scrolls, and draft tubes.)

In Fig. 2, two curves are drawn indicating the potential power of the river, the lower of which allows for an estimated use of water for the fishways. Other than this, the extensive fishway apparatus does not affect the power-plant operation. It will be seen that for about one third of the time there is not sufficient water to operate 10 turbines at their rated capacity. The deficiency in possible output is represented by the area *DEF*. At such times maximum efficiency is to be desired. Depending upon the character of the future load, much of this deficiency may not occur in practice as there is ample pondage for daily regulation at any reasonable daily load factor. Also, now that the Grand Coulee is being constructed, there is a strong probability of regulation helpful to the low water flow.

The power-duration curves show the maximum head at 65 ft, while the turbines are specified and designed for 69 ft head. After the proposed dredging of the channel below Bonneville for navigation, there are certain combinations of low tide coupled with temporary shutdown of all or some of the units that could result in a 69-ft head, and safety requires that provision therefor be made.

The curve, "possible power for 10 turbines," represents the power that could be obtained hydraulically from the ten turbines, with water passages and blades like the two now installed. Actually the present turbines are designed mechanically for only 66,000 hp under 69 ft head with usual factors of safety. This capacity is required for driving a 48,000-kva generator at unity power factor. Standard and special devices are incorporated in the governors to prevent excess output.

Bonneville was authorized in October, 1933, by the Public Works Administration and it was desired that the maximum number of men be put to work at the earliest possible date. It was a comparatively easy matter to place an earth cofferdam in Bradford Slough and commence construction of the powerhouse quickly. Therefore it was necessary that the general features of the design of the turbine water passages be settled at the earliest possible moment. Experimental studies of the shape and type of the intake and draft tube were made at the U. S. Engineers' hydraulic laboratory near Portland. From the results of these studies and a consideration of structural features, plans for an intake with two intermediate piers and two islands were drawn and the type and controlling features of the draft tube were set. Data from manufacturers were checked against the engineer department studies and indicated a center-to-center spacing of units of 82 ft. Fig. 3 shows in plan the final layout of intake, scroll, and draft tube that resulted from the model tests and studies described herein. Fig. 4 is a general cross-sectional view of the powerhouse.

MODELS AND TESTS

In the U. S. Engineers' laboratory models were constructed for developing the proper intake, scroll, and draft tube. The general shape of the intake in vertical section was more or less established to fit in with the construction of a combined trash, ice, and fingerling sluiceway across the upstream face of the powerhouse. The other properties of the intake and scroll were developed by visual analysis with transparent models on a scale of 1 to 46. Sawdust was introduced into the flowing water and moving-picture films were made and studied by the experimenters

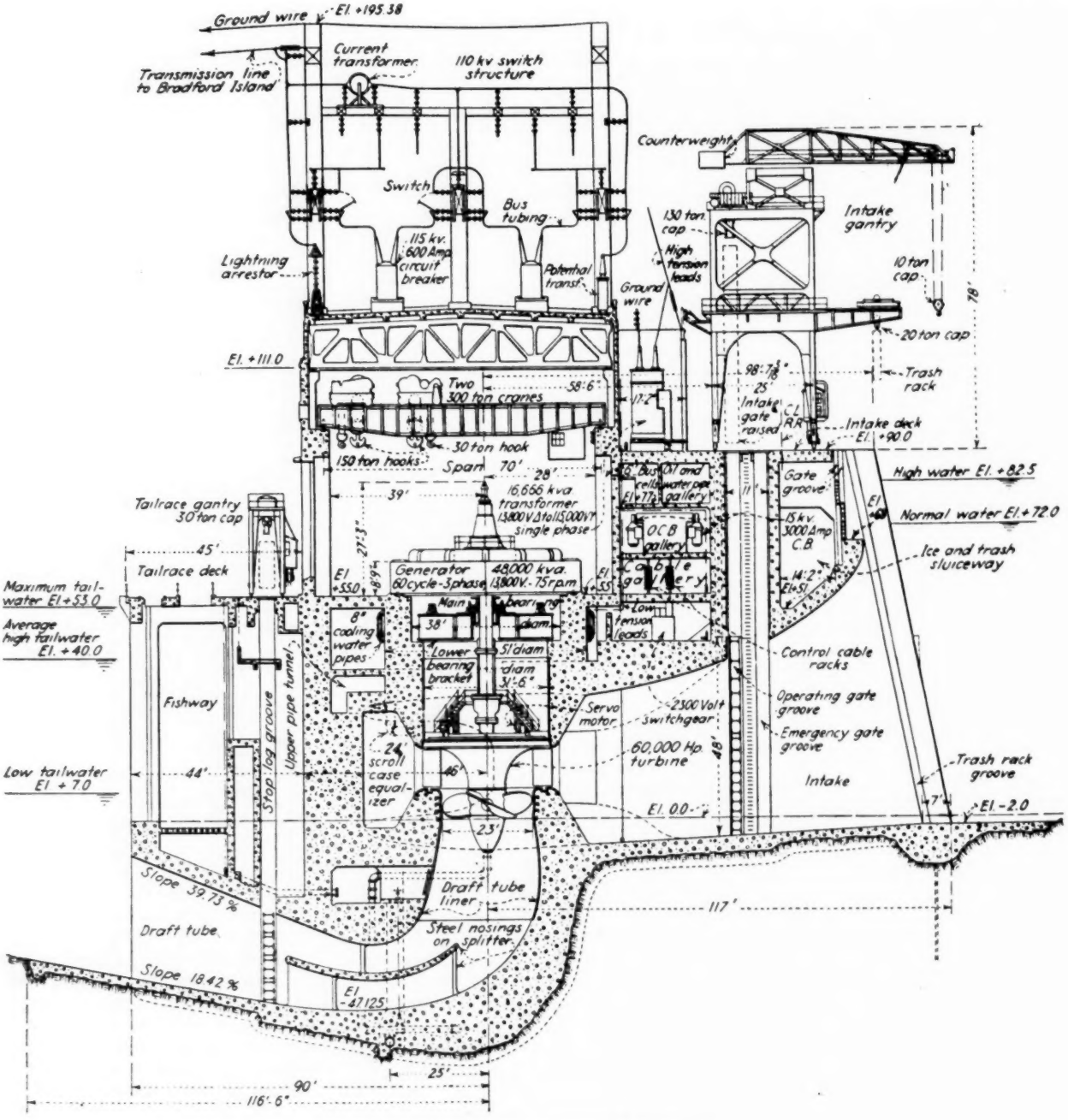


FIG. 4 CROSS SECTION THROUGH POWERHOUSE

and by the consulting engineers. From the results of many trials the best results were obtained with a constant-velocity scroll with water in the prototype flowing at 10 fps. An accelerating scroll was tried, but there was difficulty in getting a scroll in which the water would actually accelerate. A decelerating scroll was also tested but there was a tendency to form eddies, particularly at the small end of the scroll, and there was evidence of dead space. Transparent molded pyralin models of draft tubes were made and studied. The object was to get a workable draft tube to set dimensions for the structure. The most valuable result of the engineers' studies was the development of the horizontal splitter, which was found to improve conditions in most of the draft tubes investigated.

The contract for the turbines provided for acceptance of efficiency of the main units on the basis of model tests in the manufacturer's laboratory stepped up for size according to the Moody formula

$$Eff_a = 100 - (100 - Eff_m)(D_m/D_a)^{1/4}$$

Wherein Eff = efficiency, D = diameter of runner, and subscripts a and m refer to prototype and model, respectively.

The manufacturer was obligated to construct and test homologous models including intake, scroll, draft tube, and a runner of 16 in. diam arranged for substitution of blades of alternate design. From the results of the laboratory experiments at Portland, the U. S. Engineers designed an intake and scroll, also a discharge

tube, and established the dimensions of the intake up to a point within 34 ft, and of the horizontal leg of the draft tube to a point within 39 ft of the center line of the unit. Between these two points the manufacturer had a free hand for his designs.

It was not possible to use a scroll with more than slight changes from the original design, because of the islands and piers used to guide the water, sloping ceilings and floors between the piers, and the completion of a substantial amount of this work in the field prior to the conclusion of the tests. Such modifications as could be made did not indicate important changes in horsepower or efficiency and the original design of scroll was used.

The manufacturer exercised his rights under the contract and built a model draft tube laid out in so far as possible to follow the experience secured in many field and model tests. After tests in his laboratory the manufacturer elected to use his own design. This tube contains a horizontal splitter with a vertical supporting pier under it extending to its upstream edge. A heavy vertical pier is placed in the center of the horizontal leg of the tube starting at about the end of the elbow and continuing to the outlet. The splitter supporting pier is a continuation of this main pier below the splitter. The downstream end of the splitter is afforded additional support by two short piers.

The manufacturer made two sets of runner blades, selected the one giving the best results for the final tests, and used this design

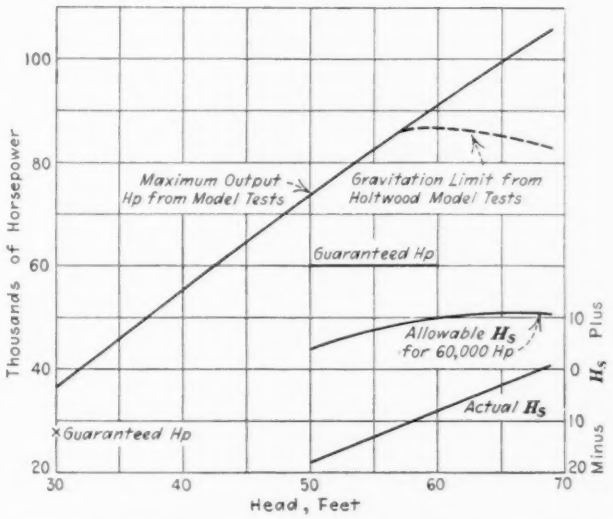


FIG. 5 SAFETY AGAINST CAVITATION AS DETERMINED BY TESTS IN THE HOLTWOOD HYDRAULIC LABORATORY

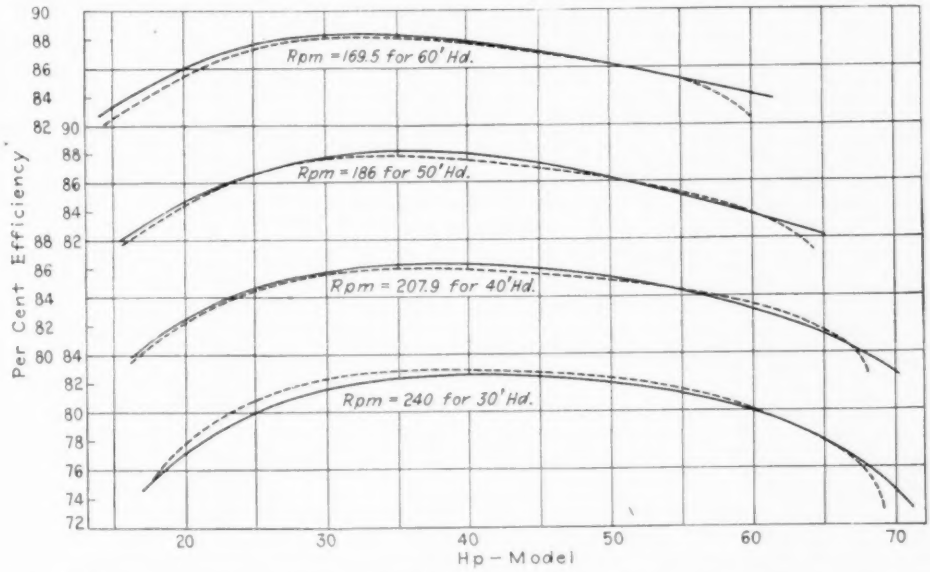


FIG. 6 COMPARISON OF RESULTS OF THE MODEL TESTS IN THE HOLTWOOD AND S. MORGAN SMITH CO. HYDRAULIC LABORATORIES (Solid curves, S. Morgan Smith laboratory. Dashed curves, Holtwood laboratory.)

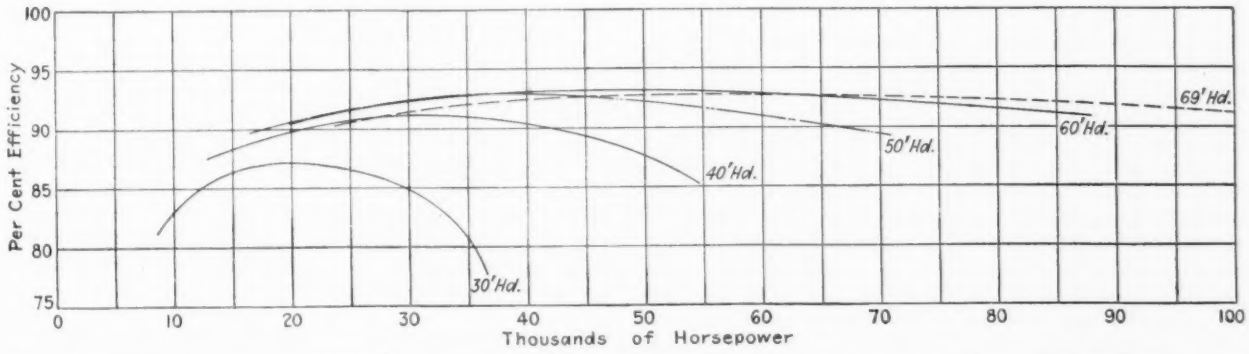


FIG. 7 HORSEPOWER AND EFFICIENCY CURVES OF THE LARGE UNITS OBTAINED BY INCREASING MODEL EFFICIENCIES BY THE APPLICATION OF THE MOODY FORMULA

for the 280-in. runners. The model runners were made of bronze and the scroll and draft tubes of welded plate steel. It was required that the models have sufficient strength for testing under a 69-ft head. The plates forming the elbow of the draft tube were die-pressed and the scroll was carefully shaped to templates. Great care was used to have the models exactly similar to the prototypes in all respects.

A model of the station-service unit was made and tested and since this unit will be field-tested, a further check between the laboratory and field results will be obtained. The acceptance tests for the service unit will be made by the Allen salt-velocity method. All of the horsepower-efficiency tests on the manufacturer's models were made in his laboratory at York, Pennsylvania.

There are many reasons for relying on model rather than field tests for a power plant like Bonneville. Because of the large cross-sectional area of the water passages of the unit, their relatively short length, and continually changing cross section in the direction of flow, it was considered to be unusually difficult to measure accurately the large quantities of water involved. Model tests as provided for by the Bonneville engineers disclose any faults there may be in design and lead the way to the elimination of the trouble at minimum expense. A choice can be made between several designs, thus securing the best possible results. Much more comprehensive data can be secured from model tests, a particularly valuable feature in a plant with a greatly varying head. Complete data are especially advantageous for Kaplan turbines. At Bonneville the model data will be supplemented by exhaustive tests using the index method for checking the best relationships between gate and blades.

The model efficiencies were stepped up according to the Moody formula to determine whether the guarantees were met and to obtain the expected field results. The model horsepower was stepped up in the usual manner without allowing for any extra capacity due to the increase in size. Fig. 7 shows the horsepower-efficiency curves for the full-size 280-in. turbine over the entire range of head. Expected and guaranteed results were exceeded by substantial margins due to improvements in the design of the turbine and the water passages. The preliminary and final investigations in the hydraulic laboratory were more than justified.

The relationship between headwater and tail water at Bonneville is such that the high head occurs with low tail water. Since the maximum operating head may be as high as 69 ft, cavitation tests become of great importance. While the elevation of the units with respect to tail water could be determined with reasonable accuracy from past experience and former tests, it was deemed necessary that additional cavitation tests be run. These were made in the Holtwood hydraulic laboratory by the testing staff of the Pennsylvania Water & Power Company. The complete model was set up, including the scroll case and draft tube, using designs which gave highest efficiencies in the manufacturer's flume. Check tests were also made on power and efficiency.

The results of the cavitation tests show a large safety whether considered in horsepower or in feet head. Fig. 5 shows that under 69 ft head, there is a safety of about 10 ft and under 50 ft head, a safety of about 22 ft. Actual H_s represents the elevation of tail water over the range of head shown. Allowable H_s is the elevation of tail water at which the turbines could develop 60,000 hp according to the cavitation tests but without any margin of safety. The center line of the turbine gates is at elevation plus 12.5 and the center line of the blades is at elevation plus 3.9167 (main units). A detailed account[4] of the procedure followed in cavitation testing at the Holtwood laboratory has been published in a previous paper presented before the Society.

Fig. 5 also shows the horsepower limits as determined by the maximum capacity of the wheel up to a head of 57 ft and then as limited by cavitation to a head of 69 ft. Since the guaranteed capacity of the turbine is 60,000 hp and the maximum load of the generator at rated capacity is 66,000 hp, ample safety is provided for any condition.

The horsepower and efficiency obtained at the S. Morgan Smith flume and at the Holtwood flume show remarkable agreement. Fig. 6 presents the comparative data reduced to 1 ft head for the model size, at unit speeds corresponding to the speed of the prototype at heads of 60, 50, 40, and 30 ft.

Turbine Design and Construction

The Bonneville turbines are much larger than any heretofore built and a thorough study was made of materials and types of construction. Because of the large size and complicated shapes of many of the parts and the consequent hazard of using steel castings, careful consideration was given to the possibility of fabricating by welding. A background of many years' experience was available and opportunities were afforded to the manufacturer to compare his practices and past experience with that of others who had used welded construction for large machinery.

It was apparent that a number of advantages could be realized by the use of welded members. There was an elimination of a large percentage of patterns, part of which advantage, however, was lost because of the necessity for making a number of dies. The parts were placed in the shop more quickly for machining and assembly, and shipping was thus expedited. There was practically absolute assurance that machine work would not expose defects with resultant rejection, replacement, and delay and, in view of the necessity for quick delivery, it was most important to be able to set and follow a precise schedule. The weights could be almost exactly figured thus making it possible to hold to accurate estimates of cost. The amount of metal allowed for finishing was reduced greatly over that required for castings and machining time also was thus cut to a minimum. The cost of welding, assembling, and handling was possible of accurate estimation, although some misgivings were felt in regard to contingency, the principal concern being the possibility of distortion.

Special additional equipment was provided at the manufacturer's shops, such as welding machines, cutting tools, tilting tables, and large machine-finished floor plates. Some idea of the task involved can be gained when it is stated that nearly 800 tons of rolled steel plate varying from $^3/_4$ in. to $3^1/_2$ in. were used. There was required about 16,000 linear feet of machine gas cutting, 24,000 linear feet of welding, and 29 tons of weld electrode for the two units. It was decided that all of the welding was to be done with the work in a horizontal position. This positioning was necessary both for economy and assurance of quality. All of the parts were stress-relieved, excepting the pit liner, which is subject to little stress.

The expected advantages of the welded construction were fully realized. There was no trouble on account of distortion and the metal to be removed by machining was remarkably uniform in depth. Time and costs were within close limits of the estimates. No cracks or defects developed in the welds or in the other parts of the structures, either before or after stress relieving. Figs. 9 and 10 show some of the welded parts in process of manufacture. Details of this welding job are available in published[5] articles.

The design of the turbine members and the work they have to perform, particularly the top structure or cover plates, are such

[4] "Cavitation Testing of Model Hydraulic Turbines and Its Bearing on Design and Operation," by L. M. Davis, Trans. A.S.M.E., vol. 57, 1935, paper HYD-57-13, pp. 455–462.

[5] "Welded Structures for Hydraulic Turbines," by George A. Jessop, *The Welding Journal*, April, 1937, vol. 16, pp. 2–6.

"Giant Turbines for Bonneville," by Curtis W. Lau, *American Machinist*, Feb. 24, 1937.

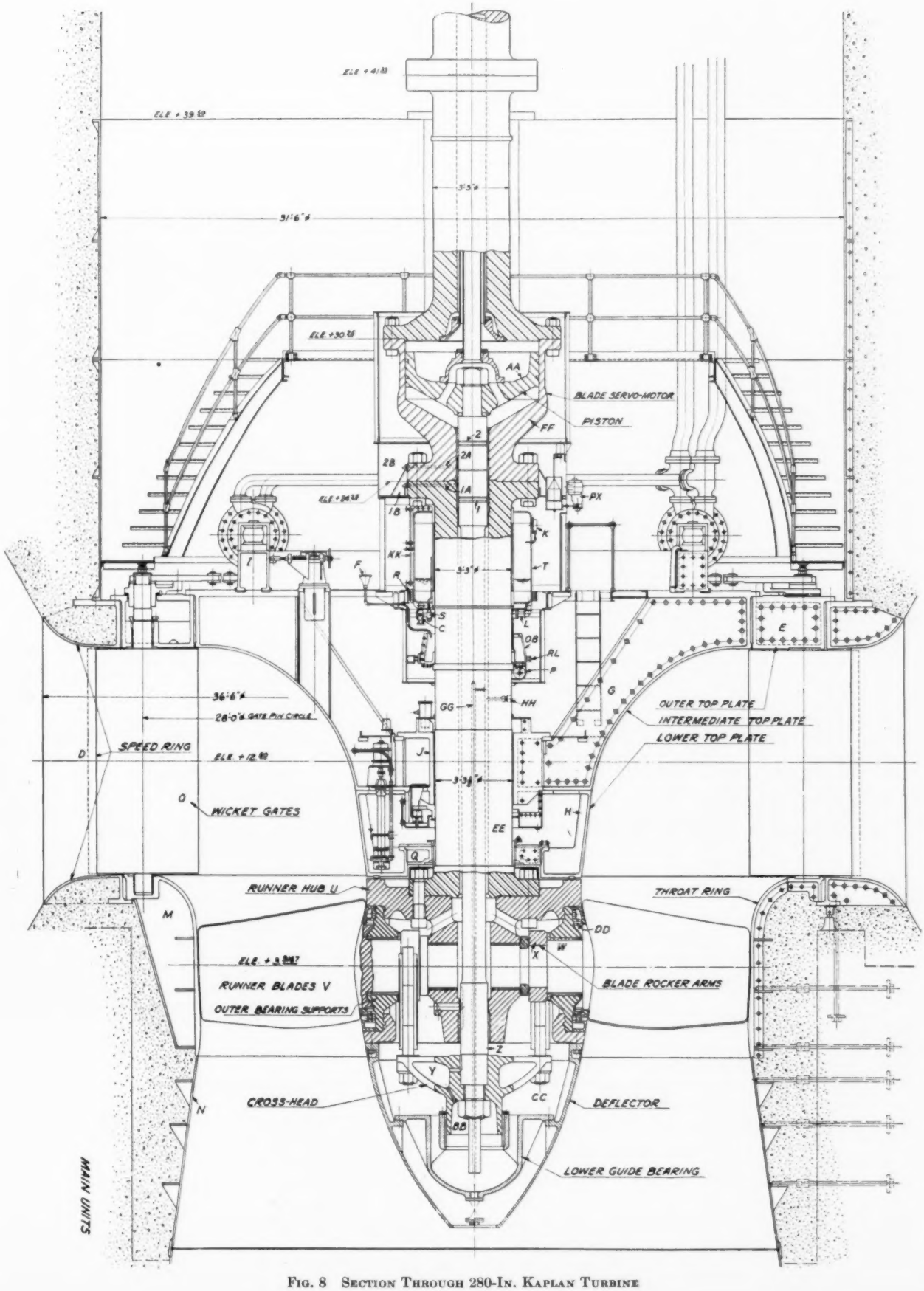


FIG. 8 SECTION THROUGH 280-IN. KAPLAN TURBINE

that an exact analysis of the stresses is impossible. From field investigations on similar installations, the loading was accurately known and an analysis showed that a test model could be constructed and loaded in such a way as to produce stresses closely comparable to those imposed on the parts in actual field service. Using a ratio of four to one, the model, Fig. 11, could be made of reasonable size with the parts of exactly the same material welded in the same manner and of the right proportional thickness. Stresses in the model were determined by means of a battery of strain gages and deflections were measured at many points. Both tangential and radial stresses were determined and concentrated stresses were carefully investigated. The information obtained from the tests proved to be of great value in designing the Bonneville turbines and the data are applicable to the preparation of plans for similar units.

The parts made of welded construction, Fig. 8, are the speed ring *D*, the outer top plate *E*, intermediate top plate *G*, lower top plate *H*, the gate or shifting ring *I*, the bearing *J*, the throat ring *M*, and a number of the smaller parts of the equipment. The

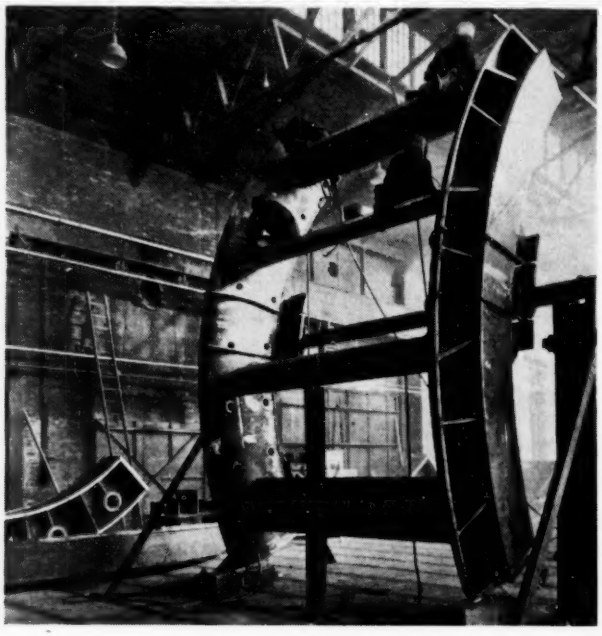


FIG. 9 BUILDING SPEED RING IN SHOP
(Welding together formed plates of rolled steel. All welding was done in a horizontal position.)

draft-tube liner, which extends to an elevation 37 ft below the center line of the gates, is welded at the upper part and riveted at the lower part. The throat ring is bolted to the speed ring and, in addition, has a continuous field weld. The draft-tube liner is field-welded to the lower part of the throat ring.

The speed ring is made in five sections, the outer top plate in four sections, the intermediate top plate in two sections, the lower top plate in one section, the throat ring in two sections, and the shifting ring in two sections. The speed ring has 20 vanes, one for each of the wicket gates. These vanes are shaped to guide the water properly toward the wicket gates but because of the special conditions of flow imposed by the supply passages, it is necessary to have a number of them specially shaped. The nose pieces or interior edges of the vanes were made from bar stock machined to the proper size and shape. The vanes act as columns and carry about 9,000,000 lb of dead load when the scroll is empty, although some of the vanes, at the large part of the scroll and at the intake, are in tension under the most severe pressure conditions. The total weight of the speed ring is about 177,000 lb.

The wicket gates *O*, are cast steel with integral stems. They are tapered toward the bottom to reduce disturbance under the gates and to allow the streams of water from the two sides of the gate to come together before coming in contact with the runner blades. The gates are supported by three bearings, one in the throat ring and two in the outer top plate.

The main guide bearing *J*, the only bearing on the turbine, is babbitt-lined and oil-lubricated. A pump, gear-driven from the

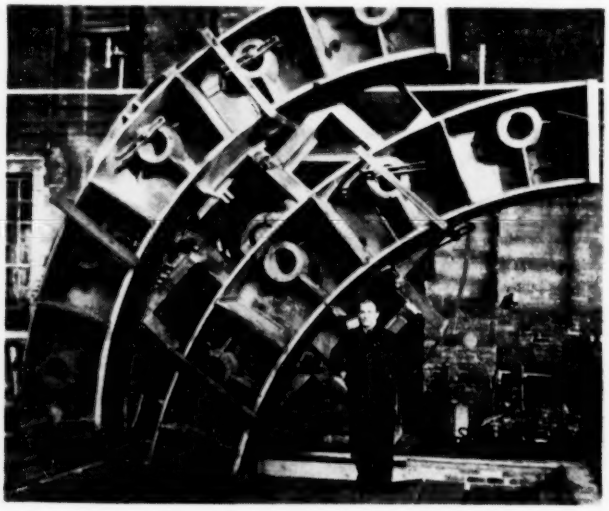


FIG. 10 SECTIONS OF OUTER TOP PLATE

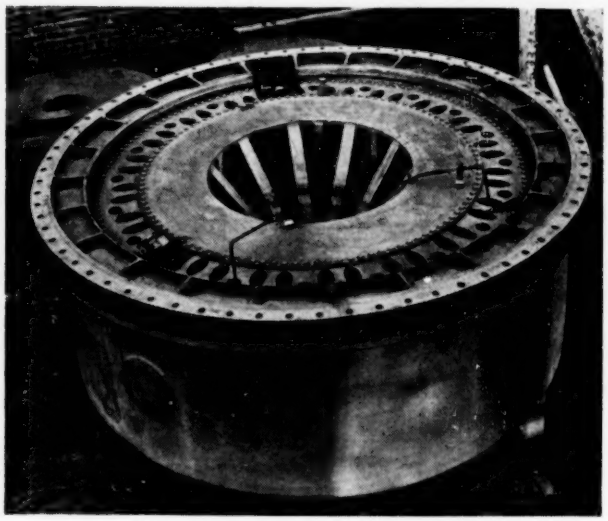


FIG. 11 MODEL HEAD COVER, ONE-QUARTER SIZE

shaft, is provided to return the oil from the bottom to the top of the bearing and there is a spare motor-driven pump, float-controlled. There is a packing box under the bearing in which an acceptable square packing is to be used. Domestic water can be applied to the packing if desired.

Below the packing box *Q* is a hollow rubber tube surrounding the shaft and normally clear of it. With unit stationary, this tube can be inflated to seal off the tail water. By this means the packing, which is always lower than tail water elevation, can be ad-

justed or renewed without lowering head gates and tail gates and unwatering the unit.

The Kaplan hub *U* is made of cast steel and weighs 100,000 lb. Each of the five blades *V* is cast steel of better quality and weighs 27,000 lb with the integral stem *W*. The outer bearings for the blades are removable and one slot for each blade is cast into the hub so that a blade can be inserted or removed with its rocker arm *X* and thrust washer attached. The rocker arms are shrunk as well as keyed to the stems and it is advantageous to be able to assemble or remove them in the open. The outer bearing supports, the rocker arms and thrust rings, the links, pins, and crosshead *Y* are all of high-strength steel.

The crosshead is moved up and down by a special steel push-and-pull rod *Z* actuated by the blade servomotor piston *AA*. This operating rod has three bearings, one under the piston, one just above the crosshead, and one just below it. The lowest of the three bearings *BB* is mounted in the cast-iron deflector *CC* and contains at the bottom a substantial reservoir for the accumulation of water should there be any leakage or gathering of condensation or moisture inherent in the oil. To prevent leakage of water in or oil out along the blade disks, a chevron-type packing *DD* is provided and held in place by a split gland. The glands can be removed and the packing replaced without dismantling the blades from the hub.

Each blade is supported by two bronze-bushed bearings. Because of the large thrust and torque on the overhanging blades, the service required of these bearings is severe, the bearing pressure is high, and the movement of the blades normally slow. Tests were made on many materials and with numerous types of grooving in order to secure the best bushings. The hub is filled with oil under low pressure and careful investigation was made to secure a lubricant with the proper film strength and lubricating properties.

The runner blades for one unit are machine-finished on a die-sinking machine to the exact shape of the layout. See Fig. 12. The blades of the other unit were given a good ground finish. Both finished and unfinished blades are prewelded with stainless steel over certain areas on the underside of the blade and also at the periphery as an extra protection against pitting and deterioration. As the units are operated, an interesting and valuable comparison can be made between the freedom from pitting and the smoothness of operation of the machine-finished compared with the ground blades. Machining by the die-sinking method produces a finish without the slightest irregularity or unevenness and is superior to any other process.

The turbine shaft is of forged steel and the blade-servomotor piston and cylinder *FF* are of cast steel. The turbine shaft, blade-servomotor cylinder, shaft extension, and generator shaft were tested in the turbine manufacturer's works for trueness by mounting vertically on a revolving table. The over-all length of the shaft was so great that it could not be tested in its entirety at one time. The lower shaft, servomotor cylinder, and intermediate shaft were first tested and then the intermediate shaft was tested with the generator shaft. The table was revolved slowly and readings were taken by means of shaft deflection gages. The vertical method of checking is much more accurate than the horizontal method.

Testing for Water in the Kaplan Hub

While every precaution has been taken to prevent the leakage of water into the Kaplan hub, it was still deemed desirable to have a positive method for checking the presence of water and for the removal of it, if found. For this purpose, the operating shaft is extended on the lower end as shown in Fig. 8 and is bored with a small hole *GG* to an elevation just above the main bearing. At the upper end a hole is drilled from the central hole to the outside of the operating shaft. In the outer shaft *EE* a corresponding hole *HH* is drilled having the same angular position as the hole in the operating shaft. The operating and main shaft always have the same relative angular position. The vertical location of the two holes is such that when the operating shaft is in its lowest position, the holes are at the same elevation. The hole in the main shaft contains a specially designed and constructed valve and plug. When the two holes are lined up, the outer valve is screwed inward until it enters the hole in the operating rod. A flexible hose is then screwed into the valve, automatically opening a passage for the drainage of oil or oil and water. The lower part of the operating rod is now close to the bottom of the reservoir in the guide bearing in the hub deflector. If there is any water in the hub, it will gather at this location when the turbine is shut down. There is sufficient pressure, as explained later, in the hub and deflector to force oil or water through the hole in the operating rod and out through the outer shaft. The hole in the rod will be filled with oil but only a few gallons must be drained away before it will be definitely known whether or not there is water in the hub. If water is found, it is necessary to let it drain until clear oil is discharged.

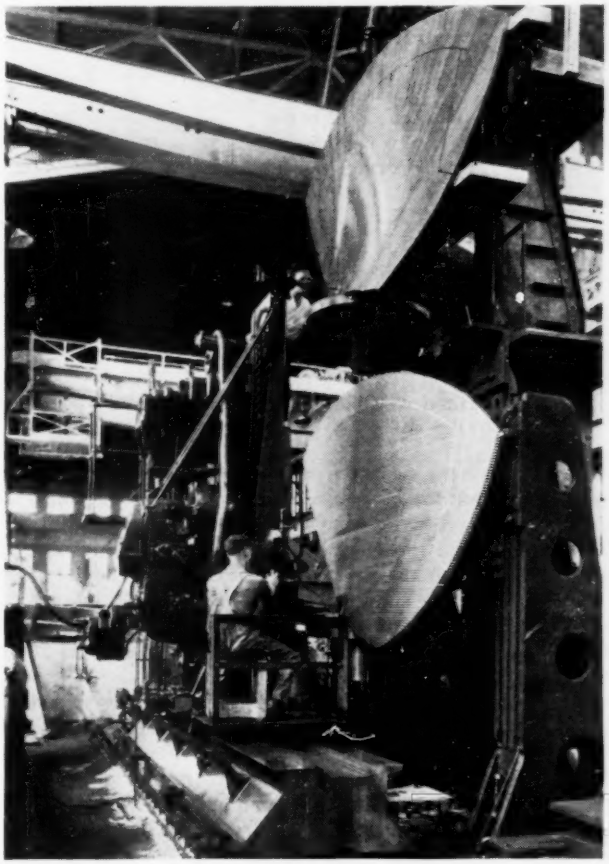


Fig. 12 Keller Profiling Machine
(Used for machine-finishing turbine blades. The top blade is a polished wooden model. The cutter on the steel blade below follows the path of the magnetic pointer above.)

For completely emptying the hub, a large drain valve is provided at the bottom of the reservoir in the lower guide bearing.

Auxiliaries

The turbine has several pieces of auxiliary equipment that are unique in the manner of their application. The Kaplan mecha-

nism inside the rubber hub must of course have lubrication. Both grease and oil have been used in the past for this purpose in Europe and in America, with oil now the usual medium, and a heavy oil is used at Bonneville. It is much more important to keep the water from leaking into the hub than to keep the oil from leaking out. The packing used around the blade shanks has worked almost to perfection in the tests, but future wear may permit a small amount of leakage. The hub is always below tail water and, therefore, when shut down, is subject to water pressure of from 3 to 50 ft. When in operation and developing power, the pressure underneath the buckets is decreased below the static pressure of the tail water and the pressure above

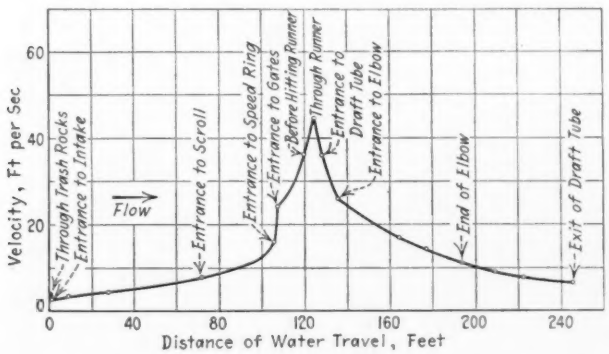


FIG. 13 VELOCITIES THROUGH 66,000 HP KAPLAN TURBINE
(For flow of 13,000 cfs at rated head and output.)

the buckets is increased by the partial closure of the blades. It is required to keep the oil pressure at a point sufficiently high to overcome the pressure above the blades when the turbine is operating and this pressure will, in fact, be sufficiently high to prevent leakage inward of water even at the maximum tail water elevation when the units are shut down. It is of interest to note that as the operating shaft for the blades moves down in the hub, oil is forced out of the hub into the pressure tank, described later, and will increase the pressure within the tank. The tank has been made of such size that the difference in air pressure in the tank between the extremes of operating-shaft travel is not of troublesome amount. The oil pressure within the hub at the level of the blade shanks, where the leakage may occur, is increased by the height of the oil column in the shaft and by centrifugal force when the unit is running. To overcome the outside pressure the oil inside the hub is kept continually under a higher pressure by means of a revolving pressure tank T, Fig. 8, surrounding the shaft and connected to the hollow space inside by the pipe L. Gage K shows the air pressure in the tank above the oil. To maintain the pressure in the tank at a predetermined desirable value, a pressure switch S operates a small compressor C, both of which are revolving and are supplied with current through collector rings R. A by-pass circuit also permits manual control of the compressor without shutting down the unit.

In case of loss of oil for any reason, as shown by the gage glasses KK, more oil can be added without shutting down the unit. The oil is poured into the funnel F, runs into a revolving oil basin OB, and a revolving oil pump P then pumps the oil into the pressure tank. The oil pump P is controlled manually by current supplied through collector rings RL.

Oil-level marks are provided on the gage glasses (KK) for both still and running conditions of the unit. If the oil level decreases, oil must be leaking out. If the oil level increases water must be leaking in and an investigation of the cause can be made by means of the testing devices described elsewhere. Prior to completion of the design of the turbine the chief engineer of the manufacturer made a trip to Europe to get the benefit of experience over there. The arrangement of auxiliaries was worked out between the manufacturer and engineers of the Bonneville project. In comparable European plants leakage is from ten to twelve gallons per month but in spite of their size the Bonneville units are not expected to leak one third as much.

The governor-pressure oil is much lighter than the runner-hub oil and the two must be kept separate. A triplicate system of piston rings is provided at the top of the blade-operating rod. Runner-hub oil that leaks past rings 1 will flow out of pipe 1A into chamber 1B and can be withdrawn. As the runner-hub pressure will never be over 20 psi, the leakage should be small. Governor-pressure oil that leaks past piston rings 2 will flow out pipe 2A into chamber 2B and be pumped up to the governor sump tank by the pump PX.

The remaining auxiliaries to the turbine are more or less conventional. Each unit has a motor-driven guide-bearing oil pump, a gear-driven companion oil pump, and two motor-driven sump pumps. The main guide bearing has two temperature

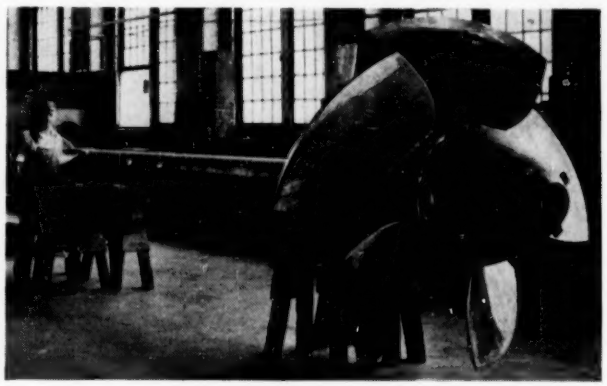


FIG. 14 KAPLAN RUNNER, 81 IN. DIAM
(For Bonneville station-service unit. Blades full open.)

indicators and two temperature relays for alarm or, if desired, for automatic shutdown.

Winter-Kennedy taps are provided in each scroll for use in making index tests of efficiency and for determining best gate-blade relations.

Provision is made for the admission of compressed air to the draft tube in event of sudden load rejection. A cam on the shifting ring opens a valve in the air line and admits air at 100 lb pressure. The valve is operated by a dashpot mechanism and in event of slow shutdown the valve gate will not open.

SETTING EMBEDDED PARTS

On account of the unusual size of the Bonneville turbines there were no parallel examples of setting from which to plan erection methods by direct experience. The speed ring is 36 ft 6 in. diam, 12 ft 9 in. high, and weighs 80 tons complete. It is made in five segmental sections. The throat ring has an outside diameter of 23 ft 4½ in., weighs 60 tons, and is made in halves. Although conservatively designed the loads are so heavy that deflections must be given careful consideration. Troubles have occurred in setting smaller units and it was earnestly desired that their repetition be avoided.

The detailed procedure of setting the embedded parts for one unit will now be described. The key numbers in parentheses refer to corresponding items in Fig. 15. Twenty rectangular concrete piers, shown as A in the figure, were poured in the original substructure within the confines of the lower supporting cone of the speed ring. Embedded in the piers at time of pour were verti-

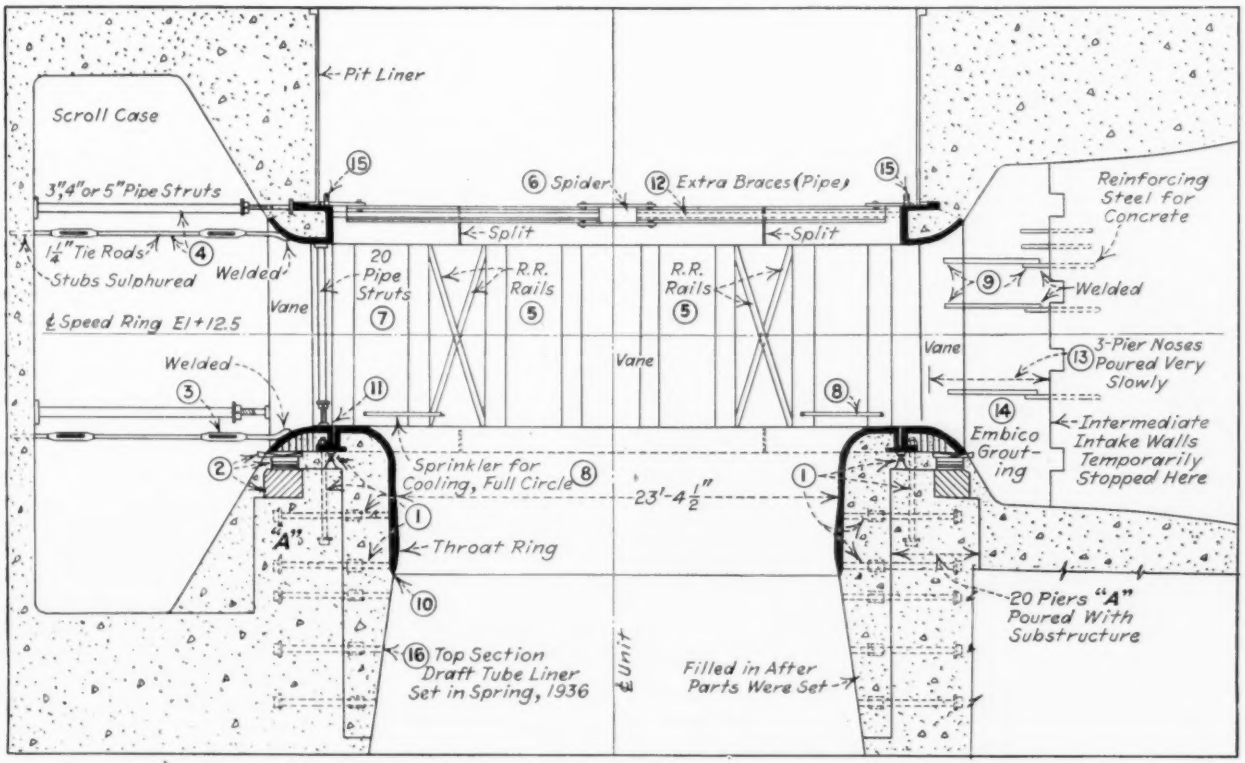


FIG. 15 SETTING EMBEDDED PARTS (EXPLANATORY DIAGRAM TO ILLUSTRATE TEXT)

cal anchor bolts (1), and horizontal jack bolts with sleeve nuts (1). Jacks (1) were set upon the top of the piers for supporting the speed ring. All of these features numbered (1) form the primary apparatus for supporting the speed ring and throat ring. To furnish additional support small concrete blocks (2) were poured on top of the piers, as shown by crosshatched area, *B*. Railroad rails (2) and wedges were used to support the ribs of the lower ring and tie rods and pipe struts (3) held it firmly in position. Similar pipe struts (4) and ties (4) were used at the top ring. To prevent racking, cross bracing of railroad rails (5), wedged and welded, was placed between the adjacent vanes of each of the five splits. A heavy steel spider (6) with eight arms, each an 8 × 8-in. H-beam, with adjusting jacks at the ends of the arms, further held the top ring of the speed ring in a truly round condition. Tie rods (9) were welded to projecting reinforcing bars and to the corresponding vanes of the speed ring, to prevent movement of the speed ring during pouring of concrete in the intake pier noses. As an added precaution, two extra pipe braces (12) were placed against the pier noses.

After the speed ring was set, lined, and leveled, the assembled throat ring was lowered by the crane, jacked into final position on top of inner ledge of lower ring of the speed ring, and then bolted firmly to the speed ring by bolts. The speed ring was again checked for line and level, adjusted as required, and when correct all wedges and railroad rails were spot-welded to prevent shifting.

Twenty vertical pipe struts (7), 4 in. or 5 in. in diam, with jack bolts, were installed between the upper and lower flanges of the speed ring. Two extra horizontal braces (12) at ends of pier noses were inserted to prevent pressure of concrete back of pier noses from locally distorting the speed-ring flanges.

Final adjustment of all braces and jacks was made and concrete poured up to and slightly above the bottom of the ribs and skirt of the bottom ring of the speed ring. A complete annular ring of sprinkler pipe (8) was placed above the throat ring and cooling water sprayed therefrom for some ten days after pouring. Later the throat ring was welded (10) to the draft-tube liner and the speed ring was welded (11) to the throat ring.

The ends of intake piers that join the speed ring were left unpoured until the speed ring was set (13). The amount of concrete was small and the height of pour was about 12 ft. Over six hours were spent in pouring these piers in order not to build up an appreciable hydrostatic head against the speed ring.

Finally, after an elapse of many days to permit shrinkage of the foundation cone, Embeco grout was poured under the speed ring to make a nonshrinking foundation. These precautions resulted in variations from zero tolerance or perfection, as follows:

	Unit 1	Unit 2
Maximum variation from round, on radius:		
Throat ring opposite runner	0.026 in.	0.027 in.
Top of speed ring	0.063 in.	0.076 in.
Maximum variations from level, range from low to high:		
Top of speed ring	0.016 in.	0.027 in.
Top of throat ring	0.018 in.	0.016 in.

These readings were taken after the superimposed foundation on top of the speed ring had been poured and cured.

Thermocouples were embedded in the cone supporting the speed ring of each unit, and readings were taken at regular intervals during the setting of the concrete, which was made with ordinary Portland cement. The couples were some two feet from the throat ring. The maximum temperature occurred on the fifth day following the pour and the indicated temperature rises varied among four couples in two cones from 50 to 57 F.

STATION-SERVICE UNIT

There is provided at Bonneville a station-service unit of 5000 hp that drives a 5000-kva generator. See Fig. 14. The service

turbine itself is homologous with the main turbine and the model, but has a plate-steel spiral scroll at the lower end of a 13-ft-diam penstock 78 ft long The draft tube is of the same general type as for the main units, but varies slightly in shape for structural reasons. The difference in specific speed between main units and service unit noted in Table 2 is due to rounding off the figures of the stated rating. For the present the blades are manually adjusted but automatically adjustable mechanism could later

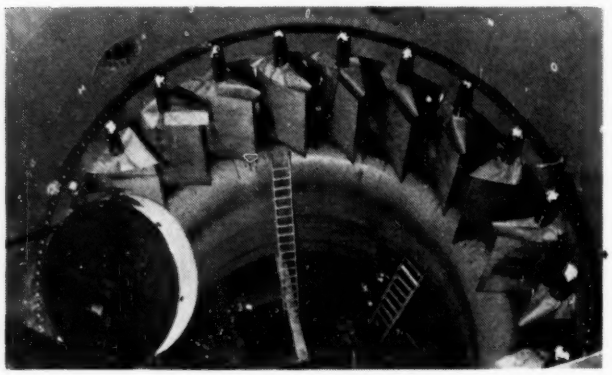


FIG. 16 TURBINE PIT FOR MAIN UNIT

be installed. The size of the service unit is larger than required for auxiliaries or even for all the load on the project including cranes, hoists, and residences, but it appeared economically advisable to build a unit as large as could be placed in one structural bay of the powerhouse. The service unit can be used to generate power for sale or can be left as stand-by for service, in which case the controls are arranged to start the unit automatically in case of failure of excitation supply for the main units. The blades of the service unit are given the usual hand-ground finish. Provision for maintaining oil pressure in the runner hub is similar to that in the main units, but is simpler in that no revolving compressors or pumps are provided. Instead it is necessary to shut down the unit for the required servicing. Provision for testing and withdrawing water from the runner hub is identical in principle with that for the large units.

GOVERNORS

The two turbines are controlled by the latest type Woodward twin-system cabinet-type governor. In one cabinet 23 ft long × 9 ft 6 in. wide on the main operating floor are located the two governor heads (one for each unit), two gate-control-valve assemblies, two blade-control-valve assemblies, motor-driven rotary-gear-type oil-pressure pumps, an oil sump with sufficient storage space for all of the oil of the system, and all of the control mechanism and devices. The pressure tanks for oil and air are located on the floor below. For each governing unit there is provided one 100-gpm oil pump and one 600-gpm pump and by means of an Eschelon control the smaller pump can be put into action first and take care of the minor regulating demand. The sump, the oil pressure tanks, and the piping can be sectionalized so that both units can be governed by a part of the equipment while other parts are isolated for inspection and servicing. The governors are equipped with the more or less standard apparatus for load limits, speed droop, emergency shutdown and alarms, remote control, remote indication, and automatic intermittent brake application. It is not within the province of this paper to describe the governors in detail, but two special features that affect turbine operation should be described.

At the higher heads both the designed physical capacity of the turbine and the rated electrical capacity of the generator can be greatly exceeded with large gate openings. To hold the output of the unit within safe limits there is provided a net-head gate limiter. Slide-wire transmitters bring headwater and tailwater indications to the control room, where through differential recording in a net-head recorder both are interpreted in terms of net head. Additional slide wires carry the net-head reading to the load limits on the governors, where the load limit is rendered ineffective if turned beyond the gate opening corresponding to the limiting power for the net head then effective. Slide wires and recorders were manufactured by the Leeds &Northrup Co.

The best relation between gate and blade varies with different heads. Careful analysis of Bonneville model data and experience with other plants led to the decision that four gate-blade cams for the entire range of heads from 30 to 69 ft would be sufficient. Cams were selected to be correct at heads of 61, 55, 50, and 45 ft. Based upon model tests, the employment of the 61-ft-head cam at 65-ft head lowers the point of best efficiency about 0.1 per cent, and the same cam lowers the best efficiency at 58-ft head about 0.15 per cent. The losses in best efficiency are about the same down to the 50-ft-head cam at 47.5 ft. The 45-ft-head cam shows a greater loss of best efficiency than this, increasing as the head lowers. However, at about 46 ft head full gate and blade are required to develop the full 66,000 hp and the turbine will go to full gate and blade on demand. At such head capacity and not efficiency is the deciding factor. The four cam curves are cut on the periphery of one cam wheel which is secured with centering pins and thumbscrews. With change of head the cam is manually rotated one-quarter turn. In normal operation change of cam position will occur only at intervals of weeks or months.

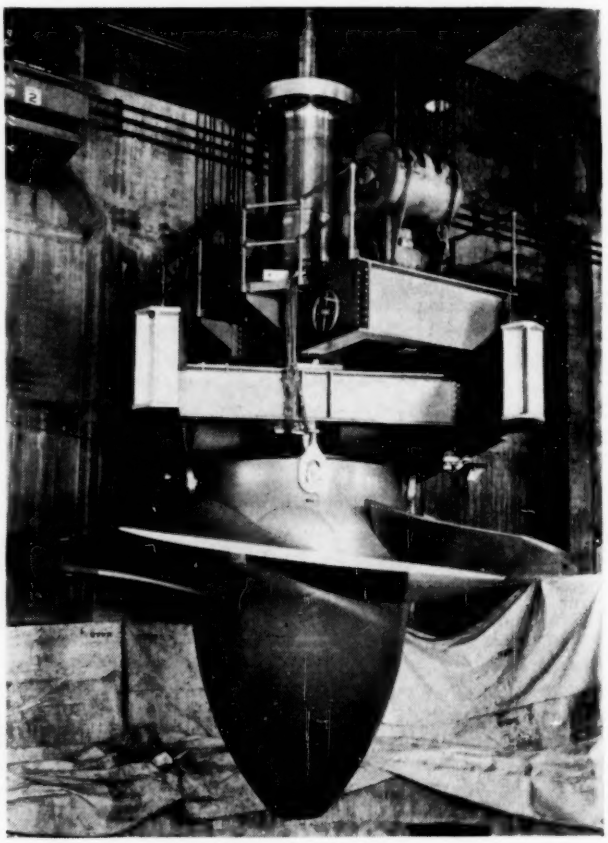


FIG. 17 MAIN TURBINE READY FOR PLACING

(Blades at 6 deg or closed. Upper frame with hand wheel is the lifting beam. The framed beams just above runner will support the turbine during erection and will then be removed. Total load 250 tons.)

Theory of Resonance in Pressure Conduits

By CHARLES JAEGER,[1] VILLARS SUR OLLON, SWITZERLAND

This paper represents one of the contributions to the Second Symposium on Water Hammer and deals with the analysis of critical conditions in pipe lines where initial surges are augmented by the characteristics of the conduit. The paper is the first treatment in English of this subject, and should prove a valuable contribution to the knowledge of water hammer.

HYDRAULIC engineers have always been interested in cases of rhythmic variations of pressure surges in conduits; surges whose periodic oscillations and resonance properties are of great importance.

All operations of total closure are transformed little by little into periodic oscillations, whose essential properties are known, thanks to the work of de Sparre and Camichel.[2] Foch has considered the case of oscillations in a conduit with an air chamber, and the author has studied the generalization of the problem for a network of conduits. One might imagine that at the present time we have sufficient knowledge of the mode of generation of these periodic oscillations and their main characteristics. On the contrary, we know very little of their determination.

The study of resonance cases has held alike the attention of Allievi, de Sparre, and Camichel, to whom we are indebted for some very fine experimental researches. Let us sum up what we know of the subject at the present time.

Allievi has analyzed, in note V of his "Theory of Water Hammer," the formation of the initial wave in a pipe of constant section, due to partial rhythmic movements of opening and closing, following each other without ceasing. His elegant proofs remain classic. Assuming that the openings η_i of the gate at odd periods are

$$\eta_1 = \eta_3 = \eta_5 = \text{etc.} \ldots\ldots = \text{constant}$$

and at even periods, are

$$\eta_0 = \eta_2 = \eta_4 = \text{etc.} \ldots\ldots = \text{constant}$$

he shows that the relative surges, even and odd, $\xi_i^2 = H_i/H_0$ approach, respectively, two limits, Z_1^2 and Z_2^2, such that

$$Z_1^2 + Z_2^2 - 2 = 0 \ldots\ldots\ldots\ldots\ldots\ldots [1]$$

Let us note that Löwy had readily given a graphical method in order to follow the formation of any resonance in a pipe of constant section. The graphical process of Löwy has subsequently been used by other authors.

Among all the possible rhythmic movements, classed by Allievi, we will take just one case for a systematic study, one which has already solicited the attention of Camichel, and to which the author has devoted a paragraph in his "General Theory." This is the case where one of the series of relative surges, ξ_i^2, tends to the limit zero. It follows that

$$Z_1^2 = 0, \quad \text{and} \quad Z_2^2 = 2 \ldots\ldots\ldots\ldots [2]$$

The other series of values ξ_i^2 thus tends to the limit 2, and we have twice the static pressure at the bottom of the conduit. Camichel has shown that this case of resonance is not possible unless the static pressure $h_0 > av_0/g$, which is almost evident. The author, applying the graphical method devised by Allievi, had shown that the limit of positive pressures, without reaching the value 2, nevertheless has very high values, when the initial opening is not zero but small.

It is fitting to mention here the good theoretical researches of Camichel and de Sparre, and the numerous experiments and calculations made at the Laboratory of Toulouse by Camichel, and those of Eydoux on high-head installations. These tests are of the greatest interest.

Camichel has shown that when one considers a conduit with variable characteristics, e.g., with variable diameter, using the "method of abrupt pressure drops"—a method which consists in causing an abrupt opening of the gates at the bottom of the conduit, followed by a closure no less abrupt at the end of the period—one records a period for the conduit which is the sum of the individual periods of each sector of constant section. We will say, following Camichel, that this period is the theoretical period of oscillation of the conduit.

If, on the contrary, we consider such a movement as the periodic return of the maximum surge following a slow closure of the gate, we verify the existence of a period differing from the preceding—in general, shorter—called the "apparent period" of the conduit.

Camichel and his followers have shown that the odd harmonics of pressure conduits have a period which is an integral fraction (e.g., $1/\eta$) of the theoretical period of oscillation of the conduit, then (also) that the period of the fundamental of a conduit having variable characteristics—which fundamental may always be produced in a similar conduit—is on the contrary equal to the apparent period of oscillation.

Assuming these experimental data, it seems that the theory for the harmonics should be quite different from the theory for the resonance of the fundamental. The latter, in contrast, should probably be linked with the studies of periodic oscillations, whose period is equal to the apparent period. The author has given the complete theory of periodic oscillations in two earlier articles in *La Houille Blanche*. The field is thus ready for the general theoretical study of resonances. We shall begin by explaining the theory for resonance of harmonics.

1—RESONANCE OF HARMONICS IN A CONDUIT WITH MULTIPLE CHARACTERISTICS

General Theorem

Let us consider a complex system of conduits, such as is shown in Fig. 1. The gate is at O, and A is any point, not necessarily a point of discontinuity. However, the sector OA is supposed to have a constant section.

Let us actuate the gate with rhythmic alternating movements, whose period is supposed to synchronize with the period of OA. We distinguish between even epochs and odd epochs; e.g., for even epochs the gate is assumed closed, and for odd epochs it will be open.

Let us suppose that a movement of resonance should have been established, and should have given rise to a permanent régime. We will write the conditions at the lower end of the conduit and

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[1] L'École Polytechnic Fédérale, Zurich.

[2] The articles published by the various authors mentioned in this paper are given in the Bibliography at the end of this paper.

Contributed by the Hydraulics Division and presented at the Annual Meeting of The American Society of Mechanical Engineers, held in New York, N. Y., December 6–10, 1937.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

inquire for what conditions in the conduit itself the régime may exist.

At even epochs $2n-2$, $2n$, $2n+2$, ..., the gate mechanism being closed, the velocity at the gate is zero, and $v_{2n-2} = v_{2n} =$

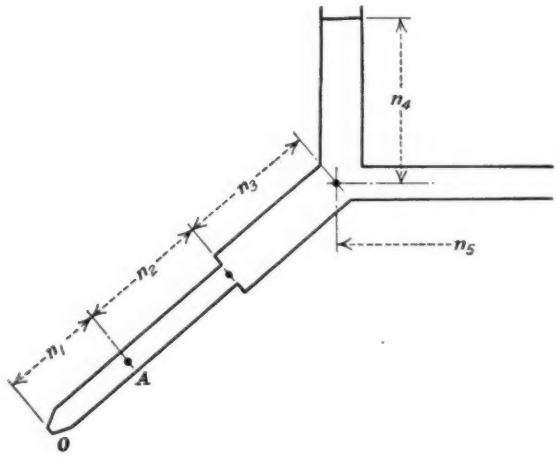


FIG. 1

$v_{2n+2} = 0$. If the permanent régime is really established, the pressure at the gate should always be the same, or

$$H_{2n-2} = H_{2n} = H_{2n+2} = \text{etc.} \quad \ldots\ldots [3]$$

At odd epochs, the gate is open. But when the régime of rhythmic oscillations is established, as experience shows, the velocity at the gate must be zero,[3] so long as one neglects the losses of head in friction, and the pressure at the same point must be equal to atmospheric.

Hence

$$v_{2n-3} = v_{2n-1} = v_{2n+1} = 0 \quad \ldots\ldots [4]$$

$$H_{2n-3} = H_{2n-1} = H_{2n+1} = 0 \quad \ldots\ldots [5]$$

Equations [4] and [5] are also connected by the Bernoulli theorem, which is considered as always valid in the theory of water hammer.

Such are the limiting conditions at the bottom of the conduit. The limiting conditions at the top will be given by the form of the conduit.

There are no other conditions. In particular, we cannot prescribe any form for the periodic curve of surge pressures at the gate. We know from the work of various authors that this curve will have sometimes a sinusoidal form, sometimes a rectangular outline (case of an automatic check valve—Camichel), or again a shape rounded at the minima and sharp-cornered at the maxima, when the conduit is provided with an air chamber (as considered by Foch).

We can write for the sector OA of the conduit

$$\left.\begin{aligned} H &= H_0 + F(t) + f(t) \\ v &= v_0 - (g/a)[F(t) - f(t)] \end{aligned}\right\} \quad \ldots\ldots [6]$$

The period of a complete oscillation in OA will be

$$\mu_1 = 2L_1/a_1 \quad \ldots\ldots [7]$$

which we take as unit time, putting $n_1 = 1$.

[3] "Étude théorique et experimentale des coups de bélier," by C. Camichel, D. Eydoux, and M. Gariel, Dunod, Paris, 1919, p. 98.

Using Equations [4] and [5], there result

$$H_{2n+1} - H_0 = F_{2n+1} + f_{2n+1} = F_{2n-1} + f_{2n-1} = \text{etc.} = -H_0$$
$$v_{2n+1} - v_0 = -g/a[f_{2n+1} - f_{2n+1}] = -(g/a)[F_{2n-1} - f_{2n-1}]$$
$$= -v_0$$

From these we obtain

$$\left.\begin{aligned} F_{2n+1} &= (av_0/2g) - {}^1/_2H_0 \\ f_{2n+1} &= -(av_0/2g) - {}^1/_2H_0 \end{aligned}\right\} \quad \ldots\ldots [8]$$

We will now use the fundamental relation which was defined at the beginning of the author's "Théorie Générale," and write

$$f_n = -\alpha_{n-1} F_{n-1} \quad \ldots\ldots [9]$$

where α_{n-1} is an unknown function, which depends, as shown in the author's "Théorie Générale," on the geometric form of the conduit and on the movements of the gate.

We obtain from our definition $F_{2n} = -f_{2n+1}/\alpha_{2n}$, and $f_{2n} = -\alpha_{2n-1} F_{2n-1}$.

Hence we write, at even epochs

$$H_{2n} = H_0 + F_{2n} + f_{2n}$$
$$= H_0 - (f_{2n+1}/\alpha_{2n}) - (\alpha_{2n-1}F_{2n-1})$$
$$= H_0 + \frac{1}{\alpha_{2n}}[(av_0/2g) + {}^1/_2H_0] - \alpha_{2n-1}[(av_0/2g) - ({}^1/_2H_0)]$$

or, on reduction

$$H_{2n} = H_0 + (av_0/2g)[(1/\alpha_{2n}) - (\alpha_{2n-1})] + {}^1/_2H_0[(1/\alpha_{2n}) + (\alpha_{2n-1})] \quad \ldots\ldots [10]$$

We may determine also

$$v_{2n} = v_0 - {}^1/_2v_0[(1/\alpha_{2n}) + (\alpha_{2n-1})] - (gH_0/2a)[(1/\alpha_{2n}) - (\alpha_{2n-1})] \quad \ldots\ldots [11]$$

As may be verified, we have for odd epochs

$$H_{2n+1} = H_0 + F_{2n+1} - \alpha_{2n}F_{2n} = H_0 + (av_0/2g) - {}^1/_2H_0 - (av_0/2g) - {}^1/_2H_0 \equiv 0$$

Consider again Equations [10] and [11]. The limiting conditions require, at even epochs, since the gate is supposed closed, that $v_{2n} \equiv 0$, whatever v_0 and H_0 may be. A necessary result of this is that

$(1/\alpha_{2n}) - (\alpha_{2n-1}) = 0$, i.e., $1/\alpha_{2n} = \alpha_{2n-1}$, and also that $(1/\alpha_{2n}) + (\alpha_{2n-1}) = 2$, whence

$$\alpha_{2n} = \alpha_{2n-1} = 1 \quad \ldots\ldots [12]$$

a relation which is true for both even and odd epochs. We deduce, for the same epochs, that $f_n = -F_{n-1}$, and

$$\left.\begin{aligned} F_{2n} &= -f_{2n-1} = (av_0/2g) + {}^1/_2H_0 \\ f_{2n} &= -F_{2n-1} = -(av_0/2g) + {}^1/_2H_0 \end{aligned}\right\} \quad \ldots\ldots [13]$$

and finally

$$H_{2n-2} = H_{2n} = H_{2n+2} = \text{etc.} \ldots\ldots = 2H_0 \quad \ldots\ldots [14]$$

Therefore, we can state that the functions F and f are represented by periodic curves, with a period of $\theta_1 = 4L_1/a_1 = 2\mu_1$, and that the head H is also periodic, having the same period. The values $H = 0$ and $H = 2H_0$ are necessarily the minimum and maximum of this latter function. Therefore, we verify that the phenomenon of the doubling of the pressure necessarily occurs, and that there is no other solution.

It has been shown in the author's "Théorie Générale" that, denoting by n_2, n_3, n_4, etc., the corresponding times of oscillation in the various sectors of the conduit, one can always express α_n in the form

$$\alpha_n F_n = a_0 + a_1 F_{n-n_2} + a_2 F_{n-2n_2} + a_3 F_{n-3n_2} + a_{10} F_{n-n_3} + a_{11} F_{n-n_2-n_3} + a_{12} F_{n-n_2-2n_3} + a_{20} F_{n-n_4} + a_{21} F_{n-n_2-n_4} + \ldots\ldots\ldots\ldots\ldots [15]$$

where $a_0, a_1, a_2, \ldots\ldots a_{10}, a_{11}, a_{12}, \ldots\ldots a_{20}, \ldots\ldots$ are coefficients depending only on the geometric form of the conduit.

Let us assume that $n_2, n_3, n_4, \ldots\ldots$ are integral multiples of n_1, and therefore integers, since $n_1 = 1$. Collect the coefficients into even and odd groups. Let A be the sum of the even coefficients, and B the sum of the odd coefficients. We know that, in every case, $A + B = 1$, as proved in the author's "Théorie Générale." Hence we may write

$$\alpha_n = A(F_{n-2}/F_n) + B(F_{n-1}/F_n) = A + B(F_{n-1}/F_n)$$

In order that $\alpha = 1$ at even and odd epochs, it is necessary and sufficient that $B = 0$ (or again that there be no function of odd order), which implies that the conduit must satisfy certain conditions of geometrical type which we will specify later with numerical examples. Recalling that A and B are, for each conduit, constants not affected by movements of the gate, and that F_n is a periodic function of period θ, such that at each instant $F_n = F_{n-2}$, we conclude (making the single reservation that the numbers $n_2, n_3, n_4, \ldots$ are integers), that for $\alpha = 1$ at even and odd epochs, it will also be unity at intermediate moments, and that we have $\alpha \equiv 1$ and $f_n \equiv -F_{n-1}$ at every instant.

This mathematical condition has a physical significance; $\alpha \equiv 1$ signifies that there is at every instant a *total reflection* of the ascending waves, and that there exists at the point A a loop of the oscillating periodic system.

But a loop is essentially stable. This would not have been so if $n_2, n_3, \ldots$ had not been integral multiples of n_1. Therefore, we must suppose that in this case it would be either impossible to have oscillations in the conduit other than fundamental—always possible, the author believes—or that the oscillations follow laws analogous to those of the fundamental, laws which we will study later.

We have seen that the limiting conditions are such that $v = 0$ at even and odd epochs. Let us study two points on the periodic curve F whose abscissas differ by $\mu_1 = {}^1/_2\,\theta$. Then

$$F_n = (av_0/2g) + \gamma_1 \text{ and } F_{n-n_1} = (av_0/2g) - \gamma_2$$

But certain curves F_1 produced by rhythmic movements, e.g., when F is a sinusoid, present characteristics of symmetry such that $\gamma_1 \equiv \gamma_2$, even if the instant n does not correspond to an instant of complete rhythm. In this case

$$v = v_0 - (g/a)[(av_0/2g) + \gamma_1 - (av_0/2g) - \gamma_1] \equiv 0$$

Hence, if the curve F shows a symmetry definable by $\gamma_1 = \gamma_2$, we have $v \equiv 0$, regardless of the instant. We know that Camichel has actually obtained discharges almost zero in the course of his experiments with the turning cock.

Statement of the Theorem. To develop over a uniform sector OA of any conduit an oscillation of permanent régime as a result of rhythmic movements of the gate, it is necessary that there be at A a stable loop. Mathematically, this means the assumption at A of the condition $\alpha \equiv 1$.

In this case, the pressure at O varies between $H_{2n-1} = 0$ and $H_{2n} = 2H_0$. This is the phenomenon of the doubling of the pressure observed by all experimenters, and whose reality had been proved by de Sparre and Allievi, for the case of resonance of the fundamental in conduits of uniform characteristics.

For these conditions to be fulfilled, and an oscillation of resonance to be possible, the conduit must satisfy certain geometrical conditions. We will illustrate the general theorem by studying a few particular cases.

Particular Examples

(*1*) *Resonance of the Fundamental in a Conduit With Uniform Characteristics, Discharging at A From an Infinitely Large Reservoir.* Referring to Fig. 2, let OA be a conduit with uniform characteristics. If the equilibrium chamber at A is very large, we have,

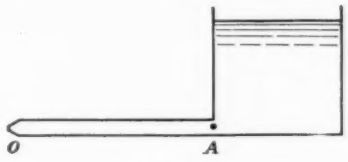


Fig. 2

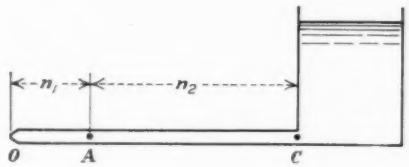


Fig. 3

according to the author's "Théorie Générale," $\alpha \equiv 1$. The fundamental resonance in such a conduit is thus a particular solution of our general theorem. There is double the pressure at O, as several authors have shown.

(*2*) *Resonance of the Odd Harmonics in a Conduit of Uniform Characteristics, Discharging at C From a Very Large Reservoir.* Referring to Fig. 3, let OA be an element of the conduit of uniform characteristics. The half period for OA is $n_1 = 1$; the half-period of AC is n_2. Since there is no discontinuity at A, we know from the author's "Théorie Générale" that the series for α reduces to a single coefficient $a_1 = 1$, with $a_2 = a_3 =$ etc. $= 0$. The series thus reduces to $\alpha_n = (F_{n-n_2})/F_n$.

In all cases where n_2 is an even multiple of n_1, F_{n-n_2} will be equal to F_n, equally at even and at odd epochs. For this case $\alpha = 1$ at every instant and there is resonance. It is seen at once that the condition of n_2 being an even multiple of n_1 gives resonance for all odd harmonics. This fact has been known since the time of the experiments conducted at Toulouse by Camichel, Eydoux, and Gariel.

It is seen at once that no other solution is suitable, since if n_2 is not an even multiple of n_1, $F_{n-n_2} \neq F_n$ and $\alpha \neq 1$. In particular, the even harmonics do not give a solution, a fact which has also been verified experimentally.

(*3*) *Generalization of the Preceding Case for Conduits Having Multiple Characteristics.* If we develop the function α_n, we see that there is at least one general solution of the problem for the case of conduits having multiple characteristics. This is when n_2, n_3, n_4, etc., are all even multiples of n_1. This is true, whatever the nature and the number of the discontinuities.

If we put α_n in the form

$$\alpha_n = A[(F_{N-2N2})/F_n] + B(F_{n-n_2})/F_n$$

it is readily seen that whenever the coefficient B of the odd terms is zero there will be a solution.

The method of procedure is thus completely outlined; in each particular case α_n is expanded in a series and the laws which govern the even and the odd coefficients are studied.

(*a*) *Case of a Single Discontinuity at A.* In the case of a single discontinuity, illustrated by Fig. 4, the series of Equation [15] becomes, as is shown in the author's "Théorie Générale,"

$$\alpha_n = r_1 + s_1 s_2 (F_{n-n_2}/F_n) + s_1 s_2 r_2 (F_{n-2n_2}/F_n) \ldots\ldots [16]$$

in which we assume that n_2 is a multiple of n_1. The symbols r and s denote the coefficients of reflection and of transmission, defined by de Sparre, Löwy, and the author, and whose properties are perfectly known. One particular property is that $s + r = 1$. For these conditions we find after carrying out all the calculations that $A = 0$ and $B = 1$. The conclusion is drawn anew. There is resonance if n_2 is an even multiple of n_1. Only odd harmonics are therefore possible for a conduit having a discontinuity at the point A.

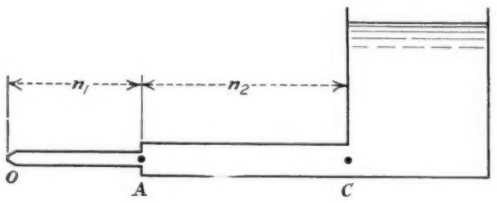


FIG. 4

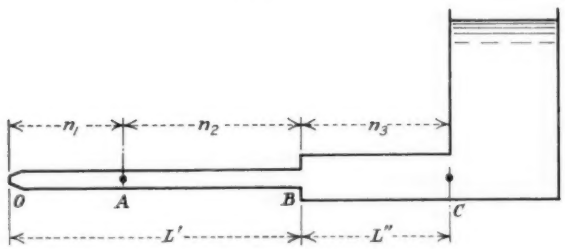


FIG. 5

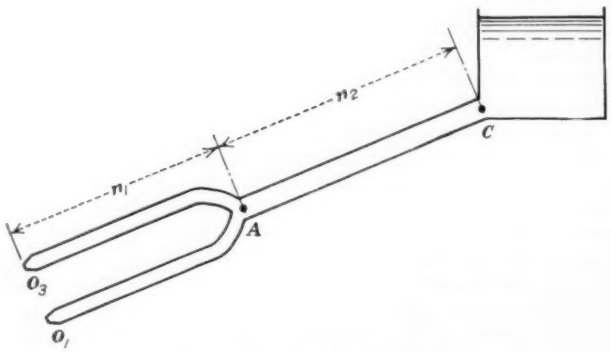


FIG. 6

(*b*) *Conduit With a Single Discontinuity at the Point B.* Referring to Fig. 5, a conduit of this type was used at the Toulouse Laboratory by Camichel. Let us study it from the theoretical point of view. At Toulouse the relation $L'/a' = 2L''/a''$ was true. Consider the point A, such that $n_1 = n_2 = n_3$, and let us ask whether it is possible in this conduit to obtain the third harmonic, as was actually observed by Camichel at Toulouse.

We find on page 107 of the author's "Théorie Générale" formula [90], giving the value of α_n for the case of two successive discontinuities, which it is suitable to use here. Let s_1, s_1', r_1, and r_1' be the coefficients of transmission and of reflection. Since there is no variation of section at point A, we have $s_1 = s_1' = 1$, and $r_1 = 0$. On developing formula [90] it is easily found that the coefficient $B = 0$. Hence $\alpha \equiv 1$, and the third harmonic can be established for the conduit under consideration.

Can the fifth harmonic also be obtained, as Camichel was able to observe? Evidently yes, since this is a particular case of the general theorem given herein by the author, where n_2, n_3, ... are even multiples of n_1.

Therefore, it appears that there is close agreement between the various observations made by Camichel and the theorems proved by the author.

We have been able to analyze by similar methods the case of conduits closed at their upper end.

(*4*) *Case of Synchronized Parallel (Twin) Conduits.* Referring to Fig. 6, the author has shown in his "Théorie Générale," as well as in an article in the *Revue Générale de l'Hydraulique*, that the function α_n, for the case of parallel conduits, geometrically identical, whose gates are operated in synchronism, is given by the expression

$$\alpha_n = r_1 - s_1 + 2s_1s_2(F_{n-n_2}/F_n) + 2s_1s_2r_2(F_{n-2n_2}/F_n) + 2s_1s_2r_2^2(F_{n-3n_2}/F_n) + \ldots\ldots\ldots [17]$$

Let O_1 and O_3 be the two synchronized gates, A the point where the principal conduit is subdivided upstream, n_1 and n_2 the half periods for O_1A (O_3A) and AC, respectively. Assume that n_2 is an integral multiple of n_1. We can write

$$F_{1-2kn1} = F_{1-(2k+2)n1} = F_{1-(2k+4)n1} = \text{etc.} = F_1$$

and

$$F_{1-(2k+1)n1} = F_{1-(2k+3)n1} = \text{etc.}\ldots\ldots\ldots = F_{1-n1}$$

These are the conditions at the lower end of the conduit O_1A if there is to be resonance. Since the conduits are synchronized, these conditions are also true for O_3A. It is easily shown that this results in the series of Equation [17] giving $\alpha_n = 1$, if n_2 is an even multiple of n_1. The odd harmonics are possible in parallel conduits under the conditions given above.

2—RESONANCE OF THE FUNDAMENTAL IN CONDUITS HAVING MULTIPLE CHARACTERISTICS

The author has shown[4] that when the function of integration $F(t)$ possesses certain properties of symmetry, properties which appear to have been realized in the course of experiments by Camichel, we have constantly at the gate

$$v_i = 0$$

But this same condition at the limits is realized after the time of complete closure in the course of the periodic phenomena. On the other hand we know that we may easily realize resonance phenomena which give at the base of the conduit pressure surges of sinusoidal form, in every respect analogous to periodic curves. Therefore, there is no cause for surprise that in these cases the resonance of the fundamental in a conduit of variable characteristics should have the same period, named "apparent," as the periodic phenomena. The identity of the conditions at the limits involves, as can be easily shown, the identity of the analytic formulas.

Camichel and Foch have shown by the experimental method that this property is, moreover, general, and that the fundamental is always in resonance with the apparent period of the conduit. Foch, in particular, in the course of resonance tests, obtained surges at the gate which did not obey at all the law of symmetry given above for the function $F(t)$, yet whose period was equal to the apparent period. The latter should therefore be a characteristic of the conduit, independent of the manner of development of the fundamental or of any other initial condition, as it arises from the final formulas themselves, which contain only the quantities L, a, and the section S of the conduit.

We may in fact ask ourselves with some reason if, with respect to the resonance of harmonics, a sector of a conduit of variable section can vibrate the same if a discontinuity occurs between the

[4] See articles by the author in *La Houille Blanche*, May-June and July-August, 1936.

node and the loop just as the fundamental vibrates in a conduit having discontinuities. Our theory of harmonics could then be generalized.

From the theoretical point of view, since periodic sinusoidal movements are identical with resonances of the same type, it will suffice to recall very briefly the theory we have developed at length, both in our "Notes à l'Académie des Sciences" of Paris and in the review *La Houille Blanche* for the case of two conduits of different sections.

The method of attack which the author followed was inspired by a work by Foch on conduits furnished with air chambers. We write down the surge at the gate in the form

$$H - H_0 = A \sin mt \quad \ldots\ldots\ldots\ldots\ldots\ldots [18]$$

where $m = 2\pi/\theta$, and θ is the apparent period of the conduit, to be determined.

In a similar manner we have at the discontinuity (wye)

$$H_1 - H_{10} = F[t - (L_1/a_1)] + F[t + (L_1/a_1)] = G(t) - G[t - (2L_2/a_2)]$$

where L_1, L_2, and a_1, a_2 refer to the two branches 1 and 2 of the conduit. We have further

$$G(t) = -\tfrac{1}{2}B \cos(mt + \phi) \quad \ldots\ldots\ldots\ldots\ldots [19]$$

Here B and ϕ are two constants easily determined.

Now we write the equations for the velocities at the point of discontinuity in the various branches, assuming sections S_1 and S_2. Application of the equation of conservation of discharge leads to the following relations:

For a variation of section

$$\tan mL_1/a_1 = \mu' \cot (mL_2/a_2) \quad \ldots\ldots\ldots\ldots [20]$$

where $\mu' = a_1S_2/a_2S_1$.

For a wye, with branch 1 discharging into 2 and 3

$$\tan mL_1/a_1 = \mu' \cot (mL_2/a_2) + \mu'' \cot (mL_3/a_3) \quad \ldots [21]$$

where $\mu'' = a_1S_3/a_2S_1$.

From Equations [20] and [21] is found the value of the apparent period θ for the conduit. With θ known, the problem of the distribution of the surges will be completely determined for the case of sinusoidal movements.

3—DISTRIBUTION OF THE PRESSURES ALONG A CONDUIT HAVING MULTIPLE CHARACTERISTICS, IN CONNECTION WITH THE RESONANCE OF THE FUNDAMENTAL OR OF THE HARMONICS

Camichel, investigating experimentally the distribution of the fundamental along a conduit of constant section, noticed that the maximum pressure at the upper third of the conduit also reached the value $2H_0$. The author has always thought that this law did not have the generality given it by Camichel, otherwise few inclined conduits could have resisted without injury the resonance of the fundamental. In examples taken from practice the author has found, quite on the contrary, distributions differing greatly from those indicated by Camichel.

The author gives for this fact the following general explanation: In all that has been said so far (with the exception of the case where we have expressly assumed for simplicity that the law of surges was sinusoidal), we have not had to make any assumption regarding the form of the curve $F(t)$, except in connection with the values of $F(t)$ at even and odd epochs. The problem thus retains a great degree of freedom.

It is easily understood that as the pressure at the gate, as well as the pressure at any other point, is always a function of $F(t)$ and of $f(t)$, at small intervals of time apart, there will be a close relationship between the curve of surges at the gate and the distribution of surges along the conduit. (This is an essential property of the integral function $F(t)$, which is used in the graphical method.)

But Camichel carried out his experiments on resonances with a turning cock which caused very abrupt opening and shutting of the discharge (hydraulic check valve); therefore, there is nothing astonishing in the fact that his recording apparatus, placed at the bottom of the conduit, should register a "notched curve" corresponding to a dangerous distribution of surges along the conduit. Some calculated examples will confirm this point of view.

Resonance of the Fundamental

(*1*) *Resonance of the Fundamental in a Conduit of Uniform Characteristic; Curve Having the "Notched Form."* Let us assume that

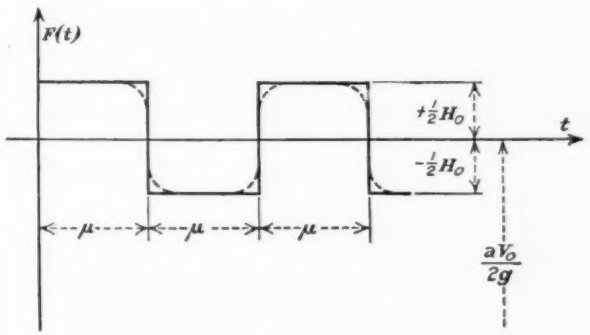


Fig. 7

the function $F(t)$ is a "notched" function, represented by Fig. 7; also, that it satisfies the conditions

$$F_{2n} = (av_0/2g) + \tfrac{1}{2} H_0$$

$$F_{2n-1} = (av_0/2g) - \tfrac{1}{2}H_0$$

It is seen immediately that

$$H - H_0 = F(t) - F[t - (2L/a)]$$

is also a notched curve. A very simple graphical construction will give the curve of surges as a function of time at a point of any abscissa whatever: $x = L/3, L/2, 2L/3$, etc. We verify in fact that the doubling of the pressure ($2H_0$) occurs as far as the immediate neighborhood of the top of the conduit (as Camichel observed in his conduit at Toulouse), because very little of the corners of the notches is rounded.

It is evident that resonances causing the formation of notched curves are very dangerous for inclined conduits. Luckily the majority of curves observed for pressure conduits are of the sinusoidal type and less dangerous, as we will see.

(*2*) *Resonance of the Fundamental in a Conduit of Uniform Characteristic; Curve Having Sinusoidal Form.* If we assume that

$$F(t) = \tfrac{1}{2}A \sin(2\pi t/\theta) + (av_0/2g) \quad \ldots\ldots\ldots [22]$$

we have clearly at any point of abscissa x

$$H_x - H_0 = A \sin(\pi/2)[1 - (x/L)] \cos\left(\frac{2\pi}{\theta} t - \frac{\pi}{2}\right) \quad [23]$$

In front of the gate we find

$$H - H_0 = A \cos\left(\frac{2\pi}{\theta} t - \frac{\pi}{2}\right) = A \sin \frac{2\pi}{\theta} t \quad \ldots\ldots [24]$$

The movement is exactly determined, at any instant and at any

point. One can always calculate the velocity corresponding to a given H_x by using

$$v = v_0 - \frac{g}{a}\left[F\left(t - \frac{x}{a}\right) + F\left(t - \frac{2L - x}{a}\right)\right]$$

It is found, on substituting for $F(t)$ its sinusoidal form, that when the pressures are maximum, they are so for the full length of the conduit and the velocities are zero. This property is general. Indeed, because of the differential equation

$$\frac{\partial v}{\partial x} = \frac{g}{a^2}\frac{\partial H}{\partial t}$$

when $\frac{\partial H}{\partial t} = 0$ (H minimum or maximum), $\frac{\partial v}{\partial x} = 0$, and the velocity is the same at this instant at every point in the conduit. It cannot be other than zero, since it is zero at the loops.

(3) *The Fundamental in Conduits Having Multiple Characteristics.* We will study only the case where the curve is of sinusoidal form. It will be sufficient here to take again the problem of periodic movements, assuming that the maximum pressure rise at the bottom of the conduit is

$$\left.\begin{aligned} H_{max} - H_0 &= A \sin(mt) = H_0 \\ H_{max} &= 2H_0 \end{aligned}\right\} \quad \ldots\ldots [25]$$

The surge at the point of discontinuity, as has been shown by the author in his article in *La Houille Blanche*, will be

$$H_1 - H_{10} = H_0 \cos(mL_1/a_1) \quad \ldots\ldots [26]$$

where $m = \frac{2\pi}{\theta}$. Knowing the extreme surges at two points in the conduit, it is not difficult to calculate them for any other point.

(4) *Case of Parallel Conduits, and the Case of the Equilibrium Chamber.* These two cases have been treated in detail by the author in his article in *La Houille Blanche.* All the developments are valid, with the condition we recall that there is double the pressure at the bottom of the conduit.

We remember that the pressure rise at the base of an equilibrium chamber is given by

$$H_1 - H_{10} = H_0 \sin\frac{\pi}{2}\frac{\epsilon}{1+\epsilon} \quad \ldots\ldots [27]$$

where ϵ is given by the formula

$$\epsilon = \frac{2/\pi}{\frac{\mu'}{(\pi/2)(L_2a_1/L_1a_2)} + \mu'' \cot\frac{\pi}{2}\frac{L_3a_1}{L_1a_3}} \quad \ldots\ldots [28]$$

where $\mu' = S_2a_1/S_1a_2$; $\mu'' = S_3a_1/S_1a_3$. This last formula, however, is no longer valid when the cotangent appearing in the denominator tends to ∞. For such a case we assume

$$(\pi/2)(L_3a_1/L_1a_3) = \pi(1 - \epsilon') \quad \ldots\ldots [29]$$

where ϵ' is assumed small; ϵ is then the root of an equation of the second degree

$$-\frac{2L_1a_2}{L_2a_1}\mu'\epsilon^2 + \left[2 + \mu'' - \frac{2L_1a_2}{L_2a_1}\mu'\epsilon'\right]\epsilon + 2\epsilon' = 0 \quad [30]$$

Case of Odd Harmonics in a Conduit—Distribution of Pressures Along the Conduit

Whatever be the form of the function $F(t)$, we have always

$$H_x - H_{x0} = F\left(t - \frac{x}{a}\right) - F[t - (2L/a) + (x/a)]$$

Let us assume that the function $F(t)$ has a period $4L/pa$, p being an integer, and that we have

$$F(t) = F[t - (4L/pa)], \text{ and } F(t) = -F[t - (2L/pa)] + (av_0/g)$$

(If the conduit is not uniform, we will assume that it is possible to substitute for any segment the expression $4L_i/p'a_i = 4L/pa$.)

The case $p = 1$ corresponds to the fundamental, already studied. To supplement this we shall consider only the odd values of p which give us the odd harmonics of the conduit.

At the point whose abscissa is $x_1 = L/p$, we have

$$H_{x1} - H_{x10} = F[t - (L/pa)] - F\{t - (L/pa) - [(2p-2)L/pa]\} \quad \ldots\ldots [31]$$

This quantity is identically zero for $p = 3, 5, 7, \ldots\ldots$ Thus, there is always a loop at the point with abscissa $x_1 = L/p$, if p is odd.

Let us study the point with abscissa $x_2 = 2L/p$. We have

$$H_{x2} - H_{x20} = F[t - (2L/pa)] - F[t - (2L/a) + (2L/pa)] \quad [32]$$

If in front of the gate we have

$$H - H_0 = \delta$$

and it may be shown that at the point with abscissa $x_2 = 2L/p$ we will have at the same instant

$$H_{x2} - H_{x20} = -\delta$$

If the pressure at the gate oscillates between H_0 and $-H_0$, it will oscillate at the point with abscissa x_2 between $-H_0$ and H_0 (a fact already known, ever since Camichel's studies, for the third harmonic).

The velocity is always given for a point at abscissa x by

$$v_x = -(g/a)[F\{t - (x/a)\} + F\{t + (x/a) - (2L/a)\}]$$

It can be shown without difficulty that at the point having abscissa x_2 the velocity is constantly zero. There is a node at these points. The analysis may be continued for the points whose abscissas are $x_3 = 3L/pa$, $x_4 = 4L/pa$, etc. In particular, at the point with abscissa x_4 the pressure is always identical and in phase with that prevailing at the right of the gate.

The Function $F(t)$ Is of Sinusoidal Form. The proof given in the preceding paragraph is quite general, and assumes solely that the function $F(t)$ is periodic. Let us suppose in addition that it is sinusoidal, which will often be true in practice, or a satisfactory approximation. It may be shown that the pressure and the velocity at any point are given by

$$\left.\begin{aligned} H_x - H_0 &= A \sin\frac{p\pi}{2}\left(1 - \frac{x}{L}\right)\cos p\left(\frac{2\pi}{\theta}t - \frac{\pi}{2}\right) \\ v_x &= -(g/a)A \cos\frac{p\pi}{2}\left(1 - \frac{x}{L}\right)\sin p\left(\frac{2\pi}{\theta}t - \frac{\pi}{2}\right) \end{aligned}\right\} \quad [33]$$

where $A = H_0$ in the case of movements of resonance. There is a maximum every time that

$$p\left(\frac{2\pi}{\theta}t - \frac{\pi}{2}\right) = k\pi$$

We have under these conditions

$$H_x - H_0 = \pm H_0 \sin p\frac{\pi}{2}\left(1 - \frac{x}{L}\right)$$

a relation which tells us how the maxima are distributed along the line.

The case $p = 1$ corresponds to the resonance of the fundamental in a conduit with uniform characteristics. The curve of distribution of pressures along the conduit is sinusoidal, and no more linear, as in the case of a linear movement of the gate.

It may be shown that there is a loop of the system

for $p = 3$, at the points $x = L$ and $x = L/3$;

for $p = 5$, at the points $x = L$, $x = \frac{3}{5}L$, and $x = \frac{1}{5}L$;

for $p = 7$, at the points $x = L$, $x = 5L/7$, $x = 3L/7$, and $x = L/7$

and so on.

At intermediate points there are "knots," nodes with $v = 0$. We do not lay stress on these calculations which are elementary.

4—CONCLUSIONS

Resonances and periodic movements obey simple laws which are not without analogies between themselves. The author's study which appeared in *La Houille Blanche*, together with this present paper, forms a complete whole.

The principal results arrived at in this paper are found to be verified in a remarkable manner by the experiments of Camichel, Eydoux, and Gariel, de Sparre, Boucher, Neeser, and Foch. These experiments have served as constant guides in the author's studies.

The author's aim has not been entirely theoretical. He has been induced to undertake his investigations by his obligation to calculate for certain practical cases the boundary curve of the surge pressures along the conduit.

With the exception of the case of resonance of the fundamental automatic check-valve type, realized by Camichel, and the resonance of the harmonics of higher order, which are moreover difficult to obtain, the cases of resonance in a conduit of uniform slope, although advisable to avoid, nevertheless cannot in general cause its rupture. Certain special cases, such as conduits with projecting points, equilibrium chambers, and branches can, on the contrary, be subject to dangerous surges. The theory enables us to study these cases.

It will be observed that the author's theory of periodic movements and of resonances, applied to the study of equilibrium chambers, furnishes us with a theory complementary to that developed in the author's "Théorie Générale." It is sometimes useful to know the values obtained by the second method. Further, it may be mentioned that the author's theory has several points of contact with the theory of sound waves developed by Constantinescu.

The author concludes this study by stating the hope that experimental research in this domain will supplement to completion the work of Camichel, Eydoux, Gariel, and Neeser.

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Typical Analysis of Water Hammer in a Pumping Plant of the Colorado River Aqueduct

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This paper presents a method of analysis of surge conditions in a pumping system by arithmetical integration and is an extension of the work done by Professor Knapp at the California Institute of Technology. The analysis utilizes the step-by-step method, includes the complete characteristics of the pumping equipment, and determines their effect upon surge conditions. The valve characteristics are also included, and this is the first presentation of such a method of analysis.

THE COLORADO River Aqueduct carries water from the Colorado River to the cities of The Metropolitan Water District of Southern California. Diversion from the river is from the reservoir formed by Parker Dam, about one hundred and fifty miles below Boulder Dam. From the point of diversion to the terminal reservoir at Cajalco, near Riverside, California, the length of the main aqueduct is 242 miles. To cross the intervening high levels in the desert area five pumping plants are required with a total lift of 1617 feet. The lifts in the various plants range from 144 to 440 feet. The aqueduct is constructed for a carrying capacity of 1600 second feet. Ultimately there will be nine 200 second foot motor-driven pumps in each plant. The present installation is three such pumps per plant. Power is obtained from Boulder Dam over a long transmission line. The delivery lines are from 800 to 2000 ft in length and are steel pipes 10 ft in diameter. Three pumps are connected to each pipe through 6-ft diameter pipes which converge a short distance from the plant. The design of these delivery lines required a careful study of the operating conditions, particularly with reference to water hammer.

The pumping system discussed in this paper consists essentially of a short inlet pipe leading from an open reservoir to the pump intake, a motor-driven centrifugal pump, a shutoff valve at the discharge of the pump, and a delivery pipe to an open reservoir at the top of the pump lift. Under normal pumping conditions the pressure throughout this system is steady. Starting a pump, shutting down a pump, failure of the power supply, or failure of certain parts of the apparatus to function properly will initiate a series of transient pressure variations in the system and give rise to more or less severe water hammer. It is the purpose of this paper to give a practical method for determining the pressure variations in such a system.

Operation of the System

A typical pumping system is shown in Fig. 1. It represents, with some simplifications, one of the pumping plants of the Colorado River Aqueduct now under construction by The Metropolitan Water District of Southern California.

In normal operation, the pump is started, brought up to speed, and the motor synchronized with the pump discharge valve closed except for a small by-pass. If the delivery pipe is empty, it is filled through the by-pass. With the pump at normal speed, and the delivery pipe filled, the pressure on the pump side of the valve is practically the shutoff pressure developed by the pump plus the inlet head, and the pressure on the delivery side of the valve is the static head from the upper reservoir. The discharge valve is then slowly opened and the pump discharges against the net head from inlet reservoir to discharge reservoir, plus the head due to friction losses in the system. In taking a pump out of service, the discharge valve is first closed slowly, the power supply to the motor is then disconnected, and the pump and motor decelerate to rest. In normal starting and stopping operations the discharge valve is moved slowly, the velocity in the delivery pipe changes slowly, and the resulting pressure variations are small.

Conditions Producing Water Hammer

There are a number of possible operating conditions which will produce rapid variation of velocity in the delivery pipe, causing fluctuations in pressure or water hammer. The effect of every such condition must be investigated, and the system made safe against failure resulting from accidental faults that are possible, even though only remotely probable.

Certain emergency and accidental conditions which may occur are the following:

1 Power failure due to electrical fault. This condition is certain to occur more or less frequently. The motor circuit breaker is tripped open and the pump discharge valve automatically starts to close.

2 Delay in tripping motor breaker. If a motor is not immediately disconnected from a short circuit, it will act for a

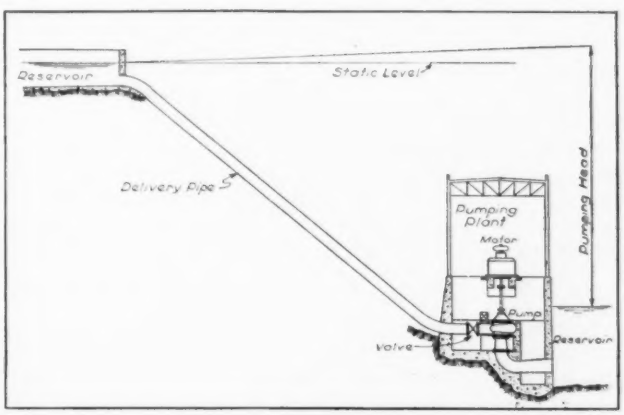


Fig. 1 Typical Pumping System

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Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y., and will be accepted until March 10, 1989, for publication at a later date. Discussion received after the closing date will be returned.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

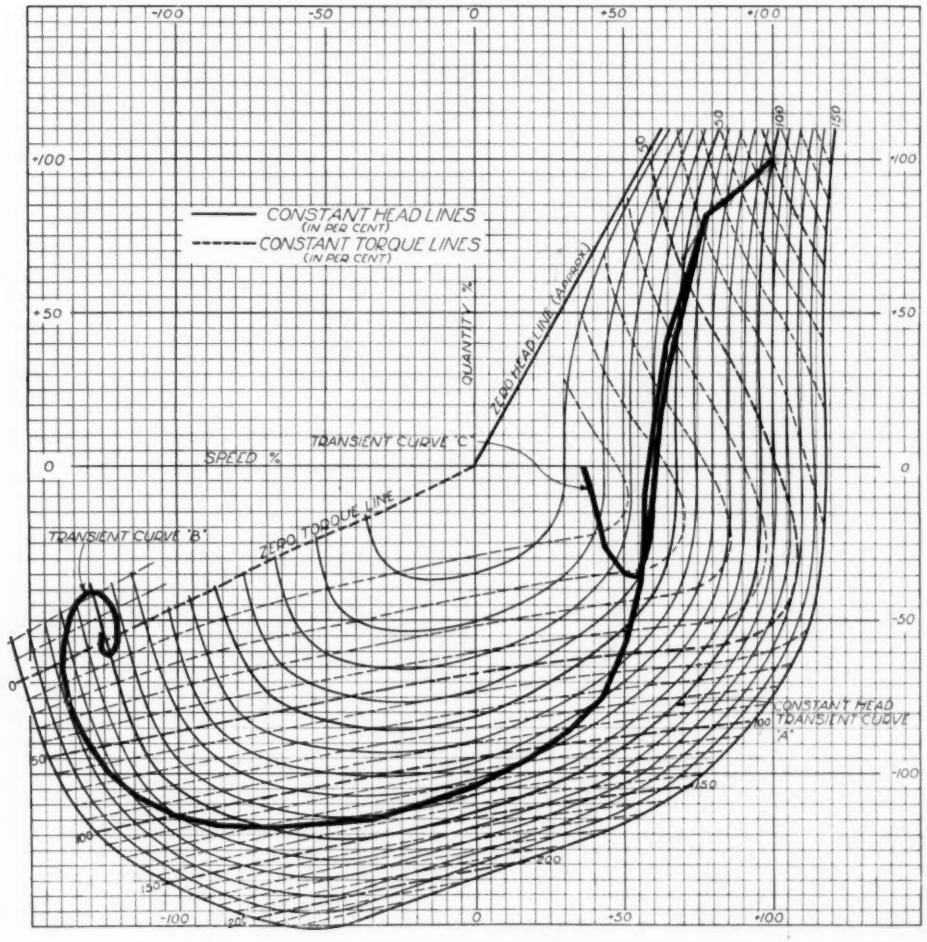


FIG. 2 PUMP CHARACTERISTICS DIAGRAM

short time as a generator, giving additional retarding torque and resulting in more rapid deceleration than normal.

3 Failure of valve to operate when power is shut off. In this case, the flow in the pipe line will reverse; the pump and motor will reverse and attain runaway speed in the reverse direction.

4 Delay in operation of valve. If the operation of the valve, in case of power failure, is delayed until full reverse flow has been established, and the valve then closes rapidly, severe water hammer will result.

5 Failure of shaft coupling. The flywheel effect of the motor is removed, the speed will drop rapidly, and the pump will reverse rotation.

6 Seal-ring seizure. The speed will drop rapidly as in 6, but the pump will not reverse.

All of these conditions, and others, must be investigated. In this paper, however, only two cases will be considered; first, power failure with the valve failing to operate; and second, power failure with the valve automatically closing at a predetermined rate.

POWER FAILURE WITH THE VALVE REMAINING FULLY OPEN

When the power supply is suddenly cut off, the only force driving the pump is the kinetic energy of the rotating parts. If the flywheel effect were large, compared to the pumping energy required, the pump would decelerate slowly. The resulting rate of change of pipe-line velocity would be gradual and the pressure variation would be small. However, the kinetic energy is generally small compared to the pumping energy and the drop in speed is rapid.

The effect of the speed drop in the first small time interval after the power is switched off is to reduce the pump discharge and to reduce the velocity in the pipe line. As a result of the sudden drop in pipe-line velocity a wave front of reduced pressure travels up the pipe and is reflected from the open end as a wave front of increased pressure. When this increased pressure reaches the pump, it adds to the pumping head then existing and further reduces the pump discharge. Soon the speed of the pump is reduced to the point where no water can be delivered against the existing head and reverse flow commences, although the pump is still rotating in the pumping direction. The speed now drops still more rapidly, passes through zero, and the pump, acting as a turbine under no load except the inertia of the rotating parts, eventually reaches a runaway speed in the reverse direction.

In analyzing quantitatively such a transient condition, it is necessary to know the head and torque of the pump for all of the variations of speed and discharge that occur during the transient. This information is completely given by a chart such as is shown in Fig. 2. The use of this chart in the study of transient phenomena has been described in a paper[2] by Prof. Robert T. Knapp. This paper gives the methods of obtaining the test data used in the construction of the chart and the method

[2] "Complete Characteristics of Centrifugal Pumps and Their Use in the Prediction of Transient Behavior," by R. T. Knapp, Trans. A.S.M.E., vol. 59, 1937, pp. 683–689.

of plotting the results. The chart gives the pump characteristics for the steady-state conditions corresponding to coordinates of speed and flow. Dr. Knapp's paper supplies experimental verification of the assumption that the instantaneous flow conditions at any point on a path of transient behavior are identical with the flow conditions for the corresponding steady state of operation. The complete characteristics diagram as given in Dr. Knapp's paper shows the pump characteristics throughout the entire field of possible pump operation. For most problems in the study of water hammer, the entire diagram is not needed and consequently in Fig. 2 the entire second quadrant, the first quadrant above the zero-head line, and most of the third quadrant above the zero-torque line are omitted. Also, it has been found more convenient to express the characteristics in percentages of the pump characteristics (flow, speed, head, and torque) at normal rating. The diagram of Fig. 2 is based on actual tests of a scale model of one of the pumps for the Colorado River Aqueduct.

Three transient paths are shown on the diagram. That marked A follows the rated-head line and is for a constant-head transient such as would occur when the power is shut off from a pump without any discharge valve, pumping directly into a large reservoir without an intervening pipe line. The speed-time relation during such a transient can be calculated from the inertia-torque relation of the pumping unit. The transient path marked B is that resulting when the power is shut off from a pump, without a valve, pumping into a reservoir through a fairly long pipe line. Transient path C is for the same condition as B, but with a valve at the pump discharge which closes at such a rate that the pump is prevented from reversing, but holds the maximum pressure rise within reasonable limits.

In determining paths B and C, it is necessary to calculate the variation with time of the head on the pump and pipe line, the discharge, the speed, and the torque. This can be done by taking small intervals of time and, by a method of trial and error, determining changes in the characteristics during the time interval such that the conditions at the beginning and end of the interval will satisfy the pipe-line characteristic equation, the pump characteristic diagram, and the inertia-speed-torque relations of the rotating parts. For transient C, it is also necessary to take into account the pressure drop through the valve. A brief development follows for the pipe-line equation and the inertia-speed-torque relation.

Notation

z = time from start of transient condition, sec
L = length of pump discharge pipe, ft
A = area of pump discharge pipe, sq ft
a = velocity of pressure wave, fps
v = velocity of flow in pipe line or through valve, fps
H = head in ft. H is the head at the lower end of the pipe line at any time, and, if the valve is fully open, or there is no valve in the system, it is equal to the head developed by the pump
H_p = pumping head. With no valve, $H = H_p$
Q = pump discharge, cfs
N = pump speed, rpm
M = pump torque corresponding to a given speed and discharge
H_v = pressure drop across partly open valve, ft
ΔH = pressure rise or drop above or below H, ft
Δv = change in velocity, fps
h = $\Delta H/H_0$
h_p = H_p/H_0
q = Q/Q_0
n = N/N_0
m = M/M_0
h_v = H_v/H_0
R = magnitude of resultant reflected wave arriving at the valve or pump at any time during a transient condition
c = coefficient of pressure drop across the valve

$$H_v = cv^2/2g$$

K = pipe-line constant, $= av_0/2gH_0$
g = acceleration of gravity
ω = angular velocity $= 2\pi N/60$
WR^2 = flywheel effect of rotating parts
I = moment of inertia of rotating parts. $I = WR^2/g$
Subscript $_0$ denotes a value for steady pumping conditions, or just prior to the start of a transient
Subscript z denotes a value at a time z after the start of a transient

The Pipe Line

The pressure variation in the pipe line is developed from the fundamental equation of change of pressure

$$\Delta H = a\,\Delta v/g \quad [1]$$

or, the change in pressure is equal to the change in velocity multiplied by the velocity of the pressure wave divided by the acceleration of gravity. The pressure change is modified by partial and complete reflections of the pressure wave from changes in section of the pipe line, any dead ends in the pipe system, and from the open end at the upper reservoir.

If subscript $_0$ indicates the start of the transient condition and subscripts $_1$, $_2$, $_3$, ..., small successive time intervals after the start of the transient, the following equations hold:

$$\Delta H_1 - \Delta H_0 = (a/g)(v_1 - v_0) + 2R_1 \quad [2]$$

$$\Delta H_2 - \Delta H_1 = (a/g)(v_2 - v_1) + 2R_2 \quad [3]$$

$$\Delta H_3 - \Delta H_2 = (a/g)(v_3 - v_2) + 2R_3 \quad [4]$$

$$\Delta H_z - \Delta H_{z-1} = (a/g)(v_z - v_{z-1}) + 2R_z \quad [5]$$

Adding, and noting that $\Delta H_0 = 0$

$$\Delta H_z = (a/g)(v_z - v_0) + 2(R_1 + R_2 + R_3 + R_z) \quad [6]$$

In the example being considered, the pipe line is assumed uniform, without any dead ends, and the only reflection is the complete reflection from the open reservoir of a wave front leaving the pump at time z, which reaches the pump again in time $z + (2L/a)$ as a wave front of equal magnitude and opposite sign. If the time intervals are equal to $2L/a$, then $R_z = -\Delta H_{z-1}$, and the equation becomes

$$\Delta H_z = (a/g)(v_z - v_0) - 2(\Delta H_1 + \Delta H_2 + \Delta H_3 \ldots + \Delta H_{z-1}) \quad [7]$$

Dividing by H_0 and noting that $v = Q/A$

$$\frac{\Delta H_z}{H_0} = \frac{a}{gH_0}\left(\frac{Q_z}{A} - \frac{Q_0}{A}\right) - 2\left(\frac{\Delta H_1}{H_0} + \frac{\Delta H_2}{H_0} + \frac{\Delta H_3}{H_0} \ldots + \frac{\Delta H_{z-1}}{H_0}\right) \quad [8]$$

and if $h = \Delta H/H_0$ and $q = Q/Q_0$

$$h_z = (aQ_0/gA_0H)(q_z - 1) - 2(h_1 + h_2 + h_3 \ldots + h_{z-1}) \quad [9]$$

or

$$h_z = 2K(q_z - 1) - 2\Sigma h \quad [10]$$

q_z is taken as positive for flow in the pumping direction and negative for reverse flow.

TABLE 1 SUMMARY OF CALCULATIONS FOR PUMP SHUTDOWN WITHOUT VALVE OPERATION

All values are percentages

Time in intervals of $2L/a$	$\frac{H_s}{H_0}$	h_s	$2\Sigma h$	$h_s + 2\Sigma h$	$q_s - 1$	q_s	n_s	$n_1 - n_2$	$m_1 + m_2$	m_s
0	100					100	100			100
1/4	86	14	0	14	6.0	94.0	93.4	6.6	188	88
1/2	75	25	0	25	10.6	89.4	87.6	5.8	165	77
3/4	66	34	0	34	14.5	85.5	82.5	5.1	146	69
1	58	42	0	42	17.9	82.1	77.9	4.6	132	63
1 1/4	56	44	28	72	30.6	69.4	73.8	4.1	119	56
1 1/2	52	48	50	98	41.7	58.3	70.2	3.6	105	49
1 3/4	49	51	68	119	50.6	49.4	67.0	3.2	92	43
2	45	55	84	139	59.2	40.8	64.2	2.8	81	38
2 1/4	42	58	116	174	74.2	25.8	61.8	2.4	68	30
2 1/2	40	60	146	206	87.7	12.3	59.9	1.9	53	23
2 3/4	38	62	170	232	98.8	1.2	58.5	1.4	40	17
3	36	64	194	258	109.9	— 9.9	57.4	1.1	31	14
3 1/4	38	62	232	294	125.3	— 25.3	56.4	1.0	30	16
3 1/2	42	58	266	324	138.0	— 38.0	55.0	1.4	40	24
3 3/4	48	52	294	346	147.3	— 47.3	53.0	2.0	58	34
4	55	45	322	367	156.2	— 56.2	50.3	2.7	77	43
4 1/2	70	30	382	412	175.4	— 75.4	42.1[a]	8.2	117	74
5	75	25	412	437	186.1	— 86.1	30.6	11.5	165	91
5 1/2	80	20	442	462	196.9	— 96.9	16.6	14.0	201	110
6	83	17	462	479	203.9	—103.9	0.4	16.2	232	122
6 1/2	88	12	482	494	210.2	—110.2	— 17.2	17.6	252	130
7	91	9	496	505	215.0	—115.0	35.5	18.3	263	133
7 1/2	95.5	4.5	506	510.5	217.3	—117.3	— 53.9	18.4	263	130
8	101	— 1	514	413	218.3	—118.3	— 71.6	17.7	253	123
8 1/2	105.5	— 5.5	515	509.5	217.0	—117.0	— 87.1	16.5	236	113
9	111	—11	512	501	213.3	—113.3	102.0	14.9	213	100
9 1/2	118	—18	504	486	206.9	—106.9	114.6	12.6	181	81
10	124	—24	490	466	198.5	— 98.5	124.7	10.1	145	64
10 1/2	127	—27	468	441	187.5	— 87.5	132.1	7.4	106	42
11	127	27	442	415	176.8	— 76.8	136.5	4.4	63	21
11 1/2	126	—26	414	388	165.2	— 65.2	138.7	1.7	25	4
12	121	—21	388	367	156.3	— 56.3	138.4	— .3	— 4	— 8
12 1/2	115	—15	362	347	148.2	— 48.2	136.6	— 1.8	— 26	— 17
13	108	— 8	346	336	143.1	— 43.1	134.0	— 2.6	— 38	— 21

[a] Change from 1/4 intervals to 1/2 intervals.

Data used in Tables 1 and 2

N_0 = 400 rpm
H_0 = 310 ft
Q_0 = 215 cfs
v_0 = 8.21 fps
hp = 8402
L = 2188 ft

a = 2856 fps
$2L/a$ = 1.533 sec
M_0 = 110,318 lb-ft
WR^2 = 465,100 lb-ft²
Δt = 0.383 sec (1/4 interval)
Δt = 0.766 sec (1/2 interval)
K = 1.175

The development just given, except as to notation, follows the one published by R. S. Quick.[3] Friction loss in the pipe line is neglected since it affects the results only slightly, unless it amounts to say, 10 per cent or more of the head.

Inertia Equation

$$M = \frac{WR^2}{g}\frac{d\omega}{dt} \quad [11]$$

or, torque is equal to the moment of inertia of the rotating system multiplied by the angular acceleration.

Let Δt be a small time interval and subscripts 1 and 2 denote values at the beginning and end of the interval.

Then approximately

$$\frac{M_1 + M_2}{2} = \frac{WR^2}{g}\left(\frac{\omega_1 - \omega_2}{\Delta t}\right) = \frac{2\pi WR^2}{60g}\left(\frac{N_1 - N_2}{\Delta t}\right)$$

or

$$N_1 - N_2 = \frac{153.6}{WR^2}(M_1 + M_2)\,\Delta t$$

Let

$$n = N/N_0 \text{ and } m = M/M_0$$

$$n_1 - n_2 = \frac{153.6 M_0}{WR^2 N_0}(m_1 + m_2)\,\Delta t \quad [12]$$

Thus, on a simple pumping system without a valve, for the analysis by successive short time intervals of a transient condition caused by removing the driving power from the pump, the following conditions must be met at the beginning and end of each time interval:

$$h_s = 2K(q_s - 1) - 2\Sigma h \quad [10]$$

$$h_{ps} = F_1(q_s, n_s) \quad [13]$$

$$M_s = F_2(q_s, n_s) \quad [14]$$

$$n_1 - n_2 = \frac{153.6 M_0}{WR^2 N_0}(m_1 + m_2)\,\Delta t \quad [12]$$

Functions [13] and [14] are given by the pump characteristic diagram. The time interval Δt should be made equal to or an even division of the time interval $2L/a$.

Table 1 shows a summary of a part of the calculations for the transient path B of Fig. 2. Fig. 3 is plotted from the data of Table 1 and shows the time history of head, flow, and speed when the pump is suddenly shut down without any valve in the system. It is to be noted from Fig. 3 that reversal of flow takes place considerably in advance of reversal of rotation. With a larger flywheel effect, and other conditions the same, the time between reversals of flow and speed would be larger and the head variation less. In a system with a small flywheel effect and a high value of pipe-line constant K, the speed and flow reverse quickly and, if the pipe line is of considerable length, the head variation is great.

Effect of the Valve

When a valve adjacent to the pump discharge is closed during a transient, the head on the pump at any instant is equal to the head on the pipe-line side of the valve plus or minus the pressure drop across the valve

$$H_p = H \pm H_v \quad [15]$$

[3] "Comparison and Limitations of Various Water-Hammer Theories," by R. S. Quick, *Mechanical Engineering*, vol. 49, May, 1927, pp. 524–530.

The plus sign is used for flow in the pumping direction, and the minus sign for reverse flow.

The head loss in the valve is a function of the position of the closing element of the valve (disk, plug, needle, or flap) and the velocity through the net area of valve opening.

$$H_v = cv^2/2g \quad [16]$$

The coefficient c varies with the valve opening, and for the various types of valves that may be used, little information about it is available.

The valves to be installed with the pumps on the Colorado River Aqueduct are of the rotating-conical-plug type, consisting essentially of a valve body or housing and a conical plug arranged to be rotated within the housing by means of an external oil-pressure-operated hydraulic cylinder. The water passage through the valve plug is circular in cross section and is tapered

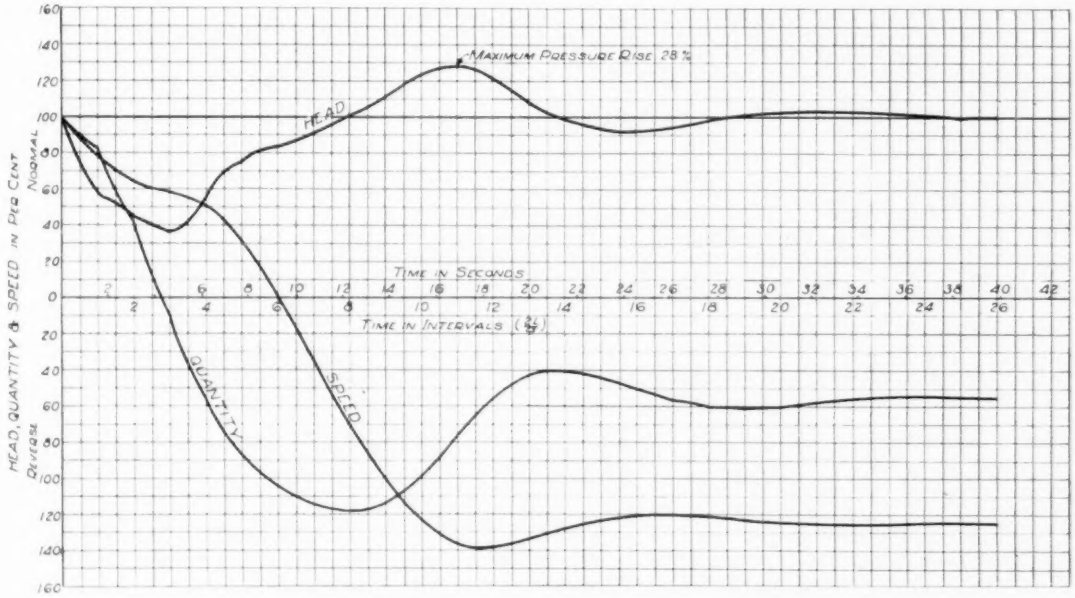


FIG. 3 TIME HISTORY OF TRANSIENT CURVE *B* OF FIG. 2
(Pump shutdown without valve operation.)

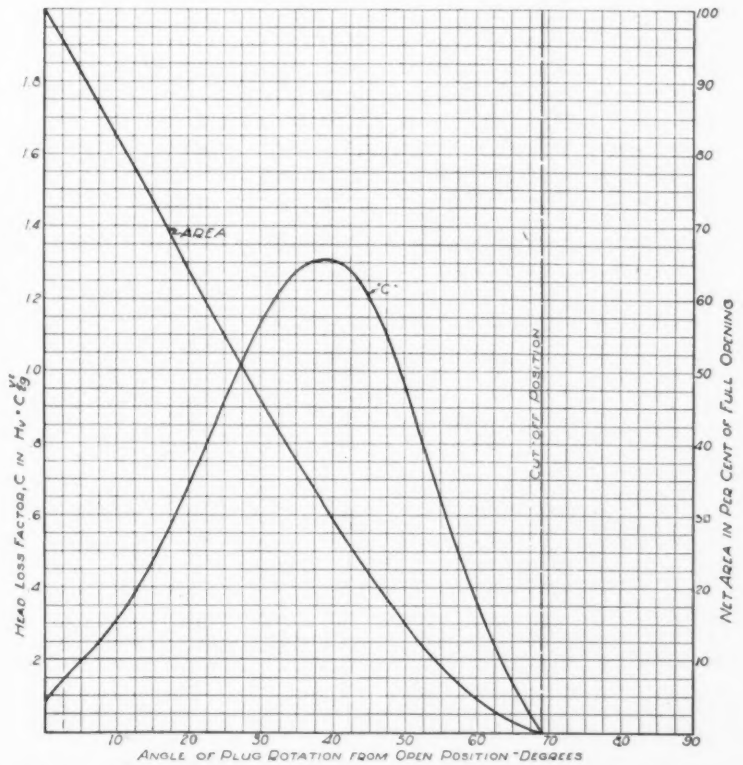


FIG. 4 TYPICAL HEAD-LOSS FACTOR *c* AND NET AREA OF A ROTARY-PLUG VALVE

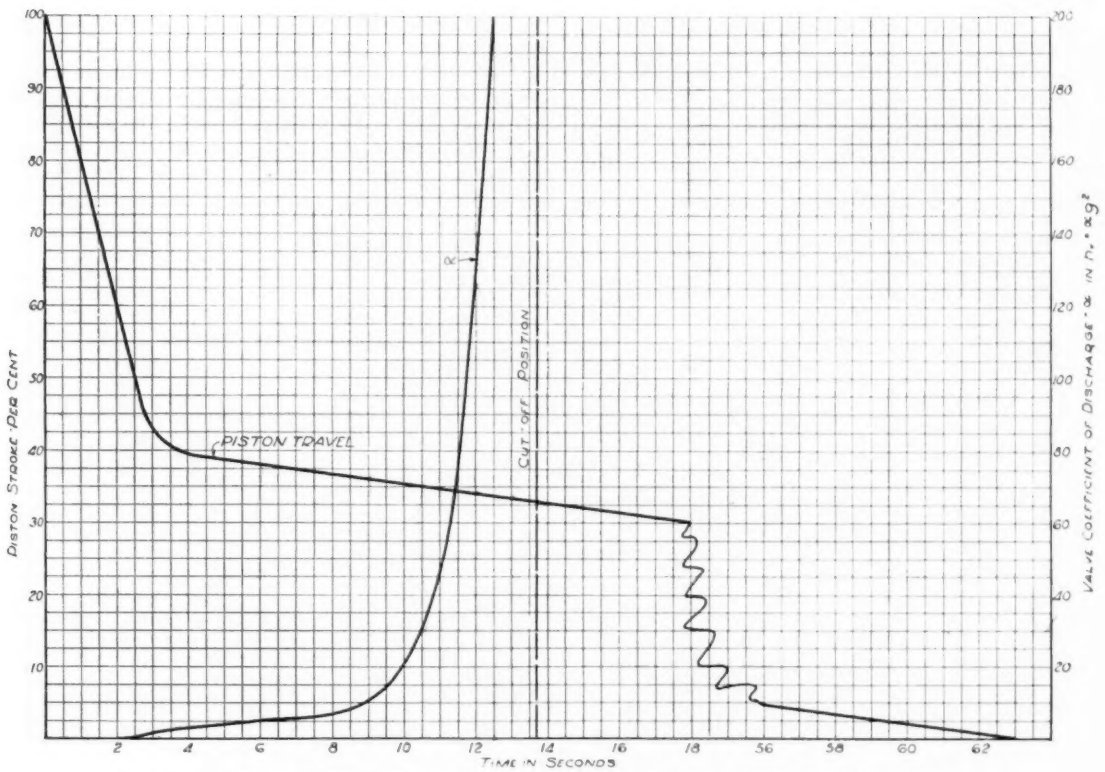


FIG. 5 VARIATION WITH TIME OF PISTON STROKE AND VALVE DISCHARGE COEFFICIENT α

so that when the valve is fully open, the water passage forms part of a smooth gradual increase in diameter from the pump discharge flange to the delivery pipe. The operating control of the hydraulic cylinder is arranged to rotate the plug slowly for the normal operations of starting up the pump or taking it out of service. In case of power failure or other emergency condition which removes the driving power from the pump, a supplementary control is brought into action automatically which rotates the plug rapidly until a considerable throttling effect is developed, after which the rate of rotation of the plug is retarded and the reverse flow through the pump is shut off gradually to reduce the pressure rise in the pipe line. The mechanism for controlling the emergency closing is so arranged that the point in the stroke where the rate changes from rapid to slow can be varied, and the rate of closure before and after this point can also be varied. Once set, the adjustments are locked and cannot readily be changed.

For predetermining the timing of the closing stroke to give minimum pressure rise and prevent, if possible, the reverse rotation of the pump and motor, the throttling characteristic of the valve must be known for all positions of the plug. Experiments have been conducted on valve models both by the

TABLE 2 SUMMARY OF CALCULATIONS FOR PUMP SHUTDOWN WITH VALVE CLOSURE

All values are percentages except for α

Time in intervals of $2L/a$	$\frac{H_x}{H_0}$	h_x	$2\Sigma h$	$h_x + 2\Sigma h$	$q_x - 1$	q_x	n_x	$n_1 - n_2$	$m_1 + m_2$	m_x	Piston travel	$\alpha \times 100$	h_v	$h_x + h_v$	h_{px}
0	100					100	100			100	0				100
1/4	86	14	0	14	6.0	94.0	93.4	6.6	188	88	7.66				86
1/2	75	25	0	25	10.6	89.4	87.6	5.8	165	77	15.32				75
3/4	66	34	0	34	14.5	85.5	82.5	5.1	147	70	22.98	0.649	— 0.5	33.5	66.5
1	57	43	0	43	18.3	81.7	77.9	4.6	133	63	30.64	2.684	— 1.8	41.2	58.8
1 1/4	51.5	48.5	28	76.5	32.6	67.4	73.8	4.1	118	55	38.30	9.767	— 4.4	44.1	55.9
1 1/2	43.5	56.5	50	106.5	45.3	54.7	70.2	3.6	102	47	45.96	29.82	— 8.9	47.6	52.4
1 3/4	33.0	67	68	135	57.5	42.5	67.2	3.0	87	40	53.62	86.48	—15.6	51.4	48.6
2	26.0	74	86	160	68.1	31.9	64.6	2.6	74	34	58.6	204.28	—20.8	53.2	46.8
2 1/4	33.0	67	125	192	81.7	18.3	62.4	2.2	62	28	60.2	305.68	—10.2	56.8	43.2
2 1/2	39.5	60.5	163	223.5	95.1	4.9	60.7	1.7	48	20	60.55	335.50	— 0.8	59.7	40.3
2 3/4	42.5	57.5	202	259.5	110.4	—10.4	59.5	1.2	34	14	60.81	360.85	+ 3.9	61.4	38.6
3	53	47	234	281	119.6	—19.6	58.5	1.0	28	14	61.06	387.69	14.9	61.9	38.1
3 1/4	65	35	259	294	125.1	—25.1	57.5	1.0	30	16	61.32	416.02	26.2	61.2	38.8
3 1/2	78.5	21.5	284	305.5	130.0	—30.0	56.3	1.2	34	18	61.58	445.84	40.2	61.7	38.3
3 3/4	99.0	1.0	317	318	135.3	—35.3	54.9	1.4	40	22	61.83	477.15	59.4	60.4	39.6
4	107.5	— 7.5	328	320.5	136.4	—36.4	53.3	1.6	45	23	62.09	511.45	67.8	60.3	39.7
4 1/2	109.5	9.5	327	317.5	135.1	—35.1	50.2	3.1[a]	44	21	62.60	591.97	72.9	63.4	36.6
5	103.0	— 3.0	313	310	131.9	—31.9	47.5	2.7	39	18	63.11	700.82	71.4	68.4	31.6
5 1/2	103.5	— 3.5	308	304.5	129.6	—29.6	45.1	2.4	34	16	63.62	863.35	75.7	72.2	27.8
6	109.5	— 9.5	307	297.5	126.6	—26.6	43.0	2.1	30	14	64.13	1185.4	83.9	74.4	25.6
6 1/2	115.0	—15.0	301	286	121.7	—21.7	41.3	1.7	24	10	64.64	1983.2	93.4	78.4	21.6
7	115.0	—15.0	288	273	116.2	—16.2	40.0	1.3	18	8	65.15	3683	96.5	81.5	18.5
7 1/2	110.5	—10.5	271	260.5	110.9	—10.9	39.0	1.0	15	7	65.66	7843	93.2	82.7	17.3
8	107.5	— 7.5	258	250.5	106.61	— 6.61	38.0	1.0	14	7	66.17	20875	91.3	83.8	16.2
8 1/2	106.2	— 6.2	250	243.8	103.76	— 3.76	37.0	1.0	14	7	66.68	64117	90.7	84.5	15.5
9	108	— 8.0	243	235	100	0	36.1	0.9	13	6	67.20				14.2

[a] Change from 1/4 intervals to 1/2 intervals.

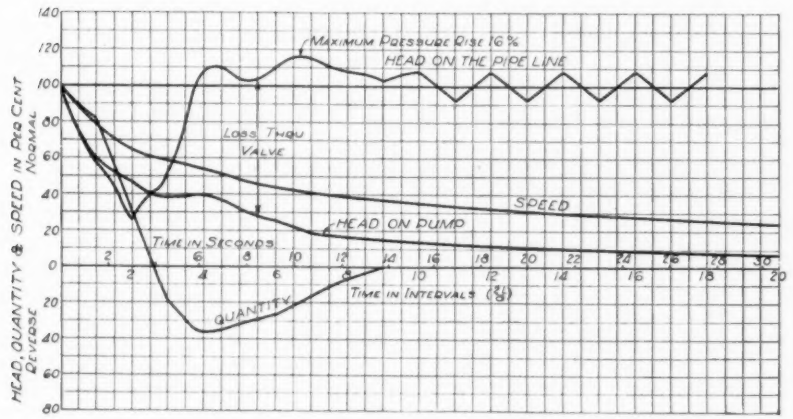


FIG. 6 TIME HISTORY OF TRANSIENT CURVE *C* IN FIG. 2
(Pump shutdown with discharge valve closing)

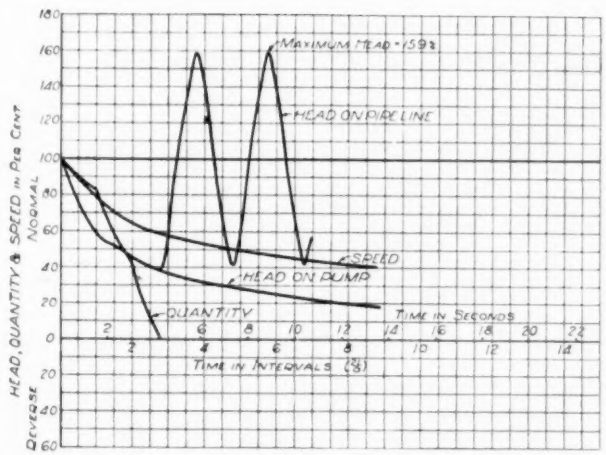


FIG. 7 TIME HISTORY OF TRANSIENT IF VALVE IS CLOSED INSTANTLY ON REVERSAL OF FLOW

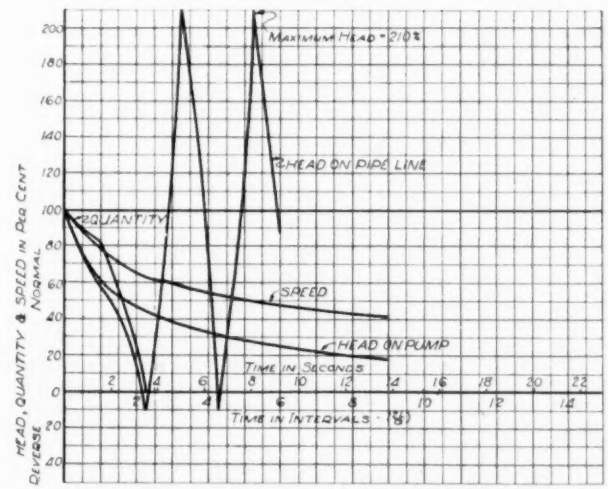


FIG. 8 TIME HISTORY OF TRANSIENT IF VALVE IS CLOSED RAPIDLY FOR ITS FULL STROKE

Metropolitan Water District at the hydraulic machinery laboratory at the California Institute of Technology, and by the valve manufacturers in their own laboratories. Tests have also been made on a scale model of the valve and pump assembled together to determine how the throttling effect of the combination is related to the characteristics of the pump and valve as determined separately.

The variation of the coefficient c in formula [16] for a typical model valve is shown in Fig. 4.

The model-valve experiments show that the head loss can be expressed by a formula of the form

$$H_v = \alpha' Q^2 \quad [17]$$

where α' is constant for each position of the valve plug.

To use this formula in the study of transients, it is more convenient to express the head loss as a ratio to the initial steady pumping head, $h_v = H_v/H_0$ and $q = Q/Q_0$ so that

$$h_v = \alpha' q^2 Q_0^2/H_0$$

or

$$h_v = \alpha q^2$$

where

$$\alpha = \alpha' Q_0^2/H_0 \quad [18]$$

The angular position of the valve plug is a function of the stroke of the piston of the operating cylinder, which for a predetermined program of closing rates is a function of the time from the start of the transient. Fig. 5 shows the variation with time of piston stroke and α as used in the illustrative example given herein.

To include the effect of the pump discharge valve in the study of transient conditions during pump shutdown, the throttling effect of the valve must be considered as affecting the head on the pump.

The head on the pump is given by

$$h_{p_2} = 1 + h_s \pm \alpha q_s^2 \quad [19]$$

The change in the different variables during each small time interval must be adjusted to satisfy Equation [19] as well as Equations [10], [13], [14], and [12]. A portion of the tabulation for calculating transient curve *C* of Fig. 2 is given in Table 2. Fig. 6 is plotted from the values given in Table 2, and shows the variation with time of pipe-line head, pumping head, flow, and speed. Note that for this condition the speed does not reverse, since the valve is closed before the reverse flow can overcome the inertia of the rotating parts.

To illustrate the effect of different valve-closure programs on the pumping system, Fig. 7 has been plotted to show the conditions resulting if the valve could be made to close instantaneously at the instant the flow in the system reverses. Such a condition is only hypothetically possible, but is approximated with some types of check valve.

Fig. 8 shows the condition when the valve starts to close at the same rapid rate as for Fig. 6, but instead of slowing down toward the end of the stroke, continues to close at the same rapid rate.

Conclusion

The behavior of a pumping system during a transient is the result of interaction between the pump, the pipe line, the check or shutoff valve, and, in many cases, also a relief valve, air chamber, or surge suppressor. If the characteristics of each of these elements are known, the conditions during the transient can be quite closely predicted. The formulas for the pipe line, even including branch pipes, changes of section, and dead ends are well supported by experimental verification, as brought out in the previous symposium on water hammer.[4] Little has been published on the reverse-flow characteristics of pumps. It appears quite probable that characteristic diagrams for pumps of a given specific speed will be found similar. However, information on complete pump characteristics is generally available for only a limited range of specific speeds. Reliable data on the loss of head in valves as a function of the valve opening are almost completely lacking.

Analysis by the methods herein outlined involves a great deal of laborious calculation, but approximate simple formulas that are generally applicable are totally lacking.

Much time and labor can be saved and a clearer picture shown of the pressure and discharge by using the graphic method of water-hammer analysis[5] instead of Equation [10] for the pipe line head variation. The pump diagram, the valve characteristic, and the inertia equation must be coordinated with the water hammer diagram. The necessity for step-by-step and trial-and-error calculations is not entirely eliminated, but the work is greatly simplified.

For important installations, the various transients possible of occurrence must all be analyzed and the system constructed to be safe for the most severe conditions reasonably probable. The cases herein used as illustrations represent approximately the condition at one of the pumping plants on the Colorado River Aqueduct. For the purposes of this paper, the effects of compound and branching delivery pipes, the inlet pipes, and other conditions peculiar to those plants have been omitted, and the examples do not accurately represent actual conditions at any of the plants.

The investigations on which this paper is based were made in connection with the pumping plants of the Colorado River Aqueduct of The Metropolitan Water District of Southern California. F. E. Weymouth is general manager and chief engineer of the Metropolitan Water District. The design and construction of the pumping plants are under the direction of J. M. Gaylord, chief electrical engineer.

[4] First Symposium on Water Hammer, The American Society of Mechanical Engineers, New York, N. Y., 1933.

[5] "Methode Graphique Générale de Calcul des Propagations d'Ondes Planes," par L. Bergeron, *Bulletin de la Société des Ingenieurs Civili de France*, Juillet-Aout, 1937; also, "Simple Graphical Solution for Pressure Rise in Pipes and Pump Discharge Lines," by R. W. Angus, *English Journal* (Canada), 1935.

Condenser-Tube Life as Affected by Design and Mechanical Features of Operation

By A. J. GERMAN,[1] WATERBURY, CONN.

Condenser-tube corrosion is found to occur as a result of many factors which, generally, are of a chemical, electrochemical, and mechanical nature. It is beyond the scope of this paper to cover the field completely; therefore, the paper is limited to those problems associated with mechanical factors affecting condenser-tube life in land and marine condensers operating on the seaboard. In such units, mechanical features of design and operation have proved to be of great importance with associated problems commonly requiring the attention of mechanical engineers rather than chemical or metallurgical engineers before a final solution is achieved.

The details of this discussion indicate a general method of approach to a solution of the problem of inlet-end tube failures, by which local occurrence the type of trouble can usually be recognized.

It should not be overlooked that many tube troubles can be minimized, if not eliminated, by a change in tube alloy (frequently without greater expense) when the mechanical alterations required are too costly or impractical. The author dismisses the choice of material, tube quality, method of production, and details of installation, all of which are of major importance in many cases, and confines his remarks to a consideration of the effect on tubes of turbulence in the circulating water resulting from design or deterioration. He gives also a brief discussion of the possible effects of high temperature on tubes while the unit is off the line.

General Considerations

IT IS known from experience that air in the circulating-water system is detrimental. Tubes carrying an unbroken column of water will last longer than if that column were intermittent. Many theories covering this question are available, but with the admission that air in circulating water is damaging, we need not consider the theoretical aspects.

Figs. 1 and 2 illustrating a ship's propeller indicate the damaging effects of air on metal parts. Relatively rapid corrosion occurs on the vacuum side on the trailing half of the blade, while the working (pressure) side is not appreciably affected. Experience has shown the necessity for streamlining the cap over the shaft nut, as shown in Fig. 2, old-style hemispherical covers having corroded through in as short a time as that required for one trip.

For the sake of brevity, the terms "turbulence" and "cavitation" as used in this paper, imply the presence of damaging air bubbles in the circulating water. The damage brought about by this type of corrosion is usually found at inlet ends of the first-pass tubes, with very little or no evidence of such action at the inlet ends of the second-pass tubes. In unusual cases, the occurrence may be reversed. Generally, trouble will be concentrated in specific zones of the condenser. Such localization of tube deterioration necessarily directs one's attention to the surrounding conditions and brings under consideration the related

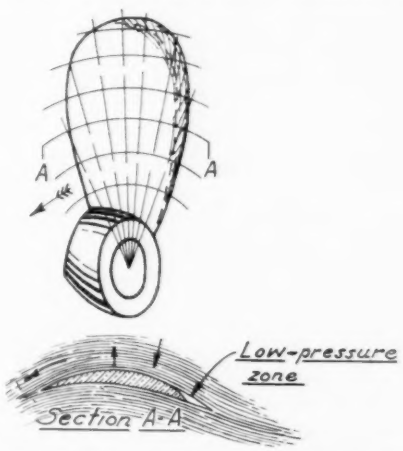


FIG. 1 PROPELLER BLADE, SHOWING WHERE CORROSION OCCURS DUE TO A LOW-PRESSURE ZONE ON THE VACUUM SIDE

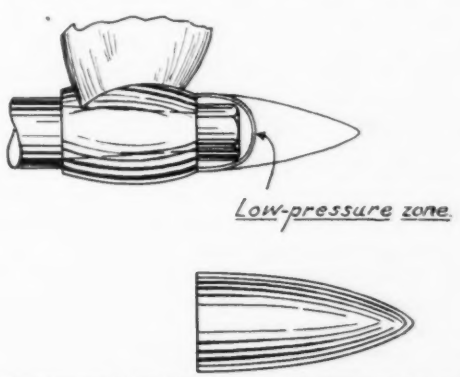


FIG. 2 STREAMLINE NUT USED OVER A PROPELLER-SHAFT NUT TO PREVENT CORROSION

[1] Chief Engineer, Scoville Manufacturing Company. Mem. A.S.M.E. Mr. German was educated in marine-engineering schools in Holland and Germany, and after graduation was employed as an engineer on the Holland-American Lines. In September, 1903, he came to the United States and held a position helping to install boilers and engines on battleships and cruisers for the Newport News Shipbuilding and Drydock Company. From this position he went to Providence, R. I., where he was chief operating engineer of the power plant of the Rhode Island Street and Suburban Railroad Company. His next position was with the Hartford Steam Boiler Inspection and Insurance Company as inspector; later he became assistant to F. B. Allen, vice-president. He joined the Scoville Manufacturing Co. in 1912 as chief engineer.

Contributed by the Power Division and presented at the Fall Meeting of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS, held at Providence, R. I., October 5–7, 1938.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

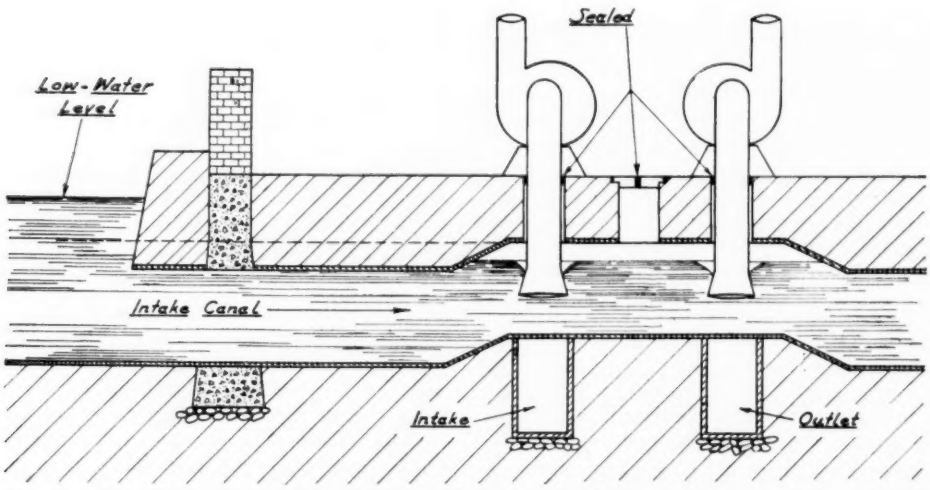


FIG. 3 ONE ARRANGEMENT FOR CONDENSER-WATER SUPPLY WHERE AIR WAS FOUND TO ENTER THE SUCTION BELL EVEN THOUGH THE ROOF OF THE INTAKE CANAL IS LOWER THAN LOW-TIDE LEVEL

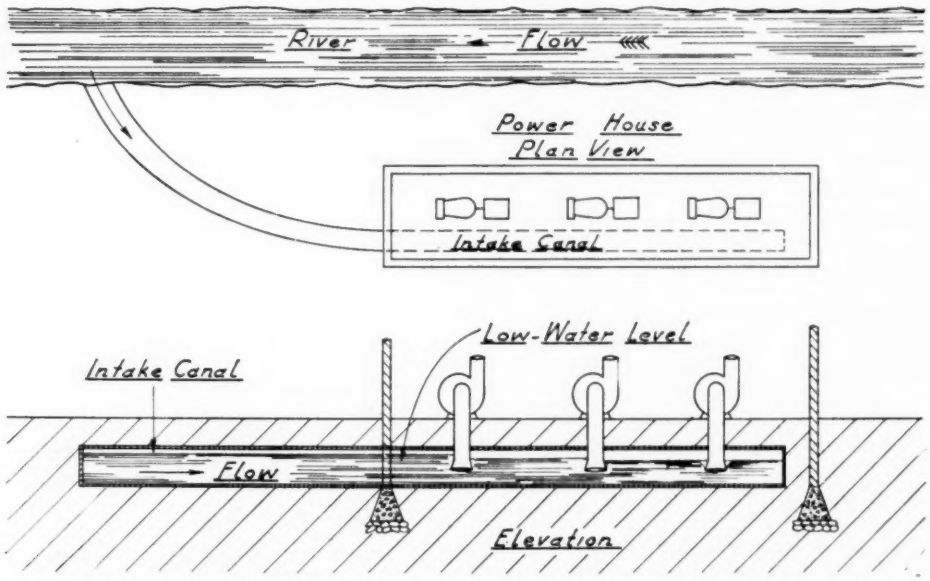


FIG. 4 ARRANGEMENT IN WHICH AIR ENTERS THE CONDENSER CIRCULATING SUPPLY WHEN THE RIVER LEVEL IS LOW AND THE PLANT IS OPERATING AT CAPACITY WITH THE THREE SUPPLY PUMPS ON THE LINE

elements of the installation as the intake screen, suction bell, circulating pump, pump stuffing boxes, valves, piping, inlet and discharge piping, water boxes, ferrules, and the conditions affecting and effected by them. Such effects are in most cases traceable to factors far ahead of the tubes themselves. The correction must be at the source. An inspection to discover the causes of tube corrosion should cover the entire hydraulic cycle, starting at the intake canal.

IMPORTANT MECHANICAL FEATURES OF DESIGN AND OPERATION

Air in the circulating water may be of two types: (*a*) Air entrained mechanically, and (*b*) air released from solution by turbulence at low-pressure areas.

Sources of and Possible Remedies for Entrained Air. The bell at the end of the pump suction pipe may not be submerged sufficiently, particularly at low tide. Such a condition obviously may permit great quantities of free air to enter. Air so introduced cannot be absorbed by the water, which is already saturated, and being carried along with the stream will damage tubes. If the suction bell cannot be lowered, due to distance limitations between the bottom of the pipe and the floor of the intake canal, one or two branches may be run from the suction pipe to act as a secondary suction. This procedure will reduce the flow at the original suction bell and prevent an external cone from forming around it.

Free air has been found to enter at the suction bell even though the roof of the intake canal is lower than low-tide level. Fig. 3 illustrates a case where such a condition was found. A new intake canal crossing over existing intake and discharge canals had been designed. Since all canals would still be below low-tide level, no objection was foreseen. Until there was an especially high tide, which flooded the basement floor because of water entering around the suction pipe of the pump and manhole under the basement floor, no trouble developed. However, in order to prevent any further flooding, the openings at the top of the new intake canal were sealed with the result that air accumulated, as shown in Fig. 3. At quite regular intervals this air accumulation reached a point in the canal where a cone formed around the suc-

tion bell and the air discharged into the circulating-water system. The remedy, of course, was to vent the canal with a pipe, the top of which was a few feet higher than the extreme high-tide level. Tube life was thereby materially improved.

When a power plant is designed with a basement-floor level dangerously close to conventional high tide, the designing of the intake canal calls for extra caution.

Many plants have a number of pumps which obtain their water from one canal. Fig. 4 illustrates a bad condition, as concerns tubes, resulting from such a layout. When the canal was originally built, future additional capacity requirements were considered, but the additional units later installed proved to require a greater capacity than was originally contemplated. The factor of safety on canal size was insufficient. After plant expansion, with low water, capacity load, and all three units on the line, the water level in the canal dropped after each suction pipe, so that air was admitted into the suction pipe farthest away from the main intake end of the canal. The remedy in such a case, when possible, is to extend the canal from the terminal end to the source of supply.

A second source of entrained air in the circulating water is found in circulating-pump stuffing boxes, water-sealed with the seal connected to the discharge chamber of the pump. Since the pressure at this point may be below atmospheric pressure, considerable air may be drawn in through the stuffing box, which is located on the suction side of the pump and always under vacuum. The water for the pump water seal should be supplied from a source with a pressure considerably higher than the pump discharge in quantities sufficient to cause a flow out of the stuffing box. Metal guards can be placed to keep excess water off the room floor.

It is obvious that leaky joints, valves, and pipes, in a system through which water under vacuum is carried, present sources of probable air entrainment.

Air Release From Solution. Under normal conditions sea water contains considerable quantities of dissolved air. Turbulence caused by obstructions to streamline flow may result in the formation of low-pressure areas and the release of this dissolved air from solution. In considering the circulating-water system from strainer to tail pipe, the following paragraphs in this section of the paper are devoted to a discussion of those portions of the equipment which may be important in this respect.

Intake Screen. The purpose of this is to keep out all trash, fish, eels, mussels, and shellfish. The screen is not always effective; failure sometimes resulting in a damaging effect on the tubes. Small shells may become lodged in the tubes and bake on while the condenser is hot or warm while off the line, due to leaking stop valves or entering drains. When the unit is replaced in service, a pit may occur directly adjacent to the shell on the downstream side, even though the circulating water does not carry entrained air. The water velocity passing over the shell, or any other foreign body, will produce a low-pressure area and cavitation directly in back of it. The released air concentrated in this zone does its usual damage. Tube samples with shells still attached, or with their impressions clearly visible, are in the author's possession and show pitting directly back of the shell on the downstream side.

Injection Strainer and Bilge Keel. The preceding paragraph, covering intake screens on stationary power-plant systems, should be supplemented by a brief consideration of injection strainers on board ship. Such injection strainers should be so designed that trash can neither enter nor collect and should be streamlined, as shown in Fig. 5, in order that undue turbulence will not be set up.

The author has observed that the rolling of a ship in heavy water may result in the entrapment of air under the bilge keel. To prevent such entrapment, it has been found advantageous to cut away a section of the bilge keel, as shown in Fig. 6.

Circulating Pumps. Old condensers of both stationary and marine types where tubes have lasted for many years are in existence, while newer installations are troublesome. The better life of old units is certainly not because tubes as now made are inferior; indeed, tube production has greatly improved.

The company with which the author is associated has in operation old pumps driven by reciprocating engines at speeds from 45 rpm to 100 rpm. Modern pumps are now driven at much higher speeds, in some cases by auxiliary turbines, without reduction gear, at speeds approaching 2000 rpm. Such excessive speeds in-

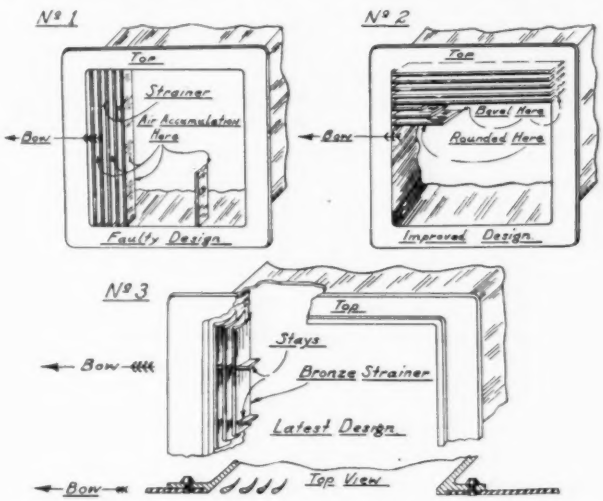


FIG. 5 STREAMLINE INJECTION STRAINERS FOR USE ON SHIPS

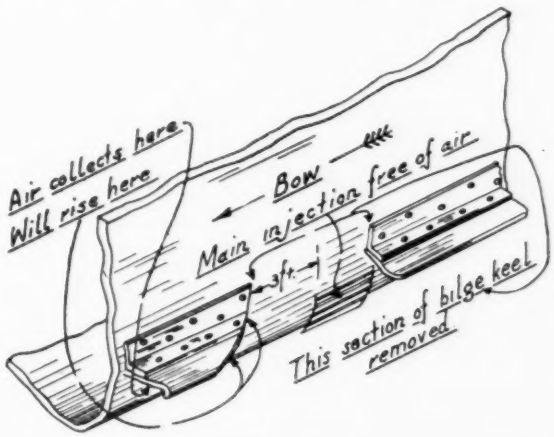


FIG. 6 SECTION OF BILGE KEEL CUT AWAY TO PREVENT AIR ENTRAPMENT

crease cavitation, cause circulating water to enter water boxes and tubes very turbulently, especially when the pumps are located close to the water box, as they frequently are. Some quite recent designs, in fact, have the pump discharge directly secured to the water box and a few have the pump integral with the water box. In such cases little or no liberated air has a chance to return to solution. The effect on tubes in such cases requires no comment.

It would seem advisable to return toward the earlier-type pump, which was larger and slower, but to incorporate improvements in design. While first costs would be greater, the author believes that the additional investment would be good, due to

power-factor correction, increased tube life, and freedom from condensate contamination, which is of increasing importance on modern high-pressure boilers operating at high ratings.

Both wear and misalignment both in impellers and pump casings, as shown in Fig. 7 will increase turbulence and are, therefore, to be avoided. In so far as pump life is concerned, it is desirable to minimize suction lift by placing the pump as near as possible to the water source.

Pump improvements will be reflected by savings on tubes.

Piping Between Pump and Water Box. In many cases, discharge piping is found with very short-sweep fittings, such as elbows and tees, which interfere with streamline flow and set up turbulence. Large divided water-box condensers, with the division plate open or closed, may require two pumps. Both pumps, however, may not be operated at low peak periods, especially with low-temperature circulating water. To provide for such an installation, the piping between pumps and inlet water box may be so designed that the water from either can reach both inlet nozzles. This is done by crossover piping and gate valves. Such design presents many sources of possible air accumulation and tube damage. Since the efficiency of the pumps and the resistance of the two sets of piping may differ, it is not safe to assume that the water supply from either pump is the same as that from the other.

In such condensers with two tail pipes, trouble may result when both tail pipes are in the same discharge canal, one entering at the canal terminal. This latter is apt to find greater resistance to flow and care should be taken that the discharge canal be ample in size, or of such design that the crowding of water at one tail pipe is not possible. If, for example, the water from both tail pipes is caused to flow into a large Y, the center or outlet of which is in line with the center line of the condenser, then the resistance to flow from each tail pipe will be equal.

Discharge piping with large check valves between the pump and the inlet water box, the top or head of which is at a considerably higher elevation than the top of the pipe, should be avoided. Gate valves are also located in such pipe lines. When such valves are in a vertical position with relation to the pipe, their tops may be considerably higher than the pipe top. Such check and gate valves are excellent air chambers, acting similarly to air chambers on pump lines, virtually as shock absorbers. This shock-absorber action is accomplished by having at least half of the chamber filled with air, such air accumulating and rising as circulating water passes through the pipe. Such collections will discharge periodically into the circulating water with the usual damaging effect on the tubes. A pipe connection should be made from the highest point on such valves to the tail pipe of the condenser, which will act as a vacuum pump. A small amount of water should be allowed to by-pass in order to make sure that no such accumulated air will enter the tubes.

Inlet Nozzles. Many inlet nozzles are so designed that flow is interrupted, as shown in Figs. 8*a* and 8*b*. The diameter of the discharge pipe is found to be greater than that of the inlet water box, which should never be the case. The nozzle is flattened, beginning some distance from the water box. Care is

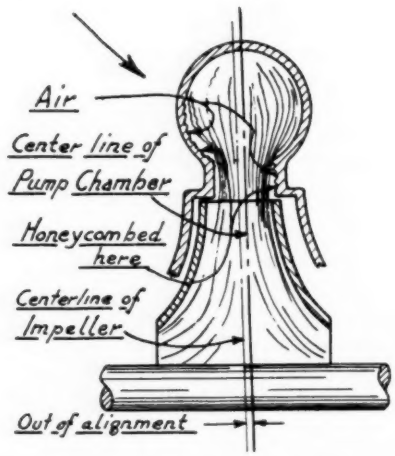


FIG. 7 MISALIGNMENT OF PUMP IMPELLERS WILL INCREASE TURBULENCE AND CAUSE CAVITATION

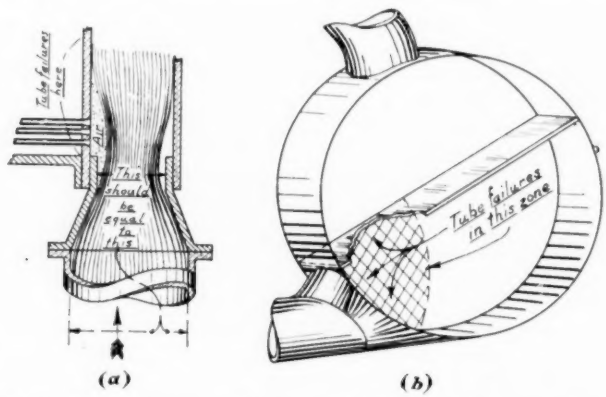


FIG. 8 INLET NOZZLES IN WHICH THE WATER FLOW IS INTERRUPTED

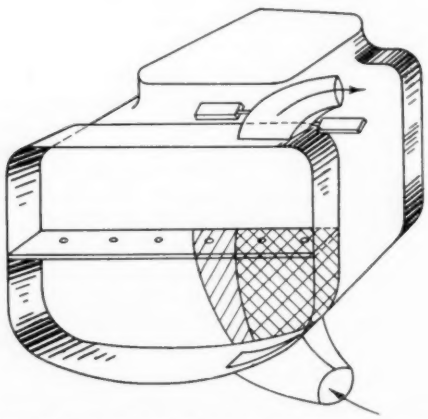


FIG. 9 CASE WHERE TOO GREAT A SPACE EXISTS BETWEEN THE INLET NOZZLE AND THE TUBE-SHEET FACE

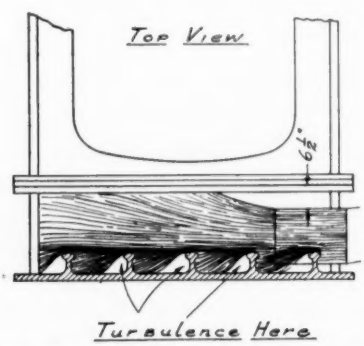


FIG. 10 IMPROPER ARRANGEMENT OF REINFORCING RIBS IN THE WATER-BOX HEAD SHOWN IN FIG. 9

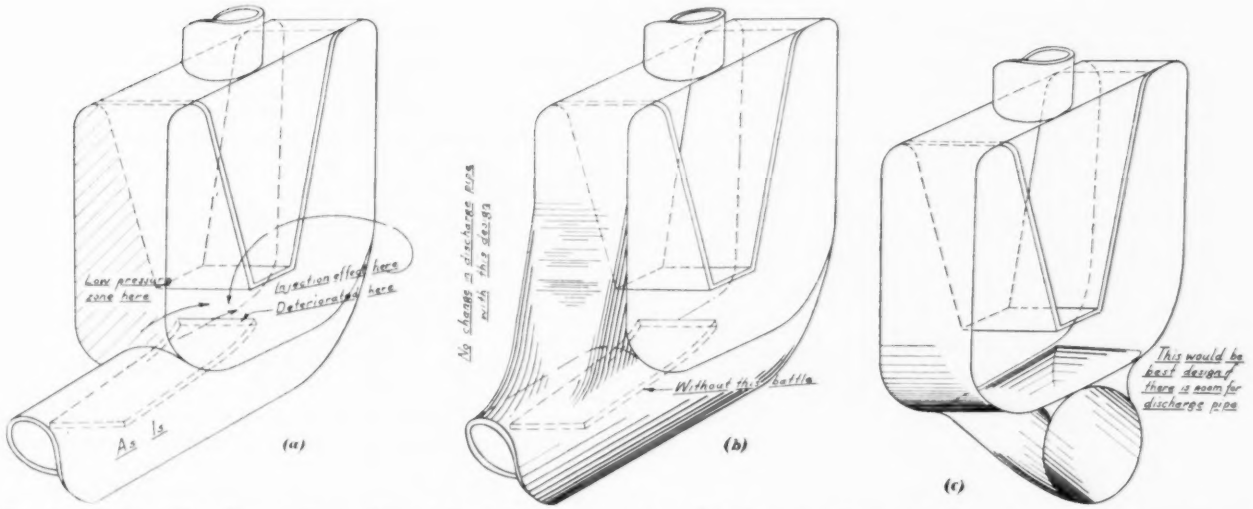


FIG. 11 A WATER-BOX DESIGN WHICH CAUSED TUBE DETERIORATION AND TWO PROPOSED DESIGNS TO ELIMINATE SUCH DETERIORATION

taken, however, to make up for this by lengthening the opening at the water box, so that its area is equal to that of the pipe area. Again referring to Fig. 8, it is seen that as water enters the water box its flow is tapered. This causes a low-pressure zone at the tube sheet in line with and on the side of the nozzle. The greater the entering water velocity, the smaller the amount of water which will pass through the zone described, which, in turn, affects tube life in that zone.

Figs. 9 and 10 illustrate a case where a space of $6^1/_2$ in. was found between the inlet nozzle and the tube-sheet face. Tubes in the low-pressure zone deteriorated rapidly. It was proved that through part of this tube bank (the shaded area), there was even a reverse flow of water, indicating that the inlet-water-box pressure in the indicated area was less than the pressure of the reverse-flow water box. This condition was further checked by temperature measurement at the outlet nozzle, which showed a difference in temperature of 6 C in various layers of outgoing water before mixing had been completed.

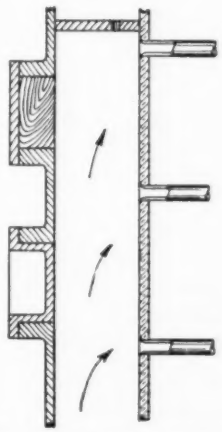


FIG. 12 WOOD-FILLED AND RECESSED INSPECTION DOORS WHICH WILL ELIMINATE WATER-BOX TURBULENCE

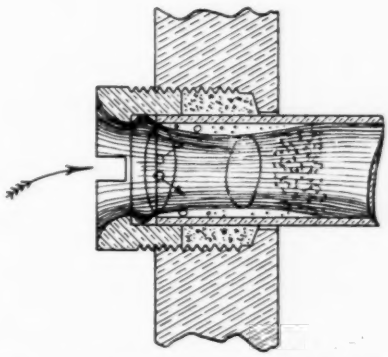


FIG. 13 CONVENTIONAL FERRULE AT THE INLET END OF A CONDENSER TUBE

Water Boxes. The author's discussion of water-box design can best be handled by a treatment of specific cases.

Fig. 10 shows a water box as well as an injection nozzle. Undoubtedly, the design of the water-box head was a contributory factor to general tube deterioration. It will be noted that reinforcing ribs were shaped like rails but were integral with the head, also that their position relative to the flow of water was at right angles. Low-pressure zones were created on the downstream sides of these reinforcements as the water struck and passed over them. The side of the ribs facing the flow and near the inlet was found to be free of corrosion. The downstream sides, however, could be cut with a pocketknife. All were affected likewise. The sides of the remaining ribs facing the flow were affected by corrosion progressing from the inlet onward.

The entire inner surface of the water box should be as smooth as possible and any necessary reinforcings should be external.

Figs. 11*a*, 11*b*, and 11*c* illustrate the effect of another water-box design on tube life. It is believed that the designer's object was to have some low-temperature circulating water pass through the upper tube rows for rapid condensation and also to have a certain percentage of the second-pass water flow through the bottom of the condenser in order to increase the condensate temperature. The condensers are on twin-screw sister ships operating under similar conditions. The condensers on one ship experienced tube trouble. The general design of the troublesome condensers is shown in Fig. 11*a*. It will be noted that the circulating water enters at one side of the water box which has an enlargement at the bottom, coming considerably below the condenser shell; this to permit easier flow to the first-pass section farthest away from the inlet. To divide the water equally between the two halves of the first pass, a horizontal baffle plate was cast integral with the inlet nozzle as a part of the water-box casting. Tube failures occurred at the upper section of that half of the first pass near the inlet circulation. The circulating water seeking the path of least resistance, resisted a change of direction of flow causing the greatest percentage to pass by the first half of the first pass and enter the half farther away from the inlet. This caused a low-pressure zone at the upper half of the first pass near the inlet, with tube failures resulting.

Reverse flow was found in the affected section of the first pass. It should be noted that inlet piping to the water box could not be seen without removing the floor plates. Upon demonstration of this evidence and submission of the recommendations shown in Figs. 11*b* and 11*c*, it was argued that the theory seemed correct but for the fact that the duplicate sister ship did not experience

similar tube failure. It later developed that the sister-ship's water-box inlet was located at the bottom and center, giving uniform distribution of circulating water and no low-pressure zone. In the second vessel tube deterioration was uniform over the entire first pass.

Much has been done to improve steam distribution with gratifying results. Circulating-water distribution is equally as important and should not be overlooked.

Inspection Doors. Practically all inspection doors are so designed that a large recess is found between the door and the main body of the water box. This causes turbulence. Such recesses should be filled with wood or replaced with doors of special design, as shown in Fig. 12.

Ferrules. With high water velocity and tubes installed with ferrules extending into the water-box inlet end, tube corrosion may be found even if the circulating water enters the water box smoothly. The conventional ferrule, with bell entrance and counterbore—to permit the tube to fit snugly against this counterbore shoulder—was designed to bring about smoother flow. Many who install tubes with this type ferrule have the opinion that a

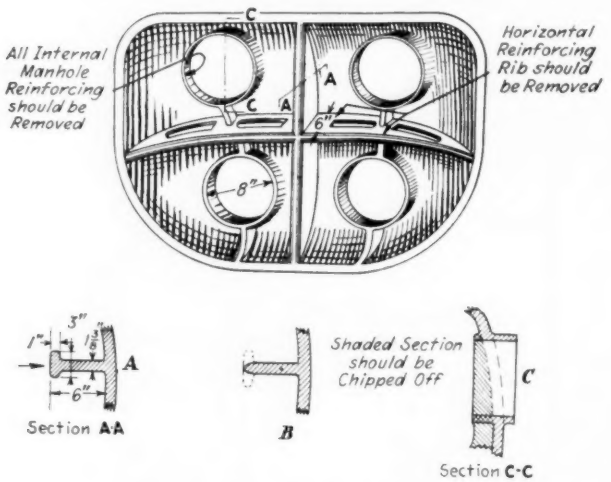


FIG. 14 COMBINATION REVERSE-FLOW WATER BOX AND HEAD

space must be provided between the end of the tube and the shoulder of the counterbore in order to have room for tube expansion.

Fig. 13 shows the conventional tube installation described. Note the clearance between the counterbore shoulder and tube end; this clearance has been found to be as much as $^3/_8$ in. Such practice requires that the ferrules extend farther into the water box (which will be shown to be detrimental), but disregards the fact that the counterbore shoulder and the tube inlet end should be in permanent close contact. All expansion and contraction of tubes installed in this way should be provided for at the outlet end. The use of a fiber packing ring in conjunction with the ferrule at the inlet end and a metallic packing with as little fiber as possible at the outlet end will accomplish this. That there are other methods allowing for expansion, such as shell expansion joints, floating heads, and a predetermined bow in the tubes when rolled at both ends, should be mentioned.

The clearance between the tube end and the ferrule counterbore shoulder forms a pocket which is ideal for causing turbulence in the water just before it enters the tube. The tube-entrance area is considerably smaller than the pocket area, causing the flow of water to assume a curvature, as shown in Fig. 13. A low-pressure zone begins to form at the tube end, which zone will vary in length, depending upon the water velocity or the length of space between the tube end and the counterbore shoulder. Air liberated in such recesses will obviously act destructively on tube ends, although it may return to solution on its way through the tube and cause no further damage. If ferrules of the type under consideration are examined after service, they frequently are found badly corroded just back of the counterbore shoulder.

Long ferrules have been used on the theory that corrosion might be absorbed by the ferrule itself. Extending well into the water box, such ferrules have in some cases been helpful. The author disagrees with the general theory of this practice, because the farther the ferrules extend into the water box, the greater will be the turbulence caused.

If ferrules must be used, there is a type that does not extend into the water box and makes no break in the inside diameter of the ferrule and tube; hence, uninterrupted flow results, reducing turbulence at the tube entrance.

The foregoing consideration of ferrules, as affecting inlet-end corrosion, permits a question as to why inlet-end corrosion also occurs where tubes are rolled into the tube sheet without ferrules. Such corrosion is found, but it should be remembered that water

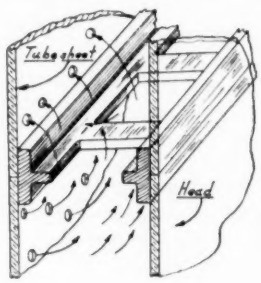


FIG. 15 REVERSE-FLOW WATER BOX WITH REINFORCING PLATE IN THE CENTER

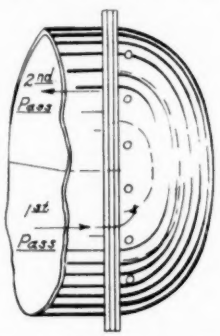


FIG. 16 IDEAL REVERSE-FLOW WATER BOX WITH TEST HOLES DRILLED AS SHOWN

may be turbulent before it reaches the tube, due to factors discussed earlier.

Failures of Second-Pass Tubes. Mention was made that occasionally condensers are found where tube failures occur in the second pass. Such troubles ordinarily result from faulty design of the reverse-flow water box, such as shown in Fig. 14. It will be noted that the water box and head are integral. This in itself would be ideal, if there were curvature instead of the conventional flat head and if there were a much larger cross-sectional area at the center; that is, between the first-pass outlet and second-pass entrance. Such ideal design, however, as in the case under consideration, is spoiled by reinforcing ribs running from left to right of the water box and at its center, since they interfere with free flow. Although four openings are provided in these reinforcings, it is apparent that water will reach the second-pass tubes in a turbulent condition. Further, the edge of the reinforcing is rail-shaped, as shown by cross section *A-A*. In addition, the designer found it necessary to extend cylinders, such as in inspection openings, into the water box, making still more turbulence. This is shown by section *C-C*. Six such condensers have come to the author's attention. The importance of having the reverse-flow water box free from obstructions to water flow is again indicated.

One reverse-flow combined water box and head was found to have been designed with a factor of safety of over 20. A calculation showed that all internal reinforcing could have been omitted with perfect safety.

Fig. 15 shows another common type of reverse-flow water box.

Again the reinforcing is placed between the tube sheet and water-box head and might be termed a division plate with an opening. The entire reinforcing is one casting, shaped like an angle iron, and bolted to the tube sheet and water-box head. Above such reinforcings low-pressure zones occur in which dissolved air accumulates, discharging at intervals into the second-pass tubes.

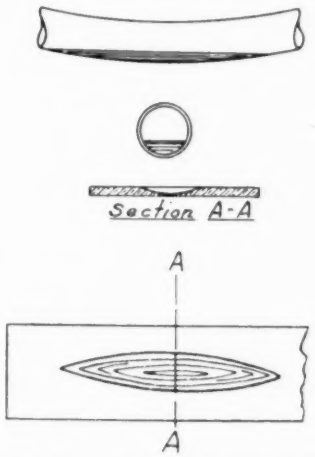


FIG. 17 CORROSIVE ACTION OF CONCENTRATING SALT WATER AND AIR ON A SAGGING CONDENSER TUBE WHEN THE CONDENSER IS OFF THE LINE

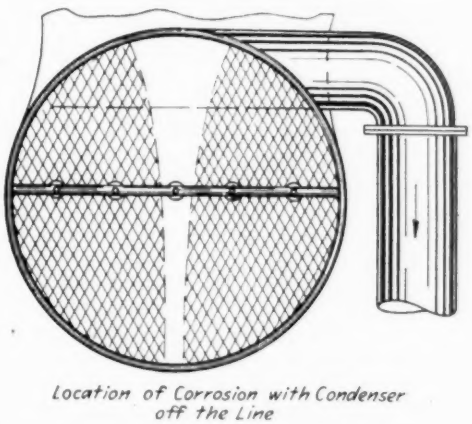


FIG. 18 CONDENSER IN WHICH CORROSIVE ACTION OF THE TYPE SHOWN IN FIG. 17 WAS FOUND

The tops of such cast-iron reinforcings are frequently found to be corroded. In a few cases the section between the tube sheet and head has disappeared, while the bottom of the reinforcing, where solid water contacts, was found sound. Stud bolts, instead of cast-iron reinforcings as described, are preferable. It is consistently noted that where there are such cast-iron reinforcings in the reverse-flow water box, tube trouble is experienced.

Fig. 16 shows what the author believes to be the most nearly perfect design for reverse-flow water boxes. It is to be noted that six openings from the bottom to the top were drilled in the side of the water box for test holes. When a colored liquid is forced into the lowest opening it will be seen to discharge from the highest opening, though not visible in the second and third openings from the top down. This indicates that, in this type of combined water box and head, turbulence is at its minimum.

OFF-THE-LINE CORROSION

Condensers are found where tubes sag between tube sheets and support plates. When such units are drained water will remain in the sagging sections. If turbine throttle and gate valves do not entirely prohibit the passage of steam the condenser will become hot and act in effect as an evaporator.

Fig. 17 illustrates the corrosive action of concentrating salt water and air entering through the turbine labyrinth packing as evaporation proceeds with ultimate penetration centrally, as at section *A-A*. This type of trouble was encountered in a plant having condensers of the type shown in Fig. 18. In this case failures were limited to those tubes above the bottom of the tail pipe. All other tubes remained full of water. To avoid this trouble condensers should be kept cool (not above room temperature) while off the line.

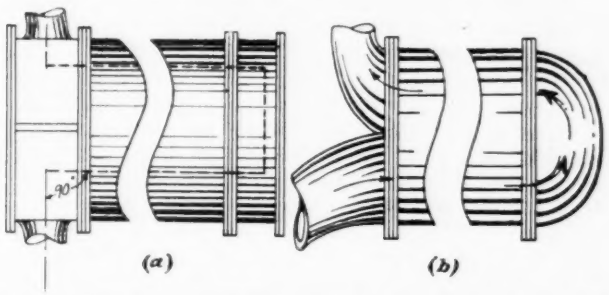


FIG. 19 TWO DESIGNS OF CONDENSER WATER BOXES, ONE (*a*) OF THE CONVENTIONAL TYPE AND ONE (*b*) IDEAL FROM A CIRCULATING-SYSTEM POINT OF VIEW

CONCLUSION

Fig. 19 illustrates two condensers; (*a*) one of the conventional design and (*b*) one ideal from a circulating-system point of view. It will be noted that in the conventional design water must make a 90-deg turn at the inlet water box, with low-pressure zones on each side of the nozzle, and an accumulation of air under the division plate. In the reverse-flow water box two 90-deg angle changes are necessary, one on leaving the first pass and one on entering the second pass. It will be noted in Fig. 19*b* that the entrance to the first-pass tubes is streamlined. There is no division plate. Provision is made for entrance of water without unnecessary turbulence. The reverse flow existing was explained earlier.

The author is of the opinion that with all conditions, except design, the same on both condensers, the tubes in the layout, as proposed in Fig. 19*b*, would last much longer than in the conventional-type two-pass condenser.

Discussion

H.N. BOETCHER.[2] Condenser-tube deterioration is one of those complicated problems which defy solution by one formula, for the simple reason that the number of factors entering into them is too large. Mr. German's paper considers one side which, at least in stationary plants, is associated closely with the tendency toward the use of higher water velocities in condensers, which in turn has resulted in shorter tube life in many cases. Lack of knowledge with regard to the potential decrease of tube life has apparently caused a condition in which a proper balance between first and maintenance costs does not always exist anymore. In some of these cases, a review of the circulating-water system on the basis of Mr. German's suggestions, with corresponding changes, may result in definite savings. In view of the extensive flood-protection work carried on in many localities, special attention should be given also to the case mentioned by

[2] Assistant to Superintendent of Steam Stations, Consolidated Gas, Electric Light, and Power Co., Baltimore, Md. Mem. A.S.M.E.

him in which sealing of the floor outlets of an inlet tunnel resulted in forming air pockets, with detrimental results.

As Mr. German's paper is based on practical experiences rather than experimental or analytical work, it may seem amiss to question his use of certain terms. On the other hand, the fact that a few authors use these terms in accordance with their own definitions has caused a condition which makes one wish for an A.S.M.E. or A.E.C.—or some other alphabetic—Committee on Technical Terms and Definitions with complete authority to fix these terms. Mr. German uses the terms cavitation and turbulence in a way which does not agree with the definitions accepted by hydraulic engineers and other workers in similar fields. Cavitation certainly is encountered in the case of propellers and pump impellers.

Air, however, is a retarding, and not an accelerating factor in this type of destruction. Turbulence is a type of flow which may be associated with, but does not cause release of air. Air dissolved in circulating water is released by general decrease in pressure below atmospheric pressure—which, though not mentioned by Mr. German, is the principal source of air of this type in many condensers—or by local decrease in pressure as caused by obstructions. In this latter case, it may be accompanied by turbulent flow.

The type of attack to which Mr. German refers is the "air-impingement" which appears to be a combination of corrosion of clean metal surfaces and a scouring action by rapid water flow which removes the products of corrosion almost immediately. The corroded surfaces usually have relatively smooth, metallically clean contours, as opposed to the pitted appearance of surfaces attacked by cavitation. The attack depends on the presence of air at high water velocities as caused by local conditions or by general design. It has been found also all along the inner surfaces of tubes when velocities are sufficiently high.

Mr. German's paper justly emphasizes the importance of designing and operating equipment connected with condensers for low air infiltration and for smooth flow. This is a field to which little hydraulic engineering has been applied so far. However, experience shows that the content of free and dissolved air in circulating water is so high even under the best conditions that relatively quick tube deterioration occurs when the water velocity is above the critical point at which turbulent flow begins. As it is impractical to reduce the air content of the water entering the system by artificial means, uniformly moderate velocities in the tubes should be used unless the tubes are made of an alloy with high resistance to air-impingement attack.

Author's Closure

The author agrees with Mr. Boetcher that a definite standardization of engineering terms is a useful project. Where "turbulence" was referred to in the paper the author pointed out the low-pressure zone caused by pressure drop, but referred to differential pressure and not necessarily lower than "atmospheric." With this one qualification it is believed that the author and Mr. Boetcher are in agreement.

Condenser Performance With Reduced Cooling Surface

By J. H. HARLOW,[1] and R. A. BOWMAN,[2] PHILADELPHIA, PA.

The authors discuss changes made in two condensers of 50,000 sq ft heat-absorbing surface, which changes provided steam lanes in the tube nests in an attempt to increase the effectiveness of the condenser surface. The heat-absorbing surface in the first condenser was reduced 20 per cent, while that in the second condenser was reduced 30 per cent. The authors present drawings showing the changes made, and explain the reasons for making them. Performance data on both condensers, before and after the changes were made, are presented in both graphical and tabular form.

AT THE Philadelphia Electric Company's Delaware Station there are six Westinghouse, two-pass surface condensers each of which originally contained 50,000 sq ft of heat-absorbing surface comprising approximately 9100 No. 18 gage tubes of 1 in. outside diameter. These condensers each serve one 30,000-kw unit and went into service during the period from 1920 to 1924. During the early portion of their life, they were operated almost continuously, but with the subsequent installation of the Richmond Steam Station and Conowingo Hydroelectric Station, they have been mainly in peak service for the last ten years. Each condenser is provided with two Le Blanc air pumps, one of which is a spare, and two circulating pumps each rated at 37,500 gpm. The normal practice is to operate with one circulating pump in service, the other being in reserve. Cooling water is obtained from the Delaware River adjacent to which the station has been built. In 1931, provision was made for chlorinating the circulating water, and, with the exception of those periods when the river-water bacteriological demand has exceeded the capacity of the chlorination equipment, all of the water used for cooling in the condensers of this plant has been treated by this equipment for the prevention of slime and scale. In general, it may be said, that these condensers have been maintained in a satisfactory condition by following this process.

About two years ago, when from two to four hundred tubes which had developed leaks had been removed or plugged in each condenser and when there was evidence to indicate that the rate of failure could be expected to increase, it was decided to begin a gradual schedule of complete retubing whereby one or two condensers would be retubed each year. It is worthy of note that each of these condensers had had a service life of over 55,000 operating hours up until the time of retubing and that all of the tubes were somewhat roughened on the inside caused by pitting.

On discussing the retubing problem with the manufacturer, it was suggested that, during the retubing process, some of the tubes be omitted and that, in this manner, provision be made for steam lanes in the tube nest. As a result of progress in condenser design since these condensers were installed some 14 to 18 years ago, it had been learned that the effectiveness of condenser surface could be considerably improved by providing free access for the steam to the cooling surface. One of several ways of doing this was by providing lanes through the tubes, and the construction of these particular condensers was such that laning could be applied merely by leaving out the proper tubes. The manufacturer made a study of the tube arrangement and suggested a pattern to be followed while replacing the tubes in the condenser.

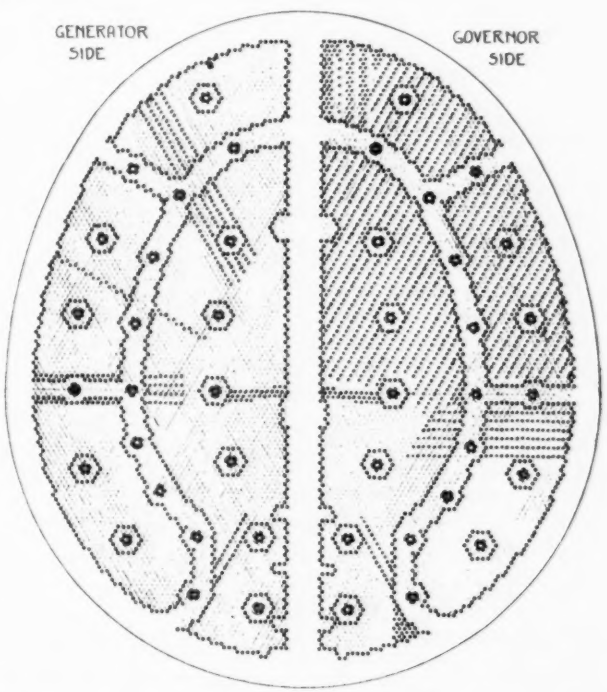


FIG. 1 TUBE ARRANGEMENT OF CONDENSER NO. 5, WITH 20 PER CENT OF TUBES OMITTED

It was concluded, therefore, to try out this scheme on No. 5 condenser which happened to be the first one scheduled for retubing. The tubes in this condenser were replaced during September, 1936, according to the pattern shown in Fig. 1. As a result, the surface in this condenser was reduced by 20 per cent or from 50,000 sq ft to 40,000 sq ft. It will be noted that the lanes were chiefly on the side of the condenser toward the turbine throttle. This was because it was felt that the velocity of the

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[1] Assistant Superintendent, Station Economy Section, Philadelphia Electric Company. Mem. A.S.M.E. Mr. Harlow was graduated in 1923 from the University of Maryland with the degree of B.S. in electrical engineering and has since been associated with his present concern in plant-test and station-economy work.

[2] Design Engineer, Condenser Engineering Division, Westinghouse Electric & Manufacturing Co. Jun. A.S.M.E. Mr. Bowman received the degree of B.S. in mechanical engineering from the University of Arkansas in 1929, and has since been associated with the Westinghouse Company in research, development, and design capacities. He is the author of several technical papers.

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NOTE: Statements and opinions advanced by authors are to be understood as individual expressions of their authors, and not those of the Society.

steam exhausting from the last-stage wheel of the turbine would tend to carry it across the exhaust connection so that the greatest burden would be on that portion of the condenser away from the turbine throttle. Therefore, more surface was allowed to remain where the loading was expected to be greatest. Further, if the laning did help the steam to enter the tube nest, it was felt that probably laning on the throttle side would tend to increase the amount of steam condensed on this side and thus evenly divide

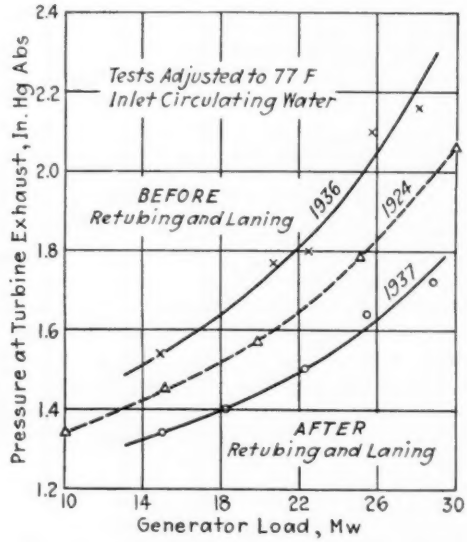


FIG. 2 PERFORMANCE DATA FROM CONDENSER NO. 5

the burden on all of the surface. It should also be realized that this was the first condenser to be retubed and a 20 per cent reduction in surface seemed to be a no inconsiderable change for a first attempt. In addition to opening up the tube bundle in order to provide better access for the steam, a small baffle was inserted to shield the air offtake connection at the center of the condenser. An inspection of Fig. 1 will reveal the location of this baffle where a small number of tubes were omitted in a symmetrical pattern on both sides of the vertical center line and slightly more than half way down from the top.

Two types of performance data, before and after retubing, were obtained to show the change in performance resulting from the alterations. The first is shown in Fig. 2 which presents the results of tests which were run on No. 5 condenser just before and just after the new tubes with laning were installed. This figure shows the relation between generator load and turbine exhaust pressure. One curve shows the condenser performance before retubing and the other after retubing. All of the tests were run under warm-circulating-water conditions and all were adjusted for comparison to the same inlet-water temperature, namely, 77 F. This temperature was selected due to the fact that it was about the average of the inlet temperatures experienced while the tests were in progress. In no case was the departure more than a few degrees, so that the required adjustment was small. The improvement in vacuum, as shown by these tests, was very substantial varying from about 0.15 in. Hg at the lower loads to over 0.40 in. at the higher loads.

Such a substantial gain as that indicated by the tests which were made just before and just after retubing caused some speculation as to whether or not all of this gain could be attributed to the laning. In order to answer this question, the record was searched and some tests were discovered which had been run in 1924 within a few months after the condensers were originally installed. The dashed curve in Fig. 2 shows the results of these early tests adjusted to 77 F inlet circulating water. This curve leads to two interesting conclusions: First, it appears that about one half of the total gain from retubing and laning resulted from the laning, the other half being the result of substituting new tubes for the old ones; and, second, it appears that the effect of the roughness on the water side of the old tubes was to reduce the vacuum as much as 0.20 in. Hg at the heavier loads and higher circulating-water temperatures when compared with new smooth surface, even though the rough surface appeared to be clean and had been subject to chlorination for several years.

A second type of comparison between the before-and-after performance is provided by the daily check of condenser operation. Fig. 3 shows the results of such a check which was made on No. 5 condenser and shows the experience for the year previous to retubing and laning and for the subsequent year. A very substantial improvement as a result of the changes can be noted as shown by the figure. Attention is called to the fact that values shown above the zero line show performance which is poorer than standard and that values below the zero line show performance which is better than standard.

These results, obviously, were very gratifying, for, in addition to reducing the cost of the retubing job, an improvement in vacuum was obtained.

The second condenser to be retubed was No. 3. The results on No. 5 had proved so satisfactory that the manufacturer suggested that the laning idea be carried still further and that the surface in this condenser be reduced by 30 per cent instead of 20 per cent. This would mean reinstalling only 35,000 sq ft of cooling surface instead of the 40,000 sq ft as replaced in No. 5 condenser. In obtaining this increased reduction in surface, it

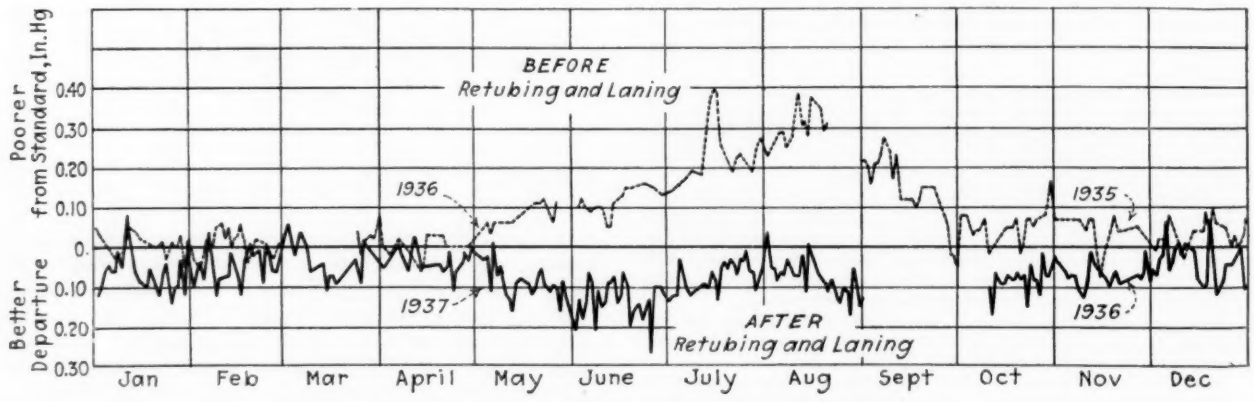


FIG. 3 DAILY CHECK OF PERFORMANCE OF CONDENSER NO. 5

was proposed to provide laning on both the throttle and the generator sides of the condenser in a balanced pattern as shown by Fig. 4. In this pattern, it will be noted that, in addition to the more balanced arrangement, a somewhat different scheme of laning was suggested from that employed for No. 5 condenser. In those zones, where tubes were not replaced in No. 5, they were omitted in every other row, thus making certain portions of the condenser more open for the passage of steam. In the case of No. 3 condenser, however, the proposed lanes through the tube bundle were more definite and wider and several open paths were provided through which the steam could penetrate deeply into the tube bundle. A baffle similar to that installed in No. 5 condenser was also to be provided. During April and May, 1937, No. 3 condenser was retubed as suggested.

Tests similar to those run on No. 5 were also run on No. 3 condenser. The tests immediately before and after on No. 3, however, were made during a period of low circulating-water inlet temperature. An analysis of the tests later, when the heat-transfer rates were being computed, developed that this was somewhat unfortunate, due to the fact that the increase in volume of the noncondensable gases, occasioned by the lower inlet temperature and consequent higher vacuum apparently resulted in somewhat overloading the air-removal equipment. The gain obtained is shown in Fig. 5, wherein all of the test points have been adjusted to 43 F circulating water, and is in the order of

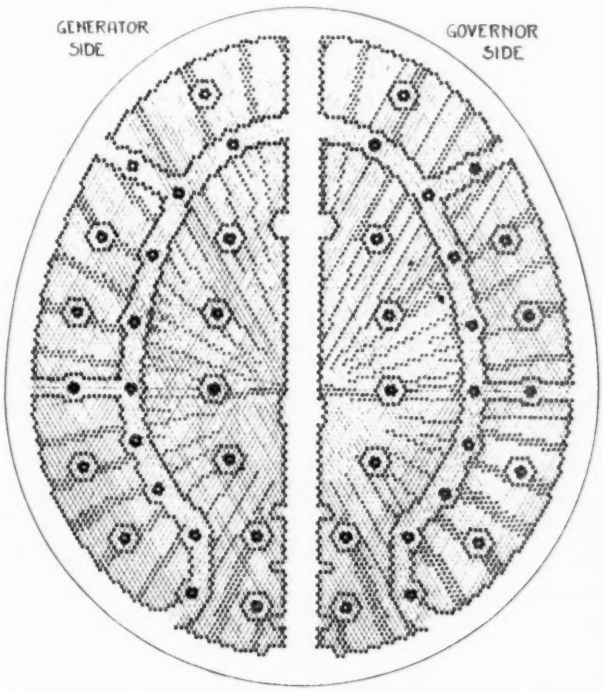


FIG. 4 TUBE ARRANGEMENT OF CONDENSER NO. 3, WITH 30 PER CENT OF TUBES OMITTED

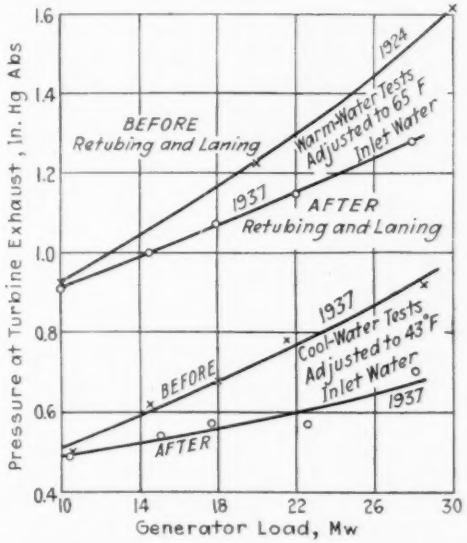


FIG. 5 PERFORMANCE DATA FROM CONDENSER NO. 3

0.25 in. Hg at the higher loads and trails off to a small value at low load. In order to obtain a comparison of before-and-after performance under warm-water conditions, it was necessary to refer to the results of a group of tests which had been made on this machine about two years after the original condenser was put into service. The results obtained from these early tests, together with the results obtained from tests run with warm water subsequent to retubing, all of which were adjusted to 65 F circulating-water temperature, are also shown in Fig. 5. The gain as indicated by this comparison is about the same as indicated by the comparison which was obtained on No. 5 condenser between the early tests and the test after retubing. Fig. 6, which is a plot of the daily check of the performance of No. 3 condenser for the year before and the year after retubing, indicates that the gain, throughout the year, was of similar mag-

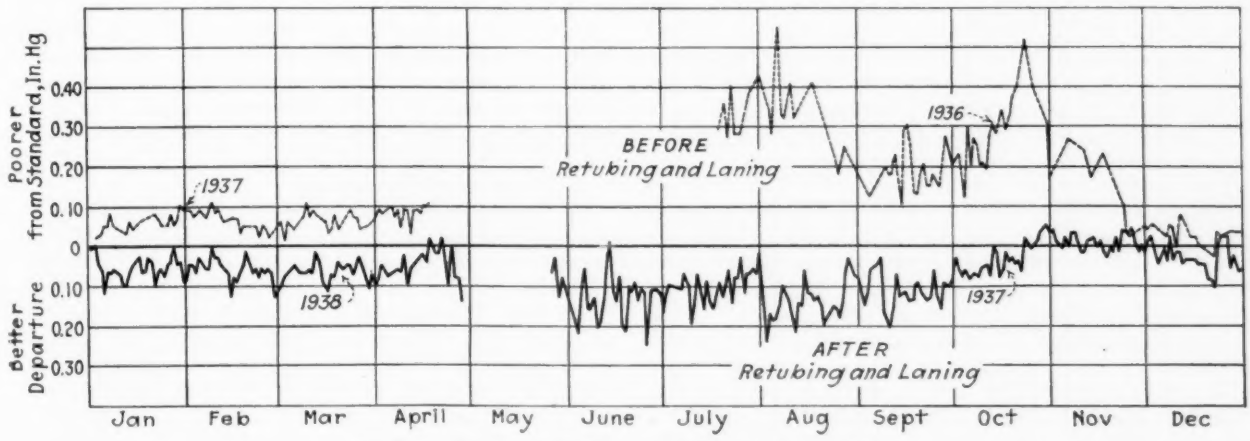


FIG. 6 DAILY CHECK OF PERFORMANCE OF CONDENSER NO. 3

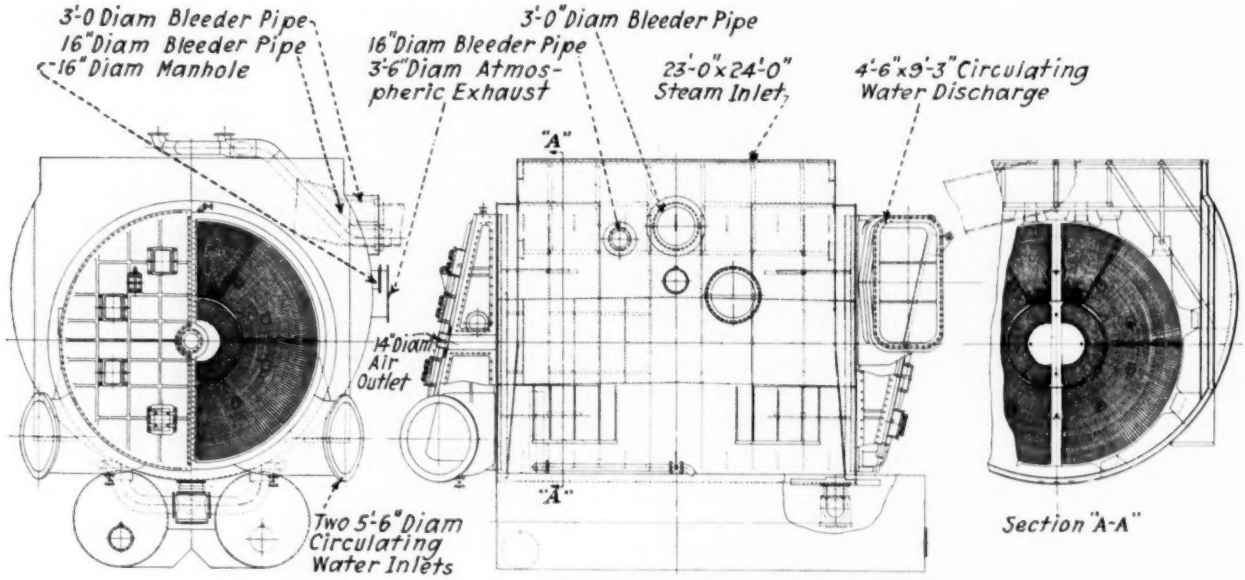


FIG. 7 CONDENSER DESIGNED SO THAT ALL THE TUBE SURFACE WILL BE SUPPLIED WITH STEAM CLOSE TO EXHAUST PRESSURE

nitude to that obtained on No. 5 condenser, as shown in Fig. 3.

Since retubing No. 3 condenser two more of the Delaware Station condensers have been retubed following the scheme used for No. 3, and with similar results.

The improved performance obtained on these condensers after the removal of tubes results from the application of modern design principles to old apparatus, and it would seem to be of interest to review briefly the developments in condenser design leading up to the present ideas on the subject.

The early condensers were no more than tubes placed in a shell and cooled by water so that steam would condense on them. The vacuum was usually limited by the air-removal pump, and the prime mover, a reciprocating engine, could not profitably utilize a high vacuum anyway so little thought was given to condenser efficiency. However, with the development of the turbine with its ability to use high vacua to good effect, and the improvement of air-removal technique, the necessity of improving condenser performance was evident.

The most obvious point of attack was the lowering of the pressure drop of the steam so that the steam would reach all the surface at the highest possible pressure and temperature. One solution was the radial-flow type of condenser of which those at Delaware Station are examples. In this type, steam is admitted all around the periphery of the bundle to reduce the velocity, and the length of flow path is limited to the radius of the circular tube bank. However, at the time the units for the Delaware Station were built, the tubes were still crowded into the shell.

In the design of the condenser under discussion, the steam may be considered to flow from the periphery to the air-removal chamber along a great many paths in parallel. The pressure at the air offtake is the same for all paths while the pressure on the periphery varies in an unpredictable manner due to the irregularities of flow from the turbine. The pressure difference available for moving the steam through the tube bank for any individual path is then the difference between the pressure which happens to exist at the start of the path and the air-removal-chamber pressure.

The pressure gradient, or rate of change of pressure with distance at any point along a path, is roughly proportional to the square of the steam velocity and to some function of the tube spacing. The amount of steam flowing at any point along a path is equal to that entering the path minus that condensed between the entrance to the path and the point. It can be shown that just enough steam will enter a path that the summation of the gradient multiplied by unit length will equal the pressure differential existing for that path. It is possible that along some paths this amount of steam will be less than can be condensed, and in this case there will be surface toward the inside of the bank which is not supplied with steam. Conversely, there will be other paths which will allow steam to pass into the air chamber and the vacuum existing in the chamber is largely determined by this steam and the characteristics of the air pump.

Since the time the condensers in question were built, the technique of arc welding has been developed; hence, today practically all condenser shells are fabricated, whereas, formerly they were cast. The resulting reduction in shell cost has encouraged the manufacturer to be more liberal in using large shells and it has been found that the extra space has paid well for itself. The tubes can now be placed on large centers with a still further reduction in pressure drop. What is more important, they can be given a very large pitch at the outside of the bank and a smaller one at the center in conjunction with a change in the air baffle to localize the air removal to the center of the bundle.

By so doing a substantial over-all flow resistance is maintained, but a large percentage of it is concentrated near the center of the bank. This means that there must be flow practically to the center of the bank to satisfy the available pressure differential and, in spite of flow inequalities and nonuniform condensing capacity, very nearly all of the surface will be supplied with steam at close to exhaust pressure.

Fig. 7 shows an example of a condenser built along these lines. It will be noticed that the main portion of the tubes are set on radial lines so that there is a gradual decrease in pitch toward the center. On the periphery, there is very little resistance to steam flow. The center group of tubes is set on a rather close triangular pitch so as to give a comparatively high resistance which assures an ample supply of steam through all the main tube bank; since lack of flow in the closely pitched center section along any path would practically eliminate the pressure drop over it and with paths in parallel this is an impossible condition.

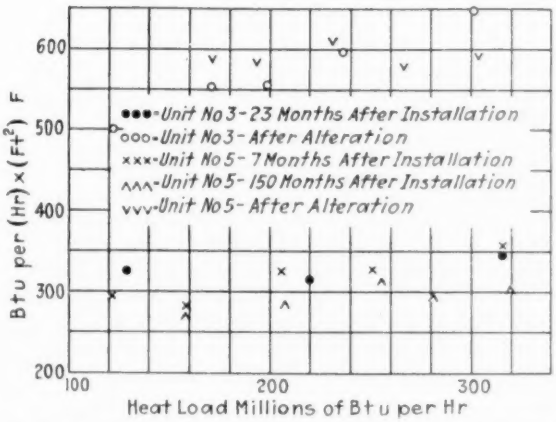


FIG. 8 HEAT-TRANSFER RATES FOR CONDENSERS NOS. 3 AND 5

The essential difference between the condenser shown in Fig. 7, and the original units at the Delaware Station is the fact that the former has a generous tube spacing near the periphery and a tight spacing near the center, while the latter is tight over the entire area. It is obviously impractical to change the tube pitch of the old unit to bring it into conformity with the modern one, but nearly the same result may be obtained by leaving out alternate rows of tubes as was done on No. 5 condenser or by introducing narrow lanes as on No. 3 condenser. Actually, the pitch on the subject condensers was not excessively close ($1^1/_2$ in.) and it seems reasonable to say that the improvement is due to introducing a nonuniform pitch more than to the increased average pitch.

In order to get a comparison of the heat-transfer rate after the changes were made, with that obtained by the same units in their original condition, the results of the tests shown in Figs. 2 and 5, together with data from some additional tests which were made shortly after the condensers were installed, have been calculated and listed in Tables Nos. 1 and 2. These rates are also given in the form of curves in Fig. 8. No data are given for tests made at water temperatures below 50 F, because the heat-transfer rates given in these cases are not consistent in that they decrease very rapidly as the load and water temperature are reduced. This indicates that the limitation on the vacuum under cool-water conditions is not heat-transfer rate, but the capacity of the air-removal pump to handle the existing leakage.

In comparing the rates after the change with the original ones,

TABLE 1 COMPARATIVE TEST DATA ON CONDENSER NO. 3

Unit No. 3 original:					
Date of test	11/19/24	10/4/24	9/23/24		
Months of operation	24	23	22		
Load, kw	30000	20000	10000		
Thousands of lb per hr	333.2	231.6	136.6		
Btu per hr, millions	316.5	220	129.7		
Vacuum—referred, in. Hg	28.926	29.005	28.980		
Saturation temp, F	81.15	78.85	79.60		
Condenser-neck temp, F	80.72	79.88	79.58		
Hot-well temp, F	78.00	74.99	79.25		
Water temp in, F	49.89	57.98	69.54		
Water temp out, F	64.48	67.55	73.52		
Log mean temp diff, F	23.15	15.5	7.96		
Heat-transfer rate Btu per hr per sq ft per deg F	274	284	325		
Heat-transfer rate corrected to 70 F Btu per hr per sq ft per deg F	344	315	327		
Circulating water, gpm	42000	46000	.....		
Unit No. 3 altered:					
Date of test	5/28/37	5/28/37	6/3/37	6/3/37	5/28/37
Months since change	1	1	1	1	1
Load, kw	28000	22000	18000	14500	10000
Thousands of lb per hr	317	249	210	181	129
Btu per hr, millions	301	237	199	172	122.5
Vacuum—referred, in. Hg	28.56	28.71	28.6	28.65	28.97
Saturation temp, F	90.35	86.88	89.46	88.3	79.9
Condenser - neck temp, F	90.5	86.2	89.7	87.3	78.8
Hot-well temp, F	90.3	86.1	89.1	87.8	79.6
Water temp in, F	68.5	68.8	73.6	74.5	69.0
Water temp out, F	82.8	80.1	83.6	83.2	75.7
Log mean temp diff, F	13.4	11.5	10.0	8.7	7.02
Heat-transfer rate, Btu per hr per sq ft per deg F	642	590	568	565	496
Heat-transfer rate corrected to 70F, Btu per hr per sq ft per deg F	649	596	557	552	500
Circulating water, gpm	45000	41200	39800	39600	36600
Water velocity fps	7.06	6.49	6.25	6.23	5.75

TABLE 2 COMPARATIVE TEST DATA ON CONDENSER NO. 5

Unit No. 5 original:					
Date of test	5/20/24	5/21/24	5/22/24	5/23/24	5/24/24
Months of operation	7	7	7	7	7
Load, kw	30000	25000	20000	15000	10000
Thousands of lb per hr	332.3	265.3	217.8	164.6	128.4
Btu per hr, millions	316	252	206.5	158.2	122
Vacuum—referred, in. Hg	28.65	28.82	28.96	29.01	29.11
Saturation temp, F	88.3	84.04	80.18	78.7	75.43
Condenser-neck temp, F	88.2	83.6	80.3	79.55	76.5
Hot-well temp, F	89.7	85.00	81.9	81.7	79.5
Water temp in, F	61.3	61.3	61.6	63.2	63.5
Water temp out, F	75.2	72.6	70.5	70.0	68.6
Log mean temp diff, F	19.2	16.5	13.65	11.8	8.85
Heat - transfer rate, Btu per hr per sq ft per deg F	330	306	303	266	276
Heat-transfer rate corrected to 70 F, Btu per hr per sq ft per deg F	354	328	324	280	291
Circulating water, gpm	45500	44600	46400	46500	47800
Water velocity, fps	5.0	4.9	5.1	5.11	5.26
Date of test	7/1/36	7/30/36	6/29/36	8/3/36	7/2/36
Months of operation	153	153	152	154	153
Load, kw	28150	25750	22500	20700	14950
Thousands of lb per hr	316	296	248	219	167
Btu per hr, millions	300	281	236	208	158.5
Vacuum—referred, in. Hg	27.97	27.73	28.35	28.12	28.56
Saturation temp, F	101.61	105.36	94.72	99.02	90.35
Condenser-neck temp, F	99.8	101.6	92.8	96.7	87.6
Hot-well temp, F	102.8	106.6	96.0	100	90.7
Water temp in, F	74.2	80.1	73.7	79.5	74.5
Water temp out, F	87.9	92.9	84.6	89.4	82.4
Log mean temp diff, F	19.15	18.15	14.4	14.0	11.42
Heat - transfer rate, Btu per hr per sq ft per deg F	313	310	317	297	278
Heat-transfer rate corrected to 70 F, Btu per hr per sq ft per deg F	306	297	312	285	272
Circulating water, gpm	43800	44000	43400	42000	40200
Water velocity, fps	4.82	4.84	4.78	4.62	4.42
Unit No. 5 altered:					
Date of test	8/24/37	8/25/37	8/26/37	8/27/37	8/28/37
Months since change	11	11	11	11	11
Load, kw	28900	25500	22300	18250	15000
Thousands of lb per hr	319	280	243	204	172
Btu per hr, millions	303	266	231	194	173
Vacuum—referred, in. Hg	28.09	28.27	28.49	28.66	28.63
Saturation temp, F	99.57	96.25	91.88	88.08	88.77
Condenser-neck temp, F	97.2	93.5	90.2	84.4	86.1
Hot-well temp, F	100	96.7	92.6	88.4	89.10
Water temp in, F	79.9	78.8	77.3	75.5	77.8
Water temp out, F	92.5	89.7	86.5	83.3	84.5
Log mean temp diff, F	12.3	11.1	9.2	8.06	7.1
Heat - transfer rate, Btu per hr per sq ft per deg F	616	600	627	600	610
Heat-transfer rate corrected to 70 F, Btu per hr per sq ft per deg F	590	576	606	584	588
Circulating water, gpm	48000	48800	50000	49700	51600
Water velocity, fps	6.6	6.72	6.90	6.85	7.11

it should be remembered that the improvement is not due wholly to increased utilization of the tube surface although that does account for most of it. Fortunately, the pumps supplying cooling water to these units have steep characteristics so that about the same amount of water is circulated now as was circulated before. As a result, the water velocity was increased in inverse ratio to the reduction of number of tubes and some of the rate increase must be credited to this increased water velocity. It might be mentioned here that if the increased resistance had caused a substantial reduction in the amount of water flowing, the increased rise in water temperature would have counteracted to some extent the effect of the higher rate on the vacuum.

It might be argued, and reasonably enough, that the gain obtained was not so much the result of laning as the result of the fact that new clean surface was substituted for the old worn out tubes. Undoubtedly a considerable gain resulted because of the new tubes, but there are several reasons as follows, for believing that a large portion of the improvement was the result of the laning:

1 The condenser surface on the water side, before retubing, although rough, appeared to be in good condition being free of slime as a result of several years of the chlorine treatment.

2 Several of these condensers were pickled with ferrocyanide and okite on the steam side a year or so before retubing, but after several years of chlorination, without showing any improvement in vacuum.

3 No. 5 condenser has been in operation for about two years since it was retubed and is still showing about the same improvement over the original installation as it did shortly after retubing. The tubes in No. 5 condenser are still smooth, however, and several years of operation will be required before a measure of the effect of roughness with the reduced surface can be obtained.

4 The heat-transfer rates on No. 5 condenser, as calculated from tests made shortly after the original installation, although somewhat higher, check closely with heat-transfer rates computed from the tests made just prior to retubing, as will be seen from Table 2.

5 As indicated in Figs. 2 and 5, a substantial improvement was shown between the early warm-water tests and the warm-water tests made shortly after retubing, in which comparison it should be expected that the tubes were in similar condition for both sets of tests.

Further Experiments in the Planing of Hardwoods

By E. M. DAVIS,[1] MADISON, WIS.

This paper is supplementary to a previous one,[2] and gives results of further experiments in the planing of hardwoods.

Results are given in tabular and graph form showing the percentage of defective pieces obtained in planing samples of twenty-two woods with six different cutting angles; seven woods with unjointed knives and with four different amounts of jointing; and sixteen woods with four different depths of cut.

ONE COULD probably devote a lifetime of study to the planing of wood without exhausting all the possibilities of the subject. About a year ago the writer presented a paper[2] describing tests recently made at the Forest Products Laboratory at three moisture contents (6, 12, and 20 per cent), two different feed and speed combinations (36 fpm at 3600 rpm and 54 fpm at 5400 rpm), and four different cutting angles (15, 20, 25, and 30 deg). Since then further experiments have been made, and this paper may be considered a continuation of the previous one. It introduces two additional cutting angles (5 and 10 deg), and presents some data on the effect of depth of cut and amount of jointing.

The machine used was a 30-in. cabinet planer of strictly modern type. This point needs to be emphasized because there are many types of planers and related machines, such as molders, planer-matchers, jointers, and flooring machines. Some of these are more flexible than the cabinet planer, some are more highly specialized for certain types of work, and each of them has its own characteristics and limitations. These machines are so similar in the type of cutter head used that it is assumed that the results obtained with one parallel the results obtained with another. However, there is no definite proof that this is so.

Judging the Quality of Planing

The "planability" of woods may be judged in several ways, all valid but all different. Recent European researches, for instance, lay great emphasis on power consumption as an index to ease of cutting. Frequently large production plants are interested in the rate at which dulling of knives occurs with different woods. When machining studies were begun at the Forest Products Laboratory about three years ago, preliminary contacts with woodworkers indicated that the quality of surface is nearly always the most important consideration in this country. For this reason and because it was necessary to limit the scope of the work, quality of surface has been used in these tests as a measure of planability.

Needless to say a mechanical method of measuring the smoothness or quality of a planed surface is desirable to eliminate the factor of personal judgment if for no other reason. Numerous devices have been made both here and abroad; some of them, according to report, can measure irregularities of a millionth of an inch in metal. Unfortunately, as far as the writer knows, no wholly practical method of measuring minute irregularities in wood has yet been developed.

One factor in the quality of a planed surface obviously is the number of knife cuts per inch. Certain German investigators consider from 16 to 25 cuts per inch necessary to a good finish, which is somewhat higher than recommendations of our own manufacturers of woodworking machinery. In this study knives were jointed lightly but often, while feed and speed were adjusted to give about thirty marks per inch. Knife marks were so uniformly good that for all practical purposes they could be ignored as a variable and this was done. Knife marks are important but they are relatively easy to control. They are always present although they may be quite obscure at times. On the other hand, planing defects may or may not be present. When present they tend to overshadow the knife marks in importance unless the latter are bad. In this paper the quality of planing is judged wholly by the planing defects. It is recognized, of course, that they are not the whole problem.

Quality of surface was judged by visual inspection. Each test piece was examined after planing for any chip marks, raised grain, fuzzy grain, or chipped grain that might have developed during the process. Any piece might contain none or possibly all of these defects, which were graded according to whether they were present in a slight or pronounced degree. As a check on the accuracy of this method it was found that a definite relationship existed between these grades and the amount of sanding necessary to eliminate the defects; the poorer the grade the more sanding. It was also found that the frequency of occurrence of a given defect in different woods gives an indication of its degree; for instance, if wood *A* has twice as many occurrences of chipped grain as wood *B*, the degree of chipped grain in *A* will average more serious than in *B*. For the purpose of this paper it was decided to combine all grades and base the tables on percentage of defective pieces. This largely eliminates the personal judgment of the grader as to just what grade should be assigned a given piece, and it eliminates a large amount of detail that only leads to the same conclusions.

A close inspection plus the fact that even a single chip mark, for instance, was enough to class a piece as defective explains in part why some of the defect figures in the tables that follow may seem high. Also, when planing with so many different cutting angles, for instance, some of the angles are not at all favorable, and this shows up in the results.

Selection of Test Material

In tests dealing with the machining of wood it often seems that the job is more than half done when the test material is properly prepared. Preparation includes selection, sizing, and condition-

[1] Senior Wood Technologist, Forest Products Laboratory, Forest Service, U. S. Department of Agriculture, Madison, Wis. The laboratory is maintained in cooperation with the University of Wisconsin.

[2] "Experiments in the Planing of Hardwoods," by E. M. Davis, Trans. A.S.M.E., vol. 60, January, 1938, paper WDI-60-1, pp. 45–49.

Presented at a joint meeting of the Wood Industries Division of The American Society of Mechanical Engineers, and the Cost and Production Division of the Southern Furniture Manufacturers' Association at High Point, N. C., September 21–23, 1938.

Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y., and will be accepted until March 10, 1939, for publication at a later date. Discussion received after the closing date will be returned.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

ing to uniform moisture content. The size chosen was 1 in. × 4 in. × 3 ft, a convenient size to handle, and, since it contains one board foot, a convenient unit of measure. The moisture content was 6 per cent, which is logical because hardwood lumber for exacting use is usually well kiln dried.

It is possible to find hand-picked samples that will prove almost anything. For example, one could prove that yellow poplar is heavier than white oak by comparing the heaviest poplar with the lightest oak. This illustrates the necessity of basing all tests on samples that are representative in quality and adequate in number, because of the fact that wood is so extremely variable.

No two pieces of wood are just alike. They vary in many respects, such as rate of growth, angle of grain, and, most important of all from the woodworking standpoint, they vary in specific gravity or density. This is the best single indication of strength properties and hardness. Hardness in turn is related to power consumption and rate of dulling of tools, and so, indirectly, the specific gravity of any test sample tells considerable about certain machining properties.

The heaviest pieces usually have about twice the density of the lightest pieces in the very same wood. For example, 1000 pieces of yellow poplar ranged from 0.28 to 0.62 in specific gravity, and 1300 pieces of white oak ranged from 0.38 to 0.80. The hardness increases as the $2^1/_4$ power of the specific gravity, so that a piece of wood that is twice as heavy as another will have nearly five times the hardness. Obviously specific gravity is a factor that deserves serious consideration in selection of samples. White oak, one of the heavier common woods, has on the average only 1.4 times the specific gravity of poplar, which is one of the lighter of the common woods. This illustrates the fact that there is often a greater difference between the extreme samples of any wood than there is between average samples of different woods.

When specific-gravity figures are graphed in a frequency curve, approximately 90 per cent of all samples are concentrated in about the middle half of the total range in specific gravity. This leaves, in round figures, 5 per cent of all samples spread over one fourth of the range at the light end and another 5 per cent covering about one fourth at the heavy end of the distribution. When a woodworker complains of difficulty with some wood it is often found that his complaint is really based on a small proportion of extremely light or extremely heavy pieces. Such pieces are frequently trouble makers because they depart so widely from the normal. They would in all probability give misleading results in test material where the number of samples is relatively small. Consequently all test samples were chosen from the 90 per cent of the pieces that lie in the middle half of the specific-gravity range.

The number of rings per inch also has some influence on the quality of the planed surface, at least in the ring-porous woods, like oak. Consequently 5 per cent of the pieces at each extreme were excluded. In other words, as far as rings per inch and specific gravity are concerned, the samples represent the log run with the freak pieces out. The angle that the rings make with the surface is another factor to be considered, since edge-grain and flat-grain lumber machine differently in some respects. Edge-grain or quartered lumber is usually a small part of the whole even in woods that recognize quartered grades, like oak, gum, poplar, or sycamore. In other hardwoods quartered boards are usually accidents. Only clear flat-grain pieces were used in the tests.

In brief, the test samples conformed to the following specification: Size, 1 in. × 4 in. × 3 ft; moisture content, 6 per cent; specific gravity, from the middle half of the range; number of rings per inch, from the middle half of the range; angle of grain, flat; number of samples, 50 per species; quality, clear.

Effect of Cutting Angles on Quality of Planing

The cutting angle appears to be the factor that more than any other affects the quality of the planed surface. To name the best cutting angle for a given wood under given conditions is difficult because of the number of factors involved.

Before discussing cutting angles in detail some definition of terms is desirable. The terms illustrated in Fig. 1 are generally accepted and have been used in this paper.

Knives 1 and 2 are held in the cutter head at a 30-deg angle with reference to the radius, and the clearance bevel *c* is the same for both, 40 deg. The cutting angle *a* is the angle between the face of the cutter and the radius. With a knife, like 1, that has only the clearance bevel, the cutting angle and the knife angle, or angle at which the knife is held in the head, are identical. Varying this latter angle is, of course, one way of changing the cutting angle, but it means changing cutter heads, which is quite a job in a cabinet planer. A more common way of changing the cutting angle is by the use of knives beveled on two sides, like knife 2, which has a cutting bevel or back bevel *b*. The

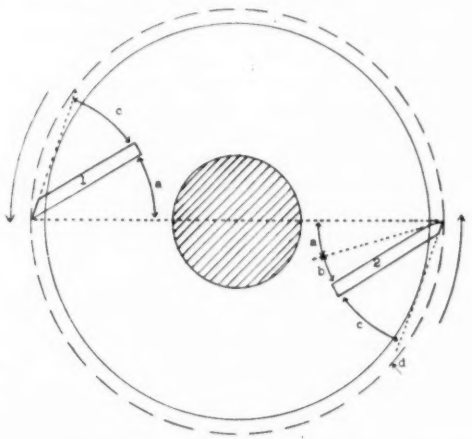


Fig. 1 Terms Used in Connection With Planer Knives (*a*, Cutting angle; *b*, cutting bevel; *c*, clearance bevel; *d*, cutting circle.)

planer used has a knife angle of 30 deg. By means of back bevels the other cutting angles used in this study, 25, 20, 15, 10, and 5 deg, were obtained. At each angle the samples were planed on one side at 36 fpm feed and 3600 rpm, then planed on the other side at 54 fpm feed and 5400 rpm. Figures used here are averages of these two feed-speed combinations.

Table 1 shows the scope of cutting-angle tests that have been

TABLE 1 PERCENTAGE OF DEFECTIVE PIECES AT DIFFERENT CUTTING ANGLES

Cutting angles, deg	5	10	15	20	25	30
Ash	31	30	28	27	21	47
Basswood	..	..	42	35	32	35
Beech	33	31	..	..	..	67
Birch, yellow	..	..	29	37	45	..
Chestnut	..	..	19	24	35	66
Cottonwood	60	63	75	73	88	71
Elm	76	76	52	67	81	82
Gum, black	58	48	53	47	57	63
Gum, red	65	34	46	49	51	56
Hackberry	63	53	25	7	46	80
Hickory	6	3	..	..	..	65
Magnolia	13	22	22	44	38	39
Mahogany	23	12	24	23	13	..
Maple, hard	..	..	44	44	49	83
Maple, soft	57	39	43	67	66	82
Oak, red	34	4	5	8	13	35
Oak, white	26	2	5	7	26	63
Pecan	22	18	24	8	5	43
Poplar, yellow	34	25	25	33	33	52
Sycamore	75	61	74	77	82	82
Walnut	..	..	36	27	50	..
Willow	68	54	50	41	54	90
Average	43.8	33.8	36.0	37.3	44.3	63.2

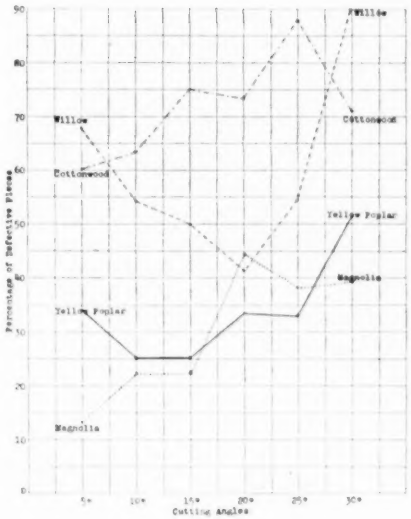


Fig. 2

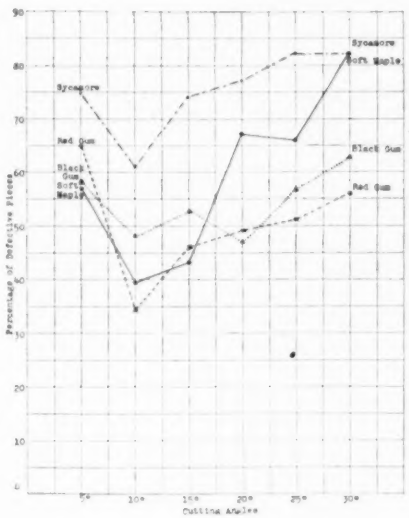


Fig. 3

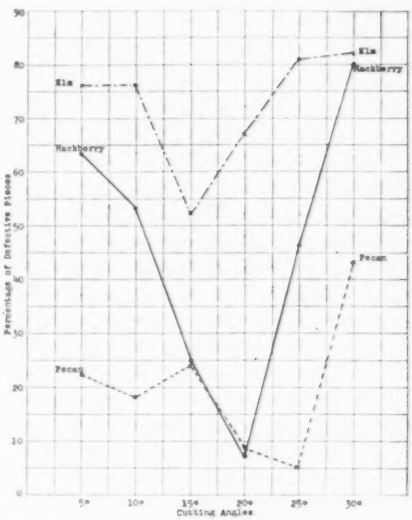


Fig. 4

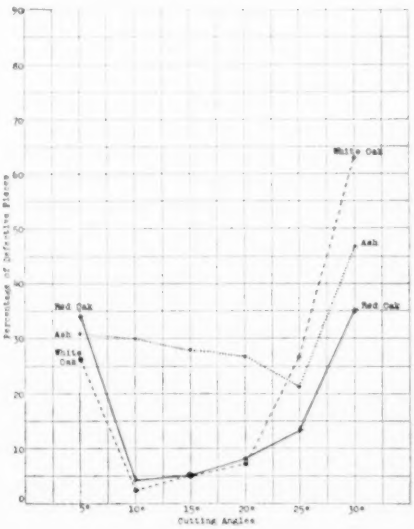


Fig. 5

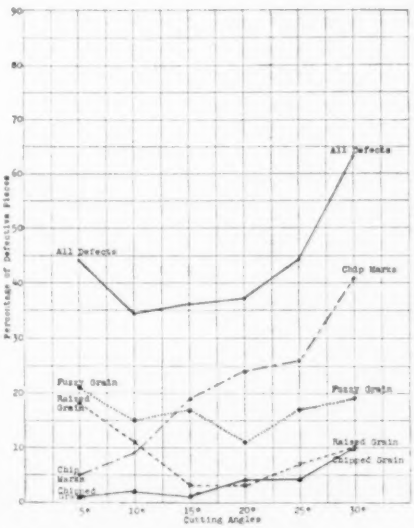


Fig. 6

FIGS. 2 TO 6 EFFECT OF CUTTING ANGLES ON QUALITY OF PLANING

made so far. These tests cannot be considered complete until the blank spaces are filled in as a minimum. It will probably be desirable to include a somewhat wider range of cutting angles and possibly a few more woods as well.

It was hoped that data on six cutting angles would reveal a consistent trend for each wood, a definite optimum with results becoming increasingly poorer as cutting angles departed from the optimum in either direction. Most of them do show a definite trend in spite of minor inconsistencies. Table 1 is rather large, except for reference purposes, and graphs have been prepared that tell the story more plainly for the fourteen woods on which data for all six cutting angles are available.

In Fig. 2 willow makes a bad start with nearly 70 per cent of defective pieces at a 5-deg cutting angle, but improves steadily until 20 deg is reached when the trend reverses sharply. Cottonwood not only starts badly but gets steadily worse until the 25-deg angle is reached, after which some improvement is made. The cottonwood curve fails to show that the optimum has been found. Possibly a 0 or a 35-deg cutting angle would give better results than any of the angles actually used. It is interesting to note that although willow and cottonwood start close together the curves cross twice. Yellow poplar is not so much influenced by cutting angles as the other woods; results fall within a relatively narrow range, except for the 30-deg angle. Taking each of the four woods shown in Fig. 2 at its best cutting angle, magnolia makes much the best showing, and yet at 20 deg it is slightly more defective than the willow. For magnolia, as for cottonwood, it is possible that the best cutting angle was not among those used so far. The testing of cutting angles at 5-deg intervals should serve to locate the optimum angle approximately, provided that enough angles are tried.

Fig. 3 shows the results for sycamore, red gum, black gum, and soft maple. These woods agree much better in their general trend than do the preceding four which are also diffuse porous. The optimum appears to lie between 5 and 15 deg for sycamore and red gum, and 5 and 20 deg for soft maple. The black gum is not wholly consistent, but even here there is a certain wider range where better results obtain than at the extremes.

Three ring-porous woods, elm, hackberry, and pecan, are shown in Fig. 4. Definite optimums appear; for elm at 15 deg, and even there the results were none too good; for hackberry, which showed the widest variation from best to poorest of any wood, at 20 deg; and for pecan, which is not entirely consistent, around 25 deg. The angles at both extremes appear to be definitely bad for all three.

An interesting contrast appears in Fig. 5, which gives the results for three more ring-porous woods, red oak, ash, and white oak. The ash runs along without being greatly affected until the optimum is reached at 25 deg. Between 25 and 30 deg it takes a decided turn for the worse. The oaks, on the other hand, show a big improvement from 5 to 10 deg and are little changed at 15 and 20 deg, but much poorer results are obtained if the knife angle is increased to 25 and 30 deg. In two distinct respects the oaks showed up better than any other wood: (1) The optimums were slightly better than the nearest rival, pecan; (2) the range within which good results were obtained was much wider than for other woods.

In Fig. 6 the effect of cutting angles on the occurrence of the different planing defects is shown. Taking all defects into account there is little to choose between 10, 15, and 20 deg although the 10-deg angle is slightly the best of the three. Some of the samples had more than one kind of planing defect, which explains why each point in the all-defects curve is somewhat lower in numerical value than the sum of the four points directly below that represent the individual defects. Fuzzy grain takes a slightly inconsistent course; it is not greatly affected, but the general trend indicates poorer results at each extreme. Raised grain improves consistently as the cutting angle increases from 5 to 15 deg and gets somewhat worse beyond 20 deg. Cutting angles have a great deal more effect on chip marks than on any other defect, increasing from an occurrence of 5 per cent with a 5-deg cutting angle to 40 per cent with a 30-deg cutting angle. Chipped grain, in general the least common of the defects, increases slightly with increased cutting angles.

Effect of Jointing on Quality of Planing

To joint or not to joint is the subject of many a woodshop debate. Some foremen maintain that one-knife finish is good enough, that one knife does all the cutting anyway, and that jointing is therefore a waste of time. Others insist that each knife can and should be made to do its share of the work, that better finish is obtained in that way, and that jointing is necessary to bring this about. The answer to the problem, as to many other woodworking problems, appears that "it all depends." It depends, for instance, upon the requirements of the job, it depends upon the kind of machine being used, and it depends, not least, upon the skill of the operator. No one answer will hold true under all the varying conditions found in the trade. This paper gives the results of a series of jointing tests

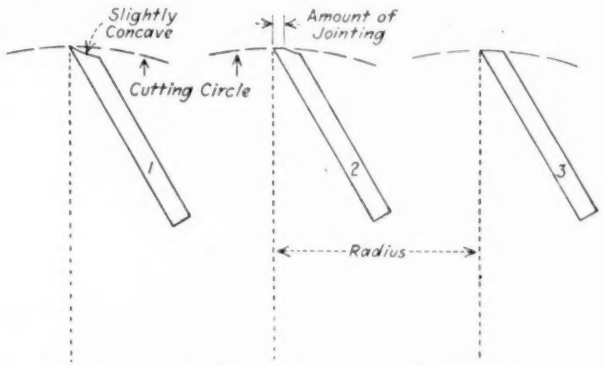


Fig. 7 Effect of Jointing Planer Knives

made with a 30-in. cabinet planer, a 20-deg cutting angle, a feed of 36 fpm, and a speed of 3600 rpm making a cut of $^1/_{16}$ in. Seven important hardwoods were tested at a moisture content of 6 per cent.

In the newly ground knife, like 1 in Fig. 7, the sharp cutting edge, has only a fine line of contact with the cutting circle, and there is ample clearance. But it is not possible to grind so that all the knives project exactly the same amount. A slight projection by one knife beyond the others causes that knife to do more than its share of the work. This often results in a one-knife finish, that is, a finish that shows only one knife mark for each revolution of the head. The first jointing is done to equal-

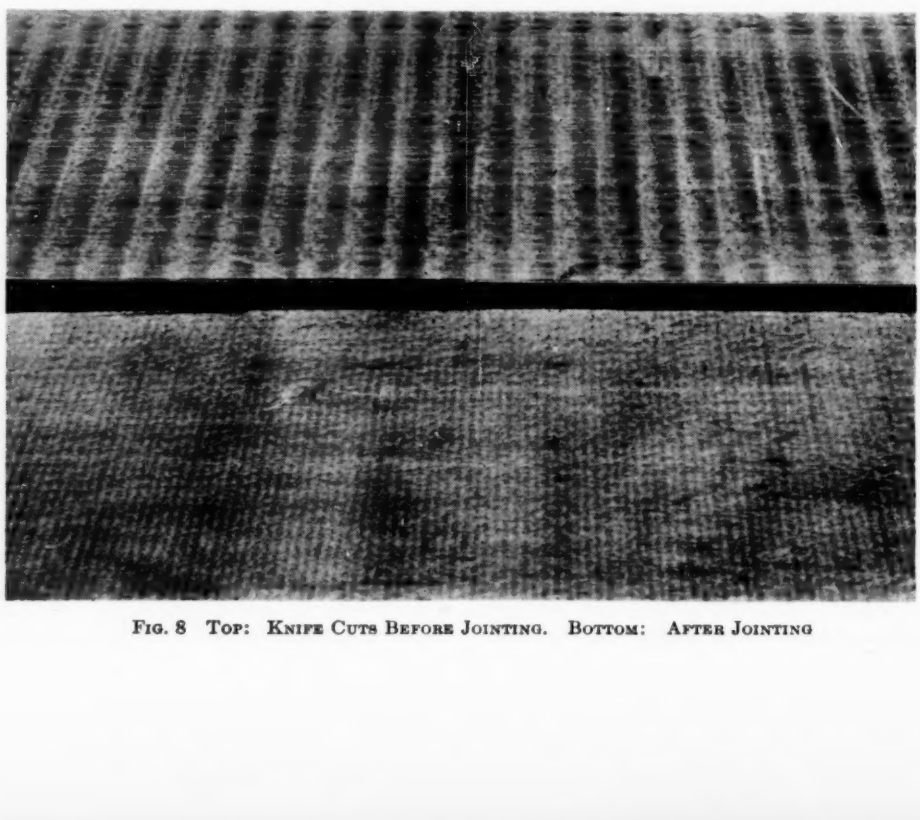


Fig. 8 Top: Knife Cuts Before Jointing. Bottom: After Jointing

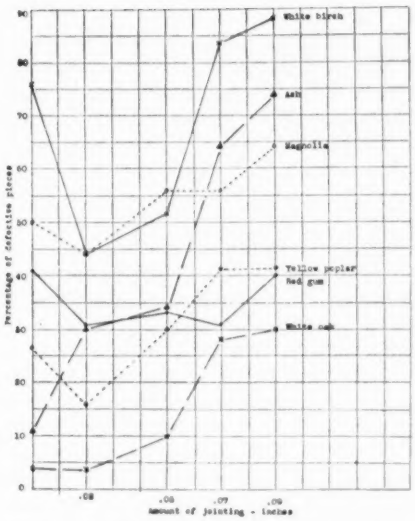


FIG. 9 EFFECT OF AMOUNT OF JOINTING ON QUALITY OF PLANING

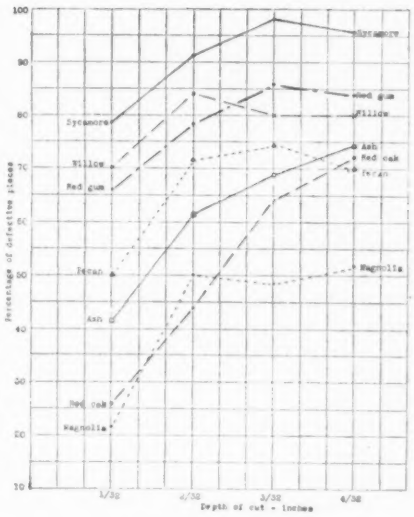


FIG. 10 EFFECT OF DEPTH OF CUT ON QUALITY OF PLANING

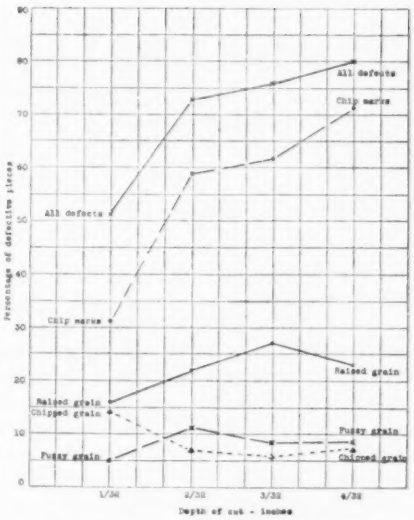


FIG. 11 EFFECT OF DEPTH OF CUT ON QUALITY OF PLANING

ize the projection of all knives and insure that each one makes its own cut in every revolution. Fig. 8 contrasts the finish before and after jointing. The same feed and speed were used with both samples, of course, and both are hard maple.

Whenever the knives become dull or nicked, jointing is again resorted to from time to time as a sharpening process. Repeated jointings gradually generate a pronounced "heel" which has no clearance because it conforms to the cutting circle. See 2 Fig. 7. Theoretically, it may be possible to joint until the entire knife end conforms to the cutting circle, as in 3. However, when an amount of heel about like that at 2 was developed a halt was called because of mechanical difficulties and because the data already obtained told the story. In practice the poorer quality of the work would probably have caused the operator to grind off most of the heel long before the condition shown at 2 was reached. A secondary effect of jointing is to produce a thicker, stronger cutting edge that offers more resistance to nicking.

Because of the knife marks the planed surface is not strictly plane but consists of a series of ripples the width of which from crest to crest is governed by the rate of speed and feed of the machine. The wider the ripples the deeper they inevitably are and the more work is necessary to prepare the surface for any fine finish. It is probably immaterial whether the ripples represent one-knife or four-knife finish provided their spacing is the same. In either case the number of marks per inch will depend upon the potentialities of the machine used and upon the requirements of the job. A finish that is wholly satisfactory for boxes may be entirely inadequate for corestock. However, the tests described here were concerned primarily with high-grade finish and were made on a machine designed for such work.

A publication of the Forest Products Research Laboratory of England gives some figures that are of interest in this connection. For a four-knife cutter head of about a 5-in. cutting circle at 3600 rpm and a feed of 30 fpm the depth of the ripples is given as 0.0000364 in. It further states that a projection of 0.0006 in. in one of the four knives removes the ripple marks of the other three. These figures seem almost insignificantly small, yet such knife marks are plainly visible to the naked eye and corresponding revolution marks can be easily felt with the finger.

The conclusion to be drawn from Table 2 is to joint the knives, but joint lightly.

The first run was made with knives that had just been ground in the machine but not jointed. For the second run the knives were jointed lightly but enough to touch the full length of each blade. Measurements made with a small steel scale and a hand lens near each end and at the middle of each blade show the joint to vary only slightly from an average width of 0.02 in. The average width of the jointed portion of the blade was increased to 0.05, 0.07, and 0.09 in. in the third, fourth, and fifth runs, respectively, as shown in Table 2.

Table 2 is based upon the occurrence of chip marks, chipped grain, raised grain, and fuzzy grain. On this basis the unjointed knives as a rule give substantially poorer results than do knives that were lightly jointed. When knife marks, as well as these defects, are taken into account every piece planed with unjointed knives may be considered defective because at this feed and speed one-knife finishing gives about eight marks per inch which requires considerable sanding in itself even if the piece is otherwise perfect. The jointed knives, on the other hand, give satisfactory knife marks regardless of the amount of joint. As the amount of

TABLE 2 PERCENTAGE OF DEFECTIVE PIECES FOR DIFFERENT AMOUNTS OF JOINTING

	Knives newly ground but not jointed	Knives jointed, amount inches			
		0.02	0.05	0.07	0.09
Ash	11	30	34	64	74
Birch, white	76	44	52	84	88
Gum, red	41	31	33	31	40
Magnolia	50	44	56	56	64
Oak, red	6	4	13	30	36
Oak, white	4	4	10	28	30
Poplar, yellow	27	16	30	42	42
Average	32	25	33	48	53

TABLE 3 PERCENTAGE OF DEFECTIVE PIECES AT DIFFERENT DEPTHS OF CUT

Depth of cut, in.	1/32	2/32	3/32	4/32
Ash	42	62	68	74
Beech	26	60	66	76
Cottonwood	62	88	80	94
Elm	94	96	100	100
Gum, black	38	62	60	66
Gum, red	66	78	86	84
Hackberry	72	90	94	96
Hickory	54	84	86	94
Magnolia	22	50	48	52
Maple, soft	60	72	70	86
Oak, red	26	44	64	72
Oak, white	42	66	78	76
Pecan	50	72	74	70
Poplar, yellow	36	64	56	66
Sycamore	78	92	98	96
Willow	70	84	80	80
Average	52.4	72.8	75.6	80.1

joint is increased, however, the quality of the planing becomes poorer. These data are shown graphically in Fig. 9, except that red oak, which follows white oak closely, is omitted.

Chip marks were the commonest defect. Chipped grain was also common with the unjointed knives. Fuzzy grain was occasionally found, particularly when heavily jointed knives were used.

Effect of Depth of Cut on Quality of Planing

The depth of the cut that is made by the planer may have a decided influence on the quality of the surface that results. There are, of course, many times when conditions leave the operator little choice in this matter. Again conditions sometimes make a preliminary roughing cut desirable and this permits some leeway in varying the depths of the subsequent cuts.

Table 3 shows the result of tests made at depths of cut of $^1/_{32}$, $^2/_{32}$, $^3/_{32}$, and $^4/_{32}$ in. Except that the shallowest cut is the best without exception, the different woods tell somewhat different stories. Some, like ash, beech, and the oaks, are greatly affected by depth of cut, while others, like elm, sycamore, and willow, run along in a relatively uniform way without much regard to this factor. On the average, the shallower the cut the better the results; however, the shallowest cut is much the best of the four, and there is relatively little to choose between the other three. Burning of knives, which often results from extremely shallow cuts, did not occur. A run of 2400 linear feet of mixed hardwoods 4 in. wide on a 30-in. cabinet planer is one thing, and an all-day run at much higher feed and speed with a molder, for instance, is a different proposition, and results would not necessarily be identical. The behavior of seven typical woods at different depths of cut is shown in Fig. 10.

The influence of depth of cut on the individual defects is shown in Fig. 11. There is a rapid increase in all defects between $^1/_{32}$ and $^2/_{32}$ in. with a gradual increase up to $^4/_{32}$ in. Chip marks closely follow this trend, in fact they are so much more common than other defects that they largely govern the trend for all defects. Raised grain shows a steady increase in occurrence up to $^3/_{32}$ in., but drops slightly at $^4/_{32}$ in. Chipped grain, contrary to the general trend, is worst at the shallowest cut and about the same in succeeding cuts. Data obtained in this test fail to indicate that depth of cut has any appreciable influence on fuzzy grain.

Obsolescence in Woodworking Machinery

By PAUL T. NORTON, JR.,[1] BLACKSBURG, VA.

The author discusses the economic aspects of obsolescence of production machinery in general and its relation to cost accounting.

With respect to the woodworking industry, index figures for recent years are given of machinery purchases for comparison with figures for orders for metal-working machinery.

The author then points out the danger to an industry as a whole of the use of obsolete equipment and continues with a discussion of the problem in its relation to profits and governmental depreciation regulations. Finally there is considered the subject of unamortized values in studies of machine replacements.

OBSOLESCENCE in machinery is in some ways similar to premature old age in human beings. If both human beings and machinery are considered merely from the standpoint of earning power, in each case there is a normal productive period and, in an increasing number of cases, this productive period ends before the actual physical life comes to an end. In the case of both human beings and machinery, it is often possible to extend the productive period through rehabilitation, but it should be remembered that in both cases the premature loss of earning power is generally caused by changes in methods to which neither the human beings nor the machinery are readily adaptable.

There is also a certain similarity between the methods which may be used to eliminate the serious economic losses due to premature old age in human beings and to obsolescence in machinery. From a purely economic standpoint there is no loss through either premature old age in human beings or through obsolescence in machinery, provided both have sufficient earning power during their productive lives to take care of their physical lives. In the case of human beings the attempt is being made to solve the problem through old-age pensions. In the case of machinery there is no economic loss, no matter how short the economically productive life may be, provided the machinery has had sufficient earning power during its economically productive period to enable it to pay back both its ordinary operating expenses and also the money invested in it plus a proper return on the investment. If this has been accomplished, the obsolete machine can be scrapped without any economic loss whatever, even though from a purely physical standpoint the machine may be just as good as it ever was. If such a machine is scrapped, or otherwise displaced, before it has reached the end of the life that was previously estimated as its economically productive period, there will almost certainly be an apparent or book loss. This apparent loss seems to be due to premature displacement, but it actually represents a charge which should have been distributed over the real economically productive period of the machine. If there had been no error in the previous estimate of productive life, all necessary charges would have been made against the machine during its real economically productive period, and there would be neither a real economic loss nor even an apparent or book loss when the machine is displaced. The economy study, which is made in order to determine whether a machine should be replaced by a more modern one, should be based on the realizable value of the present machine and should disregard completely any accounting adjustment that may be necessary because of a difference between the book value and realizable value of the present machine. The problem of unamortized value will be discussed again at greater length later in this paper.

There is no excuse for neglecting the serious economic problem caused by obsolescence of machinery and other physical assets. Theoretically the problem is easy to solve. All that is necessary is to determine the economic life of the proposed asset and then to write off the first cost of the asset from its earnings during the economic life. The real difficulty is that it is almost impossible to predetermine the economic life of an asset, but that is no reason for not making as accurate an estimate as is possible. Factors of safety are customary in both engineering and business estimates, and it would seem desirable also to use a factor of safety in making future life estimates.

Definition of Obsolescence

The Committee on Reserves for Rehabilitation of the Machinery and Allied Products Institute has adopted the following definition of obsolescence:

> Obsolescence is the loss in value of an asset brought about by exhaustion of raw materials, or the development of another asset that will better do the same thing or replace the product, before the original estimate of the life of the asset has elapsed and the cost has been recovered through depreciation; extraordinary depreciation of an asset resulting from the development of new processes or inventions.

The first part of this definition is rather unusual in that it makes obsolescence depend upon whether the original cost has been recovered through depreciation charges. This part of the definition does suggest one important point, namely, that management's willingness to replace a present machine with a more modern one depends to a large extent upon whether the investment in the present machine has been recovered through depreciation charges. As previously explained from a purely economic standpoint this should not be the case, but it is certainly true that management is faced with a serious practical problem in making replacements which involve large accounting adjustments due to unamortized book values. No matter what one's concept of obsolescence may be, it is certainly true that obsolescence presents no real problem whatever in cases where the investment has been fully recovered through depreciation charges before the machine is displaced.

Extent of Obsolescence in Woodworking Machinery

It does not seem possible to obtain definite figures, but apparently most persons who have studied the problem feel that there is even more obsolescence in woodworking machinery than in most other kinds of machinery used in the manufacturing industries. Studies have been made which give at least a rough idea of the amount of obsolescence in metal-working machinery.

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Presented at a joint meeting of the Wood Industries Division of The American Society of Mechanical Engineers and the Cost and Production Division of the Southern Furniture Manufacturers' Association at High Point, North Carolina, September 21–23, 1938.

Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y., and will be accepted until March 10, 1939, for publication at a later date. Discussion received after the closing date will be returned.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of the author, and not those of the Society.

There are also indexes which show something about the trend in the production of both machine tools and woodworking machinery. Perhaps it may be possible to form some sort of opinion with regard to the extent of obsolescence in woodworking machinery by comparing these three series.

The *American Machinist* made extensive studies of obsolescence in metal-working machinery in 1925, 1930, and 1935. These studies indicated that 44 per cent of metal-working machinery in the United States was over ten years old in 1925, 48 per cent in 1930, and 65 per cent in 1935. There is, of course, no direct relationship between age and obsolescence of machinery, but the figures just quoted do seem to indicate that there is a large amount of obsolescence in metal-working machinery, and that the condition became progressively worse during the decade between 1925 and 1935. No one familiar with the low rate of manufacture of such machinery during the years 1931 to 1934, inclusive, would be surprised at these figures, but many persons will doubtless be surprised to learn that it has been estimated that on July 1, 1937, metal-working machines over ten years old still made up 61 per cent of the total of all metal-working machinery, in spite of the much greater activity in the machine-tool industry between January 1, 1935, and July 1, 1937. It would require many years of great activity in the machine-tool industry to bring the average age of metal-working machinery back to the 1925 figure.

The machine-tool index gives dollar values of orders received and is based on 1926 shipments as 100 per cent. The index was 131.2 for 1928, 155.8 for 1929, and then dropped rapidly, being 70.9 for 1930, 41.0 for 1931, and 19.6 for 1932. It then started to climb, but was only 27.1 in 1933 and 46.2 in 1934. No wonder 65 per cent of metal-working machinery was found to be over ten years old at the beginning of 1935. The last three years have told an entirely different story. The index increased to 86.0 in 1935, to 136.6 in 1936, and to 186.9 in 1937. An unusually large percentage of these machines have been exported during the last two or three years, but the domestic orders alone would have brought the index up to 108.4 for 1936 and to 129.1 for 1937.

There is also an index for woodworking machinery but it tells a very different story during the last several years. This index gives shipments and is based on the average shipments during 1922, 1923, and 1924 as 100. This index fell to 3.2 in March, 1933, averaged only 8.4 for the year 1934, 13.8 for 1935, 24.3 for 1936, and 30.1 for 1937. To make the woodworking-machinery and machine-tool figures more comparable it seems desirable to translate the woodworking-machinery figures to a percentage of orders received in 1926 and also to use only the domestic orders for machine tools, as it seems probable that the export shipments of woodworking machinery were much less important during the last few years than was the case with machine tools. Table 1 shows the indexes with these changes.

TABLE 1 INDEXES OF MACHINERY ORDERS RECEIVED

Average for year	Woodworking machinery	Domestic machine tools
1934	16.0	36.7
1935	21.9	64.9
1936	33.6	108.4
1937	38.5	129.1

These figures are offered for what they may be worth and with no claim that they prove anything with regard to obsolescence in woodworking machinery. They do seem to indicate quite clearly, however, that during recent years the purchases of new woodworking machinery have been much less in comparison with the middle 1920's than has been the case with the domestic purchases of new machine tools.

There may easily be some reasons other than a comparatively large degree of obsolescence for what on its face would seem to be an unsatisfactory situation with respect to the equipment of the woodworking industries. In the middle 1920's large quantities of woodworking machinery were being purchased for use in factories producing wood bodies for automobiles. The woodworking industries have lost practically all of that important outlet for their productive facilities, as well as other outlets which were less important than automobile bodies but did require in the aggregate a large amount of woodworking machinery.

The great peaks and valleys in the production curves of manufacturing of all types of machinery are due primarily to the fact that the users of machinery are actively in the market for new machinery only when there is an active demand for their own products. In 1937 there was an active demand for most products which require machine tools for their production, while there does not seem to have been as active a demand for the products of the wood industries which use the bulk of woodworking machinery.

Even if it could be shown that the comparatively small production of woodworking machinery in recent years was due merely to a general decrease in the demand for wood products, and did not mean that woodworking machinery was much more obsolete than other types of machinery, there would be little cause for satisfaction on the part of the owners of the woodworking plants that are still able to operate. How soon will there be further inroads by other raw materials into those fields which are still largely supplied by the woodworking industries? The history of this country is full of examples of raw materials which have been in a large measure supplanted by other raw materials, principally because the older industries did not avail themselves of the machines and productive methods which inventive genius had made available. Some of these industries have now taken advantage of these newer methods and are regaining their former positions of importance. What will be the answer of the woodworking industries to this challenge?

Some Economic Results of Obsolescence of Machinery

Most engineers are familiar with important industries which neglected to make adequate depreciation charges and were unable to replace equipment when it became obsolete. For years individual concerns in these industries had seemed to be profitable and had paid satisfactory dividends to their stockholders, but a large part of these dividends were paid from apparent profits and not from real profits. These companies were living on their capital. What is more, both the company and the stockholders paid taxes to the federal government on apparent profits which were not really profits at all. The corporation paid these taxes on the entire amount of these apparent but unreal profits; the stockholders paid individual income taxes on that part of these apparent profits which were paid to them in dividends. As long as corporation taxes were small and in those cases where ample reserves were retained by the corporation, the tax part of the problem was not such a serious one. With present high corporation and individual taxes and with the bringing of pressure upon corporations to pay out a large part of their book profits, the situation becomes serious indeed.

From the standpoint of corporation taxes which may be levied on book profits which are not real profits, because depreciation charges have been inadequate, the situation just described has become somewhat less serious because of the recent modification of the penalty tax upon undistributed book profits, although the normal tax is now so high as to make it very costly if any of the taxable profits are not real profits. From the standpoint of the ultimate solvency of the corporation and its ability to replace obsolete equipment, the situation is still just as dangerous in cases where corporations distribute part of their capital as dividends out of apparent but unreal profits. These dividends are

apt to be spent by the stockholders (at least that part which is left after income taxes have taken what is really a capital levy) and it is not an easy matter for such corporations to get this money back from their stockholders, or indeed to obtain from anyone the funds which may be needed for machine replacements.

Recent Changes in Depreciation Regulations

Even though the penalties of the Revenue Act of 1936 have been modified considerably, Treasury Decision 4422, under which so many depreciation rates have been drastically reduced, is still in force. It is probable that there have been many cases where the present depreciation regulations have resulted in the disallowance by the federal government of adequate depreciation charges, and it should be remembered that every dollar of real depreciation which is disallowed means exactly one dollar of taxable but unreal profits. On the other hand, those who so vehemently condemn Treasury Decision 4422 should remember that there is nothing in the new regulations which will prevent the owner of depreciating assets from getting the depreciation which he considers proper, including a reasonable allowance for obsolescence, provided his rates are based upon a replacement policy which actually replaces the assets when they become obsolete. But it is no longer possible for one to depreciate at a rate of say ten per cent when he uses the assets for from fifteen to twenty years. How can anyone expect the government to allow a high depreciation rate if the actual life is so much greater than the life which is equivalent to the depreciation rate which is claimed?

One of the reasons why so many owners of machinery object to the new depreciation regulations is that most of these persons do not have adequate records with which to satisfy the Bureau of Internal Revenue that their depreciation claims are reasonable. These persons would do well to reconcile themselves to the fact that they will not be able to obtain adequate depreciation charges for tax purposes unless they do have adequate records, which means that in most cases they must keep individual records of their principal items of machinery instead of attempting to use a flat depreciation rate on a large group of machinery with different life expectancies. Many of the recent decreases in depreciation rates have been due to excessively large depreciation charges in previous years and in such cases there is nothing that the owner of the machinery can do about the matter now, except perhaps in some cases to go back to years still open and reduce the depreciation charges on those returns so as to make it possible to charge additional depreciation in the future. But even if it is reasonable for future depreciation charges on present equipment to be reduced because of excessive depreciation charges in the past, these lower rates should not be accepted without careful examination in the case of machinery which may be purchased in the future. The only safe procedure is to maintain adequate records of individual machines and to make the best possible estimates of the future life of each machine. If this is done, and if replacements are actually made when the estimated lives come to an end, there is no reason whatever why the present depreciation regulations should prevent any owner of machinery from obtaining adequate depreciation charges on his tax return.

Properly administered, the new depreciation regulations should hurt no one, but there is serious danger that the present tendency to reduce depreciation rates may in future years greatly increase the loss due to obsolescence of machinery, because there will be a tendency for obsolete and practically worthless equipment to be carried at high book values at the time such equipment would ordinarily be replaced. No company cares to write off large amounts when replacing equipment. In fact in many cases it is impossible for a company to do so and at the same time maintain its credit rating and appearance of solvency. But continuing to use obsolete equipment under these conditions will not solve the problem. Competitive conditions in an industry or between industries make it necessary for a company to have modern methods and modern equipment if it is to sell its products at a profit.

Unamortized Values in Replacement Studies

It has often been claimed that, in making a study to determine whether a machine should be replaced, any unamortized value of the present machine should be charged in some way against the proposed machine. Most of the best-known replacement formulas do this in one way or another, but it is heartening to note that in a survey made some years ago a large majority of the companies replying to a questionnaire stated that they would not think of charging this unamortized value against the future, but instead would charge it against profit and loss or surplus. It is probable that in most cases these companies were not so much interested in economic theory as in the competitive disadvantage that would result if they charged into their future costs an item which their competitors would not have to worry about if these competitors did not have to consider such unamortized values. Whatever the reason, these companies certainly gave the correct answer, both practically and theoretically.

Any difference between the book value and the realizable value of replaced equipment represents additional depreciation which actually occurred in the past during the life of the replaced equipment and which should have been charged as a part of the cost of the products of the replaced equipment. The fact that it may not have been so charged does not alter the fact that the depreciation actually occurred in the past. What is more, this depreciation would undoubtedly have been charged in the past if the estimate of the life of the replaced equipment had been correct. To attempt to charge this past cost against the future is just as unthinkable as to charge any past loss against the cost of some specific future product, when making a cost estimate for the purpose of setting the price at which this future product would be sold.

But this apparent loss, which seems to occur when there is a difference between the book value and the realizable value of a replaced machine may not be a real loss at all. The replaced machine may in reality have been so profitable that it would have shown a real profit even if the actual depreciation had been charged against it during its life. But no matter whether the machine was profitable or unprofitable, the profit or loss occurred in the past and there is no justification whatever for attempting to charge even a real loss against the operations of the future.

To show further the absurdity of the argument that any difference between book value and realizable value should be charged against the proposed machine, let it be supposed that this new machine in turn is replaced before it has had an opportunity, at the depreciation rate used, to pay back its own loss in value plus that remaining from the previous machine. This might continue indefinitely and eventually some machine might have to bear such heavy fixed charges as to make it impossible for its owners to compete with others who had simply written off such differences between book values and realizable values when these differences became apparent.

It is possible that some persons, who charge unamortized values of present machines against proposed machines in making replacement studies, would not actually make such charges on the books if the replacements were actually made. But there is certainly no logic in that attitude. The replacement study should be based on the assumption that the replacement will be made if the results of the study, including both the tangible and intangible factors, indicate that the replacement would be profitable. In making the study the present equipment should be put

in at its realizable value and depreciation and other charges assessed against the present equipment on that basis. If, however, the study indicates that the replacement would not be profitable, the present equipment need not necessarily be written down on the books to its realizable value. The very fact that the replacement has been shown not to be advisable proves that the present machine is not obsolete and therefore realizable value for some other purpose is not a relevant factor. However, the fact that consideration has been given to the replacement indicates that the equipment is probably nearing the end of its economic life, and it would therefore seem desirable to make a new estimate of future life and to set a new depreciation rate accordingly.

Conclusion

It would seem from the foregoing discussion that the problem of possible obsolescence of his woodworking machinery is one of the most important problems which the owner of a woodworking factory must face.

While no definite figures are apparently available, there are strong indications that there is an excessive amount of obsolescence of equipment in woodworking factories; a condition which may cause the woodworking industries to lose some of their present markets to industries using other raw materials.

Under present high federal tax rates and depreciation regulations, a company which does not replace equipment when it is really obsolete is quite likely to pay taxes on apparent profits which are not really profits at all, and also is likely to find it difficult to obtain the necessary funds when it is finally evident to everyone that replacements should be made.

Replacement studies should be based on values as they will be if the replacement is made, and any difference between book values and realizable values of present machinery should be disregarded completely when making such studies. Book values are apt to be much too high in industries with a large amount of obsolescence, and this condition is apt to become worse under present depreciation regulations. Much of the present obsolete equipment was to a large extent written off the books under the previous more lenient depreciation regulations. In the future an owner of depreciating assets will not be able to secure approval of adequate depreciation claims unless these claims are based on proper records and a reasonably definite replacement policy. On the other hand, there is no reason whatever why depreciation allowances under the present regulations should not be adequate, provided the owner of depreciating assets does his part by keeping proper records and by replacing assets which he considers to be obsolete.

The Mechanical Characteristics of Rubber

By F. L. HAUSHALTER,[1] AKRON, OHIO

The author presents some of the characteristics of rubber in which engineers may be interested from a design standpoint, without going into detail on every phase of the subject which might affect design. He points out that there is a vast amount of data still to be collected, especially to establish accuracy on some points that seem to be in doubt. He suggests that to obtain data accurately, it will be necessary to conduct tests in rooms where humidity and temperature are closely controlled. The author presents data showing that moduli of elasticity and the creep of shear and torsion springs are decidedly affected by temperature. Presented graphically are curves showing load-deflection, shear, and torsion characteristics of various rubber specimens including compression pads, rubber shear sandwiches, and torsion springs.

TO TREAT a subject as large as this is a tremendous task, and the author, therefore, plans only to go into phases which may add something to what has already been published, and into others, which may seem to need further clarification.

When considering rubber as an engineering material we should bear in mind that it is a vegetable product which on being coagulated from the sap of a tree becomes a closely packed mass of rubber globules or hydrocarbons separated by their adsorption film of protein, coming together with some precipitated protein and other serum substances, trapped between the particles during coagulation and drying. We should also remember that the carbon atoms of the rubber molecule lay in zigzag linkages of various lengths which straighten out when stretched, perhaps with free rotation at the points of the single bonds, for the molecule of the rubber hydrocarbon has the formula $(C_5H_8)_x$, x varying perhaps from 500 to 3000, with one double bond for every C_5H_8 group. The resistance to slippage of the molecule depends on the orientation of the linkages and on the value of the molecular cohesion, as measured by the Van der Waals forces.

Sulphur mixed with rubber mechanically combines with it chemically as a solution, at the application of heat, and even at room temperatures. The change, known as vulcanization, is not a simple chemical phenomenon, for the rate of combination increases with concentration of sulphur, and for initial concentration of sulphur the rate of combination remains constant until most of the sulphur is consumed. The rate of combination is also influenced greatly by the presence of accelerating substances, acting as catalysts.

Vulcanization greatly reduces the slippage or creep in the molecular structure of the rubber. Evidence indicates that bridges of sulphur tenaciously join together the molecular structure with an attending stiffening action. Vulcanization is undoubtedly an establishment of cross bonds, perhaps crystalline in character, between long chain fiber molecules, both chemical and mechanical in nature.

When rubber, either raw or vulcanized, is stretched it becomes warm, and when stretched even small amounts the heat generated increases sharply above 70 per cent extension. The mechanical work done on the rubber when it is stretched is only a small percentage of the total latent heat. Hence, the heat generated is something more than friction; it may be potential energy, stored by strain of an assembly of oriented single molecules, and when the rubber is stretched there is an evolution of thermal energy corresponding to a heat of crystallization. Likewise, rubber stretched to a given percentage, on being heated, contracts.

There is a tight packing of the rubber molecules as rubber is stretched, because there is a slight increase in density and a very appreciable increase in apparent hardness. The change of durometer hardness with stretch for two rubbers, one a vulcanized pure gum of durometer 40 and the other a carbon-black reinforced stock of durometer 65, is shown in Fig. 1. This closing up of the structure greatly reduces the susceptibility of the rubber to attack from oxygen, its deadly enemy. The greatest vulnerability of rubber to oxygen or ozone is just at the point of change of hardness; the effect then rapidly diminishes as the hardness increases. It has also been found that the susceptibility of vulcanized pure gum to tearing is greatest at from 50 to 150 per cent elongation.

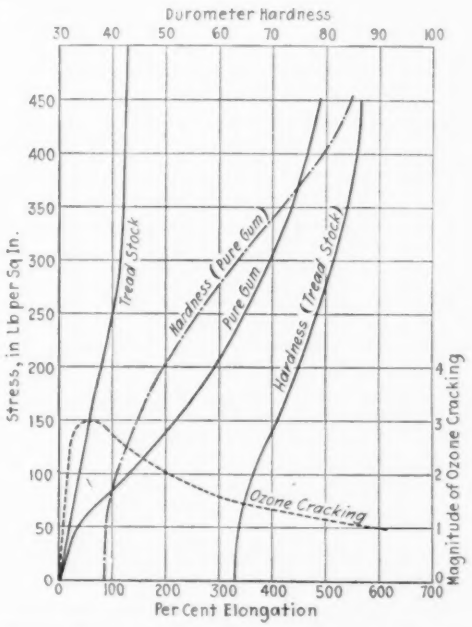


FIG. 1 STRESS-STRAIN RELATIONS AND CHANGE IN DUROMETER HARDNESS WITH STRETCHING OF TWO RUBBERS, ONE A VULCANIZED PURE GUM WITH A DUROMETER OF 40 AND THE OTHER A CARBON-BLACK REINFORCED STOCK OF DUROMETER 65

Fig. 1 also shows typical stress-strain curves for the rubbers mentioned in the preceding paragraph when tested in one-directional tension. Rubber tested in tension in one direction is in compression in the other two planes. A highly valued piece of work by Sheppard and Clapson (1)[2] showed a direct relation between compressive force and equivalent two-way tensile forces. Hollow spheres of rubber were inflated with gas pressure and elongation measurements taken in two directions at right angles

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NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their autnor.

[2] Numbers in parentheses refer to the Bibliography at the end of the paper.

to each other. The compressive force was computed from the gas pressure, and the degree of compression in the rubber computed from the reduction in wall thickness of the rubber. A continuous tension-compression curve was obtained in this manner for rubber up to a compression of 97.4 per cent. Fig. 2 shows the results of the work of Sheppard and Clapson (1) plotted with tension and compression to the same scale. For a stock of the cold-cured pure-gum variety of toy balloon, which had a tensile strength of only 570 lb per sq in. at 700 per cent elongation at break, the compressive load at 97.4 per cent compression went to 13,000 lb per sq in. of original cross-sectional area. There is a point of inflection in the tension curve but none in the compression curve.

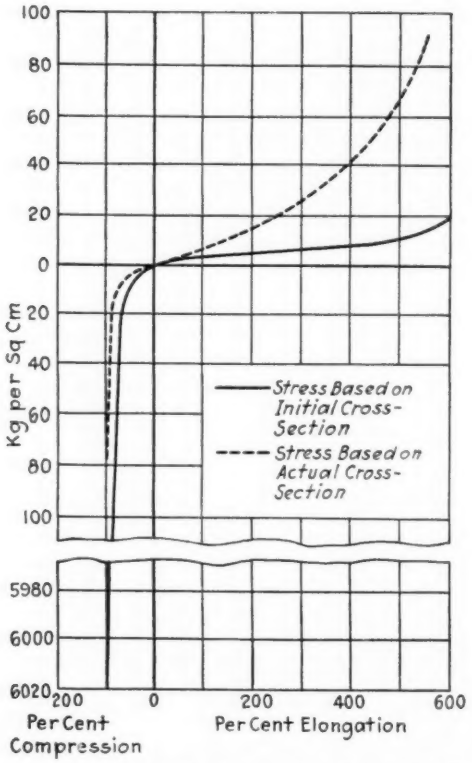


FIG. 2 CONTINUOUS TENSION-COMPRESSION CURVE FOR PURE-GUM RUBBER

Rubber is used but little in tension by engineers, although there is no good reason for not using it more extensively now that rubber can be bonded securely to metal. Rubber can be used safely in tensions up to 30 to 40 lb per sq in. and when working in this region of elongation the load-deflection curves for stocks with a high rubber content is very close to a straight line, meaning that the modulus of elasticity is very nearly a constant within this range. For a pure-gum vulcanizate of about 40 durometer, the modulus of elasticity is about 180.

Rubber has been used longest by engineers in compression, as buffers, shock absorbers, and bumpers of various types. A great number of data have been collected on compression rubber, some of the most noteworthy being by Birkitt and Drakeley (2) and J. Morrison (3). Their work gives us a basis for design of compression-rubber parts. Birkitt and Drakeley (2) investigated the influence of various lubricants on the load-deflection characteristics of compression-rubber disks of the same dimensions and recipe. The result of this work is shown in Fig. 3. What the author wishes to emphasize is that up to 20 per cent compression the curves obtained by using on the loading plates lubricants of soap, glycerine, oildag, and vaseline, practically coincide. It is the author's experience that compression rubber as a load-carrying member should be limited to 20 per cent compression under normal loading. The author wishes to amplify this fact later, but the chief reason is to keep the creep or drift in the rubber to a reasonable figure. Too often compression pads have been used under loads which very badly distorted the rubber, and the rubber took on excessive creep in a short time.

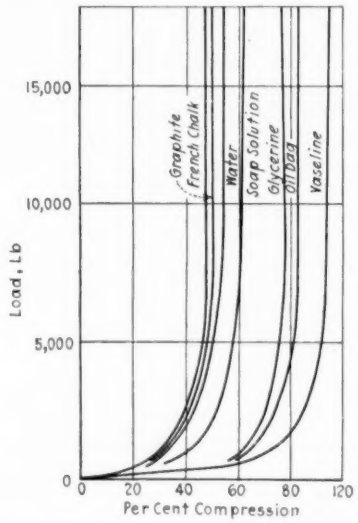


FIG. 3 INFLUENCE OF VARIOUS LUBRICANTS ON THE LOAD-DEFLECTION CHARACTERISTICS OF LIKE RUBBER DISKS

Birkitt and Drakeley (2) showed that vaseline is perhaps the best lubricant to use in investigating the laws of compression rubber. Vaseline was placed on the rubber and loading plates just prior to making the tests so that swelling and softening of the rubber would not influence the results. Using disks of various areas, from 2.4 sq in. up to 5.6 sq in. of the same thickness and compound, the rubber being a pure-gum stock of about 40 durometer, they obtained the results shown in Fig. 4. Note that even up to 30 per cent compression all the curves closely coincide. The thickness of all disks was only 0.12 in. If we figure the modulus of elasticity E from this curve as closely as

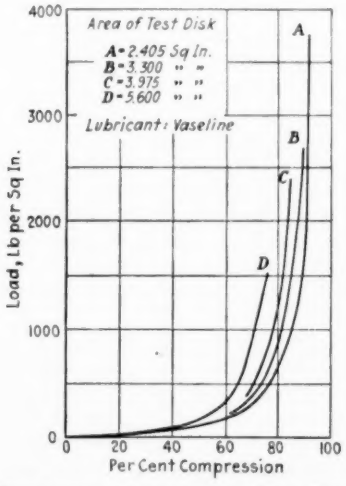


FIG. 4 LOAD-DEFLECTION CHARACTERISTICS OF RUBBER DISKS OF VARIOUS AREAS BUT THE SAME THICKNESS AND COMPOSITION, USING VASELINE AS LUBRICANT

possible, we obtain 188, which is very nearly three times the shear modulus for a stock of 40-durometer hardness.

Birkitt and Drakeley (2) also used vaseline in testing disks of the same recipe as used in the previous test, the thickness varying from 0.157 in. to 0.880 in. The area of the test piece was 2.405 sq in. Fig. 5 shows these results graphically, and Table 1 gives the actual figures obtained. The values at 41.7 lb per sq in. load for all disks lie very close to the curve drawn. If we take the average value of percentage compression at 41.7 lb per sq in., we get a modulus of elasticity of 198, which is only 5.3 per cent greater than obtained on the disks of the same thickness but with different areas.

TABLE 1 EFFECT OF VASELINE ON COMPRESSION DISKS OF VARIOUS THICKNESSES

Thickness of disk, in.	Compression, per cent, at loads in lb per sq in. of			
	41.7	417	1668	2930
0.157	23.2	55.4	72.6	80.2
0.360	19.7	56.2	70.0	74.5
0.466	21.4	62.2	75.2	80.6
0.607	21.5	61.1	74.2	77.5
0.752	20.3	61.1	74.6	78.5
0.880	20.6	62.6	76.4	80.0
Avg	21.1	59.8	73.8	78.6

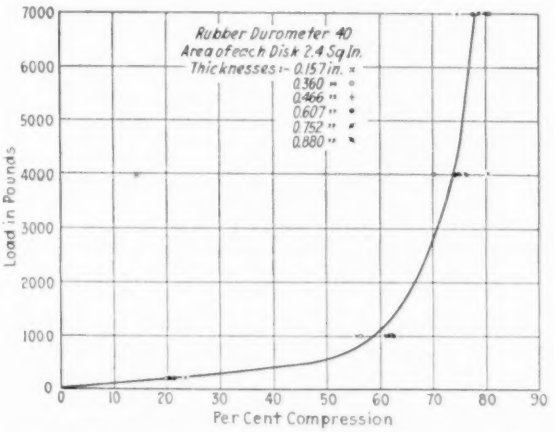


FIG. 5 EFFECT OF VASELINE IN COMPRESSION LOADING OF RUBBER DISKS, USING SMOOTH PLATES

Therefore, it appears from the foregoing that the modulus of elasticity in compression is three times the shear modulus where lubricated surfaces are used and the loading plates are smooth. Morrison (3) later confirmed the work of Birkitt and Drakeley (2) with the disks shown in Table 2. Fig. 6 shows the results of the tests without the use of a lubricant, the plates being serrated by grooves 1/16 in. deep. The rubber was of the same recipe in each case and had a durometer hardness of 55. Fig. 7 shows the results on the same size samples tested between smooth plates lubricated with vaseline. The vaseline drew the curves closely together and, with the exception of curve *A*, the curves almost coincide up to 20 per cent compression. Taking a mean value for the compression of curves *B*, *C*, *D*, and *E*, we obtain a value for *E* of 340, which is equivalent to a durometer hardness of about 57 according to our latest data on modulus of elasticity. Morrison's durometer may have read 2 points lower than the author's.

Morrison (3) took a test piece of this same rubber, 5 in. long, 1 in. wide, and 1/8 in. thick, and obtained a stress-strain curve in tension. The curve was straight up to 40 per cent elongation and within this range the modulus of elasticity in tension was 320 as compared to 340 for the compression value. This checks very closely with three times the modulus of elasticity in shear for rubber of 55 durometer.

TABLE 2 DIMENSIONS OF DISKS USED BY MORRISON (3)[a]

Curve[b]	Disk dimensions: Outside diam., in.	Inside diam., in.	Thickness, in.	Width, in.	Ratio of thickness to width
A	6 3/16	3 3/4	1	1.28	0.780
B	5 1/4	2 1/2	1	1.38	0.725
C	6	2	2	2.00	1.000
D	5	1	3	2.00	1.500
E	4 1/8	1	5 7/8	1.49	3.940

[a] Number in parentheses refers to Bibliography at the end of the paper.
[b] See Fig. 6.

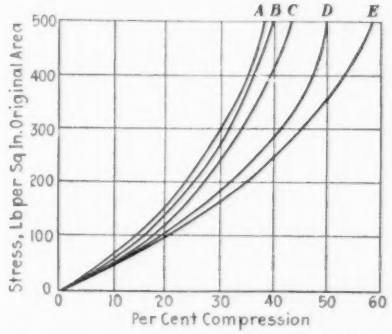


FIG. 6 LOAD-DEFLECTION CHARACTERISTICS OF MORRISON'S (3) RUBBER DISKS, USING NO LUBRICANT AND PLATES SERRATED BY 1/16-IN. GROOVES

(See Table 2 for the dimensions of the disks used.)

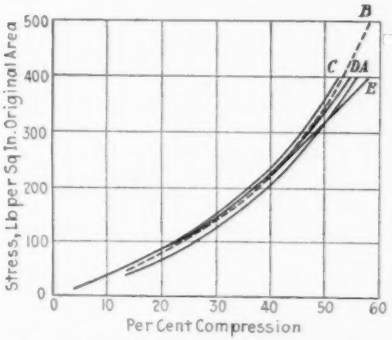


FIG. 7 LOAD-DEFLECTION CHARACTERISTICS OF MORRISON'S (3) RUBBER DISKS, USING VASELINE AND SMOOTH PLATES

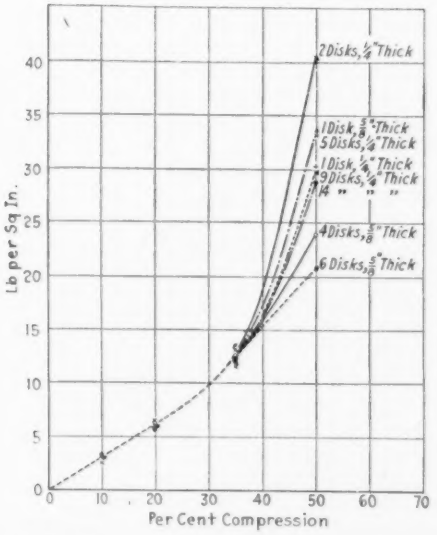


FIG. 8 LOAD-DEFLECTION CHARACTERISTICS OF SPONGE-RUBBER DISKS TESTED SINGLY AND IN STACKS

TABLE 3 VALUES OF THEORETICAL MODULUS OF ELASTICITY E AND THE MODULUS E_c CALCULATED EMPIRICALLY

	Description of article	Dur	E_c	E	Diff. %
1	Disk 6⁵/₁₆ in. OD, 3³/₄ in. ID, 1.0 in. thick	55	318	324.0	1.8
2	Disk 1³/₄ in. OD, ³/₄ in. ID, ¹/₂ in. thick	46	228	217.0	6.0
3	Disk 3³/₄ in. OD, 2.0 in. ID, ³/₄ in. thick	50	267	273.0	2.2
4	Disk 3³/₄ in. OD, 2.0 in. ID, ³/₄ in. thick	58	351	336.0	4.5
5	Disk 3³/₄ in. OD, 2.0 in. ID, ³/₄ in. thick	69	489	487.5	0.3
6	Disk 3³/₄ in. OD, 2.0 in. ID, ³/₄ in. thick	70	498	487.5	2.1
7	Hollow cylinder 2¹/₈ in. OD, 1³/₈ in. ID, 3³/₄ in. high	53	297	280.0	6.0
8	Hollow cylinder 2¹/₈ in. OD, ¹/₂ in. ID, 5⁷/₁₆ in. high	55	318	297.0	7.0
9	Hollow cylinder 3³/₁₆ in. OD, 2¹/₄ in. ID, 7.0 in. high	57	339	350.0	3.1
10	Hollow cylinder 1.94 in. OD, 1.0 in. ID, 4.0 in. high	51	276	281.0	1.8
11	1 in. cubes	38	163	162.0	0.6
12	1 in. cubes	58	350	343.8	1.8
13	1 in. cubes	60	447	461.0	3.0
14	1 in. cubes	66	372	360.3	3.4
15	Block 9 in. long, 3 in. wide, 3 in. thick	46	231	231.0	0
16	Cylinder bonded to end plates, 4³/₄ in. diam, 2¹/₂ in. high	33	130	130.0	0
17	Cylinder bonded to end plates, 2¹/₄ in. diam, 5 in. high	40	177	171.0	3.5
18	Cylinder bonded to end plates, 1³/₄ in. diam, 3⁷/₈ in. high	52	288	270.0	6.6
19	Draft-gear rubber shown in Fig. 12	62	396	384.0	3.1

The author believes that the following conclusion can now be reached: The difference between the load-compression curves, with and without the use of vaseline as a lubricant, can be attributed to the high coefficient of friction between rubber and unlubricated plates.

This conclusion is further supported by tests on sponge-rubber disks. Load-compression tests made on 1, 2, 4, 6, 9, and 14 disks in a stack are shown graphically in Fig. 8. The disks were of medium sponge rubber with a cut edge, and were 3 in. outside diameter, 1 in. inside diameter, ⅜ in. and ¼ in. thick. Up to 25 per cent compression the points all fall very nearly on the same line. The air cells collapse and there is very little consequent distortion of the rubber.

Assuming that the coefficient of friction is the factor which destroys the ratio of 3 to 1 between the modulus of elasticity in compression and that in shear, is there a straight-line relationship of any quantity between these two moduli? The author has found that such a relationship exists if certain proportions of effective width to height are maintained and deflections limited to 15 to 20 per cent of original height. This relationship is based on empirical data. The practical application to be found in such data is that compression rubber, to be used as a load-carrying member, should have its deflection limited to 15 to 20 per cent of the original height in order to stay within reasonable values of creep in the structure. All compression-rubber slabs, disks, or cylinders, whether bonded to metal end plates or not, which the author has investigated, and where the ratio of thickness to effective width of rubber is equal to 1 or a little less, the load-compression curve is practically a straight line up to 20 per cent deflection. Under these conditions it has been found that the modulus of elasticity in compression is very close to 6.5 times the modulus of rigidity or shear.

What really happens is that the high frictional engagement between the rubber and loading plate, in not permitting the rubber to slip, causes a considerable proportion of the loading force to be dissipated as a force at right angles to the direction of load application. One has only to view a compression-loading test to be convinced of this.

Effective width is defined as the width of the maximum cross section of rubber. For a 1-in. cube this would be 1 in. For a rectangle 3 × 1 in., the value would still be 1 in. For a perforated slab the distance between holes would be the effective width; for a hollow cylinder it would be the wall thickness of the cylinder, and for a solid cylinder it would be the diameter.

The factor 6.5 holds remarkably well for a vast number of sizes and shapes of rubber articles used as compression members. If we call E the theoretical modulus of elasticity in compression and E_c the modulus in compression calculated empirically by means of the 6.5 factor, Table 3 shows values of each for a number of articles of various dimensions which have ratios of thickness to effective width of 1.0 or more. It is interesting to observe that article No. 1 of this table is one of the disks loaded by Morrison (3) between grooved steel plates. Up to 15 per cent compression, the curve was practically a straight line.

TABLE 4 CALCULATED E_c AND THEORETICAL E VALUES OF MODULI OF ELASTICITY FOR SMITH'S (4)[a] DISKS

Disk dimensions: Outside diam, in.	Inside diam, in.	Height, in.	Durometer hardness	E_c	E	T/W
3.50	1.000	1.0	60	372	357	0.80
3.50	1.000	1.0	60	372	357	1.00
3.50	1.000	1.0	60	372	357	1.60
3.00	1.125	4.0	54	307	306	4.26
3.00	1.125	4.0	67	459	453	4.26
3.25	1.000	2.0	60	372	324	1.89
3.25	1.000	2.0	60	372	324	2.59

[a] Number in parentheses refers to the Bibliography at the end of the paper.
NOTE: T/W = ratio of thickness to width.

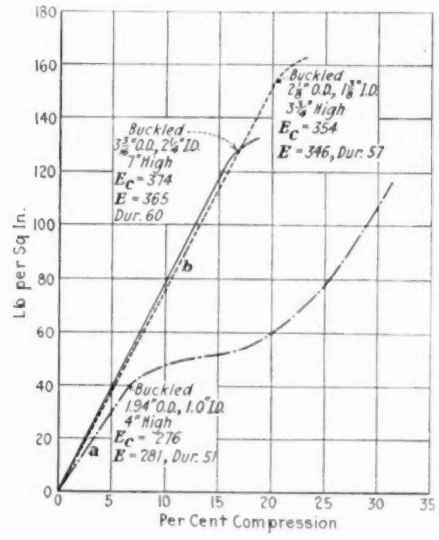


FIG. 9 LOAD-DEFLECTION CHARACTERISTICS OF RUBBER CYLINDERS LOADED AS COLUMNS

Smith has shown elsewhere (4) loading curves for various sizes of disks of various hardnesses. All of the disks, which have a ratio of thickness to effective width of 1.0 or more, can be calculated for modulus of elasticity in compression with the 6.5 factor with reasonable accuracy, as shown in Table 4. These values hold true for compressions up to 15 and 20 per cent of the original thickness. The only other curve in Smith's figure (4), which has a ratio of thickness to width of more than 1.0, does not seem to lend itself to this method of calculation. Beside the durometer hardness of 70 there is a question mark.

In loading rubber cylinders, one must maintain certain proportions of width to height and cross-sectional area in order not to get into trouble from collapse of the member when used as a column, especially when the member is to be subjected to lateral forces as well as vertical. Fig. 9 shows tests on 3 cylinders and their point of collapse. To determine what these proportions should be might well be worth the effort of an ambitious author who wishes to make a mathematical treatise of rubber cylinders, for rubber cylinders do afford a simple means of isolating machines where compliancy in both vertical and horizontal planes is desired.

Loading curves for 1-in. cubes are shown in Fig. 10 up to 30 per cent compression. The curves are practically straight up to 20 per cent compression.

Loading curves for rubber cylinders, listed as articles Nos. 8, 17, and 18 in Table 3, are shown in Fig. 11. These curves are straight lines up to 15 per cent compression.

Fig. 12 shows the cross section of a round ring type of rubber part which has been used in England for a great many years as buffers and as auxiliary springs on railroad cars and locomotives, known as Spencer-Moulton rubber springs. Holes in the steel plate tie in the rubber rings on each side. Adding together the cross-sectional area of the rings we find that the factor 6.5 applied even for this design.

In Fig. 13 is shown the effect of thickness on the compression curves of airplane shock-absorber disks. The curves for thickness of 1 in., $1\frac{1}{2}$ in., and $2\frac{1}{2}$ in. coincide up to about 25 per cent compression, indicating that where a ratio of thickness to effective width of approximately 1.0 or more is maintained the modulus of elasticity can be determined quite accurately. The factor 6.5 also applies for this case, up to the limit of recommended compression loading. Airplane shock-absorber disks for years were used at loadings up to 50 per cent compression of the rubber. High permanent set took place in these disks in the landing gears and they had to be replaced at frequent intervals. For load-carrying members, 20 per cent should be the limit for normal-load operation, exceeding this only on peak loads, for which dependence rubber is an ideal material.

Fig. 14 shows a loading curve for a cylinder of Neoprene designed to support a load of 1100 lb, also one for a rubber slab designed to support a load of 16,000 lb. The curves are straight lines in the working range. There seems to be no difference between the modulus of elasticity of Neoprene and that of natural rubber of the same durometer hardness. The rubber slab consisted of two steel plates with rubber bonded between them. A great number of holes through the slab allowed compliancy. One curve is for the first loading and the lower one for the fifth loading. This is characteristic of all compression-loading tests, but in this case the effect is somewhat exaggerated due to the network construction. Furthermore, the straight portion of the curves do not pass through the origin. This is undoubtedly due to lack of uniformity in the thickness of the slab, giving a greater apparent deflection near the origin.

In an effort to develop a formula for compression slabs of various sizes, thicknesses, and hardnesses, the author had tests

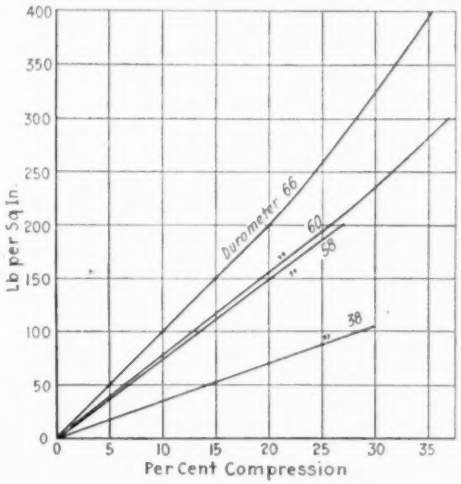


Fig. 10 Compression Loading on 1-In. Rubber Cubes

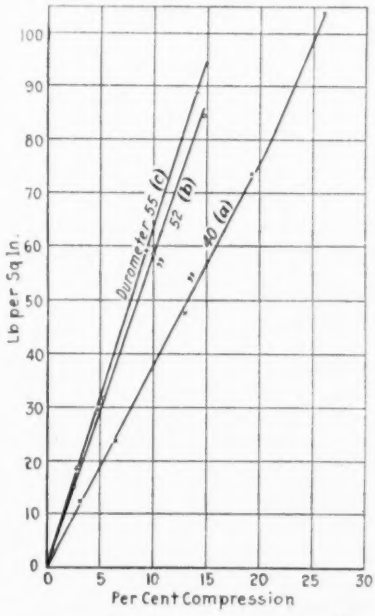


Fig. 11 Compression Loading on the Rubber Cylinders Listed as Articles Nos. 8, 17, and 18 in Table 3

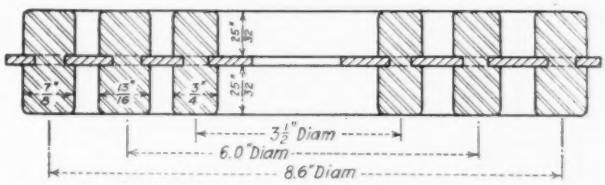


Fig. 12 Spencer-Moulten Circular Rubber Disks for Railroad-Equipment Buffers
(Durometer hardness = 62, E_c = 396 and E = 394.)

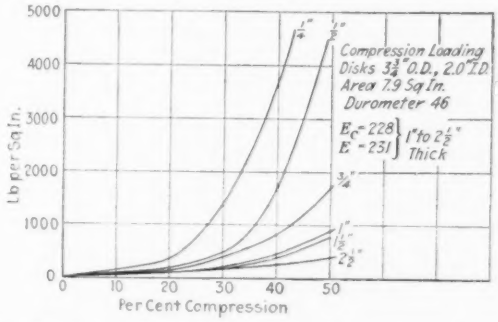


Fig. 13 Effect of Thickness on the Compression Curves of Airplane Shock-Absorber Disks

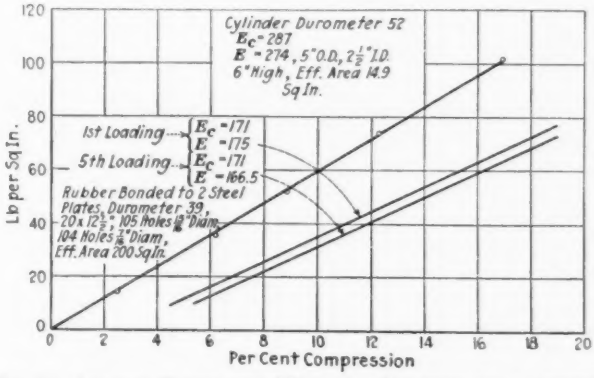


Fig. 14 Loading Curve for a Neoprene Cylinder and a Rubber Slab

conducted on slabs of the following sizes: 1 × 1 in., 2 × 2 in., 4 × 4 in., 6 × 6 in., 8 × 8 in., and 10 × 10 in., in thicknesses of 1/4, 1/2, 3/4, and 1 in., and hardnesses of 34, 40, 50, 60, and 71 durometer. The author has tabulated these data and to date can find no formula which will satisfactorily apply to them as a whole. Even the loading results on the 1-in. cubes of the various hardnesses, as shown in Fig. 15, seem to follow no definite formula beyond 20 per cent compression. It is possible that a nomographic chart can be constructed in order to make use of these data within liberal limits of accuracy.

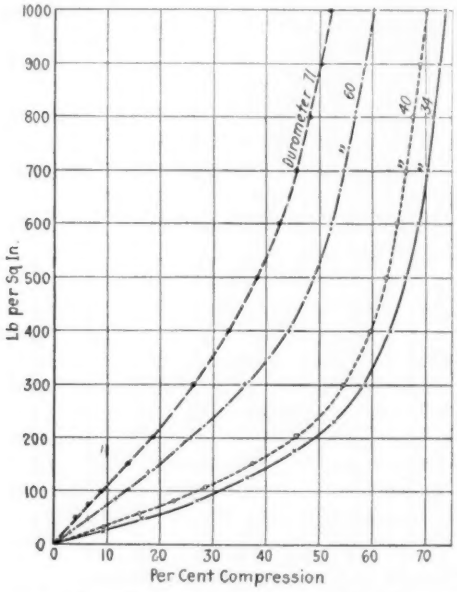


FIG. 15 HIGH COMPRESSION LOADING ON 1-IN. RUBBER CUBES

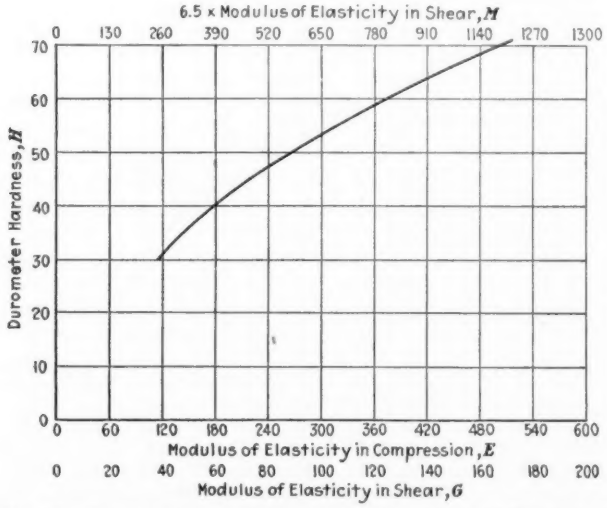


FIG. 16 RELATION OF THE MODULUS OF ELASTICITY OF RUBBER IN SHEAR TO THE DUROMETER HARDNESS

The design of shear-type rubber mountings has been quite thoroughly covered in work already published. A formula which holds reasonably well for plain shear sandwiches placed back to back, given an initial lateral compression, and loaded in vertical shear, is

$$d = WT/AG$$

where d = deflection, in.; W = load, lb; T = initial thickness of rubber in one sandwich, in.; A = area of two shear surfaces, sq in.; and G = modulus of elasticity in shear, lb per sq in.

The relation of modulus of elasticity in shear and durometer hardness is shown in Fig. 16. The data from which this curve was drawn are shown graphically in Fig. 17. The lateral compression placed on the rubber initially was 50 lb per sq in. An examination of these curves shows them to be straight lines up to 20 per cent deflection for the 60-durometer rubber and up to 40 per cent for the very soft rubbers. These data were taken after the mountings had been deflected to 1 in. deflection four times successively in order to work the rubber structure sufficiently to make it more representative of service conditions. Such preliminary loading not only loosens up the structure but removes some of the permanent set in the rubber.

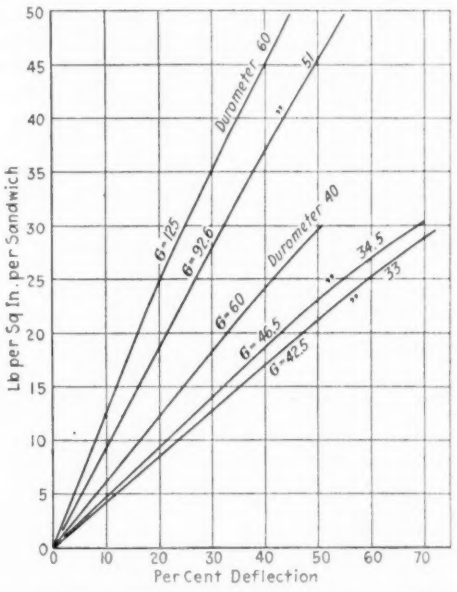


FIG. 17 VERTICAL SHEAR ON TWO 10 × 5-IN. × 1-IN-THICK RUBBER SANDWICHES
(Initial lateral compression of 50 lb per sq in. was used.)

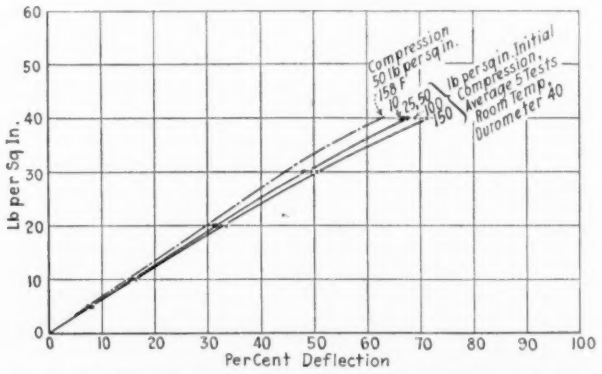


FIG. 18 EFFECT OF LATERAL PRESSURE ON SHEAR-LOADING CHARACTERISTICS OF 10 × 5-IN. × 1-IN-THICK RUBBER SANDWICHES

When the foregoing formula is used it should be noted that T is the initial thickness of rubber. It has been found that the degree of lateral compression placed on the rubber, within certain limits, does not materially affect the shear characteristics when the percentage deflection in shear is based on the original thickness of rubber. Tests made on 10 × 5-in. × 1-in-thick sandwiches, a size chosen by the Electric Railway Presidents' Conference Committee for their pioneer research in the develop-

ment of rubber springs for street-railway trucks, at initial lateral compressions of 10, 25, 50, 100, and 150 lb per sq in. showed insignificant changes in the shear loading curves when deflection was figured on the original 1-in. thickness of rubber, but when corrections for actual thickness were made for the various sandwiches, the values shown in Fig. 18 were obtained. It appears that for every value of compression except that of 150 lb per sq in., a mean curve can be drawn to give reasonable accuracy. A value of from 50 to 75 lb per sq in. compression would perhaps be a fair value to use in practice.

Fig. 18 also shows a curve for a sandwich tested under 50 lb per sq in. lateral compression at 158 F. The curve shows stiffening above that obtained at room temperature, which is to be expected because of the Joule effect. Stretched rubber, heated, undergoes a contraction in the direction of load and there is, therefore, a greater total load on the rubber in that position than before. At 30 per cent shear deflection for rubber of durometer 40, it is seen that the mounting deflects about 2 per cent less at 158 F than at room temperature. At temperatures below that of the room, the effect would be the opposite.

In Figs. 17 and 18 it is to be noted that the curves bend off toward the deflection axis after passing the point where proportionality ceases. The springs become weaker. Can this be attributed to bending of the rubber? If we assume that the rubber sandwich takes the form shown in Fig. 19, which it is reasonable to believe, and we figure the deflection of a beam fixed at both ends with an active length of beam of 2 in., for two 10 × 5-in. × 1-in-thick sandwiches of durometer 40, under 50 per cent deflection, at which point the total load is 3000 lb, we find the maximum deflection from bending to be only 0.0018 in. or 0.18 per cent when expressed in terms of original thickness of rubber of one sandwich. The curve in Fig. 17 indicates a departure of 1.7 per cent from the straight line at this place, or nearly 10 times that derived from bending.

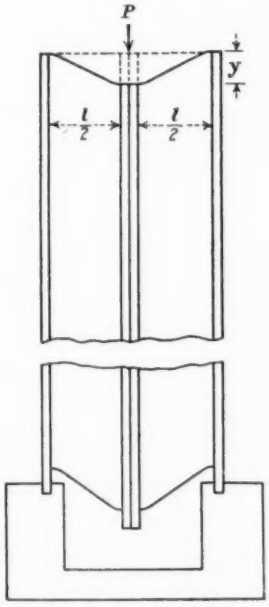


FIG. 19 ASSUMED FORM OF A RUBBER SANDWICH IN SHEAR

($y_{max} = Pl^3/192EI$, where y = deflection, in.; P = load, lb; E = modulus of elasticity, lb per sq in.; l = length of two sandwiches as shown, in.; I = moment of inertia = $bd^3/12$, where b = width of the sandwich, in.; and d = height of the sandwich, in.)

The characteristic stress-strain curve for rubber in tension perhaps accounts for most of the weakening of the shear spring after passing the point where proportionality ceases. For rubber of durometer 60 the stress-strain curve in tension starts to bend toward the deflection axis at about 15 per cent deflection while for rubber of durometer 40 it is at a point where the stretch is about 30 per cent. This softening effect in tension rubber continues up to about 100 per cent stretch for 40-durometer rubber before the stiffening action or point of inflection of the curve sets in. If stress were based on the instantaneous cross-sectional area of the test specimen during tension, the straight portion of the curve would be considerably extended. The S-shaped curve is linked inseparably with the structure of rubber; and, the theory of some writers, who think that rubber is a bundle of kinky fibers, explains this shape by saying that at the second point of inflection of the curve the slack has been taken out of the fibers. At some future time we undoubtedly will know the real reason.

If we deflect the rubber of the 10 × 5-in. × 1-in-thick shear sandwich 1 in. in shear, the elongation in a strand of rubber perpendicular to the two steel plates to which it is bonded will be 41 per cent, since the new length of this strand will be $\sqrt{2}$ in. This strand forms practically a straight line, although, as shown in Fig. 19, it is perpendicular to the plate for a short distance even when under load.

What is the practical limit of thickness of rubber to be used in shear-type springs? If, in the foregoing example of bending, we increase the thickness of rubber to 2 in., the deflection due to bending will be eight times that obtained with rubber 1 in. thick. The increased softening due to bending and the fact that the time of vulcanization of rubber varies approximately as the square of the thickness would seem to limit the maximum thickness to 2 to $2^1/_2$ in. per section. The thicker the rubber the lower the temperature of vulcanization to be used in order to obtain uniformly cured rubber throughout the mass, so the cost of curing becomes an important item where long-time cures must be used on very thick sections.

Rubber torsion bushings of the type shown in Fig. 20, can be designed by use of the formula

$$\theta = \frac{WL}{4\pi bG}\left(\frac{1}{r_1^2} - \frac{1}{r_2^2}\right)$$

where θ = angular deflection, radians; WL = torque, in-lb; b = effective length of rubber section, in.; G = modulus of elasticity in shear, lb per sq in.; r_1 = inner radius of rubber section, in.; and r_2 = outer radius of rubber section, in. In a design of this type the rubber is stressed higher near the center metal member than near the outer metal member because the volume of rubber increases with the square of the radius.

A typical loading curve of the bushing of Fig. 20 is also shown. The graph was made to show that after 120 oscillations through the loading range of 2750 to 6000 in-lb torque the spring rate becomes very nearly constant as the slope of the curve indicates; for this particular spring the rate is 100 in-lb per deg. Some hysteresis loss is shown between the loading and unloading curves even after 120 oscillations of the spring, but under a rapid cyclic test at a speed approximating spring action in service, the hysteresis loss would diminish very much below that shown. Hysteresis loss diminishes materially with temperature rise in the rubber and also with speed of oscillation. The inherent damping, therefore, of a spring of this type, using rubbers designed to keep the creep to a minimum, is of a low order, although the little there is may be beneficial under certain conditions of service.

Fig. 21 shows a double unit torsion spring of such design that nearly a straight-line characteristic is obtained up to 70 deg as shown by the graph. The heavy dot-dash line was plotted from the theoretical equation given above, showing close conformity to the empirical curve. The rubber units act in series to give

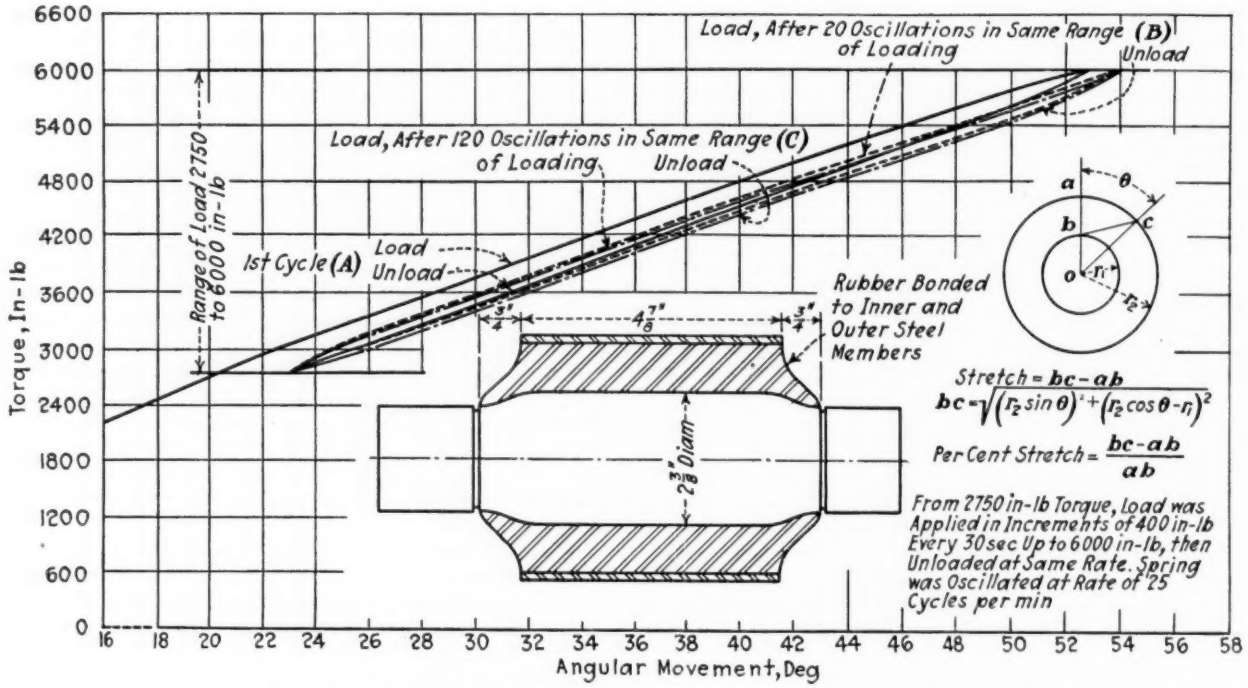


FIG. 20 RUBBER TORSION SPRING OF SPLIT-SHELL CONSTRUCTION AND HYSTERESIS-TEST DATA

twice the angular deflection for a given torque as would be obtained with one bushing. Series arrangements of this kind can be used to obtain very large distortions without going to excessive thicknesses of rubber wall.

Other constructions may be used, such as pressing a bushing, made up with 50-durometer rubber, into another bushing which has rubber of 40 durometer. These bushings are also in series arrangement for obtaining large angular movements, the hardness of rubber being graded off in order to obtain approximately equal angular movements in each bushing.

In bushings of this type the best practice is to bond the rubber to a central metal tube and to an outer shell split in halves. By doing this, full pressure can be obtained on the bonded areas during vulcanization. Inasmuch as vulcanization takes place anywhere from 270 to 320 F, the rubber on cooling, after being removed from the mold, may shrink as much as $^3/_{16}$ to $^1/_4$ in. per ft. The split shell permits this shrinkage to adjust itself. Furthermore, the split shell permits the rubber to be placed under radial compression when the bushing is assembled. It has been definitely proved that such compression very materially adds to the fatigue resistance of the rubber and reduces the amount of creep which takes place in the structure. Such initial radial compression has the added advantage of reducing the deflection of the inside metal member of the bushing in the rubber when placed under radial load. Such bushings can therefore be designed to make the rubber its own bearing with very little out-of-center movement of the inside metal member, thereby affording a complete rubber isolation between the inner and outer members of a rubber torsion spring.

Where torsion bushings are to be used as springs and the rubber is to form its own bearing without the aid of auxiliary bearings, it is important that the radial deflection of the central steel tube in the rubber be known. As the central shaft moves downward in the rubber, all that the rubber can do is to flow around the shaft, like fluid displacement. In bushings of the proportions in which the author has been working, where the ratio of wall thickness of rubber to central shaft diameter is under $^1/_3$ and the

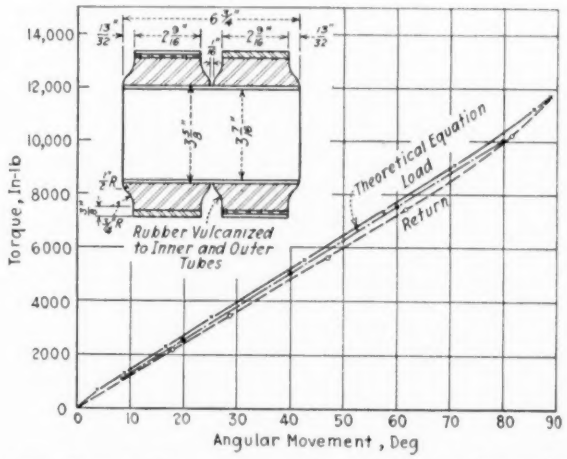


FIG. 21 TORQUE CURVE OF THE DOUBLE-UNIT OR SERIES-TYPE SPRING SHOWN
(Durometer hardness of the spring is 40.)

ratio of central shaft diameter to length of rubber section is under $^1/_4$, it has been found that

$$d = S_p/kE$$

where d = radial deflection of rubber, in.; S_p = lb per sq in. of projected area of central shaft or tube; E = modulus of elasticity of the rubber; and $k = 12$, approximately.

More data will have to be collected to determine this relation but that given previously seems to hold for a number of bushings tested, the extremes in size being for a bushing of 1 in. diameter shaft and a rubber section 5 in. long with a $^3/_{16}$-in. wall of rubber of durometer 60, the other for a bushing of $2^3/_8$ in. diameter shaft, and a rubber section 11 in. long with a $^3/_4$-in. wall of rubber of durometer 45. For this latter size of bushing, at a load of 120 lb per sq in. of projected area, the radial deflection amounted to only 0.03 in.

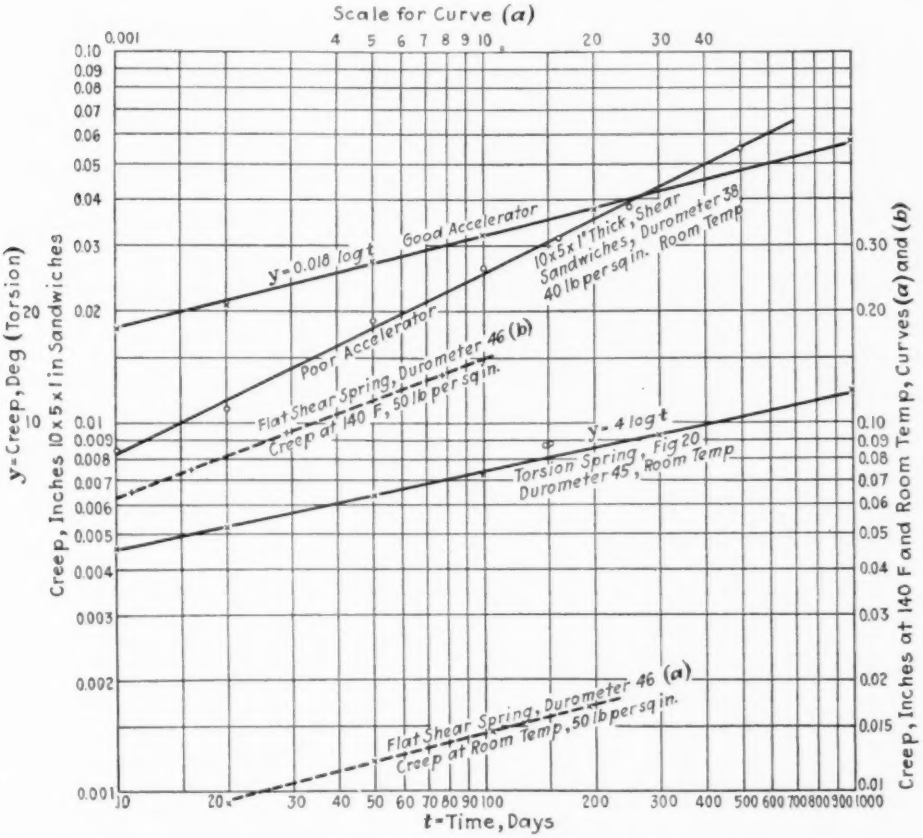


FIG. 22 CREEP IN RUBBER SPRINGS

The amount of stretch in the rubber of torsion springs is as important to control as is the stress at the area where the rubber is bonded to the central shaft, in order not to get excessive long-time creep. Referring to Fig. 20, the strand of rubber *ab* in the unstressed bushing may be said to take the position *bc* when the bushing is twisted through the angle θ. This line *bc* will have an inflection near each of the bonded surfaces, but for practical considerations it is very nearly a straight line. Therefore, we can estimate the stretch in the strand of rubber as being *bc* — *ab*. If we know the radii of the inner and outer bonded areas, then *bc* can be determined for any angle θ.

It is obvious from studying Fig. 20 that if a small r_1 is used with a thick wall of rubber, the percentage stretch in the rubber will be low but the stress at the bonded area may be dangerously high. Or vice versa, if the wall of rubber is small in proportion to r_1 the stress at the bonded area per inch length of bushing may be small but the percentage stretch in the rubber may be dangerously high. Both factors must therefore be watched in any successful design of torsion spring. The limitations of each of these two factors in design work depends to a large extent on the creep that is obtained on the particular compounds of rubber used in such springs. As indicated before, creep in the rubber is linked up with its physical and chemical structure. Merely changing the vulcanization catalyst, ordinarily called accelerator of cure, in a recipe may change the creep by 400 per cent. The problem then is to narrow down the list of compounds to be used to those of a specified range of hardness which are compounded to resist creep and at the same time will have a good flexing life. We know definitely that overcuring reduces creep, but shortens the flexing life. The creep in the best compounds designed for use in springs varies quite closely with the logarithm of the time.

A creep test on a torsion spring made up from one of the best low-set stocks of durometer 45 is shown in Fig. 22. This was obtained for the spring shown in Fig. 20. The stress at the bonded area was 110 lb per sq in., and the percentage stretch in the rubber under the constant torque load was 80 per cent. Although the maximum stress at the bonded area figured 110 lb per sq in. under 6000 in-lb torque, the mean stress in the rubber section is about 72 lb per sq in.

For the curves shown in Fig. 22, which quite closely follows the equation

$$y = n \log t$$

where y = creep in deg, and t = time in days; the torsion spring constant n is about 0.004. Also, in Fig. 22 are shown two creep curves for 10 × 5-in. × 1-in-thick flat-type springs. One is for a stock of 38 durometer with a poor accelerator for resistance to creep and the other for a stock of the same hardness which has a good catalyst of vulcanization. This curve for the latter compound follows the equation $y = 0.018 \log t$, where y is the creep in in.

In this same figure, curves (*a*) and (*b*) are for a $^5/_{16}$-in-thick flat-type spring of durometer 46, loaded to 50 lb per sq in. Curve (*a*) is for a test conducted at room temperature and (*b*) at 140 F. This would indicate that rubber springs should not be operated for long periods at elevated temperatures. The work of Lessig (5), with the Goodrich flexometer, also shows this effect. For two stocks compounded to resist creep, Lessig obtained the results, shown in the tabulation following on page 158, when pellets 1 in. high and 0.7 in. diameter were tested under 96 lb per sq in. compression at a stroke of $^1/_4$ in. and speed of 1800 rpm:

Durometer hardness	Permanent set at room temp, per cent	Permanent set at 212 F, per cent
45	0.8	8.4
33	1.4	10.16

Lessig also showed, by the same method, that high pigment loading increased the permanent set or creep in the structure. The temperature rise is a direct measure of hysteresis loss. Using a compression stroke of 0.175 in. and a loading of 140 lb per sq in., he tested four recipes at optimum cure with the only change being that in the gas black content. The results are shown in Table 5. From what the author has been able to collect to date,

TABLE 5

Compound	Volume loading of gas black	Temperature rise during flexing, F	Permanent set measured after flexure, per cent
A	0	27	1.1
B	10	37	1.4
C	20	56	3.1
D	30	89	8.8

after studying a tremendous amount of data, it appears that rubber stocks with high hysteresis effects have high creep, and vice versa. This observation is not inconsistent with the theory of elasticity, and is supported by the recent work of Kosten (6) who showed that in measuring the static and dynamic properties of rubber in compression the internal loss indicated "the elastic imperfection" of the compound. Naunton and Waring (7) recently measured the change of modulus at various frequencies by means of a special resonance type of testing machine. Fig. 23

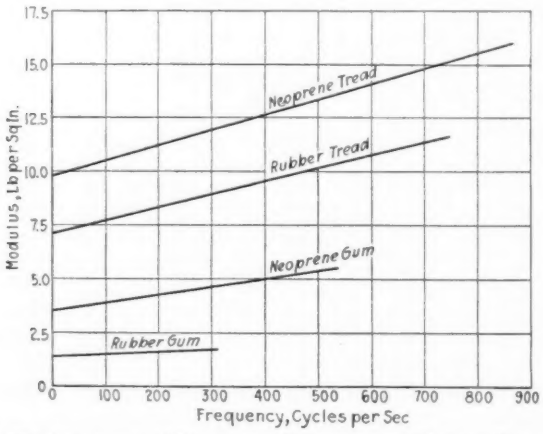


FIG. 23 CHANGES OF MODULUS OF ELASTICITY OF VARIOUS RUBBERS WITH FREQUENCY

shows some of their results graphically. The least change of slope is in the curve for rubber gum, indicating that the heat developed by hysteresis loss was least in this compound since we know that modulus stiffens with temperature rise. It would appear, then, that springs designed to incorporate appreciable damping must be worked at lower stresses than otherwise, in order not to incur trouble from creep.

It was shown previously that a shear or torsion spring has a stiffer load characteristic at elevated temperatures, due to the Joule effect. Similarly, the spring would have a weakened characteristic at low temperatures for the same reason were it not for the fact that the crystalline structure of rubber causes it to stiffen at low temperatures. The following results are for shear-type springs of rubber durometer 33 when loaded to 30 lb per sq in. stress, the results being an average of three tests on each spring:

Durometer hardness	Deflection, per cent, at Room temp	Deflection, per cent, at −20 F	Deflection, per cent, at −63 F	Stiffening, per cent, at −20 F	Stiffening, per cent, at −63 F
33	2.13	2.05	1.93	3.75	9.40
40	1.45	1.40	1.31	3.44	9.65

These tests are the result of a single load placed on each spring after the spring had been reduced to the required temperature for several hours. In service the internal working of the rubber of a spring, even though in an atmosphere of low temperature, will develop some heat and raise the internal temperature of the spring to counteract the stiffening effect.

The author has presented some of the characteristics of rubber in which engineers may be interested from a design standpoint, without going into detail on every phase which might affect design. There is still a vast amount of data which should still be collected, especially to establish accuracy on some points that seem to be in doubt. In order to obtain such data accurately it will be necessary that the tests be conducted in rooms where the temperature, and possibly the humidity, is closely controlled. The accuracy of tests to determine the moduli of elasticity and the creep of shear and torsion springs is very decidedly affected by the temperature because of the Joule effect. The previous history of a spring, as to the number of loadings to which it has been subjected, must always be considered in analyzing results. True measurements of hardness are especially difficult because of the human element which still enters into such measurements. As time goes on, rubber men will undoubtedly standardize on basic recipes for springs, accurately determine their fundamental characteristics, and together publish them.

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Discussion

Carbonaceous Zeolites—An Advance in Boiler-Feedwater Conditioning[1]

HUGH R. CARR.[2] The methods described by the author are well illustrated and the numerical values are comparable, however, the writer believes that the question of residual hardness was not covered fully.

The author stated that, following the zeolite treatment, phosphate aftertreatment is added to react to the hardness in the condensate returns. The writer has found that there is less hardness in the condensate than in the effluent from zeolites, and his experience with one acid-zeolite system and many sodium-zeolite systems has indicated that residual hardness in the effluent runs from 4 to 9 ppm as $CaCO_3$. With an acid-zeolite system the writer found that on the average, with 1,000,000 lb evaporation and 40 per cent make-up, 9 lb of disodium phosphate was required to maintain soft water and 50-ppm phosphate alkalinity in the boiler. Repeated analysis of the condensate indicated no hardness. In this case no appreciable chlorides were present in the condensate, indicating no hardness from circulating water, since the circulating water was sea water. The residual hardness must certainly be taken into account and no treating system should be designed without provision for full aftertreatment and disposal of the sludge made by such aftertreatment. Cost comparisons should account for the cost of aftertreatment of residual hardness.

The writer believes that the author takes much for granted with reference to the removal of CO_2 by the cold degasifier; apparently, he assumes a slightly alkaline water for degasification. Either method *A* or method *B* described by the author would indicate such to be the case, at least for the degasification at the feedwater heater. The writer's experience has indicated that the cold degasifier on slightly alkaline water removes very little CO_2; however, the writer believes that the performance would be far better on slightly acid water, say pH of 6.0 or less, which most feedwater heaters will handle, since the condensate from boilers treated with mono- or meta-phosphates rarely shows more pH than this, a serious fault with phosphates.

The author mentioned that the carbonaceous zeolites do not add silica to the water. It has been the writer's experience that any properly selected zeolite adds little or no silica to the water, that is, thousands of times less silica than is carried in the raw water. The author failed to mention that zeolites of any type fail to have any beneficial action on the effect of silica in boilers. The aftertreatment required to eliminate the silica content in the boiler should be taken into account when comparing zeolite systems with other types of external treatment.

In discussing permissible boiler concentration the author mentioned 2500 ppm total solids as a general limit. The writer believes that this is too broad a generalization. It has been the writer's experience that with phosphate aftertreatment, the suspended-solids concentration is the first limitation in dealing with waters of low permanent hardness and/or low nonincrustant salts, but the total-solids concentration is the limiting factor with waters of high permanent hardness and/or high nonincrustant salts. In comparing costs this should be taken into consideration. It is possible that by the use of coagulants or sludge deconcentrators, instead of phosphate aftertreatment, the suspended solids from residual hardness might be cleared away sufficiently to maintain 2500 ppm or more total boiler solids.

It should also be pointed out that the cost of softener backwash should be taken into consideration when comparing zeolite systems with other methods, such as internal treatment, lime-soda-phosphate, and external hot-phosphate systems such as are used in Germany.

AUTHOR'S CLOSURE

Mr. Carr first states that according to his experience with one hydrogen-zeolite system and many sodium-zeolite systems, the residual hardness in the effluents averages from 4 to 9 ppm as $CaCO_3$. It will be noted in Tables 7 and 8 of the paper that the average total hardness of from 0 to 2 ppm was used for the zeolite effluents. These figures are based on actual experience with several large-scale hydrogen-zeolite plants and literally thousands of sodium-zeolite units. Furthermore these figures have been checked by the gravimetric method as well as by the soap-titration method and the Blacher method. Confirmation of these facts may be found in many papers that have been published in the technical press from time to time by disinterested writers who have described their experience with such equipment. Interested persons who wish to confirm these facts may readily do so by having analyses made of the actual effluents of such plants. It can only be concluded that Mr. Carr's experience in this connection has been limited to units that were either improperly designed or operated, or were operating on waters of abnormal quality.

As an illustration, Mr. Carr states that in a given case 9 lb of disodium phosphate was required per million pounds of evaporation. It is impossible from these figures to conclude what portion of the phosphate was used for hardness precipitation since he does not indicate whether the phosphate was anhydrous or otherwise; nor does he state whether the 50-ppm phosphate in the boiler was expressed as PO_4; nor the ratio of concentration in the boiler saline compared to the feedwater. For example, if he were using $Na_2HPO_4 \cdot 12H_2O$ he would require 3.78 ppm of this phosphate for each ppm of PO_4. Assuming a concentration ratio of 20 times the concentration in the feedwater, and 50 ppm PO_4 in the boiler salines, he would require 2.5 ppm of PO_4 in the feedwater to maintain this 50-ppm PO_4 concentration in the boiler salines and replace the PO_4 lost in the blowoff. This would require $2.5 \times 3.78 =$ over 9 ppm of phosphate which would indicate that both the make-up and the condensate were free from hardness. This particular calculation may not apply in the case in point but it merely illustrates the need for detailed data in checking phosphate-consumption figures.

Under the heading, "Chemical Results on Two Typical Waters" in the paper, each of the factors influencing the amount of phosphate that must be fed to the boiler feedwater was enumerated, and Tables 7 and 8 made provision for such phosphate aftertreatment on the basis of certain definite assumptions which were stated in the paper.

With reference to the CO_2 removal performance of cold degasifiers, Mr. Carr states that according to his experience very little CO_2 can be removed by such equipment, particularly when the water is slightly alkaline. This statement conflicts with com-

[1] Published as paper FSP-60-11, by Howard L. Tiger, in the May, 1938, issue of the A.S.M.E. Transactions, p. 315.

[2] President, Mech-Chem Engineering, Inc., Newark, N. J. Mem. A.S.M.E.

mon experience in various branches of water conditioning such as the treatment of well waters high in CO_2 and iron, where aeration is commonly practiced for CO_2 removal and iron precipitation. There are any number of data to prove that degasifiers remove most of the CO_2 in the cold and there are in operation at the present time several of these H_2Z plants where initial CO_2 contents of from 100 to 200 ppm CO_2 are being continuously reduced by such equipment to less than 10 ppm CO_2. This slight residual CO_2 is then effectively removed in the feedwater heater. Such results are usually guaranteed and can be readily verified by anybody interested in inspecting such installations.

Mr. Carr is correct in stating that the carbonaceous zeolites do not remove silica from water. However, it is well known that with siliceous zeolites there may be appreciable silica pickup, particularly when the influent to such equipment is low in silica content, as is the case after proper silica-removal treatment or with waters naturally low in silica. In fact, some of the most prominent consulting engineers in the country have already authorized replacement of siliceous zeolites by carbonaceous zeolites operating on the sodium cycle for the sole purpose of avoiding such silica pickup and a number of installations of this type are operating in some of the largest steam-producing plants in this country.

The 2500-ppm total-solids concentration used for the calculations in Tables 7 and 8 were merely illustrative and Mr. Carr is quite correct in stating that one cannot generalize broadly on the maximum allowable concentration. This was well illustrated in a group of papers[3] that was recently published which showed boiler concentrations varying from 250 ppm in the case of Dayton Power & Light Co. to 1800 ppm in the case of Virginia Electric & Power Co.

Mr. Carr is doubtless correct also when he states that the suspended solids such as result from phosphate aftertreatment may in certain cases limit the concentration that may be safely carried in a boiler.

In his last paragraph, Mr. Carr states that the cost of softener backwash should be taken into consideration. Reference to Tables 7 and 8 of the paper shows that this point was specifically covered in the note under the table of chemical costs where the volumes of waste water for backwashing, rinsing, and so on, are discussed in detail. These waste-water figures were not converted into terms of dollars and cents because such conversion obviously depends upon local costs of water which may vary within wide limits. Thus an illustrative figure would be meaningless but with the data set forth in the note, the actual cost for any specific case can be readily calculated.

Influence of Pressure on Film Viscosity in Heavily Loaded Bearings[1]

H. C. Dickinson.[2] The writer does not believe that the author is justified in extending the term "oiliness" to cover effects of pressure changes on viscosity as made on page 356 of the paper in reference to the works of Herschel[3] and Hersey.[4] Herschel's original definition referred to liquids "of the same viscosity and at the same temperature, and under identical operating conditions." The writer is sure that Herschel's intent was to define conditions in which the mechanical effects of viscosity are identical. Changes in viscosity due to temperature are so well recognized that Herschel, like everyone else, recognized the need of eliminating this factor. If important similar effects of pressure had been anticipated, it is a fair assumption that Herschel would have included the same limitation here. In other words, we are accustomed to think of "oiliness" as applicable to differences between oils which do not depend upon differences in the mechanical effects of viscosity itself.

Considering the problem more in detail the classical equations deal with fluids, the significant mechanical properties of which are constant throughout the region and during the process which they represent. Any departure from the constancy of viscosity obviously entails an approximation in the use of the equations. It has long been recognized that the viscosity in the equations must be that at the bearing temperature. The same is true, of course, of pressure, except that the effects of changing pressure are smaller than those of changing temperature.

No one seems to have taken into account the fact that the temperature in any heavily stressed oil film is not constant, even through the thickness of the film. It would be difficult if not impossible to solve the equations with the inclusion of this additional factor. However, we should not lose sight of the fact that the effect of variable viscosity is included, nevertheless, on account of both temperature and pressure variations within the bearing.

If the classical equations could be so modified as to include an approximation to the effects of varying viscosity, and still be capable of solution, an important step in progress would be made. However, to solve the problem completely, we should be able to introduce both the pressure and the temperature effects on viscosity, as well as the end-leakage and tangential-flow corrections, the general effects of which are well recognized.

F. C. Linn.[5] The test results and applied theory which the author has presented in this paper[1] give information which is of extreme value for low-speed heavy-duty bearing applications. It is interesting to note that for this particular type of service a fitted liner is essential in order to obtain satisfactory bearing qualities. The question of bearing clearances is one wherein there is a great deal of misunderstanding. Some designing engineers use the same clearance regardless of speeds and loads, which, of course, is a mistake, since at low speeds and high loads the author shows that it is essential to operate with fitted liners, whereas, at high speeds, that is, velocities of 100 to 200 fps, large bearing clearances are required. A case was recently called to the writer's attention wherein the clearance of a 4-in. journal operating at 400 rpm and loadings up to 1000 lb per sq in. was 8 mils. Bearing failures occurred after a few hours' operation. Failures were eliminated by decreasing the bearing clearance to 3 mils. On the other hand, a 4-in. bearing operating at 10,000 rpm should have bearing clearances of from 12 to 16 mils.

The problem of starting and stopping high-speed journals is one with which designing engineers are continuously faced. High-speed bearings should have relatively large clearances for optimum operating conditions at speed. The large clearances are unsatisfactory at very low speeds, but shafts have to be started and stopped. This problem may be outside the scope of the paper under discussion. However, the author undoubtedly made some tests which may be applicable to this problem and a discussion of them would be beneficial. In other words, has the relationship of maximum load versus bearing clearance for various oil viscosities been determined for very low speeds?

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[3] "Power Panel Discussion on New High-Temperature High-Pressure Stations," Trans. A.S.M.E., vol. 60, July, 1938, paper FSP-60-13, pp. 411–429.

[1] Published as paper RP-60-7, by S. J. Needs, in the May, 1938, issue of the A.S.M.E. Transactions, p. 347.

[2] Chief, Heat and Power Division, National Bureau of Standards, Washington, D. C. Mem. A.S.M.E.

[3] "Viscosity and Friction," by W. H. Herschel, Trans. S.A.E., vol. 17, part 1, 1922, pp. 282–320.

[4] "Theory of Lubrication," by M. D. Hersey, John Wiley & Sons, Inc., New York, N. Y., 1936, p. 124.

[5] Turbine Engineering Department, General Electric Company, Lynn, Mass.

It is gratifying to note that the author has one explanation for the property previously known as "oiliness," and it is quite possible that as further experimental work is conducted the indefinite property of the film to withstand high loads at low speeds, which has been attributed to oiliness in the past, will be definitely understood.

A curve of calculated minimum friction (λ_a) is presented in Fig. 2 of the paper, which calculation is based upon end-leakage condition. In Fig. 9 of the paper are calculated curves of λ and λ_a based upon the conditions of $r = 0.0000755$ (lb per sq in.)$^{-1}$ and $r = 0$, respectively. Since the calculation of the coefficient of friction in Fig. 2 has been made taking into effect end leakage, the writer wonders why the author did not include the effect of end leakage in his formula, instead of basing his theoretical work upon the assumption of an infinite-width bearing.

The increase in viscosity with pressure has the effect of increasing the coefficient of friction at the higher loads and yet, when the observed values are plotted on a curve of coefficient of friction versus ZN/P_A, there is a less variation between the observed curve and the calculated curve for minimum friction at the lower values of ZN/P_A than there is at the high values. A discussion of this by the author would be appreciated. It is also interesting to note that the curve of the coefficient of friction versus ZN/P_A for minimum friction is a straight line when plotted on log-log paper. This fact has been recognized and log-log paper has been used by the writer in presenting test data on high-speed bearings. Because of being able to draw straight-line curves on log-log paper for coefficient of friction versus ZN/P_A, it is very convenient to extrapolate beyond the range of test data in predicting bearing performance.

G. L. Neely.[6] The author covers very comprehensively the subject of the effect of viscosity rise through increase in pressure in heavily loaded bearings. His data appear to be accurately determined and he presents very convincing arguments that there is an increase in viscosity of the oils in heavily loaded bearings that markedly influence the performance. The method of preparing the bearings is of particular interest, since in most work it has been found that bearings become better after running-in and are more easily lubricated after use than when freshly prepared. The fact that the opposite was true in the case of the very heavily loaded bearings is further evidence that ordinary conditions do not prevail when such loads are used.

It is true that differences between the lubricants, that might be called oiliness by Herschel's definition,[3] were found in the heavily loaded bearings, but it is doubtful that this should be called a true oiliness effect. Such results might be obtained in other testing machines such as the Almen, Timken, or other devices employing suitable friction-measuring equipment, as well as in commercial bearings. In the absence of adequate pressure-viscosity data, it would seem preferable to consider the phenomenon one of oiliness rather than to ignore it completely. On the other hand, in work on oiliness conducted in the laboratories of the company with which the writer is affiliated, we have endeavored to set up testing conditions where the effects of pressure are not measurable, that is, where the coefficient of friction is not influenced by degree of loading. This condition eliminates the necessity of correcting for viscosity or pressure-viscosity effects that cannot be adequately evaluated.

In view of the author's work and certain previous work along similar lines, such as that of Dr. Everett of The Pennsylvania State College, there seems no doubt that the viscosity of the oil in a journal bearing is influenced by the pressure conditions in that bearing. The value of this work for practical applications is such that it can be very profitably continued

[6] Standard Oil Company of California, Richmond, Calif.

G. B. Karelitz.[7] No chain is stronger than its weakest link, and no machine is sounder than its weakest detail. It is therefore sometimes strange that our machines, notably prime-mover engines, work at all. Considering, for instance, a turbogenerator or an internal-combustion engine; the operation of these depends on the behavior of an oil film in its bearings. Should the 0.001-in. or 0.004-in-thick oil film refuse to perform, a major disaster would occur to the equipment. This paradox was brought home to the writer rather forcibly when reports were made on the maiden trip of the Italian liner *Conte di Savoia*. On that trip her power-generating equipment failed due to breakage of an outboard discharge valve, and consequent flooding of the generator room. At the time the gyrostabilizers were in operation, of which the rotors weigh 110 tons each, and rotate at 900 rpm. Failure of the power supply stopped the motor-driven oil pumps, and there was no oil to the bearings. It takes four hours to stop these runners with all the braking available. The crew had an anxious time hand-pumping grease into the bearings of the gyroscopes. One can easily imagine what would happen should one of the bearings seize.

Therefore it is only appropriate that the performance of this oil film, which has such an important function, should be studied in minute detail; at least as much as the strength features of other parts of machines.

It happens that the equations involved in this study are complicated, often defying solution. However, engineers cannot be obstructed by this fact and approximate solutions have been worked out resulting in tables and graphs at the service of designers. It is the task of research engineers to bring the problems encountered to that stage of usefulness. From this viewpoint the paper is very important and timely.

The steel-mill industry is now reequipping its mills with rationally designed bearings, to save both power and the high cost of bearing replacement. The designers of these bearings will welcome this thorough and well-founded contribution, particularly so because the theoretical findings are compared with tests, or rather because the theory presented explains so well the test observations reported.

There is no doubt that the results published by the author will be of value in many other fields where heavily loaded bearings are employed. It may be wished at present that the large amount of available information on the mechanism of an oil film be brought together in a coordinated and simplified form, so that designers may use it conveniently.

Author's Closure

Mr. Linn has observed that since increase in viscosity with load increases friction coefficients to values above those calculated, it would be expected that when test results are plotted against ZN/P_A, as in Fig. 17, the agreement with theory should become closer as the load is reduced. At low values of ZN/P_A the observed friction coefficients lie above the theoretical curve because of increased viscosity due to pressure. Since the pressures are not great enough to influence viscosity at high values of ZN/P_A, closer agreement with theory should be found in this region. That such is not the case is due principally to inaccurate gage readings at low pressures. In Fig. 17, the value $ZN/P_A = 40$ corresponds to a gage reading of approximately 200 psi. The lowest value of ZN/P_A on the same chart corresponds to a gage reading of 5000 psi. Observations to the right of $ZN/P_A = 40$, forming more than half of the chart, were taken at gage readings below 200 psi. In the type of pressure gage used, inaccuracies at the ends of the scale are to be expected. At higher loads the

[7] Professor of Mechanical Engineering, Columbia University, New York, N. Y. Mem. A.S.M.E.

percentage of error is small but at lower loads it becomes appreciable.

Mr. Linn's question as to why the influence of end leakage was not included in the analytical work is readily answered by the fact that this inclusion would make a rigorous mathematical solution impossible. The question is more fully discussed on page 350 of the paper.

The author has no experimental data to offer at the present time on the effect of varying clearance at low speeds. However, reference (6) in the bibliography of the paper will throw some light on this problem from the analytical point of view.

With reference to the comments of Dr. Dickinson and Mr. Neely regarding the paragraphs on oiliness, it may be well to repeat that from the observed facts it can only be concluded that the phenomenon known as oiliness can be produced by the influence of pressure on viscosity. The author recognizes that the viscosity-pressure effect was not what Herschel had in mind when he published his definition of oiliness; and in all probability it is not the same thing that Neely refers to as "a true oiliness effect." It is unfortunate that the cause or causes of oiliness are not definitely understood. However, it was not the author's intention to suggest viscosity-pressure effects as a general explanation of oiliness, but merely to call attention to the fact that the outward signs of the two are similar. It should be remembered that the experiments indicating pressure effects on viscosity were made with complete fluid films, whereas oiliness is generally associated with incomplete or boundary lubrication.

The author is of the opinion that eventually it will be shown that oiliness is due to some property, or group of properties, yet undefined. It has yet to be proved, however, that oiliness is entirely unrelated to viscosity.

Tests of a 7 by 10½-In. Bearing at 3600 Rpm[1]

H. Steen-Johnsen.[2] In spite of the volume of data available from research investigations on bearing performance, data that pertain to the particular problems of turbine-generator bearings are still extremely meager. The author points out that information available covers the majority of applications. Unfortunately, the turbine bearing does not fall in this majority classification; however, with the tests discussed in the paper, research is taking a step in this direction, in so far as a standard turbine bearing has been tested at 3600 rpm.

In studying the author's results we find that they show closer agreement with earlier tests at lower speeds, and the theory developed from those tests, than they do with the performance observations made on turbines in commercial service. The writer notes in the test results that a larger proportion of the losses are due to the top half and the side reliefs than was found in earlier tests at lower speeds. In this connection it would be of interest if the author would elaborate on the theoretical method used for calculating the side reliefs and the top half of the bearing. The writer presumes it is done by using the expression for a guide bearing. It would also be of interest to see some of the readings obtained from the thermocouples located in the babbitt and the oil film.

In the case of the turbine bearing, the major problem that faces the designer of the lubricating system after the size and type of bearings has been decided upon is the quantity of oil required by the bearings and the amount of cooling capacity necessary in the oil cooler.

If we use the bearing-test data available, a certain kilowatt loss is obtained, which corresponds to a certain quantity of oil at the temperature rise selected. The oil requirement obtained in this manner is on the order of from one half to two thirds of the actual quantity required to keep the bearing in question cool. In other words, considerably more heat is removed from a turbine bearing than is indicated by the kilowatt loss due to friction in the oil film. This difference between calculated and actual heat removed becomes greater as the operating temperature of the unit increases.

For the majority of turbines operating at 750 F and above, the quantity of oil required becomes quite indeterminate, unless previous experience is available. The journal of the bearing in question supports a rotor which at a distance of approximately 18 in. from the journal is at a temperature of say 800 F. Naturally, heat will flow from the rotor toward the journal, and in order to maintain a suitable operating temperature of the bearing, the heat received from the rotor must be removed by the oil in addition to the heat generated in the bearing.

The writer has had occasion to check the temperature of the oil and babbitt in the lower half of a bearing operating under these conditions, and the temperatures observed were comparable to those of the bearing discharge temperature. But we do not know the temperature of the oil film that adheres to the journal, nor do we know the temperature of the journal at the surface, nor the temperature of the inside of the journal.

Shutting a unit of this type down, we know that within a very few minutes the temperature of the babbitt in the bearing reaches approximately 200 to 250 F. We also know that if the bearing be dismantled quickly and a thermocouple be held against the journal and covered with asbestos, a reading on the order of 300 to 350 F can be obtained.

The additional amount of oil required to carry away this heat conducted through the spindle to the journal is supplied to the bearing by having special grooves provided in the top half for additional oil flow, and also by increasing the oil-supply pressure to the bearing, so as to force more oil through the clearance space.

One of the first questions which presents itself in connection with the particular phase of lubrication that is experienced in turbine units is: What is the best method of supplying the additional oil for cooling the journal? Should this additional oil be provided by increased supply pressure, or by increased passages, and at what points in the bearing should the additional oil be supplied and drained, in order to have the best cooling effect and the best chance of removing the film that adheres to the journal so that oil at inlet temperature will at all times be available for the film on which the journal floats?

The heat from the rotor also brings up another problem peculiar to the turbine bearing. The average temperature along the diameter of the journal will be considerably higher than the temperature of the bearing shell. The resultant difference in expansion between the journal and the bearing will decrease the clearance ratio an unknown amount, and thus increase the losses and decrease the oil flow.

Some engineers believe with a water-cooled gland the temperature of the journal will not be much higher than the temperature of the bearing. However, it is difficult to subscribe to this opinion after calculating the rate of heat transmission from the narrow gland runner to the water that surrounds it. Most of the wall surface that surrounds the water in the gland is the gland case, bolted to the cylinder, and most of the heat absorbed is from the gland casing, conducted to it through the cylinder.

Even though the paper[1] covers tests of the type of bearing that is used in turbine design, there still is a great amount of work to be done before a clear picture can be obtained of the performance of a turbine bearing. The present tests have unquestionably

[1] Published as paper RP-60-8, by L. M. Tichvinsky, in the July, 1938, issue of the A.S.M.E. Transactions, pp. 393–397.

[2] Engineer, Turbine Division, Westinghouse Electric & Manufacturing Company, Philadelphia, Pa. Jun. A.S.M.E.

furnished desired information on the kilowatt loss in the bearing in so far as the rotational losses are concerned, but it does not touch upon the actual oil requirements of a turbine bearing in commercial service.

Design of High-Capacity Boilers[1]

M. K. DREWRY.[2] The emphasis that this paper places upon furnace temperatures and furnace heat transmission (not upon furnace volume) suggests that the formerly used poor criterion of furnace conditions, namely, Btu per cubic foot, is at last being discarded like the term boiler horsepower.

That tangential firing causes a boiler-inlet gas temperature 200 F lower than horizontal or vertical firing (all with wet-bottom design) deserves attention. This difference in thermal performance seems related, for instance, to better radiant-heat absorption with pulverized coal than with stokers. It recalls that radiant-heat transfer from burning fuel particles is much greater at the same temperature than from heated gases.

The method of firing that stores the least heat in hot gases, which is most nearly isothermal, and which facilitates the best heat absorption from the burning fuel particles, causes lowest boiler-inlet temperature. This statement suggests that the combination of dry-bottom design and of firing methods that create most combustion near heat-absorbing surfaces, will result in minimum boiler-inlet temperature.

Dependability requirements are well emphasized in the paper. The value of high reliability in reducing investment costs is usually underestimated. Designers and operators ought always to strive for higher availability, even through years of low loads.

For the ultimate in steam-temperature regulation, disposition of superheating and evaporating surfaces similar to that indicated in the paper seems inevitable. Side-by-side positioning of steam and water tubes cancels all variables and creates rugged inherent control that is truly dependable. The paper indicates that demands of higher steam pressures and temperatures have already exceeded capacity of the convection superheater. Experience with radiant superheaters will be valuable when ultimately more heat is absorbed in superheating than in evaporating.

H. J. KERR.[3] There are certain points in the author's paper which the writer would like to discuss.

Fig. 1 of the paper apparently indicates a box-header boiler. It might be misunderstood to represent a sectional-header boiler. Fig. 2 of the paper represents a sectional-header boiler of the double-deck type, though the baffling arrangement shown would hardly be used except for some very special reason. These two figures are given in the paper as representing boilers having inadequate circulation for high capacity.

Fig. 4 of the paper may be taken to represent a four-drum Stirling boiler. It is presented as a type of boiler that has inherently some circulation difficulty. Incidentally, it is referred to as having some limitations on good steam quality. In this figure the steam outlet is shown on the middle drum, an obsolete ar-

[1] Published as paper FSP-60-17, by John Van Brunt, in the August, 1938, issue of the A.S.M.E. Transactions, p. 485.

[2] Assistant Chief Engineer of Power Plants, Milwaukee Electric Railway and Light Company, Milwaukee, Wis. Mem. A.S.M.E.

[3] Executive Assistant, The Babcock & Wilcox Co., New York, N. Y. Mem. A.S.M.E.

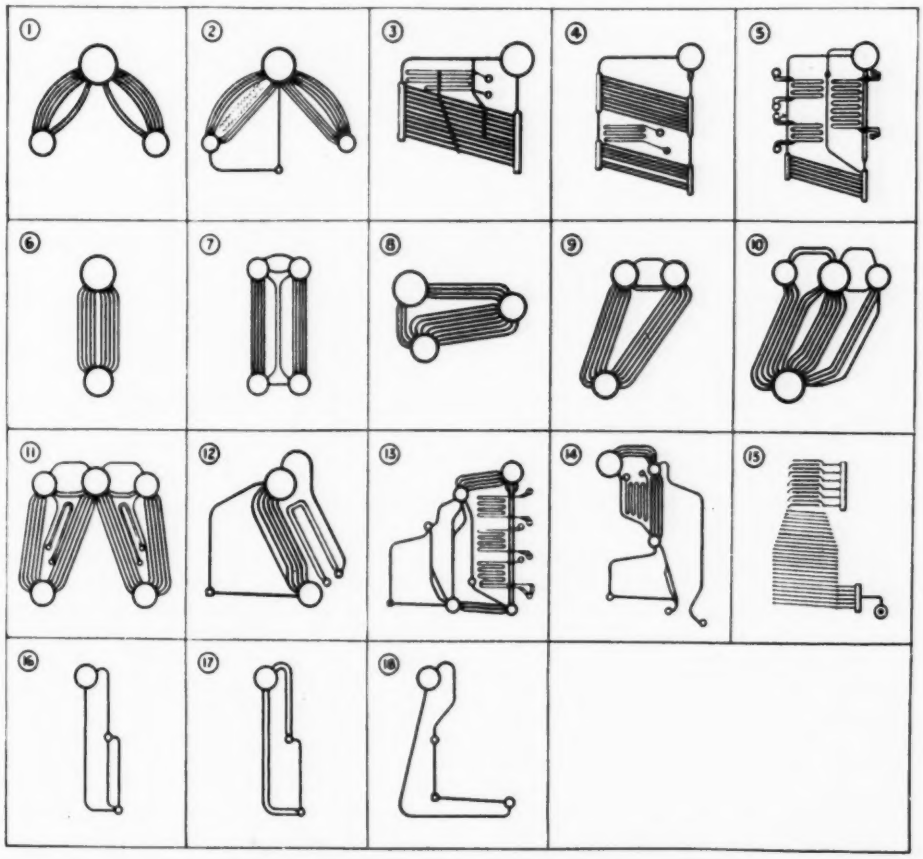


FIG. 1 VARIOUS TYPES OF BOILERS AND WATERWALLS

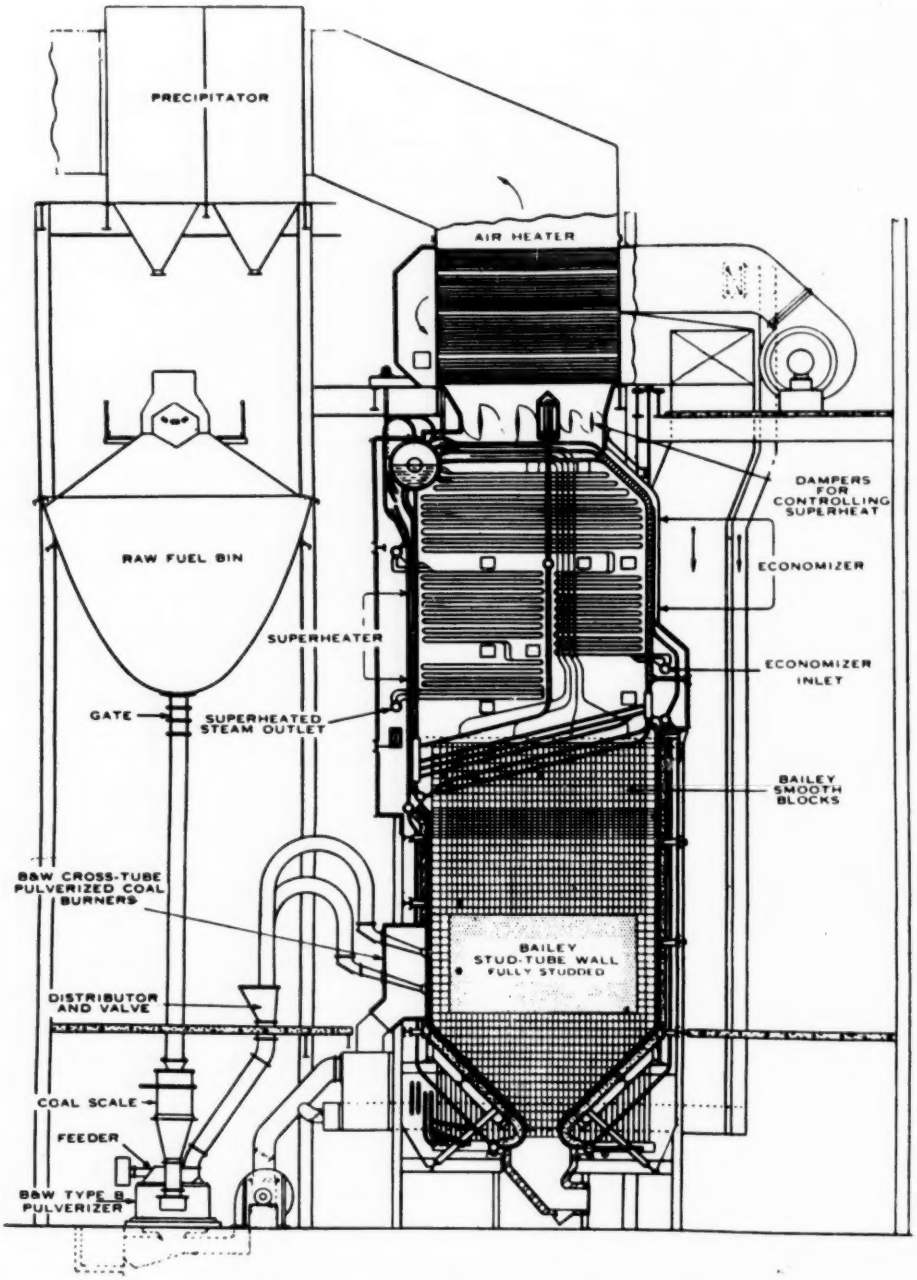


FIG. 2 RECENT INSTALLATION FOR DAYTON POWER & LIGHT CO.

rangement which disappeared 17 years ago. It is hardly believed that the author's statements on poor steam quality are intended to include this type boiler when the steam outlet is on the rear drum. The writer knows of no type of boiler capable of carrying such high concentrations as this type unit has successfully handled with high steam quality and steaming capacity.

The paper seems to indicate that the author believes certain types of boilers are immune from circulation trouble while others are not. In Fig. 1, herewith, are shown diagrammatically some fifteen types of boilers and three types of waterwalls all of which have been, or are being built by the company with which the writer is connected. The boiler types represented are familiar: The three-drum type, so closely related to Yarrow's classic test work on circulation; the express-type marine units which operate with a furnace heat release of 400,000 Btu per cu ft; the sectional-header types, many of which are in service throughout the world; the various so-called bent-tube types including the four-drum Stirling boiler; the integral-furnace boiler; the open-pass boiler of 1400 lb pressure, lately installed at West End Station in Cincinnati; the Essex type unit now being put into service; and the series-pump-circulation boiler with from 100 to 200 lb pressure differential between the ends of the circuits. The figure also shows a recirculation waterwall, a straight-through-circulation waterwall, and a furnace waterwall connected to floor tubes of the furnace. In regard to all these the writer can say definitely that each boiler has a circulation problem as is also the case with the waterwalls. It is necessary to recognize that fact and design the unit on reliable data and experience.

On page 487, referring to tangential firing, the author states that, from his data, the furnace exit temperature for a given furnace would be 200 F lower than with horizontal or vertical firing in the same furnace due to greater heat absorption, thus indicating the superiority of this type of firing in comparison with other burners and burner arrangement.

Since there is great difficulty in obtaining true gas temperatures in a furnace, it would add to the value of the paper if the author would give, in as much detail as possible, the method used in obtaining these temperatures. It would be particularly interesting if he would advise whether high-velocity thermocouples, bare-wire thermocouples, or optical pyrometers were used. It is recognized that there may be 200 or 300 deg difference obtained by these various methods of measuring temperatures in addition to the variation which will be obtained over the furnace cross section perpendicular to the gas flow.

Obviously, the conclusion reached by the author as to the superiority of this method of firing is not concurred in by others.

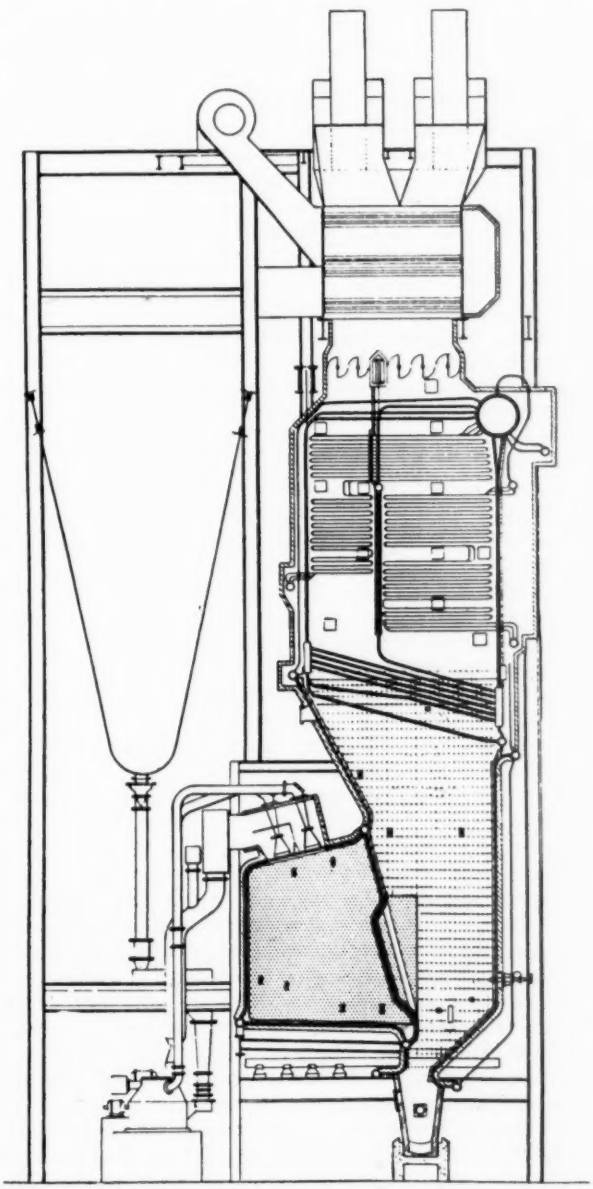


FIG. 3 SHOWING TWO-STAGE FURNACE

The writer particularly refers to the reports on tangential firing as carried out at Buffalo at about the time when slag-tap bottoms and direct mill firing became effective. The work at this station, supplemented by a great deal more experience since, may lead others to the quite different conclusion that the type and position of burners in each case must be considered and also the arrangement of furnace before a general statement can be made of the superiority of one method as compared with another.

In checking over Fig. 9 (of the author's paper) at 200,000 Btu release per sq ft of furnace envelope cooling surface, the writer understands this to mean that the average heat absorption per square foot with tangential firing is approximately 100,000 Btu. Since part of the furnace walls are necessarily slag-covered it might be expected that the local transfer rates might greatly exceed this average. Information on this point, particularly in connection with high-pressure units, would be of great interest.

One might understand the author's statements to mean that an economizer should be insured against steaming. Units in service for many years have demonstrated that the steaming economizer is an economical, dependable piece of equipment.

Dealing with waterwall design, the author states it is good practice to avoid any recirculation in waterwalls. The successful operation of a great many waterwalls with recirculation is evidence that this statement cannot mean that recirculation is bad or that absence of recirculation is superior. The opposite may be true. The writer has a considerable number of data showing that the increase in concentration in the wall in comparison with the concentration in the drum of the boiler is relatively small and the following is a typical example in terms of sodium chloride: Drum, 54.5 ppm; bottom header of the wall, 54.0 ppm; top header of the wall, 57.0 ppm. In some cases the concentration in the drum may be higher than in the wall itself. It would be of great interest if the author would present any complete data which he has contrary to these results.

Figs. 2 and 3 herewith represent two high-capacity high-pressure units, which will serve to bring out the various points which have been discussed. Fig. 2 represents two high-head units put into service last year at Dayton Power & Light Co., Dayton, Ohio. These units were designed for and operate at 375,000 lb of steam per hr at 1250 lb pressure with a constant steam temperature of 900 F from 40 per cent of load to full load. The furnace release is close to 40,000 Btu per cu ft.

(*a*) Actual measurements taken on the job show that the water flows upward in all tubes at a rate in excess of that which is required for the heat absorption and no marks whatever of water levels can be found.

(*b*) Waterwalls are designed for recirculation. There has been no difficulty whatever.

(*c*) The steaming economizers give approximately 12 per cent steam by weight and are operating without difficulty.

(*d*) Constant superheat is referred to in the paper. This is an example of what can be accomplished by a simple arrangement of parallel gas flows. The units have operated at 900 F steam temperature from full rating to below 40 per cent of capacity and in addition at full load the steam temperature has been held below 850 F.

Fig. 3 shows one of the units installed at the Rivesville Station of the Monongahela West Penn Power Company. The units were designed for 350,000 lb of steam per hour at 1250 lb pressure and have been operated materially in excess of this capacity. In general, the units are similar to the Dayton units and designed on the same principles. The primary difference is the two-stage furnace which, in this case, is operating with coal containing ash of 2100 F fusion temperature. The heat release at normal load is 36,000 Btu per cu ft.

This furnace design and the results being obtained show that

exceptionally good results can be obtained by other methods of firing.

T. E. PURCELL.[4] This paper contains specific information on existing installations, data based on scientific facts, and the opinions of the author on various designs and operating characteristics. In general the writer agrees with the paper. However, there are a few items upon which he wishes to make comment.

Unquestionably, the use of pulverized fuel has made it possible to build extremely high-capacity superboiler units. There are, however, some who will not agree that units of this type are commercially justified. In this respect each individual installation must stand upon its own merit. That the use of water-cooled furnaces is absolutely necessary where exceedingly high capacities are to be developed is generally conceded. It should be the ambition of design engineers so to proportion the boiler and furnace surfaces as to cause a complete solidification of the ash before the closely grouped boiler tubes are reached in order that troublesome slag accumulations in the gas passes may be avoided.

It is suggested that the greater development and use of radiant superheaters may offer a possible solution to the superheat problem where gas temperatures must be reduced to a low value to avoid slag accumulations in the steam-generating tubes.

The lower gas temperature reported with tangential firing when compared with vertical or horizontal firing is, of course, the result of more rapid heat transfer to the waterwalls when the flame is directed tangentially. There is, however, a definite limit in any installation to the rate at which heat can be transferred to a given amount of surface without causing its failure. Because of this limitation, there is a possibility of developing vertical or horizontal burners to utilize more effectively the entire furnace volume and obtain equally effective heat transfer with burners of either of these types.

Data with respect to friction losses of mixtures of water and steam flowing in tubes would be valuable. Determination of the flow characteristics over a broad range of conditions should make an excellent subject for study by some research foundation.

The writer believes that a certain percentage of recirculation in furnace waterwalls can be used without detriment or danger of excessive concentrations since usually, where feedwater treatment is properly controlled, the concentration of salts can be materially raised without the deposition of solids. To avoid excessive impurities in the steam, it is usually necessary to limit the concentration of the water in the boiler proper much below the point where any detrimental effects to heating surface, thin scale formation, or deposition of solids will result.

The impurities in the steam which are given at 0.7 ppm with water washers is not exceptionally good in view of the fact that there are many installations operating without steam scrubbers which are producing steam, the impurities in which do not exceed 0.2 ppm.

The author states that the limit in the range of operation of the so-called wet-bottom furnaces is often restricted to from 0.75 to 1. This ratio is far greater than the load factors on most power systems and the installation of units having such a narrow range of capacity may be a great handicap inasmuch as one of the most difficult problems faced by the operating man is that of taking care of boilers during idle periods. The greater the number of units with restricted range of capacity, the greater the number that will have to be taken out of service during low-load periods. It should be the aim of the boiler designer to increase materially the economical operating range of boiler units.

[4] General Superintendent of Power Stations, Duquesne Light Company, Pittsburgh, Pa. Mem. A.S.M.E.

E. H. TENNEY.[5] The author's reference to the difficulty of obtaining high superheat with ash of low fusing temperature and at the same time avoiding slagging of boiler tubes and damage to the superheater itself is one of particular interest to burners of some Illinois coals. It is a problem which can be met in part by careful distribution of steam through the superheaters so as to cool the tubes adequately. Our new Cahokia boilers designed for a steam release of 260,000 lb per hr have thirteen leads from the steam drum to the superheater inlet header to assist in obtaining adequate cooling effect. The author mentions that the width of a boiler furnace, which it is economical to keep down, is largely determined by questions of draft loss and rates of steam release. It might also be well to add that space for burners is frequently the limitation; that furnace width in many designs may be definitely determined by the room required to place the burners across the front wall.

Reference is made to the three T's of combustion—time, temperature, and turbulence. As the author says in effect, referring to Mr. Kreisinger's comments on certain Bureau of Mines studies, the highest temperature is not always conducive to the most rapid combustion. It has been our experience at Cahokia that the element of time may also be overstressed. With good mixture of air and coal practically complete combustion is achieved in less than 0.6 sec, which requires less than a fourth of the distance from burner tip to boiler tubes. In reference to turbulence, the writer prefers to consider the problem one of mixing coal and air which can be done in the burner where mechanical surfaces can be interposed in the coal stream to accomplish mixing effectively. It has been demonstrated that maintaining a relative velocity between a moving gas stream in a furnace and a buoyant burning particle of pulverized coal is mathematically limited to an extremely short distance and time. It is like trying to throw a feather against the wind. Thus, as the author indicates, a special interpretation of the three T's is necessary in designs of modern high-capacity boiler furnaces.

AUTHOR'S CLOSURE

The author is not entirely in agreement with Mr. Drewry's statement of the method of firing that will produce the lowest boiler inlet temperature. Tangential firing in a dry-bottom furnace will produce exit temperatures equally as low as in wet-bottom furnaces but not at the same rate of combustion. With dry-bottom furnaces and tangential firing, the burners must be located a greater height from the floor of the furnace to avoid slagging of the ash pit.

The author believes that rapid combustion in the bottom part of the furnace and a high scrubbing velocity of hot gas on the water-cooled furnace walls are the most effective methods of obtaining a low boiler-inlet temperature.

Mr. Drewry's comments on the emphasis on dependability are appreciated. Dependability should be the aim of the designer and a requirement of the buyer. If steam pressures and temperatures go much higher than are now used, radiant superheaters or reheaters will be necessary.

In reference to Mr. Kerr's comments, Figs. 1 to 5 are intended as diagrammatic representations of a few boiler types and are not intended to illustrate any particular make.

The author fully agrees with Mr. Kerr's statement that the Stirling boiler with outlet on the rear drum is capable of high steaming capacity with high steam quality.

The author does believe that certain types of boilers are free from circulation troubles up to the capacities and pressures at which such boilers have been used, and at the same time will deliver steam of satisfactory quality. Certain other types also

[5] Chief Engineer of Power Plants, Union Electric Light & Power Co., St. Louis, Mo. Mem. A.S.M.E.

have good circulation but at high capacity cannot deliver dry steam. Referring to Fig. 1 in the discussion, the author believes Mr. Kerr will agree that on types 1 and 2 it has been necessary to add downcomers outside of the gas passage in order to avoid overheated or steam-bound tubes at high capacities. Types 3 and 4 will, when operated at extremely high capacity, have partly dry tubes in the high end of the upper tubes. The author doubts that the type-8 boiler would be satisfactory from a circulation standpoint at high pressure and at evaporation rates of 15 lb per sq ft. No comment is called for regarding types 11 to 18, inclusive, except that it does seem probable that in some of them the circulation might be a problem.

As to furnace exit temperatures and method of determining them, such temperatures have been taken with a bare platinum-rhodium thermocouple. The highest reading with such couples before any slag or ash adhered was assumed to be the gas temperature subject to a radiation correction of a few degrees. The temperatures were checked by figuring back from the temperature of gas leaving the superheater to the temperature of gas entering the superheater, to which was added the probable temperature drop through the front row of boiler tubes.

As to the superiority of the tangential method of firing, no claims are made but the results obtained are satisfactory to many users. It is true that this method is not universally applicable.

Mr. Kerr's assumption that the rate of absorption of 100,000 Btu corresponds to a release of 200,000 is not entirely correct. For 200,000-Btu release the absorption may be anywhere between 100,000 and 70,000 depending on many factors. Local rates of absorption may well be in excess of 100,000.

A steaming or nonsteaming economizer may be considered a matter of preference or an unavoidable feature of design. The author's preference is for a nonsteaming economizer.

No statement was made that the use of recirculators in waterwalls is poor practice or that walls without such circulators are superior. The author stated and believes that the use of walls without recirculators constitutes good practice.

Comments on the balance of Mr. Kerr's discussion would be controversial and would serve no useful purpose. The author feels, however, that the discussion is timely and will be helpful in bringing out certain details not covered in the paper.

Mr. Purcell's suggestion of the use of radiant superheaters is logical but the problem of application in a furnace having a high rate of heat release is not simple. Port Washington[6] presents a good example of what can be done in a low-heat-release furnace.

The limit of heat absorption in waterwalls has not yet been reached. If water can be kept flowing through furnace wall tubes there seems no reason why absorption rates of 200,000 Btu per sq ft cannot be safely obtained. There is undoubtedly a limit but this limit has not been reached in commercial furnaces.

Equations for friction loss of water flowing through tubes give widely varying results. Little is known of friction losses of flowing mixtures of water and steam. Certain empirical equations are used in the design of the La Mont boiler which have given reasonably satisfactory results. A study suggested by Mr. Purcell would be valuable in the art of boiler design.

The author agrees that recirculation in waterwalls may be used without danger. Referring to Fig. 16 in the paper it will be seen that risers from side and front waterwalls completely cover and protect the upper part of the furnace. Recirculators on this unit are definitely not desirable.

As to the performance of steam washers, the accuracy of determinations of $^1/_2$ ppm to within $^1/_2$ part are open to question. The purpose of a steam washer is to remove those salts that, when present in high-pressure steam, deposit on turbine blades to an extent that definitely reduces the output of the turbine. Any washer which prevents such deposits and enables the turbine to carry its rated load indefinitely is performing satisfactorily regardless of whether tests of quality show $^1/_2$ or 1 ppm.

The range of operation of wet-bottom furnaces depends on many factors. With ash of low or moderate fusion temperature, the range may be 1 to 3, and under some conditions of fluctuating load 1 to 5.

Mr. Tenny is quite correct in stating that the width of furnace is often fixed by the number of burners required. It may be well to add here that to meet these conditions by using fewer larger burners is not always possible nor desirable.

As to Mr. Tenny's comments on time, it should be borne in mind that time in combustion is relative. Turbulence is necessary to reduce time. Change of direction of coal and air streams, impingement of flame on walls, or streams of coal and air directed against each other produce turbulence and additional mixing. By such methods most of the combustion is completed quickly, or in a short period of time. The time required for a given completeness of combustion will depend then on the type or method of firing. This is the case with either solid, liquid, or gas fuels.

[6] "Design Features of Port Washington Power Plant," by G. G. Post, *Combustion*, August, 1933, p. 11.

Investigation of Stress Conditions in a Full-Size Welded Branch Connection[1]

J. M. Aiton.[2] The writer has read this paper with great interest because he has lately carried out a series of tests to destruction on similar branches, and has formed the opinion that the type of reinforcement adopted by the authors is not the best available.



Fig. 1 Failure of Welded Branch With Reinforcing Ring

The writer's experiments were carried out on pipe $12^1/_2$ in. OD × 0.400 in. thick.

In one of the tests a reinforcing ring was used equal in thickness to the pipe wall and, therefore, in proportion somewhat wider than that used by the authors. This failed in the form of a long tear at the crotch extending into both the body and the branch. Fig. 1 shows two views of this branch after failure.

The final design of reinforcement consisted of a strap and

[1] Published as paper FSP-60-12, by F. L. Everett and Arthur McCutchan, in the July, 1938, issue of the A.S.M.E. Transactions, p. 399.

[2] Aiton & Co., Ltd., Derby, England.



FIG. 2 THE STRAP-AND-CRAVAT WELD

cravat, as shown in Fig. 2. This burst in the body of the pipe remote from any welding.

Fig. 12 in the authors' paper, which shows their manifold after test to destruction, does not appear to be a very good result, because the tear seems to start adjacent to the welding of the reinforcement in that part of the body, the structure of which may have been affected by the welding. This does not prove that the branch and the metal adjacent to it is as strong as the original. The writer's test proves conclusively that the strap-and-cravat method of reinforcement produces a branch which, together with the metal adjacent to it, is stronger than the main pipe.

F. C. FANTZ.[3] The authors of the subject paper have contributed a great deal toward the solution of the problem of reinforcing branch connections in welded piping. It is believed that certain things should be pointed out for further thought and study on the part of the authors and those engineers interested in design.

First, the paper states, "The ring type shown in Fig. 1*c* which has been used extensively because of its simplicity and the fact



Fig. 3 Fig. 4

FIG. 3 SADDLE FOR REINFORCING WELDED BRANCH CONNECTIONS

FIG. 4 METHOD OF APPLICATION OF SADDLE

that it can be made without die equipment. . . ." It is assumed that the reference is to a plate ring that could be rolled to the radius to fit the header and have the proper hole cut for the branch pipe.

Fig. 4 shows a reinforcing ring extending above the center line of the header approximately 2 in. along the side of the branch pipe. It is not clear how this reinforcing ring could be made to extend along the branch as shown, without the use of dies. Fig. 1*c*, as well as the photograph of the destroyed manifold, Fig. 12, does not show this extension along the branch but rather shows the type of ring described. It is agreed that the extension as shown in Fig. 4 is beneficial at the critical section, which is the flat side of the intersection. It is also believed that this extension should go beyond the top of the header in order to reinforce the entire flat section. It is pointed out in the paper that considerable deformation took place at this flat section above the reinforcing ring.

[3] Vice-President, Midwest Piping & Supply Co., Inc., St. Louis, Mo.

The primary object of reinforcing a branch connection in a header is to compensate for the material cut out of the header, and to take care of the irregular shape of the cross section at this point. In order to compensate properly for this irregular shape, the reinforcing ring or saddle should have a ring section encircling the branch connection to a point at least above the top of the header. Such a reinforcing saddle is shown in Fig. 3 herewith.

Fig. 4 shows the application of this saddle and the method of making the branch weld before the saddle is welded to the branch and header. After the welding of the branch, the saddle is slipped down to a fit on the header and welded for strength. It should be pointed out that the weld on the saddle is not for tightness but for strength alone.

In this connection it may interest engineers who are applying either this type of saddle or the ring type on structures that are to be stress-relieved or annealed after fabrication, to know that a hole should be drilled in the ring or saddle to prevent generation of pressure during the heat-treating process of the assembly, at which time this section is heated and is in a more plastic condition. Serious deformation has been observed when this precaution was not taken.

The saddle method of reinforcing does require the use of dies in the manufacture of the saddle. These saddles however can be obtained from stock of several manufacturers in sizes and combinations up to 24 × 24 in.

Tests to destruction have shown that the flat surface on the side of the manifold will not distort to any appreciable extent due to the reinforcing of the ring band of the saddle. This extended ring also tends to prevent stress concentration at the junction of the branch and header caused by the expansion and contraction of the members joined to the header. It can be assumed that the perfect pipe layout would be one in which all members and connections are of the same cross-sectionalstrength and degree of flexibility. Since perfection is an ideal seldom achieved the designer should refrain from too rigid sections at any one point in the line. The saddle type of reinforcing shown in Fig. 3 was designed with the thought that reinforcing would not concentrate undue stresses at any one point. A comparison of the three types shown in Fig. 5 demonstrates this principle. With the saddle type the strength weld of the saddle is not connected rigidly to the

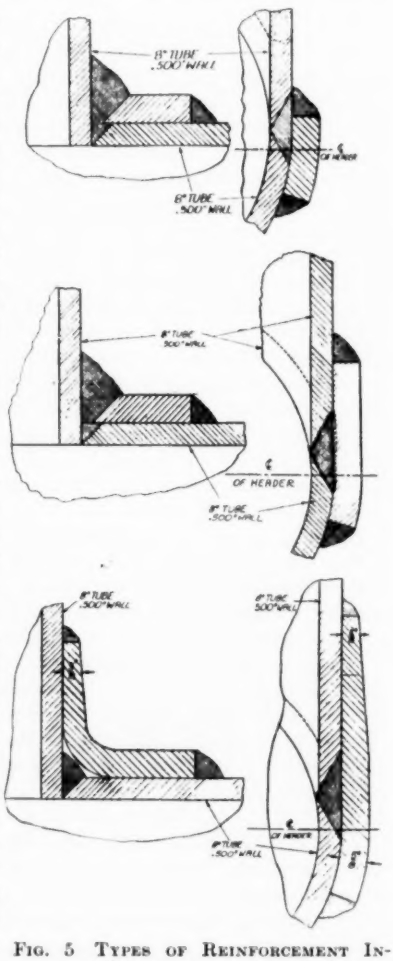


FIG. 5 TYPES OF REINFORCEMENT INDICATING FLEXIBILITY OF SADDLE TYPE

weld that holds the pressure and considerable flexibility is allowed without impairing the strength of the connection.

It is believed that the method of preparing the branch and header for the weld that is to hold the internal pressure, whether or not it is to be reinforced, could be improved. This of course will have nothing to do with the method of reinforcing or the stresses set up by the application of the reinforcing ring. However, it is conceded that the easier it is made for a welding operator to prepare the connection and deposit the metal, the better chance there is for obtaining uniform good-quality welds.

In the preparation illustrated in Figs. 3 and 4 of the paper, detail *A* shows the branch inserted to the ID of the header, and the header wall at this point beveled to 45 deg. The branch connection changes from a square cut to a 60 deg angle shown in the cross section. This method requires a very careful layout and

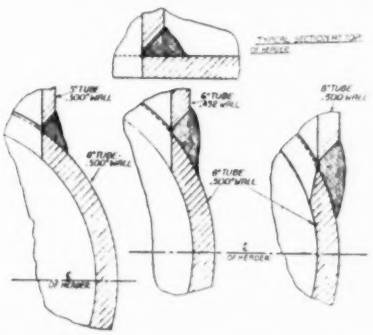


FIG. 6 SUGGESTED METHOD OF CUTTING HEADER FOR BRANCH CONNECTIONS

fit in order to have the proper welding groove. In cases where the distance from the center line of the header to the end of the branch pipe must be made to a fixed tolerance, the operator is faced with making the proper cut in both the header and the branch the first time, and cannot do any subsequent cutting to fit. This is true in a size-on-size connection and, of course, the difficulty is increased with a branch of smaller size than the header.

What is thought to be a better method of preparation for a branch connection is shown in Fig. 6. In all cases where this method is used the hole in the header is cut to the ID of the branch pipe and parallel to the side walls of the branch. This does not require any elaborate technique; in the case of small branch pipes the hole can be drilled or bored and the larger sizes can be cut with the acetylene cutting torch, guided either manually or by machine. It should be noted that considerably less material is removed from the header by this method than by the one shown in the paper.

The preparation of the branch pipe is a true development of a cylinder intersection and can either be laid out on a paper template or cut by a machine-guided torch which is capable of developing this intersection. Such machines are on the market. In the case of a hand-manipulated torch it is held at a constant angle with relation to the axis of the pipe. This manipulation is not difficult for the average welder.

It should be remembered that as the application of welding increases, angle connections will be used to a greater extent and the design and reinforcing of this type of connection is more difficult. It is also thought that the application of the ring type of reinforcement does not lend itself to this type of connection.

D. S. JACOBUS.[4] Tests of the sort described in the paper are most useful in establishing safe and dependable designs; the authors and The Detroit Edison Company should be commended for conducting so painstaking and exact an investigation.

The design of branch connections requires widely different methods, depending on the conditions and the working stresses involved. Designing the penstocks and branch connections at Boulder Dam was a far more involved problem than that of designing piping and fittings for the usual power-plant construction.

The penstocks at Boulder Dam had a maximum diameter of 30 ft and the hoop stress was made to correspond to about one half the yield strength of the material. The effect of the gravity component of the contained water had to be taken into consideration in order to limit the amount of departure from a truly circular cross section and to avoid undue stress concentrations at the points of support. As brought out in an article[5] published in *Mechanical Engineering*, photoelastic investigations were made by Prof. S. C. Hollister. Four models of branch connections were made $^1/_6$ scale and a large model of a girth joint, 15 ft in diameter using plates $2\ ^3/_4$ in. in thickness to which water pressures were applied and strain-gage measurements made to determine the stresses.

In the case of steel flanged fittings which are covered by the A.S.A. standards, the body thicknesses were determined by an approximate rule which was found to conform fairly well with the existing practice of a number of manufacturers, and this approximate rule, through extended use, has been found to give safe and consistent results. The two cases are cited to show the wide difference in the way in which the problem may be handled to meet different conditions.

In the ordinary construction of boilers unreinforced holes are used into which tubes are expanded into the drums and in some cases for attaching nozzles. The Boiler Code Committee was asked to include rules to give the allowable size of such openings. The problem was referred to a number of authorities but no rules of a form that could be embodied in the Code were suggested. The Boiler Code Committee finally published the rules referred to, which were based on data secured through the use of a great number of constructions which had given satisfaction in long-time service. As brought out in the paper under discussion, the rules were limited to a narrow field, which was, however, broad enough to cover any construction normally used in boilers. The rules have been criticized as not being based on a scientific stress analysis, and in each case the Boiler Code Committee has requested those who made the criticism to suggest other rules of a more complete and scientific nature, but so far none has been presented for the consideration of the committee. It is gratifying to note the statement in the paper that despite the limits indicated by the Boiler Code Committee, branch connections made up with reinforcements which compensate for material removed in cutting the branch opening, in accordance with the Boiler Code rules, have been found to develop the full strength of the pipe under hydrostatic tests to destruction.

G. S. LARSEN.[6] The authors are to be congratulated on starting an investigation in a field, which so far has been too little explored. Our general knowledge of the stress conditions in and around welded branch connections is extremely limited and the question of whether the neck itself constitutes sufficient reinforcement for the opening in the header or if additional reinforcement is required has been a constant problem to the piping designer.

[4] Advisory Engineer, The Babcock & Wilcox Co., New York, N. Y. Past-President A.S.M.E.

[5] "Penstocks for Boulder Dam," by C. M. Day and Peter Bier, *Mechanical Engineering*, vol. 56, August, 1934, pp. 451–465.

[6] Engineer, Pittsburgh Piping & Equipment Company, Pittsburgh, Pa. Mem. A.S.M.E.

Where reinforcement has been considered necessary, the efforts of the individual designer have led to a great many different applications as may be seen from the illustrations in Fig. 1 of the paper. As no failures in service have come to this writer's attention, it may be assumed that most of these different types of reinforcement have added sufficient strength. However, piping for the low and medium pressures has usually been designed with a comfortable margin of safety and the possibilities are that most of these necks would have given satisfactory service even without any reinforcement.

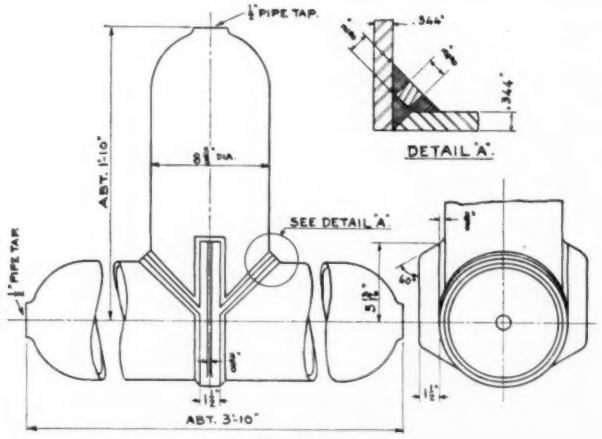


FIG. 7 DETAILS OF REINFORCEMENT SHOWN IN FIG. 8

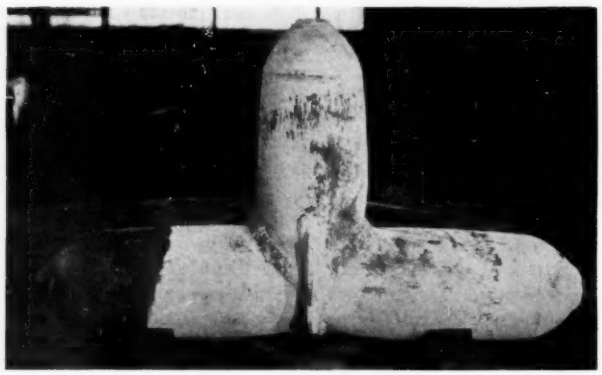


FIG. 8 MANIFOLD AFTER TEST AT 5500 POUNDS PRESSURE

It is also possible that some of the types of reinforcement may have caused increased stresses in some sections of the header or branch. This apparently was the case with the ring type of reinforcement chosen by the authors for investigation, and although this type of reinforcement in the test seemed to develop ample strength, the writer is of the opinion that it is not the best nor the most economical type.

The ring type of reinforcement has one inherent drawback, namely, the fact that unless the basic weld attaching the neck to the header is perfect, the internal pressure may be acting directly on the reinforcing ring. This would entirely change the stress distribution and cause greatly increased stresses in the ring and in the welds by which the ring is attached. It seems that it should be mandatory in the case of ring reinforcement that the basic weld be subjected to a hydrostatic test before the ring is attached. A considerable number of tests on various types of reinforcement will be required in order to develop the best and most economical type.

Some years ago the company with which the writer is connected was required to furnish a number of welded manifolds which were specified to be in accordance with the A.S.M.E. Unfired Pressure Vessels Code and of a design satisfactory to the purchaser's insurance carrier. The purchaser expressed some doubt as to the strength of the type of reinforcement proposed and it was decided to construct a test manifold in accordance with the details shown in Fig. 7. The pipe selected was $8^{5}/_{8}$ in. OD × 0.344 in. wall, made of seamless steel in accordance with A.S.T.M. Specification A-106, Grade A. Unfortunately the detailed reports on the test were destroyed during the flood in Pittsburgh a couple of years ago, but briefly the test procedure and results were as follows:

As no strain gages were available, the manifold was given a coat of whitewash to permit observation of strain lines. Hydrostatic pressure was applied by means of a triplex test pump up to a pressure of 1500 psi. Above this limit the pressure was boosted by means of an intensifier capable of producing pressures up to 20,000 psi. Some strain lines started to appear in the whitewash at a pressure slightly above 2000 psi. The pressure was gradually boosted to about 5500 psi and it proved impossible to raise the pressure beyond this point as the intensifier could furnish only 60 cu in. per stroke and the swelling of the manifold increased the internal volume by a corresponding amount. As the manifold was designed for 400 psi working steam pressure, it was felt that the safety of the neck reinforcement had been proved and the test was stopped without being carried to destruction. Fig. 8 is a photograph of the manifold after the test and it will be noted that while the outside diameter of the pipe had increased from $8\ ^{5}/_{8}$ in. to about $10\ ^{1}/_{2}$ in., little distortion had taken place at the weld.

It may also be of interest to note that the end closures, made by rolling down the pipe and closing the end with a pad welded on, showed little distortion.

On account of the large amount of welding required, the type of reinforcement used on this test manifold is not considered to be the best that can be developed but it is believed to have some advantages over the ring type of reinforcement.

E. R. SEABLOOM.[7] The authors are to be congratulated on their paper. This type of information is much in demand by fabricators and designers as there is a definite scarcity of published data, design rules, and code regulations pertaining to welding design of piping, such as we have on boilers and pressure vessels.

As stated by the authors, after the completion of their investigation, the manifold was turned over to the subcommittee on welded branch connections of the A.S.A. Sectional Committee B31 for further tests. The members of the Pipe Frabrication Institute are cooperating with this subcommittee in the fabrication and testing of manifolds to determine suitable designs of reinforcement for welded branch connections. The Crane Company's part in this program is to serve as the testing laboratory for the committee and consequently the header was sent to this company's research laboratories for the test to destruction.

This discussion is, therefore, virtually a report covering the final tests of the header, although additional data gleaned from other investigations are included to substantiate some of the authors' statements.

Test Procedure. The manifold was first stress-relieved at 1200 F to remove the effects of cold working produced by the previous tests at the University of Michigan. As the manifold had been so thoroughly explored by means of strain gages, there was no need for additional investigation of this character. It was decided, however, to measure deformations of the manifold

[7] Research Engineer, Division of Engineering and Research, Crane Co., Chicago, Ill.

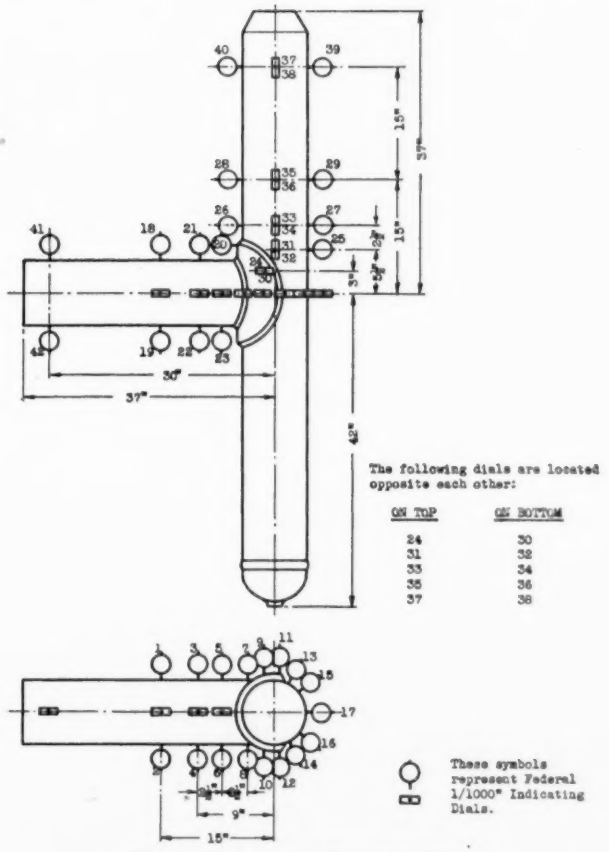


FIG. 9 LOCATION OF INDICATOR DIALS

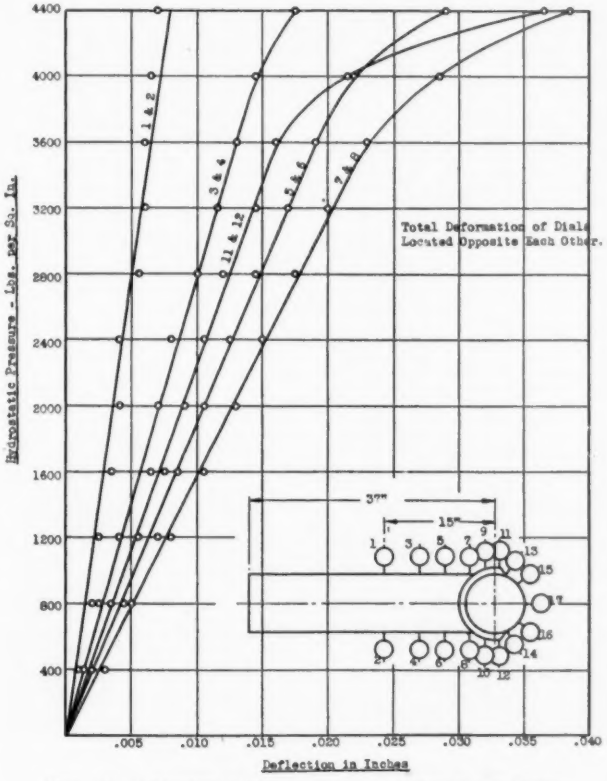


FIG. 10 DIAMETRAL DEFLECTIONS OF NOZZLE AND RING

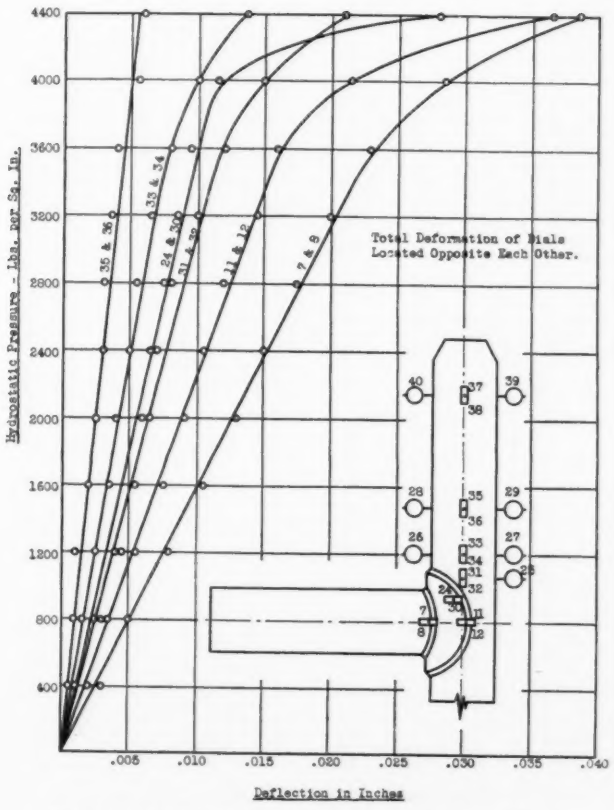


FIG. 11 DIAMETRAL DEFLECTIONS OF NOZZLE AND RING

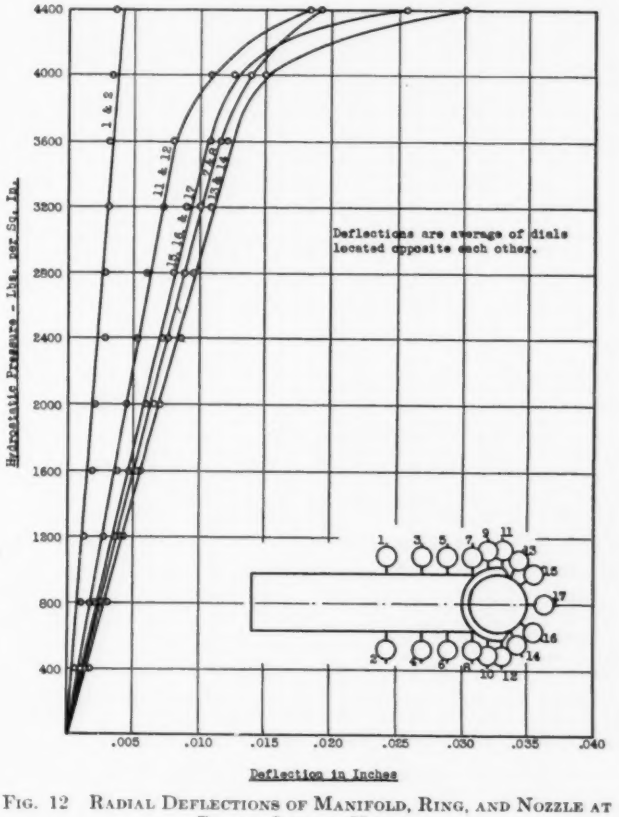


FIG. 12 RADIAL DEFLECTIONS OF MANIFOLD, RING, AND NOZZLE AT CENTER LINE OF NOZZLE

TABLE 1 DEFORMATION OF MANIFOLD UNDER HYDROSTATIC PRESSURE

Dial readings are in thousandths of an inch

Pressure, psi	Dial nos.	400	800	1200	1600	2000	2400	2800	3200	3600	4000	4400	Permanent set after 4400
Total diametral deformation of nozzle	1 & 2	1.0	2.0	2.5	3.5	4.0	4.0	5.5	6.0	6.0	6.5	7.0	—1.0
	3 & 4	1.5	2.5	4.0	6.5	7.0	8.0	10.0	11.5	13.0	14.5	17.5	2.0
	5 & 6	2.5	4.5	7.0	8.5	10.5	12.5	14.5	17.0	19.0	22.0	29.0	8.0
	7 & 8	3.0	5.0	8.0	10.5	13.0	15.0	17.5	20.0	23.0	27.5	38.5	11.5
	41 & 42	0.5	1.5	2.0	2.5	4.0	4.5	5.5	6.5	7.0	8.5	1.0	0.0
	18 & 19	0.0	1.0	1.5	3.0	3.5	4.5	6.0	7.0	7.0	8.5	11.0	1.0
	21 & 22	0.5	—0.5	—1.0	—1.0	—0.5	—0.5	0.5	0.5	0.5	0.0	—3.0	—6.0
Total diametral deformation of ring	9 & 10	2.5	5.5	7.5	9.5	12.0	14.0	15.5	18.0	20.5	24.5	36.0	13.5
	11 & 12	2.0	3.5	5.5	7.5	9.0	10.5	12.0	14.5	16.0	21.5	36.5	20.0
	24 & 30	2.0	3.0	4.5	5.5	6.0	6.5	7.5	8.5	9.5	11.5	28.0	4.5
Average radial deformation of manifold opposite nozzle	13 & 14	1.25	3.0	4.25	5.5	7.0	8.5	9.5	10.75	12.0	15.0	30.0	20.0
	15 & 16	5.0	2.0	3.5	4.75	6.0	7.25	8.0	9.0	9.75	12.5	25.5	19.75
	17	1.0	2.5	4.0	5.0	6.0	7.0	7.0	8.0	9.0	11.5	27.0	19.0
Total diametral deformation of manifold	37 & 38	1.0	1.5	2.0	3.0	3.5	4.0	5.0	5.5	6.5	7.0	7.0	1.0
	35 & 36	0.5	1.0	1.0	2.0	2.5	3.0	3.0	3.5	4.0	5.5	6.0	1.5
	33 & 34	0.5	1.5	2.5	3.5	4.0	5.0	5.5	6.5	8.0	10.0	13.5	4.5
	31 & 32	1.0	2.5	4.0	5.5	6.5	7.0	8.0	10.0	12.0	15.0	21.0	9.0
	39 & 40	0.5	1.0	1.5	1.5	2.0	3.0	3.5	4.5	5.0	6.0	8.0	2.0
	28 & 29	0.5	1.5	2.5	3.0	4.5	5.0	5.5	6.5	7.5	8.5	9.0	0.0
	26 & 27	0.0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	3.5	3.5	2.0	—4.0
	20 & 25	0.0	0.0	0.0	0.5	1.0	1.5	1.5	2.5	2.5	2.0	1.0	—2.0

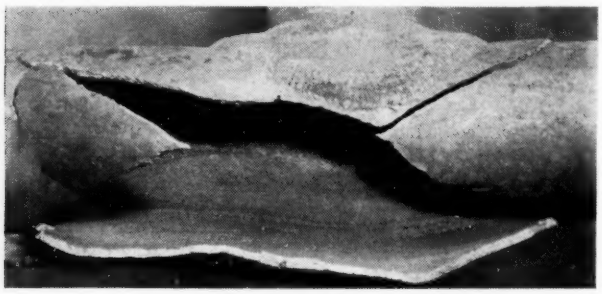


FIG. 13 SHOWING SMALL DRILL HOLES IN FRACTURE

by means of dial indicators, as this would supplement the strain-gage readings and possibly bring about a better understanding of what occurs in the highly stressed areas.

Forty-two Federal dial indicators, graduated in thousandths of an inch, were located around the manifold as shown in Fig. 9. The dials were located diametrically opposite each other, in most cases, in order to eliminate as much as possible, errors in measurement due to shifting of the manifold on its supports when expansion occurred. Internal hydrostatic pressure was applied gradually in 400-lb increments by means of a hydraulic ram, capable of building up pressure to 60,000 psi, and dial readings were recorded at each increment of pressure up to and including 4400 psi. The deflection readings obtained are tabulated in Table 1, and some of the deformations are expressed graphically in Figs. 10, 11, and 12.

It will be observed that the maximum deflection occurred on the nozzle, immediately above the reinforcing ring at right angles to the run of the manifold, as indicated by dials 7 and 8. Considerable deformation also occurred adjacent to and in the reinforced area. This is due to the deviation from a circular cross section when a full-size nozzle is attached, producing a weaker structure to resist internal pressure.

Test to Destruction. The dial indicators were removed and the internal pressure was gradually increased to 8200 psi, when serious leakage occurred through one of the drilled holes which was used for the attachment of strain gages. This leak was repaired by welding and pressure was again built up until failure occurred at 8750 psi. See Fig. 12 of the paper. The failure apparently was influenced to some extent by small drilled holes in the proximity of the reinforcing ring, which are evident in Fig. 13 of this discussion. The rupture, however, is typical of pipe failure under hydrostatic pressure, as illustrated in Fig. 14.

These pipes are a few of a series that were tested to destruction on hydrostatic pressure. They were $8^5/_8$ in. outside diameter, A.S.T.M. Specification A-106, of standard-weight lap-welded

TABLE 2 PHYSICAL PROPERTIES OF MATERIAL REMOVED FROM MANIFOLD

Standard $1^1/_2$-in. × 8-in. flat pipe-test bars

	Transverse bar		Longitudinal bar	
	As tested	Stress-relieved	As tested	Stress-relieved
Tensile strength, psi	79250	67750	79250	67000
Yield point, psi	66400	44650	62500	44000
Elongation, % in 8 in.	10.9	16.3	13.9	20.0
Reduction of area, %	38.1	33.3	31.6	43.5
Fracture[a]	A	A	A	I
Round bars, 2-in. gage length				
Tensile strength, psi	73500	71500	84500	71300
Yield point, psi	56750	44750	70250	44600
Elongation, % in 2 in.	14.0	25.5	18.0	27.0
Reduction of area, %	53.5	57.2	58.5	67.8
Fracture[a]	I	$^1/_2$C	$^1/_2$C	$^2/_3$C
Modulus of elasticity, 10^6 psi	29.4	29.8	28.2	27.9

[a] A = angular, I = irregular, C = cup.



FIG. 14 TYPICAL PIPE FAILURES UNDER HYDROSTATIC PRESSURE



FIG. 15 FAILURES OF END CLOSURES



FIG. 16 FAILURE OF STRAIGHT PIPE



FIG. 17 FAILURE OF UNREINFORCED BRANCH



FIG. 18 FAILURE OF REINFORCED BRANCH



FIG. 19 FAILURE OF HALF-SIZE BRANCH



FIG. 21 FAILURE OF CAST-STEEL WELDING TEE

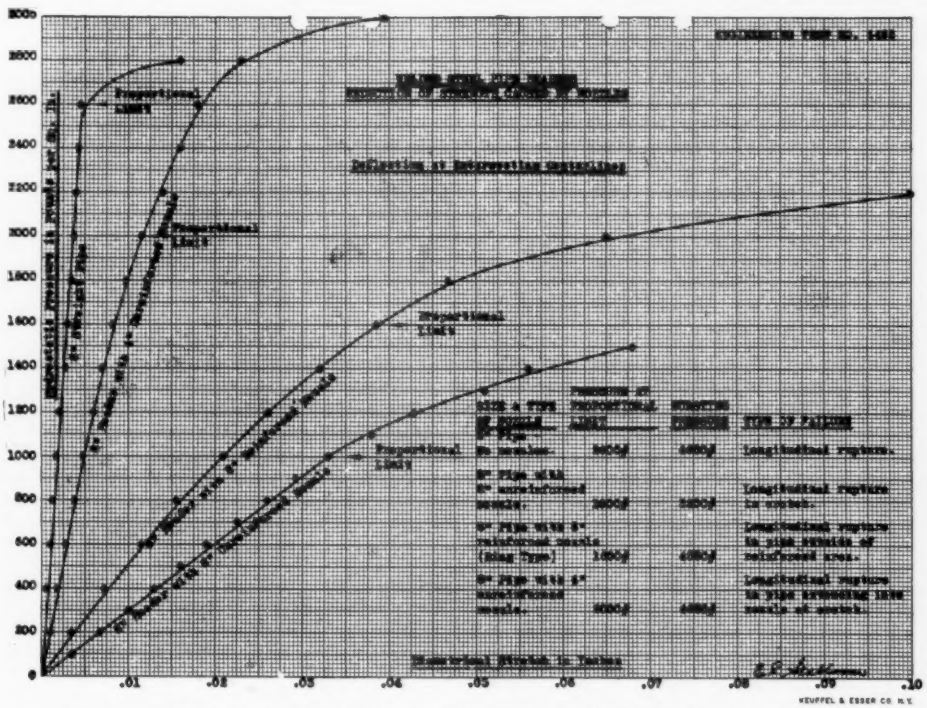


FIG. 20 WELDED STEEL PIPE HEADERS, REDUCTION OF STRENGTH CAUSED BY NOZZLES

material, 22 ft long. The failure pressures ranged from 4000 to 4400 psi and the failures represent the types generally obtained on piping.

Physical Properties of Material. After the test to destruction, longitudinal and transverse strips were removed from the manifold. One half of these were stress-relieved at 1200 F, and the other half were left in the strained state. Test bars were machined from the strips and tested to determine the physical properties of material. The results are shown in Table 2. Averaging the tensile strengths indicated by the transverse flat and round bars in the strained condition, a value of 76,375 psi is obtained. Applying this tensile value in Barlow's formula, the theoretical bursting pressure of a pipe without any openings would be 8855 psi, thus showing that the ring reinforces the manifold at the nozzle connection sufficiently to bring the failure pressure up to that equivalent to a straight length of pipe.

Test on End Closures. Sections approximately one foot long containing the three types of heads, were removed from the manifold and nozzle. The open ends were closed by means of welded-in flat heads $2^1/_2$ in. thick and the assemblies were tested to destruction on hydrostatic pressure. The failures are illustrated in Fig. 15. The first and third assemblies failed at 10,500 psi. The center assembly failed at 11,200 psi and the increase in bursting pressure over the other two is due to the shorter span of pipe between the heads. These end closures are described in the paper.

Tests on Other Manifolds. Some time ago a series of tests was organized to determine the reduction in strength caused by the attachment of nozzles to piping or manifold. One phase of this investigation was conducted on 8-in., Schedule 40, A.S.T.M. Specification A-106, Grade A carbon-steel tubing, and all of the tests were made on 5-ft lengths cut from one length of pipe in order to avoid discrepancies in results due to variations in physical properties of material.

The straight length of pipe shown in Fig. 16 ruptured at 4600 psi. The manifold shown in Fig. 17, having an 8-in. unreinforced nozzle at the center failed near the crotch at 3200 psi, or 33 per cent lower than the straight length of pipe. The manifold, with an 8-in. reinforced nozzle, employing a ring of somewhat similar design and proportions as the one investigated by the authors, fractured outside of the reinforced area at 4550 psi, which is approximately the same as for the straight length of pipe. See Fig. 18. The manifold with a 4-in. unreinforced nozzle failed in the crotch, as shown in Fig. 19, at 4650 psi, or practically the same pressure as the straight length.

Fig. 20 shows the results of deflection readings obtained on the straight length of pipe and various manifolds at the center sections under pressure. While considerably greater deformation occurred on the reinforced header compared to the straight length of pipe, both failed at practically the same pressure, thus indicating that the ring reinforces the structure as far as ultimate failure is concerned.

Stress Conditions in Crotch. The authors deduced analytically that high stresses existed in the inside wall of the manifold at the crotch. This interpretation is correct, despite the fact that the stresses on the inside cannot be satisfactorily measured, as practically all structures of this type, such as tees, elbows, and valves, generally fail in the crotch. Evidence of this is furnished by Figs. 21 and 22, which show failures of a cast-steel tee and welding elbows.

When testing large oval-section cast-iron gate valves to destruction on pressure, better observations of stress conditions are obtained, as deflection and strain-gage measurements are of greater magnitude. Fig. 23 shows a portion of a 48-in., 25-lb working pressure, cast-iron gate valve set up for test. The central area of the flat side deflected outwardly 0.154 in. at 105

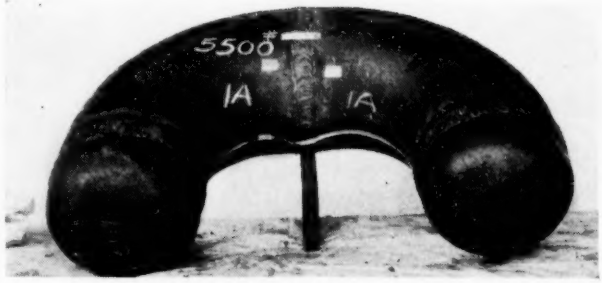


FIG. 22 FAILURE OF CAST-STEEL WELDING ELBOWS

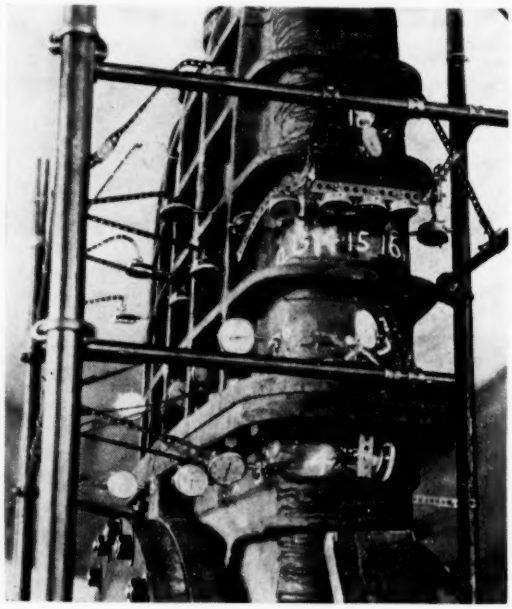


FIG. 23 TEST ON 48-IN. CAST-IRON GATE VALVE



FIG. 24 FAILURE OF 48-IN. CAST-IRON GATE VALVE

psi internal pressure, while the narrow side deflected inwardly —0.034 in. Strain-gage readings indicated that high tensile stresses existed on the outside of the flat sides, while compression stresses were in evidence on the outside of the narrow sides. Consequently, the stresses on the inside of the walls were reversed from the conditions on the outside. Cast iron is approximately four times stronger in compression than in tension, yet the valve failed on the narrow side as shown by Fig. 24, which indicates that rupture commenced on the inside of the narrow wall. We have produced further evidence of this fact by redesigning cast-iron oval-section valves in such a manner that all reinforcing ribs are placed in compression, that is, the ribs on the flat sides were located internally and on the narrow sides externally, with the result that bursting pressures have been increased over 100 per cent, even though failures occur on the narrow sides.

Reviewing all of the test data presented, it is evident that the findings and conclusions of the authors are substantially correct, and that the paper is a valuable contribution to the technical literature on this subject.

F. E. Wertheim.[8] The authors have proved the need of reinforcing a full-size welded pipe-branch connection, and have also proved the efficiency of the ring-type reinforcement, thereby furnishing a basis for sound design.

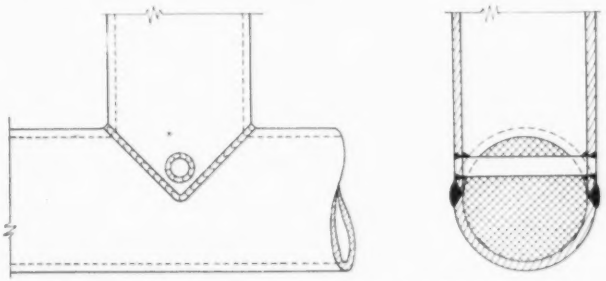


Fig. 25 Showing Use of Stay Bolt for Reinforcing

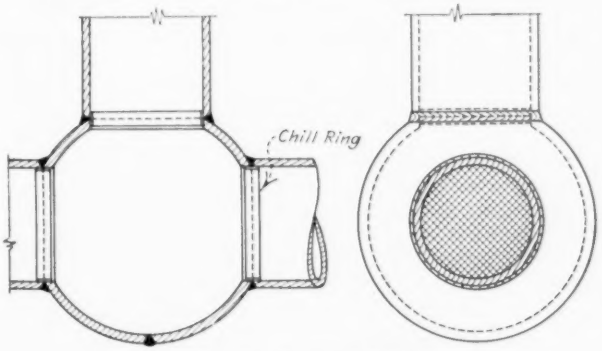


Fig. 26 Backing Ring

If this type of reinforcement receives the indorsement of the special subcommittee on welded branch connections of A.S.A., Sectional Committee B31 on Code for Pressure Piping; then it would appear advisable to add such reinforcing rings to commercial lines of welded fittings, even though, as stated, the ring can be made without die equipment.

All three divisions of these tests, i.e., (1) exploring the unreinforced weld, (2) exploring the ring-reinforced weld, and (3) bursting tests, emphasize the appreciable deformation of, and the consequent bending stresses in, the flat portion of the structure. The writer wishes to suggest the advantage of minimizing, or eliminating, bending stresses, with their tendency to create severe stresses in the crotch.

Fig. 25 shows a simple welded stay-bolt construction; the stay bolt is to be welded into the nozzle before it is joined to the header. The bolt should be strong enough to compensate, to a considerable degree, for material removed for the branch opening. Objections that the bolt obstructs flow are valid, nonetheless, stay bolts are still widely used in water legs of various kinds of boilers.

Fig. 26 shows a construction free of bending stresses, in which only simple butt welds are required, and which can be fabricated

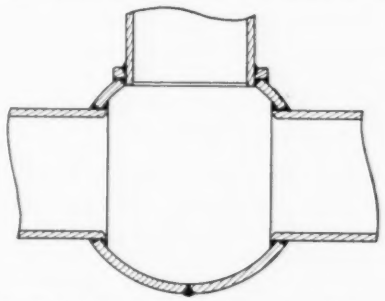


Fig. 27 Two Methods of Reinforcing Spherical Openings

from welding fittings (semispherical welding heads) to be had on the market. An alternate construction is possible by swagging a full-size pipe to form the spherical portion. Since the spherical part is shown with thicker wall than the pipe, and the openings are strengthened somewhat by the pipe and welding bead, reinforcement at the openings may not be necessary. Fig 27, however, shows two methods of reinforcing openings in the spherical portion.

These designs are submitted for the consideration of the special subcommittee on welded branch connections, with the hope that they will at least rate high enough for some tests with half-size models. No originality is claimed; the stay bolt is an ancient device; and the spherical construction has been shown previously.[9]

In Mr. Crocker's construction, the openings in the spherical end are reinforced with dished rings.

F. S. G. Williams.[10] The authors are to be complimented on the comprehensive work done in the investigation reported in this paper. The remarks by Dr. Timoshenko raised a point that is really quite vital in the study of branch connections, namely, the effect of the branch connection and its reinforcement upon the stress conditions in the area surrounding the reinforcement. Obviously Dr. Timoshenko had in mind the possibility of transferring the stress concentrations from the area immediately adjacent to the branch into the body of the header and the data submitted by the authors would indicate that such a transfer did occur in the unit under investigation.

It is well worth directing more pointed attention to it as one of the basic principles of design which must be given proper consideration in high-pressure work. It is necessary to know how much reinforcement to apply to any given connection but it is equally important to know the shape and proper distribution of the reinforcement required to produce a proper balance of the flexibility of all sections of the completed structure and to reduce

[8] Engineer, Goetze Gasket & Packing Co., Inc., New Brunswick, N. J. Mem. A.S.M.E.

[9] "Welded Fabrication of Manifolds and Special Fittings for Piping," by Sabin Crocker, *Heating, Piping, and Air Conditioning*, vol. 10, Jan., 1937, p. 18.

[10] District Manager, Taylor Forge & Pipe Works, New York, N. Y. Mem. A.S.M.E.

stress concentrations to safe figures for design purposes without creating additional stress concentrations in other areas.

Apparently there has been a tendency on the part of the authors and on others, as evidenced by the discussion submitted by Mr. Seabloom, to consider only the problem of so-called fabricated branch connections. It would leave a false impression with the casual reader if it were not pointed out at this time that the most recent high-pressure installations have, as far as possible, avoided the use of the built-up branch connections with their attendant reinforcing pads of one type or another and have employed seamless forged-steel tees in which the reinforcement has been carefully distributed to attain the desired uniform flexibility and in which the welding procedure has been transferred from the complicated problem of branch welds to the now well-established and controllable circumferential weld. It would seem, therefore, that any further investigation along the lines of this excellent work should give due consideration to the effect of the distribution of reinforcement in an effort to develop a badly needed body of data from which quantitative rules could be formulated.

Recently a hydrostatic-pressure test was made on 5-in. and 8-in. Taylor Seamless Welding Tees. The fittings tested were regular stock items and the material was in accordance with A.S.T.M. Specification A-106, Grade A, seamless. This pipe has a minimum tensile strength of 48,000 psi and, based upon this the bursting strength of the 5-in. and 8-in. standard-weight pipe is 4670 and 3700 psi, respectively. An internal pressure of 5000 psi was applied to both the 5-in. and 8-in. tees, and this was well above the theoretical bursting pressure for corresponding pipe.

Authors' Closure

The authors wish to thank those who have contributed so much to the value of their paper by their interest and discussion. The strap and cravat reinforcement proposed by Major Aiton is novel and for certain types of loading may be a desirable, though possibly expensive, form of reinforcement. The attachment of the cravat in common with all gusset types of reinforcement gives high stress concentration which under pulsation, or vibration, may result in cracking of the weld. The failure of the ring-reinforced manifold in the crotch as shown in Fig. 1 of his discussion is contrary to all tests made in this country where the cross section of the ring has been made equal to the metal removed from the header in accordance with A.S.M.E. Boiler Code rules. Possibly the fact that the ring was made in two parts with a weld at the crotch location may have had some bearing on the location of this failure.

Mr. Fantz has emphasized the advantages of a method of preparation in which the branch alone is beveled. Subsequent to constructing the test manifold, The Detroit Edison Company investigated the advantages of the construction favored by Mr. Fantz as shown in Fig. 6. To obtain a practical design for a full-size branch connection it was found necessary to make supplementary V cuts and to bevel the header at the sides, otherwise the design shown in Fig. 6 is believed to be entirely satisfactory. One advantage of this construction not mentioned by Mr. Fantz is that it permits the use of a backing ring, an improvement not feasible with other methods of preparation. Backing rings permit adequate separation of the branch and header so that the designed weld cross sections actually can be obtained at all points around the intersection.

While recognizing the simplicity of forged saddles, the authors prefer a modification of the reinforcement used in their tests. This modified reinforcement consists of a ring which extends lower on the sides than the test ring to cover the region where final failure occurred and separate side plates which, in conjunction with the redesigned ring, should prevent excessive distortion at the sides of the manifold.

The remarks of Doctor Jacobus relative to the tests and special factors involved in designing the penstocks at Boulder Dam are of interest. The authors were pleased likewise to find that the Boiler Code rules give a satisfactory basis for proportioning the crotch reinforcement.

The test data supplied by Mr. Larsen on a different type of reinforcement are a welcome contribution. This reinforcement while not as economical as might be desired apparently prevents bulging of the sides of the manifold and is in other respects entirely adequate. Since we have never had leakage occur on hydrostatic tests of high-pressure welded joints, the need for venting the reinforcing ring mentioned by Mr. Larsen has never developed but it may be good general practice. Where reinforcing saddles have been used, vent holes were provided.

The discussion by Mr. Seabloom is so comprehensive and contains so many valuable test data that it can be considered as a complete paper in itself. The authors are greatly indebted to Mr. Seabloom and to Crane Company for their cooperation in making the final tests and for their thoroughness in presenting the results.

The use of a stay bolt to prevent bulging of the sides, suggested by Mr. Wertheim, has possibilities, although the high concentration of stress where the stay bolt is welded to the manifold is undesirable. For the particular manifold tested, a stay bolt of approximately 2 in. diam would have been required which, as noted by Mr. Wertheim, would have caused some restriction to flow. The spherical construction has certain advantages, particularly for Y fittings. Commercial welding caps or heads are of ellipsoidal rather than hemispherical shape which destroys some of the economy and convenience of this method of construction. Its undoubted advantages from a distortion standpoint might offset the probable increase in cost over the conventional branch weld.

Mr. Williams' point is well taken that some types of reinforcement may result in high stress concentrations where large changes in rigidity of the composite structure occur. Where adequate thickening at the crotch and sides of welding tees is provided and if the material is comparable with the pipe material, the natural tendency would be to use ready-made tees. The relatively high cost of such tees and the present necessity of making the larger sizes by boring out solid forgings has caused particular emphasis to be placed on fabricated branch connections.

The authors are gratified that their paper has brought forth so much worth-while discussion by many competent engineers whose comments have served to clarify the presentation and augment the results.

Development of the Focke Helicopter

By HEINRICH FOCKE,[1] DELMENHORST, GERMANY

The paper gives a description of the methods used in the development of the Focke helicopter. After a short review of the analytical and experimental information regarding the helicopter in existence prior to 1932, the author states the fundamental requirements to be met and indicates the procedure adopted to satisfy them. The analytical discussion is based on the Glauert-Lock theory of the autogiro rotor, and the author shows in outline the various steps in extending the theory to cover the helicopter rotor.

Finally, the construction and testing of the first machine are described, together with its successful flights.

HIGH SPEED, the great gift which the airplane has given man, is also a curse, from which he has not yet been delivered. The inability to stand still in the air, to rise vertically, and to sink vertically, besides making necessary the use of large areas for airports, produces many dangers; dangers in taking off and on landing; dangers of poor visibility, of flight over unsuitable terrain, in mountainous regions, and over extended forests or marshes.

We all know the means which have been employed with great success in the airplane to secure minimum speed, and from the very nature of these methods it is impossible to reduce the speed to zero. This is because the necessary down-wash of air, required to produce lift in the airplane, is produced by the forward speed, and furthermore it is impossible to deflect the air through as much as 90 deg, the ideal but unattainable optimum. The third factor, the mass of air affected, can only be increased by increasing the area of the lifting surfaces, and the extent of such increase in area is governed by structural limitations.

Nevertheless it is easy to see why the airplane was the first successful type of flying machine. The reason is found in its simple construction. In the early days when all was unexplored territory in this uncommon and difficult problem it was a tempting idea to have no moving parts on the flying machine itself other than the control surfaces, and with the airplane, the moving mechanism, outside of the engine, was simple, as the propeller could be mounted directly on the motor shaft.

With the first visible successes in the years 1907 to 1909, there arose a situation, which we can now see to have been extremely regrettable, a situation that created limitations where there should have been no limitations. As soon as one type of construction was successful, that type of machine, the airplane, was demanded exclusively, and attempts at flight along other lines were forgotten. We had to wait from twenty to thirty years before we could see that the practical achievement of one solution was not proof that another solution was impossible. The obstinate adherence to the methods with which the first flights had been made, is in itself understandable, as is also the oversight of the fundamental defects of this first solution.

We know exactly what limitations the airplane has today. We allow for these limitations and adjust ourselves to them. We improve the airplane continually, within the limits of its possibilities. That is right and must be so but, in doing this, we must not forget one thing for the future: We can only achieve new fields of application by going back to fundamental principles and exploring new paths.

When the problem of making aircraft independent of the forward speed is put before us one requirement becomes paramount. We must give the supporting elements their own movement relative to the air. This requirement must be met in practice with the least possible complication. Flapping and feathering hold some technical attractions but, after sober reflection, we must conclude that one of the simplest and theoretically best-known machine elements, the airscrew, is most appropriate for this purpose.

As we all know, the idea of the power-driven airscrew on a vertical axis, is quite old, going back to Leonardo da Vinci who drew up the first direct-lift machine. The interesting subsequent history of the lifting airscrew must be omitted here because of lack of space, but we know that it is only recently that it has progressed beyond the laboratory stage.

Two impulses in quite recent times have greatly increased interest in lifting airscrews. One came from necessity. For many military, geographical-exploration, photogrammetric, and radio purposes a craft is required that can take off and land with little or no run, can hover in the air, and land or climb vertically or at any desired angle. Such a craft also opens up the possibility of extensive private flying because it can be turned and maneuvered at will and does not require a large landing field. With the helicopter, roof and garden landings are no longer Utopian.

The other impulse came from the technical side. De la Cierva has shown with his autogiro, that a large rotating airscrew is a reliable element of support, and he has so stated many times. However, his machine is not a direct-lift machine and, apart from the jump take-off, cannot rise or sink vertically or hover in the air, as the airscrew is not driven by the motor but "autorotates" freely in the air stream. The forward speed is produced in the usual manner by the motor and the propeller. The connection between the lifting airscrew and de la Cierva is only a loose one, and de la Cierva himself refused its possibilities.

Without doubt the autogiro represents an interesting transition between the airplane and the helicopter. It has no better take-off than the airplane, but it can land at a speed of around 40 km (25 miles) per hr, and has a further valuable property in that the rotating blades have their own velocity relative to the air and hence cannot be stalled, with resulting loss of control. The common feature of rotating blades has led to the general appelation for both types of rotating-wing aircraft.

The very existence of the autogiro rotor has provided theoretical information without which the helicopter would have been a long time developing. The British Air Ministry, when examination and tests of the de la Cierva autogiro were under way, commissioned aerodynamic experts like Glauert and Lock to undertake the mathematical investigation of the peculiar phenomenon of autorotation, that is, the free rotation of a large airscrew placed at small angles of attack relative to the air stream.

We shall see later that an extension of the Glauert-Lock theory can be applied to the forward flight of a power-driven helicopter rotor, down to quite small velocities. We must also add that for

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[1] Focke, Achgelis & Co.

Translated by Alexander Klemin. Mem. A.S.M.E.

Presented by W. Laurence LePage.

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NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

the helicopter lifting rotor with a purely axial air stream, that is, a rotor without forward velocity, a series of theoretical calculations and experiments had been carried out by von Kármán, Flachsbart, and Helmbold. There were also methods for obtaining the best thrust distribution along the radius, and a stability investigation by von Kármán which drew pessimistic conclusions. These constitute roughly the scope of the theoretical development of the autogiro rotor and of the direct-lift rotor up to about 1932.

In practice, by 1932 the following performances for the helicopters had been accomplished by Pescara (France), d'Ascanio (Italy), and Oehmichen (France), which were official or at least officially confirmed: distance, about 1 km; duration, about 10 min; height, about 18 meters. There had been no real sustained flight with helicopters up to that date.

On the other hand the autogiro was already in regular use, although its performance was and is today somewhat inferior to that of the airplane. To develop a true helicopter five years ago, without the attendant limitations of the autogiro, was, therefore, to develop an entirely new type of craft, not to improve an already existing machine, and, while inventive effort is worthy, sober reflection upon the aspects of the problem and a thorough investigation of each phase is a far better approach.

Fundamental Requirements of a Helicopter

The fundamental requirements of a direct-lift aircraft are, in order of their importance, the following:

Ability to Make an Emergency Landing After Failure of the Power Plant. This most important basic requirement, which has become self-evident with the airplane, no helicopter had as yet fulfilled, although theory had indicated that it would be possible to convert the lifting rotor into an autorotative system such as had been demonstrated by Cierva. To accomplish this it is necessary that the pitch angle of the blades become smaller than that used with the power-driven lifting system. This involves a mechanical complication which cannot, however, be avoided because an aircraft without the capacity to land smoothly after failure of the motor or the transmission is unthinkable in aircraft practice.

Controllability and Stability. The aircraft must be controllable, at least with normal skill, in all flight conditions including hovering. It must also have static stability about all three axes, and it is desirable to have dynamic stability about these axes. In this question of stability had been the weakest spot of all previous helicopter experiments. Helicopters up until this time had possessed little or no stability, and only by lightning-quick movements of the controls had it been possible to maintain flight and then only for a short time. For other types of helicopters, which later failed of accomplishment because of other defects, it was said that they would have been stable, but the reason for such claims of stability was never clear. Von Kármán's pessimistic predictions regarding stability led many to think that a really practical helicopter was impossible.

The autogiro has given no particular difficulty in this regard at normal speeds but theory and practice have shown that below a speed of about 40 mph, the autogiro becomes dynamically unstable about some axes at least, and to some extent unstable about all three axes. In practice this instability at low speeds had caused no difficulty, because the slowest speeds were employed only for short periods. With the helicopter, the outstanding characteristic of which is its ability to hover, this question of stability must be considered much more carefully. We must have, if not complete static and dynamic stability, at least normal controllability which must be maintained for hours, and under all weather conditions.

General Reliability of Functioning. In this respect the helicopter has had a great weakness. One could not speak of reliability of functioning when the duration of flight was of the order of a few minutes. The nonmoving structural parts are not subject to any greater stresses than are the wings of a normal airplane. The transmission parts must have at least the reliability of the motor, and these transmission parts have of necessity to be smaller than the structural elements of the wing cells. It should be understood that we are speaking of general reliability, in addition to which the requirements for emergency landing, already mentioned, must be immediately available, and must confer the ability, comparable to that of an airplane, to make an immediate forced landing.

Simplicity of Handling by the Pilot. The technical phases of design are only a part, perhaps the smallest part, of any new problem. The design must be such that the average pilot can handle and fly the machine. It is, therefore, essential to lighten the task of the pilot as much as possible. He must be provided with a control system and other operating means to which he is already accustomed in the airplane. At most only a few more manual operations must be demanded of him than are required in the operation of the airplane. The controls must not only operate in the usual directions, but also their movements must be of the usual magnitudes.

Performance Must Be Acceptable. It is of course self-evident that we cannot, at least at first, realize in the helicopter the high-speed performance of the airplane. We must pay a price for the special ability of rotating-wing aircraft in the slow-speed region. But it would reduce the practical value of a helicopter greatly if it could only hover in the air and we must seek, therefore, a performance for the helicopter which is at least comparable with that of the airplane.

Reasonable Maintenance. Naturally, all personnel handling new apparatus must become accustomed to it, yet it must be possible to maintain the helicopter structure in much the same fashion as the structure of a conventional aircraft, and the power plant should be susceptible of the same maintenance as the motor of a conventional airplane.

This summary of the most important requirements of the ideal helicopter, indicates that the solution will not be found in a remarkable patentable solution, by a so-called Egg of Columbus. There is only one path really open: To investigate many very different questions with equal care, and with equal thoroughness, and to keep them all in mind when undertaking the construction of a helicopter. Many of these questions, such as the many stability problems, lie in the region of theory, while others, such as easy handling and control, are in the region of the practical arts. In between lie the difficult constructional or mechanical problems.

In this work, as in some earlier work with a direct-lift machine, the procedure was as follows: First, many comprehensive experiments in the wind tunnel; then full-scale tests of the structural elements; and finally full-scale construction and flight tests with verification of every step with scale models. I have found that this took relatively the shortest time, and also assured the greatest success.

Let us summarize what has been said before. Prior to 1932 we had the practical autogiro, the rotors of which were not motor-driven, which afforded low minimum speeds; and, hardly beyond the laboratory stage of development, the helicopter. We saw that, for hovering, for prolonged steep-angle or vertical climbing, and for taking off and landing in the smallest possible space, the helicopter solution was still to come and would have to be combined wth autorotation as an emergency landing feature.

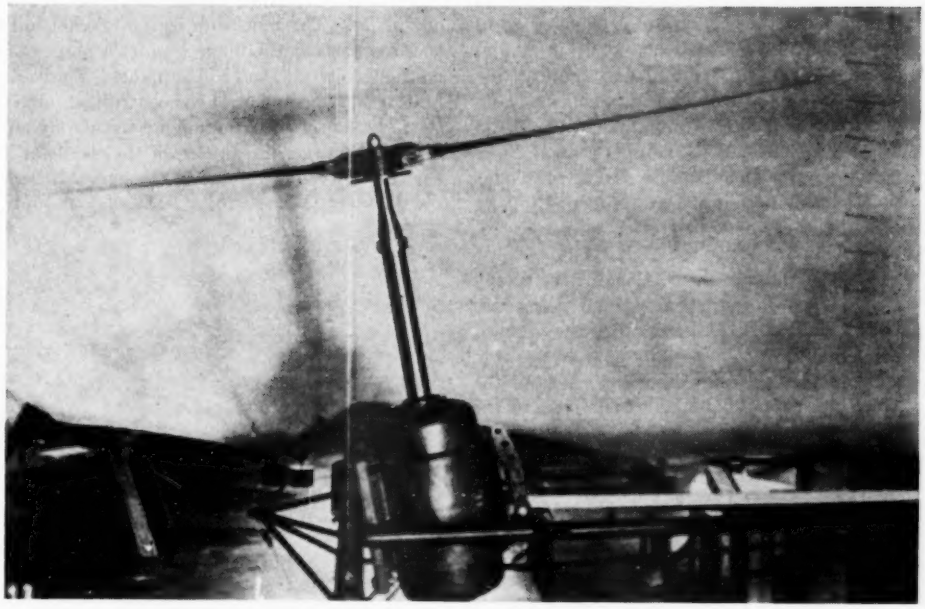


FIG. 1 MODEL OF THREE-BLADED LIFTING ROTOR WITH TEST MOUNTING IN WIND TUNNEL

LIFT PRODUCTION WITH POWER-DRIVEN AND AUTOROTATING AIRSCREWS

The creation of thrust, which is here identical with the production of lift, by an airscrew offers nothing new and I shall assume that the facts relating to the creation of lift or thrust are well known.

In the first place we must consider economy of power. We must seek the best thrust distribution along the radius of the rotor for the least power and the greatest thrust. Some compromise is necessary here, because of our fundamental requirement of emergency landing. The rotor must always autorotate when the power plant fails. If, because of this requirement, some loss in efficiency is incurred, this need not be disturbing.

At the beginning of my own helicopter investigations six years ago, the blades of all successful autogiro rotors, whose autorotation had been demonstrated, were flat (not twisted). If one did not wish to abandon a good thrust distribution along the radius, then the only method was to use strongly tapered blades, of trapezoidal form. These blades were connected to the hub by Cardan mounting, as Colonel Renard had already done in 1904 and as is current practice in most of today's autogiros and helicopters, so as to relieve them of bending moments at the root.

Naturally, these blades were constructed only after much calculation and many measurements in the wind tunnel, both as power-driven and as autorotational systems. Fig. 1 shows a model of the three-bladed screw, driven by a 3-hp electric motor, suspended from the wind-tunnel balance, which measures all forces and moments. Fig. 2 shows some of the test results. The nondimensional power coefficient k_d and also the thrust coefficient k_s are plotted against the pitch angle of the blades ϑ in degrees. According to Bendemann the efficiency of the screw with rotary motion only is given by $(k_s)^{3/2}/2k_d$. The best value of this expression is found to be when ϑ equals 12 deg, and has the value 0.71. That is a good value in spite of the flat (i.e., not twisted) blades. Let us consider the same airscrew, turning freely as an autogiro rotor. The rotor is now started by the motor, the wind tunnel is put into operation, and the rotor is set into autorotation when a certain wind velocity is reached, since a free-wheeling device operates, and disconnects the motor.

The polars can now be measured, just as in the case of the

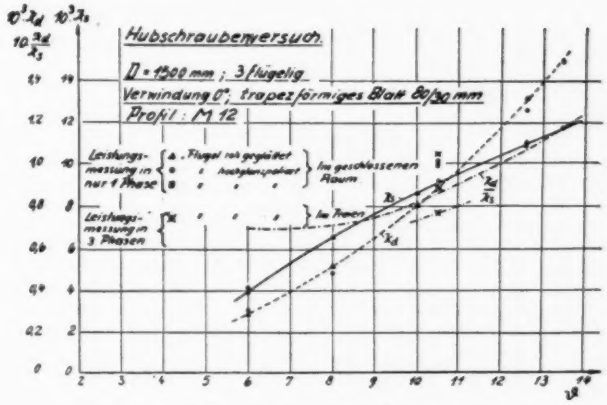


FIG. 2 SOME TEST RESULTS ON MODEL OF FIG. 1

(Power coefficient k_d and thrust coefficient k_s are plotted against the pitch angle ϑ of the blades.
Diam = 1500 mm; three blades, not twisted, tapered from a width of 80 to 30 mm.
Performance measurement in only one phase (of motor); (triangle) blades with rough polish; (circle, square) blades with high polish; in closed space.
Performance measurement in three phases; blades with high polish; unenclosed.)

autogiro. The air speed, the pitch setting of the blades, and the angle of attack of the plane of the rotor, which is termed α_T can be varied. Fig. 3 shows an example of the many polars which have been obtained with a blade pitch of $\vartheta = 4$ deg. The dotted curve in this figure will be referred to later. If the value of ϑ is increased, autorotation is endangered at a smaller value of α_T, that is, in rapid flight. When the value of ϑ is decreased, then the rotor turns faster for the same value of α_T and the polar curve becomes worse. If we keep a pitch setting of $\vartheta = 4$ deg, then we see that for transition from the helicopter rotor, for which $\vartheta = 12$ deg, to the autogiro rotor, the blades must have a pitch decrease of 8 deg, that is to say, the pitch decrease is of appreciable magnitude.

It has been assumed that the choice of the diameter of the rotor has been made beforehand. It is well known that from the point of view of the helicopter rotor, the large slowly revolving rotor is preferable. The diameter is limited only by construc-

tional difficulties. Again the ability to land in an emergency is the deciding factor. The polars of the autorotating rotor go to a maximum value of $C_a(= C_L) = 1$. If the steady minimum speed is placed at 50 km (31 miles) per hr (from American experiments the "unsteady" value of maximum lift coefficient goes to $C_a(= C_L) = 2$ which would lead to a minimum speed of 35 km (22 miles) per hr), we can use a disk loading of about 35 kg per sq m (7.18 lb per sq ft). Such loadings are also favorable for the direct-lift condition, as they permit a thrust of from 7 to 8 kg (15 to 18 lb) per hp, and the diameters are such as can be embodied in practical construction.

For the calculations of the autogiro rotor, I shall refer briefly to the principles of the Glauert-Lock papers without going into the whole development. See Fig. 4. Glauert adds the forward speed v and the rotational speed $R\omega$ vectorially at each position along the radius, and assumes that only the components of the resultant velocity which are at right angles to the span of the

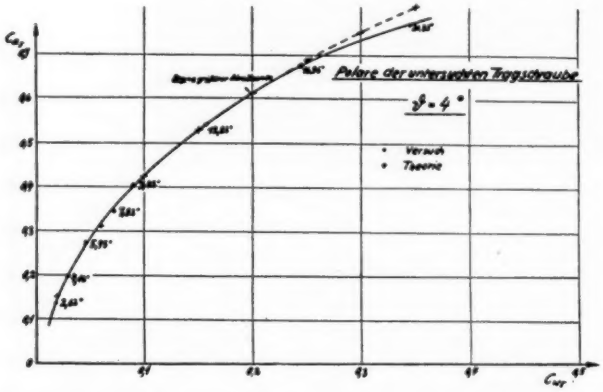


FIG. 3 EXPERIMENTAL AND THEORETICAL POLAR CURVES OF AUTOGIRO ROTOR

(Ordinates, lift coefficient; abscissas, drag coefficient; pitch angle of blades, 4 deg; degree figures on curve refer to angle α_π; dotted curve, theoretical; solid curve, experimental.)

blade need be taken into account for the computation of the air forces. Naturally the local angles of attack change in accordance with the momentary position of the blades during rotation. He designates this local angle of attack by ψ and calculates from the rearmost position of the blades as a base.

In the calculations he employs the air velocity in the form of two parameters, an axial parameter $x = v_d/R\omega$, where v_d is the axial velocity through the plane of the airscrew disk, and also a tangential parameter $\mu = v \cos \alpha_\pi/R\omega$ where $\cos \alpha_\pi$ occurs due to the inclination of the plane of the rotor with reference to the air stream. As the hinged blades are free to take up an equilibrium position, which is the resultant of the weight, the centrifugal force, and the aerodynamic force, and, therefore, bear relationship to the thrust or lift, Glauert is obliged to take the motion of the blades into his calculations. The hinging of the blades allows a forward flight of the rotor, free from the rolling moments which would be present on a rigid rotor, because of the greater lift on the advancing blade. The centrifugal force alone would keep the blades in a plane perpendicular to the axis of rotation as in Fig. 5 (*a*). Under the influence of the thrust the blades rise upward through an angle β and describe an inverted conical surface, of which the angle β may be called the "coning angle."

Glauert sets up the equation of equilibrium of the masses and aerodynamic forces about the hinge and secures the equation of Fig. 5 (*e*)

$$I_1\left(\frac{d^2\beta}{dt^2} + \beta\omega^2\right) = \int_0^R r dS_1 \ldots\ldots\ldots\ldots [1]$$

where I_1 is the moment of inertia of a blade, and S_1 is the thrust of one blade. The first term in the bracket indicates the moment of the masses of the blade against a perpendicular up-and-down motion, which we shall call flapping. The second term within the bracket is the moment of the centrifugal force of the blades. On the right side of the equation are the aerodynamic-force moments in the form of an integration of the elements of thrust. Glauert neglects the moments due to the mass of the blades, as they are

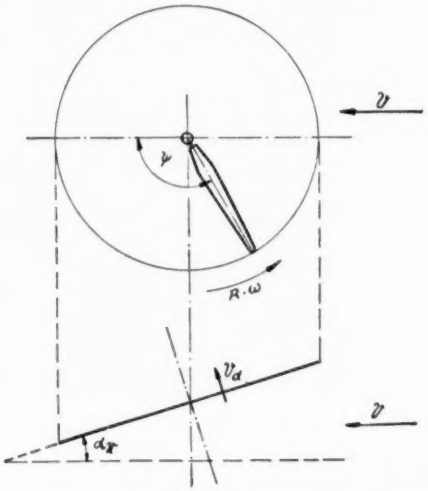


FIG. 4 DIAGRAM OF NOTATION FOR AUTOGIRO ROTOR

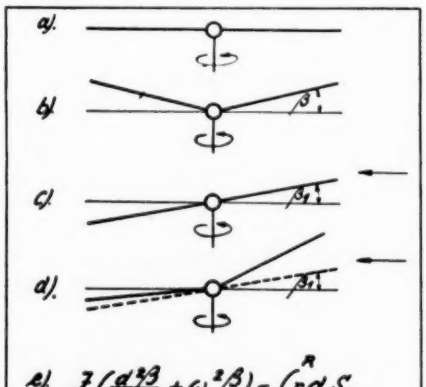


FIG. 5 DIAGRAM FOR GLAUERT-LOCK THEORY

small compared with the moments of the air forces and centrifugal forces. He then develops β and $\int r dS_1$ in a Fourier series and arrives, after some tricks and neglect of terms which I shall not discuss, at the result that the whole cone of the revolving blades is directed backward through an angle β_1 so that β changes periodically with the cycle of revolution, has its maximum value when the blade is in the foremost position, i.e., when $\psi = 180$ deg (Fig. 4), and its smallest value in the rearmost position when $\psi = 0$, when it coincides with the coning angle β. See Fig. 5 (*d*).

Glauert gives formulas only for rectangular blades, with a constant chord t. The whole Glauert-Lock theory was extended by my co-worker Bansemir for trapezoidal blades, in which

extension $t = t_0 - (t_0 - t_1)(r/R)$. Here t_0 is the chord at the root of the blade and t_1 that at the blade tip. The chord decreases proportionately to the radius of the blade. After carrying through the integration of the right side of Equation [1] we obtain, as in Fig. 5 (*f*)

$$\beta_1 = \mu \frac{x\left(\frac{t_0}{2} - \frac{t_1}{3}\right) + 2\vartheta\left(\frac{t_0}{3} - \frac{t_1}{4}\right)}{\left(\frac{t_0}{4} - \frac{t_1}{5}\right) - \frac{\mu^2}{4}\left(\frac{t_0}{2} - \frac{t_1}{3}\right)} \quad \ldots\ldots\ldots [2]$$

The tipping of the whole blade-system cone signifies, therefore, that the advancing blade rises and the retreating blade falls. Now we secure an important insight into the reason why the rotor with hinged blades experiences no tipping moments about the longitudinal axis of the aircraft. The advancing blade, because of its rising path in space, operates at a smaller angle of

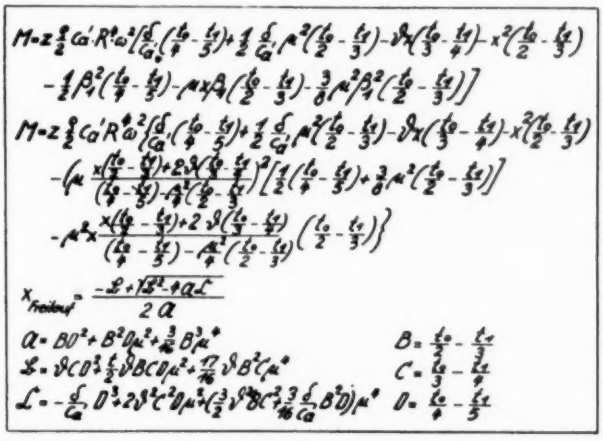


FIG. 6 CHARACTERISTICS OF AUTOGIRO ROTORS

attack, and thus the extra lift due to the vectorial addition of the forward speed and the rotational speed is eliminated. The converse is true with the retreating blade. The regulating cause of this phenomenon is found in the centrifugal force which, at the same speed of revolution and with the same weight of all blades, must be the same everywhere.

Having obtained β_1 the tipping angle, we can determine the further aerodynamic characteristics of the rotor. See Fig. 6. For the rotational moment which with the autogiro rotor must be zero, Glauert deduced the axial parameter x, the thrust S_1, and the distribution of thrust over the radius of a blade dS_1/dr. See Fig. 7. Finally, he deduced the longitudinal force H, which plays the part of the tangential force of an airfoil, and so lies in a plane perpendicular to the axis of rotation and in the direction of flight.

The rest is simple. Out of the normal force, which is in this case the thrust, and out of the longitudinal force, which is the tangential force, we can derive C_a and C_w, the lift and drag coefficients of the whole rotor. Then the polar diagram is available, of which Fig. 3 is an example. Mention has already been made of the wind-tunnel measurements. The dotted curve of Fig. 3 shows how well the English theory, in spite of its various assumptions and simplifications, agrees with the experimental results.

There is only one thing that the theory does not point out. We cannot predict our trial in terms of our first fundamental requirement, the ability to make a safe emergency landing. The dangerous possibility of cessation of autorotation is not indicated by the theory because it assumes a linear slope for $dC_a/d\alpha$ and autorotation ceases precisely because the greater part of the blade, in the region of stalling, assumes large drag coefficients coupled with small increase in lift. There is a factor of safety in the Reynolds numbers, smaller than occurs with the airplane, at which wind-tunnel experiments are made, if the boundaries of autorotation are determined in this manner. The

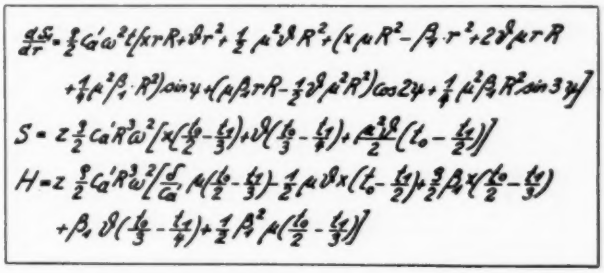


FIG. 7 CHARACTERISTICS OF AUTOGIRO ROTORS

danger zone lies in quite a narrow region of the pitch settings, With medium blade thicknesses and values of ϑ below 4.5 deg. measured from the angle of no lift, there is no danger of loss of autorotation. In the region $\vartheta = 6$ deg, there is always great danger, as unfortunate accidents have shown, particularly at high speeds and low disk loadings. We note also that the cessation of autorotation behaves in exactly the opposite fashion from the stalling of an airplane.

In our problem of a helicopter that possesses the emergency landing features of an autogiro the hazard of cessation of autorotation plays no great part. If there were any doubt as to the continuation of autorotation all that we would have to do is to reduce the pitch angle ϑ somewhat. Then we shall have completely satisfied our first fundamental requirement.

We have so far considered the production of lift only for the lifting rotor in a stationary position, and as an autogiro rotor. I have thus far purposely neglected the forward flight of helicopter airscrews. Lock has already begun to extend the Glauert considerations to this flight condition. We shall, therefore, pattern the calculations for the helicopter airscrew, in forward flight, on those of the autogiro rotor and then discuss the experiments in this condition. We only need to introduce the moment term about the axis of rotation and we have the helicopter instead of the autogiro rotor. All other considerations remain valid, and the calculations retain the same procedure. Instead of the customary lift coefficients we utilize thrust coefficients as, in the helicopter condition of hovering flight, the coefficient C_a becomes infinity. It is the relationship between the forward flight of the helicopter and its hovering flight, that is particularly interesting. According to the procedure adopted by Flachsbart, we shall, instead of C_a, take the vertical component of the thrust K_{sv}. We shall refer also to the circumferential velocity instead of the forward velocity, and substitute the horizontal component of the thrust K_{sh} for C_w.

In the helicopter condition the value x changes its sign. The flow through the rotor is downward and this remains true in forward flight. The angle of the rotor to the flight path α_π (or in the wind tunnel to the direction of the air stream) has no significance in hovering flight, but in translational flight it can have any value from zero to plus or minus 180 deg, since backward flight is also possible. It is noteworthy that, with the slowest vertical climb, it has a value of —90 deg and, with the slowest vertical descent, a value of +90 deg. In the forward flight of the helicopter, in which there is no other means of propulsion, such as a tractor propeller, the angle of attack α_π must

always be negative, as the axis of rotation must be tilted forward, in order to achieve a sufficient forward component to overcome parasite drag.

The results of the calculations for forward flight, will be compared later with the results of experimental work. The measurements of forward-flight conditions were made with the same apparatus as that employed in testing the autogiro rotor. The motor was coupled, and the performance calculations were made with the aid of electrical readings of the power required. The tests also included tachometer measurements of the rotational speed. Great care was required, as there were many chances of

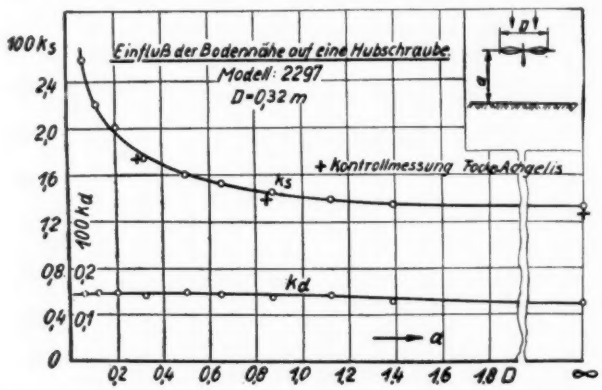


FIG. 8 INFLUENCE OF GROUND EFFECT ON A LIFTING AIRSCREW (k_s = thrust coefficient; k_d = power coefficient.)

error, which I shall not discuss in detail. I will mention, however, a single factor, the presence of the ground influence. When the ground is level over a sufficiently large area, the thrust increases, as does also the torque. This effect becomes apparent within a height above the ground equal to the rotor diameter. Thus, with a certain throttle setting, the helicopter will rise, but will not rise over a few meters without increase in the throttle setting. A helicopter with too little excess power does not rise over this "swimming level." On the other hand this phenomenon results in a welcome help on landing. In the laboratory it is unpleasant because one can never work far from rigid objects. Up to the present, few experiments have been made on the influence of ground proximity, of which the best ones are those made by Flachsbart in 1928, which our own measurements confirmed rather well. The upper curve of Fig. 8 shows this increase of thrust and the lower curve the increase in torque plotted against the distance from the ground expressed in diameters of the rotor. The three crosses indicate our own control measurements.

CONTROL AND STABILITY

No control is conceivable, unless we first of all establish an equilibrium of moments. There exists with the helicopter the well-known difficulty that in part has influenced the whole question of the direct-lift machine. The slowly rotating airscrews exercise on the aircraft a turning moment of the order of magnitude of thousands of kilogram-meters. The manner in which this torque is removed influences the whole constructional aspect of the helicopter. Many forms have been suggested as Fig. 9 indicates: (*a*) Two screws, one over the other, rotating in opposite directions (Bréguet, d'Ascanio, Pescara, Asboth); (*b*) two screws, one behind the other (Cornu), and also four placed at the corners of a square (de Bothezat, Oehmichen); (*c*) two screws rotating in opposite directions, and placed side by side (Berliner, Focke); (*d*) peculiarly enough, two screws rotating in the same direction, with axes of rotation so disposed relative to one another, that the resulting side moments have been removed (Florine, Belgian government); (*e*) a single large screw on the blades of which small screws are placed (Isacco, Curtiss-Bleeker); (*f*) on which the blades themselves perform oscillations so that a moment does not arise at all; (*g*) a single airscrew, and at the end of the fuselage one or more propellers whose thrust counteracts the torque (Baumhauer, Holländisch Rykstudiedienst); (*h*) a propeller blowing air backward against deflector surfaces (Hirtenberger Patronenfabrik, Austria); (*i*) surfaces in the slip stream of the helicopter rotor (Hafner and Nagler, Austria); and finally (*k*) a jet-reaction orifice with smaller screws and compressed air (Dornier patent, Papin and Rouilly, France).

From all these proposals we can separate out those which add appreciable drag or loss of efficiency. These are: (*e*) In which the efficiency of the small propeller reduces the efficiency of the helicopter as a whole, by about 30 per cent; (*g*) and (*h*) in which, for the production of the antitorque force, there is needed a continuous expenditure of power, which is of the order of

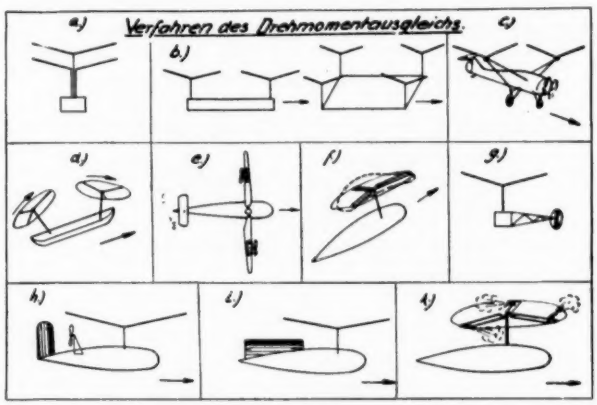


FIG. 9 PROPOSED METHODS OF BALANCING THE TORQUE IN A HELICOPTER



FIG. 10 BRÉGUET-DORAND HELICOPTER

from 20 to 30 per cent; (*f*) and (*i*) reaction and flapping drive—both are perhaps destined to play a part later on; (*d*) the inventors of which wished to retain the gyroscopic effects which are lost with rotors turning in opposite directions, offers no advantages as compared with (*b*) and (*c*). When we eliminate the use of more than two airscrews because of mechanical complication, then there remain only three cases; two screws, placed over each other, behind each other, and side by side, rotating in opposite directions. The first of these three arrangements has been most frequently employed hitherto. The helicopter which was most successful up to the last year, that of Bréguet-

Dorand in France has this arrangement as shown in Fig. 10. The disadvantages of this type are:

(*a*) Excessive vibration is set up by the passage of the blades past each other.

(*b*) The efficiency of two rotors in line is much less than with separate rotors.

(*c*) The slip stream blows over the whole structure so that the effective thrust is further reduced, and the longitudinal balance is affected.

(*d*) The saving in weight of the arrangement is balanced out by having a smaller disk area for emergency landing.

The only arrangement which entails no prejudicial interferences is the form in which rotors are disposed side by side. Vibrations of the blades disappear, both rotors offer their full disk area in emergency vertical descent, and they mutually provide high lateral stability. The efficiency is as high as that of the single rotor, and in the down-wash of each there lie only the most necessary elements of the structure. Also the difference in space requirements is not much, since what is saved in span by using two rotors placed over one another, is balanced by the greater length of the machine and above all by the greater height.

Now we can consider the stabilizing and control problems. Many helicopters have been controlled and stabilized by means of control surfaces after the manner of the airplane but, in the hovering condition, this is not possible. We can indeed think of surfaces or control surfaces placed in the slip stream of the lifting airscrew rotor, or in the slip stream of the ordinary propeller. We have ourselves tried both with little success. More suited to the nature of the helicopter is a control and stabilizing system through the action of the blades themselves. We must go back here to theoretical calculation and consider next longitudinal stability and control.

(*a*) *Longitudinal Stability and Control*

The tipping of the blades β increases, as Equation [2] showed, with the tangential parameter μ and, therefore, on the whole, with the speed. As the longitudinal blade pull is practically the only force which can be put into the hinge at the hub and as this force is the same in all blades, it is clear from considerations

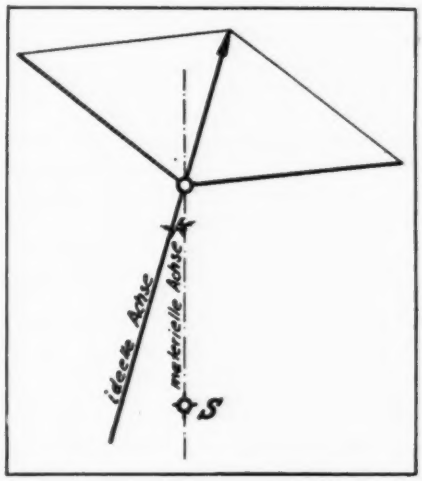


FIG. 11 DIAGRAM INDICATING ACTUAL AND VIRTUAL AXES OF ROTATION OF HELICOPTER ROTOR

of symmetry that the parallelogram of forces is a rhombus the diagonal of which is the resultant direction of force. See Fig. 11. It lies at an angle β_1 behind the actual axis of rotation. In other words, there is a virtual rotational axis which slopes downward and backward from the blade hub at an angle β_1 with the actual axis. This is only approximately correct, as the thrust distribution and the air skin friction on the blades influence its position, but the heart of the phenomenon is not seriously influenced by these considerations.

If the center of gravity of the whole aircraft lies below the airscrews, it becomes evident that there is longitudinal static stability due to the airscrew alone. An increase in speed, that is, a greater inclination of the virtual axis allows the thrust to move ahead of the center of gravity, as β_1 increases. The converse is true with decrease in speed. As the same formulas serve for the helicopter as well as for the autogiro, the principle holds true for both types of aircraft. Therefore we can calculate the longitudinal moments in both cases, or determine them

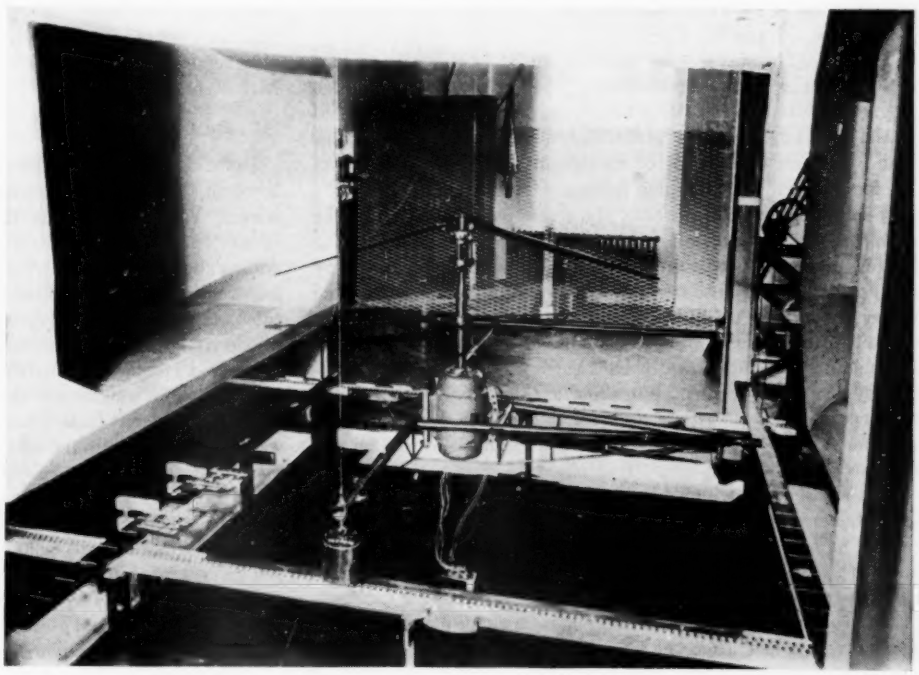


FIG. 12 ARRANGEMENT OF WIND-TUNNEL FOR STEREOPHOTOGRAPHIC MEASUREMENTS ON BLADES

by tunnel tests, in which both the magnitude and direction of the resultant can readily be obtained. As the resistance, because of many corrections and because of the size of the suspension wires, cannot be determined with great accuracy, we have been led to other experimental investigations.

First it was desired to know the position of the blades during the whole cycle of rotation, as many constructional requirements—for example the kinematics of the blade positioning—require such information. The English have carried out such an investigation with mirrors. To Professors Schering, Finsterwalder, and Flachsbart I owe the proposal and execution of a stereophotogrammetric method, which for an airscrew of $1^1/_2$ meters diam, gives the correct location of the blade at half radius with an accuracy of approximately 1 mm. See Fig. 12. Between the objectives of a stereocamera, an illuminating spark is introduced giving an illumination time of 0.000001 sec which, at a speed of 1700 rpm, allows us to see a row of white dots painted on the airscrew. With the airscrew and the wind tunnel operating, from 20 to 30 sparks are induced, giving on the exposed pair of plates a row of photographs of the white dots at random positions, but sufficiently well distributed to give a complete picture of all blade positions, after evaluation with the stereocomparator. An auxiliary system includes a measurement system on the floor of the wind tunnel. An example of such a photograph is shown

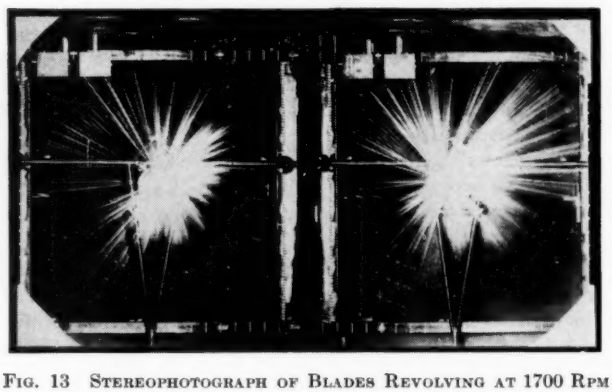


FIG. 13 STEREOPHOTOGRAPH OF BLADES REVOLVING AT 1700 RPM (Distance of white dots from the center is a measure of angle β.)

in Fig. 13. The star-shaped figure is caused by the reflections of the blades and has no significance. The method is well adapted to give information regarding the so-called change of phase, that is, the actual displacement of the highest and lowest points of the blade path out of the cone of revolution owing to the inertia of the blades during the flapping motion.

It follows that on the whole, the screw behaves not only like an airfoil with a constant center of pressure, but also like one having inherent stability. When this stability is produced by the position of the resultant, it becomes possible to control the rotary aircraft by deliberately changing the position of this resultant force. Cierva, in the C-30 autogiro, has secured control by inclining the hub forward, backward, or sidewise. Controllability has been secured, but some accidents when going into high speed, that is, with strong forces, lead to a doubt as to whether longitudinal stability is always maintained in Cierva's direct-control system.

In a small work which has been sent to interested parties, I have shown that a peculiar hazard is produced. The direction of control at high speeds changes signs or at least the control moment attained is independent of the positioning, so that the pilot has the sensation of loss of control. If the pilot decides to pull or push through even if the beginning of his control movement has the contrary effect, then the aircraft will eventually right itself. The real danger is that in the instant of apparent loss of control, such high diving speeds can be obtained that autorotation ceases. The thrust decreases, and the restoring longitudinal moment even after the successful effort, is insufficient to prevent further downward curvature of the flight path in space.

It is a mistake to think that the fixed stabilizer is the only guilty element. It was an error on Cierva's part to think that his direct control was solely a center-of-gravity control system. At high speeds, high moments are produced from the fixed stabilizing surface. I have therefore proposed that with rotary aircraft a movable horizontal surface should always be included which could work effectively under all circumstances to produce moment equilibrium. Then every danger on this score would be excluded. The D.V.L. has made independent investigations and has come to a similar conclusion. Nevertheless Cierva's direct control is not employable in the helicopter. It gives large control forces, which go into the whole aircraft, through the lever arm produced by β_1, which is not always negligible. If transmission is added to the system, then it will be seen that the large tooth and bearing loads, which change with the performance, cannot but have an effect on the control system when a tiltable hub is employed.

If, then, it is not possible to change the actual axis of the airscrew, we can change the "virtual" axis by varying β_1. We saw that the equilibrium of the air and dynamic forces prescribed the position of the flapping blades during a cycle of revolution, so that they took the corresponding angle of attack. As we must change the angle of the blades in going from helicopter to autorotative operation, we can reverse this process, and force the blades to have an angle of attack, variably controlled throughout the revolution, so that they must follow a desired path in space suitable for control.

Lock had already shown that the imposition of a sinusoidal variation in the angle of setting during each revolution produces the same effect as inclining the whole airscrew. Hence, control with the virtual or with the actual axis of rotation is equivalent, but it is important to note that control by means of varying the angle of setting, with the use of a blade having a constant center of pressure, theoretically introduces no forces into the control system. In practice, only the skin-friction forces and small mass forces are introduced. The change of the angle of attack is therefore an ideal control relay. Moreover the mechanism for the change of the angle of incidence is under little stress, and can be both light and reliable.

(*b*) *Dynamic Stability*

Up to this point everything has been relatively simple. Now, however, the most difficult question, that of dynamic stability, arises. When we began the construction of our machine, almost no information was available in this field. Nothing was known about either the helicopter or the autogiro rotor. We have followed two paths simultaneously. First we adapted the normal calculations for the airplane for use with the helicopter. Then we derived the equations for the hovering helicopter from the forces and moments involved. Numerical data were obtained in part by careful weighing of the degree of reliability of the Glauert theory, and in part from wind-tunnel work. Particular attention was paid to the stability and control processes at the instant of transformation from helicopter to autogiro flight. From these calculations we were able to give the pilot careful instructions about the movement of the controls and the setting of the stabilizer. The calculations were verified in practice, and the first test gave a perfect three-point gliding landing from a height of 400 m (1300 ft). Some two seconds after the transformation, the machine was in normal gliding flight. One can say that with this achievement of Pilot Rohlfs on May 10, 1937, the realization of

the practical helicopter was assured. The haunting thought of power-plant failure had lost its terrors.

Of our fundamental requirements we have therefore obtained both stability and calculable controllability about all three axes, both for hovering and in forward flight. We have obtained the transformation from helicopter into autogiro glide flight in a few seconds. We have secured simplicity and reliability in the arrangement of the control system, since with only two principal movements on the screw hub, there have followed four controls, two linear—transformation and rolling control—and two of a sinusoidal character—height and yawing control. Finally, the condition of simple effort by the pilot has been met because the three customary movements of hand and feet have only been increased by another operation with a lever placed at the right side of the seat. This is for the transformation from helicopter to autogiro rotor, and is used only in take-off and in special emergency cases.

I cannot conceal the fact that the theoretical and experimental effort in securing these results was very great. In particular, the stability calculations were most complicated. As one is entering unknown territory it is necessary to be very conscientious and to avoid all approximations and neglecting of terms. Where approximations could not be avoided, due to mathematical limitations, they had to be supported by special lengthy investigations. The success attained has justified this enormous effort. The first free flight of the machine lasted 28 sec, the fourth 16 min, and, although much is to be attributed to the skill of the pilot, Rohlfs, such success in the tests would have been unthinkable without a thorough technical understanding of the entire subject.

Performance

(a) *Autogiro*

In considering the polar of the autogiro rotor, Fig. 3, it is striking that the drag is large with large values of C_a, many times greater than that of a good airfoil. That is explainable not only on the ground of the low aspect ratio $\pi D^2/4D^2 = \pi/4$ which is unchangeable, but also is traceable to the C_{wp} which is a special characteristic of the autogiro rotor. On the other hand, with small values of the lift coefficient C_a, C_w is small, that is, the drag coefficient is small at high speed. This circumstance is specially emphasized by the small loading of the blades when referred to the whole circular disk. The lift-drag ratios at small lift coefficients approach those of good airfoils, and lie around 8 to 1 or 10 to 1. Even an L/D of 12 seems attainable. But these are, in contradistinction to the airfoil, the highest possible values of L/D. At high speeds therefore the autogiro rotor should not be far inferior to the fixed airfoil. The autogiro will have a maximum speed about 15 per cent less than that of the airplane.

Schrenk has given an interesting comparison between the airplane and the autogiro in the form of characteristics curves, Fig. 14. The really striking difference rests in part on the greater parasite drag, which difference could be reduced by better streamlining of the hub, starting rig, and other parts. Therefore it can be estimated that the autogiro will be some 10 per cent lower in top speed than the airplane but, on the other hand, will have about half the minimum speed.

Things look worse in the climb. Now we are working on the higher parts of the polar curve and the resistances become large. The lowest power required is much higher than with the airplane and also occurs at slower speeds, which is undesirable because of the resultant inefficiency of the propeller. That strengthens us in the conviction that the autogiro is only destined to play an intermediate part between the airplane and the helicopter. The autogiro solves less than half the problem, that of landing in a small space, but does not permit a zero velocity, while the take-off and the performance in climb are worse than before. There is, however, no inferiority from the point of view of weight. On the contrary, in large machines the freedom from bending moments should more than counterbalance the weight of the hub and starting gear. This is one of the great advantages of all rotary aircraft.

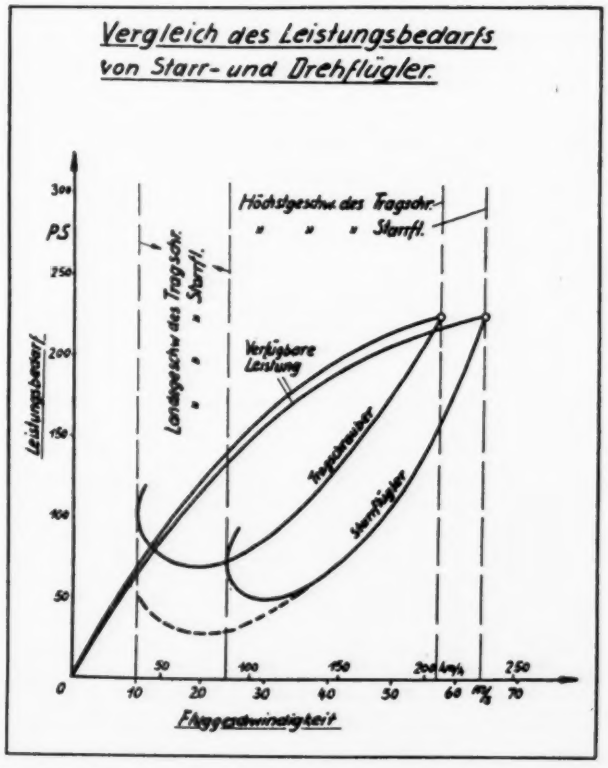


Fig. 14 Comparison of Power Requirements of the Airplane and Autogiro

(Ordinates, power required, hp; abscissas, flying speed, m per sec and km per hr; landing speed of autogiro, 10 m per sec; landing speed of airplane, 25 m per sec; maximum speed of autogiro, 57 m per sec; maximum speed of airplane, 65 m per sec; *Verfügbare Leistung = power available.*)

(b) *Helicopter*

We must make proportionately the largest sacrifice regarding weight in the helicopter. The transmission must function with complete reliability under full output of the motors, which places the helicopter in small sizes at a serious disadvantage with the airplane. Bréguet has on the other hand designed a 16-ton helicopter flying boat, and finds a saving in weight as compared to the corresponding high-speed airplane. If this appears somewhat optimistic the fact nevertheless remains that, with greater size, the weight of the transmission of the helicopter will become less of a disadvantage.

When high speeds are considered, the question arises: Is it advantageous to let a given rotary aircraft fly as an autogiro or as a helicopter without a propulsive airscrew? It is conceivable that a machine could be designed to have zero velocity and yet be flown as an autogiro. We have made careful computations, reinforced by wind-tunnel tests, with the interesting result that the same machine flown as a pure helicopter is somewhat faster, even if we deduct 10 per cent for motor cooling. Practical experience with a helicopter has fully confirmed this view. The machine after installation of a normal propeller instead of the cooling blower could be flown as an autogiro, and gave with streamlining of all tubing and parts a speed of 120 km (75 miles) per hr, as compared with the 126 km (78 miles) which had been

calculated. Streamlining for the helicopter could not be incorporated without reconstruction. The horsepower requirements of the unstreamlined bracing and parts could be determined in special tests, however, and at 120 km (75 miles) per hr, amounted to 46 hp. Thus the streamlined helicopter would be 25 km (15 miles) kilometers faster. That would give 147 km (91 miles) an hr for the helicopter as compared with the calculated value of 152 km (94 miles) per hr.

As we saw that the autogiro, in so far as top speed is concerned, is only slightly inferior to the airplane, we can conclude that the helicopter could at least equal the speed of the airplane. Here also the decrease of the parasite resistance is the chief point of further development. There is, however, a certain limitation to the helicopter. As we cannot obtain a forward-speed parameter of over 0.5, the tips of the helicopter blades will reach the speed of sound at half the forward speed of the airplane. Then the addition of speed has to be reckoned with in the advancing blade. At a speed of about 400 km (250 miles) per hr, the prospects of the helicopter become much worse than those of the airplane. Of course it must be remembered that in the same speed ranges, difficulties regarding compressibility effects also begin to occur with the conventional propeller.

The climb of the helicopter appears to be remarkable. In Table 1 are shown the initial climbs of the Fw 44 Stieglitz, of the de la Cierva C 30, and of the helicopter, flown as an autogiro and flown as a helicopter, all of them provided with the same motor, the Sh 14. In particular it should be noticed that there was no equality of gross weight. The helicopter is the heaviest, nevertheless it has the greatest climb. The autogiros are quite inferior.

TABLE 1 INITIAL CLIMBS AND WEIGHTS OF VARIOUS TYPES OF AIRCRAFT WITH THE SAME MOTOR

Machine	Initial climb M per sec	Initial climb Fps	Flying weight Kg	Flying weight Lb
Fw 44 Stieglitz airplane.....	3.5	11.5	870	1920
Cierva C30 autogiro.....	1.5	4.9	815	1795
Fw 61 helicopter as an autogiro..	1.3	4.26	950	2095
Fw 61 helicopter as a helicopter.	3.6	11.8	950	2095

I must particularly emphasize here that the helicopter does not secure its justification by the existence of superior climb or speed, relative to those of the airplane. Its special characteristics give the helicopter special duties and possibilities. It will be all the more worth while however if it shows that it is not inferior in performance characteristics.

It is hardly necessary to speak of the take-off and landing characteristics. The ideal sought for is attained, at least technically. Doubtless the full utilization of all the possibilities of the helicopter will make new demands on the skill of the pilot and the personal equation will enter. But I can report that contrary to anticipation, no acrobatic skill is needed. Miss Hanna

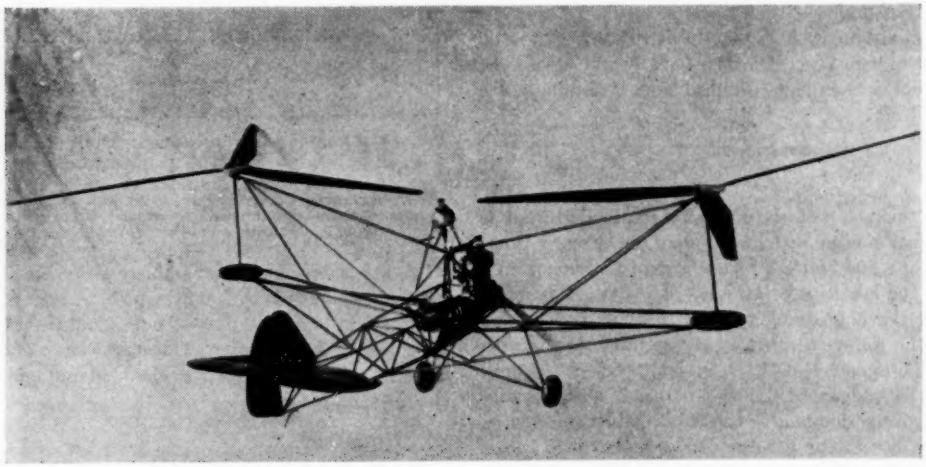


FIG. 15 FREE-FLYING MODEL OF HELICOPTER

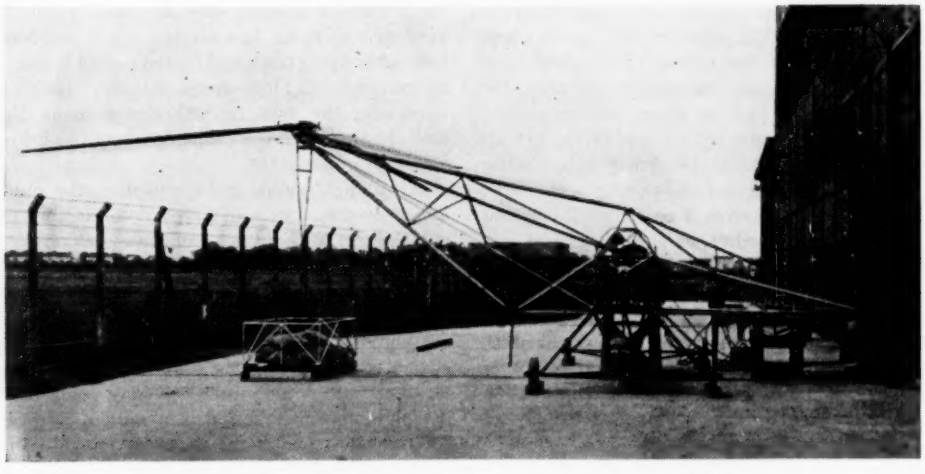


FIG. 16 HELICOPTER WITH ONE ROTOR LOADED FOR TESTING

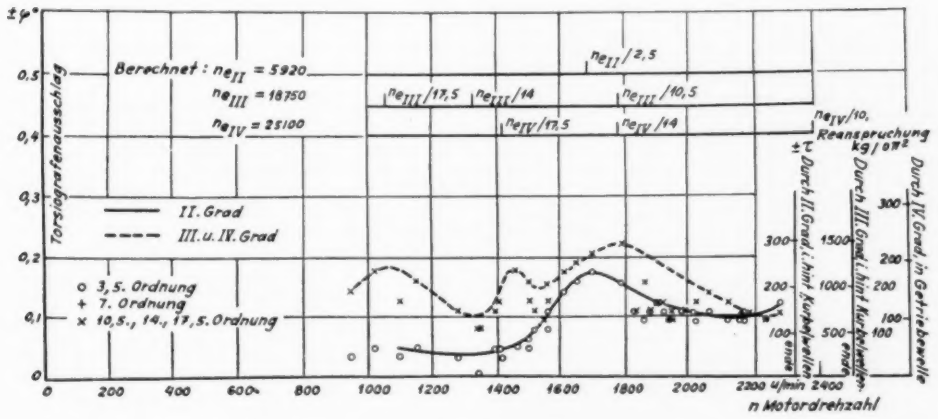


FIG. 17 TORSIOGRAM OF POWER TRANSMISSION

Reitsch and Messrs. Dipl. Ing. Francke, Ballerstedt, and Bode have flown my helicopter splendidly with short technical instruction, and made smooth landings.

Due to the accidents with autogiros caused by cessation of autorotation at small angles of attack and large pitch angle, and to the consideration that small angles of attack must inevitably occur in gusts, the opinion has arisen that all rotary aircraft are sensitive to gusts. As the connection with autorotation shows, this conclusion is not at all applicable to the helicopter, and we have seen already that with the autogiro there is no danger if an adjustable horizontal surface is provided. Besides it is to be surmised that, as a result of the flexibility of the blades, the rotor would be less sensitive to gusts. By means of simultaneous tests of an airplane and of the helicopter, it was shown that the pilot of the airplane experienced severe bumps, while the pilot of the helicopter hardly felt the gusts. At other times the gustiness was perceptible at wind velocities of 40 km (25 miles) per hr. It therefore seems probable that the character of the gust is to be considered. It is interesting to learn that, on the small factory landing field of Focke, Achgelis & Co., the helicopter was easily controllable, even in the disturbed regions of buildings, in gusts of from 6 to 10 m (20 to 33 ft) per sec.

I have so far considered only the investigations which were necessary to the development of the helicopter. I cannot emphasize too strongly in what measure pure science was the foundation on which new knowledge was based. The second and still more difficult stage of our work was in embodying in practice the information thus gained.

CONSTRUCTION

The first step was a free-flying model, driven by a 0.7 hp two-cylinder motor. See Fig. 15. It had a flying weight, with 50 g of benzene, of 4.9 kg (10.8 lb). It is easily understandable that the model was more frequently in bits than whole, nevertheless many worth-while experiments were made with it. In November, 1934, it reached a height of 18 m (59 ft), equaling what was up to that time the world's record for a helicopter with a pilot. The model is now in the Deutsches Museum in Munich.

It was decided further that the transmission, coupling, and flap control should be submitted to the same tests as a new engine. The Brandeburg Motor Werke, which under the personal direction of Director Wolff, had undertaken the difficult task of this construction and of the conversion of the Sh 14-a motors, built one side of the system. See Fig. 16. It was provided with a single airscrew and its structural supports. A Leonhard electric drive was used, so that the performance requirements of the airscrew could be measured in full scale. The thrust was measured

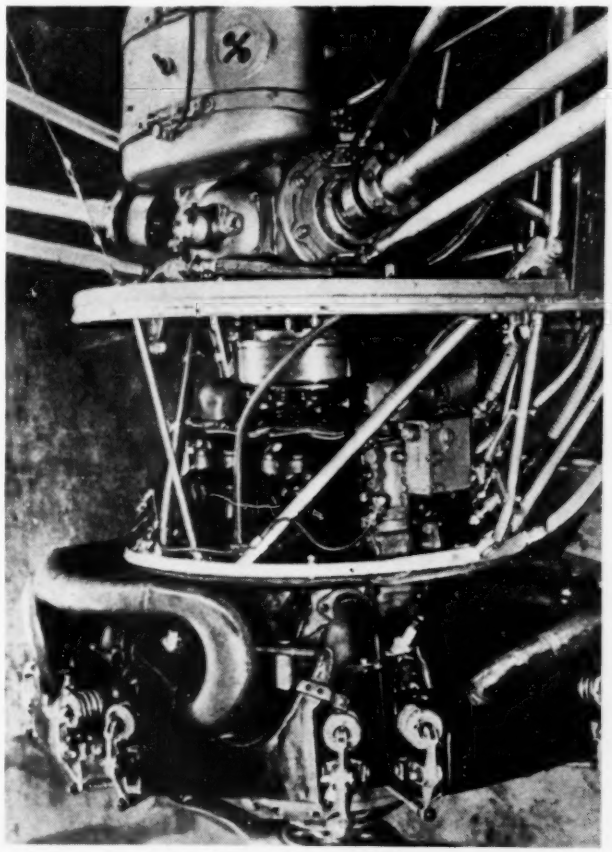


FIG. 18 SHOWING ENGINE, FRICTION CLUTCH, AND AUTOMATIC MECHANISM FOR CHANGING TO AUTOROTATION

by means of ballast and by making the fuselage rotatable about its longitudinal axis, taking due account of the ground effect. This constituted a valuable supplement to the wind-tunnel tests at full-scale evaluation. A 50-hr test was made and, after overhaul of the transmission, another 10-hr test. The controls were continuously tested at the same time.

A vibration investigation of the whole transmission system was made for the D.V.L. by Professor Lürenbaum and his associates. The calculations were confirmed by tests with the torsiograph, in which the aircraft was flown as a kite. This was all the more necessary since the electrical drive used in the first test was not

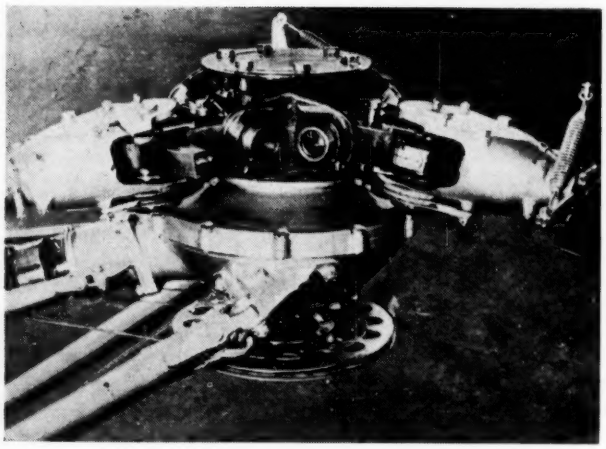


FIG. 19 ROTOR HUB WITH BLADE-PITCH CONTROLS



FIG. 20 SHOWING PROPELLER-TYPE BLOWER FOR COOLING ENGINE IN HOVERING FLIGHT



FIG. 22 PILOT ROHLFS AND THE MACHINE AFTER BREAKING ALL WORLD RECORDS



FIG. 21 COMPLETED HELICOPTER IN CAPTIVE FLIGHT

sound from a vibration point of view. Fig. 17 shows a torsiogram, of which I will only say that the most highly stressed part was within the motor and not within the helicopter. Fig. 18 shows the transmission with the friction coupling, and the safety provisions which, by failure of the motor or of the gears, or the reaching of a certain minimum rotating speed, automatically causes transformation into the autogiro state. Fig. 19 shows one of the screw hubs with controls for the blade motion.

The construction of the framework and of the rotors was undertaken as for a normal airplane. New ideas in the design office and the unconventional constructional elements made difficult work for my associate Ing. Körper. Under these conditions it is gratifying that maintenance, while not ideal, is adequate. For the new problems in stress analysis, the calculations and experiments previously made were utilized, and in some respects these had to be supplemented.

For the cooling of the motor in flight in one spot, a propeller-type blower was developed, and the cooling was investigated by cylinder-temperature measurements in cooperation with the Brandeburg Motor Werke. See Fig. 20.

Flights

The completed prototype was often flown in a captive condition, as shown in Fig. 21. Captive flights are an excellent method of investigation, as everything happens under flight conditions while one is only from $^1/_2$ to 1 meter above the ground. No further steps in the proving were undertaken until the conditions were clarified by further calculations or tests. On June 26, 1936, Rohlfs made the first flight. On June 25 and 26, 1937, he was able to bring to Germany all the world helicopter records, with fifteen times the performance existing up to that time. See Fig. 22. I have already spoken of the important glide landings. The height reached, 2439 m (8000 ft), which did not long remain the maximum height, has brought me the undisguised charge of deceit from Mr. Asboth in a foreign technical paper. The lack of foundation for this accusation is shown by the many witnesses of the flight and through the immediate recognition of all records by the Fédération Aéronautique Internationale.

In June and October, 1937, the first and second machines, respectively, were taken over by the Reich. With the latter Miss Reitsch flew, in October, 1937, from Bremen to Berlin surpassing the record between Stendal and Tempelhof by over 108 km. Bode, on June 20, 1938, surpassed this record by flying 230 km after a larger fuel tank had been installed.

Nobody had expected such performances. The future is bright.

The Interpretation of a Failure of an Ordnance Structure[1]

By G. F. JENKS,[2] WASHINGTON, D. C.

For the analysis of a failure of a war model of a 240-mm howitzer at half pressure, a fragment was furnished the laboratory. The problem was divided into three phases: First, the origin of failure; second, the mechanism by which the failure developed in the structure; third, the conditions responsible for the failure. Emphasis is placed on determining the spot at which the failure started.

A method of reading the rupture flow lines found on the surface of the fragments is described. The arrow points found in torn metal surfaces are shown as pointing to the origin of rupture. These arrow points and other surface characteristics furnish the time scale by which priority as to time may be established for tears or for ruptures which appear to be independent. Once the beginning of a failure is found, however, the laboratory investigation then becomes simple as it may be confined to a small area.

In this investigation the heat-affected zone of a weld was disclosed by acid etching. The design called for no welding at this point. Microscopic investigation showed that weld metal of an inferior quality was present, and that cracks existed in the heat-affected zone of the weld. A strip of metal of high Brinell hardness was found just outside the weld metal. The failure progressed from old cracks in this region of high hardness on the exterior surface of the gun. The tension and impact properties and the macroetch characteristics of the metal were determined and their influence upon the failure estimated. Finally, manufacturing records were checked and showed that a machine error on the surface in question had been repaired by welding.

THE design of a structure is based upon an analysis of applied stresses with provision for errors in the method of calculation and for deficiencies in behavior of materials applied under the factor of safety. Failures of structure may occur for any of the following reasons: (1) Stresses applied in excess of those for which the structure was designed; (2) highly localized stresses; and (3) unpredicted behavior of the material. Designs may be checked by various proof tests but primarily by service use. If structures fail under service conditions a complete analysis of the failure is essential to the perfection of design and processing practices.

In the study of failures of structures the first task is to find the origin of the failure; the second, the mechanism by which the failure developed in the structure. After this preliminary work the examination of the material and the analysis of stresses point to the conditions responsible for the failure.

Each structural failure is a special problem and involves different lines of approach. It is believed, however, that cases of examination of failed structures are of interest. Engineering literature is comparatively poor in this respect. There is a need for a more general disclosure of information regarding failures and their analysis for the improvement of design practice.

The gun is one of the earlier engineering structures for which careful stress analysis was made. It differs from most engineering structures in that the factor of safety is low and in that its life is limited by erosion of the bore rather than by the fatigue strength of the metal. This paper deals with the analysis of a failure which has taken place in a 240-mm howitzer of war production.

Howitzer No. 5 ruptured on the 283rd round on a half charge with a bore pressure of 15,400 lb per sq in. at the breech. The howitzer is designed for a bore pressure of 33,000 lb per sq in. with a minimum factor of safety of 1.25 in the powder chamber. The factor of safety of the muzzle section was not much over unity. Expansion of this section had been experienced under service pressures until a change in the contour of rifling was made.

Fig. 1 shows the gun and mount immediately after the failure. The fragment not shown was found 155 ft to the right rear. The muzzle was split in two sections, one of which is shown in Fig. 2. The two ruptured surfaces of this section are quite different in appearance. The lower surface is smooth in the middle region and fibrous at the ends, the coarseness of the fiber increasing toward the end. The top fracture has no smooth area except two very limited ones near the bore at *B* and *C*. The ruptures are characterized by fibers torn in the general shape of an arrowhead. In the lower surface the points of the arrows are directed toward the smooth area at the middle section. The fibers in the upper surface are coarser and the surface is rougher. The arrows of the fiber point toward *C* and also in the left section toward *B*. Fig. 3 shows the middle portion of the lower fractured surface. It will be noted that just below the center of the illustration is a very smooth area and that the apex of the arrows of the fibers point to that area. The lower edge at the picture is the outside surface of the gun. In Fig. 4 is shown one of the smooth areas of the upper ruptured surfaces of Fig. 2. The arrows of the ruptured fibers point to a small area at the lower (bore) surface of the fragment.

From general observation it has been noted that the apex of the arrows in a fractured surface point toward the origin of rupture. Fig. 2 shows three origins of rupture, one in the lower surface near the outside and two on the upper surface near the bore.

To establish priority of these origins, the nature of the surfaces must be considered. The lower surface of Fig. 2 (see also Fig. 3) is smoother and the center of propagation of rupture extends along the mid-section of the wall. There is a single system of rupture lines from one center. The upper surface of Fig. 2 (see also Fig. 4) shows the path of rupture extending along the bore surface. Fig. 4 shows that the flow lines change in appearance before reaching the upper (outside) surface. The rupture

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[1] Released for publication by the Chief of Ordnance, U. S. Army. Statements and opinions are to be understood as individual expressions of their author and not those of the Ordnance Department.

[2] Colonel, Ordnance Department, U. S. Army. Presented at the Annual Meeting of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS, held in New York, N. Y., December 5–9, 1938.

Discussion of this paper was closed January 10, 1939.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

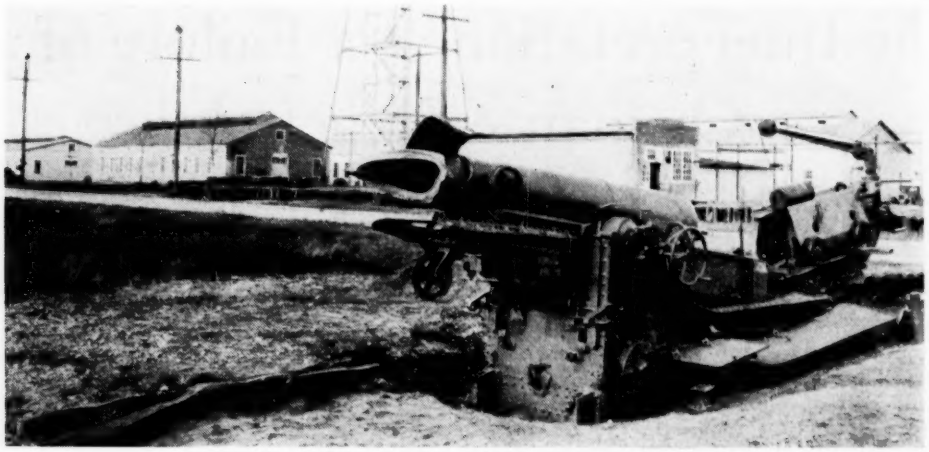


FIG. 1 FAILURE OF 240-MM HOWITZER

lines of the lower zone formed by about two thirds of width of section are traces of tearing. The upper zone has the appearance of having been pulled apart rather than torn. Near the upper edge of the upper fracture shown in Fig. 2 there is a difference in level of the fracture indicative of forces which did not exist in the rupture of the lower surface. There are at least two systems of rupture lines in this surface.

The smoothness of the ruptured surface varies directly with the speed of propagation of rupture.

The only conclusion that can be drawn is that the original rupture started near the outside surface at the point marked "start of rupture" in Figs. 2 and 3, proceeding in both directions. With the split on one side of the gun, the unsupported cylinder rotated about the outside edge of the opposite wall, causing failure to proceed from the inside surface outward. The initial fracture was more rapid. As the powder pressure was reduced, with loss of gases through the split and the increased volume of the bore, the violence of rupture decreased. Thus, the upper surface in Fig. 2 is rougher in appearance. As the two halves of gun opened out on the upper edge of Fig. 2 at a center, the rupture in this wall changed from a tear to a tensile break, as indicated in Fig. 4. In the second surface ruptured, the presence of two or more origins is natural. Both were due to stresses beyond the strength of the material induced by the failure of the opposite wall.

The appearance of the ruptured surfaces is consistent with hypothesis as to the origin of fracture. The position of fragments on the ground after the accident is also consistent with this hypothesis. The rupture flow lines shown in Fig. 1 definitely indicate the tearing proceeding from the near side, which checks with the hypothesis made from the examination of the fragment shown in Fig. 2.

A closer examination of Fig. 3 discloses a smooth, dark zone at the outer surface about $^1/_8$ in. deep and 4 in. long. An examination of this surface at low magnification, which is shown in Fig. 5, reveals a number of smooth dark areas which are interpreted to be independent crack systems of varying depth, the principal ones being near the two ends of the illustration. Near the middle of the upper edge will be noted a center from which rupture flow lines proceed in all directions and merge into the main rupture flow system. This is the localized origin of rupture at the bottom of an old crack system. It should be noted that this rupture system comes to the surface in some places and is indicative of the discontinuity and variable depth of the original crack system. Particularly to the left of the origin of rupture

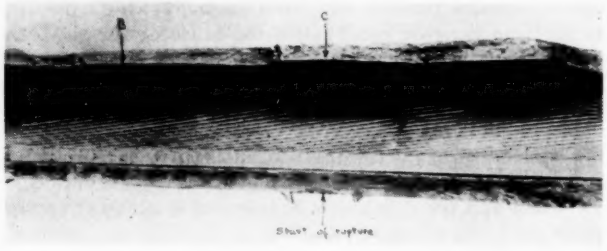


FIG. 2 ONE SECTION OF THE SPLIT MUZZLE

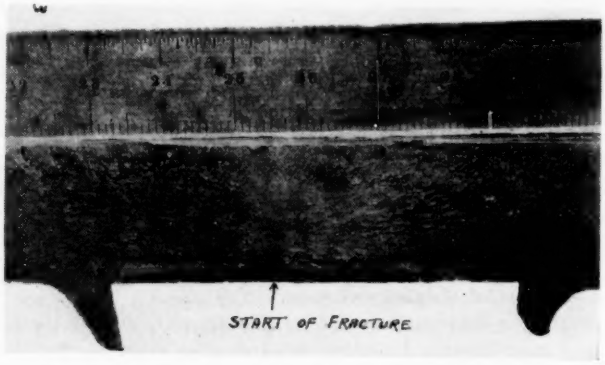


FIG. 3 MIDDLE PORTION OF THE LOWER FRACTURED SURFACE OF THE MUZZLE



FIG. 4 ONE OF THE SMOOTH AREAS OF THE UPPER RUPTURED SURFACES SHOWN IN FIG. 2

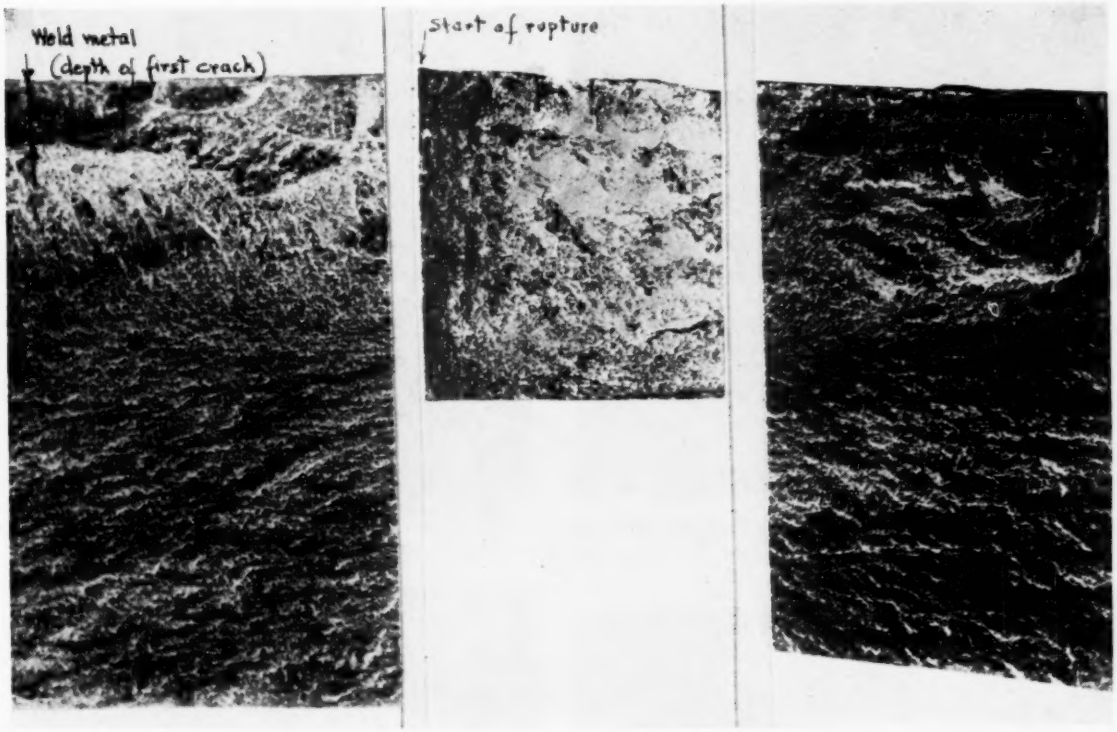


FIG. 5 LOW MAGNIFICATION OF THE SURFACE SHOWN IN FIG. 3

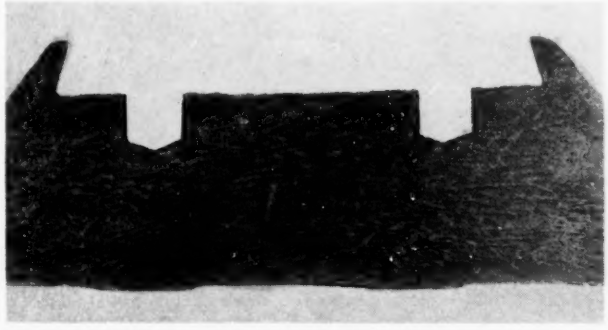


FIG. 6 MACROETCHED LONGITUDINAL SURFACE

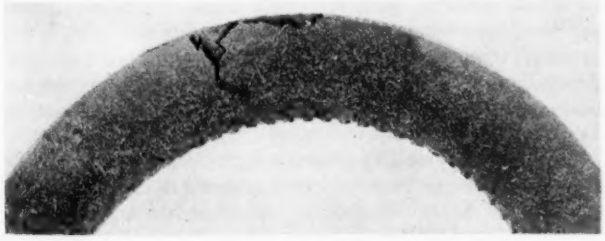


FIG. 7 MACROETCHED TRANSVERSE SURFACE

there will be observed independent flow-line systems. However, during the failure these local flow-line systems proceeded only a short distance before they met the main rupture system which was being propagated at a very high rate of speed.

Having definitely established the point from which the rupture proceeded, the next phase of the investigation is to determine under what conditions the original crack system developed and to examine the metal to ascertain why it has behaved so unexpectedly. Merely classifying the failure as one in fatigue gives no real answer.

Fourteen test specimens taken from metal of the fragments gave the results shown in Table 1, which are compared with original acceptance tests.

The lowness of proportional limits is explained in part by the cold work received by the metal in rupture and possibly in part to the use of more exact methods of test during the investigation. The difference in tensile strength indicates a difference due probably to location of specimens. The higher values for specimens originally tested may have been due to end effect in heat-treatment. That is, specimens near the end of the forging were hardened more in the quenching operation.

The average tensile impact strength is good; the low value is unsatisfactory. Two specimens gave fair values. Otherwise, the results were good. The ruptured surfaces of the tension-test specimens were characterized as laminated, with streaks of lustrous segregations and numerous checks on stems. These types of fracture and irregularity of tension impact tests are indicative of nonhomogeneity of structure.

The macroetched structure, illustrated in Fig. 6 for longitudinal surface and Fig. 7 for transverse surface, confirms the conclusions made from the appearance of fractures and the re-

TABLE 1 TEST RESULTS FROM FRAGMENTS OF THE RUPTURED MUZZLE COMPARED WITH ORIGINAL ACCEPTANCE TESTS

	Fragments	Original
Proportional limit, lb per sq in.[a]	37000–50000	63000–71000
Average	46700	.....
Tensile strength, lb per sq in.	90000–93000	95000–101500
Average	91800	.....
Elongation, per cent	20.5– 24.5	20.5– 22.0
Average	22.7	.....
Reduction of area, per cent	38.7– 49.6	37.3– 49.2
Average	45.3	.....
Tensile Charpy, ft-lb	10.7– 29.4	Not tested
Average	23.6	.....

[a] One half of the specimen showed no proportional limit, the stress-strain line being a continuous curve.

NOTE: All specimens were transverse.



FIG. 8 DENDRITIC STRUCTURE NEAR THE BORE OBSERVED AFTER ETCHING WITH COPPER CHLORIDE. ×5

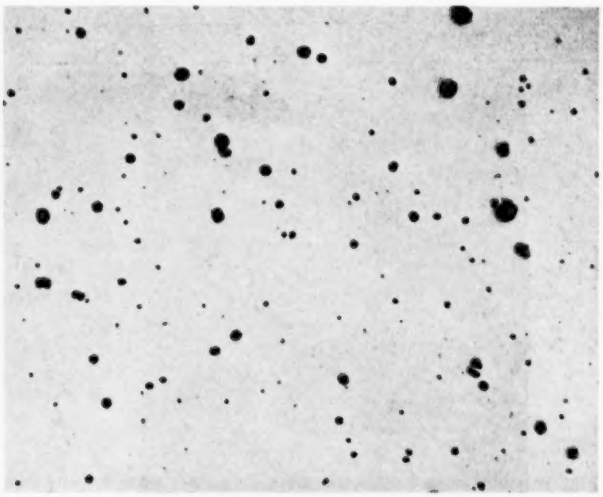


FIG. 9 UNETCHED TRANSVERSE SECTION SHOWING THE SIZE AND DISTRIBUTION OF NONMETALLICS. ×100

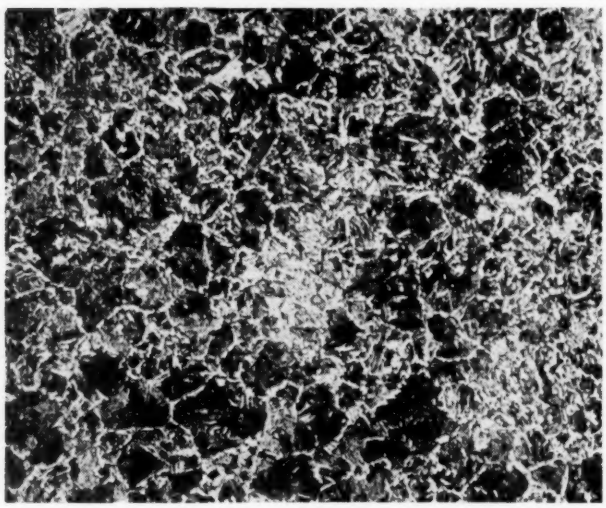


FIG. 10 THE STRUCTURE AFTER ETCHING WITH 1 PER CENT NITAL. ×100

(Normal structure of the steel is a segregated nonuniform sorbite of variable grain size. Ferrite surrounds the grains and occurs in crystallographic planes.)

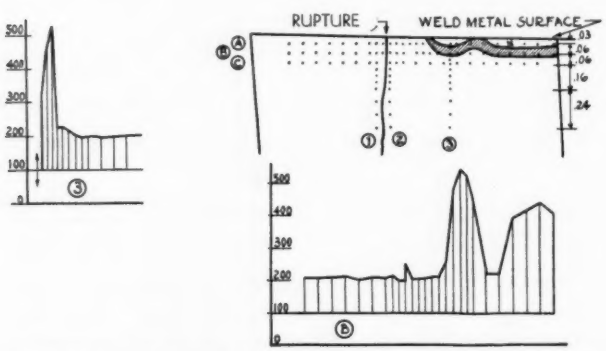


FIG. 12 VICKERS-BRINELL HARDNESSES AT VARIOUS POINTS ALONG ONE OF THE SECTIONS OF THE FRACTURED MUZZLE

(Interval of reading is about 0.08 in.)

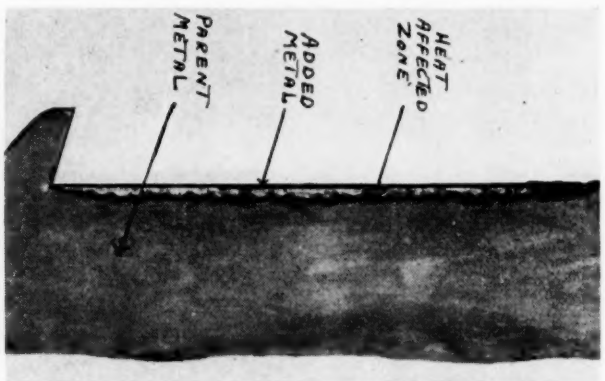


FIG. 11 SECTION PARALLEL TO THE RUPTURE AND ABOUT 1 IN. FROM IT, AFTER ETCHING WITH NITRIC ACID

sults of physical tests. The metal was of the following percentage composition: Carbon, 0.38; manganese, 0.64; silicon, 0.19; sulphur, 0.045; phosphorus, 0.061; nickel, 2.24; and chromium, 0.11.

The structure indicates inadequate hot working to break up the coarse dendritic structure of the ingot and to disseminate the phosphorus and sulphur content which was high. The structures found are favorable for propagation of crack systems. No evidence was noted, however, to indicate that any original crack system existed because of conditions of nonhomogeneity of composition or structure.

Under metallographic examination the pronounced presence of dendritic structure near the bore is shown in Fig. 8 at a magnification of 5 after a copper-chloride etch. An unpolished specimen, shown in Fig. 9, reveals the size and distribution of nonmetallics. Fig. 10 shows the structure after etching with 1 per cent Nital, which may be classed as nonuniform sorbite of variable grain size. It will be noted that the grains are surrounded by a network of ferrite and that ferrite also occurs in the crystallographic planes. The structure found correlates with irregularities of ductility and impact properties and the appearance of fractures of tension specimen. Low ductility and an increased rate of propagation of failure of the structure might be postulated from these results. As yet, however, the results do not explain the failure.

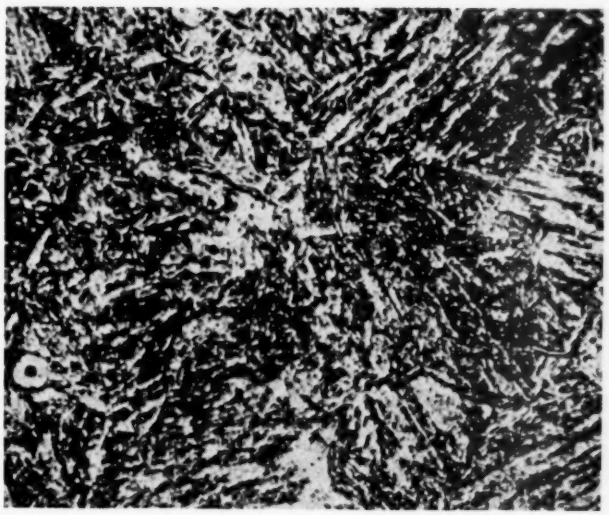


FIG. 13 SECTION NEAR THE FRACTURE ETCHED WITH 1 PER CENT NITAL. ×1000

(The structure has the acicular planes of martensite, but has been tempered by subsequent welding to troostite.)

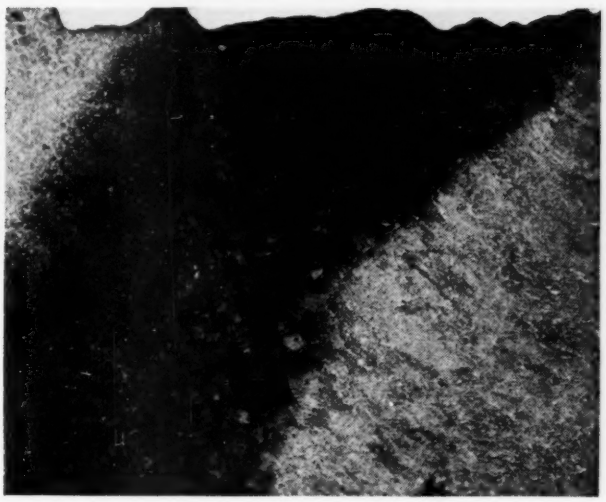


FIG. 15 VOIDS IN THE WELD METAL WHICH WERE ALSO THE STARTING POINT OF CRACKS. SECTION ETCHED WITH 1 PER CENT NITAL. ×25

(The path of rupture did not pass through the added metal at the upper right, but through the heat-affected zone in the lower section.)

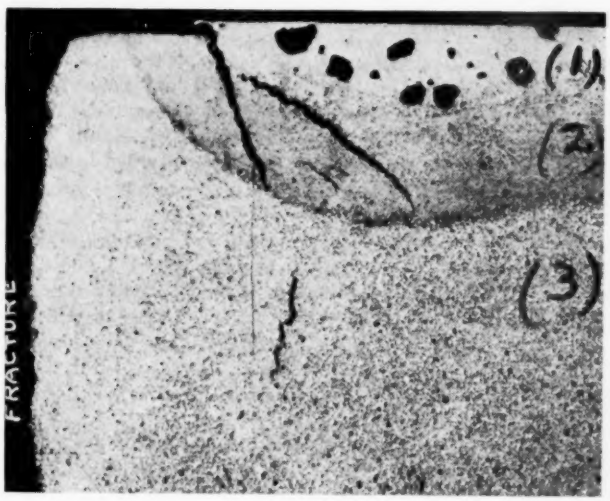


FIG. 14 PORTION OF THE WELD IN AN AREA NOT FAR FROM THE ORIGIN OF RUPTURE, AFTER ETCHING WITH 1 PER CENT NITAL. ×15

(Area 1 is the weld metal, area 2 is the heat-affected zone, and area 3 is the parent metal.)

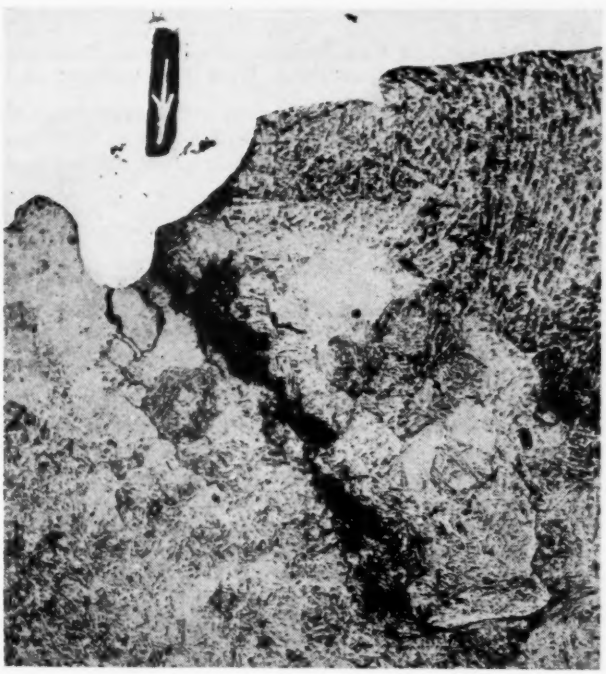


FIG. 16 ANOTHER TYPE OF DISCONTINUITY FOUND IN THE FUSION ZONE. SECTION ETCHED WITH 1 PER CENT NITAL. ×100

(The arrow points to undercut. The weld metal is in the upper right-hand corner. The crack system originated in the heat-affected zone.)

A section parallel to the rupture and about 1 in. from it was macroetched and is shown in Fig. 11. This section reveals a weld on the surface in which the rupture had its origin. A hardness survey disclosed hardnesses up to 550 Vickers Brinell in the heat-affected area. Fig. 12 shows two hardness plots—*B* taken parallel to the weld and 3 from the edge of the weld through the heat-affected zone into the parent metal. The microstructure in the heat-affected area, shown in Fig. 13, reveals acicular planes of martensite tempered by subsequent welding to troostite.

A microstudy of the weld in an area not far from the origin of rupture was made. In Fig. 14 the weld deposit is shown at (1), the heat-affected zone at (2), and the parent metal at (3). Of particular interest is the shoulder at the left edge of the weld, the crack system starting from this shoulder, and the intergranular-crack system in the parent metal.

Other intercrystalline cracks were found in the heat-affected zone near the fusion line. In some areas, weld undercuts were the starting point for intergranular cracks. Voids in the weld metal were also the starting point of cracks, as indicated in Fig. 16 which shows the weld metal at the upper right and the heat-affected zone in the lower section. In some areas cracks were transcrystalline. Most of the intercrystalline cracks are believed to have existed prior to rupture.

Another type of discontinuity found in the fusion zone is shown in Fig. 17. The fissures and voids shown are partially filled with oxides.



FIG. 17 FISSURES AND VOIDS IN THE ADDED METAL NEAR THE FUSION ZONE PARTIALLY FILLED WITH OXIDES. ×100

Fig. 15 is a view at 25 magnifications at the intersection of fracture, and the outside surface of the howitzer at the origin of rupture. It is to be noted that the rupture did not pass through the weld metal (upper-right-hand zone) but originated in the heat-affected zone just outside the weld metal. This is the region in which the ribbon of hard metal exists, as shown in Fig. 12, and which is prone to cracks and other discontinuities, as shown in Figs. 14, 16, and 17.

It is to be noted that the structure and hardness of the heat-affected zone pertains to metal which has not been stress-relieved. The defects are those expected in welding 0.40 carbon low-alloy steel of heavy cross section without preheating and with undeveloped technique. The welding does not represent current practice.

Proof is now complete that the rupture originated in the heat-affected zone just outside a weld.

The defects of the welded zone furnish the explanation of the failure of the structure. Cracks and other stress raisers were present at the outside surface where the rupture originated. The crack system gradually grew under the stress of firing until complete failure of the structure resulted. The fact that final failure took place on a half pressure round is not at variance with our knowledge of fatigue failures. All the facts related fit together in a single picture and justify the methods used in the analysis of fracture. One last check completes the story. The manufacturing records were consulted and disclosed that the area in question had been welded by metallic arc to correct an error made in machining.

ACKNOWLEDGMENTS

In preparing this paper the work of the following members of the Watertown Arsenal Laboratory Staff have been freely drawn upon: M. G. Yatsevitch, P. R. Kosting, H. C. Mann, W. C. Warner, and H. G. Carter, all of whom participated in the investigation of this failure.

The Use of the Piezoelectric Gage in the Measurement of Powder Pressures

By R. H. KENT[1] and A. H. HODGE,[2] ABERDEEN PROVING GROUND, MD.

The development of a piezoelectric gage for the measurement of powder pressures in guns is reviewed. A gage is obtained having a high natural frequency, insensitivity to shock, and accuracy. Details concerning the construction of the gage, the recording apparatus, and the method of calibration are given. Records are reproduced showing pressure waves in a gun and a closed chamber. Such records have been of value in the improvement of the design of powder charges and the determination of interior ballistic data.

NO DOUBT shortly after the invention of cannon in the middle ages, there must have been felt the need of a device for measuring pressures generated in such weapons to enable one to calculate the stresses produced by firings. However, it was not until 1857 that the first practical pressure gage was developed by General Rodman of the Ordnance Department of the U. S. Army. Rodman's gage consisted essentially of a piece of copper which was indented by a knife that was subjected to the pressure in the chamber. The pressure was deduced from the depth of the indentation.

Some time afterward Noble of England invented the well-known crusher-type pressure gage in the form which is now used. In this type of gage, the piston which is subjected to the gas pressure compresses a copper cylinder and the pressure is deduced from the amount of compression. Although such a gage should be capable of producing accurate measurements of pressures which are slowly applied and kept on, serious errors are encountered when it is used to measure the rapidly changing pressures in guns. The deformation of the copper cylinder exceeds the elastic limit and a flow of the copper takes place. It takes a certain time for this flow to be completed and thus in general, the maximum pressures indicated by the copper-crusher gage are too low.

The gages of Rodman and Noble measured only the maximum pressure in the gun. For a complete study of the generation of pressure, a gage is obviously needed which gives the pressure in the gun as a function of the time. For such a recording gage to be a satisfactory instrument for the study of powder pressures, it must satisfy the following requirements:

1 Its natural period must be small compared with the duration of the pressure-time curve. For the 14-in. gun, this duration is about 0.05 sec while for a caliber 0.30 rifle it is about 0.001 sec.

2 The gage must not be affected by the rather violent recoil and vibrations of the gun.

3 Its precision must be of the same order of magnitude as the precision of the velocity measurements.

Vieille made the first recording time-pressure gage about 1884 by the simple expedient of recording on a revolving drum the motion of the piston of the copper-crusher gage as it is compressed. Since that time a number of mechanical and electrical gages have been developed. Of these, the piezoelectric appears to satisfy the listed requirements in a much more satisfactory manner than any other.

The piezoelectric effect was discovered by J. and P. Curie in 1880. They found that certain crystals, e.g., tourmaline, quartz, rock salt, when compressed along certain axes, produce an electrostatic charge proportional to the imposed stress. Obviously such crystals by virtue of their high natural frequency are admirably adapted to the measurement of rapidly varying pressures of short durations such as occur in guns. To record the pressures one simply has to record as a function of the time the charge developed by the crystals. It was not until 1917 however, that the piezoelectric gage was invented by Sir J. J. Thomson[3] and used to measure the pressure in a gasoline engine. Shortly after this his associate Keys[4] used it in measuring the pressure of an explosion. The apparatus of Thomson and Keys consisting of tourmaline crystals and a cathode-ray oscillograph is shown schematically in Fig. 1.

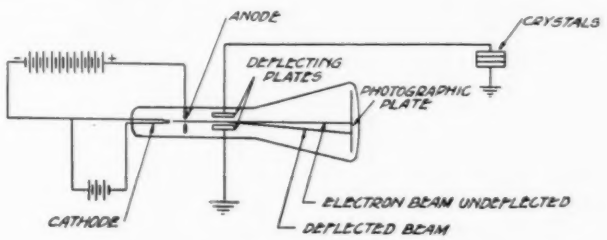


FIG. 1 APPARATUS OF THOMSON AND KEYS

As the crystals are stressed, the charges produced flow to the deflecting plates establishing a difference of potential between them. The electrostatic field thus arising causes the deflection of the electron beam which is recorded photographically by a plate at the end of the tube.

After the War, the Ordnance Department of the U. S. Army, aware of the experiments of Thomson and Keys, decided to undertake the development of a piezoelectric gage suitable for measuring pressures in cannon.

At that time, to record photographically by the cathode-ray oscillograph it was necessary to place the plate or film within the exhausted cathode-ray tube itself. To obviate this rather laborious technique various other methods were developed in turn for the Department by G. F. Hull, Karcher[5] and Eckhardt, and one of the authors (Kent).[6] Of these the methods of Hull

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Discussion of this paper was closed January 10, 1939, and is published herewith directly following the paper.

Released for publication by the Chief of Ordnance, U. S. Army. Statements and opinions are to be understood as individual expressions of their authors, and not those of the Ordnance Department or of the Society.

[3] "Piezo-Electricity and its Applications," abstract of paper by J. J. Thomson, *Engineering*, vol. 107, April 25, 1919, p. 543.

[4] "A Piezoelectric Method of Measuring Explosion Pressures," by David A. Keys, *Philosophical Magazine*, series 6, vol. 42, October, 1921, p. 473.

[5] "Piezo-Electric Method for the Instantaneous Measurement of High Pressures," by J. C. Karcher, U. S. Bureau of Standards, Scientific Paper No. 445, August 4, 1922, p. 257.

[6] "The Piezo-Electric Gage," by R. H. Kent, *Army Ordnance*, vol. 18, March-April, 1938, p. 281.

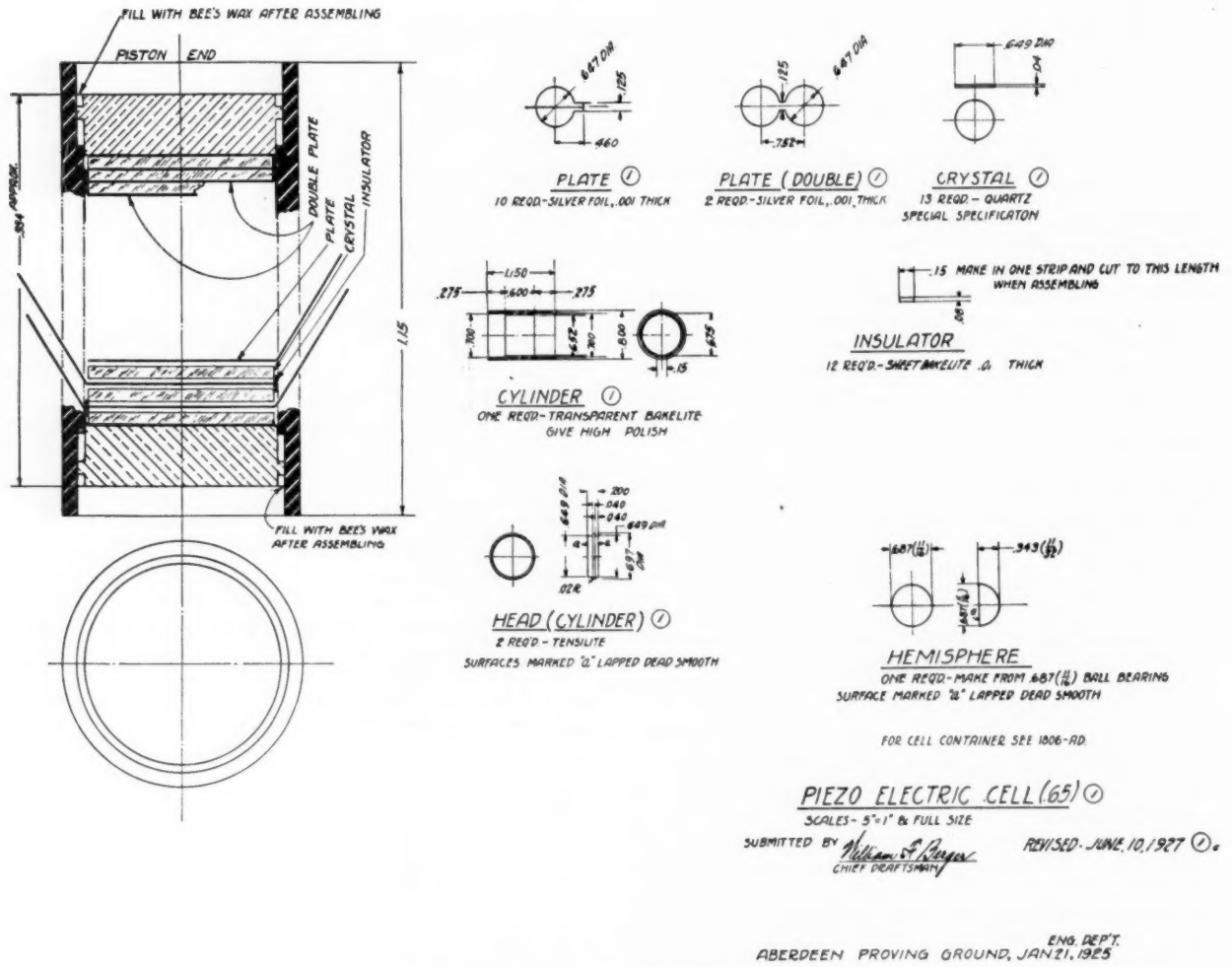


FIG. 4 PIEZOELECTRIC CELL

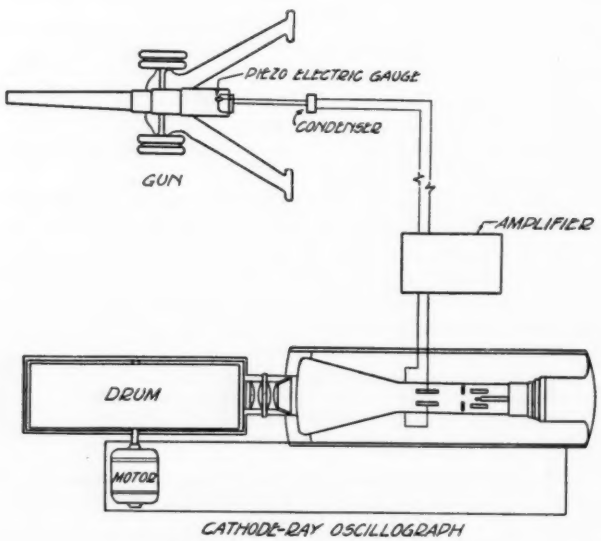


FIG. 2 DIAGRAM OF APPARATUS

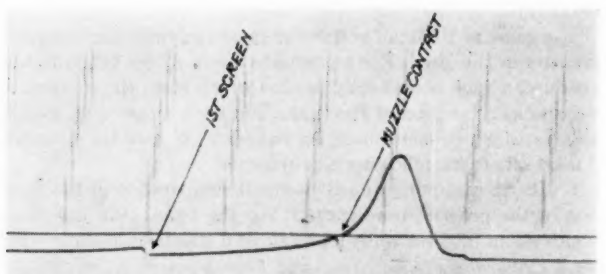


FIG. 3 PRESSURE RECORD OF SPRINGFIELD RIFLE

and Kent were essentially similar although differing in important details. The gage was connected to a condenser attached to the input of a direct-current amplifier, the output of which was recorded by a string galvanometer (Hull) or a Duddell electromagnetic oscillograph (Kent).

In 1932, Joachim and Illgen[7] developed for the Zeiss-Ikon

[7] "Gasdruckmessungen mit Piezo-Indikator," by H. Joachim and H. Illgen, *Zeitschrift für das gesamte Schiess- und Sprengstoffwesen*, vol. 27, 1932, pp. 76–79, 121–125.

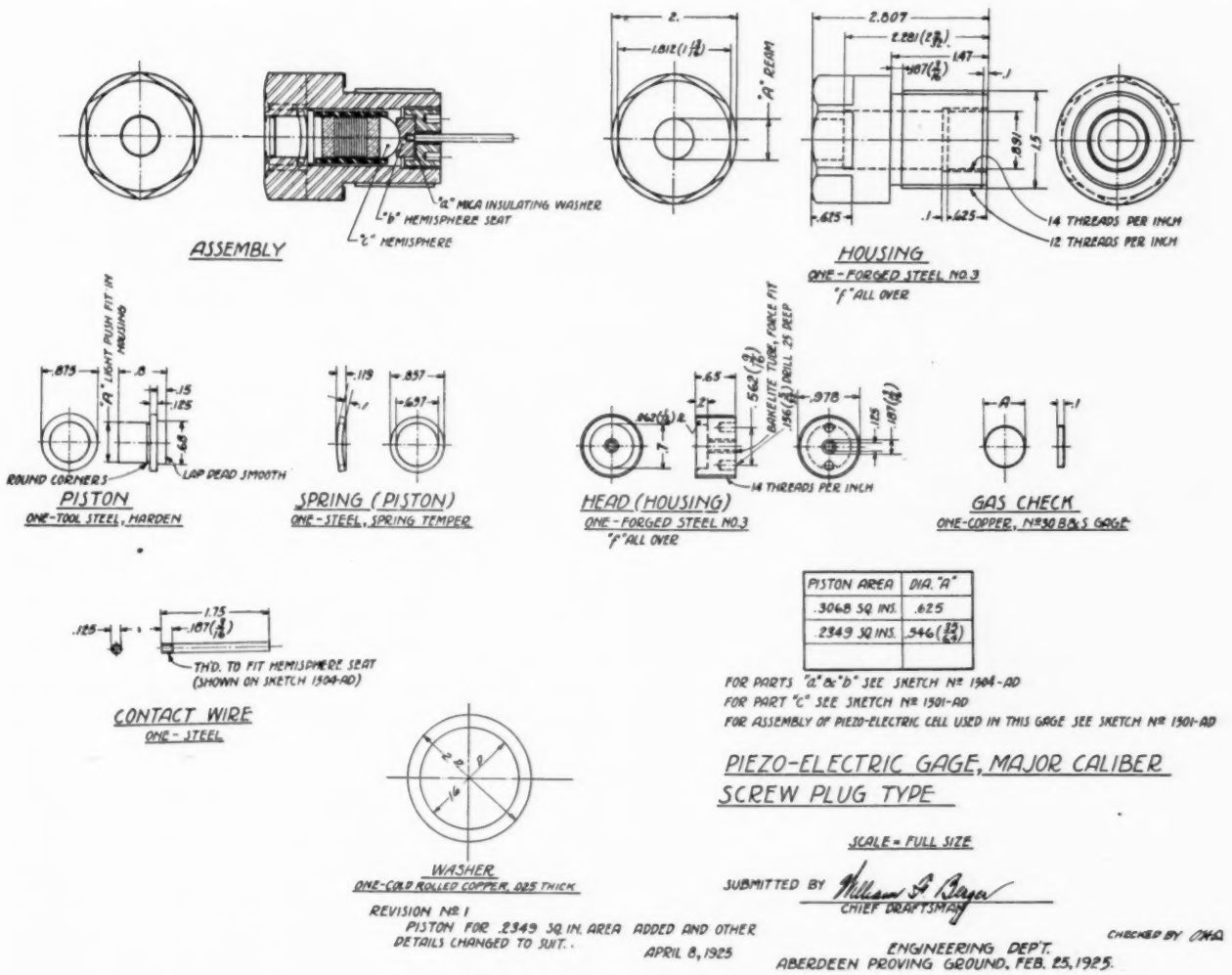


FIG. 5 DRAWING OF GAGE

Company a cathode-ray oscillograph of such characteristics that photographs of the moving spot caused by the impact of the electrons against the fluorescent coating could be taken outside of the tube. This was substituted for the electromagnetic oscillograph referred to. In this apparatus, the amplifier was retained to prevent the loss of charge that would occur with the small capacities required in the absence of amplification.

Since 1934, various other improvements have been made in the gage chiefly by one of the authors (Hodge). A schematic diagram of the apparatus as now most frequently used is given in Fig. 2. The gage consisting of a stack of quartz crystals with its housing is screwed into the chamber of the gun. It is connected to a condenser near by, which is in turn connected by an overhead line or underground cable to the input of a nonreactively coupled amplifier. The output of the amplifier impresses on the deflecting plates of the cathode-ray oscillograph a potential difference which is proportional to the potential difference of the condenser caused by the flow of electrostatic charge from the gage as the pressure changes. As the electrons impinge upon the fluorescent screen at the end of the cathode-ray tube, a luminous spot is produced. By means of a lens an image of this spot is obtained on the film or sensitized paper attached to a revolving drum. Timing lines are also placed on the sensitized paper by means of a tuning fork or other device. A photographic record is thus obtained of the pressure in the gun as it goes off. Fig. 3 is a sample record. It shows the pressure as a function of time in a caliber 0.30 Springfield rifle. The first break in the line to the left shows the time when the bullet reaches the muzzle and the second break shows the time when the bullet arrives at a screen placed in front of the muzzle. The timing lines indicate thousandths of a second. From the record it may be seen that the time from the beginning of the rise of pressure to the exit of the bullet at the muzzle is only slightly more than 0.001 sec.

DESCRIPTION OF VARIOUS PARTS OF THE APPARATUS

The gage proper consists of a piezoelectric cell and a housing in which it is placed. The cell consists of a stack of quartz crystals. The negative sides of the plates are connected together and grounded while the positive sides are connected to an insulated lead-out wire. A drawing of the cell is shown in Fig. 4.

Fig. 5 shows a drawing of the housing with the cell in place. Important features of this are the insulated stem which takes the charge from the positive sides of the plates and the Belleville spring which tends to keep the reaction between the cell and the housing constant during the small displacements of the cell allowed by the elasticity of the housing and the piston when it is subjected to the gas pressure. A photograph of the parts of the gage is shown in Fig. 6.

The gage is attached to a mica condenser of high quality placed near the gun. The capacity of the condenser is varied depending

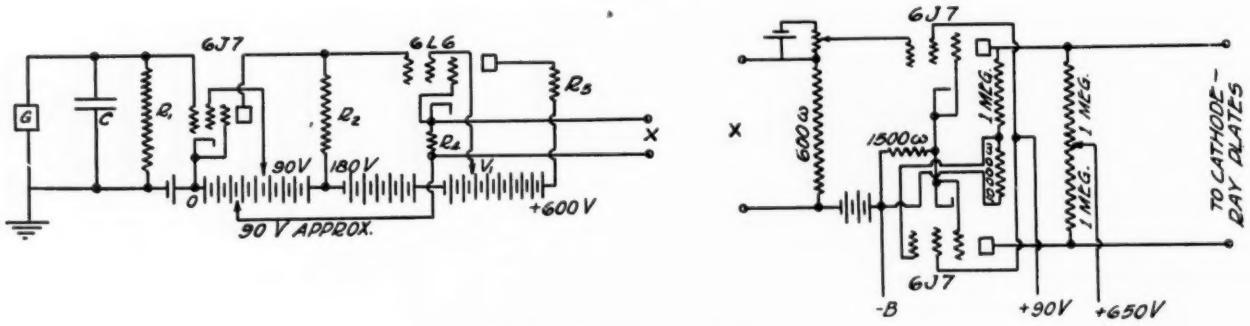


FIG. 8 AMPLIFIERS AND LINES

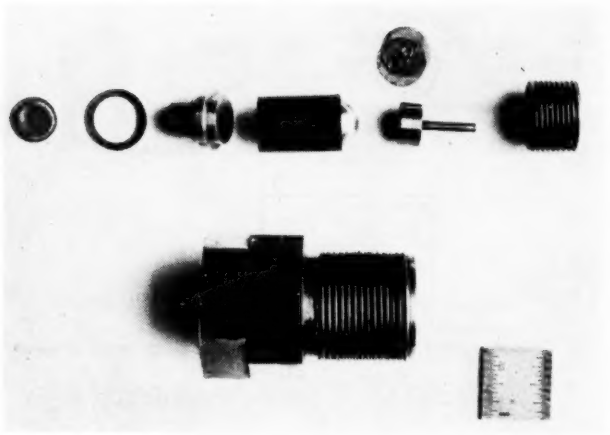


FIG. 6 PHOTOGRAPH OF PARTS OF GAGE

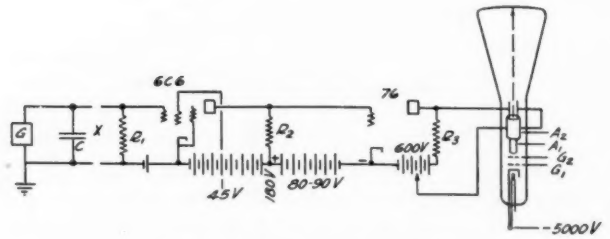


FIG. 7 DIAGRAM OF AMPLIFIER

upon the piston area of the gage used and the magnitude of the pressure to be measured. In measuring pressures in ordinary guns, the capacity is between 2 and 3 μf. The condensers are usually connected to an amplifier by means of a line or cable; in some cases a cable of a length of 2000 or 3000 ft is used without any great difficulty for all guns except small arms. In case it is desired to take pressures at more remote points, however, there is installed near the gun, an additional amplifier, which is connected to the amplifier in the laboratory, which in turn is connected to the cathode-ray oscillograph.

A schematic diagram of the amplifier currently used is shown in Fig. 7. In its design an attempt was made to keep the reactances as small as possible, since these cause distortions of the pressure-time curve, and to obtain a uniform amplification of voltages within its range of from 0 to 0.2 volt. No originality is claimed for it. The input tube is of the type 6C6 carefully selected to have a low grid current to minimize the loss of charge.

The batteries supplying the filaments and plate circuits are of such large capacity that the drift of the amplifier is slow. It can be used for a working day without appreciable change in calibration.

The cathode-ray tube is a special one having a blue short-persistence screen. In Fig. 7 the second anode of the tube is shown connected to the amplifier batteries but in case a long line is used between gage and amplifier it is better to use a separate battery for biasing the anode above ground potential.

By means of an f 1.8 lens of 1-in. aperture an image of the luminous spot of the cathode-ray tube is focused on sensitized paper attached to the moving drum.

Timing lines are placed on the moving paper by a tuning fork and an arc light.

The apparatus shown in Fig. 2 with the amplifier shown in Fig. 7 is satisfactory for the measurement of pressures in small arms and closed chambers fired near by in the laboratory and, unless the interference from power circuits and the like is large, for the measurement of the pressures in guns a half mile away. However, for the measurement of the pressures of guns situated several miles from the laboratory, the apparatus of Fig. 7 is inadequate because of the great amount of interference picked up by the long lines. To obviate this it is necessary to amplify the signal before it is transmitted.

Fig. 8 is a schematic diagram of a circuit which has been found to be reasonably good for use with these long lines. This circuit consists of two units, one of which is placed at the gun while the other is in the instrument building. The part of this amplifier which is placed at the gun consists of a high-resistance input tube directly coupled to a tube the cathode current of which passes through a 600-ohm resistor placed across the end of the line. The portion of the amplifier at the camera is a kind of cross between a single-ended and a push-pull circuit. Its input resistance is also about 600 ohms.

Some circuit constants and symbols for Figs. 7 and 8 are:

C	= 0.1 to 6 μf
G	= gage
R_1	= 5 megohms
R_2	= 0.115 megohm
R_3	= 0.1 megohm
R_4	= 600 ohms, R_5 = 5000 ohms
G_1, G_2	= control grids
A_1, A_2	= electrostatic focusing cylinders
K	= cathode
V_1	= 225 volts above cathode connection of 6L6
X	= transmission line.

The chief advantage of this circuit is that it provides an output voltage which is relatively large, 400 to 500 volts, and does not tend to cause defocusing of the cathode spot.

The 600-ohm resistors mentioned have this value to prevent reflections at the two ends of the line. Experience has shown that if these resistances differ greatly from 600 ohms, serious distor-

tions caused by reflections will occur in the recorded time-pressure curve. It is a bit difficult to obtain linear amplitude response over the useful range of this amplifier combination, but this defect can be overcome by a careful calibration.

Calibration

To enable the deflection on the oscillogram to be expressed as pressure in psi the apparatus must be calibrated. This is done as follows: (1) The gage itself is calibrated to give the relation between pressure and electrostatic charge, (2) the apparatus, consisting of the condenser, the amplifier, and the cathode-ray oscillograph, is calibrated to give the relation between the charge impressed on the condenser and the deflection of the oscillograph. The combination of the two calibrations gives an over-all calibration of the apparatus. It is possible but impractical to calibrate directly from pressure to deflection. To perform this correctly the dead-weight gage would have to be taken out to the gun. Furthermore, the calibration of the gage remains practically constant but the amplifier has to be calibrated every time records are taken.

The device which is used for calibrating the pressure gage proper is shown in Fig. 9. It is a dead-weight gage of the hydrostatic type and consists of a hand pump capable of producing 15,000 psi of oil pressure; a pressure intensifier which has a pressure ratio of 10 to 1 between the high- and low-pressure sides and an upper limit of 50,000 psi; an oscillating piston which is caused to rise off its shoulder seat by oil pressure and which raises a pan of from 50 to 500 lb weight; and a quick-release valve which is capable of releasing 50,000 psi pressure and of reducing the pressure against the gage piston to atmospheric in from 0.003

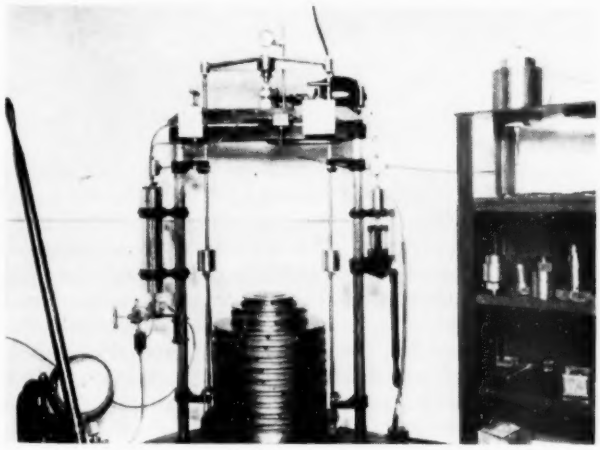


Fig. 9 Dead-Weight Pressure Gage

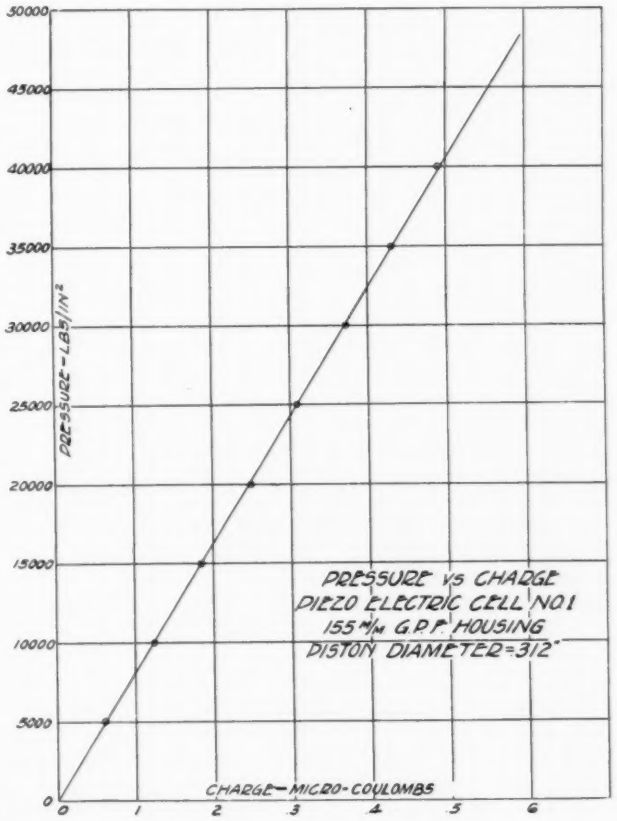


Fig. 11 Pressure-Charge Relation

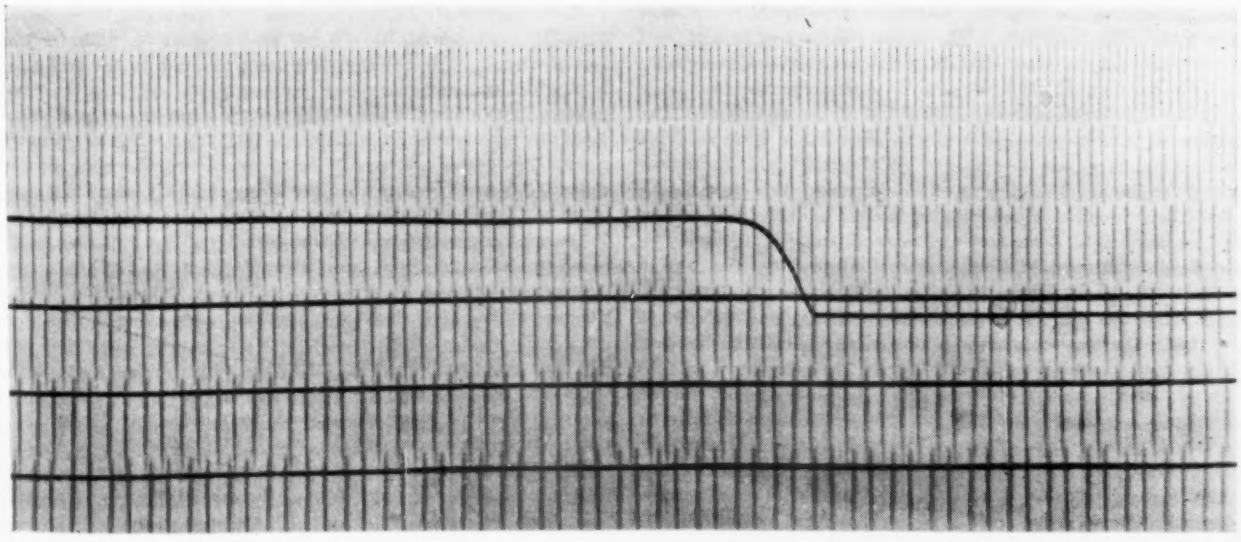


Fig. 10 Curve Showing Release of Pressure

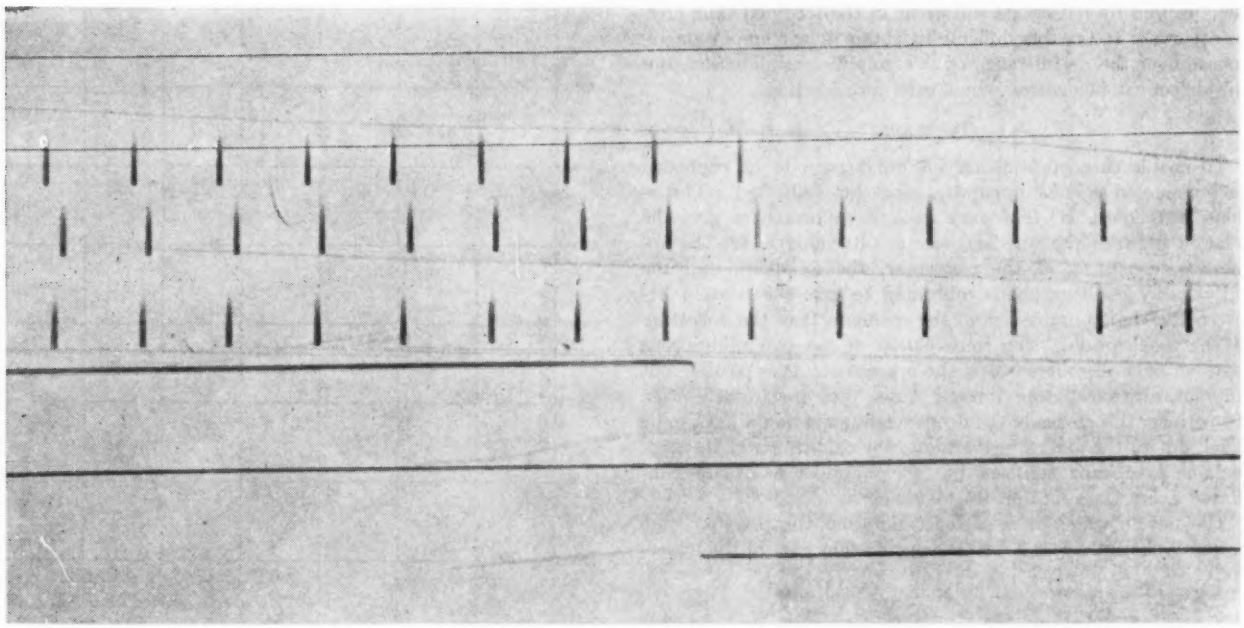


FIG. 12 CALIBRATION OSCILLOGRAM

to 0.010 sec. The charge produced by the gage on such a pressure release was used to obtain the pressure-time curve shown in Fig. 10. Thus the charge produced on the quick release of the oil pressure is used to calibrate the piezoelectric gage proper.

Fig. 11 shows the charge produced by the gage as a function of the pressure applied.

Fig. 12 is a calibration oscillogram showing the relation between the deflection recorded on the oscillogram and a charge suddenly impressed on the condensers attached to the input of the amplifier. The fact that the deflection is very sudden indicates that practically no distortion arises from the reactances of the circuits.

THE CHARACTERISTICS OF THE GAGE

As has been previously stated, a gage to be adapted for measurements of pressures in guns should have a high natural frequency, should be insensitive to the shocks produced by recoil, and should have satisfactory precision. The natural frequency of a typical piezoelectric gage is about 50,000 cycles per sec, which is amply high for most guns. If it were desired to do so the frequency could be raised considerably above this by a reduction in the number of quartz plates and in the size of the piston. This would of course reduce the sensitivity of the gage. In so far as sensitivity to the shock of the recoil is concerned, the gage has in general been found satisfactory. No trouble has been found when used in cannon of 3 in. and greater calibers. Some difficulty however has been experienced in a small-arms rifle if the gage is inserted in a weapon which is improperly balanced. The precision while not at the present time as great as desirable is tolerable. The probable error of the measurements is about 1 per cent under ordinary conditions.

In addition to meeting in a fairly satisfactory manner the requirements mentioned, the gage has some other valuable properties which should be mentioned. By reducing the capacity of the condenser, the sensitivity of the apparatus as a whole may be greatly augmented. It is thus possible to make quick shifts from an apparatus designed to measure 50,000 psi to one suitable for measuring 500 psi or less. However, as the capacity becomes smaller, the effects of leakage, referred to later, become more serious, interference increases, and the amplifier may become unstable.

The apparatus may be easily changed so that it will measure directly the rate of change of pressure dp/dt instead of p the pressure itself. For this purpose a resistor is inserted in place of the condenser shown in Fig. 2. With this form of the apparatus all of the charge has to flow through the resistor aside from the negligible amount required to charge the connecting leads. Hence if q is the quantity of charge generated, the current through the resistor is dq/dt and the potential difference between the terminals of the resistor is Rdq/dt if R is the value of the resistance. It is obvious that since q is proportional to the pressure the voltage impressed on the amplifier and hence recorded on the film is proportional to the rate of change of pressure dp/dt. This interesting and valuable use of the gage was developed by Dr. B. S. Mackey of the du Pont Company.

It has previously been mentioned that the input tube of the amplifier is chosen to have a low grid current to minimize the loss of charge. There is of course some loss due to the grid current and also some caused by leakage in the gage, in the condensers, and along the lines. However, when the gage is subjected to pressures of from 25,000 to 60,000 psi, such as occur in guns, it generates a relatively large amount of charge, approximately 0.6 microcoulomb. This large charge permits the use of a condenser of large capacity across the gage terminals. When used in connection with the amplifier of Fig. 7, the capacity is adjusted to keep the maximum input voltage to the line below 0.2 volt, which serves to keep the leakage current low. For pressure cycles which last for more than a few thousandths of a second the charge loss due to the low resistance of the input circuit may become appreciable. However, by the use of an input tube type 6C6 and of mica condensers of good quality it is possible to record pressure-time curves for all ordinary guns with negligible errors due to leakage.

Two of the most serious defects of the apparatus in its present form are: (1) Interference frequently occurs when lines half a mile long or longer are used and (2) the spot of the cathode-ray tube tends to defocus because, with the arrangement shown in Fig. 7, one deflecting plate is kept at constant potential while the

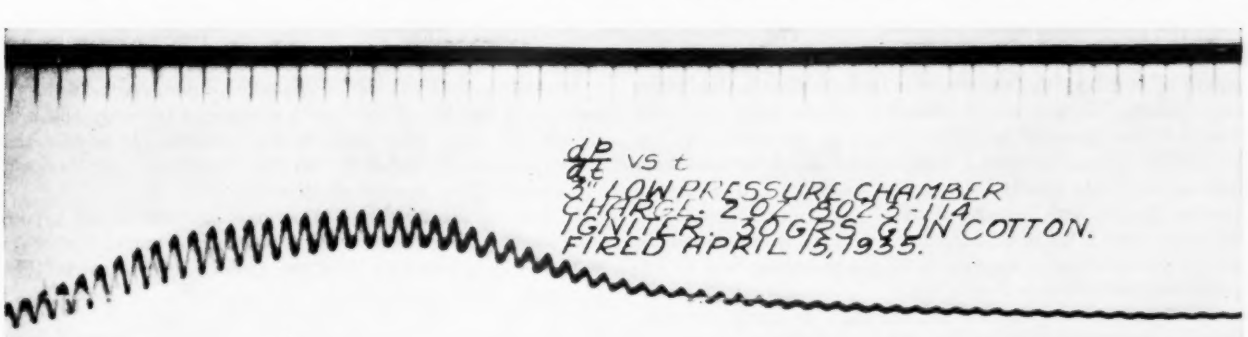


FIG. 14 SHOWING dp/dt FOR A CLOSED CHAMBER

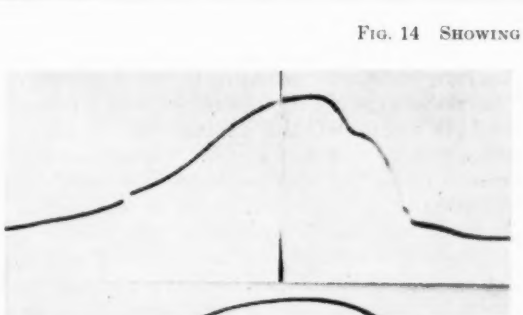


FIG. 13 PRESSURE-TIME CURVES OF 155 MM GUN

potential of the other is varied and this variation causes an appreciable change in the field accelerating the electrons. If push-pull amplification were employed the potential of one plate would be raised and that of the other lowered leaving the potential in the center of the tube unchanged. Push-pull amplification should also reduce the interference on the lines. This interference is partly caused by the unbalanced circuit. If the gage were designed to have both positive and negative terminals insulated a balanced input circuit could be obtained. With this and push-pull amplification most of the effects of interference should be eliminated. Work along these lines is now in progress.

EXPERIMENTAL RESULTS

The piezoelectric gage has been useful to the Ordnance Department in many ways. It has enabled maximum pressures in guns to be measured with much smaller systematic errors than was possible with the copper-crusher gages which had previously been used for this purpose. It has also been of great value in connection with the design of propelling charges for guns. While the possibility that there might be pressure waves of serious amplitude in guns has been known since the time of Vieille, it was not until records were obtained by the piezoelectric gage that it was known that such waves in the chambers of guns were a frequent occurrence, at least in guns using the American type of powder. Although the phenomena are really quite different, the pressure waves in a gun may be compared to the knock in a gasoline engine. If a suitable fuel and design of chamber are used, it is possible to eliminate the knock in the engine. Similarly in a gun, if the chamber, the charge, and the ignition are suitably designed, it is possible to eliminate the pressure waves and obtain a smooth pressure-time curve. Fig. 13 shows two cathode-ray oscillograms of pressures in the 155 mm gun G. P. F. For both of these oscillograms, the amount and kind of powder were the same but the arrangement of the charge was different. As may be seen, with the arrangement used to make the upper oscillogram, pressure waves of considerable amplitude are obtained while the arrangement of charge in the lower oscillogram eliminates the pressure waves completely.

When the apparatus is adapted to measure dp/dt, it is possible to get a sensitivity greatly increased over that obtained when p itself is measured. By the use of the dp/dt apparatus, it is therefore possible to record waves of an amplitude so small that they would probably escape detection if records were made with the ordinary apparatus. Fig. 14 is a copy of an oscillogram showing dp/dt in a closed chamber. From this oscillogram, the frequency of the waves may readily be obtained. From the frequency and the length of chamber the velocity of sound in the powder gas is determined. From the velocity of sound in turn, if the pressure and density are known, the ratio of specific heats γ may be calculated. It has been found that values of γ for the powder gas obtained in this way are in fairly good agreement with those calculated from the specific heats of the constituent gases based upon quantum mechanical theory.

ACKNOWLEDGMENT

In conclusion the authors gratefully acknowledge the assistance and cooperation of W. F. Berger and P. Ewing who designed many of the fittings and gage housings, R. C. Gerdom whose advice and mechanical skill have been invaluable, J. L. Sentman who has built practically all of the amplifiers and who has also taken the many records, and J. S. Durham who has cared for the gages at the guns and assisted with practically every phase of the pressure measurements.

The authors are indebted to the Ordnance Department, U. S. A. for supporting this interesting development and for permission to publish this paper.

Discussion

K. J. DEJUHASZ.[8] This paper is a valuable contribution to the literature on piezoelectric pressure indicators, which instruments have attained a considerable importance in the solution of engineering problems encountered in the measurement of rapidly changing pressures and forces. Though pressure indicators have been built also on other electrical principles (variation of resistance, inductance, capacitance, inverse Wiedemann effect, and photoelectric effect) and the last word has not been said as to which is the best principle, yet it appears at the present stage that piezoelectric indicators best fulfill the many engineering requirements.

[8] Associate Professor of Engineering Research, The Pennsylvania State College, State College, Pa. Mem. A.S.M.E.

At the Engineering Experiment Station of The Pennsylvania State College electrical indicators have been used for the investigation of pressures in Diesel-engine cylinders and in fuel-injection systems. It was found difficult to obtain fully consistent results as to quantitative calibration, due presumably to the variability of amplification. Keeping the amplification under full control would greatly increase the reliability of these instruments. In previous piezoelectric cells one terminal is grounded while the other is led to the amplifier. Insulating both terminals as the authors suggest appears to be one promising way to improve the functioning in this respect.

Development on piezoelectric indicators is being advanced by several research institutions. By some workers, namely, by Meurer[9] at the Dresden Institute of Technology, and by Boerlage, Broeze, Van Dijck and Peletier at the Proefstation, Delft,

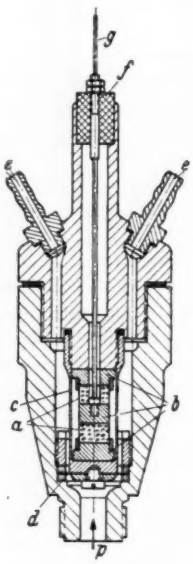


Fig. 15

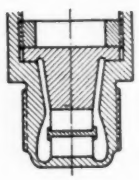


Fig. 16

FIG. 15 CELL USED AT DRESDEN

FIG. 16 CELL USED AT DELFT

Holland,[10] it has been found that mounting the quartz crystals between precompressing screws introduces hysteresis effects due to the minute slippage which occurs under stress between the male and female screw threads. Therefore the tendency is to eliminate the screw threads altogether and to mount the crystals between elastically yielding elements. Fig. 15 shows the cell used at Dresden and Fig. 16 that employed at Delft, both of which are built on this principle.

Another tendency in the construction of piezoelectric cells is to reduce the number of the quartz crystals to two only and most recent cells have been built in this manner. It is true that thereby the sensitivity of the cell itself is reduced but this can be compensated by a greater amplification.

In view of the increasing importance and use of this type of instrument, and of the progress in the construction of the cells, it would be of interest to learn the authors' experience in these respects.

B. H. MACKEY.[11] In my opinion, the importance of the instrument as described in this paper for research and development work cannot be overemphasized. The present method of measuring pressures with crusher cylinders will probably be used for routine work until the piezoelectric or some other form of pressure gage is simplified considerably, but it is quite obvious that the crusher gage has very limited value as a research tool.

It might be well to emphasize the importance of calibration in connection with the piezoelectric gage. The impression seems to exist in some places, that the piezoelectric constant of quartz is a definite quantity and that calibration is unimportant. Our experience has been that the constant for high-grade crystals can vary as much as several per cent and that each gage assembly must be carefully checked before use. It is also important that the quartz surfaces and those of the collector plates be almost optically flat in order that the gage may retain its calibration in use.

The piezoelectric gage has been used at Burnside Laboratory in essentially the form described in this paper for measuring pressure rises of comparatively long duration, i.e., 1 sec and also for pressure impulses given off by detonating explosives. In the first case, it is necessary to balance out the leakage currents with a separate supply voltage and in the second, it is necessary to damp the gage and the electrical circuit to eliminate extremely high frequency vibrations. The present paper has already indicated the wide pressure range through which this instrument gives satisfactory results.

AUTHORS' CLOSURE

As mentioned in the paper, it is necessary to place a Belleville spring between the head of the piston and the housing to keep the reaction between the housing and the piston approximately constant as the quartz crystals are compressed. This provides the elastically yielding element in the gage as developed at the Aberdeen Proving Ground.

With reference to the number of crystals, it is believed that most of the piezoelectric gages have contained not more than two plates. However, in our work, it is advisable to have many more plates because the gages as a rule are so remote from the amplifier.

[9] "Beitrag zum Bau Piezoelektrischer Indikatoren," by S. Meurer, *Forschung auf dem Gebiete des Ingenieurwesens*, vol. 8, no. 5, 1937, pp. 249–260.

[10] "Der Indikator," by K. J. DeJuhasz and J. Geiger, Julius Springer, Berlin, Germany, 1938, pp. 124–126.

[11] Burnside Laboratory, E. I. du Pont de Nemours & Company, Penns Grove, N. J.

The High-Pressure High-Temperature Turbine-Electric Steamship "J. W. Van Dyke"

By LESTER M. GOLDSMITH,[1] PHILADELPHIA, PA.

The author discusses the features of various steamship designs which were considered in the development and final adoption of the construction of the tanker S.S. *J. W. Van Dyke*. After data on steamship designs had been collected and analyzed an intensive study was made of hull form, propeller design, and power-plant requirements. The author describes the results of model tests conducted at model basins in Hamburg, Germany, and Washington, D. C.; the results of these tests were the deciding factor in selecting the propeller and hull designs. The paper includes a complete description of the steam-generating equipment, electrical equipment, high-pressure piping, controls, and miscellaneous electrical equipment which were finally selected for the ship. The author presents graphically the results of trial runs made with this turbine-electric steamship.

FOR a better understanding of the engineering development leading up to the construction of the tanker S.S. *J. W. Van Dyke*, an outline of some of the previous marine-engineering experience of The Atlantic Refining Company will first be given and may serve as a clarifying background for a ship carrying the highest steam pressure and temperature ever to go to sea from the United States.

Twenty years ago this company pioneered in the installation of Alquist or laminated type of double-reduction gears in five ships of 2600 shaft hp each, went through the entire period of this early development in reduction gears, and replaced them finally with compound turbines and single-reduction sets having solid gear wheels and embodying low tooth pressures in the design.

Fifteen years ago the company again pioneered by designing and building the first three Diesel-electric tugs and a 1200-ton Diesel-electric tanker, immediately followed by a 7500-ton tanker with the same system of propulsion. These installations were followed by others until this company had a fleet of fourteen vessels with Diesel-electric machinery, ranging in size from approximately 400 to 3000 shaft hp. At the same time, it was also operating on the same routes a number of the older steam-powered ships having both reciprocating and geared turbine machinery.

In view of this pioneering, and the experience and engineering development which followed it, together with the fact that the owners were manufacturers of petroleum products, it is evident that the company's engineers were absolutely unbiased from an engineering viewpoint, and had no other goal or object than to produce power for propulsion and transportation at the least cost per shaft horsepower, and per ton-mile.

By 1930, experience with, and observation of, Diesel-engine installations had convinced the company that this form of power had definite limitations. In other words, for horsepowers up to 3000, the Diesel engine is an undeniably economical prime mover, but above that size, doubtful, especially in view of the advent of the high-pressure high-temperature steam turbine. Having in mind that the company, within five years, would be faced with the necessity of replacing its then existing tonnage, the author inaugurated a study and thorough investigation of high-pressure and high-temperature steam plants. It was not until 1935, however, that the conclusion was reached that boilers and small turbines, using high-pressure and high-temperature steam, were suitable for marine applications. Keeping in mind the background of experience outlined, studies were made and tabulated of all available types of power plants, including Diesel-electric, direct Diesel, geared-turbine, and turboelectric.

While these data were being collected and analyzed, an intensive study of hull form and propeller design was begun. In order to correlate these studies of the power plant, propeller, and hull a description of the latter is first given.

The problem was to design a modern tank vessel to carry approximately 18,000 tons dead weight on a length of about 520 ft between perpendiculars and a maximum molded draft of 29 ft 6 in. at an average sea speed of 13¼ knots. This speed was considered the most profitable after a careful study of the conditions under which the ship was to run, and is the point at which a further increase in speed would reduce the margin between operating cost and gross profit.

A hull of the following dimensions was finally settled upon for tank testing and determination of propelling power: Length between perpendiculars = 520 ft; molded breadth = 70 ft; molded draft = 29 ft 6 in.; block coefficient (on between-perpendicular length) = 0.78; and displacement = 23,898 tons.

The Hamburg model experiment tank was chosen for model tests since it is well known to be the largest and best-equipped of its kind in the world and work on the previous ships had been done

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NOTE: Statements and opinions advanced in this paper are to be understood as individual expressions of the author, and not those of the Society.

TABLE 1 COMPARISON OF HORSEPOWER REQUIRED FOR COUNTER- AND CRUISER-STERN SHIPS

Speed, knots	Effective horsepower — Counter stern	Effective horsepower — Cruiser stern	Improvement by use of cruiser stern, hp
10	1298	1242	56
11	1742	1676	66
12	2292	2213	79
13	2944	2852	92
14	3756	3625	131

NOTE: By effective horsepower is meant the horsepower determined by towing the model; therefore, the propulsion losses are not included.

TABLE 2 COMPARISON OF TWO PROPELLER DESIGNS

Speed, knots	Effective hp	Propeller No. 1257 — Shaft hp	Prop. coef	Speed, rpm	Propeller No. 1275 — Shaft hp	Prop. coef	Speed, rpm
10	1242	1853	67.0	70.3	1890	65.6	64.3
11	1676	2490	67.4	78.0	2535	66.1	71.5
12	2213	3289	67.3	85.8	3290	67.2	78.4
13	2852	4294	66.4	93.7	4235	67.3	85.5
14	3625	5410	67.0	101.5	5280	68.6	92.5

NOTE: Horsepowers are metric; multiply by 1.0138 to convert to English units.

TABLE 3 CHARACTERISTICS OF PROPELLERS TESTED AT THE WASHINGTON, D. C., AND HAMBURG, GERMANY, MODEL BASINS

	Washington	Hamburg
Number of blades	4	4
Diameter, ft	19.500	18.170
Effective pitch, ft	15.167	16.540
Disk area, sq ft	298.600	259.500
Projected area, sq ft	118.000	.....
Developed area, sq ft	128.600	116.700
Pitch ratio	0.778	0.910
Developed area ratio	0.431	0.450
Projected area ratio	0.395	.....

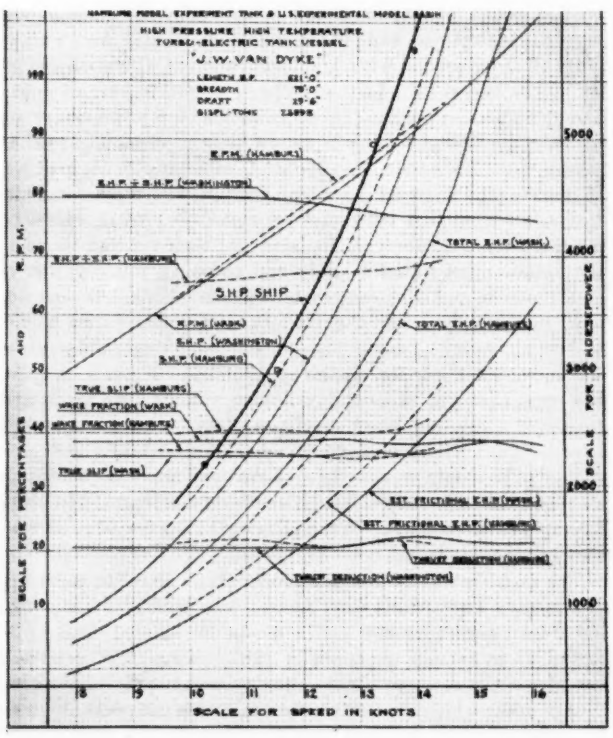


FIG. 1 COMPARISON OF MODEL EXPERIMENTS AND SHIP PERFORMANCE, WASHINGTON, D. C., AND HAMBURG, GERMANY, MODEL BASINS

there. At the time these experiments were started in March, 1934, there was still doubt in the minds of some shipbuilders as to the value of the so-called cruiser stern on the average commercial ship and, in order to obtain conclusive evidence on this point, the model was towed both with cruiser stern and the overhung counterstern, which is now almost obsolete. The results given in Table 1 leave no doubt as to the value of the former in cutting down eddy making and reducing the total resistance by an appreciable amount.

Self-propelled tests were next run with the cruiser-stern model, using propellers designed by the staff at the Hamburg model basin. Two different propellers were tried with results as given in Table 2. It can be seen from Table 2 that propeller No. 1275 gave the best results at the designed speed of 13.25 knots.

As stated previously, these tests were run in March, 1934, and it was not until January, 1936, that the company was ready to go ahead with the construction of the vessel. In view of this lapse of time and the fact that some higher propulsive efficiencies than shown in the Hamburg tests apparently had been obtained in one or two cases in this country, it was decided to have the same model experiments repeated at the U. S. model basin in Washington, D. C. This would serve not only as an interesting check on the results but would also provide opportunity for further possible improvement in efficiency. A model from the same lines was therefore tested at Washington at the same displacement, both for effective horsepower and self-propelled for shaft horsepower with a propeller designed by the staff at the Washington model basin. The results of these tests together with those obtained at Hamburg are shown in Fig. 1, and the principal characteristics of the two propellers are given in Table 3. From an inspection of the curves, it is readily apparent why the propeller developed in the U. S. model basin was adopted.

With regard to the structural plan of the vessel, the chief point of interest is the fact that her entire cargo tank and fuel bunker space, comprising a length of 353 ft, is electrically welded throughout. The bow and stern portions of the ship were riveted in the orthodox manner. The general arrangement otherwise follows more or less usual tanker practice. The ship is a poop, bridge, and forecastle type with one continuous deck and two longitudinal bulkheads and sixteen transverse bulkheads, the cargo space consisting of nine three-compartment tanks with a total capacity of 868,980 cu ft, or 6,500,000 gal. The modern tendency toward streamlining is followed to a moderate extent by rounding the fronts of bridge structure and poop and by giving the stack an ogival section. A general arrangement and profile plan of the ship are shown in Fig. 2, and her general appearance is shown in Fig. 3.

POWER PLANT

After all the analytical work was finally reduced to figures, it was disclosed that it would require 5000 hp to propel this vessel at the desired speed, as indicated by the model-basin work shown in Fig. 1. It was decided that for this horsepower, the form of power promising the lowest over-all operating cost per shaft horsepower was the turbine-electric drive. The reasons for selecting the turbine-electric drive embody more than just propulsion, for it was the request of the management of The Atlantic Refining Company that this vessel be outstanding in its ability to discharge cargo rapidly. In order to meet the requirements for discharging in less than 12 hr, pumps having a rated total capacity of 10,500 gpm at the desired pressure were selected. It was found that approximately 1000 hp would be required for these pumps when discharging in port. In addition, the trim of the vessel necessitated location of this cargo-pump room amidships. Therefore, it can be seen how simple it is to obtain this power from a standard turbine generator and operate the pumps electrically. It must also be kept in mind that not only economy of operation, but first cost and reduction of weight should be in the mind of every designer of any transportation equipment.

To meet existing rules of the American Bureau of Shipping, the Bureau of Marine Inspection and Navigation, and the require-

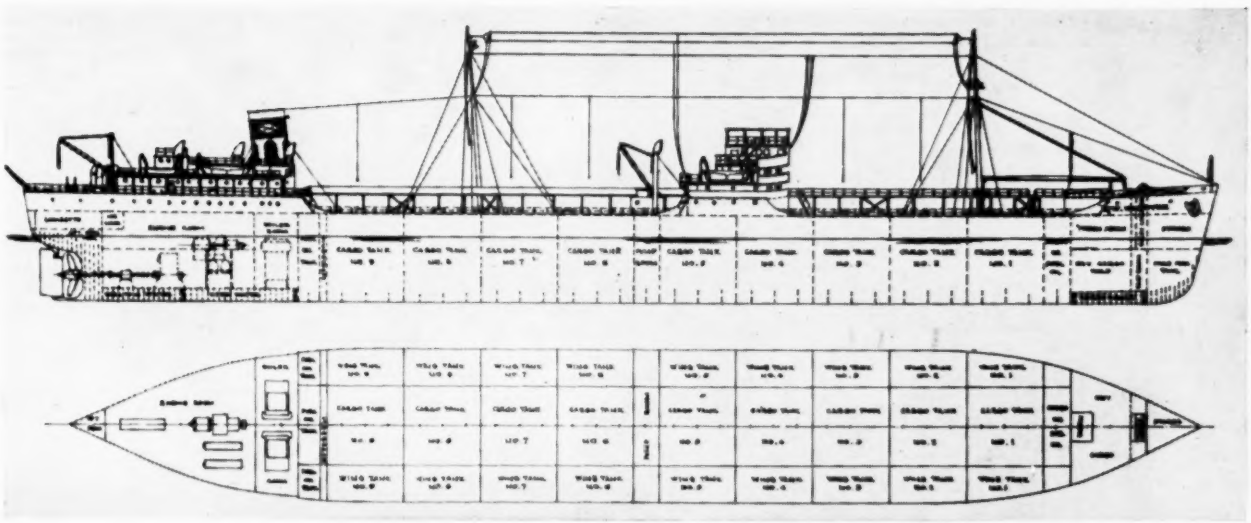


FIG. 2 GENERAL ARRANGEMENT AND PROFILE OF THE S.S. *J. W. Van Dyke*

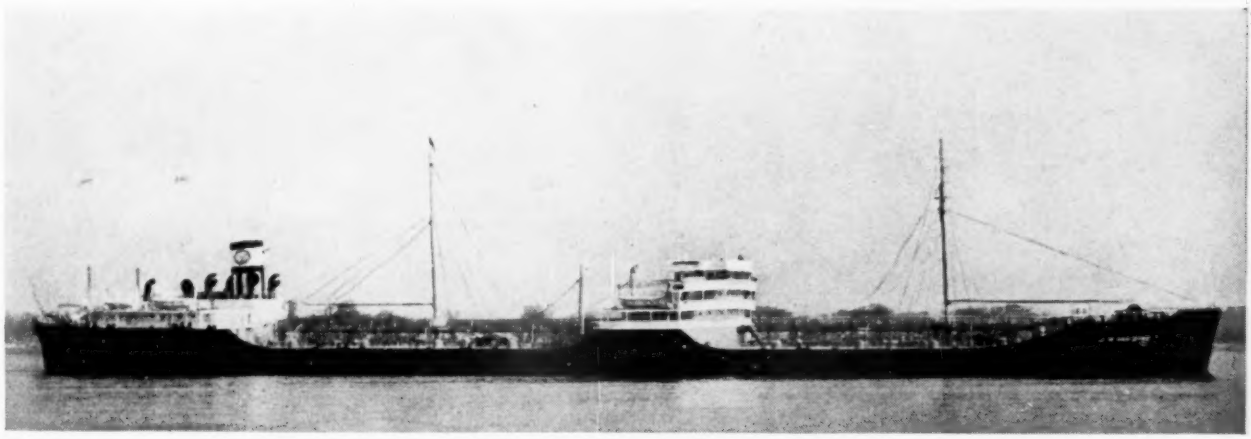


FIG. 3 THE S.S. *J. W. Van Dyke*

ments of economy in general, it was decided that the turbine should be designed for a pressure of 600 lb per sq in. and a steam temperature of 825 F. When the decision was made to go to this high pressure and temperature, one problem eliminated by the use of motor instead of gear drive was that of the reversing turbine and the question of keeping it cool.

In the tabulation of the engineering design of the previous turbine-electrically propelled vessels, both in this country and abroad, it was disclosed that none utilized a generator of standard voltage and frequency as we know them to be standard, thinking of land power plants in this country, nor did they employ the use of alternating current for auxiliary uses as is the case in all land plants. As no reason could be found why other than standard equipment should be used, engineering work was started with the idea of profiting by the principles in use in every modern land power station. A complete heat balance, as shown in Fig. 4, was first worked out in connection with the machinery manufacturers.

Three bleed stages were decided upon as giving the best compromise between the ideal and the practical complications of space requirements and piping. Another consideration borne in mind was that evaporation of feedwater should be reduced to a minimum and that every possible precaution must be taken to protect the boilers from impure water; therefore, all clean drains are collected and returned through a cooler into the main condenser while drains which are subject to possible contamination are reevaporated. The evaporators are planned to float on the line and normally only one of them, operating at much below its rated capacity, will be required. Under sea conditions, the heat of the evaporator vapor is used to supply such low-pressure steam as may be required for general heating purposes throughout the ship, and the remainder goes to the intermediate feedwater heater so that this heat is recovered. An emergency vapor line to the condensers is also provided for port use; but with proper operation this should not normally be necessary.

All three boiler feed pumps are turbine driven. Only one is required for normal operation. The exhaust is added to the second-stage bleed.

Therefore, it will be noted that under normal conditions the heat from the evaporator vapor and the feed-pump exhaust is either recovered in the feed or otherwise usefully employed.

No heat which can be put to use is rejected to the sea water except under unusual conditions.

Table 4 gives a comparison of the heat distribution as planned and as actually obtained. It will be noted that an over-all heat efficiency from burners to shaft of 22.5 per cent was expected.

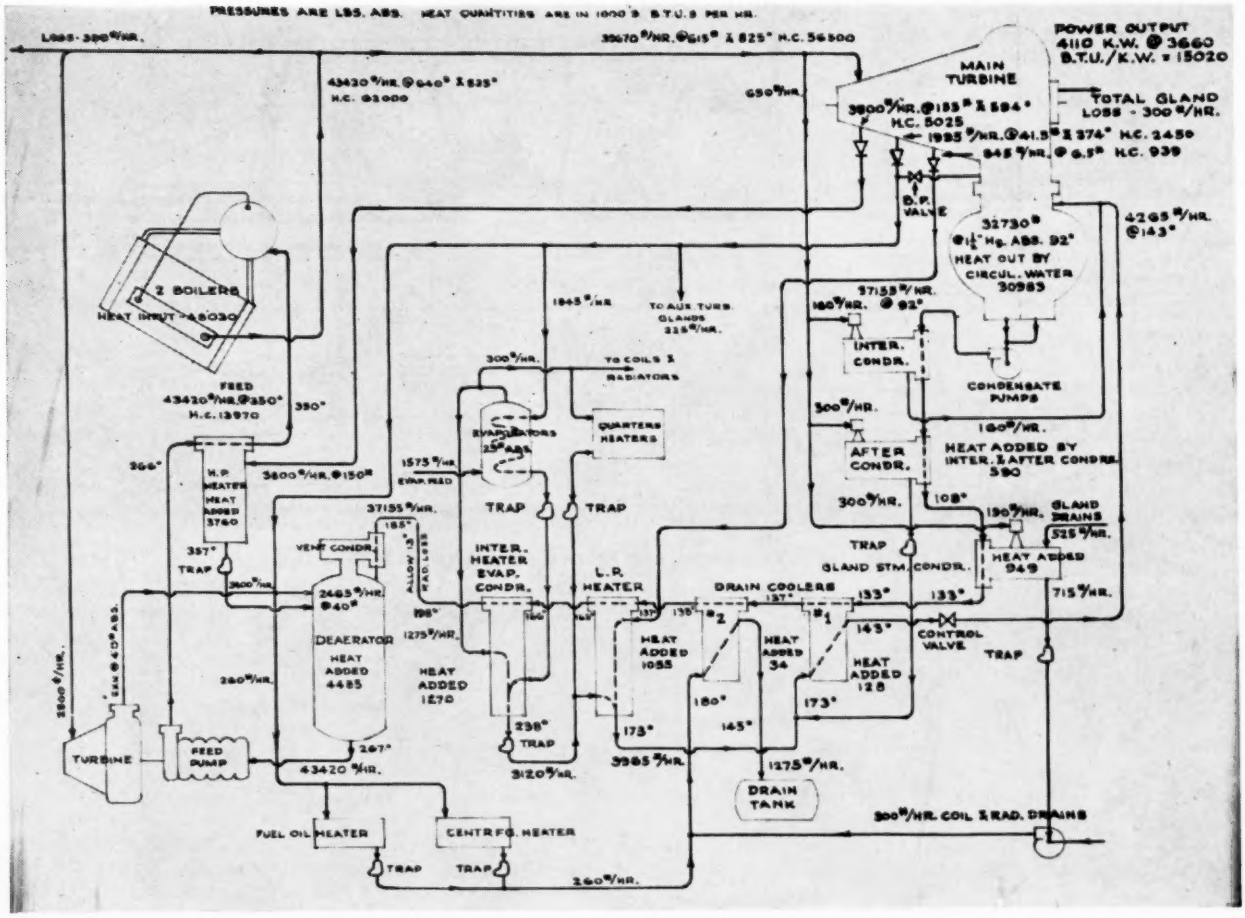


FIG. 4 HEAT-BALANCE DIAGRAM

TABLE 4 SUMMARY OF MATERIAL AND HEAT BALANCE COMPARISONS

LOCATION	DESIGN CONDITIONS		#1 FUEL RUN - 4 HRS.		#2 FUEL RUN - 3 HRS.		J.W. VANDYKE - 4 DAYS (NOV. 28 TO DEC. 2)		J.W. VANDYKE BEST DAY (DEC. 2 TO DEC. 3)		ROBERT H. COLLEY 4 HOUR TEST RUN		DERIVATION OF QUANTITIES
	QUANTITY	HEAT (1000 B.T.U./#)	QUANTITY	HEAT	QUANTITY	HEAT	QUANTITY	HEAT	QUANTITY	HEAT	QUANTITY	HEAT	NOTE - ALL INSTRUMENTS WERE CAREFULLY CALIBRATED FOR THESE TESTS.
FEED WATER TO BOILERS	43420	13970	43625	14108	44025	14130	45700	14950	48300	14780	45930	14960	BY METERS
HEAT INPUT TO BOILERS	-	46030	-	48713	-	48960	-	50150	-	49800	-	50920	FROM INSTRUMENTS
TOTAL FROM BOILERS	-	62000	-	62823	-	63090	-	65100	-	64580	-	65800	-
TO FEED PUMP TURBINES	2800	3996	2850	4260	2970	4260	2980	4250	2980	4258	3000	4290	CALCULATED FROM PUMP & TURBINE CURVES
AIR JET VAC. PUMPS	650	928	700	1010	700	1004	700	998	700	998	700	1002	FROM MANUFACTURER'S FIGURES
AUXILIARY SET	RUN AS MOTOR FROM MAIN GENERATOR.												
UNACCOUNTED FOR LOSS	300	576	375	383	175	136	470	602	320	442	120	538	BY DIFFERENCE
TO MAIN TURBINE	39670	56500	39600	57200	40180	57700	41350	59250	41300	58900	39300	56900	BY METER
FROM #1 BLEED	3600	5025	3530	4750	3720	4980	4300	5770	4170	5600	3560	4810	CALCULATED FROM TEMPERATURE RISE IN H. HEATER.
#2 BLEED	1995	2450	1820	2200	1844	2250	1232	1490	1390	1697	1835	2240	CALCULATED FROM TEMP. RISE IN INTER. HTR. & DEAERATOR
#3 BLEED	845	539	0	0	0	0	0	0	0	0	0	0	
GLAND LOSSES ETC.	300	336	300	336	300	336	300	336	300	336	300	336	MANUFACTURER'S FIGURE
TO CONDENSER	32730	32730	33950	35614	34316	35934	35728	36634	35440	36447	34000	35054	BY DIFFERENCE.
TO POWER		15020		14280		14200		14820		14910		14460	FROM M'FR'S FIGURE OF 3660 B.T.U. (3344 X MECH. EFF.) PER K.W.HR.
SHAFT HORSEPOWER	5000		4980		4940		5080		5080		NO METER FITTED		BY TORSION DYNAMOMETER
LBS. TOTAL STEAM/SHP.HR.	8.68		8.76		8.95		8.99		8.92				
% HEAT FROM FUEL = SHAFT	22.8		22.6		23.4		22.7		22.7				
LBS. TOTAL FUEL/S.H.P.HR.	.596		.594		.598		.592		.591				NOTE: THIS IS TOTAL FUEL FOR ALL PURPOSES AND AUXILIARIES. HEAT VALUE OF FUEL = 19000 B.T.U./LB.
% FUEL FOR AUXILIARIES	14.4		13.8		13.4		13.7		13.9				
LBS. FUEL/S.H.P.HR. (FOR PROPULSION ONLY)	.510		.502		.508		.511		.512				

During the trials, we obtained efficiencies as high as 22.6 per cent and in actual operation have obtained 22.7 per cent. It should be noted that these figures are that portion of the total heat supplied by the fuel to the burners which is used for propulsion, and not the gross steam-cycle efficiencies, which figures are 26.4, 26.1, and 26 per cent, respectively. Auxiliary sets do not enter into the normal heat-balance picture, since under usual load conditions they are electrically driven from the main generator.

Steam Generation

The design of the steam-generation plant is as follows:

Boilers are of the single-pass straight-tube header type as shown in Fig. 5, embodying four decks of superheat, stud-tube side walls, and air preheater.

Header-type boilers with air heaters, but without economizers, were selected with the thought in mind that the extreme simplicity of this design would be reflected in maintenance cost and, in particular, that with this type of boiler the tubes can be plugged off or even renewed without loss of ship time waiting for the boiler to cool down enough so that the drums can be entered.

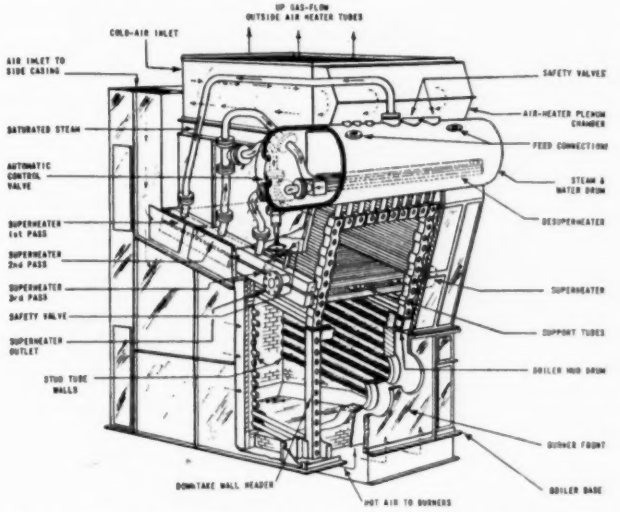


Fig. 5 Single-Pass Straight-Tube Header-Type Boiler Used on S.S. *J. W. Van Dyke*

It will be noted that the arrangement is further simplified by the fact that the boilers are so designed that the gases rise straight through, no baffles being required, thus further reducing internal maintenance.

The boilers have a rated capacity of 22,500 lb per hr each, at a pressure of 625 lb per sq in. gage and a steam temperature of 835 F, with feedwater at 300 F. They each have a total water-heating surface of 3222 sq ft, a superheating surface of 1094 sq ft, and an air-heating surface of 2412 sq ft. The furnace volume is 426 cu ft per boiler, giving a liberation rate of 66,000 Btu per hr per cu ft.

The lower two rows of tubes in both halves of the superheater are of 9 per cent chromium alloy and the boilers can be safely operated at 900 F final temperature.

They are fitted with automatic air-puff soot blowers.

The boilers are provided with complete automatic combustion control, including automatic superheat control, and the charts as shown in Figs. 6 and 7, will show how well this equipment functions. The charts in Fig. 6 are from one of the trial runs and show a continuous record of a 3-hr fuel test at constant load followed by three sets of runs at approximately full load, but under different ballast conditions. Note that no attempt was made to protect the boilers by gradual slowing or speeding up of the main turbine, but in spite of this, the superheat temperature falls off instead of rising when the load is dropped and is held at the set temperature when load is applied. At low loads the boilers are on hand operation; hence, the spread between the air-flow and steam-flow records.

When the low-load condition is to be maintained for any length of time, the burners can be changed to suit and the system again put on automatic operation.

The arrangement of the superheaters and their control is interesting. The superheater is divided into two sections, and a desuperheating coil is located in the drum which is so baffled that the entering feed passes along this coil. The saturated steam from the drum passes through the first half of the superheater and then is divided by a three-way valve so that a variable portion goes through the desuperheating coil before passing on to the second half of the superheater. The three-way valve is controlled automatically by the temperature at the superheater outlet.

It is interesting to note from precedent in this respect that today a land installation of a steam-generating plant, not embodying some form of automatic combustion control, would never be considered. Yet, in land plants, technical men, usually in the operating force and always in the supervisory force, are immediately available, while in the marine field, the supervision is only obtained at long distance. The successful operation of this type of equipment, as demonstrated on the S.S. *J. W. Van Dyke* will undoubtedly mean much greater application because of this demonstration. How well this control functions under sea conditions is shown by the charts in Fig. 7.

High-Pressure Piping

In the design of the high-pressure steam piping, the rules of the Bureau of Marine Inspection, which, at that time, not only prohibited welding, but were written without provisions for temperatures above 750 F, were somewhat of a handicap. However, through the cooperation of the bureau, rulings were obtained which permitted using stress-relieved welded joints. The main steam connections to the boilers, the boiler header, and the line to the main turbine are of 0.5 Mo, 0.2 C seamless-steel tubing with an inside diameter of 5 in. and an outside diameter of $6\frac{1}{2}$ in. This is heavier than necessary on account of the Bureau rules for thickness which did not permit taking full advantage of the greater strength of molybdenum tubing at high temperatures. In order to reduce radiation loss as well as the number of joints, the equipment was arranged to reduce the high-pressure steam piping to a minimum. The total length of the main steam lines, including expansion bends and valves, is as follows: Length of boiler risers, the 5-in. header, and main-turbine connection = 72 ft; length of the 3-in. Butterworth heater connection = 21 ft; length of the $2\frac{1}{2}$-in. boiler-feed-pump main = 22 ft; and the length of the 2-in. auxiliary-generator connections = 32 ft. All the high-pressure steam piping is of carbon-molybdenum steel. All flanges are recessed or tongue-and-grooved to give confined gaskets. All bolts, including those in the feed discharge lines, are of high-tensile class-C steel.

Main Turbine Generator

The main turbine generator is rated at 4500 kw, 60 cycles, 2300 v at 3600 rpm when supplied with steam at the throttle at 600 lb per sq in. gage and 825 F total temperature with the exhaust at 28.5 in. Hg vacuum.

The normal output for 5000 shaft hp was expected to be 4200 kw, 3850 kw going to the motor and 350 kw to the auxiliaries.

The turbine is of the impulse type, having 15 stages with bleeds after the third, seventh, and twelfth stages. The rotor is a solid forging with the wheels integral with the shaft, the first wheel carrying a double row of blades and all of the remaining wheels carrying single rows. The case consists of a high-pressure alloy fabricated steel section and a cast-iron low-pressure section, the division occurring between the tenth and eleventh stages. The case is also split horizontally to permit removal of the rotor. The blading of both wheels and diaphragms is of chrome-iron alloy; the first-stage nozzles are of steel with chrome-iron alloy partitions and the diaphragms are of cast iron or steel, depending on

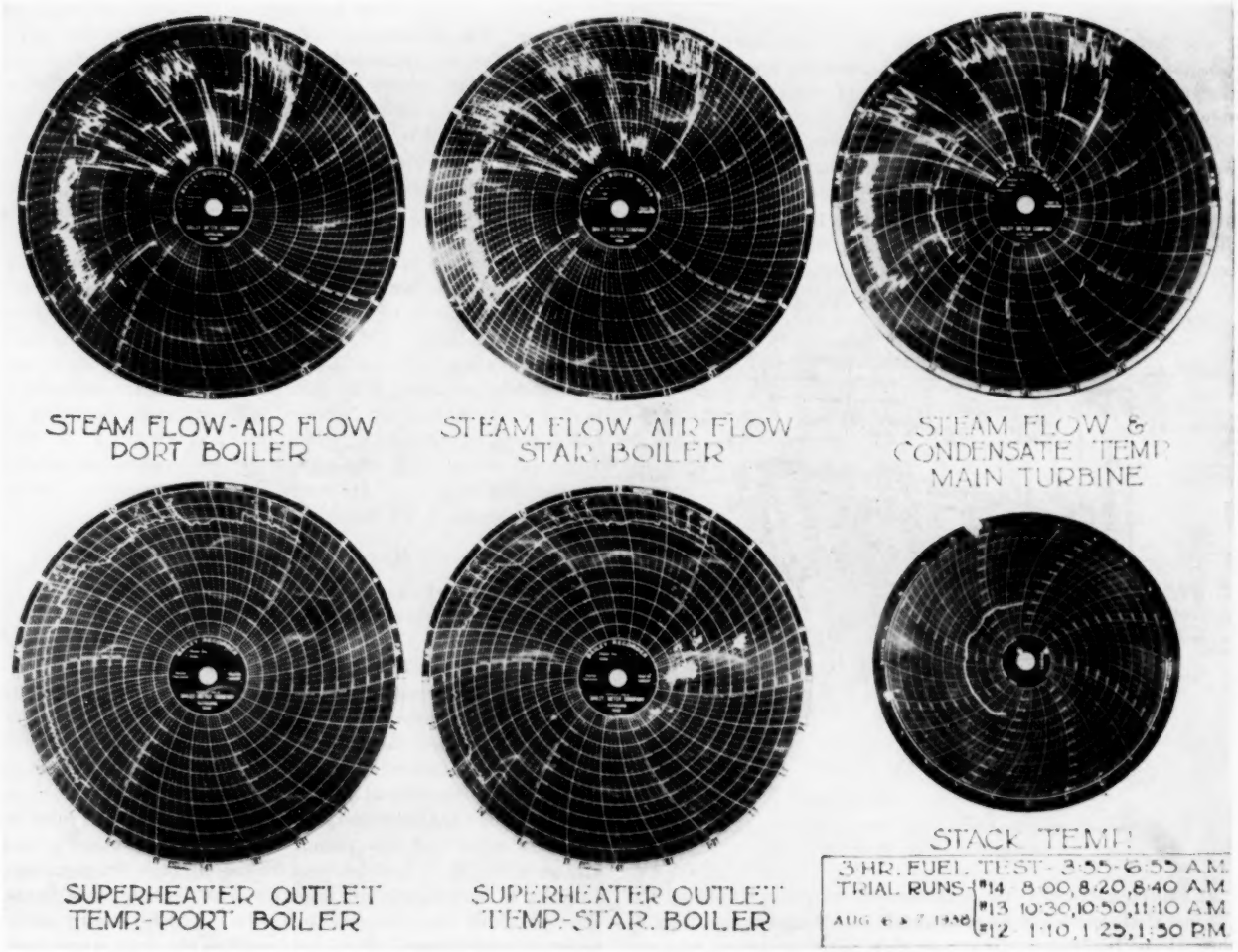


FIG. 6 CHART RECORD OF 3-HR FUEL-TEST RUN

the temperature. The packings are of the labyrinth type, and the glands are steam-sealed and provided with an automatic steam regulator.

Governing is accomplished by a valve chest containing six valves leading to the nozzle plate and so arranged that the nozzles are successively cut in or out as the turbine demand increases or falls off, thus avoiding the loss which would be caused by throttling the supply. The speed range which is obtainable is from 900 to 3780 rpm corresponding to 22.5 to 94.5 rpm of the propeller.

The generator, which is directly connected to the turbine, is of the rotating-field type. The stator consists of six coils Y-connected and brought out to six terminals. The rotor is ventilated by fans integrally mounted, which force air into its ends and out at the center; it then passes down through the air cooler, which is in turn water-cooled. This cooler is mounted directly under the generator and is housed in the same case. The bearings are of the plain babbitted type.

The combined turbogenerator set is only 22 ft 9 in. long × 10 ft 3 in. wide.

PROPULSION MOTOR

The propulsion motor is of the synchronous-induction type with 80 poles and is rated at 5000 shaft hp at 90 rpm, 2300 v, 3 phase, unity power factor.

It is only 17 ft diameter and 14 ft long over-all. This length includes the space allowed for sliding the stator aft to clear the rotor in case repairs are necessary. This arrangement avoids any need of disturbing the bearings or shaft alignment; this is the first time such an arrangement has been used.

The motor is separately cooled, but motor, fan, and air cooler are all enclosed in a tight duct system so as to give practically the effect of a totally enclosed motor.

Both the generator and motor are equipped with CO_2 connections for fire protection.

CONTROL

Figs. 8 and 9 show the main control panel with the operating levers. The method of control is extremely simple. Referring to Fig. 8, the left-hand lever (30) as you face the controls has three positions for both ahead and astern movement. As it is moved from the off position, it first closes the line contacts to the generator, then the generator field contacts so that the motor starts as an induction motor, and finally the motor field contacts, which causes the motor to pull into step and run as a synchronous motor.

Beside this lever is the turbine-governor lever (31) which permits varying the speed of the generator from 900 to 3780 rpm by adjustment of the governor setting.

In addition, there is an emergency throttle lever (32) which permits operating the generator on hand control up to 75 per cent of normal speed and an emergency trip (27).

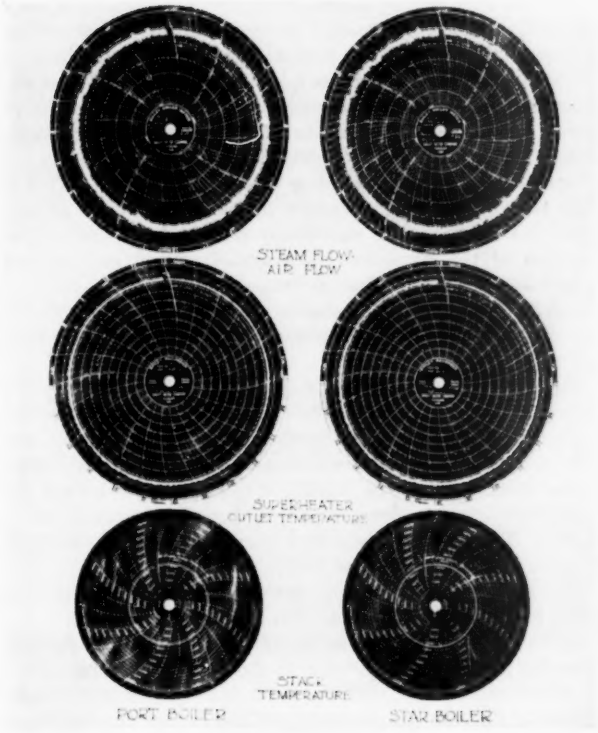


FIG. 7 CHART RECORD OF BOILER OPERATION UNDER NORMAL SEA CONDITIONS

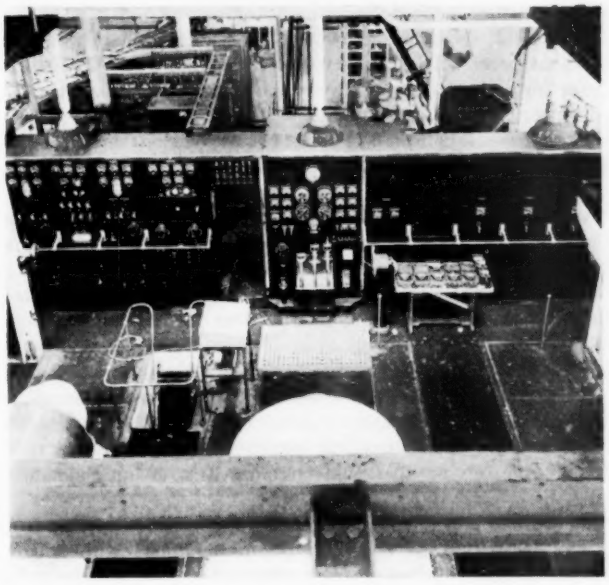


FIG. 9 CONTROL PANELS OF THE S.S. *J. W. Van Dyke*

All these levers are mechanically interlocked to prevent incorrect operation.

AUXILIARY SETS

There are two auxiliary sets, one to run and one for a stand-by unit. Each consists of a 5568/1200-rpm geared turbine driving a 350-kw 60/50-cycle 450-v synchronous generator, a 65-kw 110-v d-c exciter for the main units and a 20-kw 120-v d-c generator for excitation of the 350-kw auxiliary generator and supplying miscellaneous direct-current requirements.

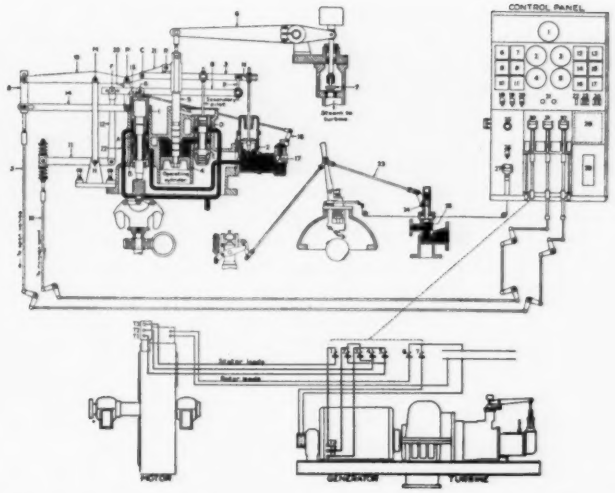


FIG. 8 SCHEMATIC DIAGRAM OF THE PROPULSION EQUIPMENT

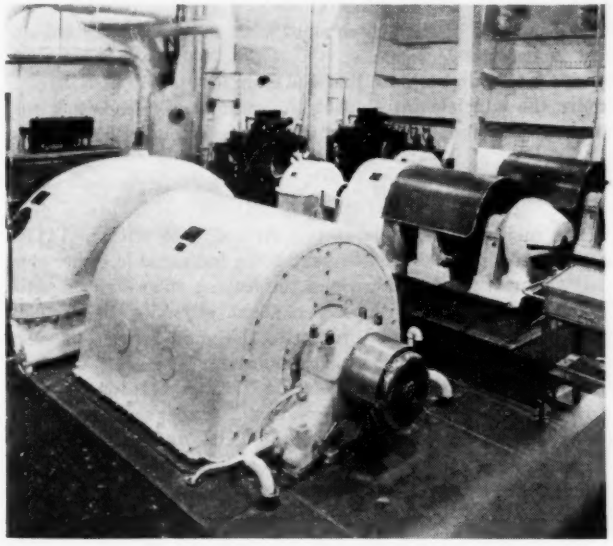


FIG. 10 THE MAIN GENERATOR AND TWO AUXILIARY SETS

The auxiliary units are so arranged that under normal conditions (60/50 cycles) the 350-kw unit operates as a motor, taking current from the main generator to drive the direct-current machines and drag the turbine. (No cooling of the turbine is required as vacuum is maintained through a connection to the main condenser.) When the speed of the main generator drops to approximately 52 cycles, the speed governor of the auxiliary set automatically starts to open the steam-admission valve so that the auxiliary turbine assumes the full auxiliary load by the time the main unit has slowed to 50 cycles. The circuit breaker between the two generators opens automatically to prevent the auxiliary generator attempting to motor the main generator. When the auxiliary turbine is to operate for any length of time, it is then brought up by hand to 60-cycle speed. When the main unit is again above 52 cycles, the auxiliary generator can be returned to motor operation as follows: The main and auxiliary generators are synchronized by means of a synchronizing device on the auxiliary-turbine governor. The circuit breaker is then

closed and the speed governor reset at its lower limit (52 cycles) which cuts off the steam, but leaves it ready to open and causes the turbine to reassume the load automatically when the main generator speed again drops below 52 cycles.

The compactness of both the main and auxiliary units and the convenience and "shipshape" arrangement of machinery made possible by electrification are clearly shown in Figs. 9 and 10.

Fig. 10 is a view from above the switchboard looking forward and to starboard across the main generator and the two auxiliary sets.

Fig. 9 is a view from the bridge across the forward end of the engine room and looks down and aft. In this view the main generator is in the center foreground with the auxiliaries to the left and the steam-gage panel board to the right. Across the center is the switchboard with the main control panel in the center, the three cargo pump panels at the right, and the auxiliary panels at the left. At the extreme left beyond the auxiliary panels may be seen the control panel for all the various motors throughout the ship. In this connection, note that equipment cannot be started from this panel, but only by means of push-button stations located at the equipment. Behind the switchboard on the left may be seen the relays and on the right the 2300-v–440-v oil-filled transformers. The table full of instruments in front of the switchboard is a temporary installation of test instruments.

It is interesting to note that the main motor is almost directly under the panel board so that the main power leads are only about 15 ft long from generator to switch panel and down to the motor.

Condensers

The main condenser is of the divided type with a divided hot well so that in case of leakage being indicated in either side, that side can be shut down and the leaking tube plugged while operation continues with only a slight reduction of vacuum. This condenser is bolted directly to the main turbine, most of its weight being taken by spring supports.

It has 4775 sq ft of $^3/_4$-in. 70–30 Cu-Ni tubes, 14 ft long between tube sheets, solidly rolled at both ends. Expansion is provided for by bellows joints in each side of the shell. It is rated at 35,300 lb per hr at a vacuum of 28.44 in. Hg with sea water at 76.8 F which is the average for the conditions under which this ship is to operate.

The auxiliary condenser has only a single bundle of 800 sq ft effective surface. The tubes are of the same material, size, and length as the main unit so that only one set of spares is required. As in the main unit, the tubes are rolled in at both ends and expansion is provided for by a bellows in the shell.

This unit has a capacity of 6000 lb per hr at a vacuum of 28.44 in. Hg with sea water at 76.8 F. This corresponds to the output of one auxiliary unit at 105 per cent load.

It can also handle the output of both units at 70 per cent load (8000 lb per hr) at a vacuum of 28.1 in. Hg.

The auxiliary condenser is connected by a header to the main condenser, and the auxiliary turbines exhaust into this header. Valves are provided to shut off either auxiliary unit and to permit the operating unit to exhaust either into the auxiliary or the main condenser.

The arrangement of the circulating-water system is unique and merits some explanation.

It has long been recognized that the principal cause of condenser-tube corrosion is entrained air. Many attempts have been made to eliminate the trouble by such means as water-box vents, streamlining of water passages, and flaring of tube ends, all of which have helped, but have not solved the problem.

Therefore, an attempt has been made to arrange the sea suction so that the entrained air is removed before the circulating water reaches the pumps. In order to accomplish this, a chamber has been provided between the tank top and the skin of the ship. The sea connections are made into this chamber. Baffles and vents are provided so that any entrained air is separated and removed before the water reaches the intakes of the circulating pumps, which are at the bottom of this chamber.

Examination of the condenser tubes in the S.S. *J. W. Van Dyke* after 6 months of operation indicates that this is proving effective as there was no sign of any pitting of the tubes. The impellers of the circulators were also examined and likewise showed no signs of corrosion or erosion.

It is believed that this arrangement will prove to be an important contribution to the protection of both condensers and circulating pumps.

The circulating pumps are of the so-called "deep-well" type, having mixed-flow type impellers at the lower end of vertical shafts driven by hollow-shaft vertical motors. Each pump normally supplies one side of the condenser, but they are manifolded so that both sides of the condenser can be supplied from one pump (when this is done, there is a loss of vacuum of about $^1/_2$ in.). These pumps also supply the generator and motor air coolers and the lubricating-oil cooler.

The remaining engine-room machinery is not described in detail, but a summary list is given in Table 5.

For those who may fear that the use of turboelectric equipment instead of gear drive adds unduly to the machinery weight, it will be of interest to note that the main turbine generator weighs 91,000 lb; the main motor weighs 107,000 lb; the air coolers and motor-cooling fan weigh 6100 lb; and the propulsion control equipment weighs 7600 lb. The total weight of this equipment is 211,700 lb.

The total dry machinery weight of the entire ship is only 423 tons, including deck machinery and cargo pumps.

The total weight of the power plant and auxiliaries only is 405 tons (wet). This equals 0.08 tons per shaft hp. The usual figure for a geared-turbine ship of this power is from 0.11 to 0.12 tons per shaft hp (550 to 600 tons); for direct Diesel, or Diesel-electric drive, it is from 0.11 to 0.14 tons per shaft hp (550–700 tons).

This low machinery weight is reflected in the unusually high ratio of dead weight to displacement obtained for the ship, the figure being 0.758, which is considered very good, especially in view of the extra-large cargo-pumping equipment installed.

Expressed in another way, the lower machinery weight alone permits the carrying of from 1000 to 2000 bbl more cargo.

Cargo Pumps

As stated earlier, it was necessary to provide cargo-pumping equipment of sufficient capacity so that the ship could be unloaded between tides. This required pumps of a total capacity of 10,500 gpm, giving an actual pumping time of approximately $10^1/_2$ hr.

In order to take advantage of the ship's generating equipment, it was decided to use deep-well pumps equipped with 2300-v motors as with this arrangement the motor room can be completely separated from the pump room except for the shafts which pass through stuffing boxes at each end of the cover pipe with bleedoffs to the suction side between them.

The pumps are three-stage centrifugals rated at 3500 gpm at 1750 rpm versus pressures of 80 lb per sq in. on gasoline and 90 lb per sq in. on crude oil, and requiring 220 and 245 hp, respectively. At maximum load on salt water, the power requirement is 300 hp. No stripping pumps are used, but instead, the cargo pumps are set in suction boxes which in turn are connected to vacuum priming pumps through float boxes so that as long as a

TABLE 5 SUMMARY OF MACHINERY USED ON THE S.S. "J. W. VAN DYKE"

Number of units	Unit	Capacity or size	Power
2	Boilers	22500 lb per hr at 625 lb per sq in. and 835 F	....
..	Boiler controls	....	Air operated
..	Soot blowers	....	Air operated
1	Main generator	4530 kva, 2300 v, 0.994 power factor 60 cycle, 3 phase	....
1	Propulsion motor	5000 shaft hp at 90 rpm, 2300 v, 60 cycle, 3 phase	....
2	Generator and motor air coolers	2300 sq ft (each)	....
1	Ventilating fan for main motor	....	15 hp
1	Main condenser:		
	Capacity	35300 lb at 28.44 in. Hg vacuum	....
	Area	4775 sq ft	....
2	Auxiliary generator sets	430 kw, 450 v, 3 phase, 60 cycle, 0.8 power factor	....
1	Auxiliary condenser:		
	Capacity	6000 lb at 28.44 in. vacuum	....
	Area	800 sq ft	....
4	Low-pressure heaters and drain coolers	40000 lb feed to 167 F	....
1	Deaerator	43500 lb feed to 267 F	....
1	High-pressure heater	50000 lb feed to 350 F	....
2	Evaporators	1400 lb per hr, each	....
2	Fuel-oil heaters	3000 lb per hr, each, to 235 F	....
1	Lubricating-oil cooler	40 gpm; 160 to 130 F	....
1	Butterworth heater and drain cooler	200000 lb per hr of water 50 to 180 per cent with 150 lb steam	....
1	Centrifuge oil heater	75 gal per hr; 60 to 140 F	....
1	Evap feed grease extractor	3000 lb per hr, fresh water	....
2	Main circulating pumps	3750 gpm each	30 hp, 1150 rpm
1	Auxiliary circulating pump	1200 gpm	10 hp, 1150 rpm
2	Main condensate pumps	90 gpm	15 hp, 3500 rpm
1	Auxiliary condensate pump	20 gpm	5 hp, 3500 rpm
3	Main feed pumps	125 gpm at 755 lb per sq in.	125 hp, 3600 rpm
3	Main-feed-pump turbines	125 hp, 3600 rpm	....
2	Evaporator feed pumps	1500 lb per hr each	1 hp, 1750 rpm
2	Fresh-water pumps	1000 gal per hr	1½ hp, 1750 rpm
2	Fire and Butterworth pumps	425 gpm vs. 100 lb per sq in., each	....
2	Engine-room and forward bilge pumps	325 gpm	7½ hp, 3500 rpm
1	Sanitary pump	90 gpm	5 hp, 3500 rpm
2	Fuel-oil service pumps	8 gpm vs. 300 lb per sq in. each	7½ hp, 1150 rpm
2	Fuel-oil transfer pumps	150 gpm vs. 100 lb per sq in. each	15 hp, 1750 rpm
1	Lubricating-oil pump	59 gpm vs. 50 lb per sq in.	3 hp, 1160 rpm
1	Heating-system vacuum pump	4 gpm and 3 cfm of air	1 hp, 1750 rpm
3	Main cargo pumps	3500 gpm gasoline vs. 80 lb per sq in.	300 hp at 2300 v (1750 rpm)
1	Pump-room bilge pump	160 gpm vs. 50 ft head	3 hp at 1750 rpm
3	Cargo priming pumps	90 cfm each at 18 in. vacuum	7½ hp at 1150 rpm
2	Air compressors	60 cfm each vs. 120 lb per sq in.	15 hp at 870 rpm
3	Forced-draft fans	8500 cfm each vs. 4.5 in. draft	10 hp at 1750 rpm
2	Boiler-room ventilating fans	11500 cfm each vs.0.1 in. draft	1½ hp at 1750 rpm
4	Engine-room ventilating fans	14650 cfm each vs. 0.15 in. draft	2 hp at 1750 rpm
1	Thrust bearing (roller type)	116000 lb thrust (16⅛ in.)	....
1	Lubricating oil centrifuge	75 gal per hr	½ hp, 1750 rpm
1	Desuperheater (for Butterworth)	22000 lb of steam per hr	....
1	Meter for steam to turbine	50000 lb per hr, max	....
1	Gland steam condenser	725 lb of steam + 210 lb of air per hr	....
1	Refrigerating machine	2½ ton	7½ hp, 1750 rpm
3	Capstans	....	25 hp, 865 rpm
2	Deck winches	....	30 hp, 200 rpm
1	Electrohydraulic windlass	....	75 hp, 1775 rpm
1	Electrohydraulic steering gear	....	Two 20 hp, 875 rpm
1	Two-unit gyro pilot system	....	....
1	Gyrocompass	....	....
1	Salinity recorder	4 point 0–10 grains	....
2	Butterworth sets	....	....
All	Auxiliary motors	....	440 v, 3 phase, 60 cycle

suction box is filled with liquid, the suction to the vacuum pump is closed by the float. If air enters, it passes into the float chamber, the valve opens and the air is drawn off through the vacuum pump, thus restoring suction to the cargo pump. The intakes of the cargo pumps being close to the bottoms of the suction boxes, the pumps continue to operate at full capacity until the tanks are practically empty as is evidenced by the fact that the *Van Dyke* has discharged her cargo in as short a time as 10 hr and 6 min, and normally discharges in less than 11 hr. Her sister ship, the S.S. *Robert H. Colley*, established a record of 9 hr and 21 min to discharge her cargo of 138,069 bbl of crude oil.

Miscellaneous Electrical Equipment

Motors. All pump and fan motors are of the class-2 marine type and are totally enclosed except (*a*) the main motor which is ventilated by a separate motor-driven fan and (*b*) motors in locations classified as hazardous, which are of the class-1 type and are explosion proof. This latter group includes the cargo pump, the cargo priming pump, and pump-room bilge-pump motors. The deck-machinery motors are also of class-1 explosion-proof construction.

All motors except the steering-gear motors are provided with overload relays.

The steering-gear motors, and the lights and galley service are



Fig. 11 Fore-and-Aft Gangway Has Solid Sides

on an automatic transfer between the two auxiliary units. Certain other motors are equipped with sequential relays to restart automatically after a power failure in the following order: (1) main lubrication oil pumps, (2) main circulators, (3) forced-draft fans, (4) main condensate, and (5) fuel-oil service.

In connection with the electrical equipment, the problem of carrying the cables forward to the cargo pump room and the midship house was given considerable thought, and it was finally decided to provide the fore-and-aft gangway on deck with solid sides instead of open railings and to run the cables along the inside of these where they are not only amply protected but are easily accessible. This gangway, with its cables, is shown in Fig. 11. The 2300-v cables are further protected by a box cover which can barely be seen where the cables turn down to enter the cargo-pump motor room in the foreground.

Steering Gear

The steering gear proper is of the conventional electric-hydraulic type with horizontal opposed rams connected to a tiller on the rudder stock and with power furnished by duplicate Hele-Shaw type oil pumps driven by 20-hp, 875-rpm induction motors. The steering control, however, is somewhat unusual in that there is no mechanical shaft or hydraulic telemotor connection between the bridge and steering-gear compartment, either for normal or emergency steering. The system used consists of two entirely independent electrical controls using Selsyn motors, and separate circuits from bridge to steering gear. One of these controls is arranged for either hand or automatic steering through the usual gyropilot and is a full follow-up self-synchronizing system. The alternate control is a nonfollow-up type operated by means of a small lever on the right-hand side of the steering stand. Any one of these three steering methods, automatic, hand, or emergency nonfollow-up is instantly available to the helmsman without moving from his position by simply shifting the three-point selection lever on the left side of the steering stand. At the afterend of the ship, for operating the steering-gear hydraulic valve, are

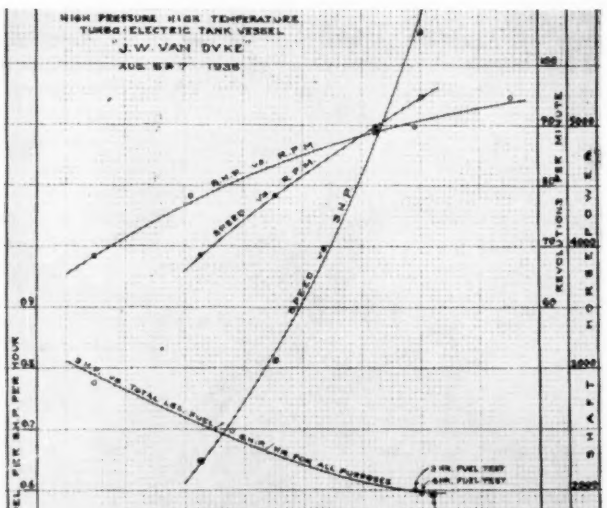


FIG. 12 PERFORMANCE CURVES FROM TESTS OF THE S.S. *J. W. Van Dyke*

two Sperry power units, one connected to the automatic and normal hand-steering control just described, and the other to the alternate nonfollow-up control. Each of these power units is connected on its output side to the hydraulic valve mechanism through a magnetic clutch operated by the selector lever on the bridge steering stand so that the unit not in use is disconnected.

In addition to the described steering controls from the bridge, the ship also has two emergency steering stations aft, one on the aft end of the poop deck house and the other on the poop deck close to the stern. The former is a mechanical rotating shaft control directly to the hydraulic valve on the steering gear and the latter is the usual hand gear operating directly on the rudder stock through spur and worm gearing and a quadrant.

Trial Results

General. On August 6 and 7, 1938, after having been in service about 5 months, the ship was put through a series of speed, horsepower, and fuel-consumption tests over a measured mile at the Delaware breakwater and off the Delaware capes. The results of the horsepower and speed tests are shown by the heavy curve in Fig. 1 and the performance curves in Fig. 12. Fig. 1 also shows the model-basin results. It can be seen that the ship-performance curve comes within the usual allowance of 12 to 15 per cent added to effective horsepower as obtained on the model, to allow for such effects as sea conditions. It should also be noted in this connection that the ship was fitted with bilge keels while the model was tested without them and the depth of water over the trial course was not as great as is generally considered desirable if speed is not to be affected. The empirical formula

$$\frac{10 \times \text{draft in ft} \times \text{speed in knots}}{\sqrt{(\text{ship length in ft})}}$$

gives a close approximation to the minimum desirable depth and, in the present case, the actual depth was about 60 to 70 per cent of the 180 ft given by this formula.

Minimum turning-radius tests were made both to port and to starboard at full speed and showed a diameter of turning circle of 0.47 nautical mile to port and 0.45 mile to starboard; the time to turn through 180 deg was 3.6 min in each case.

A test of the steering gear showed that, with the ship at full speed, in full-loaded condition, the helm could be put from hard-over to hard-over in 17.8 sec with a maximum load on one of the two 20-hp steering-gear motors of not much more than half its rated capacity.

With the ship in full-loaded condition and going full speed ahead, a full-speed-astern bell stopped her headway in 5.08 min in a distance of 0.40 nautical mile.

A similar test, in which the propulsion motor was stopped instead of being reversed, gave results in which the ship drifted for approximately 1 mile in 14.05 min before stopping.

Both of the last two tests were made against a moderate wind with a velocity of approximately 13 knots per hour and against a tidal current of possibly 1 knot per hr.

Power Plant. It was of course impractical to arrange for the measurement of every quantity and temperature throughout the entire steam and water cycles as indicated on the heat-balance diagram shown in Fig. 4, but boiler meters were provided, as was a recording and integrating flowmeter for the steam to the main turbine. Thermometers and pressure gages were installed at all important points so that it has been possible to determine fairly closely and without any unwarranted assumptions how close the actual results came to theoretical expectations.

Boilers. It was possible to determine the output and efficiency of the boilers quite closely. Fig. 13 shows the expected performance from which it will be noted that the boilers were designed to give an efficiency of 85.5 per cent at the rated load of 22,500 lb

per hr actual output at a pressure of 625 lb per sq in. and a temperature of 835 F; this corresponds to an equivalent evaporation of 26,260 lb per hr. During the trials, two fuel-test runs were made, one of 4 hr and one of 3 hr duration. During the first of these runs, the boilers were operated at a pressure of 620 lb per sq in. and a temperature of 855 F, producing 21,810 lb per hr each, or an equivalent evaporation of 25,100 lb per hr. The feed temperature was 351 F. The stack temperature was 324 F, CO_2 was 14.7 per cent, CO was 0, and oxygen 1.8 per cent. These results gave an efficiency of 86.6 per cent, a stack loss of 11.3 per cent and an unaccounted-for loss of 2.1 per cent.

The second run was at almost exactly the same rate and conditions, and showed an efficiency of 86.9 per cent with stack losses of 11.3 per cent and unaccounted-for losses of 1.8 per cent.

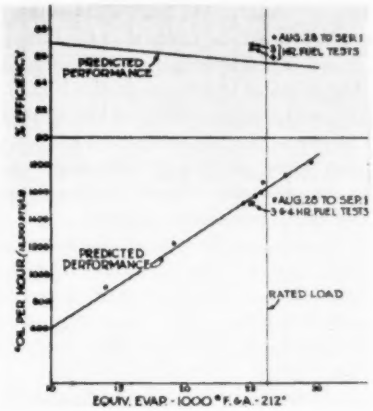


FIG. 13 BOILER-PERFORMANCE CURVES

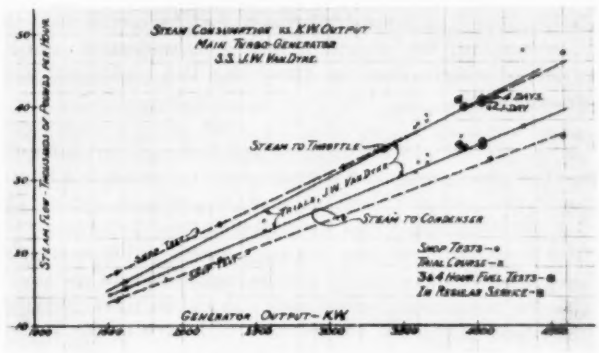


FIG. 14 STEAM CONSUMPTION VS. KILOWATT OUTPUT OF THE MAIN TURBOGENERATOR

Since these tests were made, the results under normal operating conditions have shown that these efficiencies are easily maintainable. Data taken from the log of the homeward (loaded) leg of voyage No. 12 show that an efficiency of 87.3 per cent was maintained for the four consecutive full days of this run. We believe that such efficiencies can only be obtained and maintained by the almost perfect combustion conditions made possible by automatic combustion control.

Turbogenerator. The performance characteristics of the main generator are shown by the curves in Fig. 14. The two pairs of curves show a comparison between the shop tests of the turbine and the trial results on the S.S. *J. W. Van Dyke*. In studying these curves, it should be borne in mind that the figures for the trial runs were taken over periods varying from 4 to 10 min and that the amount to the condenser was calculated from the amounts of stage bleed which in turn were calculated from the rise in feedwater temperature. However, there were sufficient data taken during the 3- and 4-hr fuel tests to permit determination of the generator performance during these runs and, in addition, points obtained by averaging the four full days of voyage No. 12 of the *Van Dyke*, as mentioned previously, are also shown. In view of the close agreement of all of these points, it is believed that the curves, as shown, are a fair representation of the turbine performance.

Table 4 gives a summary of expected results compared to those actually obtained. There are two columns under each heading, the first giving the pounds per hour, the second the amount of heat added or removed at each point in thousands of Btu per hr. Below these columns is also shown the shaft horsepower (by torsionmeter), the pounds of total steam generated for all purposes per hour per shaft horsepower, and the pounds of 19,000-Btu fuel oil used per shaft horsepower.

CONCLUSIONS

For the purpose of our conclusions, we are selecting from Table 4 the figures for four days of voyage No. 12, as we believe these to be the most accurate data. From these it will be noted that the consumptiom of fuel per shaft horsepower for propulsion is 0.511 lb and this figure is to be compared to 0.510 lb predicted; however, it can be seen from Table 4 that consumptions as low as 0.502 have been obtained. We, therefore, think it fair to say, in consideration of all the data, that the power plant in its entirety has met the economy of the design.

It will be particularly noted that in arriving at the foregoing figures from Table 4 the portion of the fuel required for auxiliaries has not been included. The reason for this is that, generally speaking, all quoted figures for engines and other types of prime movers are on a shaft-horsepower basis, due to the fact that no two owners have the same requirements for auxiliaries for a given type of vessel. Therefore, it would be misleading to compare unless we can compare like things, and so the foregoing fuel figures are based on the propulsion requirement only.

It is believed that the foregoing will reveal power-plant efficiencies not previously approached in this country in marine installations, and that when one considers the price differential between the fuel burned in these vessels and the fuel required by Diesel engines, there should no longer be controversy relative to the most efficient type of power for this size of power plant. It is altogether fitting that the results of the engineering disclosed in this paper be presented before this Society, because it is only an adaptation of well-known principles, designs, and developments previously published through the A.S.M.E. that have been applied here. It is the earnest wish of the author that more study be given and more interest shown in ship propulsion by A.S.M.E. members in the future, so that the latest developments in power-plant equipment may be available to the marine field.

ACKNOWLEDGMENTS

It is only fitting, at this time, that the author express his appreciation and admiration for Mr. J. W. Van Dyke, chairman of the board of The Atlantic Refining Company, for his extreme foresight in fathering this forward development, for his countless contributions to the engineering of these vessels, and for his continued patience and encouragement given the author; to Mr. R. C. Tuttle, manager of transportation of The Atlantic Refining Company, for his tolerance and cooperation; to the members of the engineering and construction department of The Atlantic Refining Company for their unfailing cooperation, loyalty, and devotion for the success of these ships; and to the suppliers of equipment, as a group, who have never tired of trying to make their equipment the best possible.

Discussion

E. G. Bailey.[2] The excellent performance of the tanker *J. W. Van Dyke* in regular service is a tribute to the farseeing and detailed research Mr. Goldsmith gave to every item in the power equipment of this vessel before the keel was laid.

In discussing the automatic control, the statement is made of the trials, "At low loads the boilers are on hand operation, hence the spread between the air-flow and steam-flow records." This was true at the time the steam-flow air-flow charts were made, but it has been standard practice for the last few months to continue on the automatic control when maneuvering in pilot waters, as well as in the open sea. The automatic control properly adjusts the fuel oil and air flow to meet the demand for steam. Burners are cut in and out for the major steps for large variations in steam demand, with the automatic control properly adjusting fuel and air within the oil-pressure range, which gives efficient atomization.

Mr. Goldsmith mentions the fact that one reason he selected the single vertical-pass gas-flow-type boiler was the entire absence of gas baffles which this design assures. Of equal importance from a maintenance and external corrosion point of view is the entire absence of shelves or ledges for soot to lodge. Any such ledges or other soot pockets are always potential fire or corrosion hazards with modern boiler-fuel oils with their frequently high sulphur content and minute particles of carbon which are frequently impossible to burn completely, even with relatively hot furnaces.

The use of air heaters as the medium for reducing the temperature of the gases well below that of the boiler tubes, not only assures desired high efficiencies with a minimum of high-pressure parts, but also is an important factor in providing optimum combustion conditions at all rates of operation.

The point is brought out in the paper that, with the type of boiler selected, prompt plugging or renewal of boiler tubes may be made without loss of ship's time waiting for a boiler to cool down enough for the drums to be entered. This is a particularly important feature in ships fitted with only two boilers, where the outage of one for a day or so may seriously affect the ship's ability to maintain her schedule.

Practical experience together with extensive tests and research work on both navy and merchant-marine installations have proved that, when burning bunker fuel oil, excessively low furnace-release rates have no commercial or thermal advantages. Mr. Goldsmith is to be congratulated in not handicapping his ship by utilizing excessively large furnaces under the boilers in the *J. W. Van Dyke*. The consistently excellent combustion results obtained in this installation serve to emphasize the commercial advantage of not wasting valuable floor and cubic space in the ship with unnecessarily large furnaces.

David Dasso.[3] The writer wishes to comment on the statement in the fifth paragraph, namely, "In other words, for horsepowers to 3000, the Diesel engine is an undeniably economical prime mover, but above that size, doubtful, especially in view of the advent of the high-pressure, high-temperature steam turbine." It is my belief the author is stating there the truth, the whole truth, and nothing but the truth, but only as applied to the steamship *J. W. Van Dyke*, operating on comparatively short runs between points in which cheap residual oils of the type called bunker fuels are available.

Beyond these restrictions, the author's statement is hardly in accordance with present-day experience. A large percentage of ship construction in all foreign countries is Diesel propelled. Many large navigation companies which previously bought nothing but steamers are buying more and more Diesel ships. There must be a reason why large Diesel ships in horsepowers ranging between 3000 and 30,000 are being ordered in increasing numbers. Are the purchasers making a mistake, according to Mr. Goldsmith's statement? They are not, and the reasons are fairly simple to elucidate.

The choice of the best type of propelling power depends on many factors. It is generally admitted today that the fuel consumption of an up-to-date Diesel plant does not exceed 0.38 lb per shp per hr, against a minimum under the best conditions of 0.5 lb per shp per hr for steam. While it is true that in some ports the steam vessel may avail itself of very cheap bunker fuel prices, this does not hold true for all ports of the world. Especially in the Far East, it will be found that the differential between the price of Diesel oil and the price of bunker oil is much less than it is on the Atlantic Coast and in the United States. The saving in fuel consumption in a vessel of the up-to-date type with a speed well in excess of that mentioned in this paper is very considerable and, taken on a run of 6000 miles or more gives to the motorship an additional freight-carrying capacity which in many cases makes this advantage decisive.

The best proof of this is the case of the large Dutch shipping companies which are building simultaneously steamships and Diesel ships, the former being allocated for medium runs between ports with favorable bunker-oil prices, and the latter for long runs between ports in which the differential in price between the two fuels is not great.

The foregoing conclusions have been affirmed in this country. Several years ago Robert L. Hague of the Standard Shipping Company, presented a paper[4] before The Society of Naval Architects and Marine Engineers discussing the subject in a similar manner. His conclusions were that, for service along the Atlantic Coast, the steamship was more economical, and for service in foreign waters the Diesel ship had been found more advantageous.

R. H. Davis.[5] In the design of any tanker, determining the type and arrangement of the cargo pumping system is a matter directly involved with the selection of the propulsive drive. The ideal condition from a fire- and explosion-hazard standpoint is a pump room, divorced from the engine room and located amidship, in which there are no wiring, electric current, or steam. With the pump room located amidship and no power available this immediately limits the selection of pumps to the vertical type, with the driving elements located on the upper deck.

The so-called deep-well type vertical centrifugal pump lends itself admirably to such an application. While a unit of such type had never been used previously for cargo pumping in a tanker of this size, it was selected in this case because of its many advantages.

The units are driven by 300-hp vertical explosion-proof motors, located on the upper deck, with the entire thrust load of the unit carried in the upper motor bearing, thus permitting complete freedom to the long length of intermediate shafting, cover pipe, and drop pipe. The pump bowls themselves are located at the bottom of the pump room in the so-called suction wells or boxes. The outside drop pipe is connected to these suction boxes through a flexible expansion joint, which also in-

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[3] Vice-president, American Locomotive Company, Diesel-Engine Division, New York, N. Y. Mem. A.S.M.E.

[4] "Performance of Standard Oil Tankers," by Robert L. Hague, Transactions of The Society of Naval Architects and Marine Engineers, New York, N. Y., vol. 40, 1932, pp. 357–368.

[5] Assistant manager, Marine Division, Worthington Pump and Machinery Corporation, Harrison, N. J.

corporates stops to prevent too great a movement. The drive shaft operates in a cover pipe which is filled with oil, supplied by an automatic lubricator above. The cargo flows between the cover pipe and the outside drop pipe and is discharged at the deck above. The lower portion of the drop pipe and cover pipe is made in the form of a split section so that the pump may be inspected or dismantled, if necessary, without disturbing the intermediate shafting above.

The important element in the ability of a ship to discharge its cargo rapidly is the method of stripping the tanks. In the case of the *J. W. Van Dyke*, no separate stripping pumps are used and yet this ship, together with its sister ship, the S.S. *Robert H. Colley* has established a record that never before has even been approached in the unloading time of a vessel of this size. The main cargo pumps are used as stripping pumps with the aid of vacuum priming pumps, which are connected to the suction boxes and so arranged that the cargo pumps are always completely primed regardless of the amount of air pulled through from the tanks. It can therefore be seen that the rate of stripping the tanks is greatly increased and the time required for stripping becomes a small proportion of the entire pumping period. Not only is the stripping time greatly reduced but this method produces amazingly dry tanks.

The characteristics of these main cargo pumps are such that the capacity increases with slight reductions in the head, in much the same manner as the conventional centrifugal pump with a relatively flat capacity-head curve, except that this capacity-head curve rises rapidly as it approaches shutoff. The shape of the brake-horsepower curve is such that it increases in practically a straight line until it reaches a maximum just before the design condition and then falls off rapidly at the outer end of the curve, thus providing an ideal horsepower characteristic for a unit which is called upon, not only to pump cargo, but to handle salt water during ballasting periods at relatively low heads. The units are powered to provide ample margin for any peak horsepowers when these units are used for ballasting when handling salt water.

The pump room is also equipped with a small 160-gpm unit of the same type as the main cargo pumps for bilge service, driven by a 3-hp motor on the deck above.

In connection with the feed system, it is interesting to note that the deaerating marine feedwater heater installed has made it possible to operate this vessel under all sea conditions with practically zero oxygen content at the feedwater as obtained by tests. This unit not only acts as a feedwater heater and deaerator, but provides approximately 2500 gal storage capacity.

E. H. Markley.[6] The writer's comments will be confined to the condensing plant which has some unusual features and which has had an excellent performance record. When the power plant of the S.S. *Van Dyke* was considered, the owners did not issue definite or detailed specifications for the condensing plant. The condenser manufacturer was requested to submit his recommendations on the best possible plant to meet the operating conditions, with the understanding that the final design would be determined after owner and condenser builder had studied all of the conditions; efficiency, reliability, space, weight, details of ship design, etc. The idea was to install the plant best suited to the operating requirements and the machinery to be served by the condenser. Results have proved the effectiveness and soundness of this cooperation.

As Mr. Goldsmith mentioned, the main condenser is of the divided type, having a total surface of 4475 sq ft, rated at a capacity of 35,900 lb at 28.44 in. vacuum, when circulated with 6775 gpm of cooling water at an inlet temperature of 76.8 F. It will be noted that these proportions take advantage of the benefits secured from the patented deaerator in that a 7.5 fps water velocity through the tubes, with its attendant economies, could be selected without fear of abnormal tube corrosion or erosion. So far there has been no sign of tube corrosion or erosion on the *J. W. Van Dyke*, which certainly justifies the installation of the deaerator. Another result of the use of the deaerator is that all condenser tubes are filled with solid water, thus raising the over-all heat-transferring capacity of the condenser.

It should be mentioned that the condenser tubes incorporate inlets belled into a hydraulic radius; water boxes are streamlined and proportioned for direct flow at exceedingly low velocities.

Another interesting point, particularly as it affects spare parts, is that the condenser-tube diameter and length are the same for the main and auxiliary condensers. The main and auxiliary air ejectors employ the same size elements.

Recognizing that a frequent and preventable cause of poor vacuum is air leakage into the condenser, the owner took precautions to prevent such leakage. The condenser is bolted directly to the turbine exhaust flange without an expansion joint, thus eliminating one vacuum joint. The expansion strains are taken up by providing a spring mounting for the condenser. Air ejectors are located on the condensers adjacent to the air offtake connections, thus eliminating the customary long air-suction pipe with its multiplicity of vacuum joints.

The benefits, derived from the careful engineering consideration given the problems surrounding the condenser installation, are evident in the following performance data which are average for one of the latest voyages: Steam load, lb per hr, 34,600; vacuum, in. Hg, referred to 30-in. barometer, 28.6; sea temperature, 76 F.

J. J. Nelis.[7] While this ship is an example of many progressive hull and engineering developments, there are some features of its power plant that differ from many of our newer ships. It has the highest pressure and temperature in the American merchant marine. The United States Navy is using approximately the same pressures and temperatures in some of its new ships. As regards pressure and temperature, the marine man is naturally conservative, as his equipment is confined in a small space, where failure of the pressure parts will endanger human life. That is one of the reasons why he has been slow to install high pressures and temperatures.

The straight-tube-type boiler, installed on this ship, has been in use for a great many years, both ashore and afloat. In the new ships now being built for the American merchant marine and for the Navy, more than 80 per cent of all the boilers installed and on order are of the bent-tube type as this type fits into the limited space conditions aboard ship to better advantage than the straight-tube type. The final-stage feedwater heater of the countercurrent flow type, commonly called an economizer for want of a better description, has also been installed on over 80 per cent of the new merchant-marine and naval vessels. The arrangement of the boiler room in this ship, requiring several frame spaces which could be used for additional cargo space, is different from other recent American tankers, where the boilers are placed aft of the turbines on a flat, thus saving the fore-and-aft space required for a separate boiler room.

Bent-tube boilers, coupled with economizer installations recently installed in new merchant and naval vessels, have shown stack temperatures averaging close to 280 F. These average

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[7] Vice-president, Marine Department, Foster Wheeler Corporation, New York, N. Y. Mem. A.S.M.E.

temperatures are lower than on the few new ships fitted with air heaters. There is an increase of nearly 2 per cent in the boiler efficiency, due to the lower stack temperatures with economizer-fitted boilers.

Modern ships are fitted with closed feed cycles which give an air-free feed. With rolled-tube condensers and air-free feedwater, few, if any, boiler repairs are required in the newer vessels. The bent-tube boiler eliminates the necessity for long repair periods, as it has only two gasketed joints for the manhole plates, as against the many small gasketed joints of the straight-tube-type boiler.

The boiler shown gives a liberation rate of 66,000 Btu per cu ft of furnace volume at full power. It would be interesting to learn what the Btu release is when the ship is washing holds at sea. In many modern tankers of from 3000 to 4000 shp, when cleaning the holds at sea, the steam required for cleaning tanks is equal to that for the power plant. Consequently, the Btu release in the furnace is then more than double the full-power conditions, due to a slight drop in boiler efficiency at the higher ratings.

With a straight-tube boiler and convection superheater to obtain superheat control requires considerable extra control equipment and a desuperheater in the drum. Marine drums are necessarily small. It is desirable to provide adequate superheat control without a drum desuperheater. A straight-line temperature curve may be obtained with the by-pass as shown in the paper, but a control of superheat at any temperature above saturation can be obtained without the complications of the desuperheater and the extra piping shown.

It is desirable to bring a marine turbine into service with a minimum temperature rise and to increase the temperature as the turbine is slowly warmed up. This requires complete superheat control which is now being used on other tankers.

In addition to complete superheat control, combustion control is now becoming standard on American ships. The new wide range oil burners, coming into use in conjunction with combustion control, give a steady efficiency without manual oil-burner operation both at sea, when maneuvering, coming in and leaving port, and when lying at the dock in port. Combustion control at sea is not so important on a ship as in a land plant, as a ship is essentially a base-load plant when at sea, the variation in steam demand being minor changes in some of the small auxiliaries. Many ships at sea are operating just as efficiently without combustion control as with it on account of their steady base-load conditions.

In connection with combustion, it is not necessary to design furnaces for high temperature to burn the fuel, particularly in marine work where the average fuel is oil having about 12 per cent hydrogen. Marine furnaces are small and instead of being heated by additional heat from the air heaters, they require cooling. In this case, the air is heated and additional heating surface is installed in the sides of the furnaces for cooling which is hardly logical.

There may be maneuvering and other advantages in an electric-drive arrangement. An interesting comparison of the heat-balance data, Table 4, of the paper can be had with that of a paper entitled "Heat Balance Calculations for Marine Steam Plants," presented at the 1938 annual meeting of The Society of Naval Architects and Marine Engineers, by A. S. Thaeler and D. C. MacMillan, where for approximately the same steam pressure and temperature, practically an equal fuel rate per shaft horsepower is shown with a mechanical gear drive and the same general heat balance.

The condensing system has several unique features. The air-removal chamber is new and may do the work expected of it. A well-designed marine condenser, having 70–30 per cent cupronickel tubes, properly designed water boxes, inlet and outlet piping, will operate without serious air corrosion for many years, if not for the life of the ship. The division of the condenser is an idea that has been tried many times and, for ships operating in tropic waters, has considerable value. It is not installed in many ships and has not yet become standard in marine practice.

In the past, marine operating engineers have seldom been technically trained men, but more of the mechanic class. Today with living conditions aboard ship greatly improved and with high-pressure, high-temperature, high-efficiency plants demanding expert attention, engineers of ability, many of them college-trained, have been attracted to the merchant-marine service. Eventually we may expect the staffs of our ships to be composed of high-grade engineers. Time only will tell which of the new and progressive ideas tried out in this ship are important enough to become standard in the future. There are other tankers recently built incorporating progressive ideas and for the pressures and temperatures used give a comparable fuel consumption per shaft horsepower. Comparisons of this with other new developments will in the end lead to better marine power plants.

MASON S. NOYES.[8] The extent of the engineering information in this paper is considerable and of great interest and value to those who are concerned with similar design work on government vessels.

The most notable items in the design are: (*a*) The slight step-up in average sea speed over present United States practice; (*b*) the selection of electrical speed reduction in place of the customary mechanical method between turbine and propeller; (*c*) the very short discharge period permitted for the ship's cargo; (*d*) the use of motor-driven cargo pumps; and (*e*) the employment of main steam conditions which, although no longer unusual for power plants ashore, are quite beyond the ordinary in present-day merchant vessels. Only in the merchant fleet of the progressive Germans can be found equal or higher steam conditions utilized for new ships.

Nevertheless, the boilers reflect conservatism, as well as the dictates of maintenance cost and time indicated by the author. The heat release, presently available in reliable marine boilers under the same steaming conditions, is several times larger than that quoted for the boilers on the *J. W. Van Dyke*. The $^{3}/_{4}$-in. (or more) thickness of the main steam piping seems definitely excessive and provides an element of danger. Such stiff piping imposes higher expansion thrusts on boiler and turbine connections, unless a more extensive piping arrangement is used to increase flexibility to the proper degree. The resulting low stresses reduce the creep rate below the neutral (or flat) value so that actually a decreasing creep rate may exist. Information is requested as to whether any "cold-spring" or pull-up was used in installing the main steam piping. What type of gasket and what gasket material were used? What is the actual pressure drop between superheater outlet and the main turbine?

The use of 2300-v cargo-pump motors is noted, together with the means provided to replace the usual stripping pumps and piping and to protect the 2300-v cables along the weather-deck gangway. Were any other special precautions taken to protect such cables from the effects of free water?

In his conclusion, the author mentions the very low fuel consumption obtained. This is due to the excellence of the heat-balance arrangement in rejecting the minimum amount of heat to the sea and to the high efficiency of the machinery items selected. Unfortunately, Table 5 does not give complete data, so that the exact extent of motor-driven units is not revealed. On the other hand, the effects of an advanced steam pressure and temperature

[8] Engineer, Design Division, Bureau of Engineering, Navy Department, Washington, D. C. Mem. A.S.M.E.

are reflected in the low machinery weights and spaces and the correspondingly high deadweight displacement ratio.

H. L. Seward.[9] With the experience of the author and his organization in high-temperature, high-pressure apparatus ashore, it was no extraordinary effort for them to decide to use the steam conditions, standard apparatus, and modern designs, which make these tankers so important and commendable at this time. Economy is important but dependability is more important in these vehicles, which are but a part of the machinery and equipment necessary to carry on the principal business of the company.

Those of us, whose sea experience goes back to the days when almost all of the auxiliary pumps were beam-connected onto the main engine, recall what an event it was when we got under way and started the main engine with its attached boiler feed pumps, air pump, and bilge pumps. On these tankers with a central electric-generating plant, the application of power to the propeller motor is but an incident in operation no more to be considered than the application of power to the cargo pumps or other ship's services. Aboard one of these vessels for a short trip, the writer noticed many details of design which deserve favorable comment, although not all were mentioned in the paper: Accessibility of machinery for inspection and repair; quarters for officers and men; location and type of electric cables; location of cargo-pump motors and controls; release of entrained air in circulating water; position of all motor starters on a high and protected level in the engine room; in addition to the well-known Btu savers, described in the paper, which if misapplied, may save some thermal units at the expense of over-all cost. With three fourths of the total weight of the loaded vessel as pay load, the marine industry will appreciate the achievement made in design and should watch the operating results of the next few years with great interest.

W. F. Schultz.[10] The marine industry is particularly interested in this paper, since the author not only has described in much detail the machinery installation but has also presented the highly important propulsive-design information which, in the case of privately owned ships, is somewhat rare. It is noted that, in comparing the propulsive performance predicted by the Hamburg tank, and that by the Washington tank, there is considerable divergence, although the predicted power-revolution curves are practically coincident for both tanks, and with the actual ship performance on trial. It is noted that the actual ship shaft horsepower is about 16 per cent higher than the Washington tank prediction at the designed speed of 13¼ knots. This is not unusual since the trial course off the Delaware capes is not as suitable for comparing model test data with actual ship performance as is the Rockland course. It is probable that sea and wind conditions together with depth of water and the effects of current have increased the power requirements above ideal trial conditions.

In discussing the power plant, however, there is considerable difference of opinion as to the most economical installation possible, and it is in this connection that the writer wishes to discuss the paper.

The author is to be commended for his selection of steam conditions. It is the first instance in which an American-built merchant ship has been fitted with steam pressures and temperatures above 400 lb and 750 F. These conditions are even somewhat unusual in shore plants in the same power range, although there are several. To the progressive engineer, however, the attractive features of higher pressures and temperatures make such adaptation worth while. It may be interesting to consider the economies possible through their use.

Table 6 gives a brief summary of the pertinent features of four hypothetical installations, each having different steam conditions; and each progressively higher. In this case the installation is for 6000 shp, employing multistage feed heating, all-electric auxiliaries, double-reduction-geared turbines, and boilers fitted with air heaters and economizers. An attached generator, driven from one of the low-speed pinions, furnishes the electric power for sea operation, similar to the equipment fitted in fourteen ships built by the company with which the writer is associated.

TABLE 6 SUMMARY OF STEAM CONDITIONS, TYPICAL PLANTS

Installation	A	B	C	D
Pressure, superheater outlet lb per sq in., gage	460	600	900	1200
Temperature F, superheater outlet	750	850	900	950
Feed temperature F, economizer inlet	318	332	365	400
Feed heaters, number of stages	3	3	3	4
Steam flow to condenser, ratio	1.0	0.91	0.83	0.80
Boiler heat output, ratio	1.0	0.97	0.91	0.88
Fuel, lb per shp per hr, all purposes	0.588	0.568	0.534	0.517
Fuel rate, ratio	1.0	0.967	0.908	0.88

It will be noted that the machinery installation on the *J. W. Van Dyke* is in some respects similar to installation B (Table 6). There are, however, differences that make comparison with this installation somewhat difficult. Therefore, the writer has prepared Table 7, showing the differences between an installation similar to B and that of the *Van Dyke*. Both plants have the same steam conditions and will deliver the same shaft horsepower, but for the proposed installation the writer has made certain substitutions.

The main propulsion machinery is of the cross-compound turbine double-reduction-geared type, without attached generator. The boiler plant consists of two 2-drum units fitted with air heaters and economizers. The feed system is of the three-stage type, with surface-type heaters and deaerating-type surge tank. The main feed pump is electric-motor-driven, as are the feed-heater drain and booster pumps. The electrical load requirements are provided for in two 350-kw, 230-v, d-c generators, one of which will carry the normal sea load at about 50 per cent rating. The two units would be sufficient for cargo pumping requirements in port. A contaminated steam system is included similar in design and purpose to that of the *Van Dyke*.

Based on the information given in the paper, the estimated fuel rate for 5000 shp, based on 19,000 Btu oil, is 0.596 lb per ship per hr for all purposes. Service performance has proved the author's estimates to be very accurate, speaking well for his own ability, and for the reliability of information from the suppliers of his machinery.

We are accustomed to base our performance on 18,500 Btu oil, which is a fair average value for the fuel supplied today. Referred to this value the fuel rate of the *Van Dyke* is about 0.612 lb per shp, say 0.61 lb per shp per hr for average service conditions.

Comparing this performance with an installation similar to B in Table 6, we find that a gain in economy of approximately 7.5 per cent is possible. Analyzing this gain, it is found that the reasons are generally as set forth in Table 7.

In explaining the apparent reasons for the increased economy, noted in Table 7, the writer wishes to discuss the various principal items of machinery in their order of importance.

The cross-compound turbine, as furnished by the principal machinery builders, either impulse or combined impulse-reaction

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[10] Engine Department, Federal Shipbuilding and Dry Dock Company, New York, N. Y.

TABLE 7 COMPARISON OF OPERATING EFFICIENCIES

Installation	*J. W. Van Dyke* A	Proposed B	Increased or decreased[a]
Turbine efficiency, per cent	75.6[b]	77.7	+2.7
Transmission efficiency, per cent	93.4	97.5	+4.3
Auxiliary efficiency, per cent	...	...	−0.5[c]
Boiler efficiency, per cent	87.0	88.0	+1.1
Total increase or decrease in economy			7.6

[a] Increase or decrease referred to A.
[b] Estimated from test data given in paper.
[c] Effect of operating ship auxiliaries at sea from main propulsion unit.

type lends itself particularly well to obtaining relatively higher efficiencies than the single-cylinder unit such as used on the *J. W. Van Dyke*. Higher speeds are possible, smaller shafts and smaller-diameter wheels can be used, thus reducing internal friction and bearing losses, and generally lighter units are possible. In the paper, the weight of the main propulsion unit, complete, is 211,000 lb. Recent 6000-shp turbine proposals for cross-compound 18-stage double-reduction-geared units varied from 174,000 to 180,000 lb, a saving in weight of more than 30,000 lb. It is obvious of course, that this difference in weight would indicate a substantial reduction in cost. This difference is more pronounced based on the evaluated cost, in consideration of efficiency due to higher speeds and greater number of stages. It should be noted, however, that the reduction gears proposed were of the modern single-case type, and not the extremely heavy three-case type of five years ago.

A modern double-reduction gear of the single-case type, with two bearing pinions, and tooth pressures of the order of 60 lb per in. of face per in. of pinion diameter will give a mechanical efficiency of not less than 97.25 per cent and in most cases 97.5 per cent or better, as compared with a combined efficiency of from 93 to 93.5 per cent for generators and propulsion motor. In addition, the space required for the geared-turbine unit is less than that of the electric drive for the same power.

In discussing boilers, the writer will not attempt to criticize the type used. It is generally a matter of opinion as to the type best suited for the purpose. However, it is believed that an economizer in combination with the air heater may have been used to advantage and the efficiency raised to 88 per cent, at a relatively low increase in cost, and with no appreciable increase in weight.

It is the writer's opinion that wider use of the completely water-cooled furnace, as is rather common practice in shore power plants, will prove of considerable economy to the marine field. The elimination of brickwork, especially in furnaces designed for high rates of heat release, will reduce furnace maintenance to a minimum.

The author is to be congratulated on the method of temperature control applied in this installation. It is believed to be superior to other forms, and lends itself particularly well to shipboard use. We have contemplated a similar arrangement for use in high-temperature installations for desuperheating steam to the astern turbines during maneuvering. We have introduced feedwater into the main steam line for this purpose in the past, but this method is relatively crude compared with the excellent arrangement used on the *Van Dyke*. By this method, the one principal disadvantage of the geared-turbine installation may be overcome without great difficulty.

The application of automatic combustion control is of great interest. The writer fully concurs with the author that the use of higher pressures and temperatures will further increase the use of this equipment beyond its growing application today. It is now possible to obtain from at least two sources in this country, a wide-range oil burner that will greatly augment the value of the controls for all-purpose operation. A recent installation in a tanker provides us with a very instructive case in point; the ship was able to get under way, maneuver, drop anchor, and perform all the necessary tasks including docking without use of towboats, with all burners in use and under full automatic control.

Concerning the type of current used by auxiliary drive, the author had available the main propulsion generator for supplying power at sea and, with the selection of this type of drive, the best arrangement would be the simplest, namely, alternating current at a standard voltage. However, with geared turbine drive it is believed that direct current is more suitable since the problem of speed regulation is relatively simple. It is questionable in my mind, when the cost of the controls necessary to provide speed regulation for fuel and feed pumps, forced-draft fans, and cargo pumps is included with motor costs, whether there is any appreciable difference in cost. The space required by a-c controls is considerable and would probably minimize the gain in space inherent with smaller sized alternating current motors.

Among the interesting features of this ship are the cargo-pumping arrangements. Excellent judgment has been used in adopting explosion-proof motors, since tanker practice requires the ultimate in safety. This ship is engaged in the gasoline trade and handles great quantities of this fluid at high pumping rates, requiring extreme care. All the demands of this service seem to have been amply met. It would be interesting to obtain the author's reaction to having this same type of pump installed in the pump room below decks, and providing forced ventilation from fans on the deck, with elevated intakes.

Author's Closure

In his comments Mr. Bailey has referred to the combustion-control features of the *Van Dyke*. The record of the use of the automatic combustion control on this vessel followed exactly the predictions made for it. At first it was only used when under full or nearly full load and not in pilot waters. The next step was to leave the fuel under automatic control at all times and to regulate the air supply by hand at low loads or when maneuvering. As stated by Mr. Bailey, the automatic control is now used at all times but, at low loads or when maneuvering, the air flow is set up to avoid the possibility of smoking when the load is suddenly increased.

In reply to Mr. Dasso, the author's ideas as to the relative merits of turbine versus Diesel drive are obviously based on the conditions which he has to meet. He agrees that for other conditions the selection might have been different. It should be noted, however, that, using Mr. Dasso's figures, the ratio of fuel consumptions of turbine versus Diesel machinery is 1.32 to 1, whereas the ratio of fuel cost of Diesel to bunker C oil at present varies from 1.6 to 2.07 to 1, which is a fairly wide margin in favor of steam. It should be borne in mind also that the European preference for the Diesel is to some extent due to the fact that most of the shops and shipyards abroad are unable to build modern high-pressure, high-temperature turbines. Also it is an easier step for an engineer who has been trained on a reciprocating steam engine to change over to the Diesel type.

Mr. Nelis refers to the wide adoption of the bent-tube boiler in new merchant and naval vessels in the United States. In the author's case the lighter header-type, straight-tube boilers enable more cargo to be carried than if multidrum, bent-tube-type boilers were installed, regardless of space arrangement. This is due to the fact that, while the *Van Dyke* has a tank capacity for 18,600 tons of 60 deg Bé gasoline, the cargo allowed is only 17,125 tons with a deadweight tonnage of 18,105.

Clean and uncorroded boilers are admittedly possible with proper feed preparation but this cannot always be assured, as witnessed by the recent difficulties with the boilers of a modern tanker, which might have been avoided if the tubes could have been inspected. No difficulty has been experienced in keeping

handhole covers tight. It is the author's belief that there is but slight difference in the number of gasketed joints between a header-type boiler with an air heater and a bent-tube boiler with an economizer.

The relative boiler efficiencies obtainable with an economizer as compared to an air heater depend entirely upon the size of the air heater. On a number of recent ships, stack temperatures as low as 280 F have been obtained with air heaters. During the fuel-consumption trials, the stack temperature on the *Van Dyke* averaged 323 F. In this connection it should be noted that a gain in boiler efficiency may be made at the cost of a greater loss in over-all efficiency, as is witnessed by the figures given in the paper by A. S. Thaeler and D. C. MacMillan, to which Mr. Nelis refers. It will be noted that columns A and C in Fig. 5 of that paper compare almost identical setups, except that the boiler of column A has an economizer only, with a feed temperature of 235 F. In contrast the boiler of column C has an air heater and an economizer which permits three heating stages instead of two with a feed temperature of 300 F, resulting in a fuel reduction of 110 bbl per hr, or 2.3 per cent.

The heat-liberation rate in the *Van Dyke* boilers (66,000 Btu per cu ft per hr) is extremely conservative in the light of present practice (note Mr. Noyes's comments). Even if doubled it would still be well under what has proved to be possible. This, of course, depends on the burners used and the arrangement of the combustion space and surrounding cooling surfaces. In general, it is not the combustion space but the capacity of the downcomers which limits the capacity of a boiler.

With regard to superheat control, the author fails to see that there is essentially any greater complication in controlling a by-pass valve to the desuperheating coil than there is in controlling another oil burner.

No difficulty has been experienced in obtaining all necessary control of steam temperature when starting the turbines on the *Van Dyke* cold.

It is agreed that, for perfectly steady loads, it is theoretically possible to operate efficiently without automatic control. Evidently this generally is not done, since as Mr. Nelis states: "Combustion control is now becoming standard on American ships."

The author is in agreement with Mr. Noyes, concerning the excessive thickness of steam piping, but is happy to say that the Bureau of Marine Inspection and Navigation has recently issued a ruling which will permit advantage to be taken of the higher creep strength of carbon-molybdenum tubing on the third ship now under construction. In this case the thickness of the 5-in. main will be reduced from $^3/_4$ in. to $^1/_2$ in., approximately.

In answer to specific questions by Mr. Noyes, the following information is given:

Piping was "cold sprung," approximately 50 per cent of the expansion.

Gaskets were originally of soft (Armco) iron, but these were found to be unsatisfactory due to the temperature changes incident to shutting down at weekly intervals. "Flexitallic" gaskets have successfully replaced the solid soft iron.

The pressure drop from superheater outlets to turbine throttle is 10 to 15 lb at full load.

Cables of 2300 v on the catwalk are 106,000 cir mils with 2400-v lead and armored, basket-weave marine insulation. They are fastened to the inner surface of the catwalk side by lead-covered brass clips and covered by a brass housing bolted to the side panels.

Table 2, given in discussion by Mr. Schultz, compares a hypothetical power plant with that on the *Van Dyke* and shows a gain of 7.6 per cent in efficiency, partly due to the assumption that higher turbine and boiler efficiencies might be obtained. This is no doubt possible but the author's original investigations, preparatory to selecting the equipment for the *Van Dyke*, led to the conclusion that the extra cost of so doing would n t be justified in a plant of this size. The remaining gain is based on the higher transmission efficiency of a gear drive. Granting this gain to be possible, it was felt that the advantages of the electric drive, as outlined in the paper, more than offset this difference. However, it may be noted in passing that the oil coolers generally specified for geared installations have capacity to remove a heat equivalent of more than 5 per cent of the rated power of the unit.

Regarding Mr. Schultz' request for the author's reaction to the use of deep-well cargo pumps, having vertical motors below decks with forced ventilation, it is that this greatly increases the fire hazard, requires the added cost and complication of forced ventilation, considerably increases the work of the pumper, and only gains a reduction in the length of the shaft and cover pipe, which hardly seems worth while.

Rollcurve Gears

By H. E. GOLBER,[1] CHICAGO, ILL.

In automatic machinery the movements of the various bodies are usually obtained from the motion of a follower compelled by a driver. The driver is usually assumed to rotate at a uniform speed and the follower is given some definite motion, ordinarily a definite rotation. When the follower motion is to be a rotation at uniform speed, the driver and follower are usually connected by ordinary circular gears. When the follower motion is to be a rotation of nonuniform speed, elliptical gears are sometimes used. When the follower is to move in a more complicated manner, all kinds of devices, such as cams or planetary levers, are used. Very frequently the designer does not know of any means for producing the desired follower motion. Therefore, he is compelled to use an undesirable motion which he knows how to produce, and abandons what he considers the best.

This article discusses rollcurve gears. It shows that by using such gears, the follower can be given practically any desired motion, thereby enabling the designer to produce the follower motion desired and obtain the best condition in the new machine. The article first illustrates actual rollcurve gears installed and in use on actual machines. It discusses their durability and applicability, and gives a short history of some of them. This is to assure anyone that these gears are reliable and that their production is practical.

The article next shows the production of the "speedgraf," which is a standard method developed by the author to make the desired method perfectly definite. It is then shown how to convert the speedgraf into rollcurves which may then be drawn and tested on the drawing board. The rollcurves can then be used to obtain patterns for the gear blanks. These gear blanks then have the teeth cut into them, converting them into a pair of gears. When the driver moves at uniform speed, the follower moves in the desired motion. The result is positive and reliable.

Certain features that occur in connection with the rollcurve gears are explained. Also, to show the broad applicability of the gears, a few speedgrafs, for which gears have already been made, are shown and are commented upon.

THIS article is submitted by the A.S.M.E. Graphic Arts Division because the subject matter was developed in connection with the design of printing machinery and the applications shown are upon printing presses. However, the subject is really one of applied mechanics and is equally applicable to all classes of automatic machinery.

This article deals with means and methods for producing motions that hitherto have been either impossible or impractical. The motions now capable of production are of the greatest variety; they are produced by means that the writer calls "rollcurve" gears.

Years ago it was possible to design spur gears the axes space of which was standard. The pitch diameter of the spur gear was always the number of teeth divided by the diametral pitch. If the designer were asked to make a nonstandard axes space or nonstandard pitch diameter, he would be in trouble. Finally, however, there was introduced the hobbing machine and the Fellows gear shaper, with which the designer can readily make gears before thought to be impossible. For instance, the author has made three coaxial gears, one with 8 teeth, a second with 10 teeth, and a third with 12 teeth, all simultaneously geared with a common rack. The previous cramping limitations have vanished.

Again in bevel gears, previously the two axes had to intersect and could not pass each other. Now with hypoid gears the axes may pass each other. The cramping limitations have vanished.

Similarly, designers have made use of cams and roller arms. For a reciprocating irregular motion, of say 30 deg, there was no trouble. For a reciprocating motion of, say, 90 or 180 deg, auxiliary mechanism had to be introduced. For irregular continuous rotation the difficulties were large and so the designer avoided them. It will be found that in the rollcurve gears, these cramping limitations are gone, and the designer is able to produce in a simple and positive manner almost any motion desired, regular or irregular. The possibility of design has automatically vastly expanded.

A rollcurve gear is one that has a rather uncommon rolling curve as its pitch line. In ordinary spur and annular gears, the pitch lines are rolling circles. Also gears have been made with ellipses as their rollcurves. Some suggestions have been made to use equiangular spirals as rollcurves. The most extensive treatment of the subject known to the author is the one by Robinson,[2] who gives a rough method to find the mating rollcurve to any given rollcurve. So, to a given rotation of the first, the second must rotate in a definite manner. No attempt is made in that book[2] to treat the accelerations, velocities, or displacements of the driver and follower.

About 1931 the author developed some methods for making rollcurve gears, and they were applied in practice. His method was not derived from any previous method, although at places there necessarily must be some resemblance since the subject "rollcurve gears" is the subject in common.

The basic difference between say Robinson's methods[2] and the author's is that Robinson assumed one rollcurve and then tried to find the other, while the author first assumed the motion of both the driver and follower and then found the pair of rollcurves at once. Robinson thus obtained a motion of the follower

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Discussion of this paper was closed January 10, 1939, and is published herewith directly following the paper.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[2] "Principles of Mechanism," by S. W. Robinson, John Wiley & Sons, Inc., New York, N. Y., 1910.

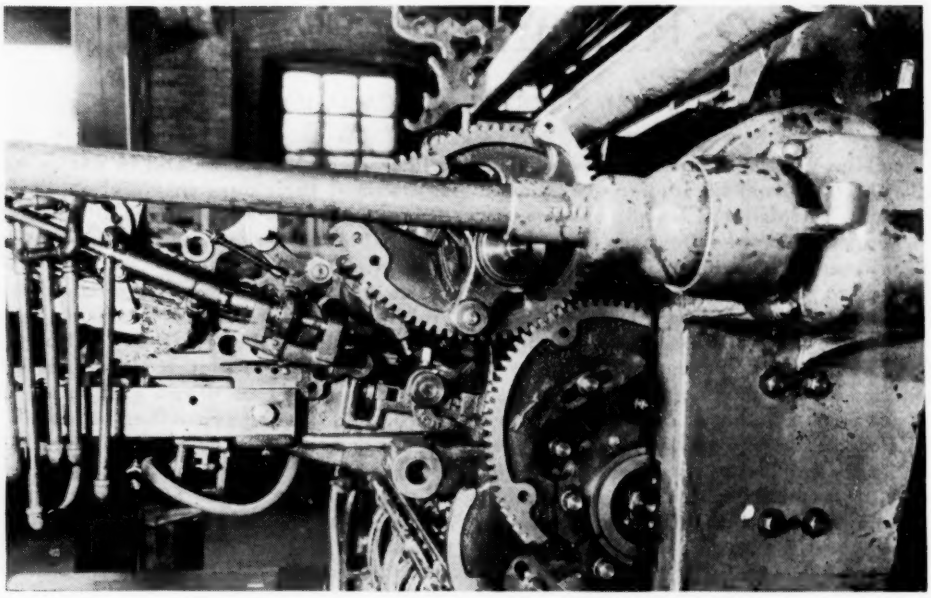


FIG. 1 THE FIRST ROLLCURVE GEARS

which depended on the properties of the assumed rollcurve. The author started with the desired motions and thereby made the rollcurves depend upon the motions. The result was that Robinson used well-known curves, say the circle, the ellipse, and the equiangular spiral. His curves were simple but his motions were complex. The author uses the circle and curves that have no names as yet. He thus obtained unusual curves, but his motions are simple. The simplicity of the motions has allowed them to be used extensively in machinery.

Observe the various illustrations of rollcurve gears. Some of them show the gears mounted upon and forming parts of machines. All are of gears for actual machines, not for models or experimental mechanisms. Many sets of gears have been made, installed, and used. They are still in use.

Immediately the questions arise: (1) How long have they been used? (2) How well do they wear? (3) How costly are they to manufacture? (4) Where should they be used? The answers in the order asked are as follows:

(1) The earliest pair was built about 1931. It is still in use. (2) They wear about the same as other gears made of similar material, cast iron or bronze. As yet there have been no replacements. (3) A first pair of gears is rather costly. They can then be reproduced quite commercially. (4) Where shall they be used? This must be left to the judgment of the designing engineer. Here he might take into consideration that by these gears, he has a tremendous broadening of the possibilities of design. He is able to produce in a positive simple manner practically any motion. Motions that otherwise are impossible or very difficult are easily made by rollcurve gears. Moreover he is able to reduce the number of links in the driver-follower chain, and thus reduce backlashes and improve register.

The author is continually called upon for more and more new rollcurve gears desired by the firm where the gears were first used. Some calls have also come in from outside.

CONSTRUCTION OF THE GEARS

We start with the driver and the follower. How these are to move depends of course on what they are to do in the machine. We assume that the designing engineer knows what he wishes to accomplish. From his knowledge he finally produces a description of the desired motion of both the driver and follower. This description the writer always finally reduces to a designer's "speedgraf."

Speedgraf. In books on kinematics we find various diagrams called "displacement diagram," "velocity diagram," and "acceleration diagram." The author refers to these as "sitegraf," "speedgraf," and "celgraf," respectively. Of these the speedgraf (or velocity diagram) is much more useful than the others and in practice he finds it is sufficient for his needs. Only very seldom does he need some additional diagrams.

In various books he has seen given a displacement diagram for the follower displacement, a velocity diagram for the follower speeds, and an acceleration diagram for the follower acceleration. Three diagrams are given. With very little training the engineer can obtain all this information at one glance upon the speedgraf only, and he needs no others. Thus looking at the speedgraf the observer finds the follower displacements, as the areas by the speedgraf; the follower velocities, as the ordinates of the speedgraf; the follower accelerations, as the slopes of the speedgraf; and the follower impulse, as the bent of the speedgraf.

From now on we shall consider the speedgraf as the fundamental graph—the start is with the "designer's speedgraf."

Speedgraf Segments. If the pair of gears is a pair of ordinary circular gears, the speedgraf is a line parallel to the speed-line zero. This is usually a horizontal line. Thus a horizontal line represents uniform rotation for both the driver and follower. When the speedgraf line is above the zero line, the driver and follower rotate in opposite directions as in a pair of spur gears. When the speedgraf line coincides with the zero line, the follower rests or dwells. When the speedgraf line is horizontal but below the zero line, the follower and driver rotate in the same direction.

It is usually assumed that the driver rotates at a uniform speed. The follower rotates at the speed and in the direction desired. When the follower speed changes, its speedgraf curve is not a horizontal. When the speedgraf curve is an oblique (straight line), the speed changes at a uniform rate. The follower motion then is like the motion of a body falling under gravity.

At first, each speedgraf was made as a polygon of oblique and horizontal lines. By such a polygon, varying motions and uni-

form motions were very nicely represented in a simple manner. But every cusp or point of the polygon spoiled the smoothness of the motion and caused trouble, first in the making of the gears and later in the finished machine itself. As these troubles were serious, they will be gone into more fully.

It was found that every speedgraf cusp produced a corresponding rollcurve gear cusp. This can be readily seen from Fig. 1. This pair of rollcurve gears is the first chronologically, being built early in 1931. The upper gear is the follower; the lower gear is the driver. The gears were made out of pieces of elliptic gears fastened together. Note that the follower gear segments are in several planes. The driver segments are similarly arranged but are covered over.

Note the rollers on the follower. One is clearly in sight, while the other at the top is half hidden. To cooperate with them is a cam, only part of which is visible at the lower left.

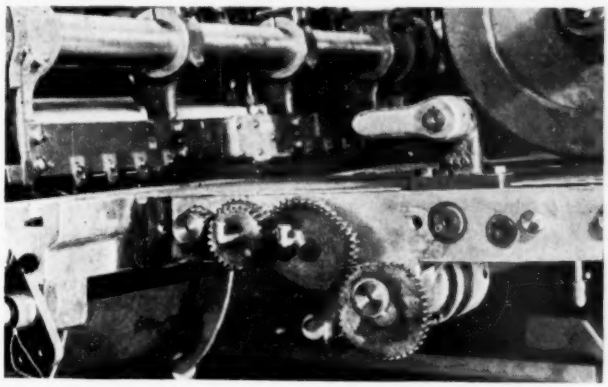


FIG. 2 TWO SMALL CONTINUOUS ROLLCURVE GEARS

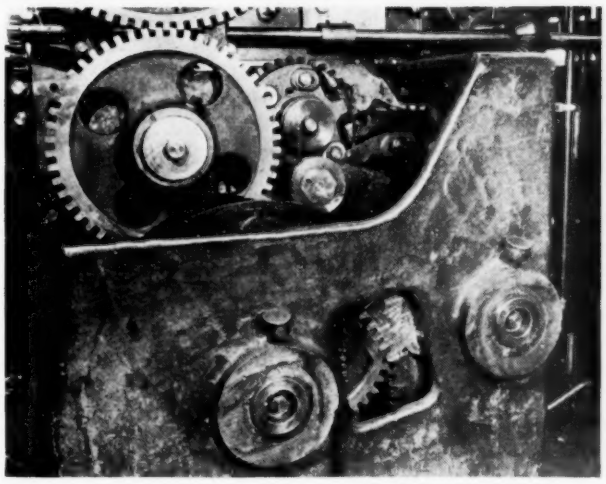


FIG. 3 ROLLCURVE GEARS, THE DRIVER AND FOLLOWER OF WHICH HAVE A CONVEX AND CONCAVE CUSP, RESPECTIVELY

This pair of gears was made before the development of the speedgraf method. In fact, it was the difficulties encountered in this pair that drove the writer to the consideration of the speedgraf and finally to the development to be described.

As the next example, consider Fig. 2. Here the two little gears are continuous and all in one plane. They are made of bronze cast with the teeth. The lower right gear is the driver. Each gear has three cusps. This spoiled the continuity of the gear rollcurve, making the cutting of the original master gears quite

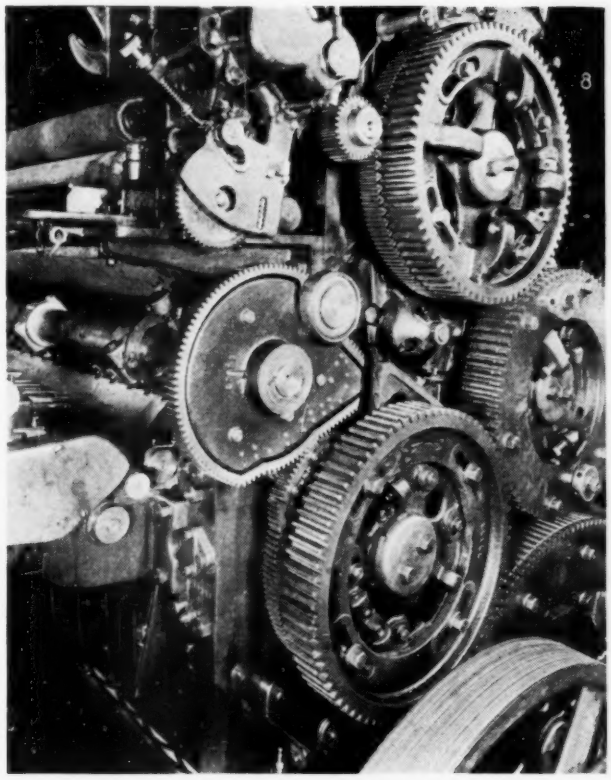


FIG. 4 ROLLCURVE GEARS, THE TEETH OF WHICH ARE ALL IN THE SAME PLANE

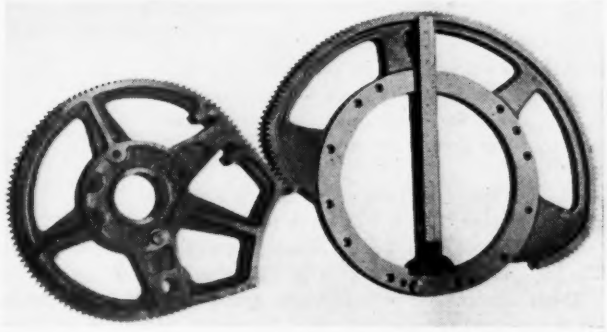


FIG. 5 ROLLCURVE GEARS WITH CUSPS ELIMINATED WHICH PERMITTED THE GEARS TO BE CUT WITH A FELLOWS CUTTER

difficult. These gears were by far the smallest, being about 3 in. in the axes space.

As the next example, consider Fig. 3. Here most of the mechanism is invisible but a portion of the driver (at the left) and the follower (at the right) are visible through the window. Note that the driver has a convex cusp and the follower a corresponding concave cusp. Here is also seen that the apex of the convex cusp has a gear space and the apex of the concave cusp has a gear tooth. This rule was found to be indispensable. Note that the follower had to be built in pieces to permit the cutting of the gear teeth. The mechanism was strong and heavy, the teeth having diametral pitch of four.

We next come to Fig. 4, in which the follower is very prominent. A portion of the driver is visible and in mesh with the follower. Note that the follower rollers have been very much

enlarged. The gear segments are still made in different pieces fastened together. However, unlike Fig. 1, the teeth are all in one plane. The teeth of the driver and follower have a pitch of eight, the big gears have a diameter of 20 inches and a pitch of four. Fig. 4 shows that the driver has quite a hump.

This was the last pair of rollcurve gears to be built in that manner. Strenuous efforts had been made to devise some scheme for the elimination of the cusps, so that teeth could be cut continuously. This was finally done successfully and the

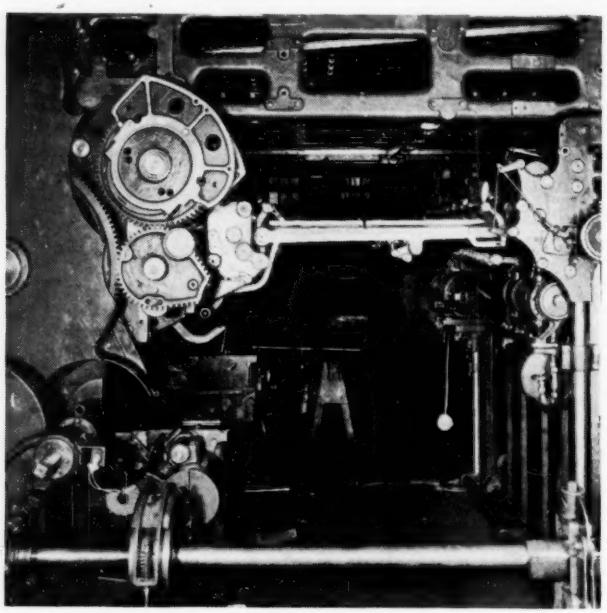


FIG. 6 LARGE ROLLCURVE GEAR

gears henceforth were cut by a single Fellows gear shaper cutter. As an example, see Fig. 5. The cusps are gone, the gears are continuous all in one plane, and easily manufactured. Moreover they are accurately machined, say to 0.001 in.

There is really nothing more that is novel to show; however, Fig. 6 will serve to exhibit the extent of the use of such gears in a large machine. The big gear at the left is 36 in. in diameter.

These illustrations will serve to prove that rollcurve gears are in practical commercial use. Moreover, attention must be called to the fact that each machine shown was not an individual machine but one of a line of machines marketed.

Returning now to the speedgraf and its cusps. One reason for objecting to cusps is the consequent cuspiness of the rollcurves, and resultant difficulty in making the gears. There are, however, other objections. Thus, upon examining the speedgraf at a cusp, we see that the slope or grade suddenly changes. That is, the acceleration suddenly changes. Now machines are assumed to be built of rigid material, say, cast iron. But there is no material absolutely rigid. Every body bends when forces are applied to it. Assuming Hooke's law that "strain is proportional to stress," then every body in a machine bends an amount proportional to the stress on it. As in a machine, the forces are proportional to the square of the machine speed; therefore, in a machine the bends of the bodies are proportional to the square of the machine speed times acceleration. At every speedgraf cusp, the theoretical bending of the bodies have two different bendings corresponding to the different grades. It frequently happened that the bodies were springy and bent sufficiently to cause a misplacement of the parts, and at high speed the machine "knocked" and did not "register." All these troubles were cured by eliminating the speedgraf cusps.

So the speedgraf polygon cusp was rounded over by a filleting arc. Here many choices could be made for the arc. It could be a portion of a circle, an ellipse, an harmonic, or a parabola. The choice finally fell upon a parabola with a vertical axis. This was because in computation it was simpler to deal with a parabola, than with a circle. True, in drawing, the circle was easier, but the parabola was never drawn except very roughly. The vertical parabola won out and is the filleting curve now used.

To summarize the foregoing, the speedgraf curves used are: (1) The horizontal straight line for uniform motion of the follower; (2) the oblique for nonuniform motion; and (3) the vertical parabola either U or $\cap$ for uncusping the speedgraf. How these are to be combined depends of course on the machine needs and the designer's best judgment. The final result is the designer's speedgraf.

Designer's Speedgraf. The designer's speedgraf takes many shapes. Fig. 7 shows a comparatively simple one. Here we see that the speedgraf is $ABCDEFGH$. As this is a full cycle, A and H are the same point. The speed-line zero is $A_0B_0C_0D_0E_0F_0G_0H_0$, so $A = A_0$, $B = B_0$, $G = G_0$, $H = H_0$. In the speedgraf, AB and GH show speed zero, the portion $G(H=A)B$ showing a follower dwell. The horizontal DE shows a follower uniform rotation in a direction opposite to the driver. The obliques BC and FG show follower increasing and decreasing speeds. The curves CD and EF are parts of a $\cap$ parabola. The points D and E are therefore vertices of the parabolic arcs.

To show that the speedgraf covers a complete cycle, the column of zone angles is given. The total $AH = 360.0000000$ deg. The column of ordinates is given, having been determined in some manner by the designer. There are other items, such

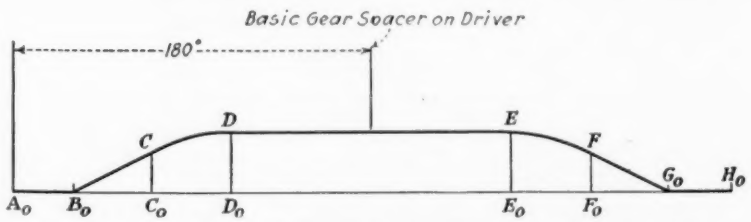


FIG. 7 DESIGNER'S SPEEDGRAF FOR A GRIPPER DRIVE

A_0B_0	= 29.6388885 deg	A_0A	= 0
B_0C_0	= 40.0000000 deg	B_0B	= 0
C_0D_0	= 40.0000000 deg	C_0C	= 1.00628931
D_0E_0	= 140.7222230 deg	D_0D	= 1.50943396
E_0F_0	= 40.0000000 deg	E_0E	= 1.50943396
F_0G_0	= 40.0000000 deg	F_0F	= 1.00628931
G_0H_0	= 29.6388885 deg	G_0G	= 0
A_0H_0	= 360.0000000 deg	H_0H	= 0

Axes distance = 16.625 in.
Arm for roller = 5.75 in.
Arms angle = 134 deg
Diametral pitch = 8
Roller diameter = 120 mm
Gear cam equal steps = 1 deg
Roller cam equal steps = 1 deg

as 180 deg = basic gear space on driver, which are also given.

As the drive part $GHAB$ is a dwell, it is necessary to provide for it in some manner. For it, a follower tooth would have to be kept at the center of the driver axle, and this would be very unsatisfactory for several reasons. Therefore, the dwell part is made not by rollcurve gears, but by cams and roller arms. Moreover, the gear portions on BC near B, and on FG near G, are not satisfactory because they unmesh, and so the cam drive is made to extend and cover the unsatisfactory parts of BC and

TABLE 1

Rectangle A_0ABB_0	$= A_0B_0 \times A_0A$	$= 29.6388885 \times 0.0$	$= 000.0000000$ deg
Triangle BCC_0	$= (1/2)B_0C_0 \times C_0C$	$= (1/2) \times 40.0000000 \times 1.00628931$	$= 20.1257862$ deg
Parabolic C_0CDD_0	$= (1/2)C_0D_0 \times (C_0C + 2D_0D)$	$= (1/2) \times 40.0000000 \times (1.00628931 + 2 \times 1.50943396)$	$= 53.66876305$ deg
Rectangle D_0DEE_0	$= D_0E_0 \times D_0D$	$= 140.7222230 \times 1.50943396$	$= 212.4109023$ deg
Parabolic E_0EFF_0	$= (1/2)E_0F_0(2E_0E + F_0F)$	$= (1/2) \times 40.0000000 \times (2 \times 1.50943396 + 1.0062893)$	$= 53.66876305$ deg
Triangle F_0FG_0	$= (1/2)F_0G_0 \times F_0F$	$= (1/2) \times 40.0000000 \times 1.00628931$	$= 20.1257862$ deg
Rectangle G_0GHH_0	$= G_0H_0 \times GG_0$	$= 29.6388885 \times 0.0$	$= 00.0000000$ deg
			360.0000008 deg

FG. The required data for the cam drive are given in the lower column.

It has been stated previously that all cusps are to be removed from the speedgraf and yet there are two left, *B* and *G*. But these two occur not on the gear portion but the cam portion, and the follower speed is very low. In some speedgrafs even the speed-zero cusps are rounded out.

The designer might have been very careful. But did he make a mistake in his computation? To some extent this can be tested from the data given and implied. It was understood that while the driver made a complete rotation, so did the follower. The driver rotated uniformly and made 360 deg per machine cycle. The follower rotated always in the same direction while starting, speeding up, speeding, slowing down, or resting. But in one machine cycle it also made 360 deg. We have already stated that in a speedgraf the area represents the displacement. For the driver the area would be 360 deg $\times$ 1 = 360 deg. For the follower, the cycle displacement must be the area by the speedgraf or the area *ABCDEFGH*, which we see is equal to the values given in Table 1.

The result for the follower area we call the area angle. It should be exactly 360 deg. It is 360.0000008 deg. We assume the agreement to be accurate enough; that is, the follower rotates once for every rotation of the driver. Our first check is satisfied.

The above accuracy seems to be silly. But as we proceed, the errors that occur, because of the inability of the author to integrate accurately for the length of the arcs, reduce the above from ten significant figures to seven, which is about right to make rollcurve gears correct to 0.001 in. on an axes space of 36 in.

We next examine the tangencies. The grade at *C* of *BC*, and the grade at *C* of *CD*, must be the same. The grade at *C* of *BC* is

$$\frac{C_0C}{B_0C_0} = \frac{1.00628931}{40.0000000} = 0.0251572327$$

The grade at *C* of *CD* is

$$\frac{D_0D - C_0C}{(1/2)C_0D_0} = \frac{1.50943396 - 1.00628931}{(1/2) \times 40.0000000} = 0.0251572325$$

Again the agreement is close enough.

At *D* the parabola is tangent; similarly at *E*. We see also that *F* is symmetric with *C*. Thus far the speedgraf checks. We assume that it is correct in all details.

The Rollcurves

Having now an accepted and tested speedgraf, the next step is to obtain the rollcurves. In Fig. 8 let *C* and *F* be the driver and follower axes, respectively, with the rollcurves *V* and *W* contacting at their tangent point on the axes line *FD*. Let *r* and *R* be the driver and follower "rays," respectively. Note the word used is "rays" and not "radius," the latter term being limited to the distance from the center of a circle. Let $d\theta$ and dw be the respective small rotations of the rollcurves. Then

$$r + R = DF = L = \text{axes space} \quad [1]$$

and because of rolling

$$r\,d\theta = R\,dw \quad [2]$$

From Equation [2] we obtain

$$\frac{dw}{d\theta} = \frac{r}{R} \quad [3]$$

However

$$\frac{dw}{d\theta} = \frac{\text{speed of follower}}{\text{speed of driver}} = \text{speedgraf ordinate}$$

Therefore, we obtain Equation [1] ($r + R = L$), and

$$r/R = \text{speedgraf ordinate} = S \quad [4]$$

Combining Equations [1] and [4], we obtain

$$r = \frac{LS}{1 + S} \quad [5]$$

and

$$R = \frac{L}{1 + S} \quad [6]$$

As *L* is the axes space for the two gears it is a known constant. And as the speedgraf is known, *S* is known for every value of θ or for every driver displacement. We can therefore find as many different values of *S* as may be required. In a drawing, the angles θ are usually taken say every 5 deg. In computation for machine work, θ is taken for every 2 deg for gears say up to 10-in. axes space and for every 1 deg for larger gears.

The computation of *S* for every θ, of course, varies in the various segments of the speedgraf. In *AB* and *GH* the *S* is 0.0. In *DE*, *S* is constant. In the oblique *BC* it is equal to $(CC_0/B_0C_0) \times$ abscissa, measured from B_0. Similarly in the oblique *FG* it is equal to $(F_0F/F_0G_0) \times$ abscissa, measured back from G_0. In the parabolas *CD* and *EF* we use a computation as in a body falling under gravity.

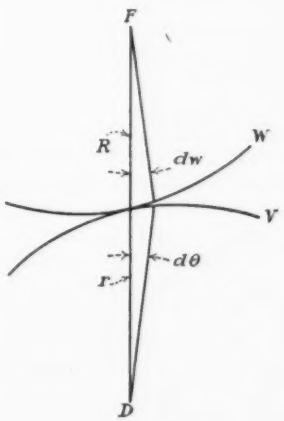


Fig. 8

We thus finally have the *S* value for every θ value and make a table for every θ, say from degree to degree.

Driver Rollcurve. We can now substitute in Equation [5] and obtain the value of the rollcurve ray for every θ. This gives the driver rollcurve mathematically accurate. The angle θ varies by equal angular steps.

Follower Rollcurve. By substituting in Equation [6] we obtain the set of follower rays corresponding to the follower sites for the driver places. The driver steps are equal. The follower steps usually are not equal and must be computed. These steps, of course, correspond to the corresponding area obtained by the speedgraf. We see that in the speedgraf the follower steps *AB* and *GH* are equal to zero. In *DE* they are constant. In *BC* and *FG* the areas

are easily obtained as they are the areas of trapeziums. In CD and EF they are the areas by parabolic arcs, and can be computed say by Simpson's formula.

Thus, finally for every driver angle θ, there is found a corresponding angle w for the follower. This combined with R of Equation [6] gives the polar coordinates of the follower rollcurve.

Production of the Driver and Follower Blanks

Patterns and rough castings can be made from drawings made from the values previously computed herein. When it comes to accurate machining the shop facilities must be taken into account. The most direct method would be to use a turntable capable of being rotated and set to 0.001 deg. The ray length can then be measured to 0.0001 in. and a hole with a definite radius be bored. This would give a blank with a series of holes whose centers are exactly on the rollcurve.

In the establishment with which the author is associated there is no machine kept up to do the accurate boring as just outlined. There is, however, a very accurate jig boring machine with slides at right angles and it is kept in fine condition. Therefore, the polar coordinates previously obtained for the driver and follower rollcurves are now converted into rectangular coordinates. These are used on the jig borer and the rollcurve blanks are bored with centers exactly on the rollcurves.

Thus, by one machine and polar coordinates, or by another machine and rectangular coordinates, a first pair of blanks are obtained. The boring is carefully done and then the ridges between borings are filed off. A careful workman leaves very faint marks showing the bottoms of the original borings. This filed master will be called master cam No. 1. It is smaller than the rollcurve by the radius of the bore, or master cam No. 1 = rollcurve — bore radius of master cam No. 1.

The master cam No. 1 is now mounted on a cam reproducing machine. Of these, there is an excellent one in the shop. It really reproduces a cam. From master cam No. 1, by means of a proper sized roller and cutter, there is now produced master cam No. 2 whose periphery coincides with the rollcurve. Similarly by another operation on the same machine, there is produced a gear blank for the actual rollcurve gear. It has the addendum of the teeth added thereto.

For another set of gears, only a new gear blank has to be machined. Master cams Nos. 1 and 2 serve for all sets.

A similar procedure is now gone through for the follower rollcurve, and its master cam No. 1, and its rollcurve or master cam No. 2 are produced. From these as many blanks as desired with the teeth addenda added are milled out.

Computation of Arc Length

At first, attempts were made to approximate the rollcurves by circle arcs and to cut the teeth as circular segments. Here great difficulties were encountered. The approximations were made on the drawing board and it was immediately found that to obtain any acceptable result, the drawings had to be larger than natural. And there always was danger. It was very easy to go outside of the rollcurve and obtain a jam, or to go too far inside of the rollcurve and obtain undesired backlash. Finally, the author, discouraged with his ill success on the drawing board to obtain usable circle approximations, attacked the problem mathematically, and evolved the method now used.

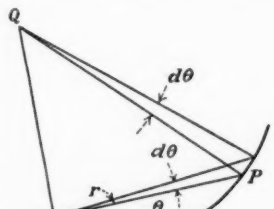


Fig. 9

In Fig. 9, let P be a point of a curve (rollcurve). Let $OP = r$, $AOP = \theta$. Let OQ be perpendicular to the ray OP, and PQ be a normal to the curve at P. Then PQ will be called the norm, and OQ the rayper(pendicular). In this place we call OQ the rayper, as being a short, expressive term.

We know from differential calculus that when θ is measured in radians (not degrees), dr and $rd\theta$ represent the ray increments along its length and transversely, respectively. This results immediately in the equation $dr/d\theta = OQ$. Moreover, the length of the arc, say AP is obtained thus

$$\text{arc } AP = \int_0^\theta \sqrt{(OP)^2 + (OQ)^2}\, d\theta = \int_0^\theta PQ d\theta$$

As the rollcurve AP was obtained from the speedgraf, and as the speedgraf was composed of horizontals, obliques, and parabolas, it seemed hopeful to obtain the polar equations of AP and to integrate for the length AP. There was no difficulty in obtaining the polar equations. The integration for the length of arc was something else. The horizontal segments gave simple integrals, the obliques gave elliptic integrals, and the fillet parabolas gave hyperelliptic integrals.

The author devoted much time to the study of the computation of elliptic and hyperelliptic integrals In this connection the best book he knows is "Simplification of Integrals," by Söderblom.[3] It is the best but is not simple enough. Therefore, the author was driven to the standard methods of approximation and used the well-known formulas, namely the Simpson $^1/_3$, the Simpson $^3/_8$, and the Weddle. These are given in various books.[4] Using these he was able to obtain the approximate values of the arcs. By computing first with the Simpson formula and then with Weddle's the author was able to make a check on the accuracy of the results. Making the original computations on a ten-place computing machine, his Simpson and Weddle results agreed to seven significant figures. These he assumed to be accurate. Moreover, they proved so in practice.

In this manner he obtained the length of the rollcurve arc of the driver from one step to another, say from degree to degree. As the lengths of the rollcurve arcs are the same for both driver and follower, he thus obtained also the length of the follower arc for degree to degree of the driver (not the follower). A table of the driver degrees and rollcurve arc lengths was then made.

The next was to select some satisfactory diametral pitch for the teeth of the rollcurve gears. This, of course, had already been done long ago when the proper addenda were added to the rollcurves to make the gear blanks. In practice the little gears in Fig. 2 had a diametral pitch of 16. For the bigger gears, there were used 8 pitch, 6 pitch, and 4 pitch, $14^1/_2$-deg involutes.

When the diametral pitch P was known, the pitch arc was known as $\frac{\pi}{P}$ and could be used as a measure to determine the number of pitch arcs.

Look again at Fig. 7. It shows the basic gear space on the driver. The start was therefore given, and all the other pitch points could be located therefrom.

Gage Points

Not all the pitch points were necessary. Only a few were required and the rest were derived from them. These were called the gage points and they had to be computed. These gage points almost never came at an exact step of the driver angle so did not fit in the table of arc lengths that had been computed. However, as the exact location of the gage points was required, the exact driver angle was computed by inverse interpolation.

[3] "Simplification d'Intégrales," by Axel Söderblom, Göteborg, Wald Zachrissons, Boktryckeri, A.B., 1909.

[4] "Numerical Mathematical Analysis," by J. B. Scarborough, Johns Hopkins Press, Baltimore, Md., 1931.

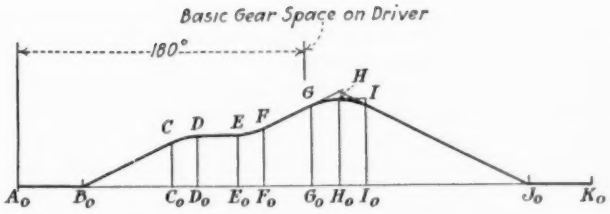


FIG. 10 DESIGNER'S SPEEDGRAF FOR A MORE COMPLICATED ROTARY GRIPPER DRIVE

A_0B_0	= 40.00000000 deg	A_0A	= 0
B_0C_0	= 56.00000000 deg	B_0B	= 0
C_0D_0	= 16.00000000 deg	C_0C	= 1.20312500
D_0E_0	= 26.00000000 deg	D_0D	= 1.37500000
E_0F_0	= 16.00000000 deg	E_0E	= 1.37500000
F_0G_0	= 30.32939447 deg	F_0F	= 1.54687500
G_0H_0	= 16.67060553 deg	G_0G	= 2.19848308
H_0I_0	= 16.67060553 deg	H_0H	= 2.37756185
I_0J_0	= 102.32939447 deg	I_0I	= 2.19848308
J_0K_0	= 40.00000000 deg	J_0J	= 0
A_0K_0	= 360.00000000 deg	K_0K	= 0

Axes distance = 14.2500 in.
Arm for roller
Arms angle
Diametral pitch = 8
Gear cam equal steps = 1 deg
Roller cam equal steps = 1 deg
Roller diameter

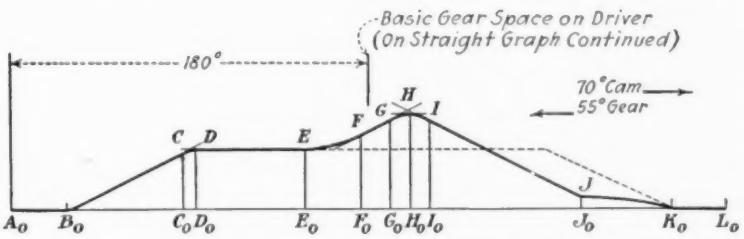


FIG. 11 SPEEDGRAF FOR A 15-IN. TAPE SPEED OF A ROTARY GRIPPER DRIVE

	Driver angle, deg	Follower angle, deg	Follower velocity	
A_0B_0	= 27.6522	0	A_0A	= 0
B_0C_0	= 60.0000	40.872131	B_0B	= 0
C_0D_0	= 6.0000	8.898361	C_0C	= 1.4327868
D_0E_0	= 55.0858	83.080223	D_0D	= 1.5081967
E_0F_0	= 28.2843	46.009840	E_0E	= 1.5081967
F_0G_0	= 14.3077	29.237877	F_0F	= 1.8636821
G_0H_0	= 10.0000	23.071194	G_0G	= 2.2233307
H_0I_0	= 10.0000	23.071194	H_0H	= 2.3490138
I_0J_0	= 76.0178	96.384139	I_0I	= 2.2233307
J_0K_0	= 45.0000	9.375000	K_0K	= 0
K_0L_0	= 27.6522	0	L_0L	= 0
A_0L_0	= 360.0000	359.999959		

PEARSON QUADRATIC

After spending a considerable time experimenting with various interpolation formulas, the author selected the Pearson quadratic as being the most suitable for this work. It can be found in "Tracts for Computers," No. 2, University of London, 1920. It is the simplest he knows of, for both direct and inverse interpolation. Moreover, both Simpson and Weddle formulas can be derived from it.

By means of the quadratic, all interpolations were made, and finally the ray lengths and ray angles were converted into rectangular coordinates by means of trigonometric tables. The values were then entered on the detail drawings.

CUTTING OF TEETH

It has been stated that a Fellows gear shaper cutter was used to cut the teeth on the gears. This was done by making a special machine in which the master cam No. 2, already described, was used. As a roller for it, rolled around the master, the cutter shaped out the teeth of the gear blank.

It was soon found that sometimes the roller had to be increased or decreased in diameter. Here the gage points came in. The cutting was started at the start gage point. If the finish gage point were hit accurately, it would be assumed that the intermediate points were correct. In practice it always worked well enough.

This first gear cut was seldom used in the drive. It was installed in the gear-cutting machine. In it then rolled a toothed circular gear, nonrotatable with the Fellows cutter. All subsequent gears were thus "fellowsed" without the possibility of slipping.

Refer now to Fig. 5. Note that the gears are both cut off. This was usually done at a place where the ray-norm-angle (of both the driver and follower) was 45 deg. If the angle were larger, the gears would unmesh. The complete drive for the speedgraf was thus unfinished. The drive for the gap was made by two cams on the driver and two rollers on the follower. These are seen in several of the gear illustrations. However, the making of roller cams for a given drive is nothing new and so it is not gone into further.

As it has been stated before that by means of the rollcurve gears almost any desired motion of the follower may be obtained for example, the author adds Fig. 10 showing a more complicated speedgraf. In making the rollcurve gears therefrom the work was slightly more extensive but no new principles were encountered.

Also, finally, the author shows an even more complicated speedgraf Fig. 11. Here the cusp J was in the cam part, not in the rollcurve gear part. Moreover, the designer checked both the driver and follower angles himself.

The foregoing gives a general description of the subject. In practice it was necessary to systematize it, and to reduce it to a method that could be used say by an intelligent high-school graduate. The details of this, however, would require an article much larger than the space taken herein, and it has been thought that sufficient has been given for the present.

Discussion

J. W. HUCKERT.[5] Spur gears having pitch curves other than circles have been in demand for a long time. One obstacle is the mathematics. It is too complicated for practical use. The rectification of the noncircular curves, as the author points out, involves elliptic and hyperelliptic integrals, and much labor is required to evaluate them either by the methods of the calculus or by the methods of approximation. If the methods of the author are simple enough so that he can compute the chordal thicknesses of all the teeth on a rollcurve gear, it may be enlightening to compare the theoretical against the measured thicknesses as a check on the spacing of the teeth.

In order to have accurate spacing of the teeth, the angular-speed ratios proper between the tool and the blank must be maintained throughout the generation of the gear. It is doubtful whether the author has met this requirement so that gears may be generated which will be suitable for all classes of machinery. In the ordinary setup of the Fellow's gear-shaper, the tool and the blank are positively rotated with respect to each other by means of a gear train. The author replaces this gear train with a friction drive during the cutting of the first gear. If slippage occurs, the spacing of the teeth on the first gear cannot be accurate. Since the first gear is employed in the drive between the tool and the blank during the cutting of subsequent gears, its

[5] Associate Professor of Mechanical Engineering, University of Maryland, College Park, Md. Mem. A.S.M.E.

inaccuracies will be reflected in all the others. Maybe accuracy in spacing is not necessary on gears for printing machinery. Neither is it so important on gears for agricultural, textile, and wrapping machinery. However, accuracy of spacing is important on gears for high-speed machinery and for machine tools, where not only the total angular displacements but also the instantaneous angular speeds must be preserved.

A. L. Ruiz.[6] This paper presents the practical aspects of a topic the writer has examined at various times in the past from a theoretical point of view. The investigations were never carried through to the stage of constructing noncircular gears, principally because of lack of an immediate application. It was with interest therefore, that the writer read the author's exposition of his method of manufacturing rollcurve gears. It is requested that the author explain further certain points and an alternative method of using these gears under certain circumstances is proposed.

With regard to cutting rollcurve gears on the Fellow's gear-shaping machine, it is rather clear that the cutter is moved in and out with respect to the blank by means of a roller bearing on a master cam. It is not so clear to the writer how the proper rotations of the cutter and the blank are obtained. In cutting the first of a particular type rollcurve gear, it appears that the cutter alone is rotated, and this, as it cuts into the blank, causes it to rotate. When one successful gear has been produced, it is used in the machine with a proper-size gear to cause succeeding blanks to rotate at a definite variable rate with respect to the cutter. Is this interpretation of the process correct?

Mention was made in the paper of the calculation of arc lengths of the rolling curves for the determination of gage points. Is this calculation extended to determine the size of the rolling curves so that an integral number of teeth is obtained in a complete circumference of each gear?

The author is apparently interested primarily in using the rollcurve gears to obtain a variable rotational velocity from a constant rotational velocity. With what degree of accuracy do the gears produce the velocity for which they were designed? For other applications one might be interested in obtaining a variable position of the follower shaft as a function of the position of the driving shaft. To what accuracy do rollcurve gears perform this service? The problem of backlash is of prime importance in any system designed to reproduce positions. This was touched on during the verbal discussion of the paper at the Annual Meeting. At that time the author stated that his rollcurve gears had backlash which increased as the ray-norm angle increased, although no magnitudes were given. The thought immediately arises as to whether methods for reducing the backlash have been tried, such as, decreasing the center distances of the gears, or using slightly enlarged blanks, or cutting thicker teeth.

The paper states that for low velocity ratios or for zero velocity of the follower gear, roller cams were used; the curved gears were designed to unmesh at a proper place, at which point the roller cam assumed the duty of transmitting the variable rotary motion. It would appear that the design and manufacturing requirements for constructing cams and gears so as to give a smooth transition with a minimum of lost motion would be rather fussy. The writer suggests the following scheme as a possible antidote for these difficulties.

Assume the designer wishes to construct a pair of rollcurve gears for a speedgraf some of whose ordinates are zero or negative. Add a constant to each ordinate so chosen that the entire speedgraf becomes positive. All ordinates may then be scaled down in proportion so that the area under the graph will be 360 deg. If the constants have been chosen properly, a pair of rollcurve gears may be constructed for this resulting speedgraf. For this pair of gears, as the driver is rotated at a constant velocity, the follower never reverses, but always rotates in a direction opposite to that of the driver, with a velocity which varies between predetermined maximum and minimum values. Then, connect the driving gear and the following gear together differentially with some form of planetary gearing, so that a portion of the driver velocity is subtracted from the follower velocity. If the gear ratios are chosen properly, the output of the differential will rotate at a velocity proportional to the ordinates of the original speedgraf. (Adding a constant to every ordinate of the speedgraf means adding a constant to the velocity of the follower gear at each point. This constant velocity is subtracted out by the planetary gearing.) The advantage of this method of construction is that the roller cam could be eliminated. The motion would be transmitted entirely by rollcurve gears and straight gearing, with a probable decrease in lost motion and an increase in smoothness of operation.

[6] Aeronautics and Marine Engineering Department, General Electric Company, Schenectady, N. Y. Jun. A.S.M.E.

Author's Closure

The author wishes to clear up some questions asked in the oral discussion of the paper at the time of its presentation. One was in reference to the backlash where the ray-norm angle is increased. The author answered that the backlash was increased. Somehow this answer led some to the idea that thereby a good deal of backlash was introduced. This is a false conclusion. If the gear is cut with a backlash of 0.0000 in. where the ray-norm angle is 0, and with similar accuracy at other places, then the backlash would be 0.0000 in. everywhere even at 45 deg. If the backlash were, say, 0.001 in. where the ray-norm angle is 0, then when the angle is 45 deg (at the limit) the backlash would be only $\sqrt{2} \times 0.001$ or 0.001414 in. The author's meticulous truthfulness and accuracy seem to have misled the listeners. For all practical purposes there is no backlash at all, anywhere. In ordinary spur gears, we work with backlash of, say, from 0.000 to 0.010 in. The rollcurve gears have been made equally tight.

A second point raised at the meeting was that in the rollcurve gear, the line of pressure of the gear tooth changes all the time and, therefore, the theory cannot be right. However, in cycloidal gears, the line of action is not straight and nevertheless the action is correct. The same is true with rollcurve gears.

The third point raised at the meeting was that the use of a friction drive at one point, obviously made the process inaccurate so that it could not be exact. Here of course the question arises as to what is meant by "exact." Must gears be correct in every dimension to an accuracy of 0.000001 in. before they are called *exact?* If so, then these rollcurve gears are not exact. But there never has been any exact gear made by anybody at any time in this world so far as the author knows. In commercial spur gears in the chord over, say, five teeth, the variations will amount to say 0.005 in., and the rollcurve gears are equally accurate.

The author does not know of any better test than by the accurate fitting of the plug in the predetermined space of the rollcurve gear, giving a location to the plug accurate in both abscissa and ordinate. It is actually correct to about 0.0005 in. and that is close enough for even the fine work such as is required for printing presses.

Professor Huckert states "Maybe accuracy of spacing is not necessary . . . . . on gears for printing machinery." Actually, printing presses require an accuracy of spacing that is beyond the commercial output of even the best of machine gear cutters. The teeth have to be corrected as by the Maag gear-tooth grinder.

The author does not understand the meaning of ". . . where not

only the total angular displacements but also the instantaneous angular speeds must be preserved." The author always assumed that if the angular displacements are always right, the instantaneous angular speeds must always be right. In any case in so far as the rollcurve gears are concerned, both the instantaneous angular displacements and also the instantaneous angular speeds are always preserved.

That the mathematics is "too complicated for practical use" raises quite a discussion. It would be spendid to find simpler mathematics for the solution of the problem. The author does not know any simpler. The fact remains, however, that it is being used by the Miehle Company to make gears on their lines of printing presses. If that is not practical use, what is?

Mr. Ruiz seems to have concluded that the rollcurve gears are cut on a Fellows gear shaper. They are not. The author nowhere said so. He did say that a Fellows gear-shaper cutter was used. The gears are cut on a special machine designed for the purpose, which it would take far too long to attempt to describe.

Complete gears, with an integral number of teeth, have been made, as in Fig. 2, and are being made regularly. There are now cuspless, continuous rollcurve gears, driving each other as desired. There is no difficulty.

It is true that the author described the driver rollcurve gear as rotating at a uniform rate, and the follower as having a predetermined variable rate. But this is simply a mathematical convenience. After the gears, have been cut and meshed, then obviously the driver may be moved and at any rate, constant or variable.

Again Mr. Ruiz also raises the question of accuracy. The gears are machined to 0.001 or 0.002 in. on the rollcurve arc and they perform their function to this degree of accuracy. No special means for decreasing the backlash is required since, when desired, the gears are cut to run metal to metal without backlash.

Mr. Ruiz intimates that it is difficult to make the node cams join the gears smoothly. "It must be a fussy job." It is a commercial job, as the line of machinery marketed by the firm shows. It gives a fine job with a minimum of backlash.

Mr. Ruiz inquires whether by the use of an auxiliary planetary gear, it would not be possible to produce a dwell and even a negative rotation. Certainly it would be possible, but the author abandoned that idea because in his opinion it would introduce not a minimum of backlash but an increased backlash. Moreover the machine, Fig. 3, produced both positive and negative rotations of the follower. To accomplish this result an intermediate (but not planetary) gear was used, and is being used today.

Moreover, when the gears need to be on a dwell in printing-press work, there should be a dwell and not a constant quiver.

Finally, the author does not doubt that improvements can be made in the mathematics and in the method of cutting. He himself has shown that it is both possible and practical to produce rollcurve gears. He assumes this is but the beginning and sees no reason why rollcurve gears should not become quite common.

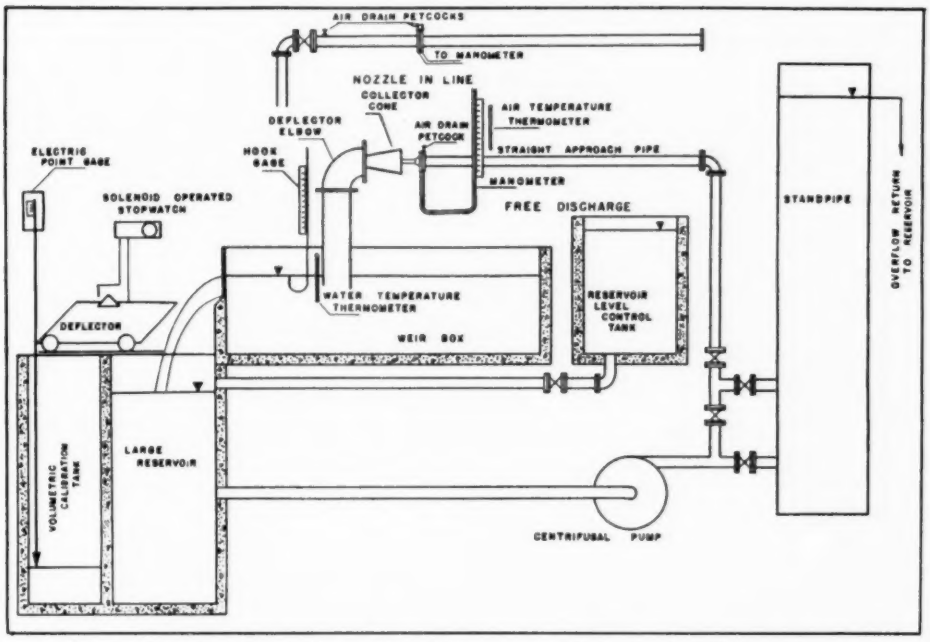


FIG. 1 ARRANGEMENT OF TEST EQUIPMENT AT THE UNIVERSITY OF CALIFORNIA FOR THE CALIBRATION OF NOZZLES PLACED IN AND AT THE END OF A PIPE LINE

Nozzle Coefficients for Free and Submerged Discharge

BY R. G. FOLSOM,[1] BERKELEY, CALIF.

This paper presents in brief form a summary of investigations made at the hydraulic laboratory of the University of California, in cooperation with the A.S.M.E. Special Research Committee on Fluid Meters, to determine nozzle coefficients for free and submerged discharge. A description of test facilities and procedure is included. The results given concern a series of A.S.M.E. long-radius nozzles for 3-in., 4-in., and 8-in. pipes tested with pipes of different wall roughness, various upstream piping arrangements, and different pressure-tap locations. Some data on I.S.A. nozzles which have been manufactured at the University shops are added.

THE INVESTIGATION reported in this paper is one of many conducted at different laboratories throughout the country with the support of the A.S.M.E. Special Research Committee on Fluid Meters to study the characteristics of the A.S.M.E. long-radius nozzle. Earlier papers have dealt with the general program as well as some particular results (1, 2, 3).[2]

The work at the University of California involves calibration of the nozzles placed in a pipe line in the normal manner, and also placed at the end of the pipe line. The latter condition of free discharge is of importance in various testing arrangements, particularly in tests of irrigation and dredge pumps. A previous paper (4) presented results of tests with orifices for free discharge.

During the investigations, it was found that if the nozzles were left in the pipe line for a few days in contact with water, material forming a sand-like surface was deposited on the nozzles. A similar phenomenon has been observed at the National Bureau of Standards (3). This deposit has an appreciable effect on the coefficient. Unless otherwise noted, all tests considered in this paper refer to nozzles with clean smooth surfaces.

EQUIPMENT AND PROCEDURE

Fig. 1 shows the arrangement of the equipment in the Hydraulic Laboratory at the University of California for calibration of nozzles placed in and at the end of a pipe line. Water was drawn from the large reservoir by the centrifugal pump and discharged to the standpipe or directly to the nozzle as desired. An additional reservoir was provided to maintain constant suction condi-

[1] Assistant Professor of Mechanical Engineering, University of California. Jun. A.S.M.E. Mr. Folsom received the degrees of B.S., M.S., and Ph.D. from the California Institute of Technology in 1928, 1929, and 1932, respectively. Since his graduation in 1928 he worked one year as part-time research assistant at the Riverside Cement Company, Riverside, Calif.; four years as teaching fellow at the California Institute of Technology; and one year as engineer for the Water Department of the City of Pasadena, Calif. He has been at the University of California since 1933.

[2] Numbers in parentheses refer to the Bibliography.

Contributed by the A.S.M.E. Special Research Committee on Fluid Meters and presented at the Annual Meeting of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS, held in New York, N. Y., December 5–9, 1938.

Discussion of this paper was closed January 10, 1939, and is published herewith directly following the paper.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

tions on the centrifugal pump when it discharged directly to the nozzle, thus keeping the discharge rate constant during the test.

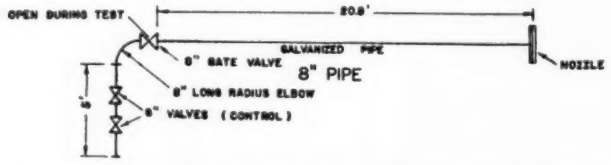


FIG. 2 ACTUAL ARRANGEMENT AND DIMENSIONS FOR TESTS WITH 8-IN. GALVANIZED PIPE WITH FREE DISCHARGE

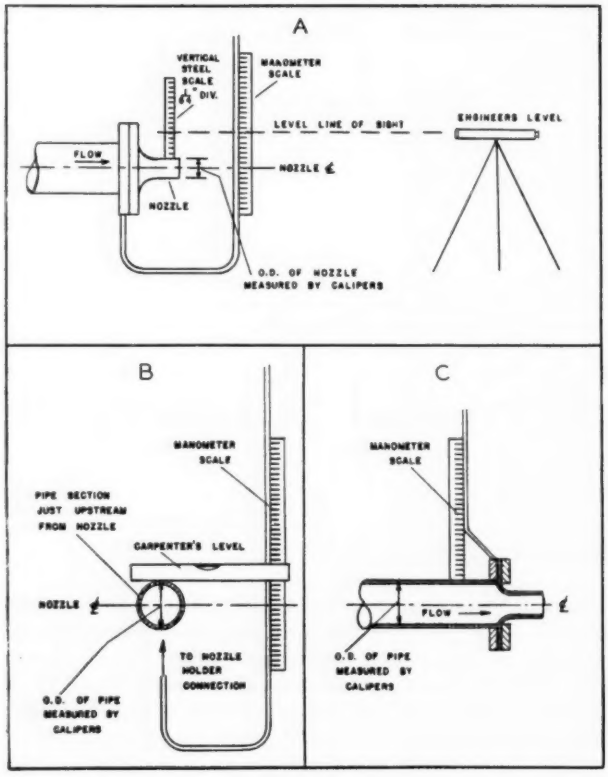


FIG. 3 VARIOUS METHODS FOR DETERMINING THE ZERO READING OF MANOMETERS

The dimensions and actual piping arrangement for a typical test setup are shown in Fig. 2.

Nozzles. The long-radius nozzles supplied to the laboratory were manufactured from brass or bronze and possessed smooth surfaces except as indicated in the following:

Nozzle No. 8.300.2 was built up by brazing parts together and then machining the meter. Since the material was not homogeneous from the standpoint of hardness, the tool tended to chatter and left a series of waves on portions of the nozzle converging section.

Nozzle No. 8.375.1 was similar in construction to that of nozzle No. 8.300.2. Blowholes were evident at the junction of the parts which also left a prominent line around the upstream face.

Nozzle No. 8.562.1 was made of steel.

Nozzle No. 8.506.1 had indentation areas in the surface, which were due to insufficient metal in the casting.

Quantity Measurements. The rate of discharge was determined from the volume of water collected in a measured period of time.

The gravimetrically calibrated volumetric tank had a capacity of about 400 cu ft and a cross-sectional area of about 46 sq ft. The rise of water level was measured with an electric point gage reading to the nearest 0.001 ft. The water-collection period was determined from a solenoid-operated stop watch with contacts for the solenoid placed on the hand-operated deflector. The stop watch was checked daily against a clock known to keep correct time within a few seconds per month.

The hook gage was read at the start and finish of each run to determine the storage or depletion of water in the weir box. The weir was not used as a flow-measuring device.

Head Measurements. All nozzles were mounted in holding rings having centering devices and with corner taps which corresponded to I.S.A. standards (5, 6). Additional pipe taps were placed at various distances upstream for the free-discharge measurements. For the nozzle-in-line position, corner taps, and pipe taps, one pipe diameter upstream and one-half pipe diameter downstream, were used.

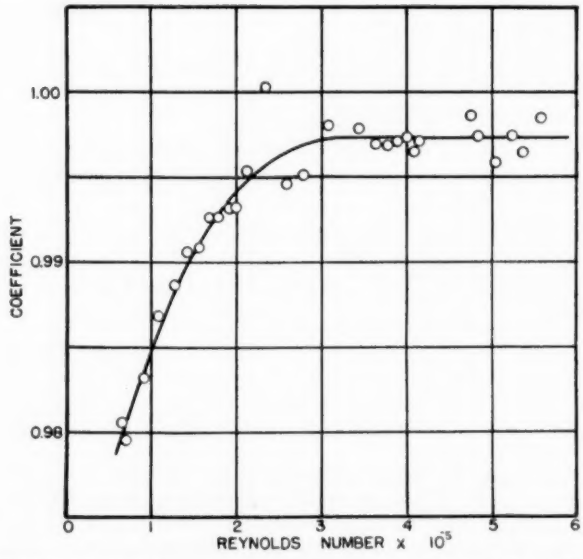


FIG. 4 TYPICAL NOZZLE-CALIBRATION CURVE PLOTTED FROM TEST DATA ON NOZZLE NO. 57195*R* WITH GALVANIZED PIPE AND FREE DISCHARGE

Vertical water columns, slant and vertical mercury-filled U-tubes were used for manometers. The scales were graduated to 0.01 ft and all readings were estimated to the nearest 0.001 ft. The inside diameters of the manometer tubes were of sufficient size to make the effects of surface tension negligible (7).

With free discharge, it is necessary to measure the elevation of the manometer scale with respect to the center line of the nozzle. Fig. 3 illustrates some common methods used for this determination. The method shown in Fig. 3*A* was employed for the tests reported in this paper.

In many cases, the room or air temperature was appreciably different from the water temperature, the discrepancy reaching as high as 30 F. For these tests, the manometer fluid was assumed to be at room temperature. Corrections were applied to express the head in terms of the fluid flowing through the nozzle.

EXPERIMENTAL RESULTS

Discharge Equation. As indicated elsewhere (4), the form of the discharge equation is largely a matter of convenience. All coefficients reported in this paper were computed from

$$Q = CA\sqrt{(2gH)} \quad \ldots\ldots\ldots\ldots\ldots\ldots [1]$$

TABLE 1 EXPERIMENTAL RESULTS WITH CORNER TAPS

			Free Discharge								In-line							
			Galvanized pipe				Black pipe (service rough)				Galvanized pipe				Black pipe (service rough)			
Nozzle no.	Nozzle diameter in.	Diam ratio, D_2/D_1	Straight pipe, no. diameters	C	Avg deviation, per cent	Min RN × 10^{-5}	Straight pipe, no. diameters	C	Avg deviation, per cent	Min RN × 10^{-5}	Straight pipe, no. diameters	C	Avg deviation, per cent	Min RN × 10^{-5}	Straight pipe, no. diameters	C	Avg deviation, per cent	Min RN × 10^{-5}	Coefficient ratio[b]
57195R	1.2240	0.400	81	0.997	0.10	3.0	..	...	..	...	..	...	..	...	..	...	..	...	...
57299	2.4476	0.800	81	1.165	0.38	2.0	..	...	..	...	..	...	..	...	..	...	..	...	...
4.150.2	1.4374	0.356	60	0.991	0.14	2.9	58	0.989	0.13	1.8	..	...	..	...	..	...	..	...	...
4.200.4	2.000	0.496	60	1.004	0.09	1.7	58	1.007	0.07	1.5	60	0.991	0.19	3.2	..	...	..	...	1.000
4.250.1	2.5007	0.619	60	1.044	0.12	3.0	59	1.045	0.10	2.5	60	1.005	0.11	1.5	48	1.009	0.08	1.8	0.999
4.275.1	2.7502	0.681	60	1.068	0.23	3.0	59	1.076	0.18	2.5	60	1.045[a]	...	5.5	..	...	..	...	0.999
4.300.1	3.001	0.743	60	1.096	0.10	2.3	58	1.116	0.13	1.6	60	1.068	0.16	2.0	..	...	..	...	1.000
4.350.1	3.5001	0.867	60	1.306	0.37	1.9	59	1.314	0.22	4.0	60	1.102	0.08	2.2	48	1.118	0.19	2.5	0.995
8.300.1	3.0012	0.371	30	0.996	0.14	1.8	20	0.993	0.13	2.0	30	0.995	0.22	2.3	..	...	..	...	1.001
8.300.2	3.1226	0.386	30	0.996	0.15	3.0	20	0.994	0.08	3.0	30	0.995	0.15	3.0	..	...	..	...	1.001
8.375.1	3.7497	0.464	30	1.005	0.09	2.3	20	1.007	0.07	2.3	30	1.003	0.16	3.0	..	...	..	...	1.002
8.450.1	4.4981	0.557	30	1.018	0.17	3.0	20	1.024	0.22	2.3	30	1.019	0.13	2.3	..	...	..	...	0.999
8.506.1	5.0585	0.626	31	1.037	0.18	4.1	20	1.040	0.20	2.8	30	1.037	0.30	4.7	..	...	..	...	1.000
8.562.1	5.6180	0.696	31	1.068	0.35	3.2	20	1.076	0.30	3.4	30	1.071	0.29	3.6	..	...	..	...	0.997
8.615.1	6.150	0.761	31	1.123	0.31	6.1	20	1.134	0.12	5.0	30	1.120	0.18	6.8	..	...	..	...	1.003
8.665.1	6.648	0.823	31	1.187	0.24	7.0	20	1.214	0.46	6.7	30	1.180	0.18	6.5	..	...	..	...	1.005
8.715.1	7.151	0.886	31	1.321	0.60	5.6	20	1.371	0.47	6.7	30	1.300	0.42	4.3	..	...	..	...	1.016

[a] The coefficient did not reach a well-defined constant magnitude. The value given is an approximate one toward which the coefficients approach.
[b] Ratio of coefficients for nozzle at free discharge to the coefficient in-line for galvanized pipes.
RN = Reynolds' number.

where Q = discharge rate, cfs; C = a dimensionless coefficient; A = area of nozzle throat, sq ft; g = weight per unit mass = 32.15 ft per sec per sec; and H = differential head across the nozzle (in-line) or head at the center of the nozzle (free discharge) expressed in feet of water corresponding to pipe-line conditions.

The coefficient C is primarily a function of the diameter ratio, upstream-velocity distribution, and the pressure-tap location and type. As the upstream-velocity distribution is often difficult to measure, the factors which influence the distribution are usually indicated as the variables. These are the Reynolds number, roughness and length of approach section, and piping arrangement upstream from the straight approach section. Other factors also influence the coefficient, e.g., roughness of nozzle, submerged or free discharge, and eccentricity of the nozzle with respect to the pipe. The linear dimension in the Reynolds number was selected as the throat diameter of the nozzles.

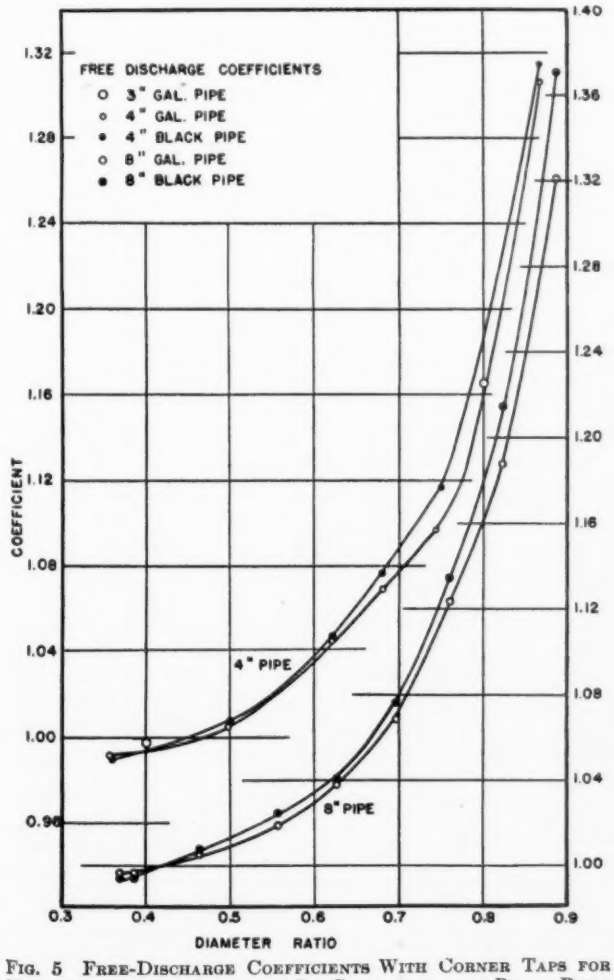


FIG. 5 FREE-DISCHARGE COEFFICIENTS WITH CORNER TAPS FOR NOZZLES IN 3-IN., 4-IN., AND 8-IN. GALVANIZED AND BLACK PIPES

Free Discharge With Corner Taps. Fig. 4 shows a typical nozzle-calibration curve, that is, the coefficient rises at low Reynolds number until a critical value is reached. At larger Reynolds numbers the coefficient is constant. The critical value is indicated as the minimum Reynolds number for a constant coefficient, in this case about 3×10^5. Coefficients used in this report refer to values in the constant-coefficient region. Insufficient data were obtained to determine a minimum Reynolds-number curve throughout the diameter ratios investigated.

The summary of tests with new galvanized and service-rough[3] or new black pipe is presented in graphical form in Fig. 5. Specific values are tabulated in Table 1. The values for the largest diameter ratios are approximate since repeat tests did not check original values. For these diameter ratios ($D_2/D_1 > 0.82$, where D_1 is pipe and D_2 is the nozzle diameter), the coefficient is extremely sensitive to conditions of nozzle surface, eccentricity with the pipe, and pipe roughness.

In Line With Corner Taps. The usual coefficients for quantity rate meters refer to installations in the middle of a pipe line. Table 1 contains a column of coefficients obtained for this type of application. Previous treatments of free-discharge conditions have expressed the coefficients in terms of the coefficients for the in-line installation. Table 1 shows this comparison as obtained from this series of tests. The 4-in. nozzles show an average ratio slightly less than unity, while the average for the 8-in. nozzles is slightly greater than unity, the average of all runs being close to unity. Thus, according to this investigation, the coefficients for the in-line and free-discharge operation differ by a negligible amount.

Pressure-Tap Location. The type and position of pressure taps depend on many factors. The experience of some laboratories has indicated that a well-made pipe tap is less sensitive to extraneous influences than corner taps. On the other hand, experience has demonstrated the difficulties involved in produc-

[3] Translation from reference (6): "A service-rough pipe may be understood to be a cast-iron pipe which has rusted internally through long use, but which has no thick encrustations on the surface."

ing a well-made pipe-tap connection under field conditions. In view of the uncertainty regarding the optimum position, investigations were made to determine the relationships of heads measured at various points.

The method selected to present the results of upstream pressure investigations assumes that if the coefficient based on the corner tap were constant, then the coefficients based on other taps investigated would be constant also. Analysis of the original data indicates this procedure to be valid. The results are expressed in terms of a ratio of heads based on the head measured at the corner tap.

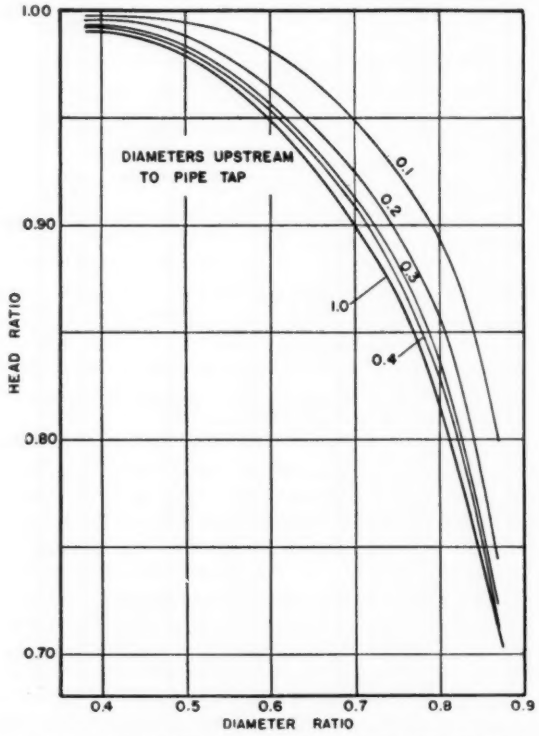


FIG. 6 RATIO OF HEADS AT UPSTREAM PIPE TAPS TO HEADS AT CORNER TAPS FOR FREE DISCHARGE

The results of head measurements in galvanized pipes at the corner tap, and taps about 0.1, 0.35, 0.5, 0.6, and 1.0 diameters upstream are shown in the faired curves of Fig. 6. The square root of the pressure ratio will give the change in coefficient at any upstream distance with respect to the coefficient based on the corner tap for corresponding diameter ratios. For example, at a diameter ratio of 0.62 and a distance of 1 diameter upstream the pressure ratio is 0.94. If the coefficient for the corner tap were 1.033, then the coefficient for one diameter upstream would be $1.033 \div (0.94)^{1/2}$ or 1.065.

The pressures measured at 1 diameter upstream from the upstream face of the nozzle and $1/2$ diameter downstream from the downstream side of the nozzle plate have been shown to be the most satisfactory (3). Several of the nozzles were tested in the line with these pipe taps. Also tests on free discharge with 1 diameter upstream pipe taps were made. The results for both series of investigations are given in Table 2 and Fig. 7.

Short Approach Lengths. In actual fluid measurements, it is frequently necessary to install a quantity rate meter with insufficient upstream straight approach pipe. Specifications for straightening vanes have been proposed (8), but the straight lengths of pipes specified are longer than desired. Data on some nozzles installed a short distance downstream from a single elbow

TABLE 2 RATIO OF HEADS FOR PIPE TAPS AND CORNER TAPS

Nozzle no.	Diameter ratio, D_2/D_1	Head Ratio — Free discharge[a] — Gal. pipe	Free discharge[a] — Black pipe	In-line[b] — Gal. pipe	In-line[b] — Black pipe
57299	0.800	0.825	...	...	...
4.150.2	0.356	...	...	...	...
4.200.4	0.496	0.979	0.981	0.985	0.980
4.250.1	0.619	0.943	...	...	...
4.275.1	0.681	0.918	...	...	...
4.300.1	0.743	0.863	0.874	0.877	0.879
4.350.1	0.867	0.768	...	...	...
8.300.1	0.371	0.992	...	...	...
8.300.2	0.386	0.990	0.991	...	...
8.375.1	0.464	0.984	0.987	...	...
8.450.1	0.556	0.960	0.957	...	...
8.506.1	0.627	0.933	0.937	0.924	...
8.562.1	0.696	0.898	0.903	0.892	...
8.615.1	0.762	0.851	0.866	0.850	...
8.665.1	0.823	0.783	0.807	0.777	...
8.715.1	0.886	0.692	...	0.706	...

[a] Ratio of head at 1 diameter upstream to that at the corner tap.
[b] Ratio of differential heads measured at corner taps to that measured with taps 1 diameter upstream and $1/2$ diameter downstream.

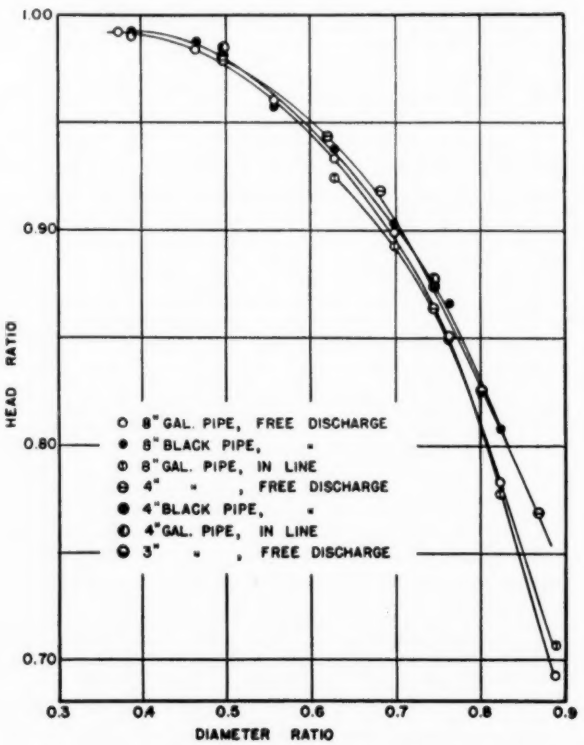


FIG. 7 RATIO OF HEADS AT A PRESSURE TAP 1 DIAMETER UPSTREAM TO THE HEAD AT THE CORNER TAP FOR FREE DISCHARGE; ALSO, THE RATIO OF HEADS AT 1 DIAMETER UPSTREAM AND $1/2$ DIAMETER DOWNSTREAM TO THE HEAD WITH CORNER TAPS FOR IN-LINE POSITION

were obtained to demonstrate the effects of distorted velocity distribution on the coefficients. All measurements were made with corner taps and black or service-rough standard steel pipe; the results are shown in Fig. 8. Throughout the diameter ratios studied, the coefficient reduces as the length of upstream pipe decreases, the reduction being greater the larger the diameter ratio.

A curve for nozzle coefficients with a highly corroded pipe is also shown in Fig. 8. As the pipe becomes rougher, the coefficient rises, the rise increasing with the diameter ratio.

I.S.A. Nozzles. Nozzles manufactured in the laboratory shops were calibrated in order to compare coefficients with standards specified in the "Regeln" (6). Bronze or brass castings were machined to required dimensions as indicated by a template and given a high polish. No measurements were made on the actual

contour of the finished nozzle. Table 3 presents the test results and the comparison with standard values. The *a* value for the 3-in. nozzle was measured about 3 months prior to the *b* value, the pipe and holder in each case being the same. During the interval, the nozzle had been used but no deposit developed and the surface was merely polished for the *b* series of tests.

TABLE 3 I.S.A. NOZZLE WITH CORNER TAPS

Nozzle no.	Nozzle diameter, in.	Diameter ratio, D_2/D_1	Coefficients: Free discharge, gal pipe	Coefficients: In-line, gal pipe	Coefficients: C, "Regeln"[a]	Coefficients: C, corrected	Ratio of free-discharge coefficient to C	Ratio of free-discharge coeff to in-line coeff
3 in.	1.890	0.62	(*a*) 1.041 (*b*) 1.033	...	1.036	1.0380	...	...
HN-7	2.493	0.62	1.064[b]	...	1.036	1.0375	...	...
PN-1	2.400	0.60	1.024	1.025	1.030	1.0310	0.994	...
PN-2	4.445	0.56	1.007	1.010	1.018	1.0180	0.992	0.999
PN-3	1.253	0.61	...	1.029	1.033	1.0360	0.993	0.997

[a] Correction of coefficients for pipe roughness. See paragraph 28 and Fig. 23*a* of reference no. 6 in the Bibliography.
[b] This coefficient corresponds to a rough nozzle.

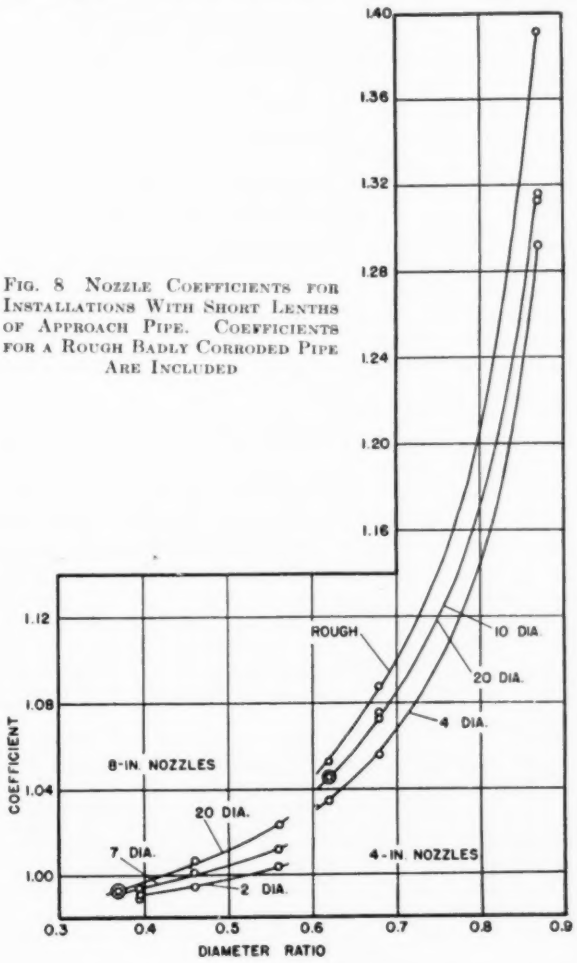


FIG. 8 NOZZLE COEFFICIENTS FOR INSTALLATIONS WITH SHORT LENTHS OF APPROACH PIPE. COEFFICIENTS FOR A ROUGH BADLY CORRODED PIPE ARE INCLUDED

Nozzle no.	D_2/D_1	Straight pipe, no. diameters	C
8.300.1	0.37	20	0.993
		7	0.993
8.300.2	0.39	20	0.994
		7	0.991
		2	0.989
8.375.1	0.46	20	1.007
		7	1.001
		2	0.995
8.450.1	0.56	20	1.024
		7	1.012
		2	1.004
4.250.1	0.62	20	1.045
		10	1.045[a]
		4	1.035[a]
4.275.1	0.68	20	1.076
		10	1.073
		4	1.056
4.350.1	0.87	20	1.314
		10	1.312
		4	1.292
Rough badly corroded pipe:			
4.250.1	0.62	40	1.053
4.275.1	0.68	40	1.088
4.350.1	0.87	40	1.392

[a] The coefficient did not reach a well-defined constant magnitude.

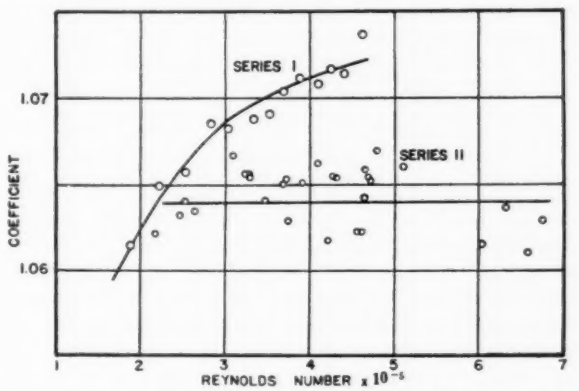


FIG. 9 CALIBRATION DATA ON NOZZLE NO. HN-7 FOR FREE DISCHARGE

(Series I tests are for a smooth nozzle surface, and series II tests apply when deposits were present on the nozzle surface.)

Fig. 9 shows the data for two tests on the HN-7 nozzle. Series I corresponds to new highly polished conditions. Series II is the calibration after the nozzle had been installed in a regular laboratory service line long enough to build up a sand-like surface. The new diameter is tabulated; the measured rough diameter was 2.4897 in. The coefficient is based on the diameter when new. Similar results on nozzle roughness have been reported elsewhere (3).

Tests on two of the nozzles for in-line and free-discharge operation indicate no appreciable difference in coefficient. This corresponds with the data obtained on the long-radius nozzles.

The comparison of the test coefficients with the values predicted on the basis of methods in the "Regeln" (6) shows satisfactory correlation except for nozzle HN-7. The difference is less than 1 per cent which is within the tolerances specified for service rough pipes. The coefficient for nozzle HN-7 is about 2½ per cent high. The reason for this difference has not been ascertained.

SUMMARY

The conclusions stated here apply to the results of this limited series of investigations with water as the fluid. Further work and comparison with data obtained by other laboratories may modify these statements.

1 For nozzles of the same area ratio with free-discharge operation with corner taps, there is no consistent difference in coefficients for the size range used in a given type of pipe. The service-rough or black pipe produces coefficients slightly higher than those corresponding to galvanized pipe.

2 Using corner taps, there is no appreciable difference in coefficients for free-discharge and in-line operation.

3 The head measured at an upstream pressure-tap location is lower than the corresponding head at the corner tap. Graphs and data present the relationships. The pipe size and roughness affect the head ratio; the ratio increases as the pipe relative

roughness increases, the amount of the increase being dependent on the diameter ratio.

4 Nozzle coefficients are appreciably reduced for installations having less than 10 diameters of straight pipe between a single elbow and the meter.

5 The nozzle coefficients become larger as the roughness of the pipe increases.

6 All other factors remaining constant, deposits or roughness on the nozzle surface change the coefficients as indicated in Fig. 9.

7 All except one of the I.S.A. nozzles constructed by the laboratory gave coefficients within the tolerance limits specified by the "Regeln" (6).

8 Diameter ratios greater than about 0.82 are not recommended as the meter is supersensitive to pipe roughness, nozzle surface conditions, and the particular installation.

Acknowledgments

The work reported in this paper was made possible through a grant from the A.S.M.E. Special Research Committee on Fluid Meters, assistance in experimental observations by the National Youth Administration workers, and computation assistance from Works Progress Administration personnel. Professor M. P. O'Brien acted in an advisory capacity representing the Fluid Meters Committee.

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Discussion

Howard S. Bean.[4] As stated at the beginning of this paper, the series of tests reported forms a part of the research program on flow nozzles sponsored by the Special Research Committee on Flow Nozzles. It is appropriate to add that the tests made at the University of California were the only ones using water as the fluid, where the nozzle was placed at the end of the pipe, thus giving free discharge into the air. In connection with this program there were a few other tests made at the Ingersoll-Rand plant with nozzles on the end of the pipe to provide free discharge, but in these the fluid was air.

From the standpoint of this particular feature, the second conclusion reported by Dr. Folsom is of particular interest. Except that corner taps were used more often at the University of California than the pipe-line tap locations, there seems to be no reason why this conclusion should be limited to corner taps. The results of the tests with air at the Ingersoll-Rand plant, as well as some earlier tests made by the National Bureau of Standards[5] agree very well with those which were made on the same nozzles within a pipe.

Author's Closure

In discussing the second conclusion of the paper, Mr. Bean calls attention to other results which demonstrate the same correspondence of free and submerged discharge coefficients. This conclusion is generally accepted when a single fluid is present, but some uncertainty exists when a free jet of liquid discharges into a gas. The surface tension has some effect, the magnitude of which is imperfectly understood. The results reported in this paper indicate that this effect is negligible for water discharging into air for the nozzle sizes investigated. The author concurs in the opinion that the pressure tap location has no influence on this conclusion.

[4] Chief, Gas Measuring Instruments Section, United States Bureau of Standards, Washington, D. C. Mem. A.S.M.E.

[5] "Discharge Coefficients of Square-Edge Orifices for Measuring the Fow of Air," by H. S. Bean, E. Buckingham, and P. S. Murphy; Research paper No. 49, United States Bureau of Standards, vol. 2, 1929, p. 561.



FIG. 1 SPECIMEN TESTED

Creep in Tubular Pressure Vessels

BY F. H. NORTON,[1] CAMBRIDGE, MASS.

The paper is a report of tests on the circumferential and longitudinal creep, under internal pressure and high temperature, in steel tubes such as are used in boilers and oil stills. A description of the testing apparatus is given and the results are shown in the form of tables and curves comparing the tubular creep with the creep of tensile specimens at the same stress.

The tentative conclusion is reached that the circumferential creep is about half that of tensile specimens. The longitudinal creep is negligible.

THIS RESEARCH was started to determine the creep properties of cylindrical pressure vessels as compared with the creep properties of tensile specimens of the same steel, at the same temperature, and under a stress equal to the circumferential stress in the cylinder. Previous work[2] has indicated on a theoretical basis that metal under combined stresses has a lower rate of flow than when under a pure tensile stress of the same magnitude, due to the reduction in shear. Some experimental work[3] has confirmed this and some has shown close agreement between the flow under tensile conditions and the flow under combined stresses. It was, therefore, desired to make careful creep tests on closed pressure vessels of the size and material comparable with tubes used in commercial installations, such as superheaters and oil stills. In the fall of 1936 this research was turned over to the Massachusetts Institute of Technology and work was at once started on the design of the apparatus to test such specimens.

SPECIMENS

The specimen shown in Fig. 1 consists of a carefully machined steel tube with welded-on hemispherical ends and a pressure tube welded to the lower end. The specimen contained a filler which allowed only $^1/_{16}$ in. of free space inside the walls for the gas used to provide the pressure. A cross section of a specimen is shown in Fig. 2 in place in the furnace. The dimensions of the specimen selected by the committee were: over-all length, $28^3/_4$ in., outside diameter, 4 in., and wall thickness, $^3/_8$ in. Whether or not the length is sufficient to eliminate entirely end effects in the center is not certain.

Two types of steel were used in these specimens, one a carbon-molybdenum steel and the other a 4 to 6 per cent chrome-molybdenum steel. The characteristics of these steels have been given in a report prepared by metallurgists of The Babcock and Wilcox Company.[4]

THE FURNACE

A drawing of the furnace is shown in Fig. 2. The base plate (1) supports an 18-8 tube (2) which carries the heating elements. This tube is closed at the ends by the disks (3) of K-30 insulating refractory, and the whole furnace is then covered with 1 in. of high-temperature pipe covering (4). Over this is placed a layer of aluminum foil (5) to reduce the radiation. The furnace tube is wound with five layers of mica tape (6) and then over this is wound No. 14 Kanthal A wire with separate end coils in order to permit a uniform temperature over the specimen. The specimen itself has lugs (8) welded to the outside to guide it centrally in the furnace tube, and the weight is taken by the supporting frame (9) which is carried on separate leveling screws so that the specimen can be accurately adjusted in the furnace.

Stainless-steel bolts (10) connect the crosspiece (11) with the upper end of the furnace tube. This crosspiece is connected in turn to the stationary platform (12) by the support (13) and the horizontal springs (14), thus holding the top of the furnace in alignment with the upper platform. A strut (20) resting in a hardened socket (19) screwed into the platform (11) transmits the expansion of the furnace tube to the lever (15) which is supported by a thin-strip knife-edge (17) between the supporting block (16) and the clamp (18). The lever (15) therefore magnifies the motion of the furnace tube approximately 50 times, thereby opening and closing the contacts (21) as the temperature changes. The lower contact can be adjusted by the knurled head (22) passing through the bushing (23) in the insulating block (24). The beam itself is insulated by fiber strips (14). The upper platform is kept at a constant distance from the base plate by two fused-quartz

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[1] Department of Metallurgy, Massachusetts Institute of Technology.

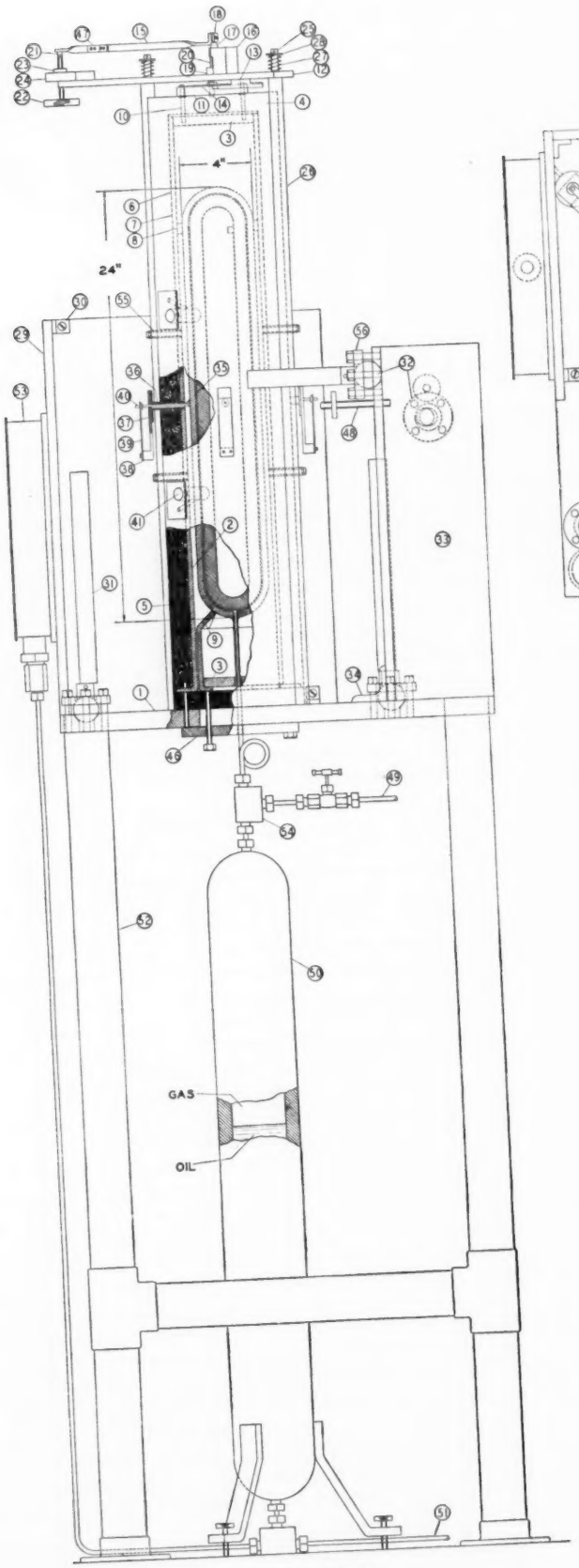
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[3] "Investigation of Creep and Fracture of Lead and Lead Alloys for Cable Sheathing," by H. F. Moore, B. B. Betty, and C. W. Dollins, Engineering Experiment Station, University of Illinois, Urbana, Ill., Bulletin No. 102, vol. 35, Aug. 19, 1938.

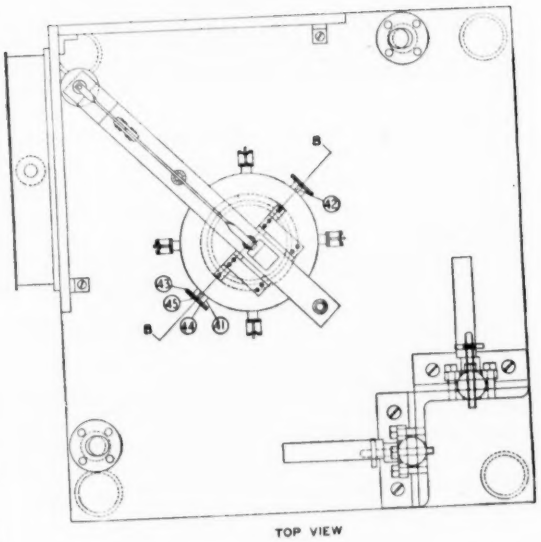
Contributed by the Joint A.S.M.E.-A.S.T.M. Research Committee on the Effect of Temperature on the Properties of Metals, and presented at the Annual Meeting of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS, held in New York, N. Y., December 5–9, 1938.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[4] "Digest of Properties of Carbon and Alloy Steel Tubing for High-Temperature and High-Pressure Service," Technical Bulletin No. 16, The Babcock and Wilcox Co., New York, N. Y., 1938.



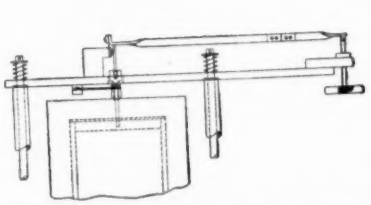
SIDE ELEVATION



TOP VIEW



SECTION ON B-B



SIDE VIEW AT RIGHT ANGLES TO B-B

FIG. 2 FURNACE AND MEASURING EQUIPMENT

tubes (26) which are held in slight compression by the long bolts (25) and the springs (27).

The principle of operation of this temperature controller has been previously described[5] and need not be discussed in detail. It might be said, however, that this method has proved particularly reliable, and the temperature generally stays within ± 1 F. The temperature of the specimen is measured by four thermocouples, one each on the ends of the gage length and two in the center at opposite diameters.

The measurement of increase in diameter is carried out by employing small fused-quartz tubes (35) which rest on a platinum fixture welded to the surface of the tube in the form of a small tack. The outer end of the fused-quartz tube is supported by a cantilever spring (39) attached to the supporting block (38) which keeps a slight pressure on the fused quartz all the time to hold it firmly on the specimen. A reference mark (40) exactly 5 in. from the center of the specimen is used as a sight for the same pair of telescopes used in the 10-in. tensile test. A small alloy tube (36) welded into the wall of the furnace serves to protect the quartz tubes from injury. The telescopes for measuring the diameter of the specimen are supported on a horizontal post (32) which has a ball-and-socket joint as well as an eccentric mounting to allow accurate alignment of the telescopes at the beginning of the test. A fine screw (48) adjusts the telescopes horizontally. Both telescope posts for horizontal measurements are mounted on the heavy angle (33) in order to give a solid support.

The vertical length measurements are made with the same telescopes supported on the post (31) which also has a ball-and-socket joint and an eccentric mounting for complete alignment. The sighting of the telescopes is carried out through the stainless-steel tubes (41) with the plate (43) welded on the face of them. On this plate is a glass window (44) held by a brass frame (45). An angle tube (42) is used to project a light beam on to the specimen at the point of sighting. The automobile headlight lamps used for this purpose are not shown on the drawing. The reference mark on the specimen is a platinum wire, spot-welded to the surface, which is our usual practice with the tensile specimens. Fine V marks on the wire serve as reference points.

As it would be impractical to employ a small pump in the laboratory capable of producing gas pressures of 10,000 psi for these tests, it was thought advisable to use bottled nitrogen gas up to a pressure of 2000 psi and then compress this gas to the desired amount with an accumulator (50) which has a connection to a small hand-operated hydraulic pump (51) capable of forcing oil into the bottom of the accumulator and thus compressing the gas ahead of it. The connection (49) goes to the tank of nitrogen through the tee (54) which connects with a short capillary tube to the specimen itself. Due to the small volume inside the specimen it is possible to compress the gas from 2000 to 10,000 psi quite readily by this method. Thanks to Dr. Keyes of the chemistry department fittings and valves were constructed which were absolutely tight at pressures up to 10,000 psi. Under no conditions did any leaks occur in the system except through the porous metal of the ends of some of the specimens themselves. This was because the cup-shaped ends were turned out of a billet with considerable longitudinal strain. The pressure itself is read by the gage (53) connected to the bottom of the accumulator. The pressure gages were calibrated before and after the test, and while some hysteresis of the spring was noted, the error was not sufficiently large to be taken into account. A second standard gage was also on hand for periodic checking of the pressure. The whole furnace is supported on a pipe-leg frame (52) to bring it up to a convenient height for reading the telescopes.

Two units of this type were constructed and set up in a separate research room especially devoted to this test, as shown in Fig. 3. As it was necessary to have constant room temperature in order to maintain a constant pressure and constant temperature of the furnace, a room-temperature controller was constructed in connection with a powerful ventilating fan which maintained the room at 85 F with variations of not more than ± 2 F. As this controller is of a simple thermostatic type, a description of it is not necessary here.

[5] "A New Device for Creep Testing," by F. H. Norton and J. A. Fellows, *Metal Progress*, vol. 24, 1933, p. 41.

Method of Making the Test

The tubular test specimen was first placed in a jig and the positions of the reference marks and strut attachments were carefully laid out with a fine scratch on the surface of the specimen. It was then taken over to a welder and the platinum attachments spot-welded in place in their correct location. The thermocouples were next put in place and bound to the sides of the specimen with Chromel wire. The thermocouple wires themselves were insulated with porcelain beads to the end of the furnace and then with asbestos tubing. The connecting nipple was then soldered on to the capillary tube with soft solder, which completed the work on the specimens.

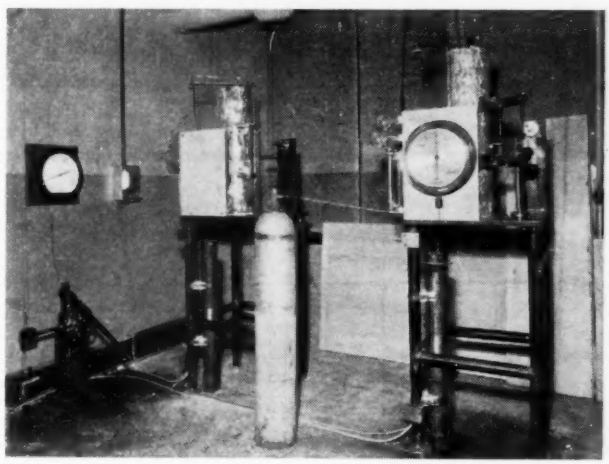


Fig. 3 Testing Laboratory With Two Furnaces

The placing of the specimen in the furnace was a rather delicate operation and it was found advisable to raise the furnace itself with a tackle supported from the ceiling and place the specimen on its supporting ring with another tackle, allowing the capillary tube and the thermocouple to pass down through the bottom of the furnace. The specimen was then adjusted in a vertical position and held there temporarily while the furnace itself was lowered carefully around the specimen, and clamped down with the three bottom bolts. Great care had to be taken during this operation to prevent dislodging the platinum reference marks, and it was found advisable to wrap the center of the specimen with soft twine which protected the platinum marks and which later burned off.

After the furnace itself had been carefully leveled, the specimen was adjusted for height and rotary position as well as possible, but the silica struts were not put in place at this time.

The furnace was heated up gradually, taking a period of several days to reach the equilibrium value. Then, during a period of several weeks, the current through the end coils was adjusted to get a satisfactory temperature distribution. Because of the heavy specimen it was necessary to wait at least 24 hr after each adjustment before the next one could be made. After the temperature had become satisfactorily balanced, the height of the

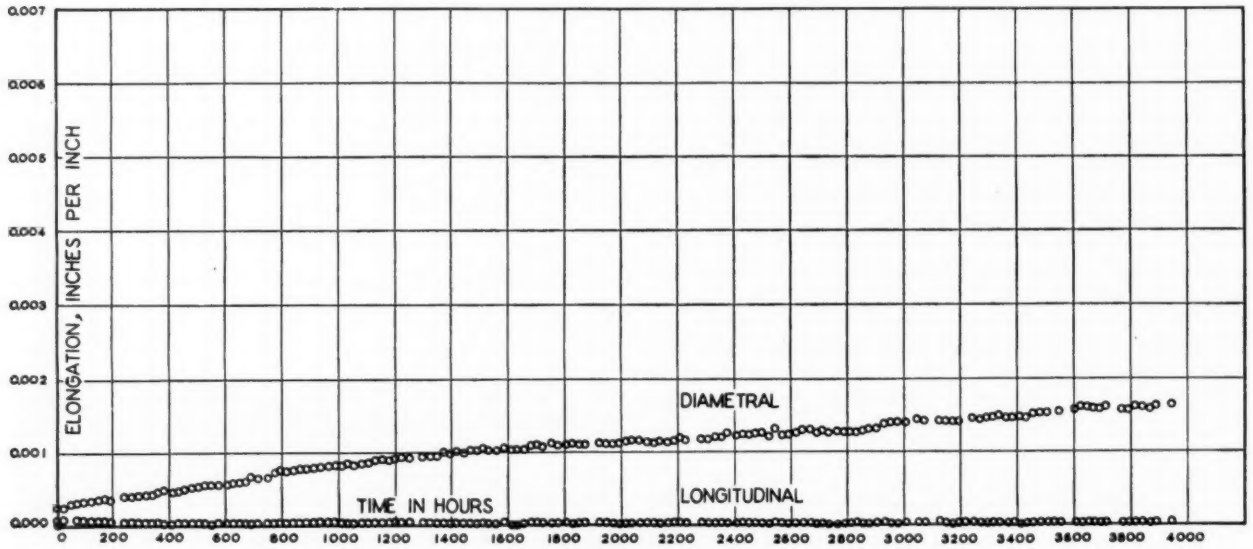


FIG. 4 TUBULAR CREEP TEST
(C-Mo steel at 1050 F and 1617 psi.)

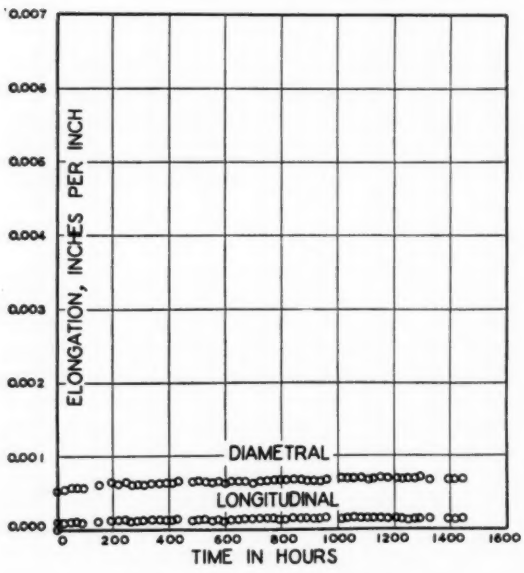


FIG. 5 TUBULAR CREEP TEST
(C-Mo steel at 800 F and 4000 psi.)

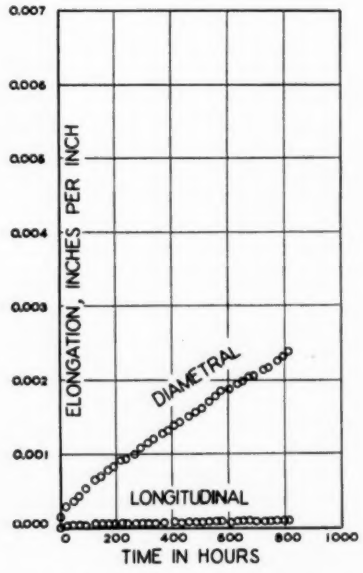


FIG. 6 TUBULAR CREEP TEST
(4–6 Cr steel at 1200 F and 650 psi.)

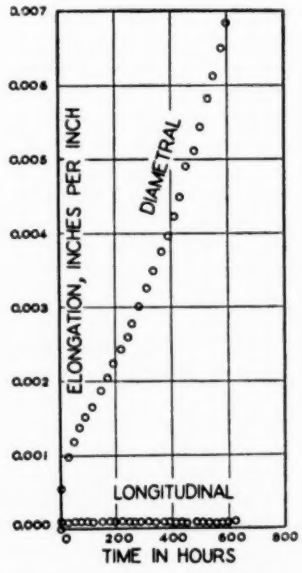


FIG. 7 TUBULAR CREEP TEST
(C-Mo steel at 1050 F and 3230 psi.)

specimen was finally adjusted with the leveling screws and the silica struts were put in place. The telescope posts were then lined up to give simultaneous forms on the two marks.

Length measurements were taken with the telescopes with no pressure, then the pressure was brought up to the working value as quickly as possible. At the lower pressures this could be done in a few minutes, but at the higher pressures it took from 6 to 8 hours' pumping to reach the desired value. Another length measurement was taken with the telescopes immediately after the correct pressure was established, and readings were taken thereafter every 24 hr. All the readings were taken by two observers and the results averaged.

In case a specimen showed a drop in pressure due to leakage, the pressure was pumped up to the correct value every morning and every night. In some cases the leakage was small; in one case there was none at all, and in one case leakage was considerable. Except in the two cases where leakage was severe the pressure was held within ± 1.0 per cent at all times.

TABLE 1 FINAL PRESSURES AND STRESSES

Steel	Temperature, F	Internal pressure, psi	Circumferential stress, psi			Longitudinal stress, psi
			f_i[a]	f_o[b]	f_{av}[c]	
C-Mo	800	4000	19430	15430	17230	7720
C-Mo	800	9240	44890	35650	39792	17830
C-Mo	1050	1620	7870	6250	6970	3120
C-Mo	1050	3230	15690	12460	13910	6230
4–6 Cr-Mo	1200	650	3160	2510	2800	1250
4–6 Cr-Mo	1200	920	4490	3568	3980	1780

[a] f_i = circumferential stress on inside of cylinder.
[b] f_o = circumferential stress on outside of cylinder.
[c] f_{av} = circumferential stress, average.

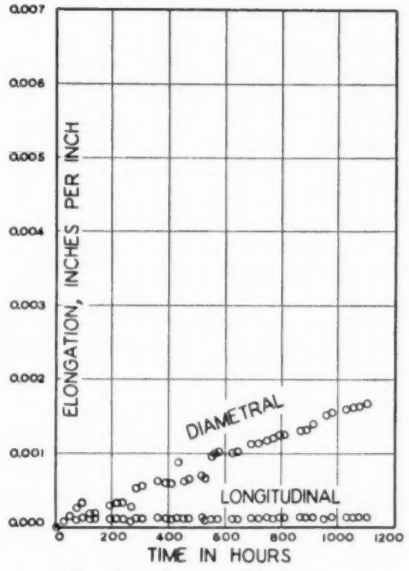


FIG. 8 TUBULAR CREEP TEST
(4–6 Cr steel at 1200 F and 923 psi.)

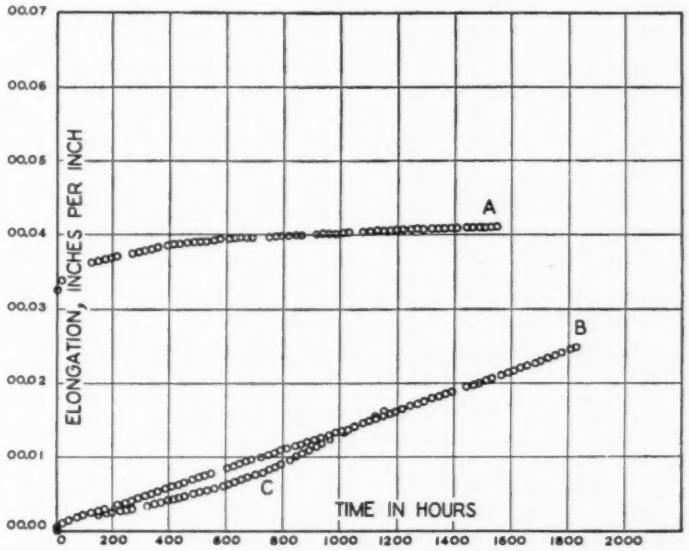


FIG. 9 TENSILE CREEP TEST

A	= C-Mo steel	800 F	39,170 psi
B	= 4–6 Cr steel	1200 F	3,982 psi
C	= C-Mo steel	1050 F	13,933 psi

CONDITIONS OF TESTING

The temperatures selected by the committee for these tests were 800 and 1050 F for the carbon-molybdenum steel, and 1200 F for the 4–6 chrome-molybdenum steel. The duration was to be 1000 hr. The pressures were selected, as nearly as could be determined from past experience on these materials, to give circumferential creep rates of 5 and 50 per cent per 100,000 hr. The loads were altered somewhat after the first test under a given condition in order to adjust the second load more nearly to the conditions wanted. The final pressures and stresses in the metal figured out by the Lamé formula by The Babcock and Wilcox Company are given in Table 1.

PRECISION OF THE MEASUREMENTS

The readings of the longitudinal creep are accurate to two parts in a million, which is equivalent to the regular 10-in. tensile creep tests. The creep in diameter, however, is good only to six parts in a million, because the gage length is only $^1/_3$ as great. In regard to the temperature, the distribution was good to ± 3 F over the gage length, and the variation in temperature from day to day was ±1 F.

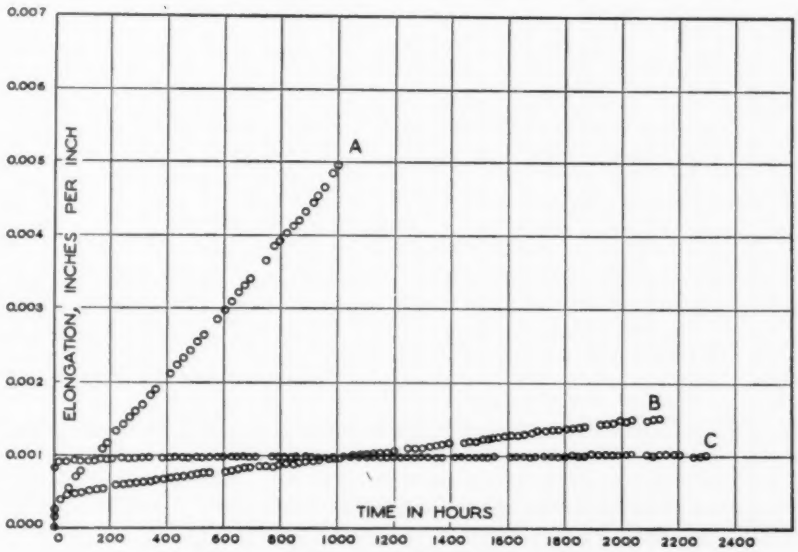


FIG. 10 TENSILE CREEP TESTS

A	= 4–6 Cr steel	1200 F	2,818 psi
B	= C-Mo steel	1050 F	6,973 psi
C	= C-Mo steel	800 F	17,226 psi

Micrometer readings of the diameter in the center of the specimen were made before and after each run. In every case the permanent increase in diameter determined in this way agreed with the telescope readings.

The first specimen was run at 1200 F and initially some difficulty was had in adjusting the silica struts to give an even bearing so that consistent readings could be obtained, but toward the middle of the run this difficulty was eliminated and satisfactory readings were obtained. However, due to the rather rapid leakage of gas through the ends of the specimen it was necessary to pump up the pressure frequently, which gave a condition of alternating stress causing the results to be out of line with the later tests. Therefore, the results are shown merely to indicate the improvement in technique during the first run. The specimen run at 800 F and 9240 psi also leaked so badly that creep rate is only an approximation. The flow curves in both a circumferential and longitudinal direction are shown in Figs. 4 to 8.

It will be noted first that the longitudinal creep is rather low in most cases and sometimes has even a negative value. The circumferential creep gives a curve quite similar to the usual tensile creep curves except there seems to be a tendency to take a longer period of time in coming to an equilibrium rate, which is due, perhaps, to adjustment of stresses in the tubular specimen. Therefore, there is some possibility that the rates given in the first runs may be a little higher than the equilibrium rate even though the tests were run as long as is generally considered necessary in the tensile test. The last specimen run was carried along

for a considerably greater period as requested by the committee at H. J. Kerr's suggestion and we find here a definite decrease in the rate after a period of around 2000 hr, a change which might have occurred in the other specimens had they been carried through this period.

Tensile Tests

Specimens were cut out of the walls of the same batch of tubing that the pressure vessels were made from and these were run in the regular 10-in. furnace as a standard tensile creep test. These were cut with their length parallel to the axis of the tube. The creep curves for these runs are shown in Figs. 9 and 10 and need no special comment.

Comparison of the Creep Rates in the Tubular and Tensile Specimens

Log-log plots of the circumferential creep rate, based on the average stress, and the creep rate in tension are shown in Fig. 11, while the specific data on the creep rates are tabulated in Tables 2 and 3.

TABLE 2 TUBULAR SPECIMENS

Steel	Temp, F	Internal pressure, psi	Circumferential creep per 100,000 hr, per cent	Longitudinal creep per 100,000 hr, per cent	Time interval from which rate is computed, hr
C-Mo	800	4000	0.7	0.6	400–1500
C-Mo	800	9240	10.0	0.0	200–400
C-Mo	1050	1620	5.8 2.2	0.0	200–1200 1800–3900
C-Mo	1050	3230	139.0	0.0	400–800
4–6 Cr-Mo	1200	650	25.0	0.9	200–800
4–6 Cr-Mo	1200	920	15 ?	0.0	600–1100

TABLE 3 TENSILE SPECIMENS

Steel	Temp, F	Load, psi	Rate of creep per 100,000 hr, per cent	Time interva from which rate is computed, hr
C-Mo	800	17230	0.3	400–2300
C-Mo	800	39170	22.0	800–1500
C-Mo	1050	6970	4.8	600–2100
C-Mo	1050	13930	195.0	800–1200
4–6 Cr-Mo	1200	2818	46.0	400–1000
4–6 Cr-Mo	1200	3980	137.0	1000–1800

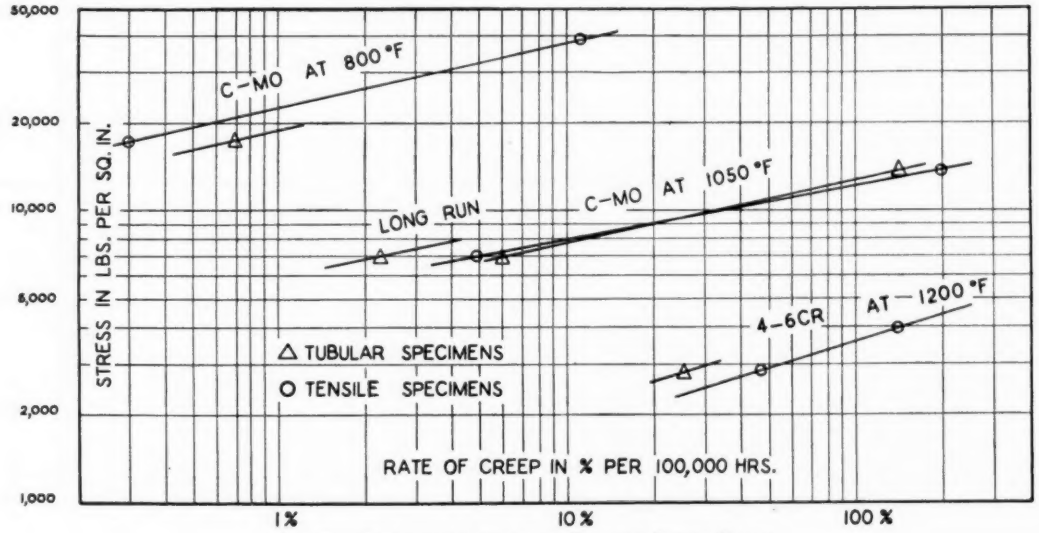


Fig. 11 Circumferential and Tensile Creep Rates

The comparison between the circumferential creep of the tubular specimens, and the tensile creep would indicate that at 800 F the tubular specimens creep faster under the same stress. At 1050 F there is little difference between the two conditions, while at 1200 F the tube creeps somewhat more slowly than the corresponding tensile specimen. The differences, however, are not greatly in excess of the experimental error and it would look in general as if the rates were substantially the same under the two conditions. However, if the tubular specimens are run for a longer period of time, which was done in the case of the carbon-molybdenum steel at 1050 F, a material decrease in the rate occurs from 5.8 per cent at the end of 1000 hr to 2.2 per cent from 2000 to 4000 hr. It would, therefore, seem, at least from this single test, that the tubular specimens take a much longer time to reach an equilibrium rate of flow than the tensile specimens. It is probable that if all of the tubular specimens had been run for from 4000 to 5000 hr the rates would have been materially lower. Some work carried out by Prof. H. F. Moore[3] on the flow of lead tubes confirmed this conclusion, as he states that the tubular specimens required a considerably greater length of time to reach an equilibrium flow than the tensile specimens. If this point of view is correct then it may be stated that the tubular specimens creep at about half the rate of the tensile specimens under the same stress.

It should be realized that it is improper to base any general conclusions on the few tubular tests carried out in this project. More data are needed to supplement the present information in order to formulate definite factors for converting creep rate in tension to creep rates of tubes under internal pressure. Especially, tests should be made with other diameters and wall thicknesses.

The longitudinal creep is in most cases small; sometimes it is slightly positive and sometimes slightly negative. It is probably safe to assume in any design problem a zero change in length of the tubular member during its life.

Acknowledgments

The author wishes to express his appreciation for the assistance received in this investigation by members of the Joint High Temperature Committee. The assistance of The Babcock and Wilcox Company, and especially H. J. Kerr, in the difficult problem of making up the test specimens and for advice and suggestions during the test are especially appreciated. Also, the help and suggestions of C. E. MacQuigg, chairman of the committee, have been most helpful. The work of John Fellows in setting up the apparatus and Donald Fellows in making the observations has been of the greatest assistance in carrying out these tests.

Discussion

C. O. Rhys[6] and M. S. Northup.[7] The purpose of the investigation is of extreme interest because of the high-temperature high-pressure equipment now in service and the certainty that temperatures and pressures of future equipment will increase rather than decrease.

The tests reported by the author carry on more completely and fully a line of investigation touched briefly by tests at room temperature made in our laboratory in 1933 on $3^1/_2$-in. OD × $^1/_4$-in. wall lead tubes to determine qualitatively any difference between creep in tension specimens and creep in tubes. It is of interest that these tests showed that in closed-end tubes the axial creep is negligible, a conclusion drawn by the author from his test results. In this connection R. W. Bailey,[8] Metropolitan-Vickers, Ltd., England, also found that axial creep need not be considered.

The range of creep rate generally used in design work is from 0.1 to 10 per cent per 100,000 hr. An examination of the tables for tubular and tensile specimens shows that most of the deformation rates on the specimens were outside this range and the writers are reluctant to discuss the results. These enormous creep rates may and probably do have a considerable effect on the creep-rate distribution through the wall.

According to experiments made by R. W. Bailey with a lead cylinder $3^1/_4$ in. OD and $^1/_8$ in. thick the diametral creep rate with no axial stress is from three to four times as much as with the axial stress set up by the internal pressure.

From experiments made in our own laboratories with a lead cylinder $3^1/_2$ in. OD and $^1/_4$ in. thick, the ratio is between 1.5 and 2.5 to 1. The theory for a very thin cylinder would give about 5 to 1. It is interesting to note that as the cylinders become relatively thinner the theoretical result is approached as shown in Table 4.

According to Bailey's experiments for these cylinders, the diametral creep rate is a minimum, when the axial stress is that due to internal pressure. It would appear from this that most pressure vessels are subjected to the least possible amount of diametral creep. By treating the $3^1/_2$ × 3-in. lead tube tested in our laboratory as a thick cylinder, the ratio of diametral creep rates at the outside surface, with and without axial pressure stress, is 1 to 2.57. This is a good check with the 2.5 found experimentally. For the steel cylinder of the paper the calculated ratio is 1.42 which is a close check with 195/139 = 1.4.

TABLE 4 RATIOS OF DIAMETRAL CREEP WITH AND WITHOUT THE AXIAL STRESS CAUSED BY INTERNAL PRESSURE

	OD/ID	Creep ratio
Standard Oil Development Co.	1.17	1.5 to 2.5
R. W. Bailey	1.08	3 to 4
Theoretical	1.00	5

Comparisons of creep rate at the inside wall of tubular specimens would be of great interest as it is this creep rate which governs design of cylindrical pressure vessels.

We agree with the author that the few tests covered by the report are insufficient to arrive at any definite conclusions. It is believed that further work along the same lines is desirable, but with deformation rates of from 0.1 to 10 per cent per 100,000 hr and different ratios of diameter to thickness, so that the results will apply to both pressure-vessel and tube design. Ultimately, for tube design, such a program should include the determination of the effects of heat transfer through the wall if any practical way can be found to do so.

Author's Closure

The comments of Messrs. Rhys and Northup give some interesting data on lead cylinders which seem to check well with the theoretical work of Bailey. We quite agree that some of the creep rates in our work were much higher than would be encountered in service, but due to the fact that reliable tensile values of the particular steels used were not available before the tests on the tubes were started, some of the pressures were set too high.

We agree also that it would be desirable to carry the test with a thermal gradient through the tube wall, but the experimental difficulties of carrying this out are so great that we can see no way of accomplishing it without extremely costly equipment.

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[6] Process Design Department, Standard Oil Development Company, Elizabeth, N. J.

[7] General Engineering Department, Standard Oil Development Company, Elizabeth, N. J.

[8] "Utilization of Creep-Test Data in Engineering Design," by R. W. Bailey, Proceedings of The Institution of Mechanical Engineers, vol. 131, 1935, pp. 131–349.

Effect of High Temperatures and Pressures on Cast-Steel Venturi Tubes

By W. S. PARDOE,[1] PHILADELPHIA, PA.

The author presents results of tests showing the expansion in the main and throat diameters of cast-steel venturi tubes under dry heat and pressure, respectively. From these data values are obtained for the coefficient of expansion and the modulus of elasticity, and the comparative effects of high temperatures and pressures are shown.

THE RESULTS of temperature tests conducted on a 5.625 × 4.25-in. extra-heavy cast-steel Simplex venturi tube at H. Brinton Company's oven on April 24, 1929, are shown in Fig. 1.

The coefficient of expansion of the throat per deg F is 0.0000108 and that of the main is 0.00000946; a mean value of 0.000010 might be used. This is greater than the usually quoted values for cast steel and about equal to that of bronze.

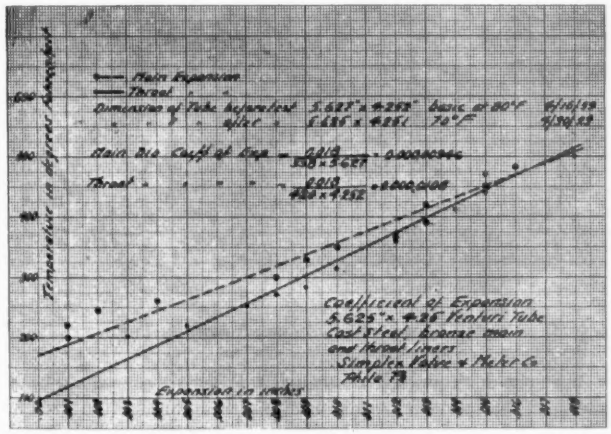


FIG. 1 EXPANSION OF VENTURI TUBE IN DRY HEAT

Since the discharge varies directly as the square of the throat diameter, the error resulting from neglecting the expansion is 0.2 per cent per 100 deg F, which is not a negligible quantity when the rise in temperature may be 800 deg or more.

The tests were conducted as follows:

The basic dimensions were taken at a temperature of 80 F. These were main diam 5.627 in., and throat diam 4.252 in. An outline sketch is shown in Fig. 2. The tube was then placed in the oven, the temperature of the oven was raised to 200 F, and held constant for 15 min. The tube was withdrawn, measured in less than 30 sec, and replaced, after which the oven temperature was again raised and held stationary for 15 min. This process was repeated for each temperature.

The high-pressure tests were conducted in the hydraulic laboratory of the civil-engineering department of the University of Pennsylvania on an 8 × 5-in. extra-heavy cast-steel Simplex venturi tube, dimensions of which are shown in Fig. 2.

The results are shown in Fig. 3 and give a modulus of elasticity (defined as unit pressure, in lb per sq in., divided by unit expansion of the throat diameter in in. per in.) of 17,000,000 lb per sq in.

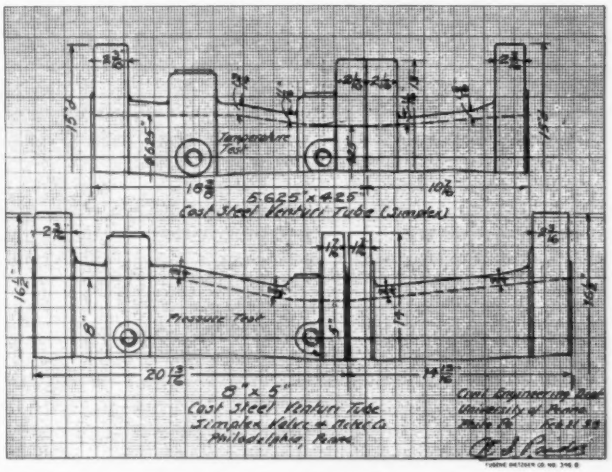


FIG. 2 DIMENSIONS OF VENTURI TUBES USED IN TEMPERATURE AND PRESSURE TESTS

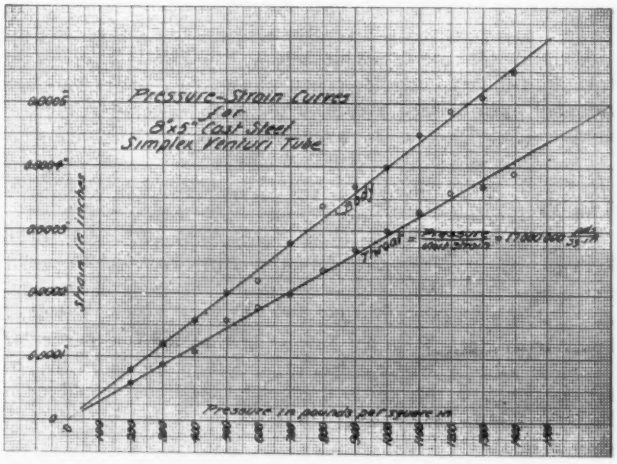


FIG. 3 PRESSURE-STRAIN CURVES FOR 8 × 5-IN. CAST-STEEL VENTURI TUBE

[1] Professor of Hydraulic Engineering, University of Pennsylvania. Professor Pardoe was graduated in mechanical engineering from the Ontario School of Practical Science and from the University of Toronto in applied science. He spent three years at marine-engine and pump drafting and was employed as hydraulic engineer for three years by the Canada Foundry Company. He has served for twenty-seven years as instructor, assistant professor, and professor of hydraulic engineering at the University of Pennsylvania, and as consulting engineer on hydraulic power, dams, and hydraulic problems.

Contributed by the Special Research Committee on Fluid Meters for presentation at the Annual Meeting of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS, held at New York, N. Y., Dec. 5–9, 1938.

Discussion of this paper was closed January 10, 1939, and is published herewith directly following the paper.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

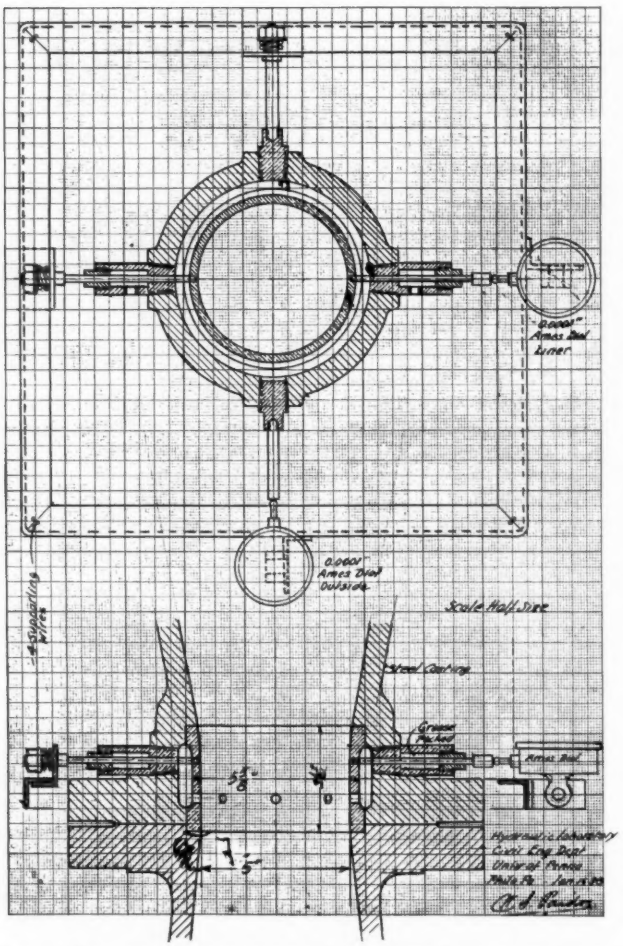


FIG. 4 DETAILS OF APPARATUS FOR MEASURING THROAT AND BODY EXPANSIONS UNDER PRESSURE

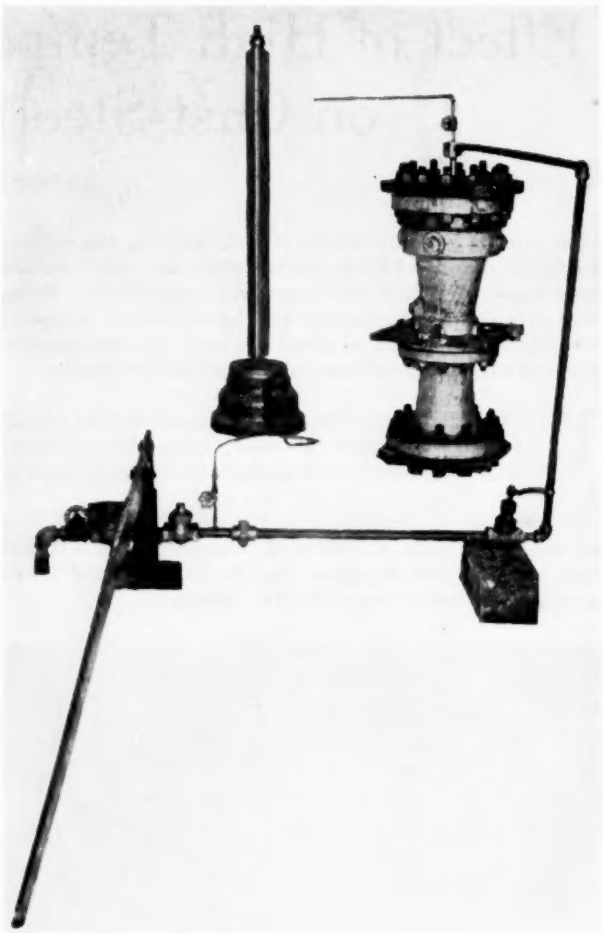


FIG. 5 APPARATUS USED FOR PRESSURE TESTS

Thus there is an expansion of 0.00025 in. for a pressure of 850 lb per sq in., which if neglected would make an error of 0.01 per cent, an entirely negligible quantity.

The apparatus for measuring the throat and body expansions is shown in Fig. 4. Steel rods of $\frac{1}{8}$ in. diam pass through grease stuffing boxes and are tapped into the bronze throat liner. A square frame of 1-in. angle iron, hung by wires from the meter flange, support two 0.0001-in. Ames dials which can be read to 0.00001 in. The meter was blank-flanged and water pumped into it with a 1-in. hand pump having a long lever. The pressure was recorded by a 5000-lb pneumercator gage (differential plunger mercury-loaded). The entire apparatus is shown in Fig. 5.

Discussion

S. R. BEITLER.[2] These tests, especially those having to do with the effect of temperature on venturi tubes, are of major importance for accurate measurements of high-temperature fluids, since such tubes are used generally for the measurement of boiler feedwater. It is not surprising that for this particular design of tube the coefficient of expansion approximates that of bronze; since the piezometer ring behind the throat would mean that the bronze throat section was not supported by the steel shell at this point.

Since the description does not tell whether there was a bronze insert at the inlet-pressure connection, it is difficult to predict whether this reasoning would follow for this section.

A paper[3] which was presented to the Society in 1931, reported on a set of similar tests for sharp-edged orifice plates and steel flow nozzles clamped in steel flanges. The tests showed that for these primary elements it was possible to calculate the increase in diameter for the coefficient of expansion of the metal (monel and 0.12 carbon metal) as accurately as it could be measured. It was also found that no change in shape or flatness was caused either in the nozzles or in the orifice plate.

L. K. SPINK.[4] The author does not draw any conclusions regarding the effect on the flow coefficient due to the measured change in dimensions caused by temperature and pressure. It is possible that warping caused by use of dissimilar metals, having different coefficients of expansion, may create effects exceeding those calculated from the changes in dimensions. It has been found that the use of 18 and 8 steel orifice plates in steel flanges at high temperatures in some cases produces undesirable warping. The present engineering trend is toward choosing orifice-plate materials to minimize warping or toward

[2] Assistant Professor of Mechanical Engineering, The Ohio State University, Columbus, O. Mem. A.S.M.E.

[3] "A Study of Primary Metering Elements in 3-In. Pipe," by S. R. Beitler, P. Bucher, and T. C. Barnes, Trans. A.S.M.E., vol. 54, 1932, paper RP-54-3.

[4] The Foxboro Company, Foxboro, Mass.

mounting the plates with means to permit expansion and contraction without warping.

Nevertheless, it is sound practice to correct for all known variables and to eliminate in so far as possible all unknown variables. The writer believes the author would have been justified in concluding that the venturi-coefficient calculations should be based upon the dimensions at operating conditions.

Author's Closure

Professor Beitler's suggestion that the high coefficient of expansion can be explained by a loose throat piece does not seem tenable. The pressure-strain curves indicate that the bronze throat expands, although subject to no difference of pressure, which would indicate that the bronze throat piece was prestressed by force fitting and expanded with the body. Also, there was no bronze liner in the inlet section and it had approximately the same coefficient of expansion, i.e., 0.00000946 main and 0.0000108 throat.

The author welcomes the reference to "A Study of Primary Metering Elements," by Professors Beitler, Bucher, and Barnes. He should have referred to this before rather than after his experimental work, but such is life: We go through it smashing in open doors. The results indicate an appreciable error in using the normal coefficient of expansion (0.0000065) in this particular tube.

Mr. Spink draws attention to the warping of large orifice plates. Professors Bucher and Barnes say there is none. This paper concerns itself with venturi tubes and the whole idea is to determine the corrections to the diameters as measured at normal temperatures and pressures when using the tubes at high temperatures and pressures.

The author desires to take this opportunity to thank those contributing written and oral discussions to his paper.

Twenty Years of Machine Activity in the Woolen-and-Worsted Industry

By A. W. BENOIT,[1] BOSTON, MASS.

The paper is a discussion of data, compiled from various sources and presented in the form of graphs and tables, tracing conditions in the woolen-and-worsted industry with a view to determining what may be expected in the future. Past performance is analyzed with respect to basic equipment, its active working time, the amount of raw materials used, and working hours.

The conclusions are reached that the liquidation of mills as a whole is nearly finished, that present spindle equipments are not excessive, but that there is still considerable excess loom capacity.

DURING the last twenty years the textile industry has been passing through a critical period, the end of which is not yet in sight. While the depression has been a contributing factor, it was not the principal cause, because the present difficulty made its first appearance about 1923. There has been a buyers' market with rare exceptions for many years, and the textile manufacturer has been at the mercy of the purchaser of his products. He has been unable to make consistent profits, burdensome legislation has dogged his steps, labor unrest has added to his difficulties, raw-stock markets have been unsteady, styles have changed frequently and radically, and, like the caged squirrel, he has gone round and round, getting nowhere.

Within the industry itself there is much hesitation and uncertainty among its members, due somewhat to the rapid political, social, and economic changes which affect its operation. The competition between mills for the business which is available is so sharp that it is destructive, and many liquidations have resulted. This is equally true of both the cotton and wool branches of the industry.

In the eleven years from 1925 to 1936, the number of cotton spindles in the United States dropped from about 38 million to 27 million, or at the rate of a million a year. At the same time, the consumption of cotton by the mills, while it had its usual annual fluctuations, averaged about the same as prior to 1925. An analysis of this situation, made in October, 1937, showed quite clearly that the increased number of hours at which spindles and looms were being operated was the greatest contributing factor and that, if the cotton mills continued to operate 80 hours or more per week, there still would be considerably more machinery in place than would be necessary to supply the country's needs.

A similar analysis of the machinery in the woolen-and-worsted industry was deemed desirable to determine, if possible, the facts as to trends and future possibilities. There are certain data prepared by various federal agencies, as well as those kept by private groups, which are available and from which tables and graphs can be made which show what has occurred. The analysis has been confined, in so far as possible, to that part of the industry which is engaged in making apparel, omitting the carpet and felt manufacturers. These latter are such a relatively small part of the whole that they affect the general picture but little.

Where it has been possible, the data on woolen and worsted machinery have been kept separate, and the reports are so made that this can be done up through the spindles. The looms, however, in many cases weave both woolen and worsted fabrics and it is impossible to make any intelligent division of this machinery.

In order to get a general picture of the industry as it is at present, with reference to its location and size, Table 1 was prepared. Worsted spindles include flyer, cap, and ring spindles, and woolen spindles include both frames and mules.

TABLE 1 UNITED STATES DISTRIBUTION OF WOOLEN AND WORSTED MACHINERY IN 1938

(Information furnished by textile-mill directories and other sources)

Section	Woolen and worsted spindles	Per cent of total	Woolen and worsted looms	Per cent of total
1	3887000	93	52500	92
2	201000	5	3600	6
3	69000	2	900	2
	4157000	100	57000	100

Section 1, north of Maryland and Kentucky, and east of Mississippi River.
Section 2, south of section 1, and east of the Mississippi River.
Section 3, west of the Mississippi River.

It will be noted that the total figures of Table 1 do not check exactly with the data from the U. S. Bureau of the Census, used later in this paper; however, they are sufficiently accurate to show distribution in percentages and, at any rate, they are the only data on this phase of the subject that the author has been able to find.

It is to be noted that the great bulk, over 90 per cent, of this industry is in the northeastern section of the country, including the New England States, New York, Pennsylvania, New Jersey, Delaware, Ohio, Indiana, Michigan, and Wisconsin. Of the total, about 60 per cent is in the New England States and 20 per cent in New York, Pennsylvania, New Jersey, and Delaware. This area is compact and homogeneous, and much alike as to climate, working conditions, labor rates, usual hours of labor, and general competitive conditions such as freight, power, and proximity to raw stock and market.

At the present time a small part of the machinery is in the South, and if this region operates under any different conditions as to hours and rates of labor from the northeastern section of the country, it will have comparatively little effect upon the general situation.

The United States Department of Commerce, Bureau of the Census, reports the monthly activity of woolen and worsted machinery, and its report as of July, 1938, shows the following machinery in possession of mills:

[1] Member of the firm and head of the textile department of Chas. T. Main, Inc. Mem. A.S.M.E. Mr. Benoit was graduated with the degree of B.S. in civil engineering from Tufts College in 1907. He has worked with Chas. T. Main and Chas. T. Main, Inc., since 1909 and in his present position since 1919.

Contributed by the Textile Division and presented at the Annual Meeting of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS, held in New York, N. Y., December 5–9, 1938.

Discussion of this paper was closed January 10, 1939.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

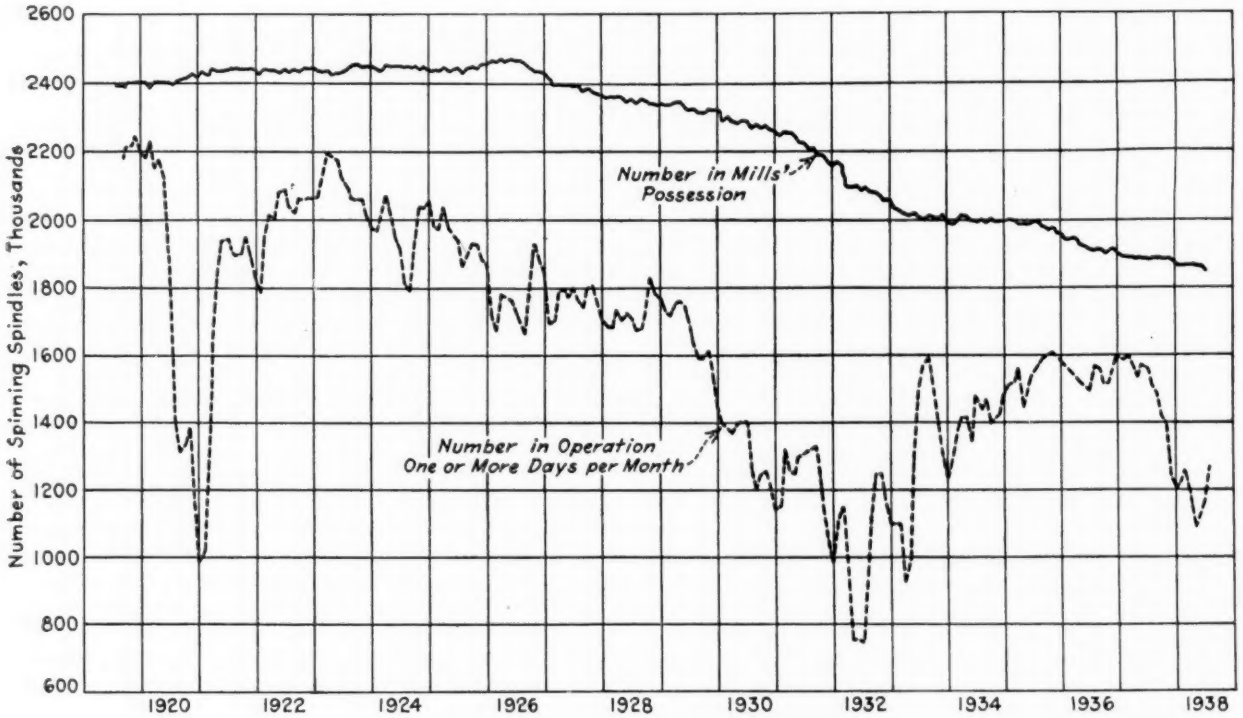


FIG. 1 NUMBER OF WOOLEN-SPINNING SPINDLES

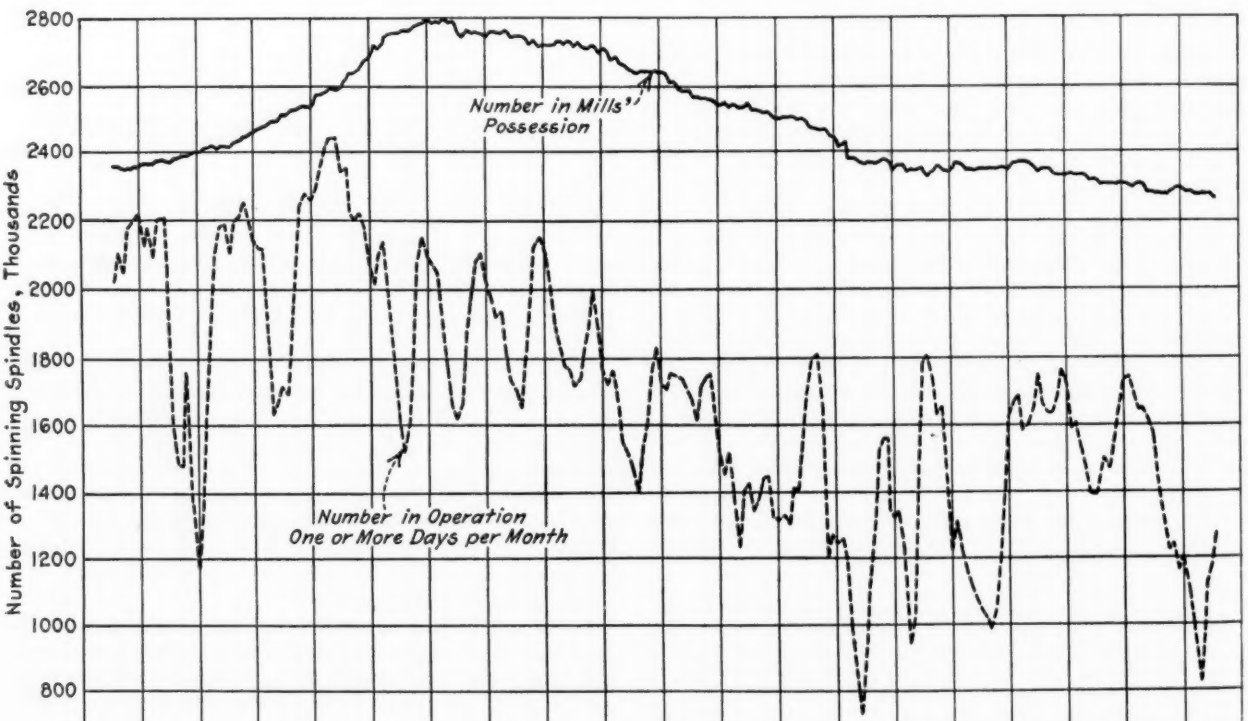


FIG. 2 NUMBER OF WORSTED-SPINNING SPINDLES

Woolen and worsted broad looms[2]	44,887
Woolen and worsted narrow looms	7,948
Woolen-spinning spindles	1,856,844
Worsted-spinning spindles	2,258,045

Of the 44,887 looms listed, 26,307 or 58 per cent are of the automatic type of one kind or another; the remainder, 18,580, are hand feed or nonautomatic. There are about 1,856,000 woolen spindles reported, of which it is estimated that about 78,000 are frame spindles and the remainder are on mules. As stated before, the worsted spindles include flyers, caps, and rings, but no information is available as to the division and all are included under one heading.

The twenty-year period being considered covers two important decades, the first of which was the rehabilitation period after the War, with its tremendous demands for textiles and other commodities. The end of this period saw the greatest boom in securities in our history, culminating in the crash of 1929. The second decade covers the worst depression this country has ever experienced, which has carried all industry, including the textile, to new lows in percentage of activity. Although the crash of 1929 marks the beginning of the decline in industry as a whole, the textile industry reached its peak back in 1923. From 1923 to 1929 there was a gradual decrease in activity and also in the amount of machinery in place, followed by a sharp decline during the depression and up to the present time.

A good picture of what has happened to the woolen-and-worsted industry can be obtained by plotting the data collected by the Department of Commerce and showing graphically the trend of machinery in place and that active. While this information is probably not 100 per cent complete or correct, and only recently has a distinction been made between automatic and nonautomatic looms, it is the best information available and has been used as the basis for these graphs.

Fig. 1 shows the woolen spindles in place from 1919 to and including July, 1938. The peak of spindles in place was reached in 1926, with a total of about 2,477,000 spindles, and since then there has been a continuous reduction year by year to a total of about 1,856,000 in July, 1938, which is a reduction of 621,000 spindles, or 25 per cent. This averaged about 52,000 spindles a year, or approximately 10 ten-set average mills per year. Viewed from this angle we get some real conception of the extent of this decline.

On the same graph is shown the number of active woolen spindles in any month. A spindle is counted as active if it has been operated at any time during the month. This gives a good graphic picture of the tremendous fluctuations in production in this industry. There is one significant thing to be noted, that, even in the boom years of 1919 and 1923, there were always 200,000 spindles idle, or about 8 per cent. Because of its many ramifications, the woolen industry never can be 100 per cent employed, but there are indications that at that time there was an excess of machinery. While the spindle activity started to drop off in 1923, the number of spindles installed increased until 1926. This graph shows that the beginning of the recession for the woolen industry was about 1923, with a gradual sliding off until 1929, and then the sharp drop with little let up, into the depths of the 1932 depression. The effect of the National Recovery Administration is shown by the burst of activity in 1933, followed by the gradual recovery up to 1935 and the see-saw activity of 1936 and early 1937. Then came another sharp decline which did not stop for practically a year, and out of which the industry is now laboriously fumbling its way.

Fig. 2 gives a similar picture of the worsted spindles in the country. It is of interest that in 1919 there were about 2,400,000 woolen spindles in the country and about 2,350,000 worsted spindles, or practically the same number.

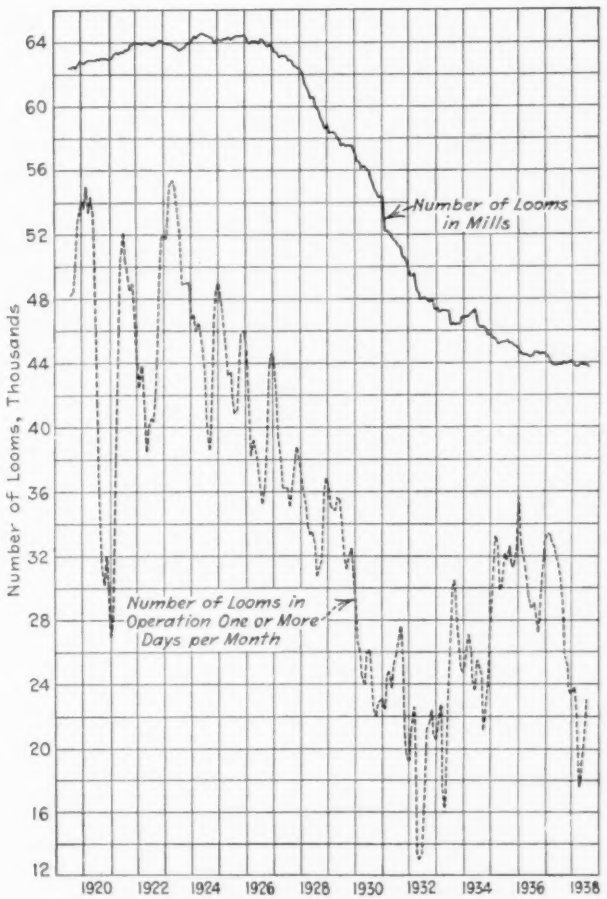


FIG. 3 WOOLEN AND WORSTED BROAD LOOMS[2]
(Includes idle mills.)

The graph shows that the number of worsted spindles increased to about 2,800,000 in 1924, after which there was a steady decline, and in 1938 the number had been reduced to a little less than 2,300,000, or a decrease of 500,000. This is about 18 per cent as compared with the 25 per cent reduction in woolen spindles. The activity curve is, in general, the same as that of the woolen spindles, except that it is considerably more erratic and the fluctuations are more violent. The year 1923 was the peak point of activity, followed by several years of erratic but always decreasing business, which finally reached a record-low-activity period in 1932. The hypodermic effect of the N.R.A. in 1933 is evident, with its sad aftermath. The partial recovery during 1935 and 1936, as in the woolen industry, was followed by the sharp recession beginning in the early part of 1937, which did not turn upward until the spring of 1938.

It is to be noted that the industry continued to install spindles after the peak of activity had been reached. The fluctuations in machinery activity are so great in the industry that it is difficult at any time to forecast future trends. In 1923 the industry came very close to running all of its spindles for a short time, but since then there has always been a half million or more idle.

This picture would not be complete without showing a graph of the looms in place and their activity. Since the same looms may use both woolen and worsted yarns, they cannot be sepa-

[2] Looms with more than 50 in. reed space are classified as broad looms.

rated as can the spindles, and so they have been grouped into broad looms[2] and narrow looms. The broad looms represent 85 per cent of the total number in place and the narrow looms are of only minor importance.

Fig. 3 shows the number of broad looms in place from 1919 to July, 1938, inclusive. The maximum number of looms was about 64,600, reached in 1924, following the great activity of 1923. There was no appreciable reduction in the number of looms in place until 1926, from which time, as the graph shows, the liquidation has been continuous and rapid until at the present time there are less than 45,000 in place. This is a decrease of 32 per cent or a reduction of about 1700 looms a year. The year of maximum liquidation was 1931, when there was a reduction of about 4600 looms.

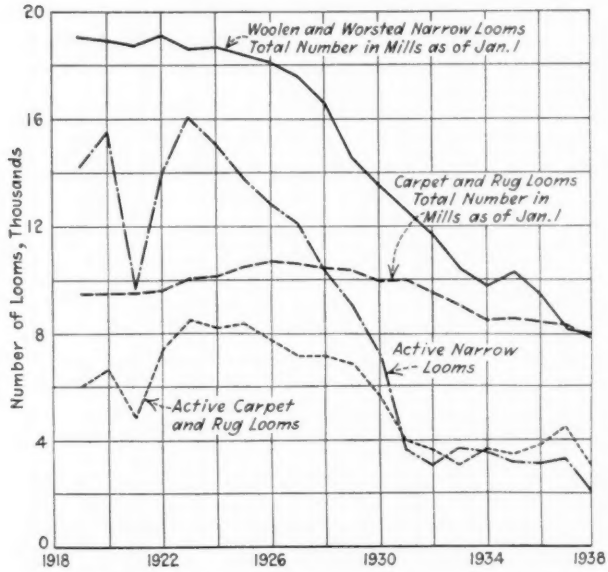


FIG. 4 WOOLEN AND WORSTED NARROW LOOMS

On the same graph is shown the number of active looms, which represents all looms which have run any part of the month in which they were reported. It is to be noted that during these 20 years, for only one period of 3 consecutive months were there more than 55,000 looms active, which is about 85 per cent of those installed. The nature of the woolen-and-worsted industry is such that except in boom periods it does not expect to operate on an average of over 75 per cent of the full capacity, which, at the maximum number of looms in place, would have been about 48,500 looms. Until 1926 the average of activity was in this neighborhood, but in 1930, with 56,000 looms in place, the activity was 25,000; in 1932, the low point, with 48,000 in place, the activity was from 13,000 to 22,000; in 1934, with 46,000 looms in place, the activity was 26,000; and in 1935, with 45,000 in place, the activity was 32,000, or 71 per cent (nearly normal); and in 1938, with 44,000 looms in place, the activity is 22,000, or 50 per cent.

This graph definitely shows a sharp division between the first and second decade of the period considered. During the ten years previous to 1928, the activity was seldom less than 36,000, while in the years since then it has never reached 36,000.

In order to complete the presentation of the loom situation, Fig. 4 has been prepared to show narrow looms in place and active. This shows that there were 19,000 narrow looms in place in 1919 and that in 1938 only 8000 of them remained. It also shows that the peak of activity was in 1923 with 16,000 looms running, and that at the present time there are only 2000 active. It is to be observed that the narrow-loom activity was between 3000 and 4000 looms from 1930 to 1937. The demand for narrow goods has been decreasing steadily and this is reflected in the loom activity. It is not possible to say just how far this will go, but it would seem from the indications that it may reach the point of almost complete extinction. It would naturally be assumed that the broad-loom activity would have been benefited by the decrease in narrow fabrics, and undoubtedly it was, but in spite of this the number of broad looms decreased steadily during this period.

On this graph also is shown the carpet looms, in place and active, in the United States. These are shown only as a matter of passing interest.

The graphs which have been presented show the extent of the liquidation of the spindles and looms in the industry during the 20-year period, and the decrease in activity which has been even more marked. The question naturally rises as to the causes for this apparent wiping out of from 25 to 50 per cent of an industry in 20 years.

A number of things may have contributed to this result, such as, (1) more efficient machinery, (2) lower consumption of woolen and worsted fabrics, and (3) a greater number of hours of operation of the active machinery.

There have been some rather radical changes in a few operations which would have had some effect on the amount of machinery required.

The spinning of woolen yarns on frames instead of mules has reduced the number of spindles required. The best information available indicates that there are about 78,000 frame spindles which, under average conditions, it is estimated would have replaced about 120,000 mule spindles, so that there might have been expected a reduction of 42,000 spindles due to this factor alone, but there was a reduction of 600,000.

The introduction of ring spinning in place of cap spinning in the making of worsted yarns might account for some reduction in this machinery, but not for the 500,000 spindles which have disappeared.

The wide adoption of automatic looms was a real factor, however, in the number of looms required. Twenty years ago probably not more than 3000 looms, or about 5 per cent were automatic. Today, there are reported to be 26,307 automatic looms, many of them of the latest models operating at much higher speeds and efficiency than the old nonautomatic looms. Some measure of this can be obtained by a study of the ratio of spindles to looms for the 20-year period. Table 2, giving the numbers of spindles and looms every 5 years, shows the trend.

TABLE 2 RATIO OF SPINDLES TO WOOLEN AND WORSTED BROAD LOOMS

(Machinery in place)

Year	Number of spindles in thousands			No. of looms	Spindles per loom
	Woolen	Worsted	Total		
1919	2393	2354	4747	62289	76.0
1924	2445	2718	5163	64111	80.5
1929	2345	2638	4983	58363	85.6
1934	1994	2368	4362	46834	93.0
1938	1866	2281	4147	44887	92.2

The average number of spindles per loom has increased from 76 to 93, or about 22 per cent, in spite of the fact that the spindle production per hour has also increased. Although there are no data available, it is quite certain that where the fabrics will permit, the automatic looms are run in preference to hand-feed looms. It is probably a conservative statement to say that the average active loom-hour today produces 20 per cent more than the loom-hour of 1923. This would account for a reduction of about 13,000 looms, whereas the actual reduction has been about 21,000.

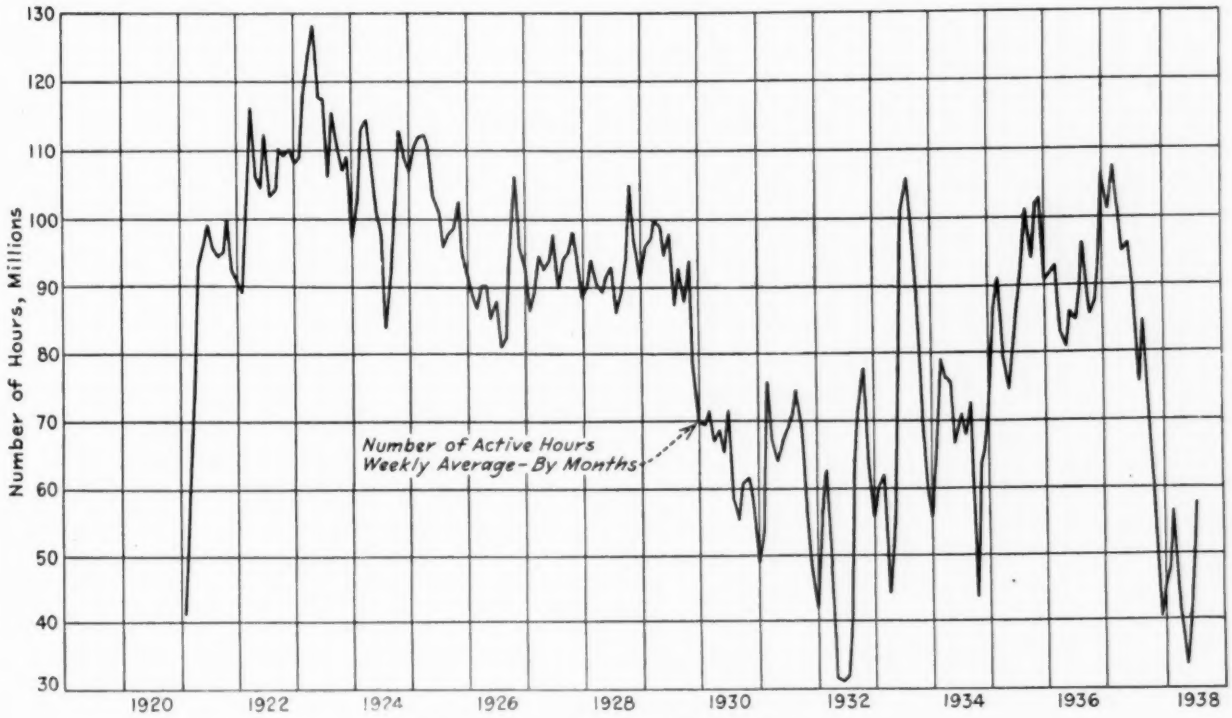


FIG. 6 ACTIVITY OF WOOLEN-SPINNING SPINDLES
(Active hours, weekly average by months.)

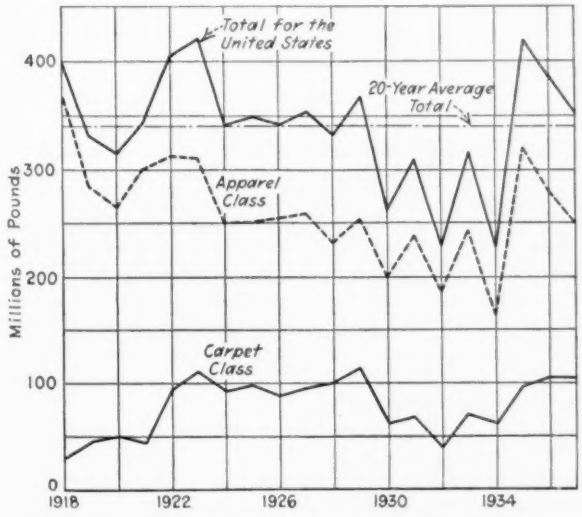


FIG. 5 RAW-WOOL CONSUMPTION BY CLASS, SCOURED BASIS

It would seem, therefore, that machinery changes alone account for only a part of the liquidation.

The question of consumption changes presents some difficulties. The worsted-and-woolen industry consumes a great variety of raw materials besides wool, such as animal hair, cotton, rayon, and silk, and in addition uses large quantities of reworked wools.

The information available from the government reports does not cover all years, and is in itself a little confusing. However, Table 3 shows the various materials used per year for the years when it was reported. The figures are probably not entirely correct for the minor items, but are reasonably so for the main ones and any inaccuracy would not affect the results materially.

The totals would indicate that there has been no general downward trend in production. The years from 1925 to 1929 were normal years and consumption was practically constant. The low production of 1931 is accounted for by the depression, and the 1935 peak by the increasing production from the 1932 low point. The average for the years reported is 523.45 millions of pounds of materials used, which is close to the more normal years from 1925 to 1929.

In this same table is given the total time which the looms were run for each year. From this was determined, at least arithmetically, the average production in pounds per loom per hour, which is given in the table. It probably is not entirely correct but it indicates that the production per loom-hour does not vary too much even with wide fluctuations in total production, and that the trend is toward higher levels, as one would expect.

Since the principal raw material of the woolen-and-worsted industry is wool, Fig. 5 was prepared, which shows the total consumption of scoured wool in the United States per year and that used by the apparel industry.

While the total consumption during the last 10 years is less than in the 10 preceding years, the 1935 peak would indicate that, under more normal conditions, consumption of wool might

TABLE 3 PRINCIPAL MATERIALS USED IN WOOLEN-AND-WORSTED-GOODS INDUSTRY

Material	Millions of pounds used in—					
	1935	1931	1929	1927	1925	1919
Wool, scoured basis	417.6	311.0	368.1	354.1	349.9	329.1
Animal hair	27.3	15.4	27.5	26.1	27.2	28.3
Cotton	12.5	14.5	20.2	22.8	26.6	17.3
Rags, clips, and recovered wool	111.4	51.8	93.0	81.6	106.5	79.6
Rayon	12.7	2.6	3.5	1.5	2.5	....
Silk and spun silk	0.65	0.45	1.2	1.9	0.6	0.4
Cotton yarns	37.3	27.0	43.0	31.5	26.8	28.7
Total materials	619.45	421.75	556.50	519.50	540.10	483.40
Yearly run, millions of loom-hours	95.6	74.4	95.6	98.6	112.8	....
Pounds per loom-hour, average	6.50	5.66	5.82	5.23	4.78	....

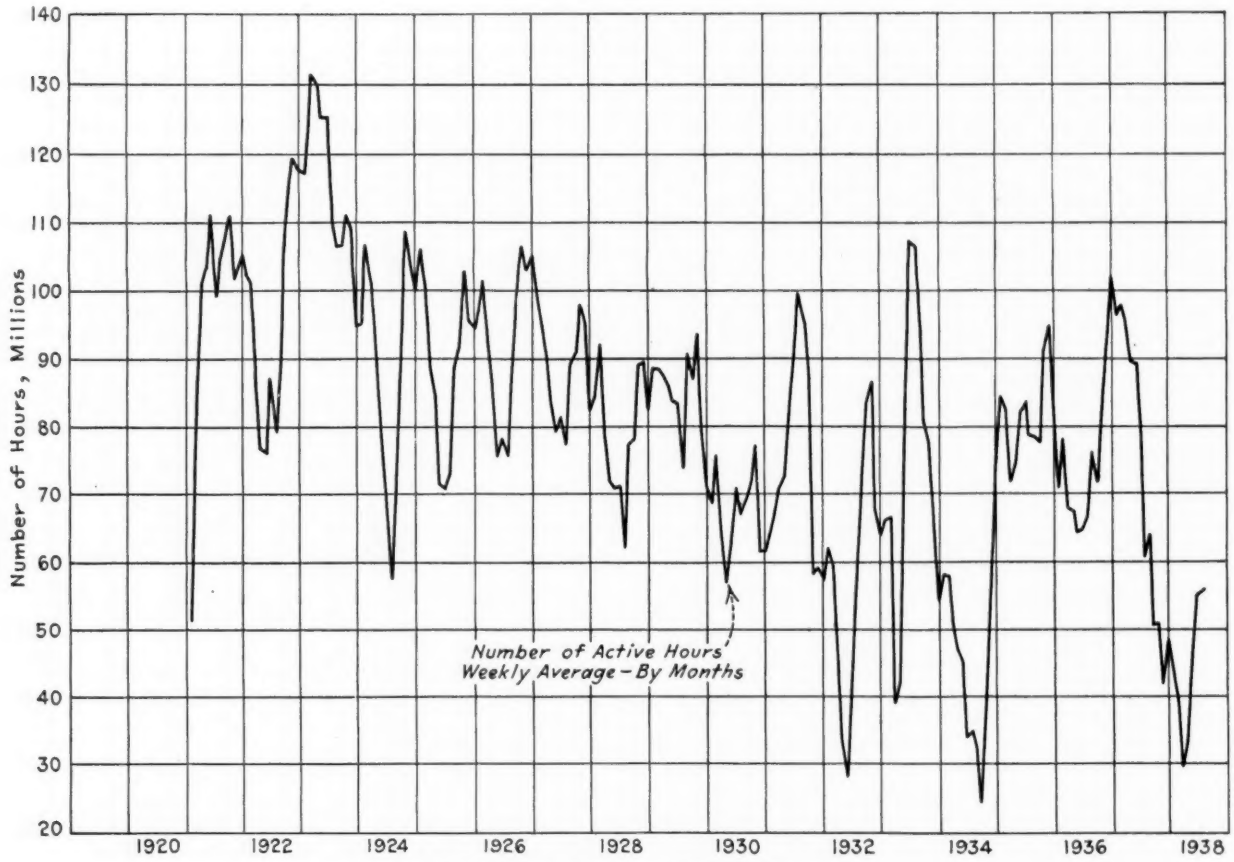


FIG. 7 ACTIVITY OF WORSTED-SPINNING SPINDLES
(Active hours, weekly average by months.)

be expected to be as much, if not more, than was consumed from 1919 to 1929. The table previously given, of all materials consumed, would lead to the same conclusion.

It is a logical conclusion that the use of higher production and more efficient machinery does not account for the decrease in machinery installed, and also that there is no permanent reduction in the quantity of raw materials consumed. In 1935, 319,000,000 lb of wool were consumed for apparel purposes as compared with 312,800,000 lb in 1922, with 534,000 fewer spindles, and 15,700 fewer looms.

And so the next point is the consideration of the hours of operation of the machinery in the industry. The actual number of spindle- or loom-hours operated per week or per year is, after all, the best measure of the condition of the industry, and a good indicator of consumption. It will be recalled that the amount produced per loom-hour varies little between periods of high and low consumption. There have been prepared some graphs of spindle- and loom-hours operated, which are interesting. These graphs show by months the weekly average for the month of the hours of operation. They do not show the maximum and minimum weeks but these would not be very different.

Fig. 6 shows the woolen-spindle activity on this basis. The graph accentuates what has been indicated on some of the previous graphs, and that is the higher general average of production of the first decade over the second. For the first time a real picture is given of the activity of recent years as compared with the period prior to 1930. In 1933, 1935, and 1937, the hours of operation reached a plane which compares favorably with the heights reached prior to 1930.

Fig. 7 shows the same information for the worsted spindles. It is to be noted that the annual fluctuations for these spindles are much greater than for woolen spindles. Peaks of activity and the depths of slack periods are more extreme than in the woolen industry but the general shape and trend are the same in both cases. It is the general impression that operations are much more erratic now than previous to 1929 and this graph would seem to bear out that impression, but it is also evident that the earlier period was not without its troubles. The greatest slump was the one that started in the middle of 1933 and continued downward for 15 months, almost without a stop.

Fig. 8 shows the average weekly number of hours of broad-loom operation, plotted by months. In general form, it follows the two previous graphs with its peaks and depressions coinciding as to time and varying as much in their swings.

The loom-hour is probably the best index of production in the woolen-and-worsted industry and because many looms use both woolen and worsted yarns, its activity is a reflection of the conditions in both branches. There is a noticeably general downward trend from 1921 to 1932 and what appears to be an upward trend since that time. The 1937 recession cast some doubt as to "where we go from here." It must be remembered that, in the period from 1926 to 1938, the number of looms in place dropped from about 64,600 to about 43,900 in spite of the fact that the 1937 consumption of raw stock was the maximum since 1919 when the keeping of such records was started.

Another interesting observation is that it required 12,750,000 loom-hours to consume 540,100,000 lb of raw stock in 1925, and in 1935 only 10,400,000 loom-hours for 619,450,000 lb of raw stock.

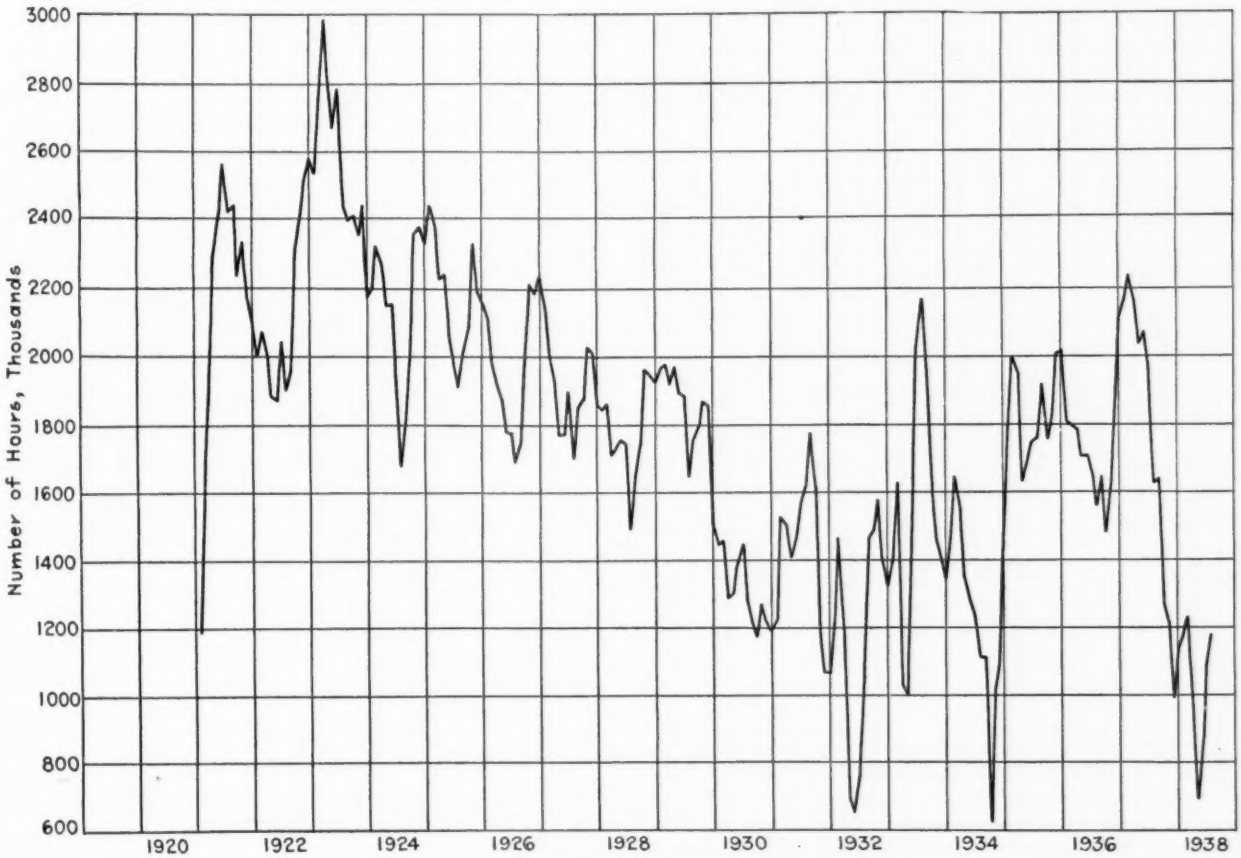


FIG. 8 ACTIVITY OF WOOLEN AND WORSTED BROAD LOOMS
(Active hours, weekly average by months.)

In order to properly evaluate such comparisons, there are tabulated in Table 4 several computations which were made of woolen and worsted spindles and looms at the beginning and end of the 20-year period. The raw-consumption figures are only available for certain years and the hours of operation only since 1921, so the years 1925 and 1935 were chosen because complete information was available for those years. However, since it covers only ten years, Table 4 does not reflect the full change in the 20 years.

TABLE 4 COMPARISON OF HOURS OF OPERATION IN 1925 AND 1935

Year	Thousands of spindles: In place	Thousands of spindles: Active	Approximate average, spindle-hours per week	Hours run per week: Spindles in place	Hours run per week: Active spindles
WOOLEN					
1925	2420	1950	92,700,000	38.4	48
1935	1950	1550	90,500,000	46.5	58.5
WORSTED					
1925	2760	1900	89,000,000	32.5	47
1935	2340	1650	82,500,000	35.3	50

Year	Looms: In place	Looms: Active	Approx. average loom-hours per week	Hours run per week—Looms in place	Hours run per week—Active looms	Raw materials, millions lb
LOOMS						
1925	64200	44200	2,168,000	34	49	540.1
1935	45200	32070	1,847,000	41	57.5	619.4

There are certain observations which can be drawn from Table 4, as follows:

1 While the number of woolen spindles in place was reduced 23.5 per cent, and the number of active spindles 20 per cent, the average weekly number of spindle-hours increased 21 per cent for those in place and 22 per cent for those active.

2 The number of worsted spindles decreased by 15 per cent of those in place and 13 per cent of the active spindles, but the number of hours operated per week for both in-place and active spindles increased only 6 per cent.

3 From the total yearly number of spindle-hours run by woolen and worsted spindles in 1925 and the raw-stock consumption of 540 million pounds per year, it is found that each spindle accounted for 2.70 lb of stock per week, whereas the corresponding figures for 1935 show that each spindle accounted for 3.76 lb per week, or an increase of 39 per cent.

4 The raw-stock consumption of 1925 divided by the loom-hours operated per year, gives 234.7 lb of raw-stock required to supply an active loom for an average week, while in 1935 an active loom, operating for its average weekly number of hours required 372.9 lb of raw stock, or an increase of 59 per cent.

The year, 1925, was quite normal for that period as to raw-stock consumption and hours of operation. The year, 1935, was a year of high consumption and a quite different setup as to hours of operation. In 1925 the mills in general operated one shift with some overtime, but in 1935 they were endeavoring to run two 40-hr shifts per week.

The figures would indicate that the production per loom had increased materially between 1925 and 1935. Eventually the 1937 figures will be available and this trend can be traced further.

The industry, because of its many ramifications in stocks and fabrics, the wide fluctuation in seasonal activity, and the relatively narrow range of yarns over which spindles can operate, is

bound to have a considerable number of spindles idle at any one time even though they all operate on an average 75 per cent of the year. It is not clear, therefore, that there will be much more liquidation of spindles except that due to the introduction of more efficient machinery.

The situation with reference to looms is somewhat different. Table 5 shows the average weekly number of hours operated per loom from 1921 to 1937. The first column is the weekly average for the year; the second column is the weekly average for the maximum month and the third column is the annual consumption of raw stock for the years in which it is available.

TABLE 5 ACTIVITY OF WOOLEN AND WORSTED BROAD LOOMS

Year	Active loom-hours, in thousands: Average week of year	Active loom-hours, in thousands: Average week of maximum month	Raw stock consumed, millions lb
1921	2140	2563	...
1922	2130	2577	...
1923	2560	2986	...
1924	2140	2389	...
1925	2170	2433	540
1926	1960	2235	...
1927	1900	2136	520
1928	1780	1963	...
1929	1840	1984	557
1930	1310	1463	...
1931	1430	1783	422
1932	1190	1584	...
1933	1550	2167	...
1934	1250	1649	...
1935	1840	2021	619
1936	1760	2119	...
1937	1710	2244	...
17 years' average.....	1810	2140	...

The noticeable feature is the steady decrease in loom-hours over the period without a corresponding decrease in raw-stock consumption. Assuming that the consumption of woolen and worsted fabrics is to continue at about the same rate as the present average year, which is reasonable, the figures would indicate that the average number of loom-hours required per week will be around 1,800,000 and the maximum not over 2,250,000. This gives a measure which can be applied to determine the number of looms which the industry can support for different numbers of weekly operating hours. Table 6 shows the results of applying this yardstick.

TABLE 6

Hours operated per week	No. of looms[a]: For 100% operation	No. of looms[a]: For 75% operation	No. of looms[b]: For 100% operation	No. of looms[b]: For 85% operation
40	45250	60400	56100	66000
54	33500	45000	41600	49000
80	22600	30000	28100	33000
108	16800	22400	20900	24600
120	15100	20100	18700	22000

[a] Based on an average number of 1,800,000 loom-hours per week.
[b] Based on an average number of 2,250,000 loom-hours per week.

On the basis of 1,800,000 loom-hours average per week, column 2 would indicate the number of active looms and column 3 the looms in place assuming 75 per cent activity for the conditions indicated in column 1. Column 4 gives the number of active looms on the basis of using 2,250,000 loom-hours per week average but it is assumed that if the activity reached this high level there would be only 15 per cent idle and column 5 gives the number of looms in place using this assumption.

It is probably useless to try to predict the actual hours of operation which legislation or other agencies will determine, but it is highly improbable that the work week will be a single 40-hour shift, because of insufficient capacity, or a single 54-hour shift because of opposition from many sources. Neither is the third shift likely to be acceptable in the long run, and operation for 120 hours per week is therefore doubtful. The most probable hours will be 80 per week, or possibly a few more. From the table it is seen that this would not require more than 33,000 looms, or about 10,000 fewer than are now in place. No prediction to this effect can be made, but a study of the graph of looms in place gives no clear indication that the liquidation has ceased. It is possible that there is no great excess of spindles but the further introduction of automatic looms must necessarily reduce the number required and the new automatic looms which can be converted to 4 × 4 box looms for pick-and-pick fabrics eliminates the necessity of keeping the older looms for that purpose, and again reduces the necessary number of looms in place.

The author is inclined to believe that the liquidation of woolen and worsted mills as a whole is nearing its close although there will be some further reduction in numbers of spindles and looms in operating plants.

The loom situation, however, is not quite as bright and there are many indications that the number of looms can be further reduced without any lessening of the amount of goods produced. This, again, does not mean the liquidation of existing weaving and finishing plants, but rather a reduction in the number of looms which they will operate.

The making of woolens and worsteds is a basic industry and its continuance at its present average rate of production is an absolute necessity. This does not insure, however, that because a plant is in existence today it must continue; only those who keep abreast of the times in their mechanical equipment, who foresee style trends, and who exercise good judgment in the management of their plants can expect to go on.

The Elastic Theory of Wood Failure

By CHARLES B. NORRIS,[1] GRAND RAPIDS, MICH.

This paper deals with various theories advanced for the failure of wood. The formulas developed by Jacoby, Howe, and Hankinson for expressing the crushing strength of wood are examined and their relative practical value indicated. Of the three the Hankinson formula, while being strictly empirical, agrees with experimental results better than the others. The author then proceeds to outline the steps of derivation of Hankinson's formula, applying the theory of failure advanced by Hencky. A second theory is analyzed which also leads to Hankinson's formula. The author points out that the methods used in deriving Hankinson's formula are very approximate but that such a formula is better than none.

THE FAILURE of wood has long been studied by the Forest Products Laboratory. Publication has been made of the strength values of woods grown in the United States when the stress is applied either parallel or perpendicular to the grain (1).[2] This study was greatly accelerated during the World War due to the extensive use of wood in the construction of aircraft. Numerous studies have also been made of the compressive strength of wood when the stress is applied at an angle to the grain of the wood. As a result various formulas have been developed, notably those of H. S. Jacoby, N. A. Howe, and R. L. Hankinson (2).

These formulas all express the crushing strength of wood N, in terms of the crushing strength of the wood parallel to the grain P, the crushing strength of the wood perpendicular to the grain Q, and the angle to the grain θ at which the stress is applied. The formulas are

$$\text{Jacoby} \ldots\ldots N = P\cos^2\theta + Q\sin^2\theta$$

$$\text{Howe} \ldots\ldots N = Q + (P - Q)\left(1 - \frac{\theta}{90}\right)^{1/2}$$

$$\text{Hankinson} \ldots\ldots N = \frac{PQ}{P\sin^2\theta + Q\cos^2\theta}$$

The Jacoby formula gives values which are too high compared with experimental results. The Howe and Hankinson formulas give very similar curves, especially for values of θ over 25 deg. The Hankinson formula is strictly empirical and agrees with the experimental results better than the other two. It seems to be the best formula but it cannot be applied to more general conditions of stress because of its empirical nature.

Considerable light is thrown upon the problem by applying the theory of failure developed by H. Hencky (3). This theory is very clearly described by A. Nádai (4). In this theory the total energy per unit volume, stored in a body due to elastic distortion, is divided into two parts, that due to the change in volume and that due to the change in shape. It is assumed that, when the elastic energy due to the change in shape reaches a certain value, the material changes from an elastic condition to a plastic one, i.e., it starts to fail. This has been found to be true of a great number of materials. The energy constant is different for each material but is independent of the manner in which the material is stressed.

The total energy stored in the material per unit volume is

$$^1/_2(s_1e_1 + s_2e_2 + s_3e_3) \ldots\ldots [1]$$

The energy stored due to a change in volume is

$$^1/_6(s_1 + s_2 + s_3)(e_1 + e_2 + e_3) \ldots\ldots [2]$$

where s_1, s_2, s_3 are the principal stresses and e_1, e_2, e_3 are the principal strains.

Subtracting Equation [2] from [1], the energy stored per unit volume, due to the change of shape, is obtained as follows

$$^1/_6(2s_1e_1 + 2s_2e_2 + 2s_3e_3 - s_1e_2 - s_1e_3 - s_2e_1 - s_2e_3 - s_3e_1 - s_3e_2) \ldots\ldots [3]$$

For simple compression $s_2 = s_3 = 0$, and Equation [3] becomes

$$^1/_6(2s_1e_1 - s_1e_2 - s_1e_3) \ldots\ldots [4]$$

or

$$\frac{s_1}{6}(2e_1 - e_2 - e_3) \ldots\ldots [5]$$

The three relations are

$$e_1 = \frac{s_1}{E_1}, \qquad e_2 = -\frac{s_1}{E_1}r_1, \qquad e_3 = -\frac{s_1}{E_1}r_2$$

where, r_1 and r_2 are the Poisson ratios in the two directions and E_1 is the modulus of elasticity in the direction of the stress s_1. Substituting these values in Equation [5], the energy per unit volume due to the change in shape becomes

$$h = \frac{s_1^2}{6E_1}(2 + r_1 + r_2) \ldots\ldots [6]$$

If this equation is applied to wood, first applying the stress in a direction parallel to the grain and then in a direction radial to the annular rings, the two following equations are obtained

$$h_L = \frac{s_L^2}{6E_L}(2 + r_{LR} + r_{LT}) \ldots\ldots [7]$$

$$h_R = \frac{s_R^2}{6E_R}(2 + r_{RL} + r_{RT}) \ldots\ldots [8]$$

At the point of failure

$$H = \frac{P^2}{6E_L}(2 + r_{LA} + r_{LR}) \ldots\ldots [9]$$

and

$$H = \frac{Q^2}{6E_R}(2 + r_{RL} + r_{RT}) \ldots\ldots [10]$$

Substituting values obtained from Equations [9] and [10] in

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[2] Numbers in parentheses refer to the Bibliography at the end of the paper.

Presented at Joint Meeting of the Wood Industries Division of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS and the Cost and Production Division of the Southern Furniture Manufacturers' Association at High Point, N. C., September 21–23, 1938.

Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y., and will be accepted until May 10, 1939, for publication at a later date.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

Equations [7] and [8], the following values of the energy stored per unit volume due to the change of shape are obtained

$$h_L = s_L^2 \frac{H}{P^2} \quad [11]$$

$$h_R = s_R^2 \frac{H}{Q^2} \quad [12]$$

The first equation applies when the stress is applied in a direction parallel to the grain and the second when the stress is applied in a direction radial to the annular rings. The value of H is the same in both equations because the theory assumes that failure will take place at the same energy in either case.

Now, if a stress s is applied to a block of wood at an angle g to the grain, the state of stress of the wood can be described in terms of a stress parallel to the grain, a stress perpendicular to the grain, and a shear stress. These relations may be expressed as

$$s_L = s \cos^2 g \quad [13]$$

$$s_R = s \sin^2 g \quad [14]$$

$$q = s \sin g \cos g \quad [15]$$

The values given by the first two of these equations may be substituted in Equations [11] and [12] to obtain that part of the elastic energy due to each of the stresses s_L and s_R.
Thus

$$h_L = s^2 \frac{H}{P^2} \cos^4 g \quad [16]$$

$$h_R = s^2 \frac{H}{Q^2} \sin^4 g \quad [17]$$

The energy caused by the shear stress is all due to a change of shape. This energy is given by

$$h_S = \frac{s^2}{N} \sin^2 g \cos^2 g \quad [18]$$

where N is the rotational modulus of elasticity.

The sum of these three last equations gives the energy per unit volume due to the change of shape of the block of wood. This sum is equal to H at the point of failure.

$$H = s^2 \left[\frac{H}{Q^2} \sin^4 g + \frac{1}{N} \sin^2 g \cos^2 g + \frac{H}{P^2} \cos^4 g \right] \quad [19]$$

It is desired to find the value of H which will reduce this equation to Hankinson's formula. If

$$H = \frac{2PQ}{N} \quad [20]$$

this condition is satisfied. Both sides of [19] can be divided by H and the expression in the bracket becomes a perfect square. Taking the square root of both sides of Equation [19]

$$1 = s \left[\frac{1}{Q} \sin^2 g + \frac{1}{P} \cos^2 g \right]$$

Multiplying both sides by PQ and solving for s

$$s = \frac{PQ}{P \sin^2 g + Q \cos^2 g}$$

The foregoing derivation of Hankinson's formula is of course an approximate one. Yet by it Hencky's theory is made to apply to wood. The relation expressed by Equation [20] is a fundamental one. This relation can be used and the theory applied to more complicated conditions of stress. The theory should be checked by tests in which two separate stresses are applied to a block of wood at the same time.

The value of H can also be obtained by the use of both Equations [9] and [10] if the values of Poisson's ratios are known. The two values obtained in this way do not agree very well. This fact might be expected since the measured values of Poisson's ratios are only virtual because of the cellular structure of wood. The energy value given by Equation [20] should be the true elastic energy due to the change of shape of the cell walls.

Another theory of failure of wood can be devised which does check very well with the virtual values of Poisson ratios determined by test. This second theory also leads to Hankinson's formula and may be equivalent to the theory just given, the difference being due to the use of the true Poisson's ratios in one case and the virtual values in the other.

The criterion of failure in this theory is that the expression $(2e_1 - e_2 - e_3)$ reaches a certain value. In examining the meaning of this term, consider a block of material subjected to stress. The material is given the unit deformations e_1, e_2, and e_3 by the application of the stress. Of these e_1 is the greatest. Assume now that stresses are added so that these three deformations become equal. The expression above becomes zero. If the extra stresses are relieved, a unit volume of the material will change. The expression $(2e_1 - e_2 - e_3)$ is exactly equal to that change in volume. The units in which it is measured are cubic inches per cubic inch. According to this, as well as to Hencky's theory, hydrostatic pressure is incapable of causing failure. Of course this is not true of a cellular structure like wood, unless the liquid enters the spaces within the cells.

The development of this theory is similar to the development of the first theory but much simpler. The change in volume, when the extra stresses are removed, is of course given by

$$k = 2e_1 - e_2 - e_3 \quad [5a]$$

If the three stress-strain relations given in the first part of this paper are applied to this equation it becomes

$$k = \frac{s_1}{E_1} (2 + r_1 + r_2) \quad [6a]$$

The method is parallel to that just given.
Then

$$K = \frac{P}{E_L} (2 + r_{LR} + r_{LT}) \quad [9a]$$

$$K = \frac{Q}{E_R} (2 + r_{RL} + r_{RT}) \quad [10a]$$

$$k_L = s_L \frac{K}{P} \quad [11a]$$

$$k_R = s_R \frac{K}{Q} \quad [12a]$$

If the two relations given by Equations [13] and [14] are applied to the last two of these equations then

$$k_L = s \frac{K}{P} \cos^2 g \quad [16a]$$

$$k_P = s \frac{K}{Q} \sin^2 g \quad [17a]$$

The shear strain does not change the volume of the block of wood, so it does not affect the value of K and is therefore omitted.

The total change in volume caused by removing the extra stresses referred to is given by the sum of the foregoing expressions. At the point of failure

$$K = s\left[\frac{K}{Q}\sin^2 g + \frac{K}{P}\cos^2 g\right] \quad [19a]$$

The volume-change constant K cancels out and it can be seen at once that the equation reduces to Hankinson's formula.

The volume-change constant K can be obtained from either Equation [9*a*], Equation [10*a*], or a third one designated as Equation [10*b*]

$$K = \frac{R}{E_T}(2 + r_{TL} + r_{TR}) \quad [10b]$$

The agreement between the values obtained by the use of these three equations is very good. The Poisson ratios for various woods are not very well known. However, C. F. Jenkin (5) has given values for spruce, mahogany, ash, and walnut with the other elastic constants and the strength values for the same pieces of wood. The values for spruce under compression will suffice to show the agreement obtained. These values are

$$K = 0.00905,\quad 0.00635,\quad 0.00962$$

Considering the difficulty in obtaining accurate test data the agreement seems very good indeed.

The author realizes that the methods used in deriving Hankinson's formula are very approximate. There are twenty-one elastic constants for a material such as wood (6). Only ten have been used in this work. However an approximate formula is better than none. If this paper stimulates interest in the theory of wood failure it will have served its purpose.

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Discussion

Energy Distribution in the Pulverized-Coal Furnace[1]

ADDENDA AND CORRECTIONS

W. J. WOHLENBERG.[2] While the use of radiant-mean positions of burning and ash particles for the purpose of approximating the equilibrium solution in Parts I to IV of the paper is valid, extension of their use for approximating the distribution of energy absorption as heat over the cold walls is not valid. The latter yields the artificial distribution shown in Fig. 9 of the paper. This could exist only if the particles were actually concentrated in their radiant-mean planes. Hence the secondary peak shown in curve No. 1 is the result of an assumed condition and has no place in reality.

It follows that Part V of the paper from its beginning to the heading "General Deductions and Conclusions" and Appendix 5 each involve such errors of assumption and should be replaced by the information submitted herewith for that purpose. Certain other omissions also are noted later in this discussion.

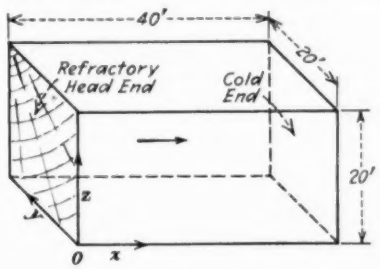


FIG. 1 CONDITIONS IN FURNACE CAVITY

(The cavity is single flow with charge entering through nozzles uniformly distributed over head-end wall. Since the cross section is a square and the longitudinal walls are alike (cold), the origin O may be taken at any one of four corners at the head end and the resulting pattern of contours of energy absorption as heat over cold walls will be the same. The fuel is carbon with 10 per cent ash, pulverized to 70 per cent through 200 mesh, and fired with 120 per cent air at 15,000 Btu per cu ft per hr. Radiant-mean temperatures from equilibrium solution are, in deg F abs: $T_a = 3460$, $T_r = 2540$, $T_g = 2465$, $T_f = 2453$, and $T_c = 1000$.)

Two of the latter affect the equilibrium solution in Parts I to IV. When these corrections are introduced the radiant-mean-temperature values change from those shown in Figs. 8 and 12b of the paper to those given here with Fig. 1. The total energy absorbed by the furnace walls as corrected is now 51.4 per cent of that supplied instead of 48.8 per cent as shown in Fig. 8 of the paper. Other details of this figure are of course also affected slightly but not sufficiently to alter either the qualitative nature of the results or the discussion with respect to them.

For the purpose of evaluating the distribution of energy absorption as heat over the cold walls of the cavity from the data obtained in the equilibrium solution it is necessary to consider the burning and ash particles in their real instead of in their radiant-mean position. The net radiating power of burning particles and ash particles is then determined with respect to each point of cold surface which it is proposed to investigate. This of course must also be done for radiation absorption at such points from the radiating gas molecules and the refractory wall. The relations and methods by means of which this is accomplished are stated later.

Referring to Fig. 1 herewith the points investigated in the longitudinal walls of the cavity $z = 0$ were chosen in lines $y =$ const with x variable. In the cold end the points were taken at $z = y$, that is, in lines connecting diagonally opposite corners. The results when plotted, Figs. 2 to 6, then form contours of energy absorption as heat for the longitudinal walls. For the cold end the points falling in the two similar diagonal contours are indicated.

The curves of Fig. 6 represent the final result sought. They are composed of information contained in Figs. 2 to 5 and so a brief discussion of each will aid in an understanding of this final result.

It is noted that in each figure the contour of lowest absorption rate is that for $y = 0$ where two walls intersect. Minimum points fall in corners at $y = 0, x = 0$ and $y = 0, x = 40$ ft. The rate of energy absorption rises more or less rapidly across the cavity from $y = 0$ to $y = 10$ for any given value of x and the contour of maximum absorption rate is in all cases that for $y = 10$. Corresponding contours are of course equally spaced, as to y, on either side of the central line $y = 10$.

Peaking of the contours in Fig. 2 is the net result of two effects which oppose each other. One of these is a geometrical influence which tends to increase the absorption rate at the point, from particles in suspension, as this point moves away from the end wall of the cavity. The other is due to the rapidly decreasing extent of radiating surfaces of burning particles as these move away from the head end of the cavity. The peaks occur in a zone close to the radiant-mean position of burning particles because in fact by far the larger fraction of burning-particle surface is concentrated in a region close to this radiant-mean position. This accounts for the close proximity to each other of contours $y = 1.0$ and $y = 10$ for $x > 5$. Contours for $y > 1$ and < 10 of course fall between those for $y = 1.0$ and $y = 10$. It is seen that the error introduced by computing the distribution of radiant-energy absorption over the cold walls from the radiant-mean position is not a large one in so far as burning particles are concerned. This however is quite the reverse for the ash particles represented in Fig. 3.

It is noted in this case that there is no similarity whatever between the shapes of the contours in Fig. 3 and curve No. 2 in Fig. 9 of the paper, which is supposed to represent the same thing. This clearly illustrates the magnitude of the error in distribution which was introduced on the former basis.

The rather abrupt rise from the left, of the curves shown in Fig. 3, is caused by the fact that as the surface point moves from the head end of the cavity it is rapidly passing into regions of greater and greater concentrations of ash particles.

Comparing Figs. 3 and 4 it is noted that the proportional spacing of contours $y = 1$ to $y = 10$ is considerably greater in Fig. 3. This is due to the difference in effect of radiant-mean thickness of the cloud in the two cases. As shown, this effect is much larger for ash particles in suspension than it is for the radiating gas

[1] Published as paper FSP-60-19, by W. J. Wohlenberg and D. E. Wise, in the October, 1938, issue of the A.S.M.E. Transactions, vol. 60, pp. 531–547.

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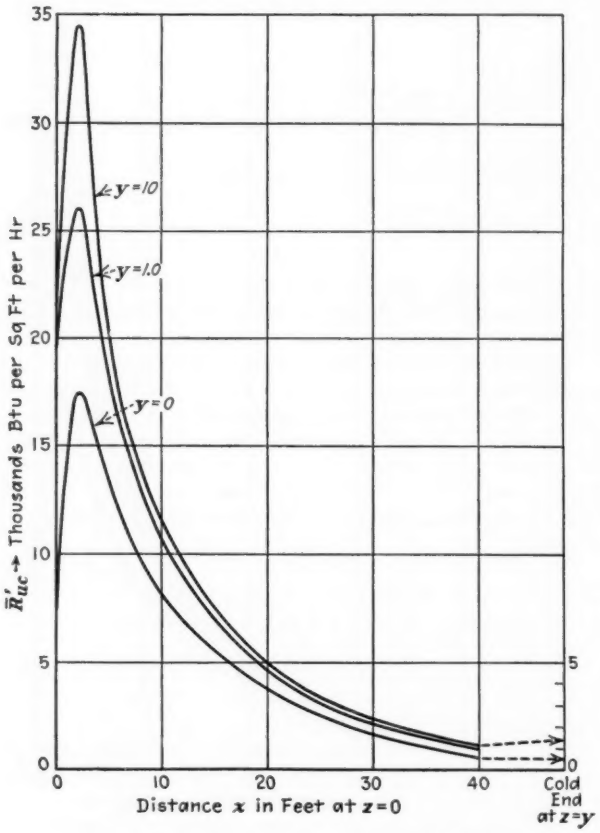


FIG. 2 $\bar{R}'_{uc}$, BURNING PARTICLES ⇆ COLD WALLS
(T_u = 3460 F abs, T_c = 1000 F abs.)

molecules in the range of thickness variations which occur in the present cavity.

In Fig. 5 all contours shown for absorption rate from refractory end, except that for $y = 0$, begin at substantially the same point for $x = 0$. That for $y = 0$ falls in a head-end corner of the cavity. For this corner the angle factor F'_{cr} subtended by refractory walls is 0.25. With $x = 0$ it requires only a small finite displacement from $y = 0$ toward $y > 0$ to increase F'_{cr} from 0.25 to 0.50 at which value it remains for all points $x = 0$, $20 > y > 0$. This accounts for the grouping of points shown on the ordinate of Fig. 5, at $x = 0$.

In Fig. 6 any ordinate of the contours of total energy absorption as heat is the sum of ordinates from Figs. 2 to 5 for the given surface point. The absorption rate over the total cold walls as computed on the basis of Fig. 6 differs by 5 per cent from the same thing as computed directly on the basis of the results, found in the equilibrium solution. This difference represents 2.5 per cent of the energy supplied by the fuel. In view of the numerous approximations, some of which are unavoidable, the check is probably as close as should be expected.

With respect to the cold end of the cavity it should be kept in mind that the absorption rates shown are due to radiation only. Hence if the gases leave through this end, the total absorption rate here will be increased considerably because of the large transfer by convection between gases and tubes. This has no influence on the temperature of the contents of the cavity because it does not occur until such contents are leaving the cavity. Thus it is seen how information furnished by the contour lines of Fig. 6 might be employed for estimating the convectional heat transfer at exit more closely.

All of the contours shown in Fig. 6, except that for $y = 1.0$, peak near the left end. Reasons for the peaking are now quite

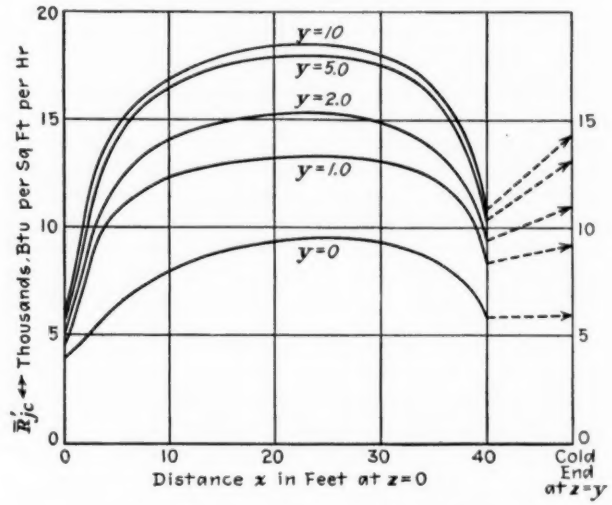


FIG. 3 $\bar{R}'_{jc}$, ASH PARTICLES ⇆ COLD WALLS
(T_j = 2453 F abs, T_c = 1000 F abs.)

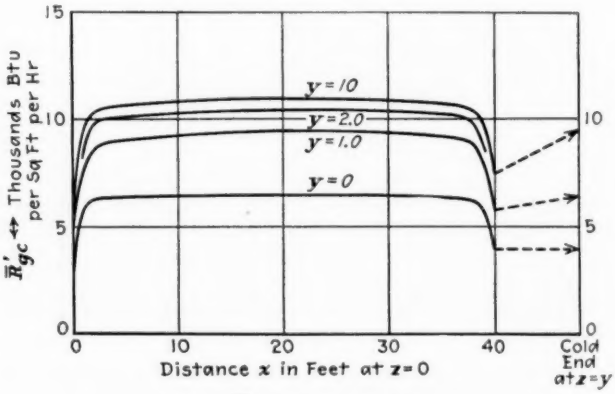


FIG. 4 $\bar{R}'_{gc}$, GAS ⇆ COLD WALLS
(T_g = 2465 F abs, T_c = 1000 F abs.)

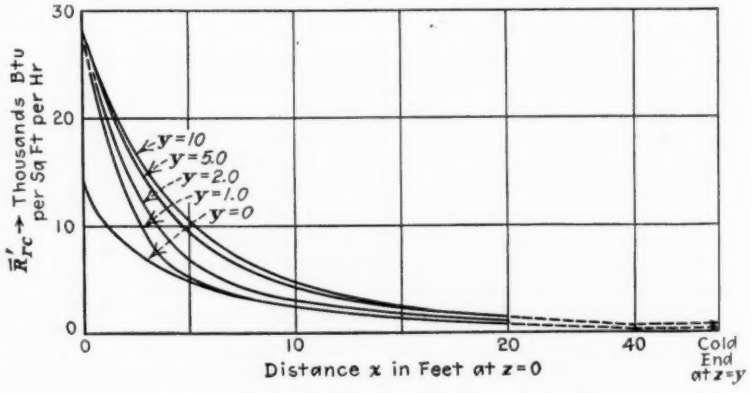


FIG. 5 $\bar{R}'_{rc}$, REFRACTORY WALL ⇆ COLD WALLS
(T_r = 2540 F abs, T_c = 1000 F abs.)

plain. Curve $y = 1.0$ does not peak because the combination of the rapid drop in the refractory curve for $y = 1.0$, Fig. 5, plus the effect of decreasing surface of burning particles more than offsets the sum total of other influences which tend to increase the absorption rate as the surface point moves away from the head wall. For $y = 0$, however, the curve again peaks because, as shown in Fig. 5, the contour for $y = 0$ starts low and so its gradient is much smaller than is that for $y = 1.0$. It follows that, for the conditions of this problem, the total contours between $y = 0$ and $y = 1.0$ except possibly those for which y is a very small quantity Δy, fall from a high point at $x = 0$ to a low point at $x = 40$ without peaking.

Relations for Determination of Radiant-Energy Absorption at Points on Cold Walls of Cavity. At any point of cold surface the radiant-energy absorption, Btu per sq ft per hr is

$$\bar{R}'_c = \bar{R}'_{uc} + \bar{R}'_{jc} + \bar{R}'_{gc} + \bar{R}'_{rc} \quad \ldots\ldots [1]$$

where index prime denotes this basis of reference. Then, with the same notation[1] except where otherwise noted

$$\left.\begin{aligned} \bar{R}'_{uc} &= \sigma\, \bar{\mu}'_{uc} F'(T_u^4 - T_c^4) \\ \bar{R}'_{jc} &= \sigma\, \bar{\mu}'_{jc} F'(T_j^4 - T_c^4) \\ \bar{R}'_{gc} &= \sigma\, \bar{\mu}'_{gc} F'\,(T_g^4 - T_c^4) \\ \bar{R}'_{rc} &= \sigma\, \bar{\eta}'_{cr} F_{cr}(T_r^4 - T_c^4) \end{aligned}\right\} \quad \ldots\ldots [2]$$

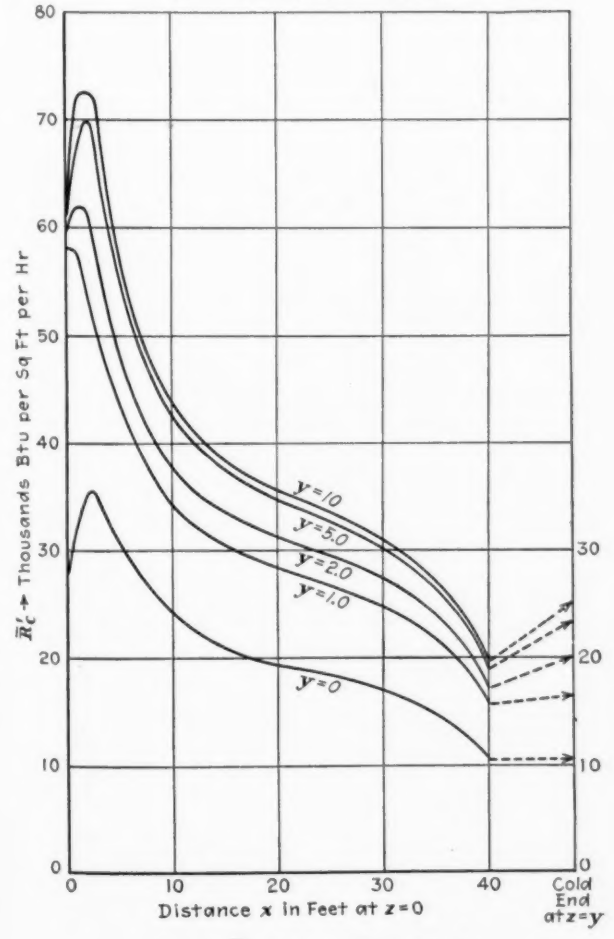


FIG. 6 $\bar{R}'_c$, TOTAL OF FIGS. 2 TO 5

FIGS. 2 TO 6 CONTOURS OF ENERGY ABSORPTION AS HEAT OVER COLD WALLS OF CAVITY
(Coordinates x, y, and z and furnace conditions as in Fig. 1.)

Angle factor F' is unity at all wall points except at an edge where two wall surfaces meet, or in a corner where three wall surfaces meet. For rectangular parallelepipeds $F' = 0.50$ along edges and 0.25 in corners. The special form on the right for $\bar{R}'_{rc}$ follows from the relation

$$A'_c F'_{cr} = A_r F'_{rc} \quad \ldots\ldots [3]$$

where cold surface A'_c is now unity. It is noted also that $\bar{\eta}'_{cr} = \bar{\eta}'_{rc}$.

Emissivity coefficients $\bar{\mu}'$ and transmissivity coefficient $\bar{\eta}'$ are each radiant means with respect to the point of cold surface under consideration. As an example, let μ' represent the value of the emissivity coefficient for differential region subtending angle factor dF from the considered point of cold surface. Then

$$\bar{\mu}' = \frac{1}{F}\int_0^F \mu' dF \quad \ldots\ldots [4]$$

where F is the angle factor of the total region from which the radiation under consideration may be thought of as converging toward the point under consideration. For an element dF of this region

$$\left.\begin{aligned} \mu'_{uc} &= (1 - \psi'_j f'_{jc})\,(1 - G''_{uc}) f'_{uc} \\ \mu'_{jc} &= (1 - \psi'_u f'_{uc})\,(1 - G''_{jc})\, f'_{jc} \\ \mu'_{gc} &= G'_{gc} - f'_{gc}\,(G'_{gc} - G''_{pgc}) \\ \eta'_{cr} &= (1 - f'_{cr})\,(1 - G'_{cr}) \end{aligned}\right\} \quad \ldots\ldots [5]$$

Here ψ'_j represents the probability that ash particles j will fall between burning particles and cold-surface point c. Factor ψ'_u is the same thing for burning particles u, relative to ash particles and cold surface. For uniform distribution relative to each other for the particles u and j, $\psi'_u = \psi'_j = 0.50$. If particles u and j are completely confined to separate zones with respect to each other along the radiant beam through both sets of particles, and radiation is from, say, particles u through particles j to wall point c, then $\psi_j = 1.00$. If under the same conditions radiation is from particles j through particles u to the wall, then $\psi_u = 1.00$. In view of these statements it is obvious that the following values of ψ apply to the conditions in the present furnace cavity:

All transverse beams, $\psi_u \approx \psi_j \approx 0.5$.

Radiation u to refractory end, $\psi_j \approx 0$.

Radiation j to refractory end, $\psi_u \approx 1.00$.

Radiation u to cold surfaces, ψ_j is between 0.5 and 1.00 and the mean value for radiation to all cold points is closer to 1.00 than to 0.5.

Radiation j to cold surfaces, ψ_u is between 0 and 0.5 and the mean value for radiation to all points is closer to 0 than to 0.5.

Factors G'' in Equations [5] are similar to factors G' but the beam length l'' varies in value between 0.5 l' and l' depending on the relative distribution between the intervening class of particles u, j, or p and the radiating gas molecules. The following range of values is seen to apply for the furnace conditions in the present cavity:

For G''_{uc} and point c near head end of cavity, $l'' \approx 0.5\, l'$. For point c at cold end $l'' \approx l'$.

For G''_{jc} and any location of point c, $l'' \approx 0.5\, l'$.

For G'_{pgc} all particles are considered as in one class p, and $l'' \approx 0.5\, l'$.

In other respects the notation of Equations [5] is similar to that employed in the paper.

In order to see how information, on which values of ψ', f', G', and G'' depend, may be furnished in such form that the results for $\bar{\mu}'$ and $\bar{\eta}'$ approximate those for radiant means of the type illustrated by Equation [4], consider the geometry shown in

Fig. 7, which applies to rectangular parallelepipeds. Here p is the surface point under consideration. For a given set of furnace conditions the radiant-mean values of ψ', f', G', and G'' throughout plane section $pqrs$, with respect to point p, may be approximated by reference to: (1) The radiant-mean beam length from p throughout the plane section; and (2) the line-mean values along x from x at p to x at q, of the concerned particle areas a_w, a_j, or a_g and of the concentration v of radiating gas molecules.

Of these line-mean particle areas between x at p and x at q, Fig. 7, that for a_w applies to f'_{wc}, that for a_j applies to f'_{jc}, and

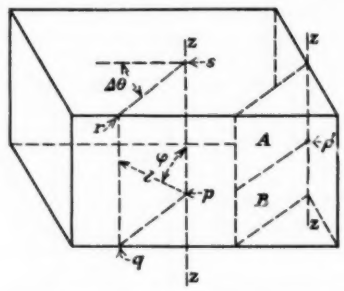


Fig. 7 Geometrical Aids in Approximating Radiant-Mean Values With Respect to a Point on a Wall

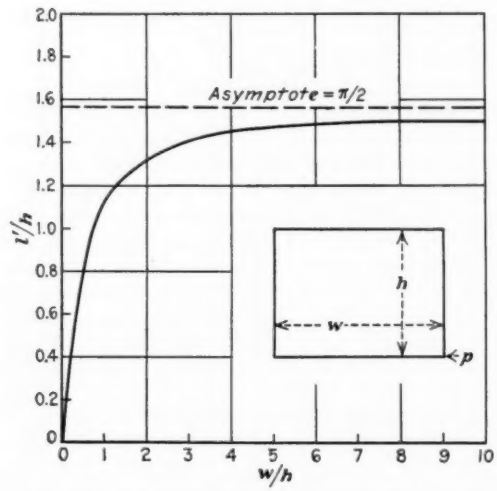


Fig. 8 Mean Beam Length l' With Respect to a Corner p for a Rectangular Plane Section

that for a_g to both f'_{gc} and f'_{cr}. The mean beam length l' from a corner p of plane section $pqrs$ is, of course

$$l' = \frac{2}{\pi}\int_0^{\pi/2} l\cos\varphi\, d\varphi \quad \text{..... [6]}$$

which on integration may, for convenience in use, be represented as shown in Fig. 8.

If p is located in the cold end of the cavity as at p', Fig. 7, the plane section is divided into parts A and B. The mean beam length l' for the whole section is a simple average of those for the two parts. Thus it is seen how values of ψ', f', G', and G'', and hence those of μ' and η' may be approximated for plane section $pqrs$ in any given position of rotation θ.

Suppose for a moment now, that instead of approximating μ' as just outlined, it had been evaluated on the basis of the relation

$$\mu' = \frac{2}{\pi}\int_0^{\pi/2} \mu\, d\varphi \quad \text{..... [7]}$$

where the emissivity coefficient has value μ along any direction l in section $pqrs$. This is its exact radiant-mean value for this plane section. On this basis the value of $\bar{\mu}'$ as found by means of

$$\bar{\mu}' = \frac{1}{\theta}\int_0^{\theta} \mu'\, d\theta \quad \text{..... [8]}$$

where edge qr of the plane section slides along the walls of the cavity, is exactly equal to that found on the basis of Equation [4]. Hence, if μ' in Equation [8] is determined on the approximate basis before outlined instead of by means of Equation [7] then Equation [8] yields a value of $\bar{\mu}'$ which closely approximates that found on the basis of Equation [4].

The integration indicated in Equation [8] is of course seldom possible or feasible for actual furnace conditions. In lieu of this, let plane section $pqrs$ occupy successive positions θ spaced by equal intervals $\Delta\theta$ from each other and covering the region θ from which radiation converges to the point p. On this basis the value of $\bar{\mu}'$ is a simple average of values found for each of the successive positions of $pqrs$.

It is noted that this procedure may be employed to find the radiant-mean values $\overline{\psi' f'}$, $\overline{G'}$, and $\overline{G''}$ of the coefficients $\psi' f'$, G', and G'' which appear in Equations [5]. When this is done the values $\bar{\mu}'$ and $\bar{\eta}'$ are computed from Equations [5] with $\overline{\psi' f'}$, $\overline{G'}$, and $\overline{G''}$ substituted for $\psi' f'$, G', and G''. They are thus products of the radiant means of the involved factors instead of the radiant mean of the product of such factors. This of course superimposes an additional approximation. The error involved is, however, a small one in most cases and the procedure is less cumbersome when this is done. For these reasons it was employed.

Additional Omissions and Corrections. In Equation [34], angle-factor term F_{wr} was omitted from both numerator and denominator. The corrected equation is thus

$$v_{wr} = \frac{\frac{1}{\varphi_s}\int_0^{\varphi_s} v_\varphi a_w F_{wr}\, d\varphi}{\frac{1}{\varphi_s}\int_0^{\varphi_s} a_w F_{wr}\, d\varphi} \equiv \frac{\overline{v_\varphi a_w F_{wr}}}{\overline{a_w F_{wr}}} \quad \text{.....for [34]}$$

In Equation [43] the radiation coefficient σ should appear in the denominator. Thus

$$G_{12} = \frac{\pi - \pi_2}{\sigma(T_1^4 - T_2^4)} \quad \text{..... for [43]}$$

Beam-length values for $\bar{l}_{wr}$, $\bar{l}_{wc}$, $\bar{l}_{wg}$, $\bar{l}_{jr}$, $\bar{l}_{jc}$, and $\bar{l}_{jg}$ in Equation [40] should have been computed on the basis that the source of the radiation is composed of *spherical* particles scattered uniformly throughout the radiant-mean plane for the class of particles indicated by the subscript. The beam length is then between this source just defined and the boundary indicated by the second subscript.

On this basis the following beam-length values should replace corresponding values in Equations [40] and [42].

$\bar{l}_{wr} = 3.7$ ft $\quad \bar{l}_{wc} = 13$ ft $\quad \bar{l}_{wg} = 9.0$ ft

$\bar{l}_{jr} = 14.5$ ft $\quad \bar{l}_{jc} = 13.5$ ft $\quad \bar{l}_{jg} = 13.7$ ft

$\bar{l}_{wpg} = 4.5$ ft $\quad \bar{l}_{jpg} = 6.8$ ft

On the basis of Equation [34] as revised and the corrected beam lengths, the corrected data for the equilibrium solution are as before indicated in this discussion.

Equation [57b] should be

$$\bar{R}_{rj} + \bar{R}_{wj} + \bar{R}_{gj} + \bar{L}_{gj} = 17 \text{ per cent} \quad \text{..... [57b]}$$

Term $\bar{R}_{rj}$ was omitted.

Combination Oil-and-Gas Burners[1]

H. H. WELLANDER.[2] The writer would like to have the author explain the method he uses to correct for sulphur dioxide in gas samples. In the burning of refinery gases rich in hydrogen sulphide, a considerable amount of sulphur dioxide is found in the flue gases. In an ordinary gas analysis, the carbon dioxide and sulphur dioxide are both measured as carbon dioxide. Therefore, the author should explain whether or not his curves are based on a sulphur-free determination of the flue gases.

D. S. FRANK.[3] The author's paper presents a timely comprehensive picture to the engineer interested in fuel. With the advent of low-priced natural gas, which is available in various territories, and the extra incentive of having burner equipment capable of taking natural gas at periods of low demands on the gas-company's distribution system, the combustion engineer is faced with the problem of selecting proper fuel stand-by equipment. This stand-by equipment must be for either pulverized coal or fuel oil, depending on the cost per 1,000,000 Btu.

The paper points out very clearly the main factors which should be taken into consideration when selecting the combination oil-and-gas burners.

The prime function of any oil burner is to properly atomize the fuel oil. Whether the atomization is obtained by pressure, as in the mechanical burner, or by steam or air, the important point is to break the oil up into particles as minute as possible. The next function is to surround properly each minute globule of oil with sufficient air to burn the oil completely, quickly, and with a minimum amount of excess air.

The problem of controlled flame length and efficient combustion is primarily one of turbulence. The importance of this one factor assuming sufficient time and temperature are present, cannot be overestimated.

Generally speaking, the greater the turbulence the shorter the flame, but also a noticeable increase in flame width on expansion through the burner throat. In view of this, the design of the combustion chamber tends to limit the choice of burner. In a long narrow firebox, the burners shown in Figs. 8, 9, and 10 of the paper would be more conducive to lower wall maintenance than the burners shown in succeeding figures. However, it appears that the burner illustrated in Fig. 16, due to the adjustable feature of the gas jets and individual air control, lends itself to practically any combustion-chamber design. This adjustable feature with ability to obtain various flame contours should tend to minimize burner troubles which have heretofore existed.

The proper selection of burner tips is also important. The writer has found that the multiple-round-hole tip is more conducive to low excess air when used with an air register than the flat flame tip. In a burner employing multiple gas jets, either the multiple-round-hole tip or the flat tip can be used with equal success. Here again, the design of the combustion chamber and spacing of the required number of burners tend to fix the correct type of tip to be used.

Ordinarily, the greater the turbulence, the higher the draft drop across the burner. In a forced-draft installation, this point can be taken care of in the fan and duct design, but in designing natural-draft installations it must not be overlooked.

Draft loss through the vanes and throat of an air-register type burner is a function of Btu release—the higher the Btu release, the higher the draft loss. For example, firing natural gas with 20 per cent excess air, the draft loss at a release of 10,000,000 Btu per hr is 0.19 in. of water, whereas at a release of 14,000,000 Btu per hr, the draft loss increases to 0.41 in. of water. Draft loss is also a function of throat diameter and the values just given by the writer are for a definite throat size. At the same Btu release on a larger throat diameter, the draft loss would be less than the values given. However, with increase in throat diameter, there is a decrease in turbulence for the same Btu release.

In burning fuel oil, the draft drop changes with the atomizing medium, as the draft loss is less for a mechanical-atomizing burner than for steam-atomizing burner, comparison being made at the same Btu release.

When designing the wind box for a forced-draft installation, considerable thought should be given to the capacity on natural-draft operation. If quickly detachable plates are incorporated in the wind-box design, and friction drop is low, the forced-draft burner can be operated on natural draft within reasonable limits. Longer flame lengths will, of course, be obtained.

The writer is familiar with the author's combustion charts and has found them very useful in making rapid furnace-efficiency calculations.

It is interesting to note that the stack-loss chart for a 10 A.P.I. residual petroleum oil can be used with very little error on any gravity fuel oil from 0 A.P.I. to 40 A.P.I. Assuming an arbitrary condition of 600 F stack temperature and 10 per cent excess air, a 0 A.P.I fuel oil shows a stack loss of 17.5 per cent; a 10 A.P.I. oil shows a stack loss of 17.7 per cent; and a 40 A.P.I. oil shows a stack loss of 18.5 per cent. Therefore, under the foregoing conditions, a maximum error would exist of 1.0 per cent. At 100 per cent excess air, the stack loss varies from 26.5 per cent for a 0 A.P.I. oil to 27.3 per cent for a 40 A.P.I. oil, or a maximum error of 0.8 per cent is involved.

Since industrial fuel oil is within the previously mentioned gravity range, and normally the excess air is also within this range, the chart can be used for rapid furnace calculations with a negligible error.

The stack-loss charts for a combination of fuels have proved to be helpful in determining furnace efficiency, and they are recommended for plants firing more than one fuel simultaneously.

R. C. VROOM.[4] In connection with the charts the author has prepared, perhaps it might be well to emphasize that those giving heat losses are in all cases based on complete fuel combustion.

Small furnaces and high rates of heat release, as the author points out, require efficient burners. The writer has fired with oil at a rate of 1300 lb of bunker fuel per burner per hr with excellent efficiency into a boiler furnace where the heat release approached 1,000,000 Btu per cu ft hr. In the U. S. Navy, heat releases exceeding 300,000 Btu per cu ft per hr with oil fuel are standard practice and the boiler efficiency is excellent. Our European neighbors are ahead of us in putting into practice in stationary plants what is possible in the way of furnace-heat releases. There is no valid reason why similar heat releases are not feasible with gas fuel using combined burners.

The author has in a sense defined the combined gas-and-oil burner as one having the ability to burn oil or gas equally well, separately or in combination. The words "equally well" are vital in this definition because there are few burners which fulfill that part of it. There are many which burn gas with a reasonable degree of economy; however, the oil feature is suitable only for stand-by purposes. In certain of these, when shifting from gas to oil, the output must be reduced and the excess air greatly increased. With some designs the use of oil alone for extended periods results in irreparable damage to the gas-burning elements.

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[1] Published as paper FSP-60-14, by O. F. Campbell, in the August, 1938, issue of the A.S.M.E. Transactions, pp. 457–467.

[2] Chief Power Engineer, Los Angeles Works, The Texas Company.

[3] Assistant Chief Combustion Engineer, The Pure Oil Company, Chicago, Ill.

[4] Chief Engineer, Peabody Engineering Corporation, New York, N. Y. Mem. A.S.M.E.

The mere ability to burn oil temporarily by no means qualifies such burners as combined burners.

The ability of a burner to operate without cleaning is of major importance where oil-refinery heaters are fired with wet gas, especially if no distillate trap is installed. Several of the special designs for refineries shown in the paper, including the open-ring burner shown in Fig. 14 (in which the operating mechanism now has been modified to avoid the mechanical difficulty mentioned), are in general applicable to one particular kind of refinery heater.

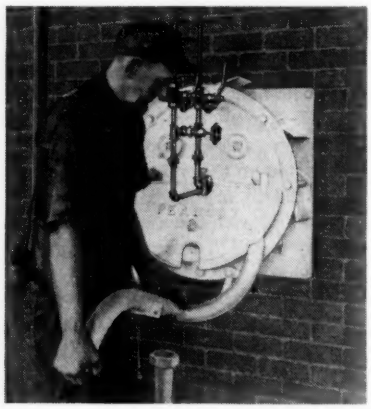


FIG. 1

However, the open gas ring which has no restrictions to cause plugging can be used with a standard air register and this assembly has been employed for firing boilers and many types of oil heaters. The removable closed-ring burner design, shown in Fig. 1 of this discussion, has found wide application in many refineries because it permits replacement of a plugged gas ring and at the same time permits oil to be burned during replacement.

In regard to liquid fuels, it is stated in the paper that any grade of fuel oil which can be delivered to a burner can be burned satisfactorily if the burner design is correct. This is true, but it is important in selecting a burner that full consideration be given to all the fuels which will have to be burned and to the results which will be considered as constituting satisfactory operation. A distillate burner, for example, could not be considered of incorrect design because it will not handle bunker *C* fuel. Neither should a burner be termed unsatisfactory if a fuel contains salts which attack the furnace brickwork. Sometimes such difficulties as unforeseen steam temperature, tube failure, high excess air in the exit gases, high stack temperature, low efficiency, and pulsation are due to faulty burner design or operation. Not infrequently the amount of superheater surface, scale in the tubes, leaky settings, fouled heating surface, and insufficient draft are largely responsible for such troubles.

In boiler firing, a combined burner is gaged by its ability to operate indefinitely with either fuel without damaging the elements provided for burning the other, and to do this over the full capacity range with complete combustion and a minimum of excess air. At the same time, the burner must be of a type which permits changing from one fuel to another almost instantaneously. Fig. 2 of this discussion shows a type of combined burner which is giving an excellent account of itself in most of the modern gas-and-oil-fired utility plants, as well as in many industrial plants and refineries, on the Pacific Coast. There are 225 burners in one station in Los Angeles. These plants are firing natural and refinery gases, bunker oil and refinery sludges, including acid sludge. For liquid fuel, some plants employ steam atomization while others use mechanical atomization. All types of boilers are being fired including integral-furnace and other recent designs.

In referring to the burner shown in Fig. 12 of the paper, the author mentions its lack of flexibility when burning oil due to mechanical fuel-oil atomization. Many engineers are of the opinion that all mechanical atomizers are greatly limited in capacity range, because such is the case with most of them. However, Estcourt[5] and others, who have had operating experience with the type of burner he describes, which is that shown in Fig. 5 of Philo's paper,[6] realize that a mechanical atomizer can have an extraordinary degree of flexibility.

In Estcourt's paper,[5] which describes what has been adopted as the regular low-load operating procedure at Station *A* of the Pacific Gas and Electric Company, San Francisco, Calif., it is shown that the automatic-control equipment is required to operate satisfactorily over a load range between maximum and less than 5 per cent of maximum capacity. With the 1400-lb boilers, each having a steam-generating capacity of 500,000 lb per hr, the

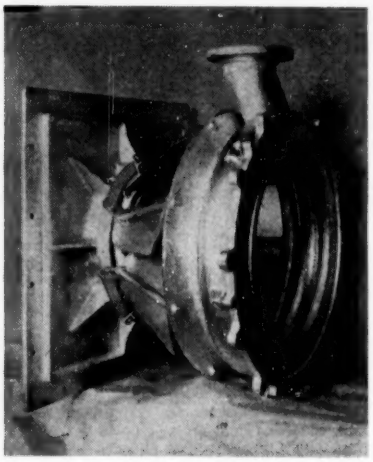


FIG. 2

minimum steam flow for satisfactory continuous operation with all burners in service is approximately 30,000 lb per hr (17-to-1 range) for gas fuel, and 65,000 lb per hr (8-to-1 range) for oil fuel. This paper[5] also states that it is entirely possible to operate (on swings) down to the same minimum load (oil fuel) with all burners in service (17-to-1 range) as is obtainable with gas fuel. In other words, the mechanical atomizers in this plant are anything but inflexible.

The paper[5] also states that "it is entirely possible that still lower loads may be obtainable. . . .(but) the figures just given are based upon a conservative opinion as to what is a satisfactory operating condition in the furnace." The burner gas pressure was 8 lb per sq in. at maximum capacity, and anyone who is interested can readily calculate what pressure would have been required for the minimum capacity. The arrangement for firing oil employed is essentially the constant-differential method of control mentioned by Philo in his paper.[6]

AUTHOR'S CLOSURE

In reply to Mr. Wellander's question regarding the flue-gas analysis to correct for sulphur dioxide in flue-gas samples, it will be necessary to know the refinery gas analysis and calculate the SO_2 produced as CO_2 and the stack-loss chart calculated on this basis. The CO_2 curve will be the CO_2 plus SO_2.

[5] "Design and Operating Problems With Gas- and Oil-Fired Boilers for Stand-By Steam-Electric Stations," by V. F. Estcourt, Trans. A.S.M.E., vol. 59, January, 1937, paper FSP-59-1, p. 7.

[6] "Technique of Burning Fuel Oil and Natural Gas," by F. G. Philo, *Mechanical Engineering*, vol. 60, April, 1938, p. 315.

The Behavior of Sodium Sulphite in High-Pressure Steam Boilers[1]

H. F. JOHNSTONE.[2] It is important to note that there is no essential disagreement between the results reported by the authors and those of Taff, Johnstone, and Straub.[3] The tests made by the latter were run at concentrations of about 4000 ppm of sodium sulphite. Under these conditions decomposition was found in every case at temperatures of 545 F and higher. These results have been verified by later tests made by Professor Straub using bombs under steaming conditions. There were two reasons for choosing the high initial concentration for the tests: (1) To explain the disappearance of sodium sulphite from a high-pressure boiler in which localized concentration was believed to exist; and (2) to permit following the reaction without using complicated semimicro analytical methods.

At the high concentrations the rate of decomposition is proportional to the concentration of residual sulphite, a relationship which is generally said to indicate a first-order reaction. In all of the tests made over a 36-hr period, however, the percentage of sulphite remaining was between 2.4 and 4.1 per cent of the original amount, corresponding to 96 to 160 ppm. Since this concentration would have been reached in a much shorter time (6 to 23 hr at 600 to 550 F), if the initial rate of decomposition had prevailed, it is evident that the reaction either reached equilibrium, or slowed down appreciably at low concentrations. The small concentrations of sulphide found in high-pressure boilers by Hitchens and Purssell,[1] using sensitive methods of analysis, show that the latter alternative is probably true. The results of the two investigations,[1,3] therefore, are not at variance and, since they were carried out under different conditions, may be considered as actually confirmatory.

The fact that the decomposition is "pseudounimolecular" rather than of true first order is only of academic interest. The practical engineer is primarily interested in knowing the answer to the question: "Does sodium sulphite decompose in actual operation?" On the evidence obtained so far, the answer is: "Yes, at appreciable rates at temperatures above 545 F and concentrations above 200 ppm. At lower concentrations, not exceeding 30 ppm and in a boiler in which no abnormal concentrations are likely to occur, the salt does not decompose at a rate to be of practical importance."

Even when decomposition does occur, little is known about the nature of the products and whether their presence would be considered detrimental to boiler operation. It would be expected that the hydrolysis of sulphides to give hydrogen sulphide in the steam would take place as readily as that of carbonates to give carbon dioxide. There is indeed some evidence that this hydrolysis does take place. First, in the laboratory tests under steaming conditions the total sulphur content decreases when the vapors come in contact with copper or clean iron surfaces. Second, in four of the tests reported by the authors[1] on high-pressure boilers, no sulphide was found in the boiler water but corresponding analyses by their sensitive methods showed sulphide in the saturated steam entering the superheater, before concentration of carry-over could have taken place.

It is obvious that the chemical investigation of this subject is not complete. In view of the complexity of the reactions of sulphur compounds as shown by the extensive work of Foerster on decomposition of sulphurous acid and its salts, further investigation would seem to be justified as a precautionary measure.

L. DREW BETZ.[4] The use of sodium sulphite for oxygen removal has assumed a prominent place of importance in the field of water-conditioning chemistry. The behavior of this salt at elevated temperatures and under pressure is therefore of considerable importance. Prior to the papers presented by Taff, Johnstone, and Straub[3] and the authors,[1] little was known of the action of sodium sulphite under boiler-operating conditions.

The authors in the paper under discussion present some very interesting data and are to be congratulated upon the thoroughness of their investigation. Particularly impressive are the methods of analysis they have worked out for the determination of sulphides and sulphites in minute quantities in condensed steam. Without having actually used the methods proposed by the authors, it is very difficult to present a constructive discussion. With such information available, however, it should lend impetus to additional research in this field.

Regarding test method *A* which the authors describe in this paper, no mention is made of the possible interference of iron. Snell,[5] in describing a method he proposes for sulphides based on the same reactions given by the authors, states that while small amounts of iron do not interfere, larger quantities may possibly cause some trouble. Also, in using this method for the determination of sulphides in boiler water, some difficulties might ensue if organic matter be present in sufficient quantities to cause a discoloration of the water. Such difficulties would be in the nature of matching the color produced with the color standards.

It would also be of material value to ascertain whether any reactions occur between sodium sulphite and organic matter at elevated pressures. An analysis, made some few months ago in the writer's laboratories, of a deposit from a boiler tube, showed the presence of sulphates in quantities greater than the amounts necessary to combine with the inorganic cations present. The deposit was high in organic matter, leading to the assumption that the sulphate was originally present in the form of sulphite and quite possibly combined with the high organic content. A reduction in the amount of sodium sulphite used, in this instance, eliminated further deposits of this character.

AUTHORS' CLOSURE

The authors agree with Professor Johnstone that there need be no essential disagreement between the results reported by himself and coworkers and those by Hitchens and Purssell since the tests were made under entirely different conditions.

It is still felt that the traces of sulphite and sulphide found in the steam of the two higher-pressure boilers mentioned in the paper did not originate from hydrolysis of sulphites and sulphides in the boiler water but rather from decomposition of sodium sulphite carried over mechanically into the steam. The evidence for this is that the sulphite and sulphide both disappeared completely in the steam with decreased load on the boilers, a situation which is untenable with hydrolysis which proceeds at a rate quite independent of the steaming rate.

The authors still feel that the extent of contamination of the steam is too minute to be of any practical significance.

It is true as Mr. Betz says that appreciable amounts of ferric iron interfere with the colorimetric determination of sulphide with lead plumbite in method *A*. In fact the two react as follows

$$2\,Fe^{+++} + S^{=} \rightarrow 2\,Fe^{++} + S$$

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[1] Published as paper FSP-60-15, by R. M. Hitchens and J. W. Purssell, Jr., in the August, 1938, issue of the A.S.M.E. Transactions, pp. 469–473.

[2] Associate Professor of Chemical Engineering, University of Illinois, Urbana, Ill.

[3] "Decomposition of Sodium-Sulphite Solutions at Elevated Temperatures," by W. O. Taff, H. F. Johnstone, and F. G. Straub, Trans. A.S.M.E., vol. 60, April, 1938, pp. 261–265.

[4] General Manager, W. H. & L. D. Betz, Philadelphia, Pa. Mem. A.S.M.E.

[5] "Colorimetric Analysis," by F. Dee Snell, D. Van Nostrand Company, Inc., New York, N. Y., 1921.

There was too little iron in the samples of condensed steam to be of importance.

In the boiler water the presence of ferric iron is improbable since the excess sodium sulphite present would reduce it to the ferrous condition. It is felt that iron did not interfere with the tests for sulphide in the boiler water since addition of definite amounts of sulphide gave colors comparable to those with distilled water, iron-free.

It is true of course that the method is applicable only to a clear boiler water. Those the authors encountered were clear. It should be feasible to filter the water before applying the test with no loss of sulphide since the excess sulphite would keep it in the reduced form. Discoloration caused by organic matter might be circumvented by removal of the sulphide with nitrogen in a faintly acid solution and collecting on a lead-acetate strip. Each such water would be a research problem in itself.

The authors have no information on the reaction of sulphite with organic matter in boiler water at high temperatures and pressures. It will react quantitatively with aldehydes and ketones at lower temperatures. The authors are much interested in Mr. Betz's findings and would appreciate hearing of any later developments in the problem.

An Oil-Bath-Lubricated Railway Bearing[1]

H. M. Warden.[2] The writer has studied prints and has seen the "Disc-Flo" journal units in operation. There are several features that are improvements over the conventional box and parts, some of which would greatly improve the performance of present bearings if they were incorporated in A.A.R. designs.

One such item is dust guards. The old wood dust guard universally used insures no certainty of oil supply as it will not prevent the loss of oil. It is frequently detrimental to good bearing and journal performance as it fails to keep water and dirt out of the box. They make a poor fit between the box and the axle, and are not moisture-proof.

The paper in my opinion is correct in its statement that "serious trouble experienced today with A.A.R. type of bearing and box is not friction." Journal friction when reduced to a minimum by use of suitable oil and waste, properly turned journals, and broached bearings is but a small part of the total rolling friction. If oil, waste, journals, or bearings are not correct, hotboxes are apt to occur; however, failures of these parts are frequently due to presence of foreign matter in the box. A poor grade of oil or unstable feeding of oil, as well as presence of dirt in the box, are causes of most failures.

A good dust guard should provide an oil seal around the axle and in the dust-guard cavity, and the design should readily fit present A.A.R. boxes. It should also be manufactured at a reasonable cost.

Another item is the necessity of providing for lateral thrust of the axle to be applied against the fillet of the journal bearing by the journal fillet. A majority of bearings show excessive wear on the collar, which, besides loosening the lining, increases lateral play on a truck which already has too much. With present A.A.R. trucks operating under bad conditions, such as poor track, it is possible for the back of the boxes to strike the hub of the wheel. Changes necessary to locate lateral thrust at the journal fillet can be made by reducing tolerances and changing dimensions of bearings and wedges.

The self-aligning fit between the wedge and the roof of the box will overcome any irregularities in the box or contained parts, but the writer believes if it were exaggerated as shown in this paper it would further cut down lateral play since it would take the place of the wedge stop. Lack of oscillating movement for which the wedge is intended will cause the brass to bind, producing uneven loading, excessive friction, and eventually a hot bearing. When the top bearing is untrue it gives uneven pressure on the journal on account of improper seating.

The writer does not believe that a flat-back bearing or that extending the sides of the bearing will materially eliminate bearing tilt caused by impacts, but would probably take care of side movement resulting from reasonable brake applications if the boss in the box were extended close to the center line. Heavy impacts and emergency air-brake applications, even at slow speed, cause the bearing, wedge, and box to raise momentarily off the journal. If impact be heavy enough to displace the packing, which may be caught under the bearing, the result will be waste grabs.

The use of hardened pedestal liners is desirable to minimize wear. When pedestals are worn, a jerky movement occurs between the box and the pedestal and rocking of the box produces a distorted bearing face in the lining. Excessive wear may cause partial locking of the box. A pedestal out-of-alignment may prevent proper seating of the bearing and produce collar friction.

If recognized methods of maintenance and application of parts be closely followed and suitable oil and waste used, the A.A.R. journal box and parts will give satisfactory service; however, if the A.A.R. would adopt some changes in design incorporated in the Disc-Flo unit there is no question but that we would obtain longer life at less maintenance and have less trouble with journal box and contained parts.

In the foregoing comments the writer has in mind freight equipment, whereas the use of the Disc-Flo journal unit as covered by this paper seems to be intended for passenger-car service and would necessitate carrying a different stock of parts at all points for protection to such equipment. It is true that our passenger-car boxes and parts, wheels, and axles are better maintained and inspected more frequently, yet at times they operate under severe conditions. The writer understands that very few units have been applied for service tests and is not familiar with such tests, but he believes additional long-time service tests should be made under all conditions so that reasonably accurate comparative figures with A.A.R. standards would be available.

G. W. Ditmore.[3] The writer is not in a position to discuss the Disc-Flo unit from a practical standpoint having had no experience with its operation other than laboratory demonstrations. Laboratory tests have shown this device to possess great possibilities toward improved car-journal lubrication, providing actual service conditions do not develop unlooked-for difficulties.

The problem of developing a satisfactory sealing arrangement at the back of the box, that will function properly for the service life of the wheels without constant policing, when solved should be of great economic value to the railroads.

The authors deserve commendation for the exacting and thorough manner with which they have approached and are working to overcome one of the difficult problems of railroad operation.

The operation of Disc-Flo lubrication units in actual service will be followed with keen interest.

Frank E. Cheshire.[4] The writer has observed much of the laboratory research and many of the service tests of the Disc-Flo journal-bearing assembly. These observations have been made

[1] Published as paper RR-60-3, by Albert Vigne and I. E. Cox, in the August, 1938, issue of the A.S.M.E. Transactions, pp. 499–506.

[2] Chief Mechanical Officer, Missouri-Kansas-Texas Lines, Parsons, Kan.

[3] Master Car Builder, Delaware & Hudson Railroad Corporation, Albany, N. Y.

[4] General Car Inspector, Missouri Pacific Lines, St. Louis, Mo.

as a railroad mechanical man. It is apparent, from approximately two years of operation under varying but representative service conditions, that a constant supply of lubricant is delivered to the contact surfaces by the device described by the authors. The problems encountered throughout these tests have not involved the lubrication as such. Alignment, control of the axle lateral play sealing the oil in and extraneous contamination out, and the difficulties arising therefrom have required correction. As long as oil has been available—and with one exception that condition has obtained—the requisite lubrication has been provided. That one exception developed the prime necessity for closer tolerances on axles new and reworked, on bearings, boxes, wedges, and truck dimensions. In this one failure the oil supply was lost due to a combination of wide tolerances permitting excessive uncontrolled axle lateral play. This condition is common in the conventional assembly and accounts for much of the loss of oil and entrance of extraneous contamination, in addition to excessive wear of the various parts of the assembly and resultant limited service life.

Since incorporation of corrective tolerances, several thousands of miles of satisfactory service indicates that this assembly will provide dependable service. This indication will be proved or qualified as the service mileage increases.

The writer believes that the inclusion of the research data in presenting this device offers a valuable contribution to advance in the science and art of journal lubrication. Speaking as a railroad mechanical man, it is encouraging to see presented a logical analysis of the fundamentals of one of our major problems. In something more than 20 years of almost daily contact with failures of journal lubrication, many of the phenomena, here broken up into causal components, have been observed. The writer has observed in service an incorporation of many of the corrections offered. The encouraging results therefrom induce a rather critical conviction that consistent improvement of considerable economic worth may justifiably be expected from an incorporation of the recommended corrections in the fundamentals of design and standards of practice. It is the writer's observation that our most difficult problems arise from the necessity for correcting faults in existing materials. The inexorable characteristics of established standards and practices have the weight of years of use. No one factor of railroad mechanical operation is more formidably surrounded by tradition than that of journal lubrication. Each railroad has followed a practice dictated by its individually traditional standard. It appears that efficiency in journal lubrication is measured by comparative reductions in transportation delays; or, in the words of an able contributor to critical investigation, "miles per hotbox is the operating bible." Such a yardstick does not include the greater economic factor, that is, the cost to attain that particular record.

It is the writer's further opinion that critical study and analysis of the data presented by the authors, and correlation thereof with service indications evident to anyone interested, will do much to remove prejudice, eliminate weak imitation, and discard outworn tradition in relation to journal lubrication. It occurs to the writer that the formulas presented by the authors for the determination of actual axle lateral movement, under any given set of dimensions, is a constructive contribution. The necessity for a definite imposition of the thrust load at the fillet end of the bearing should be obvious but has not been generally recognized. The means suggested for control of the axle lateral movement, within reduced limits, is simple and effective. Its effectiveness has been proved in service.

In the formulas, the truck dimension indicated as the inside flanges of the pedestal jaws, is assumed as a constant and constant it must be.

The increased, and increasing, demands of railroad transportation, in dependable service and economical operation, demand general recognition of the necessity for closer tolerances in both design and practice. The slight increase in initial cost to provide such tolerances is but a small portion of the higher operating expense resulting from a denial of this necessity.

Authors' Closure

The appreciated, well-qualified, and pertinent discussion raises two questions: What are the results of service tests on the Disc-Flo unit, and to what degree can the fundamental Disc-Flo improvements in journal-box and bearing design be applied to existing A.A.R. equipment.

At the presentation of this paper, 40 journal units in test service had accumulated 5,329,206 journal-bearing miles, or an average of 661,151 car-miles per car set, during which changes were made as described, incorporating lateral control, and improvement in disk drive and seal construction.

Since June, 1938, 48 journal units have accumulated 3,220,664 journal-bearing miles, or an average of 402,583 car-miles per car set. All of these journal units are in high-speed passenger service, and have given satisfactory operation with no maintenance other than routine terminal inspection.

A tender set of 6 × 11-in. journal units is being installed under a heavy high-speed passenger-locomotive tank and a car set of $3^3/_4$ × 7-in. units has just been installed in a modern high-speed streetcar, from which additional service data will soon be available.

Intensive study has been given to the application of the Disc-Flo improvements to the Standard A.A.R. journal box and bearing assembly.

A dust guard incorporating the design fundamentals of the Disc-Flo seal has been developed for the A.A.R. box. It consists of a resilient flexible sealing member sandwiched between two aluminum-alloy plates, one a seal plate, the other a follower plate. These plates are provided with bores of different diameters; the seal plate adjacent to the wheel fits snugly on the dust-guard seat of the axle, and the follower plate, having a larger bore, permits the flexible member to form an annular wiping flange which effectively seals the fit around the axle. The seal plate, due to its close fit on the axle, guides the seal assembly in following the movement of the axle in reference to the box, as the dimensions of the seal unit are such as to afford its unrestricted movement in a standard dust-guard well. The spring action of the resilient member holds the aluminum plates against the inside and outside walls of the journal-box cavity with sufficient pressure to seal against entrance of dirt and yet not interfere with the movement of the seal assembly.

Over 700 of this type of dust guard in its various stages of development are in service, and the oldest installation is still giving good service, having made a mileage of 260,000 in high-speed passenger service as of December 1, 1938.

The system of lateral control, standard in Disc-Flo design, can be applied without modification to A.A.R. axle, box, and bearing assemblies. By establishment of close tolerances on new and reworked axles, on bearings, boxes, wedges, and truck parts, and modification of dimensions of lateral-component lugs and collar of journal bearing, the evils of excessive lateral play can be overcome.

The degree to which it is desired to carry the bearing improvement as dictated by cost and departure from existing standards, requires a decision as to whether the standard steeple-backed A.A.R. journal bearing should have its collar and thrust-lug dimensions modified, or the standard wedge and bearing should be abandoned in favor of the flat-back lateral-control bearing with its modified wedge.

By modification of the collar and thrust-lug dimensions of the standard A.A.R. bearing, the desired reduction in axle free

lateral play can be secured, but to stabilize the respective running positions of the axle and the bearing and to increase the thrust capacity of the bearing the flat-back type has to be resorted to.

In answer to Mr. Warden's doubt as to the ability of the flat-back bearing with extended sides to overcome tilting, it should be pointed out that with this combination, using a flat wedge, there is no inclined plane formed by the wedge for the bearing to climb, when horizontal impact is imposed on the axle. Further the force of the horizontal axle impact against the more nearly vertical extended side of the bearing exerts relatively small lifting force on the bearing and this has to be in excess of the static vertical load before there is any raising of the bearing.

Over 4600 A.A.R. type bearings modified for lateral control have been put in service with results that justify the foregoing statements.

One hundred thirty-two flat-back lateral-control bearings have been applied in heavy locomotive-tender service, and on high-speed passenger cars, with a single brake shoe per wheel. Under this severe service, the life of the journal bearings has increased beyond all expectation.

Problems in Modern Deep-Well Pumping[1]

CARLTON W. DAWSON.[2] The author has presented a clean-cut mathematical analysis of deep-well pumping problems, and engineers who deal directly with these problems may learn much from his careful study.

Prior to this time the importance of the step-tapered rod string (the closest approach to the rod string of uniform stress) has been recognized and used almost exclusively in sucker-rod pumping at depths greater than 7000 ft. Reasonable rod service has been achieved, but production rates have been well below the maximum rates given by Equation [18] of the paper.

The author has analyzed plunger overtravel as a function only of the accelerations at the ends of the stroke. Assuming harmonic motion, the polish-rod-stroke correction factor becomes $F = 1 + [1.93\ (LN)^2/10^5]$. However, to give results in closer agreement with those obtained by field tests with the Gilbert-Sargent bottom-hole dynagraph, it is suggested that the pitman-crank-length acceleration factor be applied. With a ratio of four, the foregoing formula becomes $F = 1 + [2.4\ (LN)^2/10^5]$. This correction assumes the magnitude of acceleration at both ends of the stroke to be equal and opposite in sign. Actually, in the conventional pumping unit, the acceleration at the bottom of the stroke is due to the individual crank and pitman accelerations being additive when at 180 deg. Conversely, when at the top of the stroke the pitman overlies the crank and the acceleration of the pitman is subtracted from that of the crank. The sum of top and bottom acceleration is approximately equal to the sum if the motion be actually harmonic. The significance of this is that the first correction is the only one theoretically justified.

But, because calculations based on this formula do not agree with actual performance, it is obvious that a more involved treatment is justified. Various applications of Timoshenko's work on vibrations have been attempted; notably those of R. W. Rieniets[3] and Kendrick and Cornelius.[4] This approach gives promise of being much more satisfactory, although a great deal more correlation of field data is required to provide the essential support.

In the summary of the paper, the author states that pneumatic counterbalances are adapted to heavy loads and will give lower peak loads than beam or crank balances. This is true, but is due not to the type of balance but to the effect of the accelerations. The point of attachment of the pitman and crank on air-balanced units is between the samson post and polish rod, whereas the samson post of the conventional units is between the pitman and polish rod. As pointed out previously when the pitman overlies the crank the accelerations of each are of opposite sign, and when they are extended at 180 deg they are of the same sign. This means that the air-balanced units have greater rod acceleration at the top of the stroke where the effect is to reduce the rod load. The conventional unit has the greater acceleration at the bottom of the stroke where the effect is to increase the rod load.

AUTHOR'S CLOSURE

Mr. Dawson's criticism of the relation $F = 1 + [2.4(LN)^2/10^5]$ is well founded. This seems to fit field results more closely than the previously used relation, but its use must be based on its value as an empirical relation rather than a fundamental of crank motion.

The air balance whether directly applied as in the Sullivan head or in combination with a crank motion tends to smooth out the load measured at the polished rod. It is true that the point of attachment of the pitman is a factor, but the absence of the large mass of the counterbalance is thought by the writer to be of greater importance.

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[1] Published as paper PME-60-2, by C. J. Coberly, in the October, 1938, issue of the A.S.M.E. Transactions, vol. 60, p. 561.

[2] Engineer, Producing Department, Standard Oil Company of California, La Habra, Calif. Jun. A.S.M.E.

[3] "Plunger Travel of Oil-Well Pumps," by R. W. Rieniets, American Petroleum Institute, Drilling and Production Practice, 1937, p. 159.

[4] "The Sucker-Rod Pump as a Problem in Elasticity," by J. F. Kendrick and P. D. Cornelius, Transactions A.I.M.E., Petroleum Division, vol. 123, 1937, p. 15.

Industrial Power

By CHARLES W. E. CLARKE,[1] PHILADELPHIA, PA.

This paper covers briefly the progress that has been made in the generation of power in industrial plants. The author points out that industries and utilities are cooperating in the generation and interchange of electric energy in order to reduce waste in many manufacturing processes.

Higher steam pressures and temperatures are widely used, along with superposition of new turbines on old equipment. Only a few installations have pressures and temperatures which exceed 900 lb per sq in. and 850 F, respectively. A number of prominent industrial plants are described to indicate the application of all types of steam turbines to the three steam cycles most commonly employed. Steam-pressure-reducing and desuperheating equipment, as developed for topping units in central stations, is finding new applications in industry. Where large quantities of process steam are needed, many interesting arrangements of modern equipment can be used to reduce the cost of power generation.

The trend in design of boilers is toward larger units, higher steaming rates, and more complicated automatic controls to permit utilization of several different kinds of fuel in the same furnace. This presupposes the proper treatment of boiler feedwater, the importance of which is emphasized. Operating data from two outstanding installations are included. Among smaller plants there is a tendency toward the use of a self-contained boiler; also, there has been a revival of semi-outdoor constructions.

Progress is being made toward better utilization of unusual fuels, such as pulverized anthracite culm, lignite, acid sludges, and black liquor, the latter of which is obtained in the process of manufacturing kraft paper.

Improvements in electrical equipment for industrial plants include metal-clad switchgear, which may be located in any available space; also, air circuit breakers for voltages up to 5000 may soon be available in order to eliminate the fire hazard from using oil. Modern insulation on wire and cable offers greater resistance to high temperatures and the effects of moisture and other harmful substances.

THE art of providing suitable electric-generating facilities for industrial use requires analysis of many factors such as type of load, source of water and fuel supply, condition of existing equipment, future growth, possible steam pressures and temperatures, interconnections, and numerous other items incident to any particular project.

As a general premise it is safe to say that in industrial plants where there is a reasonable demand for process steam of moderately low pressure, it will generally be found economical to furnish generating capacity up to the requirements for this steam, and in cases where no process-steam requirements exist it will usually not be found economical to install condensing units. This premise assumes of course that there is available in the district satisfactory service from a local utility.

First let us consider cooperation between manufacturing plants and public-utility companies in generating steam and electricity. In most localities the manufacturing industry and the utility have many interests in common. There are, for example, by-products such as waste heat, coke, wood waste, coke-oven gas, blast-furnace gas, and oil-refinery refuse. In the interests of the community, as well as for conservation of resources, these products should be used. Production of steam and power is one important means of doing so. Yet in many plants using steam for process the demand for steam does not synchronize with the demand for electric energy. Seasonal demands also complicate the picture. When a tie line to the utility is available, the latter's resources of power can take care of fluctuations in demand. No condensing machines need be installed as the high-pressure turbines run exclusively for low-pressure-steam requirements. Spare equipment with its costly fixed charges can be omitted and when the turbine is out the utility supplies the electric load and the boiler the steam requirements, through reducing valves.

Power companies are now meeting this important economic problem, which frequently involves both the sale of electric energy to and its purchase from the same industrial customer. Progress along this line has been slow but is certain to accelerate in the future as the demand for cheaper industrial power becomes more determined. The two parties concerned have everything to gain and nothing to lose in disclosing all facts and marking out the correct solution. There are at the present time under construction on the west coast projects involving such cooperation between oil companies and the local utility.

Turbogenerators

Due to the great variety of conditions to be met in industrial practice, turbines for such applications are usually built to order, and the following types of turbines are available for both condensing and noncondensing service: Nonextraction, single automatic extraction, double automatic extraction, and mixed pressure. These turbines can be built for any reasonable capacity and steam condition and a wide choice of extraction and exhaust pressure is available. Except for special cases, the machines usually operate at 3600 rpm.

Although maximum steam conditions up to about 1300 lb and 950 F can often be justified, from the practical viewpoint 900-lb pressure and 850-F total temperature appear to be the sensible limits for most industrial plants. In a few cases, such as at the River Rouge Plant of Ford Motor Company, installations have been made at 1215 lb and 925 F. Incidentally, there is in the process of construction an installation which is to use 2500-lb pressure. Usually such plants approach the utility stations in size of units and skilled personnel to supervise operation. The ordinary industrial plant is not so fortunate. Here, generation of power is not a major function but only one item necessary in the manufacturing process. Until the power companies have

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Discussion of this paper closed January 10, 1939, and is published herewith directly following the paper.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

had more practical experience with the higher steam conditions and accompanying precision in superheat control, the majority of industrial plants should operate under the lower pressure and temperature conditions just mentioned.

The superposed turbine has gained wide prominence during recent years. The rapid rise in pressures and temperatures has allowed old equipment to be so modernized that it can be used for operation on the low-pressure end of a cross-compound cycle. The overall-heat rate is greatly reduced as the same amount of heat is rejected to the condenser before and after superposition.

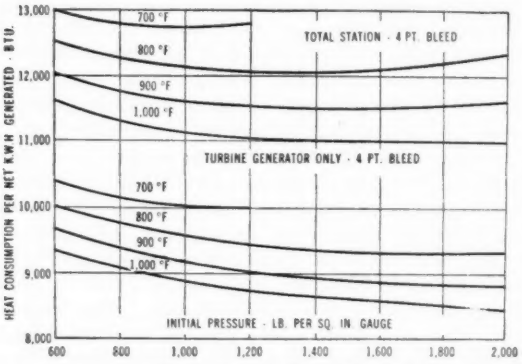


FIG. 1 ATTAINABLE HEAT RATES WITH HIGHER PRESSURES AND TEMPERATURES

Thus, obsolete and stand-by equipment become an integral part of an up-to-date installation and, in addition, the plant capacity is increased at a cost per kilowatt generally under that required for equal capacity in a condensing unit. One important item in favor of superposition is the fact that while the generating capacity of the plant is increased there is no additional demand for condensing-water facilities due to the increase in load. This is highly important in cases where the condensing-water supply is limited.

Most of the central-station topping units operate at about 1250-lb pressure and 900-F to 950-F total temperature. The effect of increased pressures and temperatures on the heat cycle are clearly indicated by Fig. 1.

As many industrial plants now operate at about 200-lb pressure, the 650-lb superposed turbine offers substantial returns on the investment and, moreover, utilizes medium pressures and temperatures. Generally speaking, industrial-plant conditions do not lend themselves to the straight superposed unit as readily as do those existing in central stations. Variations of the superposed turbine, permitting steam extraction for process use, are becoming widely used in industry as discussed later under "Steam Cycles." The common impression that superposition is the last step in modernization is quite erroneous, as 10 or 15 years hence it will be necessary in many cases to replace the low-pressure turbines now being topped. In fact, it is entirely possible that due to further experiments in the use of still higher steam pressures the units now being superposed may again be topped.

Ordinarily, the requisite pressures of process steam and the use made of it determine the most desirable conditions for the throttle steam. Usually a small amount of superheat in the process lines is desirable to ensure dry saturated steam at the point of use. To satisfy these conditions it is seldom necessary to exceed 600-lb pressure and 800 F at the turbine inlet.

A survey by *Power* of industrial plants in which 117 steam turbogenerators were installed during 1936 and 1937 discloses the classification as to types listed in Table 1. Grouping these turbines according to generating capacities, we obtain the tabulation given in Table 2.

TABLE 1 INDUSTRIAL-PLANT TURBINES INSTALLED IN 1936 AND 1937 LISTED ACCORDING TO TYPES

Throttle pressure, lb per sq in.	Total number of turbines	Types of turbines: Condensing	Condensing extraction	Back pressure	Back pressure extraction	Mixed pressure
1200	4	0	1	3	0	0
700–900	15	0	5	7	3	0
500–700	13	0	4	7	2	0
400–500	58	3	26	16	11	2
200–300	27	2	9	11	5	0
Total.....	117	5	45	44	21	2

TABLE 2 TURBINES LISTED IN TABLE 1 GROUPED ACCORDING TO GENERATING CAPACITIES

Throttle pressure, lb per sq in.	Capacity, kw: Under 500	500 to 1000	1000 to 2000	2000 to 3000	3000 to 4000	4000 to 5000	5000 to 10000	10000 to 20000	20000 and up
1200	0	0	0	0	0	0	2	1	1
700–900	0	1	1	1	4	6	1	1	0
500–700	0	0	3	2	1	2	4	0	1
400–500	5	5	13	11	4	5	13	1	1
200–300	6	8	5	3	1	3	1	0	0
Total.....	11	14	22	17	10	16	21	3	3

NOTE: Some pertinent facts gleaned from this survey (see also the data in Table 1) are as follows:

Only four machines operate at a pressure of 1200 lb per sq in. Two at the Ford Motor Company, one at the Firestone Tire & Rubber Company, and one at the Dow Chemical Company.

Pressures of 400 to 500 lb per sq in. are most popular.

Mixed-pressure turbines are still rarely used.

There are only five straight-condensing turbines.

Only two machines use steam of 950-F total temperature; four machines use steam of from 800 to 825 F; and the remainder use steam under 750 F.

All but six machines are under 10,000-kw capacity.

The speed for all but 22 of the units is 3600 rpm.

TABLE 3 EFFECT OF SIZE OF STEAM GENERATING EQUIPMENT ON PRICE

Size of steam-generating equipment, lb per hr	Cost per 1000 lb of steam, per cent
100000	Base
200000	0.700
300000	0.600
400000	0.550
500000	0.525
750000	0.500
1000000	0.485

TABLE 4 EFFECT OF SIZE OF TURBOGENERATORS ON PRICE

Rating, kw	Cost per kilowatt, per cent
1000	Base
1500	0.850
2000	0.775
2500	0.730
5000	0.635
7500	0.625
10000	0.687
15000	0.652
25000	0.623
50000	0.462

Before leaving the subject it is quite pertinent to remark that in too many new installations the block of additional power required is underestimated; consequently, the capacity of the new unit soon becomes inadequate. Future growth of electric load is a problem that deserves thorough study and analysis. Moreover, the greater the machine capacity, the less the unit initial cost and maintenance. In Table 3 are some recently published figures showing the influence of size on the cost per 1000 lb of steam for steam-generating equipment. Similarly, Table 4 indicates the effect of size on the price per kilowatt of turbogenerators. The figures in Tables 3 and 4 are for 450-lb 750-F units, but the ratios will be approximately the same for other pressures.

Encouraging this trend toward larger units has been the increase in availability of boilers. It is practically equal to that of turbogenerators. Hence, the common practice today of installing one boiler unit per turbine unit on interconnected systems or in plants having two or more units.

STEAM CYCLES

Conditions existing in each industrial plant dictate the steam cycle and type of turbine to be selected.

Steam Cycle No. 1. The simple cycle shown in Fig. 2 offers

large savings when applied to an existing power plant. The back-pressure turbine, with throttle pressure higher than the existing plant equipment, exhausts the steam into the old generating equipment or to the station steam header. A few of the industrial plants which recently have adopted this cycle are described below.

Weirton Steel Company at Weirton, W. Va., was faced with a rapidly increasing power load. There was no spare generating equipment and the 215-lb boiler plant was 26 years old. They installed a 10,000-kw superposed machine operating at 800-lb throttle pressure and 800-F total temperature. This unit exhausts at 215-lb back pressure and delivers sufficient steam to permit retirement of all the old low-pressure boilers. The result has been continuous and satisfactory operation with a saving of 500 tons of coal per day for the same load carried previous to the expansion. The two boilers, each with a capacity of 400,000 lb per hr, are fired primarily with pulverized coal, with coke-oven gas automatically spilled over into the boilers when not needed in the steel mill. The boilers operate on 100 per cent make-up water, chemically treated. As week-end shutdowns are not permitted by a steel-mill power load, these boilers have low steaming rates per foot of furnace width and ample drum space for liberation of steam.

The recent installation at Dow Chemical Company is another example of possible savings from power generation. The old plant consisted of several 400-lb 600-F boilers and turbines. Part of the 400-lb steam is used directly in the chemical processes and the remainder passes through reducing turbines, some exhausting at 150-lb pressure, some at 10-lb pressure, and the others expanding to 26 in. of vacuum, where the steam is exhausted to evaporators. Instead of enlarging their 400-lb plant they decided to purchase a 1250-lb 825-F boiler and back-pressure turbine.

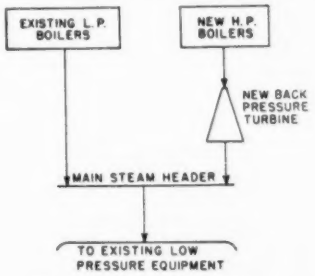


FIG. 2 STEAM CYCLE NO. 1

Some of the pertinent facts on which this decision was based were (1) favorable space requirements; (2) a 1250-lb plant, while providing additional 400-lb steam needed for process work, would allow the generation of approximately 30 per cent more power; (3) the thermal efficiency of plant, the fuel cost being relatively high; and (4) the additional cost of the development would be replaced out of earnings in a reasonable time.

Modernization of the Gulf Refining Company's power plant at Port Arthur, Texas, is another typical example. In 1917 the power facilities were designed for 200 lb and 530 F with 15-lb pressure exhaust to process. Six years later another plant was built designed for 240 lb and 580 F, and two 5000-kw condensing machines. In 1928 another 5000-kw condensing turbogenerator was purchased for a throttle pressure of 15 lb to use excess steam not required in process work. A year later they installed a 10,000-kw 500-lb 630-F turbine, exhausting to process at 130-lb pressure. Although these combined plants were generating a kilowatthour at a very low heat consumption, plans were made in 1937 to reduce this still further through increasing by-product generation. Six 225-lb boilers in use since 1917 were replaced with two modern oil-fired or gas-fired self-contained steam generators with capacities of 200,000 lb per hr at 725 lb and 650 F. Two new 650-lb 650-F steam turbines were installed to drive the existing 4000-kw generators. These turbines exhaust to process at 130 lb and 400 F. In case the turbines fail to maintain steam pressure in the process lines, three sets of pressure-reducing and desuperheating apparatus automatically make up any deficiency.

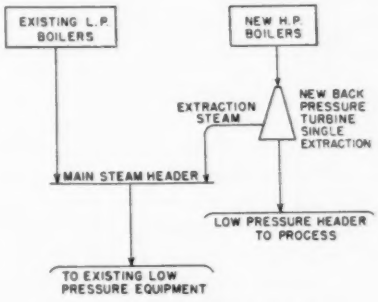


FIG. 3 STEAM CYCLE NO. 2

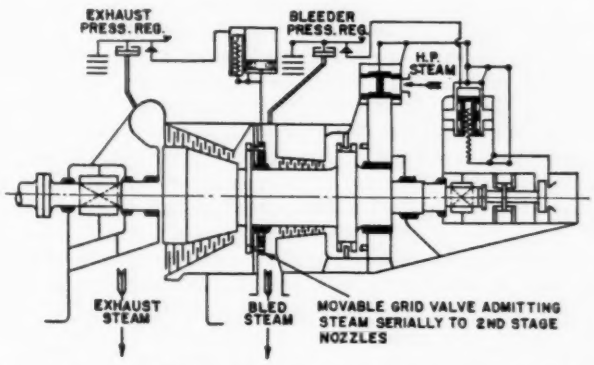


FIG. 4 CONTROL-MECHANISM DIAGRAM FOR AUTOMATIC BACK-PRESSURE BLEEDER TURBINE

A novel use of the back-pressure turbine occurs in one of America's leading plants devoted to the manufacture of rubber products. Due to a high percentage of impurities in the raw feedwater it was decided to use a closed circuit in the feed cycle. The 10,000-kw machine operates on a throttle pressure of 1250 lb and at a steam temperature of 750 F, and exhausts at 235 lb to evaporators which in turn supply steam to process at 180-lb pressure. Thus, the boiler-water problem is simplified and make-up practically eliminated. The evaporator feedwater is treated chemically in order to prevent hard scale deposits in the evaporator. In this case the cost of power is being materially reduced as compared with the cost of power from a complete condensing system, and the cost of the vapor leaving the evaporators compares favorably with that of process steam from the old low-pressure boilers. On the basis of normal-load operation the investment in the entire installation will be returned in $3^1/_2$ to 4 years.

In case of turbine outage, a pressure-reducing desuperheating station delivers steam to the evaporators from a single high-pressure boiler with a capacity of 300,000 lb per hr.

Designed to handle base load, the boiler and turbogenerator have been available 95 per cent of the time during the last year. The unit is shut down each week-end for a period of 24 to 36 hours. During this time the high-pressure boiler is bottled up and retains a pressure between 200 and 500 lb. After operating $2^1/_2$ years, the turbine was opened for inspection. It was found to be in good condition and free from deposits on the blading.

Steam Cycle No. 2 This cycle is illustrated in Fig. 3 and con-

sists of a turbine with throttle pressure higher than that of the existing plant, bled at one point only and exhausting into the old generating equipment or station header. It is ideal where the power-generating facilities already include condensing turbines. There are several industries which employ two pressures for steam process use, such as the kraft paper trade for example where this cycle is quite generally employed. The turbine-governing system is designed to maintain constant pressures at the extraction point and at the exhaust, with demands for steam varying independently. Fig. 4 shows this particular type of control.

The power facilities at Gaylord Container Corporation in Louisiana were modernized by the installation of equipment operating on this cycle—a 450-lb 710-F boiler unit and a 7500-kw back-pressure automatic-extraction turbine used in conjunction with the old 9000-kw plant of three condensing machines. The investment of approximately $400,000 produced savings in operating costs of $650 per day.

Similarly, in the food industry, General Foods Corporation at Battle Creek, Mich., recently invested about $500,000 in power-plant improvements and are obtaining annual operating savings of $100,000. Prior to 1937 all power was purchased and the five boilers operating at 150-lb pressure supplied the steam for cooking. This boiler plant is now stand-by equipment to one 100,000-lb-per-hr 625-lb 720-F steam generator supplying a 2500-kw 30-lb-back-pressure 125-lb extraction-type turbine. All electric energy needed beyond that generated by process steam is purchased from the local utility. One unusual feature which is used to provide additional flexibility is a surface condenser operated only during the summer time when there is no heating load. This plant has no spare boiler or generating capacity and an outage of the new turbine would mean throwing the entire electric load on the utility system, establishing a new demand which would cost about $15,000 for the ensuing twelve months. Outage of the high-pressure boiler would not be so serious, since about 1000 kw can be carried on the new turbine by using 125-lb steam through the extraction opening. An example in the chemical industry is the recent installation at the Columbia Chemical Division of the Pittsburgh Plate Glass Co., Barberton, Ohio. The process steam was originally supplied by exhaust from engine drives on pumps and CO_2 compressors. The balance between power and steam was obtained by generating electric energy with condensing turbogenerators, one of which was of the extraction type. For the new equipment a pressure of 900 lb was selected. At that time, 1935, this was thought to be the pressure limit at which boilers using a large percentage of treated make-up water would operate satisfactorily. In this case only a small amount of condensate returns from process. The temperature was limited to 750 F so that the turbine-exhaust temperature would about equal the temperature in the 140-lb header. A single boiler with capacity of 180,000 lb per hr serves a 6250-kw turbine bled at 400 lb for caustic drying, one of the highest bleed pressures used to date, final exhaust being at a pressure of 140 lb. This turbine, in Fig. 5, is a tandem unit with the 400-lb bleed connection between its two sections, but has only one cylinder and rotor. Automatic control of the bleed is provided for by a duplicate of the high-pressure steam chest and valves on the 400-lb to 140-lb section. This type of bleeder valve was necessary because of the very high extraction pressure, which was considered too high for the usual grid valve which at that time was limited to about 200-lb pressure. Operation indicates that approximately 25 per cent less steam is required with the high-pressure unit than with the old equipment for comparable loads. An additional unit is being installed in this plant using a newly developed water-cooled stoker which should prove beneficial as the plant operates on a relatively low grade of fuel. At the Akron plant, The Goodyear Tire & Rubber Co., a large percentage of the total steam generated is used in process work. About three years ago a study was made to determine the best plan of modernizing their old plant where boilers delivered 200-lb 530-F steam to a header supplying all powerhouse

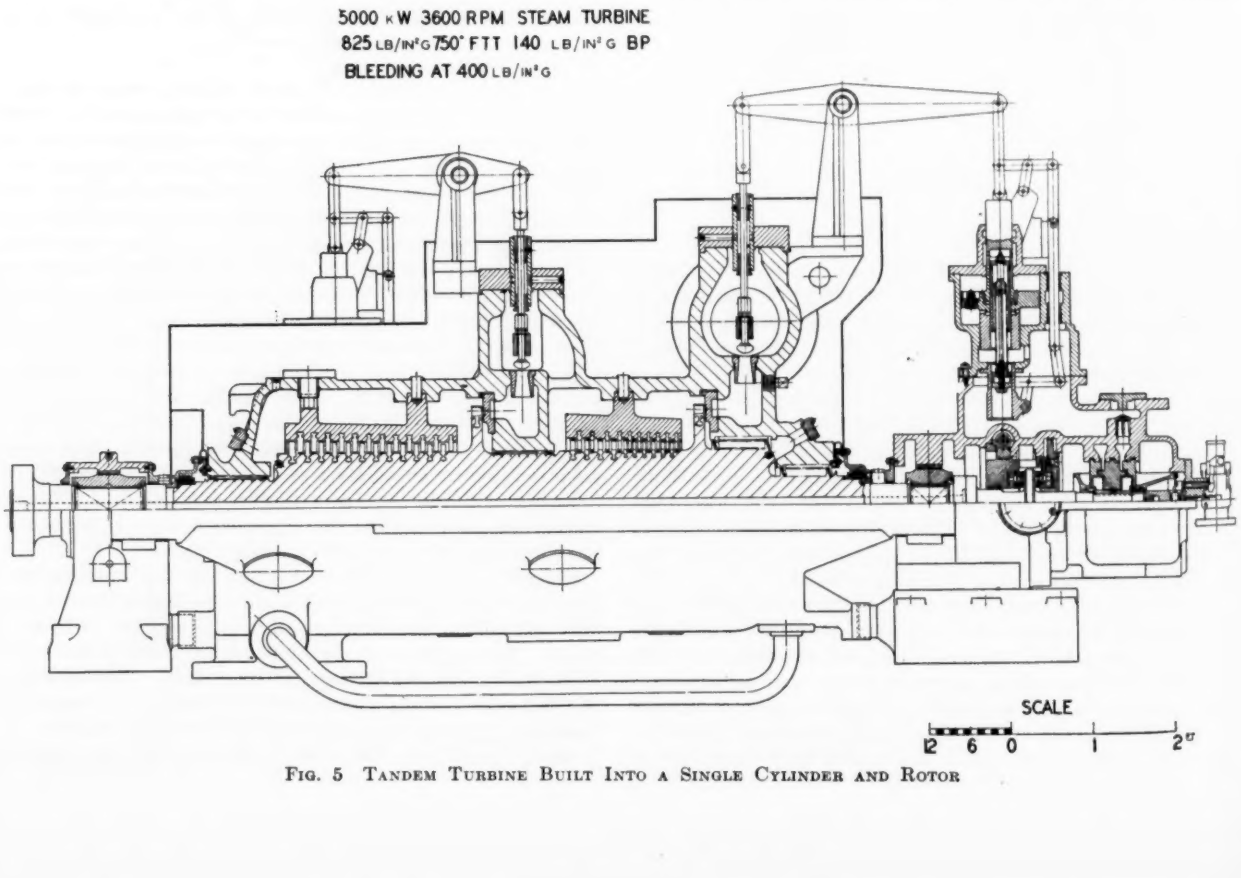


FIG. 5 TANDEM TURBINE BUILT INTO A SINGLE CYLINDER AND ROTOR

equipment. Process steam, 50 per cent of the total, was delivered through a reducing valve at a rate of 150,000 to 285,000 lb per hr. The study included pressures from 500 to 1200 lb. From these it was decided that 800-lb 750-F steam would best suit the local conditions. The new 7500-kw turbogenerator was of the noncondensing, extraction type arranged for extraction of 150,000 lb of steam per hr at 200-lb pressure to operate the old turbogenerators at full load if necessary. When no steam is extracted, all steam passes to the exhaust at 100-lb pressure for process use and the turbine operates as a straight back-pressure machine. When necessary a pressure-reducing desuperheating station supplies steam to process direct from the high-pressure system. The old boilers remain available for stand-by service.

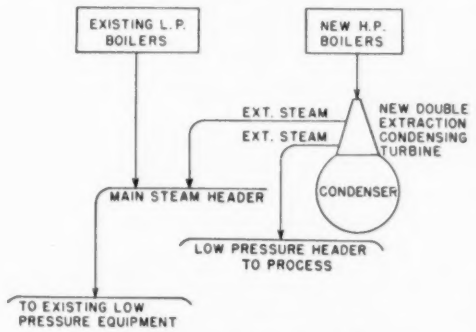


FIG. 6 STEAM CYCLE NO. 3

Steam Cycle No. 3. The double-extraction condensing turbine used in steam cycle No. 3, shown in Fig. 6, involves a more complicated control for maintaining constant-extraction pressures but it has the advantage of greater flexibility through using a condenser to balance the power demand. It is popular for new plants and has the same effect as applying Cycle No. 2 to old condensing plants. The necessity for controlling the extraction pressures and passing excess steam to the condenser requires throttling or by-passing of steam for all electric and steam loads except the one combination for which the machine has been designed. The farther the actual operating conditions depart from the design conditions the lower will be the turbine efficiency. However, a number of turbines of this type are in successful operation and have been very satisfactory, especially where the load has been fairly steady and predictable within reasonable limits. In any case it should be used where there is danger of serious lack of balance between steam or electric loads which would cause loss of economy.

Recently the R.C.A. Manufacturing Company, Inc., modernized their power plant by installing two new double-extraction condensing turbines to drive old 2500-kw generators. The two new boilers and the turbines are designed for steam at 900-lb pressure and 750-F temperature. The 175-lb extracted steam enters the station headers to operate old low-pressure turbogenerators. The 20-lb extraction opening supplies steam to process. The total cost of this reconstruction work was about $750,000 and the annual savings in the fuel bill and maintenance charges amounted to approximately $125,000.

During the last five years there has been rapid development in the kraft-paper industry throughout the southeastern section of the country. Electric power is a major item in the manufacture of paper and accordingly much engineering thought has been given to the reduction of power charges. The kraft process utilizes large quantities of steam at pressures of approximately 150 and 30 lb per sq in. gage. The double-extraction condensing turbine fits into this scheme perfectly and has been selected for such new mills as those of Union Bag & Paper Corporation at Savannah, Ga., St. Joe Paper Company at Port St. Joe, Fla., and Brunswick Pulp & Paper Company of Brunswick, Ga. In conjunction with this type of machine, and in order to get proper steam-electric balance, a back-pressure extraction turbine has been used in several of the new plants.

MIXED-PRESSURE TURBINES

The mixed-pressure type of turbine is provided with an opening which receives excess steam from process passing it through the lower pressure stages of the turbine, or should the requirements be reversed, steam is extracted to process. There are a number of special cases where such a turbine can be used to advantage, one of which is the forging plants of the automobile industry. Most automobile plants are located in regions where purchased power is available at attractive rates. Moreover, their steam requirements are generally met by low-pressure boilers without electric generation or with only sufficient power generation to balance the need for steam. The drop-hammers, using 125-lb to 175-lb steam, exhaust near atmospheric pressure to process, such as heating in winter, hot-water heating, drying ovens, and pickling tanks. Sometimes the use of motor-driven and steam-driven compressors gives control over this low-pressure steam to avoid waste to atmosphere. With these conditions in mind it becomes apparent that a combination extraction and mixed-pressure condensing turbine is an ideal method of controlling the process-steam supply and simultaneously generating some cheap power.

A case in point is one of the automobile plants in the Middle West. As first laid out, all excess low-pressure steam was absorbed in a mixed-pressure turbine. Recently the plant use of low-pressure steam has been increasing faster than the demand for high-pressure steam. This lack of balance in the steam loads has been corrected by removing a 5000-kw straight-condensing turbine and installing in its place a new 3000-kw turbine designed for either extraction or mixed-pressure operation. The old jet condenser continues to serve the new machine. This change also includes four new boilers, each with a capacity of 150,000 lb per hr, operating at 450-lb pressure.

Another ingenious arrangement of back-pressure, back-pressure-extraction, and mixed-pressure-condensing types of turbines is shown in Fig. 7, which is a line diagram of the power-generating facilities at Scott Paper Company, Chester, Pa. This plant operates 24 hours a day and prior to the installation of the new boiler and 3500-kw turbogenerator in 1936, about two thirds of

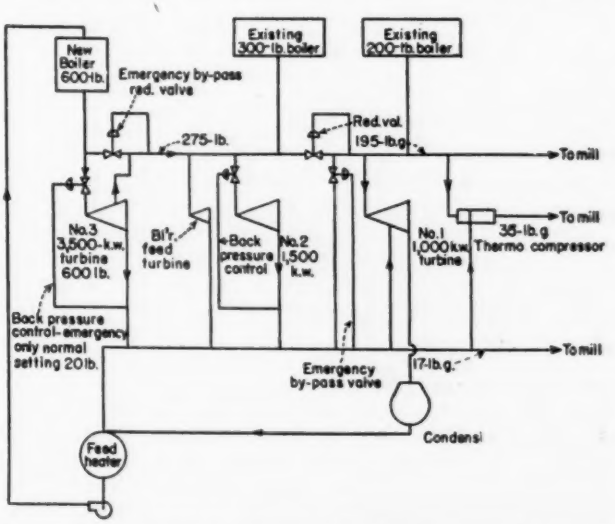


FIG. 7 FLOW DIAGRAM FOR THE POWER-GENERATING FACILITIES AT THE SCOTT PAPER COMPANY, CHESTER, PA.

the power requirements were purchased. As the system is now operated, No. 2 turbine governs the 17-lb back pressure; therefore, the power output depends on this 17-lb-steam demand. If this load falls below the control of No. 2 turbine, the emergency governor on No. 3 turbine, set at 20 lb, goes into action, automatically reduces load and, in turn, also the steam output of No. 3 turbine. All 17-lb steam in excess of normal process demands is absorbed by No. 1 turbine, running as a mixed-pressure machine either on 200-lb or 17-lb steam or both.

Steam-Pressure Reducers and Desuperheaters

The use of superposed turbines has been largely responsible for the recent rapid development in steam-pressure reducing and desuperheating equipment. In case of high-pressure outage, continuity of operation of low-pressure equipment is usually essential; therefore, availability and speed of response are of equal importance to high capacity and accurate regulation.

The two steps, reduction in pressure as well as control of temperature, are closely related and have a variety of uses. Although the heat loss is limited to radiation, the loss of potential power is considerable and for this reason such equipment should be used only for emergency operating conditions.

In a large eastern central station where the topping unit was out of service for months, the pressure-reducing-desuperheating station was in almost continuous service for about 5000 hr. Under these conditions the heat rate was 6000 Btu per net kwhr higher than normal operation. However, the use of this emergency equipment resulted in a heat rate about 10 per cent lower than that which would have been obtained by using the old standby low-pressure boilers.

As the foregoing illustration emphasizes the service reliability of this equipment, so is speed of response typified by the installation at the West End Power Station of The Cincinnati Gas & Electric Company. The requirements were that the pressure-reducing-desuperheating station must automatically react within the time consumed for closing the main-turbine governing valves, namely about $^1/_2$ sec, in order that the steam supply to the low-pressure generating equipment may not be interrupted by the failure of the high-pressure turbine. To accomplish this, two valves were used: A shut-off valve which moves quickly to its wide-open position and a regulating valve floating always in the proper position to pass a quantity of steam corresponding to the low-pressure demand. The desuperheating element is a venturi nozzle fitted with sprays and a water-storage tank located so that the water level is at the same elevation as the venturi. Thus, when steam flows through the venturi, a pressure drop is created which forces water into the hot steam and no automatic valve mechanism is required in the water line. However, in all cases, the required speed of action depends entirely on the steam-storage capacity of the low-pressure system.

Another unusual feature of this development is the ability to control extremely high quantities of steam. The flow through a 1200-lb 50,000-kw topping unit exceeds 1,000,000 lb of steam per hr, yet the pressure-reducing-desuperheating station picks up this load almost instantaneously and continues to "pinch hit" for the turbine with no interruption of steam supply to the low-pressure system.

Although the central-station industry has contributed largely to this development, the acceptance of high pressures in manufacturing plants has made such equipment indispensable there. In addition to the by-pass for a topping unit, the maintenance of process-steam pressures and temperatures under all conditions of electric load when old and new sections of a plant are joined together is another important application.

Advances in the art of combustion control have also contributed much to improvement in regulation of steam-pressure and temperature controls. There are so many applications of this equipment that its importance has assumed new proportions.

Steam-Generating Equipment

The modern steam generator bears little resemblance to the old-time boiler which was largely a vessel filled with water. Compared with its steaming capacity, the quantity of water stored in the drums and tubes was considerable. Along with the terms "boiler horsepower" and "per cent rating," the boiler itself has all but disappeared. To take its place have come elaborate designs of waterwalls, economizers, air preheaters, superheaters, and in some cases reheaters. Lack of water storage is compensated for by high steaming ability fortified by quick-acting feedwater regulating equipment and automatic combustion-control systems; all of these factors have helped to produce the present boiler unit with its very flat efficiency curve.

Contributing its share to this development has been parallel accomplishment in feedwater conditioning. Difficulties arising from deposits on the blades of high-pressure turbines, due to carry-over from the boiler, have made necessary the present-day

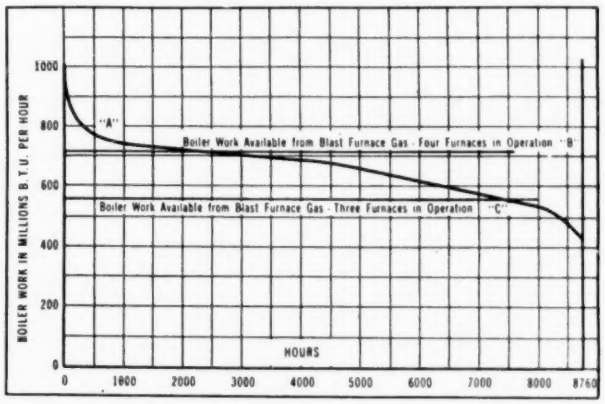


Fig. 8 Load-Duration Curve for a Steel Plant With Four Blast Furnaces

development of steam washers which are usually built inside the steam drum. However, it should be emphasized that universal use of these washers is not justified, especially where pressures are under 200 lb or where steam goes direct to process.

Local conditions affect the method of firing. The character of the coal and the size of the boiler unit are determining factors as to whether stokers or pulverized fuel shall be selected, and if the latter, whether the wet- or dry-bottom furnace shall be adopted. In some localities, natural gas and oil are cheap and abundant. Steel-mill practice dictates that excess blast-furnace or coke-oven gas receive priority over other fuels. Some boilers are arranged for three or four different fuels which come into service automatically according to some predetermined sequence. In general, where a multiplicity of fuels are to be used, pulverized-fuel firing is, for the solid fuels, the easiest method of accomplishing desired results. It is necessary that all fuels be burned in the same boiler furnace to avoid the expense of duplicate boiler equipment.

The fallacy of averages often leads designers astray in determining the size and use of equipment. This is well illustrated by Fig. 8 which is a load-duration curve for an active steel plant with four blast furnaces. In this case, the load values have been converted to heat requirement in boiler work, that is, the total quantities of heat that must be absorbed by the boilers, and are indicated by the curve marked *A*. The line *B* represents the average quantity of this heat that can be supplied by blast-furnace gas when four furnaces are in normal operation, and line *C*

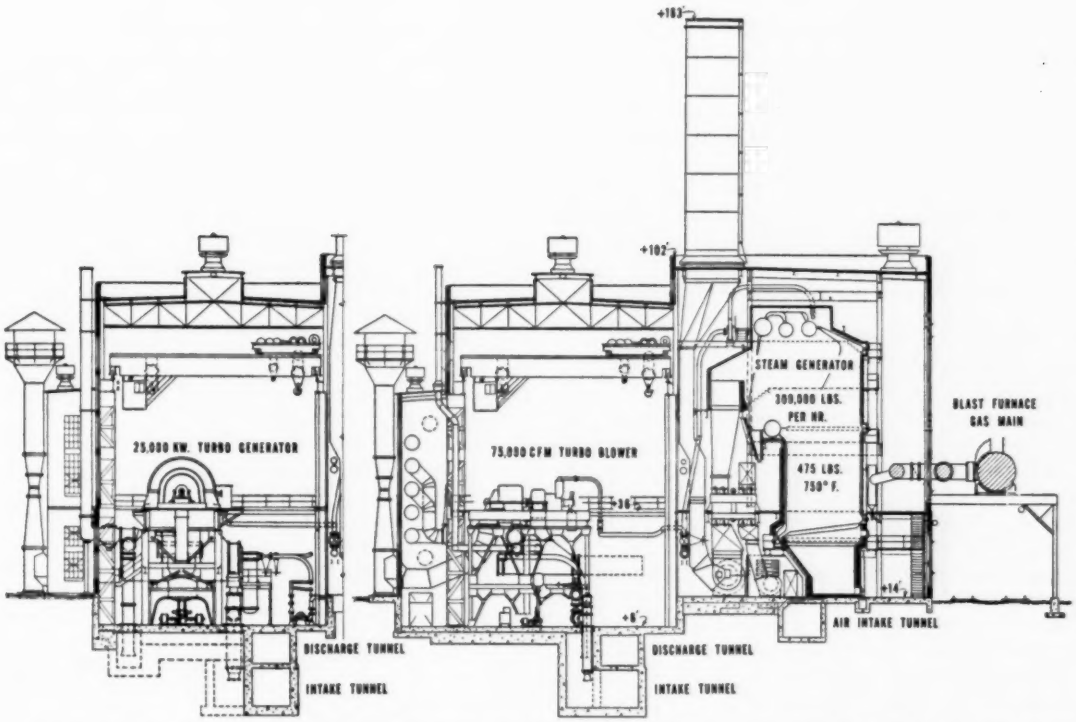


FIG. 9 CROSS SECTION OF NO. 5 POWER STATION, CARNEGIE-ILLINOIS STEEL CORP., SOUTH WORKS, CHICAGO, ILL.

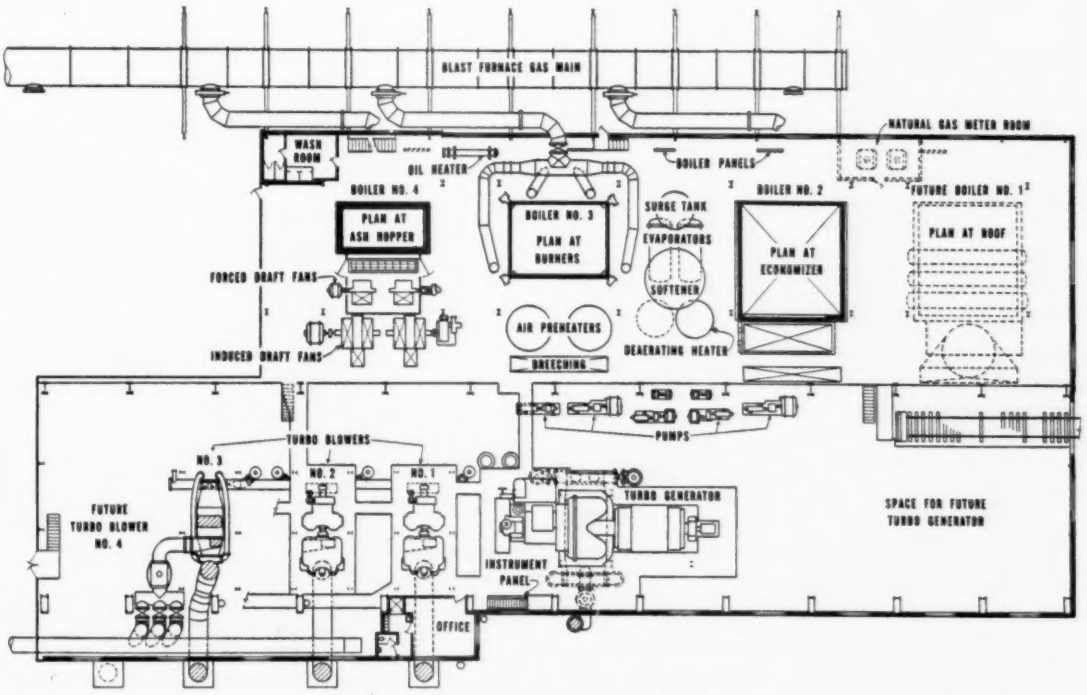


FIG. 10 PLAN OF THE PLANT SHOWN IN FIG. 9

when three furnaces are in use. It should be borne in mind that while lines *B* and *C*, covering averages, are necessarily straight lines the actual supply of gas from the furnaces is apt to fluctuate considerably. There are interruptions due to casting or furnace slips and variation in use of gas for heating purposes in the mill. In the diagram the areas under curve *A* and above lines *B* and *C* indicate the average amount of supplementary fuel which must be supplied, while, conversely, the areas below the lines indicate periods of nonuse of available gas.

An auxiliary-fuel supply must be available to supplement the blast-furnace gas and also carry the entire plant in case no gas is available. This requirement has been very strongly emphasized

in some of the steel mills in the last 4 or 5 years where operations were of such a nature that there was an active demand for process steam, although the blast furnaces were practically shut down and little or no blast-furnace gas was available.

In the case of industrial plants using steam for process work, while the average quantity of steam rejected by the generation of electrical load may balance the average requirements of industrial steam, the times at which these loads balance are apt to be seriously out of proportion and unless a very careful analysis is made the use of averages in determining these factors will be very misleading.

One of the largest industrial power plants recently completed is the No. 5 Power Station of the Carnegie-Illinois Steel Corporation at their South Works in Chicago. Figs. 9 and 10 show a cross section and plan, respectively, of this project which was placed in service last winter.

The building houses three boilers, each with a capacity of 300,000 lb of steam per hr at 450 lb pressure and 750 F temperature, one 25,000-kw condensing turbogenerator, and three 75,000-cfm condensing turboblowers. Space is also provided in the present building for one additional boiler, a future 50,000-kw turbogenerator and another turboblower.

Fig. 9 includes a cross section of the boilers which are of the four-drum bent-tube design equipped with convection-type superheaters, economizers, and air preheaters. Each boiler has two induced-draft fans and two forced-draft fans. A geared turbine drives one of each type of fan and a two-speed motor drives the other.

About 20 per cent of the total feedwater supply is make-up taken from Lake Michigan and treated by a hot-process, lime and soda softener. A deaerating heater and two closed feedwater heaters complete the feedwater-heating equipment.

An interesting feature of these boilers is the tangential firing designed to handle blast-furnace gas as primary fuel with natural gas and oil as secondary fuels when the bulk of surplus blast-furnace gas is being utilized for metallurgical purposes. Provision has also been made to use pulverized coal as secondary fuel in the future if and when the price structure warrants such a change. At each corner of the furnace is located one of the burner boxes consisting of the following: Three 11-in. × 14-in. burners for blast-furnace gas, two 6-in. diameter burners for natural gas, two $2^1/_2$-in. diameter burners for fuel oil, and two 14-in. × $4^1/_2$-in. burners for pulverized coal (future). There is a common windbox of cast iron and each burner has an individual louver damper which can be adjusted and clamped in a position to pass the correct amount of air for combustion over the complete operating range. Back of these individual dampers is the variable main damper which controls the air pressure in accordance with load requirements.

The boiler and auxiliaries are under the control of an air-operated system, the primary function of which is to supply fuel and air to the furnace in proportion to the steam demand.

A butterfly valve in the blast-furnace-gas supply line to the boiler responds first to the loading pressure set up by the master controller, which opens the butterfly valve to its maximum effective opening before any supplementary fuel is supplied. The position of the valve maintains pressure on the burners in proportion to demand regardless of fluctuations of pressure in the main. A normal pressure in the blast-furnace-gas supply main to the boiler plant is 20 in. of water. In order to avoid failure of the gas supply for metallurgical use, the regulating system is so set that should the pressure in the blast-furnace-gas main fall as low as a predetermined minimum of 14 in. the butterfly valve in the supply main to the boiler assumes a position which will not lower the pressure in the main below 14 in. Any additional fuel required to produce the requisite steam will automatically come from the auxiliary-fuel supply.

When the available blast-furnace-gas supply is insufficient for the steam demand, the secondary fuel normally brought into service next will be natural gas. Alternately, the natural-gas supply may be made the primary fuel by manual manipulation of the selector valves. In such case the blast-furnace-gas supply would be completely shut off.

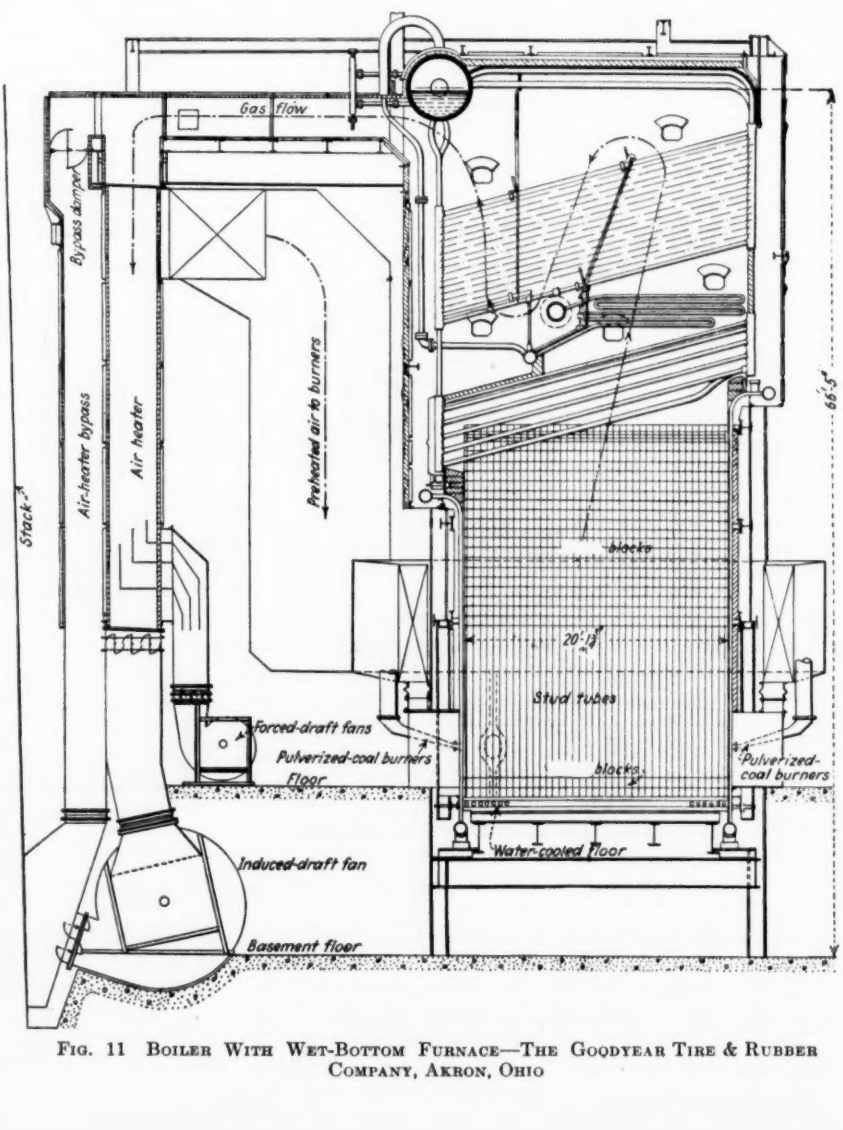


FIG. 11 BOILER WITH WET-BOTTOM FURNACE—THE GOODYEAR TIRE & RUBBER COMPANY, AKRON, OHIO

Normally the fuel-oil supply is closed off, but this fuel may be substituted for the natural gas as a secondary fuel or a supplementary fuel. Separate shut-off valves are provided in each of the two oil-supply lines and minimum stops are furnished on each of the oil-flow regulating valves. Each one of the latter controls a supply to four oil burners, one located at each corner of the furnace. By proper operation of the shut-off valves it is possible to bring first one set of four burners into service in response to the control loading-pressure change and after these burners are at their maximum capacity to cut in the second set of four burners. Thus, greater flexibility and good atomization of the oil over a wide range is afforded.

Another outstanding high-pressure boiler installation is that at The Goodyear Tire & Rubber Company at Akron, Ohio. This 800-lb 750-F boiler, the cross section of which is shown in Fig. 11, serves as the sole source of steam for a new 7500-kw back-pressure extraction machine previously mentioned in this paper, and is located in the space formerly occupied by four low-pressure boilers.

Due to the availability of Pittsburgh No. 8 coal, with its low ash-fusion temperature, a wet-bottom furnace was chosen. It is tapped intermittently from one outlet in the middle of a side wall. Satisfactory tapping at the rate of 4 to 5 tons per hr is obtained at a steaming rate of 150,000 to 350,000 lb per hr. The water-cooled furnace bottom has given no trouble from slag leaks. During three years of operation approximately seventy-five

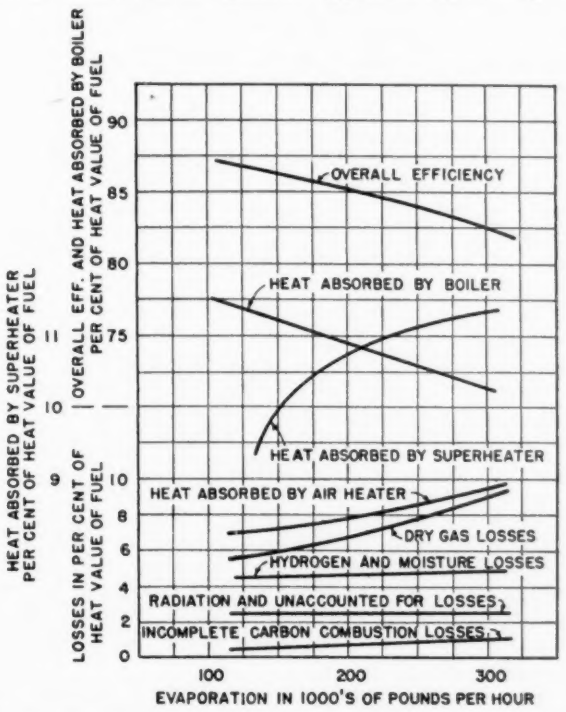


Fig. 12 Overall-Performance Characteristics of the Boiler Shown in Fig. 11

eroded side-wall blocks along the slag line have been replaced. Stud-tube construction is used on all four walls to a height of about 13 ft above the slag line of the furnace. The upper sections have smooth bare metal blocks. The large proportion of black radiating surface thus provided just below the boiler has prevented any trouble with slagging in the lower tubes of the boiler and the superheater. At maximum load the heat liberation is approximately 33,000 Btu per hr per cu ft of furnace volume and the mean furnace temperature is 2500 F.

Fig. 12 shows the overall performance of this steam generator based upon combined averages from one year's operation. The efficiency shown includes coal for banking, starting, steaming, and usual operation. The curves in Fig. 13 give interesting data

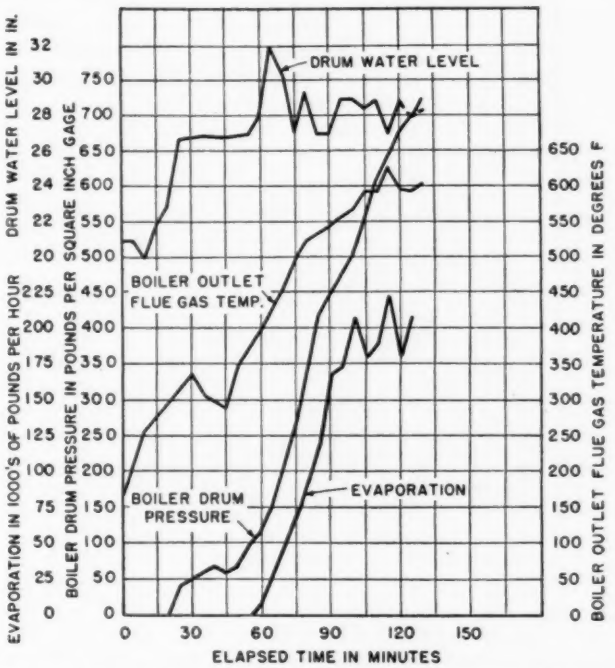


Fig. 13 Data Over 2-Hr Period While Putting the Boiler, Shown in Fig. 11, on the Line

taken over a 2-hr period consumed in putting this boiler on the line.

Self-Contained Boilers

During the last several years a new standardized steam-generating unit has come into use. Designed originally for the replacement demand in industries requiring small quantities of steam, its development has been such that today it is used in both industrial and central-station plants and for steam conditions ranging up to 700 lb and 850 F, and capacities up to about 250,000 lb per hr.

This boiler is generally of the two-drum type with the boiler tubes extended to enclose the furnace. It can be fired with oil, gas, or pulverized coal. Requiring no basement and small head room, it makes an ideal low-cost installation where a compact unit is needed to replace antiquated equipment. Due to the expansion and contraction of the furnace floor, the foundation requirements are relatively expensive but this is not a major item. The low draft loss simplifies the installation where only natural draft is available and saturated or moderate steam temperature is satisfactory.

Such a boiler fired with oil or gas develops from 80 to 82 per cent overall efficiency. An air preheater can be added to raise this to 84 or 86 per cent, but often it is not done as the increased draft loss necessitates the use of an induced-draft fan. This additional power cost reduces the saving to a point where, on small units, the greater investment often becomes unattractive. However, pulverized-coal firing requires some means of drying the fuel and an air preheater is often installed for this purpose.

In the paper-mill industry, several of these boilers have been used in connection with bark-burning stokers located in Dutch ovens surmounted by a capacious bark-storage bin as shown in

Fig. 14. The bark of hogged sawmill refuse containing about 50 per cent moisture is fed to the stoker. The fuel slides down the inclined grates, drying out rapidly as it reaches the burning zone. Combustion is completed in the main furnace. About 30,000 lb per hr of such fuel has been successfully handled in this manner. The ash and dust carried over the parapet wall and through the waterwall screen can be easily scraped or blown into the ashpit of the boiler proper. As a supplementary fuel, oil, gas, or pul-

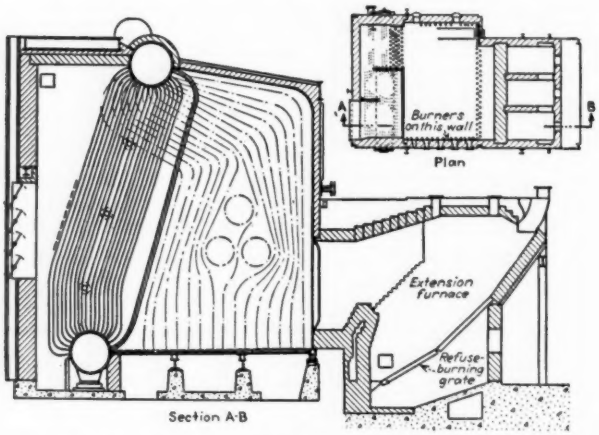


FIG. 14 SELF-CONTAINED BOILER WITH DUTCH OVEN

verized coal can be burned in standard register-type burners located in the front wall of the boiler furnace. Units of this type are in operation with steam-generating capacities as high as 80,000 lb per hr, 450-lb pressure, and temperatures in excess of 700 F.

A furniture plant using this type of boiler fired with refuse of kiln-dried birch and maple reports better than 12 per cent CO_2 on wood alone and 15 per cent CO_2 or better when using only pulverized coal. In this installation only a single furnace is used. The wood chips are introduced at the top of the furnace and on one side, so as not to interfere with the turbulence at the pulverized-coal burners. The secondary-air ducts admit forced draft near the bottom of the furnace so as to prevent the wood piling up on the floor. In this way much of the wood is burned in suspension.

While on this subject of combination coal and wood firing it may be of interest to note that in Sweden fine sawdust, bark, and shavings, which at times contain 50 to 60 per cent moisture, are burned on water-cooled inclined grates using primary air at temperatures as high as 650 F.

FORD MOTOR COMPANY

No paper on industrial power would be complete without reference to the River Rouge Plant of the Ford Motor Company. It is probably the largest industrial power plant in the world and it is also the largest single installation of 1200-lb equipment.

It was originally designed in 1920 for 240-lb steam. The original low-pressure generating plant consisted of four 30,000-kw turbines. An interesting item is that they were designed and built by the Ford organization. This plant was one of the original users of pulverized fuel and has since adhered to this method of firing.

During 1925 and 1926 four of the original 200,000-lb-per-hr boilers were rebuilt to give a capacity of 500,000 lb per hr and waterwalls, air preheaters, and radiant superheaters were added. Subsequent to 1929 this plant was rebuilt in several steps. In 1929 two of the old boilers were replaced with two 1350-lb 760-F generating units, each with a capacity of 700,000 lb per hr, and one 110,000-kw high-pressure condensing turbine was installed.

TABLE 5 OPERATING DATA ON HIGH-PRESSURE BOILERS AND TURBINES AT THE FORD MOTOR COMPANY UP TO MAY 1, 1938[a]

	Boilers			Turbines		
Unit number	1	3	5	1 & 2	3 & 4	5
Installed hours	60264	56568	15672	59332	15648	18271
In operation, hr	44242	43298	13944	50804	14088	8939
Down (no-load), hr	16022	13270	1728	8192	1560	9333
Outages, hr	0	0	0	336	0	0
Steam generated, 1000 lb	13097620	12294909	5213249	...	...	...

[a] From a statement by the Ford Motor Company.

TABLE 6 ANNUAL KILOWATTHOURS GENERATED BY THE FORD MOTOR COMPANY DURING THE LAST 15 YEARS

Year	Annual generation, kwhr
1923	260,000,000
1924	275,000,000
1925	424,000,000
1926	429,000,000
1927	377,000,000
1928	556,292,000
1929	702,210,000
1930	565,551,000
1931	385,902,000
1932	370,809,000
1933	302,615,000
1934	512,293,000
1935	657,892,000
1936	724,519,000
1937	825,202,000

In 1936 another of the old boilers was replaced by a 1350-lb 915-F boiler of 900,000-lb-per-hr capacity, and a 110,000-kw high-pressure, condensing, generating unit was installed, together with a 15,000-kw back-pressure turbine for supplying process steam. At present they are arranging for the replacement of the last of the original low-pressure boilers by a 1350-lb 930-F boiler with a capacity of 900,000 lb per hr, together with an additional 110,000-kw turbine.

Each of the 110,000-kw units has the high-pressure element mounted above the low-pressure turbine and the same arrangement applies to the separately excited 55,000-kw generators. In Units 1 and 2, the high-pressure turbine exhausts at 85-lb pressure and is reheated to 560 F by means of steam reheaters before entering the low-pressure cylinder. Higher initial temperatures in later units avoided this. Also, the generators on Units 6 and 7 will be hydrogen cooled.

The 15,000-kw machine generates all possible by-product power from process steam. The exhaust at 240-lb pressure goes to a battery of evaporators which generate 180-lb steam.

The boilers are of bent-tube design, fired with pulverized coal from a storage system which is supplemented by blast-furnace and coke-oven gas when available. About 4×10^9 cu ft of blast-furnace gas per month are normally consumed.

These new high-pressure units should produce a kilowatthour for less than 11,000 Btu. Table 5 covers the operation of the high-pressure boiler and turbine equipment from the time of the installation up to May 1, 1938. The outage of 336 hours on turbogenerators Nos. 1 and 2 was due to leaky tubes in the reheaters which utilize live steam at 1230-lb pressure. The no-load inactivity of the back-pressure unit No. 5 was augmented by building alterations.

The annual kilowatthours generated by the Ford Motor Company during the last 15 years are given in Table 6.

The figures in Table 6 serve as a barometer not only of the automotive industry but also of business in general. It is interesting to note that the output for 1937 was only slightly less than that generated by the Potomac Electric Power Comany and affiliate serving the metropolitan area of Washington, D. C.

OUTDOOR PLANTS

Semi-outdoor installations of boilers and steam-generating equipment have been receiving considerable attention for some time. Certain parts of the equipment of necessity must be enclosed so that operators may function, but in the main these in-

stallations may be called "outdoor plants." While the author personally questions the financial advisability of such developments except in warm climates, a recital of what has been done should be interesting. One of the inherent difficulties is the necessity for making repairs either to the boiler or turbogenerator during bad weather. This has become quite a problem.

In Schenectady, which has a rather cold climate, General Electric Company has a 20,000-kw mercury-vapor installation. In this plant the generating unit and the upper part of the boiler are entirely exposed. Other parts are under cover. This is a particularly favorable location for an outdoor plant, especially because of the large shop facilities which are in close proximity and which can be used for repair work.

The Utah Power and Light Company in 1936 placed in operation their Provo Station, designed as a semi-outdoor plant. The turbine room has a low ceiling and only the lower part of the boiler is enclosed. The water columns have separate enclosures. A waterproofed steel casing suitably insulated is provided for the upper part of the setting. The induced-draft fans are exposed and so are driven by totally enclosed motors which draw ventilating air from beneath the roof slab and vent downward to permit operation even though buried in snow. Special precautions were taken to prevent freezing in the water-column drum connections, feedwater regulating valves, and instrument piping.

This design is now spreading to industrial power plants, especially in the South. At their Sugar Land, Texas, Plant the Fort Bend Utilities Company has just placed in service an outdoor steam generator of 160,000-lb-per-hr capacity, fitted with an airtight steel casing, sealed at points of drum and header movement. The only housing provided is for auxiliary motors, control equipment, and operator's station.

Along this line there have been a fairly large number of synchronous condensers installed in the outdoor manner.

Unusual Boilers

Among the principal foreign boilers are the Loeffler, Velox, Benson, and Schmidt-Hartmann. The Loeffler and Velox are of the forced-circulation type; the Benson is in the so-called "once-through" class; and the Schmidt-Hartmann is the indirect heat-exchanger type. The only one of these now installed in this country is a Velox boiler operating in a plant of an eastern oil company.

The "Steamotive" boiler is being developed jointly by the Babcock & Wilcox Company, General Electric Company, and Bailey Meter Company. This unit is a complete small steam electric generating unit of relatively high efficiency and adaptable to many fields of industry. Two units, each having a capacity of 40,000 lb per hr, have been built to drive two 2500-hp turboelectric locomotives for a western railroad. They operate at 1500-lb pressure and 950-F temperature. Oil is used as fuel and the fuel-combustion rates reach very high values, namely about 400,000 Btu per hr per cu ft of furnace volume.

In the central-station industry the mercury-vapor cycle has received serious consideration and the installations at Hartford, Kearny, and Schenectady have provided valuable data for future designs. There are also a number of mercury-boiler installations in the industrial field, the most numerous being in connection with oil-refinery work, where the mercury unit is used as a heat carrier because of the possibility of very fine temperature regulation.

Fuels

As stated previously, the fuel selected depends largely on local conditions. By far the fuel most widely used is bituminous coal in pulverized form or on stokers. Bunker C oil and natural gas are available in many localities. In the steel industry blast-furnace and coke-oven gases are utilized when available. However, there remain several uncommon fuels and uncommon uses of ordinary fuels which are of interest.

Anthracite culm accumulates in the mining districts at the rate of millions of tons a year. A small quantity is burned in pulverized form, although its main use is on forced-draft chain-grate stokers or on hand-fired grates. It is low in volatile and has a high-ignition temperature. When burned in pulverized form one difficulty has been to secure sufficiently close regulation of the primary or carrier air to ensure a coal-air ratio approximating that required for maximum speed of flame propagation. Its flame is extremely susceptible to cooling until firmly established; therefore, the secondary air must be admitted to the furnace under careful control.

When burning this culm on forced-draft traveling-grate stokers, excessive sifting and carry-over to the boiler and even beyond, plus failure to follow rapidly fluctuating loads, have also retarded its popularity. In many cases it is necessary in order to secure proper operation, to mix the culm with No. 3 or 4 buckwheat.

Fig. 15 shows a direct-fired anthracite culm system in operation in a New Jersey paper mill. The boiler has been so successful that an additional unit is to be installed soon in another plant of the company. Culm containing as much as 15 per cent moisture and as little as $2^1/_2$ per cent volatile can be burned alone, unmixed with any other fuel and without other means for supporting ignition once the fuel has been ignited with the usual lighting torches.

A fan draws hot air through the pulverizer, dries the coal, and delivers it to the anthracite burners. As the air-coal ratio required for best pulverization is too high for good ignition, the burner arrangement is designed to remove about two thirds of the air in the coal-air suspension arriving at each burner and to divert this relatively clean air to the lower part of the furnace where it mixes with the secondary air entering through the front wall. In this manner the coal which drops through the vertical burners is suspended in a mixture containing the correct amount of primary air and will maintain stable ignition.

Culm containing moisture as high as 10 to 15 per cent has been dried satisfactorily by passing preheated air at temperatures of 350 to 400 F through the pulverizers. The power per ton consumed for pulverizing and firing at normal capacity of the system is about 35 kwhr as compared with 10 to 15 kwhr per ton of the usual bituminous coals. Maintenance cost on the mills is approximately 3c per ton. The overall thermal efficiency is about 3 per cent lower than that of a similar plant burning pulverized bituminous coal.

It may be of interest to note that economic conditions in England have forced the use of anthracite coal in the power industry. The large Tir John Station is using this fuel in pulverized form by means of the bin-and-feeder system. The cold boiler and setting is heated first with fuel oil. Preeconomizer air heaters are used to obtain primary-air temperatures of 700 F. Straight-shot vertical burners are installed.

Scattered through a number of our western states are approximately 2,500,000 million tons of lignite. Proximate analyses from two large sources of lignite are given in Table 7.

TABLE 7 PROXIMATE ANALYSES OF COLORADO AND TEXAS LIGNITE

State	Colorado	Texas
Moisture, per cent	27	35
Volatile matter, per cent	31	25
Fixed carbon, per cent	38	30
Ash, per cent	4	10
Heating value as received, Btu per lb	9100	7000
Ash fusion temperature, F	2000	2300

In Minnesota, the burning of lignite on traveling-grate stokers by the so-called "short-fire" method has proved successful. The fuel bed travels along undisturbed over the first zones of the stoker where surface ignition occurs and volatiles are driven off. Most

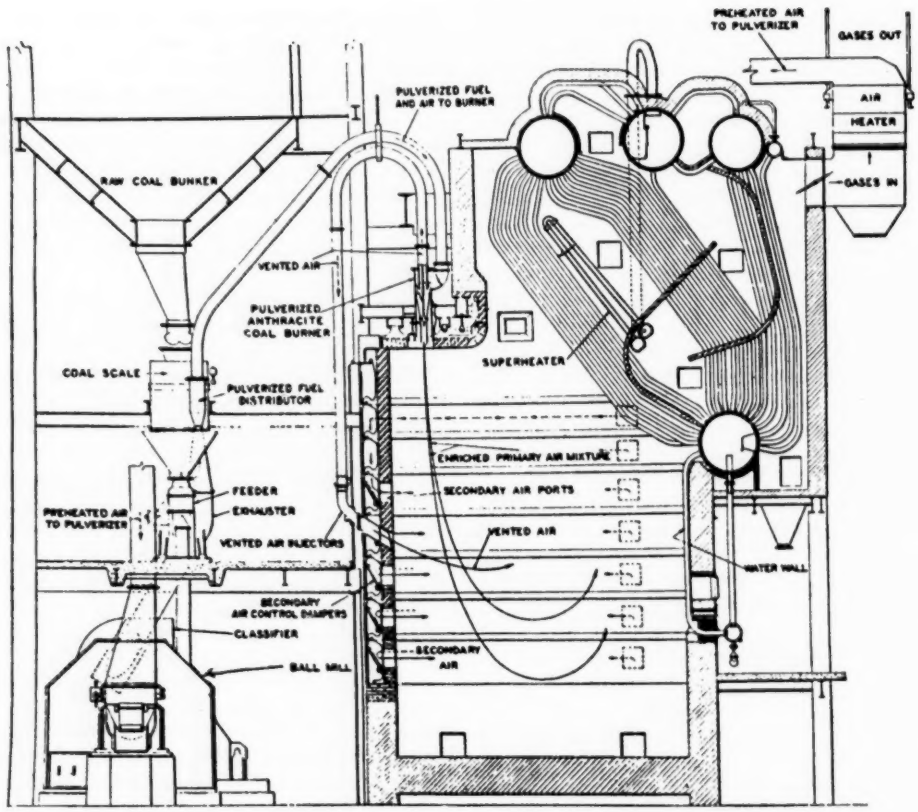


FIG. 15 UNIT SYSTEM FOR THE DIRECT FIRING OF PULVERIZED ANTHRACITE CULM

of the combustion takes place in suspension as the fuel reaches the blast zone, about half way along the stoker, where high air pressure lifts the fuel from the grate.

At Valmont Station in Denver the coal is burned in pulverized form and the flexibility of operation is much greater than with stoker firing. In this plant, four boilers, each with a capacity of 80,000 lb per hr, are equipped with air-cooled refractory walls, dry furnace bottoms, and burners of the down-firing type. The latest boiler with a capacity of 275,000 lb per hr, has water-cooled walls and a wet-bottom furnace.

The Texas deposit covers an area of 50,000 sq miles and the veins average 6 to 9 ft thick at depths of 50 to 150 ft. A number of small stoker-fired boiler plants use this fuel successfully, but at fairly low efficiency. About 12 years ago when the Trinidad Station of the Texas Power and Light Company was designed it was decided to burn lignite in pulverized form. The initial installation was so successful that the same method of firing was chosen when the plant was extended in 1931. Operating costs have been favorable compared with fuel oil and natural gas even in a region where these latter fuels have been unusually plentiful. The steam-heated driers plus the drying effect of the mill circuit reduces the moisture content to about 26 per cent. Forced-draft burners are used under straight-tube cross-drum boilers. The furnaces are water-cooled with dry bottoms. Steam is generated at 450-lb pressure and 750-F temperature. Natural draft is used which prohibits heat-recovery equipment beyond the boiler, but overall efficiencies of 75 to 78 per cent have been attained.

Another plant designed to use pulverized Texas lignite is the Comal Station near San Antonio, Texas.

Early in 1937 The Detroit Edison Company began to use in their Delray Power House surplus blast-furnace gas purchased from a neighboring manufacturing plant, the Great Lakes Steel Corporation. The gas is burned in conjunction with coal under three stoker-fired boilers as is illustrated in Fig. 16. As the gas dust and the coal ash may unite to form a mixture with a low-fusion point, resulting in serious slagging on boiler tubes, the gas is first washed and then is passed through an electric precipitator to reduce the dust loading to 0.05 grain per cu ft. Four burners, arranged for horizontal firing, are installed in the rear wall of each boiler. An intertube type of burner, protected against radiant heat when not in use, has proved satisfactory. As the combustion characteristics of coal and blast-furnace gas differ widely, a control system for combined firing was worked out for metering the gas to each boiler and also to proportion the gas and its combustion air. Elaborate precautions have been made to prevent explosions. Cleaned inert flue gas, cooled to 100 F, is used for purging the boilers and gas main. To protect men working inside the boiler, a water seal is provided in the gas main. No appreciable loss or gain in overall efficiency is chargeable to the burning of gas. The steam temperature is increased slightly. The use of gas results in an annual saving of 60,000 tons of coal, which is approximately 20 per cent of the total fuel requirements.

At the Louisiana Steam Generating Corporation, Baton Rouge, La., there are five boilers each with a capacity of 350,000 lb per hr of 620-lb 725-F steam. Natural gas makes up about 70 per cent of the fuel and the remainder is petroleum coke breeze and acid sludges. Bunker C oil is the stand-by fuel for use when there is no natural gas. Approximate analyses of the coke and acid sludges are given in Table 8. All five boilers are equipped to burn natural gas and fuel oil. Four can burn acid sludges in addition to natural gas and oil. Two boilers are able to burn coke breeze in addition to the other fuels. Normally, the procedure is

to burn coke breeze and natural gas under two boilers, acid sludges and natural gas under two boilers, and natural gas alone under the fifth unit. From flue-gas analyses there have been worked out for various fuel-burning combinations certain percentages of excess air which give the best combustion efficiency. The percentage of natural gas burned with other fuels varies from 50 to 90 per cent and is the only fuel controlled by the combustion-control system. The automatic fuel corrector adjusts the ratio of natural gas to total air in order to compensate for the quality and quantity of other fuels not under the control system.

In modernized kraft-paper mills, the old picturesque rotary recovery furnace with its rolling barrel 8 or 10 ft in diameter and 20 to 30 ft long, emitting fire and "black ash" at one end and wasting valuable heat up the stack at the other, is rapidly passing from the scene. For over 50 years this unique piece of equipment was the principal means of recovering valuable sodium compounds from the "black liquor." Black liquor results from cooking wood chips and "white liquor" (NaOH and Na_2S) in a large cylindrical digester under about 150-lb steam pressure. After 3 to 5 hr of cooking, the lignins in the wood are separated from the cellulose by action of the chemicals. The digester contents are then discharged into a diffuser where the cellulose is separated and washed prior to passing to other equipment for the manufacture of paper. The remaining liquid and the wash water form the weak "black liquor," which contains about 15 per cent total solids. On its way to the recovery furnace this liquid passes through multieffect evaporators where the solid content is raised to 40 or 50 per cent. On a dry basis these solids contain about 38 per cent carbon, 10 per cent oxygen, 7 per cent miscellaneous combustibles, and the remainder inorganic chemicals. The calorific value is 6500 to 6800 Btu per lb.

In burning the solids the inorganic chemicals are recovered in molten form. They are largely in the form of sodium carbonate (Na_2CO_3), with some sodium sulphide (Na_2S), reduced from the added salt cake (Na_2SO_4, sodium sulphate). This molten chemical is run into a dissolving tank filled with water to form a solution known as "green liquor." By the addition of lime in a causticizing operation, the "green liquor" precipitates calcium carbonate ($CaCO_3$) and forms "white liquor" for the wood-cooking process. Lime is recovered by burning the calcium carbonate in a kiln.

TABLE 8 ANALYSES OF PETROLEUM COKE AND ACID SLUDGES USED BY THE LOUISIANA STEAM GENERATING CORP.

Proximate analysis of petroleum coke:		
Volatile coke, per cent	11.06	
Fixed carbon, per cent	87.80	
Ash, per cent	1.14	
Sulphur, per cent	2.25	
Moisture, per cent	8.00	
Heating value, Btu per lb	15300	
	Lubricating acid sludge	Acid tar
Approximate analyses of acid sludges:		
Specific gravity	1.084	1.052
Moisture, per cent	39.680	47.510
Acidity of weak acid, per cent	32.580	32.210
Acidity of sludge, per cent	12.950	16.730
Weight, lb per gal	9.030	8.760
Heating value, Btu per lb	10940	9525

Until a decade ago the primary consideration of the kraft-paper industry was recovery of the chemicals in "black liquor," but developments in recovery furnaces accomplish also the generation of steam from the heat formerly wasted. Rates as high as 10,000 lb of steam per ton of pulp are not uncommon.

About 1925 the Wagner spray-type unit was introduced and enjoyed widespread use. "Black liquor" at about 65 per cent total solids is sprayed into the top of a vertical cylindrical furnace. Dehydration occurs during the fall through the furnace, and the remaining combustible with its chemicals are deposited on the smelter floor. Separation takes place and the chemicals flow out in molten form. A boiler behind the furnace absorbs heat from the gases for generation of steam.

The Murray-Waern system shown in Fig. 17 is a development involving a combination of heat-recovery apparatus and one or two rotary furnaces.

The Tomlinson furnace was first installed on a commercial scale at Quebec in 1934. Fig. 18 shows a 125-ton unit, generating 55,000 lb of steam per hr at 430 lb and 660 F. "Black liquor" at 220 F and containing about 55 per cent total solids and some suspended salt cake is delivered to the spray nozzle at a pressure of 50 lb per sq in. pressure. This nozzle produces a horizontal sheet of coarsely sprayed liquor which becomes partly dehydrated in its passage across the furnace and adheres to the opposite wall

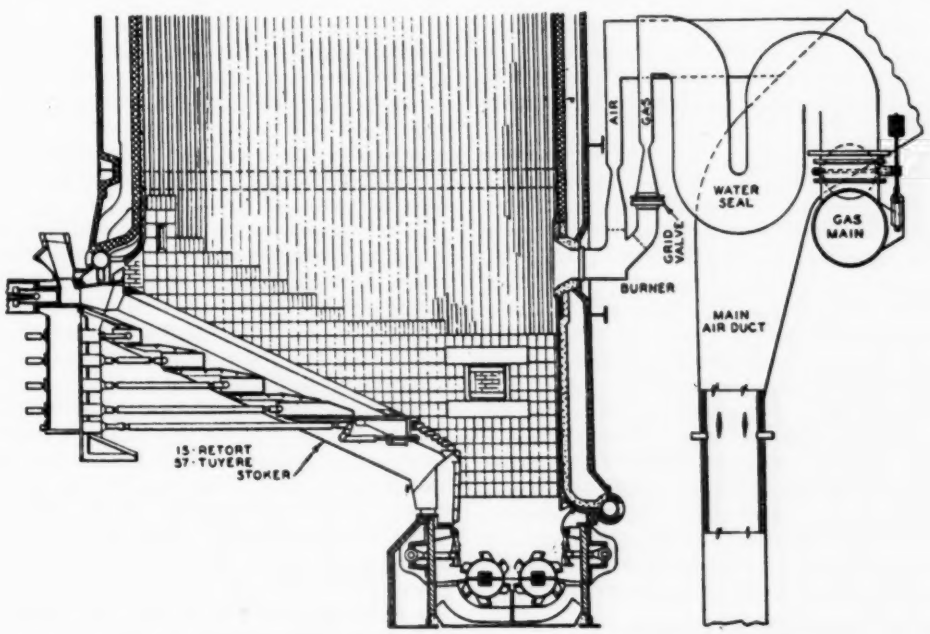


FIG. 16 ARRANGEMENT FOR BURNING BLAST-FURNACE GAS OVER A STOKER

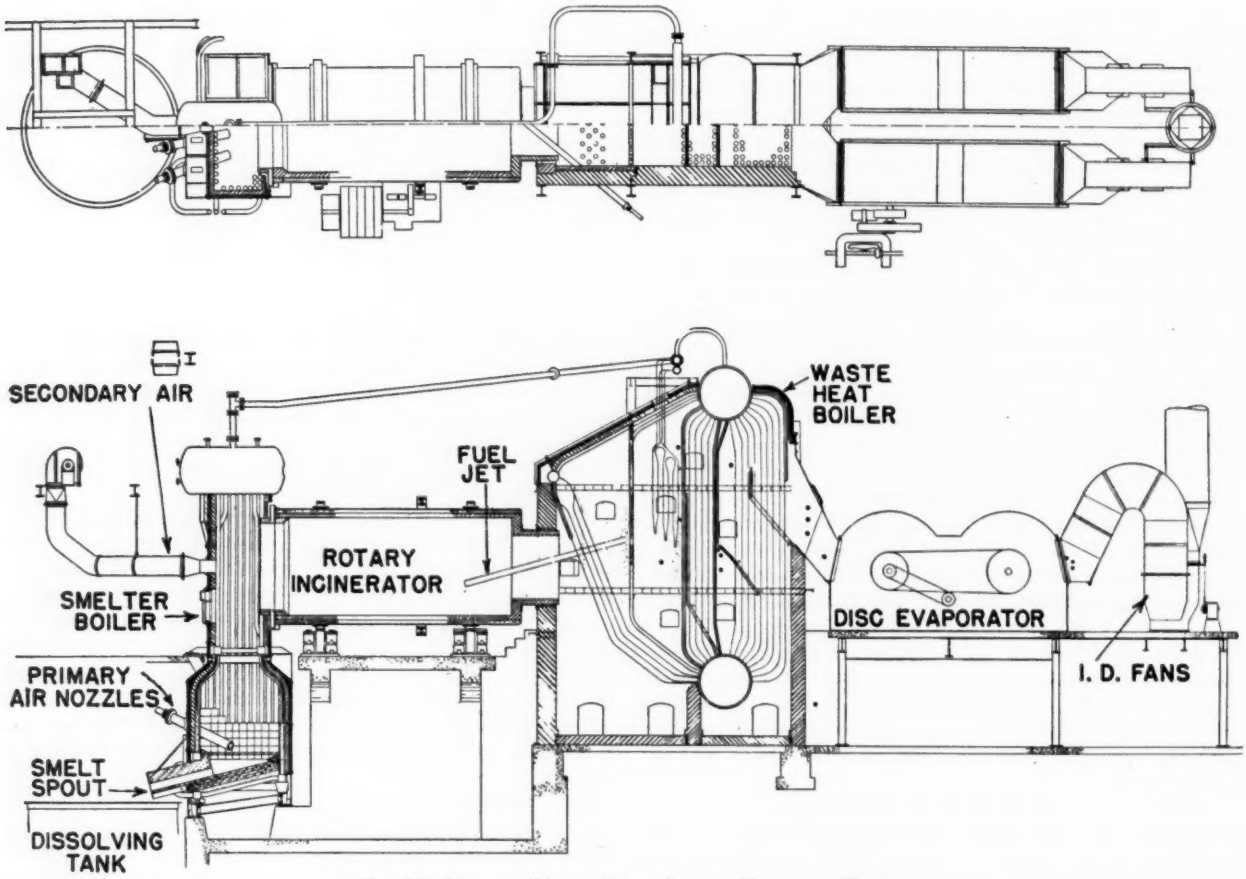


FIG. 17 MURRAY-WAERN BLACK-LIQUOR RECOVERY UNIT

in a partly wet condition. A vertical and rotational oscillating movement of the spray plate gives a paint-brush effect in its distribution of the wet liquor over the walls. Here, complete dehydration takes place as the char builds up to a thickness of 10 to 15 in. when, because of its weight, it falls to the smelter floor. Primary air is admitted near the fuel bed and secondary air higher up. As in all other types of smelter, the molten chemicals flow through a spout into a dissolving tank of water to form green liquor.

The unit shown in Fig. 18 involves a three-drum bent-tube boiler with a water-cooled furnace formed by studded walls covered with plastic chrome ore. Most of the boiler tubes are vertical and there are no baffles, thus affording easy removal of chemicals deposited on the tubes from sublimation in the smelter where the temperatures are between 2000 F and 2200 F. This type of unit lends itself to large capacities and to use with liquors from woods containing large amounts of rosin, such as southern slash pine. Results indicate that 60 per cent of the heat value in the black-liquor solids is absorbed by the boiler in the generation of steam. An air heater is standard equipment on these units since hot air has such beneficial effects in the combustion of this low-grade and high-moisture fuel. Frequently an economizer is installed to reduce heat losses, and sometimes a spray tower is used at the tail end to capture chemicals which may have escaped the boiler and heat-recovery equipment.

Another interesting feature pertaining to these furnaces is the fact that no ash remains in the smelter during the recovery cycle.

BOILER FEEDWATER TREATMENT

Coincident with the initial trend toward high steam pressures, which occurred over a decade ago, the problem of feedwater treatment attracted the attention of many able chemical engineers. The technical literature bears witness to the excellent progress that has been made in this important field. Without the accomplishments of these specialists, operation of the modern steam generator in many cases would be impossible. One of the great surprises that comes to managers of industrial plants facing expansion of the steam facilities is learning that water which was perfectly suitable for the old low-pressure boilers requires some special form of treatment for the new equipment. Yet no problem is more important to the engineer in this field than boiler feedwater treatment.

To compensate for the increased cost of equipment operating at higher steam pressures, it has been necessary to use fewer boilers designed for higher rates of steam generation per square foot of surface. Ratings of 200 or 250 per cent, once considered high, are now dwarfed by such average ratings as 500 to 700 per cent. At certain points, such as the front row of boiler tubes or waterwall tubes, the heat transfer may reach 150,000 Btu per sq ft per hr, which corresponds to about 5000 per cent rating. For ordinary carbon steel, the safe maximum continuous tube temperature does not exceed about 900 F. In a boiler operating at 1200-lb pressure, and assuming a heat absorption of 150,000 Btu per sq ft per hr, the temperature of the outer surface of the tube would be increased to 950 F if 0.01 in. of many types of scale were present on the inner tube surface. If such a condition were to exist, the tube would be likely to fail. This illustrates the necessity for using boiler waters free from scale-forming properties.

At these points of extremely high rates of steam generation, complex scale formation may develop. In this group, the one

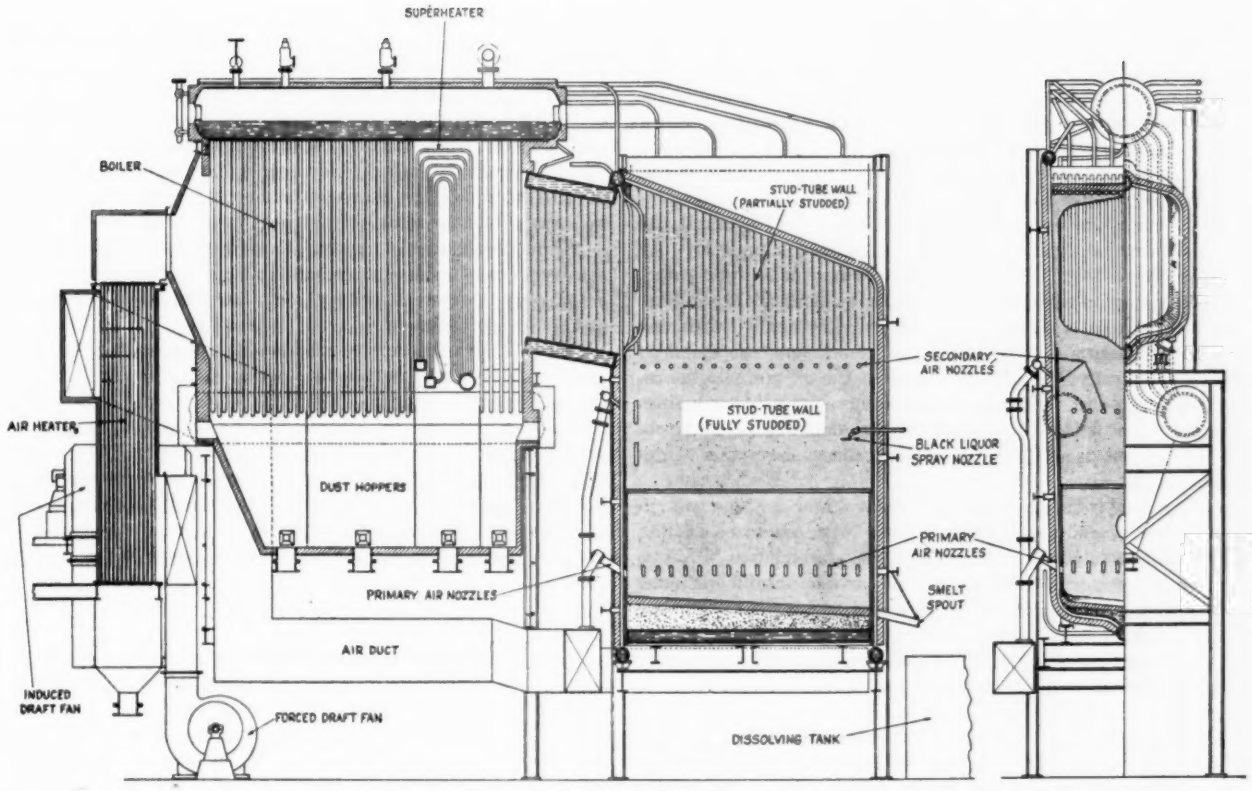


FIG. 18 TOMLINSON FURNACE WITH BENT-TUBE BOILER

which has given most difficulty is known as analcite, a complex silicate scale containing sodium and aluminum. Even the use of phosphate or other treatment cannot in all cases prevent the formation of this scale or remove it if it is once formed by the alkaline boiler water. Because of condenser leakage and evaporator carry-over, aluminum and silica cannot be kept out of the boiler water even if the feedwater consists entirely of condensate returns and evaporated make-up. The solution to this problem requires the cooperation of the boiler designer. Assurance of very rapid rates of water circulation at the localized hardest-worked sections of the boiler, plus satisfactory external and internal water treatment, is required to minimize the formation of this type of deposit.

With the elimination of scale it becomes necessary to protect the boiler metal from corrosion. The rate of reaction between the water and the steel is increased by the higher temperatures corresponding to the higher pressures. A thin coating of iron oxide integral with the surface of the boiler steel acts as a restraining influence and a protective barrier, thus prolonging the life of the thin boiler tubes. The iron-oxide coating will dissolve in highly acid or highly alkaline solution. The minimum caustic concentration which assures protection at boiler feedwater temperatures is that which corresponds to a pH of 9.6 at room temperature. For most operating conditions a boiler-water pH of 10.8 to 11.3 has been found satisfactory. In seams and at other points where restricted circulation and evaporation occur simultaneously, the concentration of alkaline solids may become very high. Under these conditions, and in the presence of silica or certain other agents, the steel may be subject to a selective attack at the grain boundaries which leads to intercrystalline cracking, better known as caustic embrittlement. Fusion welding of boiler drums has minimized this source of trouble by elimination of seams. The A.S.M.E. Boiler Operating Code requires the

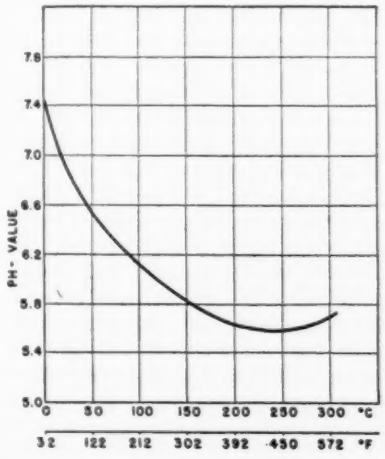


FIG. 19 RELATION OF pH VALUE TO TEMPERATURE FOR PURE DISTILLED WATER

maintenance of specific sulphate-carbonate ratios in the boiler water to inhibit caustic embrittlement. Considerable controversy has developed in regard to the adequacy of this protective treatment and other corrective measures have been suggested. These new treatments must, for the present, be regarded as in the experimental and developmental stage. It is probable, however, that some revision of the present A.S.M.E. Code may be forthcoming in the immediate future.

Many boiler feedwaters have a pH not far from the neutral point, or 7.0, and are treated to bring the alkalinity up to boiler requirements. Fig. 19 shows some interesting data on pure neutral distilled water and indicates why such untreated water in

a high-pressure boiler could be dangerous. At 0 C, the pH value is 7.44; it crosses the standard neutral point of 7.0 at 22 C, and reaches its lowest value of 5.58 at 250 C. Since the water contains no free acidity or alkalinity, its own acid and alkaline characteristics are balanced, i.e., it is neutral at each of these points. However, at the higher temperature, both the acid and the alkaline properties of the water are more than ten times as active as at room temperature.

Even when the boiler water contains sufficient alkalinity to inhibit corrosion, it is imperative to remove completely dissolved oxygen from the water if corrosion is to be prevented. The first step in removal of oxygen is mechanical deaeration, which is capable, with the most modern equipment, of reducing the oxygen content of the water to an amount too small to measure analytically. The last traces of the oxygen may be removed by the use of chemicals, the most popular being sodium sulphite. The latter should not be considered as a substitute for mechanical deaeration, but only as a secondary line of defense.

Raw waters found in this country vary over a wide range in their chemical analyses, but means have been found to condition most of them satisfactorily for high-pressure boilers. Make-up of even 100 per cent has presented no insurmountable obstacle. In deciding the form of water treatment to be employed, consideration must be given not only to providing the necessary protection for the boiler, but also to the complete inhibition of corrosion in the feedwater-treating system and equipment preceding the boiler.

The pretreatment of water in apparatus external to the boiler to make it suitable for feedwater make-up is most commonly accomplished by either the lime-soda process or the zeolite process, each of which has many modifications. The lime-soda process is more generally adaptable where the raw water is turbid or contains free acid, iron, or excessive hardness. It is adapted to the use of supplementary chemicals to accomplish various results. The lime-soda process may be carried out on either hot or cold water; since the reduction of hardness and the speed and efficiency of the process are greatly increased at higher temperatures, hot-process softeners are widely used. The less-complete degree of softening that is effected in cold water would, in many cases, be inadequate for the preparation of boiler feedwater. A recent development for treating relatively soft waters, with hardness not exceeding 3 ppm, is primary phosphate hot-process softening. This effects softening and coagulation in a single-reaction and settling tank, and zero hardness may be obtained. For very hard waters, similar results can be secured in two steps—the less expensive lime and soda in the first stage to remove the bulk of the hardness, and the economical use of phosphates in the second stage.

Zeolite systems are capable of reducing the hardness of water to a few parts per million and are suitable where the hardness varies considerably, since the softening reaction is more or less automatic and does not depend on instantaneous adjustment of chemicals to the quality and volume of the water. A clear and approximately neutral water supply must be provided for this treatment. It is sometimes combined with cold-process softening as a pretreatment and is frequently preceded by coagulation and filtration systems. The relative merits of chemical-process softening and zeolite softening will depend entirely on the operating conditions of the plant in question and the nature of the water supply. The size of lime-soda equipment is approximately determined by the quantity of water required, regardless of its hardness, although unusual problems may require additional settling capacity and special equipment. On the other hand, the size of zeolite systems is influenced by the hardness of the water, which determines the quantity of mineral that should be provided, as well as the volume of flow, which determines the total bed area of the softeners. The lime-soda process provides a water of high pH value which adequately protects equipment surfaces and minimizes the concentration of carbon dioxide. On the other hand, zeolite softeners deliver an effluent with a pH value below 8.0 and containing bicarbonates, so that carbon dioxide is evolved when the water is heated. However, some modification or combination of the chemical process and zeolite systems can usually be devised to satisfactorily treat most water supplies.

At the present time, a number of new types of synthetic zeolites are being developed, and radical changes are occurring in the art of water treatment, using these materials and older methods in various combinations.

Evaporators are generally used for high-pressure boilers where make-up constitutes only a few per cent. They produce substantially pure feedwater.

The type of apparatus and the process to be used in any particular case must be carefully analyzed to the end that initial and upkeep expense will be commensurate with the protection afforded. The engineers designing the plant should take cognizance of the need for careful investigation of the most satisfactory form of feedwater treatment which should be provided under a given set of conditions.

Evaporators

Years ago the use of an evaporator in connection with boilers was limited to seagoing vessels where distilled fresh water from seawater was used as make-up. The adoption of higher steam pressures coupled with a question as to the ability to successfully treat feedwater for the higher pressures led to the development of evaporators, and this trend has progressed to a point where evaporators of large sizes have been installed to produce low-pressure steam for process work. In such cases they are used as heat transformers generating low-pressure process steam and returning the clear condensate to the boiler-feed system. Notable installations of this sort were the evaporator plants at the Allegheny Steam Heating Company's Cecil Plant, installed in 1924; at the Youngstown Sheet and Tube Company, installed in 1928; at the Gulf States Steel Company plant, installed in 1928; and at Ford Motor Company's River Rouge plant. The Youngstown installation had a total capacity of 300,000 lb of low-pressure steam. The author believes these plants were among the earliest installations of this character.

At the River Rouge plant of Ford Motor Company, a 15,000-kw high-pressure turbine exhausts to a battery of three evaporators, generating vapor at 180-lb pressure for plant process. Most of the condensate is not returned. Process steam not supplied by the evaporators is generated in the old 240-lb boilers and delivered to the system through a pressure-reducing station.

Because of the high percentage of impurities in the raw boiler-feedwater for the plant of a large tire manufacturer, it was thought advisable to exhaust the turbine into an evaporator system rather than direct to process. The five evaporators operating in parallel produce 260,000 lb of dry vapor per hr at 180 lb per sq in. gage, when supplied with 270-F feedwater and 269,000 lb per hr of dry and saturated steam at 235 per sq in. gage. The evaporator feed is treated to prevent scale deposit rather than depend upon cracking. The tube surface has been maintained clean with a consequent steady rate of heat transfer. Results indicate that cost of vapor from the evaporators compares favorably with the cost of process steam from the old boilers.

Triple-effect evaporators are sometimes installed to supply steam demands that vary greatly. When more steam for process is required, the unit is operated on the double-effect principle.

With the adoption of the regenerative cycle and wider use of motors for auxiliary drives, the make-up requirements for

turbine-driven plants were reduced to 1 or 2 per cent. Moreover, the several extraction points on the turbines offered ideal conditions for the installation of single-effect evaporators between bleed points, with a lower-pressure feedwater heater acting as the evaporator condenser. General practice follows this arrangement today and nearly all of the 1200-lb installations in central stations use evaporators for small quantities of make-up.

In the new No. 5 Power Station of Carnegie-Illinois Steel Corporation, two evaporators were installed, each having a vapor capacity of 50,000 lb per hr. These units take steam directly from the boilers at 450-lb and 750 F and deliver vapor at 170-lb pressure to the low-pressure system. In this way the steam requirements in the old part of the plant were satisfied without the expense of adding new low-pressure boilers.

Developments in evaporator design include wide use of shells of welded-steel plate, efficient steam separators, arsenical copper or cupro-nickel steam-heating coils, and special designs of removable heads suitable for high pressures. Successful operation has resulted also from a new type of evaporator-drain pump so designed that a fixed level of water in the evaporator drain line is not required. This constant-speed pump adjusts itself automatically to capacity and head by means of submergence in the suction pipe, that is, by allowing the submergence to drop to a point where flashing in the eye of the impeller will occur in such a manner as to reduce capacity. This service is severe on the first-stage impeller. In some cases, vapor superheaters are used to remove superheat from the steam entering the unit and thus ensure dry steam for process use.

Electrical System

During the last few years it has become possible to improve the industrial power plant, from an operating standpoint, by means of recent developments in electrical equipment. The common use of metal-clad switchgear instead of the old open construction reduces the possibilities of service interruptions and also eliminates practically all of the life hazards. Furthermore, operation becomes simpler and the possibility of operating errors is reduced. Recent activity in the development of the air circuit-breaker for voltages up to 5000 may soon make it possible and practical to avoid entirely the use of oil for all circuit breakers within the station, thus greatly reducing the fire hazard.

All of the switching equipment, including auxiliaries, except that above 13,200 v, may now be metal-clad and located at any part of the station where space is available, and in large industrial plants the operating switchboard may consist of steel panels which are entirely dead-front and very compact in construction. For the small plant, the operating switchboard and the metal-clad switchgear may be combined into one structure, with the operating instruments located on steel panels forming the front of the circuit-breaker housings.

Considerable development has recently been made in the insulation of wire and cable. It is now possible to make use of cable insulation which can withstand higher temperatures than were formerly possible, and which is practically immune to the effects of moisture and various other formerly harmful substances.

Conclusion

One indispensable item in engineering of industrial plants is the economic result obtained, i.e., the return on the money invested. Most industrials, due to changes in production and processes, require returns of at least 25 per cent gross before embarking on an expenditure made solely on the score of economy. Conditions are somewhat different in the utility industry where there is a wide diversity of load and therefore a much better load condition. Consequently, utilities are not faced with changing processes which cause early obsolescence of equipment. While it is seldom found economical to replace steaming and power-generating equipment which is in reliable operating condition, there are cases where it is necessary to replace portions of the plant due to obsolescence or for other reaons and, in such cases, the replacement cost of this item should be credited to the total cost of the new development for providing both steam and power generation in order to arrive at the net cost of the power development to be justified.

Discussion

R. H. McLain.[2] In line with the excellent summation which the author has made in regard to electrical improvements in industrial plants, it should be pointed out that not only are air circuit breakers available up to 5000 v, but breakers of the type which formerly used oil are now adapted to use water and available up to 15,000 v. This still further reduces the fire hazard which the author mentioned.

It should also be interesting to know that transformers using a cooling medium which is noninflammable and nonexplosive are available instead of oil-cooled transformers. This development makes it still easier to use high voltage inside or alongside plant buildings in crowded spaces, without having to take precautions which were necessary in the case of oil-cooled transformers.

Joseph Pope.[3] Following are brief descriptions of several industrial power plants, in addition to some noted in the paper, with which the writer has had considerable personal contact. They are not all new but have never been much publicized.

The Ahmeek Mill power station of the Calumet & Hecla Consolidated Copper Company was built near a steam stamp mill, and replaced an old boilerhouse containing 16 hand-fired locomotive-type boilers of 210 hp at 155 lb per sq in. working pressure. Three 90,000-lb per hr boilers, fired by multiple-retort underfeed stokers, and fitted with integral economizers, were installed to supply steam at 425 lb per sq in. and 675 F total temperature in about equal amounts to a 7500-kw condensing turbine generator and to a 1250-kw geared back-pressure turbine generator, which exhausts through an accumulator to the old steam header at 155 lb per sq in. pressure. The accumulator does not function in the usual manner, the flow being straight through, but it does desuperheat the steam exhausted by the turbine and that by-passed through the reducing valve, and smooths out pulsations from the steam stamps. The 150-lb per sq in. pressure steam drives a triple-expansion condensing mill pumping engine, as well as the stamps. Exhaust from the stamps at 5 lb per sq in. operates a 2000-kw condensing turbine generator. These all were existing equipment. The accumulator acts as a direct-contact evaporator and requires the injection of feedwater, which concentrates and is blown off. There results a partial boiler feedwater make-up and conditioning action, since much of the steam appears later as condensate from the turbines and is fed to the 400-lb boilers. A combination straight-condensing and straight back-pressure turbine was selected rather than a single extraction unit because of less first cost and better over-all economy, under the expected load conditions. Stoker firing of coal was used instead of powdered coal, because of first-cost considerations, relative simplicity, character of coal available, and the severe climatic conditions under which it is stored.

A relatively recent addition to the power facilities of the Manville factory of the Johns-Manville Corporation consists of one 125,000-lb per hr integral-furnace boiler and one 2500-kw back-pressure-turbine generator. This boiler is fired by pulverized

[2] General Electric Company, New York, N. Y.

[3] Consulting engineer, Stone & Webster Engineering Corporation, New York, N. Y. Mem. A.S.M.E.

coal, using two direct-firing mills, and supplies steam at 620 lb per sq in. and 650 F total temperature to the turbine which exhausts at 200 lb per sq in. into existing mains. This exhaust, and any reduced-pressure by-passed steam, flows through a tubular desuperheater, through which enough feedwater is automatically diverted to effect the desired temperature control. The boiler is fitted with an air preheater and forced- and induced-draft fans. The fans and feed pumps are driven by steam turbines, with steam and exhaust apportioned between the three pressures of 600 lb, 200 lb, and 5 lb to obtain reliability and flexibility of operation. The new plant was installed within an existing building, which originally housed six 200-hp 150-lb per sq in. pressure boilers, long out of use. It operates in conjunction with the older 200-lb steam and electric-generating system. The feed to the high-pressure boiler is mostly uncontaminated process drips diverted especially to the new plant. Make-up used principally in the older boilers is river water, clarified and then conditioned by Zeo-Karb.

Two plants which use small-sized anthracite for fuel, not culm, are the Baltimore refinery of the American Sugar Company, and the St. Nicholas steam plant of the Philadelphia and Reading Coal & Iron Company. Neither is exactly new, but there is scarcely a plant in that particular field, which is more modern. The Baltimore plant houses five bent-tube boilers of 12,060 sq ft area each, equipped with an additional lower drum and tube bank of 3160 sq ft, serving as an integral economizer. It was designed to burn dredged river coal, averaging in size 70 per cent through $^{3}/_{32}$-in. round mesh and all through $^{3}/_{8}$ in., with 8 per cent volatile, 75 per cent fixed carbon, and 17 per cent ash. When it was built, large stokers and water-cooled arches were not available, and each furnace was equipped with two forced-draft traveling grates of 318 sq ft combined area having front and rear sprung refractory arches. The boilers were provided with natural uptake draft and supplied saturated steam at 200 lb per sq in. pressure to engine generators and to the refinery direct. Efficiencies of 76 per cent at about 55,000 lb steam per hr per boiler and capacities of about twice that were obtained.

The St. Nicholas steam plant is a further development of the Baltimore idea adjusted to local conditions. The boilers, of which there are two, are very similar to the American Sugar Company's units in size and type, except that they produce saturated steam at about 400 lb per sq in. pressure. However, each boiler is fired with a single traveling grate of 425 sq ft area and equipped with one long flat suspended refractory rear arch, water-cooled side walls, and mechanical undergrate and uptake draft. The coal used is small, unmarketable fresh-mined anthracite from the St. Nicholas breaker of the same owning company. The coal is pumped with water to the steam plant through 1800 ft of pipe, and there separated and drained. Ash is removed by a continually running sluice or launder. Practically all the boiler feed is make-up. The steam output from the plant is distributed through long pipe lines at boiler pressure to the various operations of the company. It is reduced in pressure where used in old equipment. The steam load is extremely variable in character, there being wide differences between idle and operating mine shifts and violent swings occasioned by operation of steam-actuated mine hoists. The new plant handles these severe conditions in a satisfactory manner. Each boiler produces up to 125,000 lb steam per hr and swings equivalent to the output of one boiler are of normal occurrence.

The Kirk Power Plant of the Homestake Mining Company in the Black Hills of South Dakota, while of the same general character as a public-utility generating station, nevertheless belongs in the industrial-power classification. The main points of interest are that it is located a mile above sea level; all turbines are of the condensing type with forced-draft cooling towers employed to dissipate the heat from the circulating water; and the three 10,000-sq ft boilers operating at 415 lb per sq in. and 700 F total temperature are fired with Wyoming lignite, burned on forced-draft spreader-type stokers.

The boiler plant of the Franklin Sugar Company in Philadelphia is an example of the reconstruction of an old plant within an existing building. Forty-eight 250-hp boilers on two floors were replaced by five 1021-hp boilers, located on one floor and fired by pulverized coal, using unit mills. The boilers are fitted with individual economizers and forced-draft fans, but uptake draft is supplied naturally by a chimney 300 ft in height. Two of the boilers are equipped with superheaters and supply steam at 225 lb per sq in. to pressure-reducing turbine generators. The remaining three boilers furnish saturated steam at reduced pressure to the operations of the sugar refinery. When this plant was reconstructed twelve years ago it was particularly notable for its pioneering in the use of the direct-firing system of pulverized coal.

The Richmond Refinery Power Plant of the Standard Oil Company of California is thoroughly modern, in every sense of the word. It is new; it employs a high steam pressure and the latest methods of feedwater treatment; it burns several fuels, natural gas, fuel oil, and large quantities of refinery sludge; the piping systems are completely welded; and it is of semioutdoor construction. Three 125,000-lb per hr boilers are installed producing steam at 850 lb per sq in. and 750 F total temperature. They are fitted with furnace side and upper front waterwalls and tubular air preheaters. The air preheaters and the motor-driven vane-controlled, duplex-forced and induced-draft fans are located outside the building. The water-conditioning plant of original capacity sufficient to supply two boilers was of the two-stage type; a cold process for silica removal, followed by a phosphate hot process for softening. Subsequent enlargement of the treating plant has taken advantage of the newly available nonsiliceous base exchange type of softening.

There are three turbine generators, each rated at 4000 kw. They are designed for future possible exhaust at 25 lb per sq in. pressure and extraction at 125 lb, but initially the low-pressure stages carry no blading and exhaust is through the future extraction opening at the higher pressure.

The author has mentioned the three projected West Coast plants as examples of cooperation between utility and industrial interests. They are all in the general neighborhood of the plant just described and undoubtedly represent a recognition of the possibility of such situations. Of course the prototype of such a joint-interest plant is that of the Louisiana Steam Corporation, now the Gulf States Utilities Company, in Baton Rouge. In addition to the apparatus already in that plant, as described in the paper, there is now being installed equipment which will about double its former capacity. Two 500,000-lb per hr boilers, for the same working conditions as the earlier ones, are now ready for use, and two 25,000-kva turbine generators are rapidly nearing availability. These latter units are of the automatic extraction condensing type. Their actual capacity depends upon the division of maximum steam flow between extraction and condensation. This plant now has contracts for steam sales at 135 lb per sq in. pressure, totaling 1,331,000 lb per hr, and for electricity at 13,800 volts to the same purchasers, totaling 53,001 kw.

There is also being added to the plant of the Scott Paper Company, referred to in this paper, a 125,000-lb per hr oil-burning boiler to deliver steam at 600 lb per sq in. and 750 F total temperature to a 5000-kw dual-extraction, dual-admission condensing turbine generator. The extraction pressures may be automatically maintained at from 40 to 80 lb per sq in. and 17 to 20 lb per sq in., respectively. The second opening serves either to extract or to admit low-pressure steam.

Organization—Design for Modern Business

By FAIRFIELD E. RAYMOND,[1] CAMBRIDGE, MASS.

Questions of organization for large-scale enterprise are of increasing importance, since existing concepts applicable to single-unit organizations are becoming noticeably inadequate. More dynamic concepts appear to be necessary to comprehend those particular activities essential to a business and to provide adequate channels of coordination in an effort to humanize management. For each degree of specialization, in ascending from the smallest to the largest size of business, the direct channels of coordination following the lines of authority need to be supplemented by appropriate channels to make mutuality of interest effective. Decentralization, in contrast to centralized authority, may not only facilitate leadership, but also provide a means by broadening the organization structure to draw administrator and worker closer together, materially enhancing their unity of purpose. Recent developments point not only to an increasing functionalization of administration, but also to the reversal of the long-accepted pyramidal structure of management where the work group becomes the unit of management and the superstructure of organization must be justified on the service it renders these units. With increasing frequency the lack of constructive research in the field of organization has been remarked upon. In these times when the effectiveness of large-scale business is being challenged, is it not appropriate to seek for more comprehensive and possibly dynamic concepts of organization?

TO AN INCREASING extent executives are heard to express concern over what constitutes the economic size of a business enterprise. Do economic and social conditions warrant the mammoth corporations established in the last decade? In the textile field a cotton mill of 50,000 spindles has for some time been regarded as marking the optimum size of plant. Is management justified, however, in basing its measure of economic size on the efficient combination of production facilities alone? Are machines or process technology the sole determinants of minimum-cost operation, the apparent sine qua non of successful and profitable enterprise?

Unquestionably, the administrators of business have shown a high order of ability in dealing with their "inanimate material" (2).[2] Marked evidence of this is to be found in the existence of the large-scale enterprises of today. Nevertheless in establishing these multiunit organizations, the business leader has brought together an aggregation of human beings whose individual endeavor must be effectively coordinated and controlled, if the potential effectiveness, material and human, of the enterprise is to be fully realized and perpetuated. Should not management preferably adopt a criterion of economic size, based upon the character of that unit which can attain maximum results from its workers and plant with the simplest structure of management?

Has it not frequently happened that the smaller, less technically perfected enterprise, whose executives and employees are highly competent and cooperative, may take business away from the larger, efficiently equipped, but ponderous and impersonal industrial giant? May it not be that the 50,000-spindle textile mill excels because it achieves the most harmonious and effective coordination of its total personnel, the closest relationship between workers and top management with the maximum loyalty and minimum personal friction, without necessarily corresponding perfection in processing and equipment?

Thus enterprise may become just too big; yet present-day business conditions seem to favor a continuing increase in size. The test will be in whether human beings have the capacity to administer wisely large enterprise, to devise means for holding together the thousands of people in their employ, and to provide at all levels satisfying incentive and opportunity. The ultimate breakdown will in all probability occur, though possibly not initially manifest, in the failure of a top-heavy superstructure of management to promote and transmit leadership and to coordinate effectively its manifold and essential activities. Are we not faced with a serious challenge to existing concepts of organization if industry is to prosper on an even larger scale of operation? History repeats itself.

Historical Implications

Mankind has organized to a constantly increasing extent since the dawn of civilization. With the beginning of cooperative effort man set himself apart from the other creatures of the jungle with whom he had long struggled single handed for existence, and demonstrated the supremacy which comes with specialization and organization in the determination of his destiny. The inseparability of these two essentials cannot be too strongly emphasized. Any protracted lack of balance between them caused by the inescapable demands to maintain progress in civilization, on the one hand, and to establish commensurately effective methods of coordinating group endeavor, on the other, would bring about a social upheaval such as could thrust mankind back into its former jungle existence.

History now reveals (3) a direct lineage of modern military organization dating from the organization created by the primitive tribes of the "fertile crescent" in the valley of the Tigris and Euphrates rivers, to protect their crops and fields from floods, which proved also effective in repulsing invasion by the nomads of the neighboring deserts. For nearly five thousand years the basic concepts of organization have remained relatively little changed, though its forms have evolved through the ages in response to changing circumstance as the efforts of larger and larger groups of people have had to be coordinated.

The process has been largely one of trial and error where, in rather long cycles, economic and social conditions eventually surpass current ability to organize, causing the overextended unit to break up into its various homogeneous parts. Such a case is well illustrated (4) by the failure of the Greek States in the long run to hold on to their far-flung colonies, a precursor to the collapse of their independent civilization. Nevertheless today we see in Great Britain's ability to maintain her empire of world proportions the results of recent advances in organization.

Present Status of Organization

The relatively compact and unified chain of activities initially constituting organization has lengthened and broadened, expanding upward and outward as well in its evolution, until the indi-

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[2] Numbers in parentheses refer to the Bibliography.

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NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of authors, and not of the Society.

vidual responsible for the administration of the whole is so far removed from those who actually perform each constituent task that his personal influence of leadership is likely to be lost before it reaches the lowest level or the most remote corner of the enterprise. Similarly his sensitiveness to the needs and personal desires not only of these workers but also of the people with whom the enterprise may do business is impaired by the inevitable reinterpretation of conditions as reported back to him through intervening executives.

The complexity and rigidity of the consequent structure with its many standardized routines have increased the impersonal character of enterprise until any one member of the personnel at large has nothing tangible or satisfying for self-identification except the work before him. He has then, beyond his immediate superior, only the symbols of administration to which to attach his loyalties. Unless such conditions of superorganization can be rectified, another point of disintegration for large enterprise may be near at hand.

Need for Dynamic Concepts of Organization

Has not a totally new situation to be met in modern multiple-unit organizations, since the principles applicable to single-unit organizations are apparently quite inadequate? Aside from certain profound studies of the abstract essentials of organization, the chief emphasis has been placed so far on schemes for classifying activities and patterns or forms for coordination. All these are static and inflexible concepts becoming less and less applicable under the rapidly changing conditions of business today. Management is unquestionably in need of a truly dynamic concept of organization which fully explains and demonstrates the differences in relationship, methods of coordination, processes of functional development, and integration for each characteristic increase in size and complexity of business, and the corresponding changes produced in the factors conducive to leadership, in the responsibilities and duties of executives and their subordinates, and in the means for maintaining a spontaneous unity and purposefulness of the whole enterprise.

The lack of truly scientific research in the field of organization is to be greatly deplored. So is the lack of attention which executives give to acquiring a proper understanding of how to deal with problems of organization. Though such problems are of daily concern to them, top executives are, for the most part, immune to the need for further study; ingrained personal convictions and a pride in accomplishment, since organization is an inherent duty of administration, seem to stand in the way. More to the point are the facts that the abstract and intangible aspects of organization are abhorred and the more tangible and immediate problems of policy and operation are naturally given precedence. Thus, organization has remained an art, the basis of which has but slowly evolved through the centuries under the pressure of necessity.

Organization

The management of enterprise will always remain an art, drawing its foundations of technique from the natural and social sciences. Nevertheless management should be systematic and orderly, thus implying a need for a more scientific basis of organization. Since business is a collective undertaking, its organization may be briefly defined as the means for articulating its parts to constitute a whole for the harmonious fulfillment of a common purpose (5). It appears then that the fundamentals of organization could be disclosed through a realistic understanding, on the one hand, of the nature and relationship of those specialized activities essential to the satisfactory fulfillment of the expressed purpose, and, on the other, of what is required to insure their harmonious and unified functioning.

The first relates to the techniques of specialization and the second to those of coordination, the two inherent processes of organization. Through specialization each essential activity can be segregated and its distinctive character specified to provide a clear basis for the assignment of duty. At the same time the natural sequences of related activities may be determined to form a basis for the organization structure. Though this latter provides a foundation for coordination, it is only mechanical and lacks cohesiveness.

In collective human undertakings leadership is essential as the motivating force to crystallize the common interest and to unite the common effort. Though in all organization there must be a predetermined purpose to give explicitness to its intent, the inherent activities alone provide no vitalized basis to promote harmonious action. Coordination becomes effective only through mutuality, that subtle quality in organization which, under the influence of wise leadership, engages both the mental and emotional faculties of the group and holds it voluntarily steadfast to its purpose. Mutuality is manifest and operative in community of interest, harmonious understanding, and faithful cooperation (6). In these are to be found the determinants of the methods and techniques of coordination.

Thus there are both physical and psychological factors in organization. Its structure, therefore, not only should facilitate management in attaining the objectives of enterprise but also should provide a suitable medium for the exercise of leadership. The difference between successful and unsuccessful organization seems to lie in the fact that only certain arrangements of the physical structure for a given purpose and set of operating conditions permit the fullest realization of the psychological aspects. So it is in nature; all sorts of combinations of elements are conceivable but only certain ones are stable under the existing conditions. To comprehend organization fully it will be necessary to seek for those dynamic concepts of coordination which will mark the conditions favorable to stability and harmonious action where different purposes and stages of specialization prevail.[3]

Specialization

The starting point should therefore be with specialization, to inquire into what constitutes the natural and logical subdivisions of activities essential to the realization of the objective of business enterprise. In general, its objective may be stated as the rendering of a satisfactory service to the community in exchange for an adequate recompense for the costs and risks involved. The responsible head of an enterprise is always confronted by a similar question. The way in which he logically solves his business problem provides then an opportunity to explore the more fundamental question relating to the concepts of organization itself. Purpose in organization has external as well as internal aspects. Since the strategy of conducting the enterprise is a matter of entrepreneurship, present interest attaches solely to the process by which the problem of how to accomplish the ends of enterprise is solved.

The head of the business has two avenues of approach; he can either be an opportunist or he can use his intelligence and thus manage scientifically. The latter method is the only one with any rational basis and corresponds in essentials with the accepted process of scientific research. The seven basic steps which comprise this process are:

1	Careful planning	
2	Thorough preparation	
3	Conscientious performance	(observation)
4	Rational disposition of the facts	(analysis)
5	Precise evaluation	(synthesis)

[3] It is anticipated that the concepts expressed in this paper can later be extended to comprehend the essentials to the organization of military, religious, governmental, and other social institutions.

6 Preservation of results for reference
7 Constantly improved technique.

These steps in turn naturally derive from those recognized by psychologists as constituting the process of reasoning (7).

Accordingly, if the businessman is rational in carrying on his business undertaking, he will naturally:

1	Evolve a plan of action	(administration)
2	Gather together the necessary materials and tools	(purchasing)
3	Produce the goods which will supply the desired satisfaction or service	(manufacture)
4	Take the goods to those who wish to benefit by them in exchange for an adequate recompense	(distribution)
5	Exercise precautions to avoid failure of the venture	(protection from risk)
6	Figure out whether the undertaking was worth while, and	(control)
7	If worth repeating, study out how it may be done better.	(research)

In this fashion the major activities of a business may be logically accounted for in their natural sequence.

Functional Components

By stripping these descriptive statements of either their research or business connotations, the basic functional components to any rational human endeavor can be set forth. These become, adopting an arbitrary but sufficiently explicit terminology:

1 Plan 2 Procure 3 Perform 4 Dispose
5 Protect 6 Evaluate 7 Improve

Study has shown that these functional components are basic to any activity whether it be one of administration or a work unit in any operation cycle. Furthermore, as any major activity can be broken down in this fashion into its constituent functions, so also can they in turn be similarly subdivided. The process of specialization repeats itself until the last act in the sequence is either to exert a force upon a thing through a tool, if its consummation requires physical effort, or to exercise thought on a single proposition, if mental effort is required. The last act, however, is always reached in seven stages from the all-inclusive activity of enterprise.

Facilities

Though functional components can thus be revealed, the resulting activities cannot as yet be made explicit. From the descriptive statement of business other important primary activities are conspicuously absent. Naturally, the accomplishment of any task requires the use of certain facilities and only by recognizing that which must be dealt with can a subordinate activity be adequately defined.

These facilities derive directly from the productive agencies commonly spoken of as land, labor, and capital. Obviously the group to be organized supplies the men through whose present labor, physical or mental, each elemental task is performed; to these however must be added the people with whom the enterprise does business. Through the accumulation of past experience accepted methods are evolved. Land furnishes the natural resources in the form of raw materials (or power) by means of which goods or services are given tangible form. Capital, in turn, as the product of past surpluses of labor, provides the tools or machines to facilitate present labor or through money commands the product of others' labor.

Functional Definition

These facilities of money, men, methods, machines, and materials (8), together with the seven functional components, provide two series of coordinates making possible precise functional definition. Table 1 shows the expansion of the activity of conducting an enterprise into its major component activities and illustrates how in their appropriate relationship these activities become defined. On this pattern the functions of treasurership personnel, comptrollership, property management, and research

TABLE 1 FUNCTIONAL DEFINITION

MAJOR ACTIVITIES OF ENTERPRISE

(Phase Activity: Enterprise. Scale: III. Technique: Trading.)

		Money	Men	Methods	Machines	Materials
		Treasurership	Parties to Exchange	Management	Agencies of Trade	Commodity
Plan	Administration	Fiscal Program	Scope of Business Operations	Organization	Integration of Enterprise	Merchandising
Procure	Procurement	Appropriation	Purchasing	Trade Agreements	Sources of Supply	Engineering
Perform	Production	Fiscal Transactions	Personnel	Exercise of Authority	Property Management	Manufacture
Dispose	Distribution	Revenue	Marketing	Trade Practices	Trade Channels	Commerce
Evaluate	Comptroll	Financial Control	Goodwill	Accountability	Position in the Trade	Product Acceptance
Protect	Safeguard Business	Reserves for Contingencies	Public Relations	Industrial Arbitration	Trade Relations	Patent Protection
Improve	Business Investigation	Better Use of Funds	Greater Business Opportunities	Better Management Methods	Growth of Enterprise	Product Research

TABLE 2 FUNCTIONAL DEVELOPMENT FROM FUNCTIONAL DEFINITION

PERFORM FUNCTIONS OF ENTERPRISE RELATED TO MATERIALS

(Endeavor = enterprise. Functional development = perform. Facility = materials.)

Scl	1st	2nd	3rd	4th	5th	6th	7th
Agy	Enterprise	Management	Department	Group	Individual	Body	Exertion
Act	Exchange	Differentiate	Implement	Utilize	Execute	Move	Force
Art	Strategy	Method	Process	Procedure	Practice	Skill	Precision
U.M.	Functional Units	Cycle	Sequence	Space	Time	Rhythm	Energy
Plan	Merchandising	Basis of Manufacture	Production Control	Scheduling Work	Standardized Technique	Approved Motion Cycle	Habit
Procure	Engineering	Inventory Control	Material Control	Bring up Material	Reach for Material	Position	Grasp
Perform	Manufacture →	Production →	Processing →	Operation →	Action →	Exertion →	Guide (Force)
Dispose	Commerce	Packing for Shipment	Internal Transportation	Take Away Product	Place in Container	Removal	Release
Evaluate	Product Acceptance	Production Efficiency	Quality Control	Inspection	Output	Examination	Watch
Protect	Patent Protection	Stockkeeping	Elimination of Waste	Temporary Storage	Rest	Avoidance of Delay	Care
Improve	Product Research	Improved Mfg. Methods	Process Engineering	Job Analysis	Time Study	Motion Study	Progress

among others of major importance take a definite place. Any one of these, in turn, may be defined by a similar expansion.

Such patterns have their use in setting forth the relationships basic to organization and in specifying the character of any activity. Their weakness lies in the present inadequacy of management terminology. Words themselves are of little consequence; it is the dynamic concept of an activity conveyed that has real significance. Within the precision of language, mutually exclusive definitions can be composed to give full expression, if need be, to the specific nature of given activities.

Functional Development

If the seven functional patterns of Table 1 presenting the full expansion of any activity, from its origin in enterprise to its final execution, are arranged in the logical sequence of functional definition, the functional development of that activity becomes manifest in the form of another pattern, Table 2, all in the plane of the facility with which it is concerned. Through functional development the specific character of the inherent act itself can be made explicit and it reveals in the functional transitions facts having a bearing on coordination. For instance, the activities based on the functions of procure and dispose provide the connecting links, up or down the line of action, respectively, between the activity of which they are a part and those adjacent of the same order. Procure, in administrative activities, reaches upward and outward to current external conditions, as is found in consumer research in connection with merchandising, Table 2. In perform activities, procure, such as inventory control as a part of manufacture, assures the correct inflow of the essential facilities. Dispose, in perform activities is concerned with the transmittal of the product of performance to its destination, as indicated by internal transportation in the case of production, Table 2. In administrative activities the dispose function is the means of directing all action, such as the merchandising authorization to those who are to put into effect an approved change to a new style or line of product. Furthermore from procure and dispose is derived the specific character of the principal activity because the perform function in no case becomes explicit until the kind of exertion can be specified at the final stage.

TABLE 3 STAGES OF FUNCTIONAL DEVELOPMENT

(Endeavor = business enterprise. Function = perform. Facility = materials or product.)

	The whole			Its parts	
Stages	Agency	Activity	Technique	Elements	Basis for coordinating parts
1	Enterprise	Exchange	Strategy	Divisions	Primary *Functions*
2	Executive Management	Manufacture	Method	Departments	Cycles of *Activity*
3	Operating Management	Production	Process	Work groups	Sequence of *Flow*
4	Supervision	Processing	Procedure	Individuals	Location in *Space*
5	Worker	Operation	Art	Actions of body	Period of *Time*
6	Limb	Motion	Skill	Effort	Synchronized *Rhythm*
7	Muscular Power	Exertion	Efficiency	Force	Economy of *Energy*

Functional Correlation

It is significant that the patterns of functional development, such as Table 2, in furnishing another plane of reference mark the stages in which all action takes place. These stages obviously correspond to the principal subdivisions of management and in so doing are characterized by the technique basic to the type of action occurring at each level of activity. See Table 3. Thus technique, column 4, provides a third coordinate for the proper alignment of specific activities defined by functional components and facilities, the basis for which is indicated in the last column of Table 3.

From a closer examination of the intent of action and the character of technique for each stage, significant groupings of the different stages in threes become evident. The first of the groups at the top of such a scale, service to society,[4] comprehends

[4] The first two stages preceding enterprise in this category have not been considered in this paper since the discussion has been purposely confined to the concepts basic to the organization of a business unit.

the activities of entrepreneurs in conducting enterprise as a profitable business venture, thus realizing the external objectives of organization. The second group, executive management (stages 2, 3, and 4), marks the stages in which the activities of enterprise are subdivided, facilitated, and undertaken in response to the internal objectives. The third group, execution of task, denotes the stages by which individuals comprising the organization perform the necessary acts, each with an efficient work cycle, in applying the requisite physical or mental effort.

Of note is the fact that the three stages which combine in each case, as well as the three resultant groups, have in the order of their occurrence the common characteristics of specialization, facilitation, and technique. The apparent unfailing repetition and its correspondence to the coordinates of rational endeavor would indicate that these three characteristics are of fundamental origin. Being the dimensions for designating explicitly any activity, it is but natural, in specifying respectively (*a*) its distinguishing features or parts, (*b*) the bonds uniting it with associated activities of the same order, and (*c*) the manner of performing its particular function, that they are the embodiment of nature, relationship, and behavior; the factors exclusively identifying any manifestation.

Functional Integration and Lateral Disintegration

A further inspection of the superimposed patterns of functional definition shows that in numerous instances occasion exists for the functional integration of service and staff activities on the basis of common techniques, to minimize duplication of managerial effort, to promote uniformity, and to bring about a more homogeneous organization. Thus a separate functional structure paralleling the primary one of operations can be reared for such activities as technical research and engineering, comptrollership or accounting, office management, quality control, and the like. It is important to note that integration causes the establishment of superimposed activities which have characteristics identical to others at the corresponding higher levels of supervisory, executive, or administrative management.

Integration may, however, go to such extremes as eventually to divorce a group of functional activities from the original enterprise where they become a distinct enterprise of their own, hiring out their services instead.

Coordination

With this sketchy, though nonetheless significant picture of the dynamics underlying specialization, the problem of coordination can be more readily approached. The author has long felt that too much emphasis has been laid on the application of fixed types of organization, such as line, line-and-staff, functional, and committee, without recognition of what really constitutes effective coordination and of the natural processes of structural evolution with the increasing complexity of organization.

Authority and Accountability

Though specialization discloses the functional relationship of all activities and the technical basis for their correlation, human nature is not so constituted as to undertake voluntarily the separate activities of an endeavor in any such logical order without some impelling force. In initiating an undertaking, the individual or group immediately assumes the responsibility for its successful outcome, from which derives directly the sole right to determine how the objective is to be obtained. In the case of an individual this right, in view of his own interests, gives him the supreme authority to carry out his undertaking as he sees fit within the confines of his expressed objective, subject only to the moral obligation incurred toward those in the community with whom he necessarily must deal. In so doing, if

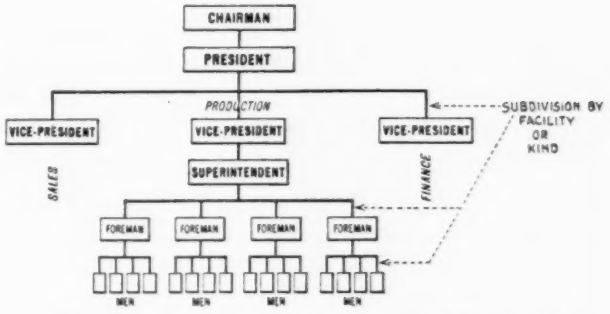


Fig. 1 Lines of Delegation for Supreme Authority

he is unable to perform every activity himself, he must delegate to those in the group he employs the authority for their separate actions. In the other case of a group, the same holds true except that, to become coherent and operative, it must delegate the supreme authority for action, resident in its combined membership, to its chosen leader. Here lies the distinction between an organization to operate and an organization to own or to attain any other objective which in its common interests has brought the group together.

Accordingly, the delegation of authority occurs upward from a group free to determine its own purpose and downward from its leader to those he designates as responsible for the performance of each essential activity. Furthermore, since failure to satisfy fully the obligations incurred in undertaking responsibility exacts its irrevocable penalties, authority, if it is not to be unrestrained, must be accompanied by complete accountability. When clearly defined on the basis of specialization, authority and accountability provide together directive and controlling forces throughout the organization, making for unity and harmony of action. Therefore, the lines of delegated authority, as tracing the natural subdivisions of responsibility, provide the primary channels for coordination.

Lines of Delegation

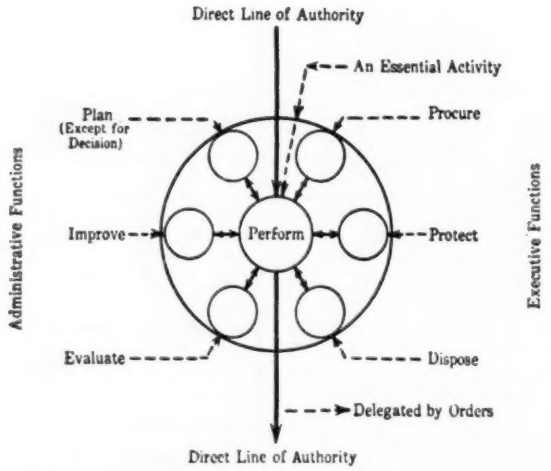
These lines of flow for authority and accountability necessarily conform to those manifest in the functional relationships inherent in specialization as a result of the requirements for orderly and efficient operation. Since the magnitude of no undertaking as yet has become so great that the complete pattern of functional development is evidenced in organization structure, on what basis then are the lines for the delegation of authority established and how completely do they facilitate coordination under varying degrees of specialization? An examination of typical structures of organization will be of value in exposing the techniques of coordination, where the delegation of authority is conditioned by the character of the activities which from their importance should be given individual status.

Undifferentiated Delegation

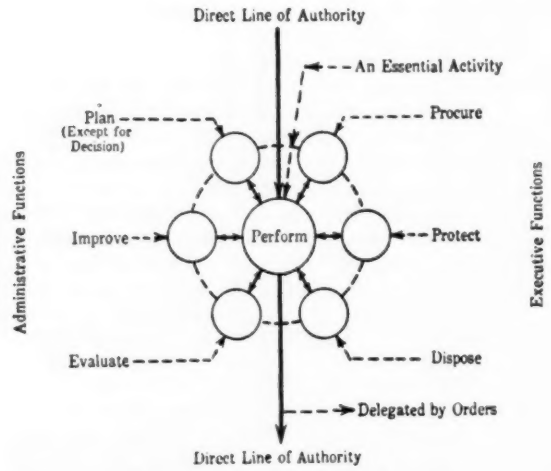
The simplest of all structures represents the subdivision of supreme authority purely in response to the number of units required, each unit composed of men functionally self-sufficient and of a size (9) conducive to effective leadership. The eventual number of levels is merely a mathematical function of these two factors, and obviously the units can only be distinguished on the basis of space or territory occupied, since all have identical objectives.

Simple Vertical Delegation

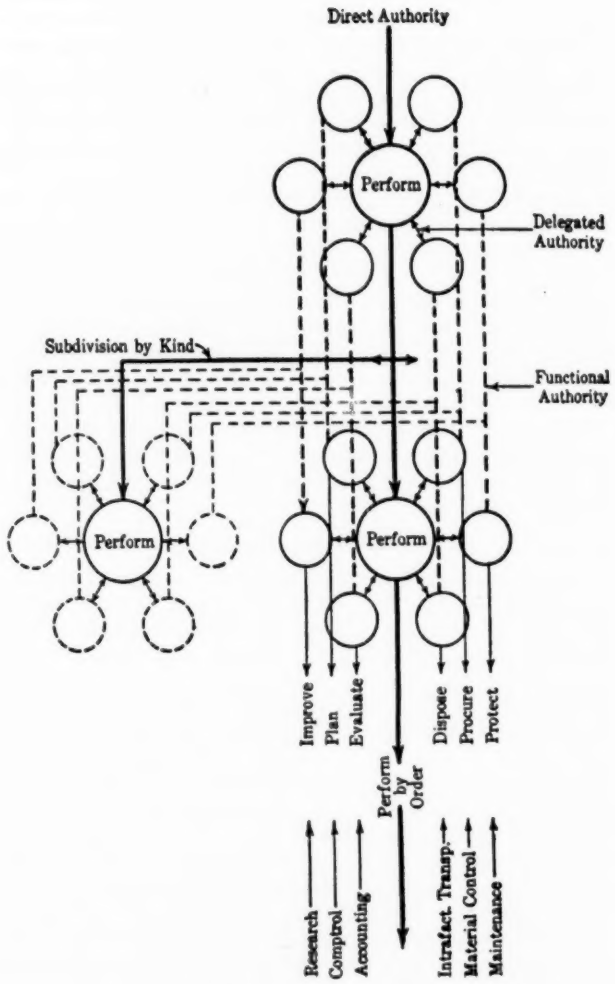
The first differentiation comes when the objective of any subdivision is conditioned, though unaltered in intent, by specific relationship to one of the *facilities* of men, methods, machines,



(a) OPERATING SPECIALIZATION (LINE)



(b) INTERNAL FUNCTIONALIZATION (STAFF)



(c) COMPLETE FUNCTIONALIZATION (SERVICE)

FIG. 2 LINES OF DELEGATION FOR FUNCTIONAL AUTHORITY

materials, or money. Subdivision occurs then according to either territory (as just shown), process, equipment, product, or markets. A second type of differentiation may be caused by subdivision according to *kind:* that is, the specific nature of one of these agencies such as passenger automobiles, trucks, tractors, or the like. The third type occurs when *functional specialization* takes place at any point and the line of supreme authority subdivides into separate lines of delegated authority supreme within their particular technique. This accounts for the usual top structure of enterprise where each unit or any of its subdivisions is still functionally undifferentiated and self-sufficient. See Fig. 1. In these three cases as well, the simplicity of the structural relationships amply facilitates leadership as the coordinating medium.

Functional Delegation, Internal

If, on the other hand, the need for further technical specialization requires functional development, the structure of coordination takes on more specific forms as the magnitude of the constituent activities increases. In the first place, when the work of the unit becomes more than one man can perform, activities divide according to duties of an administrative character and of execution. See Table 4 and Fig. 2(*a*). Obviously, this is the usual case where the man responsible for results retains the policy-forming and control functions of administering the unit and delegates the various duties of execution to subordinates. Should no greater functional specialization be required, the activities of execution subdivide either by facility or kind as previously described. Here again strict line authority provides the basis of harmonious action.

Lateral Delegation

In the second place, if functions such as procure and dispose are of sufficient importance to be set apart, as for instance material control or internal factory transportation in production, service units are created having distinct status only within the realm of the level of activity to which they are associated. See

TABLE 4 BASIS OF FUNCTIONAL DIFFERENTIATION

Constituent function	Relation	Character	
Plan	Staff	Administration	Cycle of one action. ↓
Procure	Service	Execution	
Perform	Line		
Dispose	Service		
Protect	Service		
Evaluate	Staff	Administration	
Improve	Staff		

Fig. 2(*b*). A clear delegation of functional authority has here taken place, supreme however, only within the corresponding technique.

Similarly, a planning or records clerk may be added to take over such routine duties of the executive, creating a staff unit of like character. In fact, the lateral delegation of authority may apply to all the constituent functions, except that, if he is to retain any vestige of responsibility, the executive must never be divorced from his essential acts of decision.[5]

Likewise, the executive may delegate laterally his responsibilities toward men, methods, machines, materials, and money employed by his unit, but in respect to men he must retain the full exercise of leadership or else lose all human influence.

Though these service or staff units are responsible and operative through the executive head only, they have occasion for informal contacts in carrying out their duties with those concerned with the other constituent activities of the whole unit. To facilitate coordination, the executive may provisionally authorize their dealing directly with the other associated functional or operating units in an unofficial capacity, with the final authorization of a particular action following the regular lines of authority. If desirable, a blanket but revocable authorization may be given to act in regard to all routine matters without further recourse to the executive. Since more than two people become involved in such transactions, with the possible impairment of mutuality, channels of coordination other than that of line authority must be at least informally established.

Functional Delegation, External

In the third place, efficient operation may require a similar functional development, at all lower stages, to the point of focusing on the individual executing each task. Complete functionalization, as in Fig. 2*c*, then takes place where all activities of the same character at each level are delegated to form a service or staff division, having supreme functional authority, for the benefits gained from uniformity in technique. When it is essential that such functional divisions serve other branches of the enterprise to minimize duplication, functional integration takes place, as evidenced particularly in personnel. Where functional activities now lie entirely outside the chain of operating units, the mutuality of interest, previously present in a common endeavor, has been destroyed and formally recognized channels of coordination must be established to reinstate, through structural facilitation, the incentives for harmonious association (10).

[5] That is, in respect to the formulation of policy, the approval of action, and the review of accomplishment.

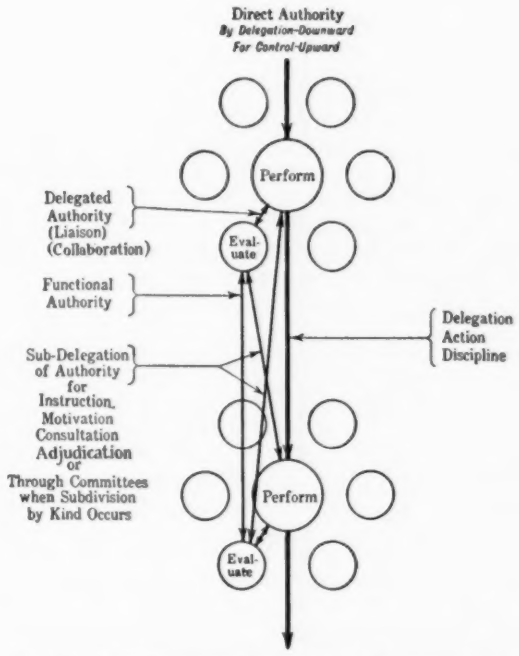


FIG. 3 SUPPLEMENTARY CHANNELS OF COORDINATION
(For contact and communication between operating and functional units, or operating and service divisions.)

Otherwise the conflict from having two masters is likely to prevail.

Facilitating Coordination

Thus with functionalization the need becomes extreme for lines of contact and communication supplemental to those of supreme authority. Authority, of course, supplies the primary means for harmonious action in that it conveys the sole right to undertake a specific action on condition that a satisfactory accounting of results follows. Hence, orders pass down the line of authority and reports ascend providing an effective bond for purposes of direction and control.

The channels thus provided for the flow of supreme authority, since it embodies the common objectives of the organization, are adequate for the exercise of leadership in fostering all aspects of mutuality. The lines of specifically delegated authority are equally effective channels of coordination since the primary

TABLE 5 PERSONAL ASSOCIATIONS ESSENTIAL TO MUTUALITY OF INTEREST

From Superiors

Nature of Facilitation	0	1	2	3	4	5	6	7	8
	Purpose	Delegation	Liaison	Collaboration	Motivation	Consultation	Instruction	Action	Adjudication
Direction of Flow ↓ Down	To Direct	Authority	Advice	Requests	Criticism	Formulation	Teaching	Orders	Discipline
↑ Up Direction of Flow	To Control	Accountability	Information	Proposals	Suggestion	Acceptance	Learning	Reports	Complaints
Form of Coordination	Line	Line	Staff	Staff	Committee	Committee	Committee	Line	Line

Plane of Reference — A Given Stratum of Organization

Usual Sequence of Occurrence

From Subordinates

objective remains unaltered though restricted in application to some one facility of a specific kind.

With functional authority, regardless of the manner of delegation, new and supplemental objectives are introduced, applicable only within the realm of its own technique. As long as functional activities form natural subdivisions of the common endeavor, leadership can still maintain mutuality, provided it encourages informal associations along paths of natural routine (11). When functional activities are given individual status, authority and leadership must be supplemented by other channels of coordination to maintain mutuality of interest.

Factors of Coordination

To supplant structurally the incentive for mutuality destroyed by functionalization requires an understanding of the means by which it is made operative. See Table 5. Its component, community of interest, is apparently fostered by liaison and collaboration, where two units turn to each other for advice and information, and work out upon the request of one a proposal for some action by the other which is of consequence to both. The second, harmonious understanding, is derived from instruction and motivation where one learns from the teachings of the other and each is stimulated to better performance by mutual criticism and suggestion. The third, faithful cooperation, is effected through consultation and adjudication where one unit takes the other into its confidence to secure advance agreement, by mutually formulating and accepting a plan of common action, and to assure fair disciplinary treatment in the event of conflict or complaint.[6]

Supplementary Lines of Coordination

Accordingly, supplementary channels for contact and intercommunication should be established along the natural lines for liaison, collaboration, instruction, motivation, consultation, and adjudication, as in Fig. 3. The first two are most likely to flow over the lines of laterally delegated authority. The second two will pass over the line of subdelegated authority from the functional unit associated with primary authority to a subordinate operating unit. The third two will operate over the line of discretionary authority between the primary unit and a subordinate functional unit, where the subordinate operating executive, in view of effectiveness, waives his right to be a party to the deliberations, reserving as always his responsibility of final decision. Thus, acceptance or disciplinary action in the end must pass over the lines of operating authority to the lower unit and then laterally to the functional unit. Between integrated functional units all six types of contacts regarding uniformity of technique are contained in the line of supreme functional authority. When interdepartmental contacts of a regularly recurring nature are essential, these channels may be effectively formalized by the establishment of a committee (12).

Facilitating Leadership

The longer and more intricate the chains of specialized activities become, the more ponderous and impersonal the organization, especially when all authority is concentrated at the top. Centralization has its advantages in unity and uniformity but is lacking in flexibility and spontaneity of action. It is, therefore, more effective in the coordination of functional divisions than of operating units, particularly when the latter are far removed from the administrative center.

Since operating units are in continual contact with new situations and constantly changing circumstance, they are more effective when authority is decentralized. Their responsible heads, being then close to the scene of events, are more sensitive to the required course for effective and prompt action and can preferably be held accountable for objective results, instead of efficiency of operation alone. Leadership, thus, is spread out over the whole organization and no longer suffers dilution in being transmitted from the top (13).

The point where the decentralization of authority should occur depends upon the determination of the level where constructive action takes place in the direction of the specific objectives of the endeavor and what constitutes adequate freedom under the conditions prevailing, in so far as all contributory prior activities can be departmentalized and rendered upon request as services to the operating units. Basically the decentralized operating unit should be self-sufficient and autonomous, provided its objectives are in no way at variance with those of the enterprise. Primarily, executives, supervisors, and employees should be concerned only with the intelligent, resourceful, and efficient attainment of results. This means, however, an alertness by all in anticipating and meeting the exigencies of each situation with a full understanding of the human, technical, and economic factors involved.

Thus the superstructure of organization could be compressed into a purely motivating, coordinating, facilitating, and evaluating medium. The opportunities for attaining such an organization lie in the understanding of how functional integration will provide a broadening of the structure of enterprise without loss of technical efficiency, and yet narrow the spread between administrator and worker for the fuller and more direct exercise of leadership.

Recent Developments in Business Organization

It appears that the influence of an executive will reach effectively through only three levels of the organization. Similarly the layers of personal insulation (14) dilute the interests of the man at the bottom in reaching further upward with consequent distortion in being relayed to the top. In fact, a new concept is becoming manifest where the work group under a foreman is regarded as the basic unit of management. All the superstructure of management in this case must be justified upon its efficiency in serving these work units. A similar development has appeared in sales organizations.

A corresponding change is being wrought by functional integration in administrative organization (15). Usually the major operating branches of enterprise stem directly from the administrative officer. However, with constantly increasing personal demands, some administrators are surrounding themselves with what may be termed an administrative council, the members of which are the former top operating executives, relieved of all their executive duties. Administration is thus also being forced to functionalize. From the character of administration as now understood, functional integration in its full application would indicate that the administrative group should comprise those executives concerned with (*a*) public and industrial relations (personnel and publicity), (*b*) technical development (research and engineering), (*c*) comptrollership (control information and reports—internal and external), (*d*) law, (*e*) finance (working funds and property), and (*f*) operations (purchasing, manufacture, and sales). This development presupposes that the administrator be himself a practical economist, an organizer, and above all a leader of men.

Accordingly, if decentralization together with functional integration continues to develop in this direction, business enterprise will become substantially, if not in fact, a group of separate enterprises, mutually dependent, serving each other

[6] Note the correspondence of the separate actions with the essential qualities of a leader, i.e., he should be a planner, a technician, a commander, a coordinator, a trainer, an energizer, and a critic. See Bibliography (7), chapter 12, p. 157.

efficiently in carrying out their common business objective and, above all, far better adapted to the fostering of individual initiative and leadership. Thus one way has manifested itself to extend the limits of optimum size in response to the continued growth of large-scale enterprise and at the same time effectively to humanize management.

Practical Aspects of Organization

The remaining aspects of organization are of a practical nature since they are concerned with making the structure operative, and derive their basis of application from the concepts which have been thus far described. Though deserving of equal treatment from the standpoint of immediate utility, space limitations permit only their brief mention to complete the present consideration of the dynamics of organization.

Departmentation is the process by which essential activities are effectively grouped to meet the needs of an enterprise. Aside from natural functional relationships, it is largely governed by the magnitude of operations and the frequency of occurrence. In the upper executive categories where coordination of policies and programs is of primary concern, activities which are naturally associated or complementary in kind should preferably be combined. In the lower categories, groupings should conform to similarity of technique or skills (16). The number of activities so grouped should not exceed that of the direct personal contacts which the executive in charge can effectively maintain (17).

In smaller enterprises the coordinating and facilitating stages may combine with either that of general management or operations, because the more direct personal contacts possible eliminate the need for their formal recognition. In the largest enterprises the stratification manifest among the stages in functional development accounts for the pyramidal structure of holding companies where their subsidiaries in descending order are concerned with the affairs of entrepreneurship, functional services, and specific operations.

The point, where specialization by functional activities should supplant that based on primary authority, occurs when two or more men, simultaneously doing the whole job, can instead do separately different portions of the job, giving a consequent increase in effectiveness, with no other change having taken place (18).

Assignment to duty comprehends the fitting of available personnel to the established departmental units in satisfying the requirement that "the man equals the job." Though one individual may be given responsibility over activities of various kinds, it merely means that he acts in different capacities at different times. Coordination is only affected to the extent that between the temporarily associated activities the single executive in charge himself provides the essential connecting link, and that otherwise separate external channels may become identical until with another individual a reassignment occurs. Though the degree of specialization is reduced, functional definition derived from it serves to clarify relationships in specifying an individual's duty and thus avoids conflicts in authority.

The character and amount of work assigned to an individual, directly and implied, whatever the necessary degree of specialization, should be governed by what fairly constitutes a full day's occupation, with due regard for the physical, mental, and nervous strains involved and for their frequency and severity under periods of peak as well as normal load. Assistance, which is the first step toward further specialization, should be granted before a point is reached where the work becomes so exacting as to impair the effectiveness of the human being (19).

Through systemization, the application of organization to method, standardized processes may be established to relieve an executive or supervisor of unnecessary repetitive detail in directing the regular work of his department, so that he may be free to devote his attention chiefly to exceptional situations, the checking of performance, or the anticipation of the needs for improvement. What happens is that, by planning once in advance, the administrative functions of a constantly recurring nature, for all activities standardized, are dispensed with until circumstances require change.

Routinization, in turn, is a similar standardizing of procedures. Technique is, of course, the standardized practice of an art based on the underlying sciences.

Complete mechanization, the extreme of systemization, contemplates not only the transfer of operator skill for the mechanical performance of work but similarly the incorporation of devices to perform the allied procure and dispose functions as well.

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Pulsating Air Velocity Measurement

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This paper includes an analysis of the weaknesses of three methods of air velocity measurement, that is, (1) the hot-wire anemometer, (2) the Thomas or Callandar meter, and (3) nozzles, orifices, or pitot tubes used with liquid manometers, and the author points out that these methods give unreliable results when the air flow is pulsating. After analyzing the influence of plenum chambers he then describes a modified proportional flowmeter in which the pressure change of a nozzle, orifice, or pitot tube produces an *air* velocity through a small orifice which is always proportional to the instantaneous velocity of the main air flow; therefore, by measuring volumetrically the *air* that passes through the small orifice, the average velocity of the main air flow can be determined. Included in the paper are test results obtained with this meter on a $3^1/_4 \times 4^1/_2$-in. single-cylinder C.F.R. internal-combustion engine.

IN A few simple cases, quantities of air are metered by direct volumetric measurement, but in general it is necessary to determine the velocity of flow. This is usually done in one of three chief ways, namely, by (1) the hot-wire anemometer, (2) the Thomas or Callandar meter, and (3) nozzles, orifices, or pitot tubes used with liquid manometers. The hot-wire anemometer indicates the velocity by the rate of heat loss from a heated wire in the air stream. In the Thomas or Callandar meter, heat is supplied at a known rate and the resulting temperature rise of the air stream is used to determine the velocity. When nozzles, orifices, or pitot tubes are used with liquid manometers, the energy necessary to produce the velocity is measured and used to calculate the velocity.

With suitable precautions, any one of these three methods will give accurate results when the velocity being measured is steady and without fluctuations, but when the flow is unsteady or pulsating in nature these accepted methods of velocity measurement are all subject to serious errors. If any method of measurement is to indicate the true average air velocity, it must depend upon some phenomenon that is influenced only by the average velocity, regardless of the variations of the instantaneous velocity above and below this average value. Since this is not true of the rate of heat transfer involved in the hot-wire anemometer and the Thomas heat meter or of the energy content involved in the use of pitot tubes, nozzles, and orifices, they do not conform to this requirement.

No mechanically induced air motion is free of velocity fluctuations, and in reciprocating engines and compressors the flow is decidedly of a periodic nature. The importance of the need for accurate flow measurement under such periodic conditions makes it seem worth while to analyze the weaknesses of existing methods of velocity measurement, and to present recent developments made in a general method of velocity measurement that was originally presented in 1934.[2]

Hot-Wire Anemometer

The hot-wire anemometer[3] is essentially an electrically heated wire of small diameter that is placed in the air stream. From the applied voltage and current, and the temperature coefficient of electrical resistance of the wire, its temperature may be determined. With the heat input and temperature known, the forced convection film conductance can be calculated after a radiation correction is made.

If this film conductance for wires were a linear function of air velocity, the hot-wire anemometer would yield the true average air velocity for any condition of flow. Unfortunately it varies as the air velocity u to the 0.41 power for small wires and as the velocity to the 0.58 power for large wires.[4] If it be assumed that on the average, the film conductance varies as $u^{1/2}$, it follows that the rate of heat loss at any instant is proportional to $u^{1/2}$ and to the temperature difference between the wire and the air.[3,5] Except for low pulsation frequencies and very small wires, the heat capacity of the wire will tend to maintain a constant wire temperature.[6] In some special work in the study of boundary-layer velocities the temperatures of the very small wires used partially follow the velocity variations, but in general equipment, the temperature difference between the wire and air may be considered as unvarying. It follows that the average rate of heat loss is proportional to the average value of $u^{1/2}$, and the velocity calculated would be the square of the average value of $u^{1/2}$. If this velocity is designated as the indicated velocity it is defined by

$$u^{1/2}{}_{ind} = \frac{\int_0^t u^{1/2}\,dt}{t} = \text{average value of } u^{1/2} \ldots\ldots [1]$$

This is the same as the average velocity for steady flow, but when the velocity is fluctuating it is something quite different. This definition gives a maximum deviation from the average and this difference decreases as the apparatus becomes more delicate and highly refined.

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Discussion of this paper closed January 10, 1939, and is published herewith directly following the paper.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

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Thomas Meter

With the Thomas or Callandar meter the weight per second of air flowing is found by determining the rate of adding heat necessary to produce a small temperature rise in the entire body of air flowing. Knowing the temperature rise, the rate of adding heat, and the specific heat of the air, the rate of flow may be found. This seems like a positive and safe method of measuring air flow under any conditions of velocity variation until the individual heat-transfer steps involved are analyzed.

Since all the air approaching the heater is assumed to be at the same temperature, the first thermometer will indicate the correct initial air temperature regardless of the velocity fluctuations present. In the heating element, which is usually an electrically heated wire grill, the temperature rise of the air passing is small compared with the large temperature difference that must exist between the heater and the air. This temperature difference may then be considered constant for all the air passing, regardless of velocity variations.

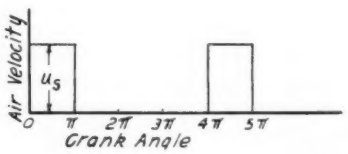


Fig. 1 Simplified Intake Wave of a Single-Cylinder Engine

A convenient method of discerning what happens in this heater is to follow an increment of air through it. If any unit of air has a velocity u, the rate of heat transfer to it as it passes the grid will be proportional to $u^{1/2}$ as in the hot-wire anemometer. The time that this portion of air is in contact with the heater varies inversely as its velocity or as $1/u$. It follows that the heat received by the element of air and consequently its temperature rise are proportional to the product of film conductance and time of contact, and will vary as $u^{1/2}/u$ or as $1/u^{1/2}$. This means that the temperature rise experienced by any portion of air as it passes the heater will vary inversely as the square root of its velocity. This may be expressed mathematically as

$$T_{air} - T_1 = K/u^{1/2} \quad [2]$$

where T_{air} is the temperature of any portion of the air as it leaves the heater; T_1 is the initial temperature; and K is a factor which depends on wire size and temperature, and air properties and temperature. For any practical thermometer, thermocouple, or resistance thermometer sizes and pulsation frequencies, the temperature T_2 indicated by the thermometer after the heater will be steady.[6] Its value will be such that there will be no net heat transfer between it and the air stream. Since the film conductance of the thermometer varies as $u^{1/2}$, it follows that

$$\int (T_{air} - T_2) u^{1/2}\, dt = \int_0^t \left[T_1 + \left(\frac{K}{u^{1/2}} \right) - T_2 \right] u^{1/2} dt = 0 \quad [3]$$

From this, the temperature rise indicated in the Thomas meter is

$$T_2 - T_1 = \frac{Kt}{\int_0^t u^{1/2}\, dt} \quad [4]$$

If u_{ind} be the velocity indicated by the meter, it would be calculated from the temperature rise $(T_2 - T_1)$ as

$$u^{1/2}_{ind} = K/(T_2 - T_1) \quad [5]$$

From Equations [4] and [5]

$$u^{1/2}_{ind} = \frac{\int_0^t u^{1/2}\, dt}{t} = \text{average value of } u^{1/2} \quad [6]$$

It is evident that the Thomas meter, like the hot-wire anemometer, averages $u^{1/2}$; consequently, this indicated velocity is seriously in error when the flow is pulsating.

Pitot Tubes, Nozzles, and Orifices

When air flow is measured by a pitot tube, orifice, or nozzle, the energy necessary to create the flow velocity at any instant is proportional to u^2 and this is measured as a pressure by a liquid manometer. Except when the manometer is of such a length as to oscillate in synchronism with the pulsation frequency, the manometer reading is proportional to the average value of u^2. Consequently, the effective velocity u_{eff} calculated from the manometer reading is given by

$$u^2_{eff} = \frac{\int_0^t u^2\, dt}{t} = \text{average value of } u^2 \quad [7]$$

Again, the pitot tube, orifice, and nozzle when used with a manometer do not give the correct average velocity when the flow is pulsating.

Pulsation Errors

The possible magnitude of the errors involved in the use of existing methods for air velocity measurement when the flow is

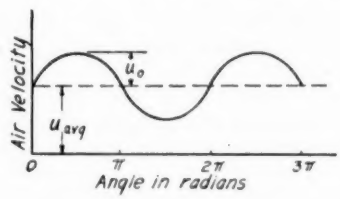


Fig. 2 Pulsating Air Flow

pulsating or periodic may be dramatized by assuming the intake air to a single-cylinder four-stroke-cycle engine to be as simple as shown in Fig. 1. For such a velocity wave

the average velocity is $u_{avg} = \pi u_s/4\pi = u_s/4$

the indicated velocity is $u_{ind} = [(\pi u_s^{\frac{1}{2}})/(4\pi)]^2 = u_s/16$

the effective velocity is $u_{eff} = \sqrt{[(\pi u_s^2)/(4\pi)]} = u_s/2$

or

$$u_{avg}/u_{ind} = 4$$

and

$$u_{avg}/u_{eff} = 1/2$$

This means that the hot-wire anemometer or the Thomas meter would give an indicated value one quarter the average, and a pitot tube, orifice, or nozzle would give an effective value two times the average.

A more comprehensive picture of these errors may be presented by representing any pulsation as an equivalent sinusoidal variation as shown in Fig. 2. The instantaneous velocity for such equivalent pulsation is given by

$$u = u_{avg} + u_0 \sin\theta \quad [8]$$

and the pulsation may be defined as

$$\text{Pulsation} = u_0/u_{avg} \quad [9]$$

For the hot-wire anemometer and Thomas meter

$$\frac{u_{avg}}{u_{ind}} = \frac{u_{avg}}{\left[\dfrac{\int_0^{2\pi} (u_{avg} + u_0 \sin\theta)^{\frac{1}{2}}\, d\theta}{2\pi} \right]^2} \quad [10]$$

$$= \frac{1}{\left[1 - \frac{1}{16}\left(\frac{u_0}{u_{avg}}\right)^2 - \frac{15}{1024}\left(\frac{u_0}{u_{avg}}\right)^4 \ldots\ldots \text{approx}\right]^2} \quad [10a]$$

and for the pitot tube, orifice, and nozzle

$$\frac{u_{avg}}{u_{eff}} = \frac{u_{avg}}{\left[\frac{\int_0^{2\pi}(u_{avg} + u_0 \sin\theta)^2\, d\theta}{2\pi}\right]^{1/2}} \quad [11]$$

$$= \frac{1}{\left[1 + \frac{1}{2}\left(\frac{u_0}{u_{avg}}\right)^2\right]^{\frac{1}{2}}} \quad [11a]$$

These variations are shown in Fig. 3.

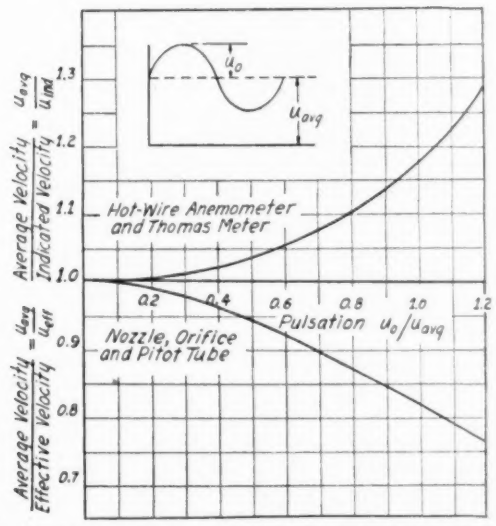


FIG. 3 PULSATION ERRORS FOR SINUSOIDAL VELOCITY VARIATION

The measurement errors for the idealized air intake of four-stroke-cycle internal-combustion engines are summarized in Table 1. The air velocity is assumed to be sinusoidal during the intake strokes and zero at other times.

TABLE 1 IDEALIZED PULSATION ERRORS FOR INTAKE AIR OF INTERNAL-COMBUSTION ENGINES

Engine type	u_{avg}/u_{eff}	u_{avg}/u_{ind}
Single cylinder	0.45	4.60
Two cylinders	0.76	2.30
Four cylinders	0.90	1.15

Errors of this magnitude are not realized in practice because of the reduction of the intake manifold, but Table 1 does illustrate the possible magnitude of pulsation errors.

PULSATION REDUCTION

The usual method used to reduce the pulsation in the air flow to reciprocating machines is to draw the intake air through a plenum chamber where the elastic action of the air in the tank reduces the velocity fluctuation in the air entering the chamber through the orifice or nozzle. The air dome on the discharge of a reciprocating liquid pump fills this same function.

If a reciprocating machine draws a pulsating flow from a plenum chamber with an instantaneous velocity V, the mass per second of air leaving the chamber is

$$\text{mass per second leaving} = c\rho VA \quad [12]$$

where ρ is the mass density of the air, A is the orifice area, and c is its coefficient of discharge. The mass per second being stored in the tank of total volume S is $S(d\rho/dt)$, and, since the air conforms to the general gas equation $PV = WRT$ and the temperature may be considered constant for the small pressure changes involved

$$\text{mass per second stored} = \frac{S}{gRT}\frac{dP}{dt} \quad [13]$$

where P is the instantaneous pressure in the tank at any time t.

If u is the instantaneous velocity of the air entering the plenum chamber through the intake orifice, a change in this velocity du will produce a change in the plenum chamber pressure of

$$dP = -\rho u\, du \quad [14][7]$$

For an intake orifice area a and coefficient of discharge c, the mass per second entering the chamber is

$$\text{mass per second entering} = ca\rho u \quad [15]$$

Since the mass per second entering the plenum chamber must leave or be stored

$$ca\rho u = c\rho VA - \frac{S}{gRT}\rho u\frac{du}{dt} \quad [16]$$

or for equal discharge coefficients

$$V = u\frac{a}{A} + \frac{S}{gRT\,AC}u\frac{du}{dt} \quad [17]$$

From Equation [17] it may be seen that u differs from $(A/a)V$ by a term which depends upon $u(du/dt)$. This term represents three physical differences between the velocity wave drawn by the engine and that entering the plenum chamber. The amplitude of oscillation is decreased, the time of maximum velocity is shifted, and the wave is distorted by the introduction of harmonics. Since the actual velocity wave drawn by a compressor or engine contains many harmonics, and since each harmonic has its amplitude and phase angle changed and is distorted, a completely general solution of Equation [17] is out of the question.

However, as will be verified later, the maximum possible effect of a plenum chamber may be predicted from a simple approximate solution of Equation [17]. Any wave form drawn from the tank may be represented by an equivalent sinusoidal variation that has the same value of V_{avg}/V_{eff}, or by

$$V = V_{avg} + V_0 \sin\omega t \quad [18]$$

When only the change in amplitude and the phase displacement are considered, and all distortion due to harmonics is ignored, the velocity wave entering the plenum chamber may be represented by

$$u = u_{avg} + u_1 \sin\omega t + u_2 \cos\omega t \quad [19]$$

Substituting Equations [18] and [19] into Equation [17], ignoring all harmonic terms in $u(du/dt)$, and solving for u_{avg}, u_1, and u_2, gives

$$u = \frac{A}{a}V_{avg} + \frac{V_0\, aAg^2R^2T^2c^2}{S^2u^2_{avg}\omega^2 + a^2R^2T^2g^2c^2}\sin\omega t - \frac{V_0 gRTcAS u_{avg}\omega}{S^2u^2_{avg}\omega^2 + a^2R^2T^2g^2c^2}\cos\omega t \quad [20]$$

[7] To obtain solvable equations, the entering and leaving densities are assumed equal. The only place where the variation in density is numerically significant is in the rate of change of mass in the plenum chamber $Sd\rho/dt$.

This may be simplified to

$$u = \frac{A}{a} V_{avg} + \frac{(A/a)V_0}{\sqrt{\left(\frac{Su_{avg}\omega}{agRTc}\right)^2 + 1}} \sin\left[\omega t - \tan^{-1}\frac{Su_{avg}\omega}{agRTc}\right] \quad [21]$$

If the tangent of the angle of phase displacement is defined as

$$\beta = \frac{Su_{avg}\,\omega}{agRTca} = \text{plenum factor} \quad [22]$$

Equation [21] becomes

$$u = \frac{A}{a} V_{avg} + \frac{(A/a)V_0}{\sqrt{(\beta^2 + 1)}} \sin\left[\omega t - \tan^{-1}\beta\right] \quad [23]$$

By the same method used in Equation [11]

$$\frac{u_{avg}}{u_{eff}} = \frac{1}{\left[1 + \frac{1}{2}\left(\frac{V_0}{V_{avg}}\right)^2\left(\frac{1}{\beta^2 + 1}\right)\right]^{\frac{1}{2}}} \quad [24]$$

and since from Equation [11]

$$\frac{1}{2}\left(\frac{V_0}{V_{avg}}\right)^2 = \left(\frac{V_{eff}}{V_{avg}}\right)^2 - 1 \quad [25]$$

Equation [24] then becomes

$$\frac{u_{avg}}{u_{eff}} = \frac{1}{\sqrt{\left[1 + \frac{(V_{eff}/V_{avg})^2 - 1}{\beta^2 + 1}\right]}} \quad [26]$$

This equation gives the maximum possible value of u_{avg}/u_{eff} for air flowing into a plenum chamber having a plenum factor β (Equation [22]) when the air flowing to the engine has pulsation represented by V_{avg}/V_{eff}. This plenum-chamber action is

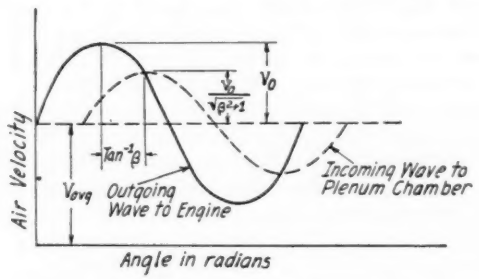


FIG. 4 THE EFFECTS OF A PLENUM CHAMBER ON AIR VELOCITY WAVES

illustrated graphically in Fig. 4, which shows the incoming and outgoing air waves reduced to the same orifice-area basis. From Equation [17] when $du/dt = 0$, the maximum or zero slope value of the incoming wave occurs at an angle $\tan^{-1}\beta$ later than the maximum value of the outgoing V wave, and the equivalent amplitude is reduced to $V_0/\sqrt{(\beta^2 + 1)}$. The effect of the harmonics that were ignored in the approximate solution of Equation [17] would be to produce a distorted wave with the same average value but having a higher maximum value and also a smaller u_{avg}/u_{eff}.

Fig. 5 shows the maximum possible values of intake u_{avg}/u_{eff} plotted against the plenum factor β for different values of initial V_{avg}/V_{eff}. It is interesting that a plenum chamber having a factor of $\beta = 5$ will reduce the pulsation error as far as it is practically feasible. Further gains in pulsation reduction would be too slight to justify the necessary increase in plenum-chamber size.

Applying this conclusion to an internal-combustion engine of N cylinders, each having a displacement P, turning n revolutions per second, Equation [22] may be written

$$S = \frac{\beta ac\, u_{avg}\, gRT}{\omega\, u^2_{avg}} \quad [27]$$

For a four-stroke-cycle engine, the cubic feet of air per second is

$$acu_{avg} = (N/2)nP\epsilon$$

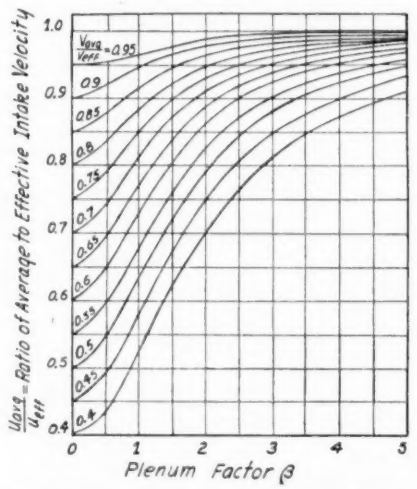


FIG. 5 PULSATION REDUCTION IN PLENUM CHAMBERS
(V_{avg} and V_{eff} refer to air drawn from the plenum chamber, while u_{avg} and u_{eff} refer to resulting air flow to the plenum chamber.)

where ϵ is the volumetric efficiency. Also, $\omega = (nN/2)2\pi$; $\beta = 5$; $\epsilon = 0.7$; $R = 53.3$; and $T = 530$ F, abs. For an average intake-orifice velocity ($u_{avg} = 100$ fps)

$$S = \frac{\beta(N/2)nP\epsilon gRT}{2\pi(nN/2)u^2_{avg}} = \frac{\beta gRT\epsilon}{2\pi\, u^2_{avg}} P = 50\ P \text{ (approx)} \quad [28]$$

This is the size usually recommended[8] for such plenum chambers, and for a two-cylinder engine for example, Fig. 5 shows that an initial $V_{avg}/V_{eff} = 0.76$, as given in Table 1, would be reduced to a $u_{avg}/u_{eff} = 0.985$. This would represent a minimum pulsation error of 1.5 per cent. However, if the engine speed is reduced one half and the plenum chamber and intake orifice are not altered, u^2_{avg} of Equation [29] will be reduced to about one fourth. From Equation [28]

$$\beta = \frac{2\pi(u_{avg})^2\, S}{gRT\epsilon P} \quad [29]$$

This would give a plenum factor $\beta = 1.25$ instead of 5, and from Fig. 5, for $V_{avg}/V_{eff} = 0.76$, u_{avg}/u_{eff} would be 0.875, which would represent a minimum possible measurement error of 12.5 per cent. This illustrates the need for proportioning a plenum chamber and its orifice for the exact conditions of use, and also the danger of depending on a simple rule that specifies only the volume of the chamber.

AN AVERAGE-VELOCITY METER

If any instrument is to measure the true average velocity of air when the flow is pulsating, it must be actuated by some phenomenon that is always directly proportional to the instantaneous

[8] "Power and the Internal Combustion Engine," by W. E. Dalby, Longmans, Green & Company, New York, N. Y., 1931, p. 111.

velocity. In 1934 the author proposed[3] a modified proportional flowmeter in which the pressure change of a nozzle, orifice, or pitot tube produced a water velocity through an orifice always proportional to the instantaneous air velocity. By collecting the water that passed through the orifice, the average air velocity was determined.

Subsequent work with this meter has emphasized the need for the elimination of the flow of water through a small orifice, and this has resulted in the evolution of the instrument shown in Fig. 6. In it the pressure drop across the intake orifice *1* causes an instantaneous air velocity through the small orifice *2* that is always equal to the air velocity through *1* as long as the pressure in *3* is maintained equal to atmospheric. This pressure is maintained by admitting water from storage *6* through the valve *5* at such a rate as to maintain the water levels in the glass tubes *3* and *4* always equal as they rise. If the time required to draw a certain volume of air from the annular space between tubes *3* and *4* is found, the average velocities through orifices *2* and *1* may be found.

For satisfactory operation, tubes *3* and *4* should be large enough to give a flat meniscus and so proportioned that their capillary heads are equal. Their lengths should be sufficiently great to lower the natural oscillating frequency of the water columns well below the minimum pulsation frequency of the air flow being measured. The orifice area *2* needs to be very small in comparison with the tube sizes to cause the water level to rise

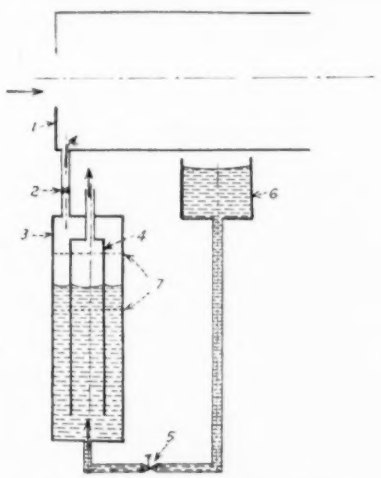


FIG. 6 AVERAGE VELOCITY METER

slowly enough to make it possible to maintain the two levels the same throughout a run, and also to minimize air-friction losses in the rubber tubing leading to the instrument.

This meter is, in effect, a combination of proportional flow and positive-displacement meters. With it the true average velocity may be determined directly for any conditions of pulsation with either intake or discharge orifices or nozzles, and when it is used in conjunction with a manometer, the value of V_{avg}/V_{eff} may be found. Fig. 7 shows the calibration curve of the experimental instrument now in use. During its development it was checked by velocity determinations where the wave shape was produced artificially and was known accurately. Using a trapezoidal wave through an intake orifice it gave $V_{avg}/V_{eff} = 0.912$ and the value calculated from the wave shape was 0.898. For a similar wave through a discharge orifice at the same frequency of two cycles a second it gave $V_{avg}/V_{eff} = 0.898$ and the calculated value was 0.895.

The accuracy of the present meter is within 2 per cent, and its

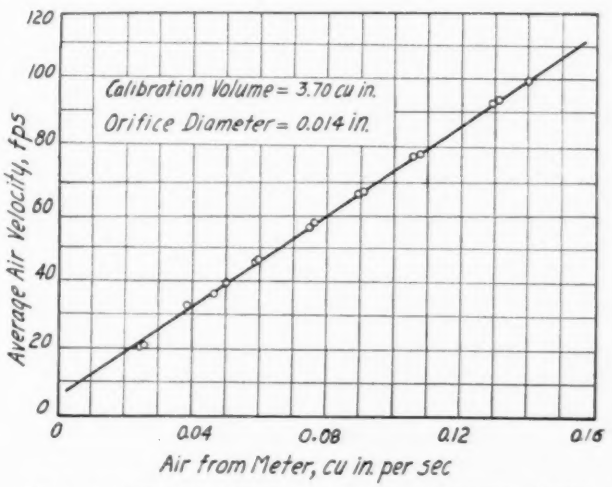


FIG. 7 CALIBRATION CURVE FOR AVERAGE VELOCITY METER

operation requires approximately the same degree and type of skill as an Orsat gas analyzer. Starting with the water level below the lower calibration mark, water is admitted at the correct rate to keep the levels equal in the two tubes. From the time required for the level to rise from the lower to upper calibration mark *7*, the average velocity may be found from the calibration curve. It is calibrated against a liquid manometer under steady-flow conditions where the average and effective velocities are equal.

EXPERIMENTAL VERIFICATION

To illustrate the operation of the average-velocity meter and to check the predicted action of a plenum chamber, tests were made on a $3^1/_4 \times 4^1/_2$-in., C.F.R., single-cylinder internal-combustion test engine operating at variable speed and a compression ratio of 5 to 1. Since a single-cylinder four-stroke-cycle engine draws air of distorted wave shape for one stroke and none for three strokes, it represents about the worst possible condition of operation for the average velocity meter. It also has an intake air-velocity wave with a wide variety of harmonics and represents the greatest variation from the simplifying assumptions made in the solution of Equation [17] and shown in Fig. 5.

When the air to the engine was drawn through a $^7/_8$-in. diameter orifice directly in the $1^1/_4$-in. intake to the engine, the values of V_{avg}/V_{eff} shown by Table 2 were found.

When the intake air was drawn through a plenum chamber $2^1/_2$ ft long with a total volume of 1.98 cu ft and a $^7/_8$-in. diameter intake orifice, the values listed in Table 3 were obtained.

Theoretical values of u_{avg}/u_{eff} calculated from the results of Tables 2 and 3, using the curves of Fig. 5, are given in Table 4. In Fig. 8 the test and theoretical results are shown graphically.

Fig. 8 brings out several points of interest. The increase in pulsation of the engine intake air shown by the decrease of V_{avg}/V_{eff} as the engine speed is increased might seem contradictory until it is remembered that an increase in speed results in a sharper-topped suction wave. It is not surprising that the theoretical values of u_{avg}/u_{eff} exceed the test values by as much as 0.085 when it is remembered that the case tested varies a maximum amount from the conditions assumed when solving Equation [17]. As the initial-velocity variation becomes less marked and more symmetrical, results would more closely correspond to test results.

SUMMARY

From the foregoing it seems safe to conclude that conventional methods of air velocity measurement are unreliable when the

TABLE 2 DIRECT AIR MEASUREMENT TO C.F.R. ENGINE

Engine speed, rpm	Manometer: Orifice pressure, in. water	Manometer: V_{eff}, fps	Velocity meter: Volume, cu in. per sec	Velocity meter: V_{avg} fps	$\frac{V_{avg}}{V_{eff}}$
600	1.32	7.60	0.0644	49.6	0.653
700	1.51	81.3	0.0666	51.0	0.628
800	1.69	86.0	0.0676	52.0	0.605
900	1.75	87.6	0.0690	52.5	0.601
1000	1.87	90.5	0.0700	53.4	0.590
1100	1.97	93.0	0.0710	54.0	0.582
1200	2.12	96.2	0.0712	54.2	0.562
1300	2.21	98.5	0.0724	55.0	0.558
1400	2.25	99.3	0.0708	54.0	0.542
1500	2.32	101.0	0.0708	54.0	0.535

TABLE 3 AIR MEASUREMENT WITH PLENUM CHAMBER FOR C.F.R. ENGINE

Engine speed, rpm	Manometer: Orifice pressure, in. water	Manometer: u_{eff}, fps	Velocity meter: Volume, cu in. per sec	Velocity meter: u_{avg}, fps	$\frac{u_{avg}}{u_{eff}}$
600	0.60	51.2	0.0557	43.8	0.856
700	0.66	53.7	0.0567	44.3	0.825
800	0.72	56.2	0.0570	44.9	0.798
900	0.74	56.8	0.0572	45.0	0.792
1000	0.76	57.8	0.0582	45.5	0.788
1100	0.77	58.2	0.0604	47.0	0.806
1200	0.79	58.9	0.0617	47.8	0.812
1300	0.81	59.5	0.0638	49.0	0.823
1400	0.82	60.0	0.0647	49.5	0.825
1500	0.82	60.0	0.0638	49.0	0.825

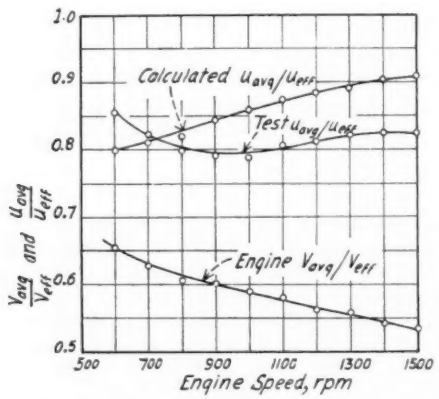


FIG. 8 TEST AND THEORETICAL RESULTS FOR $3^1/_4 \times 4^1/_2$-IN. C.F.R. ENGINE WITH DIRECT INTAKE AND WITH 1.98-CU FT PLENUM CHAMBER

flow is pulsating. This trouble may be at least partially eliminated by a properly designed plenum chamber, but there are possibilities of plenum-chamber errors not previously mentioned. Some of the apparently unproductive development work[9] of the past four years has emphasized the fact that when any dimension of the plenum chamber or intake system is such as to produce resonance of any one of the principal harmonics of the suction-velocity wave, a plenum chamber may be a menace instead of an aid.

An incorrect length of rubber air hose to an instrument or a wrong water-column length may produce resonance and the difficulty of knowing the harmonic frequencies present in an engine suction wave increases this danger. In general, it is desirable to keep all air paths short enough to have natural frequencies above any harmonic present and to use long water columns to keep their resonance frequencies very low.

Since plenum chambers have a knack of introducing unsuspected errors and since they are not feasible when testing large engines, the author believes that the average velocity meter shown in Fig. 6 used with nozzles or orifices directly in engine-intake pipes can fill a definite need.

[9] "Pulsating Air Flow," thesis by Lynn Gore, in the Iowa State College Library, Ames, Iowa.

TABLE 4 THEORETICAL PULSATION REDUCTION

Engine speed, rpm	Fundamental radians per sec ω	u_{avg}, fps	$\beta = \frac{Su_{avg}\omega}{agRTc}$	$\frac{V_{avg}}{V_{eff}}$	Calculated u_{avg}/u_{eff}
600	31.4	43.8	1.17	0.653	0.800
700	36.7	44.3	1.37	0.628	0.812
800	41.9	44.9	1.60	0.605	0.820
900	47.1	45.0	1.81	0.601	0.845
1000	52.3	45.5	2.02	0.590	0.860
1100	57.5	47.0	2.30	0.582	0.875
1200	62.8	47.8	2.55	0.570	0.885
1300	68.0	49.0	2.82	0.558	0.891
1400	73.2	49.5	3.08	0.545	0.905
1500	78.5	49.0	3.27	0.535	0.910

NOTE: $S = 1.98$ cu ft; $a = 0.00418$ sq ft; $c = 0.61$; and $T = 530$ F, abs.

The coefficients of discharge of nozzles vary so little that no fear has been felt in using the steady-flow values under pulsating conditions. The work done to date[9] on the effects of pulsation on orifice discharge coefficients indicates that here also, the steady-flow values may be used with safety. The apparent discrepancy between the average velocities V_{avg} and u_{avg} in Tables 2 and 3, when $^7/_8$-in. diameter orifices were used in both cases, resulted from the methods of installing them. When the plenum chamber was used the coefficient of discharge was assumed to lie between 0.60 and 0.61; however, Table 2 is for a $^7/_8$-in. diameter orifice used in a $1^1/_4$-in. diameter pipe and here, as in steady flow, the coefficient ranged from 0.52 to 0.56.

However, there are enough such questions to be answered that plans for the future include the investigation of orifices, nozzles, and pitot tubes. It is also planned to refine the present instrument, which is now just an accumulation of glassware, into a rugged instrument that will require less operating skill.

Discussion

J. F. ALCOCK.[10] This paper is of great interest to the writer since, having studied the problem for some years, he has arrived at an entirely different solution. In this the fluctuation difficulty has been eliminated by using the pressure drop across a viscous resistance for flow measurement. Because at any instant the pressure drop is directly proportional to the flow, the root-mean-square error disappears and pulsation has no effect.

In this viscous-flow meter the resistance element is a honeycomb of long narrow triangular passages, formed by winding alternate layers of plain and corrugated strip metal on a core, as in the "Amal" flame trap but with smaller and longer passages. The strip is of nickel silver and the passages are 0.017 in. wide. The theoretical critical velocity in these passages, for air under atmospheric conditions, is about 270 fps, but the highest velocity so far used in practice is about 22 fps, although the resistance is linear up to at least twice this figure. There is a small square-law resistance component, due to entry losses, which is automatically corrected by making the downstream manometer connection face upstream and vice versa, thus giving a small reverse kinetic head.

There are, to prevent clogging of the meter exhaust element, two air cleaners, one oily to trap dust and the other dry to trap any oil blown over from the first cleaner. Felt pads are fitted in the tubes between the meter and the manometer. This raises an interesting point. In the early tests it was found that, although the resistance characteristic on steady flow was quite linear, the readings on pulsating flow were incorrect (generally on the low side). It was suspected that the air flow in and out of the manometer tubes, due to the pressure changes in the meter and to the elasticity of the air in the tubes, was responsible. The matter was examined theoretically for a simplified pulsating flow in which the velocity was n times the mean for $1/n$ of the cycle,

[10] Ricardo & Company, Limited, Consulting Engineers, Bridge Works, Shoreham-by-Sea, Sussex, England.

with no flow in between. The capacity of the tubes was assumed to be so large, and the orifice connecting them to the meter so small, that there was negligible cyclical pressure variation therein. The resistance of the manometer correction orifice was assumed to be square-law.

This investigation showed that the pressure in the manometer tube is not, in general, the true mean pressure in the meter. Thus there is an error which is expressed by the formula

$$\frac{\text{True flow}}{\text{Apparent flow}} = \frac{n}{1 + (n-1)^2}$$

For $n = 2$ (and for other wave forms having no odd harmonics) the error disappears. The meter reads high for $n < 2$ and low for $n > 2$. At $n = \infty$ the error is equal and opposite to the root-mean-square error of the orthodox orifice meter. Thus the latter would, for example, record fairly accurately the gas discharge from a machine gun.

If, however, the connection from meter to manometer is viscous, this manometer connection error disappears for all values of n, hence the felt pads. Their exact resistance does not matter provided that it is large enough to swamp incidental square-law resistances in the connections.

Another source of error with square-law connections is that they often have different resistance coefficients for inflow and outflow, thus acting as imperfect nonreturn valves. This cannot happen with a viscous resistance element.

These connection errors would not apply to the author's meter if the air volume above the water is large enough to render negligible the cyclical pressure changes therein. The accuracy of the viscous meter on pulsating flow has been checked by tests on single-cylinder four-cycle gasoline engines in series with a gasholder, and on the delivery of a Roots blower delivering at 6 lb per sq in., where the pulsations are extremely violent owing to the absence of compression in the blower.

Under these exceptionally severe conditions, the maximum discrepancy found was 2 per cent, which is within normal meter tolerances, except over one narrow range of engine speed in the first test, where the error reached a maximum of 8 per cent low. Here the engine volumetric-efficiency curve showed the presence of a strong pipework resonance. A smoothing receiver of only four cylinder volumes reduced the error to 3 per cent. Since the reading was low the error was probably due to the velocities being outside the viscous range of the manometer connections; recent modifications have increased this range. In any case it is generally desirable on other grounds to avoid resonances of such magnitude.

These meters have been used in internal-combustion-engine research for some years. One advantage for such work is that the cyclical variations of pressure drop are less than with a square-law meter. Thus on a single-cylinder four-stroke engine the effective pressure drop during the suction stroke is only four times the mean pressure drop as against sixteen times with a square-law meter. Thus for a given degree of engine throttling, a larger head and, therefore, a less sensitive manometer, can be used. For the same reason, the sensitivity at reduced flows falls off less than with square-law meters.

COLIN CARMICHAEL.[11] The accurate control of air-fuel-mixture ratios in aircraft engines during flight is of paramount importance in securing maximum performance, particularly in regard to economy at cruising speed.[12] The impossibility of applying existing laboratory test methods to the measurement of air supply in flight has led to investigations of indirect means of estimating air-fuel ratio, such as the analysis of exhaust gases. The accuracy of such methods is admittedly of a low order, as indicated by the scattering of test points in the investigations reported. A direct method of air measurement that does not involve cumbersome apparatus should therefore be welcomed with enthusiasm by aircraft engineers in particular, as well as by automobile, railroad, and marine engineers who have to make tests in a limited space.

Since the author's average-velocity meter requires no provision for pulsation reduction, an orifice or nozzle fitted in the carburetor intake is all that is required. It would appear, if the orifice is so proportioned that the average air velocity at wide-open throttle is about 100 fps, the reduction in volumetric efficiency will be little more than 0.5 per cent. Simultaneous measurements during the same period of time of air flow and fuel flow will afford a ready and accurate check on mixture ratio and enable the correct setting to be made as quickly as possible.

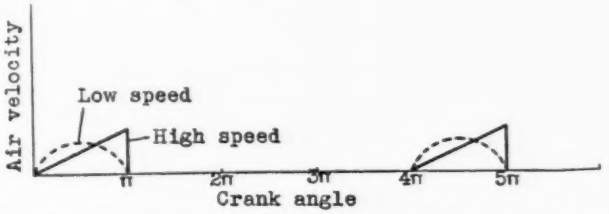


FIG. 9 SIMPLIFIED INTAKE WAVES OF A SINGLE-CYLINDER ENGINE AT LOW AND HIGH SPEEDS

While the manipulation of the author's instrument appears relatively simple after a little practice, it would seem desirable and perhaps quite feasible to introduce some means of maintaining automatically the two liquid levels equal.

Some of the author's discussion has centered around the probable shape of the velocity wave drawn by the engine. It would appear that Equation [17] is a tool which could be used for an investigation of wave shapes and volumetric efficiency, and how they are affected by speed and valve timing. In this case A is the piston area, a is the instantaneous area through the valve opening, S is the instantaneous cylinder volume and V is the instantaneous piston velocity. A step-by-step integration is of course necessary, for which purpose Equation [16] may be rewritten

$$du = \frac{gRTAdt}{S}\left(\frac{V}{u} - \frac{ca}{A}\right)$$

the coefficient of discharge at the piston area being unity. The instantaneous values of velocity through an area of the valve being known, the mass flow and velocity in the pipe may be calculated.

When the velocity is low the wave shape in the pipe will tend toward that of the piston-velocity wave for as long as the valve is open. As the speed increases, with normal timing, the restriction in the early part of the stroke due to slow opening of the valve will decrease the velocity, but later in the stroke the momentum acquired earlier will cause the velocity to exceed that corresponding to the piston velocity. The resulting curve at high speed might be simplified to a triangular wave Fig. 9, instead of the approximate sine wave found at low speed.

R. B. SMITH.[13] The problem of pulsating flow is an important, yet extremely difficult one to attack rationally, for the reason

[11] Assistant Professor of Mechanical Engineering, Rutgers University, New Brunswick, N. J. Mem. A.S.M.E.

[12] "Interrelation of Exhaust-Gas Constituents," by H. C. Gerrish and Fred Voss, N.A.C.A., Technical Report No. 616, 1938.

[13] Development Engineer, Elliott Company, Jeannette, Pa. Jun. A.S.M.E.

that we are none too sure of our fundamental premises. To be specific, the Saint Venant or Bernoulli equation, which has been employed by the author and his contemporaries, is merely a special formulation of Newton's equation as written by Euler. Now as is well known in hydrodynamics, the acceleration term may consist of both a time derivative and a space derivative. Upon the supposition that we are dealing with substantially steady-state flow, the time derivative is neglected and the integration of the equation yields, with special restrictions, the Saint Venant or Bernoulli equations.

For the extreme variations that may be met in the field of pulsating flow, there exists some question in the writer's mind regarding the legitimacy for neglecting a time differential. It is not inconceivable that an error of very serious magnitude may result and, in view of this fact, it would appear that the burden of proof rests upon the author.

In attacking the problem experimentally, the author has, in principle, attempted to approximate the total flow by a sampling scheme which embodies a means for positive-displacement calibration. Unfortunately, the size of the sampling orifice is such that we have very nearly the calibration of a kinetic meter with a capillary tube. Inasmuch as the flow relations for these two conditions are different, this would appear to offer some difficulties. For instance, referring to Fig. 7, it is evident that the Reynolds number of the sampling device varies from 0 to about 500. This performance lies in the region where there is a rapid variation of orifice coefficients with Reynolds' number, with the result that only slight variations in fluid viscosity should have theoretically at least, considerable effect on the instrument calibration.

Author's Closure

It is fortunate that the discussion of this paper includes Mr. Alcock's description of the viscous meter used in the experimental laboratories of Ricardo and Company. This viscous-flow meter, designed for high-precision laboratory use and the kinetic meter, described in the paper and designed for portable use in the field, represent two extremes in the approach to this problem. Mr. Alcock's reference to, "Another source of error with square-law connections is that they often have different resistance coefficients for inflow and outflow, thus acting as imperfect nonreturn valves," is a very important point to be remembered in the design of kinetic meters such as the one discussed in this paper.

Professor Carmichael's discussion of Fig. 9 is a very good explanation of the increase in the pulsation in an engine intake as the speed increases. This increase in pulsation is illustrated in Fig. 8 and is intimately associated with the important variations of volumetric efficiency of an engine. His suggestion that the manipulation of the instrument be made more nearly automatic is fully agreed to by the author.

From the physical point of view, the neglect of the "time derivative" referred to by Mr. Smith is concerned with the neglect of the compressibility of a gas, which results in Bernoulli's approximate equation. When a gas undergoes an acceleration, the displacement of any surface is made up of the motion that results from the actual acceleration of the gas plus the motion which is a result of the compression of the gas. If s represents the total displacement at any surface, ρ is the gas-mass density and t represents time, the inertia force on a unit volume is $-\rho(\partial^2 s/\partial t^2)$. This inertia force at any point x may also be expressed in terms of the deformation and the bulk modulus of elasticity E, as $E(\partial^2 s/\partial x^2)$.

The resulting force–balance equation is

$$E\left(\frac{\partial^2 s}{\partial x^2}\right) - \rho\left(\frac{\partial^2 s}{\partial t^2}\right) = 0 \quad \ldots\ldots\ldots\ldots [30]$$

Solving Equation [30] for the case of a velocity ($u_0 \sin \omega t$) drawn from one end of a uniform path, the velocity wave entering the other end at a distance l ft is

$$u_l = \frac{u_0 \sin \omega t}{\cos \sqrt{(\rho/E)}\omega l} \quad \ldots\ldots\ldots\ldots [31]$$

While the dangers of resonance in air paths were discussed in the summary of the paper, it may be worth while to give some quantitative limits. The amplitude of the entering-velocity wave will be approximately 2 per cent in error when $\cos\sqrt{(\rho/E)}\omega l = 0.98$, or when $\sqrt{(\rho/E)}\omega l = 0.2$ radian. For air at atmospheric conditions, Table 5 shows the path lengths that will result in approximately 2 per cent distortion for various frequencies.

TABLE 5

Pulsation frequency in cycles per sec	Path length in ft for 2 per cent wave distortion
1	35.6
10	3.56
100	0.356

As long as these limits are observed in the construction of equipment, there is nothing to worry about when gas compressibility is neglected.

Mr. Smith's second question about the viscous effects in small orifices and tubes is one of relative magnitudes. The pressure drop through any tube is partly viscous and partly dynamic. In very long tubes of small diameter the viscous pressure drop predominates, but in short tubes the situation is different. The orifices used in the meter are in reality short tubes, since construction limitations make it necessary to have the metal-wall thickness approximately equal to the diameter or 0.014 in. For a tube the acceleration pressure drop is $(\rho u^2)/2$, where ρ is the mass density and u is the air velocity. The viscous pressure drop is $8\mu l u/R^2$ where μ is the viscosity, l the tube length, and R the tube radius. The ratio of dynamic to viscous pressure drop is $\rho u R^2/16\mu l$ and, for a length of 0.014 in., a radius of 0.007 in. and an air velocity of 10 fps, this ratio is 2.4.

Since 10 ft a sec is the minimum velocity tested for Fig. 7 and most test work is at higher velocities, where the ratio is much greater, it is not surprising that no viscous temperature change in the calibration curve has ever been detected.

The Effect of Pulsations on Orifice Meters

By S. R. BEITLER,[1] COLUMBUS, OHIO

The paper describes three test setups made for investigating the effects of pulsation on orifice flowmeters for the measurement of gases. Arrangements are described for subjecting meters to pulsation from either the inlet or outlet side and determining the error in measurement.

The work so far is admittedly just a start toward a possible solution of the problem of metering pulsating flow, but the results show the extent and seriousness of the problem and point the way for further research which is of vital interest to the measurement industry.

THIS PAPER is written, with the hope that the restatement of one of the important unsolved metering problems will lead to a more complete study of it and, possibly, to its solution. It has been known for a long time that an orifice meter, or any other type of differential-head meter, cannot be used with accuracy if the flow is not steady but is pulsating. While this fact is known, no method of compensating for it has been developed, and the problem in general has been dodged by all of the experimenters on fluid flow.

The last published work on this subject was that of Judd and Pheley presented to the A.S.M.E. in 1922.[2] In the discussion of this paper, and in another paper[3] presented to the Society, John L. Hodgson presented a theoretical analysis of this type of flow and some suggestions for elimination of the effects which were interesting but which as yet have had little practical application. As a result most of the standard measurement instructions state that pulsations must be eliminated before a flowing fluid can be measured by a differential-head meter.

In the standards of the V.D.I.[4] the following statement is made: "The discharge equations apply only for steady flow. . . . . In particular there should be no pulsations in the flow. . . Very little investigational work has been done to determine the error due to nonsteady flow: This error therefore cannot be corrected."

The A.S.M.E. Fluid Meters committee states:[5] "Measurements of a pulsating type of flow are very likely to be in error, particularly if a head-type meter is used."

The foregoing statements show that the existence of the errors is well known and that it has been accepted as a limitation of the measurement of fluids with head-type meters. It has meant that practically no meters have been installed to measure the flow to steam engines, for instance, and their use has been limited in the measurement of the discharge of reciprocating compressors and pumps.

There are certain places, however, where measurements must be made close to reciprocating devices and in a great many cases meters have been installed, with a knowledge that, while the resultant measurements will not be exactly correct, they are better than no measurements. Such practice, however, is unsatisfactory and has resulted in a great deal of discussion of methods which might be used to eliminate the errors.

The natural-gas companies are much interested because of the fact that they must, in many cases, measure the gas close to either the outlet or inlet of compressor stations. It is important at times to make these measurements in order to detect leakage between stations and, if this leakage is a small percentage of the total flow, the measurements must be accurate. Because of the fact that some measurements were known to be slightly erroneous two of these companies decided in 1936 almost simultaneously, to make some investigations into this phase of measurement.

Two test setups were made, one by the United Fuel Gas Company at their Kenova compressor station, near Kenova, W. Va., and the other by the Ohio Fuel Gas Company at Brown compressor station near Mt. Vernon, Ohio. After some of the results from the experiments at these stations were analyzed, it was felt that some data should be secured with a simple setup, and a thesis was arranged under the direction of the author at Ohio State University. All of these test plants were constructed early in 1937 and investigations have been continued intermittently on them ever since. While no positive results as to methods of eliminating the error have been found, yet it is felt that the results show more of the nature of the problem and point the way toward a solution.

It was the intention in all of the installations first to find the magnitude of the error introduced by pulsations and then if possible, to devise some method of eliminating it. However, it has been found that there are so many permutations to the determination of the errors that it has not been possible to spend much time on elimination.

It has been shown by previous experimenters that the amount of error is dependent on the quantity of fluid being measured, the resistance and capacity between the disturbance and the meter, the frequency and the amplitude of the disturbance, and the viscosity and density of the fluid.

The three test setups gave quite a wide variation in these items and covered a broad range of conditions.

Test Apparatus

The general arrangement of all test stations was the same. That is, there were two orifice plates in series in a line between the suction and discharge headers of the compressors. These orifices were connected up with the proper gages so that they could be used as measuring devices. Arrangements were made in the test line so that it was possible to have pulsationless flow through either one or both of the meters, by setting up critical-

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[1] Assistant Professor of Mechanical Engineering, The Ohio State University. Mem. A.S.M.E. Mr. Beitler received the degree of bachelor of mechanical engineering at Ohio State in 1920 and of mechanical engineer in 1932. In 1920–1921 he served as apprentice engineer with the Whiting Corporation of Harvey, Ill. Since 1921 he has been at Ohio State University as assistant and instructor in mechanical engineering. From February, 1927, to date he has been consulting research engineer for the Bailey Meter Company, Cleveland, Ohio.

[2] "Effect of Pulsations on Flow of Gases," by Horace Judd and D. B. Pheley, Trans. A.S.M.E., vol. 44, 1922, paper no. 1869, pp. 853–918.

[3] "The Laws of Similarity for Orifice and Nozzle Flows," by J. L. Hodgson, Trans. A.S.M.E., vol. 51, part 1, 1929, paper FSP-51-42, pp. 303–332.

[4] "Standards for Discharge Measurements With Standard Nozzles and Orifices," fourth edition, 1937, V.D.I.

[5] "Fluid Meters, Their Theory and Application, Part 1," Report of the A.S.M.E. Special Research Committee on Fluid Meters, fourth edition, August, 1937.

Contributed by the A.S.M.E. Special Research Committee on Fluid Meters and presented at the Annual Meeting of The American Society of Mechanical Engineers, held in New York, N. Y., December 5–9, 1938.

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Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

flow conditions, either through a valve or through another orifice, between the source of the pulsation and the meter. With this arrangement it was possible to check the orifices against each other with steady-flow conditions, and then to make the same checks with one meter subject to pulsations and the other to steady flow. The pulsations could be obtained either on the outlet side of the orifice, i.e., from the suction of the compressor, or on the inlet side of the orifice, from the discharge of the compressor. Since the two meters were measuring the same fluid a comparison of the indicated flow determined the effect of the pulsations. The method of first checking the two orifice meters against each other on steady flow eliminated the chance of any error creeping in, due to difference in the primary and secondary elements of the meter. In calculating the flow through the meters the proper computations were made, taking account of pressure,

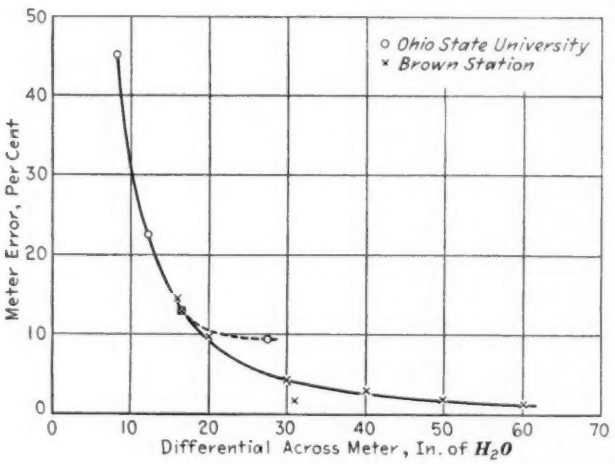


FIG. 1 ERROR CAUSED BY PULSATION ON INLET SIDE OF ORIFICE METERS
(Diameter ratio of orifices = 50 per cent.)

temperature, expansion of the gas through the orifice, and deviation of the gas from Boyle's law.

The station at Kenova had seven compressors, five 1350-hp 88-rpm twin double-acting, and two 1000-hp 125-rpm twin double-acting. The suction pressure at this station averaged about 100 lb per sq in. and the discharge pressure about 325 lb per sq in. Though the suction pressures varied from this amount considerably at times, the discharge pressure was fairly constant. The 4-in. test piping was connected directly to the discharge header and arranged so that the test flow could be discharged into either the inlet or outlet of some wet-type gas scrubbers in the suction lines of the station. The meter piping was made up according to the recommendations of report No. 2 of the Gas Measurement Committeee of the A.G.A. Orifices of from 0.25 to 2.50 in. diam were used in the tests.

The station at Brown had ten compressors, six 160-hp 190-rpm twin double-acting, three 170-hp 200-rpm twin double-acting, and one 80-hp 190-rpm single double-acting. The suction pressure was about 25 to 50 lb per sq in. and the discharge pressure about 170 lb per sq in., varying at times from 150 to 190 lb per sq in. The gas for the test line was taken from the discharge line of one of the 160-hp compressors just before it entered the station discharge header and the test line discharged directly into the suction header of the station. Meter piping and orifices were the same general design and size as used at Kenova.

In Robinson Laboratory the 3-in. test line was the direct discharge of a 12-in. and 7.5 × 9-in. two-stage double-acting compressor running at 300 rpm. The line discharged directly into the atmosphere and was arranged so that a tank with a volume of 54 cu ft or one of 3 cu ft could be placed between the compressor and the meter line. The only orifice size used up to the present was one of 1.5 in. diam.

DISCUSSION OF RESULTS

Most of the tests have been made with the pulsations on the inlet of the meter though a few have been made at Kenova with the pulsation on the outlet side. Because of this, the first part of the discussion of results is limited to meters on the discharge of the compressor, that is, to those subject to pulsations on the inlet side of the orifice plate.

Pulsations on Inlet Side. The first question which was raised was what was the total error produced by pulsations on meters of similar size at the two stations. Fig. 1 indicates this comparison. No tests were run at Kenova using a varying head on the meter and so no comparison on this basis can be made. In the case of the tests at the University and at Brown it has been necessary to make some extrapolations to determine the curves as drawn, but they seem to indicate that the errors produced by the pulsations at these two stations were about the same when

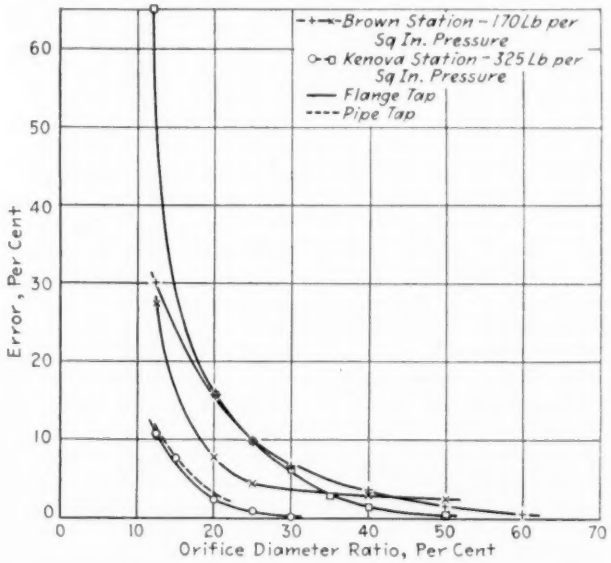


FIG. 2 ERROR CAUSED BY PULSATION ON INLET SIDE OF ORIFICE METERS
(Various values of diameter ratio at constant differential head.)

orifices of 50 per cent diameter ratio were used. It will be noticed that this error was relatively small, that is, about 1.5 per cent when the differential on the meter was 60 in. of water but that, as the differential head decreased, the error increased rapidly to about 45 per cent at an 8-in. differential head. The error of the meter was positive, that is, it made the meter read high. Some other tests which were not so complete showed the same effect. This investigation seemed to show that the effect of pulsation for any fixed condition was a function of $1/h^{1.75}$ or $1/Q^{3.5}$, where h = the head and Q = the quantity of flow. This indicates one important fact, and that is that, for a given meter, the error due to pulsation will be small if the quantity flowing is large, but that the error will be large if the flow is small; a decrease of one half in the flow will increase the percentage of error twelve times.

The next effect investigated was that of changing the diameter ratio of the orifices keeping the differential head practically constant. Fig. 2 shows some of the results of these tests. Each of the curves represents the average of the runs on several dif-

ferent orifices under approximately the same conditions. It will be seen that except for the general tendency there was no similarity of results even when the conditions seemed the same. All of the errors were different in absolute value, and the slopes of all of the curves were different. It is quite possible that this was due to the fact that different compressors were running at the time of the tests or that the valves in the discharge header were set differently so that the frequency and amplitude of the pulsations were not the same. Since these test lines were actually by-pass lines on operating stations it was impossible to control the conditions exactly and conditions for one run may have differed from another.

It will probably be necessary, before the investigation is ended, to make some tests in which the complete discharge from a compressor station is piped through a test line to determine the effect of this type of flow. However a great deal of valuable information can be secured by using the setup at the University where the complete discharge of the compressor can be sent through the test line and where conditions can be controlled within the limits of the pressures available.

The question as to the effect of frequency and amplitude of the pulsations is not answered by these tests for two reasons. The first of these is the lack of any device to indicate these values accurately and easily and the second is the fact previously mentioned, that it was impossible to control them.

Pulsations on Outlet Side. One other question which has been raised often, and which has not as yet been answered is the one

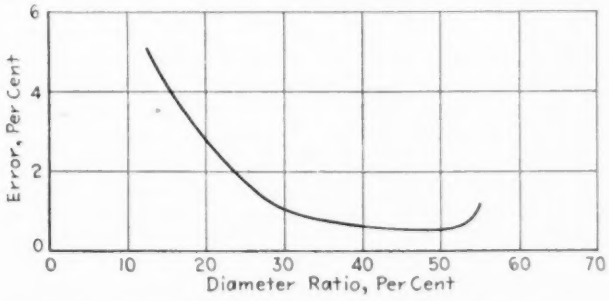


FIG. 3 ERROR CAUSED BY PULSATION ON OUTLET SIDE OF ORIFICE METERS

(Various values of diameter ratio at constant differential head.)

as to whether or not pulsations on the outlet of the meter (this is the type encountered in metering steam to a steam engine, or in metering the gas flowing to a compressor) would have the same effect as do those on the inlet. A series of tests at Kenova was made to determine the effect of suction pulsations on the meter. The results of these tests for different diameter ratios are shown in Fig. 3. It will be noticed that this curve has approximately the same shape as have those in Fig. 2 except for the fact that the end of the curve for the higher ratios seems to be turning up. This, however, is based on the results of the tests of one orifice only and does not prove that the curve has this shape.

It will be noticed that the absolute errors for this curve were much less than for those in Fig. 2. This may be accounted for by several factors. First, the nature of the pulsation was different due to the fact that the inlet valves of a compressor are open during the full suction stroke while the exhaust valves are open for only a portion of the discharge stroke, so that a much different wave form was obtained. Then the differential head on the orifice was somewhat greater during these tests, though the mass flow was about the same. Also there is always the possibility of a different condition in the line due to differences in operating conditions in the station. Some of these will be acting in all cases however, and it is predicted that the errors on the suction side of compressors will be less than on the discharge side. This line of reasoning will not apply to steam engines, however, because of the fact that the earlier cutoff of engines makes the pulsations even worse than the discharge pulsations of a compressor.

One discrepancy which appeared in this series of tests for a meter with pulsations on the outlet of the orifice plate, was the fact that the errors were positive and the dead-end reading of the differential, i.e., the reading when the inlet valve to the meter was closed so that there was no flow, was above zero. At several other stations in which observations had been made of this phenomenon the dead-end reading was below zero.

Pipe-Tap Meters. All of the tests described so far had to do with orifice meters with flange taps. Another question which was raised was whether or not the effect on pipe-tap meters would be the same. This question was answered by a series of tests the results of which are plotted in Fig. 2 as the dotted curve. It will be seen that the results of these tests were practically the same as the results plotted as curve No. 4 of Fig. 2, which were obtained at the same time and under the same conditions at Kenova. This shows that for the same mass flow and the same pulsation conditions the effect was the same on either type of tap.

Pulsation Elimination. Several elementary changes were tried to eliminate the effects of the pulsations. It is well known that resistance or capacity or both between the meter and the source of pulsation will help to eliminate the error by decreasing the pulsation. It is not practical to use too much pressure drop for most pulsation elimination because of the expense involved. Since the function of the compressor station is to produce a high pressure in the flowing gas, if much of this pressure is dissipated to get accurate measurements then the cost of measurement will be excessive. In order to determine what pressure drop would be required to completely eliminate errors due to pulsations several series of tests were run at both of the gas-measuring stations, throttling the gas flow between the compressor and the meter until the error was eliminated.

In general it was found at both stations that a pressure drop of from 2 to 14 lb per sq in. was required to eliminate the error and that the larger pressure drops were required by the smaller orifices. The results of these tests were inconclusive, however, due to the fact that they were inconsistent. In some cases the errors would be eliminated with little pressure drop and in other cases the error would never be completely eliminated, but would reach a constant value which did not decrease with increase in pressure drop. These inconsistencies were probably due to the fact that the throttling device disturbed the flow conditions to the orifice enough to create some error of measurement which the straightening vanes did not eliminate.

The effect of capacities was measured at the University and on the tests with the pulsations on the outlet of the meter at Kenova. At the University the 54-cu ft volume in the discharge line completely eliminated the pulsations, while the 3-cu ft volume eliminated most of the effect of them. When the inside of the 3-cu ft volume was treated acoustically the pulsation error was eliminated.

At Kenova tests were run with wet-type gas scrubbers between the meters and the compressors and these eliminated the error completely.

Another idea tried at the University was that of adapting the pressure taps of the secondary element to eliminate the effect of the pulsations. A large number of different types of pressure taps, with volumes, restrictions, combinations of restrictions and volumes, and acoustical treatment were tried and in all cases it was found that these devices increased the error. It would appear from these tests that the best pressure connection is the shortest and straightest one.

At the start of the tests it was felt that a good indicator to tell

when pulsations were present would be of great value and an attempt was made to design such an instrument. This consisted of a volume connected to the line in the plane of the pressure tap of the meter through a small orifice meter, which gave the dead-end flow condition mentioned previously. If pulsations were present the orifice meter would indicate a flow and this indication had some relation to the severity of the pulsations. Readings were taken on this instrument when the tests were being made for meter errors. It was found that when the readings on this instrument were below a certain value there were no errors due to pulsations. However the instrument is too sensitive to changes in line pressure, and it is very easy to blow the measuring liquid out of the manometer. An attempt is being made at present to design an electrical instrument which will not be so difficult to use and which will give a more complete picture of the pulsations.

While the results of the work done so far do not seem to have shown any definite remedy for the problem of measuring pulsating flow yet they are of great value, in the opinion of the writer, for several purposes. The first is that they show that those interested in measurement realize the problem and are working on it. They show the extent and seriousness of the problem and point the way to possible solutions which look promising.

To gain the greatest value from this work it will be necessary first to develop a device for measuring the pulsations more accurately and easily and then to continue the tests in conjunction with this device, to determine the causes of the errors and what must be done to eliminate their effects. To do this will require a great deal more experimental work, which is not the function of any one company, but which should be done by the measurement industry as a whole because all of the industry should be vitally interested in it.

Acknowledgment

This work would not have been possible were it not for the support and initiative of the two companies mentioned and the men interested in their measurement problems. A great deal of the credit for the work done by the gas industry goes to T. H. Kerr of the Ohio Fuel Gas Company and D. A. Ketcham of the United Fuel Gas Company, under whose authorization the work was done. The major part of the actual field research work, however, has been done by Edward Sackett and J. E. Overbeck and to them the most credit is due. A. M. Hutchison also cooperated. The work at the University was done by John Eagle and W. A. Daberko, and H. S. Bean has assisted with his advice.

Discussion

Howard S. Bean.[6] Probably the fact that pulsations in a fluid stream may cause considerable error in the indications of any device used for metering the rate of flow, particularly meters of the differential-pressure type, has been known about as long as fluid metering has been studied. It may seem a little strange at first, therefore, that more fundamental work on this problem has not been done. The answer to this is found when we come to consider all the possible factors and variables particularly as they occur in present-day pipe-line systems.

In all of the tests considered in Professor Beitler's paper, the effect of pulsation was to make the observed meter indication high. This is in the same direction as that generally reported by other observers. Various writers have offered mathematical explanations of the reason the indications of a meter subjected to pulsating flow are high. While it would be possible to state the essence of these explanations with few mathematical terms it would require considerable space which it seems hardly necessary to take here.

In 1931 the writer made a survey for the Measurement Committee of the American Gas Association to see how the problem of metering pulsating flows of gas was being handled by different companies in the fuel-gas industry. A report of this survey appeared in the October, 1935, issue of *Western Gas* (now published under the name *Gas*). In this report the suggestion is made, and it has been repeated many times since, that the most promising method of attacking and solving the pulsation problem would be first to build a pulsometer with which both the frequency and amplitude of the pulsation waves can be detected and measured. Then by comparing the indications of this pulsometer with the errors in the indications of orifice meters under various conditions, it would be possible to determine under what conditions the effects of pulsations may or may not be serious. Since that report was written, it has been suggested on various occasions (but originally by F. C. Morey[7]) that a suitable pulsometer can be constructed by using a piezocrystal as a pickup or pulsometer responsive unit. By introducing suitable amplification in the form of vacuum tubes the very small emf produced across the crystal by the pulsation waves can be made to operate a chronograph pen. The time element could be superimposed on such a chart, in several ways.

The writer is gratified to see from his concluding remarks, that Professor Beitler is apparently attacking the pulsation problem along these lines, although he does not give any statements about "the device it will be necessary to develop for measuring the pulsations more accurately and easily . . . . ."

C. A. Dawley.[8] Professor Beitler's paper refers to the effect of pulsations on orifice meters, by which he means fixed-orifice meters with variable head. His statement, that no method of compensating for the error produced by pulsating flow has been developed, is substantially correct for this type of meter.

In some meters of the variable-orifice, fixed-head type (Area Meters, Class 2.3, Fluid Meters Committee Report), pulsation compensators have been developed and applied to handle such severe pulsations as are met with in measuring the flow of compressed air to rock drills and similar service. Complete compensation can be effected for any one kind and degree of pulsation within reasonable limits. The same compensation will not fully compensate for some other degree of pulsation, but a meter compensated for an average degree of pulsation will have a much smaller error on some other degree of pulsation than will an uncompensated meter.

The method of compensation comprises the use of a definite restriction or damping of the motion of the meter indicator in the downward or decreasing direction, and a greater restriction, definitely related to the first, to motion in the upward or increasing direction. The same method of compensation may be applied to a water-column manometer used with a fixed orifice. It is simple in construction but so highly complex in theory as to defy mathematical analysis. The proportions must therefore be determined empirically.

The writer would define pulsating flow as flow that varies rapidly from maximum to minimum rate in recurring cycles (not necessarily identical). The common examples are the flow of fluid to a reciprocating pump, engine, compressor, or tool, or the flow of fluid from a reciprocating pump or compressor. The reasons why such flow produces error in head meters follow:

1 Head varies as the square of velocity, and average head is therefore not proportional to average velocity.

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[7] Assistant physicist, National Bureau of Standards, Washington, D. C.

[8] Chief engineer, New Jersey Meter Company, Plainfield, N. J. Mem. A.S.M.E.

2 No manometer will reliably indicate even the average head when the head is fluctuating rapidly.

3 Complications of pressure and density variations (pressure waves), set up by velocity variation, are likely to give spurious manometer readings; and

4 There may be actual reversals in the direction of flow or "alternating currents" of gas flow, as in the discharge lines of a gas compressor.

Figs. 1 and 2 of the paper show the error on the discharge or outlet flow from one or more compressors. Fig. 3 shows the error on the suction or intake flow to a battery of compressors. In Fig. 3 the errors are small for orifices with diameter ratio larger than 30 per cent. No data are given as to comparative heads and rates of flow on the various orifices, but it is likely that the greater error of the smaller orifices may be due to the comparatively small pipe-friction losses, relative to orifice heads, when the small orifices were used.

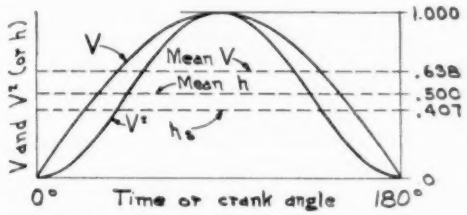


FIG. 4 THEORETICAL ERROR ON FLOW TO ONE DOUBLE-ACTING CYLINDER

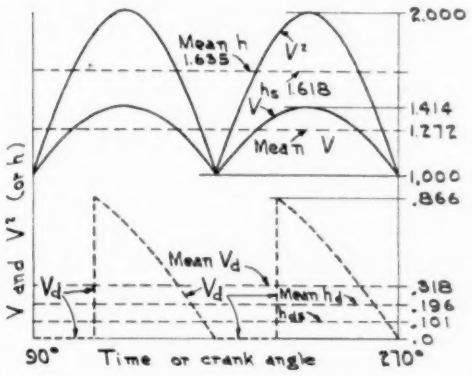


FIG. 5 THEORETICAL ERROR ON FLOW TO TWIN DOUBLE-ACTING UNIT

Most of the compressors were twin or two-cylinder units, double-acting, and presumably with cranks at 90 deg. It is interesting to compare the results reported with the theoretical error on the flow to one double-acting cylinder and to a twin unit with two such cylinders. Assuming simple harmonic motion, the piston velocity varies as the sine of the crank angle. Assuming flow in the intake pipe proportional to the piston velocity, taking the crank velocity as unity and the head produced at an orifice in the intake pipe as unity, when the piston velocity is equal to the crank velocity, we get for a single cylinder the curves shown in Fig. 4. In these curves velocity and velocity squared (or head) are plotted against time or degrees of crank angle. The mean velocity for one stroke is 0.638 and the mean head is 0.500. If the same flow took place at uniform velocity, the head h_s would be the square of 0.638 or 0.407. The ratio of the head for harmonic flow to that for steady flow is 0.500/0.407 or 1.23, and the square root of this head ratio is 1.11. In other words, a flow of this kind produces a theoretical excess indication of 11 per cent.

When two such cylinders are driven by cranks at 90 deg, the composite velocity of the two pistons at the same instant is shown by V in the upper part of Fig. 5. The value of V varies only from 1.000 to 1.414 and the value of V^2 or h from 1.00 to 2.00. This is a far closer approximation to uniform flow than that from a single cylinder. The mean velocity for 180 deg of crank travel is 1.272 and the mean head 1.635. If the same flow had taken place at uniform velocity, the head h_s would have been 1.618. The ratio 1.635/1.618 is 1.01, and the square root of this ratio 1.005. In other words the theoretical error in measuring this flow is about 0.5 per cent. This compares closely with the values shown in Fig. 3 of the paper on the larger orifices.

The discharge flow from a compressor shows much wider variations as indicated by the dotted curve V_d in the lower part of Fig. 5. The velocity curve depends on the ratio of compression The actual velocity in the outlet pipe is likely to depart more widely from the theoretical than is the case with the inlet flow. Curve V_d is based on the assumption that discharge occurs during 60 deg of crank travel or 25 per cent of stroke. For 180 deg of crank travel, the mean velocity is 0.318 and the mean velocity squared or head h_d is 0.196. If the velocity were uniform, the head h_{d_s} would be the square of 0.318 or 0.101. The head ratio for this curve as compared with steady flow is 0.196/0.101 or 1.94, and the square root of this is 1.39. In other words, this flow would give an error of 39 per cent in excess of true flow. This figure should be taken with a grain of salt as it represents an orifice directly on the compressor outlet without the benefit of any smoothing action from capacity, pipe friction, and the pressure drop of the orifice itself. It does indicate plainly that the measurement of compressor discharge is far more severe than measurement of the intake flow.

Most of the results shown in the paper are not for the flow of a single compressor, but for a by-pass from the delivery main to the suction main of a battery of compressors. It would seem that more significant conclusions could be drawn by using the entire flow of one or more compressors. The above calculations would suggest that measurement made at a little distance upstream on the intake main would show comparatively small pulsation errors. The pipe line has large capacity which, combined with the fluid friction of the line, provides good steadying action. The combination of capacity and resistance in the proper proportions is the best way of reducing pulsation.

R. E. SPRENKLE.[9] The revitalization of the pulsation-flow problem is indeed most interesting not only to the potential users of metering equipment as applied to pulsation-producing prime movers, but to the meter manufacturer as well. That a need for more information along this line still exists is quite evident when it is considered that in a great many industrial plants there are compressors, steam engines, pumps, ice machines, and other reciprocating units which are pulsation producers to which units the flow of fluids should rightfully be measured, in order to obtain best over-all plant economy. At the present time, such measurements cannot be undertaken with surety unless special precautions in the way of throttling orifices and volume enlargements are used, and in many instances such devices cannot be employed because of the impracticability of their application.

While Professor Beitler's paper draws no conclusions from his test, this should not be wondered at because the work involved has merely gotten under way and much yet remains to be done. In fact, the paper really is a progress report of the development stage in this project, more of which it is hoped will follow in the future,

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as soon as definite work and conclusions have been reached regarding any phase of this important subject.

It is interesting to note among several of the elementary things tried out that the enlargement, restriction, or the use of additional volume, or combination of such changes in the connecting piping between the primary and the secondary, serves not to reduce the effect of pulsation in so far as the meter error is concerned, but rather to augment such meter error. This is borne out by our own experience in that we have found throttling of the connecting piping between the metering orifice and the meter register does not dampen pulsation but only tends to give a false picture of the actual conditions.

There are several preliminary results which appear on the surface to be the reverse of expectations. One of these is a decrease of meter error with increasing differential, and another that smaller amounts of throttling were required for larger-diameter-ratio orifices. It is further noted, particularly with regard to the second example just stated that the results were somewhat inconsistent and therefore no final conclusions could be made.

It is hoped that this work can continue to a point where definite and correct conclusions can be drawn from the test work. Further, and this is the most important, the real answer is the accumulation of sufficient data which will enable one to prescribe definite means for the elimination of pulsations from the metering system so that the flow through the primary element will be of the normally turbulent type unaffected by pulsations. If and when such knowledge becomes available, industry will be well served by this research.

Author's Closure

Mr. Bean has suggested the use of piezoelectrical devices for the measuring of the pulsations. A device of this type has been developed and is being tried at this time. Considerable data have been taken with it but, as yet, the results have not been completely analyzed nor has the practicability been proved. However, the results to date look very promising.

Mr. Dawley has spent considerable time calculating the errors due to the pulsations and has arrived at some interesting conclusions. However, it is the author's opinion that Mr. Dawley has not taken into account the fact that the rapid fluctuation of static pressure has a greater effect on the error due to pulsation than has the change in flow. Professor Judd in his 1922 paper seems to have proved this conclusively. The author's results to date seem to indicate the same thing. For instance, Mr. Dawley has calculated that the error on a meter in the suction of a two-cylinder, double-acting compressor with the cranks 90 deg apart would be about 0.5 per cent.

The author in December, 1938, investigated a small compressing station having one two-cylinder, double-acting compressor with the following results: A meter placed about 50 ft from the compressor suction read about 50 per cent higher than did one placed about 5000 ft from the compressor discharge. This error was cut to less than 10 per cent by throttling between the compressor and the suction meter, so that at least 40 per cent of the difference in these meter readings was due to pulsation on the suction meter. The fact that the error is so much greater than the calculated error seems to indicate that it will be impossible to calculate it, even approximately, until there is a more complete knowledge of the causes of the error.

As pointed out by Mr. Sprenkle and in the paper, the attack on this problem is just started and there is yet an enormous amount of work to be done. It is hoped that the statements made will stir up more interest and lead to a more rapid solution of the problem.

The Effect of Size and Shape of Cut Upon the Performance of Cutting Fluids When Turning S.A.E. 3140 Steel[1]

By O. W. BOSTON,[2] W. W. GILBERT,[3] and L. V. COLWELL,[4] ANN ARBOR, MICH.

This paper gives the results of detailed experiments to determine the influence of each of several classes of cutting fluids on tool life when turning a steel with different sizes of cut. The tests were divided into three series: In the first, two cuts were used having the same area, in one case the depth was equal to eight times the feed, and in the other the depth was equal to twice the feed.

In the second series of tests the feed was kept constant while the depth was varied over a considerable range to give cuts of different area. All cutting fluids were tested for each combination of depth and feed.

In the third series of tests the depth was kept constant while the feed was varied to give a series of cuts of different area.

Lastly, all data are correlated to show the influence of each type of cutting fluid as a function of the size and shape of cut, as a function of the cutting speed for a given tool life, and in terms of the volume of metal removed per tool grind.

THE OBJECT of these tests was to determine the influence of several types of cutting fluids on tool life and cutting speed when varying the size and proportion of the cut in turning annealed S.A.E. 3140 steel, with a constant tool of high-speed steel.

It has been shown[5] that various types of cutting fluids materially affect the value of the cutting speed for a given tool life when turning an annealed S.A.E. 2345 steel and an annealed S.A.E. 3140 steel.

The Tests

There were three series of tool-life cutting-speed tests involved in this investigation. In the first series, each of the cutting fluids was used on deep thin and shallow thick cuts having the same area. In the second, the depth of cut was varied while the feed was kept constant, and in the third, the feed was varied while the depth was kept constant.

For each test, the tool life was measured with a stop watch and checked by noting the increase in power on the wattmeter chart. The tool life consisted of the time required from the start of the cut until the tool failed. Total failure was evidenced by the wiping off of the cutting edge of the tool. The edge broke off and produced a burnished band on the shoulder of the cut. At this time there was a decided increase in the power consumed as indicated by the wattmeter. In some cases, a preliminary failure was noticed, at which time a burnished streak would appear on the shoulder of the cut. This indicated the breaking down of a part of the cutting edge, but it did not change the quality of the machined surface, and did not cause an appreciable increase in power consumption, so that all tools were allowed to run until total failure occurred.

Cutting Tools. The cutting tools used consisted of $^3/_8$-in. square tool bits of the 18-4-1 type of high-speed steel. These bits were ground and used in a special tool holder which had a back slope of 10.5 deg. The tools themselves had working angles as follows: 8-deg back rake, 22-deg side rake, 6-deg end and side clearance, 6-deg end-cutting-edge angle, 15-deg side-cutting-edge angle, and $^3/_{64}$-in. nose radius, as used on monel metal.[6]

Equipment. The equipment used in the tests has been described previously.[5,6]

Constant Area of Cut. In the first series of tests, the area of cut was held constant at 0.00125 sq in., while the ratio of depth to feed was varied. In one set of tests, the depth was equal to eight times the feed; (depth 0.100 in. and feed 0.0125 in. per revolution). In the second set of tests, the depth was equal to twice the feed; (depth 0.050 in. and feed 0.025 in. per revolution).

The cutting-speed tool-life curves are shown in Fig. 1 as straight lines on log-log paper for each of the cutting fluids and for each of the two cuts. These results show again that the line for each cutting fluid for the deep cut with the light feed is higher than that line for the same cutting fluid obtained with the shallow cut and heavy feed. That is, a higher cutting speed for a given tool life is obtained when the ratio of depth of cut to feed is high, with other factors remaining constant. The cutting speed for a 40-min tool life V_{40}, for dry cutting, is 68 fpm when the depth is twice the feed, but is 83 fpm, or 22 per cent higher, when the depth is eight times the feed, for the same area of cut. Similarly, V_{40} for the highest curve of each group is 83 fpm for the lower group, and 101 fpm, or 22 per cent higher for the upper group. The cutting-speed tool-life lines for both cuts are higher when an oil or emulsion is used, and in both cases these curves are higher and are more nearly horizontal than are those curves for dry cutting.

In the upper group of curves, in which the depth was eight times the feed, it is shown that dry cutting gives the shortest

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[1] Report No. 8 of the Subcommittee on Cutting Fluids of the A.S.M.E. Special Research Committee on the Cutting of Metals.

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[3] Assistant Professor, College of Engineering, University of Michigan. Jun. A.S.M.E.

[4] Instructor in Metal Processing, College of Engineering, University of Michigan.

[5] "The Influence of Cutting Fluids on Tool Life in Turning Steel," by O. W. Boston, W. W. Gilbert, and C. E. Kraus, Trans. A.S.M.E., vol. 58, July, 1936, paper RP-58-11, pp. 371–378.

Contributed by the A.S.M.E. Special Research Committee on Cutting of Metals and presented at the Annual Meeting of The American Society of Mechanical Engineers held in New York, N. Y., December 5–9, 1938.

Discussion of this paper closed January 10, 1939, and is published herewith directly following the paper.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[6] "A Study of Cutting Fluids Applied to the Turning of Monel Metal," by O. W. Boston and W. W. Gilbert, Trans. A.S.M.E., vol. 58, Nov., 1936, paper MSP-58-10, pp. 685–689.

tool life for any given cutting speed, or the lowest cutting speed for a given tool life. For high speeds, that is, values of tool life up to approximately 30 min, the sulphurized-base oil *SB* gives the highest cutting speed for any tool life, but inasmuch as it has a steeper slope than the lines for the other cutting fluids, it crosses the curves for the three emulsions and falls below them for values of tool life in excess of 50 min. The 1:50 emulsions of

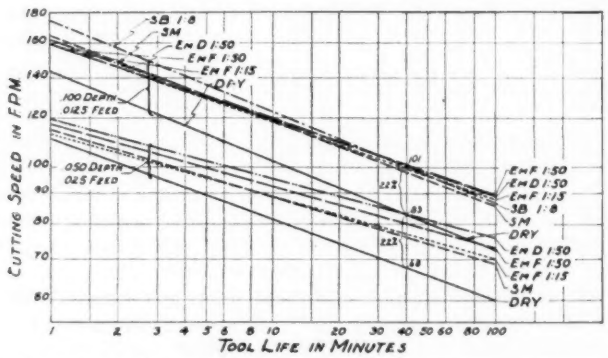


FIG. 1 THE INFLUENCE OF CUTTING FLUIDS ON CUTTING SPEED

(Turning annealed S.A.E. 3140 steel with two proportions of cut. Shape of high-speed-steel tool, 8-22-6-6-6-15-3/64. For symbols of cutting fluids see Table 1.)

soluble oils *D* and *F* give the highest cutting speeds for values of tool life above 30 min. The relative cutting speeds for all cutting fluids may be represented by the cutting speed for a 1-min tool life, which is equal to the value of the constant C in the equation $VT^n = C$, which is given in Table 3 for each oil for the cut of 0.100-in. depth and 0.0125-in. feed.

TABLE 1 DESIGNATION OF CUTTING FLUIDS USED

Symbol	
Dry	dry cutting
EmD 1:50	1 part soluble oil *D* to 50 parts water
EmF 1:15	1 part soluble oil *F* to 15 parts water
EmF 1:50	1 part soluble oil *F* to 50 parts water
SM	a sulphurized mineral oil (No. 110, formerly No. 10), having a viscosity of 110 sec Saybolt at 100 F and containing 3.3 per cent active sulphur
SB	a sulphurized lard-mineral oil, having 1 part of high-sulphur-base oil blended with 8 parts of a light mineral oil, giving a final viscosity of 110 sec Saybolt at 100 F

Soluble oils *F* and *D* represented two commercial soluble oils

TABLE 2 PROPERTIES OF MATERIAL CUT

(Test log 13 in. diam, 5 ft long, S.A.E. 3140 steel forging)

Analysis, per cent			
Nickel	3.43	Silicon	0.29
Manganese	0.74	Sulphur	0.029
Carbon	0.40	Phosphorus	0.028
Chromium	0.33		
Heat-treatment, normalized and annealed			
Heated to 1600 F, held 8 hr, air cooled			
Reheated to 1440 F, held 8 hr, air cooled			
Reheated to 1260 F, held 8 hr, furnace cooled			
Physical Properties			
Tensile strength	110,000 psi		
Yield point	60,000 psi		
Elongation	19 per cent in 2 in.		
Reduction of area	41.57 per cent		
Shore hardness	27 to 30		

TABLE 3 EQUATIONS OF UPPER GROUP OF CURVES OF FIG. 1

SB	$VT^{0.152} = 175$ — (87.0 fpm)
SM	$VT^{0.142} = 165$ — (86.0 fpm)
EmD 1:50	$VT^{0.130} = 162$ — (89.0 fpm)
EmF 1:15	$VT^{0.130} = 160$ — (88.0 fpm)
EmF 1:50	$VT^{0.125} = 159$ — (89.5 fpm)
Dry	$VT^{0.147} = 143$ — (72.5 fpm)

The values in parentheses represent the cutting speed for each oil for a tool life of 100 min, for which case the emulsion *EmF* 1:50 gives the highest value, and dry cutting the lowest. Equations for all of these curves are summarized in Table 5.

For the lower group of curves in Fig. 1, when the ratio of depth to feed is two, for which the depth is 0.050 in. and the feed 0.025 in. per revolution, several differences from the results noted above are observed. Emulsion *EmD* 1:50 produces higher speeds than those for *EmF* 1:50. Otherwise the relative values of speed produced by the several cutting fluids remain in the same order, although they are spread over a greater range. For a tool life of 1 min the cutting speed is increased from 111 for dry cutting to a maximum of 119 for *EmD* 1:50 or 7 per cent. This emulsion produces cutting speeds higher than those obtained with any of the other cutting fluids for all values of tool life. For a tool life of 100 min the cutting speed for this emulsion is 26.6 per cent higher than that for dry cutting. Emulsions, in general, are superior to the sulphurized mineral oils, except for values of tool life below 8 min. The thin emulsion, *EmF* 1:50 gives speeds higher than those for the thicker emulsion *EmF* 1:15. This seems to indicate that cooling is of greater value than lubrication. The equations for each line for the shallow thick cuts, showing the constant which equals the cutting speed for a 1-min tool life, as well as the cutting speed for a 100-min tool life, in parentheses, are given in Table 4.

TABLE 4 EQUATIONS OF LOWER GROUP OF CURVES OF FIG. 1

EmD 1:50	$VT^{0.097} = 119$ — (76.0 fpm)
EmF 1:50	$VT^{0.105} = 117.5$ — (72.5 fpm)
EmF 1:15	$VT^{0.104} = 113$ — (70.0 fpm)
SM	$VT^{0.113} = 115$ — (68.5 fpm)
Dry	$VT^{0.133} = 111$ — (60.0 fpm)

TABLE 5 SUMMARY

(A summary of experimental results as represented by cutting-speed tool-life equations when turning S.A.E. 3140 steel annealed with high-speed-steel tools of the form 8-22-6-6-6-15-3/64, with various cutting fluids, at different combinations of feed and depth. The number of cubic inches of material removed per grind for a 100-min tool life is shown also for the various cuts and cutting fluids.

$VT^n = C$, in which V = cutting speed in fpm and T = tool life in minutes)

Cutting fluid	Depth of cut, in.	Feed, in. per revolution	n	C, cutting speed for 1-min tool life	100-min tool life: Cutting speed	100-min tool life: Cu in. per grind
Dry	0.100	0.0125	0.147	143.0	72.5	108.8
EmF 1:50	0.100	0.0125	0.125	159.0	89.5	134.2
EmF 1:15	0.100	0.0125	0.130	160.0	88.0	132.0
EmD 1:50	0.100	0.0125	0.130	162.0	89.0	133.5
SM	0.100	0.0125	0.142	165.0	86.0	129.0
SB	0.100	0.0125	0.152	175.0	87.0	130.5
Dry	0.050	0.025	0.133	111.0	60.0	90.0
EmF 1:50	0.050	0.025	0.105	117.5	72.5	108.8
EmF 1:15	0.050	0.025	0.104	113.0	70.0	105.0
EmD 1:50	0.050	0.025	0.097	119.0	76.0	114.0
SM	0.050	0.025	0.113	115.0	68.5	102.8
Dry	0.0125	0.0125	**0.125**	313.0	176.0	**33.0**
	0.035	0.0125	0.111	200.0	120.0	63.0
	0.100	0.0125	0.147	143.0	72.5	108.8
	0.200	0.0125	0.110	117.5	71.0	213.0
EmF 1:50	0.0125	0.0125	0.115	330.0	194.0	36.4
	0.035	0.0125	0.113	212.0	126.0	66.1
	0.100	0.0125	0.125	159.0	89.5	134.2
	0.200	0.0125	0.107	150.0	91.5	274.0
SB 1:8	0.0125	0.0125	0.127	346.0	193.0	36.2
	0.035	0.0125	0.130	226.0	124.0	65.1
	0.100	0.0125	0.152	175.0	87.0	130.5
	0.200	0.0125	0.133	152.0	82.5	247.5
Dry	0.200	0.0125	0.110	117.5	71.0	213.0
	0.200	0.025	0.128	74.0	41.0	246.0
	0.200	0.050	0.140	46.7	24.5	288.0
EmF 1:50	0.200	0.0125	0.107	150.0	91.5	274.0
	0.200	0.025	**0.097**	90.0	57.5	**345.0**
SB 1:8	0.200	0.0125	0.132	152.0	82.5	247.5
	0.200	0.025	0.103	90.0	55.8	335.0

Variable Depth at Constant Feed. In the second series of tests, the depth of cut was varied from 0.0125 in. to 0.200 in., while the feed remained constant at 0.0125 in. per revolution. These tests were run dry, with the emulsion *EmF* 1:50, and the sulphurized-base oil *SB*. See Figs. 2–8, inclusive, and Figs. 17, 18.

The experimental data of the variable-depth tests are shown plotted on log-log paper in Fig. 2. For each depth of cut with a constant feed of 0.0125 in., cutting-speed tool-life curves were obtained with three cutting fluids, namely dry, *EmF* 1:50, and *SB*. The three lines at the top of the figure represent the cutting-speed tool-life relationship for each of the three cutting fluids

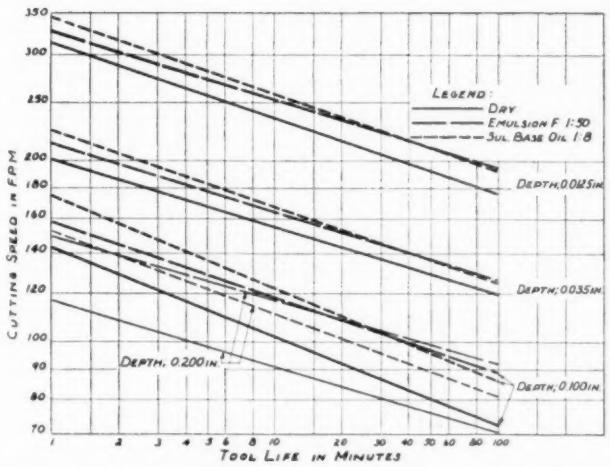


FIG. 2 A SUMMARY OF EXPERIMENTAL DATA SHOWING THE RELATION BETWEEN CUTTING SPEED AND TOOL LIFE

(Turning S.A.E. 3140 annealed steel; constant feed of 0.0125 in.; various-depth cuts. Shape of high-speed-steel tools, 8-22-6-6-6-15-3/64. Three cutting fluids used for each depth of cut.)

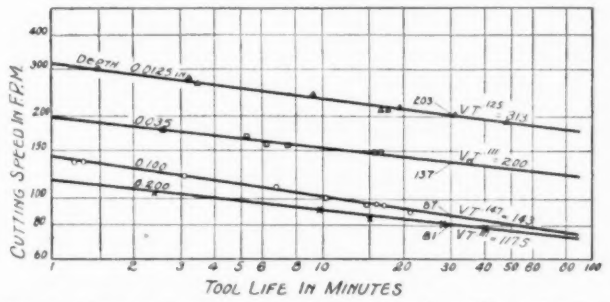


FIG. 3 LINES OF CUTTING SPEED VERSUS TOOL LIFE WHEN CUTTING DRY

(Four depths of cut; constant feed of 0.0125 in. From Fig. 2.)

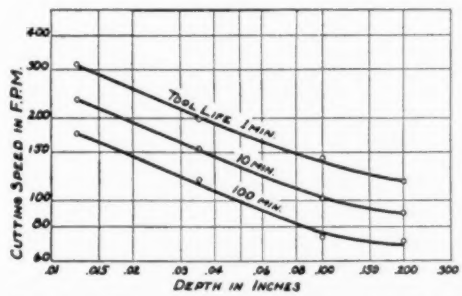


FIG. 4 RELATION BETWEEN CUTTING SPEED AND DEPTH OF CUT

(Three values of tool life when cutting dry. From Fig. 2.)

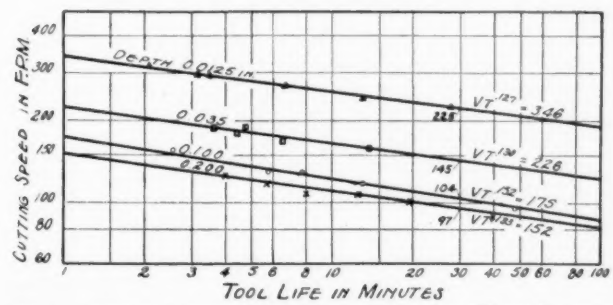


FIG. 5 LINES OF CUTTING SPEED VERSUS TOOL LIFE WHEN CUTTING WITH *SB* OIL

(Four depths of cut; constant feed of 0.0125 in. From Fig. 2.)

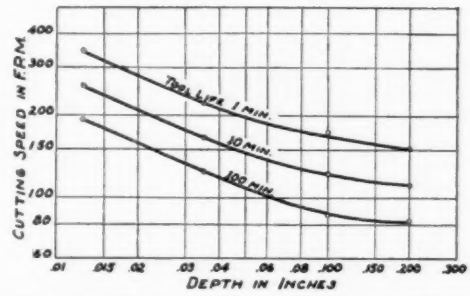


FIG. 6 RELATION BETWEEN CUTTING SPEED AND DEPTH OF CUT

(Three values of tool life when cutting with *SB* oil. From Fig. 2.)

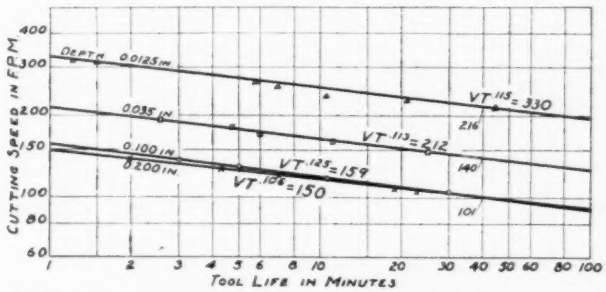


FIG. 7 CUTTING SPEED VERSUS TOOL LIFE WHEN CUTTING WITH *EmF* 1:50

(Four different depths of cut; constant feed of 0.0125 in. From Fig. 2.)

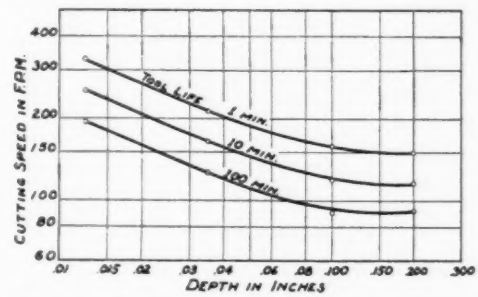


FIG. 8 RELATION BETWEEN CUTTING SPEED AND DEPTH OF CUT

(Three values of tool life when cutting with *EmF* 1:50. From Fig. 2.)

when the depth of cut was 0.0125 in. Dry cutting, as represented by the solid line, is lowest. For values of tool life below 40 min, the sulphurized-base-oil line is higher than that for the *EmF* 1:50 emulsion. These two lines cross, however, so that for values of tool life above 40 min, the emulsion gives higher cutting speed for any tool life. Similar results are observed by the intermediate set of curves for the depth of 0.035 in. Again, the sulphurized-base oil *SB* is highest for values of tool life up to 40 min, at which point it crosses the curve for the emulsion. The lines for the 0.100-in. and 0.200-in. depths of cut are shown grouped at the bottom of the figure. The lines for the 0.100-in. depth are slightly higher than the respective lines for the 0.200-in. depth. The short-dash lines for the sulphurized-base oil are highest for the shortest tool life for each depth. They are steeper, however, than the lines for the emulsions, and so for tool-life values in excess of 30 min, the emulsion gives the highest cutting speed for any value of tool life. It is interesting to observe that the curve for the emulsion for the 0.200-in. depth is even higher than that for the emulsion for the 0.100-in. depth of cut when the tool life is in excess of 30 min. The tool-life lines for dry cutting for the two greatest depths appear to cross at a tool life of 200 min. At this point the cutting speed would be the same for both cuts.

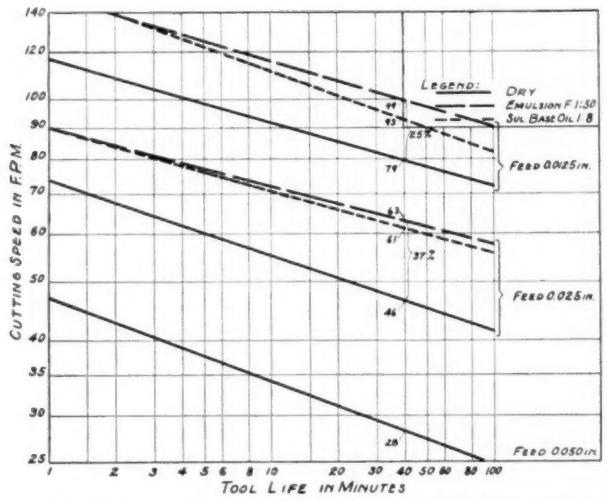


FIG. 9 SHOWING THE RELATION BETWEEN CUTTING SPEED AND TOOL LIFE

(Turning S.A.E. 3140 annealed steel; feed varied for a constant depth of cut of 0.200 in.; three cutting fluids. Shape of high-speed-steel tools, 8-22-6-6-6-15-3/64.)

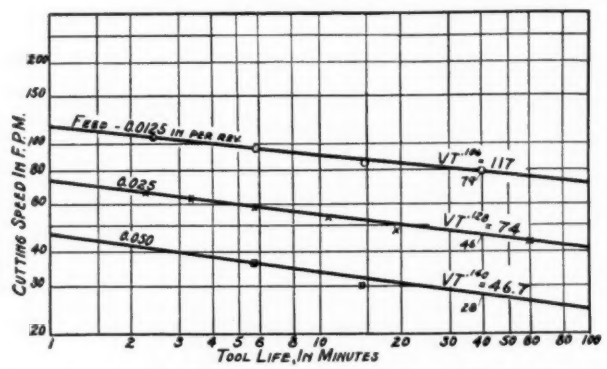


FIG. 10 CUTTING SPEED VERSUS TOOL LIFE WHEN CUTTING DRY

(Three feeds; constant depth of cut of 0.200 in. From Fig. 9.)

When the depth of cut is increased, with the feed remaining constant, the length of cutting edge in contact with the material being cut is increased. If the cutting tool had no nose radius and a straight cutting edge, the length of cutting edge would increase directly with the depth of cut, and the unit linear loading of the tool cutting edge would not be increased. This would lead to the assumption that, if the cutting tool could absorb all the heat generated in each small unit of length of cutting edge, the depth of cut would not influence the cutting speed for a given tool life. The heat developed in the tool is concentrated at the nose of the tool. When the radius of the nose is increased, the length of cutting edge is increased and the average chip thickness is decreased, thereby increasing the allowable cutting speed as the heat is dissipated over a greater length of cutting edge. These assumptions indicate that a maximum-possible depth of cut should be used. This has several practical limitations. There is a decided tendency for deep thin cuts to produce excessive chatter, which may make their use objectionable. Also, when turning small-diameter stock and using deep cuts, the extreme condition will be reached, in which the point of a tool will be at the center of the rotating work and the back of the cutting edge will be at the outside diameter of the work, so that the cutting speed will vary along the cutting edge and be a maximum at the outside diameter. Failure would occur probably at the point of this outside diameter, due to the highest cutting speed.

The advantage of varying the depth of cut is shown in Fig. 3, when cutting dry, where the depth is varied from 0.0125 in. to 0.200 in., with the feed remaining constant at 0.0125 in. per revolution. The cutting speed for a 30-min. tool life V_{30} is reduced as the depth is increased in multiples of the lightest depth, as follows:

$$1f = 200 \text{ fpm}, 3f = 140, 8f = 86, \text{ and } 16f = 80 \text{ fpm}$$

When the depth of cut exceeds 0.100 in., its influence on tool life is less, and if the two lowest lines were extended to tool-life values greater than 100 min, tools cutting at a depth of 0.200 in. would have a longer life than those cutting at 0.100 in. for the same cutting speed. This indicates that, when the depth of cut exceeds eight times the feed, a further increase in depth will not reduce the tool life to any great extent.

The cutting speed in feet per minute is plotted as ordinates against depth of cut in inches as abscissas on log-log paper in Fig. 4. There is a uniform decrease in cutting speed for each constant tool life of 1, 10, and 100 min, until the depth of cut reaches 0.100 in. For larger depths, the lines tend to flatten out, indicating that the increased depth beyond 0.100 in. has little effect upon the cutting speed and, therefore, is a desirable proportion of cut.

When using the sulphurized-base oil *SB* and varying the depth of cut, Fig. 5 shows that the greater depths produce the most advantageous cutting condition, in that the cutting speed is reduced at a slower rate than the depth is increased. The results were similar to those obtained with dry cutting except that the addition of the cutting fluid tended to make the increased depths even more desirable. In Fig. 6, the cutting speed is plotted against depth of cut for tool-life values of 1, 10, and 100 min. These curves tend to be concave upward and more horizontal as the depth of cut exceeds 0.035 in. The curve shown in Fig. 6 for a 1-min tool life did not flatten out as much as the curve for a

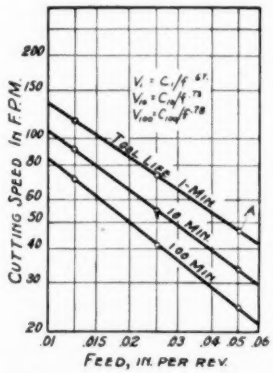


FIG. 11 RELATION BETWEEN CUTTING SPEED AND FEED WHEN CUTTING DRY

(Three values of tool life; depth of cut of 0.200 in. From Fig. 9.)

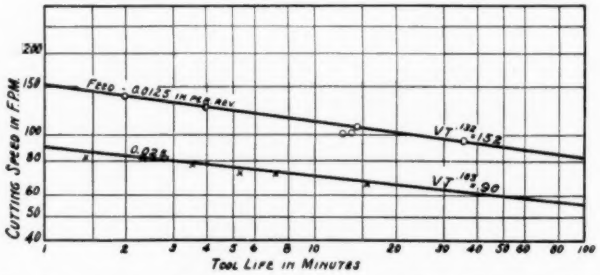


FIG. 12 CUTTING SPEED VERSUS TOOL LIFE WHEN CUTTING WITH *SB* OIL

(Three feeds; constant depth of cut of 0.200 in. From Fig. 9.)

100-min tool life. For the latter, the tool and the cutting fluid had the capacity to carry away the heat generated, regardless of the additional depth of cut, after the depth had exceeded 0.100 in.

The use of the *SB* oil allowed higher cutting speeds than dry cutting for all depths of cut, but it was most effective when taking depths of cut of 0.100 in. and greater. For the shallow depth of cut of 0.0125 in., the cutting speed for a 100-min tool life was increased 9 per cent by changing from dry to the *SB* oil. With a large depth of 0.200 in., the corresponding increase in cutting speed was 15.7 per cent, which indicates the advantage of deep cuts when using a cutting fluid.

Values of tool life for various depths of cut when using the emulsion *EmF* 1:50 are shown in Figs. 7 and 8. For short values of tool life, the *SB* oil produces higher allowable cutting speeds than the emulsion *F*. However, for tool-life values of 10 min and more, and particularly for the large depths of cut of 0.200 in., the emulsion gives higher cutting speeds.

The ability of the emulsion to carry heat from the cutting tool is demonstrated in Fig. 8. After the depth of cut exceeds 0.100 in., there is practically no decrease in cutting speed for any given tool life.

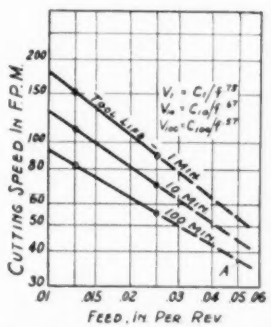


FIG. 13 RELATION BETWEEN CUTTING SPEED AND FEED WHEN CUTTING WITH *SB* OIL

(Depth of cut, 0.200 in. From Fig. 9.)

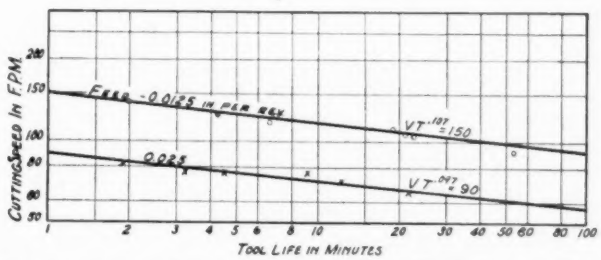


FIG. 14 CUTTING SPEED VERSUS TOOL LIFE WHEN CUTTING WITH *EmF* 1:50

(Three feeds; constant depth of cut of 0.200 in. From Fig. 9.)

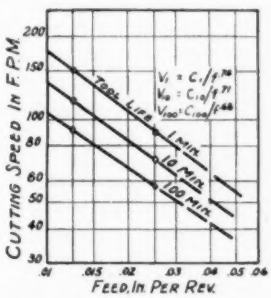


FIG. 15 RELATION BETWEEN CUTTING SPEED AND FEED WHEN CUTTING WITH *EmF* 1:50

(Depth of cut, 0.200 in. From Fig. 9.)

These curves indicate that, on the basis of highest cutting speed for a given tool life, emulsions should be used for the deep cuts and long values of tool life, whereas the sulphurized-base oil *SB* should be used for the smaller depths and shorter tool-life values.

Variable Feed at Constant Depth. In the third series of tests,

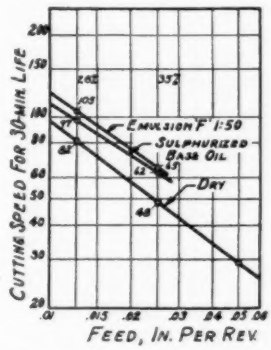


FIG. 16 SUMMARY CURVES SHOWING RELATION BETWEEN V_{30} AND FEED

(Constant depth of cut of 0.200 in.; three cutting fluids; turning S.A.E. 3140 annealed steel; form of high-speed-steel tools, 8-22-6-6-6-15-3/64.)

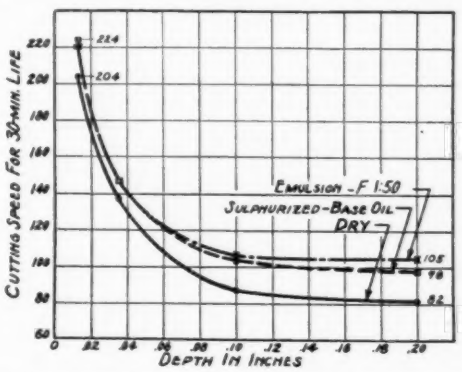


FIG. 17 SUMMARY CURVES SHOWING THE RELATION OF V_{30} AND DEPTH OF CUT

(Constant feed of 0.0125 in.; three cutting fluids; cutting S.A.E. 3140 annealed steel; form of high-speed-steel tools, 8-22-6-6-6-15-3/64.)

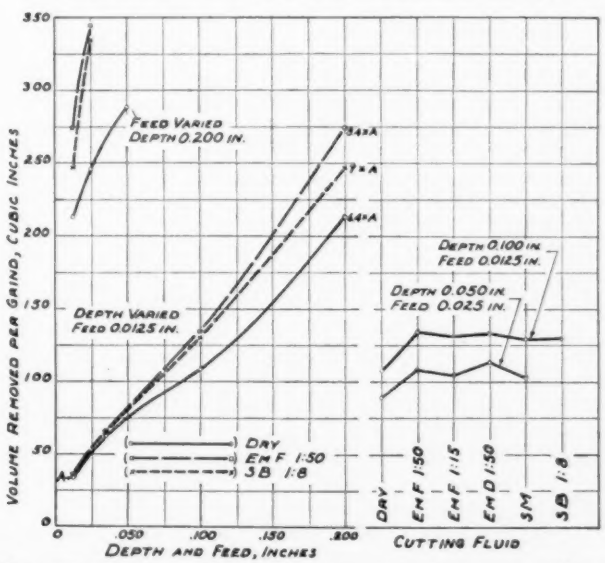


FIG. 18 SUMMARY CURVES SHOWING THE VOLUME OF METAL REMOVED PER TOOL LIFE OF 100 MIN FOR DIFFERENT COMBINATIONS OF FEED AND DEPTH FOR VARIOUS CUTTING FLUIDS

the feed was varied from 0.0125 in. to 0.050 in. per revolution, while the depth of cut was kept constant at 0.200 in. Three cutting fluids, dry, *EmF* 1:50, and *SM* were used. See Figs. 9 to 15, inclusive, and Fig. 18.

This depth of cut of 0.200 in. was selected as, in previous tests in which depth was varied, it was found that cutting fluids are more effective with large depths of cut than with small depths of cut. The results of the experimental data of these tests are represented as straight lines on log-log paper in Fig. 9. The tool-life cutting-speed line is shown for each cutting fluid for the light feed of 0.0125 in. at the top. As usual, the line for dry cutting is lowest, giving a V_{40} of 79 fpm. The line for *SB* is higher but of a steeper slope and has a V_{40} of 93 fpm. The line for the emulsion *EmF* 1:50 is highest, producing a V_{40} of about 99 fpm. The increase in cutting speed of the emulsion over dry cutting is 25 per cent. The three lines for the feed of 0.025 in. are somewhat lower, producing a cutting speed V_{40} of 46 fpm for dry cutting, 61 fpm for the *SB* oil, and 63 fpm for the *EmF* 1:50 emulsion. The increase in cutting speed of the emulsion over dry cutting for this feed is 37 per cent. Only the line for dry cutting is shown for the maximum feed of 0.050 in., for which V_{40} is 28 fpm.

The tool-life cutting-speed lines for the emulsion *EmF* 1:50 do not have such a steep slope as those for the *SB* oil. This indicates that, for longer tool-life values, the emulsion would be superior to the *SB* oil for these sizes of cut.

The lines for dry cutting of Fig. 9 are shown alone in Fig. 10 for each feed of 0.0125, 0.025, and 0.050 in. per revolution. Increasing the rate of feed lowers considerably the tool-life cutting-speed line. The slope of the line, as represented by n in the equation $VT^n = C$, is increased from 0.106 for the light cut to 0.140 for the heaviest cut. The values of C, which correspond with the cutting speeds for a 1-min tool life, are decreased from 117 to 46.7, or 60 per cent, while the feed is being increased by 300 per cent. The cutting speed V_{40} for a 40-min tool life is decreased from 79 fpm to 28 fpm as the feed is increased from 0.0125 in. to 0.050 in. V_{40} is increased from 28 to 46 fpm, or 64 per cent, as the feed is reduced from 0.050 in., to 0.025 in., or 50 per cent. It is increased from 46 to 79 fpm, or 46 per cent, as the feed is reduced further from 0.025 in. to 0.0125 in., or 50 per cent. It is increased from 28 to 79 fpm, or 182 per cent, as the feed is reduced from 0.050 in. to 0.0125 in., or 75 per cent, or an increase of feed to four times, reduces the speed to $^1/_{2.5}$ of that for the light feed.

It was difficult to obtain the exact time of total failure of the tools when the feed was 0.050 in. because a preliminary failure would often be obtained early in the run, but the tool still would cut, getting progressively duller, until the operator's judgment set the time of total failure. The preliminary failures introduced excessive loads on the tools, causing them to be broken off, particularly when using the oil or emulsion.

The relation between cutting speed and variable feed for dry cutting is shown on log-log coordinates in Fig. 11, for three values of tool life, namely, 1, 10, and 100 min. Straight lines are formed, the slopes of which increase slightly as the tool life is increased. These slopes, as represented by the exponent of the feed in the equation $V = C/f^z$, are shown on the figure. Since the tangent of the slope of these curves, that is, the exponent of the variable feed, is less than one, the most efficient cutting speed to remove this material is obtained for the large values of feed. It is seen that the exponents of feed are 0.67 for a 1-min tool life, 0.73 for a 10-min tool life, and 0.78 for a 100-min tool life. The 100-min tool life is the steepest. From this, it follows that a lower exponent is obtained for a lower value of tool life. Therefore, for dry cutting, the most efficient metal removal would be obtained for the largest values of feed when operating at relatively high speeds or short tool life. In Fig. 11, the most efficient metal removal theoretically will be obtained at the point A. For practical reasons, the tool life should be longer than 1 min. The exact time must be correlated with the time for tool replacement and so on.

When using the *SB* oil and varying the rate of feed, it was not possible to use a feed of 0.050 in. and a depth of 0.200 in. because of excessive loads on the tool after the preliminary failure. These loads would cause the $^3/_8$-in. square tools to break off, thereby interrupting the tests. The tool-life cutting-speed lines for feeds of 0.0125 and 0.025 in. per revolution are shown on log-log coordinates in Fig. 12. The slope of these lines is less for the heavier feed. This is just the opposite from dry cutting, Fig. 10, and indicates that the use of the *SB* oil would be more desirable for the heavier cuts at a long tool life. For a tool life of 1 min, the cutting speed is reduced from 152 to 90 fpm, or 41 per cent, by increasing the feed from 0.0125 in. to 0.025 in., whereas for a tool life of 100 min, the same change in feed reduces the cutting speed from 82 fpm to 56 fpm, or 31.5 per cent.

Discussion

M. KRONENBERG.[7] There has been much discussion as to the question when a tool is to be considered dull. Ripper and Burley measured the wear on the face of the tool and considered a blunting of $^1/_{16}$ in. as a criterion. Schlesinger, on the other hand, found as a scientific criterion the sudden rise in the two horizontal components of the cutting pressure, which is not always accompanied by an increase in the main (tangential) component of the cutting force. This behavior may be explained by the fact that the heat which is generated by friction between both tool face and chip and tool flank and work is mainly responsible for the wear of the tool. If the friction due to dulling increases, the horizontal components of the pressure increase too before the vertical component increases. The authors noticed that the breaking down of a part of the cutting edge did not change the quality of the machined surface and did not cause an appreciable increase in power consumption. Thus they allowed the tools to run until total failure which was indicated by the increase of power consumption. It does not seem to be advantageous to run tests until total failure due to the fact that in practice a total breakdown is uneconomical on account of the high cost for regrinding. A rapid change in the horizontal components usually reveals the instant of dulling; viz., between a partial breakdown of the edge and the total failure, provided that a time-recording device is used for measuring the horizontal cutting components.

The writer agrees with the statement that an increase in cutting speed by 22 per cent is possible in the case where the shape of chip cross section is changed from 1:2 to 1:8. Fig. 19 shows the same results for S.A.E. 1035 for various areas of chip and a tool life of 60 minutes. It will be seen however later on that for practical purposes this increase is of minor importance as to the most efficient metal removal on a lathe.

The finding that a shape factor of 1:8 for the area of chip is a desirable proportion of cut agrees with actual practice, where the ratio of feed to depth of cut (= shape factor) varies approximately between the limits of 1:2 and 1:20 and usually only between 1:5 and 1:10.

Formulas for the cutting speed V, in which the feed and depth of cut are given separately are usually presented in the form:

$$V = Cd^{-x} f^{-y}$$

[7] Research Department, Cincinnati Milling Machine Company, Cincinnati, O.

which can be transformed into

$$V = \frac{C}{A^{1/2(y+x)}\ S^{1/2(y-x)}}$$

where

A = Area of cut = $f \cdot d$

S = Shape factor = $\frac{f}{d}$

f = Feed per revolution

d = Depth of cut

x,y = exponents of d and f, respectively

C = Material constant for a definite tool life

This transformation indicates very clearly that the area A of the chip is of considerably greater effect on the cutting speed than the shape S, since the exponent of A is composed of the sum of y and x, while the exponent of S derives from the difference $y - x$.

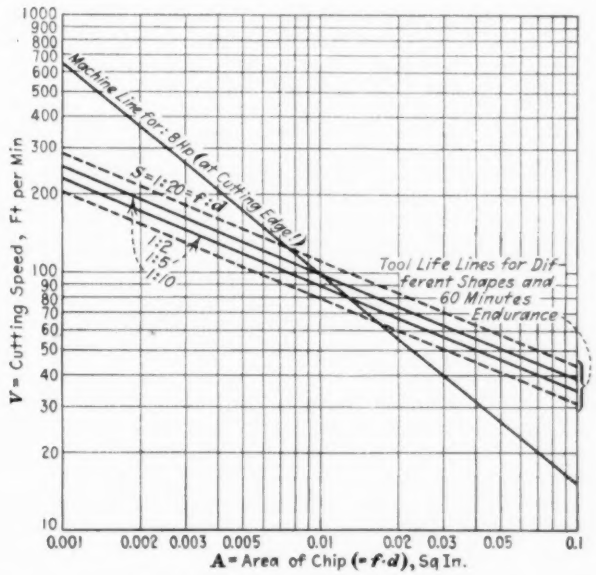


FIG. 19 CUTTING SPEEDS FOR VARIOUS AREAS OF CHIP; EFFECT OF SHAPE OF CHIP CROSS SECTION

(Example for high-speed-steel tool, [18-4-2]; S.A.E. 1035; 20-deg true rake angle; cutting fluid used. f = feed rate, in. per rev; d = depth of cut, in.; $S = f:d$ = shape of chip cross section; $A = f \times d$ = area of chip cross section.)

If as an example, the shape of chip cross section is changed in the ratio 1:4 (viz., from 1:2 to 1:8) then the cutting speed may be increased by 23 per cent, while a change of 1:4 in chip cross section (viz., from 0.004 sq in. to 0.001 sq in.) permits an increase of cutting speed by 270 per cent for the same tool life.

As to the exponent n of the cutting-speed–tool-life relationship, no definite connection with either the chip cross section, the shape factor, or the cutting fluid can be seen from the paper. It is known that the values for n vary approximately between 0.1 and 0.17 and usually between 0.12 and 0.16. The ideal condition would be an exponent $n = 0$, meaning that the tool life would be infinite; since this is out of the question the aim must be to reduce n as much as possible by cooling fluids, etc. On the other hand, care must be taken not to lay too much importance on the change of n within the limits mentioned above. A value of n of 0.12 as against 0.16 results in an increase of the cutting speed (all other conditions unchanged) of approximately only 21 per cent, while a change in the chip cross section has considerably more effect as will be shown. Furthermore the accurate determination of n is difficult. Fig. 3 of the paper indicates this. An exponent $n = 0.111$ occurs twice although no reason is to be seen why this value should be true only for $d = 0.035$ and $d = 0.200$, while the n values of the other lines are different. In the writer's opinion the four exponents shown could be averaged and the lines be made parallel. This averaging would hardly be noticeable on the diagram as it calls only for a minute turning of the lines through 1 deg or less. Thus the value of n averaged from Table 5 of the paper would be 0.127 for dry cutting as against 0.11 for *EmF* 1:50. This indicates a definite improvement, which becomes still more evident if the V_{100} values on this basis are taken into consideration and compared for the same chip cross section and shape. Then it will be seen that the improvement in cutting speed due to *EmF* 1:50 ranges between 15 per cent (for small chip cross sections) and 39 per cent for greater ones.

The most effective combination of the various variables involved in cutting with single-point tools will be explained by

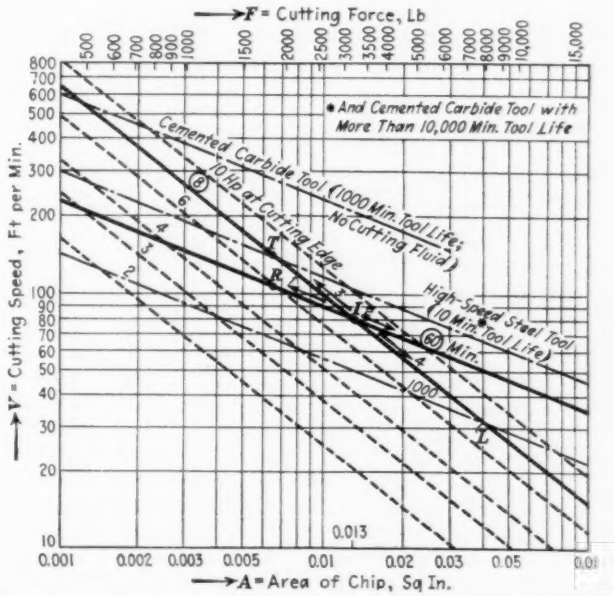


FIG. 20 DETERMINATION OF MOST EFFECTIVE COMBINATION OF CUTTING VARIABLES

(Example for high-speed-steel tool, [18-4-2]; S.A.E. 1035; 20-deg true rake angle; cutting fluid used.)

Fig. 20.[8] It will be seen that two different relationships exist between cutting speed V and chip cross section A as indicated by the lines with smaller slope (lines of constant tool life) and with greater slope respectively (lines of constant horsepower). The lines intersect, for example, in point I where a high-speed-steel tool of 60-min endurance and a machine of 8 horsepower are assumed. Increasing the cutting speed (along arrow 1) by decreasing the chip cross section in such a manner that the kind of tool and tool life is kept constant is not advisable, due to the fact that the productivity is decreased. (The combination at point I gives 12.5 cu in. per min metal removal at point R only 8.2 cubic inches per minute).

Following the tool line in direction of arrow 2 results in an increase in the chip section and a decrease in speed for a constant tool and tool life. Here the 10-horsepower line is approached, indicating an overloading of the machine.

[8] "Science and Practice in Metal Cutting" by Dr. E. Bickel, *American Machinist*, October 1934, page 693. This article is based mainly on investigations made by the writer.

Further combinations of the variables are possible by following the machine line (along arrow 3). Here we increase the cutting speed and decrease area A, but we leave the tool line of 60-min endurance. Thus the tool must be changed and a cemented carbide tool be taken. The production at point T would be 11.2 cu in. per min and the pressure 1800 lb (see scale at top). In spite of the diminution of chip volume per minute, it is often desirable to use small chip cross sections and high speeds due to the decrease of cutting force and consequently less deflection and higher accuracy (no built-up edge).

This will obviously be at some sacrifice in productivity but still considerably more advantageous than in the case of following arrow 1 (tool-life line) where the chip volume was reduced to 8.2 cu in. per min.

Following the machine line in the direction of arrow 4 leads for example to point L, where the productivity is 15.3 cu in. per min and the tool life 1000 min. Thus it seems that the use of a low cutting speed in combination with a big chip results in both a better endurance and a higher productivity. However, it must be borne in mind that the cutting force increases to 8000 lb in this example. This procedure (along arrow 4) is only advisable if roughing cuts are to be taken on stable work pieces.

In conclusion it must be said, that the selection of and changes in the cutting speed should be made in accordance with the machine line (arrows 3 and 4, respectively) and the tool should be adapted to the respective speeds. The machine line is often still unknown, although it is the best guide as to the most efficient metal removal.

Authors' Closure

Dr. Kronenberg has presented charts, the purpose of which is so to generalize the information as to make it applicable for shop use. This was not the primary object of the paper. The authors have emphasized cutting fluids particularly, which are not considered in Dr. Kronenberg's discussion.

The authors, on many occasions, have correlated cutting forces with tool wear and tool failure. They have found and have published data[9] in which the three components of the cutting force are shown to be maximum when a newly ground tool starts to cut, are gradually reduced as the tool is cupped on the face, but are increased perceptibly as soon as the newly formed cutting edge between the cup and the flank of the tool is broken off. The occurrence at this instant has been called preliminary failure by the authors, as the tool will run an additional length of time, ranging up to 60 or 70 per cent, before complete tool breakdown occurs.

By modifying the side-cutting-edge angle and nose radius for a given size of cut, preliminary failure may occur. It will be evident by a burnished ring on the shoulder of the metal being cut but in no way will it affect the size or roughness of the machined surface. The cutting-speed–tool-life line for the preliminary failure, as well as for the ultimate failure, has been determined. Sometimes the line for the preliminary failure plotted on log-log paper is parallel to, but just below, that for the ultimate failure. More often, however, this line has a slope greater than that for the total-failure line. This point is discussed to show that, by determining the time of total failure, using the increase-in-force method, a time of tool failure differing from that of the ultimate tool failure is obtained, which has no consistent relation to the latter in different sets of tests.

When high-speed-steel tools fail in machining cast iron, failure occurs by abrasion on the flank. For some tool shapes this wear is below the side cutting edge. It is gradual in its development. The cutting force will start to increase and become maximum at the time of complete tool failure, at which time there is the first evidence of change in diameter and surface quality on the machined surface. When the wear occurs under the end cutting edge, the diameter of the work gradually increases and it is hardly possible to determine a definite time of tool failure. It would be difficult to obtain consistent values of a certain definite amount of wear by the force method when turning cast iron for these reasons.

Many experimenters have tried to correlate the time of tool failure when taking light cuts with the cutting force. No satisfactory data were published until French and Digges replaced the cutting-force idea with their trailer tool. The authors have used the trailer tool, but have found, even for the 0.0125-in. depth × 0.0125-in. feed in cutting steel, that there is a very definite time of tool failure, as observed by the sudden burnishing of the work shoulder, at which time the diameter and quality of the machined surface is first affected.

The authors believe that the chip-shape factor is of more importance than indicated by Dr. Kronenberg. It is believed that, in the development of curves for guidance in general machining, this chip-shape factor should provide for ratios of feed to depth in excess of 1:5 and 1:10. This is indicated in Fig. 18. It is shown in Figs. 4, 6, and 8 that the chip-shape factor has a greater effect for ratios larger than 1:8. For high ratios, such as 1:16, the formula expressing a relation between V and d no longer holds, as shown by Fig. 3. This would mean that, for large ratios, the depth factor S would become of equal effectiveness to that of area A, as expressed in Dr. Kronenberg's second equation, inasmuch as the value of x becomes zero.

Dr. Kronenberg points out that, as the shape of the chip cross-sectional area is changed from 1:2 to 1:8 (keeping the area of cut constant), the cutting speed is increased 23 per cent, as shown in Fig. 1. He further states that a change in chip cross-sectional area from 0.004 to 0.001 sq in. (giving one quarter the area of cut) permits an increase in cutting speed of 270 per cent for the same tool life. This latter example actually shows a loss in production by reducing the size of cut.

Dr. Kronenberg's Figs. 19 and 20 are very interesting and valuable from a practical viewpoint. It is unfortunate that the material, tools, and cuts are so different from those of the paper. The authors have no way of comparing their data with those of Dr. Kronenberg, shown in Fig. 19, inasmuch as the materials are different and the areas of cut hardly overlap. In Fig. 20 Dr. Kronenberg has used an S.A.E.-1035 steel, presumably annealed, which in general calls for a machinability rating of about 68 per cent, as compared with that of 100 per cent for the S.A.E.-1112 cold-finished screw stock. The machinability rating of the S.A.E.-2340 steel annealed as generally accepted, and used by the authors, is around 56 per cent or only 12 per cent below that of the S.A.E.-1035 steel.

By referring to Fig. 2, the cutting speed for a 60-min tool life for an area of cut of 0.0125 in. by 0.100 in. (equals 0.00125 sq in.) is shown to be about 95 fpm for the emulsion. Fig. 20 shows, for the S.A.E.-1035 steel, that the cutting speed for a 60-min tool life for the same area of cut is about 210 fpm, which is 120 per cent higher than that for the S.A.E.-2340 steel. Dr. Kronenberg used 18–4–2–type high-speed steel, which might increase the cutting speed by 10 per cent over the 18–4–1 type of steel, but even so his values do not seem to agree favorably with those of the steel used in the paper.

The authors greatly appreciate Dr. Kronenberg's contribution and the many oral questions and discussions which brought out numerous interesting phases of the research and added greatly to its practical understanding.

[9] "Metals Handbook," American Society for Metals, Cleveland, Ohio, 1936 edition, p. 752.

Thin Oil Films

By WALTER CLAYPOOLE,[1] BEACON, N. Y.

A method is described for depositing very thin lubricating-oil films of any specified thickness of the order of 10^{-7} in. on polished metal in connection with experiments on boundary friction. The oil to be deposited is converted into a highly dispersed fog consisting largely of positively charged particles. The oil is readily deposited on a negatively charged metal surface at a rate depending on the nature of the oil, the potential gradient at the metal surface, and the concentration of the oil particles in the fog. The rate of deposition is best determined by observation of the progressive change in the interference color produced over a thickness range between 5×10^{-6} cm and 30×10^{-6} cm. Comparison of color with a standard stepped film of barium stearate laid down by the Langmuir and Blodgett technique gives a high degree of accuracy to the determination of film thickness.

Experiments have been made on the wearability of very thin oil films, and on the determination of the coefficient of static friction. An instrument is described which has been found especially suitable for this work. The author points out that a low value of the coefficient of static friction is a measure of the oiliness value of an oil-metal combination.

THE ENGINEER familiar with the principles of the hydrodynamic theory of lubrication has a relatively simple problem when called upon to design a bearing which will operate satisfactorily under specified performance conditions. His interest is solely concerned with the behavior of the bearing when running under a specified load at a specified speed. He thinks only in terms of full fluid lubrication and sees to it that provision is made for supplying the rubbing surfaces of the bearing at all times with oil of a suitable viscosity. Under these circumstances he is able to state with considerable confidence that the coefficient of running friction of his bearing will be a number preceded by one or two zeros. Although he has theoretical reasons for believing that a correctly designed and properly operated bearing should run indefinitely without visible wear, he is not optimistic enough to expect it to do so in actual operation.

Wear in Bearings Under Normal Running Conditions. A very brief inquiry into the underlying causes for bearing wear will show that although their normal running operation is under fluid-film conditions, the static load when the machine is shut down may reduce the oil film to extreme thinness. On starting, there is a delay in the formation of the full fluid film, and it is during this period that wear of the rubbing surfaces takes place. This wear, however, is not necessarily of a serious type, since no matter how severe the squeezing-out action, the opposed surfaces still carry an adsorbed layer which is at least one molecule thick, and as long as this layer remains attached to the metal no clean contact can take place. An examination of the rubbed areas after long-continued operation will show that the wear has been in the nature of a polishing process, the surface of the metal exhibiting a mirror-smooth patina. Submicroscopic projections have been worn down and submicroscopic valleys have been filled in. This process of running-in combines the agencies of actual wear and surface flow. The wear, however, is to be regarded merely as the result of a slow removal of minute irregularities projecting above the general level, and the total amount of actual metal lost may be very small indeed. Surface flow is generally recognized as the phenomenon responsible for the formation of the desirable Beilby layer.

After the rubbing surfaces have been thoroughly run-in, the rate of wear in normal operation falls to a negligible figure and the question then arises as to the causes for abnormal wear, the visible evidence for which is scuffing and abrasion. If we leave out of the picture such accidental agencies as the presence of abrasive material, there remains only breakdown of the oil film between the opposed rubbing surfaces as the blanket cause for bearing failure. However, oil-film rupture is not likely to occur in a well run-in and properly operated bearing, but when it once starts, the damage to the surfaces is of a very serious nature. As long as the opposed metal parts are at all times separated by at least two molecular layers of oil there can be no wear of the type known as abrasion or scuffing. If the mechanical attack at the contact areas of metals rubbing together becomes severe enough to remove the ultimate protecting film, the clean metals will seize, causing irreparable damage to the surfaces. This seizure is in fact, incipient welding, resulting in the gross removal of metal. Polish wear, on the other hand, is due to the smooth removal of surface molecules, and occurs when the separating oil film has an average thickness less than the average height of surface irregularities. Polish wear is beneficial and is the process made use of in a running-in operation. It is obvious that the better the surface finish, the thinner may be the separating film.

The ideal case, as indicated previously, would be two perfect surfaces separated by a double molecular layer of lubricant. Since perfect surfaces are unattainable, we have to recognize the fact that the working film between rubbing surfaces must be considerably thicker than this. The rise in the coefficient of friction on passing from fluid-film to boundary-film conditions is quite rapid, which means a rapid increase in the rate of dissipation of energy at the rub spots. Unless prompt steps are taken to re-establish fluid-film conditions, the ultimate protecting film adsorbed on the surfaces is likely to be rubbed off, thus bringing clean metal into contact.

What Is a Thin Oil Film? Since it is obvious that the term "thin" is very elastic, it is necessary to set limits for its dimensional value before we can investigate the properties and mechanical behavior of lubricating films of this type. The logical

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Discussion of this paper closed January 10, 1939, and is published herewith directly following the paper.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

question to ask is: How thin must an oil film be in order that the component oil molecules lose their mobility? The reason for this question lies in the fact that we propose to investigate the protective action of a thin film subjected to severe mechanical punishment. Above a certain minimum thickness we may expect oil molecules immediately adjacent to the high-spot areas, where the squeezing-out action is a maximum, to flow in as soon as the excess pressure is relieved. Below this minimum thickness the protecting film is rubbed off faster than it can be replaced by flow.

Evidence presented later, shown graphically in Figs. 6 and 7, indicates that under favorable circumstances a considerable measure of protection is afforded by films thinner than one millionth of an inch. In this investigation a thin film will be regarded as one having a thickness less than 1/1,000,000 in.

Production of Oil Films of Specified Thickness

It is obvious that if quantitative measurements of thin films be desired, we must be able at will to produce films of uniform and known thickness. Preliminary experiments indicated that the range of thickness desired lay between 10^{-7} and 10^{-6} in., and it was essential that the specified thickness should be obtained with a fair degree of accuracy.

The simple technique described by Langmuir and Blodgett[2] for the production of known thicknesses of metal stearates by picking up successive layers from a compressed monolayer on the surface of water, was unavailable, since it is not possible to produce such a layer of a lubricating oil on a water surface.

Attempts were made to obtain films of specified thickness by driving off a volatile solvent from a weak solution of the oil applied to the metal surface. It is obviously quite a simple matter to leave behind a specified quantity of oil after evaporation of the solvent, but it is quite difficult to devise a method which insures that the deposit shall be uniform. Several other methods were experimented with, but were found wanting in one respect or another.

Oil-Fog Method

The method finally adopted was to deposit the film from a fog produced by fine atomization of oil. Such a fog appears to consist largely, if not entirely, of positively charged particles. Its mechanical behavior may conveniently be observed by feeding it with minimum disturbance into the upper end of a tall, wide glass cylinder. The larger droplets will soon begin to fall out, leaving a dense cloud as a residue which very slowly moves down the cylinder under gravity. A flat metal plate, carefully cleaned and then suspended in a vertical plane in the fog, will remain clean, but if supported at an angle to the direction of motion of the slowly falling fog particles, will acquire a very thin film of oil. The usual water-flushing test serves very well to demonstrate the presence or absence of oil on the metal plate. Gravity deposition of thin oil films from fog offers little possibility, however, for accurate quantitative work, but when advantage is taken of the fact that the oil particles are positively charged, and will therefore readily deposit on a negatively charged metal surface, it is possible to devise a suitable method by which films of specified thickness may be laid down. Preliminary experiments showed that the rate of deposition from fog was different for every type of oil. The addition of a small percentage of a highly polar body to a straight mineral oil, for instance, made the rate of deposition at least ten times greater. It was also found that the rate varied with the density of the fog, with its rate of flow past the deposition surface, and with the potential gradient in the field.

The problem therefore resolves itself into a formulation of experimental details whereby a thin and uniform film of a particular oil can be laid down on a metal surface, together with a method for estimating accurately the thickness of the film. We require a film of a specified thickness, and we have to be able to prove definitely that the thickness is what we intended it to be. Two obvious methods available are as follows:

1 Determination of Film Thickness by a Weighing Method. A metal surface of known area is exposed to standard deposition conditions in the fog, and the gain of weight carefully determined over a series of measured time intervals. The weight gains are plotted against exposure times, which gives the rate of deposition for the particular oil used. Knowing the surface area of the deposition plate, and the density of the oil, we can immediately calculate the thickness of the film laid down per minute of exposure time.

Fig. 1 shows the results obtained by this method for a series of four similar straight mineral oils having different viscosities and of somewhat different chemical types. It will be seen that the rates are linear over the range covered. As an example of the practical application of this method, we see that oil *A* shows a weight gain of 0.8 mg in a total exposure time of 4 min.

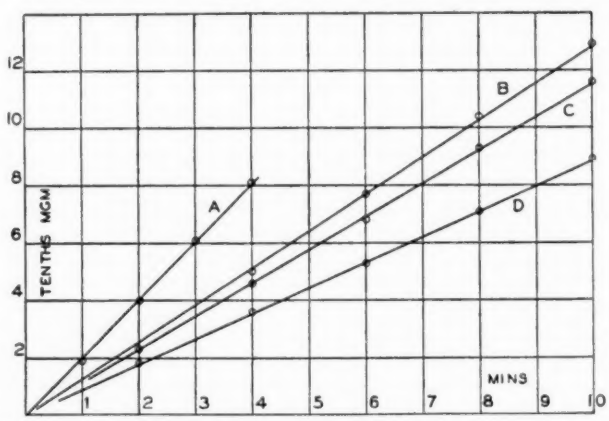


Fig. 1 Varying Rates of Electrostatic Deposition of Four Different Oils Determined by the Weighing Method

This oil had a specific gravity of 0.8735 and the area of the deposition surface was 187.1 sq cm. The thickness T in cm of the film deposited in 4 min is therefore

$$T = \frac{0.8 \times 10^{-3}}{187.1 \times 0.8735} = 4.9 \times 10^{-6}$$

If we assume that the linear relation holds below the lowest determined point, it would appear possible to lay down films, by this method, of thickness approaching molecular dimensions.

2 Interference-Color Method. If we apply to a flat piece of polished chromium plate a continuous film of oil of uniform thickness lying in the range between 500 and say 4000 Angstroms (or 5 to 40 millionths of 1 cm), we have a very accurate method of estimating film thickness by observation of the interference color developed.

In the practical application of this method to the determination of an oil film deposited from fog, very special precautions have to be taken in cleaning the deposition plate. The nature of the difficulties may best be understood by following the work of conditioning the surface. Consider that we have a well-polished

[2] "Monomolecular Films of Fatty Acids on Glass," by K. B. Blodgett, *Journal of the American Chemical Society*, vol. 56, Feb., 1934, p. 405. Also: "Built-Up Films of Barium Stearate and Their Optical Properties," by K. B. Blodgett and I. Langmuir, *Physical Review*, vol. 51, June 1, 1937, p. 964. See also references cited under footnote 4.

chromium plate to be used for the first time. Naturally, it carries an adsorbed greasy film, as shown by the fact that it will shed water. This film can easily be removed by scouring with a paste of fine optical rouge and water, or with a metallographic powder such as Shamva,[3] and water.

Now comes the real difficulty—that of removing the last trace of the polishing powder. It might be thought that scrubbing the surface with a series of absorbent cotton pads under running water would be sufficient, but very surprisingly it is found that a surface finished in this way will not serve as a base on which to lay down a continuous film from oil fog. The deposited film looks hazy, and, of course, no interference colors are seen. The probable explanation is that the surface is still covered with myriads of submicroscopic particles of the polishing powder, since it is known that such fine particles are held very strongly on a clean surface. The use of cotton pads was therefore abandoned in favor of a more vigorous scrubbing medium. The problem was finally solved by the use of rubber gloves, freed from oil by dipping in cleaning solution, followed by thorough washing in water.

The metal plate, held in one gloved hand, is scrubbed with the gloved fingers of the other under hot running water. After a final flushing with distilled water to insure that the surface when dry does not carry traces of salts from the tap water, the plate is put on edge to drain and dry naturally. It is now found that an oil film in the proper thickness range, deposited on the plate prepared in this way, shows brilliant and uniform interference colors.

The statement as to uniformity of color requires modification in the event that surface defects or contamination, however slight, are still present on the plate. The effect of scratches visible to the naked eye is especially pronounced, each scratch being traced out by a color line corresponding to a film thickness greater than that of the surrounding area. For instance, even before the main film has reached the intensity peak of the first order, yellow (thickness about 20×10^{-6} cm), a fine scratch may show as a red trace, corresponding to a film thickness of about 25×10^{-6} cm.

Another defect sometimes observed in the uniformity of the color shows itself as a mosaic pattern of two colors adjacent in the scale, and is believed to be due to some unrecognized surface condition. To sum up the first experimental condition for the deposition of a homogeneous and uniform film of oil from fog, it is necessary to use an extremely smooth and scrupulously clean metal surface.

Method of Preparing a Stepped Color Film. The first operation in the calibration of an oil for rate of deposition by the interference-color method is to prepare a standard stepped color gage by the Langmuir and Blodgett technique.[3] Table 1 gives values computed from data given by Blodgett.[4]

A stepped oil film to match exactly the standard color gage may be prepared in the following manner: A sheet of thin celluloid in which a rectangular hole has been cut, is laid on the highly polished surface of a chromium-plated disk which has been cleaned by the method described. Separated from the deposition surface by a gap of $^1/_4$ in. is another metal plate serving as the second electrode. Fog is flowed through the gap and a standard operating voltage applied for measured times. At intervals the plate is inspected for color. When the first color is brought to a perfect match with yellow-brown on the standard series, a glass slide is moved into position so as to protect a strip about $^1/_2$ in. wide from further deposition. The exposure is then continued on the rest of the plate until the next color, dark blue, is brought to a perfect match, after which the glass slide is moved up as before. In a similar manner the rest of the stepped film is built up.

TABLE 1 THICKNESS AND COLOR OF MULTILAYER STEARATE FILMS[a]

Number of layers	Thickness, cm $\times 10^{-6}$	Interference color
21	5.1	Yellow-brown
41	10.0	Dark blue
61	14.9	Light blue
81	19.8	Yellow
101	24.6	Red
121	29.5	Blue
141	34.4	Green
161	39.3	Red-yellow

[a] Based on 24.4 Angstroms for the length of a barium-stearate molecule.

A graph of film thickness against exposure time is then plotted, as shown in Fig. 2, which gives the results obtained with four oils of the same general type, but of widely different viscosities. Straight lines were drawn from the origin through the determined points, which have been omitted from the diagram for the sake of clarity. It will be noted that these four oils have very similar rates of deposition. In the thickness range in which we are interested when making wearability tests of very thin films, that is, of the order of a few millionths of a millimeter, the required exposure time would be substantially the same for each of these oils.

Assuming, as seems justified, that the linear relation observed throughout the color range continues for films thinner than

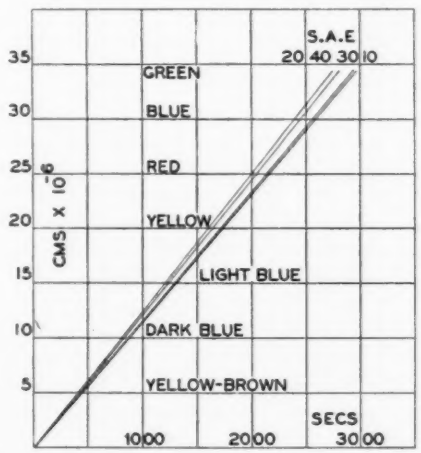


FIG. 2 RATES OF ELECTROSTATIC DEPOSITION OF FOUR SIMILAR OILS DETERMINED BY THE COLOR-INTERFERENCE METHOD

5×10^{-6} cm, the calculation of exposure time required for any specified thickness of the order of a few molecular layers is a simple matter. Suppose we find that 2500-sec exposure under standard conditions is required to match the blue at 30×10^{-6} cm, the exposure for a film three millionths of one millimeter thick will be 25 sec.

The color-interference method is subject to slight error since the refractive index of the stearate used in building up the multilayer stepped film used as a color standard differs from the index of the lubricating oil used to build up the experimental boundary film. Fortunately, the difference is too small to warrant applying a correction factor.

[3] Obtainable from Golwynne Magnesite and Magnesia Corporation, New York, N. Y.

[4] "Films Built by Depositing Successive Monomolecular Layers on a Solid Surface," by K. B. Blodgett, *Journal of the American Chemical Society*, vol. 57, June, 1935, p. 1007. Also: "Properties of Built-Up Films of Barium Stearate," by K. B. Blodgett, *Journal of Physical Chemistry*, vol. 41, October, 1937, p. 975.

Experiments reported by Langmuir and Schaefer[5] on the conditioning of surfaces show that a barium-stearate multilayer surface may be so conditioned as to be strongly oleophilic. Langmuir, Schaefer, and Wrinch have reported[6] further that increments of film thickness of the order of 3×10^{-8} cm produce a noticeable change of color when viewed with polarized light reflected at large angles of incidence from the surface of a multilayer film of the order of 1000 Angstroms in thickness. These observations suggest the possibility of direct measurement of the thickness of extremely thin oil films electrostatically deposited on multilayer stearate films conditioned in the manner described.

Some Experimental Details

1 The Oil Fog. The ideal oil fog would consist of particles alike in size and of dimensions probably of the order of 2 microns in diameter. The actual fog as delivered at the nozzle of the atomizer consists of droplets of varying diameter, as inferred by noting the behavior under gravity settling in a tall glass cylinder. In order to obtain a working fog consisting of small-diameter droplets covering not too wide a range in actual dimensions, the

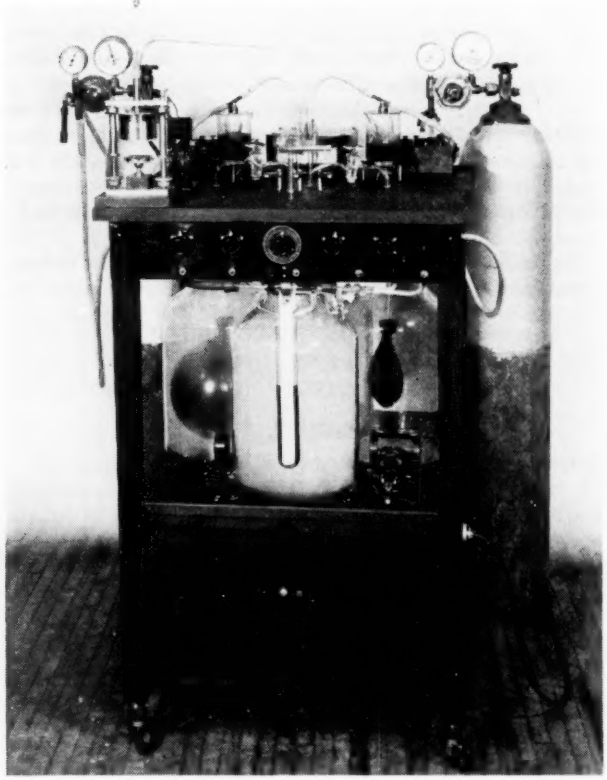


Fig. 3 Apparatus for Production of Oil Fog and for Electrostatic Deposition of Thin Films

fog is first fed into a large settling bottle in series with the working bottle. This arrangement allows the larger droplets to settle out, leaving in the upper part of the second bottle a fairly uniform fog to draw from.

In order to avoid oxidation of the finely divided oil droplets in the fog, an inert gas such as tank nitrogen is used both for atomizing the oil and for flushing out the system. Oils of differing viscosities are brought to the same viscosity while being atomized by immersing the atomizer bulb in a bath maintained at a predetermined temperature derived from the A.S.T.M. viscosity-temperature chart.

Concentration or fog density must be maintained constant. This is easily accomplished by photoelectric means. At opposite sides of the working bottle are arranged a Weston Photronic cell and a 6-volt, 32-candlepower bulb in a reflector. With an

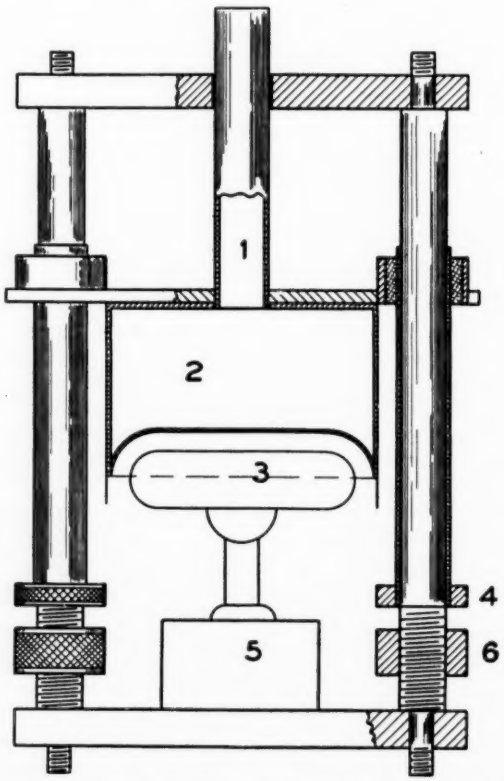


Fig. 4 Details of Deposition Electrode for Thin Oil Films

empty bottle the light is adjusted by rheostat to give full-scale reading on a microammeter. The concentration of the fog is then built up until the reading falls to a convenient, arbitrary figure, such as 40 per cent of the empty-bottle value.

In actual deposition, the fog should flow past the active-metal surface at a constant rate, in order to maintain a constant supply of charged particles in the immediate vicinity of the surface. It was found that 25 cc per sec was a suitable rate of flow, and this was maintained in all deposition experiments.

Fig. 3 shows a portable form of the complete apparatus used for oil-fog production, control, and deposition as a thin film on a polished metal test disk. Three 5-gal glass bottles are used, in two of which are rubber balloons. These are storage bottles for the fog as soon as it is produced in one of the atomizers which can be seen on the table of the apparatus. The bottle in front is the working bottle into which the fog is fed from one or other of the storage bottles, and from which it is drawn to the deposition apparatus shown on the table at the extreme left. This apparatus is sketched separately in Fig. 4 and its construction is described later. In Fig. 3, the fog chamber is shown raised out of position. At the start of a run it is lowered into position over the deposition disk, and at the same instant voltage is applied and a stop watch started.

The purpose of the balloons is to provide a means for flowing fog at a controlled rate, from the storage bottle in use at the

[5] "Improved Method of Conditioning Surfaces for Adsorption," by I. Langmuir and V. J. Schaefer, *Journal of the American Chemical Society*, vol. 59, September, 1937, notes, p. 1762.

[6] "Built-Up Films of Proteins and Their Properties," by I. Langmuir, V. J. Schaefer, and D. Wrinch, *Science*, vol. 85, January 15, 1937, p. 76.

moment, into the working bottle, and then to the deposition apparatus. To do this, nitrogen gas is fed into the balloon at a specified rate which is measured on the flowmeter seen in front of the working bottle. The expansion of the balloon serves in a very simple manner to effect the transfer of fog through the system. Stopcocks enable the operator to shift instantly from one storage bottle to the other during a long-continued calibration run with no interruption of the fog flow.

A simple and very effective means has been incorporated into the apparatus which makes it possible to keep the intensity of the light in the photoelectric density-measuring device always at its empty-bottle value. A second Photronic cell outside the bottle is illuminated by the light bulb and is connected to a second indicating instrument. Both instruments are adjusted to give the same empty-bottle indication. If, therefore, with fog in the bottle, the reading of the outside instrument is kept on this value by means of a rheostat, we can be sure that the fog density as indicated by the working Photronic cell is reliable.

2 *The Potential Gradient.* As previously stated, oil will deposit on a negatively charged metal plate suspended in an oil fog. If the plate be prepared carefully enough beforehand with respect to polish and cleanliness, interference colors will appear as the exposure continues. It will be found that the deposit is far from uniform as evidenced by the variation of color over the surface. At and near the edges the rate of deposition is a maximum, and the reason for this is clear enough when it is remembered that in this region the potential gradient is considerably higher than over the flat central portion of the plate. One obvious way of overcoming this difficulty would be to surround the actual deposition plate with a guard ring at the same potential.

Another and preferable method to follow when calibrating an oil fog for rate of deposition is to shape the deposition surface so that edge effects due to nonuniform potential gradient are eliminated. Fig. 4 shows the deposition apparatus used. Oil fog of controlled density is admitted through tube (1) into a cylindrical metal box (2), the bottom plate of which is pierced with a large number of fine holes. The circular metal disk, on which the thin oil film is to be laid down, forms the upper half of a pancake-shaped electrode (3), the edge of which has a radius of curvature of $^1/_4$ in. The flat bottom of the fog box, and the flat upper surface of the electrode can be set very exactly to a gap distance of $^1/_4$ in. by means of distance pieces inserted between the ends of the sliding members (4) and the adjustable nuts (6). The electrode (3) is carried on an insulating support (5), and is connected to the 1000-volt negative side of a direct-current power pack, the positive side of which is grounded and also connected to the metal frame. Thus, there is a potential gradient of 1000 volts over a $^1/_4$-in. gap through which the oil fog flows at a uniform, slow rate. The fact that during a calibration run the interference color over the whole of the deposition surface is perfectly uniform is evidence that the necessary operating conditions have been attained.

Rupture Tests of Thin Oil Films

Starting with the concept that a thin oil film protects the metal surface to which it is attached, by virtue of its resistance to removal under the stress of high-load rubbing, a test procedure was sought for making quantitative measurements of film strength. The following conditions were formulated:

1 All oils to be tested must be in the form of films of the same thickness.

2 The applied load must have a high and constant unit value.

3 The rubbing throughout the test must be along exactly the same track.

4 The metal base on which the film is laid down must have a high degree of polish, and the actual track to be followed must first be given a run-in treatment to bring the surface to as high a state of uniformity as possible.

5 Means must be provided for measuring the gradual degradation of the film up to the point of breakdown, so that a numerical value may be arrived at which expresses its resistance to rupture.

Considering these requirements individually:

Film Thickness. The procedure already described proved to be suitable for laying down uniform oil films of any specified thickness on polished metal. As previously mentioned, experiments showed that a suitable thickness for a film-wearability test lay between 10^{-7} and 10^{-6} in.

Unit Load. A high and constant unit load can be obtained by using as the rubbing "point" a sapphire formed to a hemispherical contour of small radius. In the tests made, the sapphire was formed to a radius of curvature of $^1/_{16}$ in. An application of the Hertz formula[7] for the case of a steel ball of this radius in contact with a steel flat, under an applied load of 100 g, gives an average value for the unit pressure over the area of the contact circle of about 93,000 lb per sq in. A search of the literature failed to disclose any values for the elastic constants of sapphire, but it is believed that an average pressure at the contact spot of the order of 100,000 lb per sq in. may be expected.

Rubbing Track. The machine built for tests of thin-film rupture made it possible to pass the sapphire rubbing button repeatedly over exactly the same circular track on a rotating steel disk carrying the thin film. Before the oil film is applied, a track about 0.2 mm wide is given a run-in treatment under a diamond which has been formed to a rounded contour and polished smooth. This operation is done dry, both diamond and steel being absolutely free from any trace of oil. Contrary to what might be expected, there is no abrasion or wear of any kind, and the coefficient of friction has a value sometimes as low as 0.02. This remarkable phenomenon does not appear to be connected with the hardness of the diamond. A highly polished boron carbide point which approaches the diamond in hardness will tear up the surface of clean steel even under light loading. Sapphire behaves similarly to boron carbide. Fig. 5 shows, at a magnification of 100 diameters, two tracks on molecularly clean glass. The faint track, which may not be visible in the reproduction, is the result of 100 passages of a diamond under an applied load of 1000 g. The slight image visible is a refraction effect due to distortion of the surface of the glass as a result of surface flow. The other track was made by a single passage of a rounded sapphire under an applied load of only 100 g and shows very severe abrasion accompanied by a skittering progress of the sapphire over the surface.

Measurement of Film Strength. The most logical and convenient method of determining the resistance to mechanical wear of a thin oil film is to follow the progressive rise of friction during the course of a test run. As the protecting film is reduced in thickness the friction rises, usually very slowly at first and quite rapidly toward the end. The initial value of the coefficient of friction for an average straight mineral oil 10^{-7} in. thick is of the order 0.130. After the value has risen to 0.200. the value rises to 0.250 during the course of a very few passages of the sapphire over the track. This value of the coefficient is

[7] Application of the Hertz formula for the case of a steel ball of radius R, in., in contact with a steel flat, under a load L, lb:

Poisson's ratio p for steel = 0.25; Young's modulus E for steel = 30,000,000 lb per sq in.; L = 100 g = 0.2205 lb; R = $^1/_{16}$ in. = 0.0625 in.; radius r of contact circle = $\sqrt[3]{[(3\pi/2)KLR]}$, where $K = (1 - p^2)/\pi E$; and the average intensity of pressure $P = L/\pi r^2$. Inserting values, we have r = 0.000867 in., and P = 93,360 lb per sq in.



FIG. 5 DIAMOND AND SAPPHIRE TRACKS ON MOLECULARLY CLEAN GLASS. ×100

arbitrarily taken as the breakdown value, and when not exceeded no damage is done to the steel surface which cannot be repaired by a light polishing process with optical rouge and water.

Fig. 6 shows the effect of varying film thickness on resistance to mechanical wear. The oil tested was a straight mineral oil of S.A.E. 10 grade, and the three film thicknesses tested were 2, 3, and 4 millionths of 1 mm. It will be noted that a film thickness of 4×10^{-6} mm affords a greatly increased measure of protection against breakdown, as compared with one only 25 per cent thinner. With regard to the molecular thickness of these thin films, it is to be realized that the dimensions and shapes of the component molecules of a material of such complexity as a mineral lubricant have no degree of uniformity. We shall not be far wrong, however, if we assume for working purposes an average diameter of the order of 10 Angstroms or 1 millionth of 1 mm. On this basis it is seen that a considerable degree of resistance to mechanical breakdown was provided in the reported experiment by a film approximately four molecules thick. It must be emphasized that the basic reason for the breakdown of these very thin films under mechanical attack lies in the fact that the mobility of the oil molecules in the area immediately outside the track is exceedingly low. With much thicker films, molecules flow in to replace those removed. Experimental evidence of this phenomenon is given by the dashed curve in Fig. 6. This was obtained in a run in another series, using an

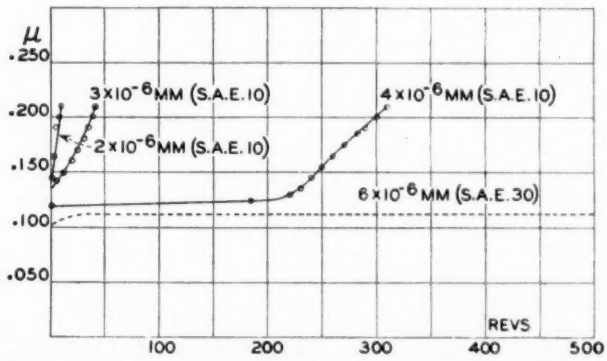


FIG. 6 EFFECT OF DIFFERENT FILM THICKNESS ON THE RESISTANCE TO MECHANICAL WEAR OF A STRAIGHT MINERAL OIL

S.A.E. 30 straight mineral oil, laid down as a film 0.25 millionth of 1 in. thick or slightly over 6 millionths of 1 mm. After a slight initial rise in the coefficient, the value was constant over the entire run, and as a matter of fact remained constant for over 2000 passages of the rubbing point over the surface of the steel disk. This result is believed due to the increased mobility of the oil molecules in the film, new molecules flowing onto the track as fast as removed by the rubbing action.

The curves in Fig. 7 show the effect of an addition agent on the

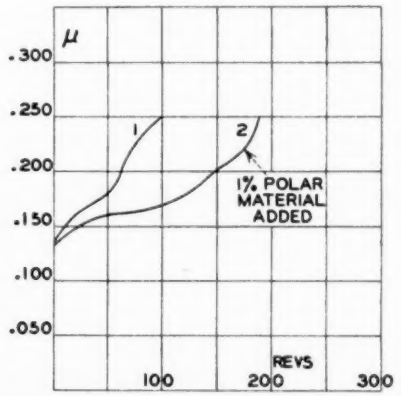


FIG. 7 EFFECT ON THIN-FILM WEARABILITY OF THE ADDITION OF 1 PER CENT OF ACTIVE POLAR MATERIAL

resistance to wear of a thin film, and suggest this means of increasing the protective action of an oil against the risk of failure when fluid-film conditions give place to boundary-film conditions.

Curve 1 shows the progressive rise of the coefficient of friction for a straight mineral oil of S.A.E. 30 grade in a film approximately two molecules thick. Curve 2 shows the effect of the addition of 1 per cent of a material of marked polar activity.

THIN-FILM-WEARABILITY MACHINE

Fig. 8 shows the machine for testing thin oil films set up for operation, and Fig. 9 shows its essential features. A balanced horizontal bar is so suspended as to have only the following two degrees of freedom: (1) It can rock freely in a vertical plane about a line defined by knife edges at right angles to the length of the bar. (2) It can move in the direction of its length, the amount of movement being limited by stops which serve also as electric contacts thus providing a means of indicating departure from a neutral or balanced position.

Near one end of the bar is a holder carrying a round-faced rubbing button which rests on the outer portion of the circular test disk, the latter being fixed on the top of a rotating table offset with respect to the center line of the bar. As the table rotates, the button traces out a true circle on the face of the test disk. The working load is a 100-g weight placed on the top of the button holder. At the other end of the balance bar is attached a silk thread which passes in a left and right direction over two frictionless pulleys to a double chainomatic device by means of which a pull P can be applied to the bar, equal in amount

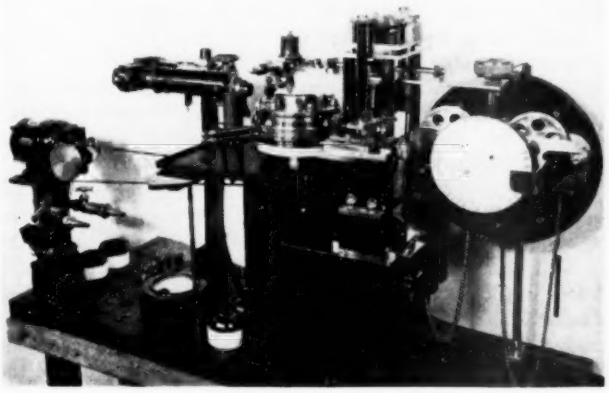


FIG. 8 TESTING MACHINE FOR THIN OIL FILMS

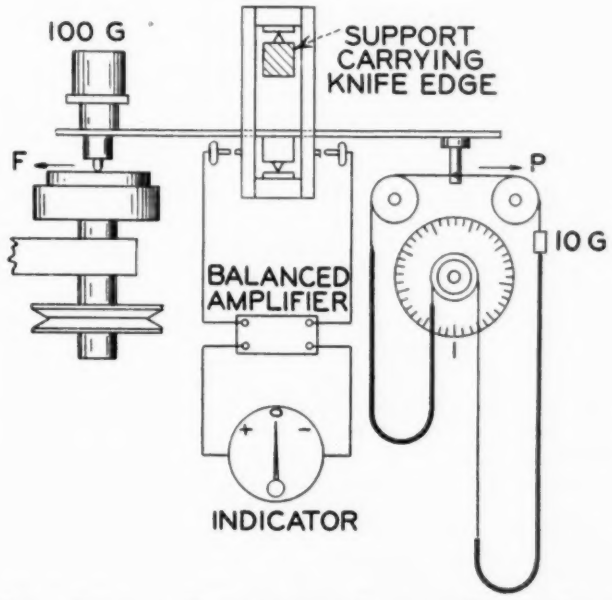


FIG. 9 DETAILS OF MACHINE FOR THE TESTING OF THIN OIL FILMS

and opposite in sense to the drag of friction F between the rubbing button and the face of the rotating test disk. If the frictional drag exceeds 10 g, a 10-g weight is hung on the outermost chain. A simple gear operated by turning the graduated dial allows the chain to be lowered on the right and raised on the left, which operation increases the pull on the silk thread in the direction opposed to the frictional drag. The machine is sensitive to an out-of-balance setting of 0.1 g, which means that with a load of 100 g the coefficient of friction can be determined to better than 1 per cent. The exact point of balance is determined by observing the needle of a microammeter which moves left or right of zero when left or right contacts are made at the limit stops already referred to. The free gap at each of these contacts, which are faced with polished tungsten, is slightly greater than 0.0005 in., and entire freedom from the effect of resistance changes at the contacts is secured by the use of a simple two-tube radiotron circuit, the balance of which is upset when either contact is made.

A scale around the circumference of the rotating table on which the test disk is mounted is divided into 10 equal parts numbered from zero to nine. This scale is useful when determining the value of the coefficient of static friction at various points around the track circle as will be described later.

DETERMINATION OF THE COEFFICIENT OF STATIC FRICTION

A very cursory search into the literature of the subject will show that the determination of the coefficient of static friction for a stated combination of metals and oil is a problem incapable of exact solution. This is hardly surprising in view of the fact that friction between solid surfaces in close contact depends on a number of factors, some of them not permitting control, and some producing effects of an obscure nature. The most difficult factors to control are those connected with the physical nature of the surfaces. No surface, for instance, can be brought mechanically to the state of perfect freedom from minute irregularities such as is exhibited by the surface of quiet mercury. The best that can be done is to reduce the scratches on a carefully flatted surface until they are no longer visible under fairly high magnification. The further we carry this process the lower will be the coefficient of friction, other conditions being the same. It may fairly be assumed that the friction between ideal surfaces would be very low, but since such surfaces can never be produced, the lowest value we can hope to obtain will be notably higher than the theoretical minimum. Another factor of outstanding importance is the change in surface structure produced by cold working. It has been definitely established in the course of the present work that a considerable reduction of the coefficient of friction is found when slip occurs on a well run-in track. Since it is noted that the friction is not only reduced, but is also made uniform around the track, it is suggested that the crystalline structure has been replaced by a Beilby layer. The fact that friction between glass surfaces separated by a boundary film is constant from point to point lends support to this view.

As an illustration of the effect of running-in, the coefficient of static friction under boundary-film conditions between a round-faced tungsten-carbide button and a flat polished steel surface was reduced from 0.132 to 0.114 after running-in dry under a diamond at extremely high pressure.

COEFFICIENT OF STATIC FRICTION—STARTING AND STOPPING VALUE

Consider the case of a rounded button of some very hard material such as tungsten carbide sliding on a surface of polished steel under a normal load of 100 g. Even with a thick film of oil on the steel the unit loading is so high that the surfaces are separated by a boundary film only. If now a gradually increasing force be applied to the button, pulling exactly in the plane of the flat surface, we shall eventually reach a value just sufficient to cause slip. The ratio of this force to the normal load is, of course, the value of the coefficient of static friction. Stated perhaps more precisely, it is the value of the coefficient at the instant of starting to slip. Whether the value of the coefficient at the instant of stopping is the same, is not immediately apparent, and to clear up this point a number of tests were made with the machine described.

Method of Making a Static-Friction Test

The most convenient method of determining the static coefficient is to measure the frictional drag at the exact instant when sliding ceases. Suppose the specimen disk is rotating at its normal speed, and that the balance pull is set for exact balance of the normal running friction. The indicator needle is, of course, at zero. Now stop the rotation and observe the behavior of the needle. As the rubbing speed decreases, there is usually a tendency for the deflection to go to the left of zero, meaning that the friction is higher for decreasing speeds than for the normal speed. At the exact instant of stopping, however, the behavior of the needle provides very exact information as to the relation of static friction to running friction, under the operating conditions. The needle may move to the left, or plus side, which means that static friction is definitely greater than running friction, or it may move to the right, or minus side, which means the reverse. In certain cases no departure of the needle from its zero position is observed from the time the motor is switched off to the instant of stopping. This, of course, indicates that the coefficient of static friction (stopping value) is equal to the coefficient of running friction at the normal rubbing speed of 1.5 fpm.

TABLE 2 LOG OF RUN FOR DETERMINATION OF COEFFICIENT OF STATIC FRICTION (STOPPING VALUE), USING THE STANDARD METHOD[a]

Station number	Balance pull P, g: 10.4	10.6	10.8	11.0	11.2	11.4
	Pointer indications					
0	0	0	0	—	—	—
1	0	0	0	—	—	—
2	+	+	0	0	0	0
3	+	0	0	0	0	—
4	+	+	0	0	0	0
5	0	0	0	0	0	—
6	0	0	0	0	0	—
7	0	0	0	0	—	—
8	0	0	0	0	—	—
9	0	0	0	—	—	—

[a] Test made with S.A.E. 30 straight mineral oil and tungsten carbide on steel.

TABLE 3 LOG OF RUN FOR DETERMINATION OF COEFFICIENT OF STATIC FRICTION (STARTING VALUE), USING DELAYED-SLIP METHOD[a]

Station number	Balance pull P, g: 10.4	10.6	10.8	11.0	11.2
	Pointer indications				
0	0	0	0	—	—
1	0	0	0	—	—
2	0	0	0	0	—
3	0	0	0	0	—
4	0	0	—	0	0
5	+	0	0	0	0
6	0	0	0	0	—
7	0	—	—	—	—
8	0	—	0	—	—
9	0	0	—	—	—

[a] Test made with S.A.E. 30 straight mineral oil and tungsten carbide on steel.

TABLE 4 EFFECT ON THE COEFFICIENT OF STATIC FRICTION OF A HIGHLY POLAR COMPONENT IN THE OIL

Oil	Coefficient of friction for Steel	Bronze	Reduction, per cent
(*A*) Straight motor oil	0.116	0.086	26
(*B*) Straight motor oil plus 0.5 per cent cp palmitic acid	0.083	0.051	39
Reduction, per cent	29	41	

NOTE: Disk metals used were of steel and bronze, while the rubbing button was of tungsten carbide.

TABLE 5 COMPARISON OF STATIC COEFFICIENTS S AND RUNNING COEFFICIENTS R FOR SEVEN DIFFERENT OILS

Oil	Tests on steel: Coefficients Static S	Running R	Difference $(R-S)$, per cent	Tests on bronze: Coefficients Static S	Running R	Difference $(R-S)$, per cent
A	0.116	0.100	−13	0.086	0.100	+16.5
B	0.083	0.093	+12	0.051	0.088	+56.0
C	0.105	0.105	0	0.103	0.106	+ (very small)
D	0.106	0.105	— (very small)	0.078	0.097	+24.5
E	0.085	0.098	+15	0.102	0.105	+ (very small)
F	0.103	0.102	— (very small)	0.089	0.100	+12.5
G	0.090	0.100	+11	0.069	0.095	+38.0

Method of Taking Observations. An estimate of the probable value of the static coefficient is made, and, while the machine is running, the balance pull is set to a value less than the corresponding frictional drag. The turntable is then stopped at station no. 0 of the divided circle, and the deflection of the needle noted. The starting switch is then closed momentarily so that the table stops at station no. 1, and the deflection again noted. After 10 observations have been made at the 10 stations around the rubbing track, the balance pull is increased by 0.2 g and the whole operation repeated. This is done for as many new settings of the balance pull as are necessary to show definitely that the pull has exceeded the value corresponding to the true static coefficient. Table 2 illustrates the method followed.

Inspection of the results of this run shows that the average value of the balance pull P around the track is 11.2 g, there being five positions where the value is slightly too great and five positions where it is not high enough to cause slip over a distance of 0.0005 in. Since the load is 100 g, the coefficient of static friction (stopping value) is 0.112.

Delayed-Slip Method. By modifying the described method somewhat, the coefficient of static friction at the start of slip can be obtained. The turntable is arrested as before at each of the marked stations around the circle, and the same procedure as before is followed in the setting of the balance pull P on the dial. The working load is the standard value of 100 g, but provision is made for adding an extra load of 20 g by lowering a weight suspended by a thread onto the 100-g weight. Just before the turntable stops at each of the ten stations, the 20-g weight is added, making a total load of 120 g, therefore increasing the frictional drag 20 per cent. The indicator needle, of course, goes to the left or friction side. After the turntable comes to complete rest, the extra weight is gently removed, thus restoring normal conditions. Whether the needle will still remain on the friction side depends on the value to which P has been adjusted. It requires a slip of only 0.0005 in. to affect the needle. After waiting five seconds, the indication, whether plus, zero, or minus, is noted and the operation repeated for the next station. Table 3 will make the method clear. Inspection of these results shows that the value of P necessary to balance the static friction (for start of slip) has a mean value of 11.0 g, so that, for a load of 100 g the coefficient is 0.110 which is a satisfactory check on the value obtained by the standard method.

Oiliness and the Coefficient of Static Friction

The much discussed property of lubrication loosely termed as "oiliness," is without question of considerable importance. However, it is a phenomenon which becomes strongly evident only when the oil film separating rubbing surfaces is exceedingly thin, probably of the order of molecular dimensions. The presence of an oiliness component in a film thick enough to obey the hydrodynamic laws obtaining in viscous-film lubrication has no effect whatever on the friction losses. Conversely, in films of molecular dimensions, viscosity is almost a negligible factor, while oiliness has a paramount effect. Oiliness has been described somewhere as a partnership property of both oil and the surface to which it is attached, and this significant statement may be definitely proved by measurement of the coefficient of static friction for various combinations of metals and oils.

As implied in an earlier statement, it is not possible to obtain an absolute value for the coefficient of friction for any specified combination of metals and oil, principally because of the uncontrollable factor of the physical state of the sliding surfaces. We can, however, with sufficient care, prepare surfaces for use in comparative tests, so nearly alike that relative values of the coefficient will give real information of the oiliness effect for specified oil-metal systems. The figures given in the tests to be

described, while reproducible to better than 1 per cent, represent only the values observed under the specific technique followed in the preparation of the surfaces.

The values given in Table 4 illustrate clearly the effect on the coefficient of static friction of both the surface activity of the metal and the molecular activity of the oil; in other words, they confirm the statement that oiliness is a partnership property of the oil-metal system.

Comparison of the Coefficients of Static and Running Friction

It is generally accepted that the coefficient of static friction for a pair of metals and a thin oil film is greater than the coefficient of running friction. The figures presented in Table 5 show, however, that this belief is subject to radical modification.

In the machine used for this investigation, the rubbing speed was quite slow, being of the order of 1.5 fpm. The values of the coefficient of running friction may, therefore, be expected to lie close to their possible maximum, thus making all the more striking the comparisons shown.

The seven oils tested were of widely different types and it is to be noted that in only one case is the static coefficient higher than the running coefficient. Another quite evident point shown by an inspection of the figures in Table 5 is the part which was played by the metal in emphasizing the difference between the coefficients.

Perhaps the most generally useful deduction is that the oiliness value of a lubricant can best be judged by determining the static, rather than by the running coefficient.

Oils *B* and *G* were known to contain active polar groups, and these two showed the greatest percentage differences between the coefficients of static and running friction.

Acknowledgments

The author wishes particularly to acknowledge valuable assistance given by E. N. Dacus, one of the author's associates, in working out some of the experimental details. Also, the author's thanks are due to L. C. Roess, of the laboratory with which the author is associated, for helpful criticism. The apparatus was built in the laboratory workshop by Allan Jamieson.

Discussion

A. S. Brace.[8] The use of extrapolated weights or color-interference patterns leaves something of a question as to the actual film thickness used.

Time permitting, it would seem desirable to make check tests on identical samples with decreasing time of exposure to oil fog, to the point where the coefficient of friction of the bare button is approached. It is appreciated that this has been done for one oil as illustrated in Fig. 6.

With the wide difference in index of refraction existing between different oils, particularly in the heavier grades, it would seem that a direct comparison of oiliness would be reliable only if a series of runs on each oil were made similar to the three runs shown in Fig. 6.

Another question which arises is that of residual adsorbed film for succeeding tests using the same steel surface. The question here is whether or not the light polishing process removes the adsorbed film or Beilby layer, if present. Shop experience indicates that the adsorbed film or Beilby layer can only be removed by removal of an appreciable part of the surface.

The conclusion with respect to static and running friction suggests that the author is working in a range of surface smoothness which has not hitherto been achieved. It is suggested that measurements of surface smoothness (profilograms) be made before and after test runs.

W. E. Campbell.[9] The author's remarks on cleanliness of surfaces are very important. He may be interested to know that we have found by the electron-diffraction method that surfaces polished with magnesium oxide contain traces of magnesium oxide. However, polish by levigated alumina, followed by scrubbing with cotton under tap water, gives a surface which is free of detectable traces of abrasive.

The results obtained with a diamond bearing are interesting. It has been shown[10] that carbon is a very strong adsorbent for organic molecules. It would appear possible that the results obtained are due to a very strongly held adsorbed layer of a long-chain organic compound which cannot be removed from the diamond surface by the cleaning method employed.

David Clayton.[11] The author obviously has gone far in the direction of obtaining even distribution of the oil, but is it quite certain that the thinnest films are uniform? With reduction of exposure time discontinuity will ultimately be reached. With the films used (2 and 3×10^{-6} mm) each drop of oil has to spread over a relatively large area, and in the process the oil approaches the limit of mobility. Extrapolation of the curves in Fig. 6 shows values of starting friction decreasing with increasing thickness of film. This might be an indication that the surfaces are not completely covered in the case of the smallest quantities of oil. The value of the factual findings is not less even if they apply to minute quantities of oil instead of to very thin films. However, the point is raised because of the direction given to ideas by a paper of this character.

It is interesting to note that the right-hand track in Fig. 5 is similar to that obtained by Tomlinson[12] with a glass bead on a glass plate, and with a hard-steel ball on a hard-steel plate. The remarkable behavior with a diamond recalls to mind the extremely small wear, resulting in low friction, obtained by Stott[13] with unlubricated diamond cups on steel pivots in an investigation of pivots and jewels for instruments and meters. In this instance there was great contrast to the behavior with sapphire cups. Consideration of Fig. 5 raises the question as to the appearance after running-in the tests to the different stages shown by Fig. 6. Was any correlation with the friction behavior apparent?

It is interesting to note from Fig. 7 that so small a proportion of molecules of the polar material in the thin film affected the friction and delayed the breakdown. In the usual use of polar additions to oil, it is considered that preferential adsorption occurs, so in the boundary layer the proportion of polar molecules is high even when the general proportion is very low. No effect of this kind can occur in the circumstances described. Thus the result, while interesting, does not represent the practical case. This would be represented more satisfactorily by similar films of

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[9] Bell Telephone Laboratories, New York, N. Y.

[10] "Quantitative Correlation of Interfacial Free Surface Energies," by F. E. Bartell and L. S. Bartell, *Journal of the American Chemical Society*, vol. 56, November, 1934, p. 2205.

[11] Engineering Department, National Physical Laboratory, Teddington, Middlesex, England.

[12] "Rusting of Steel Surfaces in Contact," by G. A. Tomlinson, Proceedings of the Royal Society of London, series A, vol. 115, 1927, p. 477.

[13] "Some Experiments on the Lubrication of Pivot and Jewel Bearings," by Verney Stott, Proceedings of the General Discussion on Lubrication and Lubricants, The Institution of Mechanical Engineers, London, 1937. American edition, A.S.M.E., New York, 1938, vol. 2, p. 148.

wholly polar material, if it lends itself to treatment by the method adopted, or at any rate a high concentration.

It would be interesting to know how closely the results in Fig. 6 were repeated in duplicate experiments; also whether or not the coefficients of friction, given in Tables 4 and 5, were obtained with an excess of lubricant or with thin films. It would seem that more information is required to support the statement that at 1.5 fpm, "the values of the coefficient of running friction may be expected to lie close to their possible maximum." Moreover, this statement does not agree with a previous one that as the rubbing speed decreases, there is usually a tendency for the friction to become higher for decreasing speeds than for the normal speed. The general idea that the running friction is less than the static friction probably arises from the fact that in most practical cases there is set up at any rate a partial fluid film when motion occurs. There is little experimental evidence of the comparison for purely boundary conditions. The author's results are therefore valuable additions to knowledge on the subject.

In drawing conclusions, however, it is perhaps necessary to consider that the running friction might be artificially high due to work being done in deformation of the material under the high-pressure conditions of the experiments. High pressure is one way of approaching boundary conditions, but it involves this danger. Further, in connection with the friction results, it is not obvious from the evidence presented that "the oiliness value of a lubricant can best be judged by determining the static rather than the running coefficient." Usually this is true because of the absence of any viscosity effect under static conditions, but if the kinetic boundary friction is different from the static value, it is surely the kinetic oiliness which is of main value for practical application. The present results suggest different degrees of oiliness for static and kinetic conditions.

It is difficult to speculate helpfully on the influence of surface roughness on friction or breakdown. The "wringing" or cohesion of block gages and optical flats, certainly does not support the view expressed by the author: "It may fairly be assumed that the friction between ideal surfaces would be very low;" though not perhaps ideal in the sense visualized, the approach to it is accompanied by enormous increase in the friction.

It is hoped that the author may be tempted to enlarge on the effect of the running-in process. The writer would certainly not have preformulated this condition, if highly polished surfaces were being used. Moreover this particular running-in process is certainly not one that can be used in practice. It would be interesting to see how Fig. 6 appeared without any running-in process. Even though the results might be less consistent, it is necessary to know how ordinary surfaces behave. It is also hoped that the author will be able to work through the range of each of the several important variables to supplement the excellent work already carried out.

G. B. Karelitz.[14] The technique of laying down a thin film of oil on a metal plate will be welcomed by experimenters. It seems that this technique is straightforward and may be easily reproduced by other laboratories. In all tests on boundary friction the researchers were handicapped by viscous action of the pool of oil at the rubbing surfaces. This is the main difficulty in using friction-testing machines of the Deeley type. With a thin film of a predetermined thickness, the difficulty seems to disappear and true coefficients of friction can be measured.

Tests conducted by various investigators have shown that films remain liquid and can be treated as liquid down to a thickness of several millionths of an inch. In actual bearings of good finish the film thickness down to 0.00003 in. was observed. There always has been doubt at what thickness the film begins to obey the hydrodynamic laws of lubrication. In this respect the tests of Claypoole are most significant. It appears that the orientation of molecules, which would prevent their mobility and flow, does not extend beyond five, or say ten molecules, or one half millionth of an inch. It is, therefore, quite legitimate to treat films occurring in actual bearings, no matter how fine, by the usual hydrodynamic theory.

With respect to the value of the static coefficient of friction, Coulomb himself knew, and several investigators have observed that, with due precaution, the coefficient of friction does not drop suddenly when motion takes place. When the speed is very slow, as was the case in the author's experiments, there should be either no increase or very little increase in the coefficient when changing from kinetic to static friction. It is the writer's belief that for this speed the kinetic and static coefficients of friction are equal. It will be of interest to mention tests with huge ball bearings loaded to several hundred thousand pounds, which were made recently for the 200-in. telescope. In these tests the bearings were started and stopped running at a speed of one revolution in 15 min. The starting friction did not differ from the running friction more than 0.75 per cent, in spite of the general rule that the starting friction in ball bearings is twice the running friction. The authors' suggestion that a more exact definition of static friction is in order should be welcomed.

R. J. S. Pigott.[15] Some of us engaged in problems connected with lubrication and bearing design have been conscious, for some time, that there should be a better and much closer cooperation between physicists, engineers, and chemists on this extremely elusive subject. The author's interesting paper is an approach mainly from the point of view of the physicist. Early in the paper the author states: "The ideal case, as indicated previously, would be two perfect surfaces separated by a double molecular layer of lubricant." It should be made perfectly clear that this statement can only apply to academic study on extremely thin films, as of course such a situation is not in the least ideal from an engineering point of view. The ideal bearing, from the engineer's point of view, is one in which the friction torque is the lowest to carry a given load at a given speed. This will certainly not correspond to very thin film conditions. In engineering the principal concern is with moderately thin films, either for flat or cylindrical bearing surfaces, with dimensions which must necessarily remain much larger than two molecules thickness, for any practical operation.

Toward the close, the statement is made: "The presence of an oiliness component in a film thick enough to obey the hydrodynamic laws obtaining in viscous-film lubrication has no effect whatever on the friction losses." The writer is in entire agreement with this statement and cannot urge upon those interested in the subject, too strong a consideration of it. This is particularly true at the present time, since many groups engaged in the manufacture of automobiles, aeronautical engines, and similar devices, have been riding the hobby of oiliness agents added to oils, where this ghostly quality can be of no value whatever except possibly at the start and stop of operation. In most of these cases, the starting and stopping loads are far below the maximum and the oiliness requirements are therefore almost absent. When it is realized that one company is insisting upon rating lubricating oils for use in its internal-combustion engines on the basis of a single bearing test, which is carried to boundary lubrication conditions and therefore benefits by oiliness, one can readily realize how far afield "hunch plays" of this sort can

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[15] Staff Engineer, Gulf Research & Development Company, Pittsburgh, Pa. Mem. A.S.M.E.

carry investigators. In this particular case, some of the oils which, by operating performance show the best results in the actual engine, would be rejected by this test. Conversely some of the oils rating very highly in this test are worthless, or nearly so, in the engine, due to sludging, lacquering, and other objectionable characteristics.

The idea expressed, immediately following the statement quoted, is heartily supported, that oiliness is a combination property of the oil and the surfaces.

With regard to the ability to prepare surfaces that are physically alike, the advent of the profilograph and other similar instruments will undoubtedly give much better means of standardizing surface conditions, since an examination can now be made on the order of millionths of an inch. The writer has found in all his lubrication testing experience, that the mechanical factors in a testing machine, such as those of operating temperature, trueness of surfaces, clearances, and surface roughness, are very many times more important than any characteristics of the oil or other lubricant. In short, he has found it is necessary to use the utmost rigor in standardizing any piece of test equipment for lubricant ratings in order to obtain results which are continuously reproducible.

This paper is valuable in its approach to the little understood problem of imperfect film or boundary lubrication. This condition obtains mainly in heavily loaded low-speed bearings or in other bearings just before scuffing or failure begins.

Author's Closure

Mr. Brace raises the question of the effect of differences in the refractive indexes of the oils used. As a matter of fact in the work reported, the measured values over a very wide viscosity range did not differ by more than 1 per cent.

An adsorbed film of oil on the surface of polished metal can be completely and readily removed by rubbing with a mild abrasive and water. It is entirely probable that the removal of the adsorbed film is accompanied by the removal of surface atoms of the metal.

The Beilby layer, the formation of which is promoted by polishing, naturally cannot be removed by a fine abrasive such as rouge.

Mr. Clayton suggests that the thinnest films deposited may not be quite uniform. The author agrees that this may be so, but would like to point out that the spreading of droplets hitting the surface is greatly influenced both by the perfection of finish and the state of cleanliness. A pure mercury surface is intrinsically free from surface imperfections and can be kept free from accidental contamination. With vertical illumination, it is possible under a microscope to follow the progress of deposition of an oil film from fog. Droplets can be seen striking the surface and flashing out into a film which quickly becomes invisible. The color presently attained is even and pure. Almost as perfect spreading is observed on a highly polished clean chromium surface. It is not possible, however, to prepare a steel surface having the degree of physical uniformity possessed by a polished surface of chromium plate. For this reason, it may be true that oil films deposited on a steel plate are not as uniform in thickness as those on a chromium plate.

The running-in technique is useful in that it reduces the variations in the coefficient of friction observed with a material having the physical structure of steel. The coefficient obtained on glass is remarkably constant from point to point of the surface.

Many tests similar to those shown in Fig. 6 were made. Some checked surprisingly well. Others were very erratic. The chief causes for poor checking were found to be the presence of accidental dust on the test plate, and wear on the sapphire button.

By "ideal surfaces," the author had in mind the case of two geometrically flat surfaces separated by a double molecular layer of oil. It is to be presumed that such surfaces should slide relatively freely over each other, since in the absence of any surface projections there is no possibility of metallic contact. The friction is measured by the resistance to shear at the slip plane of the film. The component due to the interaction of molecular forces, existing at the actual metal surfaces, is believed to be negligibly small. The case of the wringing together of gage blocks is not analogous, since the separating oil film is ruptured in innumerable minute areas by projecting asperities on an imperfect surface.

Mr. Pigott points out that the academic ideal case of perfect surfaces separated by a double molecular layer of lubricant is by no means ideal from the engineer's point of view. This is, of course, perfectly true. The engineer uses the word "ideal" as synonymous with "perfection," whereas in the paper it was used in the sense of "hypothetical or imaginary."

Wear in Lubrication Problems

By L. M. TICHVINSKY,[1] EAST PITTSBURGH, PA.

Boundary or semifluid friction takes place in bearings of practically all rotating machines when they are started or brought to a standstill. This type of friction, lasting a short period of time, is usually accompanied by the wearing of the bearing surface, the journal surface, or both. Wear takes place also in the case of antifriction bearings due to sliding, rolling, or combined sliding and rolling friction. Many other causes, such as misalignment, dirt, or inadequate lubrication, might be responsible for wearing of any type of bearing. This paper deals with the methods which are employed to obtain the wear resistance of various metals, especially those used in the design of bearings.

MECHANICAL wear, especially of metals, is as important and interesting to physicists as to engineers. The great number of variables which must be considered during this kind of dynamical testing range from very specific and unique properties of materials to the most common and usual. For this reason it is practically impossible to make a wear test during which all variables would be accounted for.

Most of the existing wear-testing machines are built so as to take care of only a few of the numerous variables. The result is that data obtained on a particular wear-testing apparatus are fundamentally related to the type of the machine used, and of course, to the conditions and methods of test.

Mechanical wear is subdivided into two major groups: (*a*) The one in which the wear is caused by abrasion of metal on metal, and (*b*) the one in which the wear is caused by abrasives on metal. A typical example of the first group is the wear of a bearing against a shaft when not perfectly lubricated. The wear of steam shovel teeth or of a grain conveyor belongs to the second group. Very often these two kinds of mechanical wear take place simultaneously. Thus, the rails and wheels of a railway car show wear due to both rolling friction and contamination such as dust and sand. Especially during wet weather the amount of contamination will be very small and most of the wear will be produced by rolling friction, but during dry weather wear due to contamination may be predominant.

These few examples show at once that mechanical wear is one of the most perplexing problems confronting the engineer, and it can be stated that any relative motion between parts in contact is usually accompanied in machines by mechanical wear.

The railroad industry was the first to be confronted with problems of wear. Adams (1),[2] Schwarz (2), Sandberg (3), and others, as long as 70 years ago, observed the wear of rails and reported upon it in current technical literature.

In 1878, Dudley (4, 5, 6) tried to correlate the known physical properties of rail materials with their resistance to wear when the total load which passed over these rail sections was measured. The wear was figured from the decrease of the rail cross section. For this purpose very accurate sketches of worn sections were made, measured by a planimeter, and compared with original cross sections. As a result of such field experiments, Dudley recommended the use of high-strength material with fine grain structure.

Further progress in the study of wear of rails was made when complete records of operating conditions were systematically gathered by railroad companies.

Laboratory wear testing was first mentioned in the literature by Bottone (7) and Bischoff (8). However, methodical experimentation was started only at the beginning of this century.

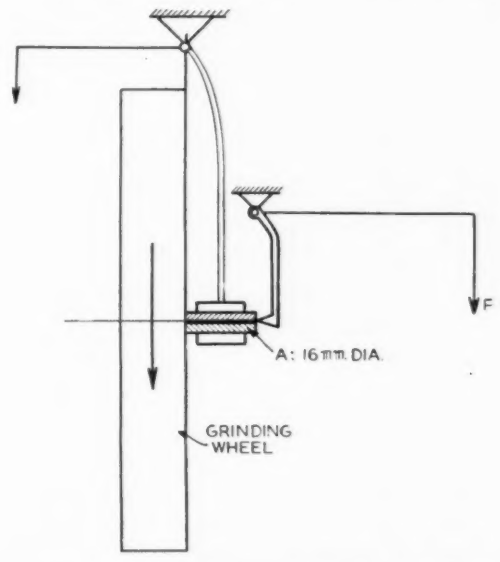


FIG. 1 SELLERGREN'S WEAR-TESTING APPARATUS (1905)—SLIDING FRICTION

In 1905 the Swedish engineer Sellergren (9) reported wear-test data obtained on his wear-testing apparatus. This apparatus is shown in Fig. 1. The test piece *A*[3] was inserted into a bushing fixed to an end of a lever, the fulcrum of which is in line with the plane of the grinding disk. The loading was obtained by another lever system and weight *P*. On account of vibration at very low loads, tests were conducted at pressures above 1 atm. Sellergren performed a great number of wear tests during which he investigated the influence of pressure, velocity, and area. He found that the loss of weight was proportional to the square of the rubbing speed.

The duration of each test was within 1 to 2 min. But during even such a short period of time the surface conditions of the wheel changed so much that no consistent results could be obtained. To some extent this defect was cured by frequent dressing of the wheel surface.

In 1909, Breuil and Robin (10) and Robin (11) obtained wear

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[1] Research Engineer, Mechanics Division, Research Laboratories, Westinghouse Electric & Manufacturing Company.

[2] Numbers in parentheses refer to the Bibliography at the end of the paper.

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Discussion of this paper was closed January 10, 1939, and is published herewith directly following the paper.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[3] In the figures of this paper all test specimens will be designated by *A*.

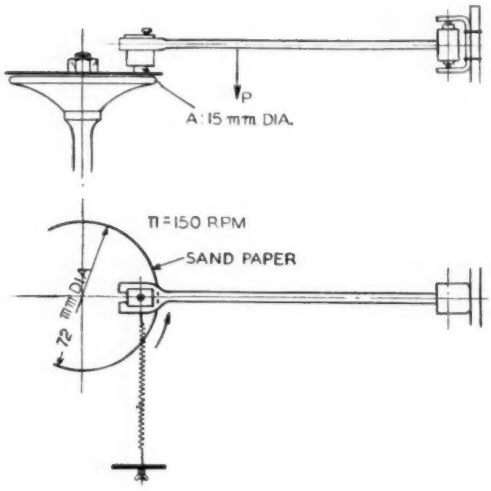


FIG. 2 ROBIN'S WEAR-TESTING APPARATUS (1909)—SLIDING FRICTION

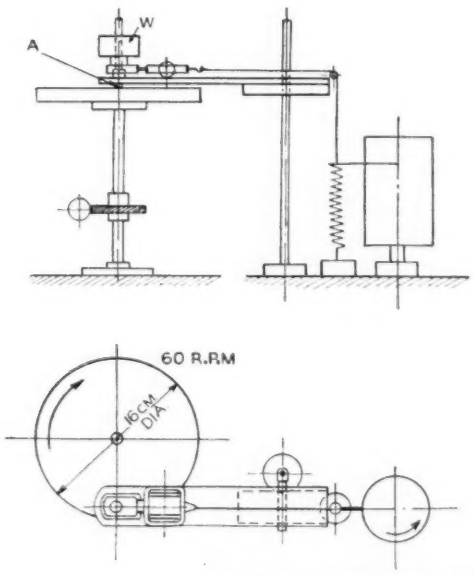


FIG. 3 HONDA AND YAMADA'S WEAR-TESTING APPARATUS (1924)—SLIDING FRICTION

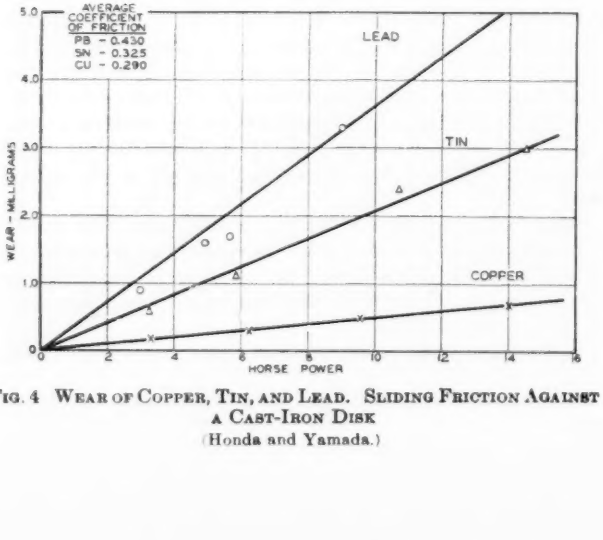


FIG. 4 WEAR OF COPPER, TIN, AND LEAD. SLIDING FRICTION AGAINST A CAST-IRON DISK
(Honda and Yamada.)

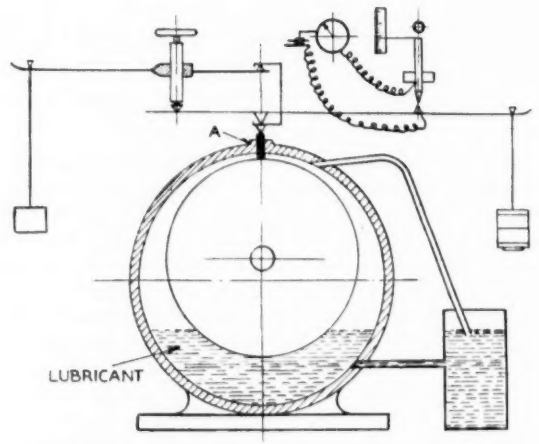


FIG. 5 DEBIHON'S WEAR-TESTING APPARATUS (1912)—SLIDING FRICTION

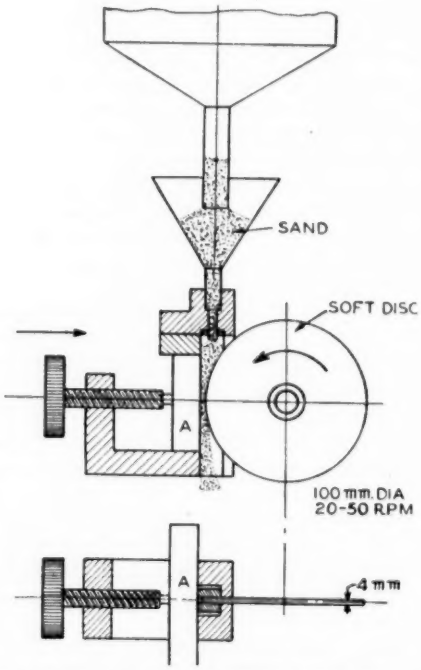


FIG. 6 BRINELL'S WEAR-TESTING APPARATUS (1921)

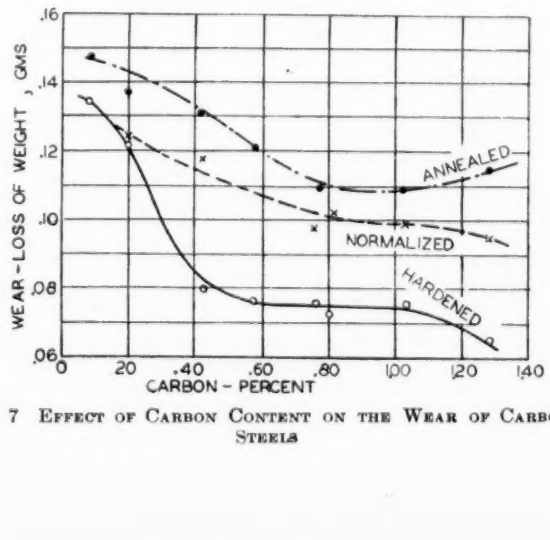


FIG. 7 EFFECT OF CARBON CONTENT ON THE WEAR OF CARBON STEELS

data on the machine shown in Fig. 2. The test piece was pressed against sandpaper placed on a rotating disk. The test piece was prevented from sliding out from the sandpaper by means of a spring. During 3-min tests, on the same piece of sandpaper, the 15-mm diameter sample showed signs of wear under a pressure of 1 kg per sq cm and a travel of 200 m. At 1-min intervals the test was stopped and the sample weighed. The amount of wear was measured as a wear number $(W_1 + W_2 + W_3)/A$ in gr per sq cm, where W_1, W_2, and W_3 are losses in weight after 1-, 2-, and 3-min periods, and A is the contact area of the specimen. Robin made numerous tests for investigating such effects on wear as the size and shape of the end of the specimen, the velocity of sliding, and the mesh of the sandpaper. The most consistent results were obtained when diamond paper was used. He came to the conclusion that test results could be compared only when test methods and conditions were similar. Among other things, Robin was seeking the relation between hardness and wear resistance. He found that a 0.4 per cent carbon steel showed more wear than others. He investigated also the wear as affected by the grain size of steel. His experimental results on rail and wheel materials were in good agreement with the performance of these steels in practice.

Based on essentially the same principle is the design of the wear-testing machine built in 1924 by Honda and Yamada (12), shown in Fig. 3; the horizontal disk covered with sandpaper, the test sample, and the weight W can be seen. The test piece is fixed in a small holder which slides in a slot in the supporting table. The holder is connected by means of a string to a calibrated spring which measures the friction force during the rotation of the sandpaper disk. Thus, on this machine, the effect of friction can be readily observed. A series of tests was made with pure metals and cast irons during which many variables were measured and subjected to changes. Wear of copper, tin, and lead thus obtained is shown in Fig. 4. During these tests the value of the coefficient

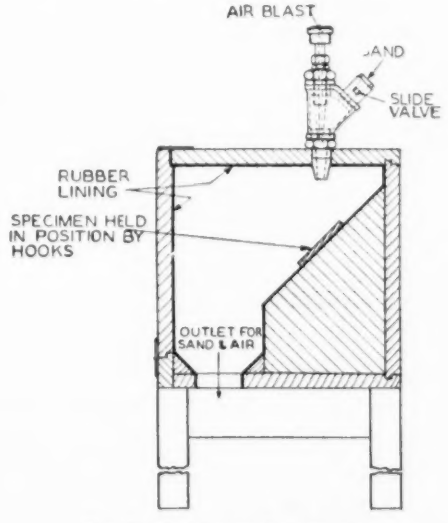


FIG. 8 SANDBLAST APPARATUS

of friction varied slightly. The position of the test piece was changed at each measurement so as to maintain a constant value of the coefficient of friction.

The machine shown in Fig. 5, on which the effect of a lubricant on wear could be obtained, was built by Derihon in 1912. In this machine, the test piece is pressed toward the outside diameter of a disk partly submerged in a lubricant. These tests, as described by Portevain and Nussbaumer (13), were made in order to obtain wear data of bronzes used in bearings. The loss of weight of the specimen was measured after 2000 km of disk travel.

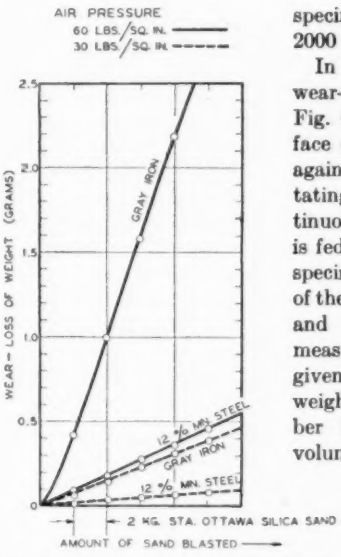


FIG. 9 EFFECT OF SANDBLAST PRESSURE ON THE RESISTANCE TO ABRASION OF IRON AND STEEL

In 1920, Brinell (14) built the wear-testing machine shown in Fig. 6, in which the plane surface of the test piece is pressed against the edge of a slowly rotating soft-steel disk, while a continuous stream of graded sand is fed between the disk and the specimen. After prescribed travel of the disk, the sample is weighed and the depth of impression measured. The wear value is given either in grams as loss of weight or in wear-resistance number $1000/V$, where V is the volume in cubic millimeters of the worn-out segment for 1 mm thickness of the disk. Brinell made a thorough investigation on the wear properties of steel of different chemi-

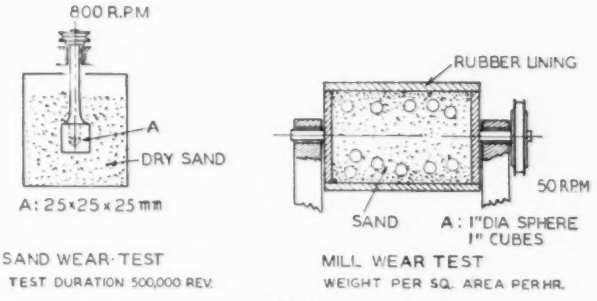


FIG. 10

cal compositions. In Fig. 7 are plotted data obtained by Rosenberg (15) on this machine when the effect of carbon content on the wear of carbon steels was studied.

The wear resistance of metals to abrasion by sand can also be obtained on some rather simple testing machines which might in some cases represent good approximation of actual working conditions. For example, the sandblast apparatus shown in Fig. 8 was used in experiments by Rosenberg (15) during which abrasion by sand of various grades of steel was studied. In Fig. 9 the wear is plotted against the amount of sand blasted at air pressures of 30 and 60 lb per sq in. During these tests gray iron was compared with 12 per cent Mn steel.

To the same class of wear-testing apparatus belong the two machines shown in Fig. 10. The sketch on the left shows a simple apparatus by means of which the wear of a specimen rapidly rotating in dry sand is measured after 500,000 revolutions. The right-hand sketch shows a slowly rotating mill filled with sand and ball- or cube-shaped specimens (15). The wear in these cases is expressed as the loss of weight in grams per square centimeter of surface per unit of time.

Another simple apparatus for similar application was described by Roesch (16), in which a small cylindrical specimen is fixed vertically at the bottom of a sand-filled container and a loaded hardened strip of steel moves back and forth over the specimen. Abrasion by this method was obtained for various grades of wear-resisting metals at pressures as high as 3000 lb per sq in.

In the same article Roesch reports on an experimental pump built with interchangeable vanes. The effect of the amount and

grade of sand added to the water was conveniently studied upon various vane materials. This is an example of a laboratory test which gives rather reliable data on the wearing qualities of various vane materials because these test conditions are very similar to those existing in field service. This small pump with a 150-mm wheel rotating at 3000 rpm was able to deliver water at a head of 35 ft.

There are several types of wear-testing apparatus on which the wear of a flat specimen is produced by the edge of a rotating disk.

Jannin (17) described interesting tests performed with the machine shown in Fig. 11, in which the test specimen was pressed toward the small rotating disk. The indentation produced by the latter on the specimen served as a measure of the wear. In experimenting with journal and bearing material, Jannin explained that (*a*) the cause of wear in bearing metals is due to insufficient shaft polish, and (*b*) the cause of wear of shafts depends greatly on the cleanliness of the lubricant used. Typical results obtained by Jannin are shown in Fig. 12. Phosphor,

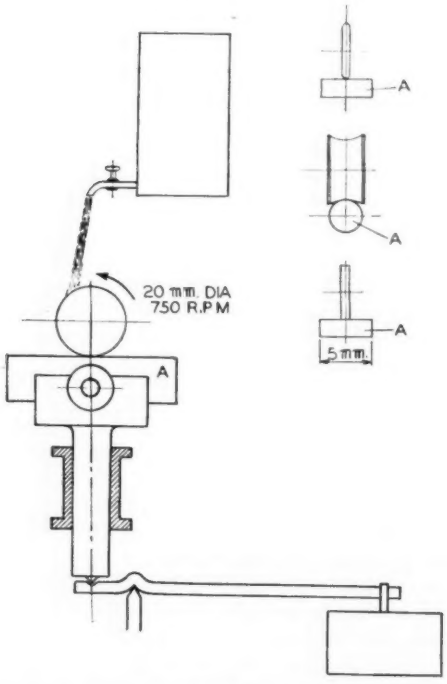


FIG. 11 JANNIN'S WEAR-TESTING APPARATUS (1922)

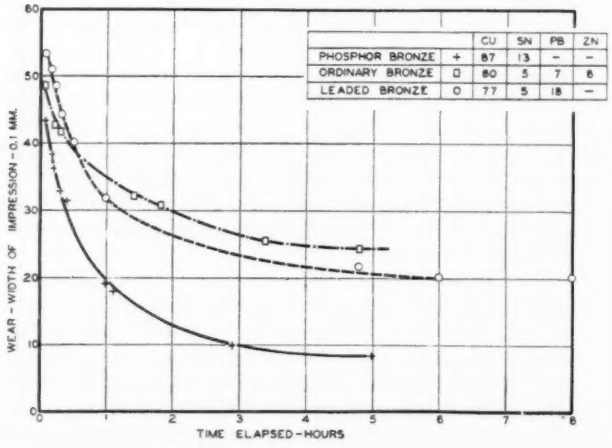


FIG. 12 WEAR TESTS OF BRONZES. ACCORDING TO JANNIN

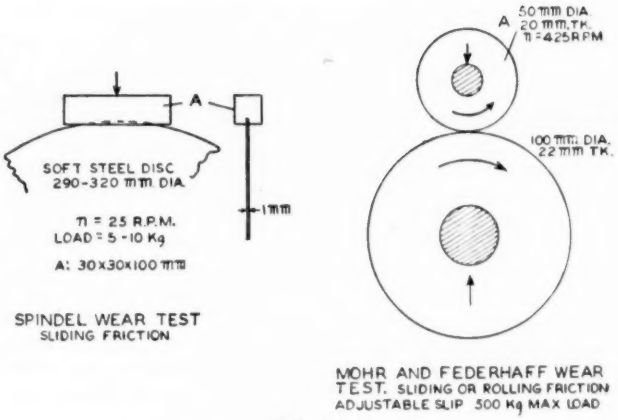


FIG. 13

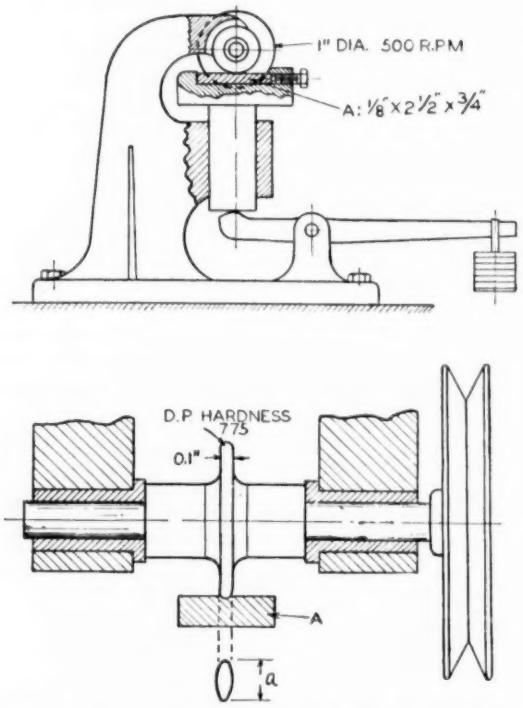


FIG. 14 BROWNSDON'S WEAR-TESTING APPARATUS—SLIDING FRICTION

leaded, and ordinary bronzes were tested and the wear measured; standard mineral oil was used during these tests. Guillet (18), using the same machine, investigated the effect on wear of such variables as speed, polish, and thermal treatment of the disk.

In 1924, Spindel built the machine shown in Fig. 13 for testing wear and grade of lubricants (19), primarily in connection with wear problems of rails and wheels. A great number of experiments made on this machine by Spindel (20), Schulz and Lange (21) for the Austrian Railroads proved to be of great value. A similar testing machine was designed by Sawin (22) for machine-tool applications; very hard materials were tested successfully on this machine.

Over a year ago, Brownsdon also reported on metallic wear (23). His machine, shown in Fig. 14 and consisting of a rotating disk pressed upon the test specimen, belongs to the same type of wear-testing machine used by Spindel. Fig. 15 shows results published by Brownsdon. The effect of various additions

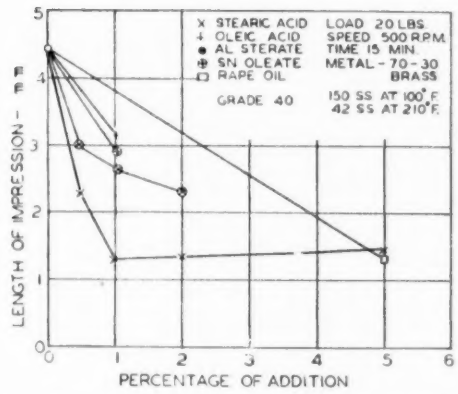


FIG. 15 EFFECT OF ADDITIONS TO MINERAL OIL ON WEAR

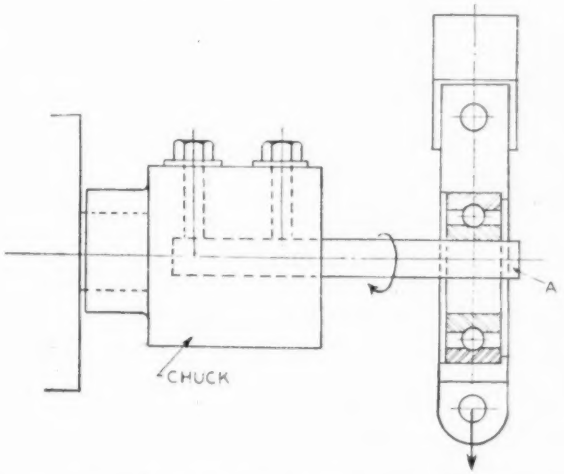


FIG. 16 SANITER'S WEAR-TESTING APPARATUS (1908)—ROLLING FRICTION

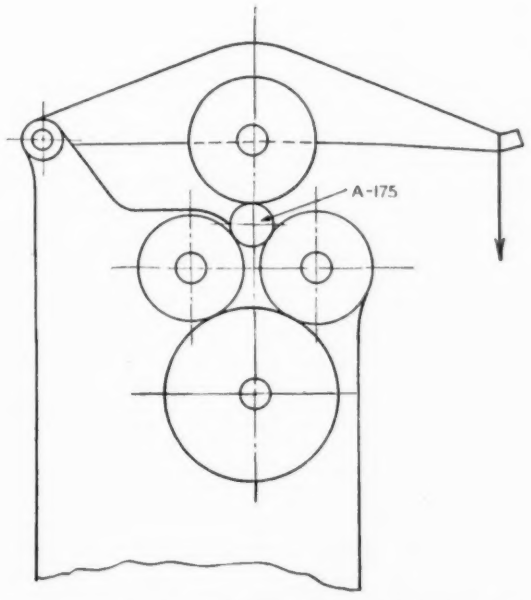


FIG. 17 STANTON'S WEAR-TESTING APPARATUS (1908)—ROLLING FRICTION

to mineral oils changes the properties of the lubricating mixture. Thus, it can be seen from the curves that the addition of all substances used resulted in a decrease in various proportions of the wear value, which is measured by the length a, shown is Fig. 14, of the impression made by the disk on the sample.

Several testing machines were built in order to produce wear due to rolling friction. Approximately at the same time, Saniter (24) and Stanton (25) made wear experiments in which a circular test specimen was subjected to pure rolling friction. Saniter's machine shown in Fig. 16, consisted of a journal specimen fixed in a revolving chuck. The hard inner ring of a large ball bearing served as the member which produced wear on the shaft. The rate of wear was obtained as the decrease in shaft diameter in ten thousandths of an inch after 200,000 revolutions and an application of a load of 205 lb. In Stanton's machine, shown in Fig. 17, the test specimen is pressed radially by three disks located 120 deg apart. The two lower disks are driven through a pair of gears which can be so selected as to produce a certain slip, in which case rolling and sliding frictions can be obtained.

The Amsler wear-testing machine is well known for its wide application when rolling, sliding, or combined rolling and sliding friction must be used during tests. Fig. 18 shows the principle of the Amsler testing machine. The machine by Mohr and Federhaff, shown in Fig. 13, is similar to the Amsler wear-testing machine, the only difference being in the manner of loading. Loading on the latter is obtained by means of a powerful helical spring, while a hydraulic arrangement is used on the Mohr and Federhaff machine.

The application of these machines is rather wide. Tests on it made by Fink (26) and later by Rosenberg and Jordan (27) were run in room atmosphere and in an inert gas. The latter tests indicate that a film of ferric oxide on the surfaces of carbon-steel rollers serves as a medium for preventing metal-to-metal contact. In the absence of these films (tested in nitrogen and hydrogen)

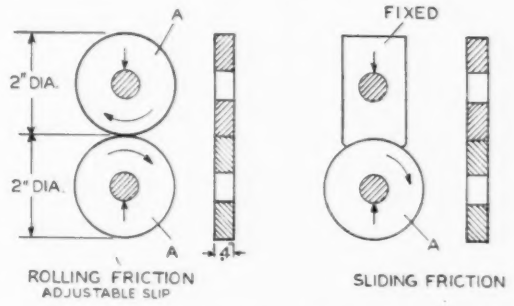


FIG. 18 SPECIMENS USED ON THE AMSLER-TESTING MACHINE
(Load = 20 to 300 kg, adjustable speed.)

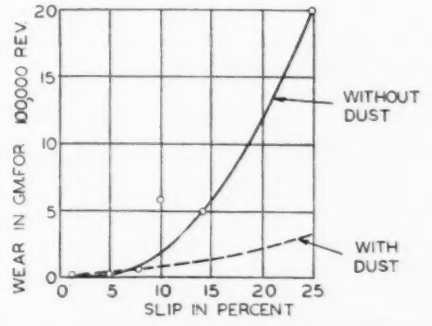


FIG. 19 AMSLER WEAR-TESTS SHOWING EFFECT OF REMOVING DUST FROM LOWER DISK
(Brinell hardness = 700.)

severe wear takes place. The effect of removing dust from the lower disk of the Amsler machine is shown in Fig. 19. This test was performed at various rates of slip (28).

The wear-testing apparatus shown in Fig. 20, using ring-shaped specimens, was built by Mayer (29) in 1925. Since the specimen is entirely submerged in lubricant it is possible to control and study the effect of temperature closely. On this machine the torque and coefficient of friction are also measured during tests. Fig. 21 shows results obtained by Kehl and Siebel (30) when pearlitic cast-iron specimens were used during tests.

Zaitzeff (31) and Suzuki (32) also designed wear-testing machines, employing ring-shaped specimens, on which they have obtained good results.

The wear-testing machine, shown in Fig. 22, was built at the Westinghouse Research Laboratories for determining rather high rates of wear. A small cylindrical rotating test specimen *A* is pressed against the 2-in-wide circumference of a large hard-steel disk *D*. The steel disk revolves and also oscillates along its axis, due to which action almost the entire disk surface comes in contact with the rotating specimen. In connection with the selection of a suitable heat-treatment for a crankshaft material, a study of wear resistance of three case-hardened steels was made on this machine; data on the hardness and treatment of these steels are shown in Fig. 23. After each 5-min period of test, the

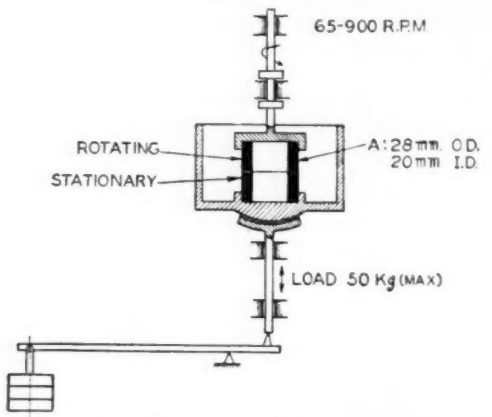


FIG. 20 WEAR TEST WITH RING-SHAPED SPECIMENS—SLIDING FRICTION
[Mayer (1925), Suzuki (1928), and Zaitzeff (1930).]

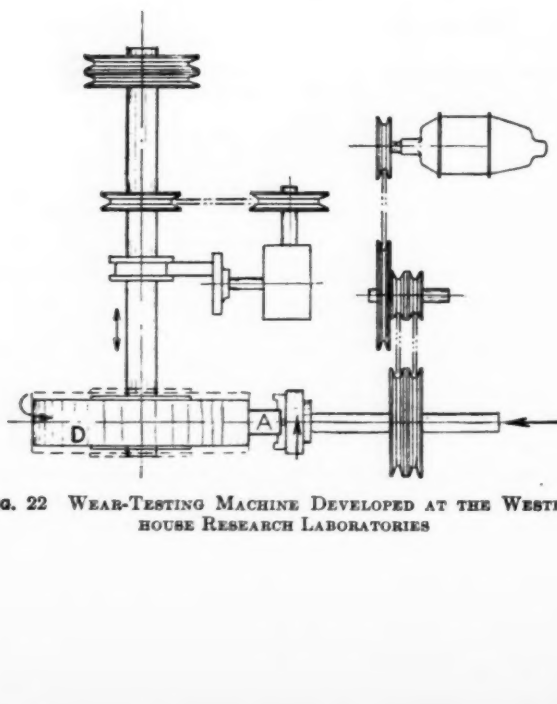


FIG. 22 WEAR-TESTING MACHINE DEVELOPED AT THE WESTINGHOUSE RESEARCH LABORATORIES

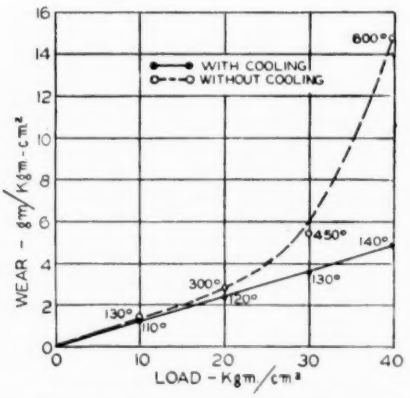


FIG. 21 EFFECT OF LOAD AND COOLING FOR PEARLITIC CAST IRON
(Dry-friction, speed = 3.7 m per sec, temperatures in deg C.)

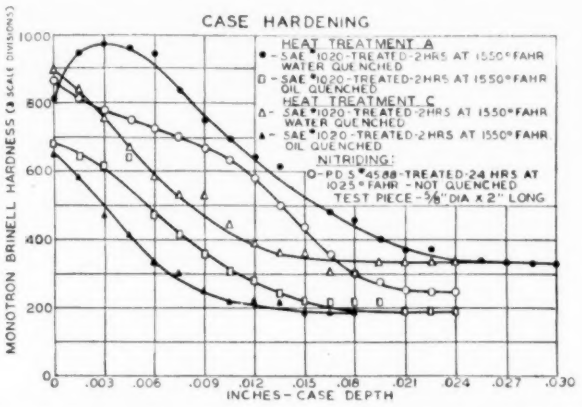


FIG. 23 BRINELL HARDNESS AT VARIOUS CASE-HARDENING DEPTHS FOR THE STEELS AND TREATMENT GIVEN IN THE LEGEND

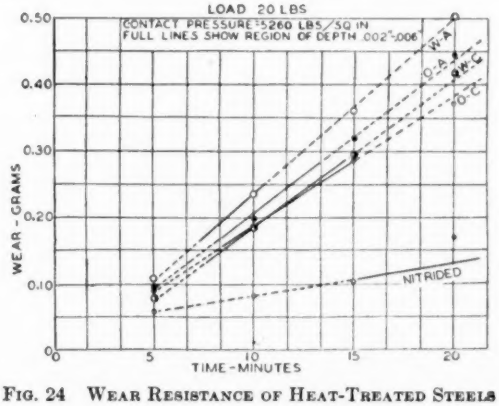


FIG. 24 WEAR RESISTANCE OF HEAT-TREATED STEELS

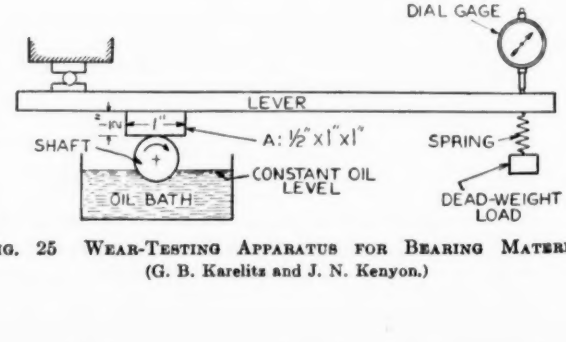


FIG. 25 WEAR-TESTING APPARATUS FOR BEARING MATERIAL
(G. B. Karelitz and J. N. Kenyon.)

samples were weighed and their length measured. As shown in Fig. 24, results obtained revealed that the material which received nitrided heat-treatment has a much greater wear resistance than the same material heat-treated otherwise. The curves also show the wear at depths between 0.002 and 0.006 in.

Karelitz and Kenyon (33) reported on tests made with the apparatus shown in Fig. 25, in which flat-faced pieces of several bearing metals were mounted to ride on a small journal lubricated by means of an oil bath. The initial lines of contact gradually developed into grooves until viscous oil films formed between the rubbing surfaces and arrested further wear.

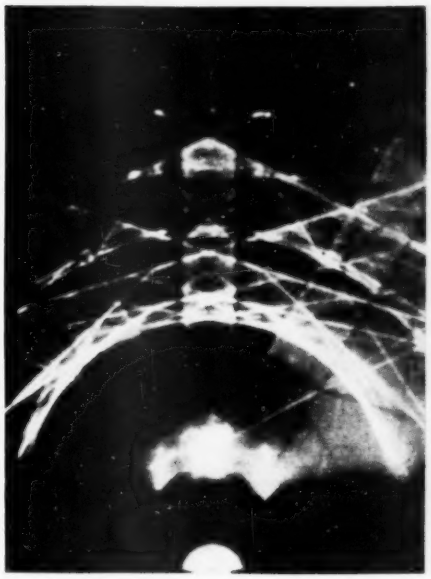


Fig. 27 Electron-Diffraction Pattern From the Natural Face of a Diamond

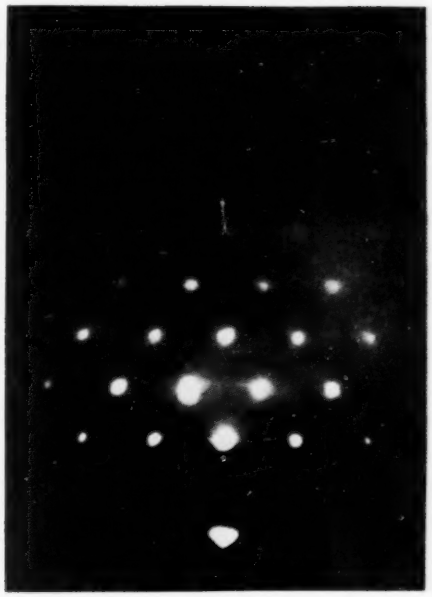


Fig. 28 Spot Arrangement of Electron Diffraction From a Rough Single-Crystal Surface

Conclusion

Although this short review of wear-testing methods and apparatus is far from being complete,[4] it shows the variety of methods of testing the wear properties of engineering materials.

All the machines mentioned were developed in view of new service conditions of various machine parts. The most frequently used fundamental methods of laboratory wear testing are shown diagrammatically in Fig. 26.

In making a choice of each type of wear testing, a careful study must be made of the performance of the testing apparatus in order to control at least a few variables or conditions which are also common in actual field application of the material. Thus,

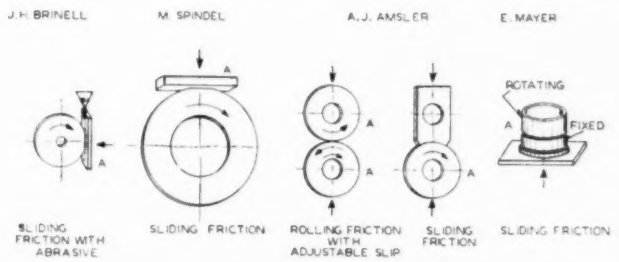


Fig. 26 Important Types of Wear Testing

for example, the sandblast method and Brinell machine will be properly selected for wear tests of materials for water-turbine blades since their wear from sand is sometimes very alarming. On the other hand, this type of test alone will be insufficient because the wearing of these blades is also due to cavitation.

The selection of methods of wear test depends in the first place on the object of the investigation. When the test results are to be applied for practical purposes, the methods must be selected so as to duplicate, as much as possible, the field conditions. When the object of the investigation is to study the behavior of the material under various types of friction, such methods must be chosen which will enable easy alteration and permutation of some of variables and study of influence of segregated factors.

It is the aim, therefore, of the individual experimenters to select such a machine or number of machines and produce such testing conditions as will cause the short-lived performance of the tested material to be representative of its performance in practice.

[4] A good Bibliography on wear, covering the period from 1864 up to 1928, inclusive, can be found in the article by Füchsel (34).

Appendix

In addition to this condensed review of wear-testing machines and methods of evaluating materials from the point of view of wear resistance, it will be interesting to mention another method peculiarly well adapted for the study of surface phenomena, namely, the electron-diffraction method (35, 36).

Unlike X rays, electrons owing to their charge are so easily deflected by the atoms in any aggregate of matter on which they are impinged that their penetration is limited to the surface layers. In practice, a beam of electrons moving at a velocity of the order of half that of light is caused to glance off the surface under examination, and the direction of the diffracted rays leaving the surface are recorded photographically as a so-called electron-diffraction pattern. Since these diffractions are determined by the manner in which the atoms are arranged in the surface layers, it is possible to derive the structure of the surface from the diffraction pattern. The pattern in its details usually reflects the degree of the order of atomic arrangement in the surface.

For example, a smooth single-crystal surface, in which all the atoms are regularly arranged at the points of a space lattice, gives rise to a pattern of spots and regularly arranged parallel pairs of black and white lines, from which the atomic arrangement can be deduced. Thus, Fig. 27 represents an electron-diffraction pattern from a natural face of a diamond. A rough single-crystal surface, on the other hand, may yield only a regular pattern of spots, as shown in Fig. 28, the sharpness of which will depend on the shape of the projections rising from the surface. The spot arrangement is characteristic of zinc blend.

As was first shown by Beilby (34), the surface finish put on a metal by polishing is quite different in its structure from that afforded by abrasive methods. In this case, the surface is smoothed to a state of high finish by smearing over the surface a molten or viscous material formed at the "high spots" as a result of intense heating due to high localized stresses. This fluid material congeals in stress-free regions so rapidly that the atoms

have no time to rearrange themselves into crystal units, but remain in disordered array just as they would be in an instantaneously frozen liquid. This noncrystalline amorphous layer, known as the Beilby layer (1921), gives rise to such a diffuse halo pattern as shown in Fig. 29, which is characteristic of a liquid surface.

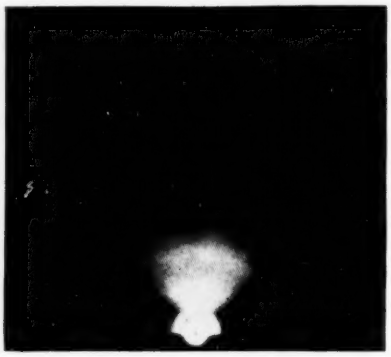


FIG. 29 ELECTRON-DIFFUSION HALO PATTERN FROM A NONCRYSTALLINE AMORPHOUS LAYER, CHARACTERISTIC OF A LIQUID SURFACE

Undoubtedly, such use of the electron-diffraction method will be as much welcomed in industry as it is in physical laboratories.

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35 "The Beilby Layer," by G. I. Finch, *Science Progress*, vol. 31, no. 124, April, 1937, pp. 609–625.

36 "The Study of Wear and Lubrication by Electron Diffraction," by G. I. Finch and F. D. Zahoorbux, General Discussion on Lubrication and Lubricants, Group IV, the Institution of Mechanical Engineers, London, October 13–15, 1937. For an abstracted report of this paper see the Proceedings of the Institution of Mechanical Engineers, vol. 136, 1937, p 153.

Discussion

W. E. CAMPBELL.[5] A very useful supplement to the present paper would be one which concerns itself mainly with a critical appraisal of the more important experimental work described. Although it is true that the electron diffraction method has shown itself to be exceedingly useful in studies of surface layers, it should be made clear that the interpretation given by the author of reflection patterns, such as that in Fig. 29, is by no means generally held by experts in the field. Kirchner[6] and Germer[7] have both described experiments indicating that diffuse reflection patterns do not prove that the surfaces are amorphous. Even the leading proponent of the author's interpretation, G. I. Finch,[8] has recently modified his views to admit that diffuse rings produced by electrons scattered from polished metal surfaces are not conclusive proof that the polish layer is amorphous.

B. F. HUNTER.[9] In this paper the author evidently has made a thorough investigation of the wear of metals not lubricated, rather than wear of metals lubricated.

It is the writer's conclusion, from experience, that wear on surfaces of lubricated metals has little relation to the wear on metals with dry surfaces.

In the operation of mechanical equipment, where the surfaces of the metals are lubricated, wear, frequently attributed to lubrication, can be traced definitely to: (1) Improper design; that is, pressures, etc., per square inch beyond the load-carrying capacities of the lubricant; or (2) wear due to an improper lubricant; or (3) improper application of a lubricant.

G. B. KARELITZ.[10] The importance of the wear problem is self-evident. The reporter of *London Engineering*, covering the symposium on wear, held in Atlantic City last spring commented on the bewildering array of testing apparatus presented there, and on the apparent wide divergence of reported results. He concurred and praised the reluctance of the researchers to generalize their findings. The practical recommendation of the author that the individual experimenter should select a machine or number of machines, which would produce such testing conditions as will cause the short-lived performance of the tested material to be representative of its performance in practice, is noteworthy. The difficulty arises of knowing what is its performance in practice.

The 1937 International Conference on Lubrication held in London was remarkable in one respect. For the first time, lubrication was discussed at a joint meeting of engineers, physicists, and chemists. Any further advance in the question of wear must come now from the physicist. It is not sufficient to know how much material is lost by friction or abrasion. As history shows, this road leads in too many directions. It seems that some order would be brought into the picture, if more were known of the physical phenomena taking place during the disintegration of material. It would then be possible to classify the wear and, probably under such classification, results of various tests of the same class could be compared. It is apparent that the author had the same feeling when he added the appendix describing the electron-diffraction method.

Tomlinson in England called attention to the phenomenon of glass being torn up by a sapphire similar to that shown in Fig. 5 of Claypoole's paper.[11] He got it even with a glass bead on a glass plate. He visualized the phenomenon as molecules being knocked out "as teeth by a dentist's chisel." He then developed his molecular theory of friction. Later Bowden presented his interesting work on microscopic zones of extremely high temperatures on the surfaces of rubbing metals. One can easily see that both these physical phenomena have much to do with the question of wear.

In the writer's experiments mentioned by the author, wear occurred under conditions of good lubrication between the rub bing surfaces. (It may be mentioned here that the machine was built earlier for wear tests by Connolly) The test specimens were prepared after the tests so that the cross section of the worn tract could be examined at the surfaces. This work was done by R. M. Sample of the Bell Telephone Laboratories. It transpired that different metals wore quite differently under the same conditions.[12]

Bronze is smeared in the direction of motion to a depth of 0.0015 in. approximately; with babbitt, the matrix is melted away and a pavement of broken tin-antimony crystals is riding on the journal, this material apparently resisting the wear; with pure silver, the crystals break up into something like shreds, these being abraded by the journal; with cast iron, there is no sign of damage to the crystals, apparently the mechanism of abrasion being very close to the "dentist's chisel" scheme of Tomlinson.

It is apparent, that only when the various mechanisms of surface disintegration are studied thoroughly by physicists and understood, that some degree of order in the question of wear testing may be expected. Meantime, such reviews as presented by the author are of real service to the art in guiding those who must give practical answers to current problems brought to them by the industry.

S. KYROPOULOS.[13] The title of the paper has been the subject of a great deal of the writer's research work for the last few years. This work aims at the exploration of the fundamental phenomena of boundary lubrication, oiliness, and wear, including the influence of bearing materials. It consists in a critical examination of existing work on boundary lubrication from the point of view of modern molecular physics, ultimately culminating in the question: Are the phenomena of boundary friction and molecular orientation on surfaces correctly interpreted and consistent with concepts of modern chemical physics, of primary importance for the technical problems of oiliness and wear?

Such studies have been supplemented by experimental studies of the process of wear on varying the physicochemical conditions, including the lubricant. This led to definite conclusions, notably on the role wear particles play in the progress of wear. Other experimental work has thrown light on the questions of polish, wear, and seizure, as influenced by the lubricant.

It may suffice to say that, notwithstanding the greatest appreciation of the experimental work on molecular orientation, etc., the writer has come to the conclusion that the answer to the above question is definitely negative.

[5] Bell Telephone Laboratories, New York, N. Y.

[6] "Electron Reflection on Polycrystalline Metal Layers and Surface Structure of Polished Metals," by F. Kirchner, Trans. Faraday Society, vol. 31, 1935, p. 1114.

[7] "Diffuse Rings Produced by Electron Scattering," by L. H. Germer, *Physical Review*, vol. 49, 1936, p. 166.

[8] "Electron Diffraction and Surface Structure," by G. I. Finch and H. Wilman, Ergebnisse der exakten Naturwissenschaften, vol. 16, 1937, p. 424.

[9] Chief Lubricating Engineer, Gulf Oil Corporation, Pittsburgh, Pa.

[10] Professor of Mechanical Engineering, Columbia University, New York, N. Y. Mem. A.S.M.E.

[11] "Thin Oil Films," by Walter Claypoole, Trans. A.S.M.E., vol. 61, 1939, page 323.

[12] "On the Mechanism of Dry Friction," Timoshenko Anniversary volume, by G. B. Karelitz, The Macmillan Company, New York, 1938. See photomicrographs on page 104.

[13] Laboratory of Physics, California Institute of Technology, Pasadena, Calif.

The title of the present paper suggests at first sight a more fundamental treatment of the matter and in an appendix two methods which contributed to the studies of surfaces are briefly outlined. This review and bibliography would appear to be useful for immediate practical needs as well as an aid in interpreting practical experience.

S. A. McKee.[14] Because of the present lack of fundamental knowledge of the effects of the various factors involved in any particular wear problem, it is necessary, as the author points out in his conclusion, to simulate actual service conditions as closely as possible. In some cases when this is done, the wear that occurs is so small or the shape of the piece is such, that to obtain a measurement of wear with significant accuracy offers a serious problem in itself. In this connection an investigation is being conducted at the National Bureau of Standards to determine the practicability of an indentation method for measuring wear. With this method the wear is determined by measuring the changes in dimensions that occur to marks or indentations placed in the surface under test. These marks are so shaped that as wear occurs their length is reduced by an amount proportional to the thickness of material removed. These marks might be considered as standardized toolmarks since for a number of years it has been common practice to use the appearance of the toolmarks on a piece of machinery as an indication of wear.

This investigation has been confined to the application of the method to the study of wear on the cylinders and pistons of aircraft engines. A cylinder or piston to be tested is placed in a special fixture and marks at any desired position on the surface are applied to a predetermined depth by means of a specially ground diamond point, which is forced by mechanical pressure through suitable cams and levers into the surface of the piece. The diamond is similar in shape to the Knoop Indentor, which has been recently developed at the National Bureau of Standards for the measurement of the hardness of materials. The mark it makes is in the form of an inverted pyramid with its base lying in the surface of the material.

The viewing and measuring apparatus consists of a conventional microscope and eyepiece scale, except that the tube is lengthened and two right-angle prisms inserted to permit use as a periscope to view the inside of the cylinders. As viewed by the microscope the base of a mark as originally made is in the form of a rhombus with its long axis about five times the short axis. For marks 0.001 in. deep the long axis is about 70 scale divisions (0.028 in.). As the surface is worn away the shape of the mark remains the same but the length decreases in proportion to the thickness of material removed.

The method has been tried in several full-scale engine tests and the results indicate considerable promise as to its practicability and usefulness. Even under the severe operating conditions present, the marks in general retain their shape well enough to provide an accurate determination of wear. The particular advantage of the method in its present application is that it provides an indication of wear only, while the usual measurements of changes in diameters of the pistons and cylinders do not differentiate between wear and such distortion as may have occurred during the test. A further advantage is that it does not require the careful technique and elaborate gaging apparatus necessary to measure the diameters with corresponding accuracy.

At the present time the study of the behavior of the marks in service is being continued. A calibration of the indentations is also being made. The changes in lengths of marks placed on flat blocks of various materials are being compared with direct measurements of change in thickness occurring when material is lapped from the surfaces.

[14] Mechanical Engineer, United States Bureau of Standards, Washington, D. C.

B. L. Newkirk.[15] This paper recalls to mind certain unreported tests made some years ago in the research laboratory of the General Electric Company of the wear of carbon blocks on smooth steel surfaces. In the first tests there was a rotating drum, carrying four rings of $3^1/_2$ per cent nickel steel $20^1/_2$ in. in diameter. Carbon brushes of various compositions, held in brush holders, were pressed by springs against these rings and the drum was run for periods of days or weeks without stopping. Spring pressure and speeds were varied. Rubbing speeds in the longer runs were 125 to 150 fps (7500 to 9000 fpm). The steel rings were well polished and at the start of each run the carbon blocks were faced off squarely and smoothed or polished on the bearing end.

Friction was relatively high in the beginning but this would fall off rapidly as a track was worn in the carbon and, in general, after a period of from 3 or 4 to 24 hr, depending on the material, wear would nearly cease. The area of the track developed by wear depended upon the composition, being smaller for brushes made of flake graphite. Eight pieces were run at a time in eight brush holders, two on each ring. The spring pressures holding the brushes against the rings were different for different brush holders, ranging from $6^1/_4$ to $7^1/_2$ lb. At the end of a run of, say, one week, the areas of the tracks were measured and the unit pressures over these areas were calculated and averaged. The figure so obtained was taken as a criterion of the wearing quality of the carbon. Values of these unit pressures ranged from 6 to 20.7 lb per sq in. In the latter case the worn and polished track was 1 in. wide and approximately $^1/_3$ in. long (in the direction of rubbing).

The friction coefficient was very low (0.001 to 0.003) and of the order of errors of determination, when the brushes were well worn in. Under this condition the load seemed to be supported by an air-film bearing. The wearing quality was determined by the manner in which abrasion occured while the brush was wearing in; i.e., presumably the size and shape of the pits left by the abraded particles.

At a later date, tests were made in which a complete (jointed) ring of graphite was run against a steel collar. In this case conditions were not favorable to the formation of an air-film bearing. With pressures of 1 or 2 lb per sq in., the coefficient of friction was around 0.1. It was found impossible to maintain a polish on the graphite with the 2-lb unit pressure.

At that time small blocks of graphite, having areas of 1 sq in., more or less, were tried as air-film bearings. It was found that they would carry loads of 5 lb or more. These air-film bearings proved, however, to be sensitive to disturbances resulting in loss of the polished graphite surfaces and consequently of the bearing power of the air film.

R. J. S. Pigott.[16] While the paper is valuable in explaining the differences in wear-testing apparatus, the title may prove a bit misleading. The behavior of metals rubbed against each other to produce abrasion, or one metal rubbed against another material to produce the same result, is only, in a very small degree, similar to what takes place in a bearing which has never had any oil or greasy substance in it. Anyone who has been engaged in turning or boring dry cast iron which strikes an oily spot will know what this means, because the tool behaves entirely differently in light cuts, sometimes refusing to enter the work where the oil patch exists.

[15] Professional Engineer, Schenectady, N. Y. Mem. A.S.M.E.

[16] Staff Engineer, Gulf Research & Development Company, Pittsburgh, Pa. Mem. A.S.M.E.

The mechanics of wear in bearings generally follows quite a different path, when there is a lubricant of any kind present, than in dry abrasion. It seems to the writer, therefore, that care must be exercised, in thinking about experimental work on dry abrasion, as to how this is coupled up with lubrication. Aside from the fact that the process of abrasion, or particularly of ordinary wear in a bearing, does not appear to be very closely related to dry abrasion, in any form it is greatly dependent upon the roughness of the surfaces involved, or in the case of a foreign abrasive material introduced between two rubbing surfaces, on the size of the abrasive particle. For example, in testing pumps for oil wells, where sand is present, it has been found that, in a closed system where the sand suffered breakdown to smaller particles, the wear first accelerated (the metal was first roughened), and, if this action was carried far enough, the rough metal did most of the remaining wrecking, regardless of the abrasive. If, however, the abrasive did not roughen the metal much, then as the sand was reduced in size by crushing, wear would again decelerate and finally come almost to a stop and the surfaces would polish instead of abrading.

The practice used thirty years ago in some of the large reciprocating-engine bearings of that day illustrates this sort of effect. After reboring and scraping a large bearing, it was quite often the custom in some of the old reciprocating power stations to feed in Sapolio with the lubricating oil. This is of course a soft abrasive which breaks up quickly, and used for the purpose mentioned, it would rapidly condition the bearing to a smooth matt surface by the time it had practically disappeared, and then would be filtered out in bag filters, practically as a sludge with no appreciable abrasive properties.

In recent accurately made tests, which unfortunately are not yet extensive enough to publish, it has been found that there is wear going on without any metallic contact and certainly with no abrasive present. Further, the places in which this wear takes place, as evidenced by measurable changes of radius of the bearing, are not the same for different speeds and different loads. There is an optimum shape which the bearing will assume in time and this corresponds with the running-in period. Finally, a constant lowest friction coefficient is reached. In some cases this coefficient has decreased to 1 per cent of the initial value and the bearing, when measured, is not round. The out-of-roundness is not a question of deflection, because the bearing housing is rugged and stiff. It is simply a case of some wear or more probably plastic flow in the high-pressure regions.

It is the writer's opinion, therefore, that the study of dry friction testing and abrasion is likely to be only of academic interest as far as lubrication is concerned. It allows us the opportunity of comparing dry frictions with the much lower values when lubricated but it does not seem to have any direct connection with the lubricated bearing.

R. G. Simard and J. C. Geniesse.[17] The author in his résumé of laboratory wear testing shows still more emphatically a truism which cannot be belabored sufficiently; namely, that significance can be attached to laboratory wear testing only in those cases where the method employed simulates closely the particular applications, or where a full analysis or knowledge is available. The latter is seldom if ever true, and therefore the test method must be carefully chosen as to type.

As a specific example of the type of data which must be correlated by careful analysis, the writers will cite some results obtained with three machines in the laboratories with which they are connected, each of a different type which shall be designated *A*, *B*, and *C*. *B* and *C* are the Falex or Cornell machine and the bearing-material machine of Karelitz, respectively. An oiliness, and a film-strength agent were tested on each of these, under comparable conditions. The results are as follows:

Machine	Film-strength agent 1	Oiliness agent 2
Type *A* (reciprocating)	Increased friction	Decreased friction
Type *B* (Cornell)	No effect on friction	Further reduction of friction
Type *C* (Karelitz)	Decreased wear; Smoother surfaces	Ineffective

Conflicting results such as these emphasize the importance of choosing conditions closely simulating those in the type of service under consideration. What is still more important, correlation must be obtained before conclusions are drawn.

Karelitz and Kenyon published data[18] obtained on a wear machine, which indicated that butyl stearate had no effect on the minimum film thickness required to support a given load without wear. This was surprising in view of the oiliness, or friction-lowering effect attributed to this compound. Evidently this is an excellent case showing that oiliness and wear reduction do not go hand in hand. Further evidence of this lack of correlation is shown in the following table wherein minimum film thicknesses, h_0, are given for two oils, one of them containing an E.P. agent. Test procedure was similar to that described by Karelitz and Kenyon.

	S.A.E. 30 oil	S.A.E. 30 oil plus E.P. agent
20-lb load		
No. 62 bronze	0.0000198	0.0000124
No. 67 bronze	0.0000197	0.0000145
Cast iron	0.0000212	0.0000165
60-lb load		
No. 62 bronze	0.0000135	0.0000073
No. 67 bronze	0.0000143	0.0000119
Cast iron	0.0000117	0.0000097

Since for a given load the track width is proportional to the minimum film thickness, the E.P. agent must very definitely reduce the tendency to wear due to excessive loads. This E.P. agent does not exhibit oiliness properties when tested in conventional machines developed for that purpose.

Author's Closure

The author is indebted to Mr. Campbell for his remarks pertaining to the interpretation of the diffuse rings produced by electrons scattered from polished metal surfaces.

As mentioned by Mr. Hunter, wear of lubricated surfaces is due to improper condition of design, lubricant and application. However, a perfectly designed bearing, performing under conditions of fluid-film lubrication, will at some time be subjected to wear, the magnitude depending upon the frequency of stopping and starting the machine. For a very short duration of time, conditions of boundary friction and sometimes even dry friction will prevail. It is the author's opinion that wear properties of practically all materials of bearing and journal combinations must be thoroughly investigated, in order to select such a pair of materials as will cause least wear under unavoidable conditions of boundary lubrication.

The experiments by Bowden, to which Professor Karelitz refers, stress the importance of boundary friction which might be associated sometimes with very high localized temperatures. The author can only support the statement of Professor Karelitz regarding a common front of investigation of wear problems by engineers, physicists, and chemists. This undoubtedly will lead to a better understanding of the complex mechanism of wear.

As pointed out by Mr. Kyropoulos, author of many papers on boundary friction, the fundamental phenomena of semifluid lubrication, oiliness, and wear, including the influence of bearing materials, are not always properly interpreted. Very often practice does not support existing theory.

Mention of unreported experiments by Messrs. McKee and

[17] Members of the Research and Development Department, The Atlantic Refining Company, Philadelphia, Pa.

[18] Bibliography of author's paper (33).

Newkirk indicate that the question of wear is engaging the attention of many institutions and investigators.

Mr. Pigott states that the study of dry friction and wear testing is likely to be only of academic interest in so far as lubrication is concerned. Regarding this, the author can stress the consideration, previously mentioned, of the effect of boundary friction upon starting and stopping. Under these conditions, wear occurs in all types of bearings which most of their life perform in the region of perfect fluid lubrication.

The importance of selecting the type and method of wear test, in order to obtain useful data, is realized by everybody concerned with the wear question. From this point of view, the example of correlating data reported by Messrs. Simard and Geniesse is worth noticing.

Discussion

Rotary Oil Meters of the Positive-Displacement and Current Types [1]

ED S. SMITH, JR.[2] The excellent performance of oil quantity meters, coupled with their moderate first cost, leads to attempts to use them over a wider field of application than can be justified from a consideration of maintenance costs, as has been fairly brought out in the paper. The writer's experience with head meters includes numerous cases involving gassy, dirty, or corrosive liquids where such meters were found more suitable than quantity meters of the displacement type.

The rotary impeller (or current) type of quantity meter promises to have wide usefulness in bridging the gap between the head and displacement meters in the oil industry. However, before it can do so properly, the analysis of performance should be related to another rational operator, in addition to the Reynolds number, instead of the present purely empirical use of the Saybolt viscosity. It is to be hoped that the discussion or closure of the paper[1] will bring out such a rational dimensionless operator.

The quantity meter can be combined with a "speedometer" of one sort or another and thus indicate flow rate as well as quantity. The wide range and practical freedom from viscosity effects make such a combination attractive, at least from a theoretical standpoint. The question of maintenance, always a real factor in practice, moreover, seems to be fairly met with the impeller (or current) meter which may provide one part of the combination more widely in the future.

CHARLES S. HAZARD.[3] The graphs in Figs. 2, 7, and 8 of the paper[1] show the variations in registration of two types of oil meters caused by variations in the rate of flow and in the viscosity of the liquid being measured. At first glance, it would seem that the combined effect of these two variables would produce errors of such magnitude as to make the meters unsuited for the service intended. A more careful analysis of this question will show that this is not the case.

At points to the right of the peak on the registration curves, any of the meters considered by the author will give consistent readings on repeated tests, the deviations being inappreciable at the scale used. A meter of either of these types, used with a liquid of reasonably constant viscosity and operated within a reasonably close range of rates of flow, and which has been calibrated to register 100 per cent at the mean rate of flow and mean viscosity of the liquid used, will give consistently uniform and accurate registration. All commercial types of meters are provided with devices for changing the calibration, consisting either of gearing to vary the ratio between the measuring element and the visible register, or mechanical devices for varying the displacement of the measuring element per cycle. The effect of all such devices is to shorten or lengthen all ordinates of the calibration curve in the same proportion, thereby making it possible to bring this curve into the best possible conformity with the axis of zero error. This will not correct the deviation of the registration curve from a straight line; but the author's results show that a meter tested on a low-viscosity liquid will, in general, show less of such deviation when calibrated for a liquid of higher viscosity.

In actual practice, the necessity for taking into account the variation in registration with variation in viscosity arises in the two following cases: (1) Where a meter originally tested by the manufacturer with a liquid of one viscosity is to be put in service on a liquid of different viscosity; and (2) where a meter which has been used on one type of liquid is transferred to another service involving a different liquid of considerably different viscosity. In the first case, the manufacturer can furnish definite instructions for adjusting the registration of the meter, using the means provided. The second case is ordinarily handled by recalibrating the meter in a standard test measure at the time of the changeover. In the handling of petroleum products there are very few installations in which one meter is called upon to measure liquids of widely varying viscosities, as this would require cross connections between piping handling entirely different products with the certainty of contaminating one or the other product.

In the case of the rotary-sliding-vane displacement meter, the author's statement that "Tests show no appreciable effect of viscosity variation within wide limits," was very interesting, as the writer's tests have shown some variation with viscosity with this type of meter. The same is true of the oscillating circular-piston type of meter, which is extensively used in the measurement of petroleum products, and which was not covered in this paper.[1] This type has a characteristic calibration curve similar to that given for the rotary-sliding-vane type.

The method used in testing the meter should be described. If a standard test measure were used, it should be stated whether tests were made with the inside surface of the test measure dry, or wet with the liquid to be measured. Particularly in the case of extremely viscous liquids, an appreciable percentage of the liquid adheres to the interior surface of the test measure, and, in effect, reduces the capacity of the standard measure. This produces an apparent underregistration on the meter which tends to compensate for the increase in registration due to the increased viscosity. Another interesting effect occurs in the case of low-viscosity liquids which are highly volatile at ordinary temperatures, such as aviation gasoline. In this case, there is an evaporation of the volatile constituents of the liquid while discharging into the test measure, which tends to produce an apparent overregistration on the meter, which compensates for the underregistration due to the low viscosity. The combination of these two effects tends to conceal the variation in the registration of the meter with change in viscosity.

This raises the interesting question as to how a volumetric displacement meter for petroleum products should be calibrated. Should a meter for viscous liquids be calibrated to record the amount which would fill a standard measure to the mark, starting with the measure dry, or should the meter record the amount which would be delivered from a standard measure, filled to the mark? Should a meter for a highly volatile grade of gasoline be calibrated to record the amount actually passing through the meter, or the visible amount delivered into a standard test measure?

M. F. BEHAR.[4] Apart from (1) the intrinsic merit of this

[1] Published as paper PRO-60-3, by Everett M. Cloran, in the November, 1938, issue of the A.S.M.E. Transactions, vol. 60, p. 617.

[2] Hydraulic Engineer, C. J. Tagliabue Mfg. Co., Brooklyn, N. Y. Mem. A.S.M.E.

[3] Chief Engineer, Neptune Meter Company, Long Island City, N. Y. Mem. A.S.M.E.

[4] Editor, *Instruments;* founder and director, The Instrumentation Manual Project, Pittsburgh, Pa.

paper[1] as a technical presentation, (2) the light which it sheds upon a field of measurement which had not been explored as thoroughly as adjoining fields, and (3) the interest attending the description of a meter which though employing a new principle is offered in a highly developed state of design and construction, this paper is of importance because what it represents may profoundly affect both the practice of metering viscous fluids in industry and the attitude of the engineering profession toward the choice of methods and the choice of meters.

About the only major criticism that one could make of this paper is that it should have been two distinct papers—one dealing with current meters and one dealing with positive-displacement meters. Each of these, of course, should deal first with the general category and then lead up to the engineering description of the improved specific type.

And yet, it may well be that the author deliberately prepared a unified treatment of related aspects of metering with positive meters and metering with current meters. Although the writer's admiration for many features of the paper and of the devices themselves tempt him to be specific, he will discuss only the broader aspects, that is, those which may have economic significance—tremendous economic significance, one may predict, not only in the metering of viscous fluids but in the automatic control of processes involving flows of viscous fluids. Removal of obstacles toward the complete automatization of these processes will affect not only the petroleum industry but many others.

For 17 years—ever since the publication of its first report—the A.S.M.E. Fluid Meters Committee has laid down a definition of positive[5] meters which specifies that "no fluid can pass through such a meter without actuating the primary device." This rigid definition has created a rather paradoxical situation among engineers concerned with specifying, using, or manufacturing quantity meters, because of the practical impossibility of making truly "positive" commercial meters of certain types that are in demand by reason of their convenience, simplicity, and low cost. In the oil and other meter-using industries, the vague classification and loose terminology of former days are giving way to engineering specifications. Not so long ago, the men who specified positive meters did not belong to engineering societies, did not know that a representative committee had laid down a scientific set of unambiguous definitions in 1922; today they not only know it but some of them have discovered that the report, like certain laws, "has teeth in it" and they sometimes demand the impossible. The greater the proportion of industrial engineers thus "armed" with the report, the more serious the situation grows. Scrupulous sales engineers, who honestly admit that some of their meters are not truly "positive," suffer under a handicap.

Strictly interpreted, the committee's definition implies that no matter how low the rate of flow through the meter, every drop of liquid or every cubic inch of gas must actuate the primary device. While some commercially produced weighing meters can meet this requirement, most of the devices in the great field of commercial, industrial, and public-service volumetric metering, which by usage and custom are called positive-displacement meters, would be excluded if the letter alone of the definition were adhered to. It obviously is impracticable to manufacture and sell against competition a meter of the rotating-disk type (for example) which will register at all when, after a period of inoperation, less than 0.1 per cent of its rated capacity is to be metered. The plain fact is that only such types as the valveless wet drum (for gases) and the tilting tank (for liquids) employ truly positive principles whereby, not only when the meter is new but after working parts have begun to wear, every cubic inch is registered and every drop caught. And even a meter in which buckets are filled and emptied is not absolutely positive, because open vessels permit evaporation, so that in the case of a volatile liquid the totalized quantity at low rates of flow is less than that delivered to the meter and more than that delivered from it. Moreover, spouts and valves are not always absolutely positive in their action.

Most positive meters are quantity meters. Their distinctive characteristic is that they register the total flow passed. If we draw the analogy to the field of speed measurement, positive fluid meters correspond to revolution counters while inferential fluid meters correspond to inferential tachometers such as those of the centrifugal and electromagnetic classes.

But this is all a matter of definition, arising from the historic use of positive meters as quantity meters, that is to say quantity counters. There has arisen lately a demand for fluid meters with primary devices of positive types and secondary devices giving direct readings of rate of flow. Such meters would correspond to the positive direct-reading tachometer—that is to say the integrating chronometric, the conjugating chronometric, and the geometric types (these last in principle only). Such fluid meters would consist essentially of a positive primary device, measuring off isolated quantities, and of a tachometric secondary.

Why not use an inferential primary device in the first place and get a rate reading directly? Because there are required—mostly in the chemical industries—dependable indications and/or records of the instantaneous values of the true volumetric rate at all rates of flow, from practically zero to the full capacity of the meter. The expression "dependable indications" refers not so much to the deflection of a pointer along a scale but to the initiation or "command" of automatic-control functions.

True enough, the new sliding-vane meter described by the author is not the first meter which makes possible the automatic control of various processes on a positive volumetric basis, but the combination of this new meter, together with the author's thorough description of its characteristics, represents a great step forward.

Now as to the piston and disk controversy. The writer was called upon about 6 years ago to act as a sort of umpire between powerful manufacturing interests in a dispute where the placing of orders hung upon the acceptance of disk meters as positive meters—not ideally positive, of course, but positive enough for the purposes in view. The trouble arose from the fact that the A.S.M.E. had rigidly defined the term "positive," but had not defined the piston and disk types. One side held that the distinction between "disk" and "piston" should be ended; the other people claimed that the disk type should be excluded from the élite "positive" class. These two sides still exist, though their personnel has largely changed.

Which is right? To some extent, both—from a realistic standpoint, neither. This is the conclusion reached by studying carefully the liquid-displacement meters that are being produced and considering also those that could be made.

If one were to scrutinize the manufacturer's own blueprints for possibilities of leakage and slippage in the various designs of meters, he would discern that there is no sharp line dividing meters that are of positive design from meters that depend entirely on small clearances between rigid moving elements and rigid walls, that is to say on the closeness of manufacturing tolerances. It will be seen that in design and construction, in theory and practice, *there are degrees of "positiveness."* It would indeed be possible to establish a continuous scale of infinitesimal gradations, from an ideally positive meter to a "displacement" meter with such large clearances that its registration curve has no "flat" portion. ("Flat" could be defined as lying between ±0.5 per cent anywhere along the volume abscissa and ±10 per

[5] This term is no longer used in the fourth edition, received by the writer after presenting this discussion. The writer does not feel that he need alter his discussion, for the influence of the earlier editions is still felt.

cent anywhere along the rate ordinates.) The basis for such a scale of the figure of merit, with regard to positiveness alone, would of course be the registration curve—not necessarily the intrinsic accuracy of the meter being graded. At one extreme would stand a design which has never been commercially produced, namely, a piston meter which could measure any fluid, even hydrogen, without leakage or slippage. The other extreme would characterize an equally nonexistent kind of meter, that is, one having a plunger, a piston, or a disk (but not a velocity-impelled rotary element) with intolerably excessive clearances—a meter, in fine, belonging neither in the "volumetric" class nor in the "current" class, therefore, a meter which is neither positive nor inferential.

All this may sound far-fetched, but a line of reasoning ad absurdum was needed to demonstrate (1) that, since the various liquid meters in question do not arrange themselves into mutually exclusive categories, any scheme of classification must be a compromise; and (2) that to discard the A.S.M.E.'s compromise classification for a purely scientific gradation would be impracticable. The outstanding fact is that no displacement meter is ideally positive. But as long as volumeters are generally considered positive, the nonpiston type seems to be as much entitled to this qualification as is the piston type. Users in the processing industries are interested in the fact that some forms of the piston type depend more on clearances between fixed walls and displaced members than do some forms of the other types.

Finally, positiveness is not the only basis of evaluation; pressure-loss characteristics, possibility of adjusting the register speed, initial and running costs, effects of changes in the physical properties of the metered fluid, the cleanliness of the metered fluid, the presence of abrasive particles, and other considerations should not be lost sight of in the flurry over pistons and other forms of reciprocating, or rotating, or nutating elements.

The author has not failed to discuss these considerations. He is to be congratulated.

Author's Closure

The interest manifested by such quantity of constructive criticism has been very gratifying. Mr. Behar's suggestion that the paper might well have been divided into two distinct papers would be acceptable if the object had been to make a detailed study of the mechanism and characteristics of each type of meter. However, the author's purpose was to treat in a general way the related aspects of viscosity effect on various types of quantity meters. In general, the effect of viscosity on registration is related to the degree of positiveness of the meter. The maximum effect is encountered with orifice measurement at low flow rates, and becomes successively smaller with approach to the final degree of positiveness, as is illustrated by comparison between (1) the free- and confined-impeller meters, (2) the impeller and disk meters, and (3) the disk and piston or rotary-vane meters, which latter closely approach the truly positive. The author avoided delving into the metaphysical problem of the positiveness of positive meters, but is indebted to Mr. Behar for his admirable discussion which emphasizes and illuminates this important point. For example, it is true that the rotary-sliding-vane meter described, or other synchronized-vane meters are positive meters and are as justly entitled to such classification as certain reciprocating-type displacement meters. In the last analysis, the criterion of positiveness is a volumetric efficiency dependent upon clearances and not upon type of mechanism or motion.

Mr. Hazard's discussion is pertinent to some of the problems confronting the users of oil meters. However, the author cannot agree with his simple practical disposition of the problem of treating the variation in registration of a meter on liquids of varying viscosity. From the point of view of manufacture and sales, a desirable meter should have characteristics which do not require special test and adjustment for different oils within wide latitude. Certainly the user would have more confidence in the meter and be put to less expense, when it is necessary to put different oils through the same meter or transfer the meter from one installation to another, if test and adjustment were not necessary within its periodic inspection and maintenance period. This is a practical and important consideration in the measurement of crude oils where viscosity varies considerably. In the case of the rotary sliding-vane meter, the author's statement, "Tests show no appreciable effect of viscosity variation within wide limits," was meant, of course, to convey the idea that the tests were made with liquids of widely different viscosities. Within the working range of the meter curve, i.e., from the beginning of the tangent point on the slip curve to the maximum allowable flow rate, no appreciable translation of the accuracy curve was observed between tests of gasoline and fuel oil. The reason for this lies in the high degree of positiveness inherent in the design. This has also been observed to be true in other designs of synchronized-vane meters. It is true, however, that in the slip region viscosity as well as density has some effect, and it is observed that on a low-viscosity liquid the slip is slightly more than on a high-viscosity liquid. Special analysis of variation in the slip region, which is outside the working range, was not given. However, this meter has a wide working range due to the low slip point, and the variation in shape and amount of the slip is a minor effect which does not materially affect its working range. Of course, test on air would show excessive slip, but on a liquid it would be a much smaller and reasonable amount. The sealing of the clearance between rotor and casing is essentially capillary, and the efficacy of the seal is dependent upon the fluid viscosity and rotor speed. A variation of the accuracy at low flow rates due to viscosity change has also been observed in some piston meters which are of the nonpacked type dependent upon a capillary seal of the clearance between piston and cylinder. Some discussion along this line will be found in a paper by Sylvan Cromer, entitled "Fundamental Principle of Quantity Meters," given at the Petroleum Fluid Metering Conference, University of Oklahoma, 1938. The oscillating circular-piston type meter is well discussed in this paper, also.

Detail description of the test methods cannot be given due to limitation of time and space except to say that the sliding-vane meter was tested by both volumetric-tank gaging and discharge-weigh methods. It is generally recognized that adherence of the liquid to the interior of the tank can have an effect on the test result; however, this can be obviated by actually weighing the discharge. Concerning the effect of highly evaporative fluid in the test result, the writer would call attention to research work done along this line on meter tank trucks as presented in a thesis by E. W. Jackson to the faculty of the Graduate College, University of Nebraska, 1938. The material in the author's paper did not embrace tests on volatile fluids and, hence, the above reference will suffice.

Mr. Smith's remarks are misleading to a slight extent in that the author did not emphasize or wish to point out that quantity meters are being applied "over a wider field of application than can be justified by maintenance costs." This may be so, and Mr. Smith's wide and authoritative experience on head rate-of-flow meters has no doubt brought him to many situations where a head meter was the proper solution. This question is one of application engineering—intelligent selection of type, study of operating conditions, and other problems. Active research is being promoted by both operators and manufacturers within the petroleum industry to improve metering. That this will be fruitful is certain and it can be confidently predicted that new meters, longer-lived and more accurate, and better operating technique

will result. For several practical reasons, the author has not considered important at this time the development of another rational operator in addition to the Reynolds number for analysis of performance of the current meter. Further intensive investigation at the present time does not seem probable as the synchronized-vane type seems to hold forth better promise for the majority of applications due to its wide-range positive characteristics. Such meters have the advantage of the simple rotary motion of the impeller meter combined with a high degree of positiveness. A second reason is the necessity of finding an improved current-impeller form which can be reproduced with the exactness of the orifice plate or venturi meter. This attained, the effect of viscosity would be uniform between different meters and thus some dimensionless operator would be useful in correlating and analyzing results to establish correction factors for viscosity effect at different flow rates. Such an operator would possibly be based on the ratios of kinematic viscosities and the pressure drop to pressure loss through the meter.

This paper was offered in the hope of making some small contribution to fill the gap between the organized detailed information available in head-type meters, such as the venturi and orifice, and the general information available on quantity meters. The A.S.M.E. Fluid Meters Report is a splendid accomplishment, giving complete and authoritative information on head meters, but treats quantity meters in a very general manner. One reason for this contrast is that in order to apply and interpret intelligently the results of head meters requires not only a complete understanding of operating principle and factors affecting performance, but a detailed knowledge of coefficients, correcting factors, and other experimental data. This is not true of the quantity meters which produce a recognizable result on the registers no matter what the operator knows about it. Further, the manufacturer will furnish instructions for installation and repair and, when special problems are met with, will willingly cooperate and work with the user to make a proper installation. Nevertheless, in the interest of extending knowledge and stimulating the progress in this art, it seems to the writer that further attempts should be made to organize available knowledge, giving due deference to proprietary interests when encountered, and stimulate further research on quantity meters. Is not this labor within the province of Fluid Meters Committee? Such meetings as the recent Petroleum Metering Conference specified by the petroleum industry will be helpful and indicate that more information will be available on metering in the petroleum industry in a short while.

Application of Automatic Control in the Oil Industry[1]

ALBERT F. SPITZGLASS[2] AND MAURICE J. ZUCROW.[3] This paper is a worthy contribution to regulation literature and will be of great value to the A.S.M.E. Industrial Instruments Committee in its effort to derive logical methods for analyzing industrial processes from a regulation standpoint. Although the author has concerned himself with a single industry, his story is in a language and form which readily permit correlating his work with problems in other industries.

One of the points of great interest to control engineers is the emphasis placed by the author upon stability for the entire process, and its relation to the individual regulation problems comprising this complex industrial process. The importance of this prerequisite is easy to understand, and it is satisfied only with regulators having specific characteristics. In the paper, nine regulators are described in detail. Of these, only the heater flow regulator is subject to variation in type, the appropriate type depending upon the kind of pump drive. The remaining eight regulators and their types are listed in Table 1 of this discussion.

TABLE 1

Number of regulators	Functional classification	Type
2	Pressure	Proportional narrow-band
1	Pressure	Proportional wide-band
3	Level	Proportional wide-band
2	Temperature	Proportional wide-band (with reset)

It is seen that six of the eight regulators are proportional-wide-band regulators. Although the popular conception is that this type is a necessary evil to be used only when the narrow-band regulator is liable to produce hunting, here the wide-band regulators were purposely chosen. A choice dictated by the problem, which is to balance "supply" and "demand" and regulate the "storage" so that it aids in absorbing shocks caused by sudden "demand" changes. Consequently, their use in this manner does not involve any loss of accuracy. This emphasizes the point that it is dangerous to judge the merits of regulators as devices apart from the particular processes which they are to serve.

In discussing the regulation of the flow of oil to heaters, the author emphasizes the importance of uniform flow; this is a deciding factor in obtaining smooth operation of the cracking unit. He surmises that although the application of the flow regulation directly to the cutoff governor of the engine has advantages it is unjustified. At first blush, the writers were tempted to question the correctness for this resigned attitude, since they have installed many systems where a pressure or flow regulator either shades the action of the governor or replaces it entirely. However, further deliberation revealed that, while this type of problem can be solved satisfactorily, the engineering required, due to lack of standardization, is usually out of proportion to the results obtained. It is unfortunate that cooperation between engine-governor builders and regulator manufacturers is in such a state that it justifies the philosophical attitude displayed by the author; this type of regulation is quite common, however, in the case of turbogovernors. The important point illustrated by this is the existence of a problem which the Committee must attack in its work to coordinate and bring some degree of standardization to the regulation art.

In connection with the regulation of combustion, one finds that a condition exists in the oil industry similar to that which formerly prevailed in the steel industry. Combustion regulation in the oil refinery has found few proponents. In the steel industry, combustion and furnace pressure regulation for open-hearth furnaces have finally proved their value, and now the soaking pit is coming into its own.

While there is a good argument against combustion regulation, the changeable heat content of the refinery gases used for the fuel making this type of regulation difficult and complicated, the argument for furnace pressure regulation is not so easy to refute.

Furnace pressure regulation is valuable because it has a stabilizing effect upon the furnace conditions and incidentally aids in maintaining a uniform combustion rate. Furthermore, it is helpful in directing the flame and in obtaining the correct distribution of gases within the furnace, an ever-increasingly important factor in modern furnaces. The pressure variations which exist in unregulated furnaces have not been apparent until recently, due to the insensitive draft gages which were used. It is only recently that industrial draft gages of high sensitivity

[1] Published as paper PRO-60-7, by Donald J. Bergman, in the November, 1938, issue of the A.S.M.E. Transactions, vol. 60, p. 651.

[2] Consulting Engineer, Chicago, Ill. Mem. A.S.M.E.

[3] Consulting Engineer, Chicago, Ill. Mem. A.S.M.E.

(±0.1 in. of water for full scale) have become generally available.

In his closing paragraphs, the author points out the direction in which the oil industry is moving. This information should be studied carefully by those responsible for the development of instruments and regulators. In cooperation with the designers of the equipment for the newer processes it will be necessary to establish those factors in process apparatus and regulator design which will produce a well-regulated plant. The new measurement problems which will accompany these newer processes must be solved. Their correct solution will benefit not only the oil industry and the instrument manufacturer but industry in general.

J. B. McMahon.[4] This paper gives a very interesting exposition of the reasons why automatic control has become an integral part of modern petroleum-refining processes. This is particularly true with respect to the discussion of how operators actually use the automatic controls which have been placed at their disposal and how far it is practical to go in the application of automatic control in such plants.

The author's discussion of the various kinds of equipment used for the various jobs involved is probably the most interesting part of this paper. With this discussion there will probably be considerable disagreement in which the writer is going to take some part. For instance, under the subject "Pressure Control on Distillate Receiver" the author states that the application of a simple air-pilot-operated pressure control to the release of the gas in the receiver gave very satisfactory and accurate results. Under most operating conditions, however, the type of controller mentioned will not produce the smooth, uniform gas discharge that can be accomplished by means of a proportional wide-band controller equipped with automatic reset. The difference is illustrated in Figs. 1 and 2 of this discussion, which are taken from an actual installation where the simple pilot-operated type of pressure regulator mentioned by the author was replaced by the latter instrument type of recorder controller. The operating conditions were identical and it is quite obvious that the over-all plant operation would be quite considerably smoothed out by having the gas flows, as well as the gas pressures, as uniform as are shown in Fig. 1 of this discussion, which were produced by the instrument equipped with the wide-band proportional-reset control mechanism.

It seems to the writer also that it is going to be necessary to clarify what is meant by "averaging" level control for discussions such as this. This term was introduced by The Foxboro Company in 1932 to designate a type of level-control results made possible by a combination of very wide-band proportional control with very slow automatic reset. Ordinary proportional wide-band control, referred to by the author, will not produce the results that are designated by the originators of this method as "averaging" control. Ordinary proportional wide-band control should not be designated as being averaging control since the valve movement for true averaging control should be considerably less than that which would be afforded by a proportional wide-band control that just took up the entire range of the instrument or, in terms of former designation, had definitely and exactly 100 per cent throttling range.

The writer does not consider that true averaging control is afforded unless the throttling range is at least several hundred per cent of the scale range in the case of differential-type instruments and several times even that amount for ball-float level instruments which do not have the advantage of the multiplication afforded by the differential method of measuring the level. A

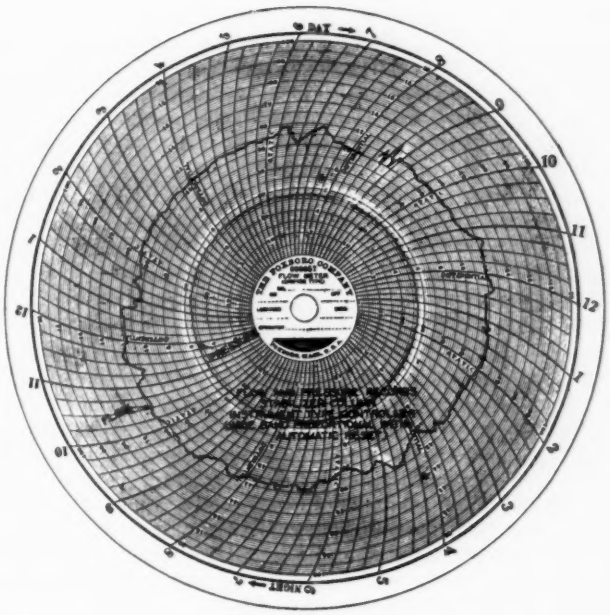


FIG. 1

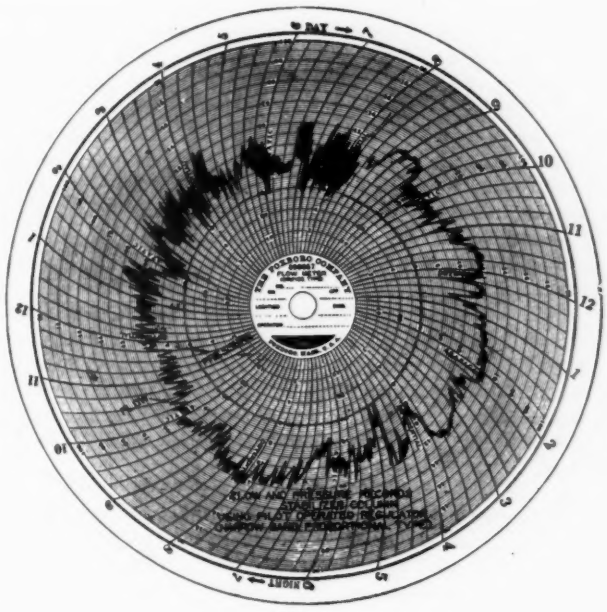


FIG. 2

discussion of this point is brought out in the paper[5] by the writer.

In this connection also, let us point out that in many cases for rather more intricate refining operations than are covered by this paper,[1] a storage capacity of considerably more than the author designates as advisable is necessary to provide the smoothness of operation that will give the best over-all operating results. It is under such conditions of storage relationships that the true "averaging" level controller is necessary and really valuable.

With respect to the control of heater outlet temperatures, the writer would like to point out that the effect of a transient condition such as is shown by the dotted curve in Fig. 5 of the author's

[4] Manager, Controller Division, The Foxboro Company, Foxboro, Mass.

[5] "Control of Liquid Level in Vessels Under Pressure," by J. B. McMahon, *Industrial and Engineering Chemistry*, vol. 29, November, 1937, p. 1219.

paper[1] would make it necessary to use rather highly specialized automatic control to overcome the effect of transfer lag in these furnaces. The automatic temperature control of pipe stills is up front among the very tough temperature-control jobs due to the requirements for very close control under very severe process conditions. This might readily be a fit subject for some of the process analysis work which has been discussed to some extent.

The same remarks with respect to level control in the flash chamber and the level control at the base of the fractionating column apply as they did to the other level-control jobs discussed. The writer would particularly like to draw attention to the fact that the author's statement, "A reset mechanism is not necessary or even desirable in this control since bringing the level back to a normal overcorrects the flow and acts opposite to the damping or averaging effect desired," evidently misunderstands the function of true averaging control. In a true "averaging liquid-level controller," which is equipped with extremely wide-band proportional mechanism and very slow automatic reset, greater variations in flow rate of the uncontrolled flow can be absorbed in a vessel than are possible where the vessel is equipped with an instrument having only proportional wide-band control mechanism. Two things must be taken care of, that is, both the instantaneous upsets and the average changes; simple proportional wide-band control mechanism which does not possess automatic reset will not be able to take care of these as well as the averaging controller since it would not be feasible to set the proportional band as wide without having some provision to take care of average change of conditions. Also, with respect to the level control in the flash chamber, it is quite evident from the author's discussion that it is desirable that the flow remain smooth but that the level should be kept within very narrow limits. Over any appreciable range of change of conditions this can be done only with proportional wide-band control with automatic reset and cannot be achieved by proportional wide-band control alone unless it is very materially assisted by manual resetting which is very much more upsetting to a continuous process than is automatic continuous resetting.

It is to be hoped that the author's paper[1] will be only a precursor for other thoughtful, well-considered discussions along the same line. All too frequently such discussions have revealed a lamentable lack of knowledge by the authors either of instrument mechanisms or of practical plant operation. Mr. Bergman's paper shows a happy combination of knowledge and appreciation of both.

B. R. Carney.[6] The author is well qualified by experience in actual cracking-plant problems involving the use of automatic controls. The title of his paper is broadly inclusive, and the text is confined to cracking-plant problems. This is probably justified by the importance of such problems and the author's preeminent qualifications along this line. Nevertheless, aside from the new processes briefly mentioned, the established operation of crude distillation, stabilization, and chemical treatment have for a long time justified the intelligent use of automatic controls.

The ideas in this paper[1] form a good general policy to guide the use of automatic-control systems. To express this policy in somewhat different words, automatic controls are not intended to replace intelligent operation, but rather are tools provided so that the operator may do a better job and work more accurately than with simple globe valves. This implies, of course, that the controls are subject to the operator's will, and can be conveniently reset to effect the changes in conditions which he finds necessary.

Considering now the specific problem of controlling the raw oil and furnace feed of a cracking unit, the system outlined implies a constant combined or furnace feed and a variable raw feed. Whether right or wrong, many of those responsible for refinery cracking operations like to think in terms of a fixed raw feed. Is it not feasible to make this the control point, absorbing variations (which should be slight) by allowing the combined feed to fluctuate? Admittedly a varying combined feed presents a problem in flow control when it is desired to divide equally the flow in a multipass furnace. For a two-pass furnace, equal division of variable flow would not seem extremely difficult, although the writer does not recall any instances of such a control system. Quite likely the instrument people already have the answer to this; if not, the problem is recommended for their spare-moment attention.

Development of the various throttling-type controls has permitted the use of automatic control in numerous cases where the on-and-off type would have been impossible. The operation of a stabilizer direct from the cracking unit, with only a few minutes surge capacity between, as explained in this paper,[1] is a very good illustration. Six to eight years ago such an arrangement would not have been considered practical. However, in spite of the capability of good controls, there is probably a practical limit to such close interlocking of refinery processes. Thus, it would likely not be desirable to extend further the combined operation by tying the subsequent chemical treatment of the gasoline directly to the stabilizer outlet if for no other reason than because the control problems of chemical treatment differ considerably from those involved in cracking and stabilization. In general, adequate supervision of over-all refinery results requires checks on quantity and quality at intermediate stages, and extensive interlocking of operations makes such checks impossible.

In connection with the control of fractionating-tower top temperature and, thereby, presumably—gasoline end point, the writer remembers an early pyrometer and electrical relay-system installation which was rather unsatisfactory. Controls using a thermometer bulb and air operation have given reasonably good results for many years on crude-distillation units.

The paper includes some pertinent and important hints about practical mechanical details of control and instrument installation. When trying to "line out" the operation of a new refinery unit, there is nothing more exasperating than failure of instruments. Besides the points mentioned the following might be stressed: (1) Instrument lines containing heavy oil have to be protected against low temperature, usually by putting such lines in an insulated conduit along with a small steam line. (2) Flow meters and flow controllers operating on a saturated gas should be protected against low temperatures which will cause partial condensation. (3) A flowmeter or rate-of-flow controller should preferably not be used on a liquid at its bubble point. These three items may seem elemental, but it is surprising how frequently they are overlooked in practice.

J. F. Luhrs.[7] Of particular interest is the author's emphasis on practical considerations in the application of automatic control in the oil industry. The paper serves to illustrate two important points. One is that there are definite limitations to any theoretical approach to the application of automatic control. The other is that empirical methods must be used in the solution of any specific control problem.

The author lists in reference to transfer-line and tower temperature control several independent variables which must be taken into consideration. The writer would like to emphasize the effect which extraneous or superfluous changes in the control agent have on these temperatures. The flow through any regulating valve is dependent upon the port area and the pressure

[6] Shell Union Oil Corp., New York, N. Y.

[7] Bailey Meter Company, Cleveland, Ohio.

differential impressed across the valve. If the port area be varied solely from temperature, changes in the rate of application of the control agent can still occur through changes in pressure differential across the valve. Hence, even though the temperature may be at the normal value, an extraneous change in the rate of application of the control agent may occur which will upset the system. Because such changes are usually quite rapid, their effect is more aggravating than that of any of the other variables listed by the author. It follows that if precise control is to be achieved, such extraneous changes must be immediately corrected, so that the rate of application of the control agent will be dictated solely from temperature.

In the case of a gas-fired oil heater, a change of 1 oz in the pressure at the burners may effect a sufficient change in the rate of fuel supply to change the transfer-line temperature several degrees. Such changes in burner pressure may occur bearing no relation to transfer-line temperature, because of changes in the pressure of the gas supply ahead of the fuel-regulating valve of corresponding magnitude. Such changes in supply pressure may be caused by sudden changes in demand by other equipment, or by changes in gas production. An independent control of the pressure of the gas supply will seldom hold the pressure within the required limits as at the relatively high pressure at which the supply is maintained a change of a few ounces is practically imperceptible. A control operating solely from transfer-line temperature will eventually correct for such extraneous changes in fuel supply, not however without an appreciable departure of the temperature from the normal value.

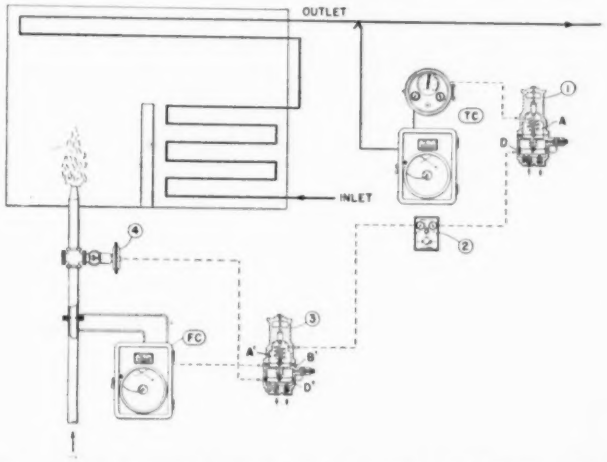


FIG. 3 TWO-ELEMENT CONTROL FOR TRANSFER-LINE TEMPERATURE

Similarly in tower control a slight change in the pressure of the reflux may effect a sufficient change in reflux flow to cause a material deviation in tower temperature from the normal value. Again such changes will ultimately be corrected, but not before an appreciable change in tower temperature has occurred.

A greatly improved control can be obtained if the port area of the regulating valve be varied not only from temperature but from the rate of application of the control agent as well. In a system of this type an element responsive to temperature establishes a desired rate of application of the control agent and a second element responsive to the control agent maintains correspondence between the desired and actual rates of application. Control systems of this so-called two-element type have been used successfully for many years, particularly in connection with boiler combustion and feedwater control. Their use in the last few years in the oil industry has been increasing.

Such a system applied to transfer-line temperature control is shown schematically in Fig. 3 of this discussion. A loading pressure corresponding to transfer-line temperature is established by the recorder-controller *TC* and transmitted to a chamber *A* of a standardizing relay 1. This device acts, in addition to relaying pressure changes, to introduce a floating action so that a loading pressure established in chamber *D* changes both in correspondence with changes in transfer-line temperature and the amount of departure of this temperature from the normal value. The loading pressure established in chamber *D* is transmitted through a selector valve 2 to a chamber *A'* of a standardizing averaging relay 3 which is similar in construction and action to the relay 1. A loading pressure corresponding to fuel flow established by the recorder controller *FC* is transmitted to a chamber *B'* of the relay 3. This device establishes in chamber *D'* a control pressure which is transmitted to a diaphragm fuel-control valve 4.

The pressures in chambers *A'* and *B'* oppose each other and for the relay 3 to be in equilibrium, that is to produce no change in pressure within chamber *D'* and hence no change in fuel flow, it is necessary that the two pressures be equal or stand in predetermined proportion. Hence, for each and every loading pressure established by the relay 1, a definite rate of fuel flow (in distinction to a definite valve position) is established. It will also be apparent that equal changes in loading pressure established by the relay 1 will produce equal changes in fuel flow (in distinction to equal changes in valve position). Any variation in fuel flow not called for by a change in control pressure will immediately be corrected by the controller *FC* before such variation causes a material change in heat liberation in the heater furnace.

The system may be readily transferred from automatic to remote manual control by the selector valve 2. This device also acts to inhibit the floating action of the relays 1 and 3 during periods of remote manual control. As a result, during such periods, if the temperature maintained be slightly different from that for which the controller *TC* is set, the pressures established by the relays 1 and 3 will not go to an extreme but will be maintained at the control pressure established by the selector valve. The transfer from hand to automatic may then be made smoothly without upsetting the system.

The author mentions a few instances where the interlocking of several control points would give improved results, but questions the desirability of so doing in view of the added complication. Such multielement systems have been in use, however, for many years, particularly, as previously mentioned, in boiler combustion and feedwater control. In the writer's opinion, the proper coordination of several variables into one system serves to simplify the control problem rather than to complicate it. It is believed that as knowledge of any particular step in a process advances many instances will occur where regulation of a control agent from several variables will be highly desirable. Specific reference is made, as an example, to the possibility of controlling fuel to cracking units from a proper correlation of time, temperature, and stock characteristics,[8] which in addition to improving heater performance would undoubtedly aid in giving improved results throughout the entire refining process.

AUTHOR'S CLOSURE

Messrs. Spitzglass and Zucrow mention combustion control. It is probable that combustion control has not had greater

[8] "The Determination of Yield per Pass, Time of Treatment, and in situ Density and Instrumentality for Their Control in Commercial Cracking Units," by Robert L. Rude, R. D. Junkins, and C. Barnes, Proceedings of the Nineteenth Annual Meeting of the American Petroleum Institute, Chicago, Illinois, Nov. 18, 1938, Section 3, Refining, pp. 149–167.

application in the oil industry because of two conditions. First, fuel gas is usually available at pressures of from 15 to 30 psi, and changes in combustion-chamber pressure which might affect the flow of gas from burners operating on 4-oz gas have little effect except with regard to change in quantity of excess air. Second, the gas usually has a variable heating value and the setting of fixed fuel-air ratios common in combustion control would not be entirely satisfactory. Furthermore, most refining processes are continuous, and have a relatively constant heat demand, as against boiler operation where demands for power may vary at the whim of a passing cloud.

Mr. McMahon points out improved operation of a gasoline stabilizer by the use of a proportional wide-band controller with reset. It should be noted that the reference in the article had to do with the control of pressure on the cracking-plant fractionating column, which has relatively greater capacity, less drop across the control valve, and probably a lower percentage variability in load, thus making the conditions more favorable to the simple control. However, the wide-band proportional control with reset is of advantage in many of these cases, though not vitally necessary.

With regard to the definition of "averaging," control, this is left to the Subcommittee on Terminology. As a suggestion, an extension of Mr. Haigler's idea might be the classification of proportional controllers, as follows, in terms of instrument scale range:[9]

Narrow band	less than 4 per cent
Wide band	4 to 20 per cent
Full band	20 to 100 per cent
Extreme band	over 100 per cent

Regarding the comments on level control, when the basic factor of flow through the heater is held constant, the entire process is stabilized, variations in other flows are minimized and the necessity of reset or other additions to simple controls is likewise minimized.

Mr. Carney questions the principle of control of furnace feed versus control of raw-oil feed, expressing a preference for holding raw-oil feed constant. It is possible in a fully automatic system to change combined feed rates as he suggests, to correspond to variations in combined-feed ratio. However, this further requires changes in the flow of fuel to the heater, and finally adjustment of the reflux return to the top of the tower to balance the increased heater load. Maintenance of constant combined feed at constant outlet temperature under control of the operator is generally felt to result in more stable and uniform plant operation.

With regard to Mr. Luhr's comments, it is surprising to note the representative of an instrument company expressing the thought that control of fuel-gas pressures to almost any desired limit is difficult, and this view is not generally shared in the industry. Constant pressure drop across a control valve and burner orifices will usually result in a satisfactory control of flow of fuel gas to meet the demands of a temperature controller. However, if it be desired to set up a proportionality with the loading pressure of a wide-band temperature controller with reset, a simple and relatively inexpensive method is the provision of an instrument-type flow controller on the fuel gas, the control point of which is adjusted by the loading pressure from the temperature controller. Such instruments are in standard production by several manufacturers.

With specific reference to the paper presented by Messrs. Rude, Junkins, and Barnes,[8] while this represents an excellent case of carrying research and development through to a satisfactory conclusion, it is probable that the comments of Messrs. Spitzglass and Zucrow on control of speed governors apply in this instance also, and the cost of equipment and engineering necessary is disproportionate to the results which can be obtained through the use of simpler standardized controls available.

With reference to the general problem of separate direct versus interlocking controls, it is noticeable that the latter have been quite generally adopted in connection with boiler control as compared to refining. Perhaps this is because, considered as a process, a boiler is certainly much simpler than an oil still. With water, pressure and temperature are definitely dependent variables, while with oil they are practically independent. Latent heat is dependent upon temperature in the case of water, while in oil it is also dependent upon molecular weight, and a variable "heat of cracking" may further complicate matters. With the greater complexity or additional number of degrees of freedom in oil, the principle is usually followed in experimental work of holding all variables but one constant and noting the results with changes of that one variable. Simultaneous changes of two or more variables cloud the issue as to which change is responsible for the results.

It is on this basis that preference is expressed for the method of taking up the "slack" with one variable. In the case of a transfer temperature controller this would be fuel to heater, while in a tower temperature controller the variable is the return of reflux to the top of the tower.

This does not mean that interlocks for safety are not desirable. For instance, in the case of a transfer temperature controller, a continued slow rate of coking of the tubes would call for an increase of fuel to maintain the transfer until such a furnace temperature is reached as to result in overheating and rupture of a tube. An interlock to prevent admitting fuel beyond a given furnace temperature would minimize such a possibility.

[9] "Application of Temperature Controllers," by E. D. Haigler, Trans. A.S.M.E., vol. 60, 1938, paper PRO-60-5, p. 633.

Pressure-Responsive Elements[1]

KENNETH S. M. DAVIDSON.[2] This paper is a valuable contribution to the literature on the subject. What impresses the writer particularly is its direct approach to the problems of pressure measurement, as that problem is encountered in the ordinary course of events in present-day engineering work. Previous contributions have usually related either to highly specialized problems of pressure measurement, or to abstract analyses of instruments. Mr. Exline covers the whole field of engineering pressure measurement, starting with a rational physical approach, which will permit special problems to be treated as extensions of the basic groundwork, rather than as isolated investigations. There has been a need for a paper of this type.

The fact that this paper treats certain types of measuring instruments more fully than other types is not a very serious matter, for it helps to set up a background into which fuller considerations of other types can be fitted. In particular, it treats of bellows-type elements almost to the exclusion of Bourdon tubes. The bellows-type element has received increasing attention in recent years, and it appears to have very desirable characteristics. It has by no means displaced the Bourdon type, however, and a paper purporting to describe pressure-responsive elements in general should certainly compare the characteristics of these two types of elastic elements. At any rate, such a comparison is a logical next step in a study of the subject as a whole.

[1] Published as paper PRO-60-4, by Paul G. Exline, in the November, 1938, issue of the A.S.M.E. Transactions, vol. 60, p. 625.

[2] Associate Professor of Mechanical Engineering, Stevens Institute of Technology, Hoboken, N. J. Mem. A.S.M.E.

The bellows-type element has enjoyed a certain advantage over the Bourdon tube from the fact that, because of its comparative newness, it has been studied by the more refined methods customary at present; the Bourdon tube on the other hand has received less recent attention. However, a not inconsiderable array of useful data regarding the Bourdon tube are available, and its analysis at this time would be interesting.

A comparative analysis of bellows and Bourdon elements would naturally consider their relative stiffness (that is, their relative ability to move indicating or recording mechanisms) as a function of the maximum stress in the element, and of the maximum pressure for which the instrument is designed.

GEORGE H. LEE.[3] The wide discrepancy between the calculated and the observed bellows stiffness apparently results from a neglect of the curved sections of the bellows and from the boundary conditions of the flat annular plate assumed by the author. By computations briefly described in the following paragraphs, the writer arrives at results in close agreement with the experimental values.

One half of one convolution of the bellows is considered; the calculation of the deflection for this section of the bellows is divided into three parts: The deflection of the flat annular plate, and the deflection of a unit width of each of the two curved sections adjacent to the plate. The curved sections considered are in effect quadrants of circles in an axial plane. The flat annular plate is subjected to the following boundary conditions: A force, normal to the plate, equally distributed over the outer boundary, with an equal but opposite force distributed over the inner boundary; a moment, due to the adjacent curved sections, distributed over the outer boundary of the plate, and a similar, but not necessarily equal, moment distributed over the inner boundary. The deflection of the inner boundary is measured relative to the outer boundary. The deflection of the curved sections of unit width is determined using Castigliano's theorem. The ends of the curved sections next to the flat plate are subjected to the same forces and moments per unit length, as the boundary of the plate; the tangents at the other ends of the curved sections remain vertical, due to the symmetry of the system, and so are assumed fixed. The deflections of the curved sections are measured from these fixed ends.

The total deflection of one convolution of the bellows then is twice the sum of the three deflections. The bellows stiffness per convolution is the applied axial load divided by the deflection per convolution.

The bellows stiffness is calculated for bellows Nos. 3 and 5 of the author's tables. These values are found to be 4100 lb per in. and 4300 lb per in., respectively, against the observed bellows stiffness of 4347 lb per in. and 4938 lb per in., respectively.

The deflection of the flat annular plate is obtained from the solution of the differential equations[4]

$$\frac{d^2\phi}{dx^2} + \frac{1}{x}\frac{d\phi}{dx} - \frac{\phi}{x^2} = -\left(\frac{1}{D}\frac{P}{2\pi x}\right)$$

and

$$\phi = -\frac{dw}{dx}$$

where ϕ is the slope of the plate along a diameter, x is the radial variable, D is the flexural rigidity of the plate, P is the total applied force, and w is the deflection. The arbitrary constants are evaluated from the boundary conditions just given.

[3] Sibley School of Mechanical Engineering, Cornell University, Ithaca, N. Y.

[4] "Strength of Materials," by S. Timoshenko, D. Van Nostrand Company, New York, N. Y., 1930, part 2, p. 491.

The moments existing at the boundary of the plate and either curved section are evaluated from the condition that the slope is the same for the curve and the plate at their common boundary.

The behavior of the pressure bellows under actual operating conditions suggests that the bellows stiffness in terms of an applied internal pressure would yield information more pertinent to the design of bellows gages than the bellows stiffness in terms of the applied axial force. The bellows stiffness in terms of the applied internal pressure could be calculated by a modification of the method outlined in this discussion. The deflection of the flat annular plate may be obtained from the solution of the differential equation[4]

$$\frac{d^2\phi}{dx^2} + \frac{1}{x}\frac{d\phi}{dx} - \frac{\phi}{x^2} = -\frac{1}{D}\frac{qx}{2} + \frac{P}{2\pi x}$$

where q is the applied internal pressure. The deflection of the curved sections due directly to the applied pressure could be neglected in comparison to the deflections due to the axial force.

ED S. SMITH, JR.[5] This paper forms an excellent basis for a "Section on Instrument Elements" for ultimate issuance under the auspices of the A.S.M.E. Industrial Instruments

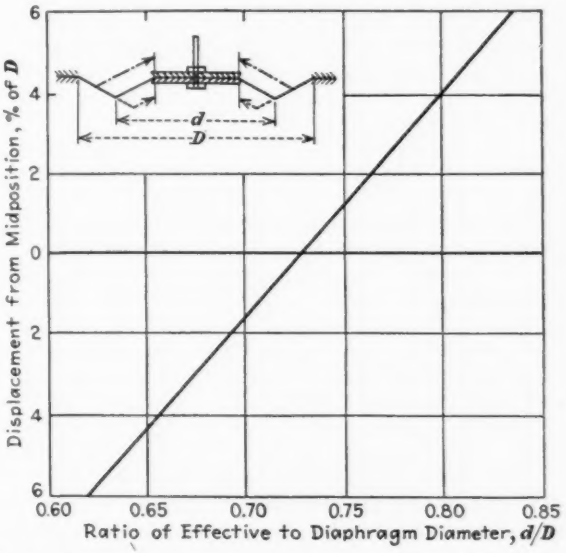


FIG. 1 PERFORMANCE OF FLAT DIAPHRAGM WITH SLIGHT SLACK

Committee, and in which the treatment of other elements, such as the Bourdon tube, would receive as full development as the present paper's section on bellows.

For bellows, and other instruments having substantially lineal response, the use of stored energy may be permissible as indicating the probable effects of friction. However, for the more general case where the response may be in other than lineal relation, it is more accurate to consider forces due to friction and to deflection of a resilient portion.

Another noteworthy point is that the maximum error of the bellows, as in Fig. 16 of the paper, is practically ten times the probable error, and that the probable error is established by practically all of the observations while the maximum error is determinable with far less accuracy due to its almost accidental values; both of which considerations lend weight to the use of the probable error in preference to the maximum error, and es-

[5] Hydraulic Engineer, C. J. Tagliabue Co., Brooklyn, N. Y. Mem. A.S.M.E.

pecially as a fair basis for guarantees. It would be interesting to have a least-squares analysis of these same data in the closure.

Under materials, (*a*) plating of resilient instrument members has been resisted in several fields, (*b*) silver is a valuable metal to use where reasonable size resilient parts must handle weak forces (W. W. Frymoyer of the Foxboro Co. is to be credited with this), and (*c*) some difficulty has been experienced in obtaining beryllium-copper bellows from some sources. It would be helpful in the treatment of instruments generally if the closure were to include a fundamental analysis of hysteresis in terms of the characteristics of materials with emphasis on the relation between low error and the economy in the use of material by the selection of one having a high yield point.

A noted omission is that of the limp flat or slightly formed diaphragm. Where a flat circular diaphragm is mounted between flanges having a slightly smaller diaphragm, and then stressed axially to take up the slack to give as much travel as possible without stretching, before clamping, such a diaphragm rolls and unrolls as a very flat cone in service thereafter within the limits of slack, accurately following a lineal relation between axial travel and the diameter of its effective area. Fig. 1 of this discussion shows the large but dependable change of effective diameter with even small travel. Although obviously such a basic characteristic should be adequately compensated for in design, still in the literature the writer, offhand, remembers only a brief note by the late J. H. Hodgson. Flat limp metal diaphragms, commonly used in diaphragm tops, have an even greater change of effective diameter with much smaller limits of travel. On the other hand, formed diaphragms having substantially cylindrical, instead of conical, shape have a nearly constant effective area and ample travel, their chief disadvantages in comparison with the slack flat limp diaphragm being that they are deformed at full travel while the latter merely roll and unroll without stretching and that they cannot be cradled as effectively as the latter which, when cradled nearly completely, can withstand surprisingly high differential pressures.

M. F. Behar.[6] In the field of measurements it has become a habit to publicize far and wide the new methods—the utilization of new physical principles—and to greet in more or less perfunctory fashion the appearance of all new instruments utilizing old methods. Sometimes this uniformity works a cruel injustice toward an inventor or a company, for whereas the majority of new instruments using old methods represent relatively small advances in the art, there are exceptional instances representing each a tremendous amount of research followed by painstaking development and culminating in a masterpiece which does advance the art.

"Another pressure recorder of the spring-loaded bellows type, and direct-writing at that!—oh, why bother?" Such would be the usual reaction.

But not in this instance!

Both this paper[1] and the new pressure recorder which it describes deserve an extraordinary reception. They mark the turn of the tide with regard to the attitude of physicists and engineers toward the various classes of pressure-measuring elements.

In considering the field of pressure instruments, one is struck by a strange paradox: On the one hand a great proportion of physicists and engineers manifest a preference for gravitational-method instruments that almost amounts to a prejudice against elastic-member instruments; on the other hand, a great majority of the instruments sold every year for pressure measurement depend on curved flattened tubes, helical tubes, diaphragms, metallic bellows, springs of various shapes, and other elastic elements that deflect in more or less exact proportion to the applied pressure.

Certain it is that easily read dials are preferable to visible liquid columns. Equally certain it is that perfect elasticity and total absence of hysteresis and variance have never been attained. All metals and alloys are imperfect. The best of springs fail sooner or later.

Evaluation of the properties of metals and alloys to resist static stress at normal temperatures has become a well-understood matter. But when it comes to methods of evaluation of resistance to repeated stress, to abnormal temperatures, or to wear, neither the methods of test nor the engineering application of the information are so well understood. And with regard to the "service life" or "accuracy life" of elastic members, as distinguished from their life until actual failure occurs, there is little exact information and there appears to be no accepted criterion. There was not even a set of accepted definitions until about three years ago.

This deadlock is nobody's fault. Users being convinced that elastic members can't be improved, they don't demand extreme-precision models and gage manufacturers don't have the necessary incentive to carry on lengthy and costly research. It has remained for a great oil company to subsidize Exline's work.[1] The demand was created by the unusual problems surrounding the measurement of bottom-hole pressures, where extreme accuracy is really worth millions of dollars to the oil industry. A great oil company comes to the rescue and the deadlock is broken.

This paper[1] greatly enriches the literature on elastic-member pressure gages. It will focus attention on the subject and some forgotten articles will be brought to light. The writer particularly likes the author's final paragraph calling attention to the importance of the energy term in mathematical analyses of elastic-member elements—a viewpoint deplorably neglected in most texts, yet one which we cannot overlook when we compare, for instance, the kinetic behavior of a small-diameter bellows and that of a large-diameter diaphragm—the former being of the "medium-force high-energy" class and the latter being of the "large-force small-energy" class.

And the extraordinary precision and accuracy of the new gage itself—exceeding the precision and accuracy of many gravitational instruments now used as secondary standards—will focus attention on the possibilities of elastic members. There will no doubt be a widespread demand for extreme-precision elastic-member gages of various other forms for various other important applications. A widespread demand having come into being, there will be the incentive; the gages will be manufactured and they will be put to important uses by physicists and engineers.

The deadlock is broken. The tide is turning. History is being made by a "development," praiseworthier than a "discovery."

Author's Closure

The author is in complete agreement with Mr. Davidson regarding the desirability of a sound exposition of the performance of the Bourdon tube and a comparison of its characteristics with other pressure-responsive elements. The Committee on Industrial Instruments and Regulators is endeavoring to secure such a paper.

Calculation of maximum stresses in both Bourdon tubes and bellows is unsatisfactory because of the incomplete state of the theory. For practical purposes an approximate value of this can be obtained by determining the pressure required to cause a given permanent deformation. Some work of this nature has been done in this laboratory on bellows but the results are so

[6] Editor, *Instruments;* founder and director, The Instrumentation Manual Project, Pittsburgh, Pa.

variable that they do not warrant publication at the present time.

Mr. Lee's method of calculating the bellows stiffness is distinctly superior to the approximate method used by the author. It is felt, however, that his statement that "the bellows stiffness in terms of an applied internal pressure would yield information more pertinent to the design of bellows gages than the bellows stiffness in terms of the applied axial force" is not true. Equation [3] of the paper represents the physical behavior of the bellows under pressure and shows that the bellows stiffness K_b is simply added to that of the restraining spring K_s. If no restraining spring is used the value of its stiffness will be zero.

Mr. Smith has suggested that consideration of the forces developed by an instrument element would be more useful in indicating the probable effects of friction than the stored energy and that the linearity of response of the element determines the applicability of the energy term. As Major Behar points out, the equivalent systems can exist using large force with small displacement on one hand and small force with large displacement on the other. Their equivalence can be shown only by considering their total stored energy or the change in the stored energy over a selected small region of their stroke. In the case of pressure elements the integral of Pdv over any part of the response curve would be equal to the integral of Fds, where F and s are the developed force and the displacement, respectively.

It is not quite clear what Mr. Smith expects in a least-squares analysis of the data presented in Fig. 16. The errors shown are deviations from true values which need not be evaluated by statistical methods. The probable error which was given was 0.6745 times the square root of the sum of the squares of all the errors divided by one less than the total number of observations. The author agrees with Mr. Smith that the probable error would be quite valuable in specifying instrument accuracies.

A discussion of the limp diaphragm was not included in the paper in order to save space for a more complete description of the bellows which the author felt was justified by the lack of other published material on this subject. The analysis of Eaton and Buckingham, reference (9) of the paper, fully covers the limp diaphragm. The author was not familiar with the work of Hodgson.

The author appreciates the work of the discussers in calling attention to the omissions and emphasizing certain points in the paper.

Corrections

Page 629, Table 2. A, a, B, and b are radii corresponding to the diameters given the same symbols in Fig. 6, except that B is the outside radius. In line 2 the values for B and b are interchanged.

Page 629, Equation [2]. The last term in the numerator should be changed to $\frac{16c^2}{c^2-1}(\log_e c)^2$.

Page 632, reference 15. The added term should be changed to read

$$+\frac{16a^2b^4}{a^2-b^2}\left(\log_e\frac{a}{b}\right)^2.$$

Pressure-Type Thermometer Systems[1]

Paul G. Exline.[2] The author presents a readable and interesting exposition which fills another gap in the broad program of the A.S.M.E. Committee on Instruments and Regulators. As with any well-written paper, this one leaves the reader with a desire for more information on the subject. In this case, the writer would like to see a treatment of the suitability of the various systems described to individual applications, together with the reasons why some ten different systems are necessary. This is by no means a criticism of the paper under discussion but a suggestion for future consideration.

The growing use of the word "spring" to designate any form of a Bourdon tube seems unfortunate to the writer. There is a large group of devices for storing and releasing energy transmitted mechanically to them, for which this term is entirely appropriate. Except for its shape, there is no more reason for calling a Bourdon tube a spring than for calling a diaphragm or bellows a spring.

The writer fully concurs with the author that there is need for further work on Bourdon tubes, both theoretical and metallurgical. Most of the theoretical work at present available is of little practical value since it deals with certain cross-sectional shapes little used in practice.

R. D. Webb.[3] From the user's point of view, this paper[1] is a very welcome one for apparently too many of us have accepted pressure-type thermometers without question. The reason for this can probably be laid to the fact that this type of instrument has been available for many years and long association has caused us to take it for granted. Too many engineers have operated their plants, taking engineering data with this type of instrument without any doubt in their minds as to its accuracy.

As we study the five major points into which the author has divided his analysis, we find that every one is of great importance to the thermometer user.

Point No. 1, the pen, which refers to the recording mechanism, is a rather fascinating one to study. The first reaction you get upon looking into the modern thermometer is that the manufacturers have retrogressed by going to the light, apparently flimsy, type of construction which so many of them now use, but years of constant observation of the heavier linkage type and of the lighter linkage type wins you over to the side of the modern trend. The light parts give the instrument a sensitiveness not otherwise obtainable, and wear is practically unknown. Where vibration is present, the lighter parts again show their superiority. The incorporation of a virtually rectilinear linkage on liquid and gas-filled thermal systems appears to be a logical means of achieving uniformly graduated charts; nevertheless, several companies produce instruments of excellent accuracy without resorting to it, although all too often three-point calibration of an instrument is difficult or impossible simply because of bad linkage angularity.

In the author's second point, the Bourdon tube, the user is naturally interested in the type of element used and, of course, the vital importance of such things as creep, set, hysteresis and endurance are recognized. The type of spring winding, whether it be spiral or helical, apparently has much to do with the linkage and its angularity just mentioned and also with the convenience of installing one or more thermal systems in a recording case. The helical winding apparently gives the high rotation necessary to use one-to-one linkage ratio between the spring and the recording-pen lever while the spiral-type winding offers the advantage of compactness. This seems slightly contrary to what the author has said in regard to the spiral spring's offering greater deflection.

The capillary connection tubing, part 3 of the paper, becomes of interest to the user mainly from one point of view, that is, durability. If of the type that uses a single heavy tube with no protecting armor, then the ability to stand flexing and also resistance to corrosion are the major considerations. The small capillary types with protecting armor usually have greater flexibility, and the use of Monel or 18–8 stainless-steel armor generally solves the

[1] Published as paper PRO-60-8, by L. G. Bean, in the November, 1938, issue of the A.S.M.E. Transactions, vol. 60, p. 657.

[2] Engineer, Gulf Research & Development Corp., Pittsburgh, Pa.

[3] Carbide & Carbon Chemicals Corp., New York, N. Y.

problem of corrosion resistance. This last type of construction seems to have a mechanical weakness usually to be found where the flexible armor is fastened to the case and to the bulb.

The subject of bulbs has been nicely covered by the author and here the user comes to realize that the main consideration is that of conveying the exact temperature of the medium being measured to the bulb; in other words, securing proper immersion. The subject naturally overlaps with point No. 5, the filling medium, simply because the size of the bulb, which greatly influences the ease of immersion, is governed by the nature of the filling medium.

The trend of the manufacturers appears to be toward smaller bulbs, this being clearly shown by the author's own company adopting the name "Small Bulb Type" for their compensated class-III system.

The second notable trend is one toward longer tubing lengths. This trend is governed a whole lot by the filling medium and the method of compensating it for tubing and case error; furthermore the use to which the thermometer is to be put in a large degree governs the type of filling that must be incorporated. If uniformity of graduations on the chart were a factor, the vapor type would be eliminated. As yet, ranges above 500 F are generally limited to the mercury and gas-filled systems. The low temperature range, that is, below —40 F, is restricted to liquid- and gas-filled instruments and here the power of the gas-filled type is questionable. The fluid in which the bulb is to be immersed sometimes becomes a factor in deciding the filling medium when separable sockets cannot be used. The mercury-filled systems are naturally restricted to bulbs made of ferrous metals, although the somewhat recent practice of making mercury-filled bulbs of 18–8 stainless steel has greatly widened the field of use for this type.

The writer agrees with the author that further work should be done on pressure-type thermometers. He names definite fields where this is most needed, but to the user the two things that are most important are accuracy and durability. Generally speaking, both of these will stand considerable improvement.

Some Fundamental Considerations in the Application of Automatic Control to Continuous Processes[1]

W. D. Wood.[2] As recognized by the authors, their paper is not an exhaustive presentation but furnishes a definite group of basic terms, which may well be acceptable to the A.S.M.E. Committee on Industrial Instruments and Regulators. Further terms could be appended from time to time as the literature on the subject grows. At present, a more complete listing would lead to confusion.

Although the paper is admirably presented, there are a few terms which warrant discussion. It is suggested, at the outset, that automatic-control terminology be divided, for clarity, into the three following classes: (1) Terms describing process or plant characteristics; (2) terms describing controller characteristics; and (3) terms describing performance. Discussion of the authors' items of terminology follow.

Item 1. The writer agrees with the authors' suggestion that this subject be referred to as "automatic-control technology."

Item 12. The term "set point" or "control setting" seems more descriptive than "normal" alone.

Item 13. This is affected by the comments regarding *Item 12.*

Items 14 and 15. The term "sensitivity" still seems to the writer to be best adapted to relate the ratio between cause and effect. It is logical and accurate to use the term "proportional sensitivity" to represent the coefficient of the second term on the right of the equation on page 647 of the paper. "Control band" is, however, an excellent descriptive term and might well be used in describing the characteristics of proportional control. The use of "sensitivity" is widespread in England and it is used here by at least two manufacturers.

Although the authors have discussed both proportional and floating control, they have neglected to present a term which describes the amount of floating-control action as their "control band" describes the amount of proportional-control action. That is, they have not described the coefficient of the first right-hand term of the equation on page 647. The term "floating sensitivity" seems logically adaptable here.

The term "sensitiveness" seems to the writer to describe adequately what the authors mean by "ultimate control sensitivity."

Item 16. "Droop" has evidently been chosen for want of a better term. There is no consensus of opinion here, but it seems to the writer that it ought not to refer equally to a proportional-control characteristic and to a two-position control characteristic. The writer suggests three terms to be used in describing this characteristic of proportional control: (1) "Set point:" the desired value of the "controlled variable" at equilibrium. (see *Item 12*). (2) "Control point:" the actual value of the "controlled variable"at equilibrium. (3) "Drift:" the difference between the actual value and the desired value of the "controlled variable" at equilibrium. "Drift" is a specific characteristic of proportional control resulting from load or demand changes. With these terms the "droop" characteristic is completely described without the use of a word which seems out of place in scientific nomenclature.

Item 17. To avoid confusion would it not be best to dispense with the term "throttling control" since it is here used to describe several types of control and has already been used to describe proportional control?

Item 19. The term "proportional control" alone seems adequate.

Items 23, 24, and *25.* Ought all these three terms to be called lags? "Velocity-distance lag" is entirely different in character from "transfer lag" or "demand-side storage lag." Each term affects the performance of a controller differently, that is, "demand-side storage lag" is generally advantageous, whereas "velocity-distance lag" is not. The writer suggests that the three terms be replaced successively by "storage response" or "capacity response," "transfer response," and "flow lag."

Item 26. "Controller response" is suggested. The word "controller" seems better here than "control" since the latter hints of a performance characteristic.

Item 27. "Measuring response" or "metering response" is suggested.

Item. 28. "Controller agent response" is suggested.

Item 30. "Controller dead time" seems preferable.

C. E. Mason.[3] The general qualitative analyses, as presented by the authors, and the recent paper by Haigler,[4] are valuable in developing an appreciation of the fundamental factors affecting the physical reactions of process lag. Although the ultimate object of the efforts of the A.S.M.E. Committee on Industrial Instruments and Regulators may be to formulate rules applicable to quantitative analysis, it is essential that all members of the Committee working on this phase of its work have a thorough and common understanding from the viewpoint of qualitative analyses.

[1] Published as paper PRO-60-6, by E. S. Bristol and J. C. Peters, in the November, 1938, issue of the A.S.M.E. Transactions, vol. 60, p. 641.

[2] Engineering Research Department, Taylor Instrument Company, Rochester, N. Y.

[3] Chief Engineer, The Foxboro Company, Foxboro, Mass.

[4] "Application of Temperature Controllers," by E. D. Haigler, Trans. A.S.M.E., vol. 60, November, 1938, paper PRO-60-5, p. 633.

It seems possible that some of the rules developed from pure qualitative analyses may require some revision or qualification after studying quantitative analyses, because the actual value of the effect will be the algebraic sum of several influences. However this possibility should not discourage the development of rules suggested by qualitative analyses alone.

The authors' definition of the term "normal" seems to have possibilities of confusion due to the presence of the word "desired." However, omitting it would apparently change the intended meaning. Controllers with "drooping characteristics" will function to maintain different values of the controlled variable if the demand or the conditions of the supply were changed. The desired value of the controlled variable does not change with these changes in demand or supply, except in a very few special cases where a temporary change may be desirable in order to effect what commonly has been called "averaging control."

It seems to the writer that two distinct terms must be used—one definitely representing the single-point mechanical setting of the controller, and one representing the actual or average final balanced condition of the variable resulting from the characteristics of the control system and certain specific conditions of the supply and demand. The authors' definition actually defines neither of these, but if the word "desired" be omitted it would define the latter. It was understood of course that these two terms would become synonymous for controllers not having a "drooping characteristic."

In order for the work of the Committee to be of material value to the industrial designing and control engineers, these qualitative analyses must be extended to quantitative form. The need of these two distinct terms, referred to previously by the writer, will be apparent when attempting quantitative analyses.

The writer is preparing, for future presentation to the Committee, a paper dealing with quantitative analyses of the effect of process lag. The results of this work indicate the desirability of introducing new definitions which are not necessarily apparent from the qualitative viewpoint. In some instances it will be desirable to reconsider definitions so as to be equally practical in both methods of analysis. It is the writer's opinion that final decision on recommended definitions be withheld until some work has been done from the viewpoint of quantitative analyses.

Reconsideration of the term defined by the authors as "normal" will affect the definitions of "deviation from normal."

"Control band" or "throttling range" seems to apply only to the proportional characteristic of a controller and becomes meaningless when other functions compensate for "drooping characteristics."

Since the term "throttling control" has been used commercially to designate a proportional control, it seems to the writer that the term should be completely discarded. However, if we disregard the possible confusion, it could be defined as "a general classification of control employing a corrective action which is capable of maintaining all intermediate values of the final control element." It seems also that this definition would be in accord with Haigler's suggestion.[4]

Storage lag retards in the sense of slowing down, but it does not effect a lost-time reaction, as might be inferred by using the word "delay." This comment also applies to the authors' definition of "transfer lag." Although the retardation is of a different nature, it is in reality the compound effect of two or more capacities separated by a potential.

"Velocity distance lag" in the same sense is not a retardation, but a definite time delay. The retardation which might be observed is due either to the storage lag which is always present, or to some small effect of transfer lag.

It must be appreciated that storage lag is the only process lag which might be considered as existing alone. Storage lag exists in all applications. The presence and proportions of the other two—which are also always present, at least to some small degree—constitute the actual difficulties encountered in automatic control.

It would seem practical to consider that the other lags defined by the authors as "control lag," "measuring lag," etc., are really one of the three major lags located in various parts of the complete system. It is the writer's opinion that these can, without confusion, all be defined by one of the first three general definitions.

Referring to Fig. 6 of the paper under discussion, it might be interesting to note that, if a sudden change had been made in the flow or temperature of water entering the tank (demand), the resultant temperature curve would be very similar to the dashed curve shown in the figure, although transfer lag does exist. This analysis is suggested only to prevent the impression that the absence of the S-curve characteristic is indicative of no transfer lag.

From a quantitative viewpoint, the curves of Figs. 13 and 14 of the paper might justify some comment; however, since they are qualitative only, they should be drawn as shown in order to illustrate the important characteristic which the authors wish to emphasize.

[The following is a discussion of the papers by E. S. Bristol and J. C. Peters[1] and by E. D. Haigler,[4] respectively. Editor.]

M. J. Zucrow.[5] The paper under discussion is in a large measure complemental to Haigler's.[4] For this reason, it is difficult to discuss it without making some reference to the one by Haigler.

The authors are to be congratulated upon their clear presentation of the fundamentals of a difficult subject. Their openmindedness in subordinating their own preference for the term "regulation" to the word "control" is praiseworthy. It raises a question, however, regarding the name which should be adopted for the art under consideration. Since the standardization of terminology is one of the main activities of the A.S.M.E. Committee on Industrial Instruments and Regulators, it might be of value to have the question of control versus regulation settled at an early date. Like the authors of the paper, the writer prefers the word "regulation," which is more consistent with the Committee's name. Furthermore, the definitions indicate that "regulation" more closely describes what is of interest, even though these words are synonyms. The definitions are as follows:

Regulation: To adjust to or maintain at a desired, required, or standard condition or action; put or keep in good order; adjust according to circumstances or requirements; as, to regulate a watch or mechanism; to regulate the ventilation of an apartment.

Control: To exercise a directing, restraining, or governing influence over; direct; counteract; regulate; as, to control the engine of a motorboat by a lever; to check by a duplicate register; verify or rectify, as an account.

The authors have treated with brevity a large amount of subject matter. To discuss all of the topics mentioned in their paper would make this discussion too lengthy. Consequently, the writer has confined his remarks to only a few of them.

The authors' discussion under the subtitle "The Automatic Control System Applied to the Process" warrants amplification to increase its generality.

Most physical processes to be regulated involve plant capacity. There is a flow of energy (or fluid) into the capacity and a flow of energy is being continuously withdrawn from it. The inflowing energy is termed the "supply" and the outflowing energy the "demand." Conditions can be such, however, that these terms should be interchanged. The conventional regulation

[5] Consulting Engineer, Chicago, Ill. Mem. A.S.M.E.

problem is one where the regulation apparatus must so vary the "supply" that it will always equal the "demand." Ordinarily, it is difficult to measure the "supply" and the "demand" directly. Instead, some new variable which is an index of the balance between these energy flows is measured. The object of the regulation is to keep this index at its proper value, because then there will be a balance between the "supply" and "demand." This index, which may be a temperature, pressure, or speed, is really a measure of the potential energy contained in the regulated system. For example, if the heat supply to an electrically heated furnace were to be regulated, this index would be the temperature of the furnace in the system. Similarly, when the steam flow through a pipe is regulated by a pressure-reducing valve, the steam pressure, used for influencing the opening of the valve, is taken as the index of balance. In this example, even though the steam flow through the reducing or supply valve equals the flow through the demand valve, the object of the regulation is not achieved unless there is no increase or diminution in steam pressure. It has become common parlance in some fields of regulation to call this index of potential-energy level the "storage" as distinct from "capacity." Using the aforementioned terms, the general statement of the regulation problem is the equating of "supply" and "demand" and the maintaining of a constant "storage." It should be noted here that there are problems where the "demand" can be measured and the "supply" adjusted directly from this measurement, but the problems discussed by the authors and by Haigler[4] are not of that type.

Owing to the frequent use of the term "storage" in the sense described by the writer, the Committee should consider the value of reserving this word for such usage. The word "capacity" could then be restricted to describe or measure the physical dimensions and characteristics of the process equipment. Capacity involves not only the physical size of the equipment but also the maximum energy supply which is available. This has not been clearly pointed out by the authors. It is a measure of the time required to produce a definite temperature change with the heat-supply valve in its wide-open position. Storage on the other hand defines the heat contained in the system when the supply and demand heat flows are equal.

Although the writer, like others, has used the term "capacity lag" indiscriminately, he now questions the appropriateness of this term unless it is given a definite meaning. Lag is the "retardation of movement for any cause;" it also means "any specific amount of such retardation." Since this property of the plant must ultimately be evaluated in a quantitative manner, some distinction must be made between the term "capacity lag" when used to denote the characteristic temperature curve of a plant and its use when referring to a definite time interval. Since the designer of the process equipment must be provided with specific values of criteria as a guide in design some simple evaluation of the foregoing factors is required. Since, as pointed out, the temperature reaction of the plant involves not only its physical dimensions, but also the maximum energy flow available, a criterion similar to the "time constant" used in the regulation theories developed for combustion control might be serviceable; this is discussed later.

It is well known that, in the usual case, when the position of the regulating valve is changed, the value of the regulated variable (temperature in this case) does not assume a new value immediately. Ordinarily, the plant temperature changes in some such manner as that shown in Fig. 1 of this discussion. When the valve is opened instantaneously from its initial opening m_0, to a new position m_1, the temperature changes slowly at first, then it increases at a more or less constant rate, and finally approaches a new equilibrium value asymptotically. The following question naturally arises: What is the "time lag" as a unit of measurement? Callender and Stevenson[6] define it as the time, in seconds, which elapses between the instant the valve was moved and the beginning of the constant rate of temperature change. They base their definition upon the fact that, with instantaneous response of the regulating valve to a temperature deviation, the deviation will grow unchecked for the duration L, defined as unit time lag. This deflection bc may be regarded as inevitable. According to this, the automatic regulator "should be able to stop a fluctuation before it has developed to more than twice its inevitable value, and subsequently damp the oscillations to comparatively small values in not more than two complete cycles." This definition of time lag is presented merely to illustrate that there is a problem still unsolved; a simple method for determining the result of the combined action of the regulator

[6] "The Application of Automatic Control to a Typical Problem in Chemical Industry," by A. Callender and A. B. Stevenson, Society of Chemical Industry (London, England), Proc. Chem. Engg. Group, vol. 18, 1936, pp. 108–117.

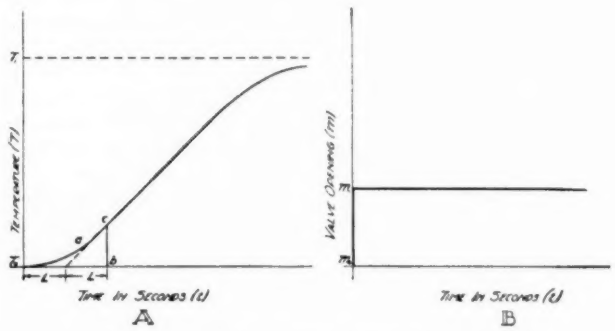


FIG. 1 EFFECT OF CHANGING SUPPLY-VALVE POSITION

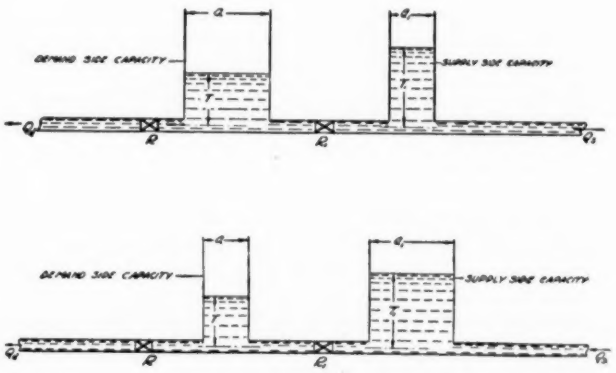


FIG. 2 HYDRAULIC ANALOGIES TO FIGS. 1 AND 3 OF HAIGLER'S PAPER[4]

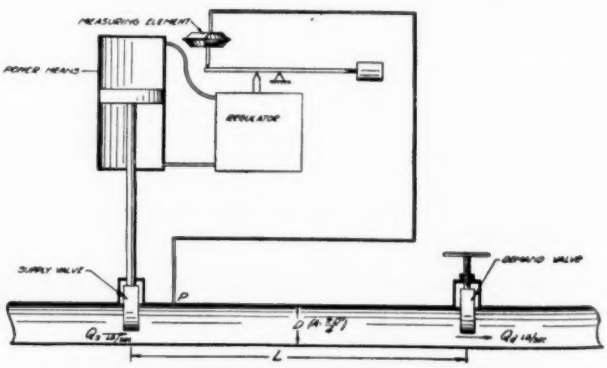


FIG. 3. PRESSURE REGULATION FOR A PIPE

and process equipment before the latter has been designed. Since the temperature response characteristic due to plant capacity and thermal resistance cannot be modified by anything in the regulator, some simple criteria must be established and means devised for evaluating the total time lag and maximum deviation, to aid the designers of process equipment.

C. E. Mason of the Foxboro Company has made an excellent beginning in this direction.[7] He utilizes the analogy between the conventional heat-flow problem and capillary fluid flow to give a picture of the process. As an example of the analogy, Fig. 2 of this discussion, which corresponds to Figs. 1 and 3 of Haigler's paper,[4] is presented. The areas of the tanks a and a_1 represent the demand-side and supply-side capacities. The resistance R_1 is the thermal resistance between the two capacities, it being assumed that the liquid flow rate q corresponding to the heat-energy flow is given by

$$q = \text{const} \times (T/R)$$

The problem is for a given change $(Q_d - Q_s)$ or $(Q_s - Q_d)$ to find the variation of the storage T as a function of time. It is unfortunate that Mr. Mason has not published more of his work.

It was mentioned earlier that in fluid-flow problems, such as are encountered in steam-power-plant practice, it is convenient to express plant capacity in terms of what is called a time constant. Stodola, who introduced this constant, calls it the "Anlaufzeit," which literally means "starting-up time" or "traversing time." The significance of this term is illustrated as follows:

Consider a conduit such as that shown in Fig. 3 of this discussion and assume that a compressible fluid is flowing through it. If the demand flow Q_d exceeds the supply flow Q_s, then the deficiency is obtained from the fluid stored in the volume LA, with a corresponding drop in the absolute pressure P. If the unbalance $Q_d - Q_s$ is 1 per cent, then the absolute pressure drops at the rate of 1 per cent per sec. Conversely, if the pressure increase by 1 per cent per sec, then the excess flow into the pipe would be 1 per cent. Consequently, if it were conceivable that this relationship continued for all flow changes, up to 100 per cent, then the time required for a pressure change of 100 per cent would be that required to add or remove the volume of fluid LA (cubic feet) from the pipe with a flow rate q; this would be given by the ratio LA/q. The dimensions of this ratio are those of time and it is a measure of the capacity of the system. If Q be that value of q which corresponds to the maximum difference between the supply and discharge flows (for example, supply = Q and demand = 0), the supply valve will have been instantaneously opened from 0 to its full-load position and for this condition the ratio is LA/Q and may be termed the "capacity–time constant."

Thus, if the "capacity–time constant" be 2 sec, it means that after a sudden change in the position of the regulating valve from zero to full-load flow, it will take 2 sec before the pressure in the pipe changes 100 per cent. This means that the rate of pressure change is 50 per cent per sec. This relationship is truly valid only for small pressure changes, since these basic relationships do not apply to large changes in pressure. However, in the quantitative analysis of regulation problems, the pressure changes are small and these relationships are applicable.

For conduits or containers which have fixed volume it is seen that the "capacity–time constant" is merely the ratio of the volume between the supply valve and the demand valve divided by the maximum flow for the system. There are cases, however, where the unbalance in flows is accompanied by a change in volume as well as by a change in pressure. Detailed discussions of this subject can be found in the books of Th. Stein[8] and Wünsch.[9]

The last part of the paper contains many definitions of important concepts pertaining to the art of regulation. The authors have given this particular phase of their material considerable thought, and these definitions should be of great help to the Subcommittee on Terminology in carrying out its program. The writer feels that the appropriateness of the definitions is something for the Committee to determine and he has, therefore, refrained from discussing them.

ALBERT F. SPITZGLASS.[10] This paper, together with Haigler's,[4] opens up the subject of qualitative approach to control problems and their relation to the processes.

While indicating a preference for the terms "automatic regulation" over the terms "automatic control," the authors have used the latter terminology throughout their paper as a concession to general practice. Such an attitude is obviously a very laudable one, but in the present case it does not seem as though the usage of either term has become well enough established to justify the selection of any but the most suitable term. The discussion by Zucrow[6] has emphasized this point and by reference to accepted definitions of these terms has indicated that we are not faced with a choice of words but a choice of ideas. A regulator is not the same thing as a controller, if we are to give any weight to the definitions in standard dictionaries, but obviously a regulator is a specific form of controller whose object is to maintain some condition at its normal value.

Therefore, if we use the term "regulation" and use it in the sense of maintaining normal values, then we do not have to distort our theory of regulation to take into consideration problems pertaining to program controllers, safety devices, interlocking mechanism, and starting-up operations. It is not that these problems do not play an important role in the general problems of control. It simply is a fact that the fundamental principles we are trying to study and classify now all have to do with the maintenance of normal value of continuous phenomena within certain limits and there is no way to stretch our theory to include this other type of problem, which is always discontinuous in some respect or other. It would not be necessary, therefore, to make the exception noted by the authors, namely, that demand-side storage may be disadvantageous when it is desired to establish promptly a new normal temperature. True enough, it is a disadvantage but that is beside the point, just as a question of the safety involved in large demand-side storage might be beside the point.

In both this paper and that by Haigler,[4] emphasis is placed on "factors favorable to control." The Haigler statement is qualified by "as reflected by smaller cycles with on-and-off control and narrower control bands with proportional control." The present paper[1] makes no definition. The conclusions of both the papers are accurate if referred to the definition of favorable factors advanced by Haigler.[4] However, there is a natural tendency to generalize from such a statement and while no such intent exists on the part of the authors, it is possible to get the impression that these factors are favorable to all types of regulation.

Since these authors[1,4] divide regulators into four general classes, it seems more logical to refer all factors, favorable or unfavorable, to the classification and the specific type of regulator, rather than to make a general rule and then make exceptions. The

[7] "Quantitative Analysis of Process Lags," by C. E. Mason, Trans. A.S.M.E., vol. 60, May, 1938, paper PRO-60-1, pp. 327–334.

[8] "Regelung und Ausgleich in Dampfanlagen," by Th. Stein, Julius Springer, Berlin, Germany, 1926.

[9] "Regler für Druck und Menge," by G. Wünsch, R. Oldenbourg, Münich, Germany, 1930.

[10] Consulting Engineer, Chicago, Ill. Mem. A.S.M.E.

TABLE 1 RECENT CLASSIFICATIONS OF REGULATOR RESPONSE

Author	Unstable regulation	Stable regulation: Self-regulating inherent follow-up: Noncorresponding	Stable regulation: Mechanical follow-up: Corresponding	Stable regulation: Mechanical follow-up: Temporary correspondence	Stable regulation: Mechanical follow-up: First-derivative combinations	Stable regulation: Mechanical follow-up: Higher (or special) derivative combination
Bristol and Peters (1937)	Two-position	Floating 1-speed Prop. 2-speed speed	Proportional	Proportional and floating	.....	.....
Haigler (1937)	On-and-off	Floating	Proportional: *a* Wide-band *b* Narrow-band	Proportional and reset	.....	.....
Ivanoff (1936)	On-and-off	Integral of deviation	Deviation	Deviation and integral of deviation	First-derivative combinations	.....
Smith (1936)	VIII and XI	Noncorresponding I, III VI IV, X	Corresponding II, V, IX	Proportional-reset VII, XII	.....	.....
Mitereff (1935)	.....	 I	II, IV	III, V, VI	VII, VIII, XI	IX, X, XII
Grebe (1933–37)	(Metered control) deviation			(Storage-lag pacer)	Rate control	Damping control (kicker)
Stein (1926)	.....	No follow-up	Permanent follow-up	Elastic follow-up		

floating regulator cannot be considered a mere exception to the rules applying to regulators in general. It embraces a tremendously important field of activity and its theory has been propounded at great length, especially in the foreign literature. Among the most able treatises on the subject of industrial regulators, and one which treats at great length on floating regulators, is that by Th. Stein.[8] The point of view that he presents has received too little consideration in this country, even though G. W. Smith and Dr. Zucrow both discussed this work at great length last year in connection with the paper by Ed S. Smith, Jr., on automatic regulators.[11]

In order to refer these qualitative statements to regulators, they should first be divided into a broad classification of a type which lends itself to a comprehensive qualitative treatment. This might be accomplished by dividing automatic regulation into the following main groups.

1 *Stabilized Regulation* (*follow-up reaction always included to give stability*). (*a*) Inherent follow-up—for systems with sufficient back pressure or flow-head reaction (self-regulation) to give inherent stability. (*b*) Mechanical follow-up—for systems having insufficient follow-up reactions to render them inherently stable.

2 *Unstabilized Regulation*. In this class the problems of expediency are controlling factors, as with two-position regulators.

For convenience, a complete tabulation of classifications presented by recent authors is given in Table 1 of this discussion to indicate the relation of this approach to that of other authors.

Authors' Closure

A considerable proportion of the discussion presented comprises different opinons with respect to terminology suggested by the authors and serves to fulfill one of the objects aimed at in the preparation of the paper. Such opinions are believed to constitute material suitable for review by the Terminology Subcommittee and it is felt that not much would be gained by attempting detailed replies to the various comments submitted. These closing remarks, therefore, refer to only a few of the more important points of difference in terminology.

Mr. Wood suggests that the term "response" be substituted for "lag" in most cases where the latter is employed in the paper. However, response of a process or control system is a reaction or effect having several characteristics such as magnitude, total duration, and lag. The latter term is merely one of the several characteristics of "response" and was broadly defined as "retardation *or* delay in response" in order to allow for just such differences as exist between "velocity-distance lag" and "demand-side storage lag." Lag may consist of an initial time delay before any appreciable response can be detected, a reduced rate of response, or both.

The authors are in complete agreement with Mr. Mason's view that both qualitative and quantitative analysis should be considered in setting up terms and definitions. The Committee will no doubt welcome further specific suggestions in this direction.

It is to be emphasized that the designer of a process may achieve marked improvements in its operating characteristics without quantitative analysis if he understands the fundamental factors involved in making it easy to control.

At present, quantitative analysis is in a nebulous state and, in the case of temperature control at least, cannot be readily applied except to the simpler cases. It is to be expected that as work continues this situation will be improved upon.

The point made by Mr. Mason in connection with the curves of Fig. 6 warrants enlargement. For purposes of discussion it may be considered that the rate of water flow, in Fig. 5 of the paper, is to be varied to maintain constant the temperature of the water within the tank, and that the degree of electric heating is an independent variable. The varying rate of electric heating may then be considered as the demand (for cooling) and the flow of cooling water as the supply. With supply and demand reversed as thus considered, the system of Fig. 5 will be at least as easy to control as that of Fig. 2. Transfer lag between heater and tank is helpful in that it retards the effect upon the temperature of demand changes. Of the response curves of Fig. 6 it may be said that, in the absence of measuring lag, the one which best indicates the relative ease of control is that obtained in response to a change in flow of the control agent. From this viewpoint, the important transfer lag is that between the point of application of the control agent and the point of measurement.

Nearly all of the curves contained in the paper, including those of Figs. 13 and 14, were taken from the results of laboratory tests with a commercial controller, using a heater of the form shown in Fig. 5. The exceptions are those of Figs. 3, 6 (dotted line), 7, 12*c*, and 12*d*.

Dr. Zucrow's conclusion that more is required of a controller than to maintain equality between supply and demand seems to hinge upon the definitions of these terms. If, in his Fig. 3, the demand is considered to be for steam at a specified pressure, the requirement of "constant storage" is automatically taken care of.

The apparent slighting of floating control noted by Mr. Spitzglass is the result of concentration on temperature control to which this type is rarely applicable. The authors are well aware of the importance of floating control in other fields. They believe

[11] See G. W. Smith's discussion of the paper "Automatic Regulators, Their Theory and Application," by Ed S. Smith, Jr., Trans. A.S.M.E., vol. 59, February, 1937, p. 125. Also, see M. J. Zucrow's discussion of the same paper, p. 132.

that a discussion of the application of automatic controllers outlining the fields in which each type is used would be well worthwhile.

The authors did not intend to imply any personal preference for "automatic regulation" as opposed to "automatic control," as inferred by several discussers.

Application of Temperature Controllers[1]

JAMES L. KIMBALL.[2] The writer believes that control problems could be simplified greatly if a definite maximum band could be agreed upon for both pressure and temperature, say 10 per cent of the total, and that such a band be specified as adjustable between zero and full range. For cases where drift is objectionable, the more expensive types of controller, involving reset, would be indicated.

Such an approach would be comparatively simple, and would relieve the manufacturer of the responsibility for unfavorable factors of the system or process over which he had no hand in the making. At the same time, it would place in the hands of the prospective user definite information as to the characteristics of the equipment manufactured in conformity to an accepted code.

A standard of this kind would give a basis for determining the degree of area when adjusted to its maximum band, whereas at present with fixed bands varying in percentage of the total, no definite comparison is possible.

These two factors, taken together, would give the complete story of any regulator. On the other hand, if the author were to rewrite his paper to apply to pressure control, it would assist materially in straightening out our present-day control problems.

J. B. MCMAHON.[3] The approach to this problem outlined in this paper[1] is a hopeful sign that our large users of automatic controllers are becoming very much aware of the coordination necessary between designers of equipment and builders of automatic controllers. Except in amazingly few instances this feature of design, designated as "controllability" by the author, has been entirely neglected by designers of processes and process equipment.

It is noteworthy that the author designates this only as an approach, not as a solution. Practical experience in the application of automatic controls soon shows that at the present time most of our analyses of control problems must of necessity be based on what amounts to almost nothing more than hunches, as it is impossible to evaluate the factors involved accurately enough to be able to call most of our analyses anything more. The conditions which the author sets up are susceptible to this inadequacy, and should not be used quantitatively on that account.

In the author's analysis of the various kinds of process lag, the effect of transfer lag is not discussed very fully, nor is a really adequate definition of transfer lag made, although it is a very bothersome factor in any problem where heat must be transferred through a wall. Transfer lag is due to two factors, i.e., thermal resistance and heat capacity. It exists only because of the simultaneous presence of these two factors, and would not be bothersome if either were absent. Any heat-transferring material possesses both. Thermal resistance makes necessary the presence of a temperature gradient in order to transfer heat. Heat capacity delays the establishing of changes of temperature gradient. The two in combination give rise to some very queer results when present in combination with other types of lag, as they almost always are.

One type of heat-exchange problem occasionally encountered is not mentioned by the author; that is, where it is desired to extract heat from a fluid without interfering in any way with its flow, which makes it necessary to control the flow of the fluid whose temperature is being controlled. The diagrammatic layout would be the same as Figs. 2 and 3 of the paper under discussion, except that the controlled valve would be in the demand line along with the temperature bulb. Generally speaking, such applications are very difficult to control, except over a very limited range, and are to be avoided wherever possible. By-passing part of the supply fluid around the exchanger by means of a three-way valve is often a happy solution to this problem.

It is to be hoped that this paper[1] will be the forerunner of many more approaching this matter from this standpoint. A series of papers such as this will hasten the time when mathematical analysis will make it possible to prove to equipment and process designers that "controllability" is just as vital a factor in design as any of the other factors with which they have to deal.

WALTER C. WAGNER.[4] While the author of the paper under discussion presents an effective approach to the general solution of a familiar subject of limited scope, the writer believes that the author has opened up a method of attack that will lead to broader applications. Although the subject specifically is applicable to process control, the writer finds in it principles involved in the elements of temperature control as applied, for example, to the related subject of temperature control of buildings purchasing steam-heating service from a public utility. The control of living conditions obviously departs widely from conditions predetermined by a process engineer, inasmuch as he determines the character of the demand for temperature, whereas in the case of meeting the varying demands of the public, a new control variable enters into the picture. Basically, however, the method of attack of a problem, based upon certain assumptions of normal average conditions, also requires instruments for research and their application to determine finally control requirements.

While there is not included in the paper[1] the recognition of the economics of the liquid or gaseous forms of supply, the ideal maximum quantity, and duration, of flow may be such as to affect the cost of supplying such heating (or cooling) service sufficiently to justify the modification of the demand for ideal requirements for such heating service. Similarly, the measurement of the quantity of flow and of its sustained demand must be measured with an acceptable accuracy. Should, for example, a change in rate of flow of steam service be great or its frequency be such as to affect this accuracy, the designing engineer must give that serious consideration.

The importance of systematic response in the control systems for process control closely parallels that for steam-service control. In general, therefore, while the writer's discussion has been concerned with the application of temperature controllers to other than a process control, it is hoped that others concerned with the subject of steam-heating service will give a wider application than intended by the author to the principles involved in his paper.

M. F. BEHAR.[5] Eight years ago someone prefaced a classification and discussion of the "objects" of measurement or control

[1] Published as paper PRO-60-5, by Edmund D. Haigler, in the November, 1938, issue of the A.S.M.E. Transactions, vol. 60, p. 633.

[2] Mechanical Engineer, Ruggles-Klingemann Mfg. Co., Salem, Mass. Mem. A.S.M.E.

[3] Manager, Controller Division, The Foxboro Company, Foxboro, Mass.

[4] Philadelphia Electric Company, Philadelphia, Pa.

[5] Editor, *Instruments;* founder and director, The Instrumentation Manual Project, Pittsburgh, Pa.

(such as, processing conditions, composition, and properties) with the following declaration: "Industrial instruments are definite means of attaining definite ends. From the viewpoint of the industrial engineer the *application* of an instrument is of far greater importance than the instrument itself." This theme was then elaborated in opposition to a school of thought—then in the majority among manufacturers and users of industrial instruments—which held that process-control devices had reached such a state of development that process-control engineers need no longer worry, for the marvelous new robots would not only do the work of human eyes watching dials and human hands operating valves, but would do it with far greater accuracy however difficult the application.

The author of the paper under discussion belonged to what was the minority school of thought eight years ago. That it is now the majority school of thought is due in no small part to the quiet, unassuming influence of the author. In this paper he has epitomized his philosophy. Wisely indeed, he has not attempted to compress into the limitations of a "paper" his approach to the analysis of controllers in general and of applications in general, but has restricted his treatment to one of the numerous entities which are controlled automatically in modern industry, that is, temperature, the most important processing condition.

The only other nonmathematical analytical discussion of temperature controllers and temperature applications considered together as one problem appeared seven years ago. In these seven years, there has been so great a spread of the recognition of the view that the application is as important as the controller, that one naturally expects a corresponding improvement in the treatment. This paper[1] comes up to these expectations. It is open to only one major criticism—it is not long enough. The author has had so much experience with the automatic control of conditions other than temperature, and with the automatic control of physical and chemical properties, that he could have written an extremely useful introduction, comparing temperature-application problems with these other purposes of automatic-control devices. There are not only interesting differences but interesting analogies. Temperature is distinguished from most other conditions by time elements which follow definite laws—a body of laws summarized by the expression "thermometric lag." This lag is so definite a phenomenon as compared with the elusive entities often met with in application engineering that the author might well have inserted a brief treatment of it in relation to automatic control. This cannot be done adequately without mathematics, so perhaps this is the reason why the author refrained from doing it in this particular nonmathematical paper.

The originality of the section entitled "Classification of Problems" is striking, and one may express the hope that this same approach may be followed in dealing with parallel subjects, that is, humidity control and the continuous control of physical properties such as viscosity. The author no doubt could do so if he had the time, but this is an age of specialists and, the author having shown the way, parallel discussions along the same line of approach would enrich the engineering literature if written by specialists in these other control problems. For example, one cannot help but notice in this paper a little slip in a passing reference to humidity measurement and control by the psychrometric method, where the author says that the dry bulb must not be shielded—which is bad advice, for it is extremely important to minimize radiation effects when using psychrometers with dry bulbs, and the "true" psychrometers are those in which both the dry bulb and the wet bulb are surrounded by two shields, that is, two pairs of concentric tubes with surfaces highly polished in order to eliminate all radiation effects.

Methods of classification can be discussed endlessly without arriving at universally acceptable conclusions. The only scientific basis of dividing a number of entities is to formulate a scheme of mutually exclusive and collectively exhaustive categories, based upon the easily recognizable characteristics of the entities and upon actual relations between them. Now then, when dealing with such entities as the kinds of behavior of automatic controllers in their active state, it is extremely difficult to formulate a valid scheme. To put it in another way: It is relatively easy to classify controllers in their inert state, as mere dead things, and it is relatively easy to classify furnaces, ovens, fractionating towers, and other applications in their inert state, but it is infinitely more difficult to classify the phenomena of what might be called "life"—the things that happen when a controller is engaged in controlling an application.

The author has chosen a compromise classification—a dual classification; he does not attempt to go into the infinite variety of these modes of behavior, but he classifies controllers as such—dead controllers, not hooked up to applications—according to five general classes of modes of which they are capable by reason of their design. Then he classifies problems (as discussed by the writer), and in so doing he classifies also the favorable factors that make for controllability. Finally, he gives examples showing how his original method of classification can be applied to various pieces of industrial processing equipment.

The writer's admiration for this original treatment is deep and sincere, for he had tried to work out in 1927 to 1930 a classification of the infinite number of kinds of behavior exhibited by "living" temperature controllers (controlling their applications) into twenty-odd classes of modes of control, starting with the on-and-off modes and working up to the mode which recognizes the rate of change of the controlled temperature. The writer is the first one to acknowledge that this qualitative and almost "popular" classification is obsolescent, but he nevertheless continues to maintain that the only perfect approach to a thoroughly scientific treatment of the problem is along the trail which he then blazed. Such a line of approach, however, is beset with mathematical difficulties, for these modes of control, the phenomena of "life," so to speak, which may be observed when a controller is at work, are relatively easy to classify qualitatively but difficult to classify quantitatively in the exact language of mathematics. That they are relatively easy to classify qualitatively is amply proved by the fact that it was done seven years ago; that they are difficult to classify into mathematical categories was demonstrated in Le Corbeiller's paper.[6]

A. F. Spitzglass[7] and M. J. Zucrow.[8] Recent years have witnessed a broad extension of the field of automatic regulators in industry. This has been accompanied by much confusion due to the lack of a common language for the art and to misconceptions regarding the purpose and abilities of automatic regulators.

In former years the actuation of the final control element, such as valve, damper, or rheostat, was done manually by an intelligent workman who based his actions upon observations and judgments derived from his familiarity with the process. The basic reasons for applying the automatic regulator was to relieve him of the many repetitive adjustments he was wont to make and to provide the process with a constant attendance independent of the human equation. These functions are usually well served by automatic regulators.

The fact that a regulator cannot exercise human judgment is frequently overlooked. As a result, many contradictory conclusions exist regarding the merits of automatic regulators.

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[6] "Sur les Oscillations des Régulateurs," by Le Corbeiller, Proceedings Third International Congress of Applied Mechanics, Stockholm, 1930, vol. 3, pp. 205–212.

[7] Consulting Engineer, Chicago, Ill. Mem. A.S.M.E.

[8] Consulting Engineer, Chicago, Ill. Mem. A.S.M.E.

This is in large part due to misguided efforts to evaluate their merits as devices per se, without taking into account the characteristics of the process equipment to which they are applied. For, to reach intelligent and correct conclusions, it is fundamental to consider the process, its equipment, and the regulator, as an entity. The differences in regulator characteristics are merely the different laws of control-valve movement built into them by the manufacturer. No such standardization exists, however, for the process equipment or the process itself. Consequently, since these latter exercise an important influence upon the controllability of the process either manually or automatically, the designers of such equipment should pay particular attention to the factors which will make the process easy to regulate. For it is wishful thinking to suppose that a process which is so designed that it is difficult to regulate with human supervision can be well regulated by a machine which has no brain.

The author, a user of automatic regulators, is to be complimented for his unbiased recognition of the importance of correctly designing the process equipment. His qualitative analysis of heat-exchange processes will do much toward pointing out to process designers the factors which must be given consideration if good regulation is to be obtained. While his paper[1] is limited to a discussion of heat-exchange equipment, the fundamental principles, which have been frequently overlooked or misunderstood, have a much broader application. Although he has limited himself to qualitative analyses only, the solutions are so clearly defined that designers should have no difficulty in applying them. Until the physical picture of the regulation of process has been correctly painted and is well understood, quantitative analyses may be of doubtful value.

The paper presents a classification of automatic regulators which is worthy of consideration. In spite of the great number of different commercial regulators on the market, it becomes obvious that the number of basic types is relatively small; therefore, the subject lends itself to basic classification quite readily. This is due to the fact that fundamentally the regulator involves only two functions: (1) The measurement of a physical variable and (2) positioning of a control element in accordance therewith.

Ivanoff[9] points out that, by classifying the methods of measurement the regulator is classified at the same time, since fundamentally the regulator positions the control element in proportion to the measurement function to which it is responsive. This is a slightly more basic method of classification than mentioned in the present paper and it might be well to compare the two, as in Table 1 of this discussion. The on-and-off or two-position methods are usually treated as a separate class as noted in this table, but actually these are limiting cases of the proportional or floating method. Thus, a very high-speed floating control and a proportional control with an extremely small band can be considered as on-and-off or two-positioning controllers.

TABLE 1 COMPARISON OF IVANOFF'S AND HAIGLER'S CONTROLLER CLASSIFICATIONS

Ivanoff	Haigler
(*a*) On-and-off method	(*a*) On-and-off method
(*b*) Actual temperature deviation	(*b*) Proportional wide-band, and proportional narrow-band
(*c*) Integral of temperature deviation	(*c*) Floating type
(*d*) Deviation plus integral (proportional plus floating)	(*d*) Proportional-reset
(*e*)[a] The rate of change (first-derivative method)	.....
(*f*)[a] The rate of change plus integral (first-derivative plus floating)	.....

[a] Ivanoff also adds these two types of specialized controllers which are not in very great usage.

The concept of supply-side capacity and demand-side capacity is important, although it only applies to a system that can be defined in terms of two capacities. The logical extension of this type of distinction is to define the system in terms of as many capacities as actually occur, with each capacity defined and bounded by the resistances which separate it from the next capacity. This has been done by recent writers and the author deserves commendation for emphasizing this concept.

In using the qualitative analysis of the author, the difficulty lies in the fact that it is impossible to think in terms of more than two capacities without some sort of mathematical definition. Furthermore, such concepts as high speed and low speed, wide band and narrow band, and large and small capacities are all relative and are only valuable in showing trends. Unfortunately, if the qualitative approach only be used, exceptions begin to be found which would not appear if the terms were defined quantitatively, as in the method of the hydraulic analogy. Until such quantitative solutions are available, an analysis such as the author's is of great assistance to the users of equipment.

S. D. Mitereff.[10] In so far as the paper elucidates the factors facilitating the application of temperature control in process industries, it is admirable. In so far, however, as it directs the attention to the necessity of "tailoring" the process equipment to make it easily controllable with the simplest types of regulators mentioned in the paper, the writer believes the paper to be somewhat misleading.

With the present tendency toward ever-increasing sizes of equipment, made often of special material, any change in either projected or existing plants is extremely costly. Moreover, the annual value of product passing through a given piece of equipment is sometimes enormous. Under these conditions, the process equipment should be designed almost entirely from the standpoint of efficiency consistent with capital cost and not from the standpoint of easy controllability. If two of these objectives be compatible, so much the better. If not, then it is up to the control engineer to select control apparatus which is "tailored" to the plant. It should be stated here that controllability is not an absolute term, depending as it does upon the type of control equipment contemplated. The process which is not controllable with one type of regulator might be, and usually is, easily controllable with some other type.

On the basis of his own study, as well as of a perfectly general theorem of Minorsky (discussed later by the writer), the writer can state a priori that there is no temperature-control problem, however involved, which could not be solved to any desired degree of accuracy by the use of the rate-of-change-responsive regulators, provided that by and large the process is one of systematic behavior. That such regulators are more complex than ordinary types should not bar their wide acceptance. The only valid criterion aside from cost, for selection of a piece of equipment is its reliability, using this term in its broad application to the process as a whole. Cost is a factor, but the incremental cost of modification of any good regulator to make it respond also to the rate of change of temperature consists of the cost of few simple additional parts and this cost is vanishingly small compared with saving in capital and operating cost resulting from the use of such advanced regulator characteristics.

Reliability of equipment depends to a great extent upon its designer. With the multitude of ideas made available recently, there is no reason why any good designer, given time, resources, and opportunity, could not design a reliable rate-of-change-responsive regulator.

The author of the paper should be congratulated on bringing to light the concept of systematic versus unsystematic behavior of controlled process. For the practical purposes investigated

[9] "Influence of Characteristics of a Plant on the Problems of an Automatic Regulator," by D. A. Ivanoff, October 16, 1936.

[10] Petersburg, Va.

in the paper this concept needs no elaboration. Scientific significance of this concept, however, merits its further discussion.

Strictly speaking, unsystematic behavior is a systematic response to such a multitude of causes, often unknown and interdependent, that the analysis of such causes becomes impossible. Although the reasoning from cause to effect is impossible in predicting an unsystematic behavior of a function, there exists another no less scientific method of prediction of such a function. This method, disclosed in a recent U. S. patent of Minorsky's, consists essentially in finding the series of higher time derivatives of a function, prefixing it by the value of the function, and multiplying it by the time interval desired. The sum of this series then becomes the value of the function which it will have at the expiration of this time interval. The number of terms in the series, and, consequently, the highest order of derivative, depends in some fashion on the number of distinct causes affecting the function.

It follows from this that the systematic behavior, by its nature, can be predicted with reasonable accuracy by the use of few terms. On the other hand, the prediction of unsystematic behavior may require the use of such great number of terms as to render this method inoperative in a practical sense.

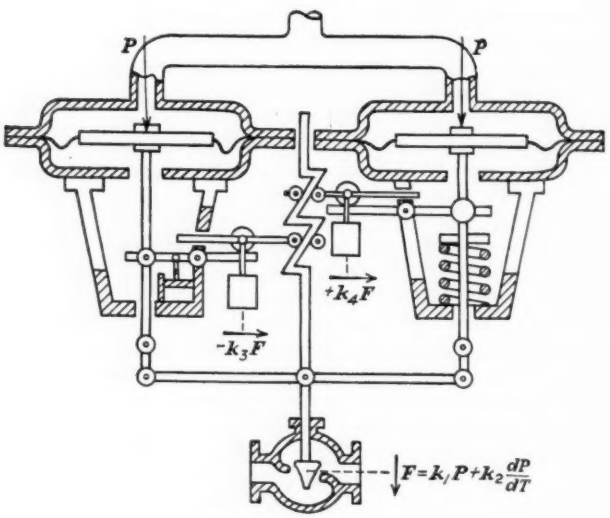


FIG. 1

The writer would like to suggest, as a research problem of some scientific institution, the development of a curve-extrapolating machine based upon this theory—this machine in effect would be a predictor of events plotted on the curve, be it weather data, or other examples of both systematic and unsystematic behavior. Since such a machine can be easily reversed, its accuracy can be checked against known past variation of the function.

In concluding his series of contributions on the subject of automatic control, the writer would like to remark that he greatly regrets the injection into the discussions of an unnecessary argument as to what is more important, theory or practice. The writer was working on the assumption that theory is just an explanation of the facts from the standpoint of basic laws of physics and mechanics and that the theory is therefore very valuable in preventing practice from straying into fallacies resulting from imperfect understanding of observed phenomena. Of course, the difference between a carefully worked out engineering theory and a mere hypothesis ought to be known to engineers.

Another value of a theory is that if it be expressed in mathematical form an advance in the art can often be made by mere manipulation of equations and analysis of the physical meaning of the results. A simple rate-of-change-responsive regulator can be designed by this method as follows: The desired characteristic equation[11]

$$F = k_1P + k_2\frac{dP}{dT} \quad [1]$$

can be rewritten as

$$(F - k_2k_4F) + k_1k_3\int FdT = k_1\int PdT + k_2P \quad [2]$$

provided $k_1k_3 = k_2k_4 = 1$.

Equation [2] can be put into the form

$$F = k_1\int (P - k_3F)dT + k_2(P + k_4F) \quad [3]$$

Examination of Equation [3] discloses that it refers to a regulator of characteristic $F = k_1\int PdT + k_2P$ provided with opposite-sign adjustment of primary impulse P in proportion to final effect F. The simplest form of such a regulator is shown, purely schematically, in Fig. 1 of this discussion.

An equivalent electric vacuum-tube circuit can easily be designed, based on Equation [3], so that adjustment of primary impulses is accomplished by use of the familiar feed-back principle.

One of a number of rather dramatic applications to electric circuits of the rate-of-change-responsive characteristics is in correcting the distortions in phonograph records made by the old-style mechanical methods. If these old records are reproduced by a carefully designed rate-of-change-responsive electrical-reproducer circuit, most of the naturalness of the original music or voice can be resurrected, especially if special compensation is made for distortion of recording caused by solid friction.

Even more spectacular results can be obtained through application of Minorsky's basic invention of the rate-of-change-responsive electric circuits to television. Depending on the degree of continuity and "systematicity" of the viewed scene, a preview of future events can be achieved in video amplifiers. In slow-changing processes, such as the growth of grass, the accurate advance in time can be as much as days or even weeks, and the importance of this invention in such cases cannot be overestimated. The field of medical prognosis and research might be interested in this invention, and it is hoped that an active development is being carried out to make its benefits available to the public.

ED S. SMITH, JR.[12] The qualitative treatment used by the author is consistent with the exploratory nature of the paper and fits in ideally with the chief aim of the A.S.M.E. Committee on Industrial Instruments and Regulators, under whose auspices it was presented, which is the cooperative sharing of experience and of knowledge of the engineering fundamentals involved, mainly for the use of those who are studying the subject without benefit of adequate personal experience.

Practically all of the paper consists of useful observations analytically coordinated; these portions will not be further dealt with in the writer's discussion. The author's main contribution in a practical sense is the example set by his own independent and courageous engineering of process conditions so as to be most favorable for control, an example that accelerates the present still-new era in successful and economic process control. On the economic side though, the writer presumes to call attention

[11] See "Principles Underlying the Rational Solution of Automatic-Control Problems," by S. D. Mitereff, Trans. A.S.M.E., vol. 57, 1935, paper FSP-59-9, p. 159, for definition of terms used in this discussion.

[12] Hydraulic Engineer, C. F. Tagliabue Mfg. Co., Brooklyn, N. Y. Mem. A.S.M.E.

to the fact that modern controllers having automatic reset are relatively inexpensive in comparison with even simple piping changes in old plants and give improved control out of all proportion to their surprisingly low cost, considering the improved performance under difficult conditions. It is of course recognized that there is no one best controller for all installations.

On the side of terminology, the paper's terms are likely to be generally understood by initiates, but it should be brought out that "corresponding" is more accurate than "proportional" and that "regulation" is possibly more widely accepted as being broader than "control" in related fields. The literature is more readily integrated if the more accurate usages are followed.[13]

Much of the paper emphasizes the fact that off-and-on control benefits by enough storage to iron out the variations due to violent attack on the controlled variable. Starting with simple assumptions as the heat H_1 equals the heat output H_2, that the load L in Btu per deg F per sec, and that the temperature change T is proportional to the heat change for a given load, the equations may be readily set up for off-and-on control in most of the heat-exchanger cases discussed in general terms in the paper and the conclusions immediately proved or disproved. It is to be hoped that the next meeting of the A.S.M.E. Committee on Industrial Instruments and Regulators will result in a subgroup that will share the work of such investigations so that the full burden will not fall on any one individual and so that the results will be available to all. This is only one of the jobs that such a group could do to the advantage of all, and with comparatively little effort. Another task would be the stating of the simple static and dynamic relations involved, so as to bring out the interrelations of capacity and other lags, relative speeds of valve travel, reset speeds, dead zones, and sensitivities.

In the author's discussion of his Fig. 1, a reference to hunting appears which seems to be based upon some unfortunately common experiences with a too-large valve. In considering the differences between off-and-on and proportional control, it seems to the writer that more care is needed in separating the effects of process and instrument lag, i.e., lag in sensing the changes of the variable, which is relatively more serious with proportional control than with off-and-on control, in view of the smaller potential changes existing with proportional control.

As to these "lags," the term itself is misdescriptive, when thought of as so many seconds between one event and its result, except in the case of transportation lag where it does apply. In the cases of metering lag and process lag due to capacity, it seems handier to the writer to use the "interval," which is the number of seconds for a value to respond to within one tenth of the ultimate change, following a sudden upset.

In the author's description of Fig. 4, first paragraph, there appears to be some implied contradiction in the average temperature differences in regard to the efficiencies of the heat-transfer surfaces.

In conclusion, it appears that the paper brings out clearly the advantages and disadvantages, from a control standpoint, of a number of common cases. This is now available as a portion of the material required for a complete treatment. However, it would seem that it might be well to take the next steps by formulating the simplest and most concise statement of the basic principles involved, and then carrying forward the mathematical analysis for both static and dynamic relations. The writer appreciates the author's courageous gesture in writing a paper for the sake of settling in open discussion what are more or less debatable points, in which he deliberately chose the *ex cathedra* presentation and with full understanding of its inherent vulnerability. The fact will remain, after the discussion is closed, that the present paper has performed an outstanding service in its field.

[13] See "Fluid Measurement and Control," by Ed S. Smith, Jr., *Chemical and Metallurgical Engineering*, vol. 44. no. 5, May, 1937, p. 178.

Author's Closure

The author wishes first of all to call attention to two typographical errors in the paper which escaped correction. On page 635, first column, line 7, the word "or" is omitted at the end of the line following "on-and-off" and preceding "narrow-band instrument" on the next line. On page 637, Figs. 9 and 10 have the correct figure numbers, but the legends are interchanged. Incidentally, while the printed text incorporates a number of minor changes from the preprint text, where misinterpretation seemed possible, it is believed that none of these changes affect the discussion.

Mr. Kimball has suggested a simplification of control problems through standardization of the specifications of controllers. It is entirely possible that, within certain fields of pressure control in which he seems to be most interested, some such standardized specifications might be workable. However, it seems to the author that, in the general field of process control and particularly in the field of temperature control, such limitations would never be possible. On the one hand, the majority of industrial control problems in most fields of application are best handled by simple on-and-off or two-position control. On the other hand, many control problems require throttling ranges of not less than from 20 to 50 per cent of convenient instrument-scale ranges, and at least one type of proportional reset controller is regularly built and used with throttling ranges as wide as 3000 per cent. Obviously, customers whose applications require only on-and-off control will not wish to pay for more complicated constructions, providing throttling range which is useless to them. Neither could users nor manufacturers be limited to any maximum throttling range, 10 per cent or any other value, if a considerable portion of the process problems could be satisfactorily solved, provided wider throttling ranges were available.

Mr. McMahon is to be congratulated on picking out and filling in one fundamental weakness in the paper. The author's experience with control problems involving considerable transfer lag had been rather limited, and this subject was barely mentioned. It is fortunate that one with so much experience in such problems has filled in this point in the discussion.

Many other arrangements of heat exchange will occur to readers in accordance with their own experience. The by-pass controlled exchanger mentioned by Mr. McMahon is certainly an important type, although one rarely met in the author's own experience.

Incidentally, it has come to the author's attention that, in the minds of some readers, the term "heat exchanger" was not sufficiently broad, and did not cover simple electrically heated units in which there was admission but no discharge of heat units on the supply side. It was intended that the term "heat exchanger" should be extremely broad, covering everything from such simple heaters—and the common steam heaters closely approach the electrically heated case, since heat losses in the condensate are relatively rather small—on to the most complicated heat exchangers.

Mr. Wagner is to be particularly thanked for discussing an application of the paper in fields far from the author's industrial experience. It is certainly true that building heating involves some of the most complicated temperature-control problems, and only the considerable tolerances permitted by the human organism have made many methods of control passable.

In regard to the question of economics raised by Mr. Wagner, it is the author's belief that, in the words of Professor Marks, "A good engineer is always aware of the economic problem."

The same economic factors are present in this temperature-control problem that exist in problems of process temperature control, but the weighting may be entirely different.

Mr. Behar's discussion opens with some statements indeed flattering to the author. It is believed, however, that they do not give due credit to the collective efforts of engineers associated with the various manufacturers. It was the author's impression as a customer that leading vendors have long been aware of the fact that wider use of instruments was limited more by application problems than by design and construction problems. Progress is in no small measure due to the collective efforts of their factory and field engineers who have been working quietly but nonetheless effectively toward enlightenment of the user.

Mr. Behar questions the author's statement that the dry bulb of a psychrometer must not be shielded, in the passage: "At the same time the adjacent dry bulb must not be shielded or spattered." It was perhaps unfortunate that the word "shielded," having also a specialized meaning, was used, since simply blocking off the representative air stream is meant. Radiation shields are, of course, frequently used in psychrometers.

The author is aware that classification of controller mechanisms without their associated processes does not tell the whole control story. Nevertheless, it is felt that the method of classification used in the paper, listing separately the control mechanisms to be purchased and the processes to be equipped, is the only one of much practical help to the user.

Messrs. Spitzglass and Zucrow have discussed at length the paper of the author and that of Bristol and Peters.[14] They point out clearly the limitations of qualitative analysis, both in regard to inexactness and in regard to inapplicability to processes involving more than two capacities; yet, at the same time, they declare the usefulness of a qualitative picture in leading to later quantitative work. The author will be entirely satisfied with the qualitative scope of his paper if it does help in understanding quantitative work which may follow.

It should be mentioned in passing that since this paper was written and presented in 1937, although only recently printed, it antedates the mathematical papers of Mason[15] and of Spitzglass.[16] Being based, therefore, necessarily entirely on observations and intuitive judgments, without benefit of exact checking by mathematical methods just now coming into use, its use for indefinite extrapolation of the conclusions far beyond the scope of the original observation is not advised without some form of checking.

Mr. Mitereff questions "the necessity of tailoring the process equipment to make it easily controllable with the simplest types of regulators," while Mr. Smith raises the same question "when modern controllers having automatic reset are relatively inexpensive in comparison with even the simple piping changes in old plants, and can improve control out of all proportion to their surprisingly low cost." The author would not for a moment advocate uneconomic changes in existing plants or even in new plants, merely to permit the use of a simpler and cheaper controller. But he does wish to emphasize that design for controllability is usually of considerable economic advantage when over-all investment and operating costs for the process are considered. It must not be overlooked that the initial response of even the best proportional reset controllers now available is the response of a simple throttling controller. While the reset mechanism will correct deviation of the control point which has occurred as a result of load changes, it does not prevent that initial deviation. Hence, any process which requires wide throttling ranges must unavoidably operate with considerable deviation following upsets. If the process is designed for improved controllability permitting a much narrower control band or throttling range, a simple proportional narrow-band, or even an on-and-off, instrument may give better control than the best reset instrument on a process improperly designed. There is no doubt in the author's mind that design for controllability will in the long run extend rather than circumscribe the application of control instruments.

[14] See pp. 359–361, and 364–365, of this issue for discussions by M. J. Zucrow and A. F. Spitzglass.

[15] "Quantitative Analysis of Process Lags," by C. E. Mason, Trans. A.S.M.E., vol. 60, May, 1938, pp. 327–334.

[16] "Quantitative Analysis of Single-Capacity Processes," by A. F. Spitzglass, Trans. A.S.M.E., vol. 60, November, 1938, pp. 665–674.

Discharge Temperatures From Pulverizers

By OLLISON CRAIG,[1] WORCESTER, MASS.

In view of the fact that the temperature of discharge from coal pulverizers was of interest because of the effect that it might have on the amount of air passing through the preheater, and on the amount of uncontrollable air through the furnace, data were taken in a number of plants covering temperature of air entering pulverizers, temperature of air in coal discharge, moisture of raw coal entering the pulverizer, moisture of discharged coal, and weight of coal pulverized per hour.

Two sets of points were plotted, one showing the temperature of discharge for given entering temperatures, and the other the temperature drop through the pulverizer for given entering temperatures. In plotting the data no consideration was given to the size of pulverizer, amount of coal pulverized, or the moisture content of the coal. The plotted points fell in a straight band having a width of approximately 65 deg. A straight line drawn through the band shows the average variation within 35 deg.

Sufficient information was obtained in the cases of two plants to calculate heat balances. From these heat balances leakage of uncontrolled air into the pulverizer was calculated. It was found that this amounted to about 2 per cent of the total air for combustion.

WHEN UNIT pulverizers are used for grinding coal and coal is discharged directly from the pulverizers through piping and burners into a boiler furnace, air is required as a conveying medium for the coal. This air is usually referred to as primary air. It would be desirable for a number of reasons to pulverize coal in a unit pulverizer and discharge it into the furnace without the use of any primary air. If this were possible the power required to propel the air would be saved; all the air of combustion except furnace leakage air would be introduced through the burner as secondary air, providing more accurate control of combustion conditions, and making possible the use of smaller discharge pipes between pulverizers and burners. However all existing designs of mills in use require air for conveying coal through the pulverizer and through the discharge pipe and burner. This primary air, being mixed with the pulverized coal, is of further value in obtaining initial ignition of the coal and in securing proper mixture of the coal with the secondary air. The fact remains that primary air is necessarily used, and primary air must be taken into account in determining performance of pulverizers and design of burners, air heaters, and air ducts.

Occasion arises in all plants when it is necessary to pulverize and burn coal containing more than a normal amount of moisture. When the coal is more than normally wet it is desirable to supply primary air to the pulverizer at a temperature higher than that of the surrounding atmosphere. The extra heat available in the hotter primary air is then available for evaporating moisture from the coal. Drying of the coal during the process of pulverizing has the effect of increasing the capacity of the pulverizer, decreasing the amount of power which is required to drive the pulverizer, and increasing the fineness to which the coal is reduced.

Many boiler installations are equipped with air preheaters and in such cases it is customary to take air from the hot-air duct and introduce it into the pulverizer as primary air. This method is satisfactory and effective, providing that the temperature of the preheated air is sufficiently high for the amount of moisture contained in the coal and for the amount of coal to be pulverized. There are, on the other hand, boiler installations which use pulverized coal but which do not have air preheaters. In these cases it is usually desirable to provide other means for heating the air going into the pulverizer. Steam-air heaters are often used for this purpose. With steam-air heaters the temperature of primary air that can be obtained is limited by available steam pressure. In other cases hot gas is taken from the back pass of the boiler or from the breeching, mixed with air, and introduced into the pulverizer. This method has the disadvantage of requiring too large a proportion of gas in the primary-air mixture, thus diluting the primary air with inert gas and interfering with ignition at the burner. Sometimes gas is withdrawn directly from the furnace, mixed with tempering air immediately outside of the furnace, and the mixture is introduced into the pulverizer as primary air. This last method has a number of advantages. Any desired temperature, subject to the limitations of the pulverizer, may be obtained. The amount of inert gas so used is usually of insufficient amount to interfere seriously with ignition.

Frequently air preheaters are designed to give rather high temperatures, that is temperatures of 500 F or more. Temperatures as high as this or higher can also be obtained by the use of hot gas from the boiler furnace. In general it would be desirable to take advantage of the highest temperature available and thus improve pulverizer performance. Oftentimes a limitation is placed on the temperature of air entering the pulverizer, or the temperature of the mixture leaving the pulverizer. This limitation may be established to protect bearings or casings but usually it is to prevent fire in the pulverizer, in the classifier, or in the coal-discharge pipe. If ignition of coal occurs, flame cannot be produced nor can there be an explosion unless the velocity of travel of the coal-and-air mixture is less than velocity of flame propagation. This is illustrated by flame action at the burner. The reason flame does not travel back into the burner is that the velocity of discharge from the burner is greater than velocity of flame propagation. In those pulverizers in which coal-and-air velocities exceed the velocity of flame propagation, for any ratio of coal-and-air mixture that might exist, ignition of coal could occur without danger of fire or explosion. This has been demonstrated by actual tests. In case the velocity of the mixture be less than velocity of flame propagation, it is necessary to take such precautions as possible to prevent ignition, since, should ignition take place, flame would result and an explosion occur.

Reference has been made to "velocity of flame propagation, for any ratio of coal-and-air mixture that might exist." Velocity of flame propagation is the rate at which flame travels through a combustible mixture and may be expressed in feet per minute. This velocity is not constant for all mixtures of powdered coal and air, but varies with the kind and condition of the coal and with the ratio of coal and air in the mixture. The curves of Fig. 1 illustrate this. Each curve is for a different coal containing a particular percentage of volatile matter and of ash. It is to be noted that the greater the percentage of volatile matter and the less the ash the greater is the velocity of flame propagation.

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NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

Each curve defines the relation between the coal-air ratio and the velocity of flame propagation. It will be noted that for each coal there is a coal-air ratio which results in a maximum velocity of flame propagation. Other factors affect velocity of flame propagation, but the factors used in the curves are of major importance and are sufficient for the purpose of this discussion.

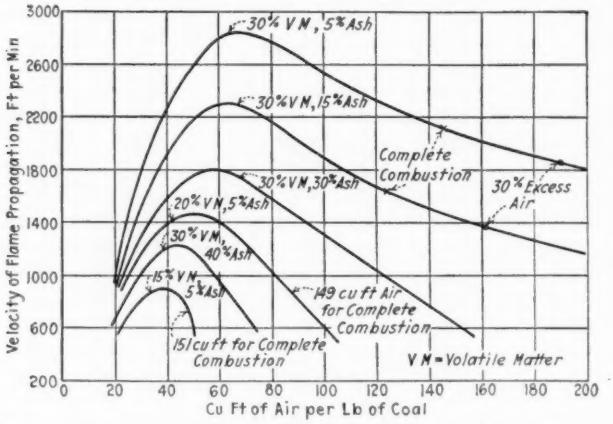


FIG. 1 VELOCITY OF FLAME PROPAGATION FOR MIXTURES OF PULVERIZED COAL AND AIR

(Coal dust through 200-mesh sieve. Curves reproduced from *Revue de Metallurgie*, November, 1922. Experiments at Taffanel Experimental Galleries, Lievin, France.)

In the course of changing rates of coal pulverizing and rates of air supply to a pulverizer, some ratio of coal and air will occur which will result in the maximum velocity of flame propagation for that particular coal. For example, assume that coal having 30 per cent volatile matter and 5 per cent ash is being pulverized. From the top curve of Fig. 5 it will be seen that if 1 lb of this pulverized coal is mixed with 67 cu ft of air, the velocity of flame propagation through the mixture would be 2820 fpm. Any other coal-air ratio would result in a slower velocity of flame propagation with this particular coal. The amount of air in this mixture is about 47 per cent of the amount of air theoretically required for complete combustion. For other coals, maximum velocities of flame propagation occur with other ratios of coal to air. In any case there is a probability of a coal-air ratio occurring in a pulverizer, at some time, which would result in the maximum velocity of flame propagation. The velocity of travel through the pulverizer must be greater than this velocity of flame propagation to avert danger of fire and explosion, or the temperature within the pulverizer must be maintained sufficiently low to prevent ignition.

The terms "explosion" and "explosive" as used here are relative. A mixture which is more explosive than some other mixture, is one in which the velocity of flame propagation is greater. Explosion is the rapid burning of a mixture in a confined or partially confined space with a resulting building up of pressure in the space. In a partially confined space, such as in a pulverizer, the faster the mixture is burned the greater is the resulting pressure and the more explosive is the mixture. The velocity of flame propagation is thus related to explosiveness.

Various manufacturers have determined from experience what temperatures of discharge from their particular pulverizers are permissible without danger of ignition within the pulverizer. In certain cases this discharge temperature is regulated by the admission of cold or room air into the pulverizer with the hot preheated air. There is of course a disadvantage in this method since the cold air, which becomes a portion of the combustion air, does not pass through the air heater. The effectiveness of the air heater is thereby reduced and consequently more expense is entailed for a larger air heater to compensate for decreased effectiveness. Also, in such cases, since all the gaseous products of combustion pass through the air heater, either the air is heated to a higher temperature than otherwise or the temperature drop of the gas is reduced.

It is probable that all pulverizers, with the exception of those which are operated under a positive air pressure, take in some cold air other than that which comes through the primary-air ducts. This leakage air may enter through joints and cracks or through the coal and feeder. Its effect is the same as that produced by tempering air which is admitted under control and is greater or less as the amount of leakage is greater or less than the amount of the controlled tempering air.

The temperature at which a mixture of coal and air is discharged from a pulverizer and its relation to the temperature of the air which is admitted to the pulverizer as primary air, is of interest, because of its bearing on the amount of primary air required, effectiveness of coal drying, amount of tempering air, and elimination of possible burning of coal within the pulverizer.

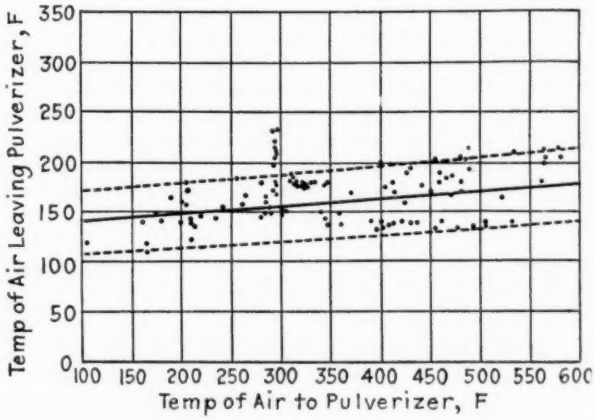


FIG. 2 AIR TEMPERATURES FOR COAL PULVERIZERS

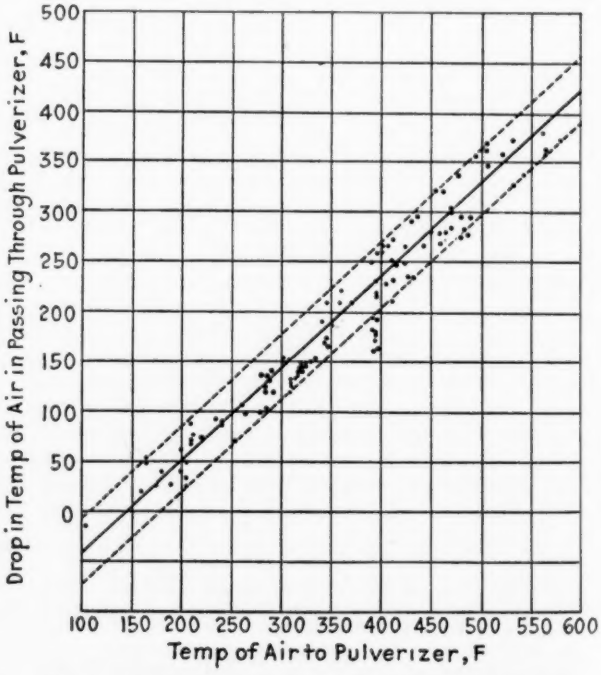


FIG. 3 AIR-TEMPERATURE DROP THROUGH PULVERIZERS

TABLE 1 COAL-PULVERIZER TESTS

Coal pulverized, lb per hr	9070	9450	9950	10850	11250	12000
Air temperature, pulverizer inlet, deg F	470	430	483	465	450	480
Air temperature, pulverizer outlet, deg F	170	195	205	185	170	185
Temperature drop through pulverizer, deg F	300	235	278	280	280	295
Moisture in raw coal as fed, per cent	3.10	3.50	3.06	3.75	2.93	3.08
Moisture in pulverized coal, per cent	0	0	0	0.57	0	0
Moisture evaporated, lb per hr	281	331	304	345	330	388
HEAT BALANCE						
Heat supplied — Thousands of Btu						
Heat from hot air	646	720	766	768	717	848
Heat from power	135	137	141	147	150	152
Total	781	857	907	915	867	1000
Heat disposed of						
Heat absorbed by coal	270	352	394	373	337	414
Heat absorbed by moisture	338	397	363	415	398	468
Heat absorbed by leakage air	145	69	107	93	104	84
Heat dissipated by radiation	28	39	43	34	28	34
Total	781	857	907	915	867	1000

TABLE 2 COAL-PULVERIZER TESTS

Coal pulverized, lb per hr	3400	3600	4300	4300	4550	4730	5000	6400	6800	6800	8000	8600
Air into pulverizer, deg F	391	394	396	394	392	395	395	392	395	395	395	392
Air leaving pulverizer, deg F	248	221	232	213	231	217	202	210	177	178	168	197
Temperature drop through pulverizer, deg F	143	173	164	181	161	178	193	182	218	217	227	195
Moisture in raw coal as fed, per cent	5.8	8.7	7.3	8.7	5.8	7.2	9.6	6.1	8.9	8.8	7.7	4.9
Moisture in pulverized coal, per cent	0	0	0	0.2	0	0	0.2	0	0.97	0.75	0.92	0.2
Moisture evaporated, lb per hr	197	313	314	365	264	340	470	390	540	547	540	403
HEAT BALANCE												
Heat supplied — Thousands of Btu												
Heat from hot air	392	460	460	504	405	477	579	510	643	643	634	543
Heat from power	113	114	122	130	127	134	142	170	193	184	218	222
Total	505	574	582	634	532	611	721	680	836	827	852	765
Heat disposed of												
Heat absorbed by coal	152	139	171	146	180	167	151	217	178	166	174	268
Heat absorbed by moisture	225	355	360	410	300	382	522	431	594	600	592	448
Heat absorbed by leakage air	91	48	16	46	17	30	20	0	49	36	62	18
Heat dissipated by radiation	37	32	35	32	35	32	28	32	25	25	24	31
Total	505	574	582	634	532	611	721	680	846	827	852	765

In order to throw more light on these relationships, test data were taken on pulverizers in a number of plants. In each case the principal data obtained were:

Rate of pulverization in pounds of coal per hour
Percentage moisture in raw coal entering the pulverizer
Percentage moisture in pulverized coal leaving the pulverizer
Temperature of air entering the pulverizer
Temperature of air leaving the pulverizer
Inlet and discharge pressures.

This information was obtained on sixteen pulverizers scattered over seven different states. Nominal capacity of the pulverizers varied from 2500 to 12,000 lb of coal per hr. More than 100 sets of determinations were made.

To indicate the primary relations in all of these tests, the temperatures of discharge from pulverizers and the temperature drops through the pulverizers were plotted against the temperatures of air supplied to the pulverizers as primary air, as shown in Figs. 2 and 3. Obviously the ordinates in Fig. 3 are the temperatures of air supplied to the pulverizers minus the ordinates in Fig. 2. In both these cases, points were plotted, giving no consideration to the size of the pulverizer, the amount of coal being pulverized, or the moisture content of the coal. The actual quantities of coal pulverized, as represented by the various points, varied from 500 to 12,000 lb per hr, and the moisture content of coal fed varied from 2 to 10 per cent. It is interesting to note that, regardless of all these variations, points in Fig. 2 representing the temperatures of air leaving the pulverizers, nearly all fell within a band having a width of about 65 deg. Also most of the points in Fig. 3 fell within a band of the same width. The writer does not attempt to explain this. It is known that those points falling largely outside this band were for conditions in which the coal was unusually wet. In Fig. 2 a line is drawn through the center of the band, representing the average discharge temperatures corresponding to the temperatures of air admitted to the pulverizers. This curve is nearly flat, changing only about 40 deg in discharge temperature over a range from 100 to 600 F in temperature of air admitted to the pulverizers. In Fig. 3 an average curve is also shown representing the average temperature drop of air in passing through the pulverizers in relation to the temperature of air supplied to the pulverizers. Because of the flatness of the curve in Fig. 2 the curve in Fig. 3 is necessarily steep.

In most of the plants in which data were taken, sufficient information was not obtained to calculate heat balances. In two plants, however, sufficient information was obtained; in one plant at six rates of coal grinding, and in the other plant at twelve rates of coal grinding. The heat balance was calculated for each rate of coal grinding in both plants, as shown in Tables 1 and 2. In principle, the calculation of the heat balance for a pulverizer is the same as that for a boiler. Primary air supplies a certain amount of heat which is determined from the weight of air, the temperature drop of the air in passing through the pulverizer, and its specific heat. A portion of the power used to drive the pulverizer is converted into heat within the pulverizer. The sum of these two quantities represents the total heat input. Heat is absorbed in raising the temperature of the coal, in evaporating and superheating the moisture, in heating leakage or tempering air, and in dissipation by radiation.

In both plants in which data were taken as a basis of these heat-balance calculations, hot air was supplied to the pulverizers from air preheaters. In neither case was tempering room air admitted to the pulverizers intentionally. Any such air entering the pulverizers occurred as leakage. In first making the heat-balance calculations, leakage was ignored, consequently the two sides of the heat balance did not check. It was obvious that the discrepancy was due to heat absorbed by air which leaked into the pulverizers. The amount of this air and the heat absorbed by it were then determined by differences. The determination of leakage then included the resultant of all errors in obtaining data. This accounts for the erratic figures in connection with leakage air. The curves in Fig. 4 give a picture of the amount of leakage

air. On the average, primary air varied from about 30 per cent of total air required for combustion at 3500 lb of coal per hr to 12 per cent at 8500 lb of coal per hr. Throughout this same range, leakage air varied from about 13 to about 11 per cent of the total primary air through the pulverizer and from about 3.5 to 1.5 per cent of total air of combustion. The average leakage was only slightly above 2 per cent of the total air of combustion. Generally in the cases of those boiler installations which have air pre-

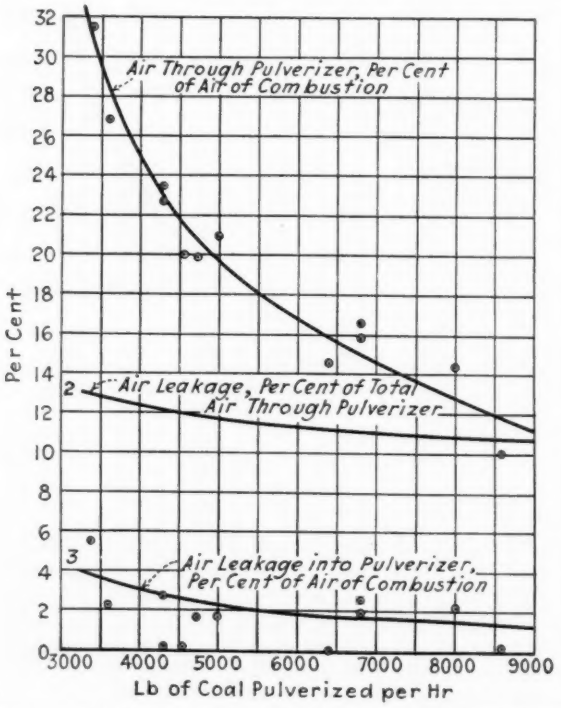


FIG. 4 AIR LEAKAGE AND TOTAL AIR THROUGH PULVERIZERS
(Data from Table 2.)

heaters, the amount of air passing through the air preheater will be from 80 to 85 per cent of the total air of combustion. From 15 to 20 per cent of the total air of combustion enters the furnace by other means. This leakage represents air which is not controllable in operation and which reduces the effectiveness of the air preheater. It is then desirable to reduce to a minimum the amount of combustion air which does not go through the air preheater. It is to be noted from Fig. 4 that the average amount of air leaking into the pulverizer is only about 2 per cent of the total air of combustion as compared to the total leakages from all causes of from 15 to 20 per cent in the ordinary installation.

It is indicated in the heat balances in Tables 1 and 2 that by far the greater portion of heat supplied to the pulverizer is absorbed by coal and by evaporating and superheating moisture. The coal used during the tests covered by Table 2 had somewhat more than the usual amount of moisture, from 5.8 to 9.6 per cent.

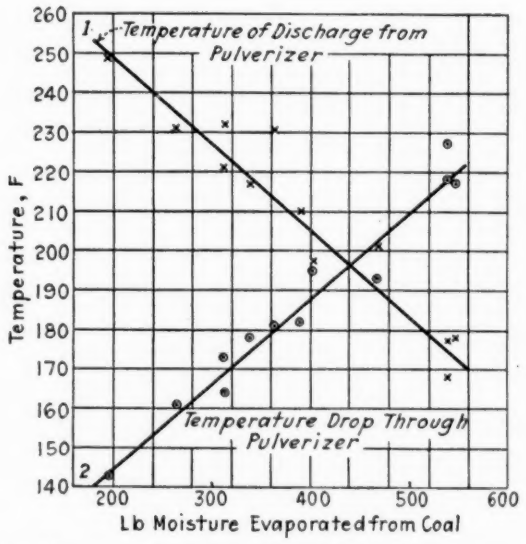


FIG. 5 MOISTURE EVAPORATED VERSUS DISCHARGE TEMPERATURE AND TEMPERATURE DROP
(Data from Table 2.)

The total weight of water evaporated from the coal varied from 392 to 643 lb per hr and the amount of heat absorbed by this moisture varied from 225,000 to 600,000 Btu per hr. Heat absorbed by moisture varied from 45 to 73 per cent of the total heat used and averaged 64 per cent. The temperature of air admitted to the pulverizer during all of the tests represented in Table 2 varied from 391 to 396 F and averaged 394 F. Since the temperature of air entering the pulverizer was so nearly constant and since the heat absorbed by the moisture was an unusually large percentage of the total heat absorbed, it would seem that in this case there was a fair relation between the amount of water evaporated, the temperature of discharge from the pulverizer, and the temperature drop through the pulverizer. Fig. 5 shows these relationships. It is to be noted that the range in temperature of discharge from the pulverizer is about 80 F although the temperature of air to the pulverizer is practically constant. A portion of the corresponding points in Fig. 2 are those which lie without the band. The reason for this peculiar arrangement of points is of course due to the large amount of heat absorbed by moisture and the large variation in total moisture evaporated.

The writer realizes that the amount of information presented in this paper is relatively meager and that no broad conclusions can be drawn from the results. It is expected however that these data will produce some interest, particularly on account of their relationship to air-preheater and boiler performance. The results indicate the need for additional tests in a larger number of plants burning a wider range of coals. The present tests were all made on pulverizers of a single make and similar information in connection with other makes would be desirable.

Atomization of Oil by Small Pressure-Atomizing Nozzles

By E. B. GLENDENNING,[1] A. R. BLACK,[2] L. H. VENTRES,[3] and W. A. SULLIVAN[4]

This paper develops the importance of atomization, carburetion, and oil characteristics for efficient combustion, and demonstrates the effect of pump pressure, oil viscosity, variation in oil temperature, and nozzle design on the rate of efflux and spray characteristics. It includes a study of hollow- and center-spray nozzles with relation to oil viscosity, aerodynamic design of the oil burner, and combustion efficiency.

THE DOMESTIC oil burner is expected to supply just the correct amount of heat to maintain constant room temperatures regardless of the outside temperature. It must do this without overloading the boiler or building too high boiler pressures. It must do this safely, economically, and with the minimum amount of attention. Economy is the feature with which this paper is concerned. Of the many factors affecting operating economy, one of the most important is atomization.

To offset physical limitations of the domestic heating plant which influence combustion, atomization and carburetion in the burner must approach perfection. In order to establish the definite relationship between viscosity and atomization and the combustion characteristics of fuel oils in domestic-oil-burner nozzles, an investigation was undertaken, the results of which are reported in this paper. All features of oil-burner design which might promote economy and efficiency of operation have been studied, particularly the construction of those parts which affect atomization of the oil and carburetion of the oil and air in correct proportions.

Principles of Oil-Burner Operation

The atomization of oil in a domestic high-pressure-atomizing burner involves a small pump, a pressure-regulating valve, and an atomizing nozzle. The nozzle comprises three essential elements, i.e., a stem with tangential slots, a conical swirl chamber, and an orifice. Atomization is the result of centrifugal action and pressure. The slotted stem imparts an initial tangential velocity which produces a rapid rotation of the oil within the conical swirl chamber. The swirl chamber converts as much as possible of the initial pressure into velocity by forcing the oil against the action of centrifugal force toward the center of the chamber. The orifice releases the oil to the atmosphere at a very small radius and at a high speed of rotation, while at the same time governing the quantity of oil delivered at a given pump pressure. Fig. 1 is a cross section through a conventional high-pressure-atomizing nozzle.

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Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors and not those of the Society.

Effect of Atomizing Pressure on Spray Characteristics

In the authors' investigation a specially devised spray-testing chamber, Fig. 2, has been used to study the effect of atomizing pressure on spray characteristics. The following sequence will illustrate this effect on the development and evolution of the spray as the pressure is raised from a very low level upward until a stable atomization pressure is reached.

As the pressure is raised slowly from 10 lb per sq in., the oil, as it issues, goes through a fairly regular set of phases, culminating eventually in the fully formed spray.

At low oil pressures, the oil forms a "bubble" or "node" as it emerges from the orifice. As the pressure is raised there is a transition of the "node" into a "cup" or hollow hemisphere. In order to develop the spray fully after the point is reached at which the cup first forms, it is necessary to raise the pressure an additional 20 or 30 lb.

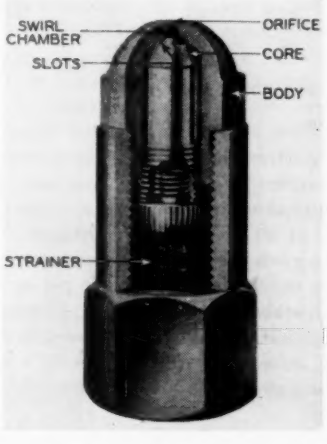


Fig. 1 Cross Section Through a Conventional High-Pressure-Atomizing Nozzle



Fig. 2 Special Spray-Testing Glass Chamber

TABLE 1 SPRAY CHARACTERISTICS OF TYPE-NS NOZZLES, USING 34-SEC-VISCOSITY OIL[a]

(Four-slot, smooth-faced disks in all nozzles; temperature of oil at nozzle 74–76 F)

Nozzle marking	Node formed	Cup formed	Full spray	Safe spray	Angle of spray at 100 lb per sq in., deg
	Lb per sq in.				
1.35[b]–60[c]	10	30	75	100	70
1.65–60	7	22	80	100	70
2.00–60	5	27	90	100	77
2.50–60	6	23	90	100	70
3.00–60	None	20	75	100	70
3.50–60	None	10	40	100	80
1.35–80	6	19	85	100	80
1.65–80	None	10	80	100	90
2.00–80	None	16	80	100	90
2.50–80	3	14	65	100	85
3.00–80	None	12	50	100	85
3.50–80	None	15	55	100	80

[a] Saybolt Universal at 100 F.
[b] Nozzle capacity.
[c] Spray angle, deg.

TABLE 2 SPRAY CHARACTERISTICS OF TYPE-NS NOZZLES USING 50-SEC-VISCOSITY OIL[a]

(Four-slot, smooth-faced disks in all nozzles; temperature of oil at nozzle 74–76 F)

Nozzle marking	Node formed	Cup formed	Full spray	Safe spray	Angle of spray at 100 lb per sq in., deg
	Lb per sq in.				
1.35[b]–60[c]	22	70	95	125	70
1.65–60	14	50	85	125	70
2.00–60	12	60	90	125	77
2.50–60	10	40	80	125	68
3.00–60	6	34	55	125	68
3.50–60	8	30	80	125	68
1.35–80	16	56	85	125	80
1.65–80	18	40	75	125	75
2.00–80	17	37	70	125	80
2.50–80	6	35	70	125	75
3.00–80	17	30	60	125	75
3.50–80	8	30	65	125	70

[a] Saybolt Universal at 100 F.
[b] Nozzle capacity.
[c] Spray angle, deg.

Table 1 contains the results of a series of tests on nozzles of various rate capacities in gallons per hour and spray angles in degrees. As a typical example consider the first nozzle, 1.35–60. The node forms at 10 lb per sq in. pressure. The transition from node to cup when the spray opens occurs at 30 lb. In this case the pressure must be raised to 75 lb per sq in. to establish the fully formed, stable-operating spray. It would be inadvisable to employ this nozzle to operate at pressures below 100 lb per sq in., since any increase in viscosity due to a slight fall in oil temperature would result in a reversion from full spray back to cup form.

Table 2 contains results of tests run on the same set of nozzles using 50-sec-viscosity oil. With the heavier oil, the nodes and cups were formed generally at higher pressures. Also, with few exceptions, full spray was developed at higher pressures. These results indicate that, in order to provide a reasonable margin of safety with these nozzles in cold weather, when the temperature of the oil is likely to drop below 50 F, a minimum pressure of 125 lb should be specified if 50-sec oil is used.

It will be noted that the margins between "full spray" and "safe spray" are not equal for all the nozzles in the group. The pressure for safe spray was selected to insure satisfactory operation for the group of nozzles as a whole on the particular grades of oil used, rather than for any individual nozzle.

Effect of Atomizing Pressure on Rate of Efflux

From the foregoing spray-chamber tests, it will be understood how pressure affects the nature and degree of atomization. It is natural to expect that pressure exerts a governing influence on the rate of efflux from an atomizing nozzle. Tests were run on a type-DR 2½ gal per hr nozzle having a 60-deg spray angle, to establish the relation between pressure and rate of efflux for oils of various viscosities. The results are given in Table 3.

TABLE 3 RELATION BETWEEN ATOMIZING PRESSURE AND RATE OF EFFLUX

(Type-DR nozzle, 2½ gal per hr, 60-deg spray angle)

Atomizing pressure, lb per sq in.	Kerosene, 30 sec[a]	No. 2 fuel oil, 34 sec[a]	No. 3 fuel oil, 42 sec[a]	No. 4 fuel oil, 50 sec[a]
50	1.70	1.85	2.02	2.27
75	1.98	2.12	2.32	2.64
100	2.25	2.44	2.65	2.94
125	2.49	..	..	..
150	..	2.85	3.13	3.38
185	..	3.15	..	..
200	..	..	3.49	3.75

[a] Viscosity, Saybolt Universal at 100 F.

The theoretical rate of discharge through a simple orifice varies with the square root of the pressure head. Experimental results on the nozzle fall short of the theoretical rates, the difference between the theoretical discharge from a simple orifice and

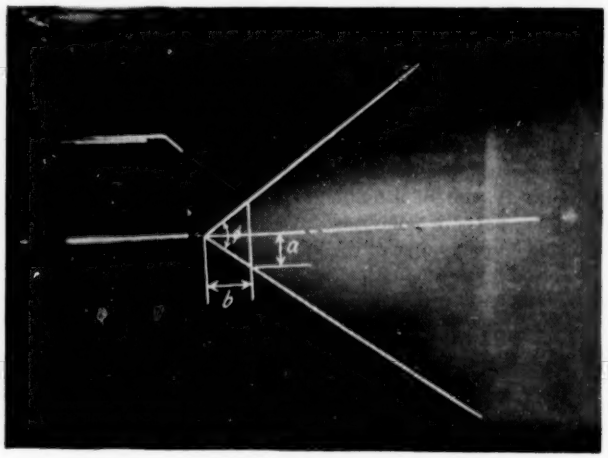


Fig. 3 Diagram for Measuring Spray Angle

the actual rate of efflux from the nozzle being more pronounced with the lower-viscosity oils.

Accuracy of Nozzle Calibration

Tests were run on several sets of nozzles in the spray chamber for the purpose of determining the accuracy of their calibration, as regards capacity and angle of spray. Table 4 shows results of

TABLE 4 SPRAY-CHAMBER TESTS, USING OIL OF 50 SEC VISCOSITY[a]

Rated nozzle capacity and spray angle	Actual gal per hr at 100 lb per sq in.	Actual angle of spray, deg	Appearance of spray
		Nozzle type-A	
1.35–80	1.66	47	Fair
1.35–60	1.56	40	Uneven—poor
1.65–80	1.98	63	Good—slight flickering
1.65–60	1.96	75	Very good
2.00–80	2.25	56	Fair
2.00–60	2.29	55	One bad opening in spray
2.50–80	2.96	83	Very good
2.50–60	3.00	61	Good
3.00–80	3.75	83	Good
3.00–60	3.62	63	Good
		Nozzle type-NR	
1.00–70	1.23	65	Good
1.20–80	1.45	63	Good
1.35–80	1.55	50–90	One-sided
1.35–80	1.52	54	Unstable
1.65–80	2.04	50–70	One-sided
1.65–80	1.94	40–70	One-sided
2.00–80	2.14	83	Good
2.00–80	2.21	80	Good
2.50–60	2.62	68	Good
2.50–80	2.62	90	Good
3.00–60	3.25	79	Good
3.00–80	3.00	83	Good

[a] Saybolt Universal at 100 F.

TABLE 5 SPRAY-CHAMBER TESTS, USING OIL OF 34 SEC VISCOSITY[a]

Rated nozzle capacity and spray angle	Actual gal per hr at 100 lb per sq in.	Actual angle of spray, deg	Appearance of spray
		Nozzle type-A	
1.35–80	1.42	83	Good
1.35–60	1.41	72	Good
1.65–80	1.75	84	Good
1.65–60	1.73	77	Good
2.00–80	2.14	88	Good
2.00–60	2.08	76	One bad opening in spray
2.50–80	2.78	90	Very good
2.50–60	2.65	72	Good
3.00–80	3.00	90	Good
3.00–60	3.18	70	Slightly irregular—fair
		Nozzle type-NR	
1.00–70	0.97	70	Good
1.20–80	1.20	77	Good
1.35–80	1.33	75	Slightly irregular
1.35–80	1.25	80	Good
1.65–80	1.70	60–80	One-sided
1.65–80	1.70	50–80	One-sided
2.00–80	1.92	85	Good
2.00–80	1.94	83	Slightly irregular
2.50–60	2.21	83	Good
2.50–80	2.25	90	Good
3.00–60	2.88	83	Good
3.00–80	2.75	90	Good

[a] Saybolt Universal at 100 F.

spray-chamber tests using an oil of 50 sec viscosity, Saybolt Universal at 100 F. The angle of spray ϕ is determined by measuring the distance a from center line to the outside of the spray cone at a distance b from the orifice, Fig. 3.

Table 5 gives the results of spray-chamber tests using an oil of 34 sec Saybolt Universal viscosity at 100 F.

The type-A nozzles ranged from 10 to 25 per cent over rated capacity on the 50-sec oil and 0 to 20 per cent over rated capacity on the 34-sec oil.

The type-NR nozzles ranged from 0 to 23 per cent over rating on the 50-sec oil and 8 per cent under to 3 per cent over rating on the 34-sec oil.

Influence of Oil Viscosity on Nozzle Capacities

From the test results given in Tables 3, 4, and 5, it will be appreciated that the capacity of a nozzle depends to a degree upon the viscosity of the oil which is being atomized. To develop more information along these lines, three different nozzles were calibrated on oils having viscosities ranging from 35 to 118 sec Saybolt Universal at 100 F. Then the slotted stem which fits inside of each nozzle was removed and the rates of flow through each of the straight nozzle orifices were determined. The same oils were used for each nozzle; a constant pressure of 100 lb per sq in. was maintained, and the temperature of the oil was held at 70 F. The resulting data are plotted in Figs. 4, 5, and 6.

The oils which were used were blends containing different proportions of No. 2 and No. 4 fuel oils. This was done for the purpose of obtaining oils of various viscosities without changing the oil temperature, thus making it possible to derive more accurate data regarding both the nozzle capacities and the angles of spray.

From Figs. 4, 5, and 6, it will be seen that the rate of flow through the straight orifices (the nozzles with the slotted stems removed) gradually diminished as the viscosity of the oil increased, which is to be expected in view of the fact that the higher-viscosity oils obviously have higher coefficients of friction and higher internal friction. The effect of viscosity changes on the output of the nozzles containing the slotted stems, however, was entirely different. Instead of dropping off, as the viscosity of the oil was increased, the rate of flow actually began to increase, until a point was reached at which the oil became so heavy that it could no longer be atomized. From this point on, any further increase in the viscosity of the oil reduced the output of the nozzles. It is interesting to note that the downward side of each of the nozzle curves takes on the general slope of the straight-orifice curves, i.e., they seem to drop off at about the same rate.

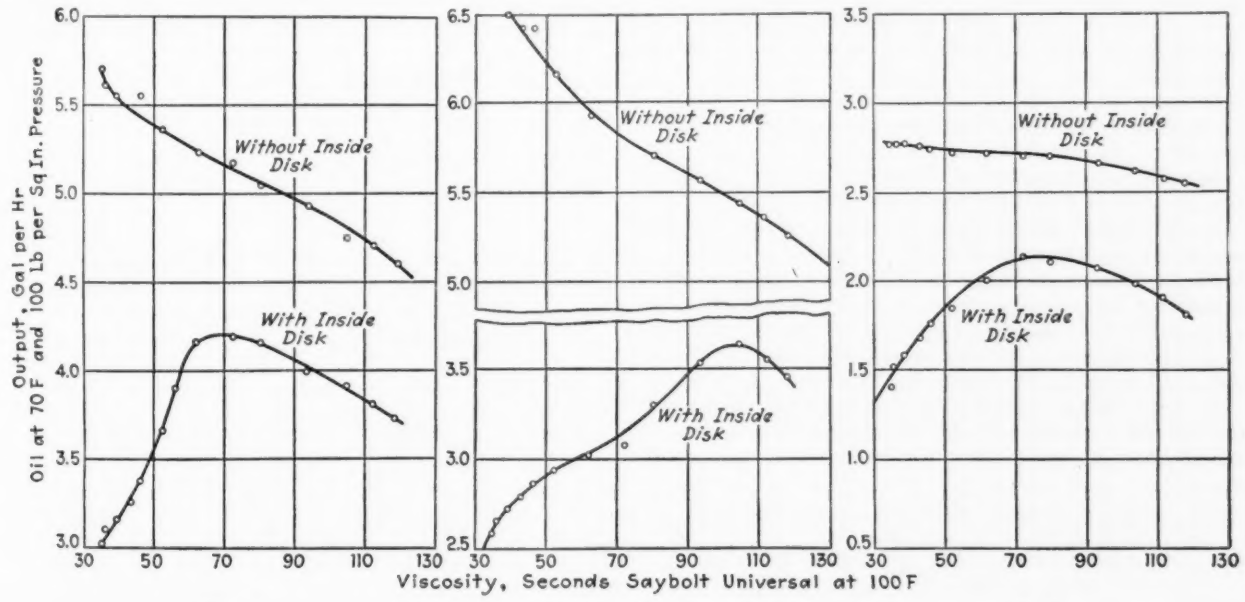


Fig. 4 Fig. 5 Fig. 6

FIG. 4 CURVES SHOWING EFFECT OF VISCOSITY CHANGES ON CAPACITY OF PRESSURE-ATOMIZING NOZZLE NO. 1
(Nozzle No. 1, rated 3 gal, 60-deg face, slotted stem.)

FIG. 5 CURVES SHOWING EFFECT OF VISCOSITY CHANGES ON CAPACITY OF PRESSURE-ATOMIZING NOZZLE NO. 2
(Nozzle No. 2, rated 3 gal, 60-deg flat face, slotted stem.)

FIG. 6 CURVES SHOWING EFFECT OF VISCOSITY CHANGES ON CAPACITY OF PRESSURE-ATOMIZING NOZZLE NO. 3
(Nozzle No. 3, rated 1.65 gal, 80-deg flat face, slotted stem.)

It should also be pointed out that all three of the nozzle curves did not break at the same point. In other words, all of the nozzles did not reach their peak output on the same-viscosity oil. The output of nozzles Nos. 1 and 3 (Figs. 4 and 6, respectively) began to drop off at 72 sec viscosity, but the output of nozzle No. 2 (Fig. 5) continued to increase until 104 sec viscosity was reached.

To understand how an increase in the viscosity of the oil causes this paradoxical increase in the nozzle output, one must consider the design and construction of the nozzle and the action of the oil as it is forced through the slots in the stem into the conical chamber, thence out through the nozzle orifice.

The slots are cut in the head of the stem at a tangent to the wall of the swirl chamber, so that the oil upon entering the chamber at high speed is given a rapid swirling motion which throws it to the sides of the chamber. This is similar to the action of any thin liquid when it is being poured through an ordinary funnel. Of course, the oil in the nozzle, being forced through the tangential slots in the nozzle stem under high pressure, swirls at a very high rate of speed. Thus, even as it leaves the cone-shaped chamber and is being forced through the orifice of the nozzle, it continues to swirl rapidly. Therefore, instead of a straight-flowing stream such as would result with an ordinary orifice, a swirling stream of oil passes through the nozzle orifice.

As soon as the stream is released to the atmosphere and is no longer restricted by the walls of the orifice, it breaks into millions of oil particles, each of which follows a straight line, the direction depending upon the relation of tangential velocity to axial velocity. The ratio of these two velocities determines the angle of the cone-shaped spray.

The lighter oils, due to their lower viscosities, are caused to swirl faster than the heavier oils, i.e., a greater proportion of the initial pressure is converted into rotative velocity. The oil, instead of filling the entire cross-sectional area of the swirl chamber and orifice, forms a hollow whirlpool moving along the walls of the chamber as a thin film, rotating at high velocity. The capacity of the nozzle is reduced due to the presence of the hollow core.

The more viscous oils, on the other hand, swirl at lower speed and more nearly approach solid or full flow in the swirl chamber and orifice. As full flow is approximated, the output curve tends to flatten out and reach a crest. When the viscosity of the oil becomes so high that it ceases to swirl in the nozzle to any appreciable extent, the resistance which the oil encounters in passing through the tangential slots in the nozzle stem and through the orifice determines its rate of flow. The resistance encountered here, of course, increases as the viscosity of the oil increases, which accounts for the fact that the nozzle output drops off with increasing viscosity, as soon as the centrifugal force of the swirling oil ceases to be a determining factor in the rate of flow.

The nozzle output increases as the viscosity of the oil increases until full flow is reached. At this point the rate of flow begins to drop off at the same rate as if it were being forced through a simple orifice.

TABLE 6 SWIRL-CHAMBER PRESSURES[a]

Viscosity of oil, S.U. at 100 F, sec	Oil pressure inside of swirl chamber, lb per sq in.	Nozzle output, gal per hr
34	58	3.90
35	57	3.91
36	55	4.05
39	51	4.13
43	48	4.37
46	47	4.50
52	47	4.59
62	50	4.50
72	52	4.41
80	53	4.37
93	54	4.32

[a] Oil pressure, 100 lb per sq in.

Swirl-Chamber Pressures

By drilling a small hole through the center of the slotted stem of a 4-gal-capacity nozzle and attaching a pressure gage (by means of a piece of small-diameter copper tubing which was inserted through the hole in the disk and soldered tight), it was found that the pressures inside the swirl chamber of the nozzle varied considerably when oils of different viscosities were used. The highest pressures were obtained with the lightest oils but, as the viscosity of the oil increased and the nozzle output increased, the pressure inside this chamber dropped off. However, when the point was reached at which a further increase in viscosity was attended by a drop in the rate of flow, the pressure in the chamber began to build up again. In Table 6 are given the oil pressures, inside the swirl chamber of the nozzles, which were recorded on different-viscosity oils.

The rapid rotation of the lighter oils inside the swirl chamber more than offsets the effect of the higher pressures, thus making it possible to force more gallons per hour of the 43-sec-viscosity oil through the nozzle orifice under 48 lb per sq in. pressure than the 34-sec-viscosity oil under 58 lb per sq in. pressure; the oil

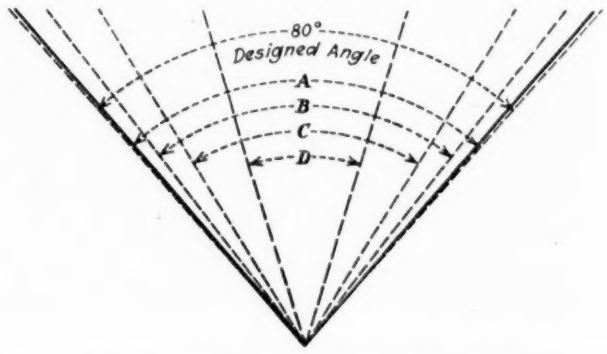


FIG. 7 NOZZLE A (80 DEG), EFFECT OF VISCOSITY CHANGES ON SPRAY ANGLE

A = 81 deg, with oils of 34 to 46 sec viscosity, S.U. at 100 deg F
B = 74 deg, with oils of 52 to 62 sec viscosity, S.U. at 100 deg F
C = 62 deg, with oils of 72 sec viscosity, S.U. at 100 deg F
D = 33 deg, with oils of 92 sec viscosity, S.U. at 100 deg F

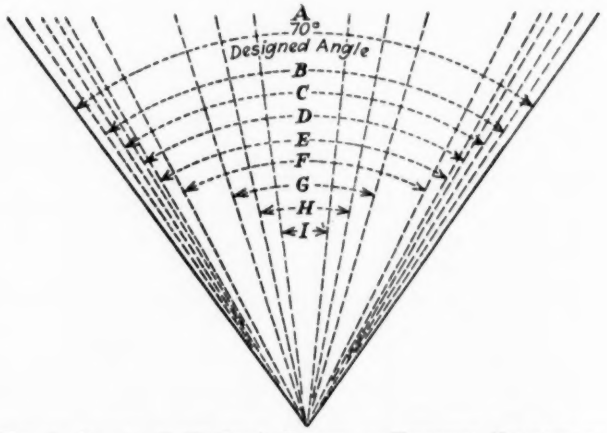


FIG. 8 NOZZLE C (70 DEG), EFFECT OF VISCOSITY CHANGES ON SPRAY ANGLE

A = 70 deg, with oils of 34 sec viscosity, S.U. at 100 deg F
B = 67 deg, with oils of 35 sec viscosity, S.U. at 100 deg F
C = 64 deg, with oils of 36 sec viscosity, S.U. at 100 deg F
D = 62 deg, with oils of 39 sec viscosity, S.U. at 100 deg F
E = 59 deg, with oils of 41 sec viscosity, S.U. at 100 deg F
F = 53 deg, with oils of 43 sec viscosity, S.U. at 100 deg F
G = 33 deg, with oils of 46 sec viscosity, S.U. at 100 deg F
H = 23 deg, with oils of 52 sec viscosity, S.U. at 100 deg F
I = 13 deg, with oils of 62 to 92 sec viscosity, S.U. at 100 deg F

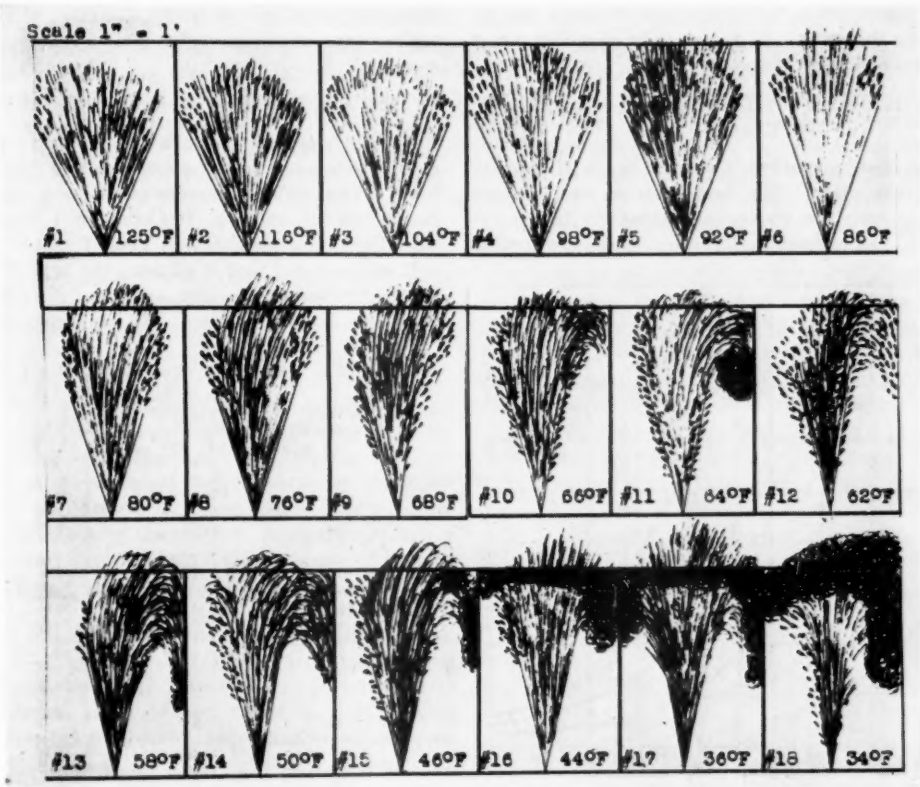


FIG. 9 GRAPHIC EVIDENCE OF EFFECT ON FLAME IN A 2.5-GAL, 60-DEG NOZZLE OF CHANGES IN VISCOSITY DUE TO TEMPERATURE
(Viscosity of oil used 50 sec Saybolt Universal at 100 F.)

pressure on the pump side of the nozzle chamber being the same in each case (100 lb per sq in.).

INFLUENCE OF VISCOSITY ON ANGLE OF SPRAY

A further study was made to determine the effect of viscosity of an oil on the angle of spray produced by pressure-atomizing nozzles. Generally these nozzles are constructed so as to produce sprays of any specified angle, from 45 to 90 deg. The angle is usually stamped on the outside of the nozzle along with its capacity.

However, it was found that the size of the spray angles produced by the nozzles tested were influenced largely by the viscosity of the oil used. Some of the nozzles were found to be much more sensitive in this respect than others depending on their design.

Eleven nozzles of different designs were tested on oils varying in viscosity from 34 to 92 sec Saybolt Universal at 100 F, with the oil temperature held at 70 F. Figs. 7 and 8 show graphically the variations in the angle of spray which took place in three of the more important nozzles of the group, as the viscosity of the oil increased. Nozzle A held its original angle of spray on oils up to 46 sec viscosity. From this point on the angle of spray became smaller as the viscosity of the oil increased. Over the entire viscosity range, the angle of spray dropped from 81 to 33 deg. Nozzle B held its original angle of spray on oil up to 36 sec viscosity. At this point there was a break and the angle of spray finally dropped to only 12 deg. Nozzle C was very sensitive. Its angle of spray varied slightly on all of the oils from 34 up to 43 sec viscosity. An increase in viscosity from 43 up to 46 sec narrowed the spray angle 20 deg. This nozzle produced its minimum angle of spray, 13 deg, on oil of only 62 sec viscosity.

It is interesting to note that both nozzles A and C attained their rated angle on 34-sec oil. However, nozzle B which was rated at 60 deg seems to have been improperly marked, since over the entire viscosity range of the oils used, its spray angles fall somewhere between those of the 70- and 80-deg nozzles. It would produce only a 60-deg spray angle when using an oil having a viscosity of approximately 48 sec Saybolt Universal at 100 F.

The explanation for this variation in the angle of spray goes back to the rate of speed at which the oil is swirling inside the nozzle and the relation between tangential velocity and axial velocity as it leaves the orifice. The construction or design of a nozzle makes it more or less sensitive to the viscosity of the oil.

The greater the ratio of tangential to axial velocity, the greater the tendency for the spray to widen, since the resultant velocity is the determining factor. As previously explained, the higher the viscosity of an oil, the lower the swirling speed inside the nozzle. Therefore, it follows that, as the viscosity of the oil increases, the spray angle decreases.

This is shown very effectively by the results obtained with nozzle C, Fig. 8. This particular nozzle is more sensitive to viscosity changes than nozzles A or B, because of the fact that the outside edge of the orifice in this nozzle is not countersunk or tapered to aid in directing the spray. Instead, the angle of spray is determined entirely by the length of the orifice bore. In a nozzle of this type, if a narrower angle of spray is desired, the length of the orifice barrel is increased. This of course, introduces a braking action on the rotation of the oil, thereby decreasing the tangential component of velocity.

The outside edges of the orifices in nozzles A and B are countersunk and tapered according to angle of spray desired. This has a stabilizing influence on the angle of spray which is quite effective when using the lighter grades of oil. But, as the viscosity of

the oil increases appreciably, the countersink becomes less effective, until finally the spray collapses and the oil comes out of the nozzle in almost a straight stream.

Influence of Variations in Oil Temperature on Character of Flame

By varying the oil temperature, similar changes in capacity and spray angle take place. The magnitude of such changes varies considerably with the viscosity-temperature characteristics of the oils used. An oil having a flat viscosity curve will cause

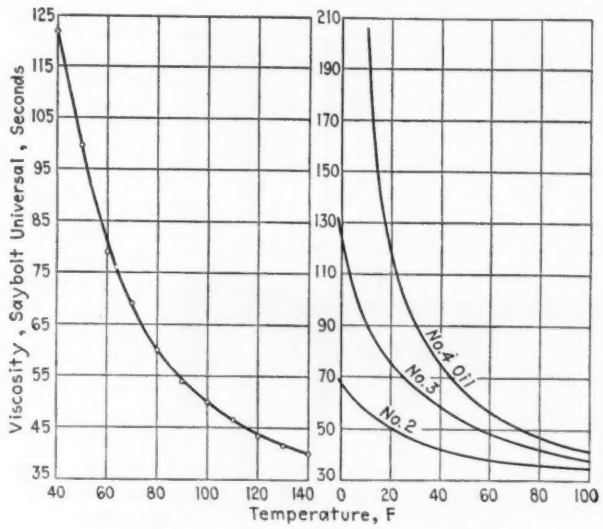


Fig. 10

Fig. 11

Fig. 10 Temperature-Viscosity Curve of Oil Used in Flame Tests

Fig. 11 Typical Temperature-Viscosity Curves for Nos. 2, 3, and Low-Viscosity No. 4 Oils

less change in nozzle capacity and angle of oil spray when the temperature varies than an oil having a steep viscosity curve.

Fig. 9 shows graphically, step by step, the extent to which the character of the flame, produced by a domestic high-pressure-atomizing burner, varies with changes in viscosity due to temperature and how the angle of spray narrows as the oil becomes more viscous. These are actual sketches of the flame, which were drawn as the tests were being conducted.

Fig. 9, sketch No. 1 shows a normal flame with oil temperature at 125 F. No. 2 shows but little change as the temperature is lowered to 116 F. In No. 3 sparks begin to appear, and in No. 4 the flame angle narrows as the flame lengthens. In Nos. 5 and 6 the condition becomes progressively worse as the temperature of the oil drops to 92 and 86 F, respectively. In Nos. 8, 9, and 10 the lengthening of the flame has progressed so far that it exceeds the length of the firebox and tends to curl around to the right. Note that the flame is becoming progressively narrower. No. 11 shows the flame beginning to smoke on the right as the temperature reaches 64 F. In Nos. 12, 13, and 14 sparks predominate. In the last four illustrations Nos. 15, 16, 17, and 18, the final breakdown of the atomization may be noted. The flame ultimately becomes long and stringy, producing clouds of smoke as the cold, poorly atomized oil impinges on the rear wall of the firebox.

The curve in Fig. 10 shows the viscosity of the oil used in this test at different temperatures. At 100 F the oil had a viscosity of 50 sec Saybolt Universal. Fig. 11 shows typical temperature-viscosity curves of three grades of light fuel oil. These curves illustrate the wide variations in viscosity-temperature characteristics of oils of similar viscosities at 100 F, which is the temperature at which light fuel-oil viscosities are generally measured.

Influence of Oil Temperature on Nozzle Capacity

Further tests were run on several nozzles of different types and sizes to determine the variation in rate of flow with oil temperature. Three different grades of oil were used, e.g., kerosene, No. 2 fuel oil, and No. 4 (Commercial Standards 12–35) fuel oil. The results are shown in Table 7.

It will be noted that in all cases the type-N nozzles showed a wider variation in capacity over the full range of temperature than the type-C nozzles. In comparing the different oils,

TABLE 7 CAPACITY TESTS ON ATOMIZING NOZZLES[a]

Oil temperature, F	Rate of flow, gal per hr — CS 12–35 No. 4[b]	No. 2[c]	Kerosene[d]
Nozzle type-C, 60°, rated at 1.35 gal per hr			
100	1.19	1.05	.85
90	1.23	1.07	.98
80	1.20	1.08	1.07
70	1.22	1.11	1.08
60	1.22	1.13	1.09
50	1.22	1.52	1.10
40	1.27	1.18	1.09
35	1.29	1.19	1.09
Nozzle type-N, 80°, rated at 1.35 gal per hr			
100	1.41	1.21	1.18
90	1.45	1.23	1.19
80	1.48	1.26	1.20
70	1.50	1.28	1.22
60	1.60	1.31	1.23
50	1.69	1.32	1.24
40	1.74	1.36	1.47
35	1.73	1.38	1.52
Nozzle type-C, 60°, rated at 1.65 gal per hr			
100	1.46	1.33	1.24
90	1.50	1.35	1.30
80	1.51	1.35	1.32
70	1.53	1.38	1.32
60	1.54	1.39	1.42
50	1.54	1.42	1.38
40	1.57	1.43	1.39
35	1.68	1.44	1.38
Nozzle type-N, 70°, rated at 1.65 gal per hr			
100	1.74	1.48	1.42
90	1.79	1.51	1.44
80	1.84	1.52	1.41
70	1.89	1.58	1.50
60	1.94	1.60	1.67
50	2.01	1.62	1.52
40	2.24	1.70	1.54
35	2.33	1.69	1.55
Nozzle type-N, 80°, rated at 1.65 gal per hr			
100	1.78	1.47	1.44
90	1.83	1.50	1.45
80	1.87	1.51	1.47
70	1.97	1.57	1.51
60	2.13	1.62	1.54
50	2.31	1.65	1.61
40	2.47	1.70	1.69
35	2.46	1.73	1.69
Nozzle type-C, 60°, rated at 2 gal per hr			
100	1.60	1.45	1.44
90	1.61	1.46	1.46
80	1.64	1.48	1.53
70	1.66	1.87	1.54
60	1.66	1.52	1.50
50	1.67	1.54	1.53
40	1.67	1.57	1.55
35	1.69	1.66	1.58
Nozzle type-N, 80°, rated at 2 gal per hr			
100	2.34	1.98	1.91
90	2.41	2.04	1.96
80	2.49	2.08	1.92
70	2.51	2.13	2.01
60	2.81	2.16	2.04
50	3.00	2.20	2.08
40	3.10	2.25	2.12
35	2.51	2.30	2.13
Nozzle type-C, 60°, rated at 2½ gal per hr			
100	2.23	1.98	1.95
90	2.28	2.02	1.96
80	2.34	2.05	1.98
70	2.39	2.08	2.02
60	2.49	2.11	2.04
50	2.53	2.15	2.07
40	2.59	2.18	2.10
35	2.65	2.22	2.10

[a] Tests conducted at 100 lb per sq in. pressure.
[b] Viscosity of No. 4 oil, 42-sec Saybolt Universal at 100 F.
[c] Viscosity of No. 2 oil, 34-sec Saybolt Universal at 100 F.
[d] Viscosity of kerosene, 30-sec Saybolt Universal at 100 F.

it will be seen that in all instances the widest variations in rate of flow occurred with the No. 4 oil while the smallest variations resulted when kerosene was employed. The reason for this will be understood after considering the temperature-viscosity characteristics of the three oils used in the test. The obviously wider variations in viscosity of the heavier oils with temperature change accounts for the differences in capacity variations previously pointed out.

Effects of Nozzle Design on Combustion

Further work was carried out to determine what effect nozzles of different designs have on combustion efficiency when used in an average-size domestic burner of good design.

One important difference between nozzles used quite generally in pressure-atomizing burners is in the slotted stem which fits inside the tip behind the nozzle orifice. One type of nozzle employs a flat-faced stem, while another employs a stem which has a recessed face, thus enlarging the size of the swirl chamber considerably. The spray produced by the nozzles using the flat-faced stem comes out in the shape of a sharply defined hollow cone, having very little oil spray in the center. The nozzle using the recessed stem produces a spray which is also cone-shaped but, instead of being hollow, it has a considerable quantity of oil spray inside the cone.

In order to get a comparison between the nozzles using the flat-faced stem and those using the recessed stem, combustion tests were conducted on ten of each type, all twenty nozzles being made by the same manufacturer. These nozzles ranged in capacity from 1.35 up to 3 gal per hr, in both 60-deg and 80-deg angles of spray.

These tests were carried out using a well-designed, gun-type burner installed in a cast-iron, sectional, oil-burning boiler. A constant stack draft of 0.06 in. of water was maintained and the atomizing pressure was 100 lb per sq in. in each case. The temperature of the oils used was 70 F. In the case of each nozzle tested under these conditions, the fan shutter was opened just enough to prevent the fire from smoking; then CO_2 readings were taken as evidence of the quality of combustion obtained.

TABLE 8 COMBUSTION TESTS[a]

Marked capacity of nozzles, gal per hr	Angle of spray, deg	34-Sec-viscosity oil[b] Rate of combustion, gal per hr	34-Sec-viscosity oil[b] CO_2 in flue gases, per cent	42-Sec-viscosity oil[b] Rate of combustion, gal per hr	42-Sec-viscosity oil[b] CO_2 in flue gases, per cent
Type-DR nozzles with recessed stem					
1.35	60	1.27	6.6	1.43	6.8
1.65	60	1.55	7.0	1.73	7.8
2.00	60	1.77	8.2	2.00	9.4
2.50	60	2.35	11.0	2.58	11.3
3.00	60	2.70	11.6	2.98	12.2
1.35	80	1.28	6.0	1.46	7.0
1.65	80	1.36	6.4	1.61	7.2
2.00	80	1.75	8.2	1.99	9.4
2.50	80	2.47	10.6	2.68	11.6
3.00	80	2.74	10.6	2.92	10.8[c]
Type-DS nozzles with flat-faced stem					
1.35	60	1.26	5.5	1.47	6.2
1.65	60	1.68	7.8	1.89	8.8
2.00	60	1.77	7.6	2.03	8.6
2.50	60	2.32	9.0	2.50	10.3
3.00	60	2.73	9.6	2.94	10.6
1.35	80	1.27	6.2	1.37	7.2
1.65	80	1.61	7.8	1.88	8.8
2.00	80	1.92	7.4	2.09	8.6
2.50	80	2.25	8.4	2.32	9.4
3.00	80	2.99	10.2[c]	3.15	11.0[c]

[a] Tests conducted at 100 lb per sq in. pressure.
[b] Saybolt Universal at 100 F.
[c] Fire impinging on sides of firebox.

The results of the tests are given in Table 8. It should be remembered that these figures are only comparative and cannot be taken as a measure of burner performance, especially in the case of the small-size nozzles. Better results could have been obtained with the small nozzles in a smaller boiler under different draft conditions, but these results give the desired comparison between the two types of nozzles.

On an average the figures given in Table 8 show that somewhat better combustion was obtained with the nozzles having recessed stems. These nozzles also produced better appearing fires. The flames produced by the nozzles using the flat-faced stems seemed to have a characteristic tendency to be stringy and lopsided, with long tips, while the nozzles with recessed stems produced shorter, bushier flames, which were more evenly spread over the firebox.

It must be pointed out that these conclusions regarding the performance of flat-faced and recessed-stem nozzles refer only to this particular class of burner. The recessed-stem nozzle producers a heavy center spray and is best adapted to a burner, the air delivery of which is such as to direct a fair proportion of the combustion air into the center of the oil spray.

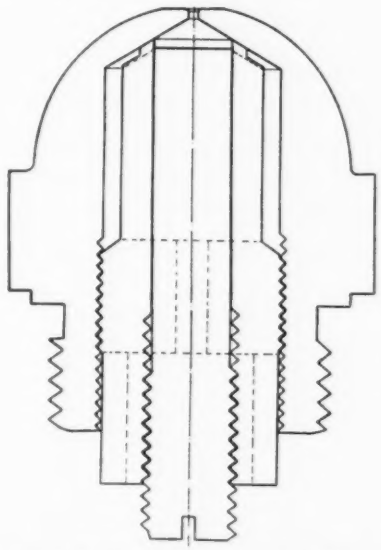


Fig. 12 Special Nozzle With Adjustable Recess in Stem

On the other hand, a nozzle with a flat-faced stem produces a hollow cone of spray which is best suited to a burner with an air tube partially blocked off in such a manner as to shield the nozzle and cause all of the air to envelop the hollow cone of oil spray. These features are now being adopted in building burners of very low capacities. The purpose is to increase air velocity, reduce air delivery, and insure proper carburetion.

The value of increasing the volume of the vortex or swirl chamber by recessing the stem was so marked that it was decided to run further tests to determine if possible the size of vortex chamber for optimum results.

TABLE 9 EFFECT OF SWIRL-CHAMBER SIZE

No. turns on plunger from flush position	Depth stem recess, in.	34-Sec-viscosity oil: Output of nozzle, gph	34-Sec-viscosity oil: CO_2 in flue gases, per cent	40-Sec-viscosity oil: Output of nozzle, gph	40-Sec-viscosity oil: CO_2 in flue gases, per cent
Small-diameter plunger in 80-deg, 2.5-gal tip					
1	0.031	2.39	9.7	2.59	11.0
2	0.062	2.56	10.6	2.76	12.6
3	0.094	2.76	11.6	2.96	13.8
4	0.125	2.88	12.6	3.15	13.8
4½	0.141	..	13.5	..	..
5	0.156	3.08	13.6	3.28	13.2
5½	0.172	..	13.2	..	..
6	0.187	3.19	13.2	..	..
Small-diameter plunger in 60-deg, 2.5-gal tip					
1	0.031	2.34	10.8	Spray collapsed	
2	0.062	2.43	12.6	..	..
3	0.094	2.56	12.2	..	..
4	0.125	Spray collapsed		..	..

Two type-D atomizers were prepared with special adjustable plungers in the stem, Fig. 12, to permit the depth of the swirl chamber to be varied from zero to 0.187 in. Table 9 shows the nozzle output and the per cent CO_2 in the flue gases for the two nozzles and two grades of oil. The tests were continued from 0.031 in. recess depth past the conditions of peak performance.

In the test of the 80-deg nozzle the recess depth for best results, using the lighter oil (34 sec viscosity, Saybolt Universal at 100 F) was 0.156 in. At this setting, the nozzle output was 3.08 gal per hr and the CO_2 reading 13.6 per cent. When the heavier oil was used (40 sec viscosity, Saybolt Universal at 100 F), the maximum CO_2, 13.8 per cent was reached between 2.96 and 3.15 (average 3.05 gal per hr) which is almost exactly the same burning rate. This would appear to indicate that there is one definite burning rate for a given nozzle which will insure optimum performance regardless of the viscosity of the oil.

As a check on this theory, a series of combustion tests was run on both of the special nozzles, using oils of viscosities ranging from 34 sec to 66 sec Saybolt Universal at 100 F.

Under set conditions of draft and atomizing pressure, the burner was adjusted for best combustion by means of adjusting the depth of the recess in the stem. After conditions of peak performance had been established, the burning rate was checked. In every instance, the rate checked within 3 per cent. Complete results of this series of tests appear in Table 10.

TABLE 10 RESULTS OF COMBUSTION TESTS[a]

Viscosity S.U. at 100 F, sec	Rate, gph	Draft	CO_2	Flame[c]
	Nozzle type-D, 80°, 2½-gal tip[b]			
34	3.04	0.04	14.0	Stable
36	3.04	0.04	14.0	Stable
41	3.12	0.04	13.8	Stable
53	3.00	0.04	14.0	Stable
59	3.05	0.04	13.8	Stable
66	3.10	0.04	13.6	Little, sparky
	Nozzle type-D, 60°, 2½-gal tip[b]			
34	2.85	0.04	13.0	Stable
36	2.78	0.04	13.2	Stable
41	2.82	0.04	13.0	Stable
53	2.83	0.04	12.4	Unstable
59	2.84	0.04	13.4	Unstable
66	2.78	0.04	13.2	Unstable

[a] Tests conducted at 100 lb per sq in. pressure.
[b] Nozzles equipped with variable-size swirl chambers.
[c] Ignition satisfactory in all tests.

Effect of Atomization and Carburetion on Combustion

Twelve recessed-stem atomizing nozzles were calibrated, examined on the nozzle-testing apparatus to determine their spray characteristics, and tested in a burner under fire to determine the relative combustion efficiency possible of attainment with each nozzle, as indicated by the percentage of CO_2 in the flue gases.

In carrying out these tests, each nozzle was calibrated under 100 lb per sq in. pressure and an examination was made of the spray in the glass testing case, Fig. 2. Then the nozzle was tried out, under actual operating conditions, in an average-size modern burner of good design, which was installed in an oil-burning boiler having a firebox 15 in. wide by 24 in. long.

All tests were carried out on two different grades of oil, one having a Saybolt Universal viscosity of 34 sec at 100 F; the other having a viscosity of 50 sec. These two grades of oil represent the lightest and heaviest, respectively, in the range of fuels ordinarily used in pressure-atomizing domestic burners.

Unusually good results were obtained with this set of nozzles on both the light and heavy oil.

It will be noticed that, although the spray of some of the nozzles appeared to be one-sided when examined in the testing case, this apparently did not affect the fires they produced to any appreciable extent.

In most cases, changing from the 34- to the 50-sec-viscosity oil reduced the spray angle by from 10 to 20 deg, but in no case did this appear to affect the ignition of the oil or the quality of combustion obtained. In fact, not a single case of late ignition or ignition failure was experienced throughout the entire series of tests. The electrodes were set flush with the tip of the nozzle.

Upon studying the test results given in Table 11, it was noted particularly that the low-capacity nozzles, namely, those from 1 to 1.35 gal per hr, did not give as high a combustion efficiency as did the higher-capacity nozzles. However, from the standpoint of the cold tests, conducted in the special glass testing case Fig. 2, it was found that these nozzles had performed very well. Upon considering this matter, it was decided that the rather poor fire test, in the case of the low-capacity nozzles, was not due to any imperfections or to the design of the nozzles but rather to the fact that it was not possible so to adjust the air controls on the large burner as to obtain good carburetion or mixing of the air and atomized oil. Therefore, further tests were undertaken on these small nozzles, installed in a smaller burner especially designed to handle the lower burning rates.

TABLE 11 EFFECT OF ATOMIZATION AND CARBURETION ON COMBUSTION

(Spray examined in glass testing case; atomizing pressure, 100 lb per sq in.; oil temperature 75 F)

Type-NR nozzle marking	Cold test: Output, gph	Cold test: Angle of spray, deg	Cold test: Appearance of spray	Fire test: CO_2 in flue gases, %	Fire test: Appearance of fire	General performance
			Using 34-sec-viscosity oil			
1.00–70	0.97	70	Good	7.8	Stringy	Good
1.20–80	1.20	77	Good	8.6	Stringy	Fair
1.35–80[a]	1.33	75	Slightly irregular	8.8	Stringy	Fair
1.35–80[a]	1.25	80	Good	8.5	Stringy	Good
1.65–80[a]	1.70	60–80	One-sided	12.0	Good	Excellent
1.65–80[a]	1.70	50–80	One-sided	11.2	Good	Good
2.00–80[a]	1.92	85	Good	13.4	Good	Excellent
2.00–80[a]	1.94	83	Slightly irregular	12.8	Good	Good
2.50–60	2.21	83	Good	12.8	Good	Good
2.50–80	2.25	90	Good	12.7	Good	Good
3.00–60	2.88	83	Good	12.7	Good	Good
3.00–80	2.75	90	Good	12.4	Good	Good
			Using 50-sec-viscosity oil			
1.00–70	1.23	65	Good	7.2	Stringy	Fair
1.20–80	1.45	63	Good	10.2	Good	Good
1.35–80[a]	1.55	50–90	One-sided	9.8	Good	Fair
1.35–80[a]	1.52	54	Unstable	9.2	Good	Fair
1.65–80[a]	2.04	50–70	One-sided	12.0	Good	Good
1.65–80[a]	1.94	40–70	One-sided	12.0	Good	Good
2.00–80[a]	2.14	83	Good	13.4	Good	Excellent
2.00–80[a]	2.21	80	Good	13.7	Good	Excellent
2.50–60	2.62	68	Good	12.5	Good	Good
2.50–80	2.62	90	Good	13.0	Good	Good
3.00–60	3.25	79	Good	13.8	Good	Good
3.00–80	3.00	83	Good	13.8	Good	Good

[a] Two nozzles of the same rated capacity and angle of spray were tested.

TABLE 12 RESULTS OF COMBUSTION TESTS ON SMALL NOZZLES

(Atomizing pressure, 100 lb per sq in.; oil temperature, 75 F; stack draft, 0.01 in. of water)

Type-NR nozzle marking	Output, gph	CO_2, per cent	Appearance of fire	General performance
		Using 34-sec-viscosity oil		
1.00–70	0.97	11.4	Good	Excellent
1.20–80	1.20	11.8	Good	Excellent
1.35–80[a]	1.33	13.0	Good	Excellent
1.35–80[a]	1.25	12.8	Good	Excellent
		Using 50-sec-viscosity oil		
1.00–70	1.23	12.4	Good	Excellent
1.20–80	1.45	13.0	Good	Excellent
1.35–80[a]	1.55	14.2	Good	Excellent
1.35–80[a]	1.52	14.0	Good	Excellent

[a] Two nozzles of the same rated capacity of spray were tested.

When a medium- or a large-size burner is used to burn a small quantity of oil it is difficult to secure the proper carburetion of air and oil. This occurs because the oil fog is blown out ahead of the large volume of slowly moving air. For good combustion it is

essential that there be an intimate mixture of atomized oil and air. This condition can be obtained only by providing air of the correct velocity and direction. Good atomization of the oil alone cannot produce the desired results.

From the figures given in Table 12, it is quite apparent that excellent results can be realized with these small nozzles, provided the burner is designed to operate efficiently at low burning rates.

Conclusions

1 One of the most important characteristics of an oil from the standpoint of atomization is viscosity. Viscosity influences nozzle capacity, angle of spray, degree of atomization, character of flame, and efficiency of combustion. Since the viscosity of the oil at atomizing temperature is of first importance, the viscosity-temperature characteristics of an oil must be taken into consideration.

2 Nozzle design has an important bearing on the character of the spray and the sensitivity of the nozzle to variations in the atomizing pressure and the viscosity of the oil. This calls for careful balancing of the sizes of the slots, vortex chamber, and orifice to produce the desired tangential and axial velocities at the exit of the nozzle.

3 Of particular importance for efficient combustion is the placement of the combustion air as it conforms with the shape and nature of the oil spray.

4 Efficiency of combustion and economy of heating with oil in domestic pressure-atomizing burners depends largely upon the selection of a nozzle having the proper atomizing characteristics, the correct aerodynamic design of the burner, and the use of an oil of correct and uniform viscosity.

High-Temperature-Steam Experience at Detroit

By R. M. VAN DUZER, JR.,[1] and ARTHUR McCUTCHAN,[2] DETROIT, MICH.

This paper is a continuation[3] of the one presented in December, 1933, in which the 10,000-kw, 1000-F turbine installed by The Detroit Edison Company, was described in detail together with the results obtained with various materials used in the construction of a small 1100-F superheater and piping system. The turbine installation was dismantled in 1937 after 26,453 hours of service and the 1100 F equipment has been continued in service for the further testing of materials.

The turbine operation has demonstrated the practicability of 1000-F operation. An examination of all principal materials used in the construction of the turbine, piping, and superheater showed that with several exceptions the alloys were in good condition. Measurements made to determine creep disclosed only small amounts.

The results of 1100-F steam-corrosion tests on 18 steels indicate that the scale formations offer material protection against subsequent steam attack and that the corrosion process is different in steam from that in air. A comparison of creep determinations made on four machined pipe sections—two in service at 380 lb per sq in. and 1100 F, and two in service at 380 lb per sq in. and 925 F—and laboratory specimens at the same temperature and stress tend to disprove the theory that total diametral elongation of a pipe subject to internal pressure is materially less than that indicated by tensile-creep tests.

IT SEEMS desirable at this time to record the final results obtained on the operation of the 10,000-kw, 1000 F turbo-generator installed in the Delray plant of The Detroit Edison Company, and to present data that have been obtained from an investigation of the materials which comprised the superheater, piping, and turbine. Results of steam-corrosion tests of various materials subjected to 1100 F steam and a field check of laboratory creep data at 925 and 1100 F which have been made in connection with the 1100 F experimental installation at the Trenton Channel plant also are presented.

Early in 1937, operation of the 1000-F installation was discontinued, as it was felt that sufficient experience had been obtained with the turbine definitely to prove the practicability of using 1000-F steam. The addition of 815-lb, 900-F capacity to the Delray Plant made it possible to have the 1000-F turbine rebuilt into a superposed unit which will use steam at the new conditions and exhaust into the existing 400-lb, 700-F system.

Since the last report of this installation in December, 1933,[3] the unit had operated satisfactorily until shut down in February, 1937. The installation had been in service a total of 26,453 hr of which 22,896 hr were with steam at 1000 F. The turbine has been dismantled and those parts of the high-pressure cylinder not reused in the rebuilt unit were cut up and examined. Samples were taken also from the piping system and different parts of the superheater.

In general, all materials examined were in good condition with the possible exception of the 18 per cent Cr, 8 per cent Ni, 2 per cent Si alloy tubing used in the superheater. This tubing, which operated with metal temperatures from 950 to 1150 F, still possessed good tensile properties. In nearly all tube samples tested, however, the impact values were extremely low and the steel was susceptible to intergranular attack, the result of chromium-carbide formation. There is no reason, however, to believe that the brittle superheater tubes would not have continued to serve for a considerable length of time. The 18 per cent Cr, 8 per cent Ni alloy pipe castings, without the high silicon addition, were in excellent condition.

The experimental 1100 F superheater and piping located in the Trenton Channel plant have been continued in operation for the testing of original materials and others which have been added from time to time. This equipment, up to Oct. 1, 1938, had been in service a total of 48,361 hr, of which 38,921 had been at 1100 F. It will be recalled that this apparatus was placed in service during 1929 to try out materials and methods of construction for later use in the design of the 1000-F turbine and superheater at Delray. Present efforts in connection with this equipment are being devoted to determining the steam-corrosion resistance of a group of steels and the creep properties of several accurately machined pipe sections. The scope of investigation was recently enlarged by lowering the steam temperature in a section of the line to 925 F.

The corrosion resistance of 18 different steels has been determined after subjection to 1100-F steam. Nine of the materials have been examined after 7500 hr of service, while the remaining number have been inspected after 3800 hr. The results so far obtained indicate that the initial scale formation offers material protection against subsequent steam attack except in the case of straight carbon steel. These data also indicate that the process of air and steam corrosion are entirely different.

The inability to secure satisfactory measurements of creep on the equipment at the Delray and Trenton Channel plants,

[1] Engineer, production department, The Detroit Edison Company. Mem. A.S.M.E. Mr. Van Duzer received the degree of B.S. in mechanical engineering from the University of Michigan in 1927. Following graduation he spent two years at the Marysville power house of The Detroit Edison Company as testing engineer and operator and was then assigned to the staff of the chief engineer of power plants. One of his assignments has been the supervision of the experimental work involved in the two high-temperature installations of the Company using steam at from 1000 to 1100 F. Other assignments have been supervision of power-plant-piping design and the operation of three small steam plants.

[2] Engineer, engineering division of The Detroit Edison Company. Junior A.S.M.E. Mr. McCutchan was educated at Monmouth College, Illinois, and the University of Michigan from which institution he received the degree of B.S. in mechanical engineering in 1926. Since graduation he has been engaged in mechanical-engineering studies with The Detroit Edison Company.

[3] For a previous report by the same title, see "High-Temperature-Steam Experience at Detroit," by P. W. Thompson and R. M. VanDuzer, Jr., Trans. A.S.M.E., vol. 56, 1934, paper FSP-56-9, pp. 497–506. See also bibliography included.

Contributed by the Power Division and presented at the Annual Meeting of The American Society of Mechanical Engineers, held in New York, N. Y., December 5–9, 1938.

Discussion of this paper was closed January 10, 1939, and is published herewith directly following the paper.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

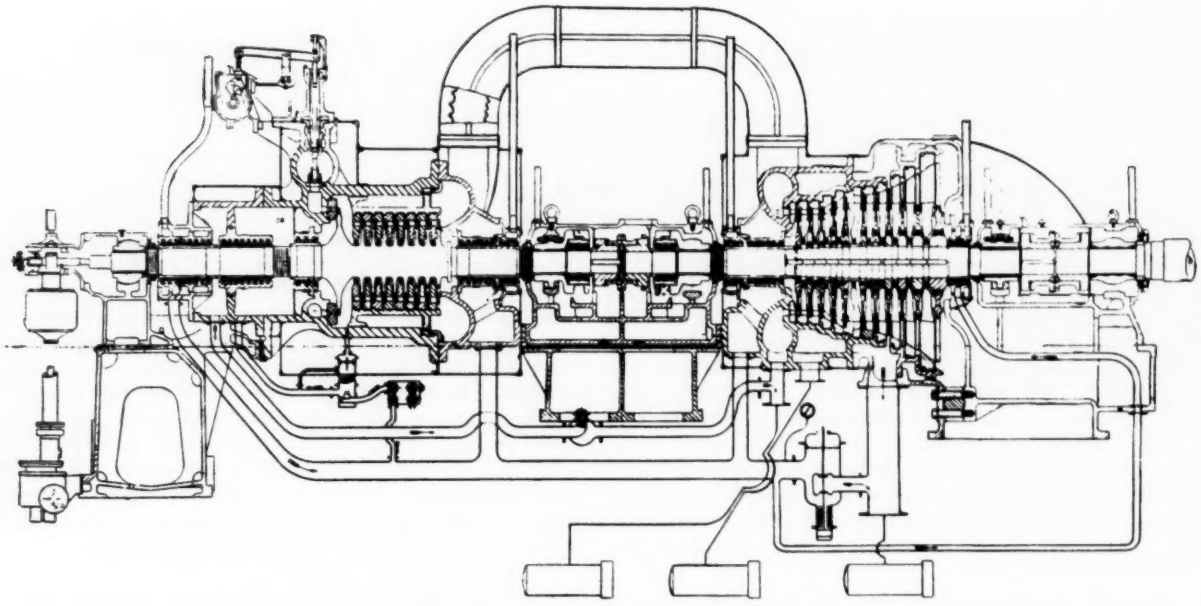


FIG. 1 CROSS SECTION OF DELRAY TURBINE

because of the low design stresses, led to the installation of four machined tube sections in which the stresses were sufficient to produce measurable creep in a fairly short period. Two of these tubes, made from Cr-Mo-Si alloy, were subjected to 380-lb, 1100-F steam, while the others of C-Mo tubing were placed in service at 925 F with the same pressure. The creep rates obtained on the first two sections have agreed fairly well with laboratory values, while those obtained on the latter sections at the lower temperature have not given such a good correlation; the diametral elongations are much greater than would be expected from laboratory results.

The subject matter of this paper has been divided into two parts; that describing the turbine installation, and that dealing with data obtained from the Trenton Channel 1100-F superheater and piping system. In the first part, the operating experiences not previously recounted[3] are detailed, as well as an account of the physical changes measured and observed. The metallurgical results obtained on twelve specimens are discussed in some detail. The Trenton Channel portion of the paper describes the steam-corrosion results obtained on a group of 18 steels, the correlation between laboratory and field creep measurements made on two low-alloy steels, and physical properties of several materials removed from service.

THE DELRAY INSTALLATION

It will be recalled that the Delray installation consisted of a two-cylinder, 10,000-kw turbogenerator, auxiliaries, connecting piping, and an oil-fired superheater utilizing 400-lb, 700-F steam which was superheated to 1000 F. The purpose of the installation was to obtain operating experience with high-temperature equipment and to determine the effect of this steam on the materials then available. A cross section of the turbine is shown in Fig. 1 in which the principal parts may be distinguished.

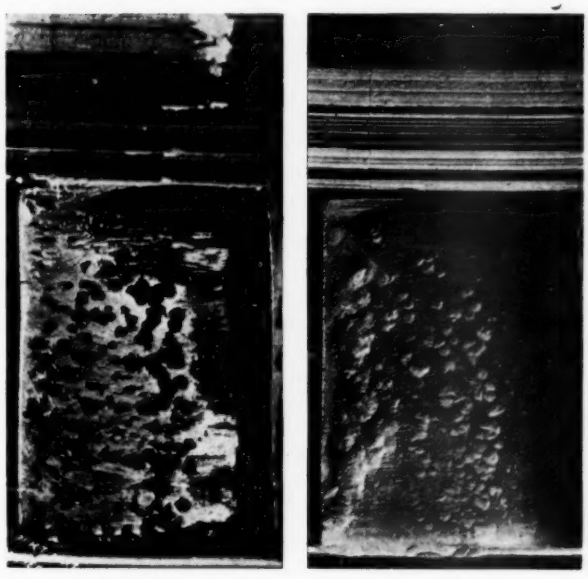
Operating Experience. The turbine was placed in service in October, 1930, with 700-F steam, pending erection of the superheater, and was first operated at 1000 F in November, 1931. From that time until the machine was dismantled in February, 1937, the unit had been under load for 26,453 hr, of which 22,896 were with steam at 1000 F.

The operation of the equipment, and particularly that of the turbine, was most satisfactory. Few of the troubles encountered with the turbine were directly traceable to high-temperature operation except in the case of the turbine stop valve. The principal repair encountered was the replacement of this valve. The bonnet flange which, because of the valve design, functioned as a line flange, was entirely too light for the steam conditions involved. The replacement valve was designed to approximate the 900-lb American Steel Flange and Fitting Standard, and used an oval ring-type gasket for the bonnet joint. This valve gave reasonably good service, although it was necessary to remake the joint twice, at about 6000-hr intervals. The nitralloy bushings of the five control valves and the throttle valve, installed in place of the original special cast-iron parts, gave no trouble and, except for a few spots where a scale 3 to 5 mils thick had formed and flaked off, the inner surfaces were in excellent condition.

Considerable trouble developed during the first two years of operation, when placing the turbine on the line, because of vibration caused by shaft deflection. This condition, worse when the machine was still warm after a short shutdown, was partly corrected by more rapidly increasing the vacuum. On several occasions vibration was so severe that it was necessary to stop the unit and allow it to cool thoroughly before restarting. Had the turbine been equipped with a turning device for maintaining a straight shaft, these starting troubles would not have been encountered.

During the latter part of 1933 the high-pressure shaft packing was examined and found severely rubbed. The packing, a high- and low-tooth segmental design, was refitted with a total diametral clearance of 18 mils. The turbine from then on ceased to be temperamental and gave no further trouble from vibration during the starting cycle. The previous method of adjusting the packing had been to allow it merely to touch the shaft and then run the turbine for a day at 200 rpm to wear a clearance.

A check of the shaft alignment during the packing refit showed the front of the cylinder to have moved 0.024 in. toward the left and 0.020 in. upward in relation to the shaft. This accounted for the severe rubbing and the excessive shaft deflection during starting, the result of localized heating. A subsequent check in 1937 showed the cylinder 0.032 in. toward the left and 0.015 in.



(a) After 1450 hr service (b) After 23,896 hr service (View with illumination from left.)

FIG. 2 FIRST-STAGE BUCKETS SHOWING INDENTATIONS, ×1.9

upward in relation to the shaft. This change in alignment leftward of 0.012 in. plus the 0.009-in. original radial clearance corresponded closely to the radial packing clearance of 0.023 in. as measured in 1937.

The condition was caused undoubtedly by warping of the front of the high-pressure cylinder, the result of uneven heating, a supposition further substantiated by measurements made on the top of the steam chest. This surface, approximately 6 ft long, was found to have bowed down from 0.012 to 0.015 in. at the center. The partial-admission governing system with which the turbine was provided allowed steam to be admitted through each of five individual nozzle groups around the periphery of the first-stage nozzle ring. Steam to each of these sectors passed from the steam chest through cored passages in the cylinder. Seventy-five per cent of the running time at 1000 F was at a load requiring steam to one nozzle section only, which probably caused sufficient thermal stress to be set up to account for the distortion observed.

No damage to the bucket and shrouding assembly was noted as the result of impact imposed on the first-stage wheel by the partial admission of steam. This is in contrast to reported experience with some of the recent higher-capacity 3600-rpm units using steam approximately three times as dense. There was evidence of slight rubbing on the inlet edge of the bucket shrouding on the second, sixth, and eighth wheels. This occurred only in spots and probably was the result of shaft deflection during starting periods.

No effort was made to keep a record of the availability of the turbine because of the experimental nature of the equipment. Regular and emergency maintenance of the other plant apparatus had precedence, and as a result, the installation on numerous occasions was out of service for periods much longer than otherwise would have been necessary. The longest intervals of continuous operation were four of 3 months each. The machine was started 50 times from November, 1931, to February, 1937; over half the starts were made during the first two years of this period.

Maintenance in the superheater was not considered excessive after insulation at the back of tubes in the bottom section was rearranged to provide more clearance for thermal expansion. The protecting tile on the fire side of the tubes did not fall off in quantities as was the case before this change was made. The superheater was not removed from service when the tiles dropped from the tubes as had been the practice before the lower part of the setting was rebuilt. There was no apparent damage to the exposed radiant tubes from this method of operation.

The joints in the piping system, not welded or provided with a welded seal, all gave some trouble, except one joint between the turbine stop valve and steam chest. There the bolts and flanges were left exposed to provide sufficient strength in the assembly at their lower temperature to maintain the joint. The joints at the center-pressure globe valve at the superheater outlet which was built to the 600-lb A.S.A. standard gave the most trouble, as was to be expected. The period between repairs of from 4000 to 6000 hr was lengthened by the use of compensating springs of a hairpin shape on each bolt. All the experience with the line indicated the desirability of welded construction to insure freedom from periodic maintenance.

A group of pictures is included here to show the condition of parts after prolonged exposure to the high temperature.

The condition of the inlet edge of the first-stage buckets is shown in Fig. 2. At (a) is a photograph showing a bucket as

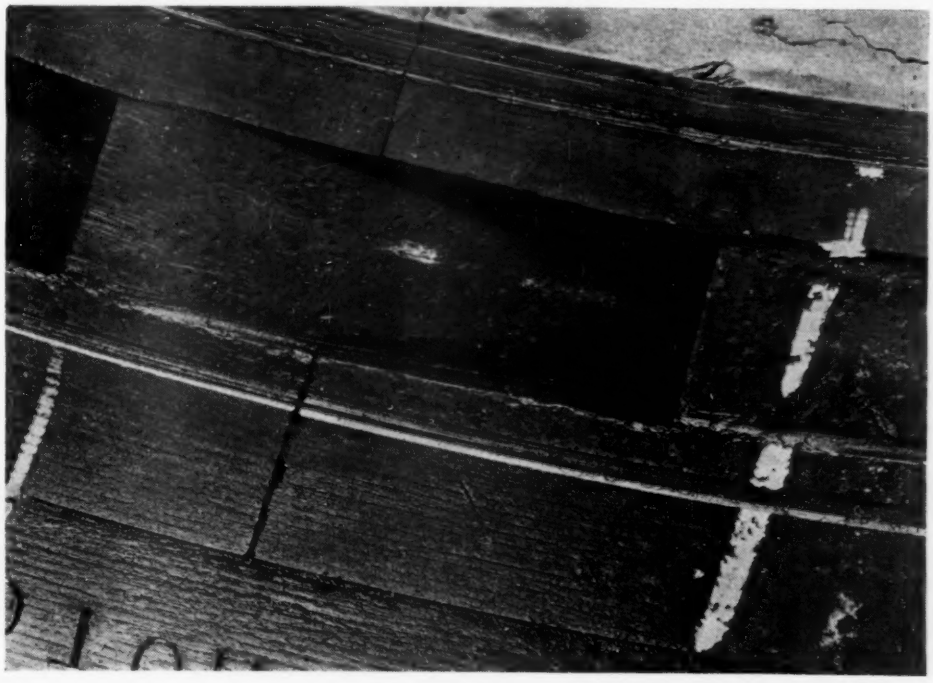


FIG. 3 SECTION OF FIRST-STAGE NOZZLE RING AFTER SERVICE, ×2.2



FIG. 4 SECTION OF SECOND-STAGE DIAPHRAGM AFTER SERVICE

cleaned in 1932 after service for 1450 hr at 1000 F. The dark areas are indentations caused by welding beads which were small enough to pass through the steam strainer. The picture at (*b*) is of the same bucket taken in 1937 after 22,446 additional hours with 1000-F steam. Here the light areas show the same indentations. The photograph shown in Fig. 3 was taken at the same time as that in Fig. 2(*b*) and shows the condition of the one partition in the first-stage nozzle ring. It will be observed in both views in Fig. 2 that the buckets are tight whereas the illustration of the nozzle, in Fig. 3, shows that the sections have tended to separate and in the upper-right corner, the seal weld has cracked. The only changes noted in the second-stage diaphragm, shown in Fig. 4, were slight indentations caused by the passage of welding beads. The composite construction used withstood service much better than the nozzle ring.

Fig. 5(*a*) shows the condition of one of the control-valve disks. The condition of this valve is representative of that of the other valve disks and seats. This material was a low alloy of the following percentage composition: 0.5 C, 1.3 Cr, 2.6 Ni, and 0.6 Mo. Typical conditions of the control-valve seats and stems are shown in Fig. 5(*b*) and (*c*). The seat material was the same as that used in the disk. The stem material was stainless steel, 18 per cent Cr and 2.5 per cent Si. The scoring shown was caused by the kinks which developed in all the stems. The scoring was not serious, however, as it did not progress to the galling stage.

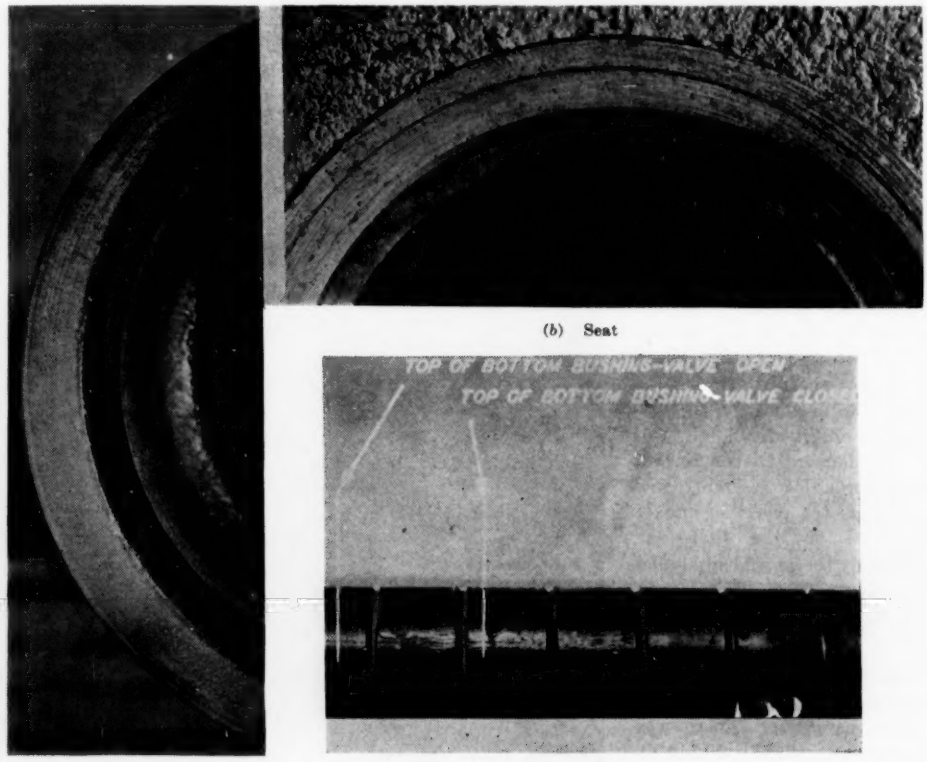
One of the causes of leakage of bolted joints using metal gaskets is plastic flow of the joint faces resulting from the high compressive stresses. Two examples of this condition are shown in Fig. 6. At (*a*) can be seen the indentation, 0.009 in. deep, in a casting joint face caused by the narrow monel gasket, 0.39 in. wide. At (*b*) is shown the groove in the vanstone lap of the adjoining 8-in. low-carbon, 18 per cent Cr, 8 per cent Ni tubing. It was necessary on several occasions to regrind the joint faces after leakage occurred because of this type of creep. No evidence was ever found in any of the leaking joints of steam cutting, in contrast to experience with lower temperatures and pressures.

A more detailed description follows of the various changes which were noted, the result of creep.

Creep Determinations. Measurements made on various parts of the equipment have shown that in all except a few cases change of shape caused by plastic flow or creep has been negligible. Deformations were so slight that differences in measurements before and after service in nearly all cases could have resulted from measuring errors.

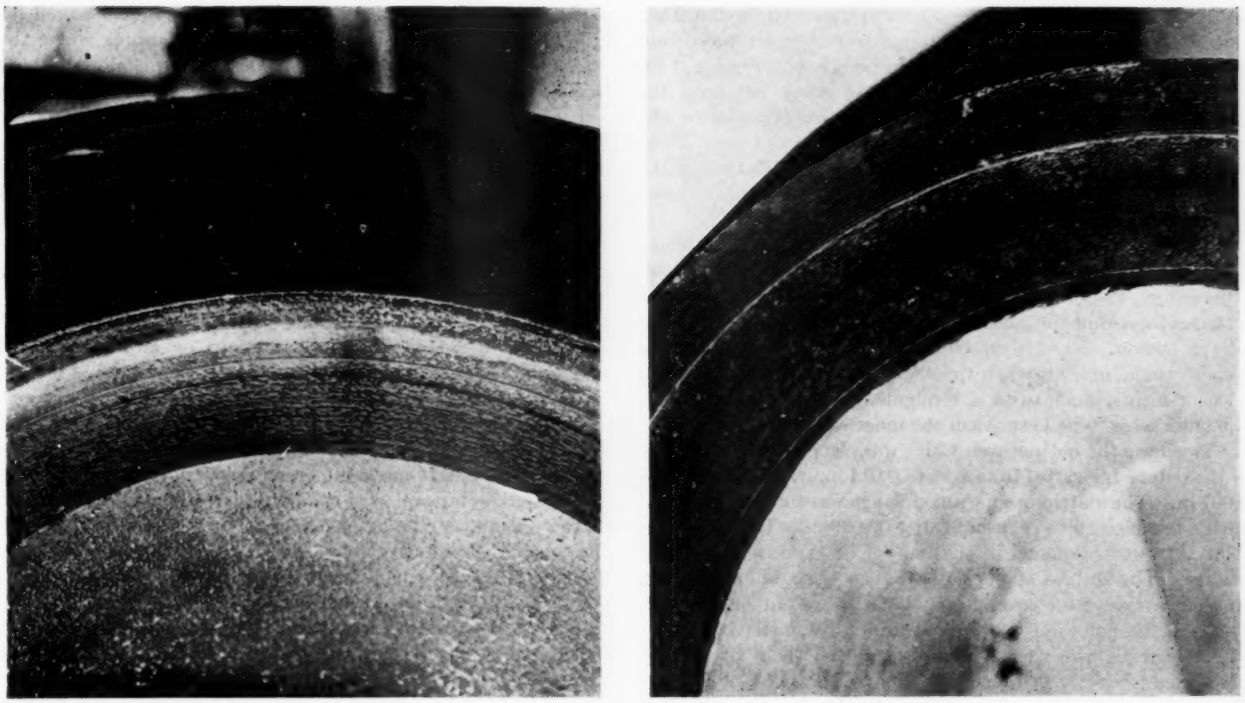
The one case in which plastic flow necessitated a replacement was in connection with the turbine stop valve, mentioned previously. Measurement extending over a period of nearly 8000 hr showed parts of the valve body to be increasing at a rate of over 6 per cent per 100,000 hr. On one occasion distortion caused binding of the valve disk and it was possible to keep the bolted bonnet joint tight only for short intervals. The replacement valve, in service for 15,000 hr and designed approximately to the standard dimensions of the 900-lb A.S.A. flange, showed a rate of increase less than 0.5 per cent per 100,000 hr.

The other cases of trouble from creep were leakage of bolted pipe joints. Leakage resulted, not only from extension of the bolts and flange dishing, but also from deformation of the flange



(*a*) Disk (*b*) Seat (*c*) Stem

FIG. 5 SECTION OF CONTROL-VALVE PARTS



(a) (b)

FIG. 6 INDENTED FLANGE FACES
(Average depth of indentations, 0.009 in.)

faces caused by compressive creep, a result of the large compression stresses imposed by the metal gaskets. The only remedy for this situation was periodic maintenance.

Numerous measurements were made in the superheater. The average rate of change in diameters of tubes in the top portion of the middle tube bank in the superheater for the total operating period of 22,896 hr at 1000 F showed the following:

Tube material, per cent	Steam temperature	Design stress, lb per sq in.	Creep rate, per cent per 100,000 hr
2½ in. 0.30 to 0.40 C, 7/16-in. wall	925 F	2000	4.0
2 in., 18 Cr, 8 Ni, 2 Si, No. 6 gage	925 F	4300	0.4

Measurements made on 1-in. steel and alloy tubes in the lower portion of this bank to determine axial change were not reliable. The alloy tubes exhibited either negative or no change while the 0.3 to 0.4 per cent C steel tubes showed positive creep rates of from 0 to 1.5 per cent per 100,000 hr. These tubes were exposed to the combustion chamber and operated with metal temperatures of about 925 F while carrying steam at from 800 to 850 F. The alloy tubes, 2 in. OD × 1/4-in. wall, in the lower section showed little change in length, roughly at a rate not over 0.25 per cent per 100,000 hr. No changes in diameter were noted in an intermediate header, 18 per cent Cr and 8 per cent Ni, at 925 F and stressed at 7400 lb per sq in. In all cases where length measurements were made, the accumulation of dirt in the measuring holes interfered with accuracy. Where diameters were obtained results were more satisfactory.

Few changes of magnitude were noted in the piping system other than at the bolted pipe joints. Changes in the 8½-in. OD, 5/8-in. wall, low-carbon, 18 per cent Cr, 8 per cent Ni tubing at the point of maximum bending stress were small. One length measurement in the horizontal plane decreased 2 mils in 20 in. and the diameter in the vertical plane increased 2 mils, indicating that a bending moment caused flattening.

The behavior of the pipe line from the superheater outlet to the turbine inlet was investigated to determine the amount of expansion and to determine any permanent change in position. The difference in position from the cold to the hot checked very well with the calculated expansion used in design. No information was obtained, however, as to any permanent change in the assembly because the line and hangers were disturbed a number of times to repair joint leaks. The whole experience with this line indicated the desirability of welded construction for trouble-free operation even though joints of the 900- and 1500-lb A.S.A. series were used for the 400-lb, 1000-F steam condition.

Changes observed in the turbine were small, and at the rate of change found, there is no reason to believe that the economic life would not have been equal to that of machines designed for lower steam temperatures.

Accurate measurement of the changes in the diameters of the first- and second-stage wheels was not obtained. The measuring points provided were damaged by foreign matter passing through the unit. The bucket connections were tight and there is reason to believe that any creep was of small magnitude. Change in length of the steam chest was at a rate of less than 0.1 per cent per 100,000 hr. Ten measurements made on the high-pressure cylinder showed both positive and negative rates of change of from −0.25 to +0.35 per cent per 100,000 hr. The negative changes found indicated slight distortion of the casting, probably caused by the relief of original internal stresses. The average unit change of these measurements was 0.0004 in. per in., neglecting the direction of change. The stresses used in the design of the casing and steam chest were under 5000 lb per sq in.

A number of the machined-joint faces in the high-pressure cylinder assembly were examined to find the amount of distortion from the original plane condition. The admission-

nozzle-ring seat, 40 in. OD × 26 in. ID, located on the inner face at the front end of the high-pressure cylinder was found warped to a varying degree around the periphery. See Fig. 1 for the location of this vertical joint. At some sections, the inner edge of the joint face was 0.005 in. lower than the outer edge. At other sections, the reverse was true. These changes undoubtedly had some part in causing cracking of the seal welds and changes found in the nozzle-ring assembly shown in part in Fig. 3.

The vertical joint face on the outside of the cylinder to which the split shaft-packing casing was bolted also was found changed. This flange, 48 in. OD × 32 in. ID × 2½ in. thick, was on a 12-in. extension of the casing and reinforced by three webs, one at the bottom and two spaced at 120 deg. The readings obtained would indicate that the flange face had been scraped. Measurements, made with a straightedge and feelers, showed the outer edge to be lower than the inner except for a section of the face near the bottom web which was plane. The depression of the outer edge varied from zero to 0.014 in. with an average of 0.007 in. The matching companion flange was substantially true with a few high spots of from 0.001 to 0.003 in. The horizontal joint between the halves of the inner split cylinder showed no change. The surface of the outlet flange of the outer cylinder was plane except for several low spots, the result of scraping. Steam at this point in the cylinder was approximately 700 F.

The changes which have been observed on the high-pressure casing may have been caused in part by the relief of internal stresses set up by the rather complicated heat-treatment given this casting. The casing was annealed and then quenched in oil from 1562 F and drawn after rough machining.

The nozzle ring was found to be distorted but not to an extent that would have impaired its reuse. The ring consisted of five nozzle sections, steam to each of which was controlled by a valve. The nozzles were mortised and doweled into inner and outer forged rings and then seal-welded. Cap screws holding the ring in place passed through each ring. The inner ring had shifted toward the first-stage wheel from 0.014 to 0.026 in. or an average of 0.019 in. In the bottom 120 deg of the assembly, the nozzles had separated from the inner edge of the outer ring an average of 0.007 in. and a maximum of 0.016 in. Practically all the seal welding had cracked. The maximum distortions occurred in the first- and second-nozzle sections which were the ones in use during the larger part of the running time. In all probability, most of this distortion was caused by the thermal stresses set up when steam was admitted to an individual section. The nozzles themselves were straight, and apparently had not changed in shape.

Some growth and distortion were found in the bushing and stem assemblies of the control and throttle valves. The control-valve bushings, of nitralloy, were made in two parts and the stems were made from a stainless steel containing 18.7 per cent Cr and 2.5 per cent Si. The bottom end of the lower bushings decreased in diameter from 1.524 to 1.523 in., while the upper end of these bushings and the bore of the upper bushings were substantially unchanged. The stems were all bowed toward the low-pressure end in varying amounts up to 0.050 in., caused, no doubt, by the stress imposed by impact. They were scored for from 2 to 3 in. at the bottom of the upper bushing where the clearance changed from 0.025 to 0.005 in. A nitralloy bushing and a 1.5-in. 18 Cr, 8 Ni stem were used in the replacement throttle valve. The bore at the end of this bushing, subjected to 1000-F steam, had decreased in diameter between 0.002 and 0.005 in., while the cold end of the bushing remained unchanged. At the time this valve was placed in service the stem clearance of 0.006 in. had been insufficient because of a decrease of 0.003 in. in the bushing diameter which occurred after 3 days' operation. The stem was ground to provide a total of 0.015 in. clearance and gave no further trouble.

The measurements made to show the changes due to creep were augmented by metallurgical examinations on all the principal materials, the results of which are given in the following section of the paper.

Metallurgical Examination. All the principal types of material were cut up and examined to determine changes in structure and physical properties. In most instances the results obtained were compared with properties of samples of the alloy in the as-received condition. Samples of materials as used in the completed parts of the installation were secured at the time of purchase, in nearly all cases, for later comparison to determine changes that occurred during service. These samples included tube ends, cast test bars heat-treated as the original part, and so on. In the case of the turbine parts reused in rebuilding the unit for higher-pressure operation, it was possible to secure samples for examination from all high-temperature materials except that of the forged Ni-Cr-Mo rotor.

Several of the samples taken from the superheater and piping system and examined are listed in Table 1. The physical properties obtained from the materials in this table are given in the tabulation of results in Table 2, except in the case of sample No. 4 from which it was impossible to cut sound specimens. Only the results of examination of one typical superheater tube are included, although several were thoroughly examined. The specimens removed from sample No. 2 were considered representative of the group of castings, 18 per cent Cr and 8 per cent Ni, at the outlet of the superheater.

The values obtained on the superheater-tube samples, 18 per cent Cr, 8 per cent Ni, and 2 per cent Si, showed that this material deteriorated in service. Examination of tubes from cooler parts indicated that the embrittlement progressed as the temperature increased, being more severe in the furnace tubes.

Samples of used and unused tubes were boiled in acidified

TABLE 1 COMPOSITION OF SUPERHEATER AND PIPING MATERIALS

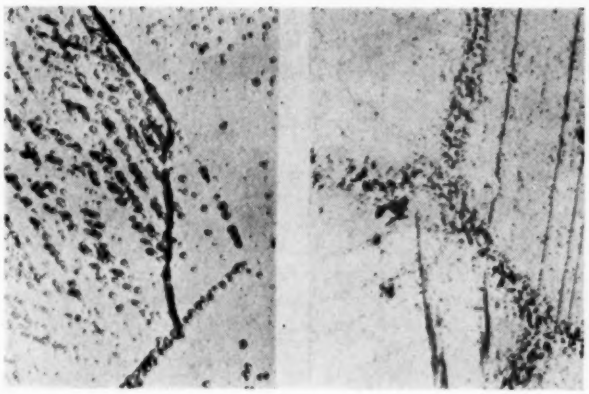
(In service 25,800 hr, 22,896 hr at 1000 F)

Sample no.	Part	Chemical composition, per cent by weight							Approximate operating temperature, F
		C	Cr	Ni	Mo	Mn	Si	W	
1	2-in. tile-covered furnace tube	0.10–0.20	17.0–19.5	8.0–10.0	..	0.65 Max	2.0–2.5	..	1050
2	4-in. cast tee	0.14	18	9	..	..	..	..	1000
3	10 × 8-in. cast reducer	0.22	..	..	0.29	0.72	0.24	..	1000
4	10 × 8-in. cast reducing ell	0.24	5.3	..	..	0.58	0.55	0.82	1000
5	8½-in. OD × ⅜-in. wall tubing	0.07 0.055–	17.9	9.26	..	..	..	..	1000

TABLE 2 PHYSICAL PROPERTIES OF SUPERHEATER AND PIPING MATERIALS

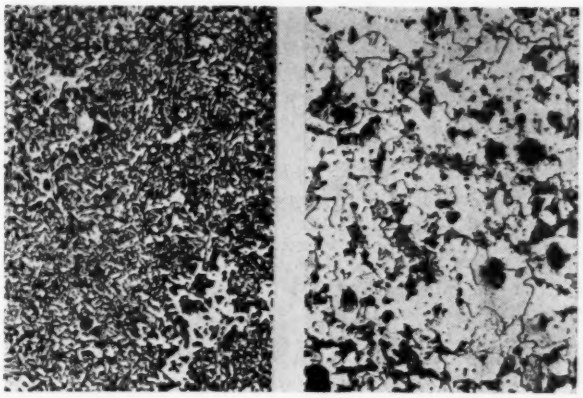
Sample no.	Condition	Yield point, lb per sq in.	Ultimate strength, lb per sq in.	Elongation, per cent	Reduction in area, per cent	Izod impact value, ft-lb	Hardness, Rockwell B
1	Unused[a]	54500	101000	59 in 1 in.	50	51	91
	Used[a]	64800	99000	22 in 1 in.	17	5	97
	Used[b]	....	77000	3 in 8 in.	3	....	..
2	Unused	35000	72000	68 in 2 in.	67	96	79
	Used	46000	82000	57 in 2 in.	56	110	83
3	Used	34500	58500	15 in 2 in.	23	21	71
5	Unused	....	....	..	..	101	75
	Used	....	....	..	..	101	77

[a] Tensile specimens ¼-in. pipe coupons; impact specimens 0.373 × 0.203 in. with Izod notch.
[b] Cylindrical section of tube.



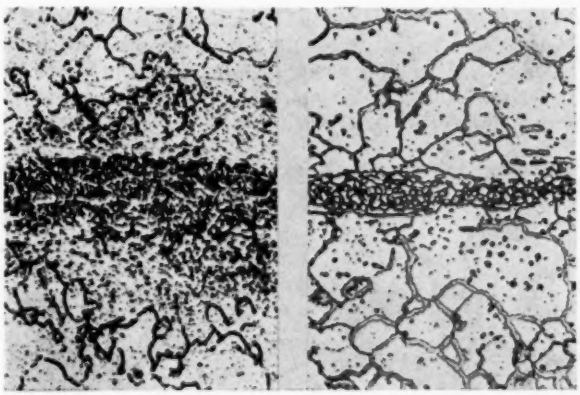
(*a*) Before service (*b*) After service

FIG. 7 SUPERHEATER TUBING, ×1000
(18 Cr, 8 Ni, 2 Si. Electrolytic etch.)



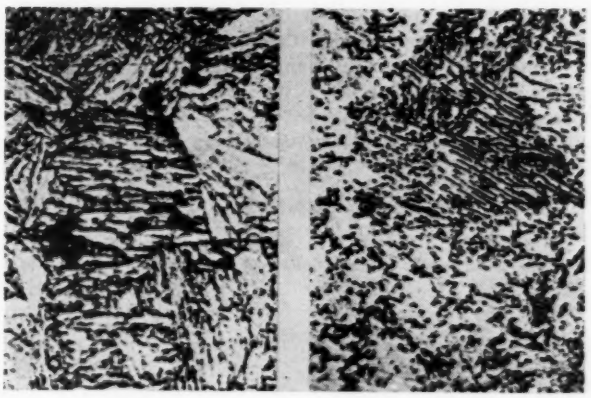
(*a*) Before service (*b*) After service

FIG. 9 CARBON-MOLYBDENUM CAST REDUCER, ×100
(Nital etch.)



(*a*) Before service (*b*) After service

FIG. 8 FLASH WELD OF TUBING, ×100
(18 Cr, 8 Ni, 2 Si. Electrolytic etch.)



(*a*) Before service (*b*) After service

FIG. 10 CAST REDUCING ELL, ×1000
(5 Cr, 1 W. Electrolytic etch.)

copper sulphate (Strauss solution) for 72 hr to determine the susceptibility to intercrystalline corrosion. All unused material showed no change in electrical resistivity whereas all the used samples disintegrated in the solution. Specimens from the casting, sample No. 2, showed no change in this test, whereas the $8^1/_2$-in. tubing showed a 58 per cent increase after the test. Previous work has shown that the low-carbon variety of 18 per cent Cr, 8 per cent Ni steel will become restabilized after prolonged exposure and entirely resist the attack of this solution. Another fairly definite proof of the embrittlement of material No. 1 was the fact that the impact fractures were intercrystalline rather than transcrystalline, as was the case in the unused material examined.

The typical structure of the tube before and after service is shown in Fig. 7, which represents the greatest change that took place during service. In connection with the tube examination, the structure of a tube flash weld was examined. Fig. 8(*a*) shows the typical structure of a weld before service, while at (*b*) is shown the condition of a similar weld after service. A definite grain structure has been formed by recrystallization. An examination of the structure of the 4-in. casting showed that the dendritic pattern present in the unused sample was changed to a distinct hexagonal-crystal structure.

Tubes of 0.35 per cent C steel surface-protected by calorizing and metal spraying had been installed in the middle section of the superheater to observe the surface protection afforded by the two coatings. The aluminum coating of the calorized tubes was still partly intact, while the sprayed 18 per cent Cr, 8 per cent Ni coating had nearly all disappeared allowing considerable scale formation and decarburization of the tube wall.

The structure of the 8 × 10-in. reducer, sample No. 3 shown in Fig. 9, is typical of that for a casting. After service, the grain had become more equiaxed and some grain growth had occurred. Likewise, considerable carbide migration had taken place. The condition of the 8 × 10-in. reducing ell is shown in Fig. 10. A number of inclusions are evident and the partial formation of pearlite can be observed in the after-service section. The microstructure of the $8^1/_2$-in. tubing underwent some change as evidenced by some grain growth and migration of chromium carbide to the grain boundaries. A summary of investigations on similar material from the Trenton Channel 1100-F installation is contained in a following section of the paper.

One of the field position welds in the 8-in. line was cut up and examined and the tensile strength of standard pipe-wall coupons taken across the weld was found to vary from 38,000 lb per sq in. at the bottom to 62,000 lb per sq in. at the top of the joint. The corresponding variation in elongation was from 2 to 6 per cent. The weld was made without a backing ring, and lack of penetration and cracks in the first bead account for the low strength. A position test weld made prior to the field weld gave corresponding tensile strengths of from 65,000 to 75,000 lb per sq in. with elongations of from 13 to 17 per cent, respectively. The Izod

impact value of the test weld was 42 ft-lb as against 22 ft-lb in the field joint. The carbon content of the weld metal in both cases was 0.07 per cent. As a safety precaution, joints of this type had been provided with bolted backing-up flanges in anticipation of weld embrittlement, which might have caused failure of the joint.

The materials listed in Table 3 are those which comprised all the high-temperature parts of the turbine with the exception of the forged rotor and bolting.

TABLE 3 COMPOSITION OF TURBINE MATERIALS

(In service 26,453 hr; 22,896 at 1000 F)

Sample no.[a]	Chemical composition, per cent by weight C	Cr	Ni	Mo	Mn	Si	Hours in service at 1000 F
6	0.30	0.25	3.10	0.25	0.50	0.26	7832
7	0.31	0.35	3.10	0.34	0.72	0.31	22896
8	0.21	...	...	0.51	0.62	0.30	15064
9	0.47	11.84	36.32	..	0.24	0.23	22896
10	0.24	14.28	0.28	..	0.52	0.34	22896
11[b]	0.35	1.24	...	0.19	0.49	0.26	7832
12	0.12	18.60	1.80	..	0.73	0.34	22896

[a] For names of parts corresponding to sample numbers see bottom of Table 4.
[b] Contains 1.80 per cent Al.

The physical properties of these materials are given in Table 4. In the case of sample No. 11, only photomicrographs were taken. Sufficient material for physical tests other than impact were not obtained from the used turbine buckets. It was possible in the case of the high-pressure casing to compare the effects of both 700- and 1000-F service.

TABLE 4 PHYSICAL PROPERTIES OF TURBINE MATERIALS

Sample no.	Yield point, lb per sq in.	Ultimate strength, lb per sq in.	Elongation, per cent in 2 in.	Reduction in area, per cent	Izod impact value, ft-lb	Hardness, Rockwell B
6[a]	114,800	131,600	17	44	45	94
6[b]	86,500	100,600	9	18	40	100
7[a]	118,300	137,300	26[g]	33	52	...
7[c]	86,500	103,600	13	22	30	98
7[d]	104,500	121,600	12	27	17	103
8[a]	36,100	68,600	31	51	9	73
8[b]	38,800	71,700	26	40	7	76
9[a]	.....	.....	..	..	23[h]	89
9[b]	.....	.....	..	..	23[h]	93
10[a]	78,500	103,000	31	64	24[h]	91
10[b]	.....	.....	...	...	37[h]	...
12[a]	108,800	121,600	17.7	53.6	54.6	28.1[i]
12[e]	95,700	113,300	20.3	52.6	9.8	27.1[i]
12[f]	66,300	101,300	9.4	10.2	1.0	23.7[i]

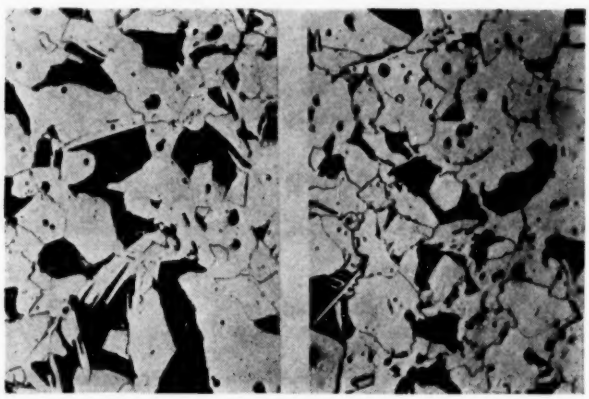
Parts corresponding to sample numbers:
6 Original throttle valve.
7 High-pressure outer casing.
8 Replacement throttle valve.
9 First-stage bucket.
10 Second-stage bucket.
11 Nitralloy bushing.
12 Control-valve stem.

[a] Before service. [b] After service.
[c] Steam chest, 1000 F service.
[d] Exhaust end, 700 F service.
[e] Air-exposed end, used.
[f] Steam end, 1000 F service.
[g] Per cent in 1 in.
[h] Unnotched specimens, 1/4 in. diam.
[i] Rockwell C.

The results obtained on the Ni-Cr-Mo steel used for the original throttle valve and casing, samples Nos. 6 and 7, are rather interesting in that a range of time and temperature conditions is given. The results indicate that the material underwent changes due to service, but that the change did not progress at a uniform rate. The properties of the throttle-valve material for instance, changed approximately the same in 8000 hr as the decrease exhibited by the casing material during 23,000 hr of service. The material from the exhaust end of the casing, while not having lost as much in tensile strength, showed about the same ductility with an appreciably lower impact strength. Part of the apparent discrepancy in the ductility values shown may be in part explained by inclusions always present to some degree in a cast structure.

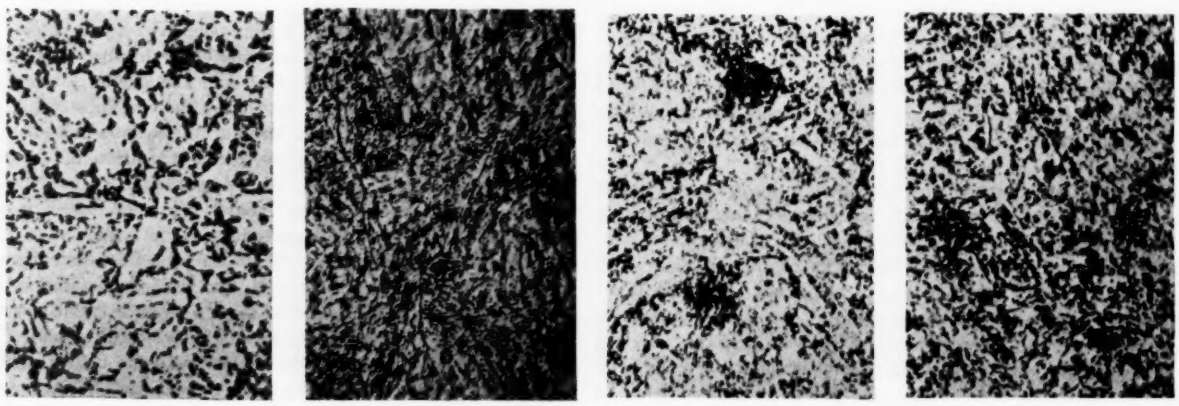
The various conditions of the structure of these samples are shown in Fig. 11. The material is extremely fine-grained both before and after service. Other sections etched to reveal carbides showed them to be evenly distributed throughout the used specimens. Their presence probably accounted for the increase in hardness shown in Table 4.

The results obtained on the C-Mo casting, sample No. 8, are of particular interest because the chemical composition is identical with that which is being used extensively in recent high-temperature-steam equipment. The differences in physical properties found in the before- and after-service condition were



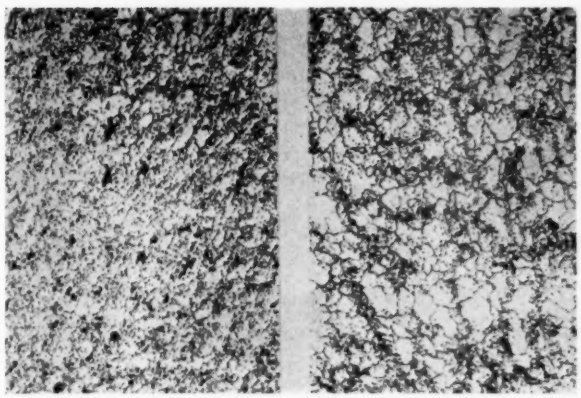
(a) Before service (b) After service

FIG. 12 C-MO CASTING FROM REPLACEMENT THROTTLE VALVE, ×100
(Nital etch.)



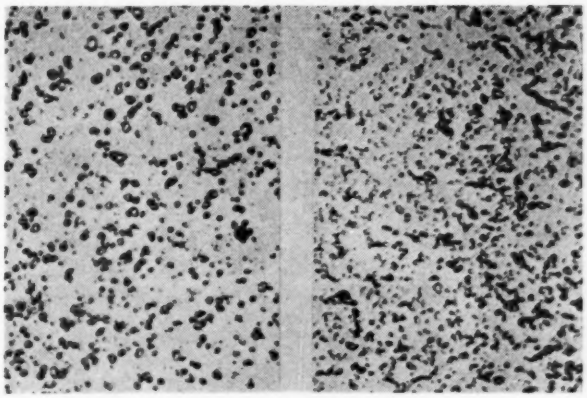
(a) Original structure (b) After 7832 hr service (c) After 22,896 hr at 1000 F (d) After 22,896 hr at 700 F

FIG. 11 CAST NI-CR-MO TURBINE MATERIAL, ×1000
(Nital etch.)



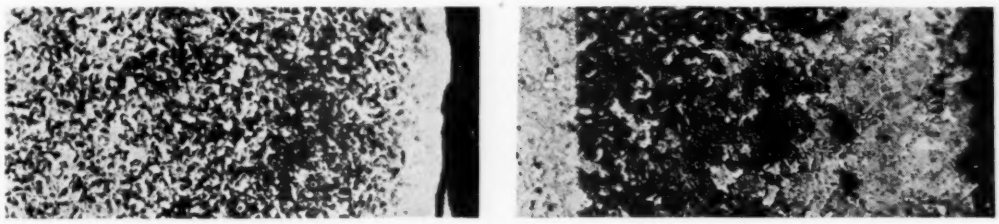
(*a*) Before service (*b*) After service

FIG. 13 MATERIAL OF FIRST-STAGE BUCKET, ×100
(Electrolytic etch.)



(*a*) Before service (*b*) After service

FIG. 14 MATERIAL OF SECOND-STAGE BUCKET, ×1000
(Electrolytic etch.)



(*a*) Cold end (*b*) Hot end

FIG. 15 NITRALLOY BUSHING FROM THROTILE VALVE, ×100
(Nital etch.)

small and may be accounted for, again in part, by the number of inclusions which were present. These undoubtedly were responsible for the low impact values obtained. The structure of this alloy before and after service is shown in Fig. 12.

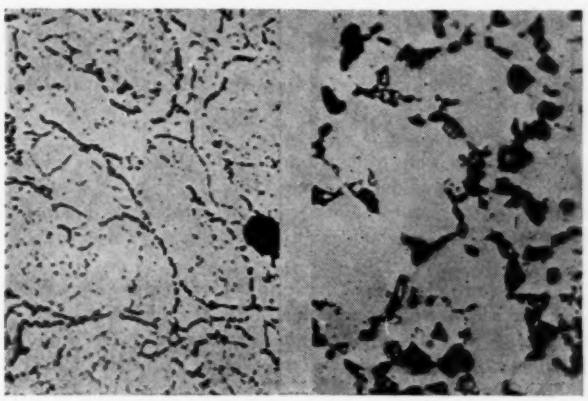
Although the impact values obtained on the new and used material of the first-stage bucket, sample No. 9, show no change in this property, the microstructure as shown in the set of pictures in Fig. 13 indicates that a change has taken place. The grain size has increased and the crystal structure in the used specimen is clearly outlined by the chromium-carbide migration to the grain boundaries. Samples of both the new and used material were subjected to the Strauss test to determine change in electrical resistivity with the result that they both disintegrated during the test. This is in direct contrast to the findings of French investigators who have reported on similar tests no attack by the copper-sulphate reagent.[4]

The condition of the second-stage bucket material before and after use is shown in Fig. 14. Apparently the only effect on the structure was that of aging which accounts for the darker condition of photograph (*b*). The material remained in a fine-grained condition and was identical with that observed in a section of a ninth-stage bucket. The impact results were somewhat erratic. Those values reported in Table 4 show a higher value for the used buckets, whereas results obtained on 0.394 × 0.228-in. notched Izod specimens gave 75 ft-lb for the new and 34 ft-lb for the used, with one of the latter specimens breaking at 13 ft-lb.

The second-stage buckets operated in a temperature range of from 790 to 900 F at from 40 per cent to full load. The corresponding range for the first stage was from 810 to 925 F. The maximum temperature at the ninth stage was 700 F.

Sections of the 18-in. nitralloy bushing removed from the original throttle valve are shown in Fig. 15. That shown at (*a*) was from the cold end exposed to atmosphere, while the section shown at (*b*) was taken from the 1000-F end. Some scale is evident on both sections and the extensive diffusion of the case, the result of long exposure to temperature, can be observed at (*b*). The hardness had dropped from a Brinell of from 690 to 730 at the cold end to approximately 560 at the hot end. This reduction had not changed the serviceability of the part.

Structural changes of the control-valve stems, sample 12, can be seen in Fig. 16. This material had deteriorated to such



(*a*) Before service (*b*) After service

FIG. 16 MATERIAL OF CONTROL-VALVE STEM, ×1000
(Electrolytic etch.)

[4] "Corrosion Fissurante du Ferronickel Type NC 4 Dans la Vapeur," by P. Chevevard, Aciers Speciaux, Metaux et Alliages, Oct., 1934, p. 341.

TABLE 5 1100 F STEAM-CORROSION SPECIMENS

Specimen no.	Chemical analysis, per cent by weight C	Cr	Mo	Si	Mn	Ni	P	S	Heat-treatment[a]
					Group A				
1	0.09	1.21	0.58	0.78	0.41	..	0.01	0.013	A
2	0.11	2.50	0.50	0.78	0.41	..	0.012	0.013	A
3[b]	0.24	0.94	..	..	0.65	..	0.02	0.021	A
4	0.13	4.96	0.52	0.32	0.45	..	0.01	0.016	A
5	0.11	...	0.50	1.35	0.19	..	0.01	0.012	A
6	0.10	1.24	0.58	1.40	0.41	..	0.01	0.013	A
7	0.07	17.6	..	0.16	0.14	10.2	...	...	WQ 2000 F
8	0.05 to 0.15 C	Typical boiler-tube quality							AR
9	0.10	4.83	0.51	1.55	0.30	..	0.009	0.016	A
					Group B				
10	0.11	12.27	Trace	0.25	0.35	..	0.016	0.308	A
11	0.3–0.4	...	..	..	0.6–0.9	..	0.04 Max	0.055 Max	AR
12	0.32	13.57	..	0.22	0.36	..	0.014	0.008	OQ 1825 F; D 750 F
13	0.3–0.4	1.0–1.5	0.15–0.25	0.3 Max	0.3–0.6	Al 0.9–1.4	0.025 Max	0.025 Max	Heated at 600 C for 6 hr; N for 30 hr at 500 C
14	0.3	...	0.49	0.34	0.4	..	0.023	0.015	Casting[c]
15	0.10	12.33	..	0.20	0.35	0.18	0.017	0.022	A
16	0.05	12.18	2.01	0.30	0.13	0.10	0.024	0.011	A
17	0.3	...	0.51	0.35	0.7	..	...	...	Casting[d]
18[e]	0.06	5.18	0.58	0.41	0.36	0.19	0.01	0.008	1400 F-3 hr AC

[a] A = annealed, AR = as rolled, D = drawn, WQ = water-quenched, OQ = oil-quenched, AC = air-cooled, FC = furnace-cooled, N = nitrided.
[b] Contains 0.18 per cent Va.
[c] 900 C 8 hr, FC to 500 C, reheated 850 C 8 hr, FC to 500 C, reheated 650 C 12 hr, FC.
[d] 950 C 8 hr, AC, 650 C 8 hr, FC.
[e] Contains Ti 0.46.

an extent that had the stems been subjected to continued stress or any shock, failure might have resulted.

The Trenton Channel Installation

The experimental work that has been undertaken at Trenton Channel during the last several years in connection with the 1100-F superheater installation, aside from the service testing of the original and replacement materials, has been that of determining the extent of steam corrosion and securing of accurate creep information on pipe. The corrosion and creep investigations were undertaken with the idea in mind that the data obtained would more nearly duplicate the results secured from actual high-temperature plant operation as contrasted with the results obtained from laboratory work. The steam conditions of 380 lb per sq in. and 1100 F or 925 F, to which the samples were subjected were not held, therefore, to the extremely narrow limits prescribed for laboratory tests. The samples also were subject to heating and cooling cycles which is not usually the case in laboratory work.

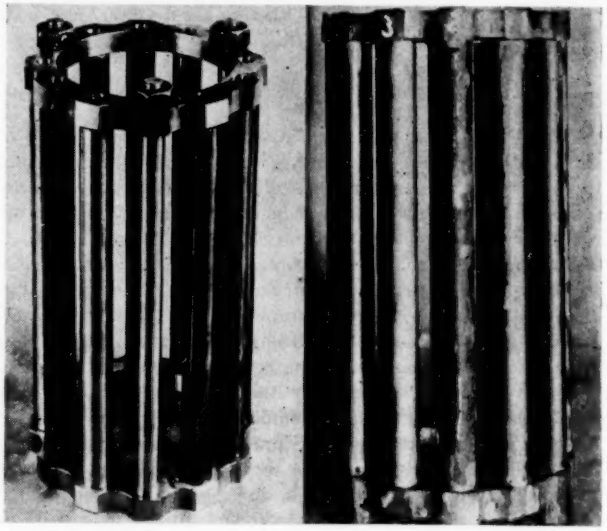
Results are included of the examination of 5 per cent Cr, Mo tubing; a 5 per cent Cr, W casting; an 18–8 superheater tube, and two grades of stellite used for valve trim.

Steam-Corrosion Experiments. Two groups of materials, designated as A and B in Table 5 and containing nine different steels each, have been subjected to the attack of 1100-F steam. Four sets of samples were made up from each group, assembled in holding cages, and the eight cages were fixed in the 1100-F steam main. The rate of steam attack will be determined as successive cages are removed at approximately six-month intervals. This has been done in the case of three cages, two from group A and one from group B.

The samples were ground bars $^1/_2$ in. OD × 6 in. and were held in the cage under no stress. Fig. 17, showing a new cage and one after service, illustrates the method of holding the samples. The steam velocity through the cages was from 1000 to 1500 fpm.

The loss in weight resulting from corrosion was determined by weighing sections of the specimens after removal of the scale and comparing with the original calculated weight. The scale from specimens in the first cage was removed in a solution of 100 parts by weight of concentrated HCl, 5 parts of stannous chloride, and 2 parts antimony oxide.[5] In the case of the stainless steel, a solution of one per cent by weight of quinoline ethiodide in concentrated HCl was used. An electrolytic stripping cell was used in removing the corrosion products from the specimens in the second and third cages. A solution of 10 per cent H_2SO_4 plus 0.1 per cent quinoline ethiodide was used in the cell with a current density of 0.5 ampere per sq in. The amount of parent material removed by both methods was too small to be weighed by an analytical balance and therefore was disregarded. The overall laboratory error involved in the weight determination was not greater than ±5 per cent. The greatest change in density of the specimens during service was 0.05 per cent.

[5] "Determination of Magnetic Iron Oxide," by R. C. Ulmer, *Industrial & Engineering Chemistry*, vol. 10, Jan. 15, 1938, p. 24.



(*a*) Before service (*b*) After service

Fig. 17 Steam-Corrosion Samples Showing Cage Assembly

TABLE 6 HOURS AT TEMPERATURE, STEAM-CORROSION SPECIMENS

Temp, F	Group A Cage 1	Group A Cage 2	Group B Cage 3
Below 900	14	28	14
950	27	63	36
1000	26	117	91
1050	15	295	280
1100	3701	7082	3381
Total	3783	7585	3802

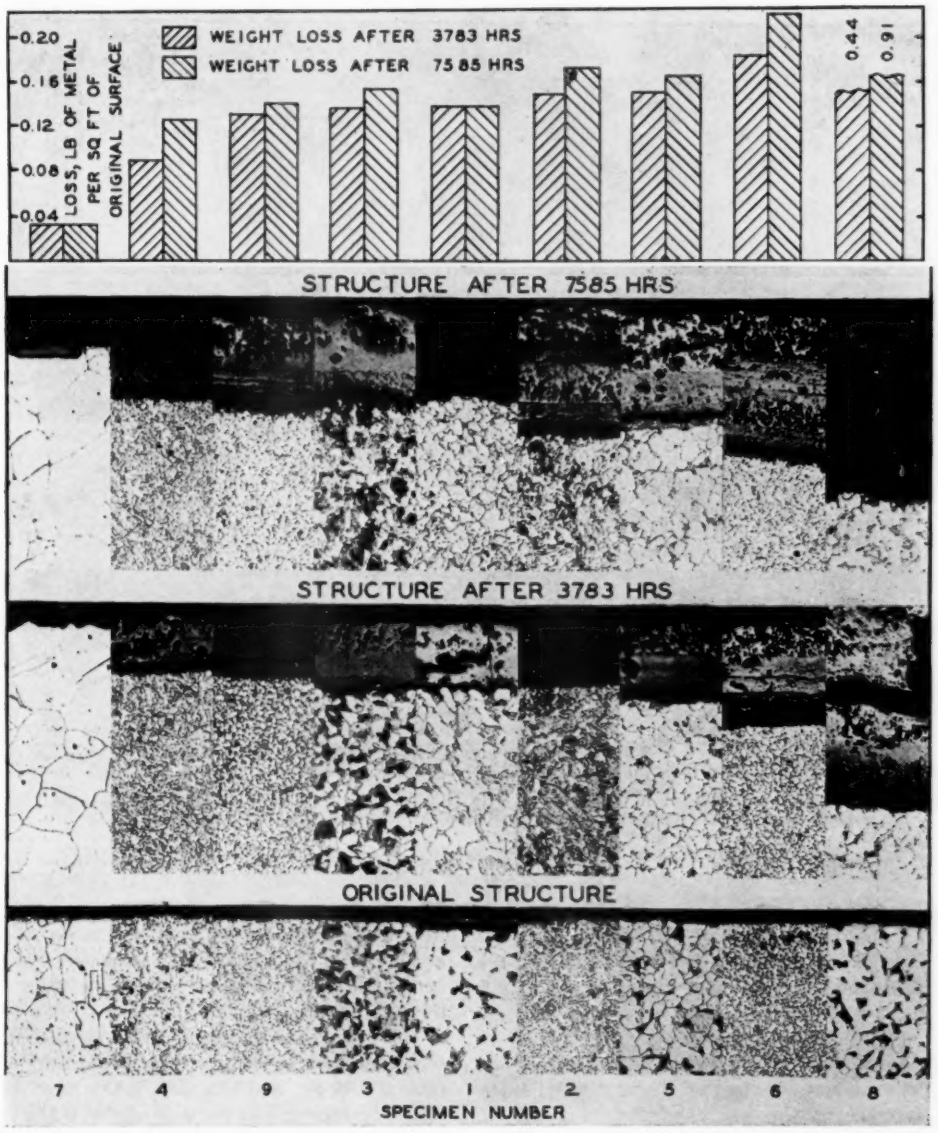


FIG. 18 MICROSTRUCTURE AND WEIGHT LOSS FOR 1100 F STEAM-CORROSION SPECIMENS IN GROUP A, ×72

The two groups of materials tested are listed as A and B in Table 5.

The number of hours at different steam temperatures to which the two groups were subjected is shown in Table 6.

The samples in cages 1 and 3 were heated and cooled six times while those in cage 2 were subjected to twelve heating-and-cooling cycles.

The results of examination of the group A materials are shown in Fig. 18. Because of space limitations, only small portions of the original photomicrographs are reproduced, but the areas shown are typical of the structures and give an idea of the scale formation except for a few specimens where the scale cracked off during the polishing operation. In the case of specimen No. 8, S.A.E. 1010, the scale layer was so thick that it could not all be included in the section shown.

Two interesting conclusions can be drawn regarding the weight-loss data shown at the top of this figure for the alloy steels; first, that the early scale layers formed, materially inhibit subsequent attack by the steam, and second, that increasing the silicon content of the Cr-Mo alloys does not increase resistance to steam corrosion, as has been expected from laboratory air-corrosion tests. In what respect the process of air and steam corrosion differ is not known, but is now being investigated. In the several cases where intercrystalline corrosion can be observed, it has not progressed deeper than one or two crystals.

The effect of even small additions of alloying elements is quite evident from an inspection of the weight-loss data. Only in the case of the low-carbon-steel sample has the corrosion progressed at a constant rate. From work at present in progress by other investigators, a boundary layer rich in the alloying elements is built up which tends to inhibit attack. It is quite probable that most of the scale formation observed on the alloy samples could have been formed in the first thousand hours or so of operation.

The effect of stress on the rate of corrosion is not known except in the case of sample No. 6, where it was possible to determine the amount of scale formed on the inner surface of a machined tube made from the same heat of steel and subjected to 7000 lb per sq in. stress during 8000 hours' service with 1100 F steam. In this case the observed amount of scaling was 16 per cent

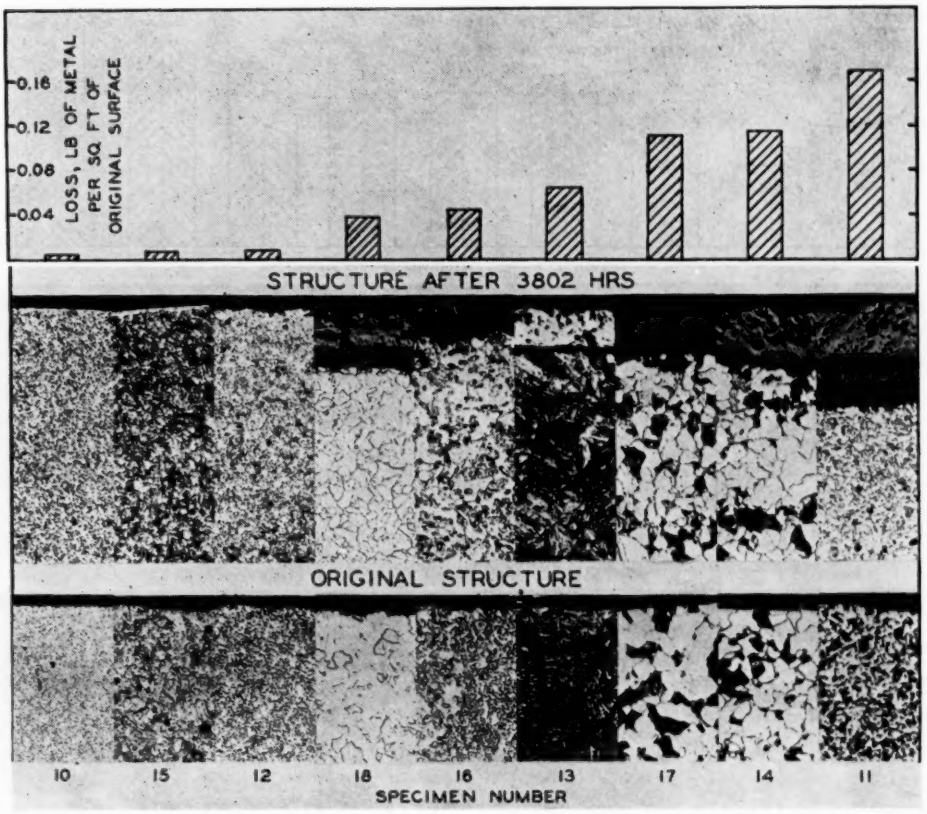


FIG. 19 MICROSTRUCTURE AND WEIGHT LOSS FOR 1100 F STEAM-CORROSION SPECIMENS IN GROUP B, ×72

greater on the tube than that obtained on the test specimen in 7600 hr when expressing the amount of corrosion in terms of pounds of Fe lost per sq ft of surface. The surface in this case is the mean of the original and the final expanded inside tube surface. It is quite possible that had a more conservative stress been used than the 7000 lb which caused creep at about the rate of one per cent per 1000 hr, the amount of corrosion would have been nearer the amount obtained on the unstressed specimen. Visual inspection of the inside surface of a 4000-lb stressed tube, still in service, shows none of the hairline scale cracks found on the inside of the 7000-lb section. See Fig. 22. The authors are led to believe that where conservative stresses are selected for steam equipment, the additional scale resulting from stress should not be a serious factor.

In the case of the stressed tube, approximately the same amount of scale was obtained on the outside surface as was obtained on the inner one. Microscopic examination showed the scale formed by steam to have a much better bond to the parent metal than the more porous scale formed on the outside of the tube, the result of air attack. Unstressed laboratory air-corrosion samples also exhibit a loose flaky scale. The microstructure of the tube and the scale formed by steam attack are shown in the after-service photomicrograph in Fig. 21. No intercrystalline cracking was evident on the inner or outer surface.

The data obtained from an examination of group B are shown in Fig. 19. Three of the four stainless steels exhibit substantially the same weight losses, while the fourth, containing 2 per cent Mo, scaled appreciably more than those containing no Mo. The results on these specimens are of interest because of the carbon range covered and the different heat-treatment. It should be observed that sample No. 10 is of the high-sulphur free-machining grade of stainless steel.

The effect of carbon in straight-carbon steel can be noted from a comparison of losses on specimens Nos. 8 and 11. The higher-carbon steel scaled only 25 per cent as much as the low-carbon sample. The reverse of this condition would be expected from air-corrosion tests.

The two groups of samples contain three alloys of the 5 per cent chromium series. Of these materials No. 18, the one containing 0.46 per cent titanium, is the most resistant to scaling. No. 9, containing 1.55 per cent silicon, is the least resistant, and No. 4, containing only the molybdenum is intermediate. Air-corrosion tests at 1250 F have shown the high-silicon composition to be much superior.

The authors would like to call attention to the fact that the corrosion results given are based on the examination of either one of two specimens of each steel and therefore they are not as conclusive as it would be desirable to have them. Then again, it is possible that by using steam with other amounts or types of impurities, somewhat different results might be obtained. These results, however, have been obtained over a long enough period to average out variations due to from 10- to 20-deg changes in temperature and relative position in the steam path and to make them indicative of the relative corrosion resistance of the various alloys tested using commercially pure and oxygen-free steam. The data already secured are of so much interest that it has seemed worth while to discuss them at some length in this paper.

Creep of Pipe Under Internal Pressure. In the hope of securing data from which some correlation might be made between actual performance of pipe and tubes in service and laboratory creep data on conventional tensile specimens, four machined tube sections were installed in the high-temperature experimental steam line at Trenton Channel. Two of these sections of Cr-Mo-Si alloy were subjected to 1100 F steam while the remaining

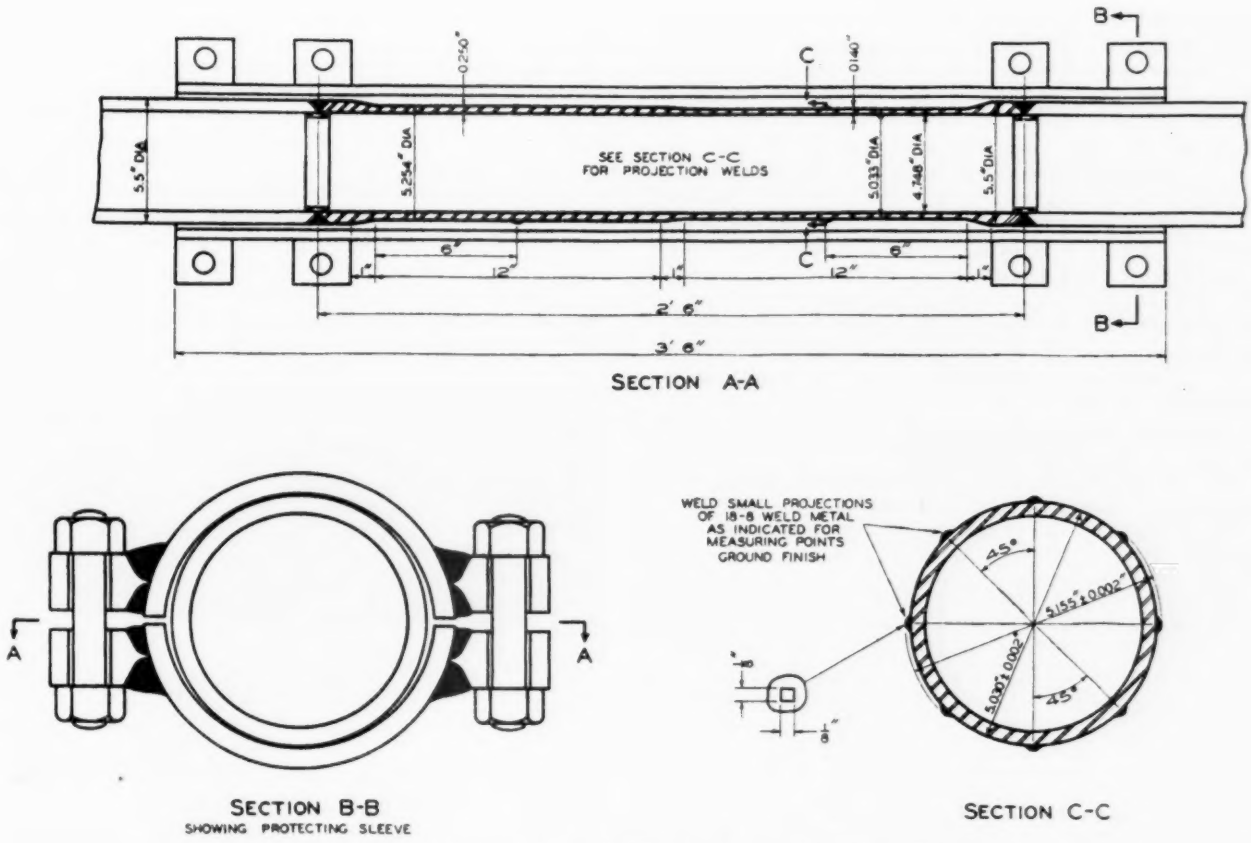


FIG. 20 DETAILS OF SECTION OF CR-MO-SI TUBING FOR CREEP DETERMINATIONS

two sections of C-Mo tubing were placed in service at 925 F. The laboratory data used for comparison have been obtained from the University of Michigan.

The tubes were machined inside and outside to give wall thicknesses such that the internal pressure of 380 lb per sq in. gage would produce easily measurable increases in diameter. Stainless-steel measuring points were welded at intervals of 45 deg around the circumference near the mid-point of each section to secure accurate measurements of diametral change and to insure freedom from effects of oxidation. The details of the Cr-Mo-Si tube sections are shown in Fig. 20.

The lengths of the test sections were only a little over two times the diameters, but it was found that the restraint offered by adjacent heavier sections did not extend more than 3 in. from each end, leaving approximately 6 in. free to expand under the internal pressure. The effect of end restraint is clearly evident in Fig. 22.

A guard sleeve shown in Fig. 20 was provided to protect the thin-walled sections from the effect of line bending moments, and in the event of failure of the thin-walled section, to restrict the steam flow and direct it in a horizontal plane. Since the use of a guard sleeve may imply that failure of the thin-walled section was anticipated, it should in fairness be stated that on the basis of information existing at the time of the installation, the working stresses were considered reasonably conservative for the Cr-Mo-Si tubing at 1100 F. The nominal stress of 4000 lb per sq in. in the section with 0.250-in. wall was thought to correspond to a creep rate of one per cent in 100,000 hr, while the 7000-lb per sq in. stress in the section with the 0.140-in. wall was considered to cause creep at the rate of one per cent in 10,000 hr. Subsequent laboratory tests showed that the material which

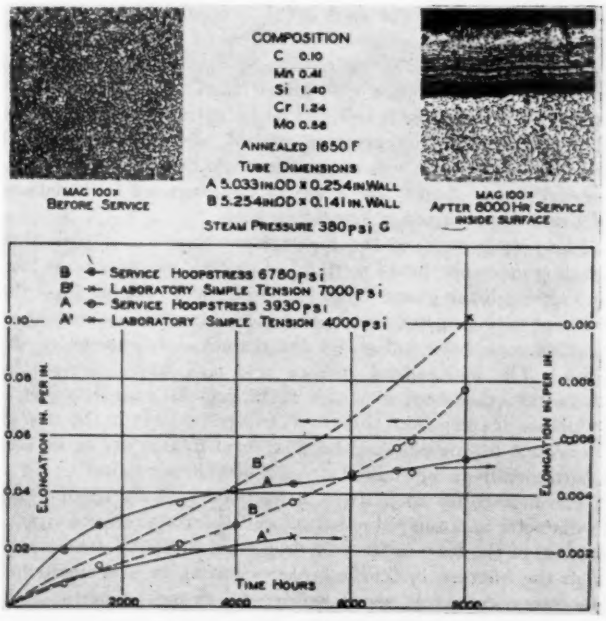


FIG. 21 COMPARISON OF LABORATORY- AND SERVICE-TIME ELONGATION CURVES CHROMIUM-MOLYBDENUM-SILICON PIPE AT 1100 F

(Magnification reduced to ×47.)

was obtained for the test sections elongated at the rate of about 4 per cent in 100,000 hr under a stress of 4000 lb per sq in. and 10

per cent in 10,000 hr under a 7000-lb per sq in. stress at 1100 F.

The diametral or circumferential elongations of the Cr-Mo-Si tubes under internal pressure are compared with results of conventional creep tests in simple tension on the same material in Fig. 21. It may be noted that the rates of creep for certain time intervals agreed quite well with tests in simple tension. The total elongations, however, were quite different. Since all measurements on the tubes were made in the cold condition with no stress acting and after creep recovery, if any, had occurred, it might be expected that the total elongation of the tube would be less than that of the tensile specimen. In the case of the tube having a hoop stress of approximately 7000 lb per sq in. this was found to be true. The lower-stressed section of Cr-Mo-Si tubing at 1100 F and the two samples of C-Mo tubing at 925 F, however, exhibited much greater elongations than the laboratory specimens.

Photomicrographs of the Cr-Mo-Si material before and after service, the heat-treatment, and chemical composition are given in Fig. 21. The time in hours at temperatures of 1000 to 1100 F was used as a basis for plotting the results.

The hours in service at the various temperatures are summarized in Table 7.

TABLE 7 HOURS IN SERVICE OF CR-MO-SI TUBES

Average wall thickness	0.141 in.	0.254 in.
Below 900 F	76	86
900 F	62	72
950	106	116
1000	870	950
1050	416	426
1100	6714	9615
Total	8244	11,265
Total, 1000–1100 F	8000	10,991

The condition of the Cr-Mo-Si tube with 0.140-in. wall after service is shown in Fig. 22. The total increase in diameter amounts to approximately $^3/_8$ in. The section with 0.250-in. wall is still in service and has been operating for the last 2900 hr without a guard sleeve. Omission of this sleeve has had no appreciable effect on the rate of diametral increase.

The method of measuring changes in diameter by means of a micrometer, while much less refined than ordinarily used for creep determinations, is believed to be entirely satisfactory for the purpose. The temperature control, allowing variations of from 10 to 20 deg F, was as good as might be expected in a commercial steam plant. These are to be compared to variations of 1 or 2 deg obtained in laboratory work.

The arrangement of the high-temperature line is such that bending moments due to restraint of thermal elongation are low, and although the guard sleeve was designed to further limit the moment acting on the thin-walled sections, a certain amount of bending may have influenced the diametral extensions of the tubes. The longitudinal stresses also may have affected the diametral extensions, although there was little evidence of a reduction in creep from this cause, except possibly in the case of the section having a normal hoop stress of 7000 lb per sq in. and a corresponding longitudinal stress of 3500 lb per sq in.

The increase in hoop stress, occasioned both by the increase in diameter and the reduction in wall thickness, caused by expansion of the tube as the test progressed, differed considerably from the increase in tensile stress occurring in a conventional laboratory creep-test specimen due to change in section. It was estimated that the original wall thickness of the tube having an original average wall thickness of 0.141 in. had been reduced 3.1 per cent by steam oxidation on the inside and 2.8 per cent by air oxidation on the outside of the tube. The increase in diameter and reduction in wall thickness caused by expansion of the tube, together with the reduction due to oxidation, resulted in an increase in the working stress in the latter stages of the test of approximately 24 per cent, or from 6780 to 8420 lb per sq in.

Two additional test sections of C-Mo tubing, similar to those shown in Fig. 20 were prepared for test at 380 lb per sq in. and 925 F, after experience with the Cr-Mo-Si tubes had demonstrated the practicability of this type of test. The wall thickness of the 4.95-in. OD tubes was 0.102 in., giving a hoop stress of 9220 lb per sq in. A comparison of the service results obtained in some 4200 hr at 925 F for the two heats of steel and the laboratory results for the same materials is given in Fig. 23. Photomicrographs of the two heats of C-Mo steel, the heat-treatment, and chemical analyses also are included in that figure.

The working stress used in these tests was only a few per cent higher than the allowable stress proposed for C-Mo pipe at

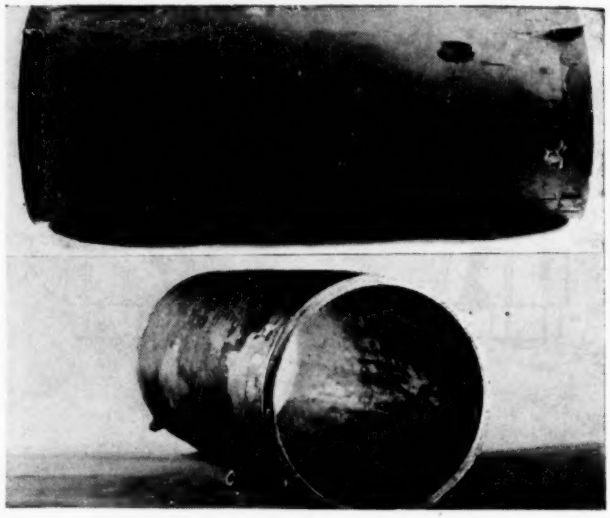


FIG. 22 INTERNAL AND EXTERNAL VIEWS OF SECTION OF 0.140-IN. CR-MO-SI TUBING AFTER 8244-HR SERVICE

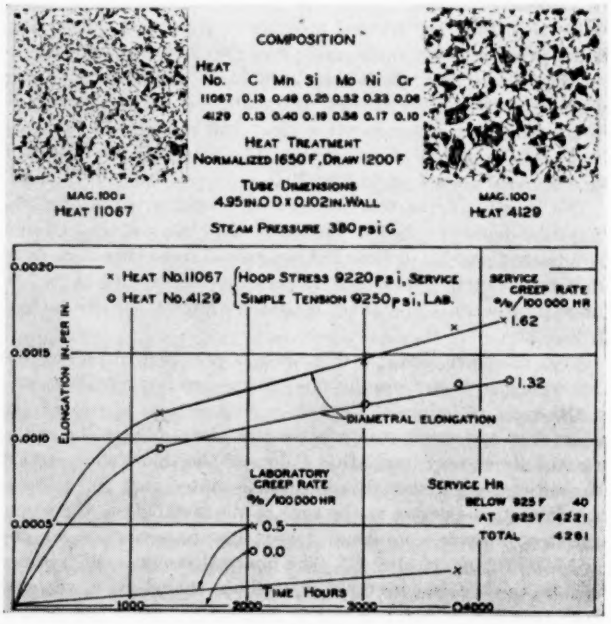


FIG. 23 COMPARISON OF LABORATORY- AND SERVICE-TIME ELONGATION CURVES CARBON-MOLYBDENUM PIPE AT 925 F
(Magnification reduced to ×47.)

925 F.[6] These data show considerably greater elongations than the results of laboratory tests in simple tension and much higher rates of creep, although the creep resistances of the two heats of C-Mo bear somewhat the same relationship to one another as in the laboratory tests. Present indications are that oxidation of the C-Mo test sections will not affect the expansion of the tubes to an appreciable extent at 925 F. The inside and outside surfaces are covered with a fine reddish-brown oxide with no evidence of scaling.

In drawing the curves of Fig. 23, the 3800-hr test points which were measured with the pipe at 138 F were disregarded. It was felt that the correction for temperature of measurement made for this point may have been in error. All measurements were corrected to 80 F, but in most cases the corrections involved less than 10-deg temperature differences.

Efforts have been made to check the effect of axial creep resulting from longitudinal pressure and bending stress in these tests. Since the means provided for measuring changes in length of the Cr-Mo-Si tubes at 1100 F had proved unsatisfactory, special precautions were taken in establishing longitudinal measuring stations on the C-Mo sections. The experimental high-temperature line in which these thin-walled sections were installed is disposed in a horizontal plane, and bending moments existing in the unusually flexible line act principally in this plane. The measuring points were located on the thin-walled sections on the inside and outside of the loop which comprised the high-temperature line.

Measurements between points on the outside of the loop showed increases in length of 0.003 in. per in. and 0.0015 in. per in., respectively, for the tubes from heat No. 11067 and heat No. 4129 after 4201 hr at 925 F. Measurements on the inside of the loop showed a decrease of 0.001 in. per in. for heat No. 11067 and an increase of 0.0005 in. per in. for heat No. 4129. Since these elongations are of the same magnitude as the diametral increases given in Fig. 23, it is apparent that despite the guard sleeve and unusual flexibility of the line, bending moments affected the longitudinal creep of the tubes. Removal of the guard sleeve from the section of Cr-Mo-Si tube with 0.250-in. wall at the end of the 8000-hr period, however, caused only a negligible change in the rate of diametral increase. The increase during the last 3000 hr was at the rate of 3.26 per cent in 100,000 hr as compared with 3.15 per cent for the 2000-hr period between 5000 and 7000 hr. This would seem to indicate that the guard sleeve had little effect on the bending moments acting on the thin-walled section, or that the bending moments had little effect on the diametral changes.

The authors wish to emphasize that the creep measurements made on thin-walled sections of pipe under service conditions cannot be considered in the same manner as refined laboratory determinations. Fluctuations in temperature, change in stress, corrosion of surfaces, and intermittent operation are factors which affect the total elongations obtained with operating equipment. The measurements under service conditions, however, do have a certain finality, and serve to bridge the gap between laboratory findings and actual plant use.

Trenton Channel Metallurgical Examination. The collection of further service data on materials at 1100 F has been confined to three materials. A section of $5^1/_2$-in. 5 per cent Cr-W tubing was removed after 18,150 hours' service, an integral test bar was removed from a 5 per cent Cr-Mo valve after 15,010 hr, and a low-carbon, 18 per cent Cr, 8 per cent Ni superheater tube was examined after 28,162 hr of service. Hardness values on stellite valve trim for different periods of service also are included.

The results obtained on the 5 per cent Cr-W tubing are summarized in Table 8.

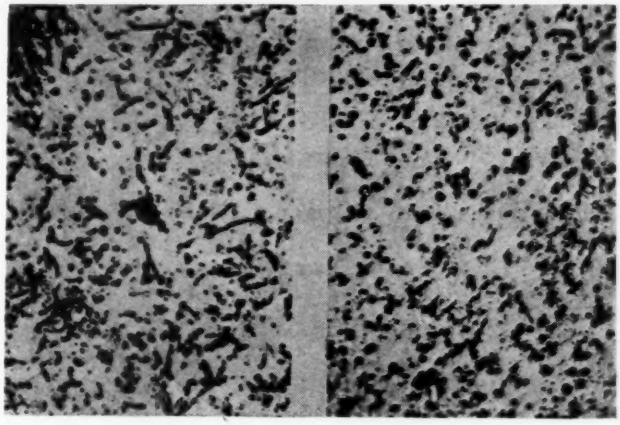
TABLE 8 FIVE PER CENT CR-W TUBING IN SERVICE FOR 18,150 HR AT 1100 F

(Composition, per cent: C, 0.11; Cr, 5.6; W, 0.94; Mn, 0.25; Si, 0.30)

Condition	Yield point, lb per sq in.	Ultimate strength, lb per sq in.	Elongation in 2 in., per cent	Reduction of area, per cent	Izod impact, ft-lb	Hardness Rockwell B
Unused...	42,800	77,800	34	71	39	80
Used.....	41,000	74,900	33	71	30	77

The structure of the material before and after service exhibited practically no change as shown in Fig. 24. The number of precipitated carbides was nearly the same, and the grain size remained the same. Apart from some softening of the material, the only noticeable change was a decrease in the impact strength.

Measurements made on welded stainless-steel points showed creep rates of 1.2 and 2.8 per cent per 100,000 hr for the horizontal and vertical diameters, respectively, during the last 9500 hours' operation of the pipe. The working stress was approximately 2500 lb per sq in.



(*a*) Before service (*b*) After service

FIG. 24 $5^1/_2 \times {}^7/_{16}$-IN. WALL TUBING, × 1000
(5 Cr, 1 W. Service for 18,150 hr. Electrolytic etch.)

Table 9 contains the data that were obtained from an examination of the 5 per cent Cr-Mo valve material. These results were secured from the examination of two cast-test bars, one of which was removed prior to placing the valve in service.

An examination of the test results given in this table and the photomicrographs shown in Fig. 25 indicate that the material

TABLE 9 FIVE PER CENT CR-MO VALVE CASTING IN SERVICE FOR 15,010 HR AT 1100 F

(Composition, per cent: C, 0.27; Cr, 5.23; Mo, 0.65; Mn, 0.59; Si, 0.55)

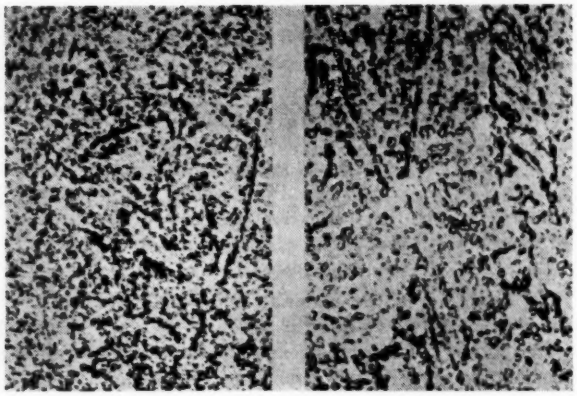
Condition	Yield point, lb per sq in.	Ultimate strength, lb per sq in.	Elongation in 2 in., per cent	Reduction of area, per cent	Izod impact, ft-lb	Hardness, Rockwell B
Unused...	68,700	101,500	22.7	53.7	39.5	97
Used.....	61,300	95,200	25.8	55.3	13.4	94

TABLE 10 18-8 SUPERHEATER TUBES

Specimen no.	1	2
Ultimate strength, lb per sq in.	113,000[a]	115,300[a]
Elongation, per cent	38.5	37
Reduction of area, per cent	57	58
Hardness, Rockwell B	94	90
Izod impact, ft-lb	22[b]	29[b]
Carbon content, per cent	0.05	0.07
Hr in service at 1100 F	15,159	28,162

[a] $^1/_4$-in. A.S.T.M. tensile specimen, $^{43}/_{64}$ in. gage length.
[b] Izod specimens 0.394 × 0.215 in.

[6] An allowable working stress of 9000 lb per sq in. at 925 F was proposed for carbon-molybdenum pipe by the A.S.M.E. Boiler Code Committee, *Mechanical Engineering*, vol. 60, no. 2, February, 1938, pp. 170–171.



(*a*) Before service (*b*) After service

FIG. 25 VALVE CASTING, ×1000
(5 Cr-Mo. Service for 15,010 hr. Nital etch.)

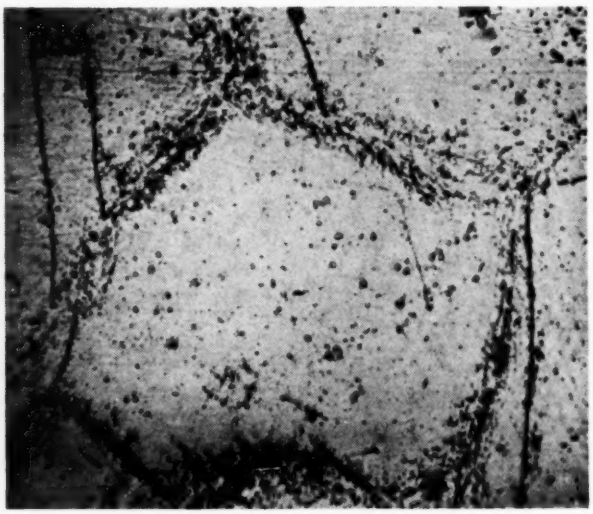


FIG. 26 SUPERHEATER TUBE, ×1000
(Low C, 18 Cr, 8 Ni. Service at 1100 F for 28,162 hr. Electrolytic etch.)

underwent a slight annealing and grain refinement. The impact-test results, however, show a pronounced reduction in resistance of a notched section to shock. The numbers of specimens available for each test were one tensile bar and six impact specimens.

The results of tests on the 18 per cent Cr, 8 per cent Ni superheater tube are given in Table 10 together with information on an adjacent tube which was removed and examined in 1933 after 15,159 hours' service at 1100 F. The latter tube is designated as specimen No. 1.

The crystal structure shown in Fig. 26 is of specimen No. 2 and is practically identical with No. 1, not shown. These results point to the interesting conclusion that the material has stopped deteriorating and that further service may even result in a continued improvement in the physical properties and, in particular, the impact value. A change in electrical resistivity of the samples after boiling in Strauss solution showed an increase of 0.9 per cent for No. 1, while the increase was only 0.6 per cent for specimen No. 2. This was a further proof of a more stable condition.

The 5 per cent Cr-Mo valve of which results were given in Table 9 was equipped with two different grades of stellite for seat and disk trim. One side of the gate and the corresponding seat ring were finished with stellite No. 1, while the other half of the valve was trimmed with the No. 6 grade. The hardness of the plug faces has been secured at different time intervals and is given in the following tabulation together with those obtained before the valve was placed in service:

	Brinell hardness	
	No. 1 stellite	No. 6 stellite
Specified by manufacturer	512	402
Before service	387–421	373–398
After 7145 hr of service	473–510	466–478
After 15,033 hr of service	472–509	427–460

Long exposure at 1100 F has had the effect of a hardening heat-treatment. The variation in hardness was probably caused by dilution of the stellite with the base metal when it was applied. The No. 6 side was in excellent shape and drop tight when the valve was subjected to a 600-lb hydrostatic test, whereas the No. 1 facing on the plug had several large cracks and a number of hairline cracks, and at one of the larger cracks, the stellite had erupted enough to prevent that side of the valve from seating. Neither of the body rings was cracked, and no corrosion had occurred on either grade of stellite.

CONCLUDING COMMENT

With the use of high-temperature steam approaching nearer to the 1000-F limit used in the Delray turbine, it is reassuring to realize that in over 4 years of actual operation, this unit and the materials of construction withstood the service conditions rather well. With the recent advances in knowledge of the high-temperature characteristics of alloys now available, and the advances made in the construction of power-plant equipment, the design and operation of a 1000-F plant is a practical thing if the savings to be gained are justified economically.

While considerable information on steam corrosion has been presented in this paper, much more is necessary before the effects of corrosion on the life of high-temperature apparatus can be evaluated. The service-creep-test data presented indicate that a better correlation between laboratory and field conditions also is necessary.

The authors wish to acknowledge the cooperation of The Timken Steel & Tube Company in the conduct of the steam-corrosion work and the creep testing of the Cr-Mo-Si tubing. Acknowledgment also is made of steam-corrosion samples supplied by the General Electric Company, The Lunkenheimer Company, and The National Tube Company.

Discussion

HANS DAHLSTRAND.[7] It is interesting to note that there was practically no difficulty in the steam turbine, which is the subject of this paper, that could be attributed to high temperature. The bucket and shrouding assembly has been trouble-free, indicating that the mechanical stresses from vibration, impact, and other forces were kept within safe limits for the materials used in their construction. The indentations shown on nozzle walls and blading indicate the importance of having the whole system, including boiler and piping, thoroughly cleaned before the plant is put into operation. While this experiment conclusively indicates that temperatures up to 1000 F are practical and may, with proper design and the correct choice of materials, be adopted for steam turbines of large capacities, nevertheless, the fact should be kept in mind that the experiment was conducted under constant conditions without any appreciable changes in temperature during

[7] Engineer in Charge, Steam Turbine Department, Allis-Chalmers Manufacturing Co., Inc., Milwaukee, Wis. Mem. A.S.M.E.

operation. It is known that the coefficient of expansion in steels is greatly increased with higher temperatures and at the same time the ductility of many of these steels is greatly decreased. Consequently the same sudden changes in temperature, when operating at high temperature, will undoubtedly be more destructive than when these same changes take place during the operation at lower temperature. It is important to realize this condition when designing power plants for high temperatures. Constant loads on the steam-turbine units cannot in all cases be maintained, therefore, the main problem will be a boiler and superheater design which will maintain constant temperature over the entire range of operating loads. Practical experience indicates the importance of maintaining constant temperature in order to realize maximum efficiency.

The experience with steam pipes in this experiment, as well as in power-plant installations, has indicated the necessity of eliminating all bolted flanges whenever possible, not only in pipes but also in all parts exposed to high temperatures.

The structural changes in many of the materials indicate that it must be known beforehand whether they are suitable for high-temperature service covering the normal life of operation of the equipment. The information given will in this respect prove valuable. It is undoubtedly true that many of these materials have been improved in later years making them more suitable for high-temperature service. This is particularly true in the case of some of the austenitic steels where stabilizing elements are used to eliminate carbide precipitations.

The steam-corrosion experiments are of special interest. Up to this time very little information has been available covering the corrosion effect of high-temperature steam on various types of steel. Some difficulties have been experienced even at lower temperatures that have been attributed to corrosion. It is realized that the effect from corrosion is influenced by several factors and therefore difficult to analyze without extensive and laborious experiments. These factors are the surrounding atmosphere, temperature, and stress conditions, each of which will have a different effect on the various types of steels. In modern power plants, the condition of the steam due to the variety of feedwater treatments and the state of the make-up water may have a different effect on different materials. This effect may be different through the expansion range caused by chemical changes in feedwater treatment compounds. It is therefore of some importance to know what this effect will be with different conditions of the steam. Differences in temperatures will naturally produce varying effects. The effect of stresses must be known, as the corrosion is likely to be more serious at higher stresses, particularly if the stresses are of such magnitude as to cause corrosion through the grain boundaries of the material.

The first corrosive covering as mentioned appears to form a protective coating which apparently prevents further corrosion quite effectively. This effect has been noted and in some cases has been utilized by giving the material a corroded surface deliberately. There is a possibility that this method of protection may be used in making materials more resistant to corrosion in the operating atmosphere by producing the protective coating in a different atmosphere.

F. O. ELLENWOOD.[8] It is gratifying to note that for all of the steels, except the straight-carbon one, the initial scale formation affords considerable protection against further steam attack. This fact, coupled with the success obtained by welding the pipe joints, should give considerable encouragement to those engineers who are now planning to use steam of a much higher temperature than they have heretofore considered feasible. The authors point out that the high compressive stresses in the bolted joints with metal gaskets cause leakage by plastic flow and that the only remedy is periodic maintenance if such joints are used. They have left to the imagination of the reader what that means in a large plant in which all of the steam mains are to be kept tight when at a temperature sufficiently high to cause the pipes to be a dull red. This still further emphasizes the importance of the welded pipe line for high temperatures.

The concluding comment states, ". . . the design and operation of a 1000-F plant is a practical thing if the savings to be gained are justified economically." This sums up the situation concisely for the use of the designing engineer, because he can usually estimate fairly well his annual capacity factor, fuel cost of energy, and the first cost of the equipment; hence, he can now design with confidence, based on the Detroit experiments, his high-temperature plant wherever he finds that his annual use factor will justify the additional cost due to the high temperature of the steam.

H. J. KERR.[9] The two high-temperature steam installations in Detroit may well in the future be regarded as the milestone in history when steam at a temperature of 1000 to 1100 F was first utilized in this country for power generation. The data in the paper give further evidence of the advancement in the manufacture of steam-generating and steam-utilizing equipment. The paper also shows clearly the progress which has been made in alloy steels.

The large steam-generating installations of the last few years show that advantage has already been taken of the experience gained from these Detroit units, as practically all of the high-pressure installations have been for steam temperatures from 900 to 950 F. None of the materials used in building these installations has actually failed and a great majority of the materials used have been found quite satisfactory for the purpose. Undoubtedly, this result would not have been obtained had it not been for the experience in the oil refineries at high temperature with alloy steels.

With high-temperature steam the oxidation problem is quite different from that encountered in the oil industry and the data on this phase of the problem in the paper will need careful study. Some of the points brought out which might be emphasized for careful consideration are:

1 The comparison of steam and air oxidation.

2 The type of oxide scale found in the case of steam oxidation and the effect of variation of temperature and stress.

3 The fact that intercrystalline oxidation effects, in the presence of steam, are materially less than in the case of air oxidation.

4 Oxidation of 0.3 carbon steel less than low-carbon steel.

5 The relatively greater resistance to oxidation (obtained by small percentages of alloys) in steam than in the case of air oxidation.

The paper gives certain data on the creep of tubular specimens in comparison with tension-creep specimens. Much of these data are at variance with those of other investigators. In other words, it is generally believed that the diametrical creep of a normally stressed cylinder will be less than that obtained with a normal tension-creep test, at least above the equicohesive temperature.

The 1100-F superheater at Trenton Channel is still in operation. In many ways it is an ideal apparatus for high-temperature testing. It is hoped the authors may continue to add to the fund of knowledge from its further use.

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[9] Executive Assistant, The Babcock & Wilcox Company, New York, N. Y. Mem. A.S.M.E.

Further data are required on three phases of the problem: (*a*) Oxidation effects, (*b*) creep of a cylinder under simple pressure and in the case of bending stresses, and (*c*) the total creep permissible on tubular members with a variation in time.

C. H. McCollam[10] and C. L. Clark.[11] The authors have added further evidence of the apparent difference in the mechanism of oxidation of ferrous materials at elevated temperatures when exposed to air and to steam. Both the type of oxidation products formed and the relative merits of the several materials tested are different in the two media. In other words, alloy additions which impart high resistance in one medium may not have the same relative merit in another atmosphere.

In order to eliminate any misunderstanding in regard to the apparent discrepancy in the originally assumed creep characteristics of the Cr-Mo-Si tubing at 1100 F and those actually obtained from both service and laboratory tests, it should be further emphasized that the composition of the steel used in the service tests differed from the steel on which the original creep characteristics, used for the basis of the authors' design stress, were obtained.

The comparison of the service and laboratory creep tests is of considerable interest. The two types of tests at 1100 F are in good agreement, while at 925 F the service tests revealed a considerably greater rate of creep than did the laboratory tests. From this it might follow that the relationship between creep in tubular sections and that in straight tension varies, depending on the relative degree of plasticity and strain hardening. For example, the Cr-Mo-Si steel at 1100 F is distinctly plastic in its behavior and the two types of test are in good agreement. On the other hand, the 0.50-Mo steel at 925 F is capable of pronounced strain hardening and, under this condition, the two types of test are not in agreement. The difference in the results at 925 F may indicate that strain hardening can occur more rapidly and to a greater extent under simple tension than under the combined stresses present in a tube under internal pressure.

E. L. Robinson.[12] Actually, this paper constitutes two separate reports; one covering the performance of the Delray turbine; and the other, the Trenton-Channel piping. Both reports contain a great many briefly stated facts of experience and there are no better precepts for guidance than such facts.

There are many hopeful conclusions: Creep distortions in the turbine were negligible. The satisfactory performance of the welded joints in the piping lead the authors to recommend this type of construction. Such scale as was formed by the steam proved to be protective in nature and more tenacious than oxides formed in the air. Scaling did not appear to be increased by the presence of such stresses as are considered allowable in good design. All in all, the authors conclude that it is quite feasible from the point of view of strength to operate at 1000 F.

It is important to note that the materials used in the Delray turbine were not the same as the materials most generally used in the United States. The turbine was built in England and except for the replacement throttle valve which was a carbon-molybdenum casting, all the materials differed in some significant respect. While the replacement throttle was made in England, it was made to the purchaser's specification and is probably reasonably representative of an American material. While, in general, the turbine materials gave a good account of themselves, American materials now available are believed to be even better.

The authors note that the enlargement of diameter in the piping, instead of being less than occurred in the tensile specimens, was fully as great or even greater. The writer would like to point out that this extra creep was not more than might occur between different tensile specimens in a laboratory. The authors express too much respect for the precision of laboratory creep tests. In order to be sure that laboratory creep tests will yield comparative results, it is necessary to cut the various samples from the same bar of material and also to examine the various samples with the microscope to see that they do not vary from place to place in the bar.

The authors have clearly pointed out that it is too much to expect that piping will enlarge on the diameter less than an ordinary tensile specimen. However, if designers have already made suitable allowance for the possible variation in creep strength of materials as ordinarily used, it does not seem necessary to discount these results any further, although it may be wise not to take credit for the theoretical 15 per cent advantage in strength which is supposed to go with the biaxial loading. These results are worthy of study in comparison with the creep tests on tubular members recently completed under the sponsorship of the A.S.M.E.–A.S.T.M. Joint Research Committee on the Effect of Temperature on the Properties of Metals.

G. B. Warren.[13] There is a question as to how generally the public appreciates its indebtedness to the utility companies and to the power-using industries for the remarkable progress which has been made over the last forty years in reducing the fuel cost of power. It is the writer's opinion that this remarkable reduction has only been accomplished by the willingness of first one and then another public-utility organization or industry to advance into regions in which, at the time, there are inadequate engineering data upon which to base such a step with perfect assurance and without risk. Progress has been made as a result of the sum of all of these individual steps.

The point which most forcibly impresses the writer, as a designer of equipment, much of which is now in service at high temperature, is that on the whole the results cited bear out the practice which has crystallized during the last few years as proper for high-temperature power plants. In other words, no failures as such have occurred even at temperatures well above those now being used in commercial practice. Particularly in the Trenton-Channel tests, the materials which are giving satisfactory and in some cases superior service are in general basically the materials which are accepted as standard practice today. For example, the use of nitralloy bushings for valves apparently has been satisfactory to an outstanding degree. The stellite-faced valves have given promise of satisfactory service. The 12 per cent chrome bucket material, now more or less standard for turbine buckets has almost the lowest rate of corrosion.

The experience with chrome-molybdenum-silicon piping has also been reassuring. It is believed the results can be interpreted as confirming other indications that the stresses, temperatures, and even improved materials, which are now being installed and used on recent high-temperature installations, are well on the side of safety.

A. E. White.[14] The corrosion resistance of most metals when at high temperatures is usually determined in air. The results of the corrosion findings reported in this paper are in an

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[12] Turbine Engineering Department, General Electric Company, Schenectady, N. Y. Mem. A.S.M.E.

[13] Designing Engineer, General Electric Company, Schenectady, N. Y. Mem. A.S.M.E.

[14] Professor of Metallurgical Engineering, Director, Department of Engineering Research, University of Michigan, Ann Arbor, Mich. Mem. A.S.M.E.

atmosphere of steam. The results are most interesting and valuable.

When high-temperature corrosion or oxidation tests have been made in air, intercrystalline cracks have been found in some steels, if the tests were carried out for a sufficient period of time. These cracks, however, have been found only when the sample was in a state of stress.

The work on the samples reported in the paper was done on specimens in an unstressed condition. What would have happened had they been in a highly stressed state? Would intercrystalline cracks have been found, especially if the time periods were of the order of 20,000 hr or more?

These are vital questions for power-plant operation, as most, if not all, of the units in a power plant should have a life of at least 100,000 hr for successful performance. It is understood that tests are contemplated or are under way which will throw light on the stress factor.

Authors' Closure

Mr. Dahlstrand has summarized very well in his discussion the various factors that enter into the question of resistance to steam corrosion. It is only when considerably more data are available that engineers will be able intelligently to design for the high temperatures that are now being encountered in portions of recent superheaters.

An example would serve best to illustrate this point. Assume that a 5 Cr-0.5 Mo tube, 1.75 in. outside diameter and 0.160 in. wall thickness, is considered for use in a superheater at 850 lb per sq in. and that the operating temperature is near 1100 F. The nominal bursting stress using Barlow's formula is 4650 lb per sq in. If a proper allowance for steam corrosion is made, so that the stress is not appreciably greater at the end of 10 years than that originally selected, an average corrosion factor is necessary. An allowance, based on the weight loss[15] observed after 1000 hr, would require 0.089 in. added to the original wall thickness, whereas the rate of metal loss observed on a sample removed from the Trenton-Channel line after 11,400 hr would indicate that 0.030 in. would be sufficient to allow for corrosion. Air-corrosion loss and the effect of stress on corrosion have been neglected in the foregoing discussion.

At the present time, tests have been started in the 1100-F line to determine the effect of stress on the scale formation.

The authors are not certain that a general statement can be made to the effect that the low alloys exhibit a greater relative resistance to oxidation and corrosion in steam than they do in air, as mentioned by Mr. Kerr. To their knowledge no air-corrosion data are available from long-time tests to compare with results given in the paper. Where additions of silicon over 1 per cent have been used in some of the low alloys, a comparison of extrapolated values of weight loss for 1000 hr exposure to steam shows a greater loss than corresponding values reported from air tests.

Mention was made in the paper of nominal variations in steam temperature at Trenton Channel of plus or minus 20 F from 1100 F. These limits have been exceeded since oil was replaced with refinery gas as a fuel for the superheater. An examination of recorded temperature for the past 3 years showed a weighed-average steam temperature of 1094 F with a variation as follows:

Temperature variation, F	Per cent of service, hr
1080–1120	64.5
1060–1140	81.3
1000–1140	96.5

The installation of a temperature controller to compensate for the sudden wide variation in heating value of the gas has been considered. One has not been installed, however, as the present method of operation more nearly duplicates plant conditions, although such wide fluctuations perhaps would not be tolerated in a regular plant installation.

[15] Unpublished data received from A. A. Potter, H. L. Solberg, and G. A. Hawkins.

The Vectorscope

By GEORGE J. DASHEFSKY,[1] NEW YORK, N. Y.

This paper describes a new instrument capable of rapidly summing up a number of vectors. The machine consists of a disk, mounted in gimbals in the manner of a nautical compass, on which are placed weights representing the vectors to be added. A spot of light on the screen indicates directly the resultant vector and its direction, as well as its horizontal and vertical components.

The mathematical principle is developed and the instrument is shown to be suitable for studying crankshaft arrangements with respect to unbalanced moments and torsional vibration. The suitability of any firing order for any crankshaft configuration may be ascertained simply and quickly.

It is shown that the instrument is inherently adapted to making harmonic analyses of periodic functions and the theory of this application is discussed. Results by this method of analysis compare favorably with other methods.

THE INSTRUMENT described in this paper was devised and built by the author early in 1936 while engaged on an engine problem which required the study of a considerable number of firing orders, particularly as they affected the characteristics of engine balance and torsional vibration. Shortly afterward in the same year, the instrument was rebuilt at the New York Navy Yard into the form shown in Fig. 1. In 1937, Prof. K. J. DeJuhasz kindly called the author's attention to a contemporary instrument[2] somewhat similar in function and used in studies of color.

Our instrument was dubbed "Vectorscope" because of its designed function of effecting rapid addition of a number of vectors. Analyses of both engine balance and torsional vibration involve vector addition and the instrument is therefore well adapted to the solution of these problems. The harmonic analysis of periodic functions may also be reduced to a process of vector addition, and the Vectorscope therefore lends itself nicely to solution of this problem. Such uses of the instrument appear to be of greatest interest to the fraternity of engine designers and the present paper is therefore limited to a discussion of these problems and the manner of effecting their solution with the Vectorscope.

Description of Instrument

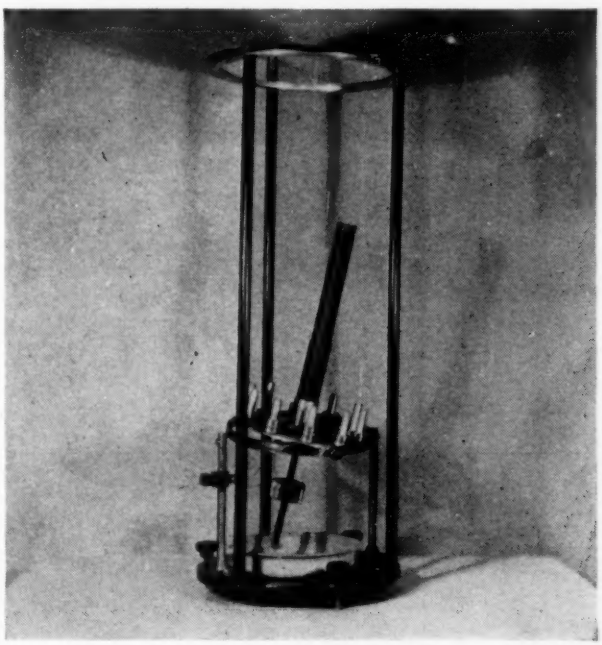
The mechanical arrangement of the instrument as now constructed is shown in Fig. 1. One of the several interchangeable disks (1) is carried by member (2), which is mounted in gimbals in the manner of a nautical compass. Member (2) is free to rotate in a pair of pivot bearings (3) carried on the ring (4). The ring (4) in turn is also mounted in pivot bearings (5) permitting rotation about an axis at right angles to the axis of rotation of the disk. The pivots are arranged so that the mutually perpendicular axes of rotation pass through the center of the disk. The ring (4) is supported on a base which may be leveled off by means of screws (6). Member (2) carries a light-projection system (7) comprising a flashlight lamp with a filament of the concentrated type. Current is fed to the lamp by very flexible leads passing through holes in (2). An image of the lamp filament is focused on a translucent screen (8), graduated in polar coordinates. The graduated screen, sandwiched between two plates of glass may readily be replaced with other screens graduated in any required manner to permit direct readings in units pertinent to the problem in hand. Deflections of the disk (1) are measured by the displacement of the light spot from the center of the screen, indicating both magnitude and direction.

A threaded rod (9), forming the lower part of (2) carries a movable weight (10), by means of which the instrument sensitivity and scale reading are controlled.

The disks (1) are provided with pegs (11) with appropriate angular spacing. Weight holders (12) carrying preweighed groups of washers (13) of 1 unit, 0.1 unit, and 0.01 unit may be shifted from peg to peg as requisite. The weights (13) representing the vector magnitudes are thus adjustable by a simple counting process rather than by weighing.

The oscillations of the disk in its gimbals are damped out by vanes (14) carried at the extremity of rod (9) and moving in a dish of mercury. The time required for the disk to come to rest is thus greatly reduced, and the instrument is rendered deadbeat.

It will be noted in Fig. 1 that the disk (1) is free to rotate about each of the mutually perpendicular axes. It may of course



THE VECTORSCOPE

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[2] "Die Scheibenwaage," by W. Ehrenberg, *Zeitschrift für Instrumentenkunde*, vol. 57, Feb., 1937, pp. 77–79.

Contributed by the Oil and Gas Power Division and presented at the Annual Meeting of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS, New York, N. Y., December 5–9, 1938.

Discussion of this paper was closed January 10, 1939, and is published herewith directly following the paper.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

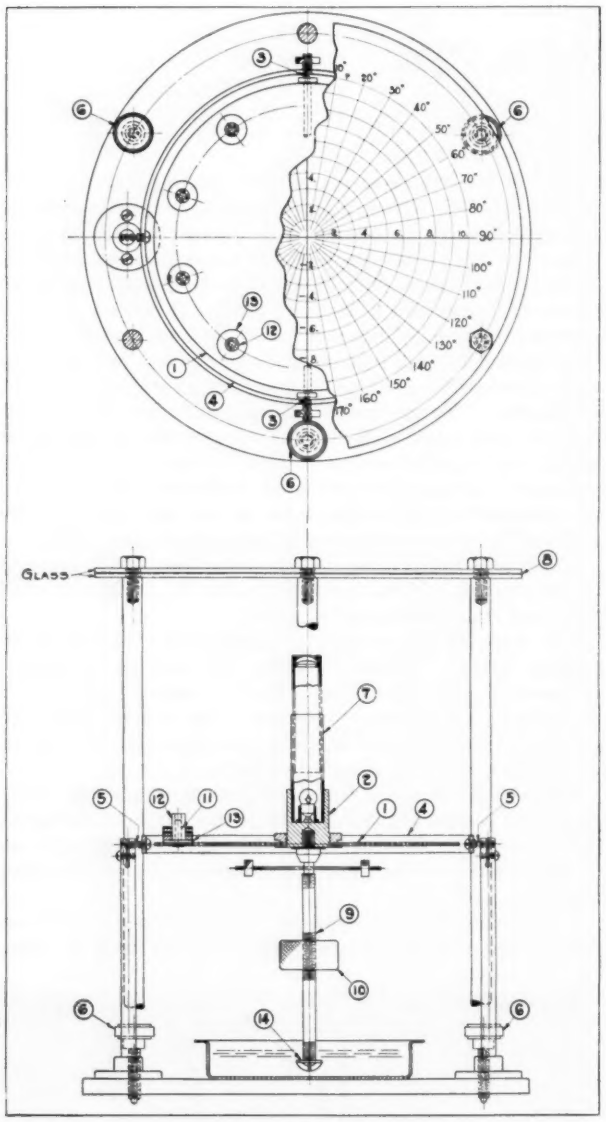


FIG. 1 THE VECTORSCOPE

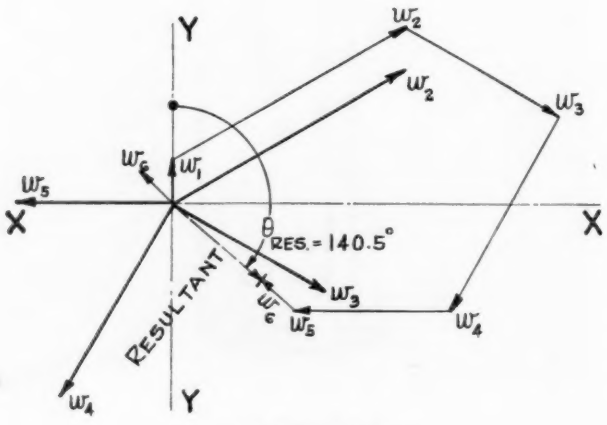


FIG. 2 A VECTOR POLYGON

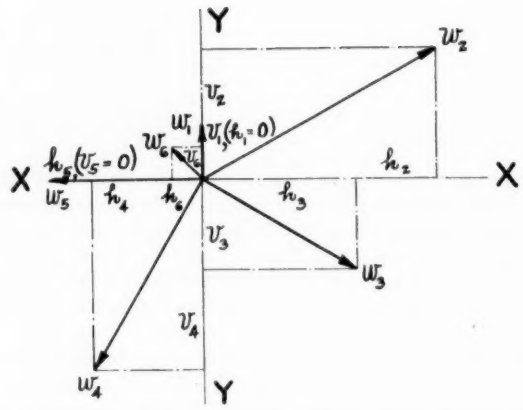


FIG. 3 VECTORS RESOLVED INTO COORDINATE COMPONENTS

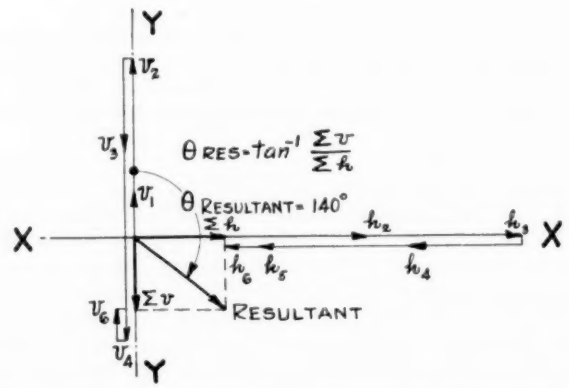


FIG. 4 RESULTANT VECTOR FROM COORDINATE COMPONENTS OF FIG. 3

rotate simultaneously about both axes. Any tilting action of the disk due to weights (13) is balanced by the weight (10), which assumes a deflected position about the two axes of rotation, in accordance with the magnitudes of weights and their disposition on the disk. Actually the weight of the light-projection system, member (2), and threaded rod must also be considered. However, there is a net weight W which represents the combined effect of all of these.

ADDITION OF VECTORS

Fig. 2 represents six vectors w_1 to w_6, inclusive, having directions as indicated. Two familiar methods for finding the sum or resultant of these vectors are shown. In Fig. 2, the resultant is obtained by closing the vector polygon in the usual manner.

A second method is shown in Figs. 3 and 4. Each vector is resolved into vertical and horizontal components, which are then added separately as shown in Fig. 4, giving the net components, Σh and Σv. The magnitude of the resultant is readily obtained by a composition of Σh and Σv.

The following example involving the addition of the vectors shown in Figs. 2, 3, and 4 will demonstrate the mathematical principle of the Vectorscope. Refer to Fig. 5 showing the Vectorscope in elemental form. Each vector is represented by a weight placed on the disk at a common radius r. These weights have the same relative values as the original vector magnitudes, while the direction of the vectors is cared for by disposing the weights on the disk in the same angular relationship as the original vectors. The weights so disposed result in a net moment about axes X-X and Y-Y, producing rotation about these mu-

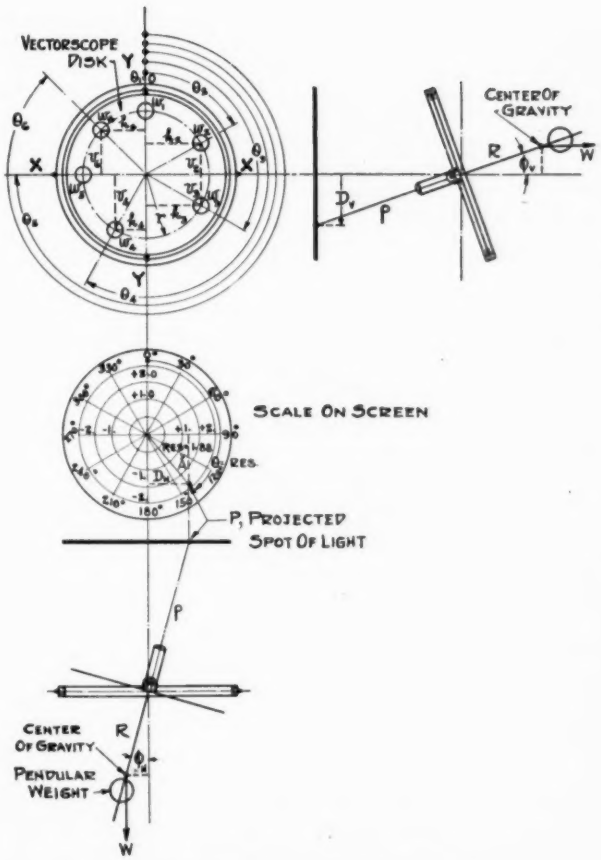


FIG. 5 ELEMENTAL FORM OF VECTORSCOPE

tually perpendicular axes, displacing the net pendular weight W from its previously plumb position. The net moments of weights w_1 to w_6 about the two axes are balanced by the moment of the displaced weight W. For this condition of equilibrium, about Y-Y

$$WR \sin \phi_H = w_1 r \sin \theta_1 + w_2 r \sin \theta_2 + w_3 r \sin \theta_3 + w_4 r \sin \theta_4 + w_5 r \sin \theta_5 + w_6 r \sin \theta_6$$

or, in general

$$WR \sin \phi_H = r\Sigma(w \sin \theta) \quad [1]$$

About X-X

$$WR \sin \phi_V = w_1 r \cos \theta_1 + w_2 r \cos \theta_2 + w_3 r \cos \theta_3 + w_4 r \cos \theta_4 + w_5 r \cos \theta_5 + w_6 r \cos \theta_6$$

or, in general

$$WR \sin \phi_V = r\Sigma(w \cos \theta) \quad [2]$$

The optical system attached to the Vectorscope disk projects a spot of light on the graduated screen. The displacements of this point of light from the zero position are

$$D_H = \rho \sin \phi_H \quad [3]$$

$$D_V = \rho \sin \phi_V \quad [4]$$

Combining Equations [1] and [2] with [3] and [4]

$$D_H = \frac{\rho r}{WR} \Sigma(w \sin \theta) \quad [5]$$

$$D_V = \frac{\rho r}{WR} \Sigma(w \cos \theta) \quad [6]$$

For small values of ϕ_H and ϕ_V, ρ may be considered constant. For deflections of 5 deg the error introduced is 1 — (1/cos 5 deg) or about 0.38 per cent. D_V and D_H are therefore measures of the net vertical and horizontal components of the original vectors.

Corresponding to displacements D_V and D_H the point of light on the screen assumes the position shown in Fig. 5. The radius to point P, therefore, represents the resultant vector, while D_V and D_H are the respective vertical and horizontal components. The direction of the resultant is also indicated directly on the calibrated screen.

Examination of Equations [5] and [6] shows that the scale or sensitivity of the instrument may be controlled by varying WR. Moving the pendular weight toward the disk, decreases R and magnifies the readings. This offers a convenient means for calibrating the scale in terms of any one of the vectors. One of the weights, say w_1, is placed on the disk. See Fig. 5. This weight has a moment about X-X, but not about Y-Y. The vertical position of the pendular weight is now adjusted until the spot of light reaches a point on the screen, which yields convenient readings in terms of the vector represented by w_1. As an example, if w_1 represents a vector of 105 units, the vertical position of the pendular weight is adjusted until a scale reading of 1.05 obtains. Subsequent readings of vector sums are therefore readily obtained by multiplying scale readings by 100.

USE OF VECTORSCOPE IN STUDY OF ENGINE BALANCE AND FIRING ORDER

The condition of engine balance is intimately associated with the choice of firing order. It is beyond the scope of this paper to discuss this problem except as it is necessary to demonstrate application of the Vectorscope to such problems.

Fig. 6 shows the crankshaft of a five-cylinder two-cycle engine. Due to the reciprocating weight and crankshaft angularity there exists at each crank an inertia force acting in the line of the piston stroke. There is no horizontal component.

The inertia force at each crank is expressed by

$$F = \frac{4\pi^2 N^2 r_c}{3600g}\left[\cos \theta + \frac{r_c}{l} \cos (2\theta) + \ldots\right] \quad [7]$$

This indicates that the inertia force is equivalent in effect to a series of harmonic forces recurring once per revolution, twice

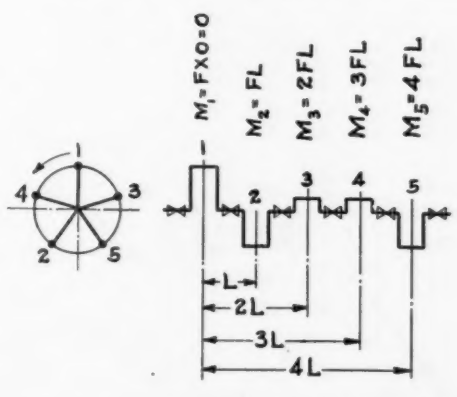


FIG. 6 CRANKSHAFT OF FIVE-CYLINDER TWO-CYCLE ENGINE (Indicating moments about cylinder No. 1 of inertia forces of reciprocating parts.)

per revolution, and so on. The first two harmonics are the most important, but the 4th, 6th, ... harmonics though progressively smaller also exist. In high-speed engines the 4th harmonic may be of importance. If the discussion is limited to the first two harmonics, only two separate forces need be considered.

$$F_{\mathrm{I}} = \left(\frac{4\pi^2 N^2 r_c}{3600g}\right)\cos\theta \quad \ldots\ldots\ldots\ldots\ldots [8]$$

$$F_{\mathrm{II}} = \left(\frac{4\pi^2 N^2 r_c}{3600g}\,\frac{r_c}{l}\right) \times \cos(2\theta) \quad \ldots\ldots\ldots\ldots [9]$$

Where

F_{I} = primary force, lb
F_{II} = secondary force, lb
N = engine speed, rpm
l = length connecting rod, inches
r_c = crank radius, inches
g = gravitational constant, inches per sec per sec
θ = angular displacement of crank from top dead center of cylinder No. 1 taken as reference position.

Take crank No. 1 as a reference point for expressing moments. Since the same force exists at each cylinder, the moment of any cylinder is proportional to its distance from the reference point. The phase relation for successively firing cylinders is 72 deg for the primary forces and moments, and 2 × 72 = 144 deg for secondary forces and moments. Assuming equal cylinder spacing the moments of cranks 2, 3, 4, and 5 are in the proportion of 1:2:3:4. The moment of crank No. 1 is zero. Vector diagrams for primary and secondary moments are shown in Figs. 7 and 8.

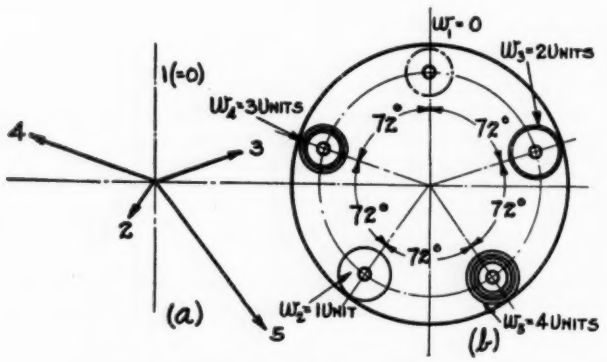


FIG. 7 PRIMARY MOMENTS OF FIG. 6
[(b) Disposition of weights on Vectorscope for evaluating the vector diagram (a).]

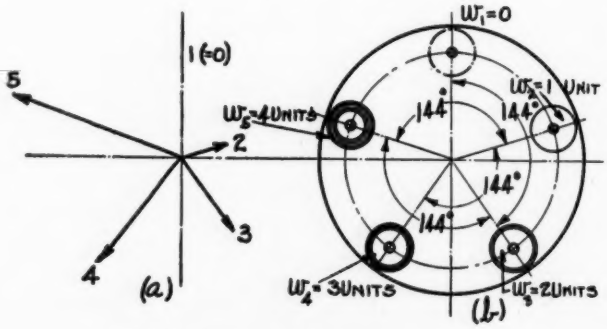


FIG. 8 SECONDARY MOMENTS OF FIG. 6
[(b) Disposition of weights on Vectorscope for evaluating the vector diagram (a).]

Consider a Vectorscope disk having 5 pegs disposed 72 deg apart, and weights in the proportion of 0:1:2:3:4 selected to represent, respectively, the moments of the cranks 1, 2, 3, 4, and 5. Each of these weight groups is marked with the cylinder number. To evaluate the vector diagrams the following procedure is followed: Temporarily place one of the weights, preferably w_2 which represents one weight unit, on the peg at $\theta = 0$, that is, the top peg shown in Fig. 7(*b*). Adjust the pendular weight (10), Fig. 1, until a convenient scale reading is obtained in terms of the primary moment

$$\left(\frac{4\pi^2 N^2 r_c}{3600g}\right) \times L$$

After the instrument is thus calibrated, place the weights on the pegs in proper phase relation with respect to firing order. There is no weight for w_1 since crank No. 1 has zero moment. For the primary moment the vectors are 72 deg (one interval on the disk) apart and the weights are disposed as shown in Fig. 7(*b*). The vector sum and the direction of the resultant are indicated directly on the screen.

To obtain the resultant secondary moment, that is, to evaluate the vector diagram of Fig. 8(*a*), readjust the scale as described for primary moments. Since the phase relation is 2 × 72 deg for secondary moments, dispose the weights, in firing sequence, on every second peg as shown in Fig. 8(*b*).

In similar manner, the 4th harmonic may be obtained by putting these same weights on every fourth peg. Higher harmonics are similarly summed up by distributing the weights by counting off each time a number of peg spaces equal to the harmonic under consideration.

Study of Firing Orders by Vectorscope. The moment of any crank is determined only by its distance from crank No. 1, and is independent of firing order. The resultant however *is* influenced by firing order. After once calibrating the Vectorscope and fixing the weights representing cylinder moments, the resultant unbalanced moments for a large number of firing orders may be quickly investigated.

For the five-cylinder two-cycle engine the number of firing orders possible is $(n - 1)!/2 = (4 \times 3 \times 2 \times 1)/2 = 12$. By systematic shifting of the weights a reading for each of the possible firing orders is quickly obtained. A similar procedure is now carried out for the secondary moments but, in reading off the firing order, successively firing cylinders are displaced by two peg spaces or 144 deg. If required, similar procedures may be carried out for the higher harmonics.

Thus far, moments due only to the inertia of reciprocating weights have been considered. These inertia forces act vertically only, having no horizontal component. Centrifugal forces due to unbalanced weights also produce moments of importance, having components in both vertical and horizontal planes of the engine. They are of primary frequency only. Higher harmonics do not exist. The response of the engine and structure in the two planes is usually different, and it is desirable and customary to determine the horizontal and vertical components, rather than the resultant unbalance. The vector diagram of moments and the procedure in the use of the Vectorscope to evaluate these moments are the same as previously described. The horizontal and vertical components may be read off directly from the screen.

A listing of Vectorscope readings permits choice of a suitable firing order. In interpreting these results, it must be borne in mind that other factors such as manifolding, bearing pressures, and torsional vibration, are also influenced by the choice of firing order.

For an engine with a large number of cylinders, the number of

TABLE 1

No. of cylinders	3	4	5	6	7	8	9	10	11	12
No. of firing orders	1	3	12	60	360	2520	20,160	181,440	1,814,400	20,958,840

possible firing orders rises rapidly, as is shown in Table 1.

From this it would appear quite feasible to completely explore the whole range of firing orders for engines up to eight and possibly even nine cylinders.

The present arrangement of the Vectorscope includes a set of disks with holes spaced as shown in Fig. 9. These disks would care for all in-line engines having from 3 to 12 cylinders. Paper masks are slipped over the disks to clearly indicate the holes to be used for a particular engine. The instrument is arranged to permit simple interchange of disks.

Disks may be arranged to suit any particular configuration of cranks. In studying the balance of the Vee engine, special disks may be made up to suit the particular configuration of cylinders. Fig. 10 represents a 12-cylinder 2-cycle Vee engine comprising two 6-cylinder banks in 90-deg Vee. Disks permitting the study of primary and secondary unbalance are shown in Fig. 11.

The Torsional-Vibration Problem

The Vectorscope is useful in solving a certain phase of the torsional-vibration problem. The input of vibration energy (inch-pounds) in a multicylinder engine is

$$K_I = (\pi^2/180) M_v \alpha (\Sigma\beta) \quad [10]$$

where α = amplitude of vibration at the free end of the crankshaft, degrees

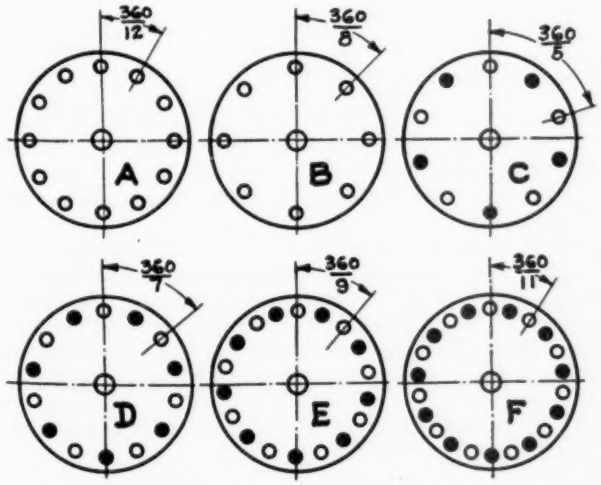
M_v = harmonic torque of cylinder for particular order of vibration under consideration, pound-inches

$\Sigma\beta$ = vector sum of the amplitudes of vibration at the respective cylinders, their relative phase being that of the successively firing cylinders.

The phase relation for a given order of vibration is

$$\phi = m\theta \quad [11]$$

where ϕ = phase angle, degrees



No Of Cylinders	3	4	5	6	7	8	9	10	11	12
Disk To Be Used	A	B	C	A	D	B	E	C	F	A

Fig. 9 Set of Vectorscope Disks

(For in-line engines of from 3 to 12 cylinders. Disks *C*, *D*, *E*, and *F* have additional holes, shown solid, diametrically opposite the basic ones. These are for negative vectors which require a shift of 180 deg.)

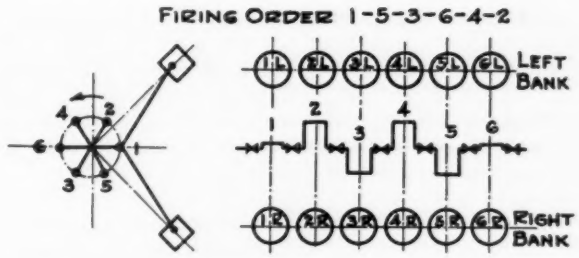


Fig. 10 A 12-Cylinder 2-Cycle 90-Deg Vee Engine

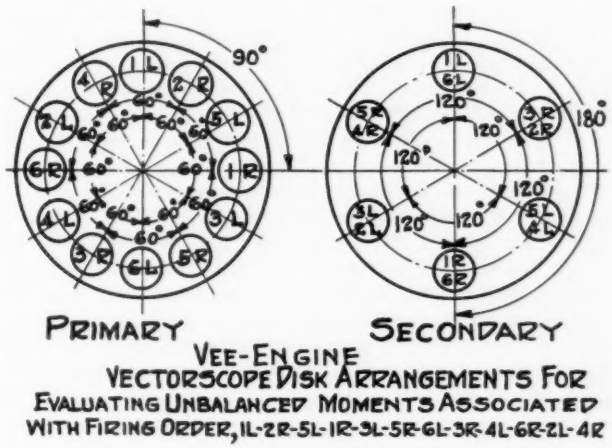


Fig. 11 Vectorscope Disks for Engine of Fig. 10

m = order of vibration (number of vibrations per revolution of engine)

θ = crank displacement between successively firing cylinders.

When in a critical speed of amplitude α, the absorption of damping energy K_D is equivalent to the energy input K_I, coming from the harmonic torques. Assume a law for damping,

$$K_D = K_d \alpha^\lambda \quad [12]$$

$$K_I = K_D$$

$$(\pi^2/180) M_v \alpha (\Sigma\beta) = K_d \alpha^\lambda$$

$$\alpha = \sqrt[\lambda-1]{\left(\frac{\pi^2 M_v (\Sigma\beta)}{180\ K_d}\right)} \quad [13]$$

It is apparent that the amplitude of vibration depends on $\Sigma\beta$. From Equation [11], as applied to the seven-cylinder engine of Fig. 12, $\theta = 720/7$ deg. When $m = 3.5$, 7, or 10.5, $\phi = 360$, 720, and 1080 deg. For these orders of vibration, known as majors, the β's add up arithmetically, since their phase relation is 360 deg or a multiple thereof. All other orders, known as minors, have a phase relation ϕ, as listed in Table 2. Groups marked with the same letter are equivalent as regards the magnitude of the resultant vector, but of different resultant phase. The $\Sigma\beta$ for major orders is independent of firing order, whereas $\Sigma\beta$ for minor orders is very much affected.

In Fig. 13 is shown a typical vector diagram for the third-

TABLE 2

	$\phi = m\theta$					
Orders of vibration, m	1/2, 6 1/2, 7 1/2, 13 1/2	1, 6, 8, 13	1 1/2, 5 1/2, 8 1/2, 12 1/2	2, 5, 9, 12	2 1/2, 4 1/2, 9 1/2, 11 1/2	3, 4, 10, 11
Phase angle[a]	1 (A)	2 (B)	3 (C)	4 (C)	5 (B)	6 (A)

[a] In terms of number of peg intervals. Each interval = 720 deg × m.

order vibration associated with the relative-amplitude curve of Fig. 12, and the firing order 1-7-2-6-3-4-5. To evaluate this vector diagram with the Vectorscope, in the now familiar manner, weights in proportion to the β values are placed on the seven-peg disk, in proper phase relation. The screen is calibrated by placing the weight representing β on the disk and adjusting the pendular weight to a position giving a suitable scale.

For each firing order, the whole range of orders may be investigated by simply shifting the weights to their appropriate angular positions. Starting with the one order, each weight, in sequence of firing, is placed on the pegs in succession, progressing in a clockwise direction. For the second order, the weights are placed on every second peg; for the third order, every third peg, and so on. Usual interest would center about the magnitude of the resultant, since the phase is unimportant, so that in the seven-cylinder engine there are only three groups of minor orders to be evaluated. The number of groups corresponding to engines with other numbers of cylinders may be found elsewhere.[3,4]

In any particular problem, the β values are fixed, and a change in firing order does not call for readjustment of the weights representing them. Thus to investigate the whole range of possible firing orders calls merely for a proper shifting of these weights. The values of $\Sigma\beta$ for a whole set of firing orders may thus be quickly and accurately determined. In the seven-cylinder engine there are 360 possible firing orders. Since there are 3 groups of minors, the complete investigation involves 1080 shifts of weights. With the aid of the Vectorscope this appears to be an entirely feasible task requiring a minimum of skill and care.

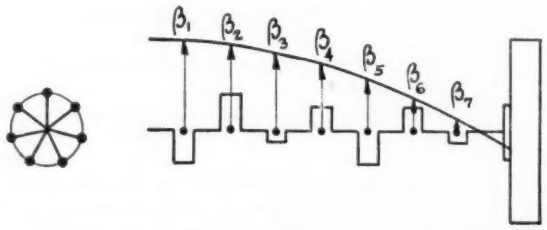


FIG. 12 TORSIONAL VIBRATION, RELATIVE-AMPLITUDE CURVE

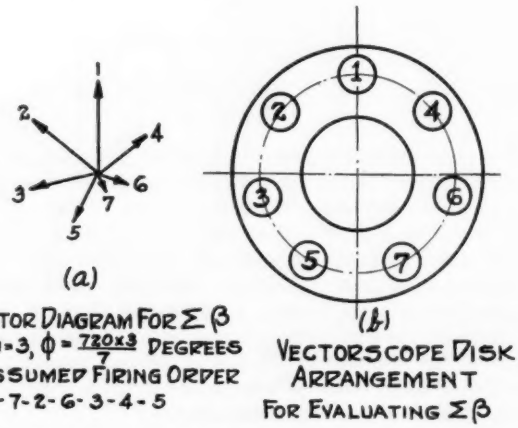


FIG. 13 DIAGRAM AND DISK FOR THIRD-ORDER TORSIONAL VIBRATION FOR FIG. 12

[3] "Torsional Vibration in the Diesel Engine," by F. M. Lewis, Trans. Society of Naval Architects and Marine Engineers, vol. 33, 1925, pp. 109–140.

[4] "The Elimination of Torsional Vibration," by G. J. Dashefsky, Pennsylvania State College Bulletin, vol. 24, no. 44, November, 1930, pp. 193–265.

RELATION OF ENGINE BALANCE AND TORSIONAL VIBRATION AS AFFECTED BY FIRING ORDER

In the usual problem met in the study of engine dynamics, it is necessary to compromise the characteristics of torsional vibration and engine balance. This means a search for a firing order which will yield small unbalanced moments primary and secondary, without aggravating the minor-order torsional vibrations, that is, $\Sigma\beta$ must also be small.

Fig. 14, indicates a more or less typical relative-amplitude curve, and shows that cylinders having large β values also have large moments about cylinder No. 1. Thus the same harmonics or orders of vibration will give similar vector diagrams. As regards unbalanced moments, only the first, second, and fourth

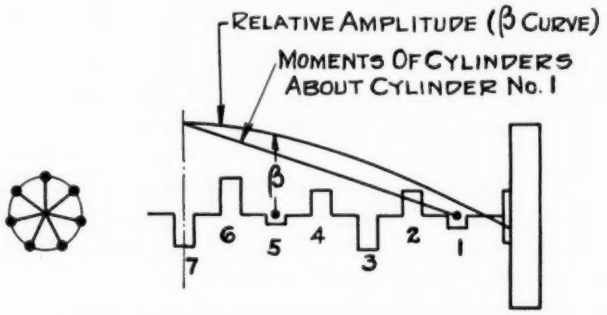


FIG. 14 RELATIVE-AMPLITUDE AND MOMENTS CURVES

harmonics are important. It is apparent that these harmonics correspond to the first, second, and fourth orders of torsional vibration. A firing order which yields a small or large value of unbalanced moment, also yields a small or large value of $\Sigma\beta$ for the corresponding order of torsional vibration. This link between unbalanced moment and torsional vibration makes it necessary to consider both of these problems as of mutual importance.

THE VECTORSCOPE AS A HARMONIC ANALYZER

Any periodic function such as the torque curve illustrated in Fig. 15 may be represented by Fourier's trigonometric series, so that

$$T = f(\theta) = A_0 + A_1 \sin\theta + A_2 \sin 2\theta + A_3 \sin 3\theta + \ldots A_n \sin n\theta + B_1 \cos\theta + B_2 \cos 2\theta + B_3 \cos 3\theta + \ldots B_n \cos n\theta \ldots [14]$$

The coefficient, A_0, represents the mean torque of the engine, while the coefficients $A_1, A_2, \ldots A_n$ and $B_1, B_2, \ldots B_n$ represent the sine and cosine harmonic components of the turning-effort curve.

These are important coefficients in many engine problems and it is frequently necessary to evaluate them. Schematic tabulations and mechanical devices are available for performing harmonic analyses. The Vectorscope seems to be well adapted to speedy determination of one or many of these coefficients by simple mechanical procedure requiring a minimum of care and concentration.

If the periodic function is considered as occurring over 2π radians it can be shown that

$$A_0 = \frac{1}{2\pi}\int_0^{2\pi} T d\theta \ldots [15]$$

$$A_m = \frac{1}{\pi}\int_0^{2\pi} T \sin m\theta \, d\theta \ldots [16]$$

$$B_m = \frac{1}{\pi}\int_0^{2\pi} T \cos m\theta \, d\theta \ldots [17]$$

A_m and B_m represent the harmonic coefficients for the mth harmonic of the Fourier series of Equation [14].

The coefficient A_0 is obtained from evaluation of Equation [15], where the integral represents the net area under the original curve. The integral involved in the expression for A_m is the area under the curve, $t = f(T \sin m\theta)$. See Fig. 15. This latter function is determined by multiplying each ordinate of the original function by the appropriate sin $(m\theta)$. The area under this curve multiplied by $1/\pi$ is the value of A_m. The values of B_m may be obtained in a similar manner, excepting, of course, that the auxiliary curve $t = f(T \cos m\theta)$ is obtained by multiplying ordinates of the original curve by the appropriate cosines of $m\theta$.

As a general matter, the process for determining the areas representing the integrals of Equations [15], [16], and [17] is

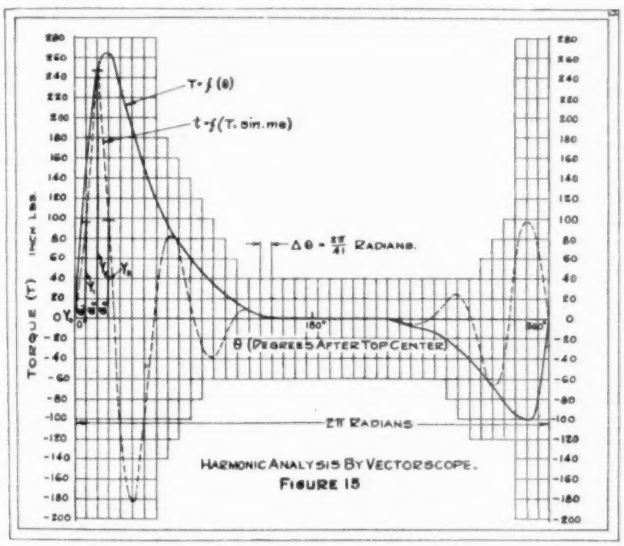


FIG. 15 HARMONIC ANALYSIS BY THE VECTORSCOPE

not important. However, in adapting the Vectorscope as a harmonic analyzer, the following method of obtaining areas is desirable.

The abscissa of the original function is divided to give r equally spaced ordinates at $(r - 1)$ equally spaced intervals of $\Delta\theta$. The ordinates at each point are multiplied by the appropriate sine or cosine value giving a new series of ordinates on the curves $t = f(T \sin m\theta)$ and $t = f(T \cos m\theta)$. See Fig. 15. The first elementary area a_1, assumed to be a trapezoid, may be closely approximated by $(Y_0/2 + Y_1/2)\,\Delta\theta$.

Adding up all of the elementary areas a_1, a_2, a_3, ... gives the net area of the curve, representing the value of the integrals of Equations [16] and [17].

The total area under the curve is therefore

$$\int_0^{2\pi} T \sin m\theta \, d\theta = \left(\frac{Y_0}{2} + \frac{Y_1}{2}\right)\Delta\theta + \left(\frac{Y_1}{2} + \frac{Y_2}{2}\right)\Delta\theta + \ldots . \left(\frac{Y_{r-1}}{2} + \frac{Y_r}{2}\right)\Delta\theta$$

$$A_m = \frac{1}{\pi}\int_0^{2\pi} T \sin m\theta \, d\theta = \frac{\Delta\theta}{\pi}\left(\frac{Y_0}{2} + Y_1 + Y_2 + \ldots . \frac{Y_r}{2}\right)$$

$$A_m = \frac{\Delta\theta}{\pi}\left[\frac{T_0}{2}\sin 0 + T_1 \sin (m\,\Delta\theta) + T_2 \sin (m\,2\,\Delta\theta) + \ldots . T_{r-1} \sin \{m(r-1)\,\Delta\theta\} + \frac{T_r}{2}\sin 2\pi\right] \ldots [18]$$

The first and last terms are zero. Similarly

$$B_m = \frac{\Delta\theta}{\pi}\left[\frac{T_0}{2}\cos 0 + T_1 \cos(m\,\Delta\theta) + T_2 \cos(m\,2\,\Delta\theta) + \ldots . T_{r-1} \cos\{m(r-1)\,\Delta\theta\} + \frac{T_r}{2}\cos 2\pi\right] \ldots . [19]$$

It will be noted that Equations [18] and [19] for A_m and B_m are of the form which can be evaluated by the Vectorscope.

The present arrangement of the Vectorscope requires that the original curve be divided into 41 equal intervals, that is, $\Delta\theta = 2\pi/41$ radians.

The use of 41 intervals, which is a prime number, avoids the possibility of the piling up on any one peg of several Vectorscope weights for certain harmonics. The disk for harmonic analysis has 42 basic pegs and 42 additional pegs diametrically opposite these to take care of negative ordinates.

The modus operandi for making the harmonic analysis is as follows: Prepare 42 Vectorscope weights corresponding to the ordinates of the curve to be analyzed, except the first and last weights which should represent only half the ordinate. Mark the weights for ordinate identification, including a definite indication of negative ordinates. Adjust the Vectorscope in the usual manner to give a suitable scale. A_0 is merely the algebraic sum of the ordinates multiplied by $1/2\pi$, and the use of the Vectorscope will not be required. To obtain A_1 and B_1 as well as their resultant and its phase, place the weights in order on the basic pegs; w_1 on peg 1, w_2 on peg 2, and so on. After the weights are distributed, the deflection of the spot of light on the screen will indicate directly the sine and cosine coefficients as well as the resultant coefficient and its phase, except for multiplication by $\Delta\theta/\pi$ which may however be incorporated into the scale reading, if so desired.

The second harmonic is obtained by redistributing the weights, placing w_1 on the first basic peg, w_3 on the third basic peg (2 peg angles apart), w_5 on the fifth peg, and so on, until all weights have been distributed.

Thus any harmonic, say the mth one, may be obtained by distributing the weights in succession, counting off m peg intervals each time.

Analyses up to the twelfth harmonic have been carried out for trial curves, indicating differences of about 2 per cent for the sixth harmonic and about 15 per cent for the twelfth harmonic, when compared with a chart method which is described by Lewis.[5]

Greater accuracy will, of course, attend the use of a larger number of ordinates, but the use of 42 ordinates should give results of sufficient accuracy for most purposes and types of curve.

SUMMARY

The Vectorscope is a simple and rugged instrument that requires little attention as regards adjustment and care. Any degree of fineness and accuracy may be built into it as dictated by the use intended. A diagram of a somewhat refined model suitable for desk use is shown in Fig. 16.

It is believed that this instrument will prove useful to the designer of internal-combustion engines in handling the problems of engine balance, torsional vibration, and harmonic analysis. Other uses may suggest themselves with further study and use of the device. Where extensive studies of firing order are

[5] "A Method of Harmonic Analysis," by F. M. Lewis, *Journal of Applied Mechanics*, Trans. A.S.M.E., vol. 57, December, 1935, p. A-137.

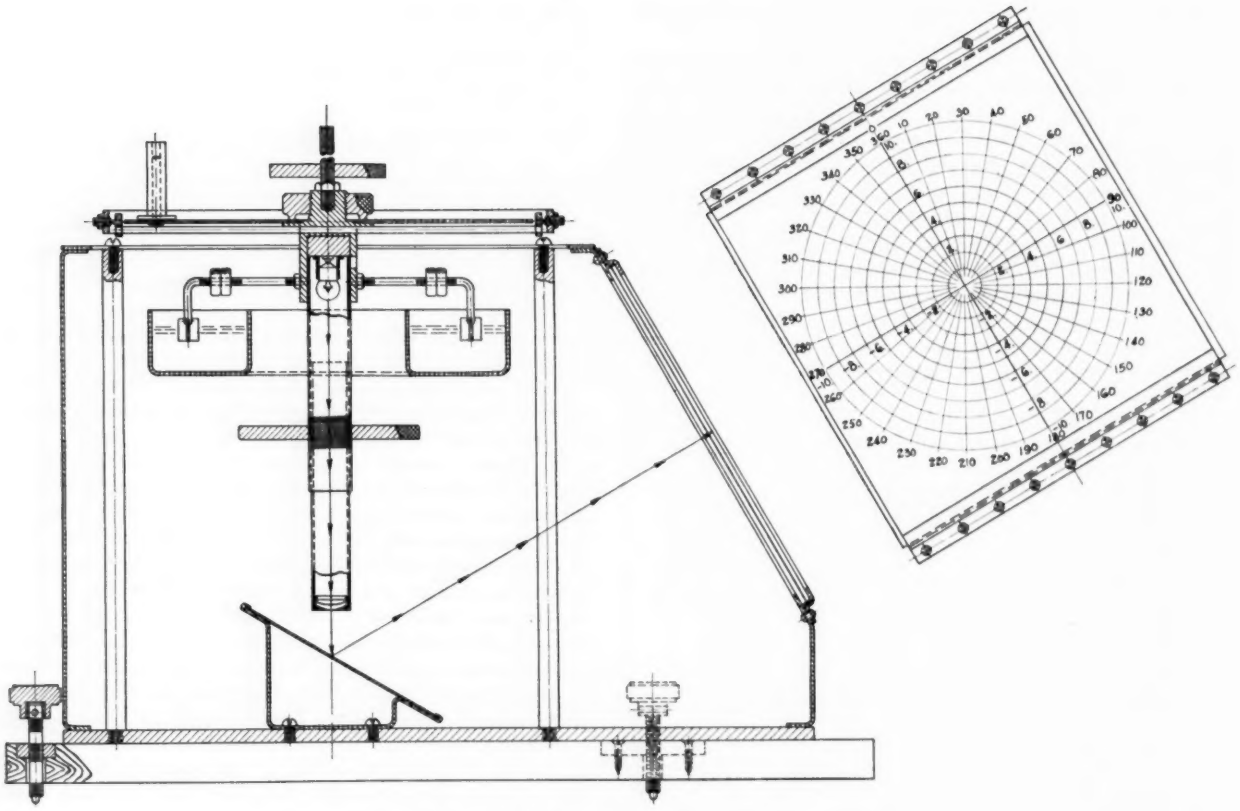


FIG. 16 A DESK-MODEL VECTORSCOPE

necessary, a considerable amount of labor may be saved. The use of the instrument is reduced to a mechanical manipulation, requiring little experience and may thus be delegated to younger members of an organization.

If considered desirable, the instrument or a bank of instruments could be used in making a complete study of all firing orders, for engines up to say nine cylinders, and listing the characteristics of engine balance and torsional vibration. For more than nine cylinders the amount of work involved for an investigation of all possible firing orders would be prohibitive. However, the brief experience of the author in using the instrument indicates that one soon recognizes the direction to shift the various weights to give small disk deflections. The sequence of the weights on the disk indicates the firing order. It is thus possible, for any configuration of cylinders, to arrive quickly at some good firing orders without investigating the whole range.

ACKNOWLEDGMENT

The author is indebted to John Dietmeier and Milton J. Berg of the Diesel-engine technical staff of the New York Navy Yard for many helpful suggestions in the design of the instrument and in preparing the present paper.

Discussion

A. D. ANDRIOLA.[6] The superiority of the Vectorscope over analytical solutions for the three types of problems discussed by the author appears chiefly as a time-saving feature. This advantage is predominant where the vectors involved are of different magnitudes as, for example, the average elastic curve met with in torsional-vibration problems. Where used for firing-order investigation for various numbers of cylinders or for harmonic analysis, a reduction in total time is not readily seen. In both of these problems, especially the latter, the matter of summation represents the appreciably smaller proportion of the work involved.

Incidentally, the instrument is subject to the same limitations that exist in analytical methods. Beyond nine cylinders the amount of work involved for a complete investigation of the range is great and almost prohibitive. Below this number of cylinders much information has been published but as yet is not complete. In this respect the instrument may prove very useful.

The writer does not feel that analysis should be discarded in favor of a mechanical device when the advantages are not too marked. This, of course, has not been implied in any manner by Mr. Dashefsky. Instead, they should be employed to supplement each other. This procedure is very desirable where complex work is involved since a check of principal results by two independent methods practically eliminates all chance for error.

Undoubtedly since the presentation of the paper the author had further opportunity to experiment with the Vectorscope. His remarks on a comparison with analytical methods for both speed and accuracy and as regards applications other than those considered in the original paper would be appreciated.

K. J. DEJUHASZ.[7] Determination of engine balance and torsional vibrations has been justifiably regarded as a bugbear of Diesel-engine design. It involves difficult and lengthy calculations in which it is easy to commit serious errors which are diffi-

[6] Electric Boat Company, Groton, Conn. Jun. A.S.M.E.

[7] Associate Professor of Engineering Research, The Pennsylvania State College, State College, Pa. Mem. A.S.M.E.

cult to detect. Graphical construction alleviates these difficulties to an important extent. Even this method, however, involves a prohibitive amount of labor, if all the great number of possible crank arrangements in multicylinder engines are to be investigated.

The Vectorscope described by the author is an invention which appears to supply the practicable solution to this complex problem. The correctness of its principle is fully proved and its arrangement and design have reached a thoroughly practicable stage.

In the present arrangement the value of the vector is changed by changing the weight and keeping its radius constant. The other alternative of keeping the weight constant and varying the radius appears to have some desirable features. Furthermore, a combination of these two methods could be used. The author has undoubtedly considered these alternatives so it would be of interest to learn the reasons for the adoption of the present choice.

Stacking the weights on the pegs introduces a slight change of the actual lever arm when the disk is tipped. While the error from this source is small, it could be fully eliminated by suspending the weights on hooks located in the plane of the virtual pivot of suspension.

The importance of being able rapidly and accurately to determine engine balance is attested to by the fact that in Europe a mechanical device has also been developed to simplify the work.[8] This however embodies a linkage arrangement. It is intended to provide balancing for the first harmonics only, and it does not appear to possess the adaptability and the accuracy of the Vectorscope.

It is interesting to speculate upon the possible further developments of this instrument. It appears to be possible to shift the weights (or, in general, to alter the relative orientation of the vectors) automatically, according to a predetermined program. Thereby the full investigation of multicylindered engines would become a practicable proposition and much drudgery would be eliminated. Far-fetched as this possibility may seem, one has to bear in mind that numerous complicated and costly mechanical calculators have been built in recent times and found useful, such as mechanical harmonic analyzers and integrators, including the apparatus at the Massachusetts Institute of Technology and at the University of Pennsylvania for solving differential equations; the apparatus developed at the University of Pennsylvania for solving algebraic equations; the wave analyzers for the prediction of tides; the automatic devices developed for the study of light spectra and of the composition of the human voice (Bell Telephone Laboratories).

Even in its present simple form, however, there can be no doubt of the usefulness of the Vectorscope for calculation, and, also, as an instructional aid serving to facilitate for a student the understanding of the complex problems of engine dynamics. The Vectorscope reflects credit on all the men associated in its development. It can be listed as one of the numerous important contributions of the United States Navy to the advancement of Diesel-engine design.

F. M. Lewis.[9] There exists a considerable error in the Vectorscope in its present form. This is due to the location of the weights, representing the vectors to be added, in a position above the plane of the gimbal supports. The correct moment of a weight is $w_1 r \sin \theta_1 + w_1 h_1 \sin \phi v$, where h_1 is the distance of the center of gravity of w_1 above the plane of the gimbals. The effect of the weights being added above the plane of the gimbals is to increase the sensitivity of the instrument, i.e., virtually to decrease R.

It would appear necessary to modify the instrument in such a way that this error is not introduced. A suggested method is to provide a disk with radial holes drilled in it, in which rods can be slid in and out. The rods could be suitably scaled. Some difficulty arises in locating the gimbals in the same plane but it is not insuperable.

It is pointed out that considerable study has already been given to the problem of balance and torsional vibration in engines up to nine cylinders. Even the Vectorscope appears to be too tedious a means for a complete investigation above that number.

The use of the instrument for harmonic analysis is very interesting and, with 41 ordinates, a high degree of accuracy should be attained, if the instrument is modified to correct the error previously mentioned. The complication of negative ordinates can be avoided by the arbitrary choice of an axis below the lowest point of the curve. This influences none of the coefficients except the constant term.

Philip Mallozzi.[10] Vector summation of forces, moments, and component amplitudes of vibrations constitutes a large portion of the task involved in a complete analysis of Diesel-engine installations. It is now admitted by engineers that serious study of the vibration problem must be made before mating a Diesel engine with other parts of the drive.

In choosing a suitable firing order for an engine having more than seven cylinders, to make a complete investigation in order to arrive at a desirable compromise between engine balance and torsional vibration is a lengthy task if the usual graphical or tabulation methods of summing vectors are used. To this end the author's Vectorscope is invaluable as a time-saving device. In operating the instrument, it can be seen at a glance how to move the vector weights in order to arrive at firing orders that will give smaller deflections of the indicator.

In summing up vectors graphically or analytically, unavoidable graphical or arithmetical errors obtain. Similarly, in summing up vectors by means of a mechanical instrument, such as the one presented in this paper, unavoidable small instrument errors are present. The predominant errors are as follows:

1 If the motion of rod No. 9 is sufficiently large, the liquid used to dampen the motion of the rod gives rise to a buoyancy component which tends slightly to increase the resultant vector.

2 If the vector weights No. 13 are sufficiently tall, the center of gravity of the system comprised of disk and vector weights will rise. This condition will tend slightly to change the true resultant moment, which moment is not accounted for in the author's Equations [1] to [6].

The foregoing comments have been made for the purpose of emphasis on the limitations of the instrument. However, in either of the two cases, the error can be made negligible for all practical applications of the Vectorscope. The error indicated in case No. 1 can be reduced by lightening and properly shaping the damper plates. The error indicated in case No. 2 can be reduced by making the disk sufficiently large and by making the vector weights low.

In Fig. 14 the author shows that the engine cylinders, which have large values of the amplitude factor $\Sigma \beta$, also have large moments about cylinder No. 1, thus yielding similar vector diagrams for the same harmonics of either unbalance moments or orders of vibration. In this connection it is desired to point out that this condition exists only for that mode of vibration which has the node at, or slightly remote from, cylinder No. 1.

[8] "An Aid for the Balancing of Internal Combustion Engines," by O. Krämer, *Zeit. V.D.I.*, vol. 81, no. 51, 1937, p. 1476.

[9] Professor, Department of Naval Architecture and Marine Engineering, Massachusetts Institute of Technology, Cambridge, Mass. Mem. A.S.M.E.

[10] Mechanical Engineer, United States Navy Yard, Brooklyn, N. Y.

L. M. PORTER.[11] The writer will discuss mainly the instrument's application to the investigation of the rocking moments produced by the inertia forces.

In discussing Equation [7] for the inertia force in each cylinder, would it not be clearer if the fact were mentioned that this equation gives the inertia force in pounds per pound of reciprocating weight? The reciprocating weight has been omitted from the equation.

The author states in his paper that the same force exists at each cylinder and, therefore, the moment of any cylinder about crank No. 1 is proportional to its distance from the reference point, taken at the center line of No. 1 cylinder. Would it not clarify the discussion to point out that it is the maximum force in any one cylinder which is constant? The instantaneous values of course are functions of the crank angle, the primary forces varying as cos θ and the secondary forces varying as cos 2θ. It is the instrument itself that resolves the maximum force in any one cylinder into a vertical component, which is proportional to the instantaneous value in the cylinder, and gives the resultant of the components multiplied by their respective lever arms. However, since there are no horizontal components of the inertia forces in an in-line engine, would it not be advisable to point out that only the vertical component of the resultant shown by the instrument should be considered in the final analysis? The diagrams shown in Figs. 7 and 8 will have a resultant horizontal component. The statement on page 406 is therefore somewhat confusing, that "the vector sum and the direction of the resultant are indicated directly on the screen." Why not leave out the direction in the present case? The horizontal component may be automatically taken care of in Figs. 7 and 8 by placing a second set of weights on the pegs in mirror similarity to the first set. This of course will double the vertical component but a suitable adjustment of the counterweight (10) will take care of this.

In Table 1 the author lists the possible number of firing orders for engines with a different number of cylinders, after stating previously that the number of firing orders is given by the expression $(n - 1)!/2$. It would appear that, in the case of a two-stroke-cycle in-line engine, there are twice as many firing orders as those listed in Table 1. In this type of engine the cranks must necessarily be evenly spaced to give an evenly spaced order of firing impulses. Each cylinder will fire as the crank passes through every top-center position. Since it is possible to twist the cranks into any desired location, it is evident that a three-cylinder engine may fire 1–2–3 or 1–3–2 instead of in only one order as given in Table 1. Similarly, for a four-cylinder two-stroke-cycle in-line engine with cranks spaced 90 deg, there are six possible orders instead of three, i.e., 1–2–3–4; 1–2–4–3; 1–3–2–4; 1–3–4–2; 1–4–2–3; 1–4–3–2; and so on for the other engines.

In the case of the 12-cylinder 90-deg V-type engine shown in Figs. 10 and 11, for the direction of rotation and spacing of cranks shown, it would appear that cylinder $3R$, and not $2R$, will fire after $1L$, inasmuch as $3R$ will be the next crank passing through its top-center position. In choosing a basic firing order for this engine, would it not have been better to use the conventional six-cylinder firing order of 1–5–3–6–2–4 instead of 1–5–3–6–4–2? The latter sequence has cylinders 2 and 1 in any one bank firing successively. It is well known that this arrangement does not produce the best manifolding conditions where the exhaust of one cylinder may directly impede the exhaust gases leaving an adjacent cylinder.

In closing, there appear to be some inconsistencies in connection with the seven-cylinder in-line engine. The firing order given on page 408 is not apparent from the diagram shown in Fig. 12, and the successively firing cylinders will again cause exhaust interference. It is also not clear whether the author is still referring to a two-stroke cycle or has changed to a four-stroke cycle. The expression for the crank displacement between successively firing cylinders given on page 407, $\theta = 720/7$ deg, would indicate that he has a four-stroke cycle in mind.

[11] Assistant Professor of Machine Design, New York University, College of Engineering, New York, N. Y.

T. C. RATHBONE.[12] The Vectorscope as described has been developed to aid in the analysis of engine balance and torsional-oscillation characteristics, where much labor is involved in the handling of vector quantities.

This apparatus would also appear to be valuable for the solution of rotor unbalance on the usual turbine generator installations having three or four bearings and three or four corrector-weight locations. The rotor-balancing problem is developed analytically in a current paper by J. G. Baker.[13]

A unit of unbalance at each location produces "unit vectors," defining the vibration at each bearing. For four bearings and four corrector locations, there are therefore sixteen unit vectors. The problem is to cancel the initial unbalance at all bearings by adjustment of the magnitude and azimuth of all the mutually interdependent unit vectors.

It would seem that four Vectorscopes, geared together, should aid materially in reducing the labor necessary to determine the group relations which affect only a single chosen bearing.

PAUL SHIRLEY.[14] There does not seem to be much that can be added to the subject matter of this paper. The explanations are clearly given. The Vectorscope should prove to be a time saver in solving the problems mentioned in the paper.

However, it might be added that the Vectorscope can be used for analysis and development of side-thrust couples and forces which are, in my opinion, just as important as the rotating and reciprocating forces and couples in arriving at good dynamic balance of engines.

P. E. SCHWEIZER.[15] The significance of the Vectorscope lies in two directions:

First, in the field of immediate demand its usefulness may be demonstrated by means of the instrument's ability to indicate the resultant vectors directly.

The determination of a suitable firing order, on account of torsional vibration and balance, has thus far been a matter of adjusting crank positions arbitrarily and calculating the results. With the Vectorscope, the minimum value of harmonic-torque input or unbalanced forces or moments may now be obtained quickly for a given set of cranks and Vectorscope weights, by means of the condition of least declination of the Vectorscope disk.

Thus, defying even the simplifying qualities of the graphic arts, the use of the Vectorscope becomes a matter of playing with weights and crank positions.

Second, if we afford ourselves a moment of leisure and review records of the past, it seems as if history again demonstrates a decided tendency toward repetition: first pebbles, then lines in the sand, and after that the flight over the runway of numbers and letters toward the higher spheres of present-day mathematics, with its complexity in form of both insolvable integrals and differential equations. Does not perhaps the Vectorscope,

[12] Chief Engineer, Turbine and Machinery Division, The Fidelity and Casualty Company of New York, New York, N. Y. Mem. A.S.M.E.

[13] "Methods of Rotor Unbalance Determination," by J. G. Baker, *Journal of Applied Mechanics*, Trans. A.S.M.E., vol. 61, March, 1939, p. A-1.

[14] General Machinery Corporation, Hooven, Owens, Rentschler Division, Hamilton, Ohio.

[15] Consulting Engineer, Diesel Engine Division, American Locomotive Company, Auburn, N. Y. Mem. A.S.M.E.

in principle at least, represent a milestone in mathematical history in the form of pebbles, but of a higher order?

Seven years ago, the location of the $5^1/_2$ order critical of torsional vibration promised to become detrimental at the speed of operation for which the unit, an eight-cylinder four-cycle engine, was sold.

Since the unit was a stock engine, it was of course not possible to raise the natural frequency in order to effect relocation of the critical sufficiently removed from the normal rating. To lower the frequency was not deemed advisable on account of increased forced vibration, likely to be encountered due to the combined effect of the $4^1/_2$ and 4 orders.

Therefore, the only alternative that remained was a change in firing order. This meant changing the relative position of the cranks arbitrarily, after which the process was solved by the slide rule. Doing this for different settings, resulted in the firing order: (*a*) 1–2–3–5–8–7–6–4–1. Checking up the question of balance for this firing order gave the result of complete balance, both for the forces and moments.

The crankshaft was ordered and the engine tested. Tests covered torsiographic measurements and comparison with those taken from a similar installation with firing order: (*b*) 1–3–2–5–8–6–7–4–1, which firing could be altered to the standard firing order: (*c*) 1–6–2–5–8–3–7–4–1 by a simple interchange of cams and fuel pipes.

The records indicate a reduction of 70 per cent over the $5^1/_2$-order amplitude due to firing order (*c*) with everything else, except the relative crank position, being identical.

It would be interesting to know whether the Vectorscope can produce a still more favorable ratio. In any case, the writer was satisfied with the results obtained and several engines are operating successfully with firing order (*a*).

Relative to the question of balance, a surprise was in store, however. Despite the fact that calculations indicated complete balance, the engine proved rough on the test floor. This consisted of two usual cast-iron flanged beams about 30 in. high and reinforced very little sideways. The engine, on the other hand, being of the individual-cylinder type, proved quite elastic in the combined base and frame sections.

Since firing order (*a*) results in the firing of two four-cylinder groups alternately, the platform, fastened to the individual cylinders, had a decided tendency toward twisting itself in the middle.

This incident demonstrated clearly that, aside from torsional vibrations and balance, torque reaction forces do exist and must also be considered in all instances in which the polar moment of inertia of frame and base combination is of small numerical value, and such is intended to be mounted on a relatively flexible foundation.

Since the engine referred to was to be mounted on a rigid concrete foundation, it was concluded that the disturbances cited would not be present in the field.

This example applied to the Vectorscope means that those who are going to use it should also consider, beside torsional vibration and balance, the foundation, the well-beaten path, and tradition of the trade.

Author's Closure

The author does not agree with Mr. Andriola that the reduction in time, when using the Vectorscope for firing-order investigations and harmonic analyses, is not marked. The study of firing orders usually involves a set of vectors of fixed magnitude and varying orientation. The Vectorscope appears particularly well adapted to this type of vector addition because, once the set of weights representing the vectors has been adjusted, each shift in position of the weights immediately yields the resultant of a complete vector diagram.

With regard to the second type of problem, harmonic analysis, it is the author's opinion that the Vectorscope will find broad application. The existing tabular methods for making an harmonic analysis lack accuracy for the higher harmonics, unless a great number of ordinates is used. The use of many ordinates involves a considerable increase in work. Furthermore, these schematic tabulations require extreme concentration as regards the algebraic signs involved. It is very easy indeed for errors to creep in. In this respect, the Vectorscope seems to have a decided advantage. After having made up the weights, representing the ordinates of the curve under analysis, the remaining procedure is entirely mechanical. The probability of error is considerably less than with the tabular solution.

The author is now constructing a Vectorscope of large dimensions, incorporating improved mechanical arrangements. In the near future he will be in a better position to make further studies of the instrument and its application to various problems.

The discussion of Professor DeJuhasz is most interesting. The objection to the arrangement of weights is noted. It has been recognized that the present arrangement introduces an error. However, the arrangement is simple and seems to afford the maximum of accessibility when shifting weights. By using weights of larger diameter, in order to reduce the height of the stacks, in combination with a larger Vectorscope disk, this error should be reduced to the point of being negligible. The suggested arrangement of combining weights, which can be shifted radially with variable weights at fixed radius, appears entirely feasible and is being considered in an arrangement now being designed.

Referring to comments by Professor Lewis, as stated above in response to Professor DeJuhasz, the location of weights above the plane of the gimbals admittedly introduces an error. In the model used for demonstration, the stacks are rather tall, compared with the disk, which is only 6 in. in diameter. An instrument intended for actual use should be larger than this, and the weights should be arranged to have their centers of gravity as close as possible to the plane of the gimbals. The radially extended rods suggested by Professor Lewis would seem to complicate the mechanical arrangement of the instrument unnecessarily. In the author's opinion it would be better to accept a slight error rather than sacrifice the mechanical simplicity. This phase of the instrument's design is being considered and improved arrangements have been devised. The author is grateful to Professor Lewis for his suggestion regarding the elimination of negative ordinates, when making harmonic analyses. This simplifies the disk as well as the distribution of weights in the process of making the analysis.

The comments of Mr. Mallozzi indicate an error introduced by the buoyancy effect of the damper. It is essential that the ratio of submerged surface to volume of the damping vanes be made large. The error due to this cause may further be reduced by using a lighter damping fluid. Use of an oil instead of mercury would reduce the buoyancy component in the ratio of approximately 1 to 15, and result in a negligible buoyancy effect.

Professor Porter's discussion regarding the force expressed by Equation [7] is well taken. The force as expressed is for 1 lb of reciprocating weight. The discusser also points out that the respective maximums of primary and secondary harmonics are used in calculating the resultant forces and moments. Discussion of these fundamentals was not deemed necessary in the paper, since it was assumed that the reader would have a knowledge of them. Rather, the object of the paper was to explain the principle and application of the instrument. There is one point upon which Professor Porter seems to have a wrong impression. In interpreting the meaning of "horizontal" and "vertical" components, he states that the moments due to reciprocating forces have no horizontal component. This is correct. Apparently, he has

overlooked the fact that the horizontal and vertical components, referred to as being indicated directly on the screen, are the two coordinate components of the resultant vector, and have no reference to their direction relative to vertical and horizontal planes of the engine. The angle of the resultant vector is merely the phase relation of the maximum value of the resultant with respect to the top dead center of the reference cylinder, No. 1, in this instance. The direction of the resultant vector is therefore quite important and it cannot be omitted from consideration as suggested. In view of the above, the complication of compensating weights, etc., suggested by Professor Porter is unnecessary.

Professor Porter questions the expression giving the number of possible firing orders. He offers proof by examples of 3- and 4-cylinder two-cycle engines, to show that there are twice as many firing orders as indicated by the formula. The two firing orders for the 3-cylinder engine are equivalents. Any vector quantities determined from these firing orders will be fully equivalent; i.e., the resultant vectors will be equal in magnitude and will differ only in direction. The vector diagrams, per se, are mirror images of each other. Likewise for the 4-cylinder engine, the following pairs of firing order are equivalent:

1–2–3–4 and 1–4–3–2
1–2–4–3 and 1–3–4–2
1–3–2–4 and 1–4–2–3

The number of firing orders is thus consistent with the formula. Another interpretation for this equivalence is that an engine having a firing order as listed at the left, upon reversing its direction of rotation, assumes the firing order indicated on the right.

Any specific firing orders mentioned in this paper were chosen merely to illustrate the various problems as related to their study with the Vectorscope. No particular effort has been made to indicate desirable firing orders with regard to engine balance, manifolding, or to any other characteristic influenced by firing order.

The author appreciates Mr. Rathbone's suggested application of the Vectorscope to dynamic-balancing problems. A study of Mr. Rathbone's paper,[16] and that of J. G. Baker,[13] referred to in Mr. Rathbone's discussion, indicates that a bank of Vectorscopes should expedite the solution of this vector problem. A study by the author indicates that gearing of the Vectorscopes involves a more or less elaborate mechanical arrangement. A somewhat less automatic arrangement utilizing a bank of manually coordinated Vectorscopes is now being worked out.

The author tenders his thanks to Messrs. Schweizer and Shirley for their comments.

In general it may be stated that inherent instrumental errors may by proper design be quite completely eliminated. The geometry of the instrument itself is correct, so that readings do accurately indicate vector sums. It is believed that instruments of any required accuracy can be constructed by observing the various precautions brought out in the discussion of this paper. An instrument now being built by the author does in fact circumvent many of these sources of error. The accuracy should be as good as that ordinarily to be expected from careful graphical solutions and probably well consistent with most engineering requirements.

Use of the instrument is not limited to any particular type of problem. Application of the instrument may be made to any problem which may be resolved into, or expressed as, a vector summation.

Use of the Vectorscope is advocated not merely as a time-saving device. It is believed that experienced calculators will be enabled to undertake problems previously slighted or only partially solved because of the large amount of labor involved. Furthermore, certain phases of calculation can be delegated to relatively inexperienced calculators and reliable results will be obtained.

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[16] "Turbine Vibration and Balancing," by T. C. Rathbone, Trans. A.S.M.E., vol. 51, part 1, 1929, paper APM-51-23.

Problems and Trends in Centrifugal-Machine Applications

By JOSEPH S. PECKER,[1] PHILADELPHIA, PA.

The paper deals with the factors that govern the design and choice of a centrifugal machine for some specific application so that a prospective user or purchaser may appreciate the scope of the problems involved in the construction of a machine that would meet his requirements.

The article describes the various types of machines that are available and specifically calls attention to the newer types of development not generally known. The author surveys the trends in centrifugal-machine development and endeavors to indicate to manufacturers, prospective inventors, developers, and users of these machines the direction in which the art may be extended and expanded by the judicious choice and application of new materials, drives, and methods of construction, as a means of securing maximum production with required safety.

Historical

CENTRIFUGAL machines for the separation of liquids from solids were used a thousand years ago in the purifying of tung oil. The first "reduction to practice" however appears to be a British patent to Seyrig in 1838 which discloses the general principles as used in centrifugals today. A patent was issued to Hurd in the United States in 1844, to Brooman in France in 1852, and to Langen in Germany in 1878.

The first filtering type of centrifugal for the clarification of sugar crystals was developed by David Weston about 1852 and the manufacture of this machine by the American Tool and Machine Company dates to approximately 1864.

Although the first cream separator appears to have been invented in France in 1876, the first nonfiltering cream separator, invented in the United States by Carl De Laval in 1878, proved successful in practical application in the United States. Cream separators were manufactured in the United States by P. M. Sharples approximately in 1887.

For some thirty years the principal applications of centrifugal machines were for cream separators, clarification of sugar crystals, and laundry driers. It is only since approximately 1915 that the use of centrifugals has been extended to a wider variety of industrial applications. There are now over four hundred such applications in the chemical, industrial, petroleum, and food industries.

It is estimated that the total value of centrifugal machines manufactured and sold in the United States within the last few years, has amounted to approximately fifteen million dollars. The greatest percentage of these centrifugals, nevertheless, are still being used as cream separators and clarifiers for sugar crystals.

Uses of Centrifugal Machines

Practically all industries at one process stage or another deal with the separation of liquids from solids, purification of crystals, recovery of by-products, the concentration and thickening of slurries, or the disposal of waste. The centrifugal machine is not a panacea for all dewatering problems. Under certain conditions, the settling tank is still the most practical and efficient method, as exemplified in papermaking and mining.

In some instances, nevertheless, centrifugal machines perform the complete dehydration process. In the main, however, centrifugal machines may be used advantageously for the removal of the bulk of the liquid or where stratification of the liquid will separate a mixture into its component parts and thus complete the separating operation, as in cream separators. Where solids must be dehydrated to nearly 100 per cent, centrifugal machines may be used as an auxiliary process with thermal drying equipment to complete the operation.

In many cases the evolution of the centrifugal machine has supplemented other equipment such as bag filters, plate-and-frame filters, and suction-leaf filters. In many applications it has surpassed the rotary-drum vacuum filter and, on the other hand, there are many applications in which the vacuum filter has proved superior to the centrifugal.

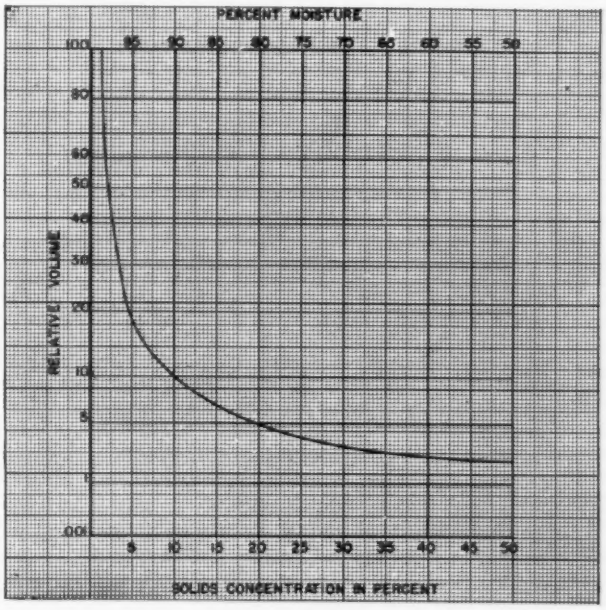


Fig. 1 Relative Sludge Volume

Where the solids in a slurry are but a small percentage of the water to be extracted, a centrifugal may be applied economically. However, although the surplus or free water can be extracted quickly, there is a point of dehydration beyond which it becomes uneconomical to use a centrifugal. A study of Fig. 1, will illustrate this point. The chart has been prepared for one special slurry and is not applicable to all slurries although generally the curve will be similar. It will be noted that a slurry with approximately 98 per cent moisture will lose its free moisture rapidly and

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[1] Consulting Mechanical Engineer. Mem. A.S.M.E.

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Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

cause a concentration of the solids down to approximately 75 per cent moisture. Beyond that the curve levels off and it will require considerable time for the centrifugal to extract 5 or 10 per cent more of the water. It is, therefore, obvious that an efficient still, evaporator, or drier may complete a dehydration process much more economically than a centrifugal.

If, however, the dewatered material is a waste product that must be disposed of by combustion, as for example in sewage disposal, it may prove more economical to retain the sludge in the centrifugal until the moisture content is such that the dewatered

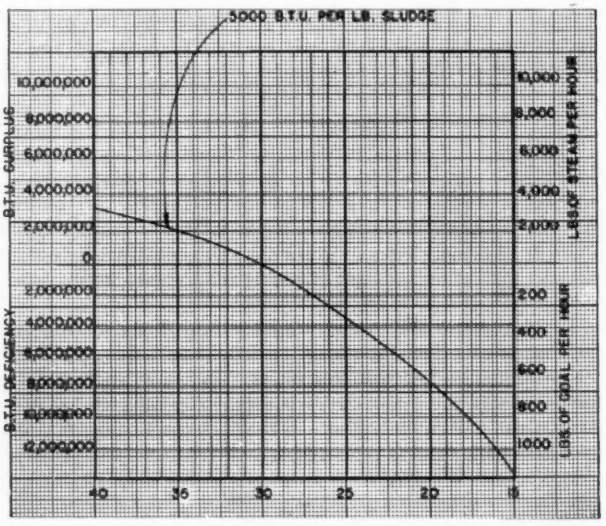


FIG. 2 EFFECT OF MOISTURE ON THE BURNING OF DEWATERED CAKE

cake will support its own combustion. This is illustrated in Fig. 2.

THE SEPARATING EFFECT

Centrifugal machines will separate any two solids when there is at least 3 per cent difference in specific gravity of the constituent parts, providing that the constituent parts are not in solution with each other. When a slurry or sludge is allowed to settle in an ordinary tank, the large particles will drop rapidly to the bottom, the lighter particles will float in suspension, and the still lighter particles, such as oil or grease, will remain on top of the water. Obviously then any materials which will separate by gravity will be separated by centrifugal machines. Materials in the state of emulsion or in extremely fine subdivision, and in some instances in colloidal suspension, that ordinarily would not be separated by gravity, although retained in a tank for an indefinite time, can be separated by a centrifugal machine.

In an ordinary tank, the rate of settling is based on Stokes' formula which may be stated as

$$V = (2/9)G\,[(s - s^1)/e]r^2$$

where V = velocity of settling
G = acceleration due to gravity
s = specific gravity of the particles
s^1 = specific gravity of the liquid
e = viscosity of the liquid
r = radius of the particle

An analysis of the formula will indicate the following outstanding characteristics:

1 If, by pretreatment prior to dewatering by a centrifugal machine, the particles can be made to coalesce by chemical treatment or other means to increase particle size, separating action will be improved.

2 If the viscosity of the fluid can be reduced, as for example by temperature increase as is now being applied in practice in sugar refineries (Stevens hot mingle), separation will be immediately speeded up.

3 If the force of gravity is increased, it is apparent that separation will increase in direct proportion and, inasmuch as gravity can be increased several thousand times in a centrifugal, the intrinsic value of a centrifugal becomes apparent.

Where the increase of temperature is impractical, the reduction of the effect of viscosity can be obtained by decreasing the depth of the liquid. If a slurry is allowed to settle in a tank of some specific depth, say 10 in., the liquids will separate at a rate of increase in almost direct proportion to the depth and if depth is decreased to 2 in. it is obvious that the settling effect will be multiplied by 5. Thus in a centrifugal machine developing an intensity equal to 2000 times gravity, where the solids are being forced through a depth of liquid averaging 10 in., if the depth of the liquid in the centrifuge could be decreased to 2 in. and the same centrifugal intensity applied for the same length of time as in the former case, the effect of centrifugal intensity would be equal to 10,000 times gravity.

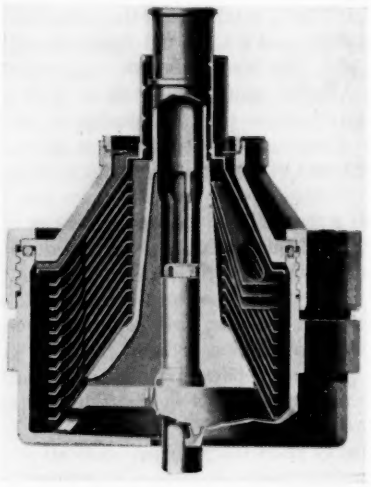


FIG. 3 CONE TYPE OF CENTRIFUGAL MACHINE

This phenomenon has been ingeniously developed in the De Laval and Sharples cream separators and oil clarifiers by the utilization of a series of cones which decrease the depth of the liquid through which a solid is acted upon while under the influence of centrifugal force. This type of centrifugal is illustrated in Fig. 3.

It is furthermore apparent that the longer the time gravity or the multiplication of gravity acts on a solid to be dewatered, the more complete will be the separation. It is, therefore, obvious that in a continuous-flow solid-bowl centrifugal, the longer the material is retained in the basket while subjected to centrifugal action, the more thorough and complete will be the separation. From this fact springs the implication that the larger the basket through which a material must pass, subjected to the same centrifugal intensity, the more complete will be the separation or the greater the production obtained.

The formula for centrifugal force is $F = KN^2R$ where F is the number of times the force of gravity is developed, N equals rpm, R is the mean radius of the material in the centrifugal, in inches, and K equals 0.0000284. It will be noted that the force of

gravity increases as the square of the speed and directly as the diameter of the basket. It is obvious, of course, that it is impossible to increase the speed of any size basket indiscriminately. The choice of speeds and diameters is a compromise depending on the extent to which force need be applied on the material to be dewatered. When that speed is known, the size of the basket that will give the greatest production can be readily determined.

Factors Influencing Choice of a Centrifugal

The outstanding object of a centrifugal machine therefore is:

1 The removal of surplus liquids from materials.
2 The separation or stratification of solids mixed with liquids.
3 The separation of materials of different specific gravities, either liquids from liquids or liquids from solids.

The choice of a centrifugal machine to perform any one or more of these conditions must meet with the following economic and operating factors:

1 The maximum degree of moisture reduction in any one pass through the centrifugal.
2 The degree of purification of the effluent.
3 The quantity output per day as compared to other processes.
4 The labor involved in operating one or several machines.
5 The economy of operation, bearing especially on the power consumed, serviceability, and maintenance.
6 Plant space required as compared to other equipment available for the same operation.
7 The processes preceding and following the centrifugal dewatering operation.
8 The flexibility of control of the machine especially in cyclical types of operation.
9 The method of feeding the material to the machine, the removal of the dewatered cake from the centrifugal, and the conveying of the clarified liquids and cake from the machine.

In addition to these factors, the acidity of the slurry, the grease content, and the particular affinity of water or liquid to the solids in which they are mixed, all contribute to the problem of selecting the proper type of centrifuge to do the work desired successfully and economically.

The ingenious designs of various manufacturers, based on the conditions as outlined, have made available to industry a choice of an infinite variety and types of machines to meet almost any conditions.

Types of Machines

Categorically, there are essentially two different types of centrifugals, namely, the solid-bowl or basket type and the filtering type.

In the solid-bowl type, the constituent materials in the slurry or sludge separate and stratify in accordance with their relative specific gravities, similar to the action which may be noted in a normal gravity settling tank.

The filtering centrifugal is one where the periphery of the basket is perforated. These perforations may take varying forms. They may be relatively large holes as in laundry machinery where the surplus water may freely pass through; the restricting effect of the walls of the basket is all that is needed to hold the large bundles of wash within the bowl.

On the other hand, filtering centrifugals may have these perforations covered with some filtering medium such as wire mesh or wire cloth, as in coal drying, or fine slots several thousandths of an inch wide, either cut into plates or built up with a series of thin plates mounted around the complete periphery of the basket.

Generally, the filtering medium is built similar to the filter surface of a vacuum filter, that is, with large holes in the basket, wire mesh next to the basket, and a fine-mesh cloth possessing high durability and wear against which the solids on the inside of the basket are deposited. Thus, the liquid surface of the material being dewatered is forced through the slots or through the filtering cloth. This type of centrifugal may be used for fine crystalline materials, and organic or fibrous slurries.

Centrifugals may be mounted on both vertical and horizontal axes. Generally centrifugals operating at high speeds are suspended on vertical axes because of the gyroscopic stability of a centrifugal mounted vertically and the facilities of feeding the material to the basket and of discharging of the clarified liquids. The driving, however, of a vertical machine becomes somewhat more difficult than a horizontal one. Larger and slower-running centrifugals are usually mounted horizontally.

Centrifugal machines both of the solid-bowl and filter types, horizontal or vertical, are manufactured to perform the dehydration in cycles, as a continuous operation, or a combination of both.

The continuous type of centrifugal performs the operation of feeding, dehydrating, and the removal of solids at a predetermined speed, usually sufficiently high to perform the dehydration satisfactorily.

With the batch machine the material is fed to the machine either when the basket is rotating at top speed or during the process of accelerating the speed, the feed is cut off and the material is centrifuged for a specific length of time. The basket is then decelerated to a lower speed, in several instances almost to a stop and the material is plowed out of the basket either by an operator who manipulates a paddle or by the operation of a plow mounted on the centrifugal and operated manually. This cycle is then repeated.

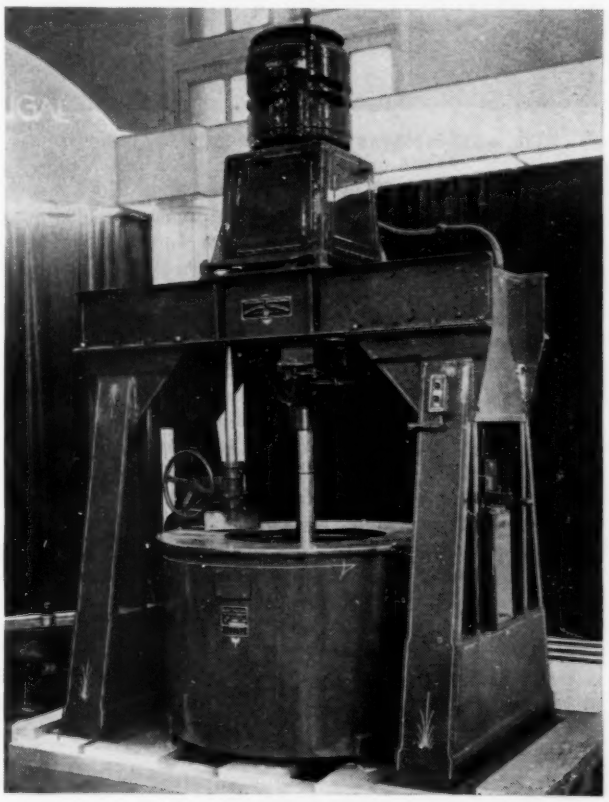


Fig. 4 A Typical Batch-Type Centrifugal

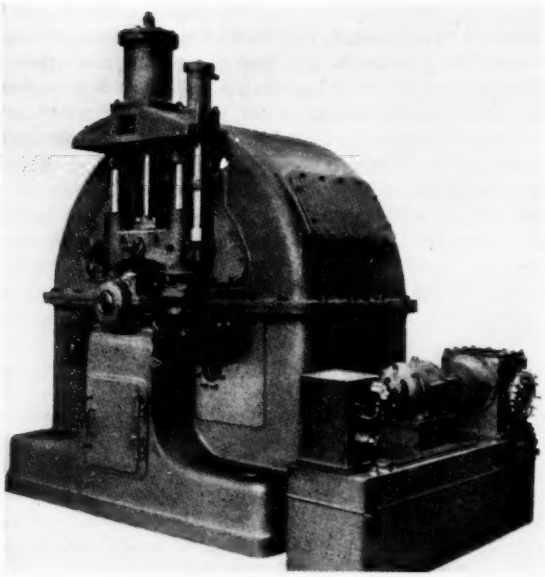


FIG. 5 LARGE-DIAMETER BATCH-TYPE MACHINE

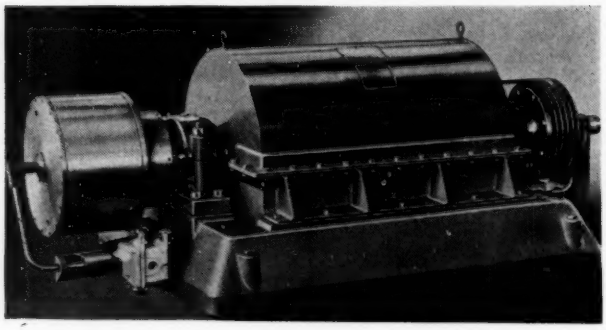


FIG. 6 HORIZONTAL CONTINUOUS-TYPE CENTRIFUGAL

On the other hand, there are batch machines which operate automatically, that is, the material is fed to the machine automatically, the solids are dehydrated, the centrifugal is decelerated, the dehydrated solids are discharged by a plow mechanically operated or by the automatic dropping of the bottom of the centrifugal, and the complete cyclical operation is repeated without the need of an operator.

A typical batch type of machine is illustrated in Fig. 4. Machines of this type are manufactured by such outstanding concerns as American Tool & Machine Company, Rochester Engineering and Centrifugal Corp., Western States Machine Company Bird Machine Company, Tolhurst Machine Company, and others.

Large-diameter batch centrifugals with perforate or imperforate baskets, operating at a continuous speed both for the dehydration and the discharge of the solids, are exemplified in the illustration Fig. 5. These machines periodically unload and reload themselves by means of a hydraulic mechanism which can be made fully automatic. The two outstanding types in this class of machine are being manufactured by Sharples Company and Baker-Perkins Company. In several instances these bulk centrifugals have their perforations not only on the periphery of the basket but also in the top and bottom plates, thus adding to the filtering area.

The continuous automatic centrifugal, where material is fed to the machine as long as the machine is operated and the dehydrated solids and the extracted liquids are continuously discharged,

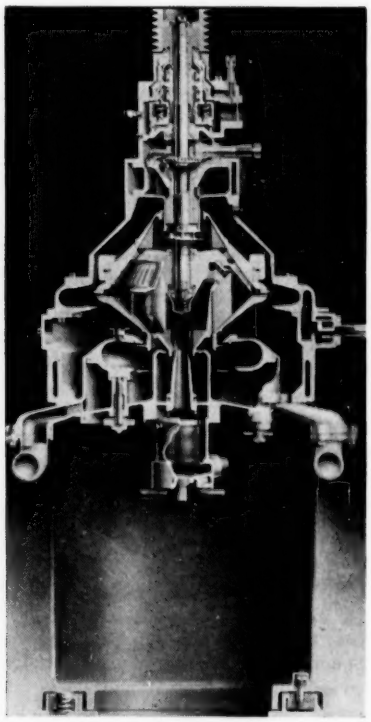


FIG. 7 VERTICAL CONTINUOUS-TYPE MACHINE WITH AUTOMATIC CONTROL OF SEPARATION

is illustrated in Figs. 6 and 7. The Bird horizontal screw type of centrifugal, as illustrated in Fig. 6, is undoubtedly the most advanced design of this kind today.

TRENDS IN DEVELOPMENT

The development of centrifugal machines is characterized by the following trends:

(*a*) *Automatic Control of the Degree of Separation.* Fig. 7 illustrates a machine of this class manufactured by the Merco Centrifugal Company. This machine is employed in various industries; in the starch industry for separating starch from gluten, and in the petroleum field for desanding and conditioning rotary-drilling mud.

The principle of operation is rather simple; the feed, after passing through a screen to remove large débris, enters the rotor at the top and immediately comes under the influence of centrifugal force. Heavy components of the feed are projected to the periphery of the rotor while the "clean" carrying fluid passes out of the rotor over the weir near the center. The solids concentrated in part of the fluid pass out of the rotor at the periphery through a series of nozzles. A large part of this discharge drops to the bottom of the housing where it is picked up by the rotor and returned to the periphery. This return furnishes means for further concentrating the solids and flushing the entrance to the nozzles. The remainder of the discharge from the nozzles passes through a specific-gravity device which regulates the density or concentration of the discharge. Once the gravity device is regulated, only material of the desired concentration can leave the machine. The cycle is continuous and operation automatic.

Another machine in this category, Fig. 8, is one manufactured by Centrifugals, Inc., which has been applied for the separation, classification, thickening, and clarification of materials. In this machine, the discharge of the solids that are dewatered is controlled by the solids themselves as they fill up the valve and cause it to open automatically for their discharge. A similar

machine has been developed by the De Laval Company who have also installed a special valve in conjunction with their standard type of oil clarifier. This machine has been recently applied in the thickening of activated sludge in the sewage-disposal field.

(*b*) *Flexibility in Operation of the Machine.* The advance in flexibility of operation, especially with regard to speeds, acceleration, deceleration, and cyclical control, is exemplified by the Roberts centrifugal which employs a new driving arrangement in which six machines are connected to a 250-hp motor and driven individually by spiral bevel gears from a constant-speed horizontal line shaft to which each centrifugal is connected at the appropriate time by means of a water-cooled friction clutch. A system of forced oil circulation and a water-cooled brake are provided. See Fig. 9.

It has been found that with the clutch shoes set to slip at 70 hp, the basket attains the speed of 1500 rpm in 45 sec; thus the

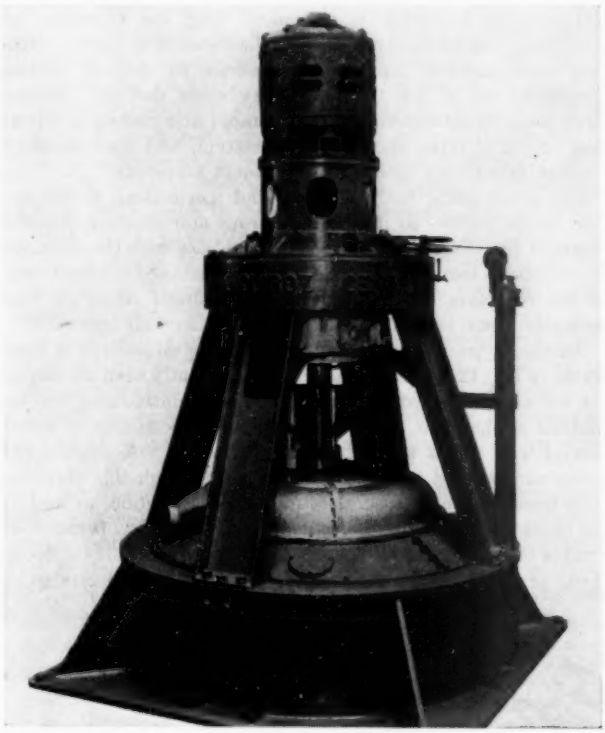


FIG. 8 VERTICAL CONTINUOUS MACHINE WITH AUTOMATIC CONTROL OF SOLIDS DISCHARGE

flywheel effect of all machines in operation contributes to the decrease in power consumption and the increase in speed with which each machine can be accelerated and decelerated through its cyclical operation.

Still another development in the method of driving a centrifugal machine is shown in Fig. 10 by the illustration of a machine manufactured by the American Centrifugal Corporation. This shows two motors mounted on a horizontal base, one used for the high or purging speed and the other for the low or basket-cleaning speed. Interconnected between these two motors is a free-wheeling coupling so that when the high-speed motor is de-energized and the low-speed motor energized, the latter continues driving the basket without the need of transferring the power from one motor to the other by a shifting clutch.

The American Tool and Machine Company has utilized the same principle in a rather ingenious arrangement as illustrated in Fig. 11. Units are available to any manufacturer.

Centrifugals manufactured by the Rochester Engineering and Centrifugal Corporation have been equipped with four-speed

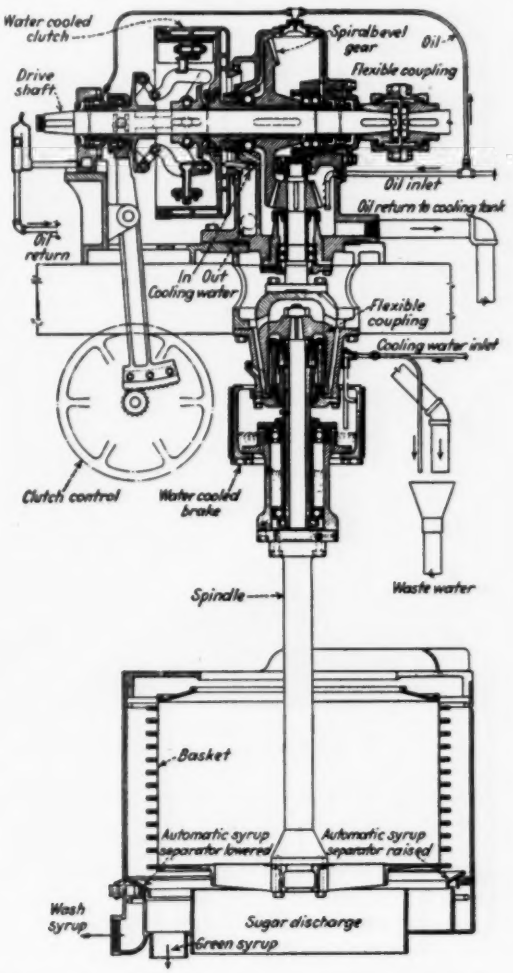


FIG. 9 UNIT WITH WATER-COOLED CLUTCH AND BRAKE

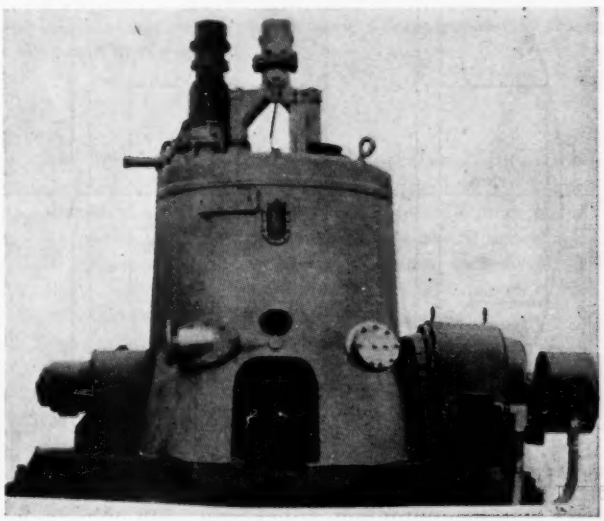


FIG. 10 UNIT WITH HIGH- AND LOW-SPEED MOTORS AND FREE-WHEELING COUPLING

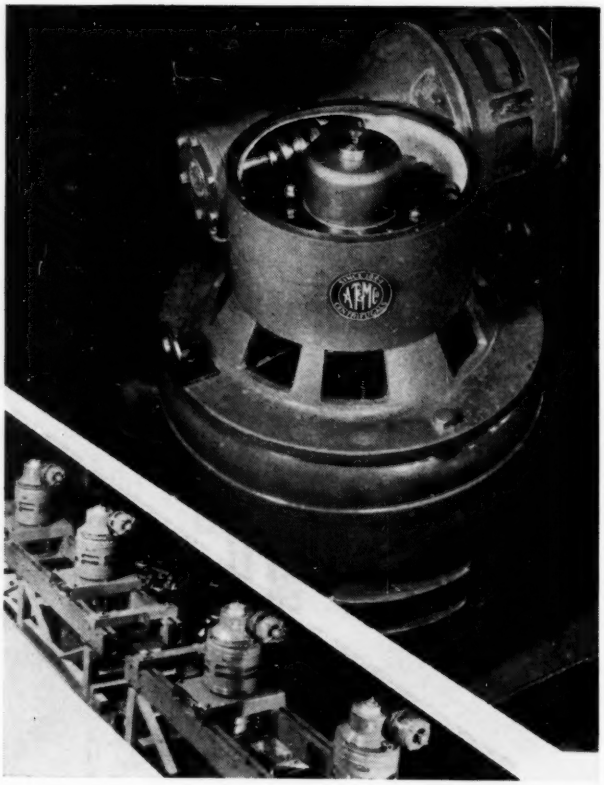


FIG. 11 TWO-SPEED FREE-WHEELING DRIVES

motors, providing, as an example, synchronous speeds of 1200, 900, 600, and 450 rpm. In addition to these four speeds, a low-speed gear ratio of 30 to 1 has been incorporated so that the machine can readily be operated at speeds of 15, 20, 30, or 40 rpm for unloading the basket. Thus, the full power of the motor may be used for high-speed centrifuging and increased torque is available at the lower speeds through the gearing arrangement.

With this flexibility of speeds, certain materials which cannot be handled by arbitrary speeds can be dehydrated efficiently. For instance, some materials of a flaky crystalline structure cannot be handled in a filter-type centrifugal basket in the average way because loading the machine while the basket is operating at top speed would cause an impounding or impacting of the crystalline materials, building up an impervious wall that would prevent the escape of the liquid.

In order to handle such a material satisfactorily, therefore, it would be necessary to load the basket at a relatively much lower speed than would be used for the handling of crystalline materials having a rhomboid or other similar shape. Thus with multiple speeds, as described, the machine could be loaded at 450 rpm, operated for a minute or two at 600 or 900 rpm to give a wringing action, and then brought up to 1200 rpm for final dehydration, after which it could be dropped to any one of the low speeds for unloading.

The unusual conditions associated with the operation of centrifugal machines, especially those subjected to batch operation have impelled motor manufacturers to develop motors specifically suited for the ideal duty cycle desired. Motors have been manufactured to provide rapid and smooth acceleration or, as is often the case, deceleration, and high constant torques at varying speed with minimum amperage.

The refinements, both electrical and mechanical, in the design of the motors to meet the rigorous and exacting requirements of centrifugal extractors have kept pace with the demands of centrifugal manufacturers and have contributed tremendously to the reduction of stresses in the centrifugal machines that normally would reduce the factor of safety in their operation.

Another type of drive which is now being considered is illustrated in Fig. 12. This drive, which has recently been developed for automotive purposes and is now being introduced for industrial applications, appears to possess a flexibility of speed control well suited to centrifugal application. A single-speed motor can be used to drive a centrifugal through this variable-ratio transmission, and acceleration and deceleration, as well as the operating speed, can be controlled through the operation of a valve controlling an oil pump which in turn controls the speed of one of the gears in a differential transmission as illustrated.

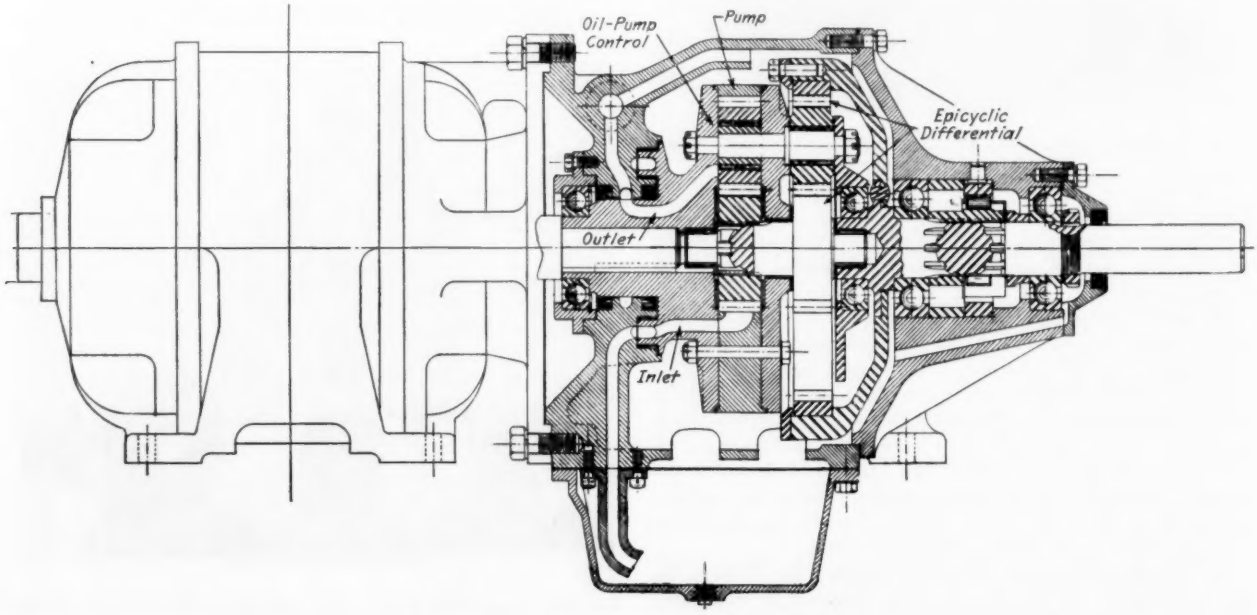


FIG. 12 SZEKELY VARIABLE-SPEED TRANSMISSION

(*c*) *Increase of Size of Centrifugal* with attendant increase of speeds and the use of noncorrosive materials and alloy steels to obtain the former with a high degree of safety.

While it is true that, in the early stages of centrifugal development, a number of baskets burst, this invariably could be traced to amateurish attempts on the part of inventors to build centrifugals without consideration of the safety factors involved in the operation of centrifugal machines.

Today centrifugal machines may be operated with considerable safety at a surface speed of approximately five miles per minute. Centrifugal machines have been built in diameters of from less than 1 in. for laboratory use to 80 in. for industrial use, and with speeds of from a few rpm to 160,000 rpm for laboratory centrifugals in which the force of gravity is increased more than a million times.

The design of a centrifugal machine to operate with safety is today a precise engineering function. For instance, in Fig. 13, there is illustrated a type of chart which may be developed for any specific size of centrifugal with a basket wall of a specific thickness and operating at varying speeds with some specific charge in the machine. The chart is for an impervious basket in which the stresses imposed on the walls are due both to the weight of the material from which the basket is made and the internal pressure developed by the material inside of the basket.

It will be noted that a curve can be drawn to indicate that, with a certain factor of safety or say a stress of 2700 psi, the particular type of basket of varying diameter can be operated only at the speeds shown. For example, a basket of 41½ in. diam operating at 2000 rpm will impose the same stress as a 49-in. basket operating at approximately 1600 rpm.

A parallel line outside of the shaded area can be drawn representing stresses of material equal to 37,000 psi. This line may be represented as a maximum stress that may be imposed on the material in question. Thus graphically can the diameters and speeds of a machine of some specific construction be chosen with safety.

The metallurgical advances within the last few years dealing with stainless alloy steels, high-tensile alloy steels, heat-treatment, rustproofing, and welding have contributed immeasurably to the increase of speed with adequate factors of safety.

The development of pierce forgings, high-tensile aluminum alloys, monel metal, and rubberizing processes has furthermore contributed toward increase of speeds of the machines with commensurate factors of safety by minimizing the deteriorating effect of corrosion.

Improvements in basket suspension, the construction of modern foundations, the use of cork or rubber in shear for supporting machines, methods of mounting ball bearings, dynamically balanced baskets, the use of spiral heat-treated gears, the design of shafts with critical speeds beyond the range of operating speeds, and improvements in drives, have contributed to the elimination of noise, vibration, and unknown stresses that are normally inherent in the operation of centrifugal machines, especially the cyclical types of machine. Thus by decreasing the factors which contribute to the unknown stresses, speeds of machines and factors of safety can be simultaneously increased.

Future Development

The rapid development in modern industry dealing with the careful and exacting preparation of foods, the advance in the art of plastics and synthetic materials, the increase in the use of paper and paper products, the need for more economical operation of the mining and metallurgical industries, and the construction of sewage-disposal plants, marks an era in which the use of the centrifugal machine is becoming increasingly important as a dewatering medium whereby the force of gravity can be multiplied many times.

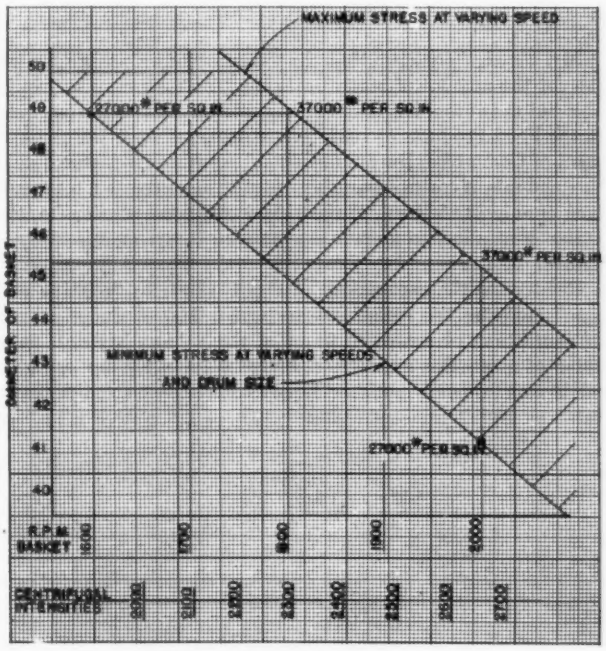


Fig. 13 Basket Speeds and Diameters for a Fixed Range of Allowable Stresses

The role of prophet is dangerous, but it is the author's opinion that the centrifugals of today are entirely too small and too slow, and that, with the judicious application of the materials and engineering knowledge available plus a little imaginative and ingenious daring on the part of engineers engaged in this field, the centrifugal art can be advanced to a degree where it might nearly supersede every other type of dehydrating equipment except the last-stage thermal drying units.

Acknowledgment

The author wishes to acknowledge with thanks the cooperation of the various manufacturers and the engineers associated with these manufacturers, in supplying necessary information and illustrations and materially assisting in the compilation of the data submitted in the paper.

Discussion

A. E. Flowers.[2] The author states that the value of centrifugal machines sold in the United States in the last few years has amounted to about $15,000,000. This estimate must be far too small for the annual value of one class of centrifuges alone, the small farm-model cream separator, is at least that much. If all classes of centrifuges are included the annual business in centrifuges would be two or three times this amount.

The author should give a further explanation of the values plotted and the meaning of the curve Fig. 1, in both the legends and titles on the chart and also in the explanatory paragraph.

In the discussion of Separating Effect, no mention is made of Brownian and other movement, limitations on settling, or of the effect of critical forces and velocities on settling rates and quantity of centrifuged solids.

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[2] Engineer, in charge of Development, De Laval Separator Company, Poughkeepsie, N. Y. Mem. A.S.M.E.

Author's Closure

Mr. Flowers' criticism that the valuation of $15,000,000 is too low for the amount of centrifugals manufactured may be justifiable but the author has taken this figure from an article appearing in the Food Products of 1937. As a matter of fact, the Department of Commerce, Bureau of Census, reports that the manufacture and sale of cream separators in 1937 amounted to a total volume of only $3,850,000. If the $15,000,000 total is incorrect, the author is at a loss to know where to turn to get more authentic figures.

Relative to the explanation of Fig. 1, the term "relative volume" indicates the volume (water and solids) of sludge at any concentration, expressed in terms of the volume having a base value of 100. For example, a sludge volume having one part solids and 99 parts liquid or 1 per cent concentration is designated as 100, 2 per cent concentration as a "relative value" of 50, 5 per cent as a relative value of 20, 10 per cent as a relative value of 10.

The use of this figure quickly gives the volume of any sludge concentration in terms of other concentrations. For example, a sludge of 2 per cent concentration has a relative volume of 50 as compared to one of 10 per cent concentration having a relative volume of 10. It is plain thereby that the latter has one fifth of the volume of the sludge having a 2 per cent concentration.

In discussing separating effect, mention of the Brownian and other movements was purposely omitted as it was not the purpose of the paper to go into a highly technical discussion of the innumerable theoretical factors that must be considered in the development of a centrifugal machine. All data given were of the simplest character in order to lay the basis for the discussion of the problems and trends in centrifugal-machine application. The paper was designed to stay more on the practical than the theoretical side.

Motive-Power Characteristics and Lightweight Equipment

By R. EKSERGIAN,[1] PHILADELPHIA, PA.

A common basis of units and ratings for the analysis of different types of locomotive power units is discussed. On the basis of these units a performance comparison of steam, Diesel-electric, and straight-electric locomotives is made. The characteristics of locomotive performance are expressed in terms of rated power and tractive force at rated speed. Consideration is given to the variation in power and tractive force in terms of a power-speed function and the tractive-force amplification relative to its rated value. The power and weight ratios of locomotive to train at different power-weight ratios for the locomotive itself are also discussed. Finally, the adhesion conditions and adhesion-weight ratio to total locomotive weight are shown to depend upon the rated speed, the tractive-force amplification and the power-weight ratio of the locomotive. The effect of power characteristics on grades and double heading are discussed. In the conclusion flexibility of ratings and economic considerations are discussed.

THE OBJECT of this paper is a preliminary survey, by a common method, of power ratings and performance characteristics of various types of power units, together with an evaluation of power savings effected by lightweight equipment, both in the train as well as in the locomotive. In order to express various characteristics of power units for performance analysis, it is necessary to reduce such characteristics to a common basis of units and ratings. With a given weight of train, and power-plant weight, definite horsepower capacity is necessary in order to meet a definite schedule.

Since both inertia and total train resistance increase with the tonnage of the train, the power required for a given performance likewise increases with the total train weight. To obtain high speeds with rapid acceleration, it is necessary either to increase the horsepower capacity of the power plant or reduce the weight of the train itself. We are confronted with a very difficult problem in increasing performance ratings with orthodox equipment. To increase the performance of the train from $2^1/_2$ hp per ton to a performance rate of 4 hp per ton, with a train at present utilizing 3000 hp, would require a power plant of nearly 5000 hp capacity. It is obvious, therefore, that to maintain the same power capacity and effect corresponding increase in performance requires considerable weight reduction of train equipment. On the other hand, where a performance schedule is satisfactory, weight reduction is obviously effective in reducing the power demand for the train. In either case, it is obvious that weight reduction has its primary effect in reducing the horsepower requirements of the power plant. Therefore, any consideration of weight reduction also requires corresponding consideration, with harmony and coordination, in the adaptation of consistent power-plant units in order to obtain full advantages of light weight.

With increasing performance demands, the weight efficiency of the power plant itself becomes an important factor. At very high power ratings, weight efficiency in the power plant becomes equally important as weight efficiency in the train itself.

Much ambiguity has arisen in the rating of locomotives. In some cases, the horsepower capacity of the unit is stated in the terms of the engine horsepower and other cases in terms of horsepower developed at the rail. Steam locomotives are frequently rated on their drawbar horsepower, and sometimes their indicated horsepower. In the following analysis, the horsepower rating of a locomotive will be considered as that developed at the rail only, so that when power ratings are stated in terms of engine horsepower, as in Diesel equipment, transmission efficiencies must be taken into consideration. Likewise, horsepower ratings in terms of drawbar horsepower for steam locomotives require corresponding corrections to rail horsepower at the driver.

Rated Horsepower

The power available for traction at the rail varies in a given speed range with different types of power units. In a steam locomotive, the cylinders, drivers, and boiler are proportioned for a given normal power output corresponding to its operating or rated speed, roughly at two thirds of maximum speed. At other speeds, either in the overspeed or underspeed zone, the power developed is reduced to a percentage of the rated output. In a self-contained Diesel-electric locomotive, the power rating depends upon the normal power rating of the engine and the transmission efficiency and occurs at a rated speed roughly 40 per cent of the maximum speed, at which the motors develop their continuous rated torque. This occurs practically at maximum rail output. Diesel-electric locomotives maintain nearly constant power over a wide overspeed range.

Characteristics of straight-electric locomotives differ essentially from that of electric drives with self-contained power plants. First, the available power is relatively unlimited, and second, the supply of power is at constant voltage. Due to this feature, electric locomotives have considerable amplification of power over their continuous ratings. This amplification of power is effected by overloading the motors and sustaining full starting tractive force to a speed limited by line voltage.

On the basis of the power variation against speed, the actual power at the rail is xH_{PR}, where H_{PR} is the rated horsepower and x is the ratio of the actual horsepower developed at V mph to the rated horsepower at rated speed V_R. The tractive force in terms of the rated output is

$$Z = \frac{375xH_{PR}}{V} \quad \ldots\ldots\ldots\ldots\ldots\ldots [1]$$

where $x = f(V/V_R)$ = the power-speed function.

The train speed is obviously correlated with the revolutions

[1] Consulting Engineer, Edward G. Budd Mfg. Co. Mem. A.S.M.E.

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Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

of the drivers per minute through the wheel diameter and gear ratios.

Performance Characteristics

If all locomotives had exactly the same variation in power over a given speed range, then the performance could be completely determined by its rated power. Due to the variation in power against speed with different types of power units, it is obvious that for a given horsepower per ton, based on its rated value, different schedules will be obtained. For a given schedule, the rated power must be greater for a locomotive with poor power-speed characteristics.

The motion of the train, including the locomotive, is governed by the relation

$$Z - R \pm 20WG = 2000\left(\frac{kW}{g}\right)a \quad \ldots\ldots [2]$$

where a = acceleration, ft per sec per sec; A = acceleration, mph per sec; R = total train resistance, lb; W = weight of train including locomotive, tons; G = gradient, per cent; k = allowance factor for rotational inertia of rotating parts, as wheels, motors, and axles; $r = R/W$ = train resistance, lb per ton.

It is more convenient to express these components on a unit basis per ton, so that Equation [2] reduces to

$$Z_t = r \pm 20G + 91kA, \text{ where } Z_t = \frac{Z}{W}$$

With $k = 1.1$

$$Z_t = r \pm 20G + 100A \quad \ldots\ldots [3]$$

Thus

$$Z_t = \frac{375}{V}\left[\frac{xH_{PR}}{W}\right] = \frac{375}{V}x(H_{PR})_t \quad \ldots\ldots [4]$$

and

$(H_{PR})_t$ = rated horsepower per ton = power rating

From this relation, it is obvious that the performance of a train depends primarily on the available power per ton of total train weight and the variation of this power throughout the speed range. Further, the tractive force falls off with increasing speed. At low speeds, since Z is large and r is small, the greater part of the tractive force is utilized in overcoming the inertia resistance $100A$ of the train. With increasing speeds Z drops and r rapidly increases, with resulting reduction in acceleration. At the maximum or balance speed, $A = 0$, so that the tractive force then just balances the train resistance.

The resistance per ton varies with different types of equipment, as well as with the condition of track and roadbed. The resistance per ton r is usually divided into (1) journal and rolling resistance, (2) flange resistance which is proportional to the speed, (3) air resistance which depends on the geometrical configuration of the train and the speed. Since the air-resistance component increases nearly as the square of the speed, it becomes of importance at the higher speeds. The air resistance itself can be divided into head and tail resistance, and side skin friction and turbulence, this latter being the greater part of the air resistance, particularly with long trains.[2]

Over an average operating schedule, with stops, grades, and slowdowns, these variations in train resistance play a relatively small part compared with acceleration and grade resistance.[3] For this reason since the latter, as well as a large component of the train resistance itself, are directly proportional to the weight of the train, the general performace is governed primarily by the available tractive force per ton over a given speed range.

The effect of auxiliary power for train equipment is either a reduction of available power for traction, when taken from the power plant, or an apparent increase of train resistance with axle-driven generator equipment. With steam air-conditioning and training heating, the steam available for traction is reduced.

When power is taken from car-axle drives, it directly reduces the locomotive power for traction so that q_pH_{PR} is the effective rated power for train performance, and $(1 - q_p)H_{PR}$ is utilized for auxiliary power. Let P_A = auxiliary power at rail per car, W_c = weight of car in tons, and W_t = weight of train not including locomotive. W = total weight of train, tons.

Then

$$(1 - q_p)H_{PR} = \frac{P_A}{W_c}W_t \quad \ldots\ldots [5]$$

Therefore

$$q_p = 1 - \frac{P_A}{W_c}\frac{W_t}{W(H_{PR})_t} \quad \ldots\ldots [6]$$

With $(H_{PR})_t = 3$, $P_A/W_c = 0.6$ for 60-ton cars and $W_t/W = 0.7$, then $q_p = 0.86$.

Power and Weight Proportions of Locomotive to Train Weight

To estimate the power requirements of a locomotive for a given train schedule requires not only its power rating, but also a correction for accounting for the variation in power at other than its rated speed. Let H_{PR} = rated horsepower per ton of train weight developed at driver; W_p = weight of power unit, tons; W_t = weight of train, tons; thus, $W = W_p + W_t$ = total train weight including locomotive.

Let q account for different power characteristics with allowance for train auxiliaries. The power requirement for meeting a given schedule is defined as performance rating, which is

$$\frac{qH_{PR}}{W_p + W_t} = h_{pt} \quad \ldots\ldots [7][4]$$

Therefore

$$h_{pt} = q(H_{PR})_t \quad \ldots\ldots [7a]$$

where $q = q_Rq_p$ is divided into q_R for comparing different power-speed functions and q_p for train auxiliaries. For example, assume a typical schedule requires 4 hp per ton power rating for a steam locomotive. With more favorable power-speed characteristics, assume a Diesel-electric requires 3.5 hp per ton for the same schedule. Then q_R accounts for the ratio of these ratings. Referring to the steam locomotive as a standard with $q_R = 1$ then q_R for the Diesel is 4/3.5. With the Diesel as a standard, then q_R for the steam locomotive is 3.5/4.

Weight Ratio. A very important constant in locomotive design is the ratio of the weight of the total locomotive in pounds divided by the rated horsepower at the drivers. This ratio will be defined as the weight ratio, the units of which are pounds per rated horse-

[2] For further discussion of train and air resistance see "Air Resistance of Railroad Equipment," by A. I. Lipetz, Trans. A.S.M.E., vol. 59, 1937, paper RR-59-4, p. 617. Also: "Design of Light-Weight Trains," by R. Eksergian, Trans. A.S.M.E., vol. 56, 1934, paper RR-56-4, p. 667.

[3] Variation in train resistance per ton where the air-resistance component is large, due to trains operating at high sustained speeds, is discussed later in connection with Equation [7].

[4] The term q can also account for deviations in train resistance for different train consists where air resistance is of importance at high sustained speeds.

power. In the case of steam locomotives, weight ratios are frequently given in terms of the locomotive without tender. In comparison with other types of power, as with Diesel-electric units, it is necessary to include the tender weight. When the weight ratio of the steam locomotive is stated in terms of the locomotive itself, then the weight ratio, as a power plant which includes tender, is increased by the ratio of tender weight to locomotive weight.

Let E = weight ratio of locomotive without tender, K = ratio of tender weight to locomotive weight, W_L = weight of locomotive alone, W_D = weight of tender, and $W_L + W_D = W_P$ = total weight of power unit. With the weights expressed in tons, then

$$\left.\begin{aligned} E &= \frac{2000W_L}{H_{PR}} \\ E(1+K) &= \frac{2000W_P}{H_{PR}} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots [8]$$

The relation can be equally applied to Diesel or electric locomotives where in this case $K = 0$. A steam locomotive weighing 120 lb per hp with a tender weighing 75 per cent of the locomotive itself, is increased to 210 lb per hp for the total power plant. Tender weights will include two-thirds maximum fuel and water capacity.

Since the performance rating is equal to the total horsepower at the rail divided by the weight of the power plant and train in tons, while the weight ratio is expressed in terms of the weight of the power plant in pounds divided by the horsepower developed at the rail, it can be shown on combining these relations that

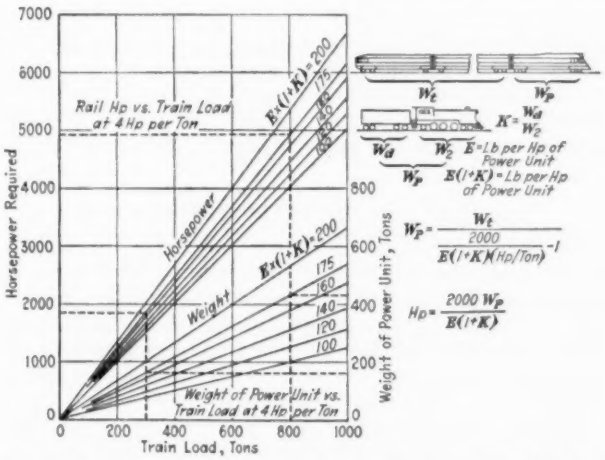


FIG. 1 RAIL HORSEPOWER AND WEIGHT OF POWER UNIT VS. TRAIN LOAD FOR A RATING OF 4 HP PER TON

the weight of the power plant increases directly as the train weight and inversely as a function depending upon the weight ratio of the power unit. Likewise, the horsepower developed by the locomotive is proportional to the train weight and the power rating required, and inversely as a function of the weight ratio of the power unit.

That is, combining Equations [7] and [8]

$$W_P = \frac{W_t}{\dfrac{2000q}{E(1+K)h_{pt}} - 1} \quad \ldots\ldots\ldots\ldots [9]^5$$

and

$$H_{PR} = \frac{2000W_P}{E(1+K)} = \frac{2000W_t h_{pt}}{2000q - E(1+K)h_{pt}} \quad \ldots\ldots [10]^5$$

[5] The term h_{pt} can be replaced by $(H_{PR})_t$ by putting $q = 1$.

Referring to Fig. 1, these relations are plotted for a relatively high power rating of 4 hp per ton, at various weight ratios against train load. Similar relations can be obtained for different power ratings. Since the size, weight, and power of a locomotive increase in direct proportion to the weight of the train for a given performance requirement, it is obvious that a reduction in train weight is most effective in reducing size of power unit. Thus, at a power rating of 4 hp per ton power rating, a 10-car train with 85-ton cars, requires, with a locomotive weighing 174 lb per hp, 5250 hp. With a like train of 10 cars weighing 45 tons each, the power is reduced to 2800 hp. In the former, double-head power would be needed with tremendous increase of operating cost.

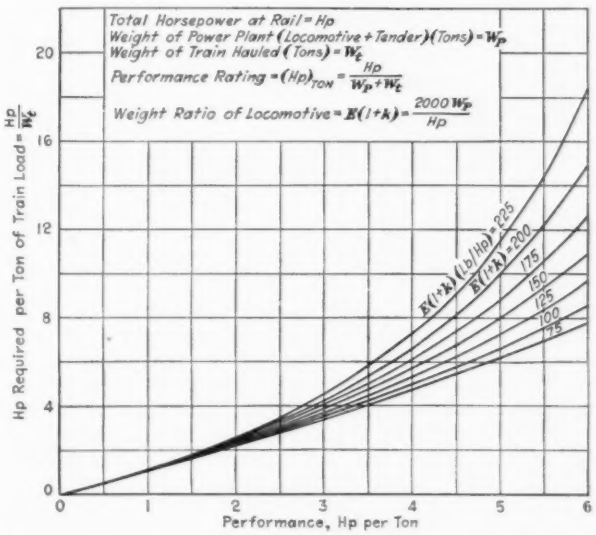


FIG. 2 TRAIN POWER REQUIREMENTS AT VARIOUS LOCOMOTIVE-WEIGHT RATIOS

While the motive power required increases directly with the train weight, it also is very much affected by the weight ratio of the power unit itself.

The locomotive power required to haul a ton of train load, from Equation [10], is

$$\frac{H_{PR}}{W_t} = \frac{2000h_{pt}}{2000q - E(1+K)h_{pt}} \quad \ldots\ldots\ldots [11]^5$$

Fig. 2 shows the effect of weight ratio of the power unit to the power required to haul a ton of train load for different performance ratings, neglecting train auxiliary power. A series of curves at various weight ratios of the locomotive are plotted, showing the horsepower required per ton of train hauled, at various performance ratings. It is to be particularly noted that the weight ratio of the power plant is relatively unimportant at low performance ratings.[6] This is almost obvious, since in considering a long freight train the effect of weight efficiency of the locomotive would have very little reaction on its performance as a whole.[6] On the other hand, as we increase toward the higher performance ratings, the power required increases considerably per ton of train hauled. In the higher power range, the weight ratio of the power plant becomes of great importance. Thus, at a performance rating of 5 hp per ton, a change from 225 to 175 lb per hp in the weight ratio of a locomotive amounts to a saving of $2^1/_2$ hp for each ton of train load

[6] Obviously, a ton of weight saved on a locomotive always means an increase of 1 ton in payload, regardless of the size of train load.

hauled. With an 850-ton train, the savings through reducing the power-unit weight from 225 to 175 lb per hp exceeds 2000 hp.

For a given train load, a given power-plant weight is necessary. The power required to pull a ton of train weight increases with its performance rating, and is also a function of the ratio of power-plant weight to train weight. Obviously, the lower the ratio of the power-plant weight to the train weight, the less is the amplification of the horsepower required per ton of train hauled at a given performance rate. Further, the ratio of weight of power plant to weight of train depends upon the weight ratio of the locomotive itself.

From Equation [9], the ratio of power-unit weight to train weight is given by the expression

$$\frac{W_P}{W_t} = \frac{E(1 + K)h_{pt}}{2000q - E(1 + K)h_{pt}} \quad \ldots\ldots\ldots [12]^5$$

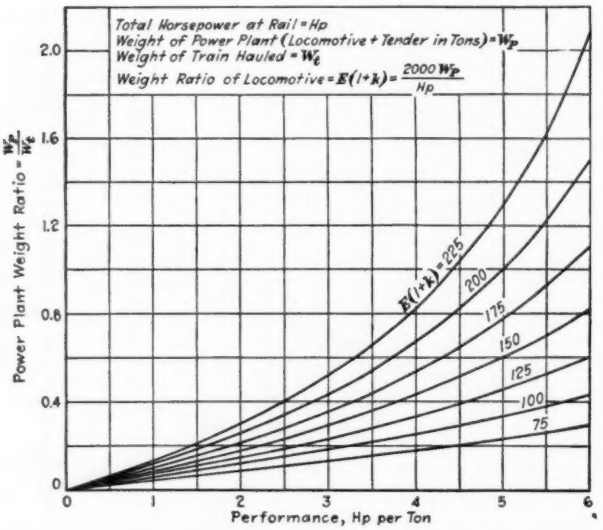


FIG. 3 RATIO OF LOCOMOTIVE WEIGHT TO WEIGHT OF TRAIN HAULED

Referring to Fig. 3, the ratio of weight of power plant to train weight hauled is plotted against the performance rating at different weight ratios of the power unit. These curves make obvious the tremendous increase of power plant to train weight in the higher performance power zone. Thus, a locomotive weighing 200 lb per hp, with a low performance requirement at say 2 hp per ton, has a weight of only 25 per cent of the train weight. On the other hand, for high-speed performance at 4 hp per ton the locomotive will weigh two thirds of the train weight. Should it be desirable to increase the power capacity to a performance rate of 6 hp per ton, as for high-speed suburban service, the locomotive weight becomes 50 per cent greater than the train load itself. Obviously, this is nearly all locomotive. This condition in the high-power-rating zone can be greatly relieved by reducing the power-plant weight ratio. Thus, if the power plant is reduced to 100 lb per hp at a power rating of 6 hp per ton, the weight of the power plant to the train weight reduces to 43 per cent. This shows why distributed electric power using electric motors throughout, as in multiunit electric trains, is the only present means of obtaining very high performance ratios. It is very difficult to reduce the weight efficiency of a self-contained power unit to anything comparable to distributed power with electrification.

Obviously, a ton of weight reduction of the locomotive is as effective as a ton of weight saving in the train hauled, since in either case the power required is equally reduced by the same horsepower-per-ton rating.

At the lower performance ratings, as with a long freight train, reducing the weight of the locomotive, no matter how much, has little effect in reducing the total weight of the train and locomotive. In this case a small percentage in the reduction of the train hauled is far more effective than a large percentage in weight reduction of the locomotive. As the performance rating increases, however, the weight of the power plant becomes a far more important factor, therefore, its weight reduction becomes increasingly important. As an example, let us compare a percentage weight reduction of 25 per cent, first, on the locomotive by reducing its weight by 25 per cent, then on the train by reducing its weight by 25 per cent.

Let the original weight ratio for the locomotive alone be E = 200 lb per hp, while W_t = original weight of train alone. With a 25 per cent reduction on the locomotive side $E = 0.75 \times 200$ = 150 lb per hp. With a 25 per cent reduction on the train side $W_t' = 0.75W_t$. The locomotive power requirements for different performance ratings, (1) by reducing locomotive weight by 25 per cent, and (2) by reducing train load by 25 per cent, separately, are as given in Table 1.

From Table 1 it should be noted that around 5 hp per ton, or

TABLE 1 LOCOMOTIVE POWER REQUIREMENTS FOR DIFFERENT PERFORMANCE RATINGS

Performance rating HP_R, hp per ton of train weight	Original horsepower per ton of train weight W_t	Horsepower required per ton of train weight W_t	
		By reducing locomotive weight 25 per cent	By reducing train weight 25 per cent
1.5	1.75	1.69	1.32
2.0	2.50	2.36	1.87
3.0	4.35	3.93	3.26
4.0	6.70	5.72	5.00
5.0	10.00	8.00	7.50

over, reducing the power-plant weight is practically as effective as reducing the train weight. On the other hand, at lower performance ratings, the gain is greater by reducing the train load.

PERFORMANCE CHARACTERISTICS OF LOCOMOTIVES

The performance of a locomotive is governed by the variation of the power in its speed range relative to its rated power at rated speed. The ratio of the power developed at any speed V mph, to the rated power at rated speed V_R mph has been defined as the power-speed function x. It can also be regarded as the performance efficiency. It measures the efficiency of power developed at various speeds relative to the rated output.

Different types of locomotives have definite power and tractive-force characteristics. The tractive force is limited by the characteristics of the transmission and the power capacity of the unit. In a steam locomotive the low-speed tractive force with late cutoff is roughly double that of the tractive force at rated speed with early cutoff. With a given steam supply, the cutoff at a given speed is fairly definite, since the cylinders are based on proportions necessary for developing rated horsepower at rated speed V_R. With Diesel-electric locomotives, the tractive-force variation depends upon the characteristics of the electric drive. The rated power occurs when the motors run at their continuous rating. Below the rated speed at which the motors develop their continuous torque, the motors are overloaded, resulting in considerable torque amplification at the lower speeds. The starting torque is based on the allowable overload current relative to its continuous-current rating. The variation of the transmission efficiency against speed with constant engine output, relative to the continuous current efficiency at rated speed, is the performance efficiency of the drive.

With straight-electric locomotives and external power, the

rated output corresponds to the continuous rating of the motors at their rated speed. The starting torque corresponds to the allowable overload torque of the motors. This torque can be sustained to a speed dependent on line voltage. Beyond this speed the motors operate on their constant-voltage characteristics with decreasing torque at increasing speeds. The overspeed range, beyond the rated speed, is further extended by weakening of the motor fields. At the speed corresponding to the motor-field change, the current loading is increased with increased torque and power output. Beyond this overspeed-field change, the motors operate on their constant-voltage weakened field characteristics.

Direct steam-turbine drives have starting torques roughly double their rated torques at rated speed. The torque decreases linearly with increasing speed. The power output, therefore, varies as a parabola with the rated power and torque at a speed corresponding to the peak of the parabola. The reciprocating-steam-locomotive drive has very similar characteristics.

With hydraulic drives, the torque converter has characteristics on the output end exactly similar to the direct steam-turbine drive. To extend the speed range, a change-over to a hydraulic coupling is effected. The speed change is then obtained through the slip of the coupling. Torque amplification with reduced speeds takes place with the converter, but no torque amplification can be effected through the coupling.

The characteristics of all locomotive drives are referred to the rated output H_{PR} at rated speed V_R. Therefore, the rated tractive force is

$$Z_R = \frac{375H_{PR}}{V_R} \quad \ldots\ldots\ldots\ldots\ldots\ldots [13]$$

The power at any other speed V is xH_{PR}, where x is the power-speed function or the performance efficiency and the corresponding tractive force, at V mph, is

$$Z = \frac{375xH_{PR}}{V} \quad \ldots\ldots\ldots\ldots\ldots\ldots [14]$$

The tractive-force amplification is defined as the ratio of the tractive force at V mph to the rated tractive force at V_R mph, that is, the tractive-force amplification is

$$\phi = Z/Z_R \quad \ldots\ldots\ldots\ldots\ldots\ldots [15]$$

On substituting Equations [11] and [12] in Equation [13] we have

$$x = (V/V_R)\phi \quad \ldots\ldots\ldots\ldots\ldots\ldots [16]$$

which shows the relation of the speed-performance efficiency with torque amplification at V mph. Thus, with a steam locomotive with a drag speed around 0.25 of the rated speed and tractive-force amplification around 2, the power output relative to its rated value is $x = 0.5$. With Diesel-electric, $\phi = 3$ when $V/V_R = 0.25$, so that $x = 0.75$.

The performance speed range is limited by the maximum allowable speed of the locomotive, V_{max}. It is not to be confused with actual peak speeds with a given train load. It is rather the limiting overspeed allowance above the rated speed consistent with not too much falling off in the power curve against speed and with mechanical limitations of the drive itself. For instance, in steam locomotives, where the rated speed occurs around 200 to 250 rpm, the overspeed range should not much exceed the diameter speed of 336 rpm. Further increase of speed will also cause a very rapid falling off in power with poor operating efficiency.

The maximum performance speed V_{max} is definitely related with the rated speed V_R for different types of locomotives. Let $\gamma_R = V_R/V_{max}$ = the ratio of rated to maximum-performance speed. For steam locomotives γ_R varies from 0.6 to 0.7. For Diesel-electric locomotives γ_R varies from 0.35 to 0.45. For straight-electric locomotives γ_R varies from 0.5 to 0.6. Direct steam-turbine drives have values for γ_R similar to steam locomotives. For hydraulic drives with converter and coupling, $\gamma_R = 0.45$ approximately for the rated output and torque of the converter.

From Equation [16] it will be noted that low values of V_R, and therefore γ_R, result in increased performance efficiencies x, in the low-speed range, so that every effort should be made to increase the overspeed range in order to lower γ_R. The more favorable overspeed characteristics of the electric drive with Diesel locomotives permit lower values of γ_R.

The tractive force can be expressed directly in terms of the tractive-force amplification ϕ by the relation

$$Z = \frac{375\phi H_{PR}}{V_R} = \frac{375\phi H_{PR}}{\gamma_R V_{max}} \quad \ldots\ldots\ldots\ldots [17]$$

By definition $\phi = 1$ at rated speed. In the low-speed range, ϕ is greater than 1, and represents a torque amplification. In the overspeed range ϕ is a fraction.

The drive, through its limitations in tractive-force amplification ϕ at a given speed V, must be consistent with the power capacity and really sets the efficiency of power performance in relation to its rated capacity. In other words, the variation of ϕ is a characteristic of the drive, or, more broadly, that of a given type of locomotive. Thus, in a steam locomotive the tractive-force variation not only depends upon the reciprocating drive, but the boiler capacity as well. However, with a relatively fixed steam supply, the performance characteristics are then more dependent upon the cylinder characteristics. In a direct steam-turbine drive, with a constant steam supply, the characteristics of tractive-force variation are entirely dependent upon the variable-speed characteristics of the turbine. For a Diesel-electric locomotive, with constant engine output, the tractive-force variation and the performance at rail output are also dependent upon the characteristics of the electric transmission. In a straight-electric locomotive the performance characteristics are dependent upon the characteristics of the motors.

From Equation [15], however, while ϕ represents the characteristics of the drive, or, more broadly, that of the locomotive itself,[7] the actual tractive-force variation is also dependent upon the selection of the rated speed $V_R = \gamma_R V_{max}$. The tractive force can be raised in the low-speed range for a given tractive-force variation ϕ relative to its rated value (a characteristic of the drive) by lowering the rated speed ratio γ_R. With parabolic power characteristics, this implies poorer characteristics and a greater dropping of the tractive force in the overspeed range. A problem in locomotive design is to improve the overspeed characteristics to a more constant rail output, thus extending the overspeed range and lowering the rated speed in order to raise the tractive force in the low-speed range.

When the speed V of a locomotive is expressed in terms of the maximum performance speed V_{max}, and if $\gamma = V/V_{max}$ = speed ratio, then, from Equation [14] $x = \gamma\phi/\gamma_R$.

Starting Tractive Force

At very low speeds in the drag-speed range the tractive force depends primarily on the characteristics of the drive rather than the power capacity of the locomotive.[8] Thus, at speeds up to

[7] In a steam locomotive, the cutoff must be adjusted to the steam supply at a given speed and this corresponds to a given firing rate.

[8] See discussion on "Adhesion Limitations."

roughly one quarter rated speed, the tractive force of a steam locomotive is limited by cylinder proportions, because at drag speeds the boiler capacity is in excess of the steam demand of the cylinders. With a Diesel-electric locomotive the tractive force is limited by the allowable overload torque on the motors consistent with heating and commutation limitations. The torque reaction at the generator does not load the engine to its capacity so that the limitation in starting tractive force is the overload capacity of the motors.

The starting tractive force can be sustained through a given speed range $\gamma_D V_R$ known as the drag-speed range. Above this speed the tractive force also becomes limited by the power capacity of the unit. In a steam locomotive not only cylinder characteristics but boiler capacity must be considered. With Diesel-electric locomotives the tractive force depends upon the efficiency of the drive with full loading on the engine. The slight falling off in tractive force in the drag-speed range is a limitation of the drive itself as, for example, wire drawing in reciprocating-engine cylinders or motor characteristics.

The starting-tractive-force amplification ϕ_i is defined as the ratio of starting tractive force Z_i to rated tractive force Z_R at rated speed V_R. That is

$$\phi_i = Z_i/Z_R \quad \ldots\ldots [18]$$

where $Z_R = (375H_{PR})/V_R$; therefore

$$Z_i = \frac{375\phi_i H_{PR}}{V_R} = \frac{375\phi_i H_{PR}}{\gamma_R V_{\max}} \quad \ldots\ldots [19]$$

For reciprocating steam locomotives and direct steam-turbine

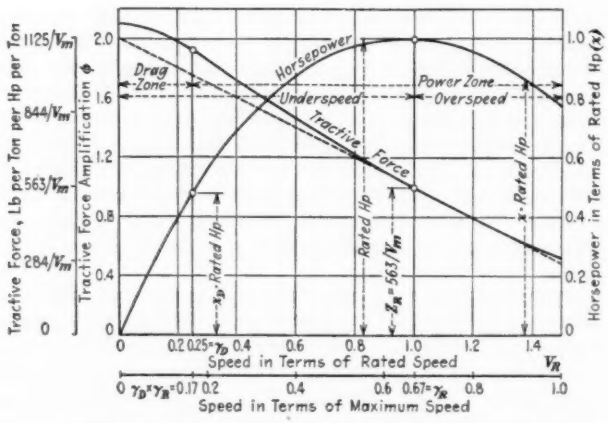


FIG. 4 STEAM-LOCOMOTIVE PERFORMANCE CURVES

drives, $\phi_i = 2$ to 2.3, with $\gamma_R = 0.6$ to 0.7. With Diesel-electric drives $\phi_i = 3$ to 4, with $\gamma_R = 0.35$ to 0.45.

The starting tractive force not only depends upon ϕ_i, but is also influenced considerably by the choice of γ_R. With fixed values of ϕ_i for a given type of drive, the only way to increase the starting tractive force Z_i is either by lowering the rated speed ratio γ_R (or V_R) or by increasing the rated output of the locomotive, except by the use of auxiliary booster engines that go out of action at the end of the drag-speed range. To lower the rated-speed ratio requires longer overspeed characteristics with improved performance characteristics in this range.

The starting tractive force must be consistent with the power capacity of the locomotive at the end of the drag speed. If the time in the drag-speed range is too small, not much benefit in over-all performance is obtained by the use of too high a starting tractive force. Usually it is desirable to maintain the starting tractive force, without too much falling off, up to approximately one quarter or one fifth of the rated speed. At the end of the drag speed, $V_D = \gamma_D V_R = \gamma_D \gamma_R V_{\max}$, where $\gamma_D = 0.2$ to 0.25 and the performance efficiency is x_D. Hence, the tractive force at end of drag speed V_D is

$$\left.\begin{aligned} Z_D = Z_i \text{ (approx)} &= \frac{375 x_D H_{PR}}{\gamma_D V_R} \\ \text{or } Z_i &= \frac{375 x_D H_{PR}}{\gamma_D \gamma_R V_{\max}} \text{ (approx)} \end{aligned}\right\} \quad \ldots\ldots [20]$$

Also the tractive-force amplification is related to the performance efficiency at drag speed V_D by the relation

$$\phi_D = \phi_i \text{ (approx)} = \frac{x_D V_R}{V_D} = \frac{x_D}{\gamma_D} \quad \ldots\ldots [21]$$

Thus, with $\gamma_D = 0.25$ and $x_D = 0.5$, $\phi_i = 2$. With a higher power speed ratio $x_D = 0.75$, then $\phi_i = 3$, approximately. Since

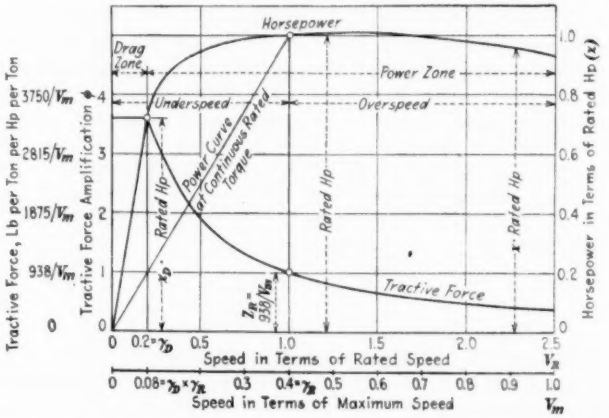


FIG. 5 PERFORMANCE CURVES OF A DIESEL-ELECTRIC LOCOMOTIVE

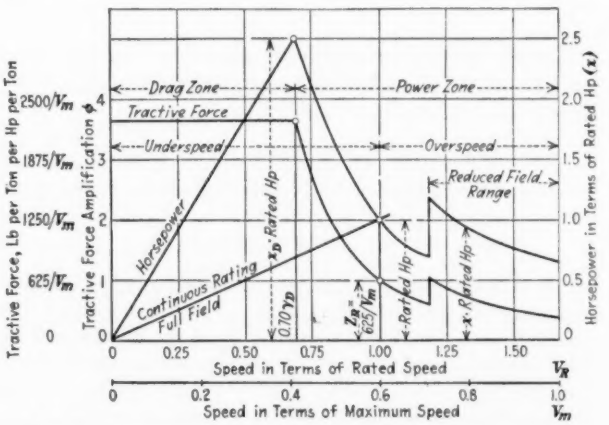


FIG. 6 PERFORMANCE CURVES OF AN ELECTRIC LOCOMOTIVE

x_D is not likely to exceed 0.8, even with the most ideal drive, the maximum tractive-force amplification should not exceed $\phi_i = 4$.

From Equations [17] and [18], the starting tractive force is definitely related with the speed-performance range of the locomotive. To maintain a high starting tractive force with a low power rating, as in switcher service, the performance range must be low. To increase the starting tractive force for high-speed road locomotives, with γ_R fixed for a given type, the power capacity of the unit must be increased. Auxiliary booster engines permit larger values of γ_R with increased starting tractive force.

In Figs. 4, 5, and 6 typical power and tractive-force relations

against speed are plotted showing typical characteristics for steam, Diesel-electric, and straight-electric locomotives.[9] The speeds are plotted in terms of rated speed V_R and maximum performance speed V_m. With unit horsepower at rated speed, the ratio of actual to rated power x, at various speed ratios in the performance range are shown. The tractive-force variation is measured either in terms of tractive-force amplification ϕ relative to the rated tractive force or directly in terms of tractive force per ton. The tractive force per ton is given by the relation

$$Z_t = \frac{375x}{\gamma}\left[\frac{(H_{PR})_t}{V_m}\right] = \frac{375\phi}{\gamma_R}\left[\frac{(H_{PR})_t}{V_m}\right] \quad \ldots\ldots [22]$$

where $\gamma = V/V_m$ = speed ratio and $\gamma_R = V_R/V_m$ = rated-speed ratio.

The tractive-force scale in pounds per ton is measured in $(H_{PR})_t/V_m$.

Fig. 4 shows the variation of power and tractive force against speed for a typical steam locomotive. The rated speed occurs at a speed two thirds of the maximum performance speed V_m. Therefore, the overspeed range is 1.5 times the rated speed V_R The ratio of actual to rated horsepower at maximum speed V_m is 0.78. The drag speed is 0.25 V_R or 0.17 V_m and the power ratio is 0.48. The starting tractive force amplification is ϕ = 2.1. The rated tractive force, in pounds per ton is

$$(Z_R)_t = \frac{375}{0.66}\,\frac{(H_{PR})_t}{V_m} = 563\,\frac{(H_{PR})_t}{V_m} \quad \ldots\ldots [23]$$

and the starting tractive force, in pounds per ton, is

$$(Z_i)_t = \frac{375 \times 2.1}{0.66}\,\frac{(H_{PR})_t}{V_m} = 1180\,\frac{(H_{PR})_t}{V_m} \quad \ldots\ldots [24]$$

Thus at a power rating of 3 hp per ton and a performance speed range to 85 mph, the starting tractive force is 42 lb per ton.

Steam-locomotive characteristics do not differ greatly from a linear variation at which the starting tractive force is approximately twice the rated tractive force. Therefore, the power curve approximates a parabola with the rated values at the peak of the parabola.

Fig. 5 shows typical power and tractive-force characteristics of a Diesel-electric locomotive. The rating of the locomotive is taken at the continuous rating of the motors. The inclined line shows the power variation of the product of rated tractive force and speed plotted against speed. Obviously, the intersection of this line with the power curve corresponds to the rated output. The rated-speed ratio is $\gamma_R = 0.4$, so that the overspeed range is 2.5 V_R. With electric transmissions in Diesel locomotives the maximum output slightly exceeds the continuous current power rating, so that, for maximum output, $x = 1.01$. The starting tractive-force amplification $\phi = 3.6$. The rated tractive force is increased by the lowering of the rated-speed ratio to $\gamma_R = 0.4$. For this case, the rated tractive force, in pounds per ton, is

$$(Z_R)_t = \frac{375}{0.4}\,\frac{(H_{PR})_t}{V_m} = 938\,\frac{(H_{PR})_t}{V_m} \quad \ldots\ldots [25]$$

and the starting tractive force, in pounds per ton, is

$$(Z_i)_t = \frac{375 \times 3.6}{0.4}\,\frac{(H_{PR})_t}{V_m} = 3370\,\frac{(H_{PR})_t}{V_m} \quad \ldots\ldots [26]$$

It should be noted that at a speed ratio $\gamma = 0.66$ the tractive force agrees with that of the steam locomotive. Below and above this speed, the Diesel-electric locomotive has superior tractive-force and power characteristics. However, it is important to emphasize that the comparisons are based on the rated output at the rail.

In the steam locomotive, the ratio of the rail output to cylinder indicated output at rated speed exceeds 90 per cent. In the Diesel-electric transmission, the efficiency of transmission, i.e., the ratio of rail output to engine output, at rated speed, varies from 75 to 80 per cent. Since the analysis is based on rail output, further corrections for efficiency allowance must be made when considering engine outputs.

Fig. 6 shows basic characteristics of straight-electric locomotives. The distinctive characteristic of this type of power unit is a large power amplification over its continuous-current rating. The ratio of maximum power to continuous-current rated power reaches a power amplification[10] $x = 2.5$ at a speed ratio relative to the performance range at $\gamma_P = 0.41$. The continuous-current output has a rated-speed ratio $\gamma_R = 0.59$. The changeover to reduced speed occurs roughly when $\gamma = 0.7$. The starting tractive force is sustained to maximum output with $\gamma_P = 0.41$. The starting tractive-force amplification[10] $\phi = 3.65$, relative to the rated tractive force at continuous-current rating. This, of course, agrees with Diesel-electric starting tractive-force amplification since in both cases the limitation is based on the motor capacity. The rated tractive force in pounds per ton, is

$$(Z_R)_t = \frac{375}{0.59}\,\frac{(H_{PR})_t}{V_m} = 630\,\frac{(H_{PR})_t}{V_m} \quad \ldots\ldots [27]$$

and the starting tractive force, in pounds per ton, is

$$(Z_i)_t = \frac{3.65 \times 375}{0.59}\,\frac{(H_{PR})_t}{V_m} = 2200\,\frac{(H_{PR})_t}{V_m} \quad \ldots\ldots [28]$$

Electric locomotives have large power amplification in the low speeds; therefore, they are well adapted for rapid acceleration from slowdowns. If the greater part of the schedule be in the slow-speed range, then the performance rating will considerably exceed the continuous-power rating. For high speeds, however, the performance rating is less than the continuous-current rating, so that the latter is more nearly the true rating of the locomotive.

Fig. 7 shows the characteristics of a typical hydraulic-turbo-transmission. The characteristics shown approximate those of the Voith turbo two-stage transmission. The transmission is assumed to operate at the normal rated speed of a nearly constant-torque Diesel engine. In the first stage, the transmission operates on the converter stage where torque amplification is effected between the engine (with impeller) and the turbine rotor through reaction stationary vanes. With constant engine torque at rated speed, the flow rate at different turbine speeds does not vary much. Therefore, the output characteristics of the converter are practically those of a direct turbine drive at nearly constant flow rate. The output torque and corresponding tractive force varies nearly linearly with an approach to a parabolic power-output curve. The characteristics in the converter range closely resemble those of the reciprocating steam locomotive. In order to extend the speed range, a hydraulic coupling (without torque amplification) comes into action at approximately 0.68 V_m, where V_m = maximum speed range. The converter performance efficiency at this speed is $x = 0.87$. At a rated speed of 0.45 V_m, the over-all efficiency is 83 per cent, so that the

[9] For a different treatment of these relations see the report of the committee on Economics of Railway Locatio.1, Proceedings of The American Railway Engineering Association, vol. 39, 1938, pp. 440–484.

[10] In average operation, the power amplification in electric locomotives is frequently less than 2. Tractive-force amplification ranges from 2.5 to 3.

over-all efficiency of the converter at the coupling entrance is 72 per cent. The efficiency of the coupling at entrance is around 96 per cent, which corresponds to 4 per cent slip.

The changeover from converter to coupling requires a sudden reduction in engine speed. Let Φ_P = primary or engine torque and ω_P = engine speed. Let Φ_S = secondary or output torque and ω_S = secondary speed. Let ϵ_R = transmission efficiency at rated output speed. Let $\epsilon = x\epsilon_R$ = over-all efficiency of converter and $1 - S$ = efficiency of coupling, with slip S. Then, at changeover speed where ω_{PR} = rated engine speed

$$\epsilon\Phi_P\omega_{PR} = \Phi_S\omega_S \quad \ldots\ldots [29]$$

for the converter; since, at this change-over speed, $\Phi_P = \Phi_S$ nearly, $\omega_{PR} = \omega_S/\epsilon$. With the coupling engaging, and since

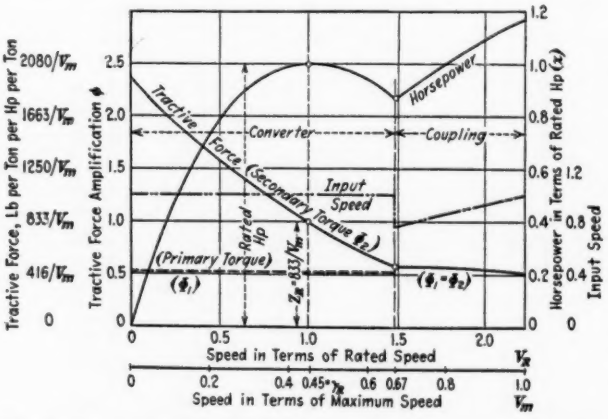


FIG. 7 PERFORMANCE CURVES OF A DIESEL-ELECTRIC LOCOMOTIVE WITH HYDRAULIC TRANSMISSION

necessarily $\Phi_P = \Phi_S$, then $\omega_{P_1} = \omega_S/(1 - S)$. Hence, the drop in engine speed is

$$\omega_{PR} - \omega_{P_1} = \left[1 - \frac{\epsilon}{(1 - S)}\right]\omega_{PR}$$

Therefore, referring to Fig. 7

$$\omega_{PR} - \omega_{P_1} = 0.25\omega_{PR} \quad \ldots\ldots [30]$$

In the second-stage operation with coupling, the engine speed increases from 0.75 ω_{PR} to its rated speed ω_{PR}. Neglecting the small slip, the engine speed increases directly with the speed of the train, with practically constant engine torque transmitted through the coupling. Thus, the power curve rises very nearly as a linear function with increasing train speed. The limiting speed occurs when $S = 0$.

The overspeed can be extended by a second coupling, the intermediate coupling having a mechanical gear for torque amplification and the second coupling a direct drive for torque amplification.

ANALYTICAL RELATIONS

Tractive-force speed relations lie between two extremes:

(1) The first relation is that where tractive force varies as a linear function of the speed, which corresponds to a parabolic power curve against speed. In this case, given the rated tractive force Z_R at rated speed V_R, then the starting tractive force is $Z_i = \phi_i Z_R$, and the tractive force and power, respectively, at speed V, are

$$\left.\begin{aligned} Z &= Z_i - mV \\ P &= (Z_i - mV)V \end{aligned}\right\} \quad \ldots\ldots [31]$$

where

$$m = \frac{Z_i - Z_R}{V_R} = (\phi_i - 1)\frac{Z_R}{V_R} \quad \ldots\ldots [32]$$

At maximum power, corresponding to the power rating, $V = V_R$ so that

$$\frac{dP}{dV} = Z_i - 2mV_R = 0$$

Therefore

$$m = \frac{Z_i}{2V_R} \quad \ldots\ldots [33]$$

and $\phi_i = 2$, so that $Z_i = 2Z_R$.

The tractive-force speed equation in terms of rated power is

$$Z = \frac{375H_{PR}}{V_R}\left(2 - \frac{V}{V_R}\right) \quad \ldots\ldots [34]$$

so that by lowering the rated speed, the tractive force is increased.

(2) The second relation is that where tractive force varies inversely as the speed, which corresponds to a constant-power curve independent of speed (except during starting-tractive-effort drag-speed zones).

ADHESION REQUIREMENTS

The adhesion weight on drivers must be consistent with the starting tractive force. The factor of adhesion must allow for variation in tractive force per revolution; that is to say, the limiting friction force corresponds to the peak tractive force, so that the mean tractive force requires a lower mean friction coefficient. Therefore, in conjunction with adhesion limitations, an important ratio is that of maximum to mean tractive force exerted at the drivers per revolution. This ratio is approximately 1.23 for two-cylinder steam locomotives with ordinary rod obliquities. With three-cylinder locomotives having cranks 120 deg apart, this ratio is reduced to 1.08. On the basis of a peak friction coefficient of 0.3, the mean value for two-cylinder locomotives is 0.245. The ratio of necessary weight on the drivers at the rail to tractive force limited by friction, i.e., the factor of adhesion, is 4.1. In other words, the weight on drivers must exceed four times the starting tractive force.

With electric motors, the effect of weight transfer is an important factor; since the motors are in series, the total tractive force is limited by the most slippery driver.

The ratio of weight on drivers to the total weight of the locomotive is defined as the adhesion-weight ratio. It determines the necessary driving wheel base to the total wheel base of the locomotive, as well as the necessary weight distribution for these component parts. Therefore, the starting tractive force is limited by the product of the adhesion-weight ratio and the total weight of the locomotive. On the other hand, the starting tractive force is proportional to the rated horsepower times the tractive-force amplification and divided by the rated speed. The weight of the power unit can be expressed in terms of the rated power and the power-weight ratio. On combining these relations we can obtain the limiting adhesion-weight ratio in terms of the rated speed, the tractive-force amplification, and the weight efficiency.

Let W_A = weight on drivers, lb; W_P = weight of power unit, lb; $E(1 + K)$ = weight ratio, lb per rated horsepower; H_{PR} = rated horsepower; V_R = rated speed, mph; ϕ_i = starting tractive-force amplification; and $\mu = 1/F_A$ = mean friction coefficient, where A = factor of adhesion, then the adhesion-weight ratio is

$$C = \frac{W_A}{W_P} \quad \ldots\ldots [35]$$

where

$$W_P = E(1 + K)H_{PR} \quad [36]$$

Therefore, the starting tractive force in terms of the adhesion limit is

$$Z_i = \mu W_A = \mu C W_P \quad [37]$$

Therefore

$$Z_i = \mu C E(1 + K)H_{PR} \quad [38]$$

But this tractive force must be consistent with the tractive force limited by the tractive-force amplification of the drive, that is, with

$$Z_i = \frac{375\phi_i H_{PR}}{V_R} \quad [39]$$

On equating expressions [38] and [39]

$$C = \frac{375}{\mu(V_R/\phi_i)} \frac{1}{E(1 + K)} \quad [40]$$

Thus, the adhesion-weight ratio varies inversely as the weight efficiency of the power unit and also inversely as the ratio of rated speed to tractive-force amplification. In terms of the rated-speed ratio $\gamma_R = V_R/V_m$ and the factor of adhesion $F_A = 1/\mu$

$$C = \frac{375F_A\phi_i}{\gamma_R E(1 + K)V_m} \quad [41]$$

which shows that the weight on the drivers to the total locomotive weight increases directly with the tractive-force amplification and inversely with the rated-speed ratio, the weight efficiency and the speed range V_m of the locomotive. Thus the operating speed range V_m has a major influence on the necessary weight on drivers. High-speed locomotives with a large operating range require less weight on drivers than locomotives in freight service which have a lower operating speed range. In switching service V_m is inherently small, so the entire weight of the locomotive is required for adhesion.

Referring to Fig. 8, the adhesion-weight ratio is plotted against weight efficiencies of the locomotive at various ratios of rated speed to starting tractive-force amplification. The mean friction coefficient is taken at 0.25. For other values of friction coefficients, it is only necessary to multiply the adhesion-weight ratio by 0.25 divided by the assumed mean coefficient of friction. The lower chart shows rated speed V_R plotted against starting tractive-force amplification ϕ_i at various values of the ratio of rated speed to starting tractive-force amplification.

Low values of V_R/ϕ_i, say from $12^1/_2$ to $17^1/_4$, correspond to low rated speeds V_R and high starting tractive-force amplifications ϕ_i. High rated speeds, in general, have large values of V_R/ϕ_i. With weight ratios $E(1 + K)$ as low as 100 lb per hp, and a low value of V_R/ϕ_i around 15, the adhesion-weight ratio $C = 1$, so that the entire weight of the locomotive is necessary for adhesion. For another example, consider a slow-speed freight locomotive with 59-in. drivers, which at 200 rpm corresponds to a rated speed at 35 mph. Let $\phi_i = 2.3$, then $V_R/\phi_i = 15.2$. Assuming $E(1 + K) = 210$ lb per hp, then by interpolation, the adhesion-weight ratio $C = 0.47$. With a tender-weight ratio $K = 0.75$, the corresponding weight ratio of the locomotive above is 120 lb per hp. Then the adhesion-weight ratio with respect to this freight-locomotive is 0.82.

On the other hand, with a high-speed passenger locomotive with 80-in. drivers and a rated speed at 60 mph with $\phi_i = 2.3$, then $V_R/\phi_i = 26.2$. Considering the locomotive power-weight ratio without tender at 120 lb, then the adhesion-weight ratio by interpolation is 48 per cent. Obviously, with a higher factor of adhesion the adhesion-weight ratio on drivers is increased.

Corresponding running gears would indicate a locomotive of the Santa Fe type for the former and an Atlantic-type locomotive for the latter.

Adhesion Limitations. More logically, the adhesion-weight ratio should be sufficient for the starting tractive force based on the limiting tractive-force amplification of the drive. On the other hand, particularly with high-speed units, tracking limitations of the running gear must be considered, which with rigid wheel bases require adequate guiding trucks. Further limitations are due to allowable wheel-base lengths and weight distribution with limiting axle loads. For these reasons the adhesion-weight ratio may be limited, so that the starting tractive force then becomes limited rather by adhesion than by tractive-force amplification.

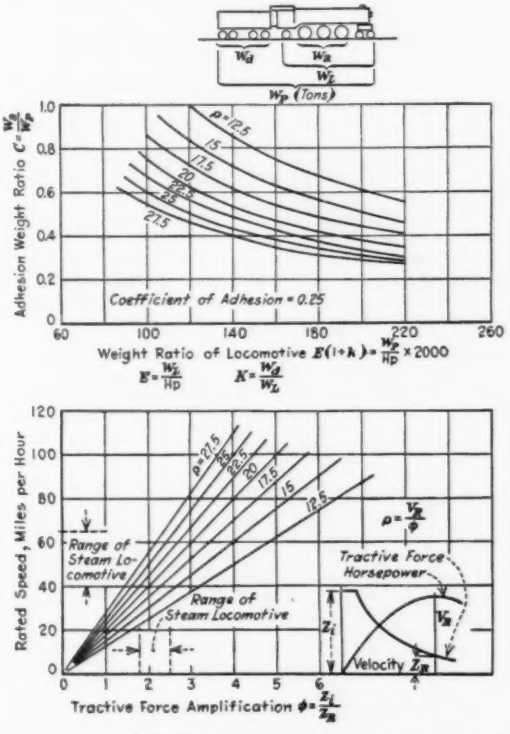


FIG. 8 ADHESION-WEIGHT RATIOS PLOTTED AGAINST WEIGHT RATIOS

The number of driving axles in a locomotive is dependent on the total horsepower required and the limiting power concentration per unit driver. This, in turn, depends upon the allowable axle loading and the ratio of rated speed to tractive-force amplification. With a limiting axle loading A the maximum power is

$$H_{PA} = \frac{\mu A}{375}\left[\frac{V_R}{\phi_i}\right] \quad [42]$$

where μ is the mean friction coefficient.

Referring to Fig. 9, the maximum power concentration per axle is shown plotted against allowable rail axle loadings at various ratios of rated speed to tractive-force amplification and with a factor of adhesion equal to 4. The lower auxiliary plot gives V_R/ϕ_i for a given rated speed and tractive-force amplification.

Axle loadings vary from 40,000 to 80,000 lb; the latter re-

quiring extremely heavy rail. Normal axle loadings range between 50,000 to 65,000 lb. Steam locomotives with high rated speeds have large power ratings per driver.

With a factor of adhesion of 4.2, $\mu = 0.24$. With a rated speed at 60 mph and a torque amplification at 2.3, then $V_R/\phi_i = 26$. With a limiting axle loading at 63,000 lb, then the limiting power per axle is 1050 hp. For a 3000-hp locomotive, three driving axles are needed. For a freight locomotive, with rated speed at 200 rpm and 69-in. wheels, $V_R = (200/336) \times 69 = 41$ mph. With $\phi_i = 2.3$ and $A = 63{,}000$ lb, $\mu = 0.24$; then

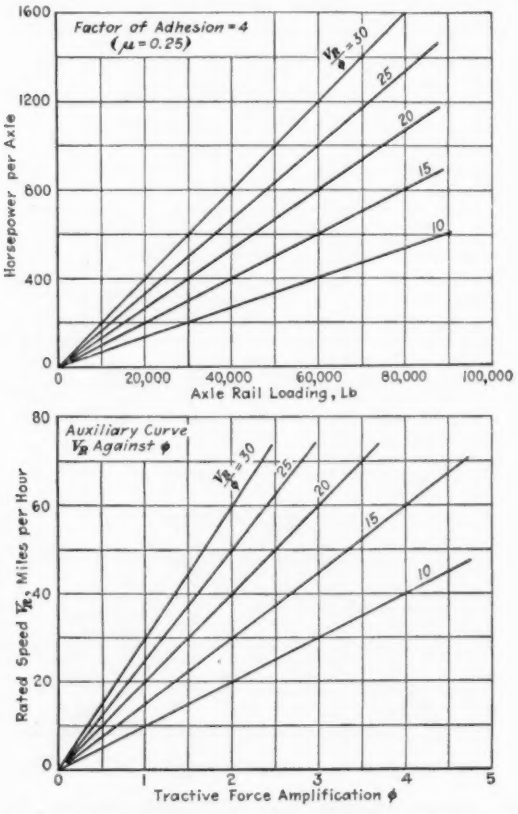


FIG. 9 MAXIMUM PERMISSIBLE HORSEPOWER PER AXLE FOR DIFFERENT AXLE LOADINGS

the limiting power per driver is 710 hp. For a 3000-hp rating, at least four drivers are necessary.

For an electric drive with $V_R = 37.5$ mph and $\phi_i = 3$, $\mu = 0.25$ and $A = 55{,}000$ lb; then, the limiting power per axle is 460 hp. For a 3000-hp locomotive at least six axles are required.

Wheel diameters D should preferably not be less than $A/1350$.

Ordinarily, the wheel diameter is fixed by the allowable revolutions per minute N_R at rated speed V_R. That is

$$D = \frac{336}{N_R} V_R \quad \ldots\ldots\ldots\ldots\ldots\ldots [43]$$

where the driving-wheel diameter is expressed in inches.

In steam locomotives, N_R ranges from 200 to 250 rpm. With electric drives, $N_R = N_m/\alpha$, where α = gear ratio and N_m rated rpm of rotor armature.

Besides these factors, constructional limitations due to boiler and allowable rigid wheel base and space limitations in motor trucks very often fix the limiting wheel diameter. Thus, in steam locomotives, maximum wheel diameters, due to boiler limitations, are limited to from 80- to 84-in. diameter. Mountain-type locomotives are limited to smaller diameters due to limiting rigid wheel bases.

POWER REQUIREMENTS FOR GRADES

Power requirements on grades usually require special consideration, particularly with low power ratings. Also, since the operating speed on a grade is low relative to the rated speed, the power developed is some fraction of the rated power. In other words, the speed-performance efficiency of the power curve must be taken into consideration in connection with the operating grade speed for estimating the required rated power.

As a simple illustration, consider a 2 per cent grade which corresponds to a grade resistance of 40 lb per ton with an additional allowance of 10 lb per ton for train and other resistances. With an operating grade speed of 30 mph, the actual power requirements at the rail is $(50 \times 30)/375 = 4$ hp per ton.

With a speed-performance efficiency of 75 per cent at 30 mph, the rated power is increased to 5.33 hp per ton. Thus, even with high power-performance ratings, heavy grades usually require additional power, so that a new formulation of power relations must be evaluated.

Obviously without additional power, the operating speed on a grade must be consistent with the tractive-force-speed characteristics of the locomotive and the balancing grade plus train resistance. Thus, it is only necessary to add to the train-resistance curves against speed the grade resistance of $20G$. Where these curves intersect the tractive force, expressed in pounds per ton of total train plus locomotive, is the balance speed corresponding to the particular grade.

With additional power, the power characteristics of the booster locomotive usually is different from that of the main power unit. Therefore, this, along with its weight efficiency, must be considered.

When a given speed up a specified grade is required, the power requirements of the booster locomotive become fixed. If Z_1 and Z_2 are the tractive forces of the main and the booster locomotives, respectively, W_{P1} and W_{P2} = the corresponding weights of the power units, and R_V = the train resistance at V mph; then, for a train weight W_t

$$Z_1 + Z_2 = (20G + R_V)(W_{P1} + W_{P2} + W_t) \ldots\ldots [44]$$

But we note

$$\left.\begin{aligned} Z_1 &= \frac{375\phi_1 H_{PR1}}{V_{R1}} \\ Z_2 &= \frac{375\phi_2 H_{PR2}}{V_{R2}} \end{aligned}\right\} \ldots\ldots\ldots\ldots\ldots [45]$$

where H_{PR1} is the rated horsepower of the main locomotive, H_{PR2} is the rated horsepower of the booster locomotive, and $E_1(1 + K_1)$ and $E_2(1 + K_2)$ are their respective weight efficiencies in pounds per horsepower. Also ϕ_1 and ϕ_2 are the tractive-force amplifications at V mph relative to their rated speed V_{R1} and V_{R2}, respectively. Hence

$$\frac{375\phi_1 H_{PR1}}{V_{R1}} + \frac{375\phi_2 H_{PR2}}{V_{R2}} = (20G + R_V)\left[\frac{E_1(1 + K_1)H_{PR1}}{2000} + \frac{E_2(1 + K_2)H_{PR2}}{2000} + W_t\right] \ldots\ldots [46]$$

Since $\phi_1 = x_1 V_{R1}/V$ and $\phi_2 = x_2 V_{R2}/V$. Then

$$x_1 H_{PR1} + x_2 H_{PR2} = \frac{(20G + R_V)}{375}\left[\frac{E_1(1 + K_1)H_{PR1}}{2000} + \frac{E_2(1 + K_2)H_{PR2}}{2000} + W_t\right] V \ldots\ldots [47]$$

expressed in horsepower, which is obviously the power equation.

The power requirement of the booster locomotive, given the main-locomotive power rating, is

$$H_{PR_2} = \frac{(20G + R_V)W_t - \left[\frac{375\phi_1}{V_{R_1}} - (20G + R_V)\frac{E_1(1 + K_1)}{2000}\right]H_{PR_1}}{375\,\frac{\phi_2}{V_{R_2}} - (20G + R_V)\,\frac{E_2(1 + K_2)}{2000}} \quad \ldots\ldots [48]$$

As an example, consider a train load $W_t = 1000$ tons on a 2 per cent grade, pulled by a main locomotive for which $H_{PR_1} = 3000$ hp with its rated speed at 60 mph and a weight efficiency $E(1 + K) = 220$ lb per hp. Let the booster locomotive have a rated speed $V_{R_2} = 35$ mph and let its weight efficiency $E(1 + K) = 200$ lb per hp.

What rated horsepower is required for the booster locomotive so that the double-headed train makes the grade at 20 mph? Assume the train resistance at this speed to be approximately 8 lb per ton.

At a speed ratio of 20/60 $V_{R_1} = 0.33V_{R_1}$, the tractive-force amplification of the main locomotive is $\phi_1 = 1.80$. At a speed ratio 20/35 $V_{R_2} = 0.57V_{R_2}$, the tractive-force amplification for the booster locomotive is $\phi_2 = 1.5$. From Equation [48]. $H_{PR_2} = 2650$ hp.

Conclusions

It is important to emphasize that all types of locomotives have more or less flexibility in rating, both as to rated speed and to corresponding power developed. Thus, in a steam locomotive, various rates of firing result in different evaporation rates. Methods of operation, for example, operating at various cutoffs, will give different road characteristics. Likewise, Diesel engines can increase their output by overspeeding, supercharging, or by increased fuel injection at a given engine speed. Steam-turbine units can augment their torques by increasing the flow rates and overloading the boilers. Therefore, the rating of a locomotive at best can be considered only as an approximation.

Certain types of locomotives are further considerably influenced in their performance ratings by modes of operation, as well as extraneous weather conditions. With electric drives, the ratings of motors are dependent upon their heat ratings so that under heavy loading at low speeds, the motors may be overloaded for a long time duration, thus requiring a reduction of performance rating for a given service. Finally, it is important to emphasize that the power-performance efficiencies at very low and at very high speeds, where these efficiencies are frequently poor, are difficult to estimate accurately due, in general, to possible overloading capacity of the power unit to meet special conditions in these ranges.

In a final analysis, the adoption of one type of power unit over another depends upon its economic justification. A fairly universal adoption of one type of power unit may have a far-reaching effect on the mode of operation of a railroad, as in electrification. On the other hand, with mixed power and self-contained units, as with steam, Diesel, and turbine locomotives where a certain degree of interchangeability must exist, it is possible to draw immediate comparisons over typical operating schedules under common maintenance conditions and hostelry.

Since the performance of a horsepower is only slightly affected by the type of power unit, a primary factor is the cost per horsepower mile. Obviously, this simply amounts to the cost per horsepower per annum divided by the mileage availability of the unit per annum. Increasing costs of superior-performing power units can only be justified to the extent of their increased availability and to where in the modes of operation of a railroad, such as in schedules, such availability can be utilized.

The primary costs attributed to the locomotive itself are (1) fixed charges, including allowance for depreciation and obsolescence, (2) maintenance charges, and (3) fuel and operating costs. These charges are for a locomotive of given rated horsepower. The charges added together per annum and then divided by the mileage availability of the unit per annum and the rated horsepower can be expressed as the cost per 1000 horsepower miles.

Thus, higher fixed charges for improved designs can be sustained provided they react in greater mileage availability per annum, with lower maintenance, fuel, and operating costs.

An immediate effect of weight savings on the locomotive or train side, is either a reduction in horsepower miles and corresponding costs or an increase of train capacity and pay load for a given schedule.

Acknowledgments

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Discussion

Lawford H. Fry.[11] The author states that this paper is a preliminary survey, and his statement of purpose shows that it covers a very wide territory. Study of the paper shows that the survey is in the nature of a sketch map, rather than a careful triangulation. This should be emphasized before someone unfamiliar with the territory tries to use the paper to plot a course.

The author proposes to describe by one common method the operating characteristics of locomotives of all species, steam, Diesel-electric, turbo-geared, and line-fed electric. All classes of service from drag-freight to ultra-high speed are to be covered. The project is extremely ambitious and would have been more successful if the analysis had been more incisive. The author writes: "To obtain high speeds with rapid acceleration it is necessary either to increase the horsepower or reduce the weight of the train. We are confronted with a very difficult problem in increasing performance ratings with orthodox equipment. . . . It is obvious, therefore, that to maintain the same power capacity and to effect corresponding increase in performance requires considerable weight reduction of train equipment."

It is hard to agree that this is obvious. No mention is made of the possibility of reducing train resistance or of increasing locomotive efficiency. Even aside from this, the statement is incomplete. The author looks at only one side of the problem and thus throws his analysis out of balance. It may, as he says, be difficult to increase locomotive performance, but it is certainly not easy to reduce train weight while keeping pace with increasing demands for comfort and safety.

It is unfortunate that the paper does not present a background of basic facts from which the conclusions are derived. Figs. 4, 5, 6, and 7 are given to represent the tractive effort and horsepower characteristics of steam, Diesel-electric, line-fed electric, and Diesel-hydraulic locomotives. A single plot is offered to

[11] Railway Engineer, Edgewater Steel Company, Pittsburgh, Pa. Mem. A.S.M.E.

cover all types of locomotives of each of the species. No information is given as to the type or size of the locomotives on which the curves are based, nor any indication as to the original sources of the curves. These four plots, Figs. 4 to 6, seem a slender foundation for a complete theory of locomotive performance.

Also, much more searching analysis is required if any effective use is to be made of the so-called rated horsepower, rated speed, and rated tractive effort on which the various plots are based.

Some thirty years ago the term rated tractive effort was introduced to designate the cylinder tractive effort computed for a mean effective pressure of 85 per cent of the boiler pressure. This was intended to represent the maximum tractive effort available at the rim of the drivers at slow speed. The term has been quite extensively used in this connection, but the rated tractive effort of the present paper is something entirely different. It seems unfortunate that a well-established term should be given a new significance. For each species of locomotive a rated speed is chosen, and the horsepower and tractive effort at this speed are designated as "rated." The rated horsepower is used as a basis for developing equations to describe operating characteristics. The drawback to this is that the author does not provide any clear-cut definition for rated horsepower and rated speed. In describing Figs. 4 to 6, the rated speed, which sets the value of the rated horsepower, is located by a different rule in each case. In Fig. 4, steam, the rated speed is $^2/_3$ of the maximum operating speed, which is assumed to be 336 rpm; in Figs. 5 and 6, Diesel-electric and line-fed electric, the rated speed is the speed at the continuous current rating of the motors.

These rather indefinite rules for the value of the rated speed, which are given in descriptions of the illustrations do not agree with those given under the subheading, "rated horsepower." When the conclusion is reached, the author says in effect, it is important to emphasize the fact that, for all types of locomotives, the rating as to speed and horsepower is more or less flexible and can at best be considered only as an approximation.

After this statement it seems legitimate to ask whether it was worth while to develop forty complicated equations on the uncertain basis of the rated horsepower.

Before answering this question, the equations themselves should be considered. Equation [3] represents the tractive effort required to overcome train resistance and acceleration,[12] while Equation [4] is an arbitrary grouping of symbols intended to represent the tractive effort made available by the locomotive. From examination of these two expressions, the author concludes: "It is obvious that train performance depends primarily on the available power per ton of total train weight;" and "over an average operating schedule. . . . these variations in train resistance (due to speed and to differences in equipment[13]) play a relatively small part compared with acceleration and grade resistance." It does not seem obvious to the writer that consideration of speed is unnecessary in estimating train resistance. Also, a question must be asked as to the meaning of "average operating schedule." How is it possible to average drag-freight and superspeed runs? The same question comes up in connection with Equation [7] for performance rating. The author assumes that a typical schedule requires 4 hp per ton for steam and 3.5 hp per ton for Diesel-electric power. "Typical schedule" is not defined, and no information is given as to how the horsepower values are obtained.

Equation [10], which forms the basis for Fig. 1, rates critical attention. The left-hand side is composed of the single symbol H_{pr}, the rated horsepower, which supposedly has a constant value for a given locomotive. On the right-hand side the numerator is the product of W_t, the weight of the train, and the so-called performance rating, h_{pt}, which is the horsepower available at a given speed and therefore varies with the speed. The variable in the numerator must be eliminated if the two sides are to be harmonious. This is done by having a similar variable in the denominator. With the variable, V, removed from numerator and denominator on the right-hand side, Equation [10] assumes a more logical form. It is then seen to be not a real equation, but an identity expressing the arithmetic fact that if values are given for train weight, locomotive weight, and horsepower, these establish values for horsepower per ton of total train and for locomotive weight per horsepower.

Fig. 1, which is plotted from Equation [10], is merely an abacus to facilitate the arithmetical computation of the relations between the foregoing quantities. In discussing the conclusions to be drawn the author says: "Since size, weight, and power of a locomotive increase in direct proportion to the weight of the train for a given performance, it is obvious that a reduction in train weight is most effective in reducing size of power unit." Omission of any consideration of locomotive efficiency and of train resistance makes this statement inadequate, to say the least.

Figs. 4, 5, 6, and 7 are discussed generally, a good deal of attention being given to the Diesel hydraulic-turbo-coupled locomotive, although this does not seem to be of any great practical importance or to have any special bearing on the main theme of the paper. Most of the discussion is very sketchy, with occasional numerical values supplied, but with no statement of origin to support such values. In many cases values are given in such broad terms as to be meaningless. To say that axle loads vary from 40,000 to 80,000 lb, with normal loads between 50,000 and 65,000 lb, does not assist the expert nor enlighten the ignorant. Under the subhead "Power Requirements for Grades," the author introduces calculations for helper locomotives. The practical value of this is not apparent, as it is not common practice to design special locomotives for helper service.

In publishing the paper the author should make clear that it is a preliminary survey; that Figs. 4 to 6 are merely illustrative, and that any values given are not absolute and may be greatly modified in practice.

Unless this is done, the paper contains elements of danger. The author would not go wrong in applying his principles and equations, nor would any other experienced locomotive engineer, but if the paper is broadcast in its present form, there is always the possibility that it may be misused.

It would be quite possible by using the present equations and selecting numerical data to arrive at some very remarkable conclusions which might be put forward as being based on the authority of The American Society of Mechanical Engineers, and the reputation of Dr. Eksergian. This contingency should be avoided.

E. E. KIMBALL.[14] The author has referred to a similar study of the characteristics of steam, electric, and Diesel-electric locomotives, described in a report of the Committee on the Economics of Railway Location of the American Railway Engineering ing Association. It may be helpful briefly to outline some of this work.

The object of the study was to develop a method for comparing the characteristics of the principal types and classes of locomotives used by the railroads of this country, in order to determine their effect upon the economics of railway location and operation.

[12] In developing Equation [3], the author estimates the rotative inertia as 10 per cent of the longitudinal. This is very high, being twice the value frequently used.

[13] Parenthetical statement by the writer.

[14] Schenectady, N. Y. Chairman, Subcommittee No. 1, Economics of Railway Location and Operation, American Railway Engineering Association.

The work of the committee largely parallels that of the author, except that it is intended to apply to specific examples rather than to a preliminary survey of a much wider field.

Fundamentally the object of any locomotive is to furnish power to propel trains over a railroad. For this reason it is necessary to consider what there is about different types of locomotives which limit their power.

Usually the power is limited in three ways:

(*a*) The amount of power which can be used is limited by the adhesion between the drivers and the rails and varies with the speed as represented by the line *OA* in Figs. 10 and 11, based on a constant coefficient of adhesion (25 per cent for steam and 30 per cent for Diesel-electric types, as explained later).

(*b*) In the case of a steam locomotive, the amount of power which is available depends upon the amount of steam the boiler is capable of generating, which is equivalent to a certain amount of horsepower delivered to the cylinders and which can be represented by the horizontal line *B'C'* in Fig. 10, designated maximum boiler capacity.

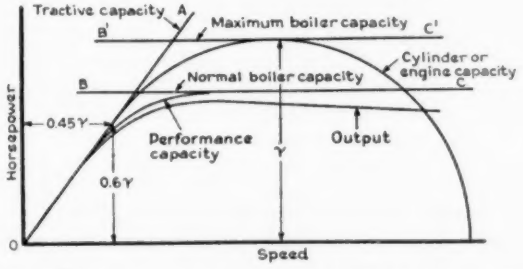


FIG. 10 DIAGRAM ILLUSTRATING TRACTIVE CAPACITY, BOILER CAPACITY AND ENGINE CAPACITY OF STEAM LOCOMOTIVES

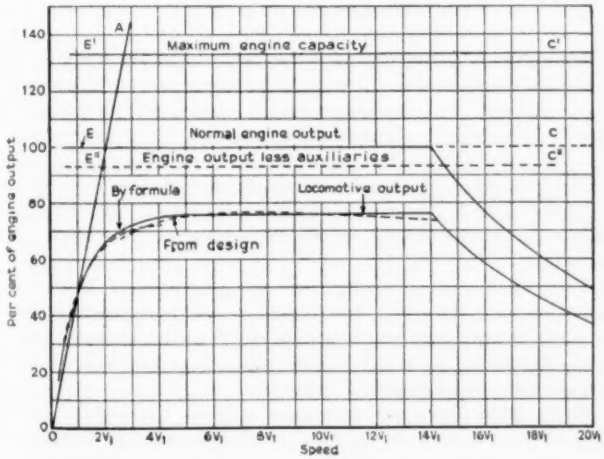


FIG. 11 TYPICAL HORSEPOWER CURVES OF OIL-ELECTRIC LOCOMOTIVES

(*c*) In order to have a locomotive, which can work up to the adhesion limit of its drivers and which can use all the steam that the boiler can generate, it is necessary to design the engine with sufficient capacity to do both.

That is, if the characteristics of the engine can be represented by a circle, then the limiting characteristics of the locomotive can be obtained by drawing a circle, tangent to the lines *OA* and *B'C'*, Fig. 10.

A circle may not represent the characteristics of the engine but it can be used for purposes of discussion. In the low-speed range, the amount of steam which can be used is primarily limited by the number of strokes per minute. As the speed increases, it is possible to use more and more steam and to develop more and more horsepower but, with full cutoff and increasing amounts of steam, the branch-pipe pressure decreases and the back pressure increases, which react to limit the amount of steam and the horsepower which can be developed with full cutoff.

By shortening the cutoff, the steam is used more economically and for a while it is assumed possible to use more steam than when

FORM FOR CALCULATING THE TRACTIVE EFFORT AND HORSEPOWER OUTPUT OF OIL-ELECTRIC LOCOMOTIVES — FORM G

_____ RAILROAD

Item
1. Type (Wheel Arrangement)_____
2. Weight on Drivers (Tons)_____
3. Weight of Locomotive_____
4. Rated Horsepower of Power Unit_____
5. Horsepower Required for Auxiliaries_____
6. *Nominal Horsepower Output of Locomotive_____
7. Horsepower per ton on Drivers Item (6)/Item (2) = K_____
8. V_1 = .395 x Item (7)_____

Normal Gearing				Special Gearing				Horsepower Output	
Speed		Driver Tractive Effort		A		B			
V	MPH	Lbs. Per Ton on Drivers	Total	MPH	Driver T.E. Total	MPH	Driver T.E. Total	Per Ton on Drivers	Total
(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(j)	(k)
0	0	600		0		0		0	0
1.00xV1		600						.638xK	
1.25 "		542						.713 "	
1.50 "		498						.777 "	
1.75 "		448						.826 "	
2.00 "		411						.865 "	
2.50 "		348						.918 "	
3.00 "		300						.950 "	
3.50 "		263						.970 "	
4.00 "		233						.982 "	
4.50 "		209						.989 "	
5.00 "		189						.993 "	
6.00 "		156						.998 "	
7.00 "		135						.999 "	
8.00 "		119						1.000 "	
10.00 "		95						1.000 "	
12.00 "		79						1.000 "	
14.00 "		67						1.000 "	
1.10xV1								(U).826 "	
1.20 "								(X).694 "	
1.30 "								(Y).592 "	
1.40 "								(Z).510 "	

*Nominal horsepower output of locomotive depends upon engine capacity and allowances for auxiliaries.
Assume 80 per cent (78-82) of Item (4) for road locomotives
75 per cent (73-77) of Item (4) for switch locomotives and rail cars including auxiliaries
(U) (X) (Y) (Z) are the horsepower outputs corresponding to 10, 80, 30 and 40 per cent over speed (after generator reaches full voltage)

Spaces A and B are provided in case it is necessary to change motor gearing to meet high speed conditions.

FIG. 12

operating with full cutoff. Hence, the horsepower continues to increase until the cutoff is shortened to such an extent that the amount of steam, which can be admitted to the cylinders, is limited by the time that the valves remain open. At higher speeds less and less steam is admitted, which means less and less horsepower input and less demand on the boiler.

Aside from these assumptions, Fry has shown that the boiler efficiency decreases as the firing rate increases, consequently it is impractical to operate steam-locomotive boilers at maximum capacity. In fact it can be shown that $^3/_4$ of the maximum capacity can be obtained if the firing rate is limited to $^1/_2$ the maximum. The committee chose this value to represent the normal capacity of the locomotive and based its estimates on a constant firing rate and boiler evaporation, as indicated by the line *BC* in Fig. 10.

On this assumption it is necessary to base the engine performance on a constant supply of steam, which is shown by the curve designated performance capacity.

The horsepower output at the driving axles of the locomotive is obtained by subtracting the power required to overcome the

FORM FOR CALCULATING THE TRACTIVE EFFORT AND HORSEPOWER OUTPUT OF TYPICAL STEAM LOCOMOTIVES USING SUPERHEATED STEAM — FORM B

_____RAILROAD

Item

1. Type (Wheel Arrangement)_____

	Actual	*Adjusted	(a) Booster
2. Weight on Drivers (Tons)_____			
3. Weight of Engine_____			
4. Weight of Tender_____			
5. Total Weight_____			
6. Ratio Item (2)/Item (3)_____			

	Based on Table 2	Based on Design	See Note
7. HP per Ton on Drivers = K_____		#	K'
8. HP Capacity of Locomotive_____			
9. Basic Speed MPH = V_1 = .296 x K_____			V_1'

Speed V	Speed MPH	Cylinder Tractive Effort Per Ton on Drivers	Cylinder Tractive Effort Total	Engine Losses 20 Lb/Ton Act.Wt. on Drivers	Driver Tract. Effort Col(d)-Col(e)	Cylinder HP Per Ton on Drivers Adj.Wt.	Cylinder HP Total	Driver HP Total
(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(i)
0	0	500		0		0	0	0
1.00 x V_1		500				.528 x K		
1.25 "		493				.649 "		
1.50 "		473				.748 "		
1.75 "		447				.824 "		
2.00 "		417				.880 "		
2.25 "		389				.921 "		
2.50 "		361				.948 "		
2.75 "		334				.967 "		
3.00 "		310				.980 "		
3.50 "		270				.993 "		
4.00 "		237				.997 "		
4.50 "		210				.999 "		
5.00 "		189				1.000 "		
6.00 "		158				1.000 "		
7.00 "		(135)						

With Booster

0		500						
1.00 x V_1'		500						
1.25 "		493						
1.50 "		473						
1.75 "		447						
2.00 "		417						

*Adjusted weight on drivers (tons) = Rated tractive effort/500

#HP = K = Item (8)/Adjusted wt. on drivers

Col.(d) = Col.(c) x Item (2) Adjusted — Col.(h) = Col.(g) x Item (2) Adjusted

Col.(f) = Col.(d) - Col.(e) — Col.(i) = Col.(h) x Col.(f)/375

TE corresponding to $7xV_1$ may be limited by cylinder capacity

'a) Note for locomotives with boosters weight on drivers equals rated tractive effort incl. booster/500. Determine K' and V_1'as above.

FIG. 13

TABLE 1 DATA FOR OBTAINING APPROXIMATE HORSEPOWER CAPACITIES OF TYPICAL STEAM LOCOMOTIVES FROM THE WEIGHT OF ENGINE AND WEIGHT ON DRIVERS, BASED ON COAL-FIRED LOCOMOTIVES

Weight on Drivers Per cent Wt. of Eng.	Wt. of Eng. & Tender	HP Per Ton on Drivers	Types of Locomotives Available			Weight on Drivers Per cent Wt. of Eng.	Wt. of Eng. & Tender	HP Per Ton on Drivers	Types of Locomotives Available		
41						71	40.8	28.2	2-10-4	2-8-2	
42						72	41.4	27.8	2-10-4	2-8-2	
43	24.0	40.0	4-4-4			73	42.0	27.4		2-8-2	
44	24.6	39.6	4-4-4			74	42.6	26.9	2-10-2	2-8-2	
45	25.2	39.2	4-4-4			75	43.2	26.5	2-10-2	2-8-2	
46	25.8	38.7	4-4-4			76	43.8	26.1	2-10-2	2-8-2	
47	26.4	38.3	4-4-4			77	44.4	25.7	2-10-2	2-8-2	
48	27.0	37.9				78	45.0	25.3	2-10-2	2-8-2	
49	27.6	37.5				79	45.6	24.8	2-10-2	2-8-2	
50	28.2	37.1	4-4-2			80	46.2	24.4	2-10-2		
51	28.8	36.6	4-4-2			81	46.8	24.0	2-10-2		
52	29.4	36.2	4-4-2	4-6-4		82	47.4	23.6	2-10-2		
53	30.0	35.8	4-4-2	4-6-4		83	48.0	23.2	2-10-2		
54	30.6	35.4	4-4-2	4-6-4		84	48.6	22.7	2-10-2		
55	31.2	34.9	4-4-2	4-6-4		85	49.2	22.3			0-10-2
56	31.8	34.5	4-4-2	4-6-4		86	49.8	21.9			
57	32.4	34.1	4-4-2	4-6-4	4-8-4	87	50.4	21.5		2-8-0	
58	33.0	33.7	4-6-2		4-8-4	88	51.0	21.1		2-8-0	
59	33.6	33.3	4-6-2		4-8-4	89	51.6	20.6		2-8-0	
60	34.2	32.8	4-6-2	2-8-4	4-8-4	90	52.2	20.2	2-10-0	2-8-0	
61	34.8	32.4	4-6-2	2-8-4	4-8-4	91	52.8	19.8	2-10-0	2-8-0	
62	35.4	32.0	4-6-2	2-8-4	4-8-4	92	53.4	19.4	2-10-0	2-8-0	
63	36.0	31.6	4-6-2	2-8-4	4-8-4	93	54.0	18.9	2-10-0	2-8-0	
64	36.6	31.2	4-6-2	2-8-4	4-8-4	94	54.6	18.5			
65	37.2	30.7	4-6-2	2-8-4	4-8-2	95	55.2	18.1			
66	37.8	30.3	2-10-4	2-8-4	4-8-2	96	55.8	17.7			
67	38.4	29.9	2-10-4	2-8-4	4-8-2	97	56.4	17.2			
68	39.0	29.5	2-10-4		4-8-2	98	57.0	16.8			
69	39.6	29.1	2-10-4	2-8-2	4-8-2	99	57.6	16.4			
70	40.2	28.6	2-10-4	2-8-2	4-8-2	100	58.2	16.0			

4 Cylinder Articulated Locomotives

67						79	49.2	22.3	2-8+8-4		
68	44.6	25.6	4-6+6-4			80	49.7	22.0	2-8+8-4		
69	45.0	25.3	4-6+6-4	2-6+6-4		81	50.1	21.7	2-8+8-4		
70	45.4	25.0	4-6+6-4	2-6+6-4		82	50.5	21.4	2-8+8-4		
71	45.8	24.7		2-6+6-4		83	51.0	21.1	2-8+8-4	2-6+6-2	
72	46.2	24.4		2-6+6-4		84	51.4	20.8	2-8+8-4		
73	46.7	24.1		2-6+6-4		85	51.8	20.5	2-8+8-4		
74	47.1	23.8		2-6+6-4		86	52.2	20.2	2-8+8-4	2-8+8-2	
75	47.5	23.5		2-8+8-4		87	52.6	19.9		2-8+8-2	
76	48.0	23.2		2-8+8-4		88	53.0	19.6		2-8+8-2	
77	48.4	22.9	2-8+8-4			89	53.4	19.3		2-8+8-2	
78	48.8	22.6	2-8+8-4			90	53.9	19.0		2-8+8-2	

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friction in the machinery, based on 20 lb per ton on drivers, from the indicated-horsepower input to the cylinders.

In the case of Diesel-electric locomotives, it is customary to assume the overload capacity of the electrical equipment is sufficient to develop tractive efforts equivalent to 30 per cent of the weight on drivers instead of 25 per cent assumed for steam locomotives. The line *OA*, Fig. 11, is drawn on this basis.

The power available on a Diesel-electric locomotive is limited by the horsepower capacity of the power unit. The maximum engine capacity is represented by the horizontal line *E'C'*, Fig 11. In the case of electric drive it is necessary to reduce the standard rating of Diesel engines, because in these cases it is possible to operate them continuously at maximum capacity, which would be injurious to the engines and cause high maintenance. For this reason the normal output of the engine is based on a much lower rating and is held constant by automatic devices embodied in the control as indicated by the horizontal line *EC*, Fig. 11.

In some cases the main engine supplies power for operating auxiliaries, air compressors, fans, battery charging, etc., consequently it is necessary to deduct this power from the output of the engine to obtain the amount of power available for traction.

Since the characteristics and capacities of the component parts of the equipment are or can be tested on the stand before assembling on the locomotive, it is possible to calculate the characteristics of Diesel-electric locomotives very accurately from design data.

The committee making this study developed an empirical formula which closely approximates the horsepowers obtained from design data and has used this formula as the basis for a special work sheet Fig. 12 (form G), for calculating the speed, tractive-effort, and horsepower-output curves of typical Diesel-electric locomotives. The first four columns of the form apply to what may be termed the speed range covered by normal gearing. If a different speed range is desired, it is necessary to change the motor gearing and adjust the speed and tractive efforts accordingly based on the horsepower outputs from column (k).

Form B, Fig. 13, is the corresponding work sheet developed for calculating the speed, tractive-effort, and horsepower curves of typical steam locomotives and is based on an empirical formula derived in a manner similar to that used in deriving the equation for Diesel-electric locomotives.

Steam locomotives are sometimes rated on the basis of road tests and sometimes on the basis of their average performance in service. In order to obtain a rational rating for steam locomotives, the committee assumed that the weight, tracking qualities and capacity depended upon the wheel arrangement or locomotive classification and compiled Table 1 based on an analysis of about 250 different designs of locomotives. This table shows the normal horsepower capacities per ton on drivers (75 per cent of maximum), corresponding to the ratio of the weight on drivers to weight of the engine, the types of locomotives available, and also the ratio of the weight on drivers to the weight of engine and tender for use in preliminary estimates.

In the final analysis or in the case of individual locomotives, it may be necessary to adjust the horsepowers in order to agree with actual design data.

Sample work sheets covering various types of electric loco-

motives together with examples showing their application will be found in the committee's report which will be submitted to the American Railway Engineering Association during its convention in March, 1939.

SIDNEY WITHINGTON.[15] Any attempt to compare power characteristics of locomotives of widely varying types on a common basis of units and ratings is indeed interesting and courageous. As the author very logically concludes, "the rating of a locomotive at best can be considered only an approximation," and there are many variables which make accurate comparison of rating data among different types exceedingly difficult.

It is suggested that, in the discussion of the characteristics of the electric locomotives, the alternating-current single-phase equipment be included, as well as the direct-current equipment. With the voltage-control methods available, the tractive-effort and horsepower characteristics of the alternating-current motor are more flexible and quite definitely more favorable, especially at the higher speeds, than those of the direct-current motor equipment.

Mention is made of the overload capacity of electric motors and the sustaining of full starting tractive force with electric locomotives. This characteristic should be recognized in the diagram, Fig. 6, in which it would seem the slope, at low speeds, of the horsepower curve should be approximately equivalent to that of the Diesel equipment given in Fig. 5, whether the electric locomotives are of the alternating-current or the direct-current type.

The limitation of the electric-motor capacity being a temperature limitation, the horsepower rating of electric locomotives can be assumed to be somewhat higher in winter than in summer, and this is favorable, since train resistance normally is higher in winter than in summer. The horsepower rating characteristic tendency is opposite in steam locomotives where cold weather tends to increase radiation losses and therefore to decrease horsepower rating at a time when increase is desirable. Furthermore, the quantity of steam required for train heating in cold weather also tends to reduce the traction rating of steam equipment at such times. With electric and Diesel locomotives the steam supply for steam air-conditioning and for train heating is usually independent of the traction equipment.

The statement in the paper that "motive power required increases directly with train weight" is open to some qualification, for there are several items of train resistance which are independent of the train weight. The resistance per ton weight of train is greater for light than for heavy trains, other things being equal.

Mention is made in this paper of the varying adhesion characteristics with different types of locomotives. It might be pointed out that, where the driving axles are not coupled, there is a tendency for the forward drivers to lift on account of the moment due to heavy drawbar pull and rail reaction. This would obviously increase the tendency of the leading driving wheels to slip and thus reduce the tractive effort of the locomotive. Coupling the driving axles by side rods of course eliminates this tendency.

While the primary costs attributed to locomotives, reduced to a mileage basis, may theoretically be based upon the "mileage available," this may not be the whole story, for the actual annual mileage may be limited, and usually is limited in practice, not by mileage available, but by traffic requirements.

AUTHOR'S CLOSURE

It is apparent that Mr. Fry has misinterpreted the author's purpose. The object of the paper was a coordination by a common basis of units and ratings for the analysis of different types of power units.

The paper is by no means a sketch map as Mr. Fry would infer, but, to the contrary, it represents a careful study of basic design limitations. Certains criticisms are very difficult to answer. For instance, his objections, "that a more searching analysis is required if any use is to be made of the so-called rated horsepower, rated speed, and rated tractive force," seem inconsistent with the actual deductions made from their use throughout the paper. The various plots, Figs. 4 to 7, inclusive, show characteristics of basic types of locomotives explicitly using these ratings. A major purpose of these plots was to show just how the various ratings were to be interpreted on a quantitative basis. Obviously, the plots do not represent the exact characteristics for all types of designs, but they can be interpreted as sufficiently typical to illustrate the method of arriving at the basic design parameters for any actual locomotive.

In stating that a steam locomotive, a Diesel-electric locomotive, and an electric locomotive have distinctive and unique characteristics appears fully as justified; as for instance, in stating characteristics of induction motors, Diesel engines, power, and torque-speed characteristics of turbines. While individual designs vary, the general characteristics are still maintained. One object of the paper was to coordinate these characteristics of locomotive power units to a common basis of comparison for train performance.

Attempts at generalizations are not usually accepted at the outset by those accustomed only to working on detail and specific designs, due to the great difficulty in presenting such generalizations to cover all details. But the advantage of generalization is very great in that it requires concentration on the fundamental limitations of a design and clearly points to the essential factors that result in a balanced design.

The author feels that the entire problem of motive power on the railways has been considered from too narrow a point of view, and that greater progress can be made with competing types of locomotive when it is more clearly realized that there are certain generalizations and limitations applicable to all types of power unit. One object of this paper was to point out the nature of these limitations and their specific applications to given classes of locomotive.

Mr. Fry's objection that the author, does not provide a clear-cut definition of rated horsepower which the author uses as a basis for developing equations for operating characteristics, clearly shows the difficulties that are immediately met in attempting generalizations. Possibly Mr. Fry raises this objection due to the particular difficulty of giving an exact power rating of a steam locomotive. For a long time steam locomotives were rated on their starting tractive force alone, but it has been appreciated for some time that the power capacity of the locomotive in its operating-speed range has even greater significance. This latter point of view was definitely introduced by the late Mr. Cole of the American Locomotive Company and revised to a rational basis by Dr. Lipetz as a method for the rating of steam locomotives.

The outstanding feature of these methods in power rating was to correlate boiler performance with cylinder performance at either a definite piston speed or number of revolutions per minute of the drivers, which amounts to a definite power output at a given speed. The author has maintained this point of view.

The power output of a locomotive varies with speed at any given firing rate. At a fairly definite operating speed, the output and the over-all efficiency reach a maximum. If this maximum efficiency is definitely limited, the locomotive then has a definite power rating and a corresponding rated speed. The over-all effi-

[15] Electrical Engineer, New York, New Haven and Hartford Railroad Company, New Haven, Conn. Mem. A.S.M.E.

ciency can be divided into two components, that due to boiler and that due to cylinders. With the cylinder efficiency fairly fixed at the rated speed, the boiler efficiency must have a definite value corresponding to a definite firing rate. Therefore in stating the rated output of a steam locomotive it is necessary to qualify this with a definite over-all thermal efficiency corresponding to that specified or desired in normal operation.

Mr. Fry refers to Equation [10] as rating critical attention. Evidently his misunderstanding of this equation is due to his misinterpretation of the symbol h_{pt}. The symbol h_{pt} for performance rating is not stated as "the horsepower available at a given speed and therefore varies with the speed." The definition of h_{pt} is actually stated as a numerical coefficient q multiplied by the rated horsepower per ton $(H_{PR})_t$ including both train weight and locomotive, and q is a correction factor for different power characteristics with allowance for train auxiliaries. Obviously the performance of a locomotive over a given schedule depends not only upon its rated power but also on the actual power developed at speeds other than its rated speeds which occur in a given schedule. Therefore, in comparing locomotives relative to their performance over some given schedule, a locomotive with unfavorable power-speed characteristics requires a higher rated power than a locomotive of more constant horsepower characteristics. For this reason a correction factor q is necessary to apply to the rated power when comparing locomotives of different power characteristics to account for the same performance in any given schedule. The correction factor, therefore, depends not only on the particular locomotive power characteristics but also on the nature of the schedule to be met. The author assumes, only as an example, that a typical schedule may require a rating of 4 hp per ton for steam and 3.5 hp per ton for Diesel-electric locomotives. The symbol q is a ratio, so that h_{pt} maintains the unit of horsepower per ton. The variable V for speed is not included or implied. Further consideration will show that Equation [10] has more significance than an identity since the rated power is stated in terms of basic parameters, which are (1) the weight of the train to be hauled, (2) the performance rating h_{pt}, and (3) the weight-power ratio $E\ (1 + k)$. It is hoped that Mr. Fry's apparent confusion of units in Equation [10] will be clarified by these remarks.

In the analysis of train performance, it is important to note that the power is measured at the drivers, so that over-all efficiencies are not considered directly. The power developed at the drivers is analogous to the brake output of an engine or motor. From a performance aspect, the effect of over-all efficiencies is to modify the weight efficiency of the locomotive $E(1 + k)$ which has been taken as a basic parameter in the analysis. The effect of variation of efficiency with speed is taken into consideration by the power-speed function. Except for balanced top speeds, possible reductions in train resistance due to better streamlining are of small order compared with inertia resistance to acceleration and grade resistance that usually occur in operating schedules. Even the balanced speed, where the tractive force balances the total train resistance, i.e., the particular speed at which the curve of tractive force crosses the train-resistance curve, is more influenced by variation of the tractive force and therefore the horsepower per ton available than by any possible reduction in train resistance. Therefore the author cannot sustain Mr. Fry's objection raised in his second and third paragraphs.

In conclusion the author is in agreement that actual performance data would be helpful in clarifying the presentation on ratings.

Mr. Kimball's discussion is of particular interest in that the work of Subcommittee No. 1, Economics of Railway Location and Operation,[9] American Railway Engineering Association parallels the author's work in certain phases of the paper. While the units and ratios for comparison differ, the agreement for estimated performance is close.

For example, in Mr. Kimball's discussion it is interesting to compare Fig. 13, for calculating the horsepower output of typical steam locomotives, and item 8, that is $V_1 = 0.396 \times$ horsepower per ton on drivers, with the author's discussion pertaining to Fig. 4. V_1 corresponds to the limit speed in the drag zone, where $V_1 = \gamma_D V_R$ according to the author's symbols. In a first approximation the author has assumed for steam locomotives, $\gamma_D = 0.25$, that is $^1/_4$ of the rated speed V_R.

With the starting tractive force sustained to the end of drag speed, then $Z_i = \mu A$ where $\mu = 0.25$ = the coefficient of friction, and A = weight on drivers in pounds. But $Z_i = \phi Z_R$ where ϕ varies from 2 to 2.3. Assume that $\phi = 2.1$. Then

$$\text{horsepower} = \frac{Z_R V_R}{375} = \frac{Z_i}{\phi} \times \frac{4V_1}{375}$$

$$\therefore \text{horsepower} = \frac{\mu A_t \times 2000}{\phi} \frac{4V_1}{375} \text{ and}$$

$$\frac{1}{V} = \frac{375\phi}{4\mu \times 2000} \frac{\text{horsepower}}{A_t}$$

With $\phi = 2.1$ and $\mu = 0.25$, $V_1 = 0.393 \times$ (horsepower per ton of weight on drivers).

With the Diesel-electric locomotive, the author uses $\gamma_D = 0.2$, so that the rated speed is 5 V_1. Mr. Kimball uses $\mu = 0.3$ for this type due to uniform torque. The torque amplification ϕ, therefore, on the basis $V_1 = 0.395 \times \text{hp}/A_t$, is $\phi = 3.16$.

This is somewhat less than the value used by the author. With high-speed road locomotives 3.5 is not excessive due to the short time period of overload current, whereas in switcher service ϕ may be as low as 2.5. A value around $\phi = 3.0$ appears a nominal average value for starting tractive amplification for Diesel locomotives relative to their continuous rated output.

Mr. Kimball gives the speed range V_m of steam locomotives as approximately $6V_1$. With $V_1 = 0.25V_R$, then $V_m = 1.5V_R$, which agrees with that used by the author in Fig. 4. It is to be noted that the ratio of power to rated power is estimated, at the end of the drag-speed range $V_1 = 0.25\ V_R$, at 0.528, which is in close agreement with the author's value in Fig. 4.

Referring to Fig. 12 for performance of Diesel-electric locomotives, the speed range is $14V_1$. In the author's Fig. 5, the speed range $V_m = 2.5V_R$. With a drag speed $V_1 = 0.2\ V_R$, then $V_m = 5 \times 2.5V_1$, a value somewhat less than used by Mr. Kimball.

Next consider Table 1 for obtaining approximate horsepower capacities for typical steam locomotives based on horsepower per ton on drivers. To obtain the actual horsepower it is necessary to consider the percentage of weight on drivers. In the author's analysis he has disregarded *types* of steam locomotives and has used the weight ratio (weight per horsepower E) of the locomotive with an additional term K to account for the tender weight relative to the locomotive weight. A basic ratio E independent of the type of steam locomotive can be further justified by referring in Mr. Kimball's table to the values, hp/A_t = horsepower per ton on drivers and C = ratio of weight on drivers to weight of engine. In Table 2 are listed the first and third columns of Mr. Kimball's Table 1, together with values of E, $E(1 + k)$, and K for different types of locomotives.

From these values, E = weight-power ratio for the locomotive alone appears fairly constant and independent of the type. Tender-weight ratios are approximately three quarters of the engine weight while $E\ (1 + k)$ = over-all weight-power ratio, range from 175 to 200 lb per hp.

The rating of a steam locomotive depends upon a normal firing

TABLE 2

C	HP/A_i	Type	E	$E(1+k)$	K
0.45	39.2	4-4-4	114	204	0.78
0.54	35.4	4-6-4	105	186	0.77
0.62	32.0	4-8-4	101	177	0.75
0.70	28.6	2-10-4	100	174	0.74
0.77	25.7	2-10-2	101	176	0.74
0.84	22.7	2-10-2	105	182	0.73
0.92	19.4	2-10-0	113	195	0.73
1.00	16.0		125	218	0.75
0.69	25.3	4-6 + 6-4	114	175	0.54
0.75	23.5	2-6 + 6-4	114	180	0.58
0.80	22.0	2-8 + 8-4	114	184	0.61
0.90	19.0	2-8 + 8-2	117	197	0.67

rate. Mr. Fry assumes the boiler efficiency to vary as a straight line against firing rate, and arrives at a parabolic curve for the evaporative capacity. These assumptions are justified from numerous tests. If x = firing rate, then the efficiency $E_B = (c - mx)/100$, the evaporation is $E = K[(cx - mx^2)/100]$, and $dE/dx = 0$; $x_{max} = c/2m$.

$$\therefore \frac{E}{E_{max}} = \frac{cx - mx^2}{c^2/4m}$$

If the normal firing rate is $x = kx_{max} = kc/2m$ then, $E = (2-k)\,kE_{max}$.

With $k = 0.5$, then $E = 0.75\,E_{max}$. If $k = 0.66$ then $E = 0.89\,E_{max}$. This shows that firing rates exceeding 50 per cent of the maximum rapidly approach the maximum evaporative capacity of the boiler. For this reason Mr. Kimball's committee's choice of a firing rate at 50 per cent of maximum corresponding to 75 per cent of maximum boiler capacity appears to be a good design value for the normal rating of a boiler.

On the basis of normal evaporation of the boiler, the performance of a locomotive then depends upon the cylinder efficiency, measured by the steam consumption per horsepower = hour h. The value of h varies with cutoff and speed, reaching a minimum at around 20 or 25 per cent cutoff. Since hp $= E/h$ and E is maintained nearly constant at the higher speeds, while h decreases to a minimum at rated speed, therefore the output reaches a maximum. With increasing speeds h increases, thus resulting in a falling off in power in the overspeed zone. For this reason there must be some falling off in power in this zone. Mr. Kimball's assumption of a constant cylinder output with a constant normal boiler capacity appears inconsistent with actual cylinder performance with a constant steam supply.

Mr. Withington raises the point that the slope of the power curve at low speeds should be the same in Figs. 6 and 5 since the permissible overload torque should be approximately the same for both Diesel- and straight-electric locomotives. This is true and, as a matter of fact, identical motor characteristics were used, though the former is on variable voltage at the high speeds, since EI = const, and the latter approaches a constant line voltage. Mr. Withington perhaps is confused by the different scales for both power and speed used in these figures. Thus the power-curve slope at low speeds for either Fig. 5 or Fig. 6 is approximately $^{0.80}/_{0.25} = x/\gamma$, where x = per cent of rated horsepower and γ = per cent of rated speed. The tractive force for any speed γV_R (where V_R is the rated speed) corresponds to the horsepower xH_{PR} (where H_{PR} is the rated horsepower). For the low speed range, the tractive force is, $Z = 375xH_{PR}/\gamma V_R$. But the rated tractive force is $Z_R = 375\,H_{PR}/V_R$. Therefore, the tractive-force amplification is $\phi = Z/Z_R = x/\gamma$ = (overload capacity of motors in starting zone). The value of x/γ in the low speed range, is practically the same in Figs. 5 and 6.

Discussion

Experiments and Calculations on the Resurge Phase of Water Hammer[1]

F. KNAPP.[2] Before commenting on the details of computations used by the author, the writer wishes to state that the type of surge discussed in the paper is only the simplest case of certain surge phenomena not yet investigated by the usual water-hammer theory.

Suppose, for example, a penstock with a projecting point as shown in Fig 1 of this discussion. Suppose further that the valve at the lower end of the penstock is opened very quickly, producing a "primary" surge drop of appreciable magnitude. This drop wave travels up the line until it intersects the zero-pressure line at section X. As the pressure in the pipe cannot fall below zero, the question arises regarding the behavior of the wave between section X and the intake. In the following discussion, this type of surge will be called type A.

Another case, type B, is represented by a primary surge rise with a steep or vertical wave front and of such magnitude that the negative secondary wave, reflected totally at the closed valve, cuts the zero-pressure line similarly as in the type-A surge.

Another case, type C, is produced by a primary surge drop such as shown in Fig. 1 of this discussion. At section Y the angle γ of the surge-drop face is smaller than the angle α of the lower portion of the penstock, but greater than angle β of the upper part.

Another case, type D, is analogous to type C with the exception that the primary surge is not a drop, but a pressure rise.

Following up the pressure and velocity conditions in the pipe line, it will be noted that only the surges of types C and D represent conditions capable of graphical analysis. In the case of type C, a vacuum is formed at section Y and the water column is separated into two independent columns. The lower column soon attains a steady state, depending on the gate movement, and all pressure waves traveling in the portion are reflected totally and with negative sign at the cavity and partially at the open gate. Consequently, the pressure and velocity conditions at section Y are known at any time.

A similar process takes place in the upper column. That part of the primary surge drop transmitted to the upper part of the line before the column broke is reflected totally and with negative sign both at the intake and at the cavity, thus accelerating the water column. If C_{1t} denotes the velocity of the lower column at section Y, and C_{2t} denotes the velocity of the upper column at the same place, then the two columns reunite when

$$\int C_{1t}dt = \int C_{2t}dt. \quad \ldots\ldots\ldots\ldots [1]$$

At the instant of the reuniting of the two columns, the velocities C_{1t} and C_{2t} are thus known. As the surge produced is proportional to the instantaneously destroyed velocity, the magnitude of the resulting pressure rise can be computed easily. Computations carried out for this type of surge revealed these pressure rises to be of surprisingly great magnitude, but of short duration in most practical cases due to the proximity of section Y to the forebay of the surge tank.

Type-C surge has great practical importance, and by it can be explained many penstock failures caused by surges of so-called "obscure origin." It is a well-known fact that the pressure distribution in the upper part of the penstock due to repeated but small movements of the governor is far from being linear but is in a curious manner approaching the distribution of surges with steep wave faces of the rapid type. Most penstock ruptures are also caused in the upper part of the line and cannot be explained by surges starting at the gate, since these pressure rises would first have broken the most-stressed lower part of the line. The events of irregular surge-drop distribution together with projecting angles of the penstock, causing surges of type C, are thus the natural explanation of such accidents.

Little is known about surges of types A and B, which at present can be fully investigated only experimentally. Gibson[3] investigated curves of type-A surges and made the following statement: "Where the pipe slopes upward toward its inlet, so that beyond a certain point in its length the absolute statical pressure is less than the drop in pressure caused at the valve by sudden opening, then on the passage of the first wave of negative pressure the wave motion becomes partially discontinuous after this point is reached and the wave travels on to the inlet with a gradually

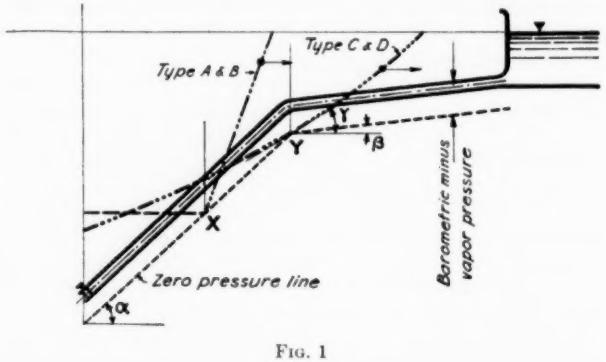


FIG. 1

diminishing amplitude. The amplitude with which it reaches the intake, and which will be probably 2 or 3 lb per sq in. less than the absolute statical pressure at the inlet, decides the state of velocity of the reflected wave. This will evidently be less than the preceding case, and under such circumstances the wave motion dies out very quickly. As the valve opening becomes greater, the efficiency of the valve as a reflecting surface becomes less, so that with a moderate opening the pressure may never even attain the pressure due to the statical head." It is to be hoped that the hydraulic laboratory at the California Institute of Technology will extend the study of water hammer to the surge types outlined in this discussion.

The author performed his calculations assuming incompressible water and rigid pipe walls. Nevertheless, his computed results compare extremely well with the experimental data. Of course, the coefficient C for the initial velocity with which the water starts away from the valve has been taken from test results.

Application of the elastic-column theory to the author's case shows that the starting and recoil velocities do not decrease and

[1] Published as paper HYD-59-12, by Joseph N. LeConte, in the November, 1937, issue of the A.S.M.E. Transactions, vol. 59, p. 691.

[2] Assistant Hydraulic Engineer, The Sao Paulo Tramway, Light & Power Company, Ltd., Sao Paulo, Brazil.

[3] "Hydraulics and Its Application," by N. R. Gibson, New York, N. Y., 1923, p. 241.

increase steadily as assumed by the rigid-column theory, but increase or decrease in steps corresponding to the reflection time of the penstock. While the author's formula for the case treated, namely, instantaneous closure, is relatively simple, it would become quite complicated or laborious for the case of a finite closing time.

Computations made by the writer based on data given in the paper gave the results shown in Table 1 of this discussion for the time t_1 of test No. 5*A* on the 2-in. line.

Although the elastic-column theory, with friction losses considered concentrated at the intake, gives results well within the precision of engineering calculations, the rigid-column theory furnished surprisingly closer results. The writer attributes this fact to the surge velocity taken as 4765 fps from test No. 4*A*. Also, the value of atmospheric pressure minus vapor pressure of water has been assumed to be 33 ft. A type of surge similar to that discussed by the author occurs sometimes in low-head pumping installations and therefore is of practical interest. The test results presented also do away with the opinion held by some engineers that the recoil of the water column takes place completely as soon as the wave reflected at the intake arrives at the gate. Basing his deductions on the elastic-column theory, the writer expected the behavior of the water column to be the same as shown by the author's experiments, and is glad to have the experimental confirmation.

TABLE 1 RESULTS FOR TIME t_1 OF TEST NO. 5*A*

	Difference between observed and computed time, per cent
Rigid-column theory (LeConte)	− 1.13
Elastic-column theory:	
Friction neglected	+16.38
Friction assumed concentrated at intake	− 4.85
Velocity V_0 = 2.707 fps	− 3.64

(The following discussion was submitted jointly with the discussion of the paper by Lorenzo Allievi.[4] For the complete discussion of the latter paper see pp. 445–450 of this issue.)

Louis Bergeron.[5] Several of the papers presented at the Second Symposium on Water Hammer referred to the graphical method of calculation for water hammer. The general equations are essentially the same as those utilized for analytical determinations of water hammer, but the various relations can be expressed graphically rather than by means of involved equations.

Numerous presentations of the graphical methods have been made in French and German, and quite recently the writer published a complete demonstration of the graphical method as applied to water hammer and surge phenomena.[6]

The general expressions for water hammer developed in the demonstration cited have a physical significance which may be expressed as follows: For an observer traveling along the pipe at a speed $\pm V_w$, the pressure H, plotted as a function of the velocity V, is always found on the straight line passing through the point characteristic of the regime existing at the time and place of his departure, and of angular coefficient tan $\gamma = \pm (V_w/g)$. This is a general law, whatever be the plane waves F and f, or their values, even if they have the particular value zero (viz., during the initial permanent regime), for then the regime is represented by the same point of the line. Likewise, the law is universal, being valid for all homogeneous mediums in which the variation of regime is propagated by plane waves.[6] Thus, it may be employed to study the transverse movements of a stretched cord, the longitudinal movements of a metal bar, the variations of regime in canals (within the limit of changes in level that are small as compared to the depth), transients in electric lines, etc.

Whatever the phenomenon to be studied, this simple relation permits one literally to follow it by the eyes of fictitious observers who travel the course of action from one extremity of the medium to the other, and to reconstruct it by means of a graph which corresponds rigorously to the realization, at slow speed, of the physical course studied.

Water Hammer With Cavitation During the Resurge Phase

Let us apply this reasoning to the problem solved graphically by Le Conte.[1] In order to find a graphical solution taking into account the head loss, this loss will be considered as concentrated at a single point close to the reservoir, as shown in Fig. 2 of this discussion. It should be noted that if one desired to know exactly the regimes at various points along the pipe it would be necessary to divide the total loss among several points between *A* and *B*.

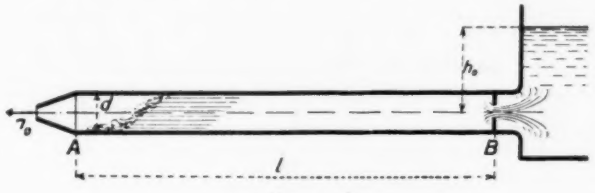


Fig. 2

In Figs. 3*a*, 3*b*, and 3*c* of this discussion, H_0 is the geometric head; H_{atm} is the head corresponding to atmospheric pressure; ψ_0 is the parabola $H = kQ^2$, relating the pressure at *A* to the discharge when the orifice is fully open; *MN* is the parabola giving the pressure at *B*, downstream of the constriction which has been calculated to create a loss of head equal to the friction loss from *A* to *B*. When the discharge is negative the pressure at the same point *B* is given by the parabola *MN'*, whose ordinates are the same as those of *MN* but with their sign changed. If the velocity head $V^2/2g$ is not negligible it should be added to the ordinates of *MN*, *MN'* remaining the same. This follows from the fact that with flow in the sense from *B* to *A* there is a negligible entrance loss in the pipe, but with flow in the opposite direction there is an exit loss (loss of kinetic energy) equal to the velocity head.

Start with the intersection O_A of ψ_0 and *MN*, shown in Fig. 3*a* of this discussion, which is the point characteristic of the initial permanent regime. For an observer leaving *B* during this regime, the characteristic line is O_AP, making with the horizontal the angle $\gamma = \tan^{-1}(V_w/gA)$. If he arrives at *A* at the exact instant of closure (which is assumed to be instantaneous) the function ψ_A is then the axis of the ordinates and the point characteristic of the regime is O_A' at the intersection of this axis with O_AP. The regime has passed instantaneously from O_A to O_A' and the excess pressure is represented by $\overline{MO_A'}$. The observer returning to *B* sees the line $O_A'1_B'$ (as yet unlimited in length) and, upon his arrival at the diaphragm constriction at the time $l/V_w = 1$ the regime point will be $1_B'$, at the intersection of *MN'* and this line.

Retracing his route the observer sees the characteristic line $1_B'=2_A'$ (as yet also unlimited). He arrives, at time 2, at *A* where the orifice is still closed, and since the pressure may only decrease to $-H_{atm}$, i.e., absolute zero, the function ψ at this time and place will be the horizontal line of ordinate $-H_{atm}$, and the characteristic point will be $2_A'$. This also shows that

[4] "Air Chambers for Discharge Pipes," by Lorenzo Allievi, Trans. A.S.M.E., vol. 59, November, 1937, paper HYD-59-7, p. 651.

[5] Professor, L'École Centrale des Arts et Manufactures, Paris, France.

[6] "Méthode graphique générale de calcul des propagations d'ondes planes," by L. Bergeron, Memoires de la Société des Ingenieurs Civils de France, July-August, 1937, pp. 407–497. Also, Société Francaise des Électriciens, Bulletin 82, October, 1937.

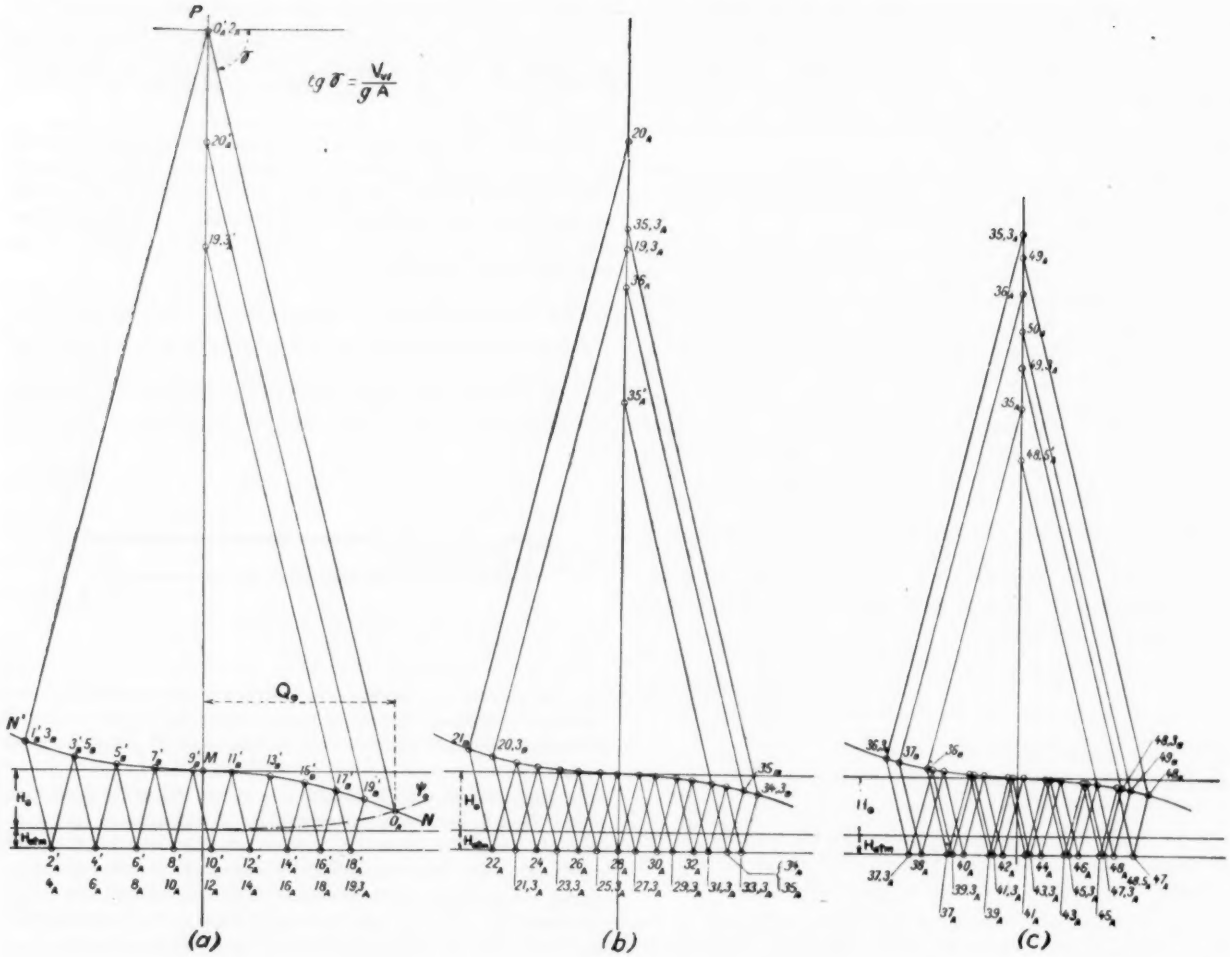


FIG. 3

the reversed flow will be equal to the abscissa of this point and is of such magnitude as to cause a cavity to form at A. Henceforth the traverses of the observer between A and B will trace the broken line $2_A'$, $3_B'$, $4_A'$, ... $17_B'$, $18_A'$, $19_B'$, whose termini are alternately on the horizontal $-H_{atm}$, at A, and $N'MN$, at B. One sees that the discharge decreases in A then recovers its initial sense, closing the cavity. By constructing the discharge curve, shown in Fig. 4 of this discussion, one easily calculates that the closure takes place at time 19.3 on a discharge given by the point $18_A'$. For an observer arriving at B exactly at this instant, the excess pressure of this discharge will give the regime point $19.3_A'$ on the ordinate axis. But first the observer, who leaves B at the time 19, seeing there the regime $19_B'$, arrives at A at the time 20 where he finds no discharge, i.e., the regime point $20_A'$ on the axis. Therefore, one sees that at A from the time 19.3 to the time 20, the pressure is the ordinate $19.3_A'$, and then it is that of $20_A'$ from the time 20 to the time 21.3, at which latter time the cavity starts to form again, as shown in Fig. 3*b* of this discussion.

Thus, from the fact that the period of oscillation of the discharge at A is not an exact multiple of $2l/V_w$, the excess pressure of the successive shocks presents an increasing series of peaks. Here is found the explanation of one of the causes of the harmonics that are seen in Fig. 5 of this discussion, which is taken from a paper by Langevin. This example also shows the precision with which the graphic construction reproduces the actual phenomenon.

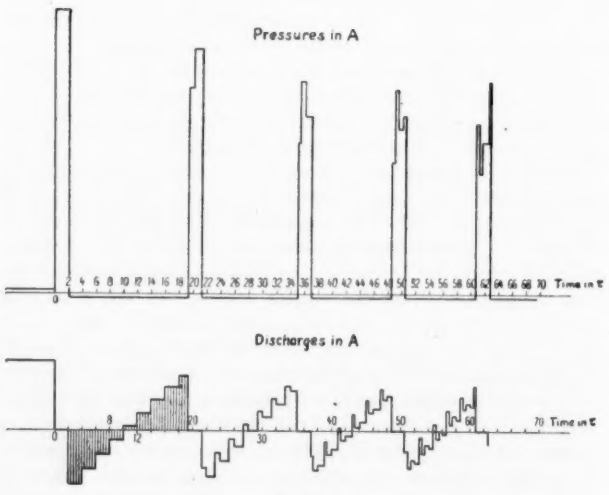


FIG. 4

AIR RESERVOIRS

Figs. 6 and 7 of this discussion relate to Allievi's numerical example *b* given on page 655 of his paper,[4] the results of which example are given in column *B* of Table 4 of the paper.[4] The numerical values given correspond to $v_0 = 1$ m per sec, $V_w = 1000$

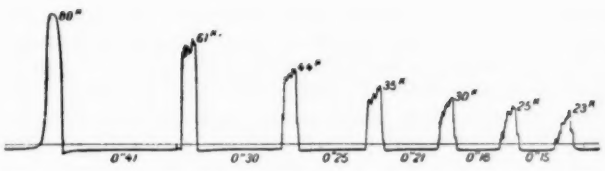


FIG. 5

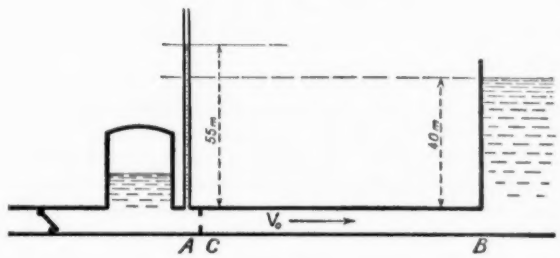


FIG. 6

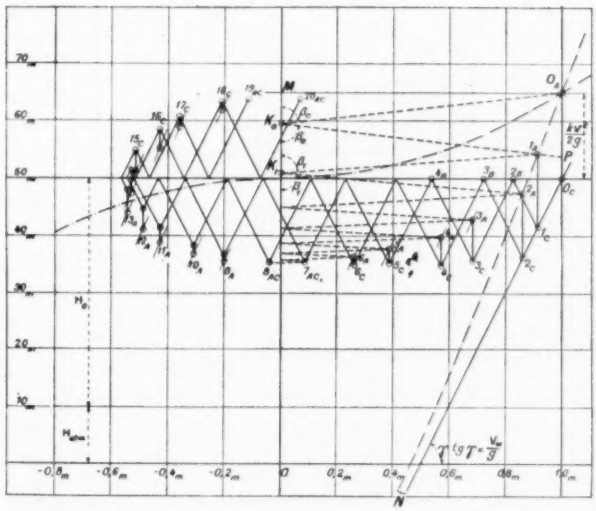


FIG. 7

m per sec, and $2g = 20$ m per sec per sec. Volume C_0 is the same as in Allievi's example *a*, where $H_0 = 50$ m, and $\sigma = 5$. To determine C_0 start with $\sigma = 5 = (1000\ C_0H_0)/W_0$ and $W_0 = 1000\ LA(v_0^2/2g)$, which gives

$$C_0 = \frac{5 \times 1000 \times (1^2/20)LA}{1000 \times 50} = 0.005\ LA$$

or, for example, for $LA = 2500$ cu m, $C_0 = 12.5$ cu m.

In Fig. 7 of this discussion let the abscissas be v to avoid distinguishing L and A in the product LA, and choose, for the time j, a fraction $1/n$ of $2l/V_w$. Then

$$\frac{v_T''' + v_{(T-j)}'''}{h'''} A = \frac{2C_T}{[(H_T''' + H_{atm}) + h'''](2l/nV_w)} \quad \ldots [2]$$

and

$$\tan \beta = \frac{C_T}{(H_T''' + H_{atm}) + h'''} \frac{nV_w}{LA} \quad \ldots\ldots [3]$$

Further, the volume $C_{(T+j)}$ becomes

$$C_{(T+j)} = C_T - \left(\frac{v_T + v_{(T+j)}}{2} Aj\right) = C_T - v_{mean}\left(\frac{2LA}{nV_w}\right) \ldots [4]$$

Fig. 7 is drawn with $V_w = 1000$, $LA = 2500$, and $n = 2$, for which $nV_w/LA = 0.8$ and $2LA/nV_w = 2.5$. The plan of the installation is shown in Fig. 6 of this discussion, B being the reservoir with water level at 40 m. The loss of head of 15 m, distributed between A and B, will be assumed as concentrated at a constriction placed at A. The air reservoir will thus have a piezometric head of $40 + 15 = 55$ m, and will be under an absolute pressure of $H_0 = 55 + 10 = 65$ m. The initial permanent regime in A is shown in Fig. 7 of this discussion by the point O_A, and in C by the point O_C. The time $j = l/V_w$ is taken as unity.

For an observer leaving B at time 0, while the regime there is still that of point O_C in Fig. 7, the characteristic line is O_CN, whose angular coefficient $\tan \gamma = V_w/g = 1000/10 = 100$. If the observer passes the constriction, the pressure increases by $15v^2$, which results in the parabola O_AN of the pressure variation in A.

A fictitious observer who leaves A at time 0, goes to the surface of the reservoir and returns at time 1, sees upon his leaving A a line O_AK_0, and on his return sees a line K_01_A, which makes an angle β_0 with the vertical, where, following Equation [3] of this discussion

$$\beta_0 = \tan^{-1} \frac{C_0}{(H_0 + H_{atm}) + h_0} 0.8$$

or, in estimating $h_0 = 11$ m, $\tan \beta_0 = [12.5/(65 - 10.5)]0.8 = 1/5.4$, which means that for $\overline{O_AM} = 1$ m, one has $\overline{MK_0} = 5.4$. This allows one to draw O_AK_0, then, since $\overline{O_AP} = 2 \times 5.4 = 10.8$ m, K_0P may be drawn, whose intersection with the parabola O_AN gives the point 1_A (of the regime in A at the time 1) sought, and, on the same vertical, the point 1_C of the regime C at the same time.

For an observer leaving B at the time 1, the regime there is still given by the point O_C and the characteristic line is still O_CN in C (changing to the parabola O_AN in A at time 2). But the observer traversing the reservoir sees the lines whose angle is now β_1, determined, in estimating h_1 as 7 m by

$$\tan \beta_1 = \frac{C_0 + (v_{mean} \times 2.5)}{54.5 \times 7} 0.8 = \frac{12.5 + (0.945 \times 2.5)}{47.5} 0.8 = \frac{1}{4}$$

From the point 1_A draw a horizontal equal to 1 m, then a vertical equal to —4 m, join the point obtained with 1_A and thus find the point K_1; repeat the construction starting from point K_1 and draw the line K_12_A, locating 2_A (the point sought) at its intersection with the parabola O_AN, and the point 2_C at its intersection with O_CN.

To find the regime in A and C at the time 3, consider again the first observer who found point 1_C; in going to the reservoir B he sees the line 1_C2_B (as yet unlimited) and on arriving at B finds the characteristic point 2_B at the intersection of this line and the horizontal which represents, in terms of absolute pressure head, the constant level of the reservoir B. Coming back to C he arrives there at time 3, seeing the line 2_B3_C (as yet unlimited) and passing the constriction he notes this line augmented by the head loss $15v^2$. There he meets the observer of the air

TABLE 2

	Z_i	V_i	K_i
1	0.73	0.850	0.16500
2	0.61	0.565	0.07400
3	0.56	0.260	0.01500
4	0.55	−0.035	−0.00028
5	0.57	−0.305	−0.03100
6	0.63	−0.485	−0.05400
7	0.73	−0.525	−0.06400
8	0.85	−0.425	−0.04150
9	0.94	−0.205	−0.00970
10	0.98	0.060	0.00083

reservoir for whom the lines have been drawn with an angle β_2, determined, in estimating $h_2 = 4.5$, as

$$\beta_2 = \tan^{-1} \frac{14.86 + (0.88 \times 2.5)}{47.5 - 4.5} 0.8 = \frac{1}{3.15}$$

These lines give the points 3_A and 3_C. The construction of the graph shown in Fig. 7 of this discussion is continued in this manner.

The points 2_A, 4_A, 6_A, etc., correspond to the times of "whole rhythm," and permit the construction of Table 2 of this discussion, established following Allievi's[4] notation.

One sees that, due to the bead loss in the pipe, there is no excess pressure at the time of the backflow. However, head losses as high as the one in the writer's example are rare; for example, if $LA = 2500$ was obtained with $L = 2000$ m and $A = 1.25$ sq m, the head loss would be only about 2 m instead of 15 m, and the air reservoir of 12.5 cu m would permit the attainment of a very high surge pressure (found in Fig. 8 of this discussion) which was made for the case of zero head loss and which corresponds to Allievi's example *a* (page 655 of the paper[4]), the results of which are given in column *A* of Table 4 of the paper.[4] However, this condition may be avoided by the use of the following artifice, previously advised by Comte de Sparre and analogous to that indicated many years ago by Johnson for surge-tank use.

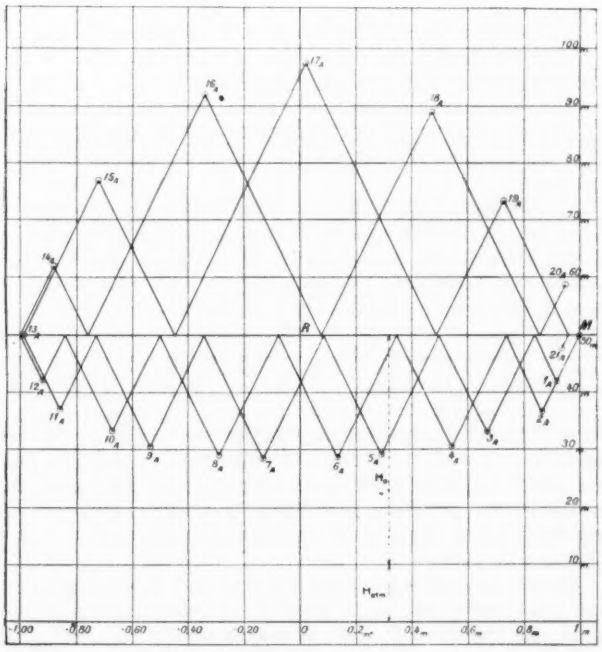


FIG. 8

The air reservoir on the pipe is fitted with a constriction as shown in Fig. 9 of this discussion. As nearly as possible this constriction is constituted of a nozzle whose coefficient of discharge, in an emptying sense, is 1, and in the sense of filling flow is 0.5; in other words, for the discharge at equal rates in the two directions the head loss is four times as much for filling flow as for emptying flow. The writer has used this artifice for ten years in installations at Rouen, Dieppe, Clermont-Ferrand, and other places in France.

Suppose now the instantaneous closure of a valve at time 0. For the observer arriving at *C* from *B* at this time the characteristic line is *MN*, in Fig. 10 of this discussion, passing through the point *M* of the initial regime. If the observer passes the constriction, its head loss is added to the line, resulting in the parabola *PN*. As the pressure in *A* could not change instantaneously, the regime point O_A will be at the intersection of this parabola and the horizontal *MR* of ordinate $(H_0 + H_{atm})$. On the same vertical, at its intersection with the line *MN*, the point O_C will be the regime point in *C* at the time 0. One sees that the velocity as well as the pressure both fall rapidly at *C*.

For the observers arriving at *C* just at the time 2, the characteristic line remains *MN*, and in *A* the parabola remains *PN*. Calculating tan β_1 and tan β_2 as before, one finds the points 1_A, 2_A, and 1_C, 2_C.

But the observer leaving *C* at time 0, i.e., point O_C, arrives again in *C* at time 2 and sees there, as the characteristic line, the line $1_B N_2$; then, passing the nozzle, he sees the parabola $P_2 N_2$. However, the pressure in *A* at time 2 (point 2_A) not being able to change, the regime point for this observer will therefore be $2_A'$ on the horizontal through 2_A and the parabola $P_2 N_2$. Thus, at the time 2 the velocity varies rapidly at *A*, and both the velocity and the pressure vary rapidly in *C*, due to the return of the wave leaving at time 0.

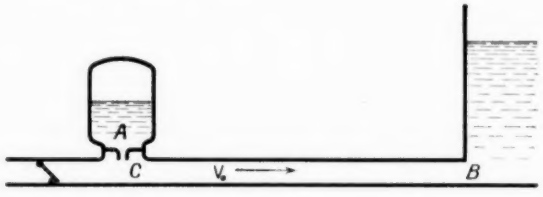


FIG. 9

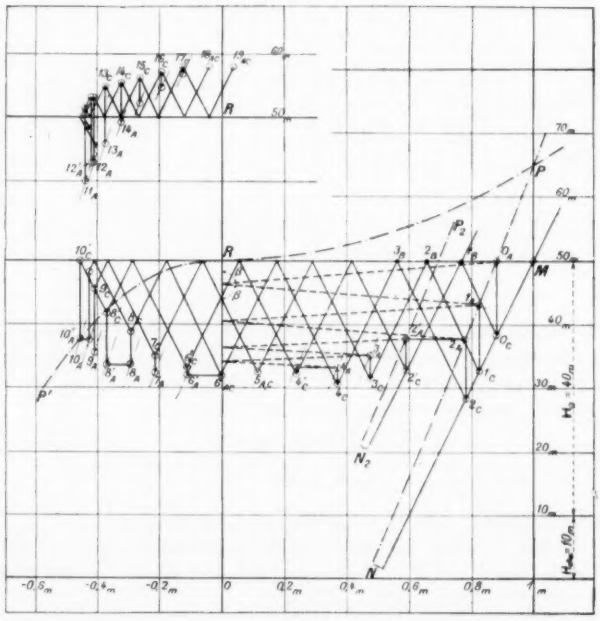


FIG. 10

Fig. 10 continues as before with the exception that at every "whole rhythm" one has anew a rapid jump with all the corresponding regime points doubled, as $4_A4_A'$, $6_A6_A'$, ... and $4_C4_C'$, $6_C6_C'$, $8_C8_C'$, etc. It is seen that the pressure ratio is only 1.15 in place of 1.95, the maximum pressure thus being very much lower; yet, note also that the maximum depression, obtained at 2_C, is only 0.57, which ratio is exactly the same as that for the air chamber without a nozzle. This nozzle arrangement is therefore not only interesting but it is indispensable—

the simple addition of the constriction, without creating a greater vacuum in the pipe, divides the surge pressure by six; in other words, it has the effect of a much larger air reservoir. It is easy to determine, for a given reservoir, the optimum value of constriction for a minimum pressure increase in the pipe by making several graphs similar to Fig. 10 of this discussion and constructing tables and curves of the results.

Table 3 of this discussion, the values in which were taken from the writer's graphs, has the same notation as Allievi's[4] Table 4 to facilitate their comparison.

Very Small Air Pocket in a Pipe

It has often been said that a very small air pocket in a pipe is extremely dangerous. The graphical method described by the writer enables one to investigate easily this question.

Suppose in the preceding case, rapid stoppage of discharge occurred with a very small air reservoir on the pipe, as shown in Fig. 11 of this discussion. For the observer leaving B at

TABLE 3

Column A, Allievi's Table 4[a]			Reservoir with constriction		
i	Z_i	V_i	i	Z_i	V_i
1.0	0.735	0.855	1	0.570	0.775
2.0	0.610	0.535	2	0.610	0.365
3.0	0.570	0.130	3	0.640	0
4.0	0.575	−0.285	4	0.775	−0.290
5.0	0.660	−0.665	5	0.965	−0.425
6.0	0.840	−0.915	6	1.065	−0.410
7.0	1.240	−0.875	7	1.110	−0.325
8.0	1.840	−0.340	8	1.140	−0.195
8.5	1.950	0.020			
9.0	1.780	0.465	9	1.150	−0.040
10.0	1.175	0.94	10	1.140	0.110

[a] Reference to Allievi's paper is given in footnote 4 of this discussion.

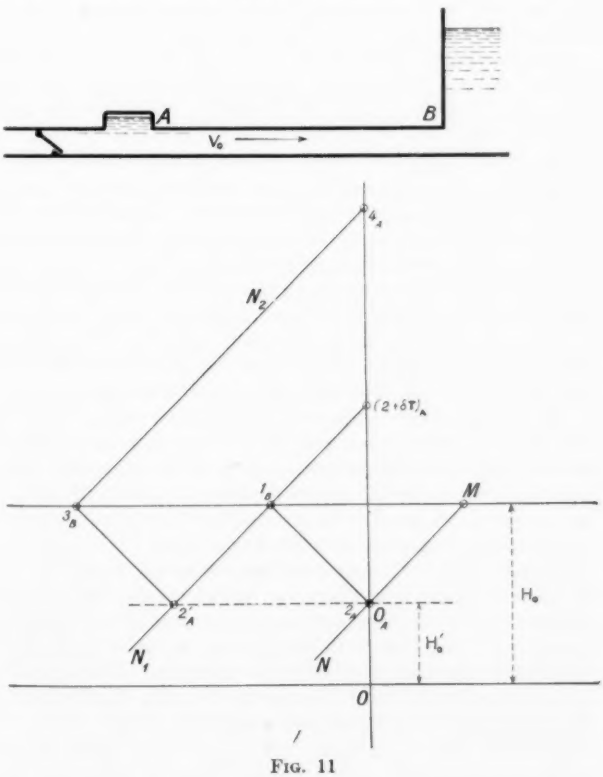


Fig. 11

the time −1 the characteristic line is MN, also shown in Fig. 11. Upon his arrival at A he finds the discharge zero, giving the regime point O_A (the small air pocket having expanded instantaneously in the ratio H_0/H_0' by reason of its very small volume), this regime existing just to the time $2l/V_w$. At this instant, the observer who left A at time 0 arrived in B at time 1 (finding the regime 1_B), returned from B seeing the line 1_BN_1, and arrived again in A. Upon his arrival, the pressure in A being H_0' exactly (because of the air pocket), the regime point will be $2_A'$ with a reverse flow equal to $\overline{O_A2_A'}$. This condition will last only an extremely short time because, of the small volume of the air pocket, which, being compressed will annul the discharge and will result in the regime point $(2 + \delta T)_A$ at the time $(2 + \delta T)$. But for the observer leaving A precisely at the time 2 the characteristic line passes through $2_A'$ and gives the regime point 3_B in B at the time 3; upon his return he sees the line 3_BN_2, but this time upon his arrival at A he will find the flow zero, giving the regime point 4_A which determines the maximum surge pressure. This maximum exists only during a time less than δT, but, as shown in Fig. 11 it is equal to $3v_0V_w/g$ or triple the pressure increase obtained when there is no air pocket.

Such is the maximum surge pressure which is approached as the volume of the air pocket diminishes; but, fortunately the conditions which permit the velocity $\overline{O_A2_A'}$ to exist throughout the pipe are rarely realized, so that the maximum is likewise rarely attained.

Author's Closure

The author is greatly indebted to Mr. Knapp and to Professor Bergeron for their discussion of his paper on "The Resurge Phase of Water Hammer." They have given a broader view of the problem, and have furnished other important methods of attack. The author's idea when the work was first taken up at the University of California was to work out a reasonable analytical solution, and back up this theory with laboratory tests. It is obvious that the recoil coefficients C have been obtained from the experimental record, but it is of interest to note that this coefficient is a constant for a given pipe line for all velocities within the range of the experiments. The problem is a most interesting one and the author hopes that similar work can be carried on at the University laboratories using pipes of different sizes and lengths.

Air Chambers for Discharge Pipes[1]

E. E. Halmos.[2] The writer wishes to call attention to a typographical error in the formula at the top ot page 653 which is confusing and requires correction. The expression referred to should read as follows:

$$\int_0^{i-1} V\partial t - 2\int_0^{i} V\partial t + \int_0^{i+1} V\partial t = \frac{C_0}{S} \cdot \left(\frac{1}{Z_{i-1}} - \frac{2}{Z_i} + \frac{1}{Z_{i+1}}\right)$$

M. A. Mason,[3] P. Danel,[4] and A. Craya.[5] Following a practice which he initiated in his 1913 paper,[6] the author

[1] Published as paper HYD-59-7, by Lorenzo Allievi, in the November, 1937, issue of the A.S.M.E. Transactions, vol. 59, p. 651.

[2] Associate Engineer, Parsons, Klapp, Brinckerhoff, and Douglas, New York, N. Y.

[3] Freeman Scholar of the Boston Society of Civil Engineers, University of Grenoble, Grenoble, France.

[4] Director, Research Laboratory, Ateliers Neyret-Beylier & Piccard-Pictet, and Professor of Hydraulics, University of Grenoble, Grenoble, France.

[5] Research Engineer, Ateliers Neyret-Beylier & Piccard-Pictet, Grenoble, France.

[6] "Teoria del colpo d'ariete" ("Theory of Water Hammer"), by L. Allievi, Atti del Collegio degli Ingegneri ed Architetti, Milan, Italy, 1913.

justly employs and stresses the importance of the use of dimensionless parameters in solving this problem and others connected with water-hammer studies. As in most of the cases arising in the applied sciences, the choice of the parameters to be used requires some care. A general study such as the author has made, involving the influence of certain factors, may require an entirely different choice of dimensionless variables if the effect of other factors is to be considered. It may be stated here that the author's use of the dimensionless parameter $\sigma\rho$ in his equations is exactly paralleled by Bergeron's use of the term

$$\tan \beta = \frac{C_T}{[(H_T''' + H_{atm}) + h'''](j/2)}$$

for if the coordinate axes of Bergeron's graphs are made nondimensional then $\tan \beta = C_0 V_w / A L v_0$, which is seen to be exactly the same as the author's $\sigma\rho$.

Unfortunately the equations given by the author for the analytical study of air-chamber problems may not be employed in all such cases without error. Their use must be restricted to those problems in which the assumptions stated as Equation [10] of the paper[1] may be considered as sufficiently accurate. This will be the case when the pressure-velocity curve is continuous, as in the examples given by the author. But when the curve has cusps, as in the examples treated by Bergeron[7] and Angus,[8] the assumption may no longer be made without introducing large error. Further, the analytical method does not itself offer any criterion as to when Equation [10] of the paper is valid and when it is not. Herein lies an advantage of the graphical method. The graph itself shows clearly whether intermediate points, i.e., other than those of the "total or whole" rhythm $2L/a$, must be considered, and, in addition, indicates just how small the phase must be divided. This indication of the need for intercalary regimes is, of course, a criterion as to the validity of the assumption expressed by Equation [10] of the paper; an indicated need for smaller time intervals shows Equation [10] to be invalid.

It can be shown that the assumption of a continuous pressure-velocity curve may safely be made for any case in which both ρ and θ, following the author's notation, are large. However, in this case, a computation made without considering the elasticity of the water and pipe gives a good approximation to the true value and is much simpler to make than the calculations with the author's equations.

Bergeron has demonstrated[7] a graphical method for the solution of air-chamber problems, in which method account was taken of the elasticity of the water, the pipe, and the air in the reservoir. The solution obtained for a general case showed that if the ratio of the pipe area to reservoir area A/A_R, the pressure head of the air in the reservoir H_{air}, the term (initial pressure head plus the atmospheric head minus the head of water in the reservoir), and the initial velocity in the pipe all have the same values for any two cases, then the relation between the velocities and pressures obtained during perturbed flow are the same in the two cases. Further, if V_0, the initial velocity, is changed, then the pressures and velocities during perturbed flow vary as V_0 in comparison to the first case cited. Finally if the term

$$\frac{A}{A_R}\left[\frac{H_0 + H_{atm} - H_R}{H_{air}}\right]$$

keeps a constant value, the pressure-velocity relation at any instant varies very little with changes in the reservoir air pressure. It follows from these statements that by setting a maximum allowable superpressure, and constructing a network of curves relating various values of this latter term to the pipe length, the proper air reservoir for a given set of conditions can be found easily.

It was also shown that in certain cases if the elasticity of both the water and the pipe are not considered, the maximum surge pressure may reach a much higher value than that computed, taking account of elasticity. For instance, in the example solved by Bergeron,[7] this maximum reached a value five times the maximum computed considering elasticity.

In cases where the difference in the maxima (as computed by methods involving or not involving the elasticity of the media) is great, Equation [10] of the paper is invalid. Nevertheless, the author's equations applied to such a case would result in a computed maximum more nearly the true value than a maximum value obtained in neglecting the elasticity. Intermediate cases, such as those computed by the author where no loss of head is involved, may be solved quite accurately by his method. Note, for example, example *a* on p. 655 of the paper.[1] Bergeron points out that the analytical method results in values very nearly the same as those obtained by a graphical computation.

Correction for Loss of Head

In the present state of our knowledge of water-hammer phenomena it is necessary to admit that it has not been possible to consider in a rigorous manner the effect on the phenomena of losses due to liquid friction. However, both the graphical and analytical methods of computing water-hammer pressures are susceptible of changes which take this effect into consideration. In the analytical method, the head loss may be considered through the introduction of a term kv_0^2 in the equations of flow, as is done by the author. In the graphical method loss of head is introduced by Bergeron, Strowger, Angus, and others through the medium of concentrated losses at local discontinuities, such as, diaphragms, or similar constrictions, with the sections of pipe between them considered in the computations as ideal pipes with no loss of head. It is not necessary to discuss here the subtle question of the necessary number and magnitude of such discontinuities as that is covered by authors dealing with the graphical method. What should be stressed is the fact that once loss of head has been introduced by this means we are no longer dealing with an actual pipe, but with an ideal pipe fitted with constrictive devices, and all computations must be made accordingly.

The artifices described have been considered legitimate by the various authors treating the subject and the author has good precedent for introducing his correction in the form employed. However, an error has been found in the final Equations [20*a*] of the paper, which, while not at all invalidating the method, may lead to large errors and should therefore be corrected.

In order to show the nature of the error let us consider the case of an air chamber located on a pipe, and let the friction losses (represented by the term kv^2) be concentrated at a constriction placed in the pipe close to the air-chamber entrance. Call Y_i the effective pressure head at the air chamber, and Y' the pressure head on the other side of the constriction, with the direction of flow being from Y_i to Y'. Then one has the relation

$$Y_i = Y_i' + kv_i^2$$

and

$$Y_{i-1} = Y_{i-1}' + kv_{i-1}^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots [1]$$

where $k>0$ if $Y>0$, and $k<0$ if $Y<0$. Substituting these values in Equation [1*a*] of the paper, one obtains

[7] "Variation of Flow in Water Conduits," by L. Bergeron, Comptes Rendus des Travaux de la Société-Hydrotechnique de France, Paris, France, no. 47, May 14, 1932, p. 615H.

[8] "Air Chambers and Valves in Relation to Water Hammer," by R. W. Angus, Trans. A.S.M.E., vol. 59, November, 1937, paper HYD-59-8, pp. 661–668.

$$(Y_i - kv_i^2) + (Y_{i-1} - kv_{i-1}^2) - 2(y_0 - kv_0^2) = a/g(V_i - V_{i-1}) \quad \ldots\ldots [2]$$

Following Allievi, let $Z_i = Y_i/y_0$ and let $W_i = V_i/v_0$, whence, by substituting into Equation [2] of this discussion, there results

$$W_i - W_{i-1} = \frac{1}{2\rho}\left[Z_i + Z_{i-1} - \frac{kv_i^2}{y_0} - \frac{kv_{i-1}^2}{y_0} - 2\left(1 - \frac{kv_0^2}{y_0}\right)\right] \ldots\ldots [3]$$

Note here that ρ is actually evaluated for an initial pressure head which is corrected for the friction effect.

Again following Allievi, let $K_i = kv_i^2/y_0$ and let $A = 1 - kv_0^2/y_0 = 1 - K_0$.

Then

$$W_i - W_{i-1} = \frac{1}{2\rho}[Z_i + Z_{i-1} - 2A - K_i - K_{i-1}] \ldots [4]$$

and adding this equation to a similar one for time $(i - 1)$, one obtains

$$W_i - W_{i-1} = \frac{1}{2\rho}[Z_i + 2Z_{i-1} + Z_{i-2} - 4A - (K_i + 2K_{i-1} + K_{i-2})] \ldots\ldots [5]$$

Finally the true forms of Allievi's Equations [20a] and [5b] are found to be

$$Z_i + 2Z_{i-1} + Z_{i-2} - (4A + K_i + 2K_{i-1} + K_{i-2}) = 2\rho\left(\frac{2C_0}{gv_0}\right)\left(\frac{1}{Z_i} - \frac{2}{Z_{i-1}} + \frac{1}{Z_{i-2}}\right) \ldots\ldots [6]$$

and

$$W_i - W_{i-1} = \frac{1}{2\rho}[Z_i + Z_{i-1} - (2A + K_i + K_{i-1})] \ldots [7]$$

from which it is readily seen that the omission of the term $A = 1 - (kv_0^2/y_0)$ in Allievi's equations is not correct.

Applying the correct form of equations, as given in this discussion, to example b on page 655 of Allievi's paper,[1] and correcting several numerical errors there involved, it is found that results obtained by the analytical and graphical methods separately, agree very well, and indicate no superpressure in place of 60 per cent as found by the uncorrected equations.

In this connection, note that the computations of surge pressures, for cases involving a correction for liquid friction, may become quite extensive. This is due to the fact, pointed out by the author, that Equations [20] and [20a] of the paper are dependent actually upon two parameters, N and μ, in place of only one, i.e., N, as appears at first glance. Since the use of the equations entails at least two series of approximate values for N, and since each of the two series should be studied for a series of fractional values of μ, then it appears that a correct determination of Z_{max} for the case involving correction for liquid friction requires a minimum of two series of approximations for each of a series of values of μ.

To conclude this discussion of Allievi's contribution to analytical methods for the solution of water-hammer problems there is place to note that an entirely new theory of water hammer in tapered pipes[9] has been developed by Dr. Henry Favre, of the hydraulic research laboratory annexed to the Eidgenossische Technische Hochschule in Zurich.

[9] "Théorie des coups de belier dans les conduites à caractéristiques lineairement variable le long de l'axe," *Révue Générale d'Hydraulique*, nos. 19, 20, and 21, January, March, and May, 1938.

R. W. Angus.[10] This paper[1] follows the procedure adopted by the author in his book[11] published by the A.S.M.E. The method used is not only simple, but elegant, in that it establishes what the author calls a "pendular system" of equations, all similar in form, and which are to be solved successively, using the numerical results from the first equation in solving the second one, and those of the second one in solving the third, and so on throughout the series. A disadvantage of this arrangement is that errors are cumulative and that it is not possible to know what happens in any interval without finding the results in all the intervals preceding it, but these defects exist in many other solutions of water-hammer problems and are not serious if the calculations can be made quickly and are not too sensitive to permit the use of an ordinary slide rule.

The author's notation interferes somewhat with convenient reading, but it is consistent with that used in his earlier papers; the frequent use of the asterisk is an easy cause of errors. The author's general Equation [1] is from the original paper[5] and the transformations down to the series of Equations [6] of the paper[1] are quite easy. The values of ρ^* given in Table 1 of the paper would cover most cases of power plants, but in pump-discharge lines, which are those to which the air chamber is primarily applicable, the table is too limited. For example, in a pumping line recently examined, the writer found a value of ρ^* exceeding 4 and, indeed, in most low-head piping, such as that on filtration work, the pipe characteristic may considerably exceed four.

The writer would agree with Equations [7], [8], and [9] of the paper, but he thinks that considerable error may result from the assumption of Equation [10], except in the first interval. The author[1] refers to a pipe line supplied by a pump without inertia, and which, therefore, stops instantaneously with power interruption; a check valve is supposedly installed between the air chamber and the pump, this valve closing instantly with power failure and preventing any return flow through the pump.

Curiously enough, that is the identical problem inadvertently selected by the writer in one of his papers,[8] and a direct check is thus easily made as to the assumption of Equation [10] of the paper under discussion.[1] In the first interval it does hold, but an examination of Fig. 5 in the writer's paper[8] shows that such an assumption cannot be made in the remaining intervals. Thus, in the second interval $v_1 = 0.29$ and $v_2 = -0.21$, but the maximum value of the velocity is at the center of the interval and is $v_{1.5} = -0.75$ and the average of all values between v_1 and v_2 is very much greater numerically than $(v_1 + v_2)/2 = (0.29 - 0.21)/2 = 0.04$, and is of opposite sign to it. Similar statements apply to other intervals. This example is one that shows an unusual pressure rise and in which the chamber is of little value, for the pressure ratio H/H_0 reaches a maximum value of 2.40 which exceeds $1 + 2\rho = 2$ due to sudden closure, therefore, the air chamber used is much too small for the system, but the solution of the problem was the method of proving this fact.

Fig. 5 of the writer's paper[8] is for a long pipe with $2L/a = 10$ sec, in which case the error is greater than for shorter pipes, and no method will be accurate unless the actual period of time used in the calculation is small; thus, the writer started with periods of 0.2 sec, lengthening later to 0.5 sec, and finally to 1 sec, in such a way as to make the velocity change small in the selected period.

With this qualification, and allowing for typographical errors, the equations of the paper[1] under discussion are correct and most helpful; Equation [18], for instance, is correct and not affected by the foregoing argument, but all of the other equations are.

The author is to be commended for his constant practice of

[10] Head of the Department of Mechanical Engineering, University of Toronto, Toronto, Ontario, Canada. Mem. A.S.M.E.

[11] "Theory of Water Hammer," by L. Allievi, translated by E. E. Halmos.

giving numerical illustrations, and he has given in Table 2 of the paper various values of Z for different pipe conditions indicated by various values of ρ and σ. From Equation [14] the product of these quantities is found in terms of the initial air volume, initial velocity v_0, the pipe area S, and $\mu = 2\,L/a$. In the writer's example $\sigma^*\rho^* = 2C_0/\mu S v_0 = (2/\mu v_0)(C_0/S) = (2 \times 10)/4\mu = 5/\mu$, so that the Table 2 of the paper[1] may be expressed in terms of ρ^* and $\mu = 2L/a$ as given in Table 1 of this discussion.

TABLE 1 VALUES OF $2L/a$ CORRESPONDING TO THE AIR CHAMBER IN ANGUS' PAPER[8] AND TO THE AUTHOR'S DATA

σ^*	$\rho^* = 0.5$	$\rho^* = 1$	$\rho^* = 2$
30	0.333	0.167	0.0833
5	2.000	1.000	0.5000
2	5.000	2.500	1.2500

The writer worked out the first two cases by the method explained in his paper[8] and found the value of Z_1 at the end of the first interval to agree exactly with that given by the author,[1] but at other intervals there was a considerable divergence, particularly in connection with the maximum and minimum values. The writer would be indebted to the author if the author would state whether there is any limit beyond which his formulas cannot be used.

The author's method of including friction is ingenious and gives another of his series of fascinating pendular equations. The writer agrees with the author that "provided the friction is neglected the air chambers are very inadequate instruments for the intended purpose of rendering innocuous the kinetic energy of the liquid column by the simple means of pneumatic accumulation." The air chamber may be used to store potential energy, but is not efficient for kinetic energy.

The author's suggestion of designing the passage between the air chamber and the pipe in such a way that the resistance for flow into the chamber from the pipe is very much less than the flow out of the chamber into the pipe, is most valuable and will result in enormously increasing the values of the air chamber.

A. A. KALINSKE.[12] The author made a notable analysis of the pressure and velocity variations which follow certain instantaneous initial conditions in a pipe equipped with an air chamber. Although the author in his various analyses and typical problems is only concerned with the behavior of air chambers on pump-discharge lines, the theory presented can be utilized, with only slight changes in some of the formulas, for the case when an air chamber is used to control water-hammer pressure caused by a quick-closing valve in a water-supply line. However, to be analogous to the case of sudden stoppage of power and coincident closure of a foot valve in the pump-suction pipe, for which the author developed his theory, it is necessary to apply his equations only to conditions where complete instantaneous closure occurs. No doubt the theory can be extended to include cases of slow closure of valves and gates.

The author's final equations for the pressure and velocity at various intervals of time $2L/a$, following the specified instantaneous initial conditions, are most unique and amazingly simple to use. The basis of the author's simplification is an important assumption which he makes and which may not be true in all cases. The assumption is that the variation of velocity during the phase interval $2L/a$ is linear; therefore, when the velocity at the beginning and end of the interval is known, the average velocity may be easily determined. Except for very long pipe lines, when $2L/a$ is large, this assumption is undoubtedly quite accurate and justifiable.

The writer has applied the author's theory to the case where an air chamber is needed to control the water-hammer pressure due to an ordinary quick-closing valve in a pipe line. In this case the pressure at the end of the first phase can be obtained from Equation [18] of the paper with a change in one sign, thus

$$Z_1^2 + Z_1 (N - 4\rho^* - 1) - N = 0 \quad \ldots\ldots\ldots [8]$$

According to the author's nomenclature Z is the ratio of the absolute pressure at the end of any specified phase interval and the initial absolute pressure. The term ρ^* is the ordinary pipe-line constant and N is a dimensionless parameter equal to the half ratio of the total initial potential energy of the air chamber and the analogous potential energy of the pipe line.

The values of Z at the end of the second, third, etc., phases can be obtained by the use of Equations [20] of the paper without any alteration. The velocity can be obtained by the use of a system of equations similar to the author's Equations [5a], except for a single change in sign. Their general type is

$$v_{i+1} = v_{i-1} - v_0/2\rho^* (Z_{i-1} + Z_{i+1} - 2) \quad \ldots\ldots [9]$$

In most instances when calculating pressure changes due to water hammer in water-distribution pipe lines, the losses due to fluid friction are large enough to necessitate their being taken into account. The author gives a satisfactory and straightforward method for taking friction loss into account by introducing the term kv_i^2. For the case of instantaneous closure of a gate the correction due to friction must, of course, be made opposite to that made for the pump-discharge line by the author. The pressure rise during the first phase is given by

$$Z_1^2 + Z_1 [N - 4\rho^* - (1 - K_1)] - N = 0 \ldots\ldots [10]$$

The term K equals kv_i^2 divided by the initial absolute pressure which for this case is equal to the absolute static pressure minus the initial friction-loss head.

The system of equations to be used for obtaining the pressure at the other phase intervals would be similar to the author's Equations [20a] with the signs on the K_i factors reversed. Unless close attention is paid to the author's nomenclature the reason for nonappearance of the initial velocity friction factor, K_0, in the author's Equations [18a] may not be apparent. Its disappearance is due to the fact that Z_0, which equals $(y_0 + h)/y_0^*$, and K_0 which equals kv_0^2, combine to give unity since $y_0^* = y_0 + h = kv_0^2$, where h is atmospheric pressure.

As the author points out, to have a basis for determining sizes of air chambers for pipe lines, in order that the pressure will not rise above a certain maximum value of Z_m, it is necessary to analyze the behavior of the air chamber and pipe line from the standpoint of energy conservation. Energy considerations reveal a fact that is usually lost sight of, that is, that the kinetic energy of the water is only a small part of the total energy stored in the air chamber at the value of maximum pressure. If no great friction losses exist in the pipe itself, or between the pipe and the air chamber, the air chamber accumulates a great deal of energy that is represented by the potential energy of the water entering the air chamber as the pressure rises, and the air is compressed. The author calculates this potential energy thus

$$\text{P.E.} = H_s w (U_1 - U_2) \ldots\ldots\ldots\ldots [11]$$

Where H_s is the absolute static pressure in feet of water, $(U_1 - U_2)$ is the volume of compression or the volume of water entering the air chamber between the initial and final conditions, and w is the weight per unit volume.

In pipe lines where there is appreciable friction loss, this potential energy would decrease by the amount of energy loss that this volume of water suffers as it flows down the pipe and into the air chamber. At any instant this energy loss is represented by the product of the weight of the water entering the air chamber at

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that increment of time, and the difference between the static pressure and the flow pressure. The total energy loss will be equal to $mw\,(U_1 - U_2)\,kv_0^2$, where m is some coefficient less than unity and usually greater than 0.75. This coefficient is nearly equal to unity because the rate of water entrance into the air chamber is high when the friction loss is high. The value of this coefficient has been checked experimentally.

Therefore, the energy stored in the air chamber is

$$W_0 + H_s w(U_1 - U_2) - mw(H_s - H_1)(U_1 - U_2) - \Delta W \ldots [12]$$

In this equation W_0 is the initial kinetic energy of the flowing water, H_1 is the absolute initial pressure before the flow in the air chamber starts, and ΔW represents the energy stored in the pipe walls and the water, minus the potential energy of the volume of water flowing into the space produced by the dilated pipe and the compressed water. If the air chamber were fairly large so as to affect a considerable pressure reduction, the term ΔW would not be very significant. For rough calculations the following equation can be used for obtaining the energy stored in the air chamber

$$W_0 + H_1 w\,(U_1 - U_2) \ldots [13]$$

Assuming that the compression of the air in the chamber is isothermal, the volume of the air chamber under initial conditions can be computed approximately from

$$U_1 = \frac{LSV_0^2}{2gH_1[\log_e r + (1/r) - 1]} \ldots [14]$$

where L is the length of the pipe line, S is its area, v_0 the initial water velocity, r the ratio of the maximum absolute pressure and the initial absolute pressure, and U_1 is the volume of air under initial pressure conditions. The actual volumetric capacity of the air chamber U_a would be obtained from the relation $U_1H_1 = U_ah$, where h is the atmospheric pressure. Thus, it is quite apparent that the size of an air chamber is influenced considerably by the initial pressure conditions in the pipe line.

Equation [14] of this discussion is valid only when no restriction exists between the pipe line and the air chamber. The author shows quite clearly how restricting the flow of water into the air chamber makes possible the use of a smaller chamber. However, the statements relating to the amount of restriction of friction loss between the pipe line and the air chamber need experimental verification. The optimum amount of restriction should be determined and experiments made checking the theoretical deductions.

There is no question but that the author's dimensionless parameters, which completely describe any pipe line and air-chamber installation, will be of inestimable value in interpreting and conducting experimental work. They will be of great aid in the presentation of the experimental data and in its extension for practical use.

Author's Closure

The following refers only to the analytical methods employed in the paper.

Some Doubts?

Some doubts have been raised by different writers concerning the approximation of Equation [18] determining Z_1 and of Equations [20] (pendular system), such doubts referring to the arbitrary assumption of Equation [10], viz., the linearity of V during and along a phase. The following observations may throw some light on the question.

Determination of Z_1 by Equation [18].

The law of the variable value Z during the first phase can be exactly obtained by means of the differential equation

$$\frac{\partial Z}{\partial t} = \frac{2Z^2}{N_\mu}(1 - 2\rho^* - Z) \ldots [\alpha]$$

which can be established by means of Equations [1a], [5], and [9], and which admits integration by a rather complicated formula[13] not available for further investigations.

But for the present object it may be considered sufficient to examine some results for a certain middle value of ρ^* corresponding to heads of from ~50 to 60 m, viz., for $\rho^* = 0.5$. With this assumption Equation [α] of this discussion becomes

$$\frac{\partial Z}{Z^2} = \frac{-2\partial t}{N_\mu}$$

whence integrating

$$\frac{1}{2Z^2} = \frac{2t}{N_\mu} + \text{const}$$

which for $t = 0$, $Z = Z_0 = 1$ gives const $= -0.5$, so that finally the foregoing equation of the curve diagram of Z during the first phase (parabolic curve) can be written

$$Z = \sqrt{\left(\frac{N_\mu}{N_\mu + 4t}\right)} \ldots [\beta]$$

and for $t = \mu$

$$Z_1 = \sqrt{\left(\frac{N}{N + 4}\right)} \ldots [\beta\beta]$$

Comparing now the exact values of Z_1 obtained from Equation [$\beta\beta$] with the approximate values obtained from Equation [18] of the paper it is seen that for a large range of values of σ^* (large range of volumes of air chamber) there is a small difference between the two series of values.

COMPARISON OF VALUES OF Z_1 FROM TWO EQUATIONS, $\rho^* = 0.5$

σ^*	N	Equation [18] of paper, Z_1	Equation [$\beta\beta$] Z_1
30	15	0.888	0.888
15	7.5	0.805	0.808
5	2.5	0.608	0.620
2	1.0	0.414	0.448
1	0.5	0.280	0.333

From this it is evident that, for middle values of ρ^* and with the exception of very small air chambers, the assumption of linearity expressed by Equation [10] of the paper can be satisfactorily accepted.

Such is not the case for small values of σ^* corresponding to small air chambers and small values of N as can be verified by the construction of parabolic diagrams represented by Equation [β], the curvatures of which are then strongly accentuated. These curvatures, probably less acute, must obviously be repeated in the diagrams of Z referring to the following phases. Alike analogous curvatures must obviously characterize the diagrams of V.

Determination of Z_1 by the Pendular System, Equations [20].

The effective law of V for small air chambers must show, in three consecutive phases $i - 1$, i, and $i + 1$, one of the two shapes shown in Figs. 1 and 2, which are apparently in contradiction with the hypothesis of linearity stated by the *arbitrary* assumption of Equation [10] of the paper.

Such is, however, not the case, because Equation [10] has not

[13] $$\frac{Z - 1}{Z(1 - 2\rho^*)} + \frac{1}{(1 - 2\rho^*)^2}\log_e\frac{2\rho^* Z}{Z(1 - 2\rho^*)} = \frac{2t}{N_\mu}$$

an expression which loses any sense when ρ^* approaches the value 0.5 and therefore Equation [β] is substituted.

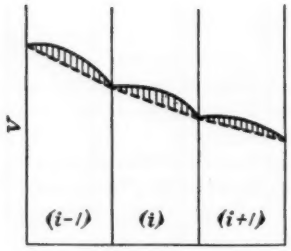


FIG. 1

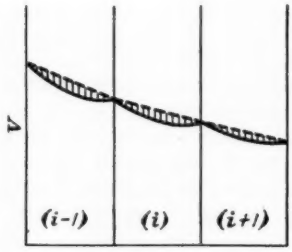


FIG. 2

been applied to single phases, but to the expression connecting three diagrams—areas of consecutive phases

$$\int_0^{i-1} V\partial t - 2\int_0^{i} V\partial t + \int_0^{i+1} V\partial t \ldots\ldots\ldots [\gamma]$$

The difference, for each phase, between the real area and the hypothetical area corresponding to linear variation is represented by the lunulae sketched in Figs. 1 and 2. It is now easy to perceive that if the areas of the successive lunulae vary with a certain uniformity, which can be plausibly assumed, their influence on the value of the foregoing expression $[\gamma]$ is insignificant and can be neglected.

These considerations fully justify Equation [16] of the paper and the plausible approximation of the pendular system, Equations [20], even for very small air chambers.

An Error?

Messrs. Mason, Danel, and Craya state that an error has been made and that the first members of the equations concerning acceleration, taking into account liquid friction, should be written

$$(Y_{i-1} - kV_{i-1}^2) + (Y_i - kV_i^2) - 2(y_0 - kv_0^2)$$

whence it appears that these writers assume that the initial head for the pumping regimen, that is, for ascending flow, has the value $y_0 - kv_0^2$ instead of the well-known $y_0 + kv_0^2$, which is an error undoubtedly, and an inexplicable one indeed.

Methods Employed to Remedy Water-Hammer Shock in Pumping Systems[1]

J. G. MILLER.[2] In the past, most papers on water hammer have treated the subject either from the theoretical point of view or as it applies to penstocks or other large hydraulic lines, delivering several thousand gallons per minute. In designing such lines, where little if any margin is allowed for contingencies, it is particularly important to know what maximum water-hammer pressures will be encountered so that rupturing of the pipe can be prevented. Consequently, this hydraulic phase of water hammer has been investigated by numerous engineers, while other places of occurrence have been more or less neglected. There are many cases of this sort where water hammer is not anticipated but is found to exist, after the pipe line is built. Then the problem is one of eliminating the trouble as cheaply and as effectively as possible. The author's paper in dealing with shock pressure from this point of view offers considerable useful and timely information.

In his paper, the author discusses the effectiveness of the so-called "recoil valve" in reducing water-hammer pressures. This valve, it seems, has been developed especially for use on water lines, where water hammer is encountered. Since the author has used it in all the cases discussed, it seems that a description of this valve is not only desirable but necessary.

H. J. Bartlett[3] suggests using an "ordinary check valve, equipped with an outside lever and weight, installed in such a way that the weight would aid in closing," for the same purpose that the author uses the recoil valve. Mr. Bartlett concludes with the statement: "In actual service such a valve will close when the pump stops, slightly before forward motion of the water ceases and thus there is no reversal of flow at all. In the many, many cases, where such valves have been used, we have yet to hear of one where they have failed to produce the desired results."

Since no description of the recoil valve has been given in the author's paper, the writer is unable to tell the difference between the recoil valve and the ordinary check valve with the weighted disk. However, in his discussion of check valves, the author makes the statement that "more severe conditions can sometimes be met by fitting loaded disks to otherwise standard check valves...." From this the writer concludes that the recoil valve is not an ordinary check valve with a loaded disk. Question is raised also as to when he deems it inadvisable to use the loaded-disk check valve and, in view of Mr. Bartlett's statement, what are the particular advantages of the recoil valve?

It is encouraging to note that both the author and Mr. Bartlett agree that the slow-closing or cushioned check valve not only fails to improve a water-hammer condition but sometimes will make it more severe.

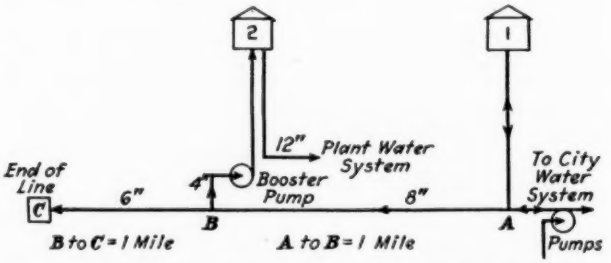


FIG. 1 DIAGRAMMATIC LAYOUT OF BRANCH LINE IN WHICH WATER HAMMER IS FREQUENTLY ENCOUNTERED

In case 4 the author uses an air chamber as a means of decreasing water hammer. This method of solution is considered effective in reducing shock pressures by most piping designers if the air chamber can be made large enough. However, in many cases, it is not practicable to install an air chamber of sufficient capacity to reduce shock pressure more than the 10 to 15 per cent usually needed to eliminate the characteristic clanking sound. If greater reduction is required, some other solution frequently must be found. Where such necessity exists, it would seem that a valve similar to the one used in case 3 offers a more general solution to problems similar to case 4 than an air chamber.

It would be interesting to learn whether the author has had experience in solving water-hammer problems in branch lines. This phase of the subject has not received much attention from the practical point of view. The writer's Fig. 1 shows a case of a branch line where water hammer is known to occur, which may or may not be serious depending on the circumstances. The height of both tanks is approximately the same. The purpose of the pump is to overcome friction in the main, thus speeding up the filling of tank No. 2 from tank No. 1, following intermit-

[1] Paper by E. B. Ball, published in the January, 1939, issue of the A.S.M.E. Transactions, vol. 61, p. 5.

[2] Engineer, Engineering Division, The Detroit Edison Company, Detroit, Mich. Jun. A.S.M.E.

[3] "Checking Up on Check Valves," by H. J. Bartlett, *The Valve World*, issues of July-August and September-October, 1938, pp. 152–158.

tent heavy use of tank No. 2. While the pump is running, the pressure at B and also at C will be equal to the pressure at A minus the friction loss from A to B (assuming no flow from B to C). When the pump is stopped, the pressure at B and C increases nearly 100 per cent above normal pressure at A, due to shock following stopping of the pump. This case is not necessarily typical of all branch-line problems, but is a type frequently encountered. A discussion by the author of a similar case or any other case of water hammer in branch lines with which he has had experience will add much to the usefulness of the paper.

Author's Closure

Mr. Miller is to be thanked for his informative contribution, and for raising several very interesting questions.

The general features of the "recoil" valve are given under the section *Remedy*, of case 1, and in the final paragraph of the article. Unfortunately a general sectional drawing would not convey the features sufficiently convincingly. The commercial check valve is a competitive product with the design controlled by cost. When water hammer, due to valve-slam, occurs, the cost of the check valve is negligible compared with the threatened expense of repairing damage. Hence a valve embodying all the greatest refinements known, verified and found out by experiment, becomes a relatively inexpensive remedy. The recoil valve is neither more nor less than this.

The question of when a loaded-disk check valve should be used is generally decided by the existence, or otherwise, of a check valve in the system. In order to avoid the removal of this, a loaded disk would be tried as a first attempt. If that failed the check valve would be replaced by a recoil valve. A loaded disk alone is regarded as a makeshift as the body design of ordinary check valves does not conform with the ideal characteristics.

In his reference to case 4, Mr. Miller overlooks the emergency possibility of a sudden shutting down of the pumping plant due to failure of the electric supply. In such circumstances it would usually be impossible to bring into service a valve of the type employed in case 3. It should also be noted that in case 4 the water hammer did not arise from the slamming of the check valve. Admittedly air chambers are not always a practical proposition and it is then necessary to fall back on some form or other of pressure-relief or concussion-relief valve. Under suitable circumstances these are quite successful in reducing, though not altogether eliminating, surge pressure. They must, however, be used with caution and understanding, and are rarely successful in short-pipe systems.

Regarding Mr. Miller's interesting branch-line problem the first essential is to determine, as precisely as possible, the origin of the water hammer, distinguishing carefully, if necessary, between shocks due to check-valve slamming and pressure surges due to changes in the rate of flow and arising from the elasticity of the water columns. As so often happens in problems submitted, sufficient data are not given to permit of a useful discussion, but the behavior suggests that there may be check-valve slamming in conjunction with the usual elastic surging of the water column on the suction side. With boosting arrangements possessing a very large water system on the suction side, the widespread disturbance set up in this system when the pumps are at work cannot settle down so rapidly as the relatively small system on the delivery side. It commonly happens that the surge pressures built up on the suction side pass through the pump by reopening the check valve on the pump delivery and thus obtain relief. If this does not happen in the case quoted excess pressures due to check-valve slamming are suggested on the delivery side. First attention should therefore be given to the check valve and, as the booster connections are small (4 in. diam) compared with the suction main (8 in. diam), further relief to the suction system should be considered, such as connecting a by-pass, with check valve, direct from the suction side of the pump to the 12-in. main leading to the plant water system.

The author desires to acknowledge the cooperation of his staff in preparing the material submitted.

Speed of Water-Hammer Pressure Wave in Transite Pipe[1]

W. S. Pardoe.[2] Professor Kessler in his paper has given a beautiful demonstration of the accuracy of Joukowsky's formula

$$a = 4670 \sqrt{\left(\frac{1}{1 + \frac{K}{E}\frac{d}{t}}\right)} \quad \ldots\ldots\ldots\ldots [1]$$

or he has an expensive method of determining the modulus of elasticity of Transite. The coefficient 4670 or the acoustic ve-

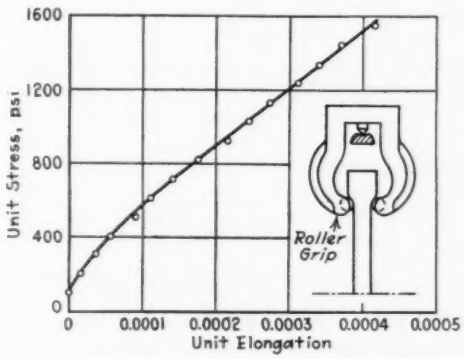


Fig. 1 Stress-Strain Curve of Cement Asbestos
(Ultimate strength = 2310 psi; secant modulus = 3,500,000 psi.)

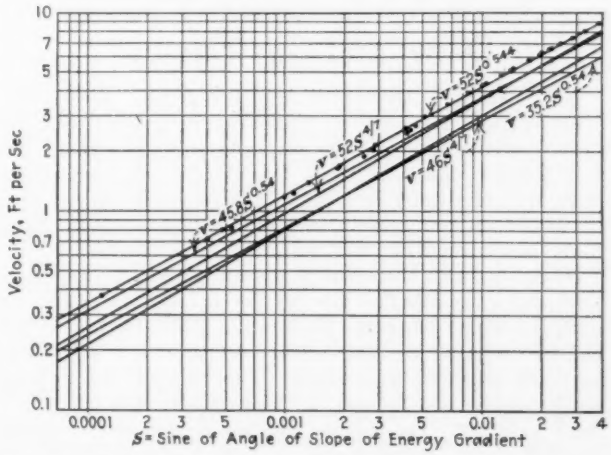


Fig. 2 Test of 150 Mm Cement-Asbestos Pipe and Comparison With Cast-Iron Pipe

Cement-asbestos	$v = 52S^{0.544}$
Hazen-Williams, clean cast iron	$v = 45.8S^{0.54}$
Hazen-Williams, tuberculated cast iron	$v = 35.2S^{0.54}$
Flamant, new cast iron	$v = 52S^{4/7}$
Flamant, slightly incrusted cast iron	$v = 46S^{4/7}$

[1] By Lewis H. Kessler, Transactions of the A.S.M.E., January, 1939, vol. 61, p. 11.

[2] Professor of Hydraulic Engineering, University of Pennsylvania, Philadelphia, Pa.

locity in water in an inelastic pipe line is obtained from the formula

$$h = \frac{aV}{g} = V\sqrt{\left(\frac{K}{wg} \times 144\right)} \quad \ldots\ldots\ldots\ldots [2]$$

If K = 294,000 psi and w = 62.3 lb per cu ft, a = 4670. Professor Kessler must have used K = 300,000 psi.

In March, 1925, the writer made some tests[3] on Italite pipe, made in Genoa, Italy, for the Asbestos Association at the civil engineering laboratories at the University of Pennsylvania. This product was called cement-asbestos pipe and was the forerunner of the Transite pipe. Fig. 1 herewith shows a modulus of elasticity of this material (sample cut from pipe laterally) of 3,500,000 psi which checks Professor Kessler's median value of 3,375,000.

The velocity of the stress wave for 14-in. class-B cast-iron pipe is

$$a = 4670\sqrt{\left(\frac{1}{1 + \frac{294{,}000}{13{,}000{,}000} \times \frac{14}{0.65}}\right)} = 3825 \text{ fps} \ldots [3]$$

and that for the Transite pipe is

$$a = 4670\sqrt{\left(\frac{1}{1 + \frac{294{,}000}{3{,}500{,}000} \times \frac{14}{1.185}}\right)} = 3360 \text{ fps} \ldots [4]$$

or 12 per cent less, which lowers the water hammer by the same percentage.

Fig. 2 shows the velocity-slope curves for cement-asbestos pipe and for various cast-iron-pipe formulas. As

$$V = 52\,S^{0.544} = C \times R^{0.63}S^{0.54} \times 0.001^{-0.04} \ldots\ldots [5]$$

or C = 142.5 in the Williams and Hazen formula. This checks the value of 140 given by Professor Kessler.

Cooling-Tower Fundamentals and Their Application to Cooling Diesel and Gas Engines[1]

E. J. Kates.[2] The writer fully agrees with the author's statement, in referring to Diesel-engine applications, that "partial recirculation is often advisable with thermostatic control, keeping off-engine water temperatures uniform without changing jacket-water circulation." So important are the benefits of such an arrangement, both as to engine and as to cooling tower, that it would be well to set forth some of the reasons.

By-passing with thermostatic control benefits the engine by producing a vigorous flow of water through the jackets and permitting a higher off-engine temperature. Also the rate of flow and the off-engine temperature do not change with engine load as would otherwise be the case. The vigorous flow prevents vapor pockets and reduces the heat-transfer skin effect from metal to water. The uniform temperature and small temperature rise reduce heat strains in the metal walls. The high temperature improves engine thermal efficiency, particularly at partial loads. Thus engine reliability is increased, maintenance is reduced, and fuel consumption is decreased.

At the same time, the recirculating system permits reducing the size of the cooling tower for two reasons: First, since some of the water discharged from the engine is by-passed around the cooling tower, the flow over the tower is reduced, thus permitting the use of a smaller tower; second, the higher temperature of the water, flowing over the tower, produces a greater temperature difference between the water on the tower and the air.

G. C. Boyer.[3] While not a designer of cooling towers, the writer would like to make the following inquiries from the viewpoint of a consulting engineer, who utilizes them as part of a Diesel power plant:

1 Does not the fine breakup in spray-type towers take the place of arrestation by decks in the deck-type tower? Does not a large contact area take the place of the time element in the deck tower?

2 What is the effect of increasing the height of a deck-type tower above 30 ft? In the paper the author limits his towers to 30 ft in height. This question is raised because it appears if the height of a tower is increased, then it is possible to decrease the length of the tower with a corresponding saving in foundation cost.

3 Is it not true that the capacity of any distribution system falls off with a decrease in pressure? Is it not, therefore, as necessary to insure maintenance of pressure on one type of distribution system as on the other? Reference in this connection is made to a comment in the paper, regarding the deck-type versus the spray-type tower.

In discussing mechanical-draft spray-type towers, the author mentions limitations to which they are subject, without specifying them, so far as the writer can discover in the text. Enlightenment on this point would be helpful.

It might be of interest to cite the performance specifications for a cooling tower, prepared by the writer. The system involved was of the closed type, consisting of a spray-type tower with the heat-exchanger coils located at the bottom. A definite requirement was established for the distribution of 300,000 Btu per hr, with a jacket-water temperature off the engine not exceeding 140 F when 76 F wet bulb and a 3-mph wind prevailed.

As a side light on this specification, it was the writer's contention in discussion a year ago on the subject of a closed cooling system, that 140-F off-engine temperature was the upper limit practicable. Exception was taken to this by Edgar Kates, who stated that 160 F was entirely permissible. In preparing the foregoing specification the writer decided to compromise between these two figures and fixed the off-engine water-temperature requirement at 150 F. The result was that the Diesel-engine manufacturers refused to tolerate a cooling-water temperature above 140 F, stating that the higher requirement, if maintained, would definitely rule them out as bidders. The temperature was, therefore, reduced to 140 F.

The answer on that point is the same as it was a year ago. When the Diesel-engine manufacturers will agree to raise the temperature off the engine with a closed cooling-water system, engineers will write their specifications accordingly.

Incidentally the same manufacturer, who objected to 140 F in this engine cooling specification, has an experimental engine running with a temperature differential across the engine of 1 degree F and a temperature off the engine approaching 160 F. The rate of putting water through this engine may well be imagined!

Author's Closure

The fine breakup in spray-type towers does take the place of filling, in so far as it provides water surface in contact with air. In a deck tower, besides the wetted deck surface, there is also

[3] "Tests of Cement-Asbestos Pipe," by W. S. Pardoe, *Engineering News-Record*, vol. 96, January 28, 1926, p. 169.

[1] Paper by Gustave J. Bischof, published in the January, 1939, issue of the A.S.M.E. Transactions, vol. 61, p. 25.

[2] Consulting Engineer, New York, N. Y. Mem. A.S.M.E.

[3] Burns & McDonnell Engineering Company, Kansas City, Mo.

drop surface due to the splash of falling water on the decks. The time element is especially important where long ranges and close approaches to the wet bulb are required. This time element can only be obtained by the use of decks to interrupt water fall in an atmospheric tower or by filling in a mechanical-draft tower. There are really two distinct phases to be considered in the evaporative cooling of water; namely, water surface and time of contact with air. Neither one can wholly substitute for the absence of the other. Careful study of the particular requirements of each installation is necessary to select the type of tower most suitable.

In regard to the most economical height of the tower, there should be no attempt to limit the height of an atmospheric-deck tower to 30 ft, even though this height has been more or less standardized for towers 12 ft wide inside of the louvers. As Mr. Boyer points out, there are times when it is desirable to go higher than 30 ft. The question is largely one of the least expensive installation in terms both of first cost and operating cost. The first cost of the combined tower and foundation may be decreased by increasing the height of the tower to about 42 ft and shortening it, but the pumping head will also be increased by the same amount. Consequently, pumping costs will go up. Beyond 42 ft, wind stresses become so large that either a wider tower or one with spread footings is desirable for stability, thus adding to the tower cost with little or no foundation savings. It might seem on first thought that a wider tower would take care of more water per foot of length. However, experience indicates that wider towers should be loaded less per square foot of deck area, as they do not cool as efficiently due to the greater obstruction to lateral air movement and progressive saturation of the air as it moves across the tower. It is evident that many factors enter into the most economical design in any particular case and it pays to study carefully all factors before the final decision is made as to dimensions.

It is true that the capacity of any distributing system is roughly a function of the square root of the pressure. If the tower operates at all times, at the definite capacity for which the distributing system was designed, then the type of distributing system is not of great importance. However, there are many installations where the distributing system is designed for a maximum circulation and is run for varying periods at much less than rated capacity. Here it becomes important to consider the part that pressure plays in the cooling of the water by means of water-surface production. In a deck tower only a small portion of the heat dissipation occurs on initial water breakup between distributing system and the top deck. Practically all the heat is dissipated during the descent of the water through the decks. Hence, it is of slight importance whether the distributing system operates at over capacity under high pressure or at under capacity with but a dribble, as the decks distribute the water and form a good portion of the total surface. In fact, the less water circulated over a deck tower, the less density of water per square foot of deck area and the better the cooling.

On the other hand, a spray-type tower without decks depends solely on fine initial breakup of water leaving the nozzles. There is in fact no other place for drop surface to form. If, therefore, a spray-type tower of a given rated capacity should be operated at less capacity or with lower pressure at the nozzles, the water is no longer broken up into drops but dribbles from the nozzles, and cooling effect falls off. The only effective way to decrease capacity in a spray tower and maintain cooling is to substitute smaller-capacity nozzles or shut off a certain portion of the nozzles uniformly throughout the tower. While the above discussion deals with atmospheric-deck towers versus atmospheric-spray type towers, the same reasoning applies to mechanical-draft towers with and without filling.

The Drafting of Steam Locomotives[1]

E. G. Young.[2] The author has discussed several matters which have been argued ever since a front-end problem was first recognized, which goes back as far as 1870.

The clearness with which the author shows the blowover effect, that is, the increase of back pressure on one side due to the exhaust from the other, is of particular importance. The indicator cards which he has used show much more of this effect than others seen by the writer, and it seems proper to ask whether the separation between exhaust passages was carried up actually into the nozzle stand, as is now recognized to be good practice, or whether the passages merged within the cylinder castings.

There are two questions with regard to the use of the four-ported Layden-type nozzle tip which arise; one of these the author has clearly answered in favor of this nozzle. The problem of "filling the stack" is one which will not down. The four-ported nozzle "fires" with first one pair (diagonally opposite) of holes and then the other. It would certainly not appear that either pair would set up a stream of steam and entrained gases which would satisfactorily fill the stack, but it becomes evident from the lower right chart of Fig. 5 of the paper that the alternate action actually does set up such a train as to fill the stack effectively at low speeds and obtain much better results than the plain nozzle. The second question is not answered, that is, the one regarding the back pressure resulting from a given rate of steam discharge. When one end of one cylinder releases, the bulk of the steam flows out through the exhaust ports in the very limited time between release and the end of the stroke, and it must flow through a nozzle-tip opening of half the area but the same perimeter as the plain nozzle for which the special nozzle is substituted. It is true that in the case of the special nozzle, the steam impulse has the opening all to itself, but it is also true, as is obvious from the exhaust-wave diagrams shown in the paper, that the interference from the opposite exhaust is very small. The expectation would be that the exhaust pressure would be considerably higher when equal amounts of steam are discharged by the four-ported nozzle.

In discussing the "characteristics and limitations" of the exhaust-pressure and draft relationship, the writer believes that there is a much more fundamental limitation on this ratio as an indication of the degree of efficiency with which a front-end arrangement operates, than any mentioned by the author. Exhaust pressure is a result of a certain weight of steam being released through a nozzle of given form and dimensions in a certain time. Draft, similarly, is an unsatisfactory unit; it results in part from the method of releasing the steam, and equally from the resistance of the path through which the air and combustion gases must travel. Paradoxically enough, the purposes of the draft-producing devices on a locomotive are not to produce draft, but to move air though the fire, and draft is only a suitable unit for comparison when a constant resistance to the gases is present. The actual problem of front-end design is to produce an arrangement which will pull an optimum weight of air through the fire for each pound of steam generated and discharged, maintaining at the same time three mechanical conditions: (1) Minimum back pressure, (2) clearing the smokebox, and (3) lifting the smoke above the train. The greatest difficulties lie in the mutual incompatibility of these mechanical requirements.

The author's case for the variable nozzle—and automatic

[1] Published as paper RR-60-4, by J. R. Jackson, in the October, 1938, issue of the A.S.M.E. Transactions, vol. 60, p. 573.

[2] Research Professor in Railway Mechanical Engineering, University of Illinois, Urbana, Ill. Mem. A.S.M.E.

operation is highly desirable—is one of the strongest which has come to the attention of the writer. The use of a fixed-area and fixed-form nozzle is a concession to the mechanical difficulties involved in the variable type. It represents a tacit assumption that it is economically more profitable (or more efficient in terms of dollars and cents) to assume that a locomotive works at a constant steam output under all conditions, and to fix upon a nozzle arrangement (including size, shape, and distance from the top of the nozzle to the top of the stack) which is most efficient for that output, and to accept inferior performance when working at any other output rate, than to use a nozzle variable in form or area. Certainly no data have been developed proving this to be the case.

A. I. Lipetz.[3] The author presented a very complete and thought-provoking paper. No conclusions are given but it will serve as a splendid introduction to a research study of drafting steam locomotives.

The historical data on the earlier research made by MacFarland with a multibladed fan driven from a live-steam noncondensing turbine is of great value and interest. The mentioning of the exhaust-steam turbine tried by Goss and the reasons given for the failure of both Goss's and MacFarland's attempts, are very important; we seldom get information on unsuccessful attempts, but they are just as valuable, for future research, as those which succeed. The Lewis exhaust ejector, which actually was a variable nozzle with a wide stack, and the partial success obtained by it is very interesting. The discussion of the tortuous development of the idea of enlarging stack and nozzle diameters is very significant and instructive. The mentioning of the necessity of smoke-lifting properties of a good drafting arrangement is appropriate, as in a great many cases the absence of this property explains the failure of some promising designs.

The discussions of the characteristics and limitations of the relationship between exhaust pressure and draft, obtained by a study of locomotive-test results of a standard single circular nozzle, together with the Lewis exhaust governor and the Layden nozzle with four round openings of constant area, are very enlightening. The idea of operating the two exhausts in order to eliminate the middle hump of indicator cards usually obtained, and to permit the ejecting effect of the exhaust of one side for the work of the cylinder on the other, should be studied more carefully. Partly unsuccessful results of tests made abroad (Germany and Russia) with the Stumpf nozzle have not been mentioned. They should be carefully analyzed.

The future investigator should, first of all, pay attention to the question of a variable versus a constant nozzle, in view of the splendid results which are now obtained on many European locomotives with the Kylchap front-end arrangement with constant nozzle. The Chapelon locomotives in France and England have shown a remarkable increase in power and the possibility of attaining very high speeds, at which the exhaust is not impaired in any way. The latest test made on the London & North Eastern, with the Sir Nigel Gresley locomotive at a speed of 125 mph at a continuous run, should set locomotive designers thinking about this subject. It seems that the only question left open to investigate is whether the Kylchap arrangement is suitable to American conditions, especially in so far as smoke-lifting properties are concerned.

In conclusion, the author enumerated certain recommendations which he would suggest to the future investigator as a result of the aforementioned thoughts. He is quite right in his statement that the energy thrown away by the exhaust steam is greater than what is required for the elevation of smoke and gases above the locomotive cab and for heating feedwater. The present drafting arrangement is probably not yet giving the maximum efficiency in itself and is not conducive to maximum power of the locomotive. Although it is over 100 years old, it is still open to improvements.

A. Giesl-Gieslingen.[4] The front ends of the modern locomotives on the Missouri-Pacific Lines are indeed examples of very sound standard practice and speak for the experience of the author under whose direction these front ends were designed or modified. Railroad mechanical engineers will be thankful for the account which the author has given of his experiences.

In many respects it is unfortunate that the ideas of D. M. Lewis were not developed further. Apart from the considerable cost of his experiments, the main reason therefore becomes apparent from Mr. Lewis' patents which reveal that good ideas were associated with a number of distinct fallacies regarding the requirements of efficient front-end action. For instance, placing the elongated nozzle crosswise to the axis of the locomotive was decidedly a mistake; Henry B. Oatley realized this about 20 years ago as evident from his U. S. patent covering an elongated stack and nozzle placed longitudinally. The Oatley design should not be missing in a discussion of Lewis' experiments and the tests made in 1933 by the Missouri-Kansas-Texas Lines unmistakably show its merits, even if the efficiency was still impaired by lack of experience in proportioning such a novel arrangement. If one considers that the elongated front-end arrangement offers the designer a still greater number of variables, it is clear that, without a correlating theory, the development of a highly efficient arrangement is likely to be very expensive and time-absorbing. From the standpoint of freedom in choosing the proper stack and nozzle areas on large locomotives, further from the standpoint of smoke-lifting power, back pressure and, last but not least, reduction of exhaust noise (this is a surprisingly pronounced feature of the elongated stack and nozzle) the Oatley arrangement is indeed worthy of most active consideration.

The effect of the noise reduction is so pronounced that those who have had experience with these stacks even cited it as a disadvantage. The engineers, accustomed to the sound of the standard exhaust, were unable to determine when the engine was working near capacity and were, therefore, induced to overwork their boilers. During the 15 or 20 years which have since passed, this argument has, of course, lost the weight which it might have had at that time. As a means of gaging locomotive capacity and efficiency of working, the valve pilot has appeared as a scientific aid, and noise reduction, on the other hand, is coming in for increasing attention by the public and by the authorities.

Another subject for the treatment of which we are indebted to the author is the utilization of the energy in the "exhaust puff" which occurs at the moment of release from any one cylinder. This energy, instead of disturbing the fire at lower speeds, can be put to work for reducing the back pressure in the opposite cylinder as the author shows in his diagrams. It is interesting that this basic idea had been patented to James Milholland as early as 1847; like a good many successors, Milholland proposed the use of concentric nozzles for this purpose but in the interest of reducing the height required for the development of exhaust passages, which is a primary necessity on large modern locomotives, the concentric arrangement is out of date and the openings should be placed side by side in some manner for which the author gives us a workable solution.

With reference to the author's Fig. 7, it might be mentioned that the relation of the curves for "steam to engines" and exhaust pressure is distinctly erroneous above a pressure of approximately 8 lb. The author's contention that "there is a

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[4] Lokomotivfabriks, A. G. Wien XXI, Germany. Mem. A.S.M.E.

limitation in the economic working range of a steam locomotive, drafted with conventional constant area nozzle" is, in the light of modern designs decidedly no longer correct within the narrow limits illustrated in Fig. 7. Tests made with well-developed standard front ends reveal that the CO_2 content can be kept within the narrow and very favorable range of 14 to 15 per cent from light loads up to a back pressure slightly exceeding 12 lb. When the boiler is forced beyond that range, the air supply per pound of coal burned will fall off considerably, and maintenance of good combustion will require reducing the nozzle area rather than increasing it; this is proved by countless experiences. Lewis' idea to keep the back pressure constant regardless of the rate of working was a decided fallacy and it is surprising that it finds the author's support. While it is true that with certain coals and at high speeds combustion can be improved by closing in the nozzle somewhat at very light loads, this observation does not signify the trend throughout the working range of the locomotive and it must be our aim to get along with a constant-area nozzle which, if the front end is to be well proportioned, will fulfill all requirements of efficient operation.

The problem of smoke lifting at very light loads can certainly not be solved by closing in the exhaust nozzle and thereby increasing the force of the exhaust; unless we are satisfied with an effect amounting to only a small percentage increase in the speed limit at which smoke trailing will begin, this method would lead to an entirely excessive air supply at light loads as can easily be demonstrated. From the theoretical standpoint, this problem could be fully solved by reducing the stack area together with the area of the exhaust nozzle at light loads and high speeds; with an elongated stack and nozzle this could be done quite easily. However, in practice it will probably be preferable to effect smoke lifting by external air streams, the force of which increases with the speed of the air against the locomotive.

H. S. VINCENT.[5] The author has had long experience with the drafting of steam locomotives and has achieved a considerable measure of success in this field. He has made a desirable contribution to our knowledge of what takes place in the cylinders and smokebox of a locomotive by the design and use of specially constructed instruments. It is to be hoped that a method will be devised for accurately registering the pressure in the exhaust cavities of the locomotive cylinder at a point immediately below the nozzle stand and that this record can be synchronized with the temperature reading taken in the same position. Such a record will afford much needed data for determining the relative effectiveness of any piece of drafting apparatus.

Discussing more particularly some of the author's conclusions, the writer believes that the complete separation of exhaust-steam passages for right and left locomotive cylinders is not essential for eliminating or reducing the "hump" in the exhaust line of indicator cards. The Pennsylvania Railroad has for a number of years on all of its locomotives provided a choke in the cylinder-exhaust passages, of smaller area than the nozzle tip, thereby partially converting the pressure energy of the exhaust "puffs" into kinetic energy. This construction eliminates or greatly reduces the "blowback" into the opposite cylinder. Its effect can be seen by reference to the indicator diagrams shown in any of the Pennsylvania test bulletins.[6]

The complete separation of the exhaust passages will, of course, eliminate the "hump" but this necessitates some form of double nozzle. The abandonment by the railways of the use of two separated nozzles was not brought about entirely by the dictum of the American Railways Master Mechanics Association as the author asserts. This form of nozzle never gained any general acceptance by the American railroads. Its incidental disadvantages, such as cross firing into the stack were the real reasons for its discontinuance.

The Layden four-ported nozzle, to which the author refers, permits a practical separation of the locomotive exhaust passages and is so designed that the steam issues from each pair of nozzles concentrically with the bore of the stack. In the construction of the Layden nozzle, two of its four apertures are available for each cylinder. The statement is sometimes made that the four apertures of the Layden nozzle are equivalent to a single nozzle of equal area. This is not correct.

One of the peculiarities of the operation of the Layden nozzle is that at the moment of the release of steam in one of the cylinders, two of its nozzles are filled with steam at the maximum exhaust pressure, whereas the remaining two nozzles are exhausting steam from the opposite cylinder at minimum pressure. Thus, at low running speed the relative weight of steam escaping through each pair of nozzles may be in the ratio of six to one. The effect of this operation on the fire and in draft pulsations is the same as if the locomotive were equipped with a single nozzle having an area about 16 per cent greater than that of the two effective nozzles.

In the tests conducted by E. G. Young[7] on a front-end model, it was demonstrated that with the same front-end setup, the four-hole nozzle, of which the Layden nozzle is assumed to be the prototype, gave an air flow 21 per cent greater than that given by a circular nozzle of equal area and with the same jet pressure. In this case the area of the four-hole nozzle was equal to the one hole of the circular nozzle. It has been assumed in some instances that this test represented the relative effectiveness of the Layden nozzle as compared with a circular nozzle of equal area. It is evident that such is not the case. Had the holes of the model-test nozzle been isolated as in its prototype, the result would undoubtedly have been quite different. We may therefore expect that the back pressure required to draft a locomotive successfully, will be about the same with the Layden nozzle as with the standard circular type.

In his second conclusion the author avers that the energy liberated from the cylinder at release opening of the valves is more than is required for drafting the locomotive. If this refers to the thermal energy remaining in the steam, the statement is correct, but no method has so far been devised for utilizing this energy. It has been nearly 40 years since Goss gave as his conclusion, from a series of tests conducted at Purdue University, that the total weight of steam passing through the cylinders of a locomotive is required to draft the locomotive when using the Master Mechanics standard front-end arrangement. Any of the energy of the exhaust steam which is diverted for other purposes must be compensated for by a corresponding increase in back pressure, with its attendant loss of cylinder power. Subsequent experience has well authenticated this pronouncement of Goss's.

The writer believes that no energy is "thrown away" in the present method of drafting a locomotive. The author's draft records show conclusively that the pressure in the smokebox of a locomotive varies widely and continuously and that these variations are synchronous with similar pulsations of the exhaust pressure. The mean of these constantly varying pressures both in the cylinder-exhaust passage and in the gas passages is the force which operates to pull the gases through the tubes. If it were practicable to smooth out the exhaust pulsations without

[5] Consulting Engineer, East Harwich, Mass. Mem. A.S.M.E.

[6] Pennsylvania Railroad Company, Test Department Bulletins Nos. 28, 29, 31, and 32, Altoona, Pa.

[7] "A Study of the Locomotive Front End, Including Tests of a Front-End Model," by Everett G. Young, Bulletin No. 256, Engineering Experiment Station, University of Illinois, Urbana, Ill., May, 1933.

loss of energy or without lowering the average pressure, the draft effect would at least be equal to and would probably be somewhat better than that obtained with pulsating pressures. No attempt should be made to lower the average exhaust pressure in the cylinders unless there is a corresponding reduction made in the resistance offered to the gas flow.

Referring again to Young's model test,[8] it was found when using a $1^5/_8$-in. circular nozzle with the standard Master Mechanics front-end arrangement that with an exhaust-jet pressure of 6 lb, the weight of air moved per hour through the tubes was 4317 lb. When this setup was replaced with a smokebox arrangement similar to that designed by W. F. Kiesel, Jr., and widely used by the Pennsylvania Railroad, while still retaining the $1^5/_8$-in. circular nozzle, a weight of 4840 lb of air was moved per hour with the same jet pressure. Thus, an increased air flow of 12 per cent was obtained solely by reducing the resistance offered by the Master Mechanics front-end arrangement.

The writer has shown elsewhere[9] that a locomotive equipped with the Kiesel front-end arrangement, including a radial ported nozzle, will move a weight of gas in unit time about 29 per cent greater than can be moved in the same locomotive when equipped with the Master Mechanics front-end arrangement and circular nozzle of equal area. Thus, a means is provided for a significant increase in the steaming capacity of the locomotive.

The author's projected design of variable nozzle and ejector are still in the experimental stage. It will be interesting to follow his experiment along this line. The exhaust card taken from his experimental equipment as exhibited in his Fig. 8, shows approximately zero back pressure at an operating speed of 5 mph. This is not an unusual record as published[10] indicator cards from a locomotive equipped with a Layden nozzle also show zero back pressure at a speed of 4.5 mph.

The writer is inclined to believe that a variable exhaust is not destined to replace the constant-area type. There have been numerous experiments made with different forms of variable nozzles but so far none have been adopted or used to any extent, probably because no decided advantage from their use has been shown. The constant-area nozzle has been the standard for over one hundred years. This successful use would seem to demonstrate that it is correct in principle.

The author states "the only justification for the variable nozzle over the constant-area nozzle is that the former will increase the economic range of the locomotive by increasing the range through which draft proportional to the rate of steam demand on the boiler is realized." He then presents his Fig. 7, to prove that the locomotive there shown, because it is equipped with a constant-area nozzle becomes less efficient in steaming capacity when operating beyond a cutoff of about 49 per cent of the stroke, at a speed of 14.7 mph. The locomotive cited by the author is one of a class designed originally for transfer service involving low speed and not requiring high boiler capacity. The test as shown by the author's Fig. 7, was made on the original design. Subsequently, this class of locomotive was equipped with a feedwater heater and other changes made to increase its boiler capacity; it was then placed in road service.

It is evident that every locomotive has an economic limit of operation beyond which it will show reduced efficiency. The locomotive as shown by the author's Fig. 7, was not designed to operate at 14.7 mph when working at a cutoff over 50 per cent of the stroke. In Fig. 1 of this discussion is shown the normal speed-effort curve for this locomotive, which is consistent with its original boiler rating. This curve indicates that at a speed of 14.7 mph the locomotive will normally develop a tractive effort of 68,000 lb. This is equivalent to 2670 ihp. The small crosses show the cylinder tractive effort that was developed in the test and the cutoff at which this tractive effort was developed. These indicate that at a speed of 14.7 mph the locomotive should operate at about 45 per cent cutoff.

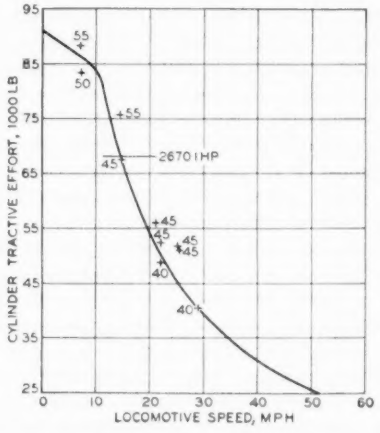


FIG. 1 SPEED-EFFORT DIAGRAM FOR PENNSYLVANIA $30^1/_2 \times 32$-IN. DECAPOD LOCOMOTIVE NO. 790

Referring again to the author's Fig. 7, it will be observed that up to a maximum cutoff of about 49 per cent, at the given speed, the locomotive functions consistently. It is only when the locomotive is pushed to a cutoff beyond the limitation of its boiler that its operation becomes relatively inefficient. In fact, below 49 per cent cutoff it will be seen that the draft curve exactly parallels the curve of coal fired, indicating that the draft arrangement is fully responsive to the demand upon it.

In the test cited, locomotive No. 790, was equipped with the Master Mechanics front-end arrangement and a Goodfellow nozzle. We now have available a drafting mechanism which in combination with a constant-area nozzle will move approximately 30 per cent more gas in unit time with the same jet pressure than was possible with the arrangement used on locomotive No. 790. It would seem therefore that we can claim to have an apparatus which in the words of the author "provides draft proportional to the rate of steam demand on the boiler."

W. F. KIESEL, JR.[11] This subject, the drafting of locomotives, is quite old; the ejector principle on which it is based was developed more than 75 years ago, and as early as 1870 the value of its fundamental principles was appreciated even if not fully understood. That is evidenced by patents Nos. 106,515 and 107,603, issued respectively to James Smith on August 16, 1870, and A. J. Dufour on September 20, 1870, on a radial-ported nozzle. Carefully organized experiments to determine the most efficient shape of stacks date back to 1860. In these tests, small round nozzles, mostly less than 1 in. diameter, were used.

The author of the paper under discussion records the "high lights of his experiments as viewed in retrospect." That seems to be an invitation for others to do likewise. Based on 50 years of experience, the writer submits the following comments on the subject.

Nozzles. The circular shape was excellent for small nozzle

[8] "A Supplementary Study of the Locomotive Front End by Means of Tests on a Front-End Model," by Everett G. Young, Bulletin No. 274, Engineering Experiment Station, University of Illinois, Urbana, Ill., May, 1935.

[9] "The Locomotive Front End," by H. S. Vincent, *Railway Mechanical Engineer*, vol. 112, January, February, and March, 1938.

[10] "Santa Fe Locomotive 5000 Shows High Sustained Power," *Railway Mechanical Engineer*, vol. 105, December, 1931, p. 572.

[11] Hollidaysburg, Pa. Mem. A.S.M.E.

areas. When these areas had to be increased for larger locomotives, the Goodfellow nozzle with four projections into the circle showed a definite superiority. For still larger areas, the ported nozzle showed similar superiority over the Goodfellow nozzle. That experience, amply proved by tests, indicated that there should be a definite relation between periphery and area.

Early experimenters determined that the nozzle area should be approximately 4 per cent of the net flue (flue and tube) area, and later writers quote the same figure; however, an examination of modern locomotives will show that their nozzle areas are almost invariably less by a large percentage. It has been definitely established by tests that the nozzle area can be increased to 4 per cent of the net flue area by a judicious redesign of the drafting appliances, thereby increasing the evaporation capacity and reducing cylinder back pressure. Experimental installations on four railroads resulted in an increase of nozzle area of approximately 35 per cent, a 35 per cent increase in evaporation capacity, and an 8-lb reduction in cylinder back pressure at an evaporation equal to the maximum performance when using the smaller nozzle. At high speed, that represents a material increase in drawbar horsepower for no greater expenditure of water and coal.

Stacks. Best steaming results have been realized when the area of the top of the stack is half of the net flue area; however, under light working conditions, the velocity of gases leaving the stack is low and the products of combustion are not propelled as high into the air as with smaller stack diameters. If the area at the top of the stack is reduced to one third of the flue area, the exit-gas velocity will be increased by 50 per cent, engendering a loss in nozzle area and correspondingly a loss of approximately 12 per cent in evaporating capacity.

The smokebox arrangement can be adjusted to even smaller stack diameters, and with the use of the ported nozzle it will show a decided advantage over the Master Mechanics front-end arrangement. In every case, the distance from the top of the nozzle to the top of the stack should be as great as the design permits—clearance restrictions prevent making that distance too great.

The foregoing indicates that in connection with locomotives used for light work there is a choice between a smoke-lifting mechanism and a reduction in stack diameter, with a corresponding small reduction in nozzle area and capacity.

In an existing locomotive it is extremely difficult to make changes that will reduce resistance to flow of gases into the ash-pan through the firebox, around the arch, and through the flues. The resistance through the smokebox and stack can be reduced by judicious design. For heavy-duty freight locomotives, the area of the top of the stack should be half the flue area. For locomotives intended for light service, the owner should state his preference of the alternatives just presented by the writer.

Cross fire of the exhaust should no longer exist on any locomotive less than 15 years old. That occurrence was cured as far back as 1900 by introducing an exhaust-passage choke vertically below the nozzle and located either in the exhaust pot or preferably in the cylinder saddle. This choke also causes the steam flow from one cylinder to induce a reduction in back pressure in the other cylinder. In this respect, Figs. 2, 3, and 6 of the author's paper show a deplorable condition of locomotives in question.

A great help in maintaining a balanced steaming condition for all variations of power utilization with proper fixed nozzle is to cut down the net opening of air passages through the grate to approximately 125 per cent of the net flue area. That also prevents stripping a light fire off the grate when the driving wheels slip or when starting with full throttle.

Since the desired steam generation, and hence the draft on which it depends, is closely proportional to the amount of steam passing through the nozzle, it is difficult to visualize how a variable nozzle can be of any benefit except to counteract some error in smokebox design that creates unbalanced results. It is much simpler to improve or correct the design than to add mechanisms which obstruct draft due to bulk; such mechanisms have thus far proved detrimental instead of beneficial and are expensive in first cost and maintenance.

Proceeding now to a review of the paper, it is noted that the author took part in, or supervised, tests of fan applications by McFarland and Goss, as well as tests of the Lewis exhaust governor. He does not record that these are of value to the locomotive, and apparently favors the Layden double-ported nozzle, which by inference from the author's statement covering tests of five different appliances on the same locomotive is the least efficient of the five.

In Fig. 8 of the paper, the author presents tentative exhaust-pot designs which he believes may correct undesired behavior found in previous tests, but which may be corrected in a much simpler way, as previously presented by the writer in this discussion.

When comparing the author's first test, using the Layden nozzle, with the fifth test, using the Kiesel nozzle, it should be pointed out that the Kiesel nozzle has approximately 30 per cent more nozzle area, hence 30 per cent more steaming capacity, and at an evaporation rate of 60,000 lb per hr (not including the feed-water heater) it will cause a reduction in cylinder back pressure of more than $7^1/_2$ lb. The author's criticism of the latter scheme is twofold: (1) It is a poor spark arrester at high capacity, and (2) it results in much trailing smoke under light working conditions.

Regarding the first criticism, the spark arrester is made of a material called "Draftac," which is approved by the Interstate Commerce Commission and used by practically all railroads. Since the author gives no reason for his opinion, it may be assumed that on a dark night more red shows at the top of the stack at 130 per cent capacity than at 100 per cent capacity, which is true because the locomotive must get rid of an additional volume of cinders at the higher capacity. However, sparks that will pass through the $^3/_{16}$-in. slits in the Draftac netting are not considered dangerous; the Draftac netting can be replaced by a louver arrangement which on test has reduced considerably the red color at the top of the stack.

Regarding the author's second criticism, if the bad trailing-smoke condition could not be corrected by a smoke lifter, the stack diameter can be reduced to provide whatever reasonable gas-exit velocity the locomotive owner may specify, with the smokebox arrangement modified to suit.

For brevity, the writer has summarized results of his experience, much of which has been obtained from actual tests. While reviewing the paper, the writer transferred the results of his tests onto the author's comparative tests to show vital economic relations which the author neglected to mention.

Opinions differ as to variable nozzles. Experimenters have tried many different types without the least success, and will continue to try more indefinitely, presumably with the same results.

Author's Closure

As discussion of the paper has brought out, the subject of drafting steam locomotives is indeed an old one. In attempting to cover the subject in a single paper, the author may be charged with having made some statements without presenting complete data in support of them.

It is difficult to see where there is basis in the paper for Dr. Giesl-Gieslingen's feeling that support is given to the idea, attributed to Lewis, of keeping the back pressure constant regardless of the rate of working. The necessity for an increase in noz-

zle pressure is fully appreciated, other things remaining equal, for an increase in steam demand on the boiler. The author's contention is the desirability for a more proportionate draft to steam demand throughout the possible range of power development for the steam locomotive than can be realized with the single nozzle of fixed area.

The author is familiar with the Oatley design of slot nozzle placed longitudinally, also mentioned by Dr. Giesl-Gieslingen in his discussion of the Lewis exhaust governor. In this connection, it may be of interest to record that an adaptation of the Lewis governor, placed longitudinally instead of transversely, was applied to a locomotive in England about 1922 or 1923 and was reported to have performed satisfactorily. It might be added that, in the author's experience, the relatively high draft efficiencies obtained with the Lewis slot nozzle and rectangular stack, either in the longitudinal or the transverse positions, were due to the increase in the exhaust-jet surface with the slot as compared with the conventional circular-nozzle design.

The point is made that the indicator cards presented in the paper are examples of poor design and bad practice as regards cross-fire effect and that, through proper design of passages and nozzle, the extreme pressure build-up against the piston in the working cylinder from the release puff in the opposite exhausting cylinder could be greatly reduced or eliminated. This is agreed to and yet the contention stands that the cards shown more nearly represent actual conditions of steam locomotives in service today than is generally suspected.

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The designs and experiments of Stumpf, mentioned by Dr. Lipetz, are familiar to the author. Full credit is accorded to Stumpf for the idea and demonstration, through the design of exhaust passages and nozzle, of utilizing the energy in the exhaust from one side to reduce the pressure on the opposite side. The Stumpf design, however, had to be carefully worked out; that is, the shape and area of the exhaust nozzle in relation to the exhaust passages had to be carefully maintained to realize the desired back-pressure control. This, in the author's opinion, is also true of the Pennsylvania Railroad or other single-nozzle constructions. In other words, very good results are to be realized from the proper relationship of exhaust passages and nozzle so long as this relationship is maintained. A change in nozzle to reduce the area, as is common practice, would be expected to cause a less satisfactory back-pressure relationship than was obtained with the ideal nozzle-exhaust-passage relationship.

After consideration of the factors involved, it appears obvious that the simplest, most efficient, and most flexible method of eliminating the blowback or cross-fire effect in the conventional two-cylinder steam locomotive is the complete separation of the exhaust passages and nozzles, yet, this is not in more general use today because:

a The double nozzle is generally considered as less efficient for drafting the locomotive, other things being equal, than a single nozzle.

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This answers the question raised by Professor E. G. Young relating to the "filling of the stack" with the Layden-type nozzle. The question regarding back pressure, resulting from a given rate of steam discharge from the conventional as compared with the Layden-type nozzle, also raised by Professor Young, involves a fundamental point apparently not generally recognized, as between drafting the same locomotive with a double as compared with a single nozzle; viz., that each of the double-nozzle areas has to be reduced over that of the single nozzle, other things remaining equal, for satisfactory steaming. The following examples are given:

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The term "smoke abatement" has really become a misnomer. Those engaged in this work are interested in the elimination of all kinds of atmospheric pollution. The aim is not only to abate smoke but also materially to reduce the discharge of solids and deleterious gases from chimneys. The new regulation in St. Louis is a forward step. This paper shows what can be accomplished by coal producers in an attempt to meet the requirements of the ordinance.

Those located in the East and accustomed to low-ash and low-sulphur coals may wonder why it is necessary to place such a seemingly high limit on ash and sulphur content. It is the writer's understanding that samples of coal sold in St. Louis are analyzed by the City Laboratory. Some samples have been found to contain as high as 28 per cent ash. A few years ago, a well-known consulting engineer in St. Louis analyzed samples of coal and found they contained 7 per cent sulphur. Certainly something should be done to cut down the discharge of solids and sulphur gases from such fuel. A few years ago an aggravated case came to the writer's attention. Called in as a consulting engineer on the case, it was discovered that strenuous complaints were being made concerning the discharge of fly ash from a pulverized-coal plant. The coal used part of the time ran about 30 per cent ash. About $^1/_2$ mile from the plant, on the edge of some timberland, was ground which had never been cultivated. At this point, the deposit of ash was $2^1/_2$ in. thick, which was the accumulation of about seven years' operation of the plant. The truck farmer had had several crops ruined by this deposit. The author has explained some of the difficulties encountered in the preparation of coal. The time is not far distant when other cities using coal of high-ash and high-sulphur content, will probably require that such coal be processed before being used.

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As an instance, $1^1/_4$-in. screenings, with an average ash content of 14.3 per cent, may vary from 10 per cent to 23 per cent in ash, and have a heterogeneity measured by the probable error of 1.4 per cent. Such a steam coal, unless cleaned, would, under the present St. Louis Smoke Ordinance, be restricted to shipping only 4 cars out of every 100 loaded at the mine into that market.

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in this country and abroad using starch, glue, lime, and caustic soda, in attempting to coagulate the finely divided clay, which stays in suspension in the washery water, causing trouble in the washery process.

In the author's discussion of the presence of sulphur in coals produced in Illinois, he has pointed out that it exists in three recognized types. For engineers, contemplating the installation of a washery process, the pyritic sulphur is about the only type that can be readily removed by any washing process. Therefore, operators may be somewhat disappointed in the small percentage of sulphur reduction in the cleaned coal after it has been subjected to preparation.

Although cleaning has its place in the reduction of smoke and sulphurous-gas nuisance in cities, too much must not be expected of it. It is, nevertheless, a step in the right direction and, probably after uniform coals have been assured, further development along low-temperature-carbonization lines may be justified to yield a truly smokeless coal for use in the cities, where the smoke-abatement problem is acute.

Theory of Elastic Engine Supports[1]

GLENN C. BOYER.[2] It appears from a study of this paper that Fig. 3 contains the "meat" of this exposition on vibration isolation. Some authors call values of the abscissa r (which is equal to f, the forced frequency of the system, divided by f_0) the natural frequency of the system. The interesting thing about this curve is that when $f/f_0 = 0$, the entire forced impulse is transmitted. When $f/f_0 = 1$, the transmissibility becomes infinity, and when this ratio increases to 1.42 the transmissibility factor again becomes unity. As the ratio f/f_0 increases beyond 1.42 the amount of transmitted force becomes less and less approaching zero as a limit.

On the basis of this curve, it appears that what one must do in designing a foundation, where springs are utilized for isolation purposes in connection with Diesel engines operating at rotative speeds of 400 rpm and less, is to be certain that the natural frequency of the spring-supported system is low. In other words, the springs used must take considerable deflection. It is also necessary that the natural frequency of the spring-supported system not be any harmonic of the forced frequency imposed upon it by the engine, in order that an excessive amount of bouncing will not result.

There is the further question of movement of the spring in other than a vertical direction. The frequency of a spring is different when movement is at right angles to the main axis of the spring helix from that when the movement is in line with that axis. In other words, there are two different frequencies to watch.

It is interesting to note that in discussing spring suspensions which have six degrees of freedom, Kimball[3] gives four independent frequencies to which such a system responds. They are torsion, sideways tilt, up-and-down bounce, and sideways shearing oscillation. Of these four independent frequencies, that of sideways shearing oscillation has the highest frequency of the four, and in some cases might be the most objectionable.

The writer is rather impressed by the fact that for engines of any size the foundation requires a massive concrete block practically equal to the weight of the engine. In so far as he can tell from the paper, it appears that the concrete foundation slab between the springs and the engine in each case is practically equal to, if not greater than, the weight of the engine placed on top of it. From this it appears that what is actually happening on some of these larger foundations is that a mass is added to the weight of the engine as a damper before the vibration gets to the springs.

The writer was rather interested in running across this statement by Slocum,[4] printed in 1931: "Steel springs have become the standard means for cushioning shock or impact as in the case of car or automobile springs, but the problem of damping nonperiodic inertia forces of this type is quite different from that of damping vibrations of high frequency and small amplitude. For this reason springs are of limited application in damping vibration where they have been tried for this purpose, as in mounting motors in automobile chassis or for supporting floors in buildings. It has been found that if the springs are stiff enough to furnish the stability required of such construction, their natural frequency is much too high to make them effective as vibration dampers."

The writer may be wrong in his interpretation, but he is wondering what would happen to some of the four-cylinder four-cycle engines, that he has seen dancing around on concrete blocks that weighed several tons, if they were mounted on springs.

It appears that it is possible by the use of springs to amplify impressed-frequency forces under certain conditions. The writer is, frankly, rather hesitant to consider the use of springs by his own organization, and he is speaking as a consulting engineer who must go out and sell this material to clients. In other words, he must design the foundation, put it in, and guarantee results.

Professor Sayre[5] states that springs can fail due to traveling waves within the spring and resonance may occur up to as high as the twenty-fifth harmonic of the traveling wave within the spring. He states that the harmonics which are the most serious usually occur below the twelfth. That appears to mean this: If a helical spring, definitely limited at the top and bottom by plates, is subjected to a certain periodic axial force on the outside of the plates, which has a period of any time from one to nearly 25 times the frequency of the traveling wave within the spring, then, over a period of time, that spring will break.

EDGAR J. KATES.[6] The author deserves our gratitude for this paper on the theory of spring supports for Diesel engines. Despite some questions in the writer's mind as to whether spring supports are needed for properly built six- or eight-cylinder engines, the effects of the bouncing action that springs frequently introduce, and the higher cost of certain types of spring isolation compared to the use of cork or other materials, there is one thing that spring isolation has definitely achieved, i.e., the satisfactory use of four-cylinder four-cycle engines in places where heretofore such engines have transmitted objectionable vibrations to the neighborhood.

The writer fears that the many impressive-looking formulas and curves may mislead some without actual experience into believing that the art of isolating Diesel engines has now been firmly established upon a scientific basis. No doubt the author himself would be the first to agree that spring application is still in the empirical stage and that spring supports designed by experts often act quite differently from what the calculations have indicated. One of the features of spring installations is the easy accessiblity for spring removal. Such removals are required not so much on account of breakages, which are rare,

[1] Paper by S. Rosenzweig, published in the January, 1939, issue of Transactions of the A.S.M.E., vol. 61, p. 31.

[2] Burns & McDonnell Engineering Company, Kansas City, Mo.

[3] "Vibration Prevention in Engineering," by Arthur L. Kimball, John Wiley and Sons, Inc., New York, N. Y., 1932, p. 98.

[4] "Noise and Vibration in Engineering," by S. E. Slocum, D. Van Nostrand Company, Inc., New York, N. Y., 1931, p. 91.

[5] "Vibration in Springs," by M. F. Sayre, Kent's Mechanical Engineers' Handbook, John Wiley and Sons, Inc., New York, N. Y., eleventh edition, 1938, Design. Shop Practice, section 10, p. 20.

[6] Consulting Engineer, New York, N. Y. Mem. A.S.M.E.

but for the purpose of testing the action of springs of different designs. In fact, this accessibility for change is, in the writer's opinion, one of the most desirable characteristics of a spring support as compared to cork. If the cork job doesn't work it can be changed only at huge expense, whereas the spring job can readily be experimented with until it is right.

If spring supports had no drawbacks they would, of course, always be used in preference to cork. The fact is, however, that many cork isolating installations are still being put in because cork is generally much cheaper.

In Figs. 3, 4, and 5 the term transmitting effect is used. Would not transmitted force be a better designation?

The author states that "damping is decidedly harmful as it increases the transmitting effect of vibrations," and the curves in Fig. 4 seem to bear him out. However, the writer believes this is another case where the theoretical study has been incomplete. Despite the author's conclusion that the low damping coefficient of steel springs contributes greatly to their effectiveness, the writer's opinion is that, in many cases, the action of spring supports would be improved by applying a certain amount of restraint and controlled damping.

The author has devoted some much needed attention to the bouncing effect caused by spring isolators. However, the writer still remains unconvinced that the bouncing action is as trivial as the author implies. It should be noted that in Fig. 6 he is comparing the bouncing effect or operating amplitude of the spring-mounted engine to that of an "ideal system" which is "an imaginary system suspended in space and provided with an infinitely elastic suspension." But what is really of interest is the bounce of the spring-mounted engine compared to one bolted down to a heavy foundation, which is just the opposite of the "ideal system." Consequently the author's statement on page 33 that, in an elastic support with a frequency ratio of 5 to 1 and with zero damping, "the amplitude of motion will be within 4 per cent of the ideal engine," means nothing as a practical matter, and the conclusion he thereupon draws that the "motion of the mass would be virtually imperceptible" is not justified. That these operating amplitudes, or bouncing actions, are not in fact negligible is shown by the figures of from 0.004 to 0.006 in. which the author himself mentions on page 35 and by amplitudes up to 0.018 in. which the writer has measured.

Spring supports are a valuable contribution to the art of vibration isolation, but much fundamental study on them still remains to be done.

K. J. DeJuhasz.[7] By concentrating on a troublesome problem of engine installation, and palliating the nuisance effect of engine vibration on adjacent structures the author performs a valuable service for the Diesel industry. This paper clarifies in many respects the theory and application of elastic supports; some aspects of this subject have, however, not been sufficiently explained.

The helical spring has a certain longitudinal stiffness, also a certain lateral stiffness which in general differs from the former. Hence the system, consisting of the mass of engine plus foundation and the spring, will have a certain natural frequency in the longitudinal direction, and another in the lateral direction. When the six degrees of freedom of the engine are considered, both the longitudinal and the lateral frequencies come into play. The author restricted his treatment to the vertical motion only (i.e., motion with one degree of freedom). In isolators with lateral motion is there a definite optimum relationship between longitudinal and lateral stiffness?

The author uses somewhat interchangeably the terms "isolator," which correctly indicates the function, and "damper," which is a misnomer as damping is undesirable as pointed out by the author.

In Fig. 3 the exact meaning of the term "transmission effect" is not given. Does it refer to amplitude, or energy?

F. L. Yerzley.[8] The writer has enjoyed Mr. Rosenzweig's paper, but does not agree with the general statement that the stiffness of rubber is from two to three times greater under a dynamic than under a static load. The statement may be true, however, under certain conditions with certain compositions.

In the literature on the mechanical characteristics of rubber one frequently encounters the statement that the modulus of rubber is a function of the speed of the test. Therefore, when rubber is to be used as an elastic support for vibration isolation, these mysterious changes would be expected to come into play. Hysteresis, one factor, logically can be expected to produce changes in the elastic moduli as a function of frequency. It is further well established that temperature changes alter the moduli to an extent dependent upon the composition of the vulcanizate. Beyond these two factors, it is not evident why dynamic and static moduli over the same load range should differ.

Rubber is used effectively and widely in both shear and compression to isolate vibration. It has also been used commercially in tension. Since its peculiar elastic properties in many instances make shape a relatively unimportant factor, simplicity and economy may be appreciable. This is especially true where the conformity of rubber to rough surfaces eliminates the necessity of expensive machined fits. The present importance of rubber is based to an appreciable degree upon its dependability after the desired initial characteristics are attained. The writer believes that the data on a range of commercial rubber compounds, given in Table 1, are characteristic of usual performance. Test cylinders $^1/_2$ in. high and $^3/_4$ in. in diameter were subjected to static and dynamic compressive loads. Unfortunately the details of the method must be discussed at some other time,[9] but the broad method of approach has been described elsewhere.[10]

[7] The Pennsylvania State College, State College, Pa. Mem. A.S.M.E.

[8] Rubber Laboratory, E. I. du Pont de Nemours & Company, Wilmington, Del. Jun. A.S.M.E.

[9] "A New Oscillograph for Routine Tests of Rubber," by F. L. Yerzley, to be presented at the Annual Meeting of the American Society for Testing Materials, June, 1939.

[10] "The Evaluation of Rubber and Rubberlike Compositions as Vibration Absorbers," by F. L. Yerzley, *Industrial and Engineering Chemistry*, Analytical edition, vol. 9, 1937, pp. 392–394.

TABLE 1 COMPARISON OF STATIC AND DYNAMIC CONSTANTS OF RUBBER COMPOUNDS

Compound 1333	Cure	Hardness, durometer A	Resilience (dynamic test), per cent	Slope of static load-compression curve at zero load, psi	Average compressive load in dynamic test, psi	Slope of static load-compression curve at stated average load, psi	Dynamic modulus calculated from frequency and inertia, psi	Frequency of test, cycles per min
30	15 min at 287 F	25	90.8	150	80	376	413	126
31		29	91.8	230	80	403	421	127
32		33	91.6	260	120	556	614	154
33		36	91.9	340	140	655	689	169
34		45	89.1	530	200	872	963	193
35		55	82.0	720	240	1100	1253	220
36		64	74.1	1100	240	1352	1580	246
37		72	65.8	1700	240	1750	1850	266

Since the load-compression curve for rubber is not usually a straight line except near the origin no single value of the modulus is adequate for all calculations. This is well illustrated by the fact that the tangent to the load-compression curve at the origin is less than that at the various average loads used in the dynamic tests. The static and dynamic moduli at the same load are for all practical purposes the same. As a matter of general interest, resilience per cycle of loading and unloading in the dynamic-test ranges is also tabulated, showing values from 91.9 to 65.8 per cent depending upon the composition.

Author's Closure

Mr. Boyer's statements may be summarized under the following headings:

1 Necessary deflection of spring
2 Isolation for all degrees of freedom
3 Necessary weight of foundation to insure minimum movement
4 Application of vibration isolators to all types of engines.

It is believed that the answer to all these points can be found in the paper. Obviously points 1 and 2 have to be considered simultaneously. On page 36 reference is made to the pendulum action of the spring isolator in providing isolation for movement of the foundation in different directions. Merely to use any spring is rarely sufficient to produce these results. As in all mechanical devices, the application of one single member, no matter how excellent it is, does not provide a satisfactory solution. It is rather the assembly of the complete unit in all its details which is of importance. In this respect the vibration isolator is no exception. It has to be designed to fit each particular case.

The necessary mass for the foundation can be easily calculated from Fig. 9 and the example given. It is the dynamic-balance coefficient (which depends upon the perfection of balance of the engine) and the permissible amplitude of the movement which control the size of the stabilizing mass, and not, as often erroneously assumed, the speed or frequency of the engine.

High-speed engines usually provide a relatively large stabilizing mass in the engine weight and therefore may require no additional concrete. This is particularly true in such engines where primary and secondary forces and moments are balanced. The omission of the concrete foundation on large engines, even of this class, is not always possible, because without it engine and generator cannot be aligned properly. In such cases, the mass is often immaterial; what is required is a stiff subbase which can be either of cast iron, steel, or concrete. Sometimes, by providing a stiff common subbase, it is possible to mount engine and generator directly on vibration isolators.

However there are many cases of large and slow-speed engines where concrete foundations are required not only for alignment but also for the purpose of providing room for the springs. To place springs from 10 to 15 in. long under the subbase is not desirable, especially when they can be easily assembled within the subbase or foundation. With these foundation assemblies, spring units and satisfactory isolation can be provided and yet raise the foundation but $^1/_2$ in. above the substructure.

Many small three- and four-cylinder engines are mounted directly on vibration isolators without any concrete foundation. Larger engines are provided with the necessary foundation weight and all of them operate satisfactorily for years. But even though the amplitude of motion is greater than with unisolated foundations it is, in the author's opinion, better to let an engine move with an elastically supported foundation instead of letting it strain and damage itself because of a rigid and unyielding support.

The author's experience with a six-cylinder engine is interesting in this connection. Two identical engines were mounted on one foundation. Operation of the first engine was practically perfect, with just a slight quiver to the foundation. When the second engine was started the movement of the foundation became excessive, perhaps as much as 0.05 in. After a careful investigation, an unbalance in the flywheel was detected. When this was corrected by means of a lead weight, the excessive movement of the foundation was completely eliminated. In this case an unbalanced flywheel, probably furnished with many an engine, upset the stability of the entire foundation assembly. If that engine had been mounted on a rigid foundation serious vibration transmission would have resulted as well as damage to the engine, the cause of which might have baffled the operating engineer. The yielding assembly and the springs, however, immediately indicated the presence of unbalance.

To summarize, vibration isolators can be applied to all types of engines using one of the different methods described in the paper. The necessity of providing a stabilizing mass in the shape of a concrete foundation is controlled by the arrangement, design, and type of engine. Practical experience has indicated that the following relative stabilizing masses give satisfactory results to be checked, of course, by the mathematical calculations:

Single-cylinder engine	20
Two-cylinder engine	10
Three-cylinder engine	5
Four-cylinder engine	3
Six- and eight-cylinder engines	0

The author is familiar with the statements relating to the value of springs in Mr. Slocum's book referred to by Mr. Boyer, and is certain that Mr. Slocum, in the light of the new developments, has changed his opinion since 1931.

Correctly designed springs will last indefinitely and break only when there is a defect in material. The reason for this is twofold, as far as vibration isolators are involved: The stress at which the springs operate is low—55,000 psi on an average—and furthermore the change in stress under operating conditions is extremely small. From his own experience in thousands of installations, the author can most assuredly state that spring failures are practically nonexistent.

The experience of Mr. Kates in one plant is illustrative of the statements made in the paper. Three Diesel generator sets are installed in this plant in accordance with his specifications. The foundations of the six- and five-cylinder engines are isolated by means of Korfund cork and that of the four-cylinder engine by means of vibrodampers of the U type. As the engines are running side by side and the operating conditions are identical, an excellent comparison is afforded. Vibration transmission from the six- and four-cylinder engines is imperceptible, and slightly noticeable on the five-cylinder engine which, on account of its primary unbalanced moment, was to be expected. It was considered inconsequential, however, as the plant is not located in a high-class residential district. The bouncing action, as Mr. Boyer and Mr. Kates call it, was not perceptible on the six-cylinder unit. It was barely perceptible on the five and the four. The engines have been giving satisfactory service for almost a year.

It would be possible to work out on paper a correct solution of spring isolation for every case if the engine manufacturers could furnish absolutely correct data on balance, and if a perfect vibration isolator were available. Such perfection is impossible to attain and therefore, as Mr. Kates so correctly stated, the possibility of adjustment after installation is of much importance, so easily accomplished with springs, and so insufficiently attainable with organic materials.

The expected movement of an isolated foundation can be easily calculated and one is referred to the explanations given in the paper in connection with Figs. 6, 7, and 9. If one assumes that an unisolated foundation operates along the 0.6 abscissa of Fig. 6, the correctly isolated foundation will have an oscillation 50 to 100 per cent in excess of the former, which means that oscillation can be restricted to a permissible maximum.

The statements of Professor DeJuhasz in regard to the stiffness of a coil spring in a longitudinal and a lateral direction are quite correct. The stiffness of this type of spring in a lateral direction is such as to make efficient vibration isolation impossible for all six degrees of freedom. Therefore, other means have to be provided in the isolator beside the spring. The pendulum action obtainable in some of the designs referred to in the paper has proved highly satisfactory and efficient for this purpose.

The term "isolator" to designate the isolating medium is as Professor DeJuhasz correctly states preferable to the term "damper." The latter was used in the paper in connection with an isolator known to the trade as "Vibro Damper."

Figs. 3, 4, and 5 indicate the relative transmitted energy of the exciting forces and Fig. 6 the operating amplitudes of isolators for different frequency ratios.

The author is not aware of any fatigue of concrete foundations due to the continuous movement when mounted on springs. There have been a number of cases though, where mill foundations cracked every few months due to excessive vibrations, in which the trouble was overcome by isolating the foundation properly.

The experience with these mill foundations fully substantiates the statement made previously that it is far better to allow a machine to move with an elastically supported base, instead of letting it strain and damage itself on a rigid foundation. The flexible joints of the human body are constantly used for a similar purpose. The slight movement of a foundation may still be called objectionable but it is certain that within a few years it will be looked upon as a matter of course. Opinions have changed about many things which at one time were considered undesirable and even dangerous, but are now accepted as satisfactory and correct.

The Mechanical Characteristics of Rubber[1]

J. F. Downie Smith.[2] For many years it was difficult to get from the rubber companies any data which would help the mechanical engineer in the design of rubber products. In most cases it was necessary to state the problem completely to a representative of a rubber company who would, after consulting his engineering staff, submit a recommended design. This led to a great amount of wasted effort, for frequently the application would not permit the use of rubber, ordinarily because of space limitations, but the designer could not tell about this beforehand.

Within the last few years, however, the attitude of some of the rubber companies has changed, and now one or two of the manufacturers will readily supply upon the request of a responsible designer the information he needs, if it is available. As a sample of this present spirit of helpfulness there is the paper by F. L. Haushalter, who has supplied information greatly needed by the engineer interested in applying rubber products to industry.

It ought to be perfectly clear to the rubber companies that the less mysterious the properties and limitations of each product the more readily will it be accepted by mechanical engineers. At present much opposition to its use is based upon ignorance of its properties.

In a previous paper,[3] the writer quoted data[4] to show that Poisson's ratio varied only a few per cent from 0.5, even for elongations of 700 per cent, and thus claimed that the modulus of elasticity in compression should be three times that of the modulus of elasticity in shear. It is gratifying to find that the data quoted by the author on the compression of rubber between lubricated surfaces bear out this belief.

It is obvious that more data must be taken on rubber loaded in compression if a general theory is to be proposed for this case. The adoption by the author of a modulus of elasticity in compression of 6.5 times the modulus of elasticity in shear is helpful to the designer, but the limitations of thickness to effective width prohibit its use in many applications.

The author states that a simple shear sandwich can be calculated from the equation

$$d = WT/AG \quad [1]$$

where d = deflection, in.
W = load, lb
T = initial thickness of rubber in one sandwich, in.
A = area absorbing load W, sq in.
G = modulus of elasticity in shear, psi

A slightly more accurate formula is

$$\text{strain (deg)} = 57.3W/AG \quad [2]$$

Of course, d/T is approximately equal to the strain in radians, so that Equation [1] gives a first degree of approximation.

Again, the author states that rubber torsion bushings can be designed by use of the formula

$$\theta = \frac{WL}{4\pi bG}\left(\frac{1}{r_1^2} - \frac{1}{r_2^2}\right) \quad [3]$$

where θ = angular deflection, radians
WL = torque, in-lb
b = effective length of rubber section, in.
G = modulus of elasticity in shear, psi
r_1 = inner radius of rubber section, in.
r_2 = outer radius of rubber section, in.

Equation [3] is again only a first degree of approximation; it is the first term of an infinite series. A more accurate equation is

$$\theta = \frac{WL}{4\pi bG}\left[\left(\frac{1}{r_1^2} - \frac{1}{r_2^2}\right) + \frac{1}{9}\left(\frac{WL}{2\pi bG}\right)^2\left(\frac{1}{r_1^6} - \frac{1}{r_2^6}\right) + \frac{2}{75}\left(\frac{WL}{2\pi bG}\right)^4\left(\frac{1}{r_1^{10}} - \frac{1}{r_2^{10}}\right) + \cdots\cdots\right] \quad [4]$$

F. L. Yerzley.[5] Mr. Haushalter is to be commended for the many contributions he has made to the improvement of mechanical structures by the application of rubber. His present paper based upon successful practice adds still another important chapter to the growing volume of literature on its mechanical characteristics. In connection with the paper, it seems worthy of

[1] By F. L. Haushalter, published in the February, 1939, issue of the A.S.M.E. Transactions, p. 149.

[2] Executive Engineer of Research, E. G. Budd Mfg. Co., Philadelphia, Pa.

[3] "Rubber Mountings," by J. F. Downie Smith, *Journal of Applied Mechanics*, Trans. A.S.M.E., vol. 60, March, 1938, p. A-13.

[4] "Change of Volume of Rubber on Stretching: Effects of Time, Elongation, and Temperature," by W. L. Holt and A. T. McPherson, Research Paper No. RP-936, National Bureau of Standards, *Journal of Research*, November, 1936, pp. 657–678.

[5] Physicist, E. I. du Pont de Nemours & Co., Wilmington, Del. Jun. A.S.M.E.

emphasis that a correlation between durometer hardness and modulus is presented for the range of values useful in mechanical applications. It is no less important that the author has defined limits for maximum permissible loads.

In this paper, one is reminded, however, of gaps in present knowledge which still remain to be filled. The relationship between durometer readings and moduli affords a useful basis for preliminary calculations. Yet it is not possible at the present time to generalize the properties of rubber to the point where it may be expected that a compound of a certain durometer hardness will be suitable for any use requiring a compound with the modulus indicated by the relationship. Compounded and vulcanized rubber is a heterogeneous mixture of multifarious pigment particles, immense organic molecules, and other substances in various degrees of chemical and physical combination. The relative states of the various phases are subject to disturbance by different factors including temperature, degree of strain, chemical instability, and penetration of surrounding media. Underlying principles are still evasive. The end result of any design, therefore, for rubber as for most other engineering materials, must still be confirmed by stringent service tests.

Author's Closure

Dr. Smith is perfectly right in his analysis of the formulas for both the simple shear sandwich and for the rubber torsion spring. For angles up to 30 deg the correction is almost negligible, and it is in this range that most of the applications of rubber springs will be found. For greater deflections there is another factor which must be considered.

It was pointed out by the author in referring to Fig. 17 that the value of the shear modulus G was constant only up to about 20 per cent deflection for 60-durometer rubber and up to about 40 per cent deflection for the very soft rubbers. The real reason for this is that the stress-strain curves of these rubbers in tension are straight only up to those values, then bend toward the deflection axis, indicating a softening of the rubber. It is chiefly for this reason that it is best to limit the deflection in shear-type springs to the limits of the straight portion of the stress-strain curves, for in most applications of springs it is not desirable to have a decrease in the rate of the spring coming in at such a place.

Obsolescence in Woodworking Machinery[1]

E. Dillon Smith.[2] This paper suggested many instances in the woodworking industry where management found equipment obsolete before the physical property had been fully depreciated on the books. Of course such examples are not unique to this industry alone. In fact air lines have had to deal with the obsolescence problem to a greater degree than perhaps any other industry. A brief comment on the methods used by some air lines to handle this problem may, therefore, be of general interest, such being, by inference, applicable to the woodworking industry.

Until recently, when the Civil Aeronautics Authority was created, the air lines had to deal with at least the following governmental bodies when considering obsolescence and depreciation: Bureau of Air Commerce of the Department of Commerce, Post Office Department, Interstate Commerce Commission, and the Treasury Department. If the woodworkers have had difficulties in handling obsolescence before one Federal body, they should consider themselves lucky in comparison to the air lines.

Fortunately for the air lines, however, they have kept complete records on each and every piece of physical equipment. Different depreciation rates are charged on each item, rather than one rate on all equipment. Irrespective of this practice, the air lines have found that some equipment was either still in operation after it had been completely depreciated on the books or that it was obsolete before it had been fully depreciated. In some of the air lines, the management has rather successfully avoided this difficulty by having current analytical studies made of policies and practices, and by always being informed on the tendencies being exhibited in the art of management and equipment design. Thus, some air line managers have learned to be sufficiently alert in their field as to guard almost wholly against being caught with bad equipment. Sometimes this is done by making advantageous sales of old equipment at a net book profit rather than at a distress-sale price, and then buying new and more modern equipment. It would seem, therefore, that one way in which some of the air lines have avoided being caught with obsolete physical equipment has been to employ an alert and informed management.

Author's Closure

In the last sentence of his excellent discussion, Professor Smith strikes directly at the heart of this important and perplexing problem. What is needed is "an alert and informed management." In that connection, however, it should be noted that a management may seem to be both alert and informed and still not realize the increasing importance of the obsolescence problem. This is more apt to be the case in the older industries, where obsolescence was formerly not so important, than in such industries as the air-transportation industry, where obsolescence has been of great importance from the very beginning of the industry.

Professor Smith refers to methods used in the air-transportation industry to minimize the bad effects which often result from the general inability of any industry to predetermine the useful life of its depreciating assets. While the details may differ, every industry could well afford to follow the plan mentioned by Professor Smith, namely, "by having current analytical studies made of policies and practices, and by always being informed on the tendencies being exhibited in the art of management and equipment design."

[1] By Paul T. Norton, Jr. Published in the February, 1939, issue of the A.S.M.E. Transactions, vol. 61, p. 145.

[2] Professor of Mechanical Engineering, College of the City of New York, New York, N. Y. Mem. A.S.M.E.

Pitot Tubes in Large Pipes

By EDWARD S. COLE[1] and E. SHAW COLE,[2] NEW YORK, N. Y.

Comparative calibrations with two different types of pitometer, which reacted differently to angular flows, indicated the absence of angularity effects in large pipes. To check this the authors made actual measurements in two large pipes and found the angularity to be less than 3 deg.

Comparisons with other methods of flow measurement in two large pipes are presented together with velocity traverses in these and other conduits. The velocity traverses show the smooth flow pattern found in large pipes at good gaging points. The authors also discuss the value of pipe factor, i.e., the ratio of mean to center velocity, for determining the suitability of a gaging point as well as for making short tests quickly and accurately, or for making continuous-flow measurements.

IN A previous article[3] one of the authors discussed the history and development of the pitot tube as means for measuring the flow of water in a closed conduit. Comparative tests were conducted in a 12-in. pipe line, and calibrations were made under a number of different conditions to determine the possible source of errors, and to establish the degree of accuracy with which flow could be measured in pipe lines.

For many years the principal source of error has been thought to be angular flows past a pitot tube and, while the comparative tests in the 12-in. pipes proved that the effect of these flows was negligible, it was thought that this work should be extended to pipe lines of larger diameter.

Two types of combined pitot tubes were used for the tests described in this paper, namely, the reversible pitometer widely used in waterworks analysis, and the heavy-duty pitometer frequently used for measuring the speed of ships. Figs. 1 and 2 show these instruments.

[1] President, The Pitometer Company, Engineers, and Pitometer Log Corp. Mem. A.S.M.E. Mr. Cole received the degree of M.E. from Cornell University in 1894 and from that date until 1902 served on the staff of John A. Cole, consulting engineer, Chicago, working on the design, construction, and management of city waterworks plants in the Middle West. In 1896 he developed the pitometer. As a member of the Department of Water Supply, Gas, and Electricity of New York, N. Y., he was in charge of water-supply studies with the pitometer during 1902 and 1903. Since that time he has conducted pitometer investigations throughout the country.

[2] Field Engineer, The Pitometer Company. Mr. Cole was graduated with an A.B. degree from Dartmouth University in 1930, and received his C.E. degree from the Thayer School of Civil Engineering at Dartmouth in 1931. From that date until 1934 he was connected with the Pitometer Log Corp. and worked on various development problems. In 1934 he joined the field staff of the Pitometer Company, Engineers, and has been engaged in pitometer surveys of waterworks throughout the country.

[3] "Pitot-Tube Practice," by Edward S. Cole, Trans. A.S.M.E., vol. 57, August, 1935, paper HYD-57-8, pp. 281–294.

Contributed by the Hydraulic Division and presented at the Annual Meeting of The American Society of Mechanical Engineers, held in New York, N. Y., December 5–9, 1938.

Discussion of this paper closed January 10, 1939, and is published herewith directly following the paper.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

Scope of Tests

The program of test and investigation described in the paper is divided into four sections as follows:

1 Determination of pitometer coefficients for still-water conditions or equivalent, and the effect of angularity upon registration of the instrument.

2 Tests in a 40-in. pipe in which both types of pitot tubes were used, and in which the discharge was measured by weighing tanks as a means of checking the discharge measurement established by the pitometer. Additional tests on the angularity of flow and its effect upon pitot-tube coefficients were also included in the studies of the 40-in. conduit.

3 Angularity tests were made in a 40-in. and in a 78-in. diameter steel penstock to determine whether the flow pattern in these larger conduits differed from that found in smaller pipes.

4 Velocity traverses were made in a number of large pipe lines in connection with flow measurements. These curves with their pipe factors (ratio of mean velocity to center velocity) are an indication of smoothness of velocity distribution in such conduits.

Calibrations

The previous paper[3] described three methods of calibration for the reversible pitometer and showed that still-water ratings on the revolving boom at the Alden Hydraulic Laboratory were practically identical with pitometer coefficients obtained in the 36 × 16-in. venturi meter. It was found that no correction for the small projected area of the rod was required for the calibration in the venturi throat; such differences as were found amounted to a small fraction of 1 per cent and were within the experimental error.

When check calibrations were made in a straight 12-in. pipe, which was free from venturi effects, the rod correction was found necessary and investigation showed that its magnitude was a direct function of the projected area of the rod. Several different methods of evaluating this correction were employed and an average correction curve established.

Angularity Characteristics

With still-water ratings or their equivalent, the pitometer orifices are set in the direction of flow. When the instrument is rotated through various angles, changes in registration occur. To establish these relations on both the reversible and the heavy-duty pitometers, tests were made in the 16-in. throat of the venturi meter, observing the change in coefficient caused by various departures of the axis of the instrument from the normal direction of flow.

Basing the values on velocity readings, the change in registration with angularity would indicate slightly higher velocities than true values or would require a decrease in coefficient.

Fig. 3*A* presents the characteristics of the reversible pitometer. The two parts making up the total error of +0.75 per cent at 5 deg will be noted. One of these parts is the change in the pitometer readings, while the other is the cosine curve which is the theoretically correct reading, the sum of the two being the full error in reading the axial component ($V \cos a$) of an angular flow a.

The characteristics of the heavy-duty tube, with a $^5/_{16}$-in. tip, were obtained by the same method and are represented in Fig. 3*B*. Here the error is shown as the difference between the

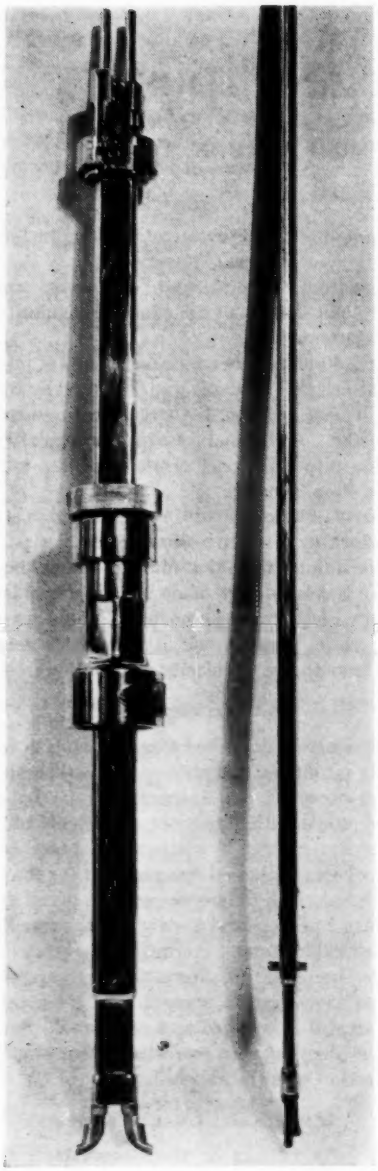


FIG. 1 REVERSIBLE PITOMETER

pitometer readings and the cosine values. As will be seen, this type of tube follows the cosine value closely to 6-deg angularity.

It will be noted from Fig. 3 that the two types of pitometers have substantially different angularity characteristics and should the average angularity of flow in a normal conduit reach as much as 6 deg the discharge measured by the two types under the same conditions should show measurable differences.

To investigate this possibility, tests were made in a 40-in. pipe with both meters and the flow was checked by a calibrated weir. A description of the tests and their results is included herein.

STUDIES MADE IN 40-IN. PIPES

At the Alden Hydraulic Laboratory, Worcester, Mass., a number of tests were made on a 40-in. pipe line to check the coefficients of two of the reversible-type pitometers for comparison with the coefficient of the instrument determined in the 16-in. throat of a venturi meter. Similar calibrations were made with a heavy-duty pitometer, using the same procedure.

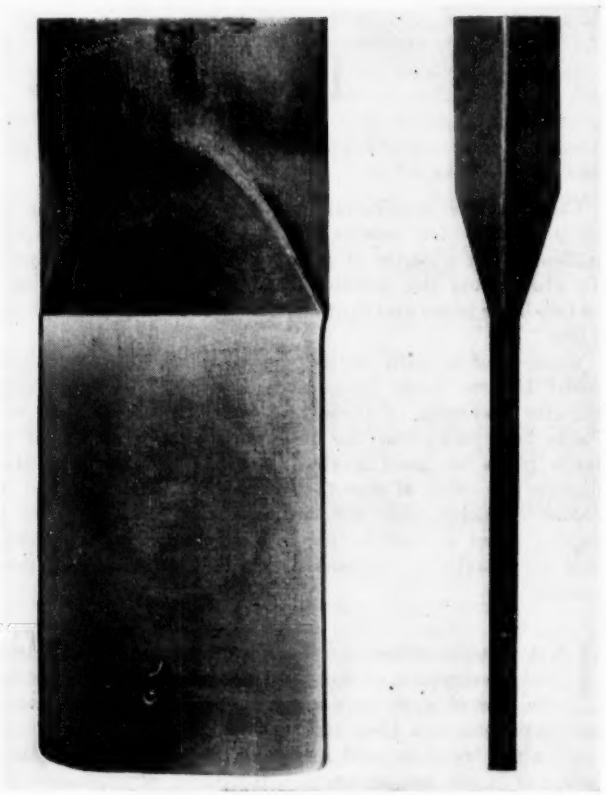


FIG. 2 HEAVY-DUTY PITOMETER

The Reversible Pitometer. The gaging station for this reversible pitometer was located 100 ft below and 40 ft above any departure from a straight section. For traversing, the tube was inserted through 1-in. cocks 90 deg apart, and for calibration the orifices were located at the center of the pipe. The flow was measured over a 10-ft contracted weir which had been calibrated by incremental measurements using a weighing tank.

The first step was to determine the pipe factor from double traverses of the pipe. Sample curves for the reversible pitometer are shown in Figs. 4*A* and 4*B*. Readings were taken at each desired point on the diameter and converted to velocities using the corresponding instrument coefficient established in the 16-in. venturi throat. The plotting of this velocity determination

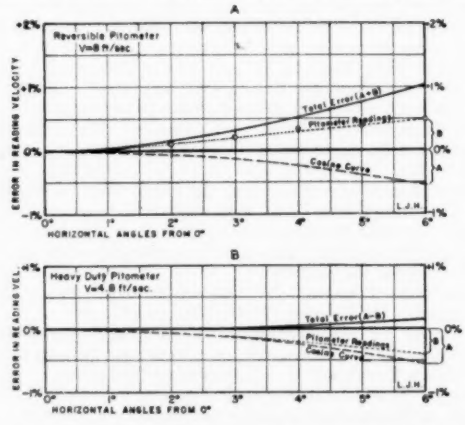


FIG. 3 ANGULARITY CHARACTERISTICS OF REVERSIBLE AND OF HEAVY-DUTY PITOMETERS IN A 16-IN. VENTURI THROAT
(Alden Hydraulic Laboratory, Worcester, Mass., 1934.)

instead of the square root of the deflection takes into account changes in the instrument coefficient at different rates of flow.

This discharge measurement was then checked by another set of runs using the 40-in. pipe with the pitometer at the center of the pipe and the flow measured by the previously calibrated weir. By applying the pipe factor to the center velocity readings a new coefficient of the instrument was secured.

The procedure was as follows: After the discharge had been set to a desired value, about 10 or 15 min were allowed to elapse to secure steady-flow conditions in the penstock and over the weir. The U-tube deflections with the orifices at the center of the pipe were then read in feet of water. Ten readings per minute were made by an experienced observer, and the figures averaged on a calculating machine. Ten readings of head were taken for each position of the orifices shown in Fig. 1, that is, first with one tip upstream and then with the other tip upstream. The head on the weir was read and then the discharge was adjusted to a new value for the next test.

The Heavy-Duty Pitometer. Tests with the heavy-duty pitometer having the $^5/_{16}$-in. tip were carried out in the same 40-in. steel penstock using the same procedure as with the reversible pitometer. The gaging station was at a slightly different location due to the necessity of using much larger entrance connections for the heavy-duty tube.

The pipe factor shifted about 1.5 per cent, averaging 0.808 for the reversible tube and 0.796 for the heavy-duty pitometer. The double traverses are shown in Figs. 5*A* and 5*B*. These discharge measurements were also checked in the 40-in. pipe against the calibrated weir in the same way as the reversible tube.

Correction for Projected Area of Rod. In utilizing the test readings of velocity, it is necessary to apply a correction for the area of the pitometer rod projecting into the pipe. Several methods may be used as determined from tests[3] made at Worcester Polytechnic Institute and at the University of Pennsylvania in 1934 and more recently from additional tests[4] at Worcester.

In general, tests were run in various sizes of pipe, holding the diameter of the rod constant, or in the same pipe with different diameters of pitometer rods. The change in coefficient was related to the reduction in area caused by the rod when the orifice was at the center of the pipe.

The comparison of the corrections for the reversible rod as obtained by the two different procedures in relatively small test sections (4- to 16-in. diameters) shows a spread of about 1 per cent at the 12-in. size, at which point the average change in the coefficient is 2 per cent from the still-water rating. The area occupied by the rod in this case is about 1.6 per cent of the pipe

[4] "Investigation of Errors in Pitot Tubes," by C. W. Hubbard, presented at the Annual Meeting of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS, New York, N. Y., December 5–9, 1938. See pages 477–506 of this issue.

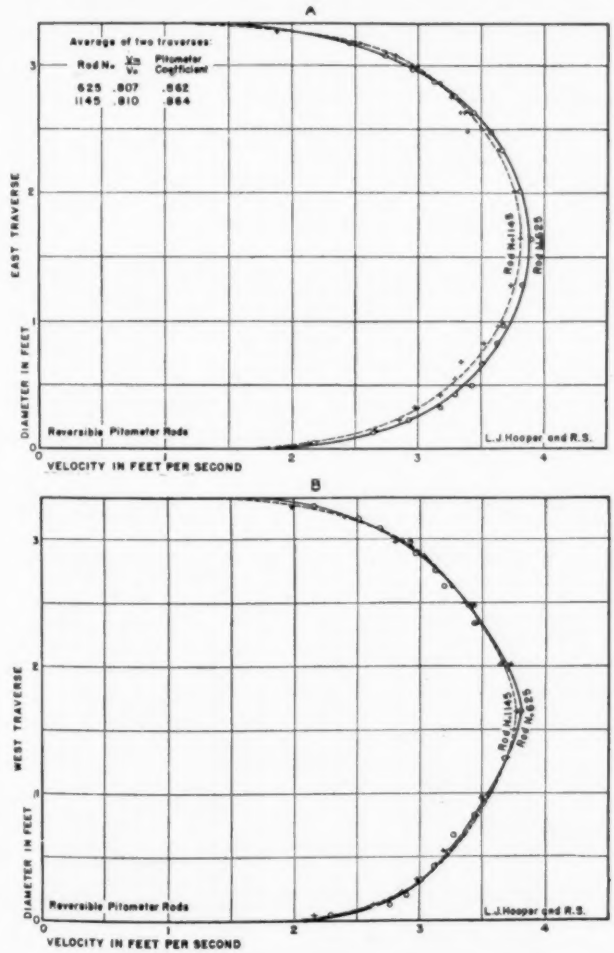


FIG. 4 VELOCITY TRAVERSES IN THE 40-IN. RIVETED STEEL PENSTOCK OF THE ALDEN HYDRAULIC LABORATORY, FOR REVERSIBLE PITOMETER, 1935

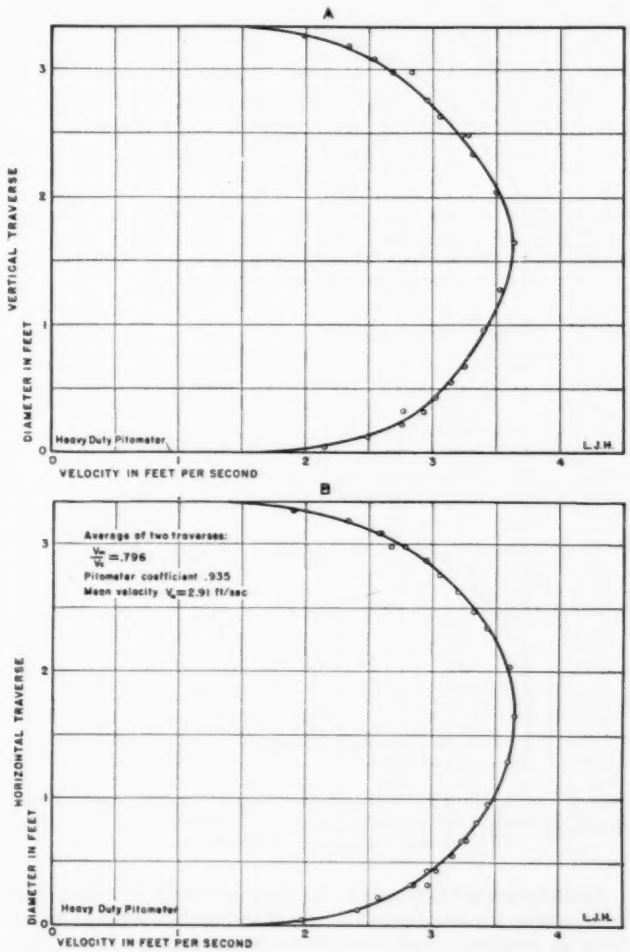


FIG. 5 VELOCITY TRAVERSES IN THE 40-IN. RIVETED STEEL PENSTOCK OF THE ALDEN HYDRAULIC LABORATORY, FOR HEAVY-DUTY PITOMETER, 1935

area; however, as the diameter of the pipe increases, the area decreases to 0.5 per cent in the case of the 40-in. pipe, adding about 0.6 per cent to the coefficient.

With this smaller correction the spread between the two methods of determination is about one fifth that with a 12-in. pipe, or expressed another way the two methods differ from the average curve by only 0.1 per cent, which is within the limits of accuracy of these tests.

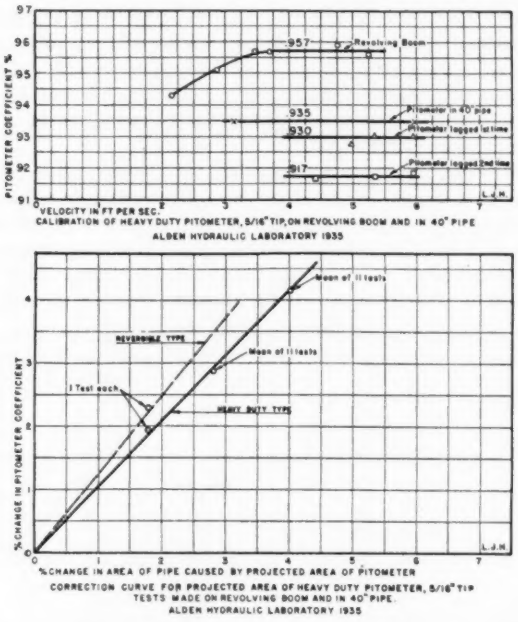


FIG. 6 TOP: CALIBRATION CURVE OF HEAVY-DUTY PITOMETER TIP. BOTTOM: CORRECTION CURVE FOR PROJECTED AREA OF HEAVY-DUTY PITOMETER—MEAN CORRECTION CURVE FOR REVERSIBLE PITOMETER ADDED FOR COMPARISON

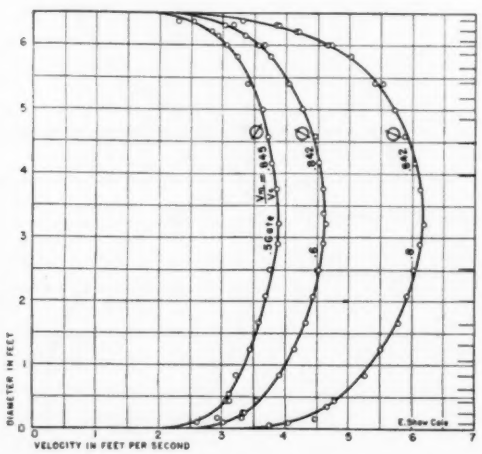


FIG. 7 VELOCITY TRAVERSES IN THE 78-IN. WELDED STEEL PENSTOCK AT THE PLANT OF THE CENTRAL VERMONT PUBLIC SERVICE CORP., RUTLAND, VT., 1937

Similar studies were made in the 40-in. pipe with the heavy rod by changing the rod diameter. Fig. 6 shows the results of these tests as compared with still-water ratings and also gives the calibration correction curve for use with the heavy-duty pitometer. The previously determined correction curve for the reversible pitometer is shown for reference. The two relations do not differ greatly and serve to confirm still further the accuracy of this method.

Due to the large diameter of the heavy-duty rod the area occupied is greater than with the reversible rod, being 1.7 per cent of the total area of the 40-in. pipe. From the correction curve, this will add 1.8 per cent to the coefficients.

Results. Applying the rod correction to the coefficients of the two types of pitometers and comparing them with the still-water rating or its equivalent, the results shown in Table 1 are secured.

From studies made by Allen,[5] Hubbard,[4] and one of the authors[3] of this present paper angularities, ranging from 6 deg down to 2 deg, have been measured in the 40-in. pipe by various means. The coefficient corrections from Figs. 3*A* and 3*B* corresponding to these degrees of angularity are shown in Table 2 compared with the results of the calibration runs. It will be seen from this comparison that either the angularity has little or no effect on the coefficient of the pitometer, or else the amount of angularity

TABLE 1 COEFFICIENTS OF REVERSIBLE AND HEAVY-DUTY PITOMETERS IN 40-IN. PIPES COMPARED WITH STILL-WATER RATINGS OR EQUIVALENT

Reversible pitometer (coefficients obtained at about 3 fps):	
Uncorrected coefficient of instrument in 40-in. pipe	0.863
Addition for projected area of the rod	0.006
Corrected coefficient of instrument in 40-in. pipe	0.869
Coefficient of instrument in 16-in. throat of a 36-in. × 16-in. venturi meter (equivalent to still-water rating at the revolving-boom station)	0.870
Difference from still-water rating, per cent	0.100
Heavy-duty pitometer:	
Uncorrected coefficient of instrument in 40-in. pipe at velocities of about 3 fps	0.935
Addition for projected area of the rod	0.018
Corrected coefficient of instrument in 40-in. pipe	0.953
Coefficient of instrument in still water from boom calibration at velocities varying from 2.1 to 5.3 fps	0.957
Difference from still-water rating, per cent	0.400

NOTE: Both of these check calibrations in the 40-in. pipe agree very closely with the still-water rating or its equivalent.

TABLE 2 COEFFICIENT CORRECTIONS FROM FIG. 3 CORRESPONDING TO DEGREES OF ANGULARITY MEASURED BY ALLEN,[5] COLE,[3] AND HUBBARD[4] IN THE SAME 40-IN. PIPE

Investigator	Measured obliquity, deg	Coefficient corrections as obtained from Fig. 3*A*, per cent: Reversible pitometer	Heavy-duty pitometer	Coefficient corrections from actual tests, per cent: Reversible pitometer	Heavy-duty pitometer
Allen[5]	6, max	−1.00	−0.10		
Cole[3]	5, avg	−0.75	−0.10	−0.10	−0.40
Hubbard[4]	2, mean	−0.20	0		

TABLE 3 READINGS AT THE GALLOWAY PLANT OBTAINED BY PITOMETER TRAVERSES AND BY THE SALT-VELOCITY METHOD

By pitometer	Test values, cfs: By salt-velocity method	Mean	Deviation of pitometer readings from mean, per cent: Individual run	Group	Group avg
Unit No. 1:					
1431	1444.9	1438.0	−0.49	−0.40	
1435	1443.6	1439.3	−0.30		
1192	1189.2	1190.6	+0.12	+0.24	
1197	1191.6	1194.3	+0.23		
1198	1188.9	1193.4	+0.38		
1063	1043.4	1053.2	+0.93	+0.74	+0.08
1060	1046.1	1053.0	+0.67		
1056	1042.9	1049.4	+0.63		
1324	1333.1	1328.5	−0.34	−0.24	
1326	1331.2	1328.6	−0.20		
1324	1328.5	1326.2	−0.17		
Unit No. 2:					
1249	1243.3	1246.2	+0.22	+0.39	
1252	1242.0	1247.0	+0.40		
1258	1244.2	1251.1	+0.55		
1310	1306.8	1308.4	+0.12	+0.13	+0.24
1311	1307.2	1309.1	+0.15		
1312	1309.3	1310.6	+0.11		
1469	1464.6	1466.8	+0.15	+0.20	
1471	1467.6	1469.3	+0.12		
1474	1464.2	1469.1	+0.33		

[5] "How Water Flows in a Pipe Line," by C. M. Allen, *Mechanical Engineering*, vol. 56, February, 1934, pp. 81–84.

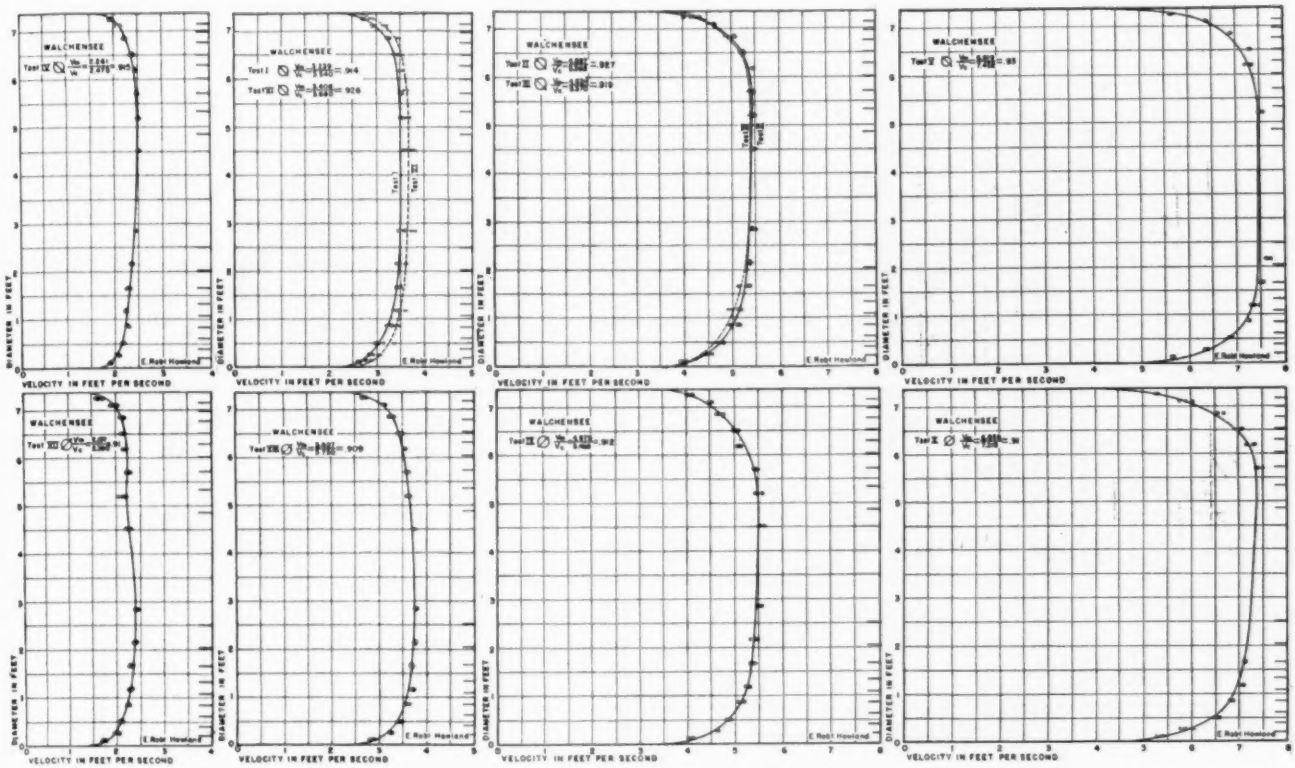


FIG. 8 VELOCITY TRAVERSES IN THE 7.5-FT PENSTOCK AT WALCHENSEE, GERMANY, 1935

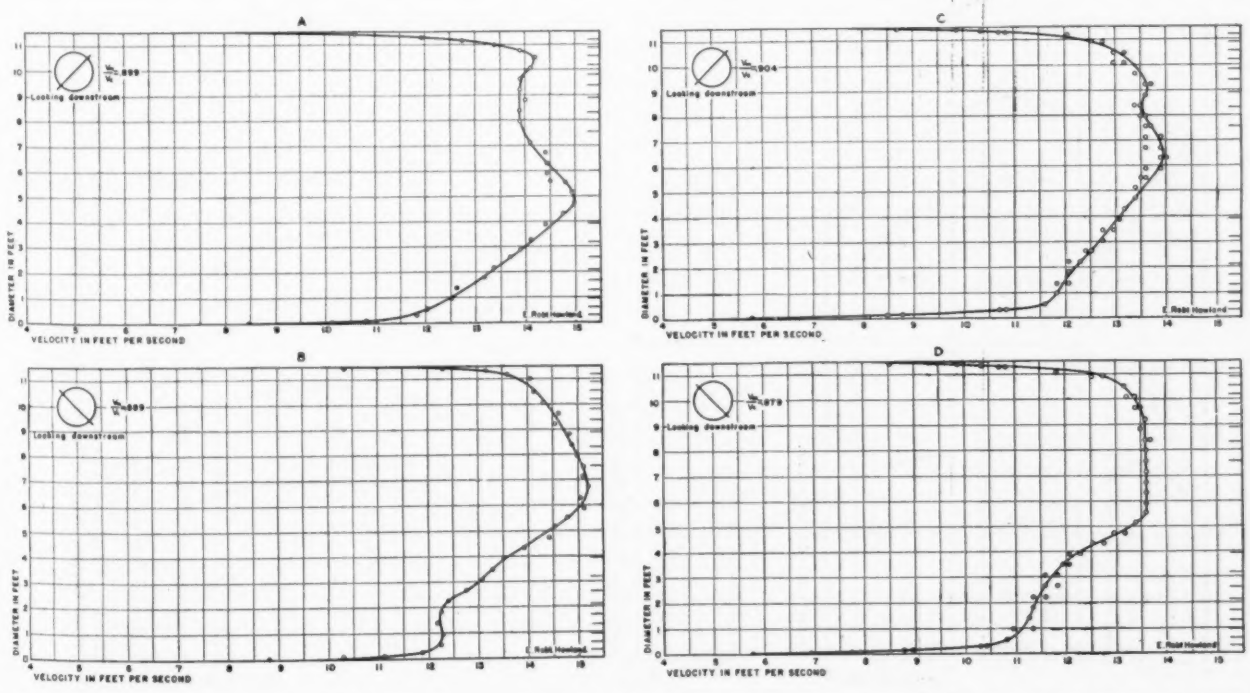


FIG. 9 VELOCITY TRAVERSES IN THE 11.5-FT RIVETED STEEL PIPE LINES OF THE GALLOWAY POWER CO., TONGLAND, SCOTLAND, 1935
(Figs. *A* and *B* are for pipe line No. 1, while Figs. *C* and *D* are for pipe line No. 2.)

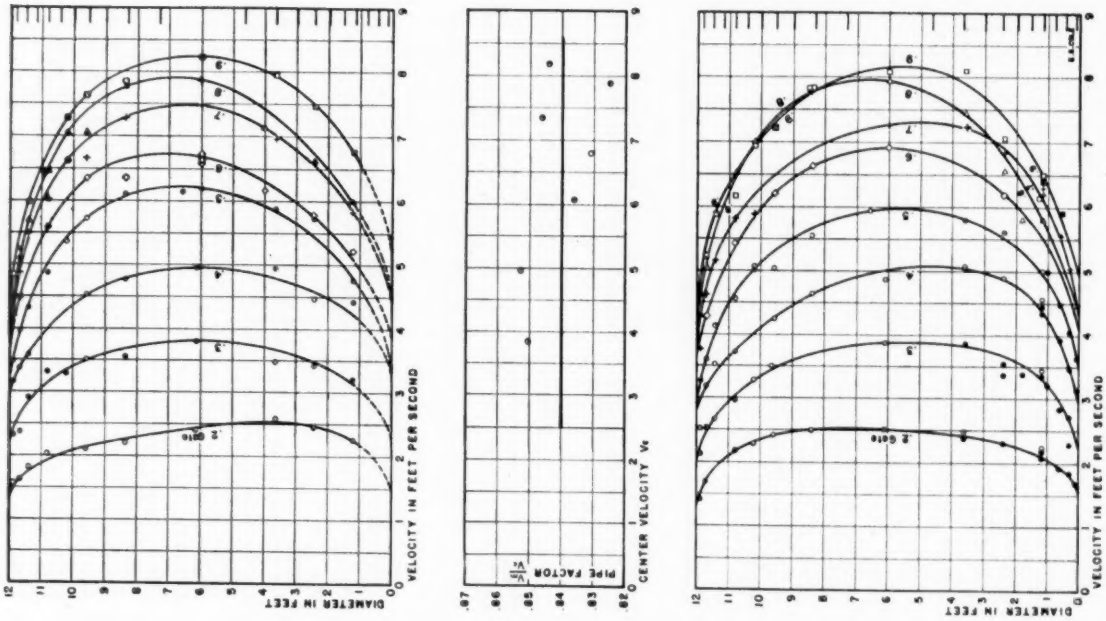


FIG. 11 VELOCITY TRAVERSES IN THE 12-FT RIVETED STEEL PENSTOCK OF THE GREAT NORTHERN PAPER CO., MILLINOCKET, ME., 1917

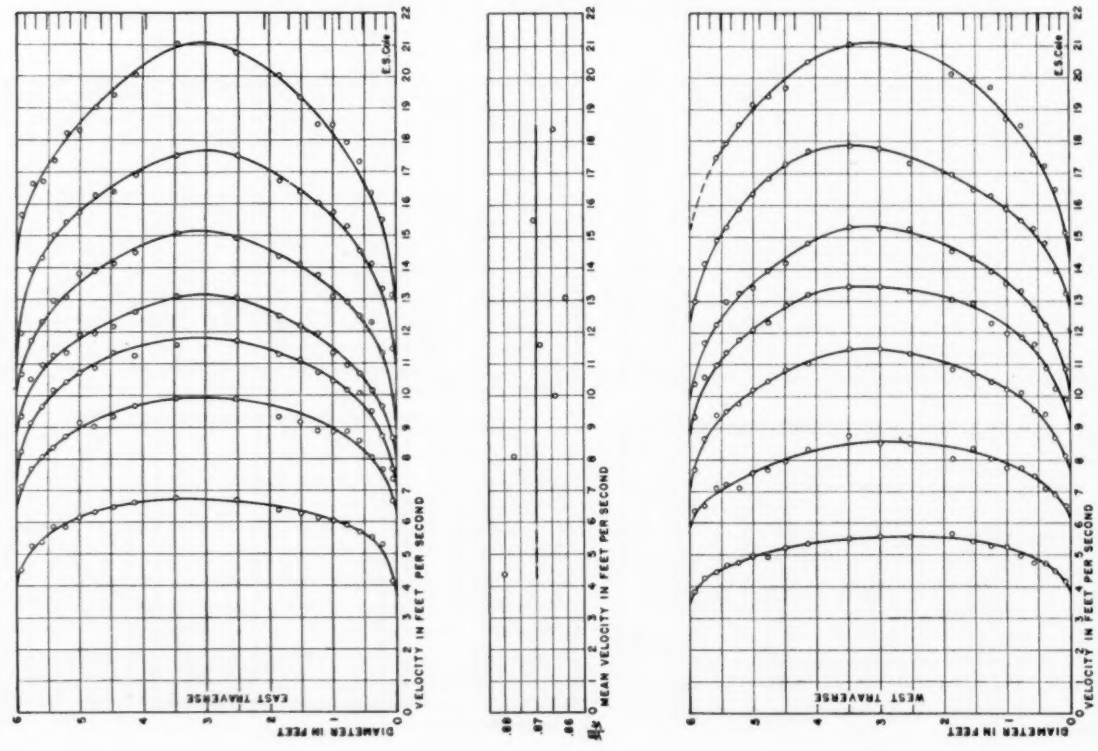


FIG. 10 VELOCITY TRAVERSES IN THE 6-FT WELDED STEEL PENSTOCK OF BIG CREEK NO. 8 PLANT OF THE SOUTHERN CALIFORNIA EDISON CO., 1922

present is much less than the 5 to 6 deg measured previously by Cole[3] and Allen,[5] respectively.

To check these conclusions, another series of tests were made to measure the angularity existing in the 40-in. pipe and in a 78-in. conduit.

Angularity Measurements in Large Pipes

A variety of means have been suggested for measuring the angularity of flow in a pipe line. For the most part these are composed of small vanes which give either a mechanical or electrical indication outside of the pipe. In spite of refinements, most vane measurements have been subject to error due to friction as well as disturbances caused by the shape and size of the vane.

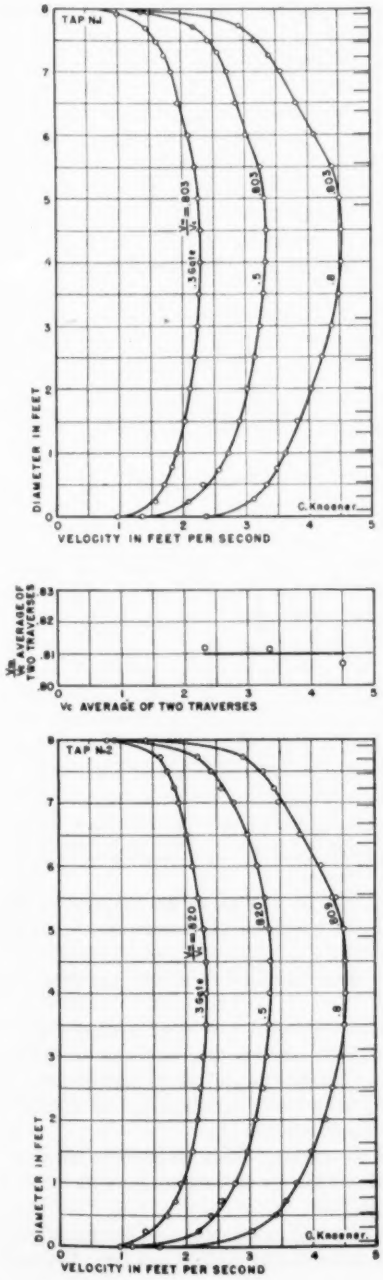
Previous to this time, a study of the dispersion of salt as it passed down a pipe was the only method which did not include mechanical errors. This method however, gave maximum angles and it was impossible to determine the average.

Hubbard[4] recently designed and constructed an angularity indicator composed of a small plow-shaped vane that was electrically connected to a chronograph. The electrical impulses were analyzed by means of the Law of Probability and the mean angle was thus determined.

Hubbard's first instrument was not adapted for field use and in order to carry out more extensive tests one of the authors of this present paper contributed a second instrument of improved design. Subsequently, Hubbard and one of the authors made angularity measurements in two large pipes where the flow characteristics had been determined by pitometer traverses.

The 40-in. pipe at the Alden Hydraulic Laboratory, mentioned previously, was used for one of these tests. Measurements at various distances from the center gave average angles that were less than 2 deg.

A 78-in. steel penstock was also tested. This pipe, located at the Glen Station of the Central Vermont Public Service Corporation, Rutland, Vt., is constructed of welded steel, with telescopic sections, and has plates $^5/_{16}$ in. thick. The gaging station where



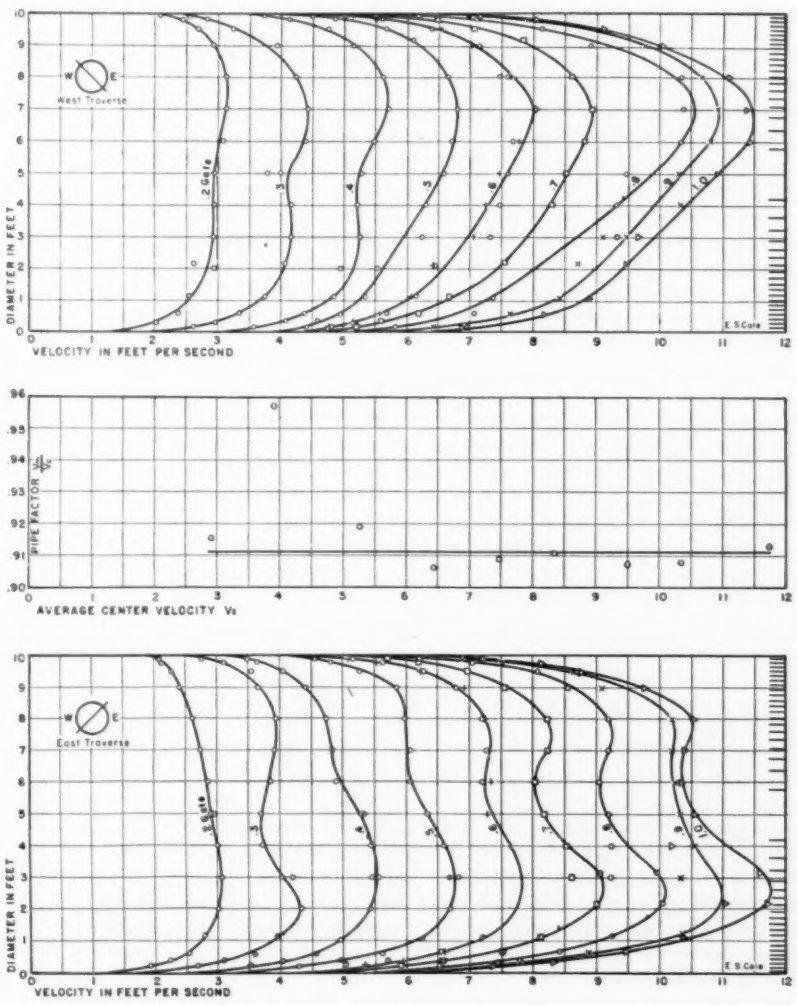


FIG. 13 VELOCITY TRAVERSES IN THE 10-FT WOOD-STAVE PENSTOCK OF THE CENTRAL MAINE POWER CO., RICE RIPS, ME., 1919

FIG. 12 (LEFT) VELOCITY TRAVERSES IN THE 8-FT WOOD-STAVE PENSTOCK OF THE MONGAUP PLANT OF THE ROCKLAND LIGHT & POWER CO., MIDDLETOWN, N. Y., 1938

the angularity tests were made was located approximately 400 ft below and 200 ft above any departure from a straight section; the velocity distribution at this point is shown in Fig. 7. The mean angularity obtained was 1.5 deg.

A 12-in. pipe was also tested in connection with Hubbard's paper[4] and it is interesting to note that the angularity in the 78-in. pipe was about the same as in the 12-in. pipe and slightly less than in the 40-in. pipe. In every pipe that was tested the angularity was found to be less than 3 deg.

Referring again to Table 2 and the angularity curves shown in Figs. 3*A* and 3*B*, it is evident that the close agreement between the tests in the 40-in. pipe and the still-water rating or its equivalent is due to the small amount of angularity; hence, corrections (on the order of 0.1 per cent for the reversible type) for angularity can be neglected as long as the gaging point is properly located and the pipe traverses show stable conditions.

With the heavy-duty pitometer, the difference of 0.4 per cent in the coefficient as determined in the 40-in. pipe line is greater than would be indicated from any angularity correction shown in Fig. 3*B*. However, it is still within the experimental accuracy of test work (probably 0.5 per cent) and since this type of pitometer is relatively unaffected by angularity, as compared to the reversible type, the difference probably can be attributed to other causes.

With the experimental confirmations just described, it can be seen that stable flow is present in large pipes where suitable gaging stations are used, and that the flow conditions do not differ measurably from those found in small pipes. This at once removes any previously expressed doubt as to the accuracy of pitometer flow determinations in large conduits.

Comparisons With Other Methods

Comparative flow measurements by the pitometer method and other methods have been made in three large conduits, i.e., the 40-in. line at Worcester, the Walchensee tests in Germany, and the tests which were made recently at the Galloway plant in Scotland.

In the 40-in. line at Worcester, the agreement was within 0.1 per cent in one case and 0.4 per cent in the other. At Walchensee, Germany's test conditions were far from ideal and the first reports were found to be in error. After the original data were checked and the results recalculated, it was found that the agreement between the salt-velocity method and the pitometer runs was remarkably close, the difference being about 0.5 per cent at half and three-quarter loads. Velocity traverse curves plotted from pitometer readings are shown in Fig. 8.

TABLE 4 ADDITIONAL PLANTS WITH LARGE-DIAMETER PENSTOCKS WHERE VELOCITY TRAVERSES WERE MADE

Plant	Diameter, ft	Pipe factor	Fig. no.
Big Creek No. 8, California	6	0.870	10
Millinocket, Me.	12	0.840	11
Mongaup plant, Middletown, N. Y.	8	0.810	12
Rice Rips, Me.	10	0.910	13
Glens Falls, N. Y.	12	0.915	14

The results obtained at the Galloway plant are given in Table 3 and the traverses are shown in Fig. 9. These tests were made by E. Robert Howland in August, 1936. The differences between the salt-velocity method and the pitometer runs ranged from +0.7 per cent to —0.40 per cent, and averaged 0.1 per cent on one unit and 0.24 per cent on the other.

In addition to the examples just cited and the 78-in. conduit at Rutland, Vt., where angularity readings were taken and for which velocity traverses are shown in Fig. 7, five other groups of traverses are included in this paper to show the uniformly smooth consistent results obtainable in pitometer gagings in large conduits when suitable test sections are employed. The plants where the traverses were taken are listed in Table 4, together with the pipe diameter and pipe factor at each plant, as well as the figure number in which the velocity traverse for each plant is shown. In each case the traverses could be repeated and gave consistent results. Their pipe factors (ratio of mean to center velocity) remained constant over a wide range of velocities. The readings and traverses were easily reproducible even in the case of Figs. 13 and 14, where upstream bends distorted the flow distribution.

The unusually good agreement obtained in the Galloway plant between the salt-velocity tests and the pitometer runs, results of which are shown in Table 3, shows that unsymmetrical flow distribution has no measurable effect on accuracy.

Simplified Gagings

As a result of this research program and its coordination with field test work, a major simplification can be made in pitometer flow measurements in large conduits without impairing the accuracy of the results.

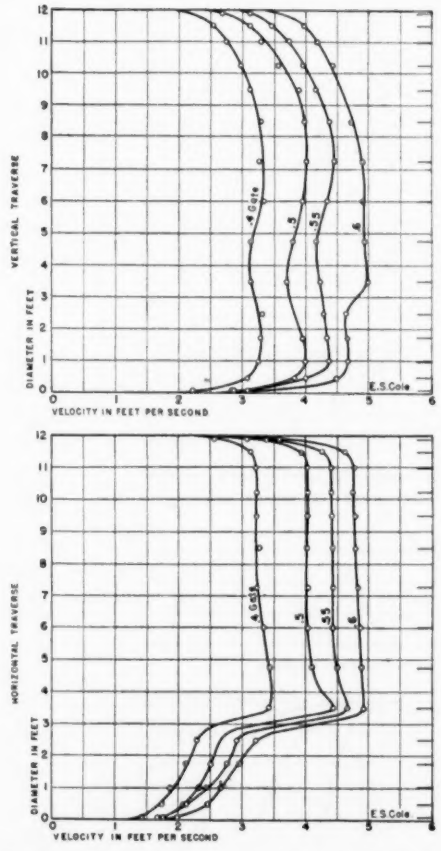


Fig. 14 Velocity Traverses in the 12-Ft Riveted Steel Penstock of the International Paper Co., Glens Falls, N. Y., 1918

It has been shown that with stable flow, the traverses are consistent and that for a given conduit the pipe factor remains constant for a wide range of discharge. It is then possible to determine the pipe factor on a given installation, and thereafter take readings only at the center of the pipe.

This provides means for making short tests with speed and accuracy and also will permit the use of the pitometer for securing a continuous record of the flow.

The Galloway tests were made in this way and the remarkably close agreement with the salt-velocity method has been discussed.

Conclusions

The value of velocity traverses and pipe factor, that is, the ratio of mean to center velocity, as a means of determining the flow pattern in a conduit has not been fully appreciated, and any theory regarding turbulent flow must take into account the behavior of this pattern. For good gaging points, traverse curves are smooth and their pipe factors constant over a wide range of velocities.

Smooth traverses do not have "humps" or irregularities, and in a rough pipe they take the general form of a parabola. Irregular curves have been found in pipes of all diameters and have been an indication of disturbances a short distance upstream. Upstream contractions give flat velocity curves which are smooth if the contraction is gradual as in a venturi throat. They are irregular if abrupt changes in section are present above the gaging section.

Accurate tests can be made when the velocity distribution is carefully determined, for at a good gaging point the pattern will have a form which remains constant even though unsymmetrical.

Although the pipe factor under such conditions remains constant and can be reproduced on different days, as was done for most of the traverses in the large pipes, it is of course closely related to pipe roughness and for this reason should be checked if too long a time elapses between tests. The change in roughness is usually a slow process but in one unusual case where a pipe had recently been cleaned an appreciable change in pipe factor was observed in less than two months.

Where stable flow exists at the gaging section, pitometer measurements of discharge can be made with a high degree of accuracy in large pipes. It is possible to use either the conventional velocity-traverse method at each discharge or to determine the pipe factor and make subsequent measurements of the center velocity only.

Discussion

H. S. Bean.[6] Since the two papers deal with the same general subject, the following is a combined discussion of the present paper and the paper, "Investigation of Errors in Pitot Tubes," by C. W. Hubbard, (see page 477 of this issue).

From their manner of presentation, the authors give the impression that the relative reliability of pitot-tube indications when used in large pipes or conduits as compared to their use in smaller pipes had been questioned. It had never occurred to the writer to question the application of pitot tubes to large pipes, at least from the standpoint of fluid dynamics. Indeed, it has always been his understanding that the pitot tube is particularly well suited to the making of flow measurements in large pipes, conduits of rectangular or irregular cross section, and open channels.

Of course, the use in large conduits presents the problem of making the tube sufficiently stiff to prevent it from vibrating. The meeting of this requirement for stiffness and, at the same time, limiting the size of the supporting leg make the design and construction of the pitot tube rather difficult. One solution is to avoid inserting the tube into the pipe beyond about 0.4 radius (in the case of circular conduits). This has the effect of decreasing the cantilevered end of the tube with a proportionate increase in its relative stiffness. By making partial traverses from each end of two normal diameters, the velocity distribution probably can be determined as reliably as if a full diameter is traversed from one end. This assumes that at the traverse section the pipe is accessible from all sides, which is not always the case.

[6] Chief, Gas Measuring Instruments Section, National Bureau of Standards, Washington, D. C. Mem. A.S.M.E.

Another method of providing sufficient stiffness is to have a support rod extend across the conduit. This is not as simple a solution as at first it may appear. Not only does it appreciably decrease the area of the channel cross section but it complicates the problem of bringing out the pressure leads.

Referring to the question of pitot-tip vibration, it is noticed that this possible source of error has not been treated by Mr. Hubbard. In one case observed by the writer this was believed to have been the major source of an 8 per cent difference between the indications of a pitot-static tube and an orifice. It is suggested, therefore, that it might be well to study the effects of vibration of the tip of the pitot tube and of a pitot-static tube. It seems possible that such vibrations, even if they are not noticeable from the outside, may be the source of some errors, particularly when a tube is cantilevered a few feet into a stream with a relatively short stuffing gland as a support.

The latter paper not only indicates several possible sources of error and the approximate magnitude of their effects, but also some of the precautions which should be taken in the use of pitot tubes. The conclusion reached, that the impact tip correctly registers the true dynamic head, but that the pressure piezometers may easily be in error, agrees with that reported by Dr. A. F. Zahm,[7] thirty-five years ago.

The remarks (oral discussion) by L. J. Hooper on the difficulties of making and using side-wall piezometer holes are not altogether surprising. In connection with the flow-nozzle research, variations have been observed in the pressures measured from several piezometer holes in a common cross section. Such differences are more likely to occur in sections across which the fluid velocity is quite high, as in a flow nozzle or venturi throat.

E. A. Hitchcock.[8] This paper and its discussion were of special interest to the writer because of a former experience in determining the velocity curves for an 8-ft penstock connected to a 10,000-hp water turbogenerator. This work took place during the summer of 1914 and was carried on in detail by H. C. Lord, professor of astronomy at The Ohio State University under the writer's general supervision.

The pitot tip selected for this purpose had been designed by Horace Judd, professor of hydraulics at The Ohio State University, to give with water a unity coefficient. It was made of 1-in-square brass, about 4 in. long with the upstream end having the form of a four-sided pyramid; the opposite faces including an angle of 20 deg. The dynamic opening was at the vertex of the pyramid and the static openings were in the two opposite vertical faces of the pyramid.

It was decided that this tip should be tested under a variety of velocities. This was done by mounting the complete traversing instrument so that it was 2 ft in front of the bow of a motorboat with the tip held 12 in. below the water surface. The boat was operated under constant speed at many different velocities over an accurately measured course of 1786.2 ft. This calibration test was carried out on an arm of Lake Ocoee at Parksville, Tenn. Fourteen runs were made (180 readings taken) with velocities ranging from 6.54 fps to 10.7 fps. The average tip coefficient was determined to be 1.015 and showed no certain variation with the different velocities.

The unit to be tested was located in the No. 2 Ocoee River powerhouse of the Tennessee Electric Power Company.

The traversing instrument had a net 4-ft travel, but readings with generator loads in excess of 4000 kw could not be obtained at the center of the penstock due to tube vibration. The pen-

[7] "Measurement of Air Velocity and Pressure," by A. F. Zahm, *Physical Review*, vol. 17, 1903, p. 410.

[8] Dean Emeritus, College of Engineering, The Ohio State University, Columbus, Ohio. Mem. A.S.M.E.

stock had a straight run of approximately 330 ft to the point where the pitot instrument was attached.

Six trials were made with generator carrying loads from 2000 kw to 7500 kw. During each test the turbine was under hand control. The total net head was 255 ft.

From the traverses it was noticed that velocities were practically constant for a 2-ft radius. Therefore, since the penstock internal diameter was found to be 7.896 ft, its area was divided into two zones, an outer one between the radii 3.948 ft and 1.948 ft, over which the velocity was variable, and the inner one, from the center to a radius of 1.948 ft, over which the velocity was considered constant.

From the runs made, velocity curves were plotted and these gave the velocity values for the ten equal areas of the outer zone. Table 5 was therefore established and the velocity factors computed.

TABLE 5 VELOCITY VALUES FOR 8-FT PENSTOCK, CONNECTED TO A 10,000-HP WATER TURBOGENERATOR

Load on generator, kw	2000	4000	4000	6000	6000	7500
Station	Velocities, fps					
1	2.12	3.85	3.90	5.23	5.22	6.56
2	2.37	4.13	4.11	5.64	5.64	7.04
3	2.63	4.43	4.35	6.05	6.08	7.50
4	2.83	4.70	4.58	6.44	6.48	7.97
5	2.96	4.91	4.78	6.80	6.83	8.35
6	3.08	5.12	4.97	7.10	7.13	8.66
7	3.16	5.27	5.15	7.35	7.33	8.93
8	3.22	5.37	5.30	7.55	7.50	9.14
9	3.25	5.44	5.43	7.70	7.63	9.31
10	3.27	5.47	5.52	7.77	7.70	9.44
Mean	2.889	4.869	4.808	6.763	6.754	8.288
Constant velocities, center zone	3.27	5.47	5.53	7.77	7.73	9.45
Volume per sec, center zone	38.982	65.208	65.923	92.626	92.149	112.653
Volume per sec, outer zone	107.035	180.392	178.132	250.562	250.229	307.062
Total volume of flow per sec	146.017	245.600	244.055	343.188	342.378	419.715
Mean velocity, fps	2.98	5.02	4.98	7.01	6.99	8.57
Ratio of velocities or V_m/V_c (pipe factor)	0.911	0.917	0.900	0.902	0.904	0.907

It is most interesting to note how closely the values given in Table 5, computed now for the first time, agree with those obtained on a 7.5-ft penstock at Walchensee, Germany, in 1935, twenty-one years later.

W. M. Lansford.[9] The problem of measuring large volumes of moving water with satisfactory accuracy is difficult. The authors have done much to develop one method of measurement, describing the use of pitot tubes in large pipes. A problem confronting every user of a pitot tube is that of calibrating the instrument to insure reliable results under the given service conditions. The effect of calibrating a pitometer, in a condition of flow unlike that in which the flow is to be measured, was brought out in a careful study of the velocity distribution at four locations in a 24-in. spiral riveted pipe line in the hydraulics laboratory of the University of Illinois, a portion of which is pertinent to this discussion.

The water was supplied to the pipe line by a 20 × 20-in. centrifugal pump, driven by a 200-hp synchronous motor. From the pump the water was conducted through a valve, an elbow, about 6 ft of pipe, and a second elbow to a straight reach of pipe 62.83 ft long in which was located pitot-tube traverse station No. 1; 60.46 ft downstream from the nearest upstream elbow and 2.37 ft upstream from a third 90-deg bend or elbow. Pitot-tube traverse stations Nos. 2, 3, and 4 were located in another straight run of pipe 111.10 ft long, extending from this third elbow. The location of station Nos. 2, 3, and 4 were at distances 20.12 ft, 56.13 ft, and 101.88 ft, respectively, downstream from the third elbow. The rate of flow was regulated by means of a gate valve near the discharge end of the pipe line approximately 40 ft downstream from station No. 4. The range of velocities was approximately from 2 fps to more than 11 fps. The discharge was measured volumetrically in a calibrated pit which had a capacity of 2500 cu ft. The time was taken by means of a stop watch. Horizontal and vertical traverses were taken at each of the four stations by a reversible pitometer. The coefficient of the instrument (ratio of actual average velocity determined by volumetric measurements to the theoretical velocity $\sqrt{2gh}$ indicated by the pitometer) was computed from the data taken at each station by the equal-area method, described by most textbooks on hydraulics, in which twenty equal areas were used. No correction was made for the projected area of the rod. The results of these tests are summarized in Table 6.

It will be noted that the highest coefficient was obtained at the station located farthest from any upstream disturbance. By using the deviation from smoothness of the velocity-distribution curves plotted from the data as a criterion of disturbance, it was found that the stations arranged in the order of decreasing degree of disturbed flow were Nos. 2, 1, 3, and 4. At the higher velocities there seemed to be as much disturbance, indicated by the irregularity of the velocity distribution curves, at station No. 1 as at station No. 2. All curves obtained from the pitometer reading at stations Nos. 3 and 4 were smooth and regular. The data herein reported were taken very carefully, with many check readings being made.

The results of these tests indicate that the accuracy of the pitometer measurements depends on the use of a calibration factor (coefficient), found by calibrating the instrument at a location in which the velocity distribution or disturbance is similar to that at the location in which the velocity distribution is to be determined.

TABLE 6 SUMMARY OF RESULTS OF PITOMETER TRAVERSES IN A 24-IN. SPIRAL RIVETED PIPE

Station no.	1	2	3	4
Distance from upstream elbow, ft	60.46	20.21	56.13	101.88
Distance from downstream elbow, ft	2.37	90.89	54.97	9.22
No. of horizontal and vertical traverses used in calibration	16	16	12	36
Coefficient $\frac{V_{avg}}{\sqrt{2gh}}$	0.845	0.835	0.859	0.864

Authors' Closure

H. S. Bean raises a question as to the effect of rod vibration which received little attention in the paper. This seeming omission was not due so much to oversight as to the belief that vibration was practically eliminated by the selection of a rod of proper stiffness to meet the field conditions of pipe diameter and velocity.

The authors have used rods of four different strengths, one of which is shown in Fig. 1; another is of 1-in. circular cross section; a third type is of steel with a 2 × 1-in. elliptical section; and the fourth is the 3 × 1¼-in. elliptical heavy-duty rod shown in Fig. 2. Experience usually indicated the type of rod required for a given set of conditions.

Where opposite taps were available in a penstock it was of course easy to make a simple check of vibration effect by inserting two rods so that their tips stood close to each other. One rod projected more, and the other less than the pipe radius. For example in the 12-ft Millinocket pipe there was little difference in the readings of the two rods even when one projected 11 ft and the other but 1 ft, thus indicating that the vibration effect was negligible at that gaging point.

As Mr. Bean points out no precise studies of vibration effects have been published but the authors much prefer to avoid excessive vibration by the use of a suitable rod than to attempt

[9] Assistant Professor of Theoretical and Applied Mechanics, University of Illinois, Urbana, Ill.

to correct for it. The use within a pipe of a support for the rod has not proved necessary or satisfactory in field practice.

W. M. Lansford's contribution of calibrations made at the University of Illinois is greatly appreciated. This investigation of the effect of upstream elbows, a subject on which there are few data, involved a large amount of work as evidenced by the many traverses which were made, but it is believed that the test data are too limited to warrant the conclusion that the pitometer coefficient changes with the type of velocity distribution.

The authors wish that Mr. Lansford had submitted traverse curves of the velocity distribution at his various gaging points, and also pipe-factor curves, for it is their belief that these definitely indicate the suitability of a gaging point. The coefficients obtained at stations Nos. 3 and 4, which were located 34 and 61 diameters, respectively, below one upstream 90-deg bend, agree within less than 0.6 per cent. This reasonably good agreement is to be expected at a normal gaging point where the velocity distribution is fairly symmetrical. Stations Nos. 1 and 2, however, which were located 38 diameters below two 90-deg bends and 12 diameters below one 90-deg bend, respectively, would be considered poor gaging points where a high degree of accuracy would hardly be expected with the usual test methods. No mention was made of straightening vanes and without them the velocity distribution must have been greatly distorted.

In view of other tests which have been made, it is doubtful if traverses on only two diameters were sufficient to give an accurate integration of the distorted flow at stations Nos. 1 and 2. In a previous article[3] calibrations made in a 12-in. pipe at the Alden Hydraulic Laboratory were described. The gaging point was located 38 diameters below two 90-deg bends but the velocity distribution was partially controlled by means of straightening vanes. In spite of these, some distortion of the traverse curves persisted. For this case traverses on two diameters were sufficient as was proved by check calibrations at the University of Pennsylvania.

Calibrations made recently at the Alden Hydraulic Laboratory in another 12-in. pipe and presented in the discussion of C. W. Hubbard's paper "Investigation of Errors of Pitot Tubes," showed that traverses on two diameters did not give an accurate determination of the mean velocity when the velocity distribution was distorted by 12 ft of wire screen which terminated 2 ft 3 in. upstream from the gaging point. When four diameters were traversed the mean velocity was determined with sufficient accuracy to show that the pitometer coefficient had the same value for the distorted flow as for the smooth flow which occurred when the screen was not present. In view of this it would seem that Mr. Lansford's calibrations at stations Nos. 1 and 2 were in need of further confirmation.

The authors are convinced from the results of many calibrations that the type of velocity distribution usually encountered in pipe lines does not materially change the pitometer coefficient. The accuracy of flow measurements made with a pitometer, however, depends on a number of factors other than the coefficient of the instrument. These factors which have previously been discussed in detail include the accurate determination of the mean velocity from traverses. At a good gaging point traverses on two diameters will give the mean velocity with sufficient accuracy for practical purposes, while at a poor gaging point more extensive traversing is required to obtain the same accuracy. The proof of this statement is clearly indicated by the results of Mr. Lansford's calibrations.

The work of E. A. Hitchcock is most interesting and this addition to flow measurements in large pipes was gladly received.

The authors gratefully acknowledge the assistance of Professors C. M. Allen, C. W. Hubbard, and L. J. Hooper of the Alden Hydraulic Laboratory, in the preparation of this paper. They are also deeply indebted to S. Logan Kerr for the time he so generously gave in editing material, and to the Central Vermont Public Service Company, and the Rockland Light and Power Company for permission to use their data.

Investigation of Errors of Pitot Tubes

By C. W. HUBBARD,[1] WORCESTER, MASS.

In late years some hydraulic engineers have come to distrust the pitot tube as an accurate instrument for measuring the flow of water in pipes. It has been found that some pitot tubes do not have the same coefficient for all conditions of flow. There is, therefore, general disagreement as to the coefficient which must be applied for a particular set of conditions to obtain the velocity within 1 per cent or less of the true value. The following experiments were made in an effort to find the reasons for this variation of the coefficient.

Tests made on an 84-ft rotating boom in still water showed the coefficient of the impact tip to be unity. The pitot deflection was balanced against a centrifugal head.

A method of measuring the angularity of turbulent flow in pipes by use of the law of probability was devised. The measurements made in pipes 12, 40, and 78 in. in diameter check the Prandtl and von Kármán theories of turbulent flow closely and show the mean angularity to be about 2 deg. The effect of the pitot support rod on the reading of the pressure-head piezometers and impact tip was determined. Tests were also made to determine the effect of pulsations.

Tests made to determine the effect of the pitot support rod showed that the support rod did not cause an error in the impact-tip reading, but did affect the pressure measured by the wall piezometer. The rod was also found to affect the reading of the pressure orifice of the pitot-static tube. It follows that any form of combined type of pitot tube in which the error could be determined by calibration of the instrument alone would give accurate results.

The average magnitude of pulsations was found to be too small to cause an error in reading of the pitot tube.

The conclusions drawn from these tests were that the impact tip correctly registered the true dynamic head whether the flow was smooth or turbulent, but that the pressure piezometers were likely to be in error. Since the pressure piezometer of the pitot-static tube was a part of the instrument and calibrated with it, this instrument was considered more reliable than the simple pitot tube using wall piezometers.

THE EXPERIMENTS of W. M. White, Lewis F. Moody (1),[2] and others have demonstrated that the coefficient of the impact orifice of a pitot tube is unity in nonturbulent water. However, examination of past test results shows that the individual values of the coefficient varied from 1 to 3 or 4 per cent from the mean of all the tests.

The rotating current-meter rating station at the Alden Hydraulic Laboratory of the Worcester Polytechnic Institute is particularly fitted for tests to obtain the impact coefficient to a greater degree of refinement than by ordinary methods. This apparatus consists of an 84-ft rotating boom constructed of structural steel. It is set on a foundation consisting of a large rock located in a 2-acre pond.

Calibration of Impact Orifices on Rotating Boom

When these tests were made the boom was driven through a rope drive and gearing by a cylinder-gate water wheel operating under a head of about 5 ft. The maximum speed of the boom was about 5 rpm, which gave a maximum peripheral speed at the outer end of about 20 fps. The speed of the boom was varied by changing the gate opening of the water wheel and by two shifts of gears. The velocity of the instrument under calibration could also be varied by the selection of the radius at which it was mounted on the boom.

This boom was constructed under the direction of Prof. C. M. Allen in 1910 for the purpose of calibrating current meters and has been described by him (2). The station has been in use since then for both student and commercial testing. A sketch of the apparatus is shown in Fig. 1.

The usual method of calibration of an instrument has been to measure the time during which the boom rotates through an exact number of revolutions and compute the actual average velocity of the instrument from this time and the distance traveled. The actual average velocity was then compared with the indicated velocity. Obviously, the accuracy of the tests was directly affected by the precision of the time measurement and the steadiness of the velocity of movement. Linear measurements such as that of the radius of rotation of the instrument were easily made to a much greater precision than the time measurements.

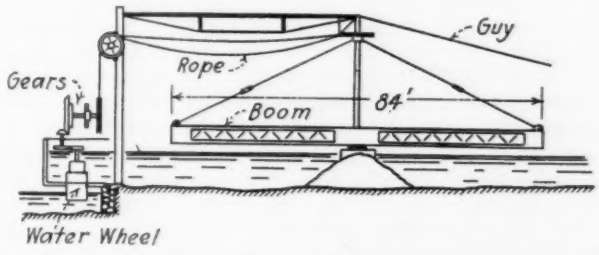


Fig. 1 Arrangement of Boom for Calibrating Flowmeters

However, if the velocity of rotation of the boom could be measured by some application of centrifugal force, an instantaneous indication of velocity would be obtained without any time measurement. Such a method would be particularly applicable to the calibration of pitot tubes since both the instrument deflection and centrifugal force vary as the square of the velocity.

The first method which was attempted was simple in theory but had some practical disadvantages. This method was based on the principle that the centrifugal acceleration acting on the leveling bubble of an engineer's level located near the end of the boom, would require the line of sight to be brought normal to the resultant of the accelerations of gravity and centrifugal force to bring the bubble to the center of the vial. By using a hand level with a prism, it was possible for the operator to observe both the cross wires and the bubble simultaneously. The level was mounted on a frame near the end of the boom at a measured

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[2] Numbers in parentheses refer to the Bibliography.

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Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

radius. An initial reading was taken on the leveling rod fastened to the mast at the center of the boom, with the boom stationary. When the boom was revolving at a constant speed the observer depressed the level until the bubble was central and observed the rod reading. Using the law of similar triangles, the change in rod reading was to the radius of the leveling instrument as the centrifugal acceleration (V^2/R) was to the acceleration of gravity. The principle of this method is shown in Fig. 2.

The method gave good results on one test of a Cole pitometer, but required three or four men to observe and record the data. It was also necessary that the boom rotate in an exactly level plane, a difficult condition to maintain with long guys.

A more effective method was to balance the pitot deflection directly against the centrifugal pressure generated in a horizontal radial pipe and measure the error in the pitot deflection. This was accomplished as indicated in Fig. 3. A pitot tube was clamped to the boom at a distance R out from the axis of rotation, about 40 ft. The pitot tip was carefully set so that it was tangent to its path at the forward end. A second pitot tube was mounted as close to the center of the boom as possible which was at a radius of about 8 ft. A line consisting of two lengths of $^1/_2$-in. brass pipe was then run radially outward from the inside pitot tube to the outer one which was to be calibrated. A differential U-tube containing a liquid having a specific gravity of 1.1 was

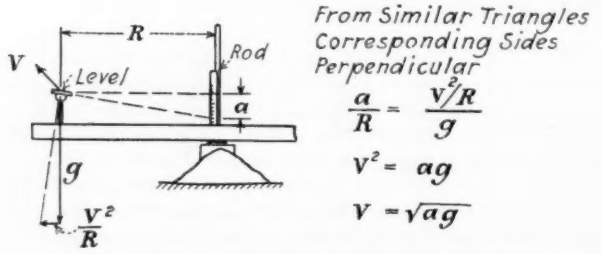


FIG. 2 LEVEL METHOD OF CALIBRATION WITH BOOM

connected between the outer end of the radial pipe and the impact tip of the outside pitot tube. When the system was filled with water the combination of the inside pitot tube and the radial pipe generated a head just equal to the true velocity head at the outside pitot station. In this way the pitot deflection was compared directly with a pressure equivalent to the true velocity head.

It can be shown that the difference in pressure caused by centrifugal force between the two ends of the radial pipe is equal to the difference in the velocity heads. This difference in pressure is not only theoretically correct but changes instantly with any momentary change in speed of rotation. Since there is no continuous motion of the water in the radial pipe, there is no friction to modify the calculated centrifugal pressure.

Since the velocity at any point along the boom is proportional to the radius, the centrifugal-pressure difference is then equal to

$$\frac{V^2 - v_1^2}{2g} = \frac{V^2}{2g}\left(1 - \frac{r_1^2}{R^2}\right) \quad \ldots\ldots [1]$$

With an inner radius of 8 ft and an outer radius of 40 ft, the pressure difference becomes

$$\frac{V^2}{2g}\left(1 - \frac{8^2}{40^2}\right) = 0.96\,\frac{V^2}{2g} \quad \ldots\ldots [2]$$

The total pressure existing at the outer end of the radial pipe equals the centrifugal head plus the velocity head of the inner pitot tube minus the elevation of the pipe. Since the inner velocity head represents only 4 per cent of the total reference pressure, the effect of an error in registration of the inner pitot tube is multiplied by 0.04, producing a negligible error in the reference pressure.

The Moody type-A pitot tip was used at the inside station as Professor Moody's tests, as shown in Fig. 5A of his article (1), showed a coefficient well within one per cent of unity in still water.

A sensitive differential manometer was connected between the outer end of the radial pipe and the impact tip of the pitot tube under test and the error in deflection of the outer pitot tube was read directly by the manometer, no accurate measurements of the actual velocity then being required. It can be shown mathematically that the location of the manometer along the radius makes no difference in the reading. In the early tests the position of the manometer was shifted over a radial distance of 3 ft while the boom was turning and no change in reading was noted. The manometer consisted of a standard Cole U-tube of $^5/_{16}$-in. glass containing monochlorbenzine having a specific gravity of 1.1, magnifying the pressure difference by about ten times. It was necessary to set the U-tube in a plane normal to the radius so that the centrifugal force on each leg was the same.

If the coefficient of the pitot tube under test were unity, the pitot-tube pressure would just balance the reference pressure and the deflection of the U-tube liquid would be zero. If the coefficient were not unity, then the U-tube would not balance by a small amount equivalent to e feet of water.

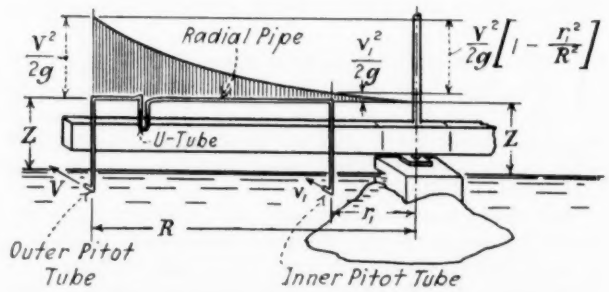


FIG. 3 METHOD OF CALIBRATION USED IN PRESENT TESTS

Expressing the error in reading in the more familiar form of a coefficient

$$V = C\sqrt{(2gH)}, \text{ or } V^2/2g = C^2H \quad \ldots\ldots [3]$$

where

V = true velocity, fps; C = coefficient; and H = velocity head indicated by pitot tube. If the error in dynamic head is e ft of water at a true velocity of V fps, then

$$H = (V^2/2g) + e \quad \ldots\ldots [4]$$

Substituting this value of H in Equation [3] for true velocity head

$$\frac{V^2}{2g} = C^2\left(\frac{V^2}{2g} + e\right), \qquad 1 = C^2\left(1 + \frac{e}{V^2/2g}\right)$$

or

$$C = \frac{1}{\sqrt{\left(1 + \dfrac{e}{V^2/2g}\right)}} \quad \ldots\ldots [5]$$

Method of Test.

Two types of pitot tubes were tested on the rotating boom. These were the round-nose tube and the impact tip of a Cole pitometer. The details of the form of both of these tubes are shown in Figs. 4 and 5 and a photograph in Fig. 5A. The pitot tips were submerged 6 in. or more below the surface of the pond during the tests.

Before starting a series of tests the specific gravity of the U-tube liquid was determined. The U-tube was then connected between the pitot tube and radial pipe and all air pumped out of the piping system. If the U-tube balanced when the boom was stationary, the boom was started rotating at some constant outer pitot velocity between 5 and 20 fps. One observer on the boom read the U-tube deflection at approximately every eighth of a revolution, the other observer on shore timed the run with a stop watch and recorded the U-tube readings on an adding machine. A single run consisted of from 2 to 3 revolutions of the boom. During a test the U-tube reading was constantly changing from small plus to small negative readings (about 0.01 ft of water).

The average of the U-tube readings during the run was taken as the mean error in pitot deflection. The average velocity of the pitot tube was obtained within 1 or 2 per cent from the stop-watch readings. Since the term $V^2/2g$ has little weight in the formula for coefficient, an error of 2 per cent in velocity measurement did not appreciably affect the results.

Experiments showed that the pond was of sufficient size so that there was no stirring action of the pitot tube which would cause the whole body of water to rotate. The effect of unequal temperature of the water in the pond and in the radial pipe was

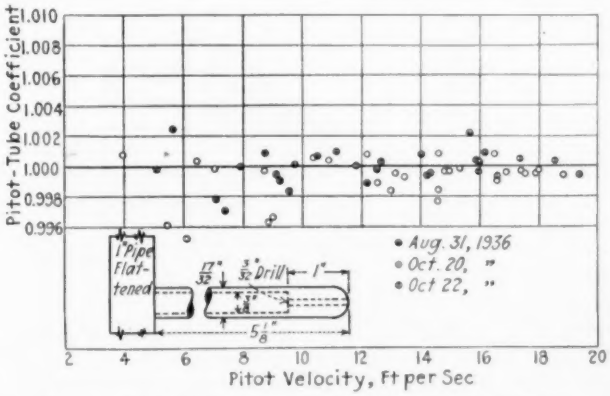


FIG. 4 TEST OF PITOT IMPACT ORIFICE ON ROTATING BOOM
(Average coefficient = 0.9995.)

negligible. The difference required to cause an error of 0.1 per cent was computed to be 16 F.

Test Results.

The test results in Figs. 4 and 5 show that the coefficient of the impact tip of a pitot tube is unity in quiet water.

The coefficient obtained by averaging all the tests for the round-nosed tube was 0.9995.

ANGULARITY OF FLOW IN PIPES

There has been much discussion among hydraulic engineers about the effect of angularity of flow in turbulent water upon the accuracy of tests made with current meters or pitot tubes. By angularity of flow is meant the constant wandering of particles of water back and forth across the average path of the stream in a straight conduit. Estimates of average angles of flow vary greatly and few reliable data are available.

Various experiments have shown that the impact orifice of a pitot tube correctly registers the cosine component of velocity at angles as large as 5 deg, but if the water approaches the pitot tip at angles of from 5 to 10 deg or more, an appreciable error will be obtained.

When water or any other fluid is flowing through a long straight pipe the velocity is known to be greater at the center than at the walls. At low velocities of flow the layers of fluid telescope or shear smoothly by each other with apparently no disturbance of the various particles. If the rate of flow is increased to normal values above the critical velocity, the rate of shearing apparently becomes so rapid and the shearing force so great that the water in one layer "seizes" on some of the water in the adjacent layer and tears it out of position, causing a whirl or eddy to start.

These eddies, as they move along with the bulk of the water around them would cause a sudden "side slap" against a small stationary vane. The front of the eddy would move the vane in one direction and the rear of the eddy would move it in the opposite direction. The combined rotation and forward motion of the particles within the eddy would make them appear to approach a vane at a constantly changing angle. When the

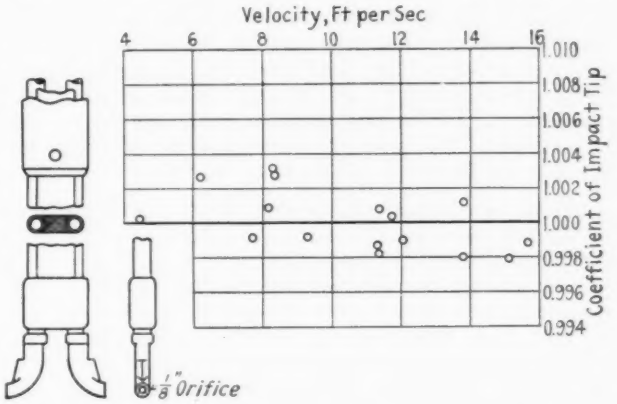


FIG. 5 TEST OF IMPACT TIP OF COLE PITOMETER ON ROTATING BOOM
(Average coefficient = 1.0000.)

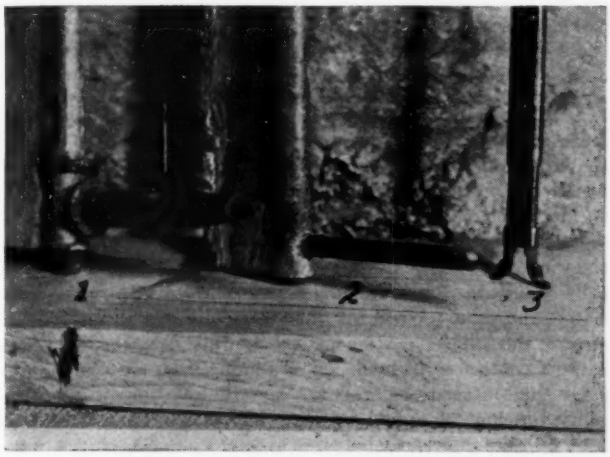


FIG. 5A PITOT TUBES USED ON BOOM
(1, Inner pitot tube, Moody type-A; 2, outer pitot tube; 3, Cole pitometer.)

outer or slower whirling particles of water struck the vane, they would appear to approach from a rather small angle because the velocity of rotation within an eddy or vortex varies approximately inversely as the distance from the center. If the more rapidly whirling core of the eddy should happen to be split by the vane, there would be a short violent side slap which would appear to be a flow from some greater-than-average angle.

Such eddies as they move along the pipe would collide with other eddies and the path and velocity of whirl of the various particles of water would be affected. The causes of such events are so complicated that it can safely be assumed that they are a

matter of accident, and that at no two instants are conditions exactly the same, but over a period of time a certain average effect is produced. This would suggest the application of the law of probability which can be used to predict the frequency of accidental events if sufficient observations of past similar events are made. If some mathematical law such as the law of probability can be found which seems to fit actual test data, conclusions can be drawn from comparatively few tests and average effects computed.

Application of Probability to Angles of Flow.

Probability is defined as the ratio of the number of times which an event occurs to the number of chances of occurrence. If an event occurs in the same way at every trial, the probability is certainty or one. If the chances of occurrence and failure are even, the ratio of probability is one half. When the law is applied to events which may occur in an infinite number of ways, actual observations of a large number of events must be made; the greater the number of observations, the greater the accuracy of the results. The following statements are the basis of the application of the law of probability to angles of flow:

1 The angle of flow will lie between certain wide limits found by test, and the probability that the angle is within these limits is practically unity.

2 In a straight unobstructed pipe the chances that water will approach a vane from the left are equal to the chances of approach from the right, so that the probability of some left or right angle is one half, since the average angle in a straight pipe must be zero.

3 A study of other accidental events indicates that small angles of flow should occur more often than large angles. If angularity is caused by passing eddies, most of the water in the eddy is in the outer slower-moving part, and most angles of approach would be small. Only rarely would the rapidly whirling core hit the vane.

4 In a large number of tests, a particular angle of flow should occur about as frequently from the left as from the right which means the probability curve should be symmetrical.

If a small weightless vane were supported near the front edge so that it would be free to flutter in a turbulent stream, the vane would be constantly moving through varying small angles. If the fluttering motion were restrained on one side by the tail of the vane striking against an electrical contact, the number of times the contact was made in a given interval of time could be graphically recorded. Assume that the contact was placed on the right side of the vane. When the vane was pointed directly into the stream, the eddies pulsating against the vane from the left would cause a contact to be made, those coming from the right would cause the contact to be broken. According to the law of probability the contact should be made half of the time if the test interval were sufficiently long.

If the support carrying the vane and its contact were turned a small angle to the right, the majority of the impulses would come from the left and the vane would rest against the contact more than half of the time. Conversely, if the support were turned to the left, contact would be made less than half of the time. Thus, such an instrument would measure the probability of occurrence of all angles to the left of the support angle.

If the support be turned further and further to the right, the contact would be made over a greater percentage of the time until some angle would be reached where the vane would rest against the contact practically all of the time, indicating that the probability of an angle of flow being to the right of the vane angle was slight.

The mathematical probability curve is represented in Fig. 6 by curve A. The abscissa scale x is general for all probability curves. To find the particular curve which fits a given set of data, the abscissa x is replaced by a parameter $h\theta$, where the value of h is a constant multiplier determined by experiment for the particular set of test data.

The probability of occurrence of an angle to the left of an arbitrary angle θ, i.e., between $-\infty$ and θ, is represented by the shaded area under curve A, or by the probability integral. Curve B of Fig. 6 shows the integral of the probability curve, and the ordinate scale represents the probability of occurrence of the event x. The integral curve transformed into a straight line by a nonuniform probability-ordinate scale is shown by curve C.

The values of the probability integral at various positive values of x may be found in mathematical tables. When it is desired to use both positive and negative values of x, the desired probability P may be converted from the tabular value p, by the following simple formula

$$P = (p + 1)/2 \quad \ldots\ldots\ldots\ldots\ldots\ldots [6]$$

The mean abscissa of curve A irrespective of sign, representing the desired mean angularity, has a definite mathematical value and always occurs at a definite value of probability. The mean value of x equals $1/\sqrt{\pi}$ and occurs for positive values at $P = 0.7875$ and for negative values at $P = 0.2126$. To determine this mean value from a given set of data, it is only necessary to plot the test results on a straight-line probability chart and draw a straight sloping line best representing the average of the test values as curve C of Fig. 6. This straight line is taken as the probability - integral curve best fitting the test points. The mean value of the abscissa θ is read from the chart where curve C crosses the horizontal line at $P = 0.7875$.

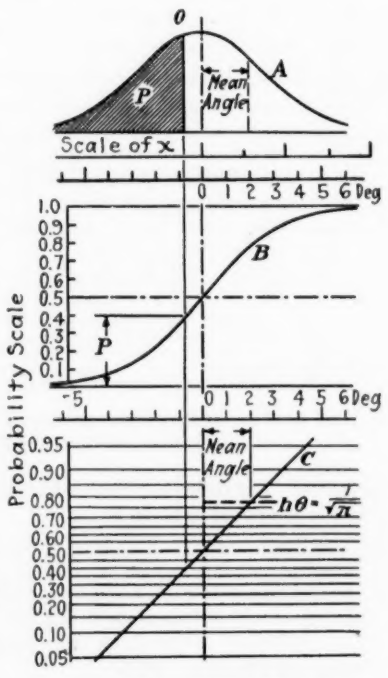


FIG. 6 PROBABILITY CURVES

If the zero of the instrument is not set correctly, curve C will be displaced from the origin. This does not affect the results, since the mean value of θ is represented by the abscissa distance between the curve intercepts of the chart lines at $P = 0.50$ and $P = 0.7875$. The slope of the line representing the average of the test points, determines the mean angle.

Description of Apparatus and Method of Test.

The first angularity indicator which was used for these tests was a plow-shaped vane made of brass shim stock 0.005 in. thick mounted on a shaft made of a common pin. The plow shape was preferred to a flat vane because unstable eddies were found to be set up on the low-pressure side of the flat vane. The indicator

was inserted through a hole in the top of the pipe so that there was no weight component influencing the results.

This angularity indicator was attached to the end of a Cole pitometer sheath which was equipped with a sliding stuffing box which could be screwed onto a $1^1/_4$-in. pipe nipple. The standard tips were removed; one piezometer tube of the sheath was used to bring out wires from the vane contacts and the other was taken up with a brass rod which turned the indicator through vertical angles. The details of construction are shown in Fig. 7.

The vane contacts were connected through a chronograph to a battery as in Fig. 8. A chart was moved at constant speed under two magnetically operated pens. When there was no flow of current in the vane contacts, the pens drew a continuous line on the chart; when one of the pens was actuated by a flow of current, a jog was made in its line. Each pen was connected to one of the indicator contacts. An interrupter was put into the circuit to give fairly rapid breaks in the current from the battery. This made the interpretation of data easier and reduced the troubles caused by polarization occurring at the contacts in the water.

The interrupter was driven by a synchronous motor to make and break the current supply five times a second so that the contact was closed 0.1 sec and open 0.1 sec.

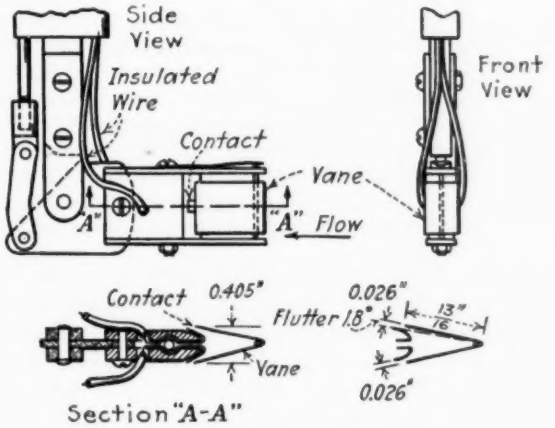


Fig. 7 Two-Contact Angularity Indicator

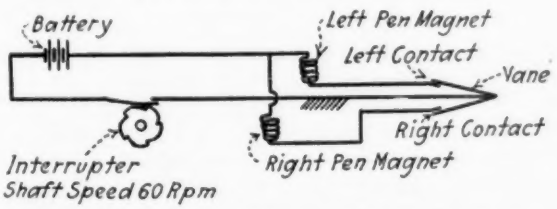


Fig. 8 Electrical Diagram of Angularity Indicator

It was necessary in constructing a practical instrument to have some movement of the vane between the left and right contacts, since the vane should not touch both contacts simultaneously. It was also necessary to space the contact from the vane sufficiently so that in the "off" position there would not be enough current leakage through the water between the contact and vane to operate the pen.

Micrometer measurements showed that the vane used could flutter through a total angle of 3.6 deg or 1.8 deg either side of center between the contacts. This effectively made the apparatus into two separate instruments the zero positions of which were 3.6 deg apart. In the early tests correction to the angles measured by the protractor for the flutter angle was made so that the data from both the left and right contacts would plot on the same curve. This was abandoned when it was found that

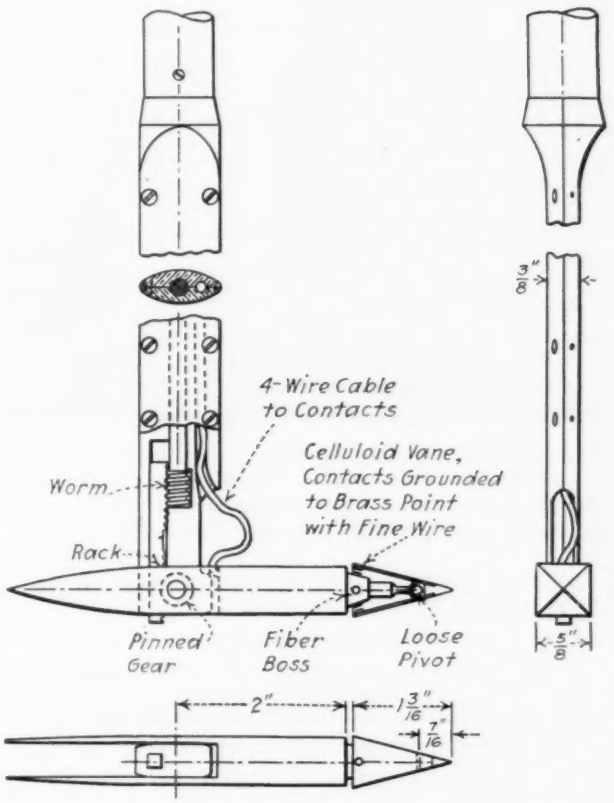


Fig. 9 Four-Contact Angularity Indicator

with the larger contacts required in the four-contact instrument, the measured flutter angle was modified by electrical leakage through the water.

In order that the angularity might be measured in a radial direction as well as tangentially, a four-contact instrument was constructed by the Pitometer Log Co., as shown in Fig. 9. This indicator was provided with a pyramid-shaped vane supported by a swivel near the point.

Since it was necessary that the vane should balance when in water, it was constructed of celluloid which is only slightly heavier than water and would require little correction. The point was of brass and so designed that the unbalanced weight of the celluloid and four contacts was neutralized. The contacts were made of platinum to avoid trouble with electrical corrosion encountered with brass or silver contacts.

A worm drive was provided so that the vane was adjustable through vertical angles. A gear connected to the top of the worm shaft was calibrated as a protractor to indicate the radial angular position of the vane. The tangential angular position with respect to the pipe axis was measured by a protractor attached directly to the shaft of the instrument.

The data and vane angle for a typical test run are shown in Fig. 10. Only a portion of a full run is shown; the usual number of possible contacts in a run was between 500 and 600. The number of possible contacts was determined by counting the number of jogs per inch of chart when one of the contact pens was actuated continuously. The chart speed was constant within 1 per cent so that it was only necessary to measure the length of chart for a particular test and multiply by the number of possible contacts per inch to calculate the total possible contacts. Although the support rod was pointed one degree to the right of the pipe axis, because of the flutter angle of 1.8 deg, the actual angle

of the vane with the pipe axis was 2.8 deg, when the right-hand contact was on, and —0.8 deg when the left contact was made.

In Fig. 10, the probability of an angle of flow to the left of —0.8 deg is 40/100 = 0.40. The probability of an angle of flow to the left of 2.8 deg is 1 — 15/100 = 0.85. These points may be plotted on the straight-line probability chart. The plot of a complete test of tangential angularity is shown in Fig. 11, and one of radial angularity in Fig. 12.

Tests were made with both the two-contact and four-contact instrument in pipes varying in diameter from 12 to 78 in. With the four-contact instrument, both the radial and tangential angularity were determined. The results of the tests are shown in Table 1. The conclusions drawn from them are the following:

1 No regular cycle of flow occurred.

2 The tangential and radial angularity were about equal.

3 The mean angularity was practically constant across the pipe, increasing only slightly toward the walls.

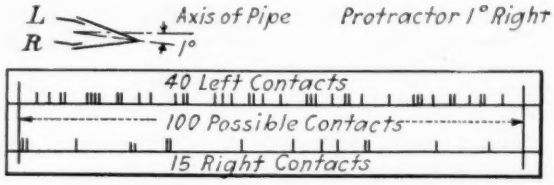


FIG. 10 SAMPLE CHART MADE BY ANGULARITY INDICATOR

TABLE 1 SUMMARY OF ANGULARITY TESTS

Pipe diam, in.	Mean velocity, fps	Pipe factor, mean velocity / center velocity	Wall surface	Mean angle at center line, deg	Mean angle off center line: Radius, in.	Mean angle off center line: Mean angle, deg	Mean angle off center line: Direction
12	5.9	0.78	Rough upstream	1.2	3	1.7	Radial
12	3.5 4.9 7.4	0.95	Smooth	1.6			
12	4.5	0.90	Smooth	1.3, 1.4			
12	6.2	0.87	Medium	0.65			
16-in. venturi throat	9.2	0.95	Smooth	0.50			
36-in. expansion below 28-in. butterfly valve	1.2			4.5, 5.5			
40-in. laboratory penstock	2.05	0.87	Medium	1.5	16.5	2.0	Radial
78-in. penstock at Rutland, Vt.	2.78	0.84	Smooth	1.3	31	1.5	Radial

4 The mean angularity was less than 3 deg and too small to cause an error in reading of the impact orifice of a pitot tube.

ANGULARITY AND THEORY OF TURBULENT FLOW

It was desirable to verify the results of the angularity tests by theory, if possible. Prandtl's theory of turbulent flow, as described by Professor Bakhmeteff (4), was found to have a direct application to angularity.

This theory was compared with test results obtained in the 40-in. diam riveted steel penstock at the Alden Laboratory. The test section was approximately 320 ft downstream from the head gates; the pipe walls were tuberculated and very rough for about 270 ft of this length. The remaining 50 ft just ahead of the test section had been scraped and coated with a waterproof paint which smoothed its surface somewhat, but there were still irregularities approximately $^1/_{16}$ to $^1/_8$ in. deep in the walls where the rust had been removed. The walls in the test section had a de-

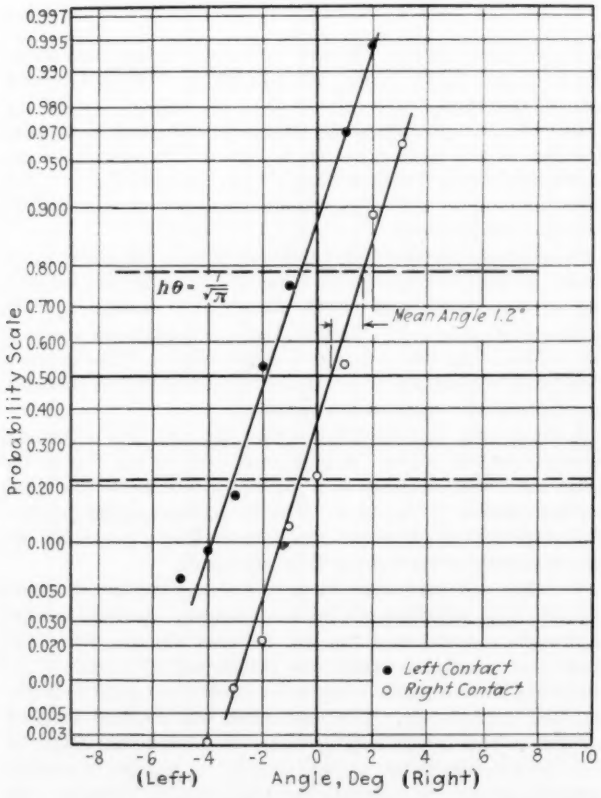


FIG. 11 RESULTS OF TANGENTIAL-ANGULARITY TEST AT CENTER OF 12-IN. ROUGH PIPE

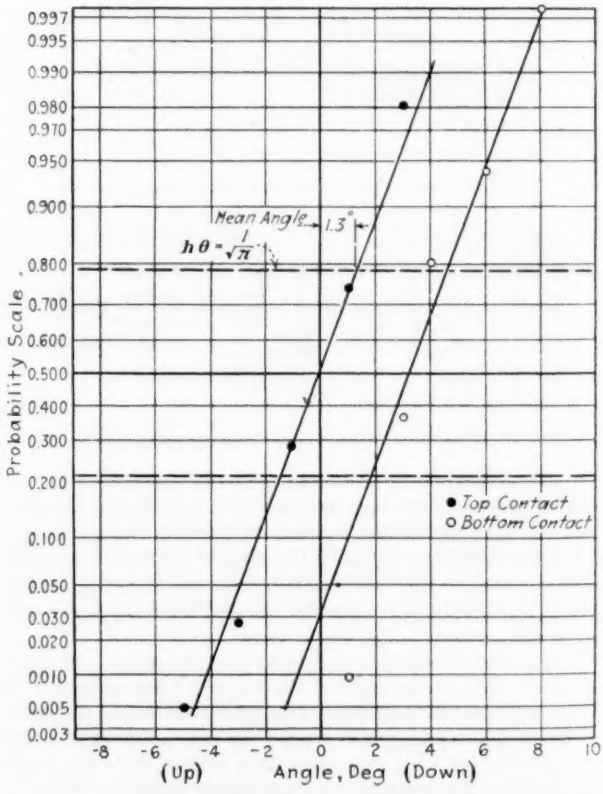


FIG. 12 RESULTS OF RADIAL-ANGULARITY TEST AT RUTLAND, VT. (Indicator 26 in. in from wall of 78-in. pipe.)

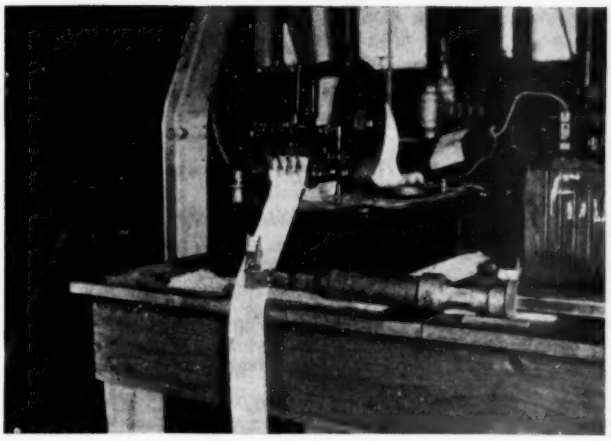


FIG. 13 TWO-CONTACT ANGULARITY INDICATOR AND CHRONOGRAPH

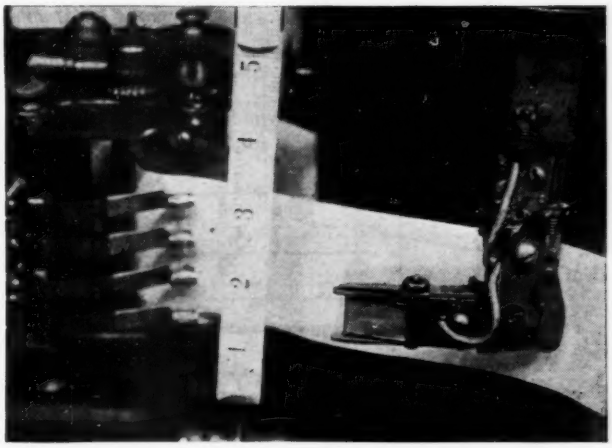


FIG. 14 TWO-CONTACT ANGULARITY INDICATOR WITH CHART

cided pock-marked appearance. The tubercles in the 270 ft further upstream gave the appearance of rivet heads, from $^1/_4$ to $^3/_4$ in. high and spaced about as closely together as the tubercles can grow.

A velocity traverse was made across the test section with a Cole pitometer. The tests are shown in Fig. 15. The curve plotted through the test points was based on the general von Kármán equation

$$u = u_m - 5.75\ U \sqrt{(f/8)}\ \log_{10}(R/y) \quad \ldots\ldots\ldots [7]$$

The values of f necessary to make this equation fit the test points was 0.0184. For some reason, probably because of the change in roughness upstream from the test section, the portion of the velocity curve near the center was unstable and the center velocity appeared to be too low. The center velocity computed from the von Kármán equation which fitted the outer test points was 1.5 per cent greater than the measured center velocity. The velocities used in the following theoretical computations were taken from the smooth curve representing the von Kármán equation.

Prandtl's Theory Applied to Angularity.

The following notation is used:

- f = friction factor in Darcy pipe-friction formula
- l = Prandtl mixing length, see text for definition
- r = radius of inner cylinder
- R = radius of pipe
- y = radial distance of point in pipe from wall
- v = mean radial or cross velocity normal to axis of pipe over a long period of time
- u = velocity at any point parallel to axis of pipe
- u_1 = velocity parallel to axis of pipe, at mixing length from boundary of inner cylinder, radius $= r + l$
- Δu = acceleration from mixing length to boundary of inner cylinder $= u - u_1$
- u_m = maximum or center-line velocity parallel to pipe axis
- U = average or bulk forward velocity = discharge/pipe area
- θ = mean angularity of flow over a long period of time.

When there is turbulent flow in a pipe, the water is known to waver sidewise as it moves along the pipe. The degree of wavering at any instant varies greatly, but over a long period of time there is some average or mean amount of wavering from the axial motion which may be expressed by the mean angularity of flow.

Since the velocity varies across the pipe, from a maximum at the center to a minimum at the wall, any water which wavers from one layer to another has its forward velocity changed. Thus, water moving from an outer layer to an inner layer has its forward velocity increased. Prandtl realized that this increase in velocity given to the wavering particles of water would require a longitudinal force and would account for the friction force exerted by the water not in direct contact with the pipe walls.

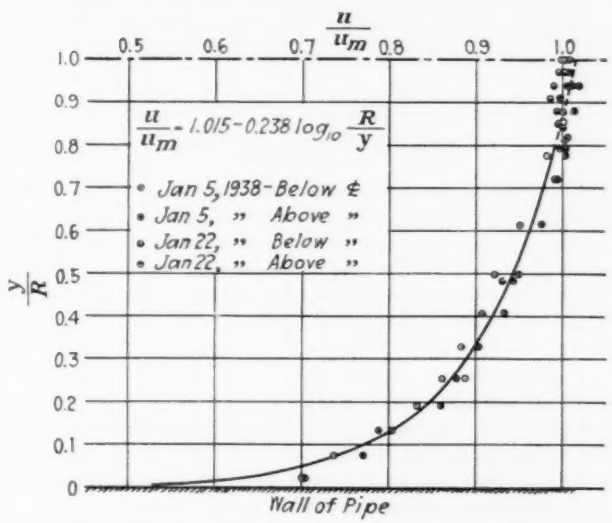


FIG. 15 PITOMETER TRAVERSE IN 40-IN. PENSTOCK WITH VON KÁRMÁN EQUATION

The longitudinal force exerted on an inner cylinder of the water in the pipe must be proportional to the mass per second of wavering water entering the inner cylinder and the increase in its velocity. Of course, when a small amount of water flows into the inner cylinder an equal amount of water is forced outward into the outer layer. This outward-flowing water can be neglected if what happens only in the inner cylinder is considered.

A diagrammatic sketch showing an inner cylinder of water flowing through the outer layer next to the pipe walls is shown in Fig. 16. Suppose that consideration be given to what probably happens at the boundary between these two cylinders.

The mass of water fluttering back and forth across this boundary is small compared to the main body of water in the inner cylinder, so that the entering water is quickly accelerated to the forward velocity near the boundary. Since the radial or cross flow indicated by the vector v in Fig. 16 is pulsating, the long-time

average of the flow into the inner cylinder must be used. This inward-flowing water is assumed to start from some point a distance l out from the boundary. Prandtl has given this dimension l the name mixing length.

Since the water entering the cylinder presumably does not start always from the same fixed point, the mixing length l prob-

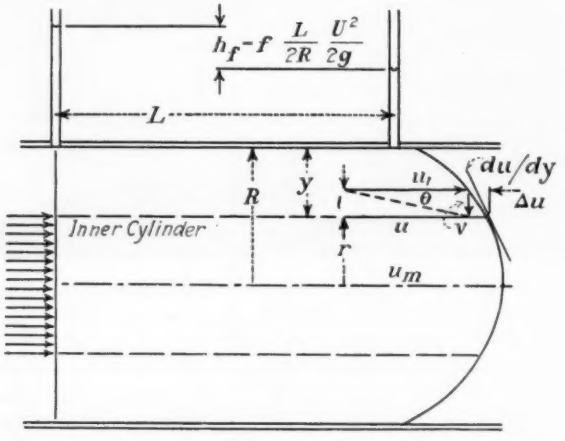


FIG. 16 DIAGRAM FOR THEORETICAL ANGULARITY IN PIPE

ably is a long-time-average distance. Referring to Fig. 16, the change in velocity, Δu, is equal to the forward velocity at $(r + l)$ subtracted from the forward velocity u at the radius r. The average true velocity of the water entering the inner cylinder would be made up of the components u and v, so that the tangent of the mean angle of flow would be v/u_1, or

$$v = u_1 \tan \theta \quad \ldots\ldots [8]$$

The experiments which have been described indicate that the value of this mean angle is about 2 deg all across the pipe. Thus, the value of the mean radial or cross velocity v is approximately 0.030 u_1, where u_1 is the forward velocity at a distance out from the boundary of the inner cylinder.

The volume of water per second which wavers into the inner cylinder is equal to the mean radial velocity multiplied by the boundary area of the cylinder, or

$$M = (62.4/g)\, v\, 2\pi r L \quad \ldots\ldots [9]$$

where M = mass, slugs per sec; Density of water = 62.4 lb per cu ft; g = acceleration of gravity; v = mean radial velocity, fps; r = arbitrary radius of inner cylinder, ft; L = length of section of pipe, ft.

Substituting for v its value $u_1 \tan \theta$

$$M = \frac{62.4}{g}\, u_1 \tan \theta \times 2\pi r L \quad \ldots\ldots [10]$$

The force required to accelerate this mass per second, an amount Δu fps, where $\Delta u = u - u_1$ is

$$F = M \Delta u = M(u - u_1) \quad \ldots\ldots [11]$$

The force equals the pressure difference acting on the inner cylinder multiplied by the area of the cross section. Since the pressure head is assumed constant across a plane normal to the axis of the pipe, from Fig. 16

$$F = 62.4\, \pi r^2 h_f \quad \ldots\ldots [12]$$

where hf is the head difference in the length L caused by friction. Using the Darcy formula for pipe friction

$$h_f = f \frac{L}{D} \frac{U^2}{2g} = f \frac{L}{2R} \frac{U^2}{2g} = \frac{fLU^2}{4Rg} \quad \ldots\ldots [13]$$

where f = friction factor, usually about 0.02; L = length of section of pipe, ft; R = radius of pipe, ft; U = average or bulk forward velocity, fps.

Then, from Equations [12] and [13]

$$F = 62.4\, \pi r^2 fLU^2/4Rg \quad \ldots\ldots [14]$$

Equating the force to the product of mass and acceleration, and using the values given in Equations [10], [11], and [14]

$$62.4\, \pi r^2 fLU^2/4Rg = (62.4/g) u_1 \tan \theta \times 2\pi r L\, \Delta u$$

or, eliminating common factors and rearranging

$$u_1 \tan \theta\, \Delta u = \frac{fU^2}{8} \frac{r}{R} \quad \ldots\ldots [15]$$

It was necessary to obtain a relation between the mean transverse velocity, v, and the acceleration, Δu. Since the flow is pulsating in an axial direction as well as across the stream, Prandtl assumed that the axial and transverse pulsations were proportional to each other at the same point in the pipe. But if a mass of water traveling with an axial velocity u_1 were suddenly moved transversely into a region where the velocity was u, then there would be a momentary negative axial pulsation proportional to $u - u_1$ while the mass of water was being accelerated to the velocity u. Thus, the acceleration Δu is proportional to the mean axial-velocity pulsation which in turn was assumed to be proportional to the mean transverse pulsation, v.

It may be seen from Fig. 16 that for small values of the mixing length, l

$$\Delta u = l\, du/dy = \text{constant } (v) = \text{constant } (u_1 \tan \theta) \quad \ldots [16]$$

If the value $l\, du/dy$ is substituted in Equation [15] for both Δu and $u_1 \tan \theta$ with the understanding that l no longer is a physical distance only, but also contains the constant of proportionality between Δu and $u_1 \tan \theta$; then Equation [15] becomes Prandtl's equation

$$(l\, du/dy)^2 = \frac{fU^2}{8} \frac{r}{R}, \text{ or } l = \frac{U\sqrt{\left(\frac{f}{8} \frac{r}{R}\right)}}{du/dy} \quad \ldots\ldots [17]$$

This enables l to be computed at various values of r/R. The mixing length for the 40 in. penstock is shown by curve A in Fig. 18. When l is known, Δu and u_1 can be determined from the velocity-traverse curve.

If Equation [15] be solved for $\tan \theta$, we obtain

$$\tan \theta = \frac{\frac{fU^2}{8} \frac{r}{R}}{u_1 \Delta u} \quad \ldots\ldots [18]$$

But from Equations [16] and [17]

$$\Delta u = l\, du/dy = U \sqrt{\frac{f}{8} \frac{r}{R}} \quad \ldots\ldots [19]$$

Substituting this value for Δu in Equation [18]

$$\tan \theta = \frac{U}{u_1} \sqrt{\frac{f}{8} \frac{r}{R}} \quad \ldots\ldots [20]$$

The theoretical angularity calculated from Equation [20] is shown by curve A in Fig. 17. The measured angularity shown by curve B in Fig. 17 agrees with the theoretical angularity in

magnitude but shows a more uniform distribution across the pipe. The probable reason for the discrepancy at the center of the pipe is that angularity near the center is induced by eddies away from the center and cannot be accounted for by the Prandtl theory alone.

The results of tests by Nikuradse, on page 53 of "The Mechanics of Turbulent Flow" (4) have shown that the velocity-traverse curve is more blunt at the center of the pipe than is indicated by the von Kármán Equation [7], and that the mixing length is finite at the center of the pipe. The mixing length as determined by Nikuradse for high Reynolds numbers is shown by curve *B* in Fig. 18, reproduced from Fig. 39, p. 54 of "The Mechanics of Turbulent Flow" (4).

The curve of Fig. 19 showing the cross velocity computed from tests plotted against distance from the wall indicated a nearly constant value of v. It seemed quite reasonable to assume that this curve was a horizontal line, or that the cross velocity was constant at a value of 0.0289 across the pipe. This assumption

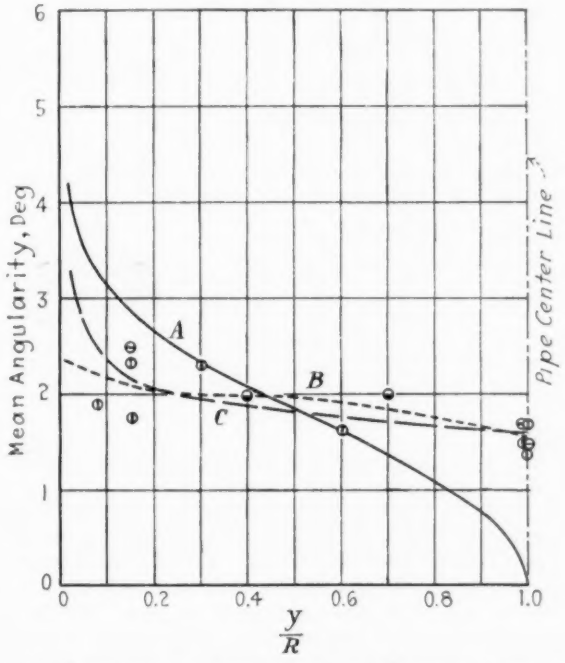


FIG. 17 COMPARISON OF THEORETICAL AND MEASURED ANGULARITY
(Curve *A*, theoretical angularity; curve *B*, mean of measured angularity; curve *C*, angularity computed using constant cross velocity of 0.0289 u_m.)

would give a means of computing the theoretical mean angle to check experimental values. Since it was necessary to have some method of computing the mixing length when the mean angle of flow was not known, the Prandtl Equation [17] was used. The value of f was taken as 0.0184. $U = 2.05$ fps, $u = 0.867u_m$. To compute the value of θ for a given value of r/R, substitution of r/R and du/dy are made in Equation [17] and the value of l obtained. This value of l when added to r gives the radius on the velocity-traverse curve at which u_1 occurs. Dividing the assumed-constant value of v, 0.0289, by the value of u_1 gives tan θ. Curve *C* of Fig. 17, showing the values of mean angle computed in this way, is in good agreement with the experimental values.

Fig. 20 shows the results of angularity tests in various pipes plotted against the pipe factor. The theoretical angularity, as computed from the von Kármán velocity formula for rough pipes, is also shown; the value of f was varied to obtain results at different pipe factors. The angularity was computed for a point midway between the center and wall of the pipe, since the tests in the 40-

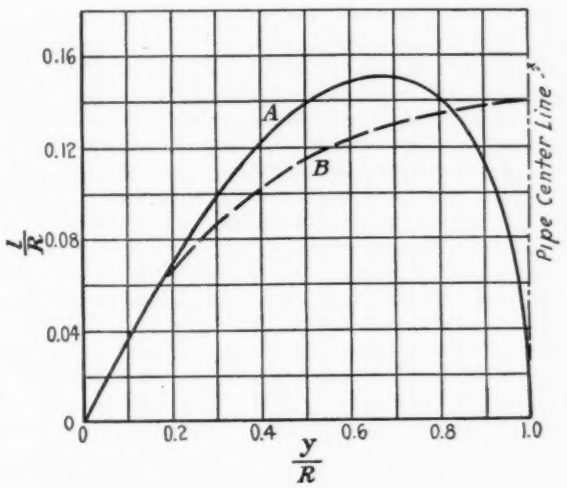


FIG. 18 MIXING-LENGTH CURVES
(Curve *A*, computed by Prandtl formula, Equation [17]. Curve *B*, from measurements by Nikuradse.)

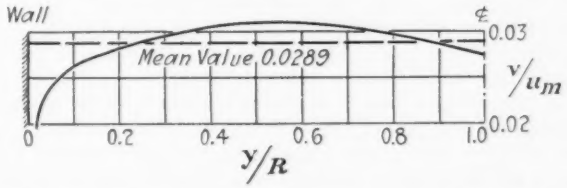


FIG. 19 RATIO OF CROSS VELOCITY TO CENTER VELOCITY COMPUTED FROM MEASURED ANGULARITY IN 40-IN. PIPE

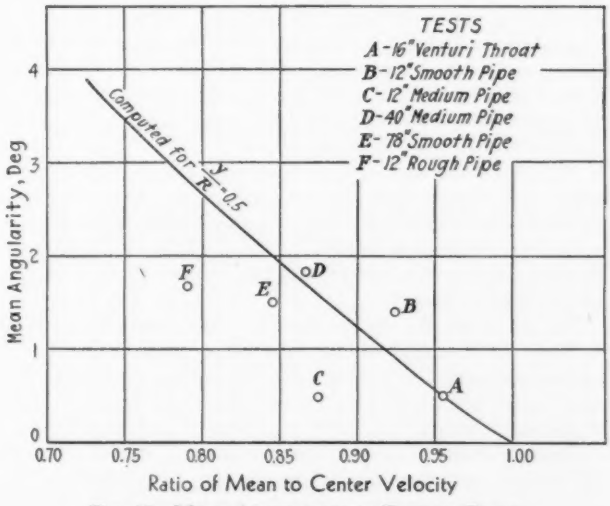


FIG. 20 MEAN ANGULARITY OF FLOW IN PIPES
(Curve computed from the von Kármán velocity formula for rough pipes.)

in. penstock showed a close agreement between theoretical and experimental values at this point.

The comparison of the theoretical with the experimental values of angularity is close in most cases. There is sufficient evidence to indicate that there is a relation between turbulence and angularity and that angularity must necessarily be only a few degrees in stable pipe flow

PULSATION TESTS

In turbulent flow where the velocity at a point in the pipe constantly varies above and below the average, the manometer

connected to a pitot tube records a constantly changing pressure proportional to the velocity head. In computing the average velocity at a point in the pipe from a number of individual manometer readings, the arithmetic average of the manometer readings is used. Since the readings are proportional to the squares of the velocities, it is theoretically possible for the average velocity computed from the square root of the average pressure to be greater than the true average velocity. This would cause an error which would make the measured velocity appear too large, for the average of the square roots of a group of numbers is less than the square root of the average. The amount of error from this cause would depend on the magnitude and duration of the pulsations in velocity and pressure.

A system to measure pulsations was devised which utilized the principles of the law of probability in a manner similar to that described for the angularity tests. The piezometer in which pulsations were to be measured was connected to one side of a thin rubber diaphragm of 1 in. diam. The other side of the diaphragm was connected to an air reservoir in which the pressure could be varied at will. An electrical contact consisting of a strip of 0.005-in. shim-stock brass was laid on the air side of the rubber diaphragm so that when the pressure was slightly greater on the piezometer side, the diaphragm would bulge a few thousandths of an inch and the brass strip would be pressed against an insulated contact. The displacement of the diaphragm was estimated to be less than 0.001 cu in.

If the pressure pulsations varied constantly in an accidental manner so that they followed the law of probability, with a constant air pressure acting on one side, the variable pressure on the water side would cause the diaphragm to move and the electrical contact to be made and broken continually. If the air pressure were increased to an amount greater than the highest pressure pulsation, the diaphragm would always be held away from the contact and no record would be obtained. If the air pressure were reduced slightly, the contact would be made only by the highest of the pressure surges. The number of diaphragm contacts divided by the total possible contacts in a given test interval would determine the probability of a surge equal to or greater than the arbitrary air pressure.

The diaphragm contact was connected in series with an interrupter and chronograph pen exactly as in the angularity tests. By making tests at a number of different settings of the air pressure, it was possible to plot the results on a probability chart and the mean pulsation was determined in exactly the same manner as the mean angularity.

Fig. 21 shows the arrangement of the apparatus. In order that the pulsations would be modified as little as possible by friction and bulging of rubber hose, the diaphragm was connected close to the piezometer or pitot rod by a 2-in. length of $^1/_4$-in. high-pressure hose.

The air pressure was measured by an open mercury U-tube. An arbitrary zero was obtained by opening a by-pass between the air reservoir and the water side of the diaphragm, as this pressure was close to the average pressure. The by-pass was closed during the tests. The mercury deflection from the arbitrary zero was measured to 0.001 in. by a micrometer with a pointed rod which caused a flashlight to signal when contact with the mercury surface was made. The air pressure was varied by means of a small hand pump.

The tests shown in Fig. 22 were made with the diaphragm connected to the impact tip of a pitot tube located at the center of a 12-in. rough pipe and then to a wall piezometer of the same pipe. The center velocity in the pipe was 9.2 fps, and the mean velocity 7.2 fps. The results indicate that the mean pulsation is 0.048 in. of mercury or 4.1 per cent of the center-velocity head for both the wall piezometer and impact orifice. This amount of pulsation is too small to cause an appreciable error in the mean square root of deflection, since it can be shown that the mean pulsation must be about 25 per cent of the velocity head to cause an error of one per cent.

Dynamic Effect on Manometer Pulsations

It is well known that the dynamic force exerted by a jet impinging against a normal flat plate is equal to twice the force which would be exerted on the same area by the static head acting on the jet (5). Although the water in the jet cannot rise above

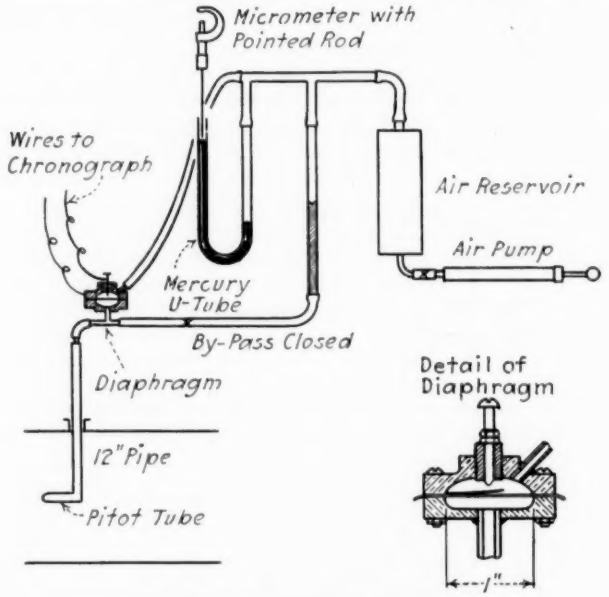


Fig. 21 Pulsation Apparatus

the level of the head causing flow, it can resist the tendency to flow of a more powerful opposing head.

In turbulent flow the pressure and velocity are not constant, but pulsating, so that the level in the manometer connected to the pitot tube is constantly shifting. After a positive surge, the water forced into the manometer must flow back through the pitot tube against the flowing water in the pipe. If this backflow is resisted by the dynamic effect of the stream in the pipe, then the tendency would be to emphasize the positive surges and to give too great an average deflection.

It was decided to test the backflow resistance by placing a pitot tube in a stream and measuring the rate of flow through the pitot tube at various artificially created surges. The pitot tip was mounted on a sliding frame so that a lengthwise traverse could be made of the jet discharging from a 4-in. orifice under a head of about 1.5 ft. The pitot support was so made that the tip could be projected through the jet 6 in. upstream from the plane of the orifice without seriously obstructing the flow. The pitot tube was connected by 5.5 ft of $^1/_4$-in. and 2.5 ft of $^1/_2$-in. rubber hose to a hook-gage stilling pot mounted on a firm support. The water level in the hook-gage pot was read by a standard hook gage to thousandths of a foot and estimated to half thousandths.

The brass pitot tip was 1 in. long, with a hemispherical nose of $^1/_4$ in. outside diameter and a $^1/_8$-in. hole drilled throughout its length. This tip was soldered to a support rod of thin brass tubing to which the hose was connected. Before the test the tip was plugged and the hook gage observed for 5 min to insure that there was no leakage between the tip and the hook-gage pot.

Before the tests were started, the discharge from the orifice was allowed to run for more than a half hour to insure constant condi-

tions. Hook-gage readings were taken for several minutes with the pitot tip at the vena contracta and also with it projected 6 in. into the forebay. With a head on the orifice of about $1^1/_2$ ft, the hook gage read 0.001 ft lower with the tip at the vena contracta than with the tip in the forebay. This close check was probably due to the fact that the edges of the orifice were rounded to about $^1/_8$ in. radius, causing little loss of head in the jet.

Excess water was poured into the hook-gage pot to create an artificial positive surge. The rate at which the level in the pot was lowered was obtained by timing a drop in level of about 0.020 ft with a stop watch. The rate of drop was a measure of the discharge through the pitot tube. In a similar manner the rate of rise after a negative surge could be determined by removing water from the hook-gage pot and allowing water to flow in through the

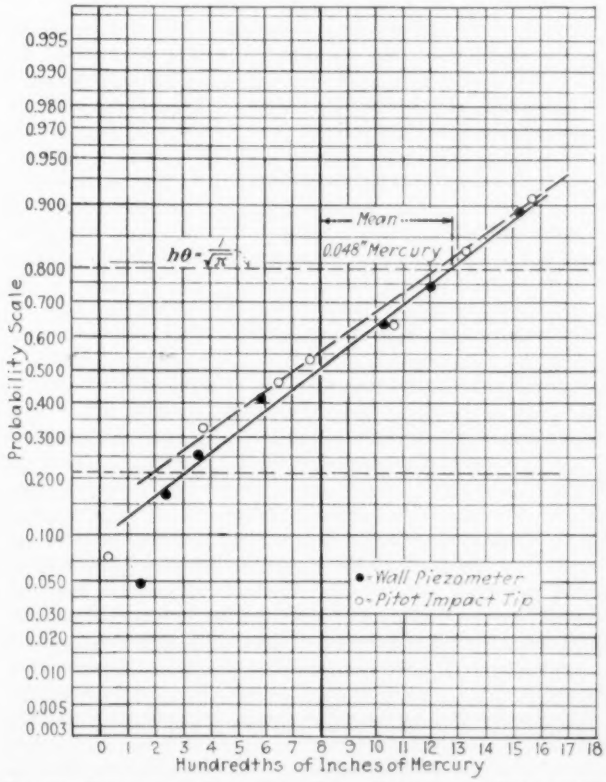


FIG. 22 RESULTS OF PULSATION TESTS PLOTTED ON PROBABILITY CHART

(Center velocity = 9.2 fps; velocity head = 1.165 in. mercury; mean pulsation = 4.1 per cent of velocity head.)

pitot tube. The results of tests with rising and falling hook-gage levels both in the vena contracta and in the forebay are shown in Fig. 23. Fig. 24 is a photograph of the apparatus.

If the dynamic effect of the stream caused a resistance to the backflow of a surge, then the discharge through the pitot tip should be less for a dropping head when placed in the vena contracta than for a rising head. Inspection of the test data in Fig. 23 shows the resistance to outward flow is slightly less than to inward flow. This indicated that there was no dynamic resistance tending to cause the impact tip to read too high in turbulent flow.

Tests similar to those just described in which the flow in and out of a wall piezometer in the side of an open flume was measured, showed no appreciable difference in flow through the piezometer. The velocity head near the piezometer was 0.25 ft. It is apparent that surges in the manometer do not affect either the impact-orifice or wall-pressure readings.

EFFECT OF PITOT-TUBE SUPPORT ROD ON VELOCITY READING

The essential measurement which must be made in order to determine the velocity of flow with a pitot tube is the difference between the dynamic head at the impact orifice and the pressure head in the fluid itself at the impact orifice. If, then, there is an

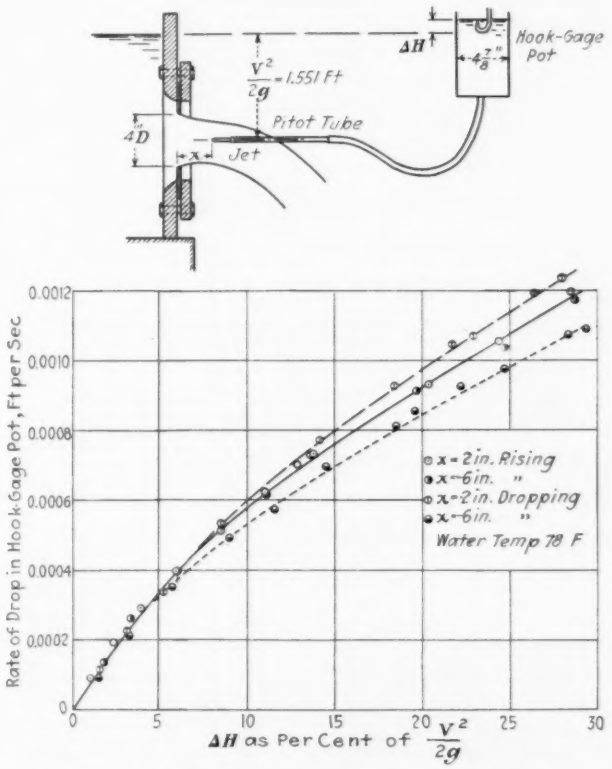


FIG. 23 TESTS TO DETERMINE DYNAMIC EFFECT ON MANOMETER PULSATIONS

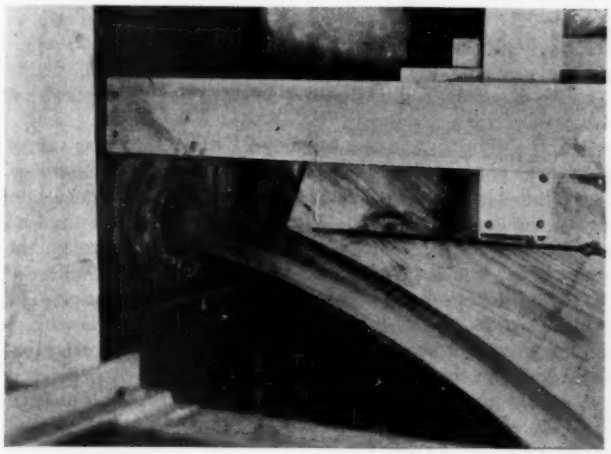


FIG. 24 ORIFICE AND PITOT TUBE USED TO TEST DYNAMIC EFFECT ON PULSATIONS

error in measurement of either the dynamic or the pressure head, the difference between them will be in error and the pitot tube will not register correctly.

In this paper a pitot tube is considered as an L-shaped tube. One leg, called the tip, points into the stream and has an orifice at

its upstream end termed the impact orifice. The other leg called the rod, is across the stream and serves to support the tip and provide a means of conducting the impact pressure outside the conduit. The rod usually passes through some form of stuffing box and allows motion of the tip across the pipe in traversing.

Tests were made to show whether the presence of the supporting rod behind the impact orifice would cause an error in reading. The apparatus and results of this test are shown in Fig. 25. The same tip used on the rotating boom (see Fig. 4) was mounted on a supporting pipe passing through a stuffing box so that the tip could be projected varying distances through a dummy rod without changing the flow. In order that small errors might be detected, the pitot tip was connected to a U-tube, the other side of which was connected to the upstream tip of a Cole pitometer. The pitometer tip was 0.2 ft away from the pitot tip and gave nearly the same deflection. With the pitometer tip fixed in position, the pitot tip was moved back and forth through the dummy rod over a distance of 3 in. One hundred readings of the U-tube deflection were taken at each position of the tip. The liquid used in the U-tube was monochlorbenzine having a specific gravity of about 1.10, so that the pressure difference between the pitot tips was magnified ten times.

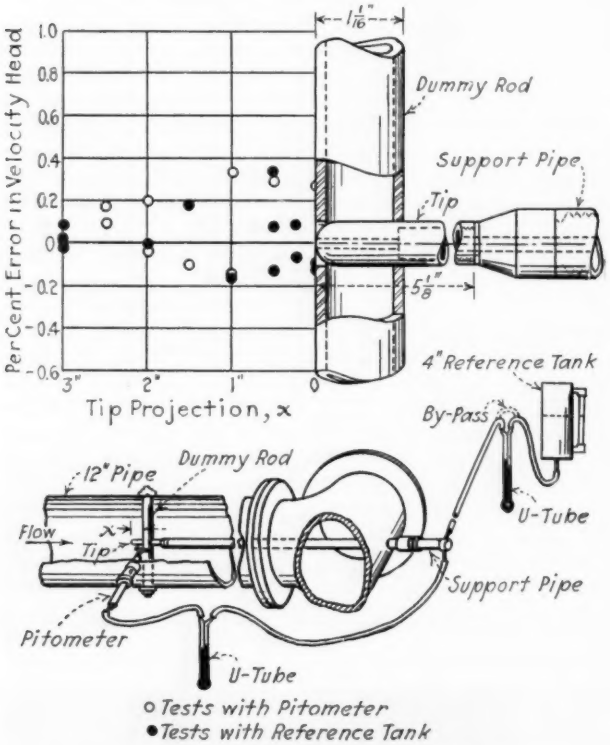


FIG. 25 EFFECT OF TIP LENGTH ON IMPACT-TIP READINGS

A second series of tests was made with the pitot tip connected to a U-tube which in turn was connected to a 4-in. reference tank having air trapped above the water surface. A by-pass was connected to the U-tube so that when the tip protruded 3 in. ahead of the dummy rod, the reference pressure could be made equal to the impact pressure. With the by-pass closed, the position of the tip was changed and one hundred readings of the U-tube deflection were taken. Monochlorbenzine was used in the U-tube. The inside diameter of the U-tube legs was $^5/_{16}$ in.

The results of these tests are shown in Fig. 25 and indicate that the length of the tip upstream from the rod has little effect on the impact orifice pressure. The tests were made at a mean velocity of 8 fps.

The previous tests have indicated that the impact tip has a coefficient of unity and is unaffected to any appreciable degree by the angularity or pulsations of turbulent flow or by the presence of the supporting rod. It seems, therefore, that any error in pitot-tube readings must be caused by an error in obtaining the correct pressure head.

Whereas the dynamic head at the impact orifice can be easily and accurately measured, it is practically impossible to measure the true pressure head at any point in the fluid. At the present time the only method of measuring pressure head in a pipe is by a piezometer in some solid surface. Such a surface cannot be placed in the stream without disturbing the flow and therefore affecting the pressure some unknown amount.

Ordinarily the pressure head is measured in one of two ways. For the simple pitot tube the pressure head is measured at the wall of the pipe by piezometers normal to the wall and abreast of the impact orifice. The second method is to have the pressure-head piezometers at some point on the tip itself, the instrument in this case being called a pitot-static tube. The errors in the reading of the pressure head will be described in a following section.

It is assumed for this discussion that the wall piezometer will be correctly made and that it reads the true pressure head at the point on the wall of the pipe where it is located. A complete discussion of the errors in wall piezometers will be found in the A.S.M.E. Transactions (6).

While traversing a pipe with a pitot tube it has been discovered, perhaps by many investigators, that the pressure reading of a wall piezometer changes with the position of the pitot tube as it is moved across the pipe. It is only natural to suppose that since the presence of the supporting rod reduces the cross-sectional area at the plane where it is inserted, the water must be speeded up as it passes the rod and the pressure at the wall be correspondingly reduced according to Bernoulli's theorem. Since the water cannot be accelerated instantly, it is necessary to find how far upstream from the plane of the supporting rod a wall piezometer should be located to avoid this reduction in pressure. It is also necessary to investigate how far around the circumference of the pipe from the point of insertion of the rod the piezometer should be located to avoid extreme local effects caused by the presence of the rod.

The errors caused by the rod were expected to be small, and the difficulty of making two or more piezometers which would read exactly alike was fully appreciated. It was, therefore, decided to determine the effect of a movable dummy rod with respect to a particular piezometer called the test piezometer. This method would eliminate the errors between several piezometers due to in accuracies of construction, since the change in the reading of a particular piezometer due to change in position of the rod could be measured. The change in reading of the test piezometer would be observed by measuring the difference in pressure between the test piezometer and a reference piezometer which read nearly the same pressure. The reference piezometer would have to be located reasonably close to the test piezometer so that the pressure difference would be small, but sufficiently far away so that its reading would not be affected by changes in position of the dummy rod near the test piezometer.

Apparatus.

A convenient section of an existing 12-in. line was selected for the tests. The section selected had several advantages. At the head end of the line just downstream from the connection to the penstock was a 16 × 8-in. venturi meter which could be readily calibrated by diverting the discharge through the test section into a 50,000-lb weighing tank, assuring accurate measurement of

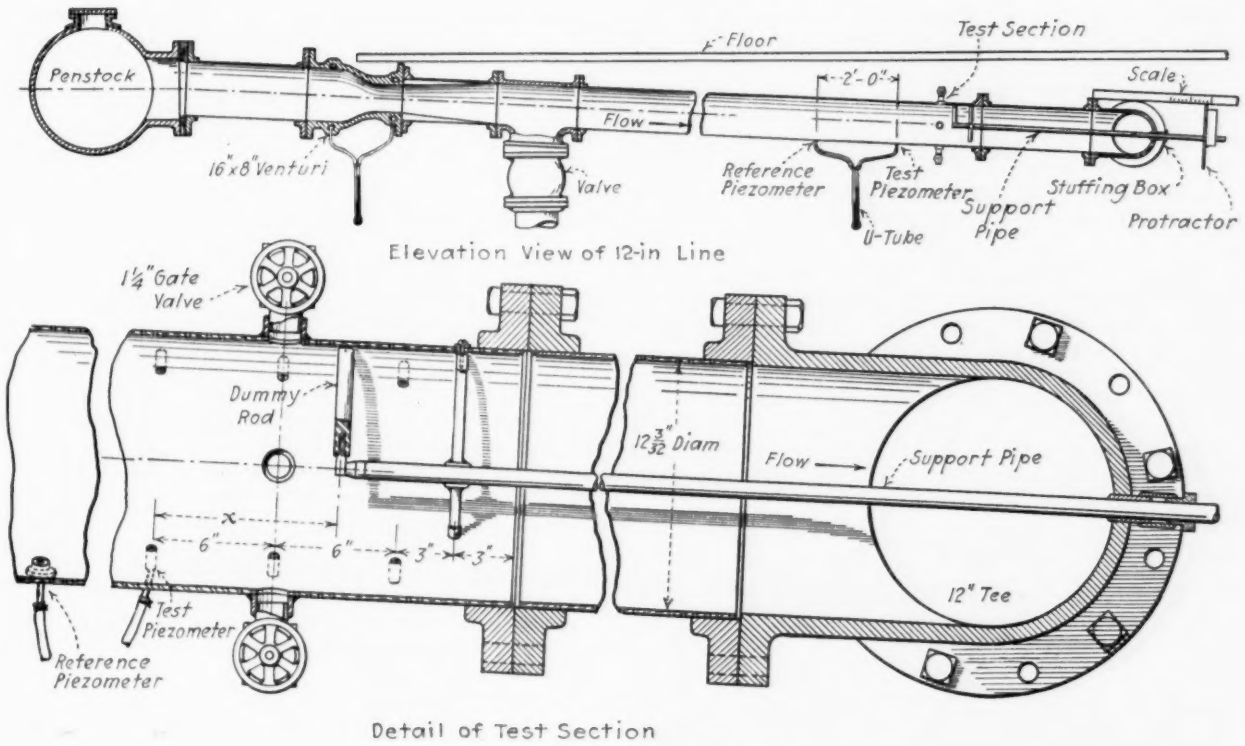


FIG. 26 DIAGRAM OF 12-IN. TEST PIPE

flow. Just downstream from the point selected for the test section was a short length of pipe which could be easily removed to make changes in the test section located near the end of a 15-ft length of pipe. The test section was close to a 12-in. tee so that apparatus for holding and moving the dummy rod could be easily set up. Because of the arrangement of the meter and piping turbulent flow unquestionably existed. The test section is shown in Fig. 26.

The 15-ft length of pipe containing the test section was painted with a smooth waterproof paint to prevent rusting and change of surface roughness while tests were in progress. Twelve brass piezometer plugs with $^1/_8$-in. holes were installed in three rings of four, 6 in. apart. These piezometers were carefully located at the ends of 45-deg diameters, but not exactly in line with the corresponding piezometers of the other two rings. They were carefully lined up and finished flush with a small portable grinder mounted on a wooden jig. All burrs and scratches were removed and the final inspection made with a magnifying glass. Although these piezometers were made with all possible care, the error shown by comparing one with the other eleven was about 1 per cent of the mean velocity head. It is obvious that the construction of the wall piezometers is important.

A reference piezometer was constructed by inserting a bronze plug $2^1/_2$ ft upstream from the center of the test section. This piezometer was too far from the end of the pipe to be finished in place, but its error was found to be constant and its value as an arbitrary reference was not impaired. The details of the piezometers and test section are shown in Fig. 26.

Four tapped connections 90 deg apart but 45 deg from the piezometers were installed in the test section. These connections were made by brazing half a $1^1/_4$-in. pipe coupling to the wall of the pipe. The connections were accurately lined up so that rods could be inserted through them from opposite directions if so desired.

A spider having three legs of $^1/_8 \times ^1/_2$-in. flat iron was accurately centered at the downstream end of the test section. This spider supported a $^1/_2$-in. brass pipe used to hold the dummy rod. The supporting pipe projected through a stuffing box in the 12-in. tee so that a protractor 19 in. in diam could be clamped to it. A suitable fitting was attached to the upstream end of the support pipe to receive the various dummy rods. This fitting was so made that the rods could be changed quickly by inserting them through one of the $1^1/_4$-in. tapped connections and screwing them into place. The rods were carefully finished to clear the pipe walls by not over $^1/_{32}$ in.

The piezometer used for most of the tests was one near the bottom of the pipe in the upstream ring. This ring was farthest from the spider and would be least influenced by it. A glass U-tube was connected by rubber hose between this test piezometer and the reference piezometer located 2 ft upstream from it. The liquid used in the U-tube was monochlorbenzine having a specific gravity of approximately 1.10. The specific gravity of the U-tube liquid was checked each day of test. Care was taken that the U-tube liquid had perfect menisci, otherwise the readings would be in error due to surface tension. To prevent sticking of the meniscus to the glass, the liquid was removed and filtered after each series of tests and the U-tube cleaned and filled with dilute nitric acid.

The flow was measured by the calibrated 16 × 8-in. venturi meter. The meter coefficient had been determined with an accuracy of about 0.25 per cent.

Description of Tests.

A complete test with a given size of dummy rod at a fixed velocity was performed as follows: The desired discharge was set and the deflection of the venturi meter measured and recorded against time of day. Ten readings were taken of the piezometer U-tube with the rod at an angle of 180 deg from the test piezometer and 0.95 ft downstream from it to obtain the initial deflection. The dummy rod was then moved to another position

and the effect on the wall piezometer determined. At each station tests were made at a sufficient number of angular positions to enable a smooth curve to be drawn through the test points.

A complete test occupied about an hour so that the room temperature affected the liquid in the U-tube and slightly changed its specific gravity. The change was gradual so that by taking all readings against time of day and correcting them to standard conditions before computing results, little trouble was encountered from this source. The rate of flow stayed constant during the test.

Tests showed that the presence of the dummy rod exerted no influence on the test piezometer when in the initial or far position. The effect of the rod at any other position, therefore, was taken as the change in the U-tube reading at the new position. Thus, if the difference in pressure between the piezometers were 13 per cent of the mean-velocity head with the rod at the initial position and 14 per cent at some other rod position the effect of the rod at its new position would be a change in pressure at the test piezometer of 1 per cent of the mean-velocity head.

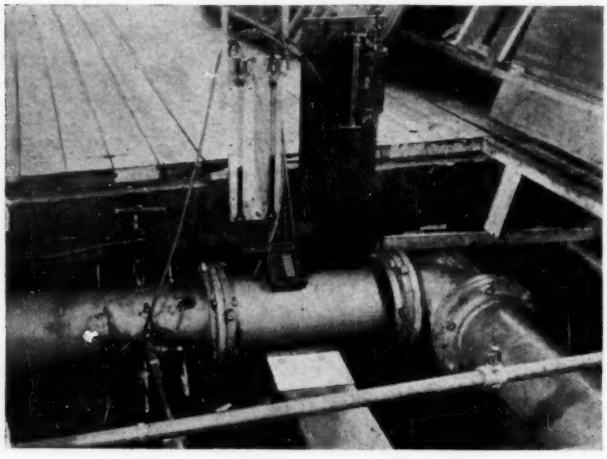


FIG. 27 TEST SECTION OF 12-IN. LINE

Tests were run with dummy rods having diameters of $^1/_2$, $^3/_4$, and $1^1/_{64}$ in. in the 12-in. pipe. Each of these sizes of rod was tested at two or three velocities. A test was also run at two different velocities with no rod which showed that the effect of the support alone was negligible.

Results of Tests.

Examination of the test data showed that the change in the test-piezometer reading was dependent on the relative position of rod and piezometer for a given size of dummy rod. For the same relative positions this change was apparently a constant percentage of the velocity head at all velocities.

In order that the data for different sizes of rods might be correlated, the assumption was made that the piezometer error, as a percentage of the mean-velocity head, was proportional to the reduction in the stream cross-sectional area for two different rods at the same position.

According to Bernoulli's theorem the pressure abreast of the rod should differ from the pressure just upstream from it by the difference in mean-velocity heads. The mean velocities at the two sections are to each other inversely as the cross-sectional areas. Thus, if the pipe area were 1 sq ft and the projected area of the rod a sq ft, then the net stream area abreast of the rod would be $(1 - a)$ sq ft. The mean velocity V_1 in the plane of the rod would be $1/(1 - a)$ times the mean velocity just upstream.

The difference in pressure between the two sections would be

$$h - h_1 = (V_1^2 - V^2)/2g \quad [21]$$

but

$$V_1 = V[1/(1 - a)] \quad [22]$$

then

$$V_1^2 = V^2[1/(1 - a)^2] \quad [23]$$

$$h - h_1 = \frac{V^2}{2g}\left[\frac{1}{(1-a)^2} - 1\right] = \frac{V^2}{2g}\left[\frac{2a - a^2}{(1-a)^2}\right] = K\,\frac{V^2}{2g} \quad [24]$$

For ratios of projected rod area to pipe area less than 0.05 the term a^2 becomes small and the term $(1 - a)^2$ is close to unity, so that K is practically equal to $2a$.

The percentage error for three different sizes of rods at the same location was divided by the factor K, and the result was called the relative percentage error. The relative percentage error was not the same for the different rods at the same location, but appeared to be the same if they were compared on the basis of axial distance in terms of rod diameters. Thus the same relative effect on the test-piezometer reading was obtained with a 1-in. rod 5 in. downstream from the piezometer as was obtained with a $^1/_2$-in. rod $2^1/_2$ in. downstream, when both rods were at the same angular position relative to the piezometer.

In order that all the tests for all sizes of rod and at all velocities might be plotted on the same sheet, the relative percentage errors are plotted as contours on the development of half the pipe wall. These results are shown in Fig. 28.

For convenience in plotting the data for various sizes of rod the piezometer errors were expressed in terms of the mean-velocity head. Essentially, we are more interested in the error in deflection, or what is desired is the apparent coefficient of the pitot tube combined with wall piezometers.

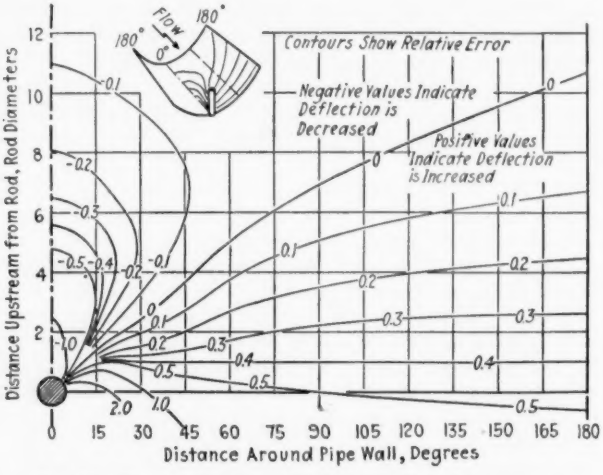


FIG. 28 EFFECT OF SUPPORT ROD ON WALL PIEZOMETER

In all the tests the dummy rod has simulated a pitot rod projecting half across the pipe with the tip at the center of the stream. The ratio of areas then has a simple relation to the ratio of diameters of rod and pipe. If the rod diameter be expressed as d, and the pipe diameter as D, the projected area of the rod will be $Dd/2$ and the area of the pipe will be $\pi D^2/4$. The ratio of areas will be $2Dd/\pi D^2 = (2/\pi)(d/D) = a$. The piezometer error in deflection is proportional to $2a$. Because of the square-root relation between velocity and deflection, the error in velocity is very closely one half the error in deflection. Thus the percentage error in velocity is proportional to a. Then the wall-piezometer error in per cent of mean velocity should be equal to the contour value in Fig. 28 multiplied by $2d/\pi D$.

As an example, suppose that a simple pitot tube having a rod 1 in. in diam is to be used in a 12-in. pipe. What would be the effect of the rod if the wall piezometers are situated 90 deg from the stuffing box and $1^1/_2$ in. upstream from the rod center line?

Fig. 28 shows that a relative error between 0.3 and 0.4 or about 0.36 exists when the piezometer is located 90 deg and $1^1/_2$ rod diameters upstream. The error in mean velocity would then be $0.36(2d/\pi D) = 0.36(2/12\pi) = 0.019 = 1.9$ per cent of the mean velocity.

If the pipe factor is 85 per cent, the piezometer error will be 0.85×1.9 or 1.6 per cent of the center velocity.

The effect of the rod on the wall piezometers will cause the indicated center velocity to be 1.6 per cent too large.

It is apparent from these tests that the wall piezometer should be located at a point little affected by the presence of the pitot rod. If two piezometers are connected together, they should be located so that the rod affects both an equal amount.

Pressure Piezometer of Pitot-Static Tube

In the combined or pitot-static tube one or more pressure piezometers are located on the tip itself. Research performed by Merriam and Spaulding (9), of the Worcester Polytechnic Institute on pitot-static tubes with air shows that the pressure along the tip is affected by the presence of the supporting rod and by the nose of the tip itself. This work was performed in the jet of a 20-in. wind tunnel. The pitot tube recommended by Merriam and Spaulding based on their tests, was one with a tip twenty-four times as long as its diameter, and with the pressure-head orifices located eight diameters back from the hemispherical nose. Thus, for a tube of $^1/_2$ in. diam, the tip would be 12 in. long. Such a tube would be satisfactory for measuring the flow of gases, but would be much too awkward to use with liquids flowing in pipes under pressure.

In the Merriam and Spaulding tests the pressure piezometers varied from two to eight arranged in a ring around the tip. With the shorter tip, more convenient for work in pipes, the pressure at each piezometer would vary greatly and the average would be appreciably affected if some should become clogged.

It was, therefore, decided to study the effect of the supporting rod on a single tip pressure piezometer in a manner similar to that used for the wall piezometers. The test section was located in the 12-in. pipe shown in Fig. 26. The test piezometer hole, $^3/_{32}$ in. in diam, was carefully drilled through the side of a $^1/_2$-in. brass pipe. A solid hemispherical nose was soldered to the upstream end of the $^1/_2$-in. pipe. The downstream end of the brass pipe which projected through the stuffing box in the 12-in. tee was capped and a connection made through the cap for a $^1/_4$-in. rubber hose. The brass pipe rested on a fixed dummy rod which was accurately shaped to receive it. With a large 360-deg protractor fastened to the projecting end, it was possible to rotate and slide the small pipe so that the piezometer was moved with respect to the dummy stem.

The effect of the hemispherical nose of the tip, as determined by Merriam and Spaulding's tests, is shown in Fig. 29. In order that the pressure at the piezometer might not be modified by this effect, the piezometer was located 11.5 diameters back from the nose.

The arrangement of the apparatus is indicated in Fig. 31. The brass piezometer pipe was connected to one side of a differential U-tube, and the other side of the U-tube was connected to a wall piezometer 12 in. upstream from the dummy rod. The U-tube contained a liquid which magnified the difference in pressure ten times.

Tests were made with a center velocity of about 8 fps. The piezometer was moved axially with respect to the dummy rod and also rotated through angles from 0 to 180 deg from the plane of the rod. The change of U-tube deflection with position of the pipe piezometer was a measure of the rod effect.

In computing the results of these tests, there was evidence that the reference pressure measured by the wall piezometer was not constant, but changed with the axial movement of the brass pipe. The dummy stem was removed and the brass pipe supported by a spider located 2 ft downstream from the reference wall piezometer. A U-tube containing monochlorbenzine was connected between the wall piezometer and the reference piezometer 2.2 ft upstream

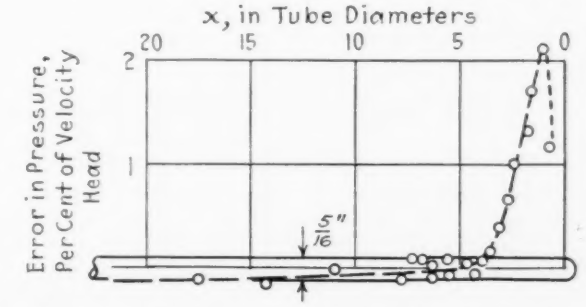


Fig. 29 Effect of the Nose on Pressure Piezometer Located on the Tip

(Flow around nose causes low pressure readings. Tests by Merriam and Spauld ng.)

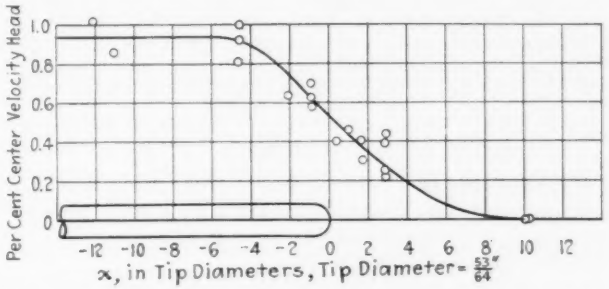


Fig. 30 Effect of Tip on Wall-Piezometer Pressure in 12-In. Pipe

(Venturi effect causes wall piezometer to read low.)

as used in the wall-piezometer tests. (See Fig. 26.) When the brass pipe was moved by the wall piezometer, the U-tube deflection was affected.

The change in the reference wall piezometer corresponded with the increase of velocity due to the presence of the $^1/_2$-in. pipe. The outside diameter of the brass pipe was 53/64 in., and the projected area equal to 0.47 per cent of the area of the 12-in. pipe. The change in wall-piezometer reading caused by moving the pipe abreast of it, was 0.94 per cent, exactly corresponding to the venturi effect obtained by reducing the area. The results of this test, shown in Fig. 30, indicate that when a large pitot tube is used in a small pipe a correction for the projected area of the tip should be made.

The results of the tests to determine the effect of the support rod on the tip piezometer are shown in Fig. 31. These results have been corrected for pipe friction between stations and venturi effect caused by the presence of a large tip. The error in pressure reading is shown in per cent of center velocity head. Because of the square-root relation, the error in velocity would be one half the error in deflection. It is apparent from Fig. 31 that piezometers located within 2 or 3 diameters of the rod will be unstable and, therefore, inaccurate. To obtain a stable coefficient near unity, the tip should be at least 10 diameters in length with the pressure piezometers located 5 diameters back from the nose.

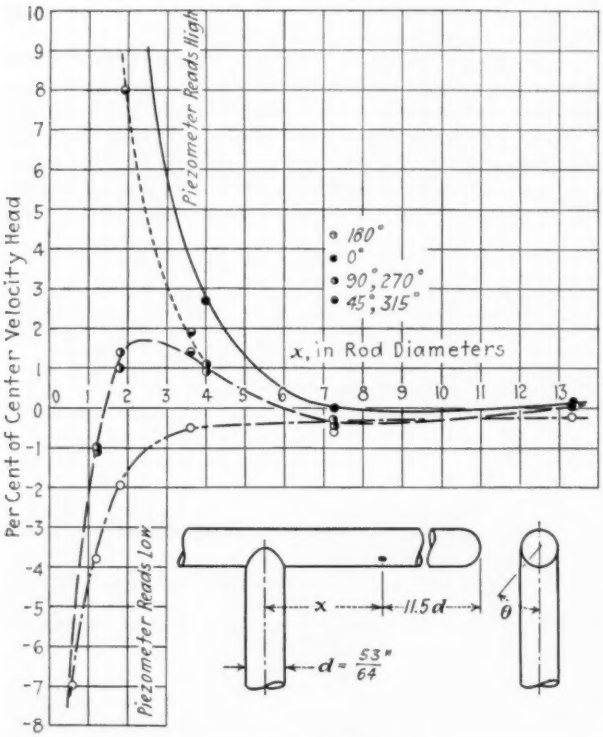


FIG. 31 EFFECT OF DUMMY ROD ON PRESSURE PIEZOMETER ON TIP OF PITOT-STATIC TUBE

CONCLUSION

If a pitot tube does not measure the true velocity head at a point in a turbulent stream, either the impact orifice reads the dynamic head incorrectly or the pressure piezometer does not read the true pressure head, or both.

The tests made in still water with the rotating boom show that the impact orifice registers the true dynamic head within one part in a thousand. If this perfect registration is changed by turbulent flow in a pipe, the change must be caused either by the turbulence or by the confining action of the pipe walls. The only conceivable ways in which turbulence can affect the reading of the impact orifice is by the angularity with which the water approaches the tip or by causing an error in registration of the average of pressure and velocity pulsations. The confining action of the pipe might possibly cause the rod to influence the pressure at the impact orifice.

Angularity. The results of the tests show that none of these effects is sufficiently great to cause an appreciable error, for in the case of angularity:

1 The mean angle of approach is less than 3 deg.
2 It is nearly constant across the pipe.
3 The radial and tangential angularity are equal.
4 There is no noticeable regular cycle.
5 The angularity is independent of the size of pipe.
6 The measured angularity agrees well with the Prandtl theory of turbulent flow.

Pulsation. The mean pulsation measured in a 12-in. rough pipe was only 4 per cent of the velocity head. It can be shown that a mean pulsation of 25 per cent of the velocity head is required to cause a 1 per cent error in velocity due to averaging deflections instead of their square roots.

The mean pulsation was the same for both impact orifice and wall piezometer.

Dynamic Effect of Stream. The dynamic pressure of the stream against the tip does not resist outward flow from the tip and cause surges to be incorrectly averaged.

Support Rod. The support rod does not affect the reading of the impact orifice.

It is concluded from these facts that the impact orifice correctly registers the true dynamic head in both smooth and turbulent flow.

Wall Piezometers. The errors obtained by experienced engineers when using the pitot tube must then be caused by an incorrect measurement of the pressure head. The results of the tests show that when the pressure is measured by wall piezometers: The rod causes an error in pressure reading.

It is obvious that if, for any reason, the pressure is not constant across the pipe, the wall piezometer does not measure the pressure head which exists at the tip of the pitot tube.

Tip Piezometers. If the pitot tube is the combined type in which the pressure head is measured by tip piezometers:

1 The pressure reading is affected by the relative size of tip and pipe, causing a venturi effect which is appreciable if the tip is too large for the pipe.

2 The pressure reading is affected by both the shape of the nose of the tip and by the support rod. These effects are not serious if the piezometer is at least 5 tip diameters back of a hemispherical nose and 5 or 6 tip diameters ahead of the support rod.

The conclusion drawn from these tests is that the most reliable pitot tube is one of the combined type using a tip-pressure piezometer. This piezometer or group of piezometers should be located on the tip at a point where the rod effect is stable and affects all piezometers in the group about the same amount. Such an instrument may not read the true velocity head, but its error will be constant for all ordinary velocities and sizes of pipes, so that it is capable of being calibrated and used under different conditions of flow if necessary. Special types of combined pitot tubes such as the Pitometer having both the impact and reference orifices on the instrument itself give accurate results since both orifices are calibrated together.

Grateful acknowledgment is extended to Mr. Julien White and Mr. Shaw Cole for their assistance in making some of the tests and to Mr. E. S. Cole for his kindness in providing the four-contact angularity indicator. The helpful suggestions and criticism given by Prof. C. M. Allen, Prof. L. J. Hooper, Prof. W. S. Pardoe, and others is also acknowledged.

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Discussion

R. W. Angus.[3] The experiments recorded in this paper are of unusual interest and give quantitative values for the ideas long held regarding the nature of hydraulic flow. The author has used a most ingenious method for getting the average angle of flow of the particles within the pipe.

The writer is not surprised at the statement that the impact orifice in the pitot tube correctly registers the dynamic head, for his own experience is that the tube gives remarkably accurate results. Errors may easily result in the use of the tube due to the incorrect readings of the differential manometers used to indicate the velocity head, for if rubber hose is used between the tube and manometer, air may easily collect in it in such volumes as materially to affect the specific gravity of the fluid in the hose. This may be readily avoided, of course, if water is frequently blown through the connecting hose, but sometimes the pressure of the water being measured is low and then water cannot be blown through quickly enough.

The writer has not used the wall piezometer for the pressure element of the pitot tube, but would expect some inaccuracies due to it, because it is unusually difficult to measure pressures very accurately. Any irregularities in the pipe wall around the measuring holes will vitiate the readings. In order to study this problem, the writer made a careful series of experiments some years ago and published the results.[4] In this case, the writer primarily wished to know the accuracy with which pressure readings, measured at the outlet end of an elbow, could be taken. Additional studies were also made. The pipe was 6-in. standard steel pipe supplied with water from a tank 8 ft square, there being straightening vanes at the pipe entry. Four piezometer openings were made in the downstream flange of the elbow, two of the holes being in the plane of the bend and two on the other diameter. In all cases the holes were carefully drilled normal to the pipe wall. All holes were drilled $^1/_8$ in. with the inner end rounded to $^1/_{16}$ in. radius. Four open-topped glass tubes enabled the pressure at each piezometer opening to be read with accuracy. The water was delivered by the elbow into a 6-in. pipe attached by flange to the elbow.

Traverses by pitot tube, on two diameters of the supply pipe above the elbow, showed a good velocity distribution for pipe velocities varying from 6 fps to 17 fps. The four piezometer tubes showed considerable differences in pressure, the one in the plane of the bend and at the outside of it usually showing the greatest pressure. For a long-radius elbow the hole diametrically opposite the former (that is, on the inside of the bend) gave the next highest reading, while those at the sides of the bend gave the lowest readings. The average of the four readings differed by 0.2 ft from the true pressure at high velocities.

To measure the effect of obstructions close to the piezometer openings, the pipe into which the elbow delivered was first set concentrically with the opening in the elbow, the face of the elbow connected to the pipe being $^1/_2$ in. downstream from the piezometer openings. This was the condition for the above readings. When the eccentricity was $^1/_4$ in. there was a variation between the extreme piezometers of 4.5 ft, for a mean pipe velocity of 17.29 fps (for which the velocity head is 4.65 ft) as against a corresponding variation of 1.5 ft under the same flow when the pipe was concentric. These experiments proved that, even when the piezometer openings were carefully made, any irregularities in the pipe wall, even below the openings, would cause considerable errors in the pressure readings. By the same reasoning the ordinary roughness of the pipe wall would affect the reading to some extent.

With a pipe velocity of 6 fps the velocity head is 0.56 ft. Even if the impact opening of the pitot tube gave a correct reading, an error of 0.028 ft in the pressure reading would correspond to an inaccuracy of 5 per cent in velocity head. The writer's experience is that pressure measurements on the ordinary pipe wall cannot be read much more closely. Wherever the pitot tube is used the writer believes that the pressure element should be close to the impact element and on the same tube with it. This naturally enlarges the whole tube, but there is no difficulty in keeping the outside diameter of the tube down to a small proportion of the pipe diameter in cases occurring in ordinary practice.

The author might well have included in his bibliography a reference to a paper by A. Fage.[5]

J. E. Christiansen.[6] The author has answered many questions regarding the behavior of pitot tubes in pipes that have long remained unexplained. Especially noteworthy are his unique methods of studying the different characteristics of pitot tubes, as ordinary tests on pitot tubes are of little value in this respect.

The characteristics of pitot-static tubes in a wind tunnel were thoroughly studied by Ower[7] at the National Physical Laboratory, London. He found that the pressure determined by piezometers along the tip of a pitot tube was too low for a considerable distance back from the nose, but that the error became small in about 6 diam. He also found the rod to have the opposite influence on the piezometer pressure and for even a greater distance along the tip. These conclusions were later substantiated by the work of Merriam and Spaulding (9),[8] referred to by the author. Marks[9] has also contributed valuable data concerning the characteristics of pitot tubes, including the first published data that had come to the attention of the writer concerning transverse pitot tubes. All of these studies, however, have dealt primarily, or wholly, with the flow of air. Can the conclusions reached in these studies be broadly transferred to the case of flow of water in pipes, where confining boundaries, incompressibility of fluid, different density, and viscosity may have an influence on the magnitude of the effects produced by different pitot-tube designs? For example, does the influence of the nose or rod on the piezometer pressure of a combined tube extend as far along the tip in the case of water in a pipe as it does in air? The author has only partly answered this question.

The agreement between the measured velocity distribution across the pipe and the von Kármán equation (Fig. 15) is very good, considering that this equation is derived by dealing only with the distance from one wall. In the central region of the pipe the velocity distribution is undoubtedly also affected by the opposite wall. One should expect a velocity-distribution curve across the pipe that is continuous and symmetrical with respect to the center line; i.e., that du/dy at the center must be zero. This is not true of the von Kármán equation which indicates a finite value of du/dy at the center, and therefore an abrupt break in the curve at this point. A very good discussion of this subject is presented by Rouse (10), who comments that the von Kármán equation, "is valid for the wall regions in either case (referring

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[3] Professor of Mechanical Engineering, University of Toronto, Toronto, Ontario, Canada. Mem. A.S.M.E.

[4] "Errors in Pressure Readings Due to Wrong Gauge Connections," by R. W. Angus, *Journal of the American Water Works Assoc.*, vol. 29, April, 1937, p. 459.

[5] "The Estimation of Pipe Delivery From Pitot-Tube Measurements," by A. Fage, *Engineering*, vol. 145, June 3, 1938, pp. 616–617.

[6] Assistant Irrigation Engineer, College of Agriculture, University of California, Davis, Calif.

[7] "Measurement of Air Flow," by E. Ower, Chapman and Hall, Ltd., London, 1933.

[8] Numbers in parentheses refer to the authors' bibliography.

[9] "The Determination of the Direction and Velocity of Flow of Fluids," by L. S. Marks, *Journal of The Franklin Institute*, 217 (2), February, 1934, pp. 201–212.

to both smooth and rough walls) and, moreover, gives good results over the entire central portion of the flow."

Regarding curve *A* Fig. 18, stated to be computed from the Prandtl formula, Equation [18], should not this read Equation [17]? If du/dy is assumed to approach 0 as the center is approached (as evidenced by the author's test points as well as measurements by Nikuradse), the mixing length l need not become zero according to Equation [17] and may have a finite value, as indicated by curve *B* Fig. 18. There appear to be no physical reasons for assuming $l = 0$ at the center. In fact, there are good reasons why it should have a finite value, probably a maximum value at this place.

The writer has been primarily interested in the pitot tube as an instrument for measuring the flow from irrigation pumping plants, especially farm turbine pumps with horizontal discharge pipes ranging from 6 in. to 12 in. diameter. Pitot tubes more nearly approach the ideal instrument for this purpose because (*a*) they can be used in nearly all cases encountered, (*b*) they are readily portable and easily installed for test purposes, and (*c*) they are sufficiently accurate for most practical purposes.

Since the inauguration of a free pump-testing service by the major power companies in California, pump companies are depending more and more upon these tests as acceptance tests on pumps sold under performance guarantees. For such tests accuracy is essential, and differences of 2 or 3 per cent in computed efficiencies may mean the acceptance or rejection of a pump. Accuracy within these limits is difficult to obtain with field tests where conditions are frequently far from ideal for testing.

Transverse pitot tubes are widely used for testing irrigation pumps, although little has been published concerning them. With limited testing facilities available, studies were undertaken at the College of Agriculture, University of California, Davis, to determine some of the characteristics of transverse pitot tubes in pipe lines. The preliminary results of these studies have been published.[10] In these tests, the mean velocities determined with transverse tubes were compared with those computed from the flow measured with a free-flowing orifice which previously had been calibrated by volumetric measurements. In addition to transverse tubes in sizes ranging from $^1/_8$ in. to $^5/_{16}$ in. diameter, a number of other kinds of pitot tubes have been tested. Of particular interest are the tests of a conventional-type pitot tube made according to the suggestions of Ower,[7] who claims such a tube to have a coefficient of unity. This tube was made from $^5/_{16}$-in. diameter brass tubing, has a hemispherical nose, and six piezometer holes on the tip 6 diam from the nose and 8 diam from the rod. This tube can be inserted through a $^9/_{16}$-in. drilled hole, tapped for a $^3/_8$-in. pipe-thread stuffing box. The flow determined with this instrument from a 10-point traverse, each point representing equal areas, at a position in the pipe line about 20 diam below a partly opened gate valve and about 7 diam above the free-flowing orifice, averaged from 2 to 3 per cent greater than that determined with the orifice. Traverses have been made in horizontal, vertical, and both diagonal directions with consistent results. Extreme care was taken to set the tube at the correct position for each point. A water-air manometer was used in all of the tests.

At a position in the pipe about 6 diam below the partly opened gate valve, the flow determined with this pitot-static tube averaged from 6 to 8 per cent greater than for the orifice. This is explained by the fact that for the extremely turbulent flow the transverse currents produce a negative piezometer pressure, and therefore increase the indicated head. At this position, the velocity distribution across the pipe was unsymmetrical, the velocity being greatest near the top and to one side, whereas the gate opening was at the bottom.

Similar tests were made with a $^5/_{16}$-in-diameter transverse tube with two orifices exactly opposite each other. The flow determined with this instrument, based on 10-point traverses in two directions, averaged about 2 per cent greater than indicated by the orifice, but there was no difference in results, 6 diam or 20 diam from the valve.

Earlier studies of transverse tubes with single orifices showed that the velocity head obtained with the orifice facing upstream agreed very closely with that obtained with a conventional-type pitot tube and, therefore, indicated the true velocity. With the orifice in the downstream position, the mean velocity computed from the negative head agreed very well with the mean velocity past the net section, the area of the pipe less the projected area of the tube, even in the exaggerated case of a $^5/_{16}$-in. tube in a 1-in. pipe. The total indicated head for a transverse tube with both impact and trailing orifices is, therefore, slightly greater than twice the true velocity head for the full pipe section. The mean velocity determined from a traverse across the pipe agrees very well with the average velocity corresponding to an area equal to the area of the pipe less half the projected area of the tube, considering the manometer head equal to twice the velocity head. This correction, when applied to tubes of different sizes, brings the results into almost perfect agreement. It was therefore applied to all tests with transverse tubes.

In field work, where it has been necessary to use the instrument close to the pump head (elbow) it has been noticed that the conventional pitot-static tube indicates an appreciably greater flow than the transverse tube. As the result of these studies, the writer believes the transverse pitot tube to be a more reliable instrument for field tests than a conventional pitot-static tube. A transverse tube is not sensitive to direction and can be rotated about 10 deg from the correct position before any effect on the velocity head is noted. Upon further rotation, the manometer head decreases rapidly, becoming negative at approximately 63 deg.

The writer is in full agreement with the author's conclusion that it is easy to measure accurately the true dynamic head with the impact orifice, but that "it is practically impossible to measure the true pressure head at a point in the fluid." This applies also to a transverse tube. Nothing seems to affect the velocity head produced by the impact orifice, but the trailing orifice is extremely sensitive and the negative head is appreciably affected by the position of the stuffing boxes with respect to the inside of the pipe wall. When the stuffing box is back in the pipe wall, the cavity around the tube at the wall produces some disturbance that creates a large negative head near the wall and a lower negative head nearer the center of the pipe. This effect, illustrated in the paper,[10] was completely eliminated by soldering couplings to the outside of the pipe for the stuffing boxes and drilling holes through the pipe wall the size of the tube. The burrs on the inside of the pipe were carefully removed. Stuffing boxes protruding slightly into the pipe have very little effect on the trailing orifice. The true velocity distribution, across the pipe, however, cannot be obtained with a transverse tube with both impact and trailing orifices but can be determined by using only the impact orifice and wall piezometer.

Incidentally, where one has access to the inside of the pipe, very satisfactory wall piezometers can be made by soldering or brazing small pipe couplings or nipples to the outside of the pipe, and then drilling a small hole (about $^1/_8$ in.) through the pipe wall, being very careful to remove completely the burr on the inside of the pipe. For field tests, however, one seldom has access to the inside of the pipe, and a combined form of pitot tube is essential.

[10] "Characteristics of Transverse Pitot-Tubes," by J. E. Christiansen and O. C. French, *Agricultural Engineering*, vol. 18 (1), January, 1937, pp. 21–24.

E. Shaw Cole.[11] Distrust of the accuracy of pitot tubes in measuring the flow of water in pipes has grown in recent years because there seemed to be no law of variation of the coefficients less than unity which were obtained in pipe calibration.

It has long been known that the impact tip does not correctly register the cosine component of velocity when the water approaches the tip of angles greater than five or ten deg. The most frequent explanation of the low coefficients obtained in pipes has been the existence of angular flow, caused by turbulence.

When the author's method of measuring the mean angularity of turbulent flow indicated even less angularity than was reported by E. S. Cole in 1935 (8), it was apparent that some other explanation for the discrepancies in simple pitot-tube coefficients must be found. A study of the author's investigation of the effect of the support rod shown by his Fig. 28 did not explain Prof. W. S. Pardoe's and Prof. W. B. Gregory's results as reported in their discussion of "Pitot Tube Practice" (see Table 2), although the value of the rod correction studies reported in 1934 by Cole were confirmed.

TABLE 2. PITOT-TUBE COEFFICIENTS OBTAINED IN VARIOUS SMALL PIPES HAVING DIFFERENT PIPE FACTORS, AT UNIVERSITY OF PENNSYLVANIA AND TULANE UNIVERSITY

Experimenter	Size of pipe, in.	Pipe factor	Tube coefficient
W. S. Pardoe	8	0.971	0.995
W. S. Pardoe	8	0.875	0.980
W. S. Pardoe	4	0.858	0.982
W. B. Gregory	10	0.83	0.976

Pardoe in his discussion gave the results of calibrations of three different shapes of impact tips combined with wall piezometers in an 8-in. pipe. These tips are shown in Fig. 32. The counterbored tip showed a change in coefficient from 0.980 to 0.995 when the 22-ft length of test pipe was reversed so that the pitot station was located just downstream from a converging section. The wall piezometers were so located that the rod effect in the author's Fig. 28 did not apply.

It was, therefore, decided to test Pardoe's tip shapes in the same 12-in. pipes used by the author in order that some correlation might be obtained. In addition to the four different tip designs shown in Fig. 32, a single pitometer tip was also tested as a simple pitot tube. The standard pitometer using the customary impact and trailing tips was used to represent the combined form of pitot tube. A sketch of the pitometer is given in Fig. 5 of the author's paper.

The calibrations presented in Fig. 32 were made in three different types of flow in 12 in. pipes. The smooth pipe was the same as used by the author in his "Rod Effect Studies" and is shown in Fig. 26 of his paper. This pipe had been painted with a smooth waterproof paint which caused the pipe factor (ratio mean to center velocity) to have the rather high value of 0.925. The same test section was used for the rough pipe tests, the pipe being lined ahead of the test section with $^1/_8$-in. mesh wire screen (see Fig. 34) for a distance of 12 ft. The lining of screen terminated 2 ft 3 in. upstream from the pitot test section. The pipe factor in this pipe was 0.774. The third section was located in another pipe line which was located along the west wall of the laboratory. This pipe had a pipe factor of 0.874 which was intermediate between the smooth and rough sections described above.

In all cases for the simple pitot-tube test the wall piezometers were located at 45 deg to the axis of the pitot tubes and 6 in. upstream from the test section so that all "rod effect" would be avoided. The piezometers were carefully finished according to the recommendations of Allen and Hooper in their paper "Piezometer Investigation." (6)

[11] Field Engineer, The Pitometer Company, New York, N. Y.

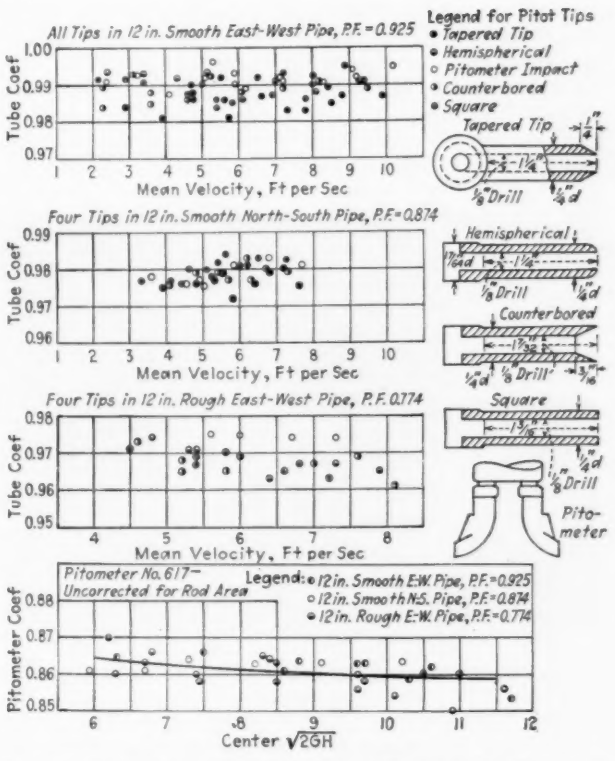


Fig. 32 Pitot-Tube Calibrations of Various Tip Shapes in Three Different 12-In. Pipes

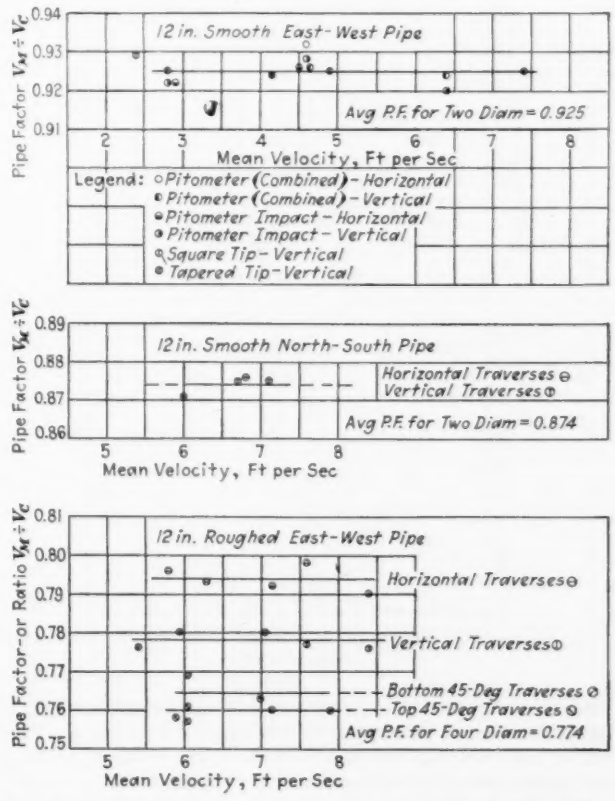


Fig. 33 Pipe Factors ($V_M \div V_C$) for Three Different 12-In. Pipes

Traverses to determine the pipe factors shown in Fig. 33 were made in all three test sections at several rates of discharge with both the simple pitot tubes and the pitometer.

These traverses, samples of which are plotted in Fig. 35, were made on two diameters 90 deg apart except for the case of the rough pipe, when irregularities in the screen lining made it necessary to traverse on four diameters 45 deg apart. In traversing,

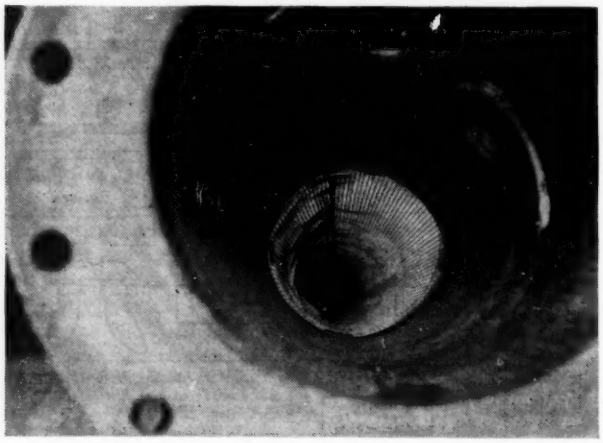


FIG. 34 VIEW LOOKING UPSTREAM IN 12-IN. ROUGHED EAST-WEST PIPE

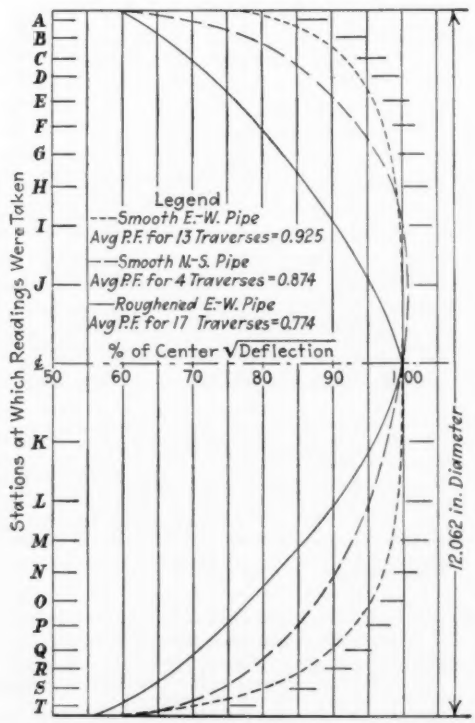


FIG. 35 SAMPLE VELOCITY TRAVERSES FOR THREE DIFFERENT 12-IN. PIPES

readings were made at 20 points on each diameter, these points being situated at the centers of ten rings of equal area. Five sets of forty readings were taken at the center of the pipe during the traverse of a single diameter.

The flow was measured by calibrated venturi meters which could be checked by a 40,000 pound weighing tank while the pitot tests were going on. Several check tests were made on these venturi meters.

These tests involved a large number of individual readings. The total number of readings in the smooth-pipe tests was more than 5000. In the rough pipe the traverses and calibrations at the center required more than 7000 readings. About 1600 readings were required for the traverses and tests at the center of the third or intermediate test section.

RESULTS

Results. The results of the calibrations are presented in Table 3 and Fig. 36 which presents a plot of pitot-tube coef-

TABLE 3 PITOT-TUBE COEFFICIENTS OBTAINED IN 12-IN. PIPES, HAVING DIFFERENT PIPE FACTORS, AT THE ALDEN HYDRAULIC LABORATORY, WORCESTER POLYTECHNIC INSTITUTE

Type of tip	12-in. unusually smooth E.-W. pipe P.F. = 0.925	12-in. smooth N.-S. pipe P.F. = 0.874	12 in. rough E.-W. pipe P.F. = 0.774
Simple tube with wall piezometer:			
Square	0.989	0.980	0.967
Counterbored	0.988	0.979	0.967
Tapered	0.985	0.976	....
Hemispherical	0.991	....	0.967
Pitometer impact	0.991	0.979	0.975
Combined-type pitot tube:			
Pitometer No. 617	0.862	0.862	0.860

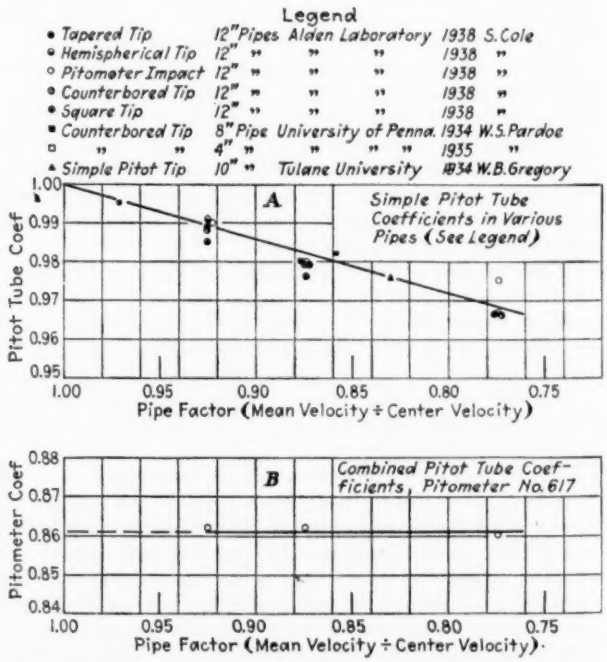


FIG. 36 RELATION OF PITOT-TUBE COEFFICIENTS TO PIPE FACTORS IN VARIOUS PIPES

ficients against pipe factor. It will be seen in the case of simple pitot tubes, Fig. 36*A*, that five tips of widely different design gave practically the same coefficient under similar flow conditions. On the other hand, the coefficient for each type of flow became smaller as the pipe factor decreased so that for a pipe factor of 0.774 the tube coefficient was 0.967. In addition the coefficients obtained at the University of Pennsylvania and at Tulane University were plotted with the others after computation had been made of the sample traverse curves presented in the discussion (8). The remarkable agreement of these points, as shown in Fig. 36*A* leads to the conclusion that simple pitot-tube coefficients in pipes vary directly with velocity distribution.

Wide experience in gaging the flow of commercial pipes and penstocks has demonstrated that the pipe factor for a straight run of pipe usually lies between 0.86 and 0.79. Referring to Fig. 36*A* it will be seen that these values correspond to pitot-tube coefficients ranging from 0.973 to 0.978, which have been so often reported in engineering literature.

Fig. 36*B* also presents the calibration results obtained with a combined form of pitot tube such as the pitometer. It will be seen that practically the same coefficient was obtained for all three types of flow distribution. The fact that the pitometer coefficient did not vary with the type of flow pattern demonstrates that no appreciable angularity was present in these three different types of flow.

Conclusion. Although the simple pitot-tube coefficient changed in pipes of different flow pattern, Fig. 36*A* shows that the pitometer coefficient remained constant in all three of the 12-in. pipes. Since about three quarters of the pitometer deflection is caused by the impact tip, if the change in the pitot coefficient were caused by pulsation effects, then the pitometer coefficient should have changed nearly as much as did the pitot-tube coefficients in the different pipes. Furthermore, if the change in coefficient had been caused by angularity effects the pitometer coefficient should have changed more than the pitot-tube coefficients. It can only be concluded, therefore, that the variation in pitot coefficient with change in pipe factor was due entirely to a decrease in the wall pressure reading, this decrease apparently depending on the type of flow distribution, caused by the pipe roughness, or the location of the gaging point.

The writer wishes to acknowledge gratefully the help and interest extended by Professors C. M. Allen, C. W. Hubbard, and L. J. Hooper.

E. A. Hitchcock.[12] To the writer certain of the discussions implied a vague knowledge of the early uses of the pitot tube. The following results obtained over a period of twelve years, 1900 to 1912, in the use of the pitot tube in measuring the flow of air and natural gas may therefore be of value.

From a partially written paper of the writer's, commenced more than thirty years ago but never completed, the following is quoted: "The first application of the pitot tube for natural-gas measurements in this country was made by Prof. S. W. Robinson, during the year 1886, when the gas wells opening up in Ohio had so high a capacity as not to admit of measurement by means of the gas anemometer which had been in use previous to that time. In 1890 the Ohio Geological Report contained a chapter written by S. W. Robinson on the Measurement of Natural Gas wherein is illustrated his first form of instrument for pipe-line flow, having what he called the combined static and dynamic tip.

"There were two forms of Robinson instruments, differing only in the design of tips, to be inserted in the line; i.e., the combined or expert pitot tip where both static and dynamic openings are in a combined form and the permanent or double-tip type, each tip of such a length as to reach the center of the line. The static tip was a duplicate in shape of the dynamic one but end closed and with parallel sides having the static openings."

Professor Robinson exhibited a setup of the application of the two forms to pipe lines at the Chicago Columbian Exposition of 1893. This display was accorded a Certificate of Award.

Years of commercial use and experimental work with the pitot tube convinced the writer that its accuracy depended largely upon an intelligent application and use. For example, the Robinson expert combined form, as furnished by a supply company, was found to give results 13.9 per cent too high. On the other hand a test pipe 2.1 in. in diameter, equipped with separate tips made of $^1/_4$-in. OD brass pipe, reaching to the center, and placed in a line 38 diameters or more from a fitting, gave correct results under accurate calibration by known quantities of air. Thirteen tests of this pitot pipe requiring over five hundred readings gave an average deviation from the true value of 0.1 per cent with a plus or minus of 1.45 and 1.88 per cent, respectively. The writer used this form of test pipe in the Ohio and Canadian gas fields extensively, in connection with commercial processes. For main gas-line work with lines up to 12 in. diameter, the writer used a traversing dynamic tip with a separate static tip, 6 in. following and $1^1/_2$ in. from the pipe wall. This tip was of $^1/_4$ in. outside diameter brass tubing, tapered to a point $2^1/_2$ in. upstream with $^1/_8$-in. static openings 2 in. from the point. In no case did he ever use an opening in the pipe wall for static pressure on account of pipe roughness. The above arrangement permitted a velocity curve for every case investigated.

It was in 1908 that Eugene Coste, Toronto, Canada, and the writer were called upon by the Union Natural Gas Company of Pittsburgh to determine the coefficients for three drawn-brass test tubes or pipes having 2 in., 3 in., and 4 in. diameters, each 12 ft in length. The static openings were in the side wall 9 ft 4 in. from the entering ends, and the dynamic tips, made of $^1/_2$-in. outside diameter steel pipe were located 10 in. beyond. The pipe interiors were exceptionally smooth. The calibrations were done by natural gas, the measuring element being a gas holder about 81 ft in diameter and having a net capacity of 5246.5 cu ft per ft of rise. All trials were carried on during the night in order to obtain constant temperatures about the holder. In all, 30 runs were made during the nights of June 3, 4, and 5, 1908, and the pitot pipe coefficients obtained were as follows:

TABLE 4 PITOT-PIPE COEFFICIENTS

4-in. tube	3-in. tube	2-in. tube
5134	3036	1345
5203	3073	1353
5207	3032	1368
5157	3065	1364
5190	3054	1355
5155	3061	1357
5156	3055	1362
Average 5171	3043	1381
	3067	1345
	3038	1369
	3044	1384
	Average 3051	1372
		Average 1363

These coefficients are tabulated in the order of gas flow from minimum to maximum which were as 64, 150, and 100 per cent, respectively, for the three tubes. These coefficients were determined by the following formula and are for cubic feet of natural gas per hour at a flowing temperature of 40 F, storage at 50 F, and 16.3 lb per sq in. absolute pressure. The specific gravity of the gas was 0.668.

$$\text{Coef} = \frac{\frac{5246.5}{16.3} \times L \times (Br \times p) \frac{510}{T_2} \times \frac{60}{\text{Time}}}{\sqrt{h(Br + P) \frac{500}{T_1}}}$$

L = lift of holder, ft
Br = pressure by barometer, lb
p = pressure in holder, lb
T_2 = temperature absolute of gas in holder, F
h = dynamic head of water, in.
P = pressure of flowing gas, lb
T_1 = temperature absolute of flowing gas, F

These pitot tubes or pipes were then used for standardizing the permanent pitot installations at measuring stations, a system in quite common use at that time.

[12] Dean Emeritus, College of Engineering, The Ohio State University, Columbus, O. Mem. A.S.M.E.

L. J. HOOPER.[13] The experimental work mentioned in the discussion by E. Shaw Cole suggested that the variation in coefficient of a simple pitot tube might be caused by the fact that the wall piezometers do not measure the mean pressure existing in the test cross section. This might conceivably arise from several causes such as faulty construction of piezometers or a variation in pressure across the traverse.

Piezometer errors were checked in a 12-in-square tilting flume 22 ft long. The entrance of the flume was made bell mouthed and a roof was provided for a length of 3 ft to obtain smooth flow conditions. A sliding gate at the downstream end of the flume was used as a backwater control. In operation the slope of the flume was adjusted for each discharge until the water surface and the bottom of the flume were parallel. The piezometer section was located 5 ft from the downstream end of the flume. The piezometers were made up in brass plates 3 in. wide inset into the flume wall so that the resulting wall surface was flush. Five piezometers were located in the bottom plate and three each in the two side plates. The orifices were made $^1/_8$ in. in diameter and the construction followed the recommendations made in a previous investigation.[14] Each piezometer was connected to a 3-in. stilling well. The water surface elevation in the wells and in the flume was measured by a traversing-point gage, mounted on a horizontal straightedge.

Thirteen series of tests, separated into three groups, were made with mean velocities ranging from 3.0 to 7.05 fps. For the first group of tests, the entire inside surface of the flume, including piezometers, was covered with galvanized-iron-wire screening of $^1/_2$ in. mesh. The average piezometer error as compared to the mean water surface was found to be —0.4 per cent of the mean velocity head. In the second group of tests, the wire screen was cut away for a radius of $^3/_4$ in. around each piezometer. The average piezometer error was found to be 0.8 per cent of the mean velocity head. In the third group of tests, the wire screening was entirely removed, leaving the flume walls smooth painted surfaces. The average piezometer error was found to be 0.7 per cent of the mean velocity head. Therefore, these tests indicated that the piezometer errors alone would not account for the low coefficients of simple pitot tubes as reported by E. Shaw Cole.

A number of tests were made to try to compare the pressure at the center of the pipe with that measured by a wall piezometer. A comparative calibration was made between a pitometer and a pitot tube in the 12-in. flume when running open and, with a roof on, as a pipe. The test results were inconclusive because of the difficulty of computing the rod-area correction factor for the open flume. Then a piezometer was mounted in a disk on the end of a rod and the 12-in. flume and a 12-in. pipe were traversed. It was found that the velocity errors of the rod were much larger than any differences of pressure that were present. The same was found to be the case for a traversing rod, equipped with a pair of piezometers in a wedge-shaped tip. Finally, a set of three geometrically similar pitot reference tubes was made up. The tips were 0.25 in., 0.54 in., and 0.84 in. in diameter and were mounted on the same $^1/_2$-in. traversing rod. A 12-in. pipe was traversed at various velocities with each rod. It was hoped that the test results would plot in a consistent manner so that a curve could be extrapolated to zero tip diameter but such was not the case. A study of the results indicated that the performance of the tip was affected by the point velocity past the tip, the change in area, and the change in velocity distribution during a traverse, so that the results contained variables which could not be readily evaluated.

These tests indicate that the measurement of pressure within a flowing fluid is a most difficult task. Either more refined instruments must be developed or a different approach used, before it can be proved experimentally that the pressure across a section of pipe is constant and equal to the wall pressure, as measured by a piezometer.

The fact should be emphasized that, even if it is found the indications of the wall piezometer differ from the mean pressure in the cross section by an amount sufficient to account for the reduction in the coefficient of the simple pitot tube, the amount of the error is still small and only important when the piezometer is used in a way that does not permit of calibration with the instrument. For instance, an error of about 5 per cent of the mean velocity head would be required to account wholly for a pitot coefficient of 0.975. If an error of 5 per cent of the mean velocity head were made in measuring the head on a water wheel, it would be negligible, since the entire velocity head only represents about 1 per cent of the total head acting on the wheel. However, this does not detract from the importance of solving this entire problem concerning simple pitot-tube coefficients.

It was believed possible that the diameter of the impact orifice might have an appreciable effect upon the velocity indication of the tube. For instance, in pipe flow the change in velocity is very rapid near the walls of the pipe so there can be a difference of velocity of several per cent across the diameter of a relatively large impact orifice. In such a case an average pressure effect is communicated to the manometer but it may well be that this does not represent the true average velocity for that interval.

Therefore, traverses at three different velocities were made in a 12-in. pipe with impact tips of three different diameters. The length of the tip was made the same in each case and the same support rod was used for all three tips. Two reference piezometers were used located 6 in. upstream and at 45 deg and 135 deg with the traverse diameter. The results of these tests are presented in Table 5.

TABLE 5 RESULTS OF TESTS IN 12-IN. PIPE AT THREE VELOCITIES AND WITH THREE DIFFERENT TIPS

Impact tube			
Outside diam, in.	Bore of tip, in.	No. of traverses	Pipe factor
0.094	0.063	6	0.810
0.297	0.258	3	0.811
0.540	0.490	3	0.816

NOTE: Pipe factor is the ratio of the mean velocity to the center-line velocity.

Thus it is seen that a $^1/_4$-in. impact tip used in a 12-in. pipe will give a value for the pipe factor which is within 0.2 per cent of the figure given by a tube of zero diameter. From this it appears that the size of the impact orifice has little effect upon the accuracy of the traverse for pitot tubes of normal design and reasonable size.

In conclusion, the tests mentioned in this discussion show that:

1 Properly constructed piezometers provide pressure indications which check the elevation of the free water surface within 1 per cent of the mean velocity head in an open channel.

2 Present laboratory technique and equipment do not allow the pressure within a moving fluid to be measured accurately.

3 The accuracy of the traverse is within 0.2 per cent for pitot impact orifices of normal design and reasonable size.

A. A. KALINSKE.[15] The writer is particularly interested in the information presented by the author regarding turbulence. The use of the angularity indicator is ingenious, and if it responds to the small velocity fluctuations should prove to be a useful instrument for the measuring of mean turbulence. The author's as-

[13] Assistant Professor, Department of Mechanical Engineering, Worcester Polytechnic Institute, Worcester, Mass. Mem. A.S.M.E.

[14] "Piezometer Investigations," by C. M. Allen and L. J. Hooper, Trans. A.S.M.E., vol. 54, 1932, paper HYD-51-1.

[15] Instructor and Research Engineer, Iowa Institute of Hydraulic Research, State University of Iowa, Iowa City.

sumption that the angles of flow are distributed according to the normal-error law has been checked by the writer in a photographic study made of the transverse velocity component at a point in a turbulent fluid.[16] Whether the indicator will respond and indicate accurately rapid velocity fluctuations, needs further investigation; this probably can be done by comparing results given by it to those obtained by the photographic method.

The author's derivation of the equations relating angularity and the velocity fluctuation components to pressure loss in circular pipes is not exactly correct. First it should be pointed out that the von Kármán equation of velocity distribution for turbulent flow in a circular pipe does not apply at the center of a pipe. Von Kármán pointed this out, when he introduced his assumption of similarity of the turbulent-flow pattern, in deriving the velocity-distribution equation.[17]

The author's Equation [14] for the shearing force on a cylindrical element of fluid is an expression for the average force. Actually, this force varies from instant to instant. If the expression for the average force is to be used, an expression must be found for the force at any instant and then the whole expression averaged. The elements composing that expression, such as the fluctuating velocity components v and Δu should not be averaged individually, but rather their product is averaged as it occurs in Equation [15]. The Reynolds equation for unit shear stress due to turbulence is

$$\tau = -\overline{\rho u v} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots [25]$$

ρ = unit density

The terms, u and v, are the longitudinal and transverse velocity components due to the turbulence (the term u corresponding to the author's Δu). In Equation [15] if both sides are multiplied by the density ρ, we have the equation for average unit shear. However, the quantity, $(u \tan \theta \Delta u)$, must be averaged as a product and not by individual terms.

The average shear at the center of the pipe is zero, since the mean velocity gradient is zero. This means that the mean product, $(u \tan \theta \Delta u)$, is zero; however, none of the individual terms needs necessarily be zero. Experimental data have verified this, particularly the data of Wattendorf.[18]

It should be pointed out that Equation [17] for the mixing length only plots as curve A in Fig. 18, when von Kármán's velocity-distribution Equation [7] is used to compute the mean velocity gradient. We know that this equation is not valid in the center of the pipe. Equations [18] and [20] are valid only for giving the general order of magnitude of $\tan \theta$, since the product $(u \tan \theta \Delta u)$ cannot be split and the mean value of each individual term used alone. At the center of the pipe v and Δu are uncorrelated, thus the mean product $(\overline{v \Delta u})$, becomes zero. Also, the term Δu need not always be positive near the center; thus near the pipe center Equation [18] for $\tan \theta$ is quite invalid.

The curve in Fig. 19, showing the distribution of cross velocities in a circular pipe, does not correspond to the data obtained by others. Wattendorf[18] for instance obtains, by means of hot-wire experiments in air, a curve that has a maximum value in the region of y/R equal to 0.1 to 0.2, and the data indicate a much more pronounced variation in the mean cross velocity between the center and a point near the wall. Experiments by the writer, in water using photographic methods to record the magnitude of the cross velocities, check in general the data of Wattendorf.

Regarding the effect of the pulsation of the manometer, it should be pointed out that, if the resistance to flow, in the manometer and connections thereto, in one direction is materially different from that in the other direction, an error can be introduced which might be larger than the error due to averaging the squares of the velocities instead of the actual velocities. Another way of indicating what effect the fluctuating velocity has on the manometer reading is to note that the reading at any instant is proportional to $(U + u)^2$, where U is the mean velocity and u is the variation, either plus or minus, about the mean. The arithmetic-mean value of u is zero, therefore, the average manometer reading is proportional to $(U^2 + \overline{u^2})$. If the value of $\sqrt{\overline{u^2}}$ is about 15 per cent of U, then the average value of the velocity as computed from the average reading of the manometer will be 1.011 U, or about 1 per cent too large. The value of $\sqrt{\overline{u^2}}$ hardly ever gets over 5 per cent of U, in ordinary pipe flow.

Regarding the use of tip piezometers on pitot tubes the writer wishes to refer to work done by Goldstein[19] and Fage [20] on the measurement of static pressure in a turbulent fluid. The reading of the manometer connected to tip piezometers is effected by the magnitude and frequency of the cross velocities in addition to being effected by the design of the tip and the number, size, and arrangement of the static-pressure holes. Fage shows that the static-pressure manometer reading is equal to

$$H = \bar{h} + K/g\,(\overline{v^2} + \overline{w^2}) \quad \ldots\ldots\ldots\ldots\ldots [26]$$

where $\bar{h}$ = the true static head
K = experimental constant
$\overline{v^2}$ and $\overline{w^2}$ = mean square cross velocities

For a circular pipe, Fage's experiments indicated the value of K as being about 0.28.

Since the longitudinal velocity fluctuation as measured by $\overline{u^2}$ tends to increase the reading from the impact orifice, and the cross velocities $\overline{v^2}$ and $\overline{w^2}$ tend to increase the reading from the pressure orifices, when the two readings are subtracted in order to obtain the true velocity head, the effects produced by the fluctuating velocities due to turbulence tend to cancel. Therefore, the author's conclusion that a properly designed combined type of pitot tube is the most reliable seems to be quite in order.

K. G. Merriam.[21] In a paper,[22] presented before the Society in 1903, written 173 years after the invention of the pitot tube, W. B. Gregory stated in his opening paragraph: "The object of this paper is to call attention of the members of the Society to a simple, efficient instrument for measuring the velocities of fluids."

Despite the alleged simplicity of the instrument, which, as described, is now termed a pitot-static tube, there seemed to exist some confusion of thought in respect to its performance. For example, in the paper mentioned (quoting from the reply to a discussion of a paper by Williams, Hubbell, and Fenkell, presented before the American Society of Civil Engineers in 1902) is found the following statement:

"Regarding the question of the proper formula for the pitot tube, the writers are not yet entirely satisfied and, while fully

[16] "Application of Statistical Theory of Turbulence to Hydraulic Problems," by A. A. Kalinske and E. R. von Driest. Presented at the Fifth International Congress of Applied Mechanics, Sept. 12–16, 1938, Cambridge, Mass.

[17] "Turbulence and Skin Friction," by Th. von Kármán, *Journal of the Aeronautical Sciences*, vol. 1, January, 1934, p. 7.

[18] "Investigations of Velocity Fluctuations in Turbulent Flow," by F. L. Wattendorf, *Journal of the Aeronautical Sciences*, vol. 3, April, 1936, p. 200

[19] "Measurement of Total Head and Static Head in Turbulent Streams," by S. Goldstein, Proceedings of the Royal Society of London, vol. 155, series A, 1936, p. 570.

[20] "On the Static Pressure in Turbulent Flow," by A. Fage, Proceedings of the Royal Society of London, vol. 155, series A, 1936, p. 576.

[21] Professor of Aeromechanics, Worcester Polytechnic Institute, Worcester, Mass. Mem. A.S.M.E.

[22] "The Pitot Tube," by W. B. Gregory, Trans. A.S.M.E., vol. 25, 1904, pp. 184–211.

appreciating Frizzell's effort to clear the matter up, they yet prefer to leave the question where they left it before, that, practically, it makes little difference in reduction of results whether a tube reading actually is $V^2/2g$ or V^2/g. The discussion presented by Seddon gives excellent reason to believe that there is a large account of internal forces to be balanced before the laws of flow will be fully understood, and until these forces have been more fully investigated, too rigid laws had best not be laid down."

Although thirty-five years later, the time is not ripe to balance the account and lay down too rigid laws, the coefficient of ignorance pertaining to pitot-static tube performance becomes smaller. The investigations which have illuminated the subject have been performed in several countries, using both water and air as fluid media. Much of the stimulation for these investigations came from the airplane and the ventilating fan. From necessity rather than from choice, air-flow problems associated with these devices have involved the use of the pitot-static tube as a basic instrument. Hence the vital need for a knowledge of its performance, especially in air applications.

Attempting to supply this knowledge, many experimenters have labored. In the Reports and Memoranda of the British Aeronautical Research Committee and in the N.A.C.A. Technical Reports, Notes and Memoranda, are numerous papers on the subject. In a valuable treatise by E. Ower[23] much space is devoted to pitot-static tube design.

From what has been written and, also considering the fact that experiments are generally conducted more conveniently and quickly with air than with water, one might infer that the author's work could consist only of verification of conclusions already arrived at as a result of wind-tunnel and air-duct research.

To a slight extent only is this true, although most of the tests described could have been performed with air. The author's ingenious and painstaking experiments, particularly those relating to turbulent flow, represent a valuable contribution to aerodynamics and general fluid mechanics as well as to hydraulics.

LEWIS F. MOODY.[24] The writer recommends this paper to students as a model of the way experimental research should be carried out and presented. Few experimenters in the hydraulic field are able to command the facilities for such a thorough investigation covering so large a number of tests of so many phases of a problem, with the assistance of so able a group of observers and on apparatus of ample capacity; but the method of analysis and manner of presentation can be followed by others to advantage. The author has not only given us clearly the physical results, answering by established facts the questions he has propounded, but he has analyzed the problem rationally and compared the results with well-digested theories.

The use of probability plotting in analyzing turbulence in flowing water is in itself a striking use of a mathematical tool admirably suited to the problem. The same method has been found most useful in hydrology in plotting duration curves of runoff, for instance, and in analyzing errors of observation in surveying. There are often occasions for using it in other engineering problems dealing with quantities controlled by so great a number of indeterminate factors that they are said to be governed by chance. Here the fact that the frequency summations of obliquity of flow actually plot on straight lines on probability paper substantiates the validity of the application.

The writer approves this paper and its conclusions in full realization that it considerably modifies his own early conceptions as to the magnitude of turbulence effects under ordinary flow conditions. At one period investigators, including the writer, were greatly concerned with errors in pitot tubes due to turbulence in the stream and carried out rather elaborate tests to find the effects; but there were few attempts to answer the difficult problem of how much turbulence usually exists, except by comparing over-all results of quantity measurements. Here few opportunities were found for full-scale comparisons, and then only with inexact or indirect means.

Although the author has convincingly demonstrated that the usual degree of turbulence in a good measuring conduit with stable flow is of a lower order than some of us had realized, so that the reduction of the pitot coefficient below unity cannot be justified on this ground alone, it cannot be concluded that a flow measurement by pitot tube should employ unity coefficient. The results of these investigations appear to absolve the pitot impact tip itself from suspicion of appreciable error due to either obliquity of flow or pulsations, but do not definitely account for the cause of the reduction of coefficient which must be applied to the flow measurement as a whole, in order to make it agree with a simultaneous determination of the discharge by an absolute method or one of established accuracy. Whether the reduction is due to a combination of factors or mainly to a particular factor has not been fully answered, but suspicion is directed pointedly to the static-pressure piezometers. This phase of the problem offers a fruitful field for a continuation of this line of research.

The discussion of the paper presented at the 1938 Annual Meeting of the A.S.M.E. contained some valuable supplementary material to which the writer would call particular attention. Professor Hooper has discussed the behavior of the piezometers and possible sources of error due to them and has described some significant experiments. Mr. E. Shaw Cole has analyzed the variation of the over-all coefficient applicable to a pitot measurement using a wall piezometer, as well as that applicable to measurements using Cole pitometers. His discovery of a simple and consistent functional relation between coefficients applicable with wall piezometers and the so-called "pipe factor" or ratio of mean to center velocity, is a notable contribution which brings order out of confusion. The curve representing this relation, which is simply an inclined straight line, seems to supply exactly the information which is needed to reestablish this method as a dependable means of water measurement, and should prove of great assistance, for example, to those A.S.M.E. Power Test Code Committees dealing with hydraulic prime movers and centrifugal pumps.

The value of this paper goes beyond the immediate purpose of the investigation, to analyze and evaluate pitot-tube errors, and extends into the field of the phenomenon of turbulence. One of the most convincing features to my mind is the comparison of the results with the Prandtl and von Kármán theories. Dr. von Kármán has expressed to the writer his dissatisfaction with his own theory as a final solution, with respect to its requiring zero mixing length at the pipe axis and a discontinuity in the velocity curve at this point. This limitation is clearly discussed by Professor Bakhmeteff (4). The methods and results of these experiments may throw further light on such fundamental questions. The writer suggests that by continuing this line of experimentation useful and far-reaching conclusions may be possible.

W. J. RHEINGANS.[25] Some of the author's tests confirm the results of previous investigations and conclusions, while others present entirely new ideas. He found that the coefficient of the impact tube is unity in quiet water, which confirms the results of

[23] "The Measurement of Air Flow," by E. Ower, Chapman and Hall, Ltd., London, 1927.

[24] Professor of Hydraulic Engineering, Princeton, N. J. Mem. A.S.M.E.

[25] Assistant Engineer, Hydraulic Turbine Division, Allis-Chalmers Manufacturing Company, Milwaukee, Wis. Mem. A.S.M.E.

numerous tests on impact-tube coefficients originated by Dr. W. M. White in 1901.[26]

The interesting feature of the present test is that a round-nosed impact tube was used. This confirms the theory advanced by Dr. White that any impact tube whose impinging surface is one of revolution will have a coefficient of unity. It would have been of added interest if the author had made a test on the effect of angular flow on the round-nosed type of impact tube.

The determination of the angle of flow in a pipe line was extremely interesting. However, the writer calls attention to the fact that while the mean angle was found to be only about 2 deg, there were momentary maximum angles of 8 to 10 deg. Since the impact coefficient for angular flow is not a straight-line function of the angle but approaches unity asymptotically, the average error for angular flow is slightly greater than the error at 2 deg.

Equally interesting were the author's tests on the effect of support rods on wall piezometers. Unfortunately in the analysis of these tests he omitted a clear definition of the symbol a which he used in Equations [21] to [24]. In the text he states that a represents the projected area of the rod in sq ft. If the reader accepts this statement and if the author used this definition in his work, it can readily be shown that K in Equation [24] is not $2a$, and that Fig. 28 is therefore in error.

However, if the author assumes that a is the ratio of the rod area to the pipe area, then his subsequent formulas and Fig. 28 are correct. Since no original data are given, it is impossible to tell from the text which definition of a was used in arriving at Fig. 28. There is a clew in a subsequent paragraph as to what the author probably intended a to represent. It would have eliminated confusion if he had definitely defined a as being the ratio of rod area to pipe area and then simply stated that the wall-piezometer error is equal to the contour value in Fig. 28, multiplied by the ratio a for the rod and pipe under consideration.

In attempting to use Fig. 28, further confusion arises from the interchangeable use of relative error and relative percentage error by the author. For nstance, the contours in Fig. 28 supposedly show relative errors. However, the text immediately following states, "the wall-piezometer error in per cent of mean velocity should be equal to the contour value in Fig. 28 multiplied by $2d/\pi D$." What the author apparently means is that the contour value in Fig. 28 multiplied by $2d/\pi D$ multiplied by 100 will give the piezometer error n per cent of mean velocity. Since no original data are given, it s impossible for the reader to check back and determine which is correct.

The above observations may seem hypercritical, but Fig. 28 is of such potential value in the determination of possible errors in pitot-tube tests that all doubts as to the exact values should be eliminated.

Although Fig. 28 indicates that the support rod affects the reading of the wall piezometer, this does not necessarily mean that the measured deflection between the pitot-tube point and the wall piezometer is in error by an equal amount, nor do any of the author's tests definitely show that this might be the case. Fig. 28 indicates that due to an increase in velocity near the support rod, the pressure at the wall piezometer drops. Now there are several theories supporting the assumption that, when the pressure at any point in a cross section of a closed conduit changes, there is a similar change at all other points in that cross section. If the pitot-tube impact point is located in the same section as the wall piezometer, any change in pressure at the wall will be accompanied by a similar change in pressure at the pitot-tube point and thus tend to eliminate any error in deflection.

On the other hand, Fig. 28 indicates an apparent increase in pressure on the wall piezometer for certain locations of the support rod in relation to the piezometer. Just why this is true is not immediately apparent. Is it possible that the support rod lowers the velocity of flow immediately upstream and for a short distance on either side; that this lower velocity causes an increase in pressure, and that this increase in pressure is not transmitted across the entire section? It would seem that further investigations along this line would be advisable in order to get a true explanation of the varying piezometer readings.

The writer calls attention to the fact that the piezometer errors indicated in Fig. 28 are of relatively small magnitude when the size of the support rods is kept within practical limits, and if the method usually applied to traverses in large pipes is used, which consists of inserting the pitot-tube rod from the opposite sides of the pipe up to the center only. The author's subsequent example, for instance, indicates an error in center velocity of 1.6 per cent. However, in measuring the flow in a circular pipe the center velocity has relatively small influence on the mean velocity. Thus, as the support rod is pulled back toward the wall of the pipe, where the greatest amount of flow occurs, the projected area of the rod is reduced. The author has shown no data indicating the effect of a reduced rod area on the piezometer error, where the reduced area is occasioned by a shorter support rod; i.e., a rod not reaching to the center of the pipe. However, since he has plotted the piezometer errors on the basis of the ratio of the rod area to the pipe area, it is natural to assume that the error is proportional to the percentage rod area, even when the rod is partly withdrawn.

Making this assumption, it can be shown that the actual traverse error due to piezometer error would be in the neighborhood of 0.60 per cent instead of the 1.6 per cent indicated for the center velocity for the conditions assumed in the author's example. Furthermore, in an actual test it would be more in line with actual practice to use a $^1/_4$-in. streamlined rod in a 12-in-diameter pipe than the 1-in-diameter rod used in the author's example. With a $^1/_4$-in. rod, the wall-piezometer error for a complete traverse, as obtained previously and for conditions and location of piezometer similar to that used in the author's example, would be less than 0.2 per cent. In fact, a 1-in-diameter support rod would be feasible in a 6-ft-diameter pipe, in which case the piezometer error for the foregoing conditions would be less than 0.1 per cent.

Thus, while at first glance the piezometer error indicated by the author and by Fig. 28 may seem of considerable magnitude, in actual practice it is negligible.

The author concludes from his tests that the most reliable pitot tube is one of the combined type using a tip pressure piezometer, and that special types of combined pitot tubes such as the pitometer having both the impact and reference orifices on the instrument itself, give accurate results, since both orifices are calibrated together. This conclusion does not quite check with other statements made by the author. For instance, he states that it is apparent that "piezometers located within 2 or 3 diameters of the rod will be unstable and, therefore, inaccurate."

Also, the author's Fig. 25 indicates that, with the impact tube close to the support rod, erratic results are obtained. For instance, with the impact tube 1 diameter from the support rod variations of 0.5 per cent were obtained. The illustration shows that the impact tube should be at least 3 (rod) diameters from the support rod to give consistent results.

Since most pitot tubes of the combined type, especially the pitometer, have both the impact orifice and the piezometer orifice located near the support rod, it would seem, according to the author's own statements and tests, that these types are subject to inaccurate results in spite of their calibrations. Furthermore,

[26] "The Pitot-Tube; Its Formula," by Wm. Monroe White, *Journal of the Association of Engineering Societies*, vol. 27, July-Dec., 1901, pp. 35–79.

it would seem that pressure orifices located on the support rod are subject to the same type of errors as indicated for the wall piezometer in Fig. 28 because, if the support rod introduces a change in the pressure at the wall, why should there not be a similar change in pressure at different points across the cross section?

It is also hard to understand why it should be easier and more accurate to measure pressures in the center of a pipe than it is to measure them with wall piezometers. Certainly angular flow, turbulence, etc., must have varying effects on piezometer orifices located within this flow, which is not taken care of by calibrations on combined pitot tubes in quiet water.

It would therefore seem that this divergence of opinion suggests several further lines of investigation, as follows:

1 The effect of angular flow and turbulence on the static pressure orifices of combined types of pitot tubes. (Although Cole has made tests along these lines[27] further tests are needed to obtain more detailed information.)

2 Determine whether pressure orifices located on support rods in the flowing stream or wall piezometers will give the most consistent and accurate results.

3 The effect of a short support rod (such as when measuring velocity near the walls of a conduit) on wall-piezometer readings.

In conclusion, the writer calls attention to the fact that the entire paper indicates that the pitot tube, if properly used, is an accurate means for measuring flow of water.

HUNTER ROUSE.[28] Pitot tubes, like piezometers, have long been used and misused, and often subjected to analytical and experimental scrutiny. Not until the rise of modern aerodynamics, however, did the pitot tube take on the aspects of a precision instrument. The requirements of fluid research in this field have led to very careful studies of pitot-tube characteristics for a wide variety of shapes and flow conditions. The author will find, for instance, an excellent résumé of results, similar to many of those which he now reports, in a translation of a German work by Peters.[29]

Although the tests described in this and similar aeronautics publications were conducted in air, pitot-tube performance in water should obviously be similar to that in air under dynamically similar flow conditions. In this regard, however, one must not lose sight of the geometric scale relationship between the dimensions of the instrument and the dimensions of the turbulence in the measured flow. For instance, the degree of wind-tunnel turbulence is usually expressed by the ratio of mean absolute velocity of fluctuation to the mean velocity of flow, a factor akin to the angularity discussed by the author. Nevertheless, research men in aerodynamics do not ignore the fact that the linear scale of the turbulence, i.e., the "mixing length," or the size of the eddies involved, is also of fundamental importance. The angularity caused by the turbulent mixing process can have an appreciable effect upon a measuring instrument only if the linear scale of the turbulence is at least of the same order of magnitude as the scale of the instrument; likewise, an instrument can correctly measure such angularity only if its scale is far smaller than that of the turbulence.

[27] "Pitot Tubes in Large Pipes," by E. S. Cole and E. Shaw Cole, Trans. A.S.M.E., 1939, vol. 61, p. 465.

[28] Associate Hydraulic Engineer, Cooperative Laboratory, Soil Conservation Service; Assistant Professor of Fluid Mechanics, California Institute of Technology, Pasadena, Calif.

[29] "Handbuch der Experimentalphysik," vol. 4, part 1, by L. Schiller: Translation in manifold copy form of section from vol. 4, part 1, entitled, "Pressure Measurement," by H. Peters, in which static-pressure and dynamic-pressure instruments are discussed. Copies are on file at the Engineering Societies Library, at the National Hydraulic Laboratory, and at the United States Waterways Experiment Station.

The author has devised an intriguing type of apparatus for measuring angularity. Such equipment is particularly needed in the study of liquid motion, for the hot-wire anemometer, so valuable in the study of turbulence in air, does not lend itself so readily to use in water. The angularity indicators described by the author must be considered commendable first steps in this direction; but they cannot always be expected to yield accurate quantitative results. That is, while the best hot-wire anemometers used in wind tunnels are but a small fraction of a millimeter in size, the author has developed an indicator with vanes approximately 1 in. long for use in pipes where the eddies are often much smaller.

With respect to the author's analysis of the turbulence mechanism, certain amplifications are in order. The equations of Prandtl and von Kármán to which the author makes reference are specifically means to an end, namely, the formulation of simple, usable, and fairly general expressions for velocity distribution and resistance, and not rigorous statements describing at once the internal mechanism of flow at all parts of the cross section. The pipe measurements presented in this paper, as well as in the references cited, include three distinct portions of the flow section—the boundary zone, the central zone, and a zone between the two. Prandtl's equation for the velocity distribution was developed for the boundary zone (this zone is not to be confused with the very thin boundary layer). That this equation also yields good velocity results nearly to the center of the section in no way insures the validity of the mixing-length distribution curve derived therefrom (curve *A*, Fig. 18) for more than the boundary zone. Von Kármán's similarity hypothesis (i.e., constant correlation between radial and longitudinal velocity fluctuations) has been shown by hot-wire measurements to be correct only for the intermediate zone. The resulting equation (Fig. 15) does not apply near either the wall or the central axis. The fluctuations show no correlation at the axis of the pipe, approaching in this zone the random nature typical of isotropic turbulence in the developed wake of a rack or grid; at the axis, moreover, the mixing length should attain its maximum value.

Evidently, these velocity relationships may be expected to yield the probable distribution of the mixing length and the velocity of fluctuation only in the zones for which they were originally developed. A better indication of true conditions over the cross section was found by Nikuradse through determining the values of l and of $\epsilon = vl$ from measured velocity profiles by means of the following relationships

$$l = \sqrt{\frac{\epsilon}{\frac{du}{dy}}} \qquad \epsilon = \frac{\frac{\tau}{\rho}}{\frac{du}{dy}}$$

$$\frac{\tau}{\rho} = \frac{\tau_0}{\rho}\left(1 - \frac{y}{R}\right) \qquad \frac{\tau_0}{\rho} = \frac{8U^2}{f}$$

In the diagram Fig. 37 will be found dimensionless plots of l, ϵ, and $v = \frac{\epsilon}{l}$ determined in this manner, containing, to be sure, the unknown proportionality constant mentioned by the author. As was shown in Fig. 18, this curve for the mixing length agrees with that found from Prandtl's velocity equation only in the boundary zone for which the equation was developed. It will also be noted that the magnitude of v increases steadily toward the boundary, contrary to the author's measurements in Fig. 19. One must recall, however, that Prandtl's original assumption upon which these curves are based $\left(\text{that } v \text{ is proportional to } l\frac{du}{dy}\right)$ is approxi-

mate, but not exact, and cannot be expected to hold even closely at the wall or at the axis.

Nevertheless, actual hot-wire measurements of the root-mean-square velocities of fluctuation[18] also indicate that these velocities continue to increase toward the wall, the form (though not the absolute position) of a measured distribution curve being shown by the broken line. It will be noted that v changes only slightly in the central (isotropic) zone and increases almost linearly throughout the intermediate zone of constant correlation. Indications are that it reaches a maximum a short distance from the wall, then drops off rapidly. More recent measurements in the wind

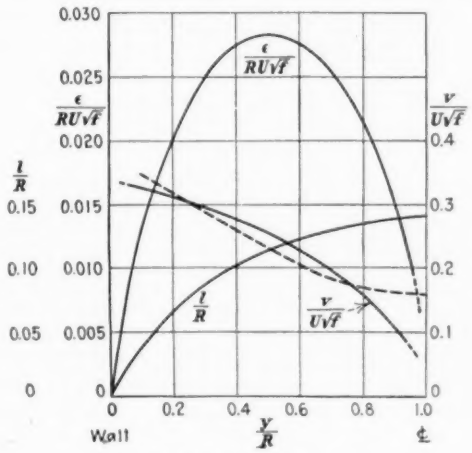


FIG. 37 CURVES FOR l, v, AND ϵ

tunnel, as well as an analysis of older studies in the Mississippi River,[30] prove the general validity of this trend.

It should be evident that the author's "theoretical" curve for angularity Fig. 17 shows the greatest discrepancy, even according to sound theoretical analysis, at the central axis, while his measurements are probably closer to the actual values in this region than in any other. Since the mean velocity decreases radially, moreover, it follows that the angularity must increase toward the wall even more rapidly than does the velocity of fluctuation, and not remain essentially constant as the author's measurements would seem to indicate. Finally, one cannot well state that any given mean angularity is typical of flow in pipes; for fully developed turbulence, the form of the v/U curve would seem to be constant, while that of the u/U curve definitely is not, varying with both the Reynolds number and the relative roughness.

The failure of the author's measurements to agree with actual conditions can be traced directly to the fact that his angularity indicator is too large to be affected by the average eddies in all but the central regions of flow. In other words, although the angularity increases steadily toward the pipe wall, the indicator is not sensitive to such variation because the average size of the eddies decreases toward the wall in direct proportion to the mixing length. Obviously, however great the angularity, it will have no marked effect upon either an angularity indicator or a pitot tube so long as the instrument is enough larger than the eddies involved.

The writer does not mean to disparage the author's development of such an instrument. Indeed, he considers the effort to perfect a device of this nature by far the most valuable portion of the investigation now under discussion, for the research world will be everlastingly indebted to the man who produces satisfactory equipment for measuring the degree of turbulence in flowing water.

[30] "Turbulence," by Th. von Kármán, *Journal of the Royal Aeronautical Society*, vol. 41, 1937, pp. 1136–1137.

R. B. SMITH.[31] The author's creditable study of pitot-tube characteristics points to the fact that difficulty with instruments of this type is generally associated, as is frequently the case in hydro and aerodynamics, with the measurement of static pressure.

The problem of static-pressure measurement has been attacked numerous times but, generally overlooked in this country, is the very interesting work of Fuhrmann,[32] who investigated the in-

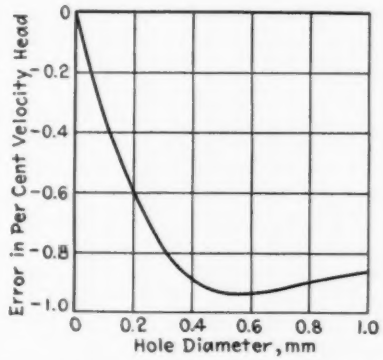


FIG. 38 EFFECT OF HOLE SIZE ON STATIC PRESSURE MEASUREMENT

fluence of static hole size. The results, Fig. 38, indicate that for nearly all commercial holes, the static-pressure reading is too low by about 0.8 per cent. This conclusion applies to all hole sizes larger than $^1/_{16}$ in. and is not in contradiction to the work of Hooper and Allen (6) inasmuch as the studies by them were made comparatively, with holes larger than $^1/_{16}$ in. Inasmuch as the head and velocity vary in a quadratic relation, an error of 0.8 per cent in head is sufficient to explain a pitot-tube coefficient of 0.996.

W. M. WHITE.[33] The determination of the coefficient of the pitot-tube point seems to have been carefully done. It is a long way from the formula for the pitot tube given in Kent's Handbook in 1900 as $V = \sqrt{gh}$ to the present-day accuracies.

In 1900 the writer determined that the pitot-tube point had a coefficient of unity by submerging the point in front of a boat traveling at known velocity. Reference has been made to a paper[26] by the writer. It was shown by the paper that the pitot-tube point would read correctly without unity coefficient, provided the point was a surface of revolution in respect to the axis of the pitot-tube point.

The second section of the author's paper is particularly interesting since he has shown that angularity of flow is of such small magnitude as not to affect seriously the coefficient of the point of the tube.

The section on pulsation tests is of particular interest, because possible errors in the reading of the pitot tube, due to the pulsating effect have been known. In pitot-tube work, where great accuracy was required, we have inserted in the connections between the pitot-tube point and the glass gage, a small double-ended venturi meter having a small hole of say $^1/_4$ the diameter of the hole in the tube itself, the shape at each end being the same, so that the pulsation effect on the reading of the tube was thus eliminated.

[31] Development Engineer, Elliott Company, Jeannette, Pa. Jun. A.S.M.E.

[32] "Theoretische und Experimentelle Untersuchen au Ballonmodellen," dissertation by Fuhrmann, Göttingen, Jahrbuch Motorluftschiffstudiengesellschaft, 1911–1912, vol. 5, p. 63.

[33] Manager and Chief Engineer, Hydraulic Department, Allis-Chalmers Manufacturing Company, Milwaukee, Wis. Mem. A.S.M.E.

The section of the paper dealing with the effect of the rods or supports, which are necessary to be used inside the pipe upon the reading of the pitot-tube point and piezometers, is an excellent contribution to the art. It is unfortunate, however, that an error has been made in the formula on which the results in Fig. 28 are based. This error was discovered by Rheingans who has suggested a correct formula. When Fig. 28 is corrected and is properly used in connection with the obstructions actually used in making pitot-tube tests on 12-in. pipe and larger, the correction to be applied is so small as to be negligible.

It is unfortunate that the author used such a large rod in the 12-in. pipe to arrive at the results plotted in Fig. 28. Probably a small rod comparable to the actual rods used in pipe would have shown such small deflections as to be unreadable. If we step up the 1-in. rod in the 12-in. pipe to a test in a 10-ft-diameter pipe, we should then have a 10-in. pipe for supporting our pitot tube which, of course, is absurd. When we apply the corrected data for Fig. 28 to tests which we have heretofore made by pitot tubes using point and pipe piezometers and taking into consideration the actual supports used, together with the locations of the piezometers, we compute from Fig. 28, the effect on the readings to be less than 0.25 per cent.

The writer is interested in the conclusions to which the author has been led by his tests, and particularly his statement: "To obtain a stable coefficient near unity in a tip-pressure tube, the tip should be at least 10 diameters in length with the pressure piezometers located 5 diameters back from the nose."

This conclusion confirms the writer's long held opinion that the heretofore used tip-pressure pitot tubes were liable to sources of considerable error. In fact, the unreliability of this type of tube, which was used by the writer in 1899, led to the investigation resulting in the paper.[36] It is certainly comforting to know that the careful data secured by the author leads us to a more rational design of a tip-pressure pitot tube.

The writer is not in accord with the conclusion drawn by the author in respect to the tip-pressure tube, and cannot see from a consideration of the observed data, how he can make the statement: "The conclusion drawn from these tests is that the most reliable pitot tube is one of the combined type using a tip-pressure piezometer. This piezometer or group of piezometers should be located on the tip at a point where the rod effect is stable and affects all piezometers in the group about the same amount. Such an instrument may not read the true velocity head, but its error will be constant for all ordinary velocities and sizes of pipes, so that it is capable of being calibrated and used under different conditions of flow, if necessary."

The writer agrees with the author that the source of error lies in the securing of a proper pressure reading. This pressure reading should be secured in a region where the velocities are least, which is at the walls of the pipes and not by piezometers on the tube itself in the high velocities, especially at the center of the pipe. The piezometer 5 diameters in front of the rod will be influenced by the position of the tube in the pipe, particularly in the vicinity of the pipe wall, where the area is of considerable moment in determining quantity. Its coefficient must be affected as a greater and greater amount of rod is introduced into the pipe in traversing a diameter of the pipe.

The author has certainly given enough data to eliminate from further serious consideration those types of tip-pressure tubes which have the pressure opening in line with or downstream of the supporting rod.

The coefficient as determined for the pitot tube with wall piezometer was seriously affected when the wire mesh was inserted in the pipe, due to the location of the wall piezometers within the influence of the sudden enlargement from the diameter of the pipe with the wiring mesh to the normal diameter.

Everything the author has developed confirms the writer's opinion that the most reliable method of making quantity determinations by means of the pitot tube is by that method described in the A.S.M.E. Hydraulic Prime Movers Power Test Code, approved June, 1938.

Author's Closure

The author is indebted to those who have discussed this paper and added so much useful information to it.

The comment by Mr. Christiansen concerning a correction for the area of the rod in small pipes is interesting in the light of tests on the pitometer reported by E. S. Cole (8).

The discussions by Mr. Kalinske and Professor Rouse concerning the method of deriving the equations for the so-called theoretical angularity are enlightening. However, the author attempted what was intended to be a simpler if less exact derivation of the theory of turbulence which would form a framework for those who have not had the time to study the subject of turbulence in detail.

Mr. Rheingans and Mr. White have called attention to the section of the paper dealing with the effect of the support rod on the wall-piezometer reading. While the wording may not be clear, there is no mistake in the equations as given in the paper. The use of the relative errors as given in the example explains the method of correction. These tests were intended to show where piezometers should be located with respect to the rod to minimize the rod influence and therefore are exaggerations of what would be considered good practice for field work. It is unfortunate that lack of space made it necessary to omit the cross plots of the original test data from which the curves of Fig. 28 were derived. The tests at three different velocities with three sizes of rod checked within 1 per cent of the velocity head.

Professor Rouse's comments add greatly to the theoretical discussion of turbulence. He may have been led to doubt the measured angularity in the 40-in. pipe by the fact that the wall roughness was not constant along the whole pipe, so that the tests were made in a section where the flow conditions were in the process of transition from rough to smooth pipe. As described in the paper, the upper portion of the pipe was extremely rough, the portion just above the test section was comparatively smooth. It is possible that the higher turbulence generated in the rough section persisted at the center well into the smooth section, while the turbulence near the wall in the rough section had decreased to a lower value after entering the smooth section. This led the author to draw the conclusion that in large pipes the angularity is nearly constant across the pipe.

Recently the author has made tests with a smaller vane in 12-in. pipes. While this vane appears large when compared with the hot-wire anemometer used in air measurements, its dimensions are only 2 per cent of the diameter of the 12-in. pipes in which the tests were made, or about twice the impact-orifice diameter of most of the pitot tips which were used.

This smaller vane was made of cork, $^1/_4$ in. high by $^5/_{16}$ in. long. Its shape and mounting are shown in Fig. 39. The weight of the grounded platinum contact on the vane was sufficient to make the whole movable element weightless in water. The testing was simplified by connecting the platinum contacts behind the vane, one at a time, to a commercial electric stop clock having a clutch action capable of moving in 0.005 sec. The stop clock integrated the total time in which a particular contact was "made" in a given measured time interval, usually taken as 2 min. The mean angularity was computed from plots on probability graphs as described in the paper. With this instrument, approximately one third the size of the one described in the paper, the results obtained checked well with the tests originally reported. The tests, which are shown in Fig. 40, show an increase in angularity

toward the pipe wall, but not enough to influence the reading of a pitot impact tip.

With the present knowledge of turbulence it is difficult to establish relations between turbulence, wall roughness, and velocity distribution across large pipes, for most pipes in which tests have been made do not have a sufficiently long uniform straight section for the flow conditions to become stable. For example, the



FIG. 39 SMALL ANGULARITY VANE

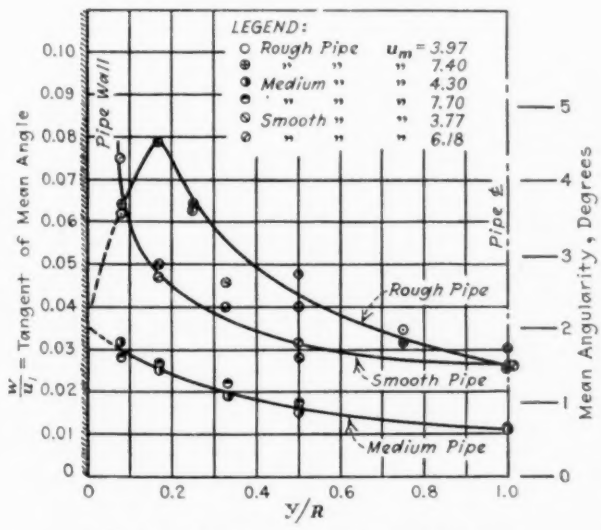


FIG. 40 MEAN ANGULARITY IN 12-IN. PIPES
(Measured with 1/4 × 5/16-in. vane.)

measured angularity in the smooth 12-in. pipe is greater than would be expected with the high pipe factor found by traversing. This may be caused by the venturi meter only 15 diameters upstream from the test section as shown in Fig. 26. The rough pipe, which was the same pipe lined with wire screening, has the same turbulence at the center, but increased turbulence near the walls. The sharp reduction in turbulence close to the wall in the rough pipe was probably due to the fact that the wire screening terminated about one foot upstream from the test section. The medium pipe, which had an intermediate pipe factor, showed the least turbulence of all. This may have been caused by straightening vanes located in an elbow approximately 30 diameters upstream from the test section.

The interesting results of the tests reported by Mr. Cole in his discussion certainly add much to the information concerning pitot tubes, although the exact reason for the variation of the wall-piezometer reading with pipe factor is not evident. The apparent reason is that turbulence varies uniformly with the pipe factor, but the angularity measurements of Fig. 40 made in the same pipes do not show this to be so.

The author has recently made tests in the same pipes that were used by Mr. Cole, both with a pitometer and also with a pitot-static tube constructed according to the recommendations mentioned in the conclusion of the paper. Mr. Cole's pitometer results were checked within 0.5 per cent, although some of the pipe factors had changed because of roughening of the pipe walls. However, it was disconcerting for the author to find that the coefficient of the pitot-static tube varied from 0.956 in the center of the rough pipe to 0.992 in the medium pipe. The pitot-static tube was calibrated in the center of each of the three pipes.

It was finally found that a small tight-fitting ring placed ahead of the pressure or static orifice on the tip of the instrument not only changed the coefficient of the pitot-static tube, but in certain positions reduced the discrepancy between the coefficients obtained in the various pipes. In fact, with the ring immediately ahead of and tangent to the upstream edge of the pressure piezometer, a nearly constant coefficient of about 0.89 was obtained in all the pipes. The results of tests with and without the ring are given in Table 6. The pitot-static tip with the ring moved away from the piezometer is shown in Fig. 41.

TABLE 6

12-in. pipe	Pipe factor	Mean angle at center, deg	Average pitot-static coefficient	
			Without ring	With ring
Rough	0.77	1.56	0.956	0.889
Medium	0.83	0.70	0.992	0.897
Smooth	0.92	1.65	0.980	0.885

Tests made at velocities of from 4 to 8 fps at center of pipe.

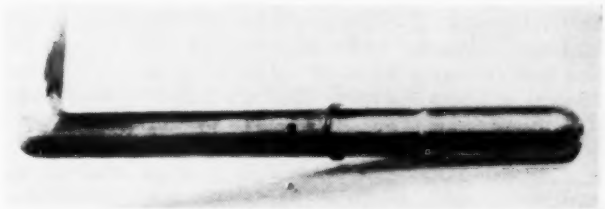


FIG. 41 PITOT-STATIC TIP WITH RING

The coefficients obtained in each pipe were constant within 1 per cent over the velocity range tested. There does not seem to be any consistent relation of the coefficient to either the pipe factor or the measured turbulence. The results certainly do not agree with the formula proposed by Fage given as Equation [26] in Mr. Kalinske's discussion.

Thus far only the velocity pulsations across the stream have been measured in connection with these tests. Since the turbulent pulsations have three degrees of motion, it is possible that some of these phenomena will be explained when measurements of the velocity surges in the direction of flow as well as the cross pulsations are made.

The information given in this paper and the discussions show definitely that the error of the impact orifice of a pitot tube is negligible in ordinary turbulent flow. The errors occur in measuring the pressure head or some other reference pressure. These errors are not a constant function of the velocity head, but are associated in some manner with turbulence. The coefficient of

the instrument is therefore likely to change in different conditions of flow.

The effect of the support rod on pressure piezometers can be largely avoided. However, until more has been learned about the effect of turbulence on pressure measurements, it will be necessary to use the coefficients for simple pitot-tubes suggested by Mr. Cole as given in Fig. 36 of his discussion. If a combined form of instrument such as the pitometer, or the pitot-static tube with a break in the surface near the pressure orifice as suggested by the author is used, then the instrument should be calibrated in at least two different types of flow conditions to insure that it has a reasonably constant coefficient. It is hoped that further research will suggest one or several forms which will have the same coefficient in all sizes of pipe under all ordinary kinds of flow conditions.

Changes in a High-Pressure Drum to Eliminate Recurrence of Cracks Due to Corrosion Fatigue

By ALBERT E. WHITE,[1] ANN ARBOR, MICH.

This paper gives the results of an investigation which was undertaken for the purpose of determining the cause of the cracks found in the No. 5 boiler at the Edgar Station of the Boston Edison Company. It also discusses the cause of the cracks found in the water columns and in certain of the superheater fittings and headers of this same boiler. The boiler in question is one of the first high-pressure units installed in this country and, therefore, it was quite essential that the cause of the cracks be determined and suitable steps be taken to prevent their recurrence. The study sets forth the changes which were made.

As a result of the investigation it was not felt that any of the difficulty could be attributed to faulty metal. The material was of the proper chemical composition and had suitable physical properties.

The primary cause of the cracks was believed due to corrosion fatigue resulting from stresses set up by changes in temperature on the surface of the metal in combination of a relatively low pH value of the water which was at times in contact with the metal that developed cracks.

As a result of the removal of the cracked metal and the changes in design which would prevent fluctuating temperatures on the surface of the drum at the boiler feedwater inlet, as well as other design changes which would raise the pH value, particularly in the water columns, it is not anticipated that further cracking will take place.

WHEN BOILER No. 5 at the Edgar Station of the Boston Edison Company was removed from service in February, 1938, for rebuilding after being in almost continuous operation since September, 1927, it was carefully examined to determine the extent it had been affected by operating conditions. This boiler had operated for approximately 60,000 hr at a rating of from 180,000 to 220,000 lb of steam per hr. It was one of the first high-pressure units installed in this country operating at 1400 lb pressure and 725 F steam temperature. It was suspected that cracks would be found in certain parts of the unit because in September, 1936, cracks were found in the feedwater connections in No. 3 boiler which is similar to No. 5 boiler.

The cracks which were found in No. 3 boiler at the time it was again examined in 1937 were not of such magnitude as to require the removal of the boiler from service. In fact, with the approval of the boiler inspectors, it was again placed in operation for a period of 6 months, after which time it was then again examined and returned to service for another 6 months, as there had been no visible increase in the extent of the cracks.

The cracks in No. 5 boiler were of greater magnitude than those in No. 3 boiler and, in consequence, a thorough investigation was made for the purpose of determining the cause and extent of the cracks, and likewise for the purpose of arriving at a decision as to what should be done.

The cracks which were found were in the walls of the boiler drum forming part of the feedwater inlets, in the lower connections of the water columns, and in the forged-steel tee fittings at the inlet and outlet of the superheater, as well as in the flanges of the inlet and outlet superheater headers.

In presenting this subject, reference is first made to a sectionalized view of No. 5 boiler which shows the drum, the feedwater inlet, the water columns, and the superheater inlet and outlet headers. These are all shown in Fig. 1.

Boiler Drum

A cross section of the boiler drum and the feedwater and water-column connections is shown in Fig. 2. Attention is called to the fact that the feedwater enters through a forged-steel nozzle which is attached to the boiler drum by stud bolts, then passes through a hole in the drum to the internal feedwater distributing pipe, which pipe is in turn attached to the inside of the drum by stud bolts. This arrangement results in the bore in the drum becoming part of the channel through which the feedwater enters. This is an important point, as the major cracks which were found existed on the face of the bore. One of the problems, therefore, was to determine the cause of these cracks and arrive at a decision with respect to repairs and changes to prevent a recurrence of the condition. Also in this general connection, it was necessary to arrive at a similar decision on cracks found in the lower water-column connections and in the superheater inlet and outlet headers and fittings.

As the first step in the investigation, the cracks were examined briefly and records taken of their appearance by photographs which are presented in the section headed Visual Examination. The metal around some of the cracks was then removed so that a metallographic examination might be made to determine the manner in which the cracks proceeded into the metal, and to arrive at some conclusion as a result of such an examination as to their nature and cause.

This phase of the examination is discussed under Metallographic Examination. It showed that the cracks were transcrystalline, relatively narrow at least at their ends and, in some cases though not all, were terminating in a snub-nosed section in the center of the crystal. This finding precluded the possibility

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Contributed by the Power Division and presented at the Annual Meeting of The American Society of Mechanical Engineers, New York, N. Y., December 5–9, 1938.

Discussion of this paper closed January 10, 1939, and is published herewith directly following the paper.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

that the cracks were due to caustic embrittlement, because cracks due to caustic embrittlement are intercrystalline rather than transcrystalline. They were not due to corrosion alone, for had they been they would have taken on more the character of zones of corrosion. Yet because of their snub-ended structure, it was evident that corrosion was a factor. In fact, the metallographic examination showed that the cracks resulted from accelerated corrosion accompanied by stress and, because of the sharpness of the cracks, especially at the lower ends, it was evident that the stress was of a repeated character.

In arriving at the conclusion that failure was due to corrosion fatigue, it was necessary to find some explanation for a stress which would be of a repeated character. This was found in the boiler operation itself; that is, the temperature of the surface of the metal in the feedwater inlets varied from 250 F to 587 F with an average temperature of 350 F. This was due to the fact that the temperature of the boiler feedwater usually varied from 250 F to 350 F. There were times, however, when the feedwater regulator shut off and on such occasions the temperature of the water at the inlet would be the temperature of the water in the drum, 587 F. Therefore, by using the bore of the drum as part of the channel for the boiler feedwater, the surface of the drum, used as the channel, was subjected to repeated heatings and coolings at a rapid rate which, on an estimated basis of ten such

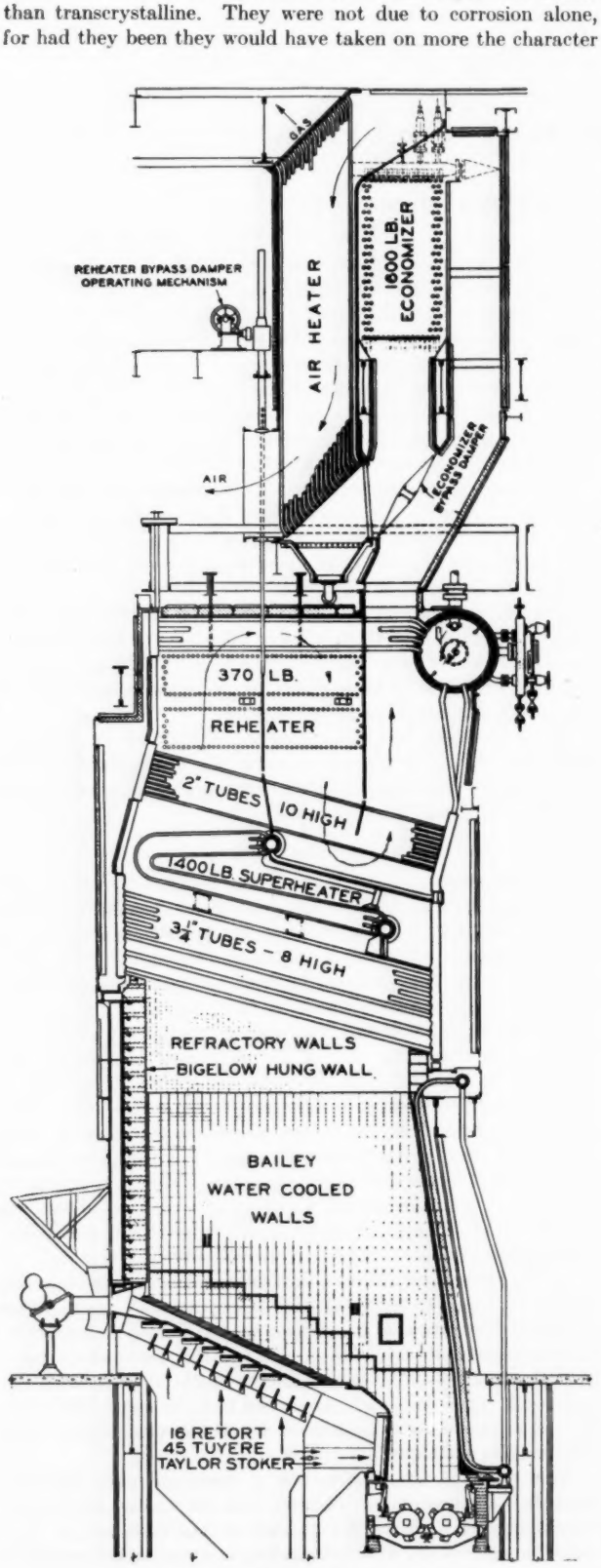


FIG. 1 CROSS SECTION OF NO. 5 BOILER

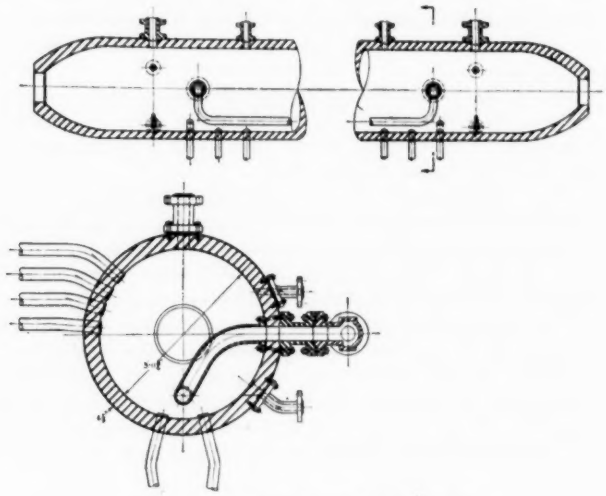


FIG. 2 CROSS SECTION OF BOILER DRUM

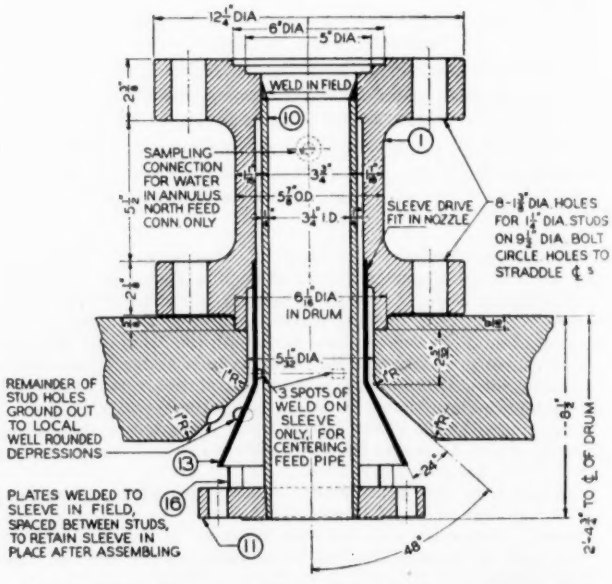


FIG. 3 CROSS SECTION OF NEW INLET FOR BOILER FEEDWATER

changes per hour, would amount to as much as 600,000 changes during the operating life of the boiler. These changes in temperature would introduce stresses into the surface of the metal which as they were repeated, would result in surface cracks and, with continued temperature changes, would cause them to grow and develop to the extent found when the boiler was examined.

In view of these conditions, it was felt that the solution to the problem involved changes in design so that the metal in the drum would not come in contact with the boiler feedwater during its passage into the drum and, therefore, not subject metal surfaces in the drum to changing temperatures, resulting in fluctuations in stress.

The design, shown in Fig. 3, was developed to accomplish this purpose. It, of course, required the boring out of the cracked sections of the drum. Essentially, it consisted in attaching a tube to the outside flange of a new nozzle in such a manner that the walls of this tube would not come in contact with the walls of the drum. Also, to prevent any possible drip from the outside wall of this tube onto the wall of the drum, an apron attached to the inside of the nozzle was installed. In this way, the boiler feedwater enters the drum without coming in contact with any part of the drum wall and, in consequence, removes the possibility of the stress conditions due to changes in temperature resulting from the variations in temperature of the boiler feedwater.

Attention is particularly called in this new design to the fact that, because of the space between the apron and the drum wall

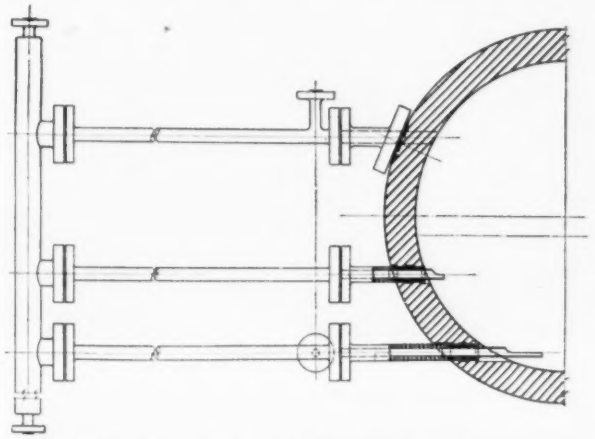


FIG. 4 CROSS SECTION OF NEW WATER-COLUMN CONNECTIONS

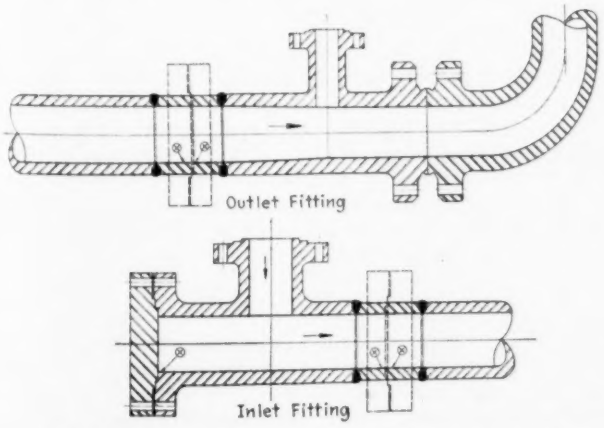


FIG. 5 SECTION OF SUPERHEATER FITTINGS AND HEADERS
(Above, outlet fitting; below, inlet fitting.)

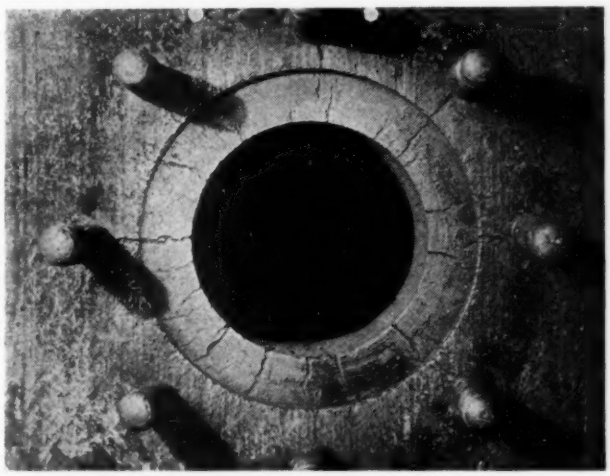


FIG. 6 FEEDWATER INLET, INTERIOR OF DRUM



FIG. 7 FEEDWATER INLET, EXTERIOR OF DRUM

and the space between the tube, and the apron and the nozzle, the temperature in the drum wall should be relatively constant and at the temperature of the water in the drum. Due to the space between the tube and the nozzle, the nozzle will be at practically the same temperature as the drum and therefore there will be no cooling action on the outside surface of the drum.

One of the possible contributing factors to the corrosion was the fact that the pH value of the water as it entered the drum varied from 6.9 to 7.2, while the water in the drum itself showed a pH value varying from 11.0 to 11.7. The change in design to reduce temperature stress also resulted in freeing all sections of the boiler drum from contact with water with a low pH and substituted alkaline water with a pH value from 11.0 to 11.7.

WATER COLUMNS

The corrosion-fatigue cracks which were first found in the base of the water columns in 1935, and in the boiler connection of the water columns of No. 5 boiler in 1938, were believed due to the same causes as those which produced the cracks in No. 5 boiler drum. To correct this condition, the inside edges of the male and female joints were rounded off for the purpose of removing stress concentrations. The major change, and what is believed to be the most important one, was the installation of a new connection

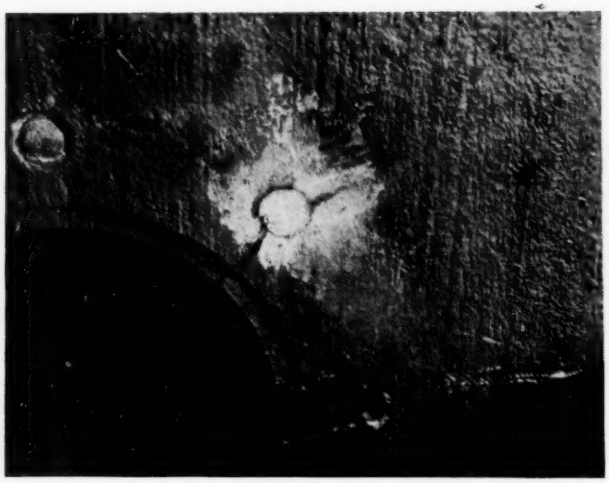


FIG. 8 INTERIOR, INLET BORED TO $5^1/_{32}$ IN.
(Note crack extending beyond stud hole.)

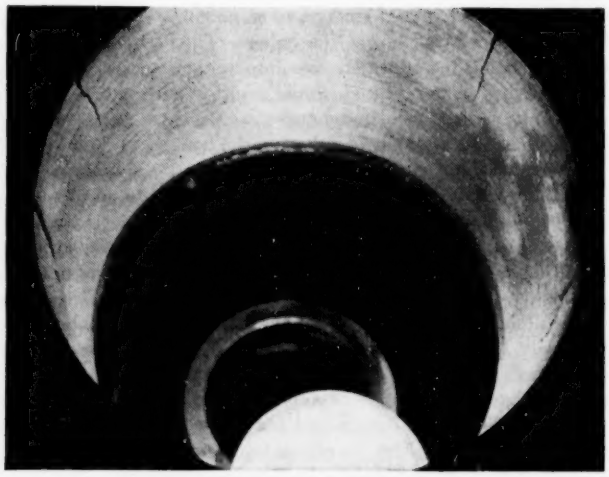


FIG. 9 INTERIOR, INLET BORED TO $5^1/_{32}$ IN.
(Showing cracks in upper half of inlet hole.)

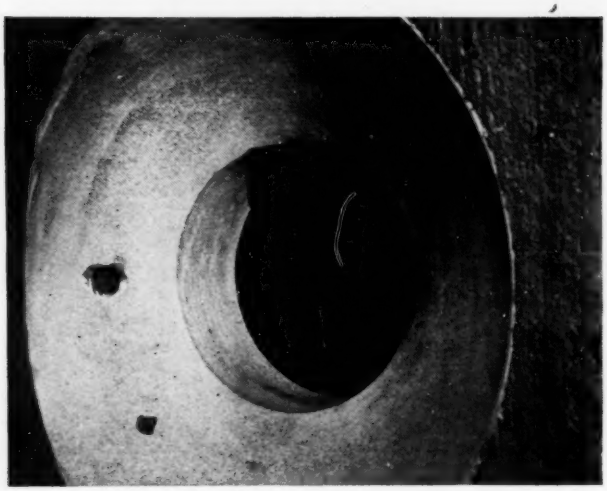


FIG. 10 INTERIOR, NORTH SIDE OF COMPLETED HOLE AFTER POLISHING, ETCHING, AND MAGNAFLUXING

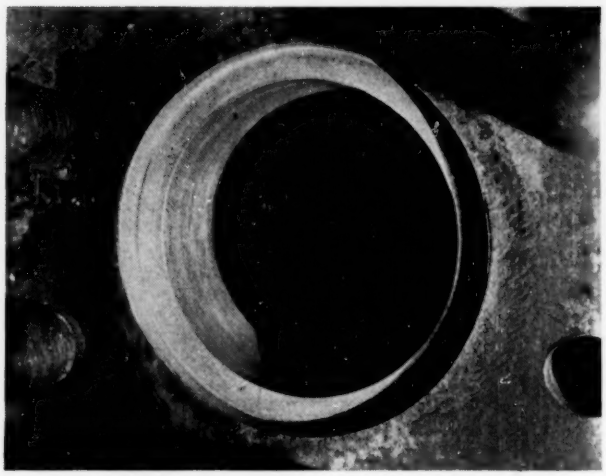


FIG. 11 EXTERIOR, SOUTH SIDE OF COMPLETED HOLE AFTER POLISHING, ETCHING, AND MAGNAFLUXING

12 in. above the lower connection to provide a continuous circulation of the water in the lower connection. With this arrangement, shown in Fig. 4, the pH value in the lower section of the water column and in the connections to the water column would be raised from values as low as 6.0 to values substantially the same as found in the drum, namely, values ranging from 11.0 to 11.7.

This change would also bring about a relatively constant temperature in the lower connections approximating that found in the boiler, namely, 587 F. Therefore, because of the changes made, overcoming troubles due to repeated changes in surface stress resulting from fluctuating temperatures, amplified by the presence of water with too low a pH value, it is not expected that further cracks will be found in the water columns.

SUPERHEATER FITTINGS AND HEADERS

Small cracks were found in some of the flanges in certain of the forged-steel tee fittings on the inlet and outlet superheater headers and also in the flanges of the headers themselves. It was believed that these cracks were due to stress reversals in the presence of condensed steam. At these points there were pH values of probably less than 7.0 which would produce a corrosive condition.

The original design of these parts is shown by the dashed lines in Fig. 5.

It was felt that the correction for this problem would be a change whereby there would be no possibility of the collection of condensed steam, accompanied by the removal of stresses by changing from a flanged to a welded construction. This opinion also was substantiated by the fact that on the outlet-tee-fitting end, jointed by a Sargol joint, there was no evidence of cracks.

Therefore, forged-steel filler pipes were welded between the two fittings and their respective headers, after the flanges had been eliminated, as shown in Fig. 5.

VISUAL EXAMINATION

The cracks which were found in the feedwater inlets of No. 5 boiler, in the lower connections of both water columns, and in the inlet and outlet superheater tee fittings, as well as in the superheater inlet and outlet headers were carefully examined visually. The cracks in the fittings and headers were not pronounced. Also, no cracks were found on the end of the outlet tee that

formed part of the Sargol joint, or in the side outlets of both of the tees. These latter were in a vertical position.

The cracks in the feedwater inlets into No. 5 boiler drum were present in both the north and south inlets. In each case, they were distributed more or less uniformly over all the exposed drum metal in the inlets. Some of them extended into the drum metal to a depth of about 2.5 in. A crack of this depth was found on the inside of the drum. The cracks were much deeper on the inside than on the outside.



FIG. 12 SOUTH WATER COLUMN SHOWING CRACKS IN BOTTOM OF LOWER CONNECTION



FIG. 14 SUPERHEATER-INLET STEP-JOINT T-FITTING, BLANKED-OFF END



FIG. 13 SAME AS FIG. 12 AFTER REMOVAL OF CRACKS

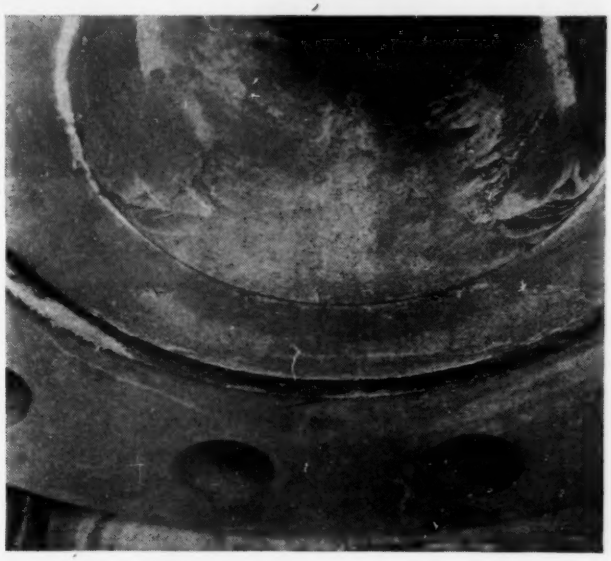


FIG. 15 SUPERHEATER-INLET STEP-JOINT T-FITTING, HEADER END

Photographs showing the nature and character of these cracks are given in Figs. 6 and 7.

Photographs showing the appearance of the inlets during and after machining are given in Figs. 8, 9, 10, and 11.

Fig. 8 shows the extent of a crack in the north feedwater inlet after boring had been completed to $5^{1}/_{32}$ in.

Fig. 9 shows the cracks in the upper half of the bore of the same connection after boring to the same diameter.

Fig. 10 gives the appearance of the hole after polishing, etching, and magnafluxing.

Fig. 11 gives the appearance of the finished bore from the outside.

Figs. 12 and 13 give the appearance, respectively, of the bottom of the lower connections in the boiler drum for the south water column before and after removal of the cracks. The appearance of the lower connection of the north water column was similar.

The inlet superheater tee fitting showed small cracks on the flanges forming the run of the tee as shown in Figs. 14 and 15. Also, as shown in Fig. 16, small cracks were found on the flange of the inlet superheater header.

The outlet superheater tee also developed cracks on one of the flanges forming the run of the tee as well as on the flange of the header which faced this flange. See Figs. 17 and 18.

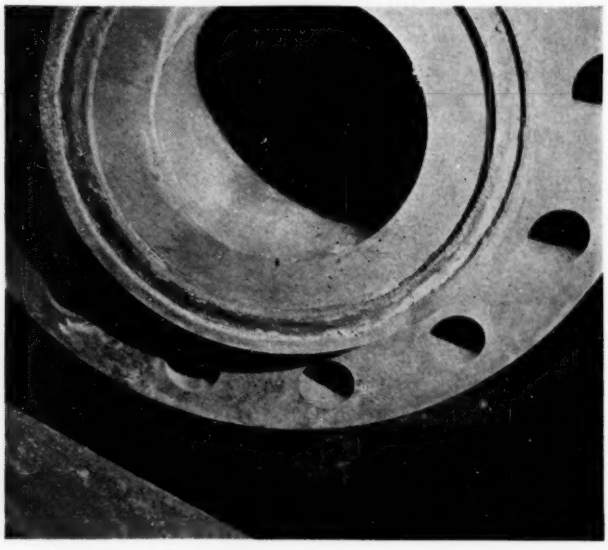


FIG. 16 SUPERHEATER INLET HEADER
(Facing flange shown in Fig. 15.)



FIG. 17 SUPERHEATER OUTLET STEP-JOINT T-FITTING, HEADER END

CHEMICAL COMPOSITION

In connection with the examination that was made of the No. 5 boiler, it appeared desirable to ascertain if there were any irregularities with respect to the chemical composition. Table 1 gives the composition of the metal in the drum as obtained from mill analyses and from the analyses of drillings from the actual metal in the drum.



FIG. 19 TYPICAL UNETCHED SECTION FROM DRUM METAL SHOWING A MODERATE NUMBER OF INCLUSIONS, ×100

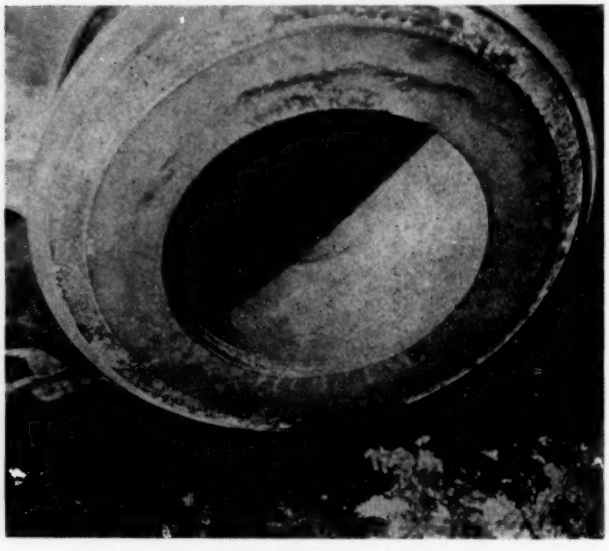


FIG. 18 SUPERHEATER OUTLET HEADER
(Facing flange shown in Fig. 17.)



FIG. 20 UNETCHED SECTION FROM DRUM METAL SHOWING THE WORST INCLUSIONS FOUND, ×100



FIG. 21 SAME SECTION AS FIG. 19, ETCHED, ×100



FIG. 22 ANOTHER SECTION OF DRUM METAL, ETCHED, ×100



FIG. 23 SAME SECTION AS FIG. 22, ×1000

These analyses show that the metal in the drum was made from killed steel. The chromium and nickel contents which were found are of no significance. They represent residuals which are oftentimes found in steel. The analyses show that from a chemical-composition standpoint, the metal was of excellent quality and conformed closely to that which was desired.

Also, there is satisfactory agreement between the analyses taken at the mill and the analyses taken from drillings from the drum itself.

TABLE 1 ANALYSES OF DRUM METAL

	Mill analyses, per cent	Analyses of drillings, per cent
Carbon	0.26	0.25
Manganese	0.53	0.52
Sulphur	0.024	0.036
Phosphorus	0.03	0.022
Silicon	0.22	0.27
Chromium	..	0.05
Nickel	..	0.14

Physical Properties

An examination was made of the physical properties of the metal in the drum to determine its characteristics. Tests were made on specimens taken from the drum and compared with tests made at the mill. Results of these tests are given in Table 2.

TABLE 2 PHYSICAL PROPERTIES OF DRUM METAL

	A.S.M.E. Code S-4	Tests at mill, longitudinal sections		Tests on drum specimens, radial cores	
		Top	Bottom	North end[a]	South end[b]
Tensile strength, lb per sq in.	60000	63000	63500	56250	64500
Yield point, lb per sq in.	0.5 T.S.	33000	33500	..[c]	32000
Yield stress (0.2%), lb per sq in.	...	...	...	24000	...
Elongation in 2 in., %	26.0	28.7	28.0	14.0	30.0
Reduction of area, %	42.0	46.3	41.0	21.3	42.8

[a] Test at University of Michigan.
[b] Test by Babcock & Wilcox Company.
[c] Not secured. Yield point would be higher than reported yield stress.

It would appear as if there were little agreement with respect to the physical properties between the core specimen tested at the University of Michigan and that tested by the Babcock &



Fig. 24 Crack in Boiler Drum, ×100



Fig. 25 End of Crack Shown in Fig. 24, ×1000



FIG. 26 CRACK IN BOILER DRUM, ×100

Wilcox Company, or the longitudinal specimens tested at the mill. The difference, however, is easily explained by the following interpretation which is based on the manner in which the specimens were secured.

The core obtained from the steam drum has its longitudinal axis parallel to and 9 in. from the horizontal center line of the drum. The testing of a cylinder in such a direction is unusual as it places the stress during the tensile test at right angles to both the longitudinal and the hoop stresses which are developed in operation and parallel to the direction in which the operating stress is zero. It is well known that minute laminations frequently occur in both plate and forgings and the plane of such laminations is parallel to the faces of a plate or the cylindrical surfaces of a forging; hence, it can be seen that, in a tensile coupon prepared as were these coupons, small laminations, if they are present, are at right angles to the longitudinal axis of the tensile coupons and hence they are planes of weakness within the coupons. Under these conditions, it is not unusual for such coupons to develop physical properties that are somewhat abnormal as compared with coupons cut longitudinally from the same material.

This was found to be the case with respect to the specimen which was tested at the University of Michigan, although it was not the case with respect to the specimen which was tested by the Babcock & Wilcox Company. That is, in examining the photomicrograph of Fig. 20, taken from the drum metal subjected to the physical test at the University of Michigan, a zone was found



FIG. 27 END OF CRACK SHOWN IN FIG. 26, ×1000

in which there were a considerable number of inclusions. The test specimen broke in such a zone because of these inclusions and therefore, resulted in the low values secured. On the other hand, it is believed that the test coupon broken by the Babcock & Wilcox Company happened to be in a section which was free from such inclusions or planes of weakness, or free to the degree shown in Fig. 19, and therefore their test coupon showed superior physical properties.

It is appreciated that the test values on the coupons, because of the fact that they were radial rather than longitudinal or transverse, are of no significance and contributed in no manner to the cracks that were found.

Physical tests were also made on specimens taken from each of the superheater headers. These both showed metal of a good quality. The results are given in Table 3.

TABLE 3 PHYSICAL PROPERTIES OF SUPERHEATER HEADERS

	Specimen no. 1,	Specimen no. 3, outlet	
	inlet	U. of M.	B. & W.
Tensile strength, lb per sq in.	60250	57750	58500
Yield point, lb per sq in.	35000	35000	36800
Elongation, % in 2 in.	33.5	37.5	36.0
Reduction of area, %	65.4	65.6	60.2

Metallographic Examination

A metallographic examination was made of sections from the drum for two purposes: One was to determine the character of the metal in the drum itself, and the other was undertaken for the purpose of determining, in so far as these examinations would disclose, the nature and character of the cracks.

Quality of Metal in Drum. Figs. 19 to 23 show metallographic structure of the metal in the drum. Fig. 19 is an unetched section at 100 diameters, representative of the specimen under examination. It shows a relatively small number of inclusions. Fig. 20 shows another portion of the same surface in which a considerable number of inclusions are shown, although there are no more of these than are frequently found in metal of this mass. It is an unetched section at 100 diameters. Fig. 21 is an etched section corresponding to the unetched section in Fig. 19. It shows the metal to be of low carbon content which is confirmed by the analyses. It shows a number of dull-gray particles which represent the inclusions in the unetched section. The grain size is somewhat irregular, due without doubt to the influence of forging. Fig. 22 shows the metallographic structure of an etched section at 100 diameters of another specimen from the drum. This shows a somewhat cleaner metal than was disclosed in Fig. 21 and a more uniform grain size. Fig. 23 shows the structure disclosed in Fig. 22 but at 1000 diameters. It is made up of ferrite and sorbitic pearlite. All of these photomicrographs show the material to be of an acceptable quality, as disclosed from a metallographic examination. The same is especially true with respect to the structure of Figs. 22 and 23.

Nature and Character of Cracks. Figs. 24 to 27, inclusive, are given for the purpose of showing the nature and character of the cracks.

Fig. 24 shows a crack in its entirety at 100 diameters. It indicates clearly that the cracks are transcrystalline.

Fig. 25 shows the end of this crack at 1000 diameters. It shows also that the crack not only passes through the crystals themselves but even directly through the carbide-carrying zones.

Fig. 26 shows the appearance of another crack. It too is transcrystalline.

Fig. 27 shows the end of this crack and brings out more clearly its transcrystalline characteristics.

Attention is called to the fact that all of these cracks are transcrystalline and blunt-nosed. The blunt-nosed condition is generally indicative of a crack which has been affected by corrosion. Also, all of these cracks are transcrystalline, which is a condition found in cracks resulting from fatigue. In fact, there is no evidence that any of these cracks are due to caustic embrittlement because, if they had been due to this cause, they would have been intercrystalline rather than transcrystalline.

Though no metallographic examination of the cracks in the failed water column and superheater fittings and headers was made there is no doubt but that they would have been of the same character and type as the cracks found in the boiler drum.

Therefore, in view of the evidence shown by this metallographic examination, there can be no other conclusion reached than that these cracks were the result of corrosion fatigue.

Conclusion

As a result of the studies which were made, none of the difficulty could be attributed to faulty metal. The material was of the proper chemical composition and had acceptable physical properties.

The primary cause of the cracks was believed due to corrosion fatigue resulting from stresses set up by changes in temperature on the surface of the metal in combination with a relatively low pH value of the water in contact with the metal.

As a result of the changes which have been made in design, it is not anticipated that further cracking will take place.

Acknowledgment

Participating in this investigation were also: W. C. Schroeder, U. S. Department of the Interior, Bureau of Mines; Babcock & Wilcox Company; Hall Laboratories, Inc.; Mutual Boiler Insurance Company of Boston; and Boston Edison Company.

Discussion

M. K. Drewry.[2] Because 100 F temperature difference of component steel parts causes 20,000 lb per sq in. stress, there seems a probability that the 337-F rapid temperature changes mentioned in the paper caused drum cracking from simple stress alone. Attaching more significance to thermal stresses and less to corrosion fatigue seems possibly more helpful in this and other similar cases.

The original 1300-lb per sq in. pressure boiler at Lakeside, with 325 F feedwater directly contacting the 5-inch-thick drum, has operated 70,000 hr without developing any cracks at any point. Feedwater-flow rate is always coincident with steam-flow rate, being regulated manually with the aid of water-flow steam-flow meters, thus preventing sudden metal temperature changes.

The lower half of all high-pressure water columns has been insulated since learning of drum-cracking troubles, in order to increase return-water temperature from 50 F below boiling temperature to within 10 F, so that thermal stresses from once-per-shift blowdown are rendered unimportant. No need of raising alkalinity of column returns appears necessary, for in addition to the apparent unimportance of corrosion, data and experience are available showing that high alkalinity is not as favorable to preserve protective coatings as is low alkalinity.

S. C. Hollister[3] and G. J. Schoessow.[4] This is a progress report of an investigation under way to determine the reduction in bursting strength of a cylindrical pressure vessel due to a circular hole cut in its side.

[2] Assistant chief Engineer of Power Plants, Wisconsin Electric Power Company, Milwaukee, Wis. Mem. A.S.M.E.

[3] Dean, College of Engineering, Cornell University, Ithaca, N. Y. Mem. A.S.M.E.

[4] Engineer in Charge Mathematical Analysis in Design, The Babcock & Wilcox Company, Barberton, O. Mem. A.S.M.E.

The investigation thus far has included three materials; fully annealed steel, cold-drawn brass, and Pyrex glass. The tubes had a nominal outside diameter of 1 in. and a length of $7^1/_2$ in. Specimens have been tested with holes varying in diameter from $4^1/_2$ per cent to about 28 per cent of the outside diameter of the tube. In each case, the bursting strength was compared to that of a tube of the same size and material without a hole.

The tests were made on the tubes without closing the ends. Stresses, therefore, were due to hoop tension only. Hydraulic pressure was applied to the tube which was lined with a rubber sack. Closure of the ends was made by the use of close-fitting plugs which were prevented from moving outward by a yoke frame around the specimen. Closure of the hole in the side of the tube was made by inserting, from the inside, a small plug having a head conforming to the inner surface of the tube at the periphery of the hole. The rubber sack bore directly against this head. A thin rubber washer was used between the plug head and the tube to absorb any small irregularity in the fit. Pressure was supplied by means of a simple plunger pump placed in a standard testing machine. Fig. 28 shows the specimen in the partially assembled yoke with the end closures and the closure for the hole already in place, and with connection made to the pump. Fig. 29 shows the entire assembly ready for testing.

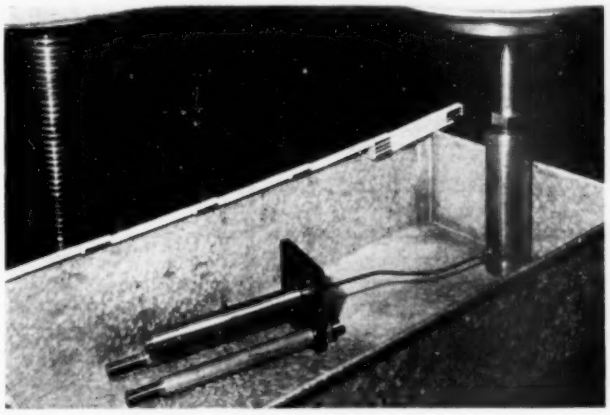


FIG. 28 SPECIMEN IN PARTIALLY ASSEMBLED YOKE WITH END CLOSURES AND CLOSURE FOR HOLE IN PLACE AND PUMP CONNECTION MADE

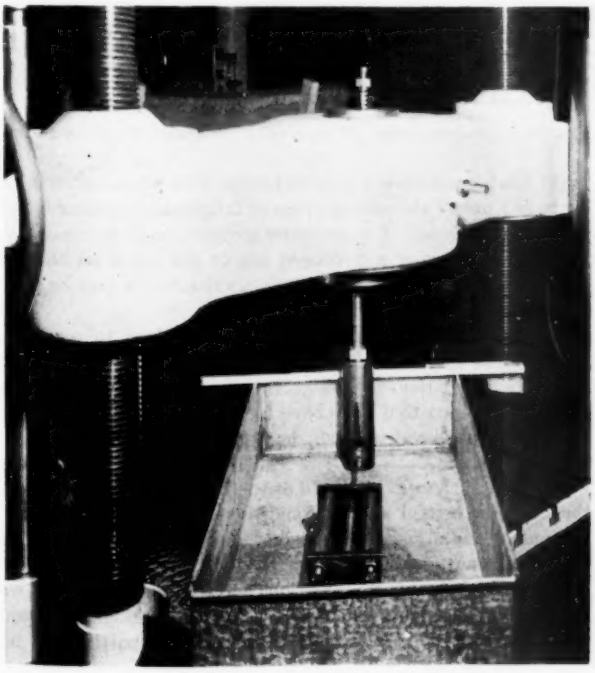


FIG. 29 ASSEMBLY READY FOR TESTING

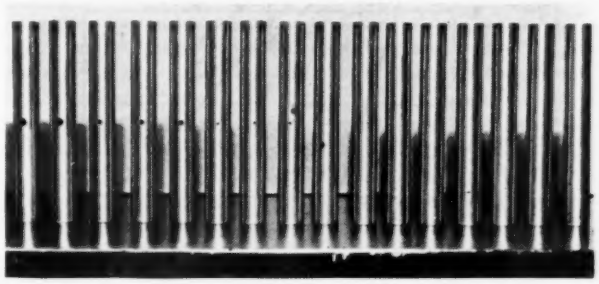


FIG. 30 TYPICAL SERIES OF TUBES BEFORE TESTING

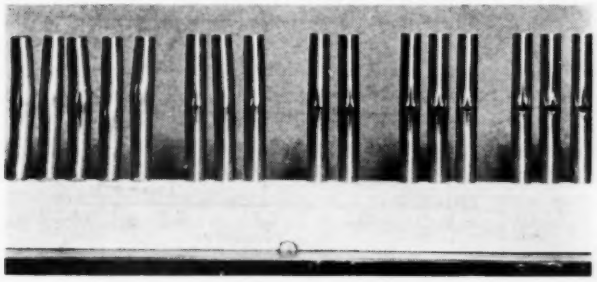


FIG. 31 SAME SERIES OF TUBES IN REVERSE ORDER AFTER BURSTING

Fig. 30 shows a typical series of tubes before testing, while Fig. 31 shows the same tubes after bursting.

Four thicknesses of steel tubing were used, varying from $2^1/_2$ per cent to $8^1/_2$ per cent, approximately, of the outside diameter of the tube. The brass tubing had a thickness of approximately 4 per cent of the tube outside diameter, while the Pyrex-glass tubing had a thickness approximately 6 per cent of the outside diameter of the tubing.

The results of the pressure tests on the various tubing samples are shown in Fig. 32, in terms of the reduction of bursting pressure (hence reduction in efficiency) caused by the hole, when compared to that of a solid tube of the same material. It will be noted that the ductile materials are grouped relatively close together at the top of the diagram, while the results of the tests on the glass specimens are much lower.

If one were to draw a 45-deg line downward and to the right from 100 per cent efficiency at 0 hole diameter, such a line would pass through 90 per cent efficiency at 10 per cent hole diameter; 80 per cent efficiency at 20 per cent hole diameter, etc. Such a line represents a value somewhat below the average of the metal specimens so far tested. It suggests a simple method of determining the efficiency of a vessel with a hole in the side. If, for example, a hole with a diameter 10 per cent of the outside diameter of the vessel reduces the efficiency in hoop tension by 10 per cent it suggests that one may use for efficiency the ratio $(D - d)/D$ where D is outside diameter of vessel, and d, diameter of hole. The diagram Fig. 32 shows that such an approximate relation holds up to hole diameters of 28 per cent of the outside diameter of the vessel, with thick walls lying well above such diagonal and thin walls lying somewhat below it.

The tests made thus far indicate that the results with glass

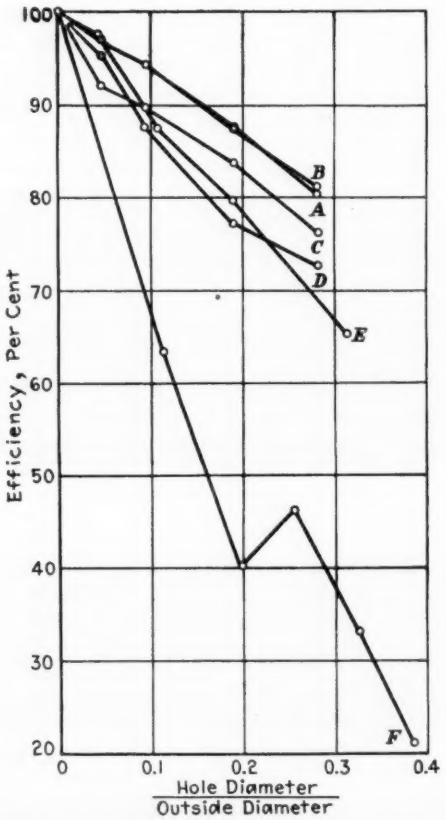


FIG. 32 RESULTS OF PRESSURE TESTS ON TUBING SAMPLES

A, 1.005 in. OD × 0.084 in. thick × 7½ in. long seamless steel tubing fully annealed
B, 1.003 in. OD × 0.068 in. thick × 7½ in. long seamless steel tubing fully annealed
C, 1.000 in. OD × 0.042 in. thick × 7½ in. long seamless steel tubing fully annealed
D, 1.004 in. OD × 0.026 in. thick × 7½ in. long seamless steel tubing fully annealed
E, 0.938 in. OD × 0.037 in. thick × 7½ in. long seamless brass tubing cold drawn
F, 0.979 in. OD × 0.060 in. thick × 7½ in. long Pyrex-glass tubing

tubing show a loss in efficiency of two to three times the loss in efficiency of steel or cold-drawn brass tubing.

This investigation thus far has been sponsored by the Babcock & Wilcox Company. It is a part of a more extended program on the study of the effect of holes in pressure vessels.

The results presented here apply to static-pressure conditions and should not be applied to conditions involving either creep or fatigue.

The results of tests so far made in this investigation, when applied to the steam drum covered in the author's paper, would indicate that 14.4 inches is the maximum diameter safely permissible for an unreinforced hole at the feed connection. Later data may modify this conclusion.

D. S. JACOBUS.[5] The conclusion by Hollister and Schoessow that an unreinforced hole 14.4 inches diameter would be safely permissible at the feed connection of the steam drum, covered in the author's paper, is consistent with the rules given in the Boiler Code, which are based on experience.

The Boiler Code contains a diagram for determining the maximum allowable size of an unreinforced opening. The method[6] of deriving the formula on which the diagram in the Code is based was published some years ago. As stated in this article, the formula gives smaller openings in the larger drums than would be sanctioned by the law of geometric proportions. The maximum allowable size of an opening obtained by the formula used in preparing the diagram in the Boiler Code is 12.2 in. The diagram in the Code was not extended to cover openings over 8 in. in diameter, as it was thought that no hardship would be involved in limiting the size to 8 in., and that it would be well not to extend the curves any further than necessary.

J. B. ROMER.[7] The author has presented the story of the cracking of the steam and water drum of the No. 5 boiler at the Edgar Station of the Boston Edison Company in an able and painstaking way, and has shown the necessity of guarding against the phenomenon known as corrosion fatigue. The generally accepted theories which are involved in a failure of this type are:

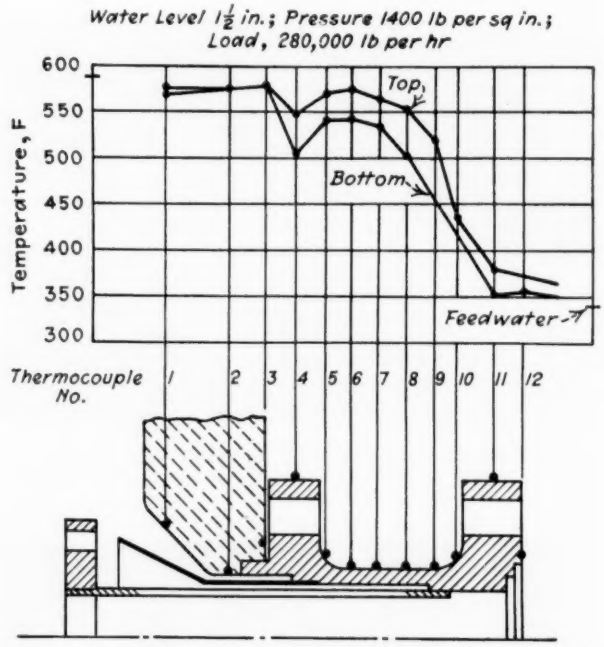


FIG. 33 TEMPERATURE AT JUNCTION OF FEED PIPE AND DRUM

(1) Carbon steel has a definite fatigue limit when tested in dry air in any one of the several types of fatigue-testing machines.

(2) The presence of a corrosive medium, such as condensate or feedwater having a hydrogen ion or pH value on the acid side, reduces the endurance limit to a value which may be below the normal stresses existing in a boiler.

(3) Sudden changes in temperature may lead to conditions favorable to the action by raising the stresses to a figure appreciably higher than the design stress.

It may happen that the above factors, especially if combined at some point in a drum, will lead to results such as described in the paper.

Cracking such as experienced at the Edgar Station emphasizes the need for careful and thorough inspections of high-pressure equipment and also the necessity of a correct design and proper operating conditions.

Reducing the corrosive conditions and the thermal stresses will slow down or stop the development of such cracking. Acid conditions in the condensate or feedwater can be partially, if not

[5] Advisory Engineer, The Babcock & Wilcox Company, New York. Past-President A.S.M.E.

[6] "Openings in Cylindrical Drums," by D. S. Jacobus, *Mechanical Engineering*, vol. 54, May, 1932, pp. 368–370.

[7] Chief Chemist, Babcock & Wilcox Co., Barberton, Ohio.

fully, overcome by feedwater treatment, and thermal stresses may be minimized by employing special devices such as were installed at the Edgar Station.

It might be interesting to readers to learn of the temperature of the metal of the steam and water drum at the feedwater inlet after installing the inner sleeve as shown in the author's Fig. 3. The temperature was measured after the boiler was returned to service by means of a specially developed spark-plug-type thermocouple.

The temperature of the drum connection nozzle for the feedwater was measured in a similar way. Fig. 33 shows the location of twelve thermocouples.

A corresponding set of thermocouples was located at the bottom of this opening; that is, 180 deg around the nozzle. The readings from both sets of thermocouples, obtained during one set of tests, are given in the curves at the top of Fig. 33.

A review of the test data, taken at different steam flows and at different water levels, indicates that the maximum temperature variation between the saturated steam and the metal of the drum at the feed opening, thermocouples Nos. 1, 2, and 3, will be of the order of 15 F, and will be less than this under most conditions of operation. The difference is much less than that which existed before installing the sleeve. It would seem that, with proper feedwater conditions, the drum should be free from further cracking.

At thermocouple No. 6, where the highest temperature in the nozzle was noted, the maximum variation between the bottom and top of the nozzle is of the order of 70 F, whereas, the general variation at this point is of the order of 35 to 50 F. Under most conditions of operation, the variation is of the order of 10 to 40 F. Thermocouples Nos. 8 and 9 show a little higher general average possibly of the order of 25 to 30 F, but do not show as high swings as were noted in No. 6 thermocouple. At the outside end, where the feedwater enters thermocouple No. 12, a maximum range of 30 F was noted with an operating range from 5 to 10 F. From this it can be seen that sudden changes will be small and that the change from the cold feedwater at the outer end to the saturated temperature at the drum wall comes in that part of the nozzle which can best withstand this change. In general, it is a rather gradual change.

Similar measurements on the water column with its new connections indicated that its temperature was at all times close to that of the saturated steam and, hence, it is not subject to strains due to temperature differences.

The physical properties of the steel in the drum are given in Table 2 of the author's paper. Emphasis should be given the point brought out in the paper regarding the direction of the axis of the cores which were obtained in trepanning the holes for the water-column connections in the north and south ends of the drum. It has been our observation that a test coupon cut so that its longitudinal axis approaches the radius of a forged drum, or extends from face to face of a rolled plate, may give test values which are not consistent with companion coupons nor with standard coupons taken in a longitudinal direction parallel to the faces of the plate. This difference usually takes the form of low ductility but in the case of plates, particularly when the coupon is cut in a normal direction to a lamination, all the test values may be lower than they should be. In some cases the area of the lamination may be greater than the cross-sectional area of the coupon and in such a case all of the test values will fall to 0.

Apparently an inclusion of some type was present in the coupon taken from the north end of the drum and as a result the ductility was lower than it would have been for a specimen taken in the standard way. As pointed out by the author, the stress applied in testing the specimen was in a direction in which there is very little, if any, working stress. The lack of ductility shown in the test contributes in no manner to the cracks that were found.

Author's Closure

The author wishes to express his appreciation for the splendid discussions presented by M. K. Drewry, Dean S. C. Hollister, G. J. Schoessow, Dr. Jacobus, and J. B. Romer. The discussions were all pertinent and have added materially to the subject matter of the paper.

To the Question of Balancing Reciprocating Weights in Steam Locomotives

By A. I. LIPETZ,[1] SCHENECTADY, N. Y.

The Association of American Railroads has been recommending since 1933 cross-counterbalancing of revolving weights for main wheels of steam locomotives. For coupled wheels the old method of static balancing is still being considered as sufficiently accurate. For reciprocating weights the static method is also retained by the new Manual of the A.A.R. The author takes exception to this scheme. He is of the opinion that for high-speed locomotives, and for these locomotives especially, greater clearness should be introduced in this subject. The dynamic augment, which plays such an important part in all high-speed calculations, should not exceed the limit allowed for the strength of the rails and the stiffness of the roadbed. An accurate evaluation of the dynamic augment for each wheel is possible only when the revolving weights are correctly balanced for each wheel, coupled as well as main. If this is not done, some of the wheels are dynamically deficient and a uniform distribution of dynamic augments is impossible. This destroys the effectiveness of proper balancing of reciprocating weights.

The author discusses the percentage of balancing the reciprocating weights in steam locomotives and how this depends on the method of calculating counterbalancing, whether static or dynamic. He shows, on the basis of existing locomotives, that a smaller total percentage of counterbalancing reciprocating weights of a locomotive is quite adequate for modern locomotives.

The author further calls attention to the existence of the steam component on the main axle and suggests that no reciprocating balance should be placed on main wheels, even if this should somewhat reduce the total percentage of balancing. He would rather increase the dynamic augment on coupled wheels in order to spare the main wheels from excessive blows. In his opinion the disregarding of the action of the steam component is responsible for the objection of some railroads to cross-counterbalancing of locomotives. This attitude should disappear when the problem is clarified.

In conclusion the author gives an example of a correct calculation of cross-counterbalancing of all weights, revolving and reciprocating, and for all axles, main and coupled, and proves that this method involves no more work, even less, than the static method, when angling of counterbalances on wheels is attempted.

HISTORICAL REVIEW

COUNTERBALANCING of steam locomotives is not a new subject. It has been practiced since the first days of building locomotives. The first locomotive builders in England and other countries realized very early the advantages of counterbalancing. Thomas Rogers, founder of the Rogers Locomotive Works in America, patented in 1837 an application of counterbalances opposite the cranks, of sufficient weight to counterbalance the cranks and connecting rods, thus introducing counterbalancing of revolving parts.

Up to 1845 it was customary to balance only revolving weights; the balancing of reciprocating weights was not recognized until later, when investigations of the problem of balancing were started by locomotive engineers. The disturbances caused by the unbalanced inertia forces were first demonstrated by Nollau (1)[2] in Germany who made special tests with a locomotive, suspended by chains, although previously to that Fernihough in England, in October, 1845, suggested and apparently tried (2) the use of weights for counterbalancing main rods, pistons, and other reciprocating parts.

Le Chatelier in France, in 1848, made tests similar to Nollau's and enunciated a very complete theory of balancing which is now known as cross-balancing. He published his theory in 1849 (3) and shortly afterward his theory was further developed by various investigators, mostly French (4). Later the correct method of cross-balancing was popularized for the English-reading public by Daniel Kinnear Clark in his classic book "Railway Machinery" (5). This treatise had a tremendous effect on the locomotive-engineering literature of the world.

Since then, practically all locomotive textbooks (6, 7, 8, 9, 10)

[1] Chief Consulting Engineer, in charge of research, American Locomotive Company, and nonresident professor of locomotive engineering, Purdue University. Fellow A.S.M.E. Mr. Lipetz was educated at the Warsaw (Poland) Polytechnic Institute, from which he received the degree of engineer technologist (mechanical engineer) of the first grade in 1902. In 1903 he entered railway service in Russia as an apprentice on the Moscow-Kiev-Voronesh Railway, later serving as fireman, locomotive driver, inspector, and assistant master mechanic. From 1906 to 1909 he was assistant professor of thermodynamics and railway mechanical engineering at the Kiev Polytechnic Institute, also passing examinations required for the degree of adjoint of applied mechanics. For three years he held administrative positions on the Tashkent Railway, and for the three years following he was chief of the locomotive department, Ministry of Railways, Russia. From 1915 to 1920 he served the Russian Railways in the United States, first as representative of the Russian Ministry of Railways and then as assistant chief and, later, chief of the Russian Mission of Ways and Communications in the United States. Since 1920 he has been connected with the American Locomotive Company. He is the author of many papers on steam and Diesel locomotives, and was the reporter for America on locomotives of new types at the International Congress held at Madrid, Spain, in 1930. He has been awarded The Melville Medal for 1938 for his paper "The Air Resistance of Railroad Equipment," presented at the 1937 Semi-Annual Meeting of the A.S.M.E. in Detroit, Mich., May 17-21, 1937.

Contributed by the Railroad Division and presented at the Annual Meeting of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS, held in New York, N. Y., December 5–9, 1938.

[2] Numbers in parentheses refer to the Bibliography at the end of the paper.

Discussion of this paper closed January 10, 1939, and is published herewith directly following the paper.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

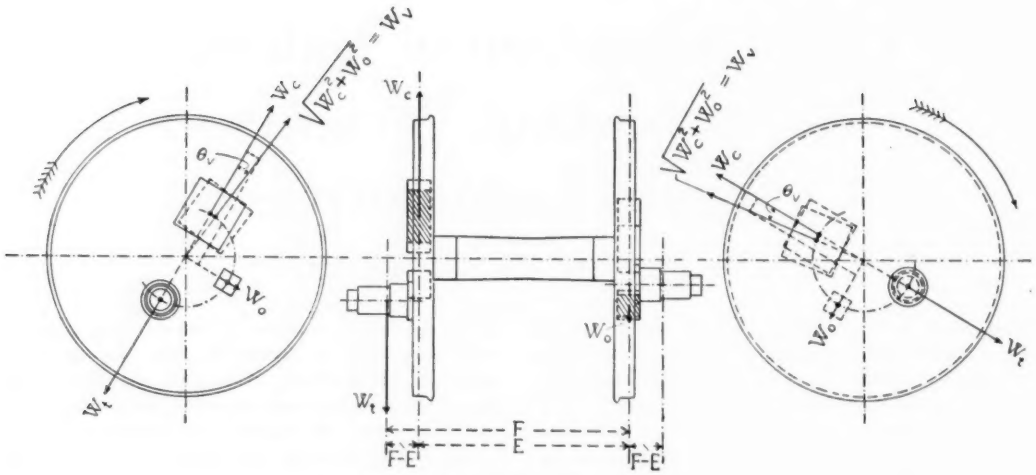


FIG. 1 MAIN DRIVING WHEEL PAIR

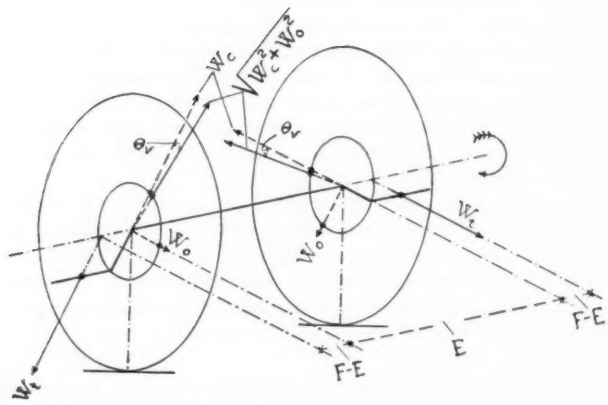


FIG. 2 PERSPECTIVE PROJECTION OF A WHEEL PAIR

have followed the Le Chatelier-Clark method as the only correct way of balancing locomotives. Locomotive builders of all countries except the United States adopted this method of counterbalancing. Some American books (11) also expounded the cross-balancing theory, although they did not recommend it for practical use. Consequently, in the United States the old method, called static balancing, in distinction from cross-balancing, sometimes called the dynamic balancing, is still in use on American railroads. In England, Professor Dalby (12) developed very convenient graphical charts for cross-balancing, which are now frequently used when cross-balancing is wanted. It is not amiss to mention here that the term "static balancing" is a misnomer and is misleading. In a moving locomotive there can be only one kind of balancing, that is, dynamic. What is meant to imply by these two terms, static and dynamic, is that the first is the balancing of forces while the other is the balancing of forces and couples. For a locomotive with forces in different planes, only balancing of forces and couples is complete. The balancing of forces alone is incomplete and, strictly speaking, cannot be considered balancing at all, although in some cases it may be sufficient and useful for locomotives. However, in this paper the old terms will be used for the sake of tradition.

The situation has been changed lately in the United States in view of the fact that the increase in size and power of steam locomotives, and especially in speed, and their effect on the maintenance of both locomotives and roadbed, called for a revision of the American practice. Several very valuable papers have been presented and discussed in various engineering societies, including those by Eksergian (13), Fry (14), and Riegel (15), the last of which recommended cross-balancing for the main wheels as the method for balancing locomotives, thus modifying A.A.R. previous recommendation (1915) for static balancing of all driving axles of locomotives. At present the recommendation of cross-balancing was incorporated in the A.A.R. Manual for 1938 (16) and has become the practice on some, not all, American railroads for high-speed locomotives.

The author believes that the reason for such a delay in adopting the universal practice of cross-balancing is probably due to the fact that for American locomotives, cross-balancing was not so important. After all, for the heavy-weight and long-base American locomotive, complete balancing was not necessary, especially for moderate speeds, and the practice which has been used for years was satisfactory. When the proper time came, the scientifically correct cross-balancing method was resorted to, after the more simple static method of balancing, requiring practically no mathematics, outlived its usefulness. Fry (14) should be complimented for using simple arithmetic instead of elaborate mathematics. If properly applied, the counterbalancing method becomes very simple. Leaving out for the present the counterbalancing of reciprocating weights and limiting ourselves to the balancing of revolving weights only, the cross-counterbalancing is fully represented in Figs. 1 and 2 and the mathematics can be easily evolved, following Fry's (14) or the A.A.R. Manual (16) formulas.

In Figs. 1 and 2 W_c, in agreement with the Manual, is the resultant of all equivalent revolving weights of one side of the wheel, actuating in different planes, and reduced to the plane of the counterbalance of the adjacent side; W_t is the total sum of the actual (not equivalent) revolving weights on one side of the wheel; $W_o = W_c - W_t$ is the component of all equivalent revolving weights on one side of the wheel, actuating in different planes and reduced to the plane of the counterbalance in the wheel of the opposite side; and θ_v is the offset angle, the tangent of which is equal to the ratio

$$\frac{W_c - W_t}{W_c} = \frac{W_0}{W_c} = \frac{F - E}{F}$$

where F and E are distances between planes of revolving weights and counterbalances as shown in Figs. 1 and 2.

The mathematics can be replaced by simple vectorial charts, such as shown in Figs. 3 and 4, which Dalby (12) so elegantly worked out. Fig. 3 is a diagram of moments of centrifugal forces

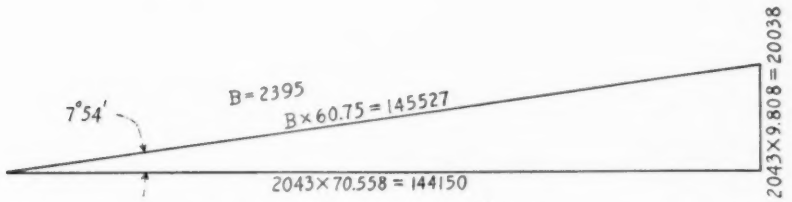


FIG. 3 DALBY'S VECTORIAL DIAGRAM OF MOMENTS

referred to the plane of one counterbalance, while Fig. 4 is a diagram of centrifugal forces themselves. In both cases the forces are, for convenience, replaced by weights, reduced to the crank radius, and the magnitudes of revolving weights so reduced, or their moments, are drawn in the direction of their centrifugal action. The reader is referred to Dalby's book (12) for particulars. Figs. 3 and 4 determine the magnitude of the counterbalances and of the offset angle.

It should be noticed that in all cases, whether with inside or outside cylinders, with the right-hand crank leading or trailing, the offset angle on one wheel is always turned to the crankpin of the wheel on the opposite side (see Fig. 12); in other words, the two wheel centers of one axle (right and left) should be cast from one pattern.

The author has no intention of going over the ground already covered by Eksergian (13), Fry (14), and Riegel (15); therefore, if the author takes the liberty of discussing this question again, it is simply for the reason that one point has been left by these authors (13, 14, 15) unanswered, namely, the balancing of the reciprocating weights.

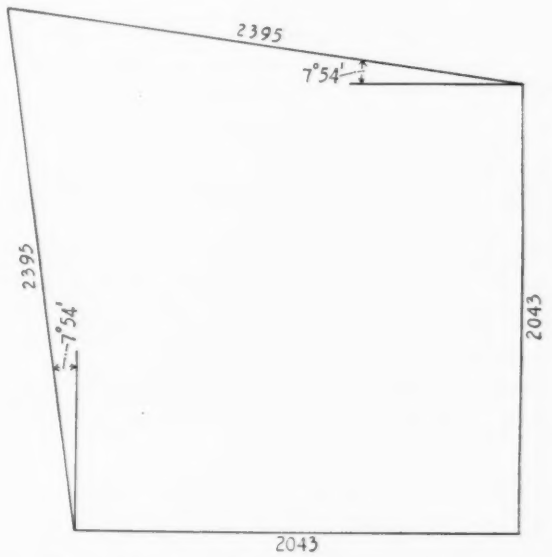


FIG. 4 DALBY'S VECTORIAL DIAGRAM OF FORCES

THE PROBLEM OF BALANCING RECIPROCATING WEIGHTS

The cross-balancing of revolving weights is an exact way of balancing; in other words, when a wheel pair is properly counterbalanced as a unit, the revolving weights (pins, crank hubs, side rods, eccentric cranks, and a part of the main and eccentric rods) are completely balanced by revolving counterbalances in accordance with the rules of dynamics. Such a wheel pair, if properly manufactured, should give no disturbing forces of any kind when rotated, even at high speed. The difficulty begins only when the reciprocating weights, consisting of pistons, crossheads, main rods, eccentric rods, also have to be considered.

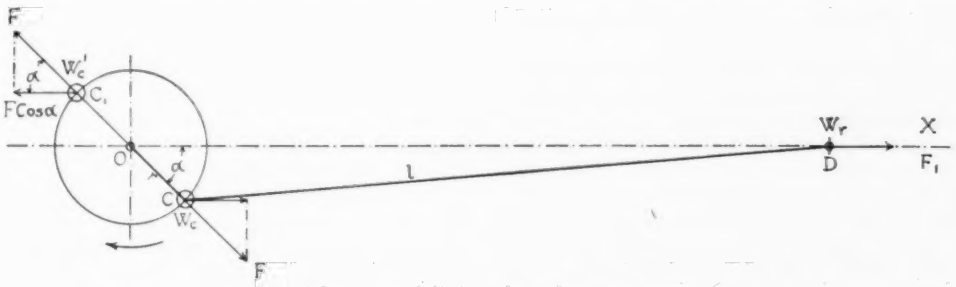


FIG. 5 INERTIA FORCES OF A ROD-CRANK MECHANISM

Propositions for balancing these weights by counterweights with oppositely directed reciprocating motion, so-called "bob weights," have been suggested at various times for reciprocating engines (17). Other suggestions, consisting of several shafts rotated at various speeds (18), have been made for automobiles and engines for cars (19), but none of these propositions have found any extended application in practice. As a result, the present universal way of balancing reciprocating weights in locomotives is by revolving weights only, built as counterbalances in driving wheels. As the reciprocating weights are producing horizontally directed forces of inertia (strictly speaking, forces in the direction of the cylinder axes), they can be balanced by horizontal components of centrifugal forces of revolving balances. These will be in addition to the counterbalances which are required for balancing the revolving weights. There will remain naturally unbalanced vertical components of the balances for reciprocating weights which act in a locomotive as disturbing forces and which cause what is known as "hammer blow" of rotating counterbalances, injurious to the track and bridges.

A few simple mathematical formulas will clarify the matter. Referring to Fig. 5, let OC be the crank of a length r, which makes an angle α with the cylinder axis X, assumed to be positive from center O to the right in the direction of the arrow. Let CD be the main rod of length l. Assume that angle α is positive in the clockwise direction of the rotation of the wheel. The centrifugal force F, in lb, of a revolving weight W_c, in lb, attached to pin C is equal to

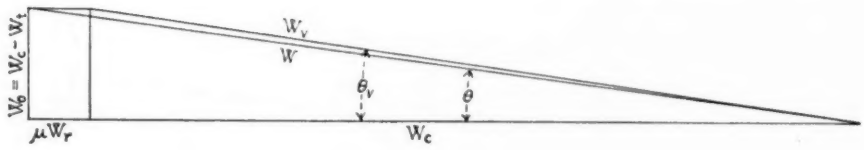


FIG. 6 DIAGRAM OF ANGLING IN LOCOMOTIVE DRIVING WHEELS

$$F = \frac{W_e}{g} r\omega^2 \ldots\ldots\ldots\ldots\ldots\ldots [1]$$

where r is the crank radius, ft; ω is the rotating speed of the wheel, radians per sec; and g is the acceleration of gravity, equal to 32.2 ft per sec per sec. It can also be expressed

$$F = 0.0000284\ W_e r n^2 \ldots\ldots\ldots\ldots\ldots\ldots [2]$$

where n is in revolutions per minute and all other notations are the same as given previously, except r which is expressed in inches.

At diameter speed, i.e., at a speed of as many miles per hour as the driving wheels (tires) have inches in diameter (of rolling circumference), the rotating speed $\omega = 35.17$ radians per sec, and $n = 336$ rpm. The centrifugal force lies on the prolongation of the crank radius and is directed from center O.

If a reciprocating weight W_r were attached to the small end of the rod in D, its inertia force, if the rod may be considered of infinite length, or $r/l = 0$, would be

$$F_1 = \frac{W_r}{g} r\omega^2 \cos\alpha \ldots\ldots\ldots\ldots\ldots\ldots [3]$$

In the dead center, when $\alpha = 0$, force $F_1 = F$; in all other positions, when α is different from 0, the inertia force is less than F. It may be seen, from a comparison of formulas [1] and [3], that in all cases the inertia force F_1 is the horizontal projection of the centrifugal force.

Further, it is evident that if a weight W_e' is placed in C_1 opposite to C, in the same vertical plane, and at the distance r of the radius crank from the center O, its centrifugal force will balance centrifugal force F of weight C_1 if these two weights, as the case always is in locomotives, will rotate at the same speed. On the other hand, the horizontal component of the counterweight will balance the inertia force of a reciprocating weight of the same mass, if the revolving counterweight and the reciprocating weight are connected by a crank-and-rod mechanism. This is the principle of balancing reciprocating weights in a locomotive.

It is now evident that the vertical component of the counterweight remains free. While the horizontal component can be taken up by the inertia force of the reciprocating weight, there is nothing to cancel the vertical component. This force, remaining free, exerts a vertical blow on the rail in the plane of the balance. Its maximum value is attained twice every revolution: First, when the counterweight is in its lowest position and the component is directed downward and acts as a blow on the rail, and second when the counterweight is in its highest position and the component is directed upward and has a tendency of lifting the wheel. Thus, this method of balancing reciprocating weights by a revolving balance, while very simple and effective, bears in itself an evil and, therefore, a compromise is necessary.

Shortly after the balancing of reciprocating weights was recognized, Crampton in the middle of the last century built a number of locomotives with total (100 per cent) balancing of reciprocating parts. These locomotives seemed to have very good riding qualities; consequently, they were very popular for a short while in England and on the continent of Europe. However, after several derailments with casualties, the effect of the hammer blow and the vertical disturbing force were taken notice of and, in order to diminish the effect of these forces, the partial balancing of reciprocating weights has become universal practice on locomotives, and has been so ever since. For a long time, following the Crampton mishaps, the percentage of balancing had been reduced to $66^2/_3$ per cent, meaning that two thirds of the reciprocating weights were balanced. This practice was followed in Europe in conjunction with cross-balancing of revolving weights, and also in America, with the difference that here only static balancing of revolving weights was made. Sometimes in America, following Henderson's suggestion, another figure, $^1/_{400}$ of the total weight of the locomotive, was taken as the part of reciprocating weights to be left unbalanced. This rule is now not being followed.

For European countries, in accordance with the Middle-European Technical Agreement, the following rules are being observed at the present time:

1 All revolving weights are totally balanced (i.e., cross-balanced) as much as possible.

2 From 25 to 60 per cent of reciprocating weights are balanced, depending upon the length of the wheel base and its relation to the total length of the locomotive. In modern long and heavy locomotives it is usually as low as 15 per cent.

3 Dynamic augment on each driving wheel should not exceed 15 per cent of the static weight on that wheel. This rule is strictly observed on a good many European railroads.

Moreover, the practice of figuring the counterbalance is also different from ours—the weight to be balanced is assumed to be in the plane of the cylinders and this amount is added to the corresponding revolving weight; in other words, the reciprocating weights are also cross-balanced.

In this country the opinion as to cross-balancing of all revolving weights and balancing of reciprocating weights is only now being crystallized. Eksergian seems to be in favor of cross-balancing revolving weights on all drivers, although in conclusion 2 of his paper he states: "In view that no gain is effected in cross-balancing for the reciprocating parts alone, the static balance of coupling drivers is permissible" (13). Fry is very definite in his negative attitude, when he states: "Cross-balance is therefore not necessary for the overbalance of the reciprocating parts" (14, p. 436); so is Riegel in his report (15), and the A.A.R. Manual (16) in recommending the counterbalancing of main wheels only.

As to the balancing of reciprocating weights, all three American papers (13, 14, 15) recommend the static method, which consists of adding algebraically that portion of the reciprocating weights per wheel W_r which is to be balanced (μW_r) to the revolving-balance component W_e before the component from the other side $W_0 = W_e - W_t$ is added geometrically, as shown in Fig. 6. Thus, the reciprocating balance μW_r is directed opposite the crank radius ($\theta_r = 0$) in the plane of the balance, and the resultant offset angle θ is slightly different from θ_v, the offset angle of the revolving parts alone. The reciprocating balance per wheel μW_r is now recommended by the A.A.R. Manual to be chosen so that the total for all axles is nearer 30 per cent of the reciprocating weights of one side of the locomotive when cross-balancing of revolving weights is applied, which is equivalent to 50 per cent

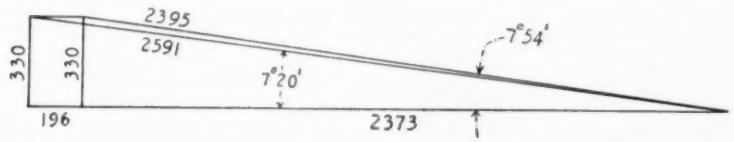


FIG. 7 DETERMINATION OF ANGLING IN A LOCOMOTIVE DRIVING WHEEL, WITH STATIC BALANCING OF RECIPROCATING WEIGHTS

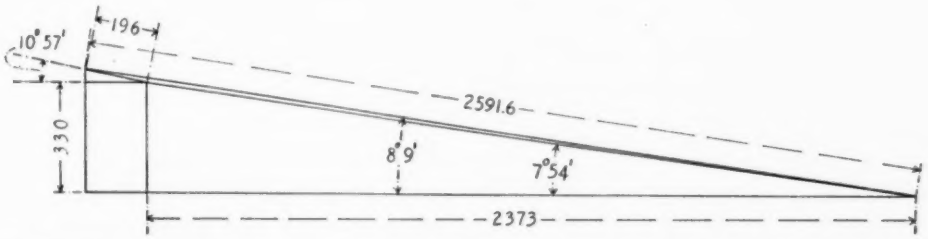


FIG. 8 DETERMINATION OF ANGLING IN A LOCOMOTIVE DRIVING WHEEL, WITH CROSS-BALANCING OF RECIPROCATING WEIGHTS

of the same weights when the 1915 A.A.R. static method of balancing was in use (20). In application to a modern locomotive, for which a Dalby diagram of revolving cross-balancing is shown in Figs. 3 and 4, the addition of the reciprocating balance would be as shown in Fig. 7. In this case the locomotive has three drivers. The weight of reciprocating parts on one side is 1681 lb, and the selected percentage for this locomotive (35 per cent) amounts to a reciprocating balance per wheel of $\mu W_r = 196$ lb. According to the A.A.R. Manual this vector should be added to component $W_c = 2373$ lb of the revolving cross-balance in the direction of the radius crank. Adding at a right angle the component of the other side $W_0 = 330$ lb, we shall get the total cross-balance $W = 2591$ lb at an offset angle of 7 deg 20 min, instead of a balance $W_v = 2395$ lb at an angle of 7 deg 54 min, for total balancing of revolving weights alone. All these balances are referred to the crank radius; in this case, $r = 15$ in.

If the reciprocating weights were also to be cross-balanced, the principle of the method remains, as shown in Figs. 1 and 2. The reciprocating balance would not be placed opposite the crank radius, since $\theta_r \neq 0$. Remembering that the reciprocating weights are in the cylinder plane, the offset angle for the reciprocating balance should be

$$\tan \theta_r = (F - E)/F \quad [4]$$

where F is the distance from the cylinder plane on one side of the locomotive to the counterbalance plane on the other side, and E is the distance between counterbalance planes.

For the modern locomotive, which we have been considering, $E = 61.5$ in., $F = 76.25$ in., and

$$\tan \theta_r = \frac{76.25 - 61.5}{76.25} = 0.1934$$

or

$$\theta_r = 10 \text{ deg } 57 \text{ min}$$

The correct cross-balance angling for the reciprocating weights for the main wheel would, therefore, be as given in Fig. 8. The offset angle of the whole balance would be slightly greater, namely, 8 deg 9 min, compared with 7 deg 20 min, as obtained in Fig. 7, but the total balance will be about the same, namely, 2591.6 lb, as against 2590.6 lb for the case of the static balancing of the reciprocating weights. The dynamic augment at diameter speed will also be the same, that is, 9408 lb, because the revolving weights are completely balanced, and only the centrifugal force of the reciprocating balance of 196 lb gives the blow in either case. The direction of the blow is slightly different in the case where the static balance is just opposite the pin; in the cross-balancing case it is 8 deg 9 min later or sooner, depending on the side—right or left. Due to the predominance of the revolving cross-balances and the smallness of the reciprocating balance, the difference in these phenomena is negligible, but in the percentage of actual balancing for the reciprocating weights the difference is considerable, as we shall presently see. In the foregoing example it is 28 per cent, instead of 35 per cent as assumed.

PERCENTAGE OF CROSS-BALANCING RECIPROCATING PARTS

This phase can be better demonstrated on the foregoing example of a 4-6-4 locomotive, which was specially studied. Curve X in Fig. 9 shows the effect of the inertia forces of all reciprocating weights (1681 lb actual weight on each side) in a counterbalance plane of one side, say A, and curve Y in the counterbalance plane of the other side, say B; in other words, referring to Fig. 10, Fig. 9 shows the horizontal reactions X and Y in the planes of counterbalances A and B as a result of two forces of inertia F_1 and F_2 on both sides. Curve A in Fig. 9 is the centrifugal force of a revolving counterbalance, in the same plane A, of a magnitude of 196 lb in order to insure the same dynamic augment as in the case of static balancing. The reciprocating balance is properly angled at 10 deg 57 min according to the rules of cross-balancing, although the total angling of the whole balance is only, as we have seen previously, 8 deg 9 min. Curve B shows the remaining unbalanced force of inertia (shaded area).

The inertia forces of reciprocating weights are usually assumed to be as determined from formula [3], which is approximately correct, and is precisely true when the ratio $r/l = 0$. In our locomotives r/l is about 1/8, and the inertia forces are about 12 per cent more in the front dead center, and 12 per cent less in the back dead center, than the centrifugal force of the same mass at the crankpin. In other words, formula [3] should be replaced, for the sake of accuracy, by

$$F_1 = \frac{W_r}{g} r\omega^2 \left(\cos \alpha_0 + \frac{r}{l} \cos 2\alpha_0 \right) \quad [4]$$

where α_0 is the angle of the right-hand crank.

Fig. 9 was drawn for the case $r/l = 1/8.6$ and to formula [4]. The forces of inertia, their distribution, and balancing by a balance of 196 lb have been accurately figured.

Fig. 11 gives similar curves for the case $r/l = 0$ and also for cross-balance of 196 lb at $\theta_r = 10$ deg 57 min. As can be seen, the difference in the curves is negligible. As practically only counter-

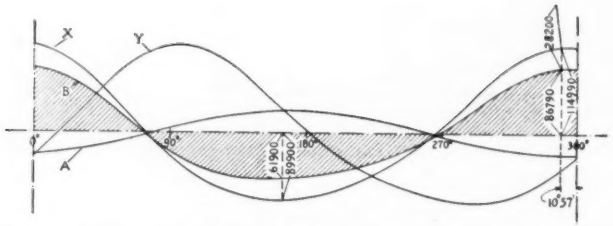


FIG. 9 INERTIA REACTIONS OF RECIPROCATING WEIGHTS, PARTIAL BALANCING; FINITE ROD

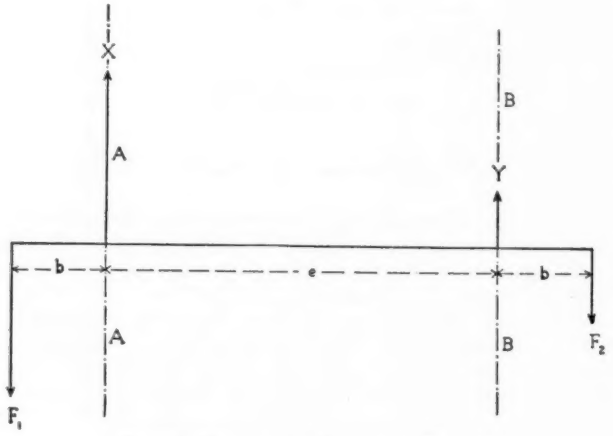


FIG. 10 DIAGRAM OF PLANES OF FORCES

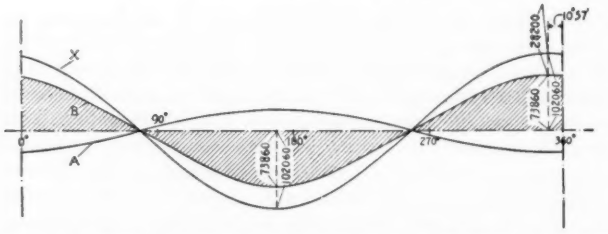


FIG. 11 INERTIA REACTIONS OF RECIPROCATING WEIGHTS, PARTIAL BALANCING; INFINITE ROD

balances for the primary forces of inertia, which result from the simple harmonic motion of the piston, assuming $r/l = 0$, can be built in a locomotive, we might, therefore, forget the more accurate formula [4]. The secondary forces of inertia, which result from the finity of the main rod cannot be balanced even in a four-cylinder locomotive. They would require counterbalances with a complicated motion and are impracticable. The study made in Fig. 9 had the object of ascertaining the error due to neglecting the finity of the main rod. It was thought that by changing the offset angle and the counterbalance some compromise could be found.

The percentage of balancing can be figured as the ratio of ordinates of curves A and X in Fig. 9. If the average of the ratios of the maximum and minimum ordinates in Figs. 9 and 11 be figured, the percentage of balancing for the locomotive taken as an example, is 27.6 for both cases. More accurately, this can be done by planimetering the areas below curves A and X and figuring their ratios. It then comes out 28.2 and 28.9 per cent for Figs. 9 and 11, respectively, indicating that it is not necessary to pay much attention to the finity of main rod for the sake of counterbalancing.

Curves have also been drawn for the same inertia reactions, but for counterbalances set at $\theta_r = 0$, opposite to the crank radius, as required by the A.A.R. Manual. The percentages for these cases were also 28.2, when $r/l = 1/8.6$, and 28.9 for $r/l = 0$, that is, for an infinite rod. This confirms the previous opinion that consideration of the length of the rod has little influence on the question of counterbalancing. In all cases the actual percentage of balancing the reciprocating forces was about 28 per cent, and in no case did we get a balancing of 35 per cent.

Thus, there is no justification for taking the percentage of balancing simply as the ratio of reciprocating balances to the reciprocating weights $(3 \times 196)/1681 = 0.35$, even after the cross-counterbalancing effect of the deficiency in balancing has been accounted for, because the actual balancing, no matter what method of figuring is used, is much lower.

This brings us to another and most important point: What percentage of balancing reciprocating weights should be used?

The object of balancing the reciprocating weights is, of course, to reduce the parasitic motions of a locomotive, of which there are as many as a free suspended body on springs may have, namely six. In a modern locomotive on good track only two need be considered, especially from the point of view of counterbalancing; these are (*a*) nosing, which is the rotation of a locomotive around a vertical axis through its center of gravity, and (*b*) the to-and-fro movement of a locomotive in the direction of its longitudinal axis. Both are caused by the unbalanced forces of inertia acting along the cylinder axes. First (nosing) is the result of a couple tending to rotate the locomotive around the vertical axis; second (to-and-fro movement), is the direct result of the application of these forces.

When a locomotive runs on a track having clearances between the rails and the flanges of the tires and, consequently, having some lateral freedom, a pure to-and-fro movement is impossible. The actual movement is a result of both nosing and the longitudinal translation of the locomotive. In any case, the inertia forces exert a couple effect which becomes apparent mainly in nosing. Therefore, to ignore the couple effect of the inertia forces, as if the locomotive were restrained laterally, is not right. When Henry, chief mechanical engineer of the French P.L.M. Railway, attempted in 1889–1890 to study the parasitic motion of a locomotive separately, he suspended a locomotive from the top and supported it by six horizontal springs laterally to restrict the movement of the locomotive sidewise (21). This, of course, is a special condition to which he was driven in order to omit the natural couple effect when he wanted to study the to-and-fro movement of the locomotive, but such a restraint does not exist in locomotives in service.

We can understand why Crampton attempted to obtain total balancing of reciprocating weights. This reduced the parasitic movements of his locomotives, but this reduction had also other effects. For practical reasons a compromise had to be made, and $66^2/_3$ per cent of balancing had been considered for a locomotive as standard for a long time. This was further reduced, until now it is much less.

We can easily see the reasons for this trend in the evolution of the steam locomotive. The power and the reciprocating weights of two-cylinder locomotives have gone up tremendously since the time of Crampton; also, the rotative speed of the wheels has increased. Consequently, the dynamic augment, notwithstanding the refinements in design of reciprocating parts, called for heavier rails, and this, in many cases, has reached the limit. Simultaneously, the locomotives have grown heavier, the rigid base longer, the moment of inertia of the locomotive has become greater, and the guiding resistance of the leading and rear trucks has become better adjusted to the service, requiring a smaller effort on the part of the guiding wheel to turn the locomotive. The riding qualities of the locomotive have improved, the importance of the unbalanced forces has decreased in value, and the percentage of balancing the reciprocating weights could

have come down without impairing the riding qualities of the locomotives. At the same time, a better understanding of the counterbalance problem, cross-counterbalancing being an example, kept pace with these developments, and permitted the industry empirically, by the method of trial and error, to achieve practical solutions.

Let us see what percentages of balancing we find in locomotives which have been running satisfactorily at high speeds on American roads, some of them for years.

A 4-6-4 locomotive with 79-in. driving wheels has been in service on one road since 1927, with maximum speeds up to 90 mph. The locomotives were originally balanced by the old static method; consequently, there was a deficiency in the balance of the main driving axle to an extent that instead of a calculated static overbalance of 271 lb per driving wheel, a dynamic overbalance of only 6 lb was attained. Likewise, on the coupled axles, overbalances of only 191 lb on the front and 190 lb on the back wheel were actually obtained, instead of 221 and 218 lb, respectively. Thus a total of

$$\left(\frac{191 \times 61\frac{1}{2}}{74\frac{3}{4}} + \frac{6 \times 61\frac{1}{2}}{75\frac{1}{4}} + \frac{190 \times 60\frac{1}{2}}{74\frac{3}{4}}\right)\frac{1}{1791} = 0.175$$

or 17.5 per cent of balancing reciprocating parts was obtained instead of expected balancing of

$$\frac{221 + 271 + 218}{1791} = 0.396 \text{ or } 39.6 \text{ per cent}$$

when using the static method of balancing of both the revolving and reciprocating weights.

The actual dynamic augments at diameter speeds are: 8550 lb on the front wheels, which is 28.2 per cent of the static weight; 269 lb on the main wheels, which is 0.9 per cent of the static weight; and 8500 lb on the back drivers, which is 28 per cent of the static weight. These values compare with a dynamic augment of 12,160 lb on the main drivers, calculated by the static method. After operating this 4-6-4 locomotive for several years, the railroad cross-balanced a great number of this type, making the actual percentage of balancing the reciprocating weights 28.6, but up to this conversion the locomotives ran with only 17.5 per cent balancing.

In a locomotive of the 4-4-2 type, which was cross-balanced, the reciprocating balance per wheel was made 173 lb and the total reciprocating weights on one side of the locomotive were 988 lb. Taking the averages for the distances of different axles, the percentage of balancing is approximately

$$\frac{173 \times 2 \times 63\frac{1}{8}}{70\frac{11}{16}} \times \frac{1}{988} = 0.312, \text{ or } 31.2 \text{ per cent}$$

The dynamic augment at diameter speed is 7750 lb per driver, or 21.5 per cent of the static weight.

This locomotive is known for its outstanding riding qualities and for its quiet running at high speeds, thus proving that 31.2 per cent of balancing reciprocating weights should not be considered excessive.

In a 4-6-4 locomotive, built about a year ago, the percentage of balancing reciprocating weights is

$$\frac{136 \times 61}{73} + \frac{101 \times 61.5}{75\frac{1}{4}} + \frac{131 \times 61}{73} \times \frac{1}{1000} = 0.307 \text{ or } 30.7 \text{ per cent}$$

The dynamic augment at diameter speed is 6300 lb, or 9.6 per cent of the static weight per wheel.

In still another 4-6-4 recently built locomotive, which was used earlier in this paper as an example, the reciprocating balance per axle is 196 lb, and the total reciprocating weight on one side is 1681 lb. Thus the percentage, as already found from Figs. 9 and 11, is

$$\frac{196 \times 3 \times 61\frac{1}{2}}{76.25} \times \frac{1}{1681} = 0.282, \text{ or } 28.2 \text{ per cent}$$

instead of 35 per cent, as it was expected.

The dynamic augment at diameter speed is 9400 lb, or 13.1 per cent of the static weight per wheel.

It should be remembered that these locomotives are supposed to run at speeds of 90 mph and higher, and that their dynamic augments may be then about 30 to 50 per cent higher than at diameter speeds.

It should also be borne in mind that some modern, especially powerful, locomotives with small weight to power ratios are liable to slip at high speed, and that then the wheels may spin at rotative speeds much in excess of diameter speeds, sometimes 40 per cent higher, which would give a dynamic augment twice as great as that calculated herein by the author. For locomotives with such tendencies, it is advisable to make the reciprocating balances (overbalances) as small as possible, even if this should reduce the total percentage of balancing of reciprocating weights. The author does not know another alternative—either less percentage of balancing, or an increased dynamic augment.

The foregoing examples would tend to show us that from 20 to 30 per cent of balancing reciprocating weights above the cross-balancing revolving weights is permissible and conducive to quiet running. Although in one of the examples the locomotive has only two driving axles, the dynamic augment in this case is not excessive, that is, only 21.5 per cent of static weight per wheel (at diameter speed). In figuring counterbalance, the dynamic augment should always be watched, and for this figure alone, it is very essential to use the correct cross-balancing method for revolving and reciprocating weights, because otherwise the dynamic augment and the percentage of balancing reciprocating weights are not accurately known. Actually the dynamic augment is the first datum necessary to start intelligently the counterbalancing calculation, and then do it by the correct, which is the cross-balancing, method. No simplifications or saving a few hours of calculation can excuse manipulating with erroneous figures in such important a matter in the design of a locomotive as counterbalancing.

As to the reciprocating weights in modern high-speed locomotives, the numerical value of the reciprocating balances per wheel is rather small, although the damage which can be done at high speed may be very large. Tests have been made on some roads which indicated that the hammer blows at high speeds, and especially when the locomotive slips, may account for rail breakages, kinking of rails, and other phenomena which take place under these circumstances. Forced vibration of a locomotive wheel on a flexible roadbed, with its own natural frequency, would call for further study (22) and experimental research (23). That counterbalances have much to do with this problem can be learned from the experience of one road which, by reducing counterbalances on main wheels of a certain locomotive by 100 lb, eliminated the kinking and breaking of rails.

The question of slipping at high speeds would also need special attention. There are indications that there is a limit to power which can be transmitted to wheels on rails through the medium of adhesion, and that this phenomenon is different at high speeds from adhesion at low. This question should be of importance to both steam- and electric-locomotive builders, and has not yet been sufficiently studied (24, 25).

Suggestions

No conclusions are offered by the author as a result of the

above study, merely the following suggestions for further study of the problem are made.

1 For high-speed locomotives the correct evaluation of the dynamic augment is essential, and this should be done not only for the main wheel, but for the coupled wheels as well. The neglect to cross-balance the coupled wheels, when main wheels are cross-balanced, and more so, when statically balanced, introduces an irregular distribution of reciprocating balances, just contrary to the object of dividing the reciprocating balance equally among drivers.

2 There always is a certain piston thrust component on the main wheel, which is nonexistent on the coupled wheels. At high speeds and short cutoffs the component is small, but not negligible, and when the full piston thrust is as high as 140,000 lb, the component may be in the neighborhood of 10,000 to 12,000 lb. It is directed downward twice every revolution; and once, when the counterbalance is in the lower quarter, the angle at which the component takes place is not far from 90 deg to the horizontal. For this reason it is advantageous to have the main wheel angled for cross-counterbalancing, because then the hammer blow takes place at $\alpha = 90$ deg $+ \theta_v$, and these two blows (of the reciprocating balance and of the steam component) are spread apart.

This may also explain why cross-counterbalancing on some railroads, when introduced for the first time, caused rail breakages and was discarded. Locomotives with static balancing had in the past an actual deficiency in the dynamic action of wheel on rail and possibly no hammer blow at all. The effect of the piston thrust component was less, because instead of two blows in rapid succession per revolution, there was only one blow. When a cross-counterbalanced locomotive is placed in service for the first time on the road, these two blows might be sufficient to break a rail. It would be still better to have no counterbalance hammer blow at all and, therefore, to eliminate the reciprocating balance from the main wheels. This will also be helpful in case of the tendency of lifting the wheel from the rail at high speed.

There should be no misgiving about running a locomotive without reciprocating balance on one axle. Many locomotives with deficiencies in dynamic action had practically no counterbalance in the main wheel, and sometimes even a negative balance. They have been running for years at high speeds without an actual reciprocating balance on main wheels. In the author's first example of the 4-6-4 high-speed locomotive, the actual reciprocating balance in the main wheel was only 6 lb.

3 Although it does not make much difference whether we add the reciprocating balance exactly opposite the crank radius or at an angle θ_r in accordance with Fig. 8, it is better for the sake of uniformity and correctness to cross-balance the reciprocating weights. Especially, it is essential for figuring the percentage of balancing of these weights by taking into account the difference in planes, which is the principle of cross-counterbalancing. Moreover, as far as calculation is concerned, it is simpler to cross-balance the reciprocating weights on the main wheel than not to. For instance, when the A.A.R. Manual rule is followed and the revolving weights have been cross-balanced, it is necessary to find, first, the offset angle for the revolving weights θ_v; second, to add a static reciprocating balance to one component, as shown in Fig. 7; and, third, to add geometrically the component W_0 from the other side, shown in Fig. 6. This will determine the final θ, which should take the place of θ_v. When the reciprocating weights are also cross-balanced, the calculation can be made either by following Fig. 8, which does not add any work to the previous procedure, or by including in the calculation of the revolving balance the reciprocating balance per axle W_r adding this to the weights in the center plane of the cylinder. The total weight of the counterbalance W and the offset angle will then be directly determined.

Another alternative for the calculator would be to add the reciprocating balance to the revolving balance vector —196 lb at 7 deg 54 min (Fig. 8) instead of 10 deg 57 min in the same example; thus, making $\theta = \theta_v$, which is approximately correct.

Examples of Cross-Balancing Computations

As an illustration, the following calculation is given for the cross-balancing of a 4-6-4 locomotive, which in this calculation has no reciprocating balance on the main wheel and about 17.8 per cent balancing of reciprocating weights in the remaining wheels:

We shall start, as it was stated previously, from the dynamic augment. According to formula [2] the centrifugal force of a weight W_r is equal to

$$F = 0.0000284 W_r r n^2 \quad \text{[2a]}$$

where F is force, lb; W_r is weight in reciprocating balance (overbalance), lb; r is radius of crank, in.; and n is the driving-wheel

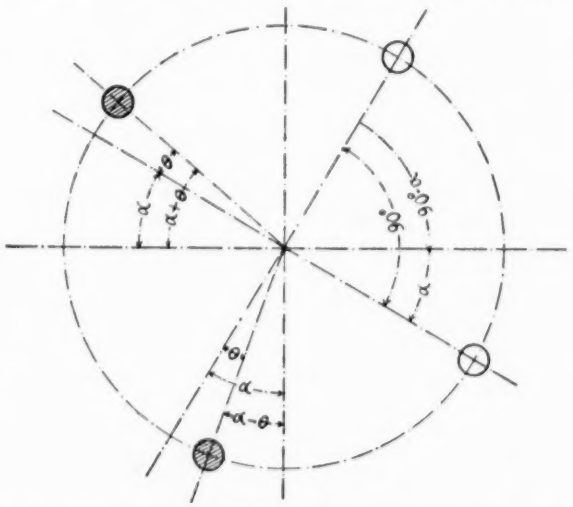


Fig. 12 Diagram of Offset Angles When Locomotive Is Cross-Balanced

speed, rpm. For this 4-6-4 locomotive, $r = 15$ in. and n, at diameter speed, is 336 rpm, thus, the formula is

$$F = 48.4\ W_r$$

Assume first that from previous experience with locomotives of this type and dimensions, or from a special study of the locomotive characteristics, we know that the locomotive will have no tendency to slip. In this case we may take the permissible dynamic augment as 25 per cent of the static weight per wheel. If the weight of this locomotive per axle is 72,000 lb, the dynamic augment for all driving wheels, except the main, i.e., the front and back wheels, can be made $36{,}000 \times 0.25 = 9000$ lb, assuming that there will be no reciprocating counterbalances on the main wheels. From the foregoing formula, we can easily find the overbalance W_r; thus, $9000 = 48.4 W_r$, or $W_r = 186$ lb.

As the locomotive is supposed to run occasionally at 100 mph, we might wish to check the dynamic augment at that speed as compared with diameter speed. The dynamic augment is $9000 \times (100/84)^2 = 12750$ lb, or $100(12750/36000) = 35.4$ per cent of the static weight, which is not excessive.

Now we can figure the percentage of balancing. From the

design of the parts of this locomotive we know that the reciprocating weights on one side of the locomotive are 1681 lb, and that the ratio of the distance F (from the center line of the cylinder to the balance plane on the other side) to the distance E (between the center planes of the balances) is 1.24. The percentage of balancing will be $100[(2 \times 186)/(1681 \times 1.24)] = 17.8$ per cent, which is sufficient.

The calculation of the main-wheel cross-counterbalance (without reciprocating balance), following the A.A.R. Manual (20) and its notations, is as follows:

W_1 = weight of the crankpin hub with included part of crankpin = 383 lb

W_2 = weight of the side rods with included part of crankpin = 750 lb

W_3 = weight of the back end of the main rod with included part of crankpin = 779 lb

W_4 = weight of part of the eccentric crank and the eccentric rod with included part of crankpin = 131 lb

W_t = actual weight of revolving parts = $W_1 + W_2 + W_3 + W_4 = 383 + 750 + 779 + 131 = 2043$ lb

$$W_c = \frac{(383 \times 61\,1/8) + (750 \times 67\,3/4) + (779 \times 75\,7/8) + (131 \times 82\,9/16)}{60\,3/4}$$

$$= \frac{144150}{60\,3/4} = 2373 \text{ lb}$$

$$W_0 = W_c - W_t = 2373 - 2043 = 330 \text{ lb}$$

The total balance will be $W_b = \sqrt{(2373^2 + 330^2)} = 2395$ lb, and the offset angle, as shown in Fig. 7, will be 7 deg 54 min.

For the front coupled axle the calculations are as follows:

W_1 = weight of the crankpin hub with included part of crankpin = 255 lb

W_2 = weight of part of the side rod with included part of crankpin = 257 lb

W_3 = part of the reciprocating weights to be balanced = 186 lb

$W_t = W_1 + W_2 + W_3 = 255 + 257 + 186 = 698$ lb

$$W_c = \frac{(255 \times 61\,1/2) + (257 \times 67\,7/8) + (186 \times 75\,7/8)}{61\,3/4}$$

$$= \frac{47239.13}{61\,3/4} = 765 \text{ lb}$$

$$W_0 = W_c - W_t = 765 - 698 = 67 \text{ lb}$$

The total balance will be $W = \sqrt{(765^2 + 67^2)} = 768$ lb, and for the offset angle, $\tan \theta_1 = 67/765 = 0.08758$, or $\theta_1 = 5$ deg 1 min.

Likewise, for the back driving axle, $W_1 = 263$ lb, $W_2 = 309$ lb, $W_3 = 186$ lb, and $W_t = 263 + 309 + 186 = 758$ lb. Also

$$W_c = \frac{(263 \times 61\,7/8) + (309 \times 68\,3/4) + (186 \times 75\,7/8)}{62\,1/4}$$

$$= \frac{51604}{62\,1/4} = 829 \text{ lb}$$

and

$$W_0 = W_c - W_t = 829 - 758 = 71 \text{ lb}$$

The total balance $W = \sqrt{(829^2 + 71^2)} = 832$ lb. For the offset angle, $\tan \theta_2 = 71/829 = 0.08565$, or $\theta_2 = 4$ deg 54 min.

However, if some precedents, or a study of tractive-effort characteristics lead to the suspicion that the locomotive might slip at high speed, it is advisable to cut the reciprocating balance. In other words, assuming that the speed of spinning may go up to 141 per cent of diameter speed, as is the case usually, the reciprocating balance should be halved, if we want to retain the same dynamic augment. In the foregoing example it should be made $186/2 = 93$ lb.

The percentage of balancing will be then reduced, but after what we have seen it would do no great harm. When larger percentage of balancing reciprocating weights is desired, greater dynamic augment must be allowed and the roadbed strengthened accordingly.

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Discussion

T. V. BUCKWALTER[3] AND O. J. HORGER.[4] One of the main disadvantages of the conventional two-cylinder steam locomotive over some other forms of power is the inability to properly balance the reciprocating weights. The usual heavy weight of reciprocating parts and the use of conventional balancing methods penalize both the track and locomotive in that the dynamic stresses are increased and the riding characteristics of the locomotive leave much to be desired.

This problem becomes of such major importance because of the weight in pounds of the reciprocating parts. Since the reciprocating parts may never be completely balanced the practical solution to this problem would be the reduction of reciprocating weight. Road experience is available on about sixty high-speed passenger locomotives, where the weight of reciprocating parts is reduced, so that it is only about one half of the conventional weight. These applications have permitted both a considerable reduction in dynamic augment as well as a reduction of the shaking forces.[5] In this respect these lightweight reciprocating parts will better permit the use of much lower reciprocating overbalance than is now the practice. This is in keeping with the recommendations of the author.

The author discusses in considerable detail the beneficial influence of reducing overbalance in order to produce less dynamic augment, but little is said about the effects of the unbalanced portion of the reciprocating weights which determines the shaking forces. These shaking forces, causing (*a*) fore-and-aft motion and (*b*) turning moment of the engines on the rail, are reflected in track and locomotive maintenance and locomotive riding. The tangible dollar value of any reduction in shaking forces as it influences reduced maintenance is unknown, but tests have been made where the overbalance was reduced to such a point that the increased shaking forces produced a rough-riding engine. In some cases this roughness was reflected in the riding of the first few cars of the train. Just where the dividing line exists for the proper selection of allowable shaking forces for minimum track and locomotive maintenance has never been fully appraised. For these reasons, it is difficult to understand how the author proposes to approach zero overbalance or to bring American practice within the European limitation of dynamic augment, in order not to exceed 15 per cent of the static weight on the wheel and, at the same time have a favorable condition of shaking forces, without coming to the conclusion that a considerable reduction must be made in weight of reciprocating parts.

The existing method of balancing does not give consideration to the proper division of back- and front-end weight of the main rod as to how much should be considered as rotating and how much as reciprocating weight. Ordinarily the scale-pan weight of the back end of the main rod is considered as rotating and is so balanced. Actually the division of the main-rod weight into rotating weight and reciprocating weight cannot be properly made, unless the moment of inertia of the rod about the center line of the crosshead pin bore is known. This may be determined experimentally by swinging the rod as a pendulum. The result is that a larger overbalance exists than is calculated by the usual procedure, by an amount of about 80 to 130 lb in the counterbalance plane at the crankpin radius, as determined by pendulum tests on several designs of rods investigated. Obviously this introduces a considerable percentage of error, especially so when a low overbalance is already used. Consideration of this error at diameter speed increases the dynamic augment by 4000 to 5000 lb over what is conventionally calculated. There are, however, a few designs of main rods having heavy back ends which minimize this error over that found on conventional rod designs.

The author cites an example of a 4-6-4 locomotive with 79-in. driving wheels as having been in road service since 1927 with maximum speeds up to 90 mph. It is stated that the original static balance of 271 lb per driving wheel calculates dynamically as actually being only 6 lb overbalance. The author then gives the dynamic augment on main wheels as actually being 296 lb instead of the previous 12,160 lb obtained by static method. While the full details of this engine are not given, it is known that in cases of a similar nature the dynamic augment opposite the crankpin is considerably reduced when cross-counterbalancing methods of calculation are applied to engines balanced statically. The writer wishes to indicate, however, that the maximum dynamic augment on statically balanced drivers invariably does not occur in a position nearly opposite the crankpin because of the great effect of the unbalanced component from the opposite wheel. For this reason the question is being asked if such consideration was given in the example submitted by the author.

Forced vibrations of locomotive driving wheels on a flexible roadbed are only briefly mentioned by the author as being subjects for study and research. This phase of locomotive operation will become increasingly important as speeds are increased and will be equally as important in the future as dynamic augment and shaking forces. Even at existing operating speeds, such forced vibrations have required rebalancing and reduction in overbalance in order to prevent wheels leaving the rail in a vertical direction, which has resulted in considerable rail damage.

In order to study this subject further, tests were made where six different kinds of locomotives were slipped at high speeds. It was shown conclusively that the phenomenon of forced vibration does exist and that the main pair of driving wheels leaves the rail at a speed much lower than would be indicated only by a consideration of the inertia forces. This study was made with high-speed moving-picture cameras. The results of tests will be shown before the New York and Chicago Railway Clubs.

Here again in order to overcome this vibration phenomenon it will be found that, unless the weight of reciprocating parts is considerably reduced, while at the same time lower overbalances are used to prevent rail damage, the increased shaking forces will leave an undesirable effect on locomotive and track.

The slipping mentioned by the author may in some cases be attributed to vibrations and/or as a result of large variation in rail pressure developed by large overbalance weights. The unbalanced portion of the reciprocating weight is also believed to contribute to slipping.

The author refers to the piston-thrust component, existing on the main wheel, but does not mention a similar component which also exists as a result of the inertia of the reciprocating parts. Neither of these components varies according to a sine law but, neglecting this, their frequency is twice that of the dynamic augment, so that their effect in causing vibrations is of much less importance than that of the dynamic augment.

R. EKSERGIAN.[6] The counterbalancing of reciprocating parts is necessarily a compromise as to the permissible unbalance in the plane of reciprocation and the allowable dynamic loading at the rail. The disturbing factors causing poor riding of a locomotive are not only due to reciprocating unbalance, but very often

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[4] In charge of Railway Engineering and Research, The Timken Roller Bearing Company, Canton, O. Mem. A.S.M.E.

[5] "Modern Locomotives and Axle-Testing Equipment," by T. V. Buckwalter, O. J. Horger, and W. C. Sanders, Trans. A.S.M.E., vol. 59, 1937, paper RR-59-1, pp. 225–238.

[6] Consulting Engineer, Edward G. Budd Manufacturing Company, Philadelphia, Pa. Mem. A.S.M.E.

and to a much larger extent to torque reaction, to variation in tractive force per revolution, and to vehicle oscillations, such as self-induced lateral and nosing vibrations. For this reason the question of permissible unbalance in the plane of reciprocation is difficult to appraise. The writer some years ago recommended values around 35 per cent for the balancing of reciprocating parts. The author's observations that satisfactory riding conditions are maintained with a 30 per cent balance are very pertinent.

The disturbance due to reciprocating inertia forces may be classified into four disturbances: First, a fore-and-aft vibration; second, a nosing moment; third, an inertia torque and vertical force on the drivers, with a corresponding reaction torque and vertical reaction on the spring-borne parts; and fourth, associated with the inertia torque on the drivers, a varying tractive force, causing additional fore-and-aft vibration. The latter two cannot be eliminated by any balancing method and therefore will be dismissed in so far as the counterbalancing problem is concerned. On the other hand, in a proper appraising of the total disturbance, they augment those due to steam thrust and track disturbance.

As to balancing reciprocating parts, it is further important to differentiate between the relative importance of fore-and-aft and nosing-moment unbalance. Dynamic balancing presupposes the same percentage of balance for these two components. In static balance a greater percentage of fore-and-aft balance is obtained. However, the effects on the locomotive have an entirely different interpretation. The linear fore-and-aft displacement for the fore-and-aft unbalance depends also on the mass of the locomotive, while the angular amplitude corresponding to the moment unbalance depends upon the polar moment of inertia of the locomotive and lateral constraints. By tying in the tender through buffers spaced laterally between the engine and tender, considerable additional lateral constraint is effected. Large locomotives, in modern power, have large polar moments of inertia, which with friction and leading-truck resistances, greatly reduce the importance of nosing unbalance. Experimental data as obtained from the Indian Bridge Commission tests have indicated no correlation between nosing vibrations and reciprocating-motion frequencies. In all cases, nosing vibrations at high speeds have been correlated with track disturbances and self-induced vibrations. This condition has also been observed on some American railroads, particularly at high speeds, where the balancing problem is of greatest importance. The author also more or less concurs on this point not only in this paper but also in his discussion of the writer's paper.[7]

In the writer's paper emphasis was given to the importance of cross-balancing revolving parts, particularly for main driving wheels which are considerably out-of-plane for these weights. While the writer completely concurs with the author as to a complete balancing of revolving weights, where they only exist, it is important to note that the very feature of introducing reciprocating balance sets up a new vertical disturbance. Relief of the dynamic augment can be effected by introducing couples at quadrature, which in no way affects the balancing of the reciprocating parts fore and aft, but does increase the nosing-moment unbalance.

This method of balancing was introduced by the writer in the closure of his paper.[7] It consisted of cross-balancing only partially the revolving parts, resulting in a minimum dynamic augment at the rail. By this method the dynamic rail pressure can be reduced by roughly 30 per cent.

The essence of the method is shown in Figs. 13, 14, and 15, where W_r = any revolving weight, spaced a distance h from the wheel on the balance plane. Let a percentage k of the revolving weight be balanced statically, so that a balance weight kW_r is placed in the wheel center opposite the crank, as in the usual method of balancing the revolving weights statically. The remainder $(1 - k)W_r$ should be cross-balanced in the orthodox method of dynamic balancing, so that $(1 - k)W_r$ has no effect at the rail or at the pedestals.

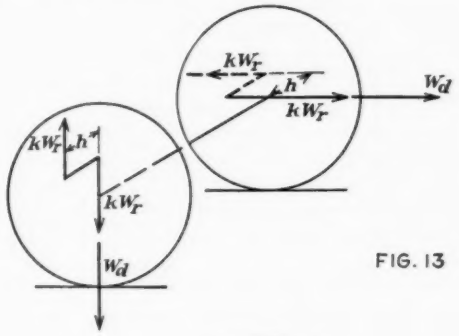


FIG. 13

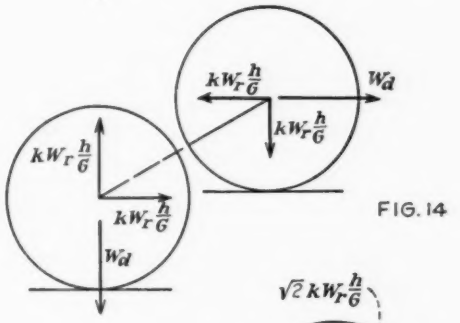


FIG. 14

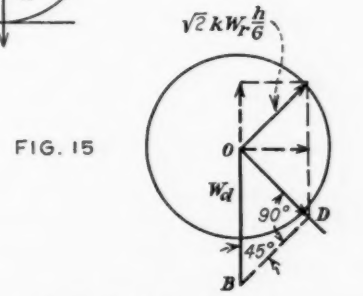


FIG. 15

FIGS. 13, 14, AND 15

From Fig. 13, obviously the couple kW_rh reduces the effect of the dynamic augment W_d, required for reciprocating balance, thus reducing the rail pressure. In Fig. 14, the couples kW_rh at either wheel are reduced to equivalent couples in the forces in the wheel (or balance) plane kW_rh/G at quadrature. These forces compound into a resultant $\sqrt{2}kW_rh/G$, making an angle of 45 deg with respect to the vertical. A little consideration will show that the minimum rail pressure occurs when OD makes a right angle with BD, Fig. 15. Therefore, since ODB is a right angle, and $OB = W_d$ and $BD = \sqrt{2}kW_rh/G$, then

$$BD = OB \cos 45 \text{ deg} = OB/\sqrt{2}$$

$$\sqrt{2}(kW_r)h/G = W_r\sqrt{2}$$

so that the percentage of revolving weight to remain statically balanced is

$$k = \frac{1}{2}\frac{W_d}{W_r}\frac{G}{h}$$

When the reciprocating parts are cross-balanced, if W_d = the required weight in the wheel center, for the reciprocating balance, and if α = the phase angle of the balance, then

[7] "The Balancing and Dynamic Rail Pressure of Locomotives," by R. Eksergian, Trans. A.S.M.E., vol. 51, part 2, 1929, paper RR-51-55.

and to a much larger extent to torque reaction, to variation in tractive force per revolution, and to vehicle oscillations, such as self-induced lateral and nosing vibrations. For this reason the question of permissible unbalance in the plane of reciprocation is difficult to appraise. The writer some years ago recommended values around 35 per cent for the balancing of reciprocating parts. The author's observations that satisfactory riding conditions are maintained with a 30 per cent balance are very pertinent.

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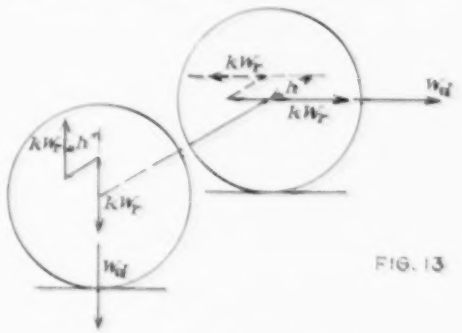


FIG. 13

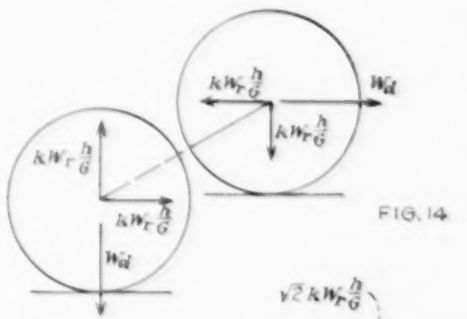


FIG. 14

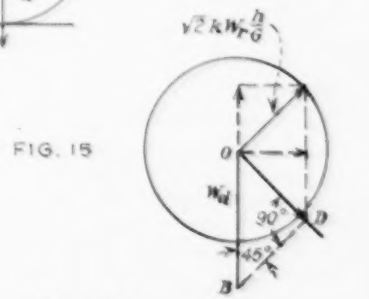


FIG. 15

FIGS. 13, 14, AND 15

From Fig. 13, obviously the couple $kW_r h$ reduces the effect of the dynamic augment W_d required for reciprocating balance, thus reducing the rail pressure. In Fig. 14, the couples $kW_r h$ at either wheel are reduced to equivalent couples in the forces in the wheel (or balance) plane $kW_r h/G$ at quadrature. These forces compound into a resultant $\sqrt{2}kW_r h/G$, making an angle of 45 deg with respect to the vertical. A little consideration will show that the minimum rail pressure occurs when OD makes a right angle with BD, Fig. 15. Therefore, since ODB is a right angle, and $OB = W_d$ and $BD = \sqrt{2}kW_r h/G$, then

$$BD = OB \cos 45 \text{ deg} = OB/\sqrt{2}$$

$$\sqrt{2}(kW_r)h/G = W_r \sqrt{2}$$

so that the percentage of revolving weight to remain statically balanced is

$$k = \frac{1}{2} \frac{W_d}{W_r} \frac{G}{h}$$

When the reciprocating parts are cross-balanced, if W_d = the required weight in the wheel center, for the reciprocating balance, and if α = the phase angle of the balance, then

[7] "The Balancing and Dynamic Rail Pressure of Locomotives," by R. Eksergian, Trans. A.S.M.E., vol. 51, part 2, 1929, paper RR-51-55.

$$P_R = \frac{W_d'}{W_r}\frac{G}{h}\cos\frac{(45-\alpha)}{\sqrt{2}}$$

Obviously the remaining part $(1-k)W_r$, of the revolving weights should be cross-balanced. Except for the main driver P_k usually considerably exceeds unity. For this reason coupled drivers should be completely statically balanced. In the case of main drivers a partial cross-balancing is necessary. With a reduction in the percentage of reciprocating balance in modern power and if the percentage of this required reciprocating balance is reduced on the main driver then the condition for full cross-balancing on the main driver is approached.

It is important to emphasize that this method of balancing is postulated on the premise that an increased nosing moment over a complete cross-balancing is permissible. On the other hand, this method results in nosing moments less than by static balance in virtue of still maintaining a considerable cross-balance for the main driver.

In the final analysis, in normal operation, the greatest concern is the rail stress and not necessarily individual rail pressures per driver. If the revolving parts are completely cross-balanced throughout and the reciprocating balance then distributed with equal dynamic augment per driver, a series of parallel vector loadings along the rail results. In this case, with equal wheel loadings, the maximum stress occurs on end drivers. Eliminating the dynamic augment on the main driver further increases these stresses. The author calls attention to steam thrust, with a downward component on the main driver. At high speeds it is questionable whether this component is sufficiently high to justify the elimination of reciprocating balance on the main drivers. The main driver can suffer greater loads due to the negative bending moments of adjacent drivers.

The advantages of the reduction in individual rail pressure by the method of partial balance (usually complete static balance for coupled drivers) more than offsets the somewhat more undesirable phase angles, even when considering rail stress.

A peculiar condition arises in locomotives running at very high speeds. Due to the elasticity of the rail, the heavy wheel-axle assemblies, and the spring-supported parts, the first approximation is that of a double-spring pendulum, consisting of an equivalent rail spring, an intermediate mass, that of the drivers, and then the driver springs connecting to the spring-borne parts.

The system has two modes of vibration with their two natural frequencies, one low and one high. The disturbing force is the unbalance in the wheel center.[8] At very high speeds the second mode can be excited, which may cause actual lifting of the driver from the rail, with consequent slipping. This phenomenon was first observed in connection with bridge vibrations. In this case the elastic rail is replaced by the elastic deformation of the bridge structure. While the actual system is more complicated with additional degrees of freedom, the cure is best obtained by a reduction of the disturbing force, particularly at the main driver; for this condition reduction in dynamic augment at the main driver may be necessary.

In conclusion the writer recommends the retaining of static balance on coupled drivers. The main driver should be partially cross-balanced on the basis of retaining at least a certain percentage of reciprocating balance with large wheels. The final design should be based on the condition of minimum rail stress. With several drivers and smaller wheels complete elimination of reciprocating balance for main drivers may be desirable, due to steam load and increased dead weight. In this case the revolving weights on main drivers are completely cross-balanced, with partial cross-balancing for the intermediate drivers with heavy revolving weights.

[8] Another disturbing force is torque reaction and vertical thrust between the main driver and spring-borne parts which may account for lifting effect on main driver and not on coupled wheels. This condition is independent of balance.

L. H. Fry.[9] Before discussing the cross-balancing of the reciprocating parts, attention is called to the fact that the author measures the amount of balance as a percentage of the weight of the reciprocating parts. If thought on the subject is to be clear, it is important that the balance should be measured in definite terms and not as a percentage of the mass of the reciprocating parts. In the case of a 350,000-lb locomotive, if the weight of the unbalanced mass of the reciprocating parts is 750 lb, the riding of the locomotive will be the same whether this is 100 per cent of lightweight reciprocating parts weighing 750 lb, 50 per cent of parts weighing 1500 lb, or 25 per cent of very heavy parts weighing 3000 lb. The longitudinal disturbance of the locomotive will depend on the mass of the locomotive and on the mass left unbalanced, not on the relation of the unbalance to the mass of the reciprocating parts. Recognition of this fact will do much to simplify the discussion of balancing problems. It is particularly desirable at the present time because of the increasing use of special lightweight reciprocating parts.

No novelty can be claimed for urging that reference to the percentage of the mass of the reciprocating parts be given up. G. R. Henderson suggested this as long ago as 1895, though the reference generally given is to his 1904 book.[10]

Henderson suggested that 1/400 of the mass of the locomotive might remain unbalanced on each side; that is, 750 lb for a 350,000-lb locomotive. However, it seems probable that satisfactory riding qualities can be obtained with a larger proportion of the mass of the locomotive left unbalanced. The author speaks of a 4-6-4 locomotive which apparently rode well with 1450 lb on a side unbalanced, which is surely more than 1/400 of the mass of the locomotive. Many supporting opinions can be quoted: Chapelon[11] refers to satisfactory results obtained in Germany with greatly reduced longitudinal balance and suggests that designers should break away from the "hypnotic influence" of the balancing of reciprocating parts; D. C. Brown[12] shows vibration diagrams taken at 76 mph, 350 rpm, from a locomotive. In one case 66 per cent of the reciprocating parts were balanced, in the other case none. It is concluded that there is little difference in fore-and-aft motion.

It is believed that existing locomotives should be studied and tests run to find out how much reciprocating balance is really needed to give good riding and how much is being put in to satisfy tradition.

Every pound of overbalance put in to balance the reciprocating parts contributes to the dynamic augment. It is, therefore, desirable to keep the overbalance down to a minimum. It is believed that this is the fundamental problem in balancing the reciprocating parts. When it is satisfactorily solved, the question of whether or not to cross-balance the reciprocating parts will prove to be of secondary importance.

Referring to Fig. 16, it is assumed desirable to balance an equivalent weight of 100 lb at the crankpin. Fig. 16 (*A*) shows the unbalanced equivalent weight with 62 in. between the planes of counterbalances and the plane of the left-hand main rod 73 in. from the plane of the right-hand counterbalance. The arrangement in Fig. 16(*B*) balances the longitudinal force, but

[9] Railway Engineer, Edgewater Steel Company, Pittsburgh, Pa. Mem. A.S.M.E.

[10] "Locomotive Operation," by G. R. Henderson, Simmons-Boardman Publishing Company, New York, 1904.

[11] "La Locomotive à Vapeur," by André Chapelon, 1938, p. 754.

[12] "Counterbalancing and Its Effect on Locomotives and Bridges," by D. C. Brown, *Journal*, Institution of Locomotive Engineers, Jan.-Feb., 1938, p. 90.

leaves a nosing moment of 1100 in-lb. The equivalent weight producing dynamic augment is 100 lb. Fig. 16(*C*) shows the effect of cross-balancing the balance. The longitudinal force and the nosing moment are zero, but the equivalent weight producing dynamic augment has been increased to 118 lb. It seems improbable that the reduction in the nosing moment is of sufficient advantage to offset this increase in dynamic augment.

In weighing the importance of nosing moment, it should be remembered that the disturbing inertia forces, as well as the balancing components, originate in the driving wheels and

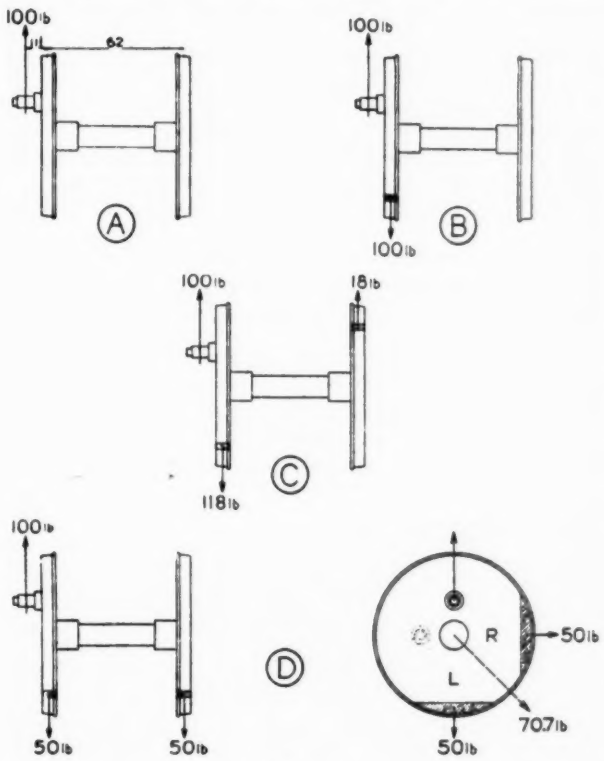


FIG. 16

attached engine parts. Until these parts move, the forces cannot reach the locomotive frame and superstructure. The group of driving wheels cannot rotate about a vertical axis until its friction on the rail is overcome. With a long locomotive the frictional moment-resisting nosing is large. Even if slippage takes place, the large mass and the length of the frame and boiler offer a high polar moment of inertia to resist nosing.

Taking advantage of this, the system of balance for reciprocating parts shown in Fig. 16(*D*) has been used with success. In this the dynamic augment is reduced at the expense of the nosing moment. To balance the equivalent weight of 100 lb at the left pin, equivalent weights of 50 lb each are added in the left- and right-hand wheels. When this is done for both sides of the engine, each wheel receives two equivalent weights of 50 lb each set 90 deg apart. The resultant, producing dynamic augment, is $50 \times 2 \times 0.707 = 70.7$ lb. Comparing the results of the three methods of balance, it is seen that in Figs. 16(*A*), 16(*B*), and 16(*C*), balance for 100 lb equivalent weight of reciprocating parts is provided on each side of the locomotive. With method (*A*), the dynamic augment is due to 100-lb equivalent weight and the nosing moment is due to an equivalent-weight moment of $100 \times 11 = 1100$ in-lb. With method (*B*), cross-balance of the reciprocating parts, the dynamic augment is produced by an equivalent weight of 118 lb, while the nosing moment is zero. With method (*C*) the equivalent weight, producing dynamic augment, is reduced to 70.7 lb with a nosing moment due to an equivalent weight moment of $50 \times 11 + 50 \times 62 = 3600$ in-lb. For any driving-wheel diameter and speed the actual value of dynamic augment and nosing moment is found by multiplying the equivalent weight or the equivalent-weight moment by

$$0.0000284\ Rn^2$$

which is derived from the author's Equation [2*a*].

Compared with direct balance, cross-balance for the reciprocating parts increases the dynamic augment by 18 per cent, while method (*D*) reduces it by 29.3 per cent. Method (*D*) was suggested in 1909 by H. H. Vaughan,[13] then superintendent of motive power of the Canadian Pacific Railway. He reported that the method was giving good results. It has also been adopted by the New Zealand Government Railways with satisfactory results, as indicated in a personal letter to the writer, dated January 10, 1935, from S. H. Jenkinson.

In conclusion attention is drawn once more to the desirability of stating the final condition of each driving wheel in simple and accurate terms. The statement that a pair of wheels is dynamically balanced is a complete and definite description. The wheels will rotate at any speed without any dynamic augment. On the other hand, the statement that a pair of wheels is statically balanced has no definite meaning. As the wheels rotate they will be subject to a varying couple, tending to rotate the axle in a vertical plane at right angles to the direction of motion. As a result each wheel is subject to a varying dynamic augment. The values of the couple and of the dynamic augment are unknown, until the equivalent weight of the rotating parts is determined and the relative positions of the vertical planes in which the rotating parts and the counterbalances revolve. If a pair of wheels is not dynamically balanced, their condition can be described in a completely satisfactory manner by saying that in each wheel there is an unbalance represented by an equivalent weight having a definite value and a definite position. Take for example the wheels in Fig. 16(*D*). Each wheel of the pair has two equivalent weights of 50 lb each in addition to the dynamic balance. The net result in each wheel is an unbalance represented by an equivalent weight of 70.7 lb acting at 145 deg from the crankpin. See also Fig. 2 in a former paper by the writer.[14]

The final condition of the wheel should be described by a single equivalent weight of definite value and position. It is immaterial whether this final unbalance arises from imperfect dynamic balancing or from the addition of an overbalance to a pair of dynamically balanced wheels, or from a combination of the two. The final net resultant unbalance should be stated as a single equivalent weight.

If this is done, a great many ambiguities will be cleared away. For example, the conditions described in L. B. Jones's discussion of this paper would be shown in their true light. In his discussion it is stated that, on one of the Pennsylvania Railroad locomotives, the balance was such that any change, either an increase or a decrease, in the amount of balance added for the reciprocating parts produced an increase in the dynamic augment. This sounds paradoxical, but is easily explained if the single resultant method is used to describe the conditions. In the main wheels in question the combination of rotating parts and of counterbalance added supposedly to produce so-called static balance and to balance the reciprocating parts is such that

[13] "Locomotive Counterbalancing," by H. H. Vaughan, Proceedings, Canadian Railway Club, vol. 8, no. 6, Sept. 26, 1909.

[14] "Locomotive Counterbalancing," by Lawford H. Fry, Trans. A.S.M.E., vol. 56, 1934, paper RR-56-2, p. 434.

the standard condition of the wheels is that shown in Fig. 17. In each wheel there is an unbalanced equivalent weight of 282 pounds acting at A in the right- and at B in the left-hand wheel. It is easily seen that if more overbalance is added at D, in the right-hand wheel, to balance a greater proportion of the reciprocating parts, the net resultant of the equivalent weights at A and D will be greater than the original 282 pounds at A and the dynamic augment will be increased. Similarly, if the overbalance at D for the reciprocating parts is reduced, the effect will be the same as an addition of overbalance at B and as before the dynamic augment will be increased.

It is also obvious that the dynamic augment could be reduced

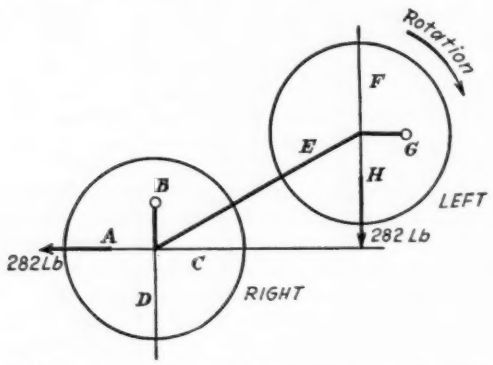


FIG. 17 NET UNBALANCE IN MAIN WHEELS OF PENNSYLVANIA RAILROAD LOCOMOTIVE

by adding overbalance at C. This would reduce the longitudinal balance opposing the inertia of the reciprocating parts because the existing unbalanced equivalent weight of 282 pounds at A in the right wheel is opposing the inertia force of the reciprocating parts acting on the left pin G.

L. B. JONES.[15] The application of the methods, suggested by the author, as well as any of the accepted counterbalancing

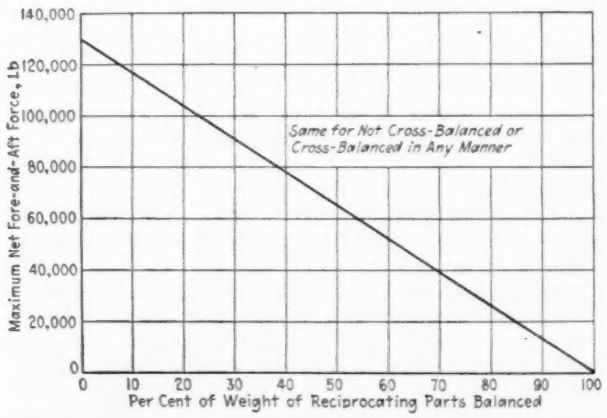


FIG. 18 GRAPHIC ANALYSIS OF FORE-AND-AFT FORCES ACTING ON PACIFIC-TYPE LOCOMOTIVE, COMPUTED AT 400 RPM

formulas, requires a large amount of calculation if the designer cares to explore carefully all the possible effects of any particular counterbalance. In operating locomotives at high speed at the locomotive testing plant, it has been found necessary to make a number of counterbalance analyses and, as a result, resort has been made to a graphic analysis by means of which it is more convenient to survey all phases of the subject.

The diagrams, Figs. 18 to 21, were computed at a speed of 400 rpm for a Pacific-type locomotive, having approximately 1500 lb of reciprocating weight per side, 28-in. piston stroke, 80-in. driving wheels, and 89-in. cylinder spread. The overbalance due to reciprocating parts is distributed equally among the three pairs of drivers. Since the percentage of reciprocating weights to be balanced is the determining factor in counterbalance behavior, all of the diagrams have this as a base, with

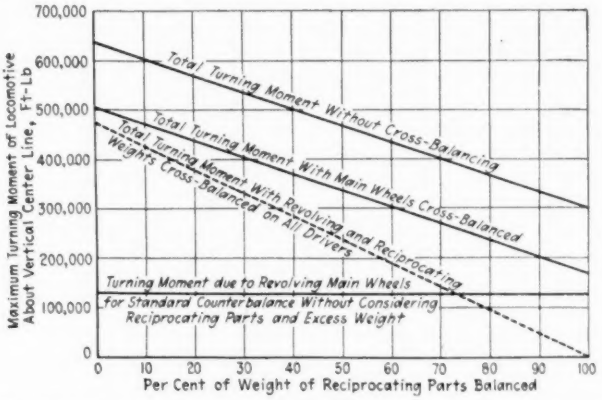


FIG. 19 GRAPHIC ANALYSIS OF TURNING FORCES, COMPUTED AT 400 RPM

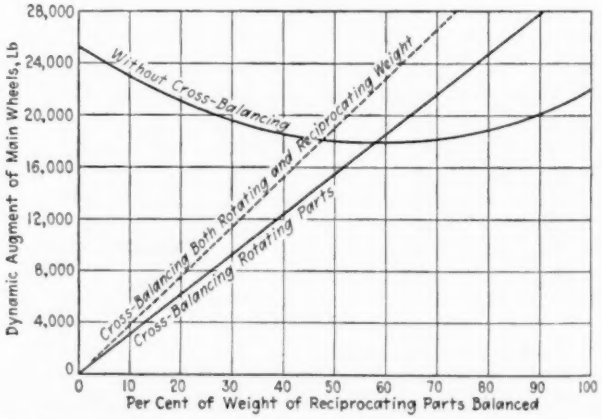


FIG. 20 DYNAMIC AUGMENT, MAIN WHEELS, COMPUTED AT 400 RPM

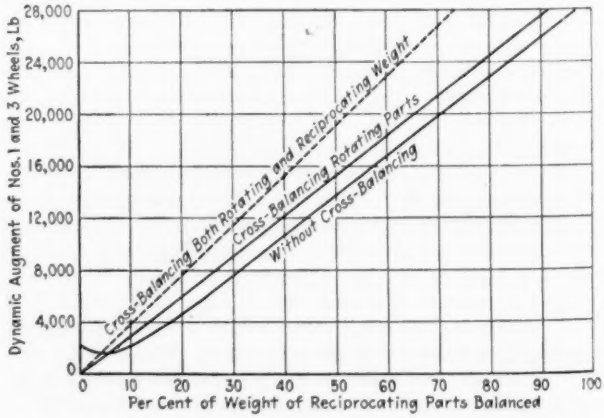


FIG. 21 DYNAMIC AUGMENT, COUPLED WHEELS, COMPUTED AT 400 RPM

[15] Engineer of Tests, The Pennsylvania Railroad, Altoona, Pa. Mem. A.S.M.E.

resulting forces as ordinates. The effect of the angularity of the main rod has been omitted, as it is relatively small, but if the designer desires to include it, he can do so after he has made his selection of the more important weights and dimensions.

Fig. 18 shows the behavior of the fore-and-aft forces, starting at 0 with 100 per cent overbalance and getting into large figures with little or no overbalance. These are the forces which tend to produce shake or vibration in the locomotive which, under certain conditions, are transmitted through the tender and couplers to the leading cars in the train. This diagram applies to any form of counterbalance—static balance, rotating cross-balance or reciprocating cross-balance.

Fig. 19 illustrates the behavior of the turning forces around the vertical axis of the locomotive. In this case, there is a difference between the effects of static balancing and cross-counterbalancing; and a further difference, as a result of cross-counterbalancing the reciprocating parts, which is illustrated by the dotted line. For high-speed counterbalancing, the effect of turning forces can probably be ignored, as the frequency is too high to produce any nosing effect of the locomotive.

Fig. 20 shows the behavior of the dynamic augment in the main wheels. The difference in behavior of the dynamic augments for static and rotating cross-counterbalance is quite marked. A peculiarity of static balancing, which is illustrated by the curve, is the fact that any decrease below or increase above the proper amount of counterbalance, will result in an increase in dynamic augment. With cross-counterbalancing, the curve is a straight line starting from 0 at 0 overbalance. A peculiarity of this curve is that it crosses the static-balancing curve at its minimum point. Above this intersection, the dynamic augment with cross-counterbalancing is greater than with static balancing and increases rapidly. It will also be noted that the static curve is relatively flat in the center and a considerable variation in overbalance produces only a small variation in dynamic augment; whereas the dynamic augment due to cross-balancing is relatively sensitive to the amount of overbalance.

In Fig. 21 the effect of cross-counterbalancing the reciprocating parts, according to the method suggested by the author, is shown by a dotted line. It is evident, for the particular locomotive under analysis, that it would be a mistake to cross-counterbalance the reciprocating parts, due to the rapid increase in dynamic augment.

Figs. 18 and 20 illustrate the necessity for a careful compromise in counterbalancing. Generally speaking, the effect of dynamic augment has been found to be greater on light rail than on heavy rail. Therefore it would appear consistent to keep the dynamic augment as low as possible in locomotives which are to operate on light rail. On the other hand, Fig. 18 shows at a glance what can be expected in the way of fore-and-aft forces, if the overbalance for reciprocating parts is kept too low.

The same form of graphic analysis can be applied to the coupled drivers, as in Fig. 21. As with the main drivers, the curves for static and cross-counterbalancing cross, but much lower down, because the plane of the counterbalance is much more nearly in the plane of the forces. For the locomotive in question, it is evident that better results will be obtained by static balancing the coupled wheels; which will also simplify the design of the wheel center.

G. F. STARBUCK.[16] It is to be regretted that the author made no reference to the paper by McCall.[17] This is the most informative paper on the subject that has appeared in many years. It includes a discussion of the steam-pressure effect and it also touches upon the cross-balancing of reciprocating weight.

It seems to be the inference that, for perfect balancing of the rotating weights, the main wheels will be just alike. That is true for the crank hubs and pins, side rods, parts of the main rods, and large ends of the eccentric cranks. It is not true for the small ends of the eccentric cranks and the back ends of the eccentric rods. These weights although small are located at considerable distances from the plane of the rail and cannot be ignored for the best counterbalancing.

While insisting that counterbalancing is a dynamic problem, the author adheres to the common practice of actually balancing the main rod statically, that is, balancing the static weight of the back end of the rod. The motion of the rod is translation combined with rotation about the crosshead wrist pin. The vertical component of the force at the crankpin, due to the inertia of the rod, is the force required for this rotation. It is, therefore, most accurately balanced by an equivalent weight of $\frac{Wk^2}{l^2}$; where W = weight of rod, k = distance from crosshead pin to center of gyration, and l = length. R. Eksergian in a recent paper[18] states: "It is difficult to see physical justification for the formula Wk^2/l^2 for an equivalent inertia mass at the pin." Justification is to be found in any treatise on the dynamics of rotating bodies. The application is not limited to the condition of a fixed crosshead. Of course there is a horizontal component, but its excess over the horizontal component of the weight determined as above, is reciprocating weight. In a typical locomotive the mistake of balancing the static weight of the back end of the rod results in a dynamic augment of about 5000 lb or, say, 15 per cent of the weight on the wheel.

There are many mathematical treatments of the subject of rotating bodies. The following illustration will help non-mathematical designers to see the mistake in the common method. If the front end of the rod moved in just such a circle as the back end, instead of moving to and fro in a straight line, the vertical effect at the main pin would be exactly as now assumed. The vertical acceleration of all points on the rod would be the same, just as in the side rods. The vertical accelerating forces for all elementary equal masses would be the same and would be resolved into forces at the two ends, according to the locations of the masses. If weights of the masses are used instead of accelerating forces the resolution of the forces between the two ends would result in the same proportions. The components would be called the static weights of the two ends. If, however, the front end of the rod moves in a horizontal line, the vertical amplitude and consequently the acceleration and accelerating force of almost every one of the equal, elementary masses will be less than in the first case and the sum of the components at each end will be less.

The unbalanced reciprocating weight will have a to-and-fro effect combined with a tendency to nose, which tendency is produced by a couple having as its arm one half the cylinder spread. Reciprocating weight simply balanced has no to-and-fro effect but has a tendency to nose, the arm of the couple being the distance between the planes of the cylinder and of the nearer counterbalance. The smallness of this tendency will be appreciated when it is realized that its arm is 13 or 14 in., compared with a locomotive length of 35 to 45 ft and a radius of gyration of 10 to 13 ft. Excepting the small amount of free lateral, there is very high frictional resistance to nosing besides the inertia resistance. To-and-fro disturbances are often transmitted to the train and can be serious if they synchronize with the draft and buffing gears.

[16] Mechanical Engineer, Waltham, Mass. Mem. A.S.M.E.

[17] "Cross or Simple Counterbalancing," by K. F. McCall, *Railway Mechanical Engineer*, August, 1936, p. 346.

[18] Discussion of: "The Design of Railway Axles and Locomotive Crank Pins," by R. Eksergian. Trans. A.S.M.E., vol. 60, Nov., 1938. See author's closure, p. 693.

Now let the reciprocating balance be angled without change in amount. The angling is equivalent to adding a 90-deg weight. Thus for each effective component opposite the crank, there will be an opposing component in the opposite wheel reducing its effectiveness as a to-and-fro balance. The resulting increase in unbalanced reciprocating weight will give rise to a nosing couple more than one half that eliminated by the angling. There has been an increase in to-and-fro rough-riding with but slight decrease in nosing, which was negligible before. If the balance weight had been increased when it was rotated, according to the customary method of cross-balancing rotating weight, the dynamic augment would have been increased.

The effect of merely angling the weight can be clearly seen by constructing diagrams similar to Fig. 9, but with these modifications—in the first diagram plot curves as follows:

(*a*) Longitudinal inertia effect of right-hand reciprocating weight.
(*b*) Longitudinal inertia effect of left-hand reciprocating weight.
(*c*) The sum of (*a*) and (*b*).
(*d*) Horizontal component of the centrifugal force of the weight opposite the crank, provided for the reciprocating weight, in the right-hand wheel.
(*e*) Same as above but for the left-hand wheel.
(*f*) The sum of (*d*) and (*e*).

Now make a similar diagram for balances angled at 10 deg 57 min. The curve *c* shows the total effect of the inertia of the reciprocating weights on both sides of the engine. The curves *f* show the effectiveness of the balances. In the first diagram the curves *d* and *e* are 90 deg apart, and 35 per cent of the reciprocating weight is found to be balanced. In the second diagram the curves are 111 deg 54 min apart. Because these curves are more nearly opposed to one another, their sum is less, as shown by less amplitude of curve *f* and the percentage is much less. This explains why so many railroads, having locomotives with main wheels only large enough for balances for the rotating weight, have found them to ride worse when the weight was angled.

In including the piston load in counterbalance calculations, it must be remembered that at slow speeds, when the load is greatest, the dynamic augment is negligible. At high speeds most of the piston pressure is used up in accelerating the reciprocating weights. The greatest axial thrust in the main rod occurs well beyond the 90-deg crank angle, after compression has begun and not far from the time when the pressures on the two sides of the piston balance. It is due of course to the inertia of the reciprocating parts. The curve of augment produced by this thrust has no resemblance to a sine curve.

When it is desired to determine the best balance for a high-speed locomotive, formulas are useful for a first approximation only. In order to minimize to-and-fro motion ("jumping," as enginemen call it) and wheel weight, while keeping within the limit of dynamic augment, nosing would best be ignored. Curves of augment, due both to inertia and steam pressure, would be plotted and weights adjusted by trial to give the best result. Accelerations are most easily found graphically. In deciding what is the best result, consideration must be given to the areas under the curves, the maximum ordinates, the elasticity of the track, and the relative crank angles of all drivers at maximum ordinates. The balances on the two sides of the engine will be different and, probably on one side, even the revolving weights will not be completely cross-balanced. It is the writer's belief that, when the main rod is dynamically balanced, nothing less than 40 or 45 per cent balancing of the reciprocating weight will be satisfactory.

It has been found that the front and rear drivers are subjected to large augments by their part in guiding the locomotive. On that account it seems inadvisable to load onto them all the reciprocating balance.

W. H. WINTERROWD.[19] In his discussion L. B. Jones mentioned the fore-and-aft or horizontal shake of the locomotive, due to the unbalanced reciprocating masses. It may be of interest to note how the effect of such horizontal shake can be effectively damped.

It is well known that much of the so-called hard riding of locomotives is due, first, to inability to eliminate entirely the slack or lost motion between engine and tender; and second, to inability to damp the horizontal unbalanced forces of the reciprocating masses.

Heretofore, the mechanism used between engine and tender has been the wedge-type radial buffer or spring-buffer type whose inadequacy became pronounced with the advent of higher speeds.

Through scientific research, embodying many tests, there has been developed and placed in service on many locomotives, an automatic buffer mechanism utilizing two principles: First, the entire elimination of slack between engine and tender; and second, the utilization of spring-operated wedges to provide high resistance to the compression forces between engine and tender, thereby enabling the tender practically to become an integral part of the locomotive.

Application of the new design of buffer has resulted, not only in greatly improved riding qualities of the locomotive, but has brought about a very material reduction in maintenance costs of such items as loose cabs, broken piping, displaced arch bricks and wear on drawbars, drawbar pins, and chafing plates.

Recently, some exceedingly interesting comparative tests between the old- and new-type buffers were made in passenger service on a prominent western railroad. The results of these tests are shown in Fig. 22.

The information contained in this comparison was obtained by the use of a highly accurate piece of scientific apparatus known as a shock-and-vibration recorder. This type of instrument is well recognized in the railroad field as an apparatus for securing accurate data in connection with the riding qualities of railroad rolling stock. As part of its development this instrument was tested for accuracy on the shake tables at Purdue University.

The comparative tests were made with the same locomotive, a 4-8-2 with 74-in. driving wheels. On these tests the shock and recording instrument was secured to the deck of the locomotive below the engineer's seat box. The tests were conducted with the same number of cars, over the same track, and on the same operating schedule. No work was done upon the locomotive between tests other than a change of buffers. Track conditions were exceptionally fine. The first test was conducted with the locomotive equipped with the old-style wedge buffer which was in excellent condition and properly adjusted.

Fig. 22 contains comparative and representative portions of the records made by the automatic recorder. The upper half shows the performance with the old-style wedge buffer. The lower half shows the performance with the improved radial buffer and clearly indicates the benefits obtained. These portions of the record were selected for comparison because the speeds over the same section of track were identical. It will be noted that there was only 0.1 mph difference in speed.

These comparative graphical records are not only interesting but highly significant. They show without question the great improvement that can be made in the riding qualities of a locomotive, particularly with reference to the smoothing out or damping effect of the new buffer on the resulting action of the unbalanced forces of the reciprocating masses.

[19] Vice-President, Franklin Railway Supply Company, Inc., Chicago, Ill. Mem. A.S.M.E.

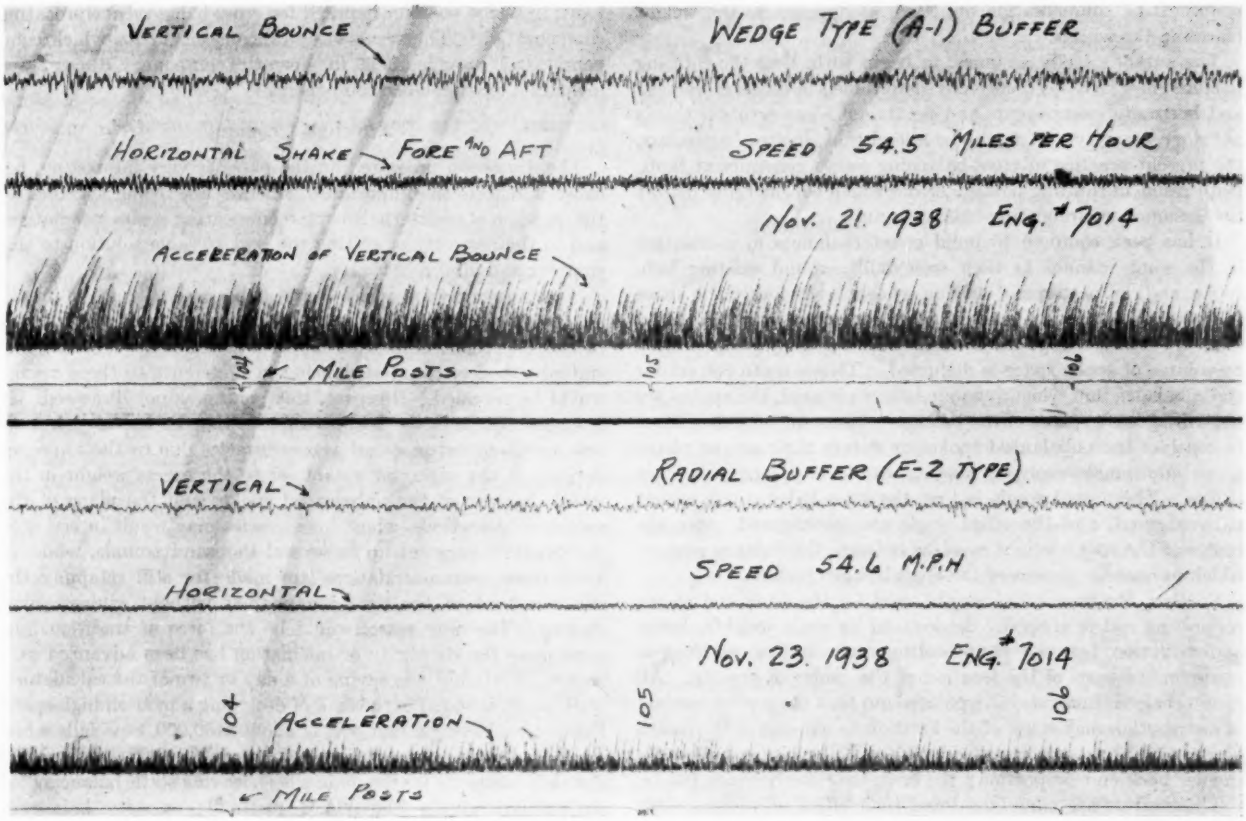


FIG. 22 RESULTS OF COMPARATIVE TESTS BETWEEN OLD- AND NEW-TYPE BUFFERS

The upper line on the record, designated "Vertical Bounce," shows one third of the actual vertical movement of the locomotive. With the old buffer a maximum vertical bounce of 0.45 in. is shown by measuring the record as compared with a maximum of 0.22 in. on the test with the new buffer. This is a reduction of 50 per cent in favor of the new buffer.

The second line on the chart represents the fore-and-aft horizontal shake due largely to the unbalanced forces of the reciprocating masses and inherent factors of steam distribution. The width of the horizontal shake line represents approximately one half of the actual fore-and-aft shake of the locomotive. A representative average width of this line on the test with the old-type buffer is approximately 0.20 in., as compared with a width of 0.06 in. with the new buffer or a reduction of approximately 66 per cent. This indicates that the damping properties of the improved design of buffer have been successful in absorbing or diminishing the horizontal shake to the point where it is less than one third of that obtained with the old buffer.

The third line on the chart, designated "Acceleration of Vertical Bounce," has a definite relation to the upper line and is really an indication of the velocity of the vertical movement or, in other words, the shock or impact obtained in conjunction with the vertical movement. A pencil movement of 1 in. represents an acceleration of G (32.2 ft per sec). A comparison of the maximum deflection of this line shows 1.40 G with the old buffer, and 0.53 G with the new buffer, a reduction of approximately 62 per cent. If it were possible to determine, it is believed the average would be still less and would probably represent only about one fourth that obtained with the old buffer.

The best possible counterbalancing of a locomotive is highly essential for the best practical and economical results from a locomotive and track-structure standpoint. Even when that is accomplished, the test data described herein indicate how a highly and scientifically developed buffer, making the tender an integral part of the locomotive, can absorb the effects of unbalanced forces, while at the same time providing increased comfort for engine crews, train passengers, and affording the means for greatly reduced maintenance costs.

In addition, it has even been suggested that the damping effect afforded by the new buffer opens up the possibility of still further reduction in the percentage of locomotive reciprocating weights to be balanced which, in turn, will decrease dynamic augment and the resulting track stresses.

E. G. YOUNG.[20] The writer particularly appreciates the clear manner in which the author has demonstrated some of the methods of counterbalancing which result in only a fraction of supposed reciprocating compensation being obtained, as well as his suggestion that, in a cross-balanced locomotive, it probably is not advisable to complicate the stresses produced under the main driving wheels by placing reciprocating balance in these wheels. With regard to the specific material of the paper, the writer would offer but a single suggestion: A formula which involves a decimal coefficient with several ciphers after the point is always treacherous. Using the same nomenclature as that in the paper, the simpler expression $3.2\,rW$, gives the centrifugal force at diameter speed, readily converted for any other speed by multiplying by the square of the speed ratio. This paper develops the need for a new technical term, representing the actual

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reciprocating compensation obtained, as opposed to the weight placed in the wheels.

The author's study assumes, to begin with, that the rotating balance shall have been correctly determined in terms of vertical and horizontal components and points out some errors if this is not done properly. It might be added that, in two particulars, the present practice in cross-balancing seems generally at fault. Both result in rotating-balance errors which considerably distort the assumed reciprocating-balance results.

It has been common to build counterbalances in calculation in the same manner as they were built around existing hub, spoke, and rim patterns of driving wheels. Obviously the space taken up by the spokes was not filled with counterbalance metal and, when the value of the counterbalance is estimated, the area or volume of spoke space is deducted. This is quite correct for static balance but, when dynamic balance is used, the spokes are not in the center plane of the balance. Consequently the failure to consider the unbalanced spokes or web in their proper plane, as an additional revolving weight, results in incorrect balance values. The actual result is that the cross-balance component is overfigured, and the offset angle also overfigured, since the center of the spoke weight must lie between the balance centers, which necessarily decreases the cross-balance factors.

Further, the amount of weight used for the back end of the connecting rod is generally determined by scale weights, hence a distribution between reciprocating and rotating portions is made on the basis of the location of the center of gravity. All counterbalance analysts have pointed out that the correct method of distribution makes use of the location or amount of the radius of gyration about the crosshead pin, resulting in a considerably smaller back end proportion; the error may easily reach 125 lb.

These errors are cumulative and their effect on reciprocating balance may readily be seen by the following example: Table 1 illustrates the results of these errors, the first column representing a balance calculated in the conventional way, and the second shows the refined calculation.

TABLE 1 CALCULATION OF RECIPROCATING BALANCE BY APPROXIMATE METHOD COMPARED WITH REFINED METHOD

Locomotive part	Approximate method	Refined method
Eccentric crank and part of crankpin:		
Equivalent weight, lb	90	90
Distance outside hub face to mean plane of revolution, in	14	14
Connecting rod and part of crankpin:		
Equivalent weight, lb	750	670
Distance outside hub face to mean plane of revolution, in	10	10
Side rods and part of crankpin:		
Equivalent weight, lb	620	620
Distance outside hub face to mean plane of revolution, in	3	3
Crank hub and part of pin:		
Equivalent weight, lb	350	400
Distance inside hub face to mean plane of revolution, in	3	3
Spokes or web of wheel:		
Equivalent weight, lb	...	180
Distance inside hub face to mean plane of revolution, in	...	4
Center plane of rotating weights, outside hub, in	5.29	4.03
Center of balance, inside hub face, in	3.25	3.25
Offset, balance plane to weight plane, in	8.54	7.28
Distance between balance centers, in	63	63
Offset ÷ balance centers (= cross-balance factor), per cent	13.5	11.6
Total weight of parts balanced, lb	1810	1960
Cross-balance required, vertical and horizontal, lb	244	227
Vertical component of rotating balance, lb	2254	2187
Reciprocating balance put in, opposite crank, lb	200	200
Total balance = vectorial sum of components, lb	2268	2398
Angle of offset, deg-min	6-11	5-27

In Table 1, with an offset of 6 deg 11 min, the vertical component of the balance figured by the approximate method, 2254 lb, allows but 67 lb excess over the rotating requirement figured correctly. There is an excess horizontal component available for reciprocating balance of 244 — 227 = 17 lb. The resultant overbalance is 70 lb, acting 14 deg 10 min from the vertical, and 7 deg from the position required for cross-balanced reciprocating compensation. The actual reciprocating weight for which compensation is provided is 60 lb, since the component 70 cos 7 deg includes a cross-balance component.

Author's Closure

The discussion presented on the paper is very interesting, because it reflects the importance which is now being attached to the question of counterbalancing reciprocating steam locomotives and to the necessity of getting the best possible solution to this complex and vital problem.

Since the principal object of the paper was to develop intelligent discussion of this problem, it had been hoped that the inherent, and perhaps not quite apparent, reasons for retaining the static method of counterbalancing in preference to the dynamic would be revealed. However, this point was not discussed. It is inconceivable to the author why, on the one hand, such matters are discussed in detail as inaccuracies due to the improper division of the main-rod weight, or to the excess weight in the wheels because of the unbalanced spokes and the plane of the spokes of the wheels, which inaccuracies may result in errors in the dynamic augment up to several thousand pounds, while, in some cases, recommendations are made for still retaining the static instead of the dynamic method without giving sound reasons. The only reason could be the force of tradition. In some cases the simplicity of calculation has been advanced as a reason. Certainly, the saving of a day or two of the calculator's work is not essential in the task of designing a modern high-speed locomotive, having a unit cost of about $150,000, especially when more than one locomotive is on order. Some discussers suggest dynamic balancing for the main wheel, leaving static balancing for the coupled wheels (the A.A.R. method); or static balancing for all wheels (Pennsylvania method); or a mixture of static for a part of the revolving weights and dynamic for the remainder (Eksergian's suggestion). In the author's opinion nothing better can be conceived for high-speed locomotives than the orthodox cross-balancing method for all of the revolving and part of the reciprocating weights. It is high time that an advocate of the static method of balancing come to the defense of his opinion and prove it, either theoretically, if he can, or by tests.

As the question of the distribution of the weight of the rod between revolving and reciprocating weights is the simplest one and seems to have gotten hold of the minds of some discussers, the author thinks it advisable to start by replying to discussers who touched on this matter (Starbuck, Buckwalter and Horger, Young), leaving the other questions to a later reply.

G. F. Starbuck regrets that K. F. McCall's publication[17] on cross or simple counterbalancing was not referred to by the author. This occurred simply because the author agreed fully with Mr. McCall and, finding no fault with his presentation of the question of counterbalancing, overlooked placing his article in the Bibliography. The author duly apologizes for the omission, it being understood that reference[17] compensates for the lapse.

Because of a 4000-word limitation placed on the paper, the author was obliged to confine his treatment of the subject of balancing reciprocating weights to a presentation of the principles of the methods of balancing, without devoting space to details of weight computation, i.e., what should be considered as revolving and what as reciprocating weights. This has been done by others and is presented in a number of textbooks to which references have been made. Mr. Starbuck refers to them himself when he says, "there are many mathematical treatments of the subject of rotating bodies," and gives a general explanation for the benefit of the nonmathematical designer.

Referring to the question of the division of the main-rod weight between its ends; generally speaking, this can be done only ap-

proximately. Older textbooks favored the division of the main-rod weight between reciprocating and revolving weights in accordance with the location of the center of gravity of the rod, which is known as the scale method. In this method, a rod is placed on two scales and the weights of both ends are thus registered. It also can be done by calculation, dividing the weight of the rod in inverse proportion to the distances of the center of gravity from either end.

Professor Low (26)[21] states that in many cases the practice is to consider two thirds of the weight as revolving and one third as reciprocating. Recent books (27) and as recent an author as Den Hartog (28), seem to agree to the division of the main-rod weight by the center of gravity. Professor Thurston (29) and John Perry (30) suggested establishing this ratio as half-and-half, meaning 50 per cent of the weight taken as revolving and 50 per cent as reciprocating.

Professor Dalby has analyzed the dynamics of the main rod and outlined the pendulum method for determining the two equivalent weights of a main rod. On page 247 of his book (12), he has given a formula covering the revolving part of the rod, to be assumed as attached to the crankpin

$$W_v = \frac{Wap}{l^2} = W\frac{k^2}{l^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots [5]$$

where W = weight of main rod

a and p = distances of center of gravity and center of percussion from center of crosshead pin, respectively

K = radius of gyration of main rod around center of crosshead pin

l = length of rod between crankpin and crosshead pin.

The remainder of the rod ($W - W_v$) is to be assumed as the crosshead pin. Thus Mr. Starbuck's formula $W_v = \frac{Wk^2}{l^2}$ is correct, provided k is radius of gyration referred to the crosshead pin.

The $p = \frac{K^2}{a}$ may be determined by calculation, or by swinging a rod as a pendulum which gave the name to the method. If T is the time in seconds of one double oscillation of the pendulum, the distance p in feet is determined from the known pendulum formula $T = 2\pi\sqrt{p/g}$ where g is the acceleration of gravity 32.2 ft per sec^2.

Professor Dalby's formula is not exact for all positions of the main rod. On pages 242 to 247 of the 1902 edition (12), he gives the inertia forces of the equivalent parts of the main rod, and on page 247, Fig. 169, he indicates the discrepancy between the actual curve and that obtained from the above formula. The discrepancy is very slight.

G. R. Henderson, on page 57 of his treatise (11), gives a more complicated formula

$$W_v = W\frac{k^2 - (a/l)r^2}{l^2 - r^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots [6]$$

where the notations are the same as above, the r is the crankpin radius. However, this has been derived for one position only of the crank—at 90 deg from the horizontal—and is coincident with the 90-deg point on the Dalby curve. The author, therefore, accepts Dalby's formula as the more general for all positions of the crankpin circle.

Professor Lanza (32) also gives a formula which actually is identical with Professor Dalby's, as a simple mathematical manipulation would show. Thus, both seem to have agreed that the division of the main rod between revolving and reciprocating weights should be made in accordance with Equation [5].

Henderson's Equation [6] should be considered as an approximate formula, if it is applied to the whole circle. Kenneth Cartwright[22] made tests with wooden models of full-size rods and, from these tests, found that Henderson's formula for the main-pin revolving weight is in very close coincidence with the scale-weight figure, if it is multiplied by a coefficient $^7/_8$; in other words, he suggests that

$$W_v = \frac{7}{8}W\frac{a}{l} \quad \ldots\ldots\ldots\ldots\ldots\ldots [7]$$

Before these tests, K. F. McCall[17] and, still earlier, Dr. R. Eksergian in his paper (13), page 79, made the same suggestion.

Of course, these formulas give different results, depending upon the relative weights of the crankpin and crosshead ends of the rod and, therefore, depend upon the design. In order to check the foregoing formulas for modern rods, a pendulum test of a full-size steel rod was recently made in the Schenectady shops of the American Locomotive Company. A rod of modern design although not very light in weight, was suspended on knife-edges located in the center line of the crosshead eye. From these tests, values were established which are summarized as follows:

	Total weight main rod with bushings, lb	W_v, lb	Per cent
Dalby-Lanza Equation [5]	834	461	55.2
Henderson Equation [6]	834	461	55.2
McCall-Cartwright Equation [7]	834	463	55.5
Schenectady swing test	834	461	55.2

These figures demonstrate the fact that all the recent formulas namely, Equations [5], [6], and [7], at least for the rod which was tested, give practically identical results. For calculation purposes while designing, before the rod is finished in the shop, Equation [7] can be used as the simplest.

It is interesting to note that for modern rods it more closely approximates the 50–50 percentage distribution, as suggested by Professors Thurston and Perry years ago, than does the more recent center-of-gravity method.

Having finished with this important matter of the influence of the main rod on the question of counterbalancing and wholeheartedly subscribing to Dalby's Equation [5] for general use, as a result of a very elaborate scientific investigation and determination of W_v, and to Equation [7] as the most accurate and so far the best empirical formula for modern locomotive rods, the author now turns to the discussion as such.

T. V. Buckwalter and Dr. O. J. Horger first raise the question of reducing reciprocating weights. Nothing can be said against this, as the reduction of reciprocating weights has always been the aim of locomotive designers, both in this country and abroad as evidenced by channeling-out rods, making them of alloy (vanadium, chrome-nickel, high-tensile steel), using lightweight forged pistons, etc. With the increase in speed, the recent adoption of more refined methods in counterbalancing and, what is more important, with the progress in metallurgy which made available high-tensile steels, the lightening of reciprocating parts became quite natural. Again, since the paper was only to discuss the methods of counterbalancing on the basis of actual weights, without considering what those weights might be, the author did not state that the reduction in weight of reciprocating parts is an aim for counterbalancing, although throughout his paper he stressed the point that the overbalance, which is in direct proportion to the reciprocating weights, should be kept as low as possible. A statement that the reciprocating parts should be

[21] Numbers in parentheses (26 to 33 inclusive) refer to the Bibliography at the end of this closure; all previously numbered references are to the Bibliography at the end of the paper.

[22] Mechanical Engineer, New York, New Haven & Hartford Railroad, New Haven, Conn.

made light was to him a self-evident, long-known truth and, therefore, no emphasis on his part was necessary. The author, therefore, cannot understand the discussers' remark in the third paragraph of their discussion regarding the author's neglect to mention the necessity of reducing the reciprocating weights for the sake of the shaking forces. This is the more surprising as earlier, the discussers, speaking on the same question, remarked that their views are in keeping with the author's recommendations.

The aforementioned writers take exception to the author's suggestion of leaving out the overbalance on the main wheel. The author did not suggest leaving out the overbalances entirely on all wheels; he is conscious of the necessity of some balancing of the fore-and-aft movements in a locomotive, but he is not at all convinced that American locomotives require as much reciprocating balancing as they actually have. Therefore, he would like to see a test with a locomotive properly cross-balanced, having as little overbalance as possible, and maybe leaving it out entirely, so that conditions of balancing would be accurately known. American locomotives are subjected to all kinds of disturbing forces, when the influence of the opposite side is taken into consideration. This condition is mainly due to static balancing. A test similar to that made by the Madras & Southern Mahratta Railway in India (33) is suggested. Possibly zero is not attainable, but reducing this percentage to a very small figure would be found quite sufficient for American locomotives, which are long and heavy.

On the Madras & Southern Mahratta Railway two cross-balanced locomotives, XB/1 and XB/1/RO, were tested. The first locomotive was cross-balanced with 66 per cent balancing of reciprocating weights, while the second had no balance for reciprocating weights. According to a statement made by Mr. Lean, chief mechanical engineer, in a private letter to the author, no difference could be found between these two locomotives, either in the wear between the driving boxes and pedestals, or between their riding qualities, at three inspections for the second locomotive, during a mileage of 232,000 miles. In an unpublished interdepartmental typewritten report (35), it was recommended that "a deduction of 33 per cent should be made in the formula laid down in the Bridge Rules of 1935" and that "the beneficial effects of rebalancing locomotives should at once be brought to the notice of all Indian railways." In view of this, it is natural that a test with "nil balancing" should have been recommended by the author for American railroads, as he has always advocated as little balancing of reciprocating parts as possible (13).[23]

It is also stated by Mr. Lean that on some 4-8-0 meter-gage locomotives, converted to superheat, no overbalance could be placed in the small wheels on the drivers when the converted engine became heavier. These were, therefore, omitted and the reciprocating balances were redistributed over the coupled wheels. That permitted placing the locomotives in successful operation—as the author suggested in the paper.

The beneficial effect of reducing the reciprocating balance by 100 lb on each side of a high-speed locomotive has been proved by Kenneth Cartwright (36).

One point must be remembered: Messrs. Buckwalter and Horger talk about the necessity of neutralizing the shaking forces up to the point where maintenance of the locomotive and of the track becomes excessive. This is probably about 25 to 30 per cent. In this case, 70 to 75 per cent of the unbalance still remains. Why is it possible for a locomotive to stand 75 per cent of the shaking forces and why would it not be possible to stand 95 per cent—an increase of only 28 per cent? Without a test to the contrary, it is not logical by pure reasoning to decide one way or another, in view of the prevailing evidence. The author only suggested that such a test be made to determine this very question for American locomotives. Of course, the locomotives to be tested should have as light reciprocating weights as possible and practically no reciprocating balances. His reference to the 4-6-4 locomotive, which actually had only 6-lb overbalance on the main wheel, was cited as an indication that many locomotives were riding with very slight balancing of reciprocating parts. His point was that, in a great many American locomotives, the old figures of overbalancing, which were necessary for shorter and lighter locomotives, do not obtain now and, therefore, there is no reason to insist on 30 or 40 per cent balancing as is practiced in many cases. After the dynamic balancing of rotating parts has been carried out and referred to the plane of the cylinders, 12 to 18 per cent balancing of reciprocating weights may prove quite sufficient.

For the information of the discussers who inquired whether or not the influence of reciprocating weights from the other side of the locomotive was taken into account for the 6-lb overbalance, the author states that it was not. If it had been, the numerical value might have been larger, but the angular direction of the augment would be such that its value would be of very little use. This statement will be clearer from the author's reply to the remarks of L. B. Jones.

The tests on slipping, as reported by Messrs. Buckwalter and Horger before the New York and Chicago Railway Clubs, are of great value. This difficulty has been recognized by the author in the paper. It must be remembered, however, that the problem is in its infancy and before drawing conclusions, more experimental work will be needed.

While the piston-thrust-component effect may not be important in some cases, this fact does not lessen the necessity of giving it attention. This is especially true from the standpoint of demonstrating why the main wheel should be made without an overbalance, provided the dynamic augments on the remaining wheels can be kept within reasonable limits. The effect of the steam component is not less than several thousand pounds, in some cases greater than the effect of improper division of the main rod.

Dr. Eksergian in his discussion refers again to a suggestion which he made several years ago in his paper (13) concerning a mixed method of balancing. A portion of the revolving weight should be statically balanced and the remainder of the revolving weight should be cross-balanced by the dynamic method. He tries to prove that ultimately, if the reciprocating balance is also added, it would result in a smaller dynamic augment of the main wheel. This may be true, in so far as dynamic augments are concerned, but the balancing of reciprocating weights would be thus disturbed by the effect of revolving parts of the other side.

It is easy to prove by pure reasoning that Dr. Eksergian's proposition is a fallacy because no smaller dynamic augment, due to the counterbalance of revolving weights of both sides, can be obtained than by complete dynamic balancing of these weights. Then the dynamic augments on both wheels are equal to zero, if the counterbalances are properly angled. The dynamic augment due to the reciprocating balance, when properly angled, is as large as necessary for the balancing of these parts and the sum of zero and the dynamic augment will be minimum for maximum effective balancing. As stated at the beginning of this closure, no better system than the orthodox cross-balancing method of all revolving weights and part of the reciprocating weights has yet been devised for two-cylinder locomotives. Any change in this system will lead to greater dynamic augments or a lesser balancing of reciprocating weights, or both. This fact will be clearer after studying the statement in reply to Messrs. L. H. Fry and L. B. Jones.

In conclusion, Dr. Eksergian recommends retaining the static balance of revolving weights on coupled drivers, the mixed cross-balancing of revolving weights, with static balancing of the rest

[23] Bibliography (13) of paper; discussion, pp. 84–85.

of the revolving weights on the main drivers, and static balancing of a certain percentage of the reciprocating weights. The mixed balancing of the main drivers will involve just as much work as the total cross-balancing of the same drivers. The static balancing of the coupled drivers would probably reduce the work of the calculator 1 or 2 hr for a new locomotive design, but would also introduce new disturbing forces in the locomotive and a confusion in the mind of the operator. No advantage can result from this mixed procedure.

In substance, Mr. Fry's discussion calls for very littly reply. Referring the unbalanced force rather than the balancing force to the weight of the locomotive is a logical thing to do, if it can be done conveniently and practically. With the rapid changes in the design of steam locomotives, this coefficient varies. It will be different if a friction-damping coupler, such as the Franklin Railway Supply E-2 coupler, is used. This eliminates the propagation of shaking forces to the cars of the train and makes them unnoticeable (see the reply to Dr. Winterrowd's discussion later); also with the improvements in the guiding of locomotives; with increase in length and moment of inertia of the locomotives, this percentage will be continually dropping. Nevertheless, it will always be necessary to watch the dynamic augment; in other words, double work, always properly done, would be required. Therefore, if there is any objection to the use of dynamic balancing because of the complication introduced, more objection may be leveled at the proposition of referring the unbalanced force to the weight of the locomotive. However, the author does not see any objection to doing this or why balancing of locomotives from this angle should not be investigated.

However, as to Mr. Vaughan's proposition of substituting for overbalances in one wheel, two half-size balances in each wheel, no advantage can be gained toward balancing the locomotive.

Referring to Fig. 16 of Mr. Fry's discussion, scheme (*B*) represents static balancing of a reciprocating weight of 100 lb with a static balance of 100 lb. The shaking force is zero, the nosing moment, when the left-hand crank is at its dead center, is $100 \times 73 - 100 \times 62 = 1100$ in-lb. Scheme (*C*) represents dynamic balancing. Under the same conditions as above, the shaking force is $100 + 18 - 118$, or zero, and the nosing moment $100 \times 73 - 118 \times 62$, which is also zero. Scheme (*D*) represents the Vaughan method. It is true the shaking force is then $100 - 50 - 50$, or zero, but the nosing moment is $100 \times 73 - 50 \times 62 = 4200$ in-lb, or 3.82 times greater than in (*B*), and the dynamic augment, as given by Mr. Fry, is 70.7 lb, multiplied by the coefficient as in the paper. In other words, better results can be accomplished by balancing *statically* 29.3 per cent of the 100-lb reciprocating weights. In this case the nosing moment would have been $100 \times 73 - 70.7 \times 62 = 7300 - 4383 = 2917$ in-lb, or only 2.65 times greater than in (*B*), instead of 3.82 times greater as in Vaughan's method of counterbalancing. The dynamic augment would be 70.7 lb, as in Vaughan's method. In other words, by ordinary static balancing, with the same dynamic augment, a smaller nosing moment would result. Of course, as good balancing would not be derived as in (*C*) dynamic, when both shaking and nosing are zero. Next in quality of balance is (*B*) orthodox static balancing. This proves the contention of the author that for a two-cylinder locomotive, nothing better as yet has been devised than cross-counterbalancing of 100 per cent of the revolving weights plus partial cross-balancing of reciprocating weights to the limit of dynamic augment the track can stand.

Mr. L. B. Jones introduces his discussion with a statement to the effect that the author's application of the cross-balancing method requires a great amount of calculation, if the designer wishes to explore carefully all the effects of counterbalancing. The author is satisfied that by the numerical example given in the paper, he has demonstrated how simple the correct cross-balancing of any locomotive actually is. Such a calculation is certainly worth carrying out, because the subsequent corrections, after the locomotive has been built, namely, changing counterbalances, replacing kinked or broken rails, and the less visible maintenance of locomotive and track, result in much greater costs than the original calculation. Mr. Jones has not advanced a reasonable argument; a complete exposition of the matter would be necessary to establish proof that his suggestion gives better results.

Mr. Jones introduces, in his discussion, curves which represent an analysis of a theoretical study made of a K4s Pennsylvania locomotive before it was tested at the locomotive-testing plant in Altoona. The most interesting curve is Fig. 20, which gives the component in the plane of one wheel, say, right, of centrifugal forces of all revolving weights, including the revolving balances in both wheels, as a function of the percentage of balancing of reciprocating parts. While the locomotive is statically balanced, the analysis is made on the basis of the theory of dynamic balancing; i.e., taking into account the various planes, studying both couples and forces. In other words, the analysis is more complex than the counterbalancing itself, as any one who tries to make such an analysis will discover. If certain rules, previously familiarized, are followed, cross-balancing is much simpler.

Referring again to Fig. 20, when the percentage is zero and there is no reciprocating balance, the action of the revolving weights of the locomotive on both sides is 24,314 lb. In Fig. 23 of this

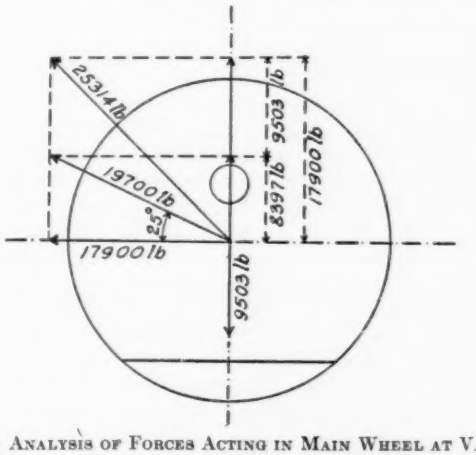


FIG. 23 ANALYSIS OF FORCES ACTING IN MAIN WHEEL AT VARIOUS PERCENTAGES OF RECIPROCATING BALANCE, COMPUTED AT 400 RPM

closure, the small circle on the vertical radius represents the upward position of the right-hand crankpin. The static balancing of the right side will result in a deficiency of 282 lb, a figure to which Mr. Fry also refers in his discussion. The centrifugal force of the right-hand revolving weights is 17,900 lb at 400 rpm, as a result of the deficiency. At the same time, the left-hand revolving weights also give a component at the right-hand wheel directed horizontally backward (the left-hand pin leading) of 17,900 lb, which, together combine into a centrifugal force of 25,314 lb at an angle of 45 deg as shown in Fig. 23. So far it has been assumed that there is no balancing of reciprocating parts whatever (zero per cent of balancing). Now suppose that we have 30 per cent static balancing of the reciprocating parts. As the total reciprocating weights on one side of the locomotive are 1500 lb, 30 per cent balancing for one wheel (150 lb) represents, at 400 rpm, a force directed downward of 9503 lb. Consequently, we still have in the right wheel a force directed upward equal to the difference of the two, which is 8397 lb, as shown in Fig. 23. As the action of the revolving weights from the left side results in 17,900 lb directed horizontally, the compound action gives 19,700 lb at an

angle of 25 deg. The magnitude of this deficiency of 19,700 lb is correctly given by Mr. Jones in his Fig. 20; only he calls it dynamic augment, although it is not directed downward and will have the tendency to lift the wheel.

At 56.4 per cent balancing, a centrifugal force of the balance in the right wheel is again directed downward and is greater than before. It is equal to 17,900 lb, which cancels the centrifugal force of the revolving-weight component directed upward. At the same time, from the left side the action of revolving weights gives a horizontal, backwardly directed component of 17,900 lb and thus the total resultant of all revolving weights and the counterbalance in right wheel is 17,900 lb directed horizontally.

It is easy to see that, when the pin on the right side is in the highest position, the reciprocating weights on the right side are in the middle position and the inertia force is, therefore, equal to zero. Consequently, when the horizontal force is 17,900 lb, there is no reciprocating force on the right side to be balanced. There is only a shaking force of 17,900 lb at the maximum percentage of balancing (56.4 per cent). At the same time, the left pin is in its extreme forward position. The combined effect of the left-hand-revolving weights, left-wheel balance, and the action of the right-hand side on the left wheel results likewise in a force, which is now a vertical force of 17,900 lb—a real dynamic augment. The left-hand inertia force of 56.4 per cent of the reciprocating weights of one wheel (500 lb) has at that time its maximum, also 17,900 lb directed forward, with no horizontal balance to take care of the force, because the balance has been used up, so to speak, for the action from the other side.

Thus, combining the two sides, for one axle two horizontal and oppositely directed forces result, which, true enough, will be balanced in so far as shaking is concerned. However, taking place simultaneously on the right and left sides of the locomotive, they will give an additional couple to the unbalanced couple around the vertical axis through the center of gravity, or a total nosing couple with a moment of $31{,}700 \times 44.5 + 17{,}900 \times 31 = 1{,}965{,}000$ in-lb. In this instance 31,700 lb is the inertia force of 500-lb reciprocating weights on the left side, and 17,900 lb is the above component of all centrifugal forces in the right wheel. The dynamic augment on either wheel will be 17,900 lb.

If the Pennsylvania locomotive were cross-balanced with the same maximum dynamic augment of 17,900 lb and the reciprocating weights, as in the example just given, were 500 lb per wheel, the total percentage of balancing reciprocating weights referred to the cylinder center planes, would be 46.0 and the unbalanced moment around the vertical axis through the center of gravity of this cross-balanced locomotive would be only 755,620 in-lb. The high figure of 56.4 per cent, which is necessary for these locomotives on the Pennsylvania Railroad, is probably due to the peculiar system of balancing which results in forces directed opposite to what they should be, causing more disturbance than balancing. If the locomotive were cross-balanced, probably not more than 30 per cent balancing of reciprocating weights would be required. Then a dynamic augment of 11,590 lb would not be exceeded and a moment around the vertical axis through the center of gravity of only 987,190 in-lb would be obtained.

The foregoing results in reference to the Pennsylvania K4s locomotive are summarized as follows:

Method of balancing both revolving and reciprocating weights	Reciprocating balance, per cent	Dynamic augment, lb	Unbalanced nosing moment, in-lb
Static	56.4	17900	1965000
Dynamic	46.0	17900	755620
Dynamic	30.0	11590	987190

The most astonishing and probably most unexpected result of the above static balancing is the following:

The inertia forces of the reciprocating weights on one side of the locomotive are not balanced by the reciprocating balance over the revolving weights, placed in the adjacent wheel on the same side of the locomotive, but by the resulting dynamic reactions in the wheel of the other side. The arms of the balancing moments are, therefore, very large; they reach almost the entire width of the locomotive.

The foregoing applies to 56.4 per cent balancing of reciprocating weights, where the straight line intersects the curve in Mr. Jones's Fig. 20. For zero per cent balancing, the right-wheel reaction is greater, namely, 25,314 lb, Fig. 23, with the same horizontal component, but 45 deg out-of-phase with the inertia forces of the reciprocating weights. At 30 per cent balancing, it will be 19,700 lb with 25 deg out-of-phase.

It is quite different in the case of dynamic balancing when the angling of reciprocating counterbalances in each wheel places the balancing force in a position exactly opposite the crankpin in each side. By this method, the best possible balance of the reciprocating weights on each side of the locomotive is obtained, at an arm from the wheel plane to the plane of the cylinders. This results in smaller arms for balancing moments, and smaller stresses being transferred from one side of the locomotive to the other through the driving axles. Consequently, the axle stresses will be lower than in statically balanced locomotives. It also reduces the unbalanced nosing moments, as can be seen from the table summarizing the Pennsylvania K4s results, without any increase in dynamic augments.

Dr. Winterrowd's contribution is very interesting as it shows how the mass of the tender can be made to take part in the total mass of the locomotive and thus reduce the shaking effect of reciprocating weights. If this can be conveniently done and the horizontal and vertical vibration reduced as shown on his charts by the application of the E-2 spring-controlled-friction radial buffer, the percentage of balancing reciprocating weights can be reduced, with resulting decrease in the dynamic augment and the consequent track stresses.

Prof. E. G. Young's discussion is very pertinent. His suggestion regarding the calculation of revolving weights in so far as wheels are concerned, is much to the point and has been for years taken care of by locomotive builders. His complaint about the miscalculation of the distribution of weight of the main rod is well taken, and already has been discussed in this closure.

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The Resistance to Relaxation of Materials at High Temperature[1]

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Relaxation tests at high temperature give direct information on the permanent tightness of bolted joints and shrinkage assemblies. They also give direct measurements of the effectiveness of strain-relief anneals. Indirectly they give, in relatively short time and small furnace space, reliable information on long-time constant-stress behavior although direct computation of either type of performance from the other is uncertain. In relaxation tests, total strain is definitely limited, whereas, creep rates based on constant-stress tests may, but do not necessarily, correspond to excessive amounts of strain, thus giving too optimistic indications of strength. Although advantageous, relaxation tests do not take the place of constant-stress tests. A simple formulation for relaxation is given which holds good for considerable periods although rather sudden changes of trend suggest the possibility of physical changes in the material which are not easily predictable in advance. Test results are given on six materials at various temperatures.

THE CONDUCT of relaxation tests under the sponsorship of the "Joint Committee"[1] was authorized in December, 1936, and the Subcommittee[3] for Project 16 was organized early in 1937. Relaxation tests relate directly to the applicability of materials for service involving elastic action at high temperature. Such service includes all types of bolted joints and all sorts of assembly depending on a shrink fit or a pressed fit for tightness. Such tests also contribute information of a less specific nature as to the general metallurgical behavior of materials at high temperature.

This project formed a part of the larger program of the Joint Committee at a time when its activities were being expanded to include, in addition to the accumulation of results on creep strength in simple tension, information on performance in pure shear and on the important case of compound stress existing in pipes and boiler drums where the hoop stress is twice the axial stress.

Therefore in order to facilitate useful comparisons as to the performance of a material under different types of service, the Subcommittee on Relaxation has scheduled tests on each of the various materials being sponsored by the Joint Committee regardless of its special suitability for bolting or other elastic service. However, certain other tests have been included on their own merits.

Table 1 gives a complete schedule of relaxation tests so far completed as a part of Project 16.

Table 2 summarizes the chemical compositions, heat-treatments, and physical properties of the six materials tested. Of these Nos. 1, 4, and 5 were regular Joint Committee materials more completely described in the references accompanying Table 2.

No. 2 was a wrought carbon steel tested by the General Electric Company. This was included because the original program of the Joint Committee drawn up in 1936 had contemplated testing carbon steel at 750 F. However, the need of keeping the program within the resources of the committee led to the abandonment of such tests. Although material No. 2 is not identical with any other material tested by the Joint Committee it represents a composition about which information is desired and hence is included for comparison with the other materials.

No. 3 was S.A.E. 4140 steel furnished by the Westinghouse Electric and Manufacturing Company and No. 6 was a steel of similar composition furnished by the General Electric Company. These were the only materials tested by Project 16 which were especially adapted for bolting material and only a stress-relief-anneal test is given for No. 6.

Thus the subcommittee would like to reiterate that the materials on which results are presented were not chosen for their high creep strength in elastic service. It is possible that the relaxation-test program may be extended in the future to include materials more especially intended to maintain elastic tightness at high temperature.

General Characteristics of Relaxation Tests

Several methods of running relaxation tests are represented by the results presented in this report. However all of them have functional significance and, although this method of testing lends itself to the rapid accumulation of a bulk of results at various stresses, the tests reported herewith were not intended solely to shorten the time of testing. Extrapolation, under any guise, requires caution.

There are a number of good reasons for the conduct of relaxation tests regardless of their direct applicability to machine performance. In the early stages of creep some form of progressive strengthening appears always to take place so that, even without relaxation of stress, the rate of flow decreases with time. This is frequently called "the first stage" of creep as distinguished from the "second stage" where the rate stays constant for a relatively long time. Relaxation tests with direct applicability to machines are usually of necessity completed within the range of total deformation where progressive strengthening is still taking place. There is thus no danger of basing results on specimens which have been subjected to strains which never would be allowed in service.

Of course, constant-stress tests may be extrapolated by plotting the stress against time to attain a specified total extension on log-log paper. Such an interpretation of constant-stress data avoids the objection just raised of basing conclusions on tests in which the material has been overstrained. Relaxation tests have the definite virtue of automatically limiting total extension while in their later stages they approach more and more closely

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[1] Progress Report by Subcommittee for Project 16 of the A.S.M.E.–A.S.T.M. Joint Research Committee on the Effect of Temperature on the Properties of Metals. Presented at the Annual Meeting of The American Society of Mechanical Engineers, New York, N. Y., Dec. 5–9, 1938.

[2] Turbine Engineering Department, General Electric Company. Mem. A.S.M.E.

[3] Membership of Subcommitteee for Project 16: Ernest L. Robinson, General Electric Company; N. L. Mochel, Westinghouse Electric and Manufacturing Company; J. J. Kanter, Crane Company. Also cooperating: H. J. Kerr, Babcock and Wilcox Company; F. H. Norton, Massachusetts Institute of Technology.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

TABLE 1 SCHEDULE OF RELAXATION TESTS CONDUCTED IN COOPERATION WITH PROJECT 16

	MATERIAL		LABORATORY	TEMP °F	INITIAL		ELASTICITY FACTOR "b"	ELASTIC MODULUS LB/IN² 10⁶	RATE-STRESS INDEX "n"	RESIDUAL STRESS		NOMINAL CREEP STRENGTH LB/IN² FOR RATE OF 10⁻⁶	NOTE	FIG.
					STRAIN	STRESS LB/IN²				LB/IN²	TIME HR			
1	ANNEALED 0.35C STEEL "K20"	1-2	W. CO.	850	.00025	5000	1.03	20.0					1	
		3-4	W. CO.	850	.00050	10000	1.03	20.0					1	
		5-6	W. CO.	850	.00100	20000	1.03	20.0					1	
		14A5	G.E.	850	.00150	30000	1	20.0	4.7	1700	10000	6200	2	2
		342	CRANE	850	.00050	10000	5	20.0	4.5	1260	10000	5130	2	3
2	WROUGHT 0.58C STEEL	462	G.E.	750	.00200	54400	1	27.2	5.6	6400	10000	16400		4
3	S.A.E. 4140	368	CRANE	850	.00125	30000	5	24.0	8.5	12800	10000	23000	3	5
			W. CO.	850	.00208	50000	1.03	24.0	7.8	15350	10000	27860	3	6
			W. CO.	930	.00227	50000	1.03	22.0	7.0 3.75	3550	10000	13970	3	7
4	C. MO. TUBING	370-2	CRANE	800	.00100	25000	5	25.0	—	20000	10000	—	3	8
		370-3	CRANE	950	.00091	20000	5	22.0	10.5	10200	10000	16900	3	9
		370-1	CRANE	1050	.00050	10000	5	20.0	4.55	1400	10000	5500	3	10
			B.&W.	1050	.00200	40000	1	20.0	9.2 3.5	1530	10000	8000	3	11
5	4-6 CR MO. TUBING	3-1	G.E.	1200	.00200	38800	1	19.4	4.2 2.5	125	1000	1110		12
		3-7	G.E.	1200	.00097	15600	3	16.2	5.0 2.25	135	1000	1240		13
6	S.A.E. 4140	550	G.E.	1112	.00200	40000	1	20.0	5.3	4000	10	3000	4	14

NOTE

1 Trans. A.S.M.E., vol. 59, July, 1937, RP-59-8, pp. 453–455.
2 Trans. A.S.M.E., vol. 59, July, 1937, RP-59-7, pp. 451–452.
3 Values for modulus of elasticity assumed.
4 Stress-relief-anneal test.

Initial Strain and Initial Stress. When both quantities have been measured directly, the modulus of elasticity is found by dividing the stress by the strain. Where only one of the two has been measured, the other is computed from an assumed modulus of elasticity.

The elasticity factor b is the ratio of the total elastic deformation of the system to that of the bar subjected to creep. Thus, an elasticity factor of 1.0 represents the performance of a bolt in an unyielding flange, while an elasticity factor of 5.0 represents the performance of a bolt in a flange possessed of four times the elastic deformation of the bolt so that the entire system has five times the elastic deformation of the bolt alone. See Fig. 1.

The rate-stress index n is the slope of the line resulting on log-log paper when the stress is plotted against the rate of creep. See Figs. 16, 17, 18. The slope of the stress-time lines given in the log-log plots of Figs. 2 to 14 is $(n-1)$.

The residual stress for a certain number of hours is a measure of the bolt performance of the material. At the lower temperatures the residual stress corresponding to 10,000 hr has been given. At the higher temperatures where the tests approach the conditions of stress-relief anneal rather than bolt performance shorter comparative times have been selected. While this makes direct comparison difficult, it was felt that there was no particular object in comparing the performance of the high-temperature materials with those used at lower temperatures. The residual stresses given in the table, in all cases, refer to performance with an elasticity factor of 1.0. In cases where the test was run with elastic follow-up the actual residual stresses would be higher and may be determined by reference to the several diagrams.

The nominal creep strength is expressed in pounds per square inch for a creep rate of one millionth of the length per hour, that is, a rate of 1 per cent per 10,000 hr. For comparison with similar values determined from constant-stress tests, see Table 3.

to constant-stress conditions. Less caution is required in drawing conclusions from such tests than from well-made high-strain tests which are prone to show the material better than it actually is as used.

Step-Down Tests at Constant Extension

At the laboratory of the General Electric Company relaxation tests are run by what is ordinarily referred to as the "flow-rate" method, because individual rates are measured during each of the several steps. A similar procedure was used at the Massachusetts Institute of Technology in running the test for The Babcock and Wilcox Company except that the load was reduced in very small decrements. This more nearly approaches continuous relaxation although direct measurements of rate are not so easy.

These tests are run in the same furnaces ordinarily used for constant-stress work in which the load is applied by weights on a lever. After the furnace is brought up to temperature the stress is applied by adding weights and the elastic extension is noted as each weight is applied. The modulus line is plotted as elastic extension occurs and any deviation indicating the appearance of plastic flow is immediately evident.

As soon as the predetermined total extension corresponding to the initial stress in the bolt has been reached, the stress is reduced and the relaxation test begins. Creep occurs at constant stress to the predetermined limit after which the stress is again reduced and so on in successive steps. Each step suffices to give a rate corresponding to the applied load, and the entire process approximates the performance of a bolt in an unyielding flange, with the entire elasticity of the system being included within the member in which the plastic action occurs.

When large steps are taken, there is usually a period of a day or two during which a certain amount of recovery or shortening of length occurs. If credit is taken for this recovery in analyzing the results of the test the corresponding computed bolt strength is likely to be somewhat higher than if no credit is taken for such recovery.

Bolt Tests in Test-Block Assembly

At the South Philadelphia Works of the Westinghouse Electric and Manufacturing Company, some relaxation tests are made by assembling an actual bolt in a test block having approximately 30 times the cross section of the bolt.[4] The assembly is placed in the test furnace and the actual change of length measured after an elapsed period of time. While this method does

[4] At the Westinghouse Research Laboratories, East Pittsburgh, Pa., an automatic relaxation machine has been used. See "Relaxation of Steels at Elevated Temperatures—A New Automatic Relaxation Machine," by A. Nádai and J. Boyd, *Journal of Applied Mechanics*, Trans. A.S.M.E., vol. 60, September, 1938, p. A-118.

TABLE 2 MATERIALS USED IN RELAXATION TESTS CONDUCTED IN COOPERATION WITH PROJECT 16

	MATERIAL	CHEMICAL COMPOSITION							HEAT TREATMENT	PHYSICAL PROPERTIES				REFERENCE
		C	MN	CR	MO	SI	P	S		TENSILE STRENGTH LB/SQ IN.	ELASTIC LIMIT LB/SQ IN.	EL. %	R.A. %	
1	ANNEALED "K20" 0.35C STEEL	.35	.55			.19	.016	.03	1	68000 70000	21500 24000	36.5 39.0	62.3 65.8	1
2	WROUGHT 0.58C STEEL	.58	.86			.18			2	104900	64000	28.6	58.3	
3	S.A.E. 4140	.37	.63	.98	.26	.14	.015	.02	3	133000	55000	20	59	
4	C. MO. TUBING	.15	.53		.56	.21	.016	.016	4	63700	40000	36.5	70.1	2
5	4-6 CR. MO. TUBING	.09	.39	4.85	.53	.28	.010	.016	5	64900	28000	42.0	60.2	2
6	S.A.E. 4140	.45		1.0	.25				6	131000	98000	23.5	66.6	

SCHEDULE OF HEAT-TREATMENTS

AC = air-cool; FC = furnace-cool; SC = slow-cool; OQ = oil-quench

1 Heat to 1550 F in 2 hr, hold 1 hr, FC to 1000 F, AC.
Reheat to 1280 F in 4 hr, hold 2 hr, FC to 1000 F, AC.
Cold-straighten, heat to 950 F, AC.
Repeat all except cold straightening.

2 Heat to 875 C, OQ; 675 C, FC; 700 C, FC.

3 Heat to 1600 F, hold 3 hr, AC; heat to 1575 F, hold 3 hr, OQ.
Heat to 1030 F, hold 3 hr, FC; rough-machine.
Heat to 1000 F, hold 3 hr, FC.

4 Heat to 1350 F, hold 1 hr, FC 50 F per hr to 1100 F, AC.
Heat to 1200 F, hold 1.25 hr, FC to 180 F in 27 hr.

5 Heat to 1525 F, hold 2 hr, FC 100 F per hr to 1225 F, AC.
Heat to 1375 F, hold 2.25 hr, FC to 300 F in 24 hr.

6 Heat to 825 C, OQ; 650 C, OQ; 425 C, SC.

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1 Trans. A.S.M.E., vol. 58, February, 1936, RP-58-4, pp. 97–101.
A.S.T.M. 1938 Preprint 29, pp. 56–58, appendix VI.
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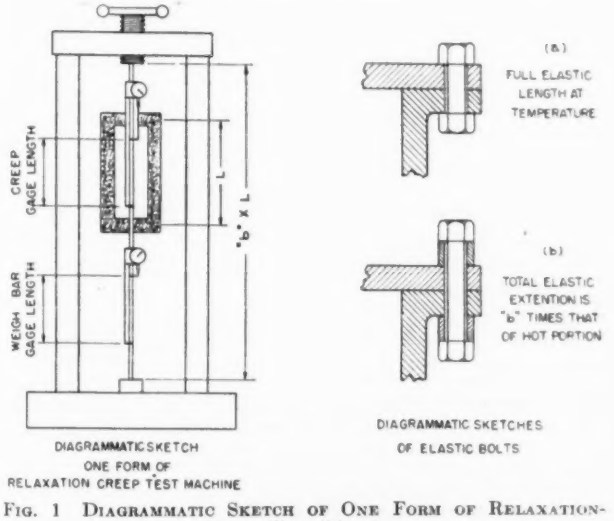


FIG. 1 DIAGRAMMATIC SKETCH OF ONE FORM OF RELAXATION-CREEP-TEST MACHINE
DIAGRAMMATIC SKETCHES OF ELASTIC BOLTS

not facilitate readings of extension at intermediate intervals, equivalent information is secured by starting a series of identical assemblies in test together and withdrawing them after successively longer periods of time. This test has the merit of reproducing actual conditions with fidelity. The total elasticity of the system is practically identical with that of the bolt subject to plastic action although other arrangements are possible.

TESTS WITH EXTRA ELASTIC FOLLOW-UP

At Crane Company laboratory tests have been run in a machine which simulates the action of a bolt in a flange possessed of considerable elasticity such that the stretching of the bolt only partially results in relaxation of the initial stress.[5] In the Crane machine the total elasticity of the system is five times that of the specimen in which the plastic action occurs. Continuous readings on a "weigh bar" at room temperature indicate the elastic stress effective at any time. Fig. 1 illustrates this type of test diagrammatically. At (*a*) and (*b*) are shown bolts without and with extra elastic follow-up.

Tests with extra elastic follow-up are also practicable using the methods employed by the General Electric Company, by The Babcock and Wilcox Company at Massachusetts Institute of Technology, and by the Westinghouse Electric and Manufacturing Company. Of the two tests on No. 5 material, 4-6-Cr–Mo tubing, by the General Electric Company, one was run with a system elasticity factor of 1.0 and the other with a factor of 3.0. This result is approximated in the step-down method by noting the elastic recovery after each stress reduction and requiring three times as much plastic extension before the next stress reduction.

FORMULATION OF RELAXATION BEHAVIOR

In order to facilitate application, the results of ordinary relaxation tests can usually be interpreted satisfactorily by the following fairly simple formulas which relate elapsed time with residual stress or elastic extension.

$$t = \frac{b}{(n-1)\,r_0}\left(\frac{S_0}{E}\right)^n\left[\left(\frac{E}{S}\right)^{n-1} - \left(\frac{E}{S_1}\right)^{n-1}\right] \quad \ldots\ldots [1]$$

$$t = \frac{b}{(n-1)\,r_0}\,x_0^{\,n}\left[\frac{1}{x^{n-1}} - \frac{1}{x_1^{\,n-1}}\right] \quad \ldots\ldots [2]$$

NOMENCLATURE

t = elapsed time, hr
b = ratio, total system elasticity to that of creep bar
n = per cent increase in creep rate per per cent increase in stress
E = modulus of elasticity, psi
r_0 = nominal rate of creep; 1 millionth of length per hr
S_0 = nominal strength, psi to cause r_0
S_1 = initial stress, psi
S = residual stress, psi after time t
x_0 = elastic extension corresponding to nominal strength S_0
x_1 = elastic extension corresponding to initial stress S_1
x = residual elastic extension after time t

These formulas simply represent the usual test results in which a stress-time plot on log-log paper, shown in the lower half of Figs. 2 to 14, is a straight line of slope $(n-1)$. This behavior is consistent with the usual straight line for the rate-stress plot on log-log paper of slope n but the value of n is not the same as given by a series of constant-stress tests. It is likely to be larger, corresponding to a flatter log-log line than the constant-stress test although the results given in Figs. 17 and 18 do not show this trend.

Examination of these formulas shows that they represent the difference of two time quantities. For an initial elastic extension which is large in comparison with the residual extension, the second term may be omitted. Hence, for year-long bolt performance the following expressions are satisfactory

$$t = \frac{b\,S_0^{\,n}}{(n-1)\,r_0\,E S^{n-1}} \quad \ldots\ldots [3]$$

$$S = \sqrt[n-1]{\left[\frac{b\,S_0^{\,n}}{(n-1)\,r_0\,E\,t}\right]} \quad \ldots\ldots [4]$$

[5] "Interpretation and Use of Creep Results," by J. J. Kanter, Trans. American Society for Metals, vol. 24, Dec., 1936, pp. 901–903.

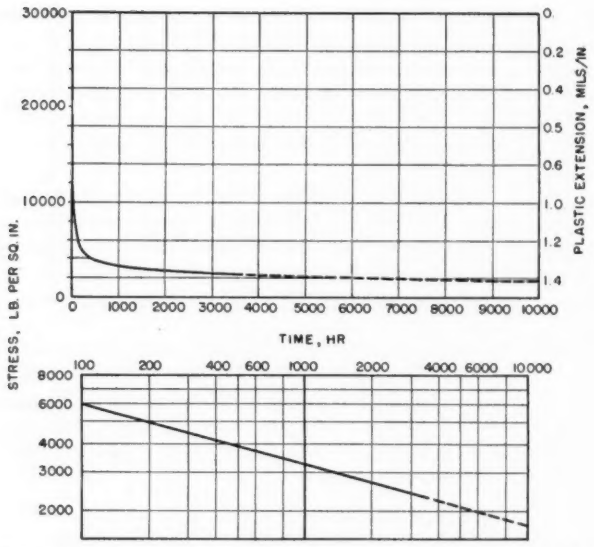


FIG. 2 GENERAL ELECTRIC STEP-DOWN TEST. MATERIAL 1, BAR 14A5, ANNEALED 0.35 PER CENT CARBON STEEL K20, 850 F

Measured elastic modulus E = 20,000,000 psi.
Constant elastic-plus-plastic extension = 1.5 mils per in.
Elasticity factor b = 1. Stress-rate index n = 4.7.
Computed 10,000-hr residual stress = 1700 psi.
Creep strength for rate of 1 millionth per hr = 6200 psi.

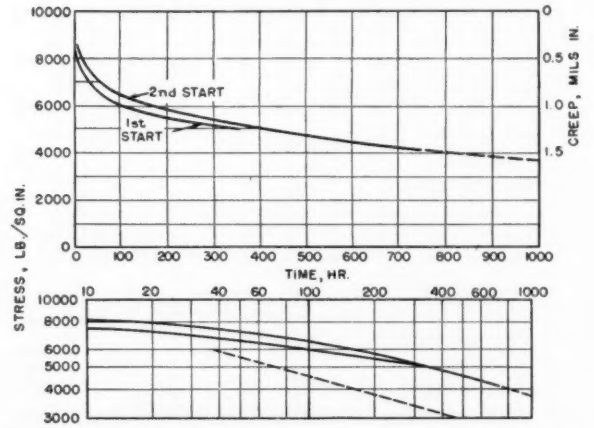


FIG. 3 CRANE CONTINUOUS RELAXATION TEST. MATERIAL 1, BAR 342, ANNEALED 0.35 PER CENT CARBON STEEL K20, 850 F

Measured elastic modulus E = 20,000,000 psi.
Constant elastic-plus-plastic extension = 2.5 mils per in.
Elasticity factor b = 5. Stress-rate index n = 4.5.
Computed 10,000-hr residual stress, (b = 1) = 1260 psi.
Creep strength for rate of 1 millionth per hr = 5130 psi.

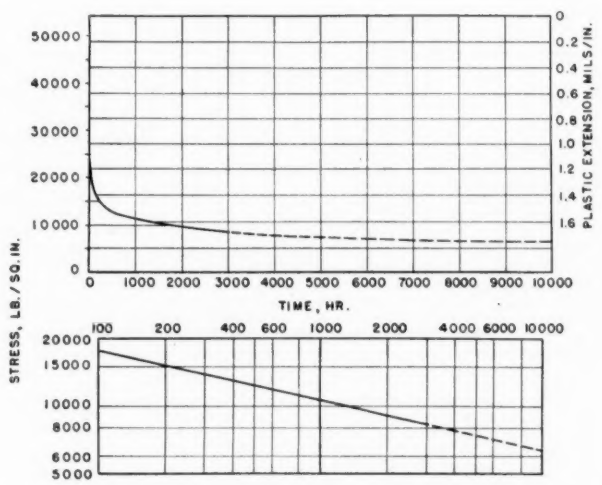


FIG. 4 GENERAL ELECTRIC STEP-DOWN TEST. MATERIAL 2, BAR 462, WROUGHT 0.58 PER CENT CARBON STEEL, 750 F

Measured elastic modulus E = 27,200,000 psi.
Constant elastic-plus-plastic extension = 2.0 mils per in.
Elasticity factor b = 1. Stress-rate index n = 5.6.
Computed 10,000-hr residual stress = 6400 psi.
Creep strength for rate of 1 millionth per hr = 16,400 psi.

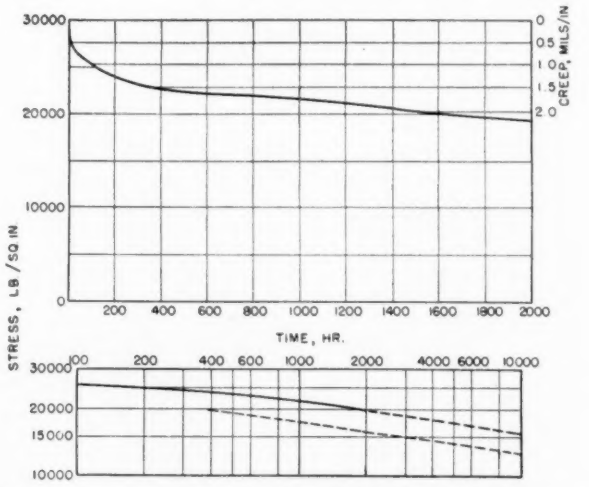


FIG. 5 CRANE CONTINUOUS RELAXATION TEST. MATERIAL 3, BAR 368, S.A.E. 4140, 850 F

Assumed elastic modulus E = 24,000,000 psi.
Constant elastic-plus-plastic extension = 6.25 mils per in.
Elasticity factor b = 5. Stress-rate index n = 8.5.
Computed 10,000-hr residual stress, (b = 1) = 12,800 psi.
Creep strength for rate of 1 millionth per hr = 23,000 psi.

Formula [3] is also satisfactory to give the time required for an annealing cycle in order to assure elimination of all stresses above a prescribed value.

Relaxation tests in various forms have been used to determine a nominal creep strength for comparative purposes. One of the objects of Project 16 is the comparison of such values with values determined in other ways. For this purpose formula [3] may be rearranged thus:

$$S_0^n = (n-1)\, r_0 E t S^{n-1}/b \quad \ldots\ldots [5]$$

TEST RESULTS

In presenting the results of the various tests sponsored, it has not seemed desirable to give elaborate records showing every test point in detail. Such a procedure would have emphasized the differences of method; whereas it seemed better to show the derived performance given by all tests according to some uniform scheme which would facilitate comparison and eliminate differences in so far as possible except such as characterize the actual behavior of the test bars.

In each case the stress-time relaxation curve is given using ordinary scales with uniform divisions. At the right of the diagram the plastic extension or creep is shown, increasing as relaxation of stress occurs. In all cases the bottom of the diagram is the zero for stress and in all except Figs. 12 and 13, the top of the diagram is the zero for creep.

Immediately below the relaxation curve, the stress-time relation is plotted on log-log coordinates in order to show the agreement with or departure from the rules represented by the formulas.

Where the elasticity factor is greater than 1, a dotted line has been drawn on the log-log plot to represent the corresponding behavior with unit elasticity. According to the formulation,

when the system elasticity is b times that of the part subject to creep, it takes just b times as long to relax to any particular stress level. The corresponding lines are parallel and separated by a distance equal to the logarithm of the ratio b.

The test results not only illustrate the characteristic behavior of materials when called upon to maintain elastic tightness at high temperature but also show a number of characteristic vagaries. The formulation corresponding to the straight line on the log-log plot is amply justified over extended periods of time but it cannot be extrapolated indefinitely in either direction. The direction of the line is likely to change. Material may appear to strengthen or to weaken as time goes on. Both the rules and the deviations from them are evidenced by the results presented on different materials. Furthermore different laboratories working in quite different ways agree both as to general trends and as to the deviations that may be expected.

Strengthening. Both material 3, S.A.E. 4140 at 850 F tested in a series of test blocks (Fig. 6), and material 4, carbon-molybdenum tubing at 800 F tested in continuous relaxation (Fig. 8), show apparent "creep limits" below which further relaxation fails to occur. It is possible that more precise methods of testing would show some relaxation still going on but the evidence as it stands points to a definite strengthening after several weeks.

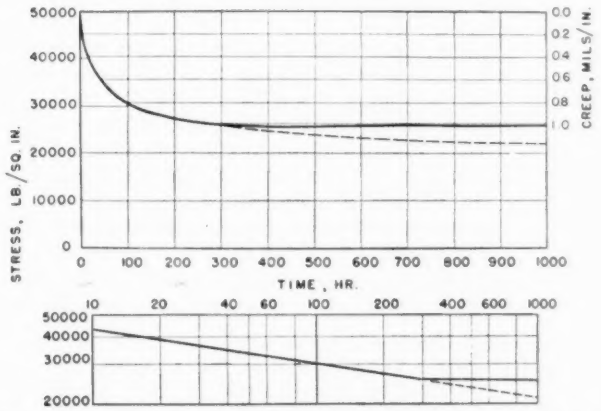


FIG. 6 SERIES OF TESTS IN WESTINGHOUSE TEST-BLOCK ASSEMBLY. MATERIAL 3, S.A.E. 4140, 850 F

Assumed elastic modulus E = 24,000,000 psi.
Constant elastic-plus-plastic extension = 2.15 mils per in.
Elasticity factor b = 1.03. Stress-rate index n = 7.8.
Computed 10,000-hr residual stress = 15,350 psi (dotted line).
Creep strength for rate of 1 millionth per hr = 27,860 psi.
Apparent creep limit = 25,800 psi.

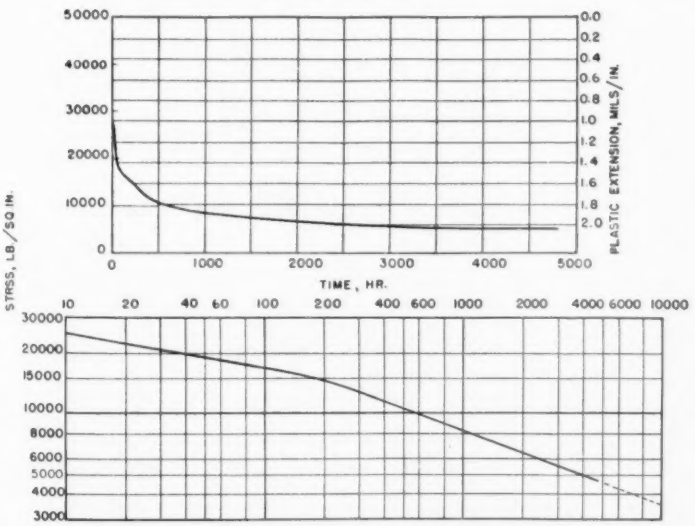


FIG. 7 SERIES OF TESTS IN WESTINGHOUSE TEST-BLOCK ASSEMBLY. MATERIAL 3, S.A.E. 4140, 930 F

Assumed elastic modulus E = 22,000,000 psi.
Constant elastic-plus-plastic extension = 2.15 mils per in.
Elasticity factor b = 1.03.
Stress-rate index n = 7 above 15,000 psi and 3.75 below.
Computed 10,000-hr residual stress = 3550 psi.
Creep strength for rate of 1 millionth per hr = 13,970 psi.

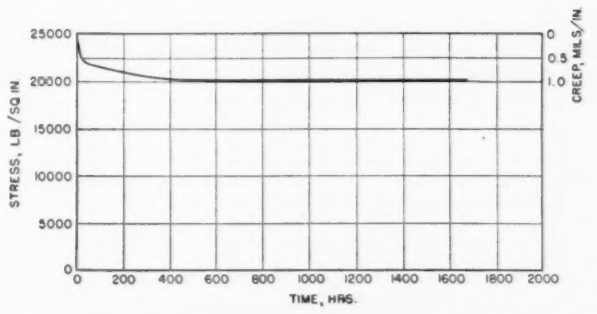


FIG. 8 CRANE CONTINUOUS RELAXATION TEST. MATERIAL 4, BAR 370–2, CARBON-MOLYBDENUM TUBING, 800 F

Assumed elastic modulus E = 25,000,000 psi.
Constant elastic-plus-plastic extension = 5.0 mils per in.
Elasticity factor b = 5.
Apparent creep limit = 20,000 psi.

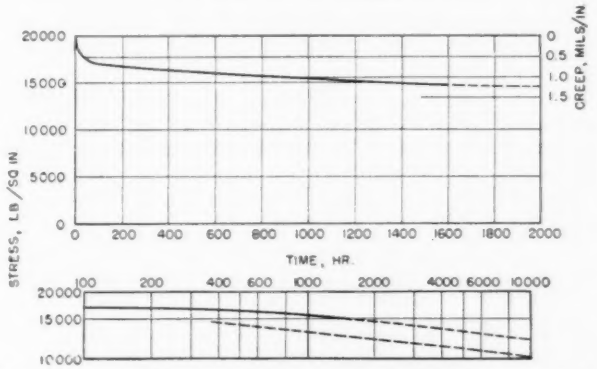


FIG. 9 CRANE CONTINUOUS RELAXATION TEST. MATERIAL 4, BAR 370–3, CARBON-MOLYBDENUM TUBING, 950 F

Assumed elastic modulus E = 22,000,000 psi.
Constant elastic-plus-plastic extension = 4.55 mils per in.
Elasticity factor b = 5. Stress-rate index n = 10.5.
Computed 10,000-hr residual stress, (b = 1) = 10,200 psi.
Creep strength for rate of 1 millionth per hr = 16,900 psi.

Another important type of strengthening appears in connection with successive tightening. A bolt is almost always more creep-resistant on its second tightening than at first. This effect is illustrated in Fig. 3 by the higher residual stresses after the second start.

Probably for similar reasons, higher initial stresses generally result in higher subsequent residual stresses. Another way of saying this is that tests run with larger values of specified constant elastic-plus-plastic extension give higher residual stresses after a certain stated time interval and indicate higher nominal creep strengths corresponding to a specified rate of extension. These effects certainly represent some form of progressive strengthening associated with the greater amounts of strain involved.

While the complete formulation [1] includes the initial stress and therefore appears to take account of various initial stresses, examination of the various test results is convincing that this formula cannot be used over a wide range without great discretion. Thus, with a test made with a certain total elastic-plus-plastic

extension, formula [1] may be used to compute behavior between any two stress levels covered by the test. But for service with a different total elastic-plus-plastic extension it is preferable to have a different test.

The only series of tests enumerated in the report that were run on the same material at the same temperature and the same elasticity factor but with several different initial stresses, that is, total elastic-plus-plastic extensions, are those indicated in the first three lines of Table 1, on 0.35 per cent carbon steel at 850 F. The effects mentioned were noted by the Westinghouse Company in the references cited with Table 1.

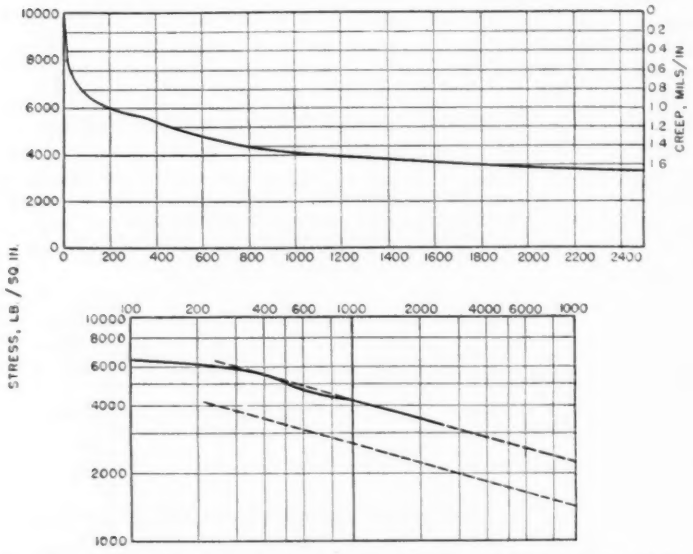


FIG. 10 CRANE CONTINUOUS RELAXATION TEST. MATERIAL 4, BAR 370–1, CARBON-MOLYBDENUM TUBING, 1050 F

Assumed elastic modulus E = 20,000,000 psi.
Constant elastic-plus-plastic extension = 2.5 mils per in.
Elasticity factor b = 5.
Stress-rate index n = 4.55 below 5500 psi.
Computed 10,000-hr residual stress. (b = 1) = 1400 psi.
Creep strength for rate of 1 millionth per hr = 5500 psi.

Weakening. Material 1, annealed 0.35 per cent carbon steel K20 tested in continuous relaxation (Fig. 3) shows an almost continuously curving line on the log-log plot. If all the results were like this, it would have to be considered typical and a different formulation sought. However, since the straight line is more typical, this curving over may be thought of as a progressive weakening tendency relative to a straight line at a former slope. Eventually the line in the lower half of Fig. 3 attains pretty nearly the same slope as in Fig. 2 and, if the elasticity factor had been somewhat smaller in Fig. 3 the two tests would have agreed even more closely.

More typical tendencies to shift to a new and weaker trend are shown by material 3, S.A.E. 4140 at 930 F tested in a series of test blocks (Fig. 7); material 4, carbon-molybdenum tubing at 1050 F tested in continuous relaxation (Fig. 10); and in the step-down test with many small steps (Fig. 11); and material 5, 4-6 Cr–Mo tubing at 1200 F tested in the step-down test with unit elasticity (Fig. 12) and with an elasticity factor of 3 (Fig. 13). In all these cases a fairly definite trend maintained for a period of some days, then shifts to a new trend with something of a knee in the log-

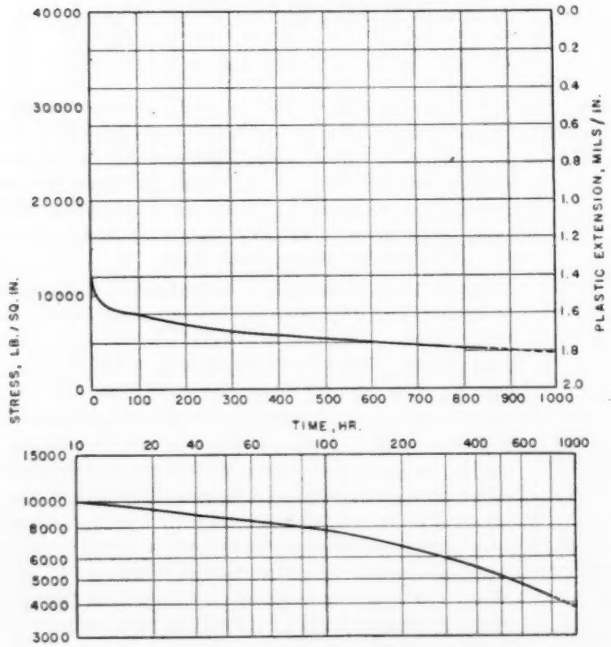


FIG. 11 BABCOCK & WILCOX STEP-DOWN TEST. (MANY SMALL STEPS.) MATERIAL 4, CARBON-MOLYBDENUM TUBING, 1050 F

Assumed elastic modulus E = 20,000,000 psi.
Constant elastic-plus-plastic extension = 2.0 mils per in.
Elasticity factor b = 1.
Stress rate index n = 9.2 above 6800 psi and 3.5 below.
Computed 10,000-hr residual stress = 1530 psi.
Creep strength for rate of 1 millionth per hr = 8000 psi. (Based on trend at lower stresses.) This test is being continued beyond the 850 hr shown here and future results may require revision. (See Fig. 26.)

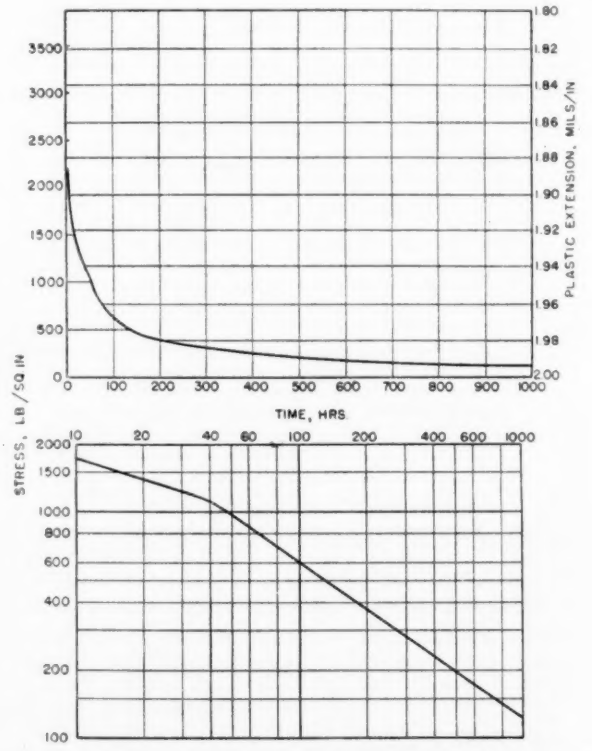


FIG. 12 GENERAL ELECTRIC STEP-DOWN TEST. MATERIAL 5, BAR 3–1, 4–6 CR–MO TUBING, 1200 F

Measured elastic modulus E = 19,400,000 psi.
Constant elastic-plus-plastic extension = 2.0 mils per in.
Elasticity factor b = 1.
Stress rate index n = 4.2 above 1120 psi and 2.5 below
Residual stress after 1000 hr = 125 psi.
Creep strength for rate of 1 millionth per hr = 1110 psi.

log plot. This change may appear in the ordinary relaxation plot in the upper halves of the diagrams as a sort of flattening when one hyperbolic curve gives way to another or it may appear as an actual dent as in Fig. 13.

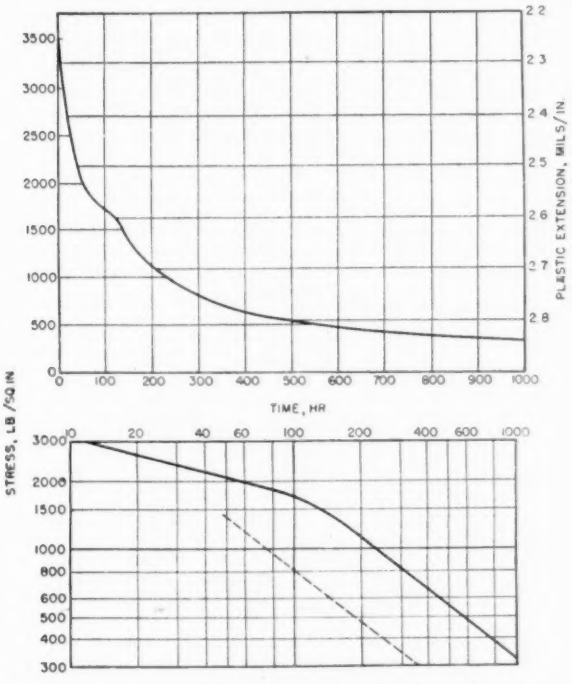


FIG. 13 GENERAL ELECTRIC STEP-DOWN TEST. MATERIAL 5, BAR 3-7, 4-6 CR-MO TUBING, 1200 F

Measured elastic modulus E = 16,200,000 psi.
Constant elastic-plus-plastic extension = 2.9 mils per in.
Elasticity factor b = 3.
Stress rate index n = 5.0 above 1670 psi and 2.25 below.
Computed 1000-hr residual stress, (b = 1) = 135 psi.
Creep strength for rate of 1 millionth per hr = 1240 psi.

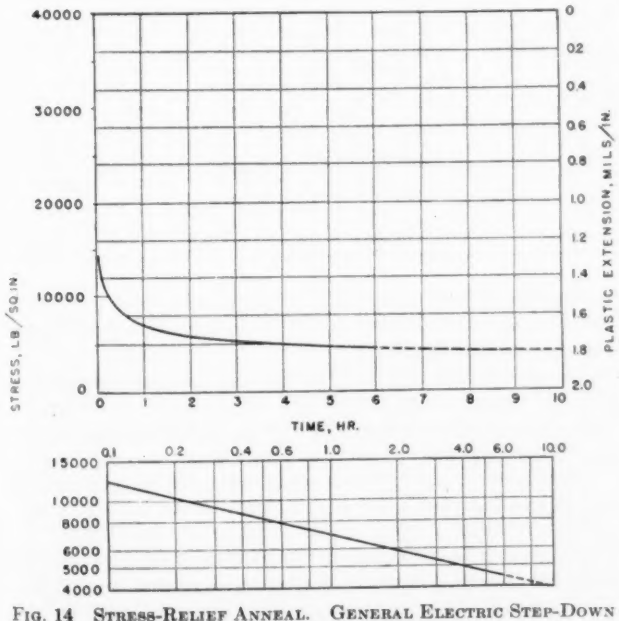


FIG. 14 STRESS-RELIEF ANNEAL. GENERAL ELECTRIC STEP-DOWN TEST. MATERIAL 6, BAR 550, S.A.E. 4140, 1112 F, 600 C

Measured elastic modulus E = 20,000,000 psi.
Constant elastic-plus-plastic extension = 2.0 mils per in.
Elasticity factor b = 1. Stress-rate index n = 5.3.
Residual stress after 10 hr = 4000 psi.
Creep strength for rate of 1 millionth per hr = 3000 psi.

It is a fair question whether this alteration represents some metallurgical transformation or physical change within the material or whether the analytical relations chosen are too simple and some more complicated relationship would be better.

STRESS-RELIEF ANNEAL

The process of developing materials to resist creep at high temperature results actually in the development of materials which are difficult to anneal and hence require higher temperatures or longer periods of time or both to accomplish the same degree of stress relief. Thus if a modern creep-resisting material is good for 200 deg higher temperature, it will require just that much hotter anneal to accomplish stress relief in the same period of time. If there are good reasons why the annealing temperature should not be raised in accordance with the strength, then greater time periods are necessary.

Relaxation tests are, in fact, quantitative measurements of stress relief and, in their early stages if run at annealing temperatures, are no more nor less than direct measurements of the effectiveness of the anneal.

The test on material 6, S.A.E. 4140 at 1112 F, 600 C, shows the early stages of relaxation or, in other words, the effectiveness of the annealing action. This test was not run long enough to see if the trend which characterizes the first few hours of behavior would continue unchanged. It is just as important to use a short-time test to predict short-time performance as to use a long-time test to predict long-time performance.

COMPARISON WITH CONSTANT-STRESS TESTS

A few comparisons are possible at this time between relaxation tests and constant-stress tests. (It is to be hoped that some time in the future more complete comparisons will be prepared for the several materials sponsored by the Joint Committee showing how they behave under different conditions of loading and different methods of test.) The prediction of bolt tightness or the determination of adequate time for stress relief is difficult from constant-stress tests. Similarly it is difficult to predict long-time constant-stress behavior from a relaxation test.

In its final stages a relaxation test is likely to approach constant-stress conditions but long-time behavior at the higher stresses is not so well indicated because of the short periods at which such stresses are sustained.

It should also be pointed out that a relaxation test with an elastic follow-up is intermediate between a relaxation test with constant elastic-plus-plastic strain and a constant-stress test.

Fig. 15 shows the relations between constant-stress tests and relaxation tests for material 1, annealed 0.35 per cent carbon steel K20 at 850 F. Specimen 10A1 was tested at Battelle Memorial Institute under a constant stress of 10,000 psi. Specimen 342 was tested in continuous relaxation at Crane Company with an initial stress of 10,000 psi and an elasticity factor of 5. This test duplicates Fig. 3. Specimens 3 and 4 were tested in test-block assembly by the Westinghouse Electric and Manufacturing Company with an initial stress of 10,000 psi and an elasticity factor of 1.03.

Fig. 15 illustrates the manner in which the relaxation method of running creep tests protects the specimen from excessive strain. It is plain that the constant-stress specimen has stretched in the brief period of test as much as would be allowable in many applications and much more than would be allowable in some. All of the curvature of this line is due to progressive strengthening usually attributed to strain and hence the indicated strength is greater than is likely to be experienced in service. On the other hand, the curving over of the relaxation tests is caused for the most part by the automatic decrease of stress.

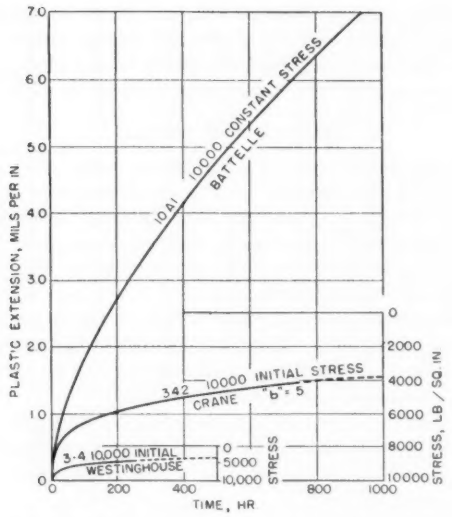


FIG. 15 COMPARISON OF CONSTANT-STRESS TEST WITH RELAXATION TESTS HAVING ELASTICITY FACTORS OF 5 AND 1. MATERIAL 1, ANNEALED 0.35 PER CENT CARBON STEEL K20, 850 F

For constant-stress test see "Long-Time Creep Tests of 18 Cr 8 Ni Steel and 0.35 per cent Carbon Steel," by H. C. Cross and F. B. Dahle, Trans. A.S.M.E., vol. 58, February, 1936, RP-58-3, pp. 91–96.

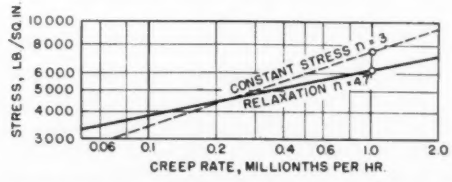


FIG. 16 COMPARISON OF CONSTANT-STRESS AND RELAXATION-TEST RESULTS. MATERIAL 1, ANNEALED 0.35 PER CENT CARBON STEEL K20, 850 F

Run by General Electric Company in same furnace at same time under identical conditions except loading.

A relaxation test comes to an end when the true allowable strain has been reached. Consequently creep rates measured during relaxation are not "inflated" estimates.

Fig. 16 for this same material shows how, at the higher stresses, the constant-stress test gives lower creep rates than the relaxation test. On the other hand, when tested at low stresses, constant-stress tests are likely to give results more nearly in line with the results of relaxation tests.

On three of the materials included in Table 1, results of constant-stress tests are available and these are listed for comparison in Table 3. Examination of this table shows that the nominal creep strength for a rate of one millionth of the length per hour is likely to be appreciably smaller when determined by the relaxation test. However, this is not necessarily always true. When the value of n is larger there is the possibility of equal values for a lower rate of creep. Thus in the case of material 1, Fig. 16 shows that the constant-stress and relaxation tests would agree for a rate of 0.22 millionths per hr. Furthermore, for the rate of 0.1 millionths per hr the relaxation test shows a higher nominal strength of 3800 psi as compared to 3500 for the constant-stress loading. Figs. 17 and 18 give similar comparisons for materials 4 and 5, respectively.

A Possible Acceptance Test?

It is a noticeable characteristic of relaxation tests that their most important stages of creep occur in the early periods of the test. After a few hours or days, a "knee" is passed after which further relaxation occurs very slowly. The rapid initial creep is what occurs in the well-known annealing process. On the other hand, after a brief period, a stress level is reached at which

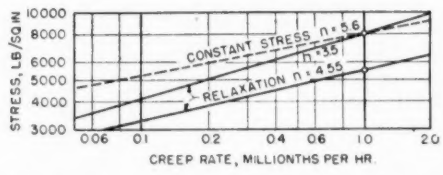


FIG. 17 COMPARISON OF CONSTANT-STRESS AND RELAXATION-TEST RESULTS. MATERIAL 4, CARBON-MOLYBDENUM TUBING, 1050 F

Constant-stress tests run by Joint Committee at Massachusetts Institute of Technology.

Upper relaxation test, multistep, by Babcock & Wilcox at Massachusetts Institute of Technology.

Lower relaxation test, continuous machine, by Crane.

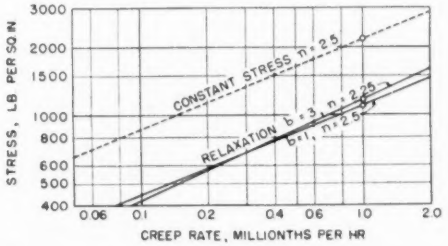


FIG. 18 COMPARISON OF CONSTANT-STRESS AND RELAXATION-TEST RESULTS. MATERIAL 5, 4–6–CR–MO TUBING, 1200 F

Constant-stress tests by Joint Committee at Massachusetts Institute of Technology.

Both relaxation tests, step-down, by General Electric.

TABLE 3 COMPARISON OF CONSTANT-STRESS AND RELAXATION-TEST RESULTS

	MATERIAL	TEMP °F	SPECIMEN	TIME HR.	PLASTIC EXT. MILS/IN.	RATE-STRESS INDEX τ_L	NOMINAL CREEP STRENGTH LB/IN² FOR RATE OF 10^{-6}	TYPE OF TEST
1	ANNEALED .35C STEEL "K20"	850	14A5	100	1.2	4.7	6200	RELAX.
			14A1-2-4	1000	2.1	3	7500	CONST. S.
4	C. MO. TUBING	1050	370-1	400	1.15	4.55	5500	RELAX.
			B. & W.	100	1.6	3.5	8000	RELAX.
			C-MO-3-2	2000	1.2	5.6	8000	CONST. S.
5	4-6 CR. MO. TUBING	1200	3-1	40	1.94	2.5	1110	RELAX.
			3-7	175	2.68	2.25	1240	RELAX.
			4-6CR-1-2	1800	2.5	2.5	2200	CONST. S.

creep proceeds very slowly indeed. This stress level is different for different materials and a bolt will stay tight if higher stresses than this are not required.

A test of from 2000 to 3000 hr can usually be extrapolated to give a 10,000-hr residual stress with a high degree of confidence. Often a much shorter test suffices if no sudden changes in the log-log representation of results occur. Such changes occurred as early as 40 hr and 1.95 mils per in. extension on specimen 3-1 of material 5 at 1200 F and after as much as 800 hr and 1.6 mils per in. extension on specimen 368 of material 3 at 850 F.

It must still be concluded that no short-time creep test can be regarded as very trustworthy for long-time predictions. A material which is of superior strength for one day or for 100 hr may alter its trend of relaxation so as to have an inferior residual strength after 1000 or 10,000 hr.

Discussion

JOHN BOYD.[6] Inasmuch as the committee's report is essentially a collection and a comparison of various relaxation tests on several materials it may not be out of place to offer some additional test data on one or two of the same materials and to make a few comments in connection with an interpretation of the results.

The data presented herewith have been obtained from automatic relaxation tests. This method has been described previously[4] but since it has been published only in abstract form it might be well to say briefly that it consists of keeping the gage length of the test specimen at a constant extension by automatically decreasing the load. A diagrammatic sketch of the apparatus is shown in Fig. 19. Fig. 20 shows a typical set of relaxation curves obtained by this method.

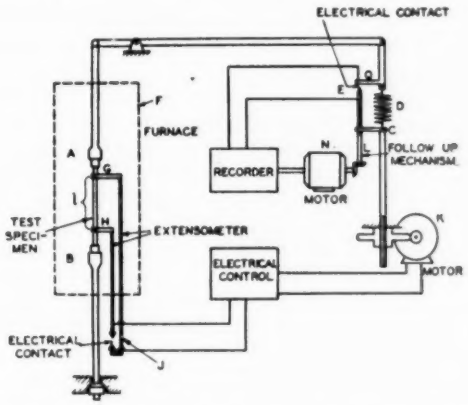


FIG. 19 DIAGRAMMATIC SKETCH OF AUTOMATIC RELAXATION TESTING MACHINES

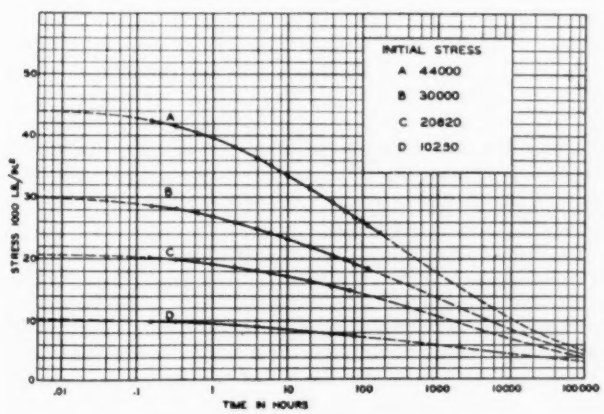


FIG. 20 AUTOMATIC RELAXATION TESTS
(On four samples of Cr–Mo–W steel at 850 F.)

TABLE 4 COMPARISON OF AUTOMATIC TESTS

Material		Laboratory	Initial stress, psi	Remanent stress, 10000 hr, psi: From Table 1	Remanent stress, 10000 hr, psi: Automatic tests
1 K20	14A5	G.E. Co.	30000	1700	...
	342	Crane Co.	10000	1260	1000
3 S.A.E. 4140	368	Crane Co.	30000	12800	11300
	.....	W.E.&M. Co.	50000	15350	14500

In Table 4 a portion of Table 1 has been reproduced for comparison and the last column shows the results of automatic relaxation tests on K20 carbon steel and S.A.E 4140 Cr–Mo steel at 850 F.

In comparing the various results it is well to remember that all of the remanent stresses shown in Table 4 have been obtained by extrapolation but it may be seen that in each case the results of the automatic tests are the lower. It may be said that the values for the Cr–Mo steel agree as well as might be expected considering all the factors which are involved. More should be said however about the case of the carbon steel K20.

Previously, the remanent stress in K20 steel after 10,000 hr had been estimated at about 800 psi on the basis of automatic tests. However more recent work has led to revising this figure upward to about 1000 psi. This value is still less than that of 1260 shown for the Crane Company tests. This is readily understandable. As a matter of fact the two figures should not be compared because the value shown is for a test piece which has been loaded twice (see Fig. 3) and experiment has shown that relaxation is less pronounced if a specimen is reloaded after having previously relaxed. Fig. 21 shows the results of a test on K20 steel in which the load was raised several times to approximately its initial value. An increase in resistance to relaxation is apparent.

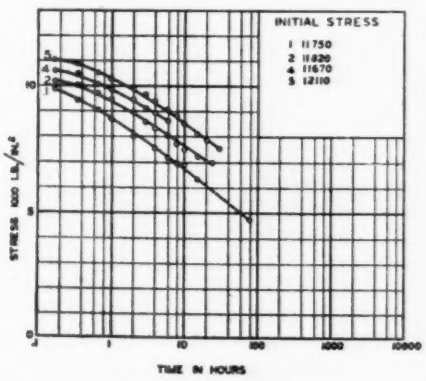


FIG. 21 EFFECT OF RELOADING ON THE RELAXATION OF STEEL K20 at 850 F

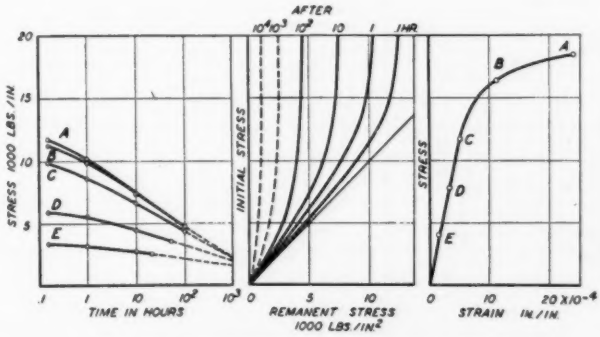


FIG. 22 RELAXATION OF K20 STEEL AT 850 F
(*a*) Relaxation tests
(*b*) Initial stress versus remanent stress
(*c*) Stress versus strain during loading period

In the case of the test at 30,000 psi initial stress no direct comparison with automatic tests could be made. In attempting to secure higher initial stresses for K20 than those shown, difficulties were encountered. In attempting to apply the load at a uniform rate, as is usually done in bolting applications, it was found impossible to increase it fast enough to keep the strain within the measuring capacity of the machine. In fact the initial extension of 0.0015 shown in Table 1 was reached before a stress of 20,000 psi was applied. That this is what might be

[6] Westinghouse Elec. & Mfg. Co., East Pittsburgh, Pa.

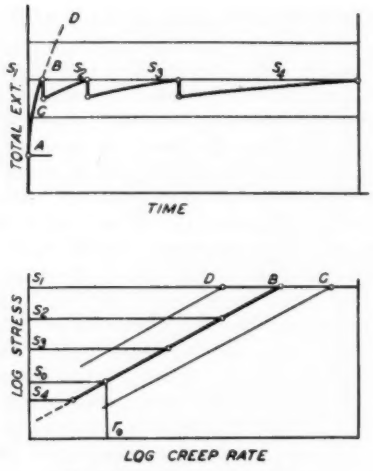


FIG. 23 FLOW-RATE TEST

Fundamental assumptions

$$\frac{dx}{dt} = \left(\frac{S}{S_0}\right)^n r_0 \qquad \frac{dx}{dt} + \frac{1}{E}\frac{dS}{dt} = 0$$

Also the expression for S given in Equation [4].

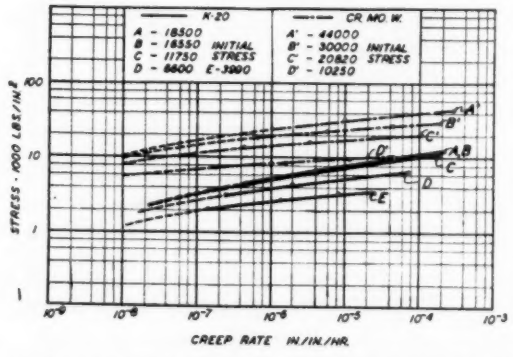


FIG. 24 CREEP RATE VERSUS STRESS FOR TWO STEELS AT 850 F

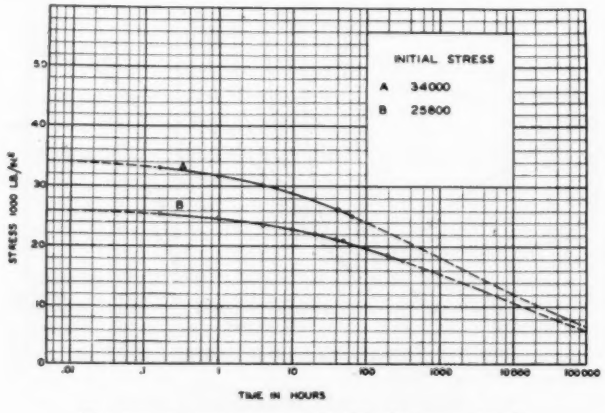


FIG. 25 RELAXATION OF CR–MO STEEL AT 850 F

expected may be seen if reference be made to the Progress Report of Subgroup D on the short-time tensile tests on K20.[7] Here it will be found that the 0.2 per cent yield point occurs at a stress of about 20,000 psi when tested under these conditions. It would seem therefore that a serious practical difficulty would be encountered in running any kind of relaxation test (except the type used by Mochel where the load is applied cold) in which an initial stress of 30,000 psi was used. These experimental troubles may have been avoided in the case of the published data by the scheme which seems to have been used in the reference mentioned as note 2 of Table 1. This appears to consist of using a smaller initial stress of say 10,000 psi and allowing the extension to increase until a strain of 0.0015 has been obtained before the load is stepped down in a manner which is calculated to keep the extension constant. Inasmuch as a true relaxation test in which the initial stress was actually 30,000 psi would undoubtedly be a test in which an extension of many times 0.0015 would be held constant, it seems that it would be necessary to show that such an experimental procedure gives results that are the same as those of the true case just mentioned. This does not follow a priori.

Several relaxation tests on K20 are shown in Fig. 22(*a*). Fig. 22(*c*) shows the stress-strain relations for these tests during the loading period and Fig. 22(*b*) shows the remanent stress plotted as a function of the initial stress. These curves bring out the fact that as the initial stress is raised above about 12,000 psi the relaxation curves tend to fall on top of one another so that the remanent stress is not greatly affected by changes in the initial stress. This fact seems to be the basis for the statement sometimes made that the remanent stress is independent of the initial stress. That this statement is misleading can easily be seen by consulting Figs. 22(*b*) and 20.

Relaxation Equations

The equations developed in the report follow directly from certain fundamental assumptions. The first of these is that the creep rate is a power function of the stress, and the second expresses the fact that the creep rate is exactly equal to the rate of decrease of the elastic strain. The general expression, Equation [4], is of course obtained by neglecting the term S_1 which is the initial stress. If any one flow-rate test, corresponding to any one particular initial stress, be extrapolated by the use of this equation it is the same as drawing a straight line on the log-stress–log-time plot. This appears to be a reasonable procedure although it naturally will not take account of any hardening or softening that may occur.

When any one such flow rate or other test is used to predict the remanent stress for a different initial stress from that already used it immediately raises the question as to whether or not the value of S_0 corresponding to the rate r_0 is constant. In general, experiment does not show this to be true as will be seen immediately from Fig. 24. That this should be expected may be made clear from Fig. 23. Let the heavy line through point B in the stress–creep-rate plot be obtained by maintaining the extension corresponding to B in the plot of extension against time. If instead, the extension C were held constant it is clear that the creep speed found for the stress S_1 would be higher and conversely if extension D were held constant the speed would be lower. This indicates that in general a unique value of S_0 should not be expected and that the creep rate is a function of the strain as well as the stress.

F. H. NORTON.[8] At the time this report was presented in December, 1938, the test on material No. 4 at 1050 F, being conducted at the Massachusetts Institute of Technology, was only about 1000 hr old. Before it was stopped this test had been run for approximately 5000 hr and the rate of relaxation had increased noticeably over what was indicated in the results available at the time as presented in Fig. 11.

Fig. 26 presents the test results for the entire 5000 hr. Since

[7] "Short-Time Tensile Tests at 850 F of the 0.35 Per Cent Carbon Steel Material K20," Trans. A.S.M.E., vol. 58, 1936, pp. 97–101.

[8] Department of Metallurgy, Massachusetts Institute of Technology.

it was necessary to use a counterweight, the loading system was not particularly sensitive to such low loads, and therefore it was thought to be impractical to carry the test on at stresses below 1000 psi. It is noteworthy that at the time the test was stopped the log-log plot of the time-stress relationship had attained a slope of 45 deg corresponding to a value of $n = 2$ for the relation between stress and creep rate. While at the time of the December report the 10,000-hr residual stress appeared to be approximately 1500 psi, the load had relaxed to 1000 psi in half that time with every indication that there would be not more than 500 psi remaining at the end of 10,000 hr.

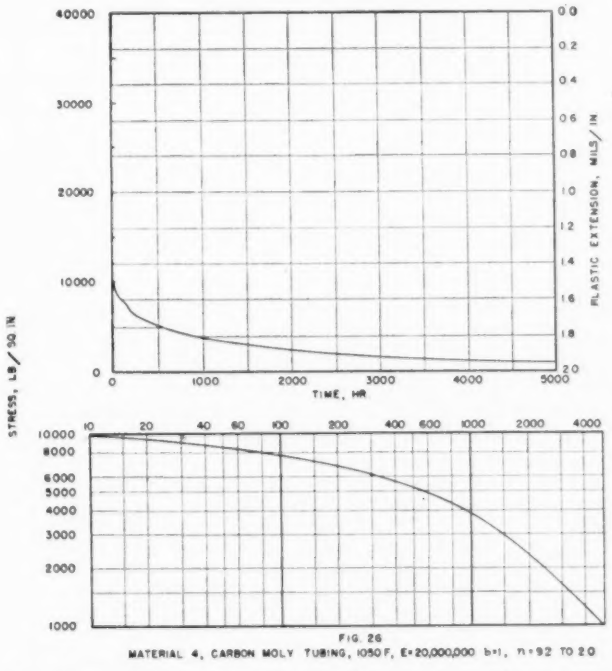


FIG. 26 BABCOCK & WILCOX STEP-DOWN TEST. (MANY SMALL STEPS.) MATERIAL 4, CARBON-MOLYBDENUM TUBING, 1050 F. (CONCLUSION OF TEST SHOWN IN FIG. 11.)

Assumed elastic modulus E = 20,000,000 psi.
Constant elastic-plus-plastic extension = 2.0 mils per in.
Elasticity factor b = 1
Stress rate index n = 9.2 above 6800 psi decreasing to n = 2.0 below 2000 psi and after 2500 hr.
Based on 5000-hr test, computed 10,000-hr residual stress = 500 psi.
Based on trend during last 2000 hr of test, creep strength for rate of 0.1 millionth per hr = 3200 psi. The actual creep rate at the conclusion of the test under a load of 1000 psi was 0.01 millionth per hr.

Whereas the nominal creep strength for a rate of one millionth per hour is shown in Table 1 as 8000 psi, and during the first week the stress to cause a rate of 1 per cent per 100,000 hr was 6400 psi, at the conclusion of the test this value had been reduced to 3200 psi and the specimen with a load of 1000 psi was actually creeping at a rate of 0.1 per cent per 100,000 hr.

The report of the Committee points out that relaxation tests are subject to changes of trend. However, such a marked alteration of strength as shown here constitutes more than an ordinary alteration of trend and makes it necessary to raise the question as to whether or not other materials may not suffer similar loss of strength after a period of time at high temperature.

The thought occurs to the writer that a relaxation test might actually come down to a zero stress or even a negative stress provided the specimen itself showed a gradual permanent expansion. Such an expansion may possibly have taken place in this particular test due to structural changes in the steel. It would be interesting to run a specimen with no load and measure any length changes which might occur. Thus, it is quite possible that relaxation is a combination of creep and structural expansion.

Creep as ordinarily measured may be a combination of extension due to stress and a permanent expansion which would occur anyway without stress. If so, it may be important in the future to measure expansion under stress-free conditions as well as extension due to a number of definite stresses.

AUTHOR'S CLOSURE

Mr. Boyd discusses the differences between the relaxation tests as run with a prescribed constant extension and a "true relaxation test" in which care is exercised really to secure a prescribed initial stress. His doubts of similarity are entirely justified and the Committee fully agrees that such a test would be quite different from one obtained by the experimental procedure used in the tests presented by the Committee.

The relaxation tests sponsored by Project 16 might well be called "tests at constant strain," that is, tests run with a total elastic-plus-plastic extension held constant, since that is a more precise way of describing the conditions obtaining during relaxation than by reference to an initial stress.

The initial stresses given in the report should, in every case, be interpreted as referring to equivalent calculated values, actually present only at the lower temperatures where the elastic limit is not exceeded.

The reason why the Committee has used this procedure is because the constant quantity in a bolted flange, a press fit, or a shrunk-on bushing, is the total strain, that is, the elastic-plus-plastic extension. The actual presence of the equivalent initial stress depends on the conditions at the start of service, and in any event, these conditions are highly transitory. On the other hand, it is important to note that in any actual machine, as in the Project 16 tests, the total extension is what is definitely known and limited rather than the stress.

Furthermore, an important consideration is the likelihood that the permanent strain, which the Committee carefully limits, is of greater metallurgical significance than the transient presence or absence of a high stress for a brief period.

The Committee fully agrees with Mr. Boyd's concluding remarks. In fact, even though not universally true, a preference has, on several occasions, been expressed for assuming that for a given temperature the nominal strength, S_6, is uniquely determined by the strain and the stress.[9]

The final results of the test on carbon-molybdenum tubing at 1050 F given by Professor Norton in Fig. 26 show a falling off in strength after several months at temperature which is very unusual although worse cases have been known. This is a widely used material tested about 100 F hotter than the maximum temperature commonly used. Even if only occasional samples are subject to any such loss of strength, it is a matter of importance to operators and worthy of investigation by metallurgists.

Professor Norton's suggestion that structural expansion might be taking place irrespective of creep due to stress, is an interesting possibility. If so, the constant-extension tests sponsored by the Committee represent not only constant elastic-plus-plastic extension, but rather a condition in which the sum of three quantities is constant; (1) elastic extension, (2) plastic extension due to stress which is commonly called creep, and (3) structural expansion due to temperature regardless of stress. While an ordinary bolt would never pass beyond a condition of zero stress, a bushing shrunk on a shaft and confined within a wheel, experiencing such

[9] "Metals at High Temperature—Test Procedure and Analysis of Test Data," by Ernest L. Robinson, *Journal of Applied Mechanics*, Trans. A.S.M.E., vol. 55, 1933, paper APM-55-17.

"The Creep of Metals—II," by A. Nádai and E. A. Davis, *Jour. Applied Mechanics*, Trans. A.S.M.E., vol. 58, 1936, p. A-7.

growth might well pass beyond the zero condition, and find itself in compression due to its external constraint, in case the latter were not subject to similar growth.

In the ordinary tests as run by the Committee such behavior would lead to zero stress after which expansion beyond the prescribed limit would occur. No results of this nature have so far come to the attention of the Committee, although of course, no special investigation has been directed at this question.

Discussion

Operating Methods and Problems of a Combined Hydro- and Steam-Electric System[1]

ALBION DAVIS.[2] The authors have presented a paper that is constructive in every respect. It covers a wide field, yet is accurate in its statements. It shows the complexity of the problems that arise in the operation of a large interconnected system and how futile it is for anyone to attempt to give simple answers to economic questions involving the balance between hydro and steam. The writer's comments are intended to supplement their paper and bring out angles of operation which are of importance on combined steam and hydro systems that have a considerable amount of seasonal storage available.

FORECASTING

The first point to be emphasized is the value of forecasting. The contribution of forecasting to smooth operation and system economy is often overlooked, but it is basic. If someone will predict specifically what the flow and the load are going to be, planning economic operation becomes a matter of curves, loading tables, computations, and personnel. Good forecasting, anticipating load conditions—foresight rather than hindsight—and timely reports are essential parts of good operation.

In many cases it is nearly as serious to overestimate runoff from a given rain as it is to underestimate. In the first case too much low-cost energy will be dropped from the steam plants and in the second case the reservoir may fill and actual waste of water occur which otherwise could have been used for generation. There are systems where a 5-hour gain in a forecast of spill may mean as much as $1000.

NEED OF A SYSTEM PLAN

The second point is the need of a well-grounded system plan for allocation of plant loadings. This would include not only a daily and weekly plan but, where storage is available, a seasonal plan. These general load-allocation plans should be sufficiently flexible so that there is an opportunity for each person concerned in actual operation to contribute the best of his experience. Where several plants of both hydro and steam are involved, the economic balance between hydro and steam is continually changing and it takes the combined intelligence of a lot of individuals to get the finest results. Automatic devices can be considered as aids but hardly as substitutes for engineering in operation. Too often elaborate schedules of operating are devised and gradually forgotten. Rigid schedules and too specific regulations do not secure the best results.

Good forecasting, a flexible system plan that will give the widest possible opportunity for intelligence to be applied all along the line, plus a follow-up are the essential elements for securing the best results.

CAPACITY USE OF HYDRO

Hydro has two values to a system, a capacity value and an energy value. Its capacity value depends on how many kilowatts of the annual system peak it can carry with minimum river conditions. Its energy value depends on the steam-plant operating costs it can replace with the average river conditions that will occur over a period of years. Distinction between these two values is more pronounced where a system has a large amount of seasonal storage available, making peak value the dominant one.

Hydro will be used for maximum capacity when all plants of the system are taxed to their utmost to supply the system load, as will be the case just before the bringing in of a new major unit of system capacity. When used for maximum capacity, storage reservoirs will seldom be drawn appreciably until the steam plants are operating at or close to their maximum output. By using reserve steam capacity when it is available, additional water can be held in storage for use during the period of actual outage of the largest unit, thereby supplying an additional amount of system capacity during the period of greatest need.

In providing reserve capacity for short system outages, the leaning will be toward hydro on account of its quick availability as compared to steam. For rolling reserve capacity also the leaning will be toward hydro because it is usually far cheaper than steam. On the other hand, for the long-continued outages where quantity production is required, the emphasis will be on steam capacity.

ENERGY USE OF HYDRO

Hydro will be used to yield its maximum energy value when there is an ample margin of capacity elsewhere on the system. Securing a maximum energy value from the hydro plants is a long and detailed story, most elements of which have been covered by the authors. There is, however, one point which the writer would like to stress, namely, the necessity of hour-by-hour analysis of the use of hydro in the load curve.

System conditions are continually changing during the day. A plant may be operating at perfect efficiency as a unit by itself yet be considerably out of balance with the best system plan. At one hour of the day the system object may be to obtain the maximum amount of capacity regardless of efficiency; at another hour of the same day the objective may be to save all possible kilowatthours, even sacrificing some efficiency to build up a reserve against more expensive steam generation a month or two later; at still another hour of the day efficiency of generation may be the rule. And there are all shades of operation between these limits. For such a day the average efficiency of any one plant, as the term is ordinarily defined, may be poor, yet as a unit of the system, its operation may be perfect. Rather than its being a simple undertaking, operation is one of the most complicated.

The exact proportion of hydro used in the top of the system load curve during low-water periods will depend largely on the difference between on-peak and off-peak steam increment costs. When hydro is flush and it is operated on base load, the steam plants may be operating at minimum loadings particularly during nights and Sundays. Under such conditions, increment steam costs are low and the value of the last few kilowatthours squeezed from hydro may be small.

INCREMENT COSTS

Since the value that can be attached to kilowatthours generated by hydro is measured by the steam-plant production costs they save, a clear knowledge of what these steam increment costs are

[1] Paper by H. L. Harrington and E. B. Strowger, published in the February, 1939, issue of Transactions of the A.S.M.E., p. 89.

[2] Chief Hydraulic Engineer, Union Electric Company of Missouri, St. Louis, Mo.

for all types of loadings and for all hours of the day is essential to all determinations of the balance between hydro and steam.

Fig. 1 of this discussion shows increment costs of steam generation for a hypothetical system supplied in part by hydro plants that have seasonal storage available. The curves show how increment costs might increase with the steam-plant loadings. It will be noted from the drawing that each unit coming on the load adds a block of constant losses which are a function of time only. Once the machine is on the load and carrying its minimum permissible load, from there on up, increment costs are nearly constant until the unit is pushed up close to its maximum output.

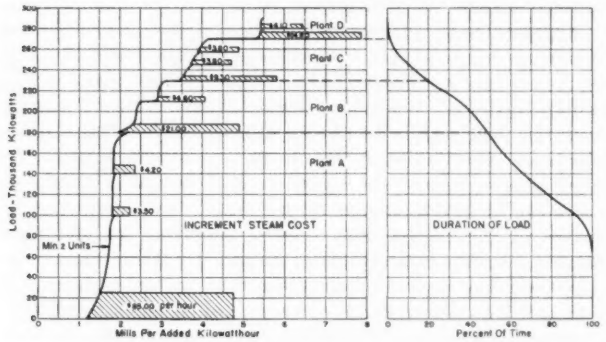


FIG. 1 INCREMENT COSTS OF STEAM GENERATION ON A HYPOTHETICAL SYSTEM SUPPLIED IN PART BY HYDRO
(A typical load-duration curve for such a system is shown at the right.)

When a plant is brought on load there is a lump-sum cost to get the plant ready for service. Then to operate the plant there is the fixed minimum hourly cost for operating labor plus the fixed cost of starting each unit.

At the right of the drawing there is shown an annual duration curve of load that might be typical for the same section of the hypothetical system. It will be noted from the increment-cost curve that if all the load is to be carried on steam some high-cost generation will result, as the top increment costs run perhaps three times the lowest. At the same time, the duration of such generation would be quite limited. The periods of low-cost steam generation are extended. If the available hydro generation can all be used in the top of the load curve the value of the water power will be large. If it all must be used in the base, its value will be small.

The object of any system operating plan would be to so fit hydro generation into the daily and weekly plan that a maximum amount of the high-cost steam generation could be saved. These steam-plant costs saved may include the shutdown of an entire plant. Where sufficient storage is available on a hydro system this use of hydro to avoid starting up inefficient steam plants is one of the main objectives to strive for in the system plan.

DIVIDING LOAD FLUCTUATIONS

The thesis that hydro is most useful in the top of the load curve operating at low load factor is true only up to a certain point. Fluctuation of hydro loads and low-load-factor operation reduces the amount of generation which can be obtained from a given use of water. This effect is exaggerated in the low-head plants where the reductions in head caused by rising tail water are a larger proportion of the total head. Increased fluctuation of load means less generation at the higher heads during the night hours and more generation at the lower heads during the day period. The combination means less total generation from a given river flow than would be secured by uniform operation throughout the day. Such losses increase very nearly as the square of the kilowatthours transferred from the night to the day period. When there is added to these losses those due to the additional turbine starts and stops that are required and the losses due to the load passing through more inefficient part-gate zones of operation, the total losses may easily run up to 8 or 10 per cent.

Fluctuation of the load carried by a steam plant likewise increases unit production costs. Fluctuation may mean additional boiler starts and machine starts or it may mean carrying excess rolling or stand-by capacity during the light-load periods, all of which increase unit costs. And yet there is a practical limit to the gains to be made in steam-plant operation by improved load factor. Few steam plants are designed for operation at load factors above 80 per cent. The last few increments of load-factor increase may yield insignificant gains.

Disregarding some minor practical considerations, the broad statement may be made that all plants should share the load swings. Just how much of a share each plant should take, however, will vary from plant to plant. The proportion of the daily swings of load which each plant should take may be determined by evaluating the losses each plant would be subject to by reason of typical fluctuations. The results may be summarized in tabular form for each plant. The best division of the load swing will be that in which the total loss due to fluctuation of load on all plants is a minimum.

SOME LONG-RANGE USES OF STORAGE

First in importance on systems where seasonal storage is available would be the planning of reservoir levels for the different periods of the year, both to yield the most energy during low water for use in the top of the load curve, and to catch excess flood waters for building up the total generation.

The many factors involved in such planning make it desirable to develop "rule curves" which will sum up experience equivalent to operating the system through all the river conditions of the past. These rule curves will show the economic drawdown of the reservoir prior to any regular spring or fall rains that may be characteristic of the drainage. The rule curves serve merely as a summary of much detailed engineering analysis. They should be looked on not as rigid schedules but merely as guides.

Usually it will be necessary to develop two rule curves; the first for energy operation where maximum economy is the guiding aim, the second for capacity operation when ability to carry peak is the first and economy the second consideration. The first rule curve will utilize greater reservoir drawdown to catch more flood waters and generate the most energy, while the second rule curve will be less concerned with catching flood waters and will aim to conserve storage primarily to yield maximum output during the extreme low-water periods. Reservoir drawdowns in the second case will be considerably less than in the first.

With these two rule curves available it is possible to judge fairly closely the proper position of the reservoir for any type of operation at any season of the year. A new plant coming on the system or a new transmission facility, or changes in the seasonal characteristics of the system load curve, will alter these curves for system use of storage.

A second use of seasonal storage is in reducing the fluctuations of top average steam loads from week to week, thus in effect substituting more low-cost generation for that of high cost. Such use may often avoid the high cost of starting up another stand-by steam plant.

Seasonal storage may also be used in anticipating fuel shortages at the steam plants, and in connection with planned outages of major equipment. Many times by planning ahead, sufficient storage may be conserved to handle such outages without loss. When the outage comes it may be handled from storage without making it necessary to use inefficient generating capacity.

Judicious use of storage sometimes makes it possible and eco-

nomical to shut down a plant entirely for extended periods and transfer the operating force to maintenance work on other parts of the system.

Scheduling Plant Maintenance

Outages of steam turbines for inspection and maintenance are often a serious tax on economic operation, particularly where the units are large and efficient. When they come down, their load must be supplied by other units which usually will carry higher top-increment costs. With large units this may represent quite a spread in generating costs. Such outages can be very expensive. Outages for maintenance of efficient steam units during periods of low hydro may require generation on other steam units at double the cost.

On a system where this is the case, it may be quite advantageous to take steam-turbine outages ahead of their regular schedule if hydro conditions at the time are favorable. If these conditions are not favorable and continue unfavorable it may be feasible to postpone the outages until there is improvement. It may be desirable to postpone the outage six months beyond the due date as, usually, water conditions will get better within a one-year period.

By scheduling steam maintenance in this manner, the steam turbines may be serviced on the average oftener and the direct cost for maintenance will be increased, but the energy cost of the outage will be reduced far more and the total cost resulting from maintenance will be substantially reduced.

System Governing

Considerable attention was given in the paper to the important problem of frequency control and its relation to tie-line loadings. The writer would like to add a further word suggesting that many of the difficulties in connection with this part of the problem can be eliminated by improving the performance of governors. On an isolated system these irregularities of governor action show up as short-period swings of system frequency. On an interconnected system, irregularities of governor action show up as swings of tie-line loadings. It is believed that proper system coordination of governor action can do much toward smoothing out the short-period swings of system speed and make it much easier to maintain frequency by either manual or automatic control.

J. H. Harlow.[3] The authors of this paper have undertaken a broad subject and have admirably succeeded in pointing their finger at a great number of system operating problems. In attacking a subject of this type, one is forced to decide whether the approach should be from the broad and inclusive point of view, or whether it would be better to confine the discussion to a few selected aspects of the problem and thereby be able to go into greater detail. From either point of view, the subject is timely and worthy of presentation. However, it is to be regretted that time did not permit them to go into a more extensive discussion of the load-allocation problem for both steam and hydro; of the manner by which transmission-line losses and capacities are considered in these allocations; and of the type of data supplied to the load dispatchers as tools for the efficient and safe operation of the system.

Of these problems, the one of steam-station load allocation is probably the most widely encountered. It would seem well, therefore, to discuss this phase of the system operating problem briefly.

In this connection, it is not necessary to go into a comparison of methods or a theoretical discussion of the proper fundamental basis for solving the load-allocation problem. Suffice it to say that it is now generally recognized that the base-loading scheme, which was frequently used a few years ago, was erroneous and that the true economical scheme of load allocation among steam stations is on the basis of incremental costs. Under this scheme, the load is allocated in such a manner that the incremental costs of energy, within the limits imposed by the system, are the same for all sources. With this premise established, the problem of economical load allocation among steam stations reduces to one of discovering a ready and accurate means by which the incremental costs can be determined and of being sure that all pertinent factors are considered.

In determining the true incremental cost of energy from any given source, there are at least three items of cost which must be considered, namely, fuel, maintenance, and transmission loss. The first two items are involved because they each have components which are functions of the station load. The third item is important because it enters into the determination of the effective value of the others. In some cases, it may be found that labor and expense costs have components which are functions of the load, but, in general, these items of cost do not enter into the problem because they are essentially constant regardless of the magnitude of the plant load and, therefore, do not enter into the plant increment cost.

By far the largest single item which is included in the total incremental cost is the incremental cost of the fuel. This item, therefore, must be determined with great care, and can be expressed most readily in units of fuel consumption per net kwhr, this, in the preponderance of cases, being a determinable ratio which is not influenced by the fuel price.

The incremental fuel rate at any given load may be defined as the change in the rate of fuel input resulting from a unit change in the load. Or, in the terms of the input-output curve of the plant, it is measured by the slope of the curve at the load for which the increment rate is to be determined expressed in units of fuel per unit of net output. Thus, in order to establish the incremental-fuel-rate curve of a plant, the most reasonable procedure is to begin by preparing a curve showing the relationship between net load and rate of fuel input. The basic data for such a curve should be obtained from actual performance tests on the various units of plant equipment such as boilers, turbogenerators, condensers, pumps, fans, and the like, because no curve of this type can be any more accurate than the groundwork upon which it is built. Where several similar units are installed, the testing of one unit, which may be selected as being typical of all, is permissible, but separate tests should be conducted to determine the performance of all units of dissimilar type. Over-all plant tests are not acceptable, due to the fact that, during such tests, it is impossible to be sure that all plant conditions are properly controlled and the effects of operating various numbers of units will not be determined. Also, small items, which enter into the proper operation of the equipment will be missed completely. It is much better to design the plant operation by building up from tests on the individual units.

The input-output curve can be simplified to a great extent by separating the total fuel into three components, namely, (1) the so-called generating fuel, which includes the total input less the banking or no-load boiler fuel with all of the boilers in service; (2) the so-called adjustment fuel, which comprises adjustments to the generating fuel resulting from the operation of fewer boilers, but does not include the banking fuel; and (3) the banking fuel, which is a constant dependent upon the number of boilers in service. The effect of circulating-water temperature can be shown best by means of a family of generating-fuel curves.

Parenthetically, it may be noted that the usefulness of a curve, such as has been described, is not limited to a study of load allocation. It is extremely valuable for use in checking up on the

[3] Assistant Superintendent, Station Economy Section, Philadelphia Electric Company.

quality of the plant performance, when converted into a daily input-output curve by coupling it with data covering typical load shapes for various daily outputs. In fact, an operating factor indicating the departure from standard performance which will be obtained through checks of this type is a desirable refinement to the determination of the load-allocation data, and is a necessity if incremental-cost data are to be used in pricing economy power transferred under pooled-power agreements.

An increment-fuel-rate curve can be developed from the input-output curve by plotting the slope of tangents to the input-output curve at various loads against their respective loads. Such a curve provides an accurate basis for load-allocation work, and is simple in its application. Similar curves are required for each steam station. The development of such a curve will reveal several facts, to which attention should be called. First, in most cases, the generating-fuel-increment rate shows a sudden increase of from 15 to 20 per cent at about 80 per cent load because at this point the overload valves on the units open. This is indicated on the input-output curve by a slight break and change in slope. Because of the increase at this point, it is not usually desirable to generate in the overload-valve region, but rather to hold this capacity for reserve. Second, there is a family of generating-fuel-increment curves resulting from various circulating-water temperatures. Third, the adjustment-increment rate, which is obtained from the adjustment input-output curves in a manner similar to that used for obtaining the generating-increment rate, must be added, as conditions require, to the generating-fuel rate. Fourth, constant components of the input, such as banking fuel, plant no-load fuel, and machine no-load fuel, do not appear in the increment rate.

As noted, the incremental fuel-rate data are expressed in terms of units of fuel per net kwhr. These fuel units multiplied by the fuel price per unit will give the desired increment-fuel cost.

The second item to be considered in determining the total incremental cost is the increment cost of maintenance. This may be determined from an analysis of the actual maintenance costs which have been experienced over a number of years of operation. A study of this subject leads to the conclusion that the total maintenance cost (and incidentally all costs of station operation other than fuel) can be expressed by a general formula in the following form

$$\text{Cost} = C_1 + C_2 \times \text{kw capacity} + C_3 \times \text{kw peak prepared for} + C_4 \text{ kwhr generated} \quad [1]$$

The constant C_4 in this formula is the desired incremental cost.

If the capacity and the peak which a station has been prepared to carry have not changed materially over a period of years, this formula can be reduced to

$$\text{Cost} = K + C_4 \times \text{kwhr generated} \quad [2]$$

In such a case C_4 can be determined simply by establishing the relationship between cost and generation. However, if the capacity or the peak prepared for have varied from year to year, it is necessary to make a more exhaustive study of the costs and to divide them into the various components. This can be done by setting up in advance certain conditions of plant operation to be covered by the various components and allocating to each component its proper share of the costs actually charged to each accounting subdivision. Although this requires the exercise of judgment to a considerable extent, it is not particularly difficult to establish the proper formula if the conditions are properly defined.

The increment fuel cost and the increment maintenance cost, which have been discussed, when combined give the total increment cost at the station bus. For load-allocation work, however, the true basis is the incremental cost at some selected point on the system, with the incremental costs of power from all sources adjusted to this common point. This cost may be greater, less, or equal to the incremental cost at the station bus depending upon the effect of transmission-line loss. For instance, if power is being fed out of the station over a transmission line and the load is increased on the station, the load and resultant loss on the line will be increased and the effective increment cost at the end of the line will be greater than the increment cost at the station bus. Conversely, if power is being fed into a station and the station load is increased with a resultant reduction in line loss, the effective increment cost at the end of the line will be less than that at the station bus. The increment line-loss factor, therefore, can be expressed best as a percentage to be added to, or subtracted from, the station increment cost and, for this purpose, may be defined as the change in line loss corresponding to a change in line load expressed as a percentage of the change in the line load. It is, therefore, the first differential of the curve of line loss against line load. On a three-phase line, this results in the following

$$\text{Per cent increment line loss} = 0.2r \frac{kva}{kv^2} \quad [3]$$

Where r = the single-phase resistance of the line in ohms; kva = line load at which the per cent increment loss is to be determined; kv = operating voltage of line.

Knowledge of the incremental line loss, being dependent upon line load, must be supported by a knowledge of the line loading which, in turn, is a function of the station load and the manner in which the system load is divided among the various distribution centers.

It is not necessary to illustrate the procedure to be followed in applying the data, the determination of which has been discussed. Suffice it to say that, from this point on, the load-allocation problem is a cut-and-try one necessitating increasing or decreasing station loads here and there until the desired result of balanced incremental costs is obtained.

A permanent allocation chart of the type described based on normal system conditions should be developed, but it should also be realized that such a chart is subject to error in its application due to the variations which occur from time to time on the system. Such variations may be caused by outages of units, outages of transmission lines, operation of more or fewer boilers than were used in the computation of the chart, changes in fuel-price differentials among stations, and the like. In order to make the most effective use of the increment-cost data in load-allocation work, therefore, the employee who is responsible for distributing the load should be thoroughly educated in the method of proper economical load allocation and should be supplied with all of the basic increment-cost data on fuel, maintenance, and transmission loss. This is important, for only in this way can load be correctly allocated under the many variations which occur from day to day. That such a procedure is entirely practical has been demonstrated by several years of experience.

The Kaplan Turbines at Bonneville[1]

C. I. GRIMM.[2] It occurs to the writer, who prepared the general designs for the Bonneville dam and power plant which were used as a basis for the project adopted by Congress, that further data relating to factors governing the selection of type and size of turbines may be of interest.

[1] By Paul L. Heslop and George A. Jessop, published in the February, 1939, issue of the A.S.M.E. Transactions, p. 97.

[2] Head Engineer, U.S.E.D., North Pacific Division, Portland, Ore.

Variation in head at the Bonneville plant is extreme. It is of primary importance that the plant have the best attainable capacity at heads around 30 ft, which are of infrequent occurrence and short duration. It is also important that the plant operate efficiently under the usual head of about 60 ft. For these conditions the adjustable-blade, propeller-type turbine was found to constitute the most economical and practical power plant. Even with this type of turbine, a material loss of capacity during high floods cannot be avoided. The two turbines installed have sufficient capacity to run the generators under full load (60,000 hp each) at a head of 44 ft. This corresponds to a river flow of 650,000 cfs. During 60 years of record, the river has been above this stage an average of $9^1/_2$ days per year, or 2.6 per cent of the time. During 43 of the 60 years, it has remained below this stage. Since the high flows occur in the summer and the peak load in this area occurs in the winter, some loss in peaking capacity during floods, even after the plant is fully loaded, is of no consequence. Six floods of about 750,000 cfs have occurred, which flow reduces the head to 40.8 ft and the turbine capacity to 54,000 hp. Three floods of about 800,000 cfs have occurred, which flow reduces the head to 39.6 ft and the turbine capacity to 52,000 hp. The effect of reductions up to this extent causes no concern. Three floods of about 900,000 cfs have occurred, which flow reduces the head to 31.6 ft and the turbine capacity to 37,800 hp or 63 per cent of the generator capacity.

As stated by the authors, the capacity of a turbine would be reduced to 27,400 hp during a flood of 1,170,000 cfs, which has occurred once during the period of record, in the year 1894.

High floods on the Columbia River are caused by melting snow and always occur about June 15. Their magnitude can be estimated by March 15. It is therefore practicable, when there is danger of plant capacity being reduced below demand, to arrange either for curtailing the load for a brief period or obtaining surplus power from other plants.

The Bonneville plant was designed with the expectation that it would be interconnected with the power network, as is now being done. Under this condition small units are not necessary for flexibility. Large units have been found to result in minimum costs per unit of capacity. The turbines and generators were therefore made about as large as available manufacturing plant and practice would permit.

Based on costs to date, the cost per kilowatt of installed capacity of the Bonneville power plant with six main units will be as follows:

Substructure	$22.80 per kw
Superstructure	3.40 per kw
Turbines	17.20 per kw
Generators	17.60 per kw
Transformers and switching	10.00 per kw
Total	$71.00 per kw

The cost per kilowatt of installing four additional units will be about the same.

J. F. Roberts.[3] The writer has watched with interest the development of welded construction for turbines of this type and feels that the statements submitted by the authors of the paper are in fact rather conservative. The use of welded-steel construction instead of steel castings offers many attractive advantages over and above those mentioned by the authors. They do bring out one of the important points, however, and that is the reliability such welded structures offer, as compared with the doubtful quality of some steel castings. Furthermore, the lack of distortion in such welded structures as compared with cast-steel members might also be mentioned. Frequently, cast-steel members will change their shape some months after being machined, with the result that, when reassembling the parts in the field, or even after being placed in operation, the turbine parts may be thrown out of line, requiring a great deal of trouble and considerable expense to correct.

One point which seems inconsistent is that the manufacturer of the Bonneville turbines uses a riveted draft-tube liner, while at the same time using welded-steel structures throughout for such important items as the speed ring, the head cover, the gate-shifting ring, and other parts which are subject to heavy strains and carry the most important loads during operation. Even granting the desirability of riveted connections in the draft-tube liner for the field joints, where they cannot be stress relieved, the writer would like to ask why the shop joints in the draft-tube liner could not have been welded in a manner similar to the other welded parts of the turbine? This would result in a smoother surface of the interior of the draft-tube liner and should be stronger than the riveted joints.

The method of checking the turbine and generator shafts on a vertical table in the manufacturer's shops certainly was an ingenious development. However, it is noted that, on account of lack of head room, it was impossible to test the entire generator and turbine shafts together. The turbine shaft was first checked with the intermediate shaft, then the turbine shaft was removed and the intermediate shaft was checked with the generator shaft. This procedure, the writer feels, might offer possibilities for future trouble, since with the expense involved in preparing the equipment, it would have been well worth while to make a complete check on the entire assembly of turbine, intermediate, and generator shafts combined.

It is noted that the engineers responsible for the design of the Bonneville structures made preliminary tests with models on which they based the design of the intake, the scroll case, and the draft tube. It seems to the writer that the design of the scroll case and the draft tube is a matter which should be left primarily to the designer of the water wheel. This is particularly true in the case of the Kaplan type turbine, since the action of the scroll case and the draft tube is so closely related to the operation of the turbine itself that only the manufacturer can determine adequately what form these parts should have to offer the best coordination with the turbine. While there are many structural designers who feel they are competent to design draft tubes and scroll cases, there is serious doubt if they should be allowed such freedom. Several cases are known where draft tubes, designed by structural designers and upon which the manufacturers were required to put their water wheels into operation, have caused serious disturbances and even vibration at certain outputs of the hydraulic turbine.

The outline of the method used in erecting the Bonneville turbines, especially the stiffening of the members to be embedded in concrete, should be very interesting to the engineer who is doing any work of this kind. It will be of great assistance to engineers planning the installation of similar or even smaller equipment, as such information has to the writer's knowledge never been published heretofore. One point that might be added to this discussion is the advisability of attaching the head cover in order to stiffen the upper flange of the speed ring during the pouring of concrete. If the head cover is available, it should be bolted into the upper flange of the speed ring and wedged solidly with steel shims in the clearance space around it. To the writer's knowledge, this was done with very beneficial results in at least two cases. While it did not materially improve the level condition of the upper flange of the speed ring, it held that upper flange to a true circle much more closely than the result given by the authors for the Bonneville turbines.

[3] Mechanical Engineer, Knoxville, Tenn. Mem. A.S.M.E.

The method used for supporting the turbine runner and shaft in the turbine pit, until the weight of those parts could be carried by the generator thrust bearing, was ingenious. In the writer's opinion, however, it certainly resulted in some rather heavy work in removing the large I-beams from between the guide vanes and speed-ring ribs after the load of the runner and shaft was taken by the thrust bearing. The high cones of the Moody spreading draft tube had one advantage during erection in that the runner and shaft could be set on the high cone until coupled up to the generator. Another system that is now being used extensively is to provide a small flange on the turbine shaft and to arrange the head cover of the turbine so that it can carry the load of the turbine runner and shaft until the load can be taken by the generator. In this case the runner, shaft, and head covers are assembled on the erection floor. Then the combination is lowered into the turbine pit in one piece.

The results of the efficiency tests and cavitation tests as given by the authors are interesting and will be helpful to many engineers, as they give much more information than is usually found in published data.

Authors' Closure

Paul L. Heslop. Extensive index tests on the units have been made since publication of the original article. Such tests do not reveal absolute efficiency but they do measure actual output, which was found to exceed predictions. An examination of the shape of the index-test curves demonstrates that model tests are a reliable means of predicting turbine performance in advance.

The net-head gate limiter described in the article was found to be unnecessary and will not be furnished on additional units. The inherent strength of the machines plus other devices present furnish ample protection.

George A. Jessop. Mr. Roberts suggests that a draft-tube liner wholly or partially welded is preferable to one entirely riveted. There is no practical advantage to a welded tube from the hydraulic standpoint when the riveted tube is made with butt straps and the plates are flush at the joints. There are small spaces between the plates forming the riveted tube, whereas with the welded construction these spaces are eliminated. The spaces between the sheets are so small that disturbances to the flow are not set up.

It is considered undesirable to have some joints welded and some riveted in the same structure. The shop assembly of such a large tube with its formed plates is a difficult and expensive process if the plates are to be joined by welding because of the necessity of having close and exact fits to enable the welding to be done according to the best methods.

The writer agrees with Mr. Roberts that it is highly desirable to allow the turbine manufacturer to design the scroll case and draft tube for the turbine. Even where model tests are made by the power-plant engineers, it is seldom that a runner and gate case of the proper characteristics are available. The turbine manufacturer has access to more previous tests and has had experience with more plants than the engineers. The manufacturer can be depended upon to submit designs which will assure the best results and not require unjustifiable dimensions or construction. The manufacturer's designs, in any event, are subject to review by the engineer.

Mr. Roberts mentions the desirability of attaching the head cover to the speed ring to afford additional rigidity during the pouring of the concrete. For turbines as large as those at Bonneville the head cover is divided in such a way that there is an outer and inner cover, the former of which contains the two sets of bearings supporting the upper gate stems. It is seldom, if ever, that the entire head cover is available on such large turbines at the point in the construction schedule when the concrete is poured around the speed ring. There have been instances where the outer head cover was used. It is pointed out that when this is done it is not safe to eliminate bracing inside this structure as the forces set up by the concrete are so great that this part itself may be distorted. The outer head cover must remain perfectly true because if it does not the wicket gates will not be in proper alignment.

Mr. Grimm offers valuable data in regard to stream flow and turbine output. The figures on costs per kilowatt are interesting as would be similar figures for the dam and the fishways.

Modern Boiler Furnaces

By E. G. BAILEY,[1] NEW YORK, N. Y.

The paper considers the fusing temperatures of various kinds of coal ash and slag, and their significance with respect to furnace temperatures, operation, and design.

A method of comparing furnace operating results is suggested consisting of (1) taking accurate gas temperatures throughout a furnace, (2) then calculating from them the rates of heat absorption in different zones, (3) calculating an "equivalent heat-receiving-surface temperature" for each zone, (4) taking samples of ash and slag from different portions of the furnace and determining the fusing-temperature range from such samples, (5) plotting these data as a "gas-ash temperature graph" against heat-absorbing surface, (6) comparing the data as shown on such graph with observed behavior of ash and slag in the furnace, and (7) comparing different furnaces, different coals, and methods of operation through the use of such graphs. Examples of the use of the method are given.

There is also included a comparison of methods for measuring furnace temperatures. Colored motion pictures illustrating some phenomena relating to slag in furnaces were shown at the time of presentation of the paper.

ASH AND SLAG from the burning of coal have long been recognized as important factors in boiler-furnace operation. Hand-fired boilers were often limited in output by clinkers in the fuel bed, with resulting excess burden on the firemen. Certain kinds of stokers were ill-adapted to handling clinkering coals, while others were developed to cope with the same coal much better. Forced-draft stokers extended the clinker trouble, formerly restricted largely to the fuel bed, to the furnace walls and boiler tube banks, where "bird nesting" of slag was a serious trouble. Pulverized-coal firing, as first applied to slightly modified furnaces, did the same, but to a greater degree, as a higher percentage of ash was in suspension.

Water-cooled furnaces have overcome the erosion of the walls, and aided in lowering the temperature of the gases before entering the boiler tube bank. The spacing of the boiler tubes has been increased to minimize the tendency of ash and slag to accumulate. Turbulent pulverized-coal burners, resulting in more complete combustion with shorter gas travels and low excess air, have also been developed.

It seems that as soon as there are solutions for one set of conditions, new problems of economic importance arise which can only be solved by further study of the factors which enter the problem.

Some of the new problems of today are found in the design of steam-generating equipment for higher capacities which will produce steam at high pressures and at temperatures of from 900 to 950 F under close control, and which can be erected within restricted building spaces available. In fact, the steam temperature must be under accurate control because of the requirements of the turbine and the approach toward safe limits of tube temperatures at the high pressures. Superheaters must absorb more than 25 per cent of the total heat produced in the furnace, and to do this with any reasonable amount of surface they must receive gases at fairly high and dependable temperatures. Furthermore, the economics of superheater size and construction calls for high steam velocities within the superheater elements, hence relatively small and closely spaced tubes for high gas-mass flow through the tube bank. The distribution of steam inside and of gas velocity and temperature outside to the different portions of the superheater should be quite uniform, to prevent localized overheating.

The greatest single difficulty to be overcome in accomplishing all of this is the ash in the coal being burned. Not so much ash as such, but ash converted into slag through high-temperature combustion; slag that is inevitable, even from the lowest possible combustion temperature when using many of the low-fusing-ash coals, which are available today. Ash constitutes the main problem; ash in the form of molten slag, plastic slag, sticky slag, sponge ash, and fly ash.

Dry ash removal from pulverized-coal-fired furnaces is satisfactorily accomplished with a wide range of ash-fusing temperatures, using turbulent burners, water-cooled furnaces, and rates of combustion from 25,000 to 35,000 Btu per cu ft per hr. Such units are suitable for steam outputs up to about 200,000 lb per hr, and for moderate steam temperatures. For larger units, the ash and slag limitations call for lower rates of liberation and/or higher fusing temperature of ash, and perhaps less turbulent burners. In many localities there is available only a limited amount of coal of high ash-fusion temperature and therefore large furnaces for dry ash removal are being used less extensively.

The nature of the ash in the coal is of greatest importance in designing a furnace for dry ash removal. The intense heat resulting from turbulent burners causes temperatures sufficiently high to fuse almost any ash locally within the active-combustion zone. For satisfactory operation with dry ash removal, the places where the ash discharges to the ashpit, and the gases to the tube bank, must be far enough away from the burner zone to permit the necessary cooling of the ash before leaving the furnace, or else it will adhere to the hopper or other cooling means at the bottom, and to the boiler tubes at the top. In any event, some ash will adhere to waterwalls of any known construction, around the middle of the furnace within the burner zone, as shown diagrammatically in Fig. 1.

If mechanical cleaning is applied to this area, and it is kept reasonably clean, the furnace may be too cold for stable ignition and efficient combustion, especially at lower rates. The rate of radiant-heat absorption may also be too high for the safety of the tubes, if the water conditions are not perfect. Experiences with gas- and oil-fired boilers bear out these points, and some protection of bare water-cooled walls is necessary to obtain complete combustion and protect the walls against tube losses.

With coal firing, the ash or slag accumulation in this zone is uncertain, and troublesome because of its uncertainty. With a combination of ash characteristics and rate of combustion which produce a sticky but not liquid ash formation on the walls, the thickness of sponge ash may increase and spread both up and down to greater areas, insulating the heat-absorptive surface so

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NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

that gas temperatures rise. Higher gas temperatures further accentuate the spreading and thickening until unduly high temperatures reach the boiler tube bank and the bottom of the ash discharge and cause slagging troubles in one or both of these locations. Thick accumulations of slag, sometimes quite bulky, may fall intermittently from the upper part of the furnace, causing damage to the furnace structure.

If the ash characteristics and temperature to which it is subjected are such that the slag is of a molten plastic nature,

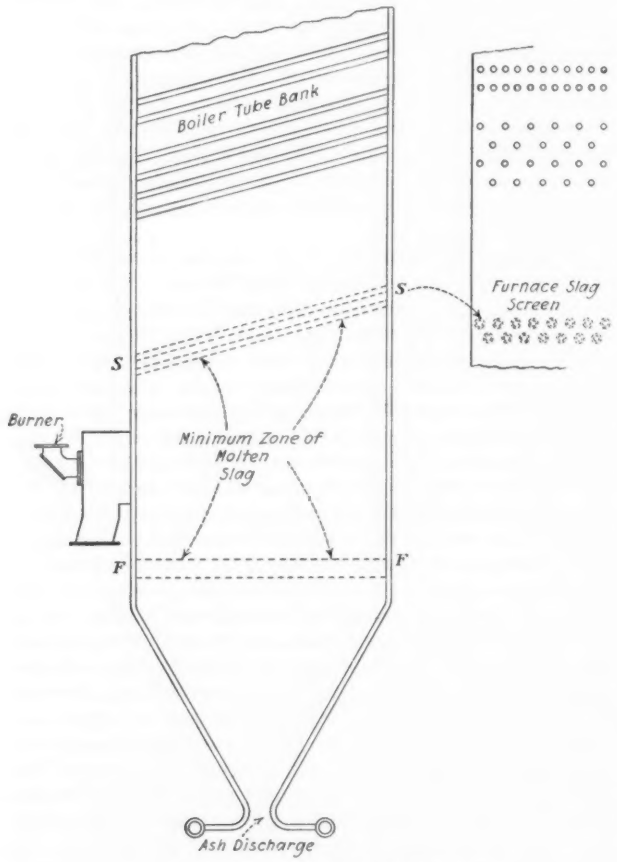


FIG. 1 DIAGRAMMATIC FURNACE

the covered area will also spread downward and upward, necessitating some cleaning to keep the coated area within bounds and also cleaning of the tube bank and ash discharge openings.

Another approach to the problem is to make the furnace of Fig. 1 with a floor at *FF*, which will accumulate molten slag, and to tap it out continuously or intermittently. This raises the furnace temperature in this zone so that the ash on the walls melts and thins down to an equilibrium thickness, with the net result that a higher rate of heat absorption per square foot of the lessened area takes place than existed in this zone before the floor was installed. The temperature in the slag zone is high enough to cause extending of the slag coating upward toward the boiler tube bank. This may call for cleaning of the walls in the upper part of the furnace, as the tube bank would otherwise have to be at an unreasonable height to prevent a gradual extension of slag, which would further insulate the wall and increase the gas temperature as the slag coating extended upward into the tube bank.

As radiant heat from the hot burner zone reaching the upper part of the furnace and boiler tube bank contributes to keeping the slag particles hotter than they would be if shielded, it is obvious that the placing of a screen *SS*, shown as consisting of water tubes coated with a slag-resisting refractory held on to the tubes by a suitable means, across the upper part of the furnace, making a two-stage furnace, serves a useful purpose. (1) The arch effect further increases the temperature of the combustion zone in the primary furnace, which increases the rate of heat absorption per square foot of remaining wall area and makes tapping of slag possible for higher-fusing-temperature ash, and for lower rates of combustion with a given ash. (2) The temperatures of ash and slag above the screen are reduced. (3) Any cleaning of the walls or boiler tube bank in the upper or secondary furnace is more effectively done in this cooler zone, and there is a more definite zone within which cleaning is possible. The screen cleans itself by the melting of the ash, which drips back into the primary furnace. (4) A larger percentage of the ash of the coal is collected than is recovered in a furnace for dry ash removal.

In actually building such furnaces the general scheme of Fig. 1 was carried out on the first installation at State Line Generating Co.[2] in 1929, except that the burners were changed to vertical firing, in order to utilize the furnace volume to better advantage and to collect a larger percentage of ash, especially the coarser particles.

There is full realization of the great importance of going into this problem thoroughly and obtaining sufficient data to make possible the most economical designs of boiler furnaces; designs that permit the operators to cope with the conditions of combustion and the removal of ash in the most satisfactory manner. Some of the results already obtained from these studies are so interesting and instructive that it seems best to present them now in a brief preliminary form.

TEST RESULTS

Table 1 gives, for comparison, certain data pertinent to the operating conditions and results from three different designs of boiler furnaces as shown in Figs. 2, 5, and 8, and plotted in Figs. 3, 6, 7, 9, and 10. As this work was undertaken primarily for gas-temperature studies and the comparison of different temperature-measuring devices rather than to check performance results and efficiency, complete test data were not taken. The heat input has been determined from the steam output, and the heat losses include heat returned in preheated air for combustion.

The nature and behavior of the ash and slag have also been observed during operation, samples have been taken of ash from the coal being burned, and samples of slag and the different kinds of ash from various parts of the furnace, slag screens, and tube banks. The fusing-temperature ranges of these samples have been determined with a view to learning whether or not the ash and slag samples, and their behavior at known temperatures in the Laboratory would add to the knowledge of actual temperatures in different zones of a furnace.

The gas temperatures were determined by a high-velocity thermocouple described by Dr. Mullikin,[3] and these readings were used to calculate the rate of heat absorption in the primary furnace, secondary furnace, and the boiler generating tubes. In the primary furnace the heat absorbed is the difference in sensible heat in the gases between the adiabatic temperature,

[2] Prime Movers Committee Report on Combustion, 1936, Publication D-7, Edison Electric Institute.

[3] "The Accurate Measurement of High Gas Temperatures," by H. F. Mullikin, *Power*, vol. 78, 1934, p. 565. This high-velocity thermocouple is inserted into the gas stream at any desired point in the furnace, and by an aspirator, gas is drawn across a shielded thermocouple at a sufficiently high velocity to bring the couple close to the true gas temperature, free from surrounding radiating influences.

TABLE 1. DATA RELATING TO GAS-ASH TEMPERATURE GRAPHS

	DATA RELATING TO:	FIG 2 AND 3.		FIG 5 AND 6.		FIG 5 AND 7.			FIG 5 AND 9			FIG 5 AND 10.			
1	MAXIMUM CONTINUOUS STEAM OUTPUT, LB. PER HR.	600,000		450,000		315,000			300,000			300,000			
2	STEAM OUTPUT, LB. PER HR.	600,000		450,000		330,000			225,000			348,000			
3	STEAM PRESSURE, LB. PER SQ. IN.	480		470		1,240			1,260			1,225			
4	STEAM TEMPERATURE, DEG. F.	850		935		900			923			910			
5	FEEDWATER TEMPERATURE, DEG. F.	315		330		330			330			350			
6	PROXIMATE COAL ANALYSIS AS FIRED — MOISTURE, PER CENT.	3.3		2.3		2.2			3.0			2.8			
7	VOLATILE, PER CENT.	30.0		33.9		36.5			35.5			36.0			
8	FIXED CARBON, PER CENT.	60.0		52.0		55.5			53.2			54.0			
9	ASH, PER CENT	6.7		11.8		7.8			8.0			7.2			
10	SULPHUR, PER CENT.	1.69		1.2		2.0			0.5			0.7			
11	HEATING VALUE AS FIRED, BTU. PER LB.	14,180		13,080		13,500			13,300			13,470			
12	ASH FROM SAMPLE OF COAL; INITIAL DEFORMATION, DEG. F.	2,200		2,335		2,030			2,180			2,350			
13	SOFTENING TEMPERATURE, DEG. F.	2,350		2,460		2,110			2,245			2,515			
14	FLUID TEMPERATURE, DEG. F.	2,810		2,740		2,410			2,660			2,680			
15	TOTAL COMBUSTION AIR AT BOILER OUTLET, PER CENT.	117		125		114 [4]			120 [5]			120 [5]			
16	TOTAL WEIGHT OF GASES, LB. PER HR.	770,000		590,000		407,000 [4]			282,500 [5]			436,000 [5]			
17	TOTAL HEAT INPUT ABOVE 80°F, MILLION BTU PER HR. [1]	839		602		453			292			464			
18	HEAT LIBERATION, PRIMARY FURNACE, BTU PER CU. FT. PER HR.	74,000		90,500		95,300			61,600			98,000			
19	TOTAL FURNACE, BTU PER CU. FT. PER HR.	31,800		38,800		40,250			32,600			52,100			
20	TOTAL HEAT INPUT DIVIDED BY TOTAL PROTECTED SURFACE OF ALL FURNACE WALLS PLUS BOILER TUBE SURFACE BEFORE SUPERHEATER; BTU/SQ. FT./HR.	65,400		72,500		70,400			56,100			69,500			
		PRIMARY FURNACE	SECONDARY FURNACE	GENERATING TUBES	PRIMARY FURNACE	SECONDARY FURNACE	GENERATING TUBES	PRIMARY FURNACE	SECONDARY FURNACE	GENERATING TUBES	PRIMARY FURNACE	FIRST OPEN PASS	SECOND OPEN PASS	CAVITY BELOW SUPERHEATER	PRIMARY FURNACE	FIRST OPEN PASS	SECOND OPEN PASS	CAVITY BELOW SUPERHEATER
21	TEMPERATURE, GAS ENTERING; DEG. F.	3740 [3]	2,960	2,550	3530 [3]	2,900	2,600	3,690 [3]	2,840	2,500	3,560 [3]	2,920	2,540	2,070	3,650 [3]	2,950	2,670	2,410
22	TEMPERATURE, GAS LEAVING; DEG. F.	3100	2,550	2,120	3,010	2,600	2,300	2,940	2,500	2,120	2,920	2510	2,070	1,695	2,950	2,670	2,410	2,250
23	HEAT ABSORBED, MILLION BTU. PER HR.	185	117	102	114	68.5	55.5	120	51.9	45.7	645	34.8	37.6	14.7	103.5	40.0	36.4	21.5
24	PORTION OF TOTAL LIBERATION, PER CENT	22.2	14.0	12.2	15.9	11.4	9.2	26.7	11.5	10.1	22.1	11.9	12.9	5.0	22.2	8.6	7.8	4.8
25	HEAT ABSORBED, BTU PER SQ. FT. PER HR.	78,000	30,500	15,600	65,700	28,200	16,000	83,600	21,000	18,160	35,500	30,900	29,000	15,350	51,000	35,600	28,200	28,400
26	EQUIVALENT HEAT RECEIVING SURFACE TEMP.; DEG. F.	2,820	2,612	2,225	2,750	2,645	2,350	2,975	2,570	2,175	2,775	2,530	2,060	1,800	2,715	2,645	2,360	2,170

[1] Includes heat in preheated air, less heat required to evaporate moisture and less heat in unburned fuel.

[2] The equivalent temperature of heat-receiving surfaces within a zone is calculated according to the Stefan-Boltzmann law, using the mean of observed gas temperatures and the rate of heat absorption within the zone, assuming black-body radiation. This may not represent the true average temperature of the tube with its coating of mill scale, dust film, sponge ash, or slag, due to undetermined values of such factors as emissivity and the like, but it is suggested as a method of comparing results which undoubtedly will lead to further knowledge regarding heat absorption in boiler furnaces.

[3] Adiabatic.

[4] Secondary-furnace outlet.

[5] Bottom second open pass.

as given in Table 1, and the observed temperature of gases leaving the primary furnace. The latter temperature was taken as the average of gas temperatures just before and after passing through the slag screen between the primary and secondary furnaces. A Leeds & Northrup optical pyrometer was also used in the primary furnace only, as it checks the high-velocity thermocouple (HVT) closely near 3000 F. The rate of heat absorption in a zone divided by the total absorption surface of the zone gives the heat absorbed, Btu per sq ft per hr, for that zone. See Table 1, item 25.

Item 26, found in Table 1, is the "equivalent heat-receiving-surface temperature" and is shown on the gas-ash temperature graphs as a straight line for each furnace zone. It is calculated according to the Stefan-Boltzmann law, using the mean of observed gas temperatures and the rate of heat absorption within a zone, assuming black-body radiation with both the radiating and receiving emissivities as 1.0 and the angle factor as 1.0. For the heat absorbed per sq ft per hr in each zone, determined from gas temperatures in and out, there is a black-body equivalent temperature of heat-receiving surfaces, given in Fig. 4.

With the mean gas temperature in the cavity as the radiating body, and the rate of heat absorbed per sq ft per hr by the projected surface of the water-cooled walls and the tube banks, determined by the HVT temperature readings entering and leaving the cavity, it is simple to calculate the equivalent black-body-surface temperature of the receiving-wall boundary areas of the cavity. It is, of course, known that the emissivity is not constant and may be much less than 1.0. This temperature, which may not be a real temperature of any physical body, is then compared with the determined fusing-temperature range of the slag, and some useful conclusions can be drawn about ash deposits on walls and tubes, as shown later. The latter temperatures are determined from actual samples of slag taken from the walls, tap spout, and tube banks throughout the unit and tested in the laboratory according to the A.S.T.M. method.

It is noted in Table 1, items 21, 22, 25, and 26, that the primary-furnace adiabatic temperature in Fig. 3 is 3740 F, the gas temperature leaving is 3100 F, the rate of heat absorption is 78,000 Btu per sq ft per hr, and the equivalent heat-receiving-surface temperature is 2820 F. The slag on the walls, and that being tapped from the primary furnace is about 2600 F and observation showed the water-cooled walls to be covered with a relatively thin film of molten ash that flowed down to the floor continuously and was readily tapped intermittently. The ash-softening and initial-deformation temperatures have little significance in the primary furnace.

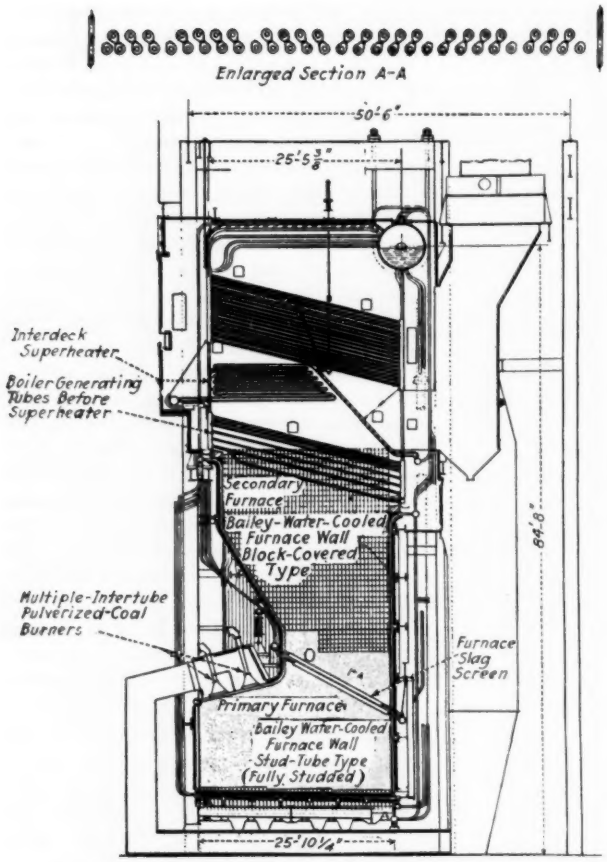


FIG. 2 BOILER UNIT WITH TWO-STAGE FURNACE
(See Fig. 3.)

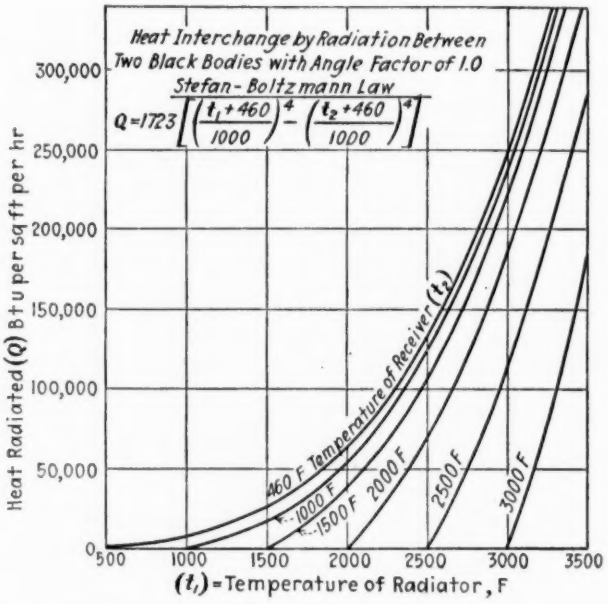
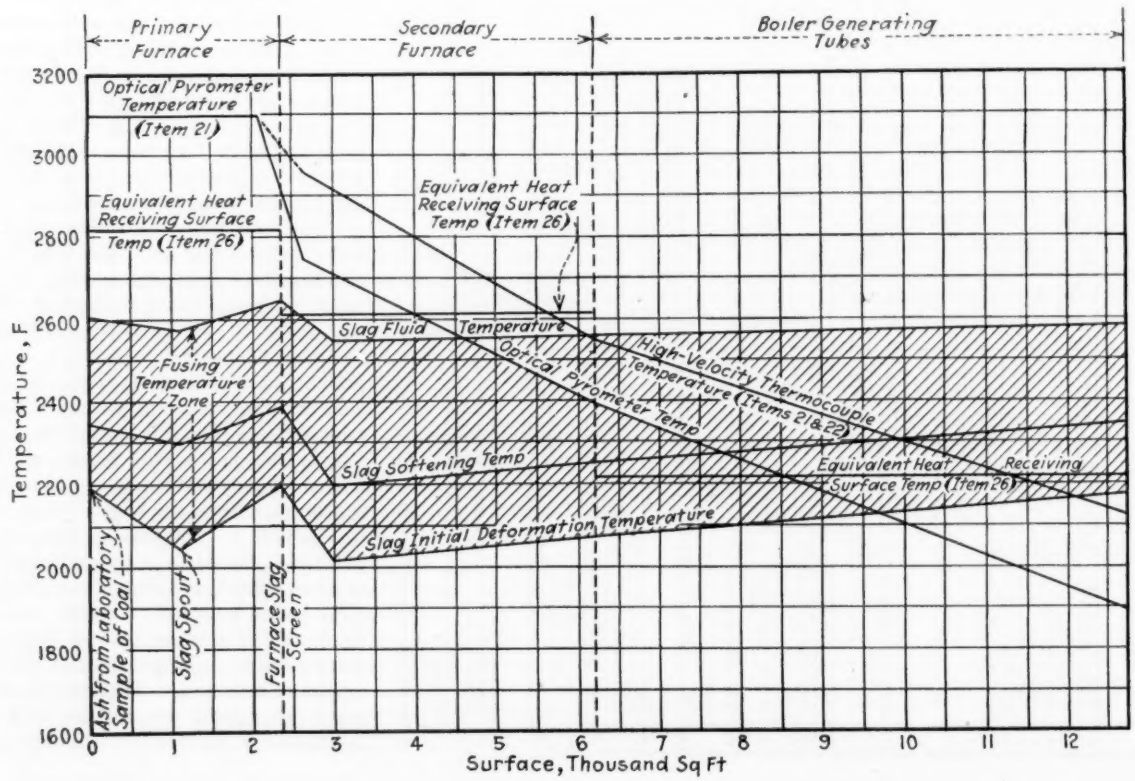


FIG. 4 HEAT INTERCHANGE BY RADIATION BETWEEN TWO BLACK BODIES WITH ANGLE FACTOR OF 1.0

FIG. 3 (BELOW) GAS-ASH TEMPERATURE GRAPH FOR UNIT SHOWN IN FIG. 2



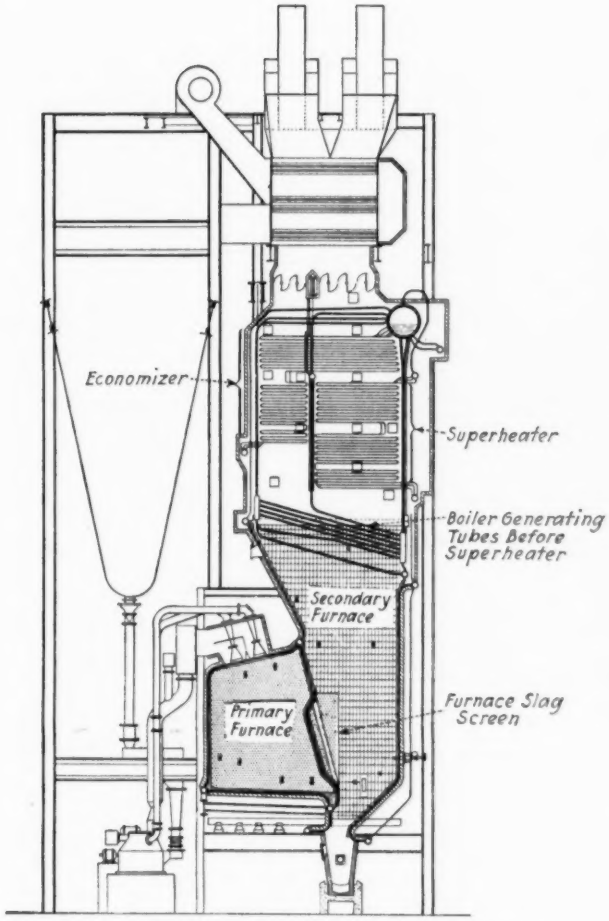


FIG. 5 BOILER UNIT WITH TWO-STAGE FURNACE
(See Fig. 6.)

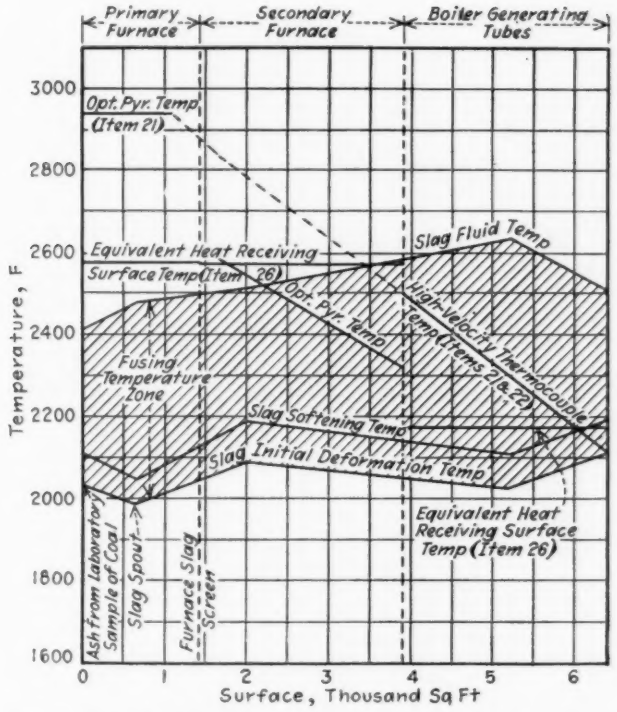
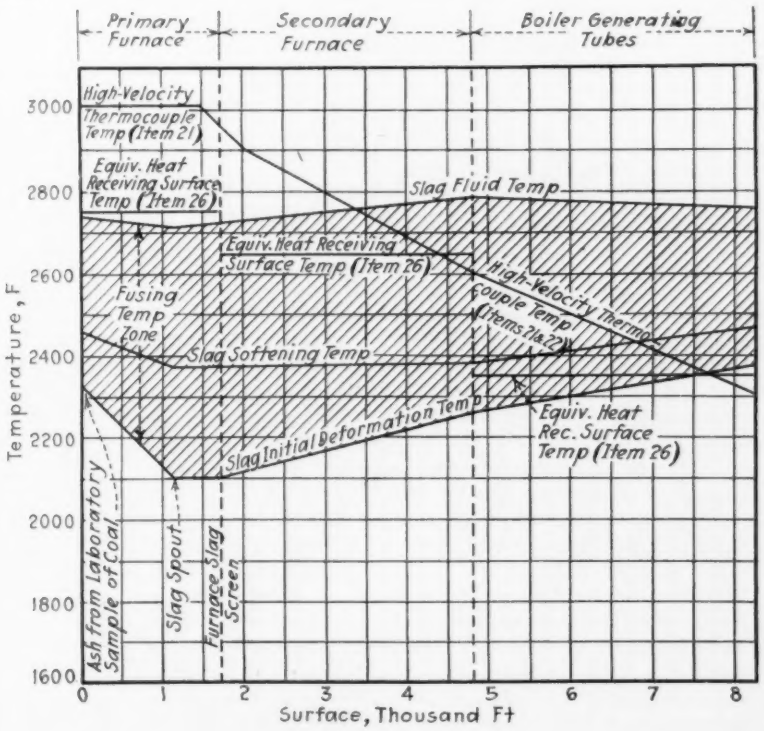


FIG. 7 GAS-ASH TEMPERATURE GRAPH FOR UNIT SHOWN IN FIG. 5

FIG. 6 (RIGHT) GAS-ASH TEMPERATURE GRAPH FOR UNIT SIMILAR TO FIG. 5



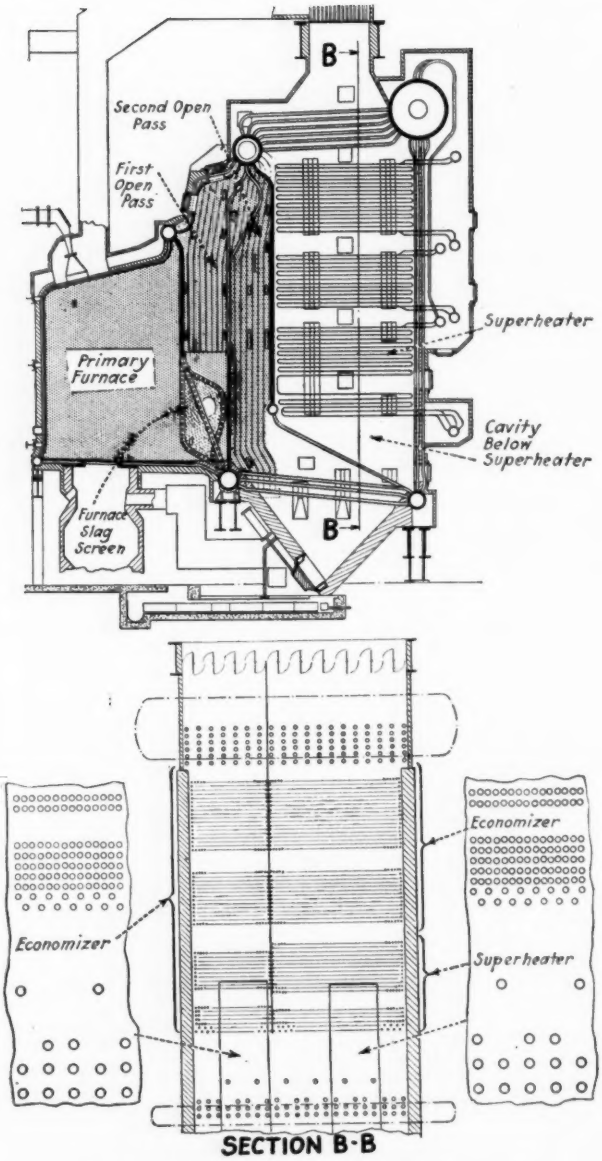


FIG. 8 BOILER UNIT WITH OPEN-PASS FURNACE
(See Figs. 9 and 10.)

In the secondary furnace similar calculations show an equivalent heat-receiving-surface temperature, item 26, of 2612 F and a slag fluid temperature of 2550 F. As a check in the actual furnace, the slag forms a sheet covering over the water-cooled wall and a slow flowing process takes place, maintaining an equilibrium condition without cleaning. The slag falls off in sheets from 1/2 in. to 1 in. thick when the boiler is removed from the line or sometimes when the output is reduced. The gas enters the boiler generating-tube bank at 2550 F, slightly below the fluid temperature, and leaves at 2120 F, slightly below the initial deformation temperature of 2170 F for the slag collected from that point.

In other words, the temperatures throughout the boiler tube bank are within the sticky range of the ash which collects there, and mechanical cleaning by means of telescopic deslaggers and retractable jets using steam-water mix are needed and are used regularly to keep this bank clear for gas flow and to keep it absorbing enough heat to pass gases of the correct temperature to

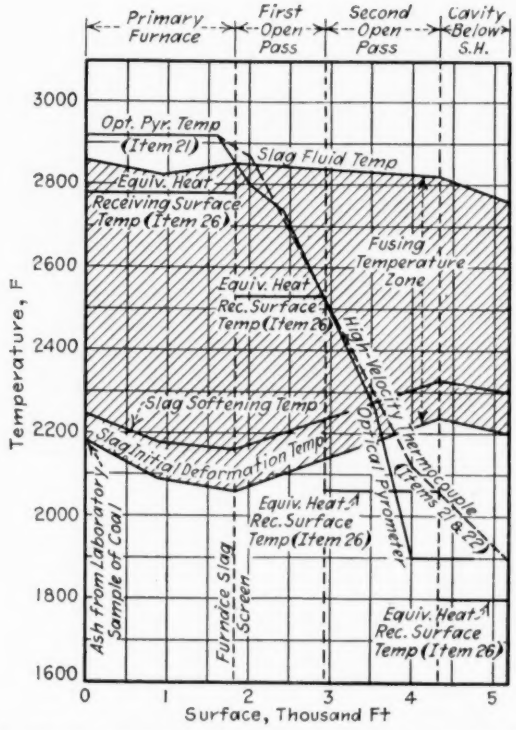


FIG. 9 GAS-ASH TEMPERATURE GRAPH FOR UNIT SHOWN IN FIG. 8, AT MEDIUM RATING

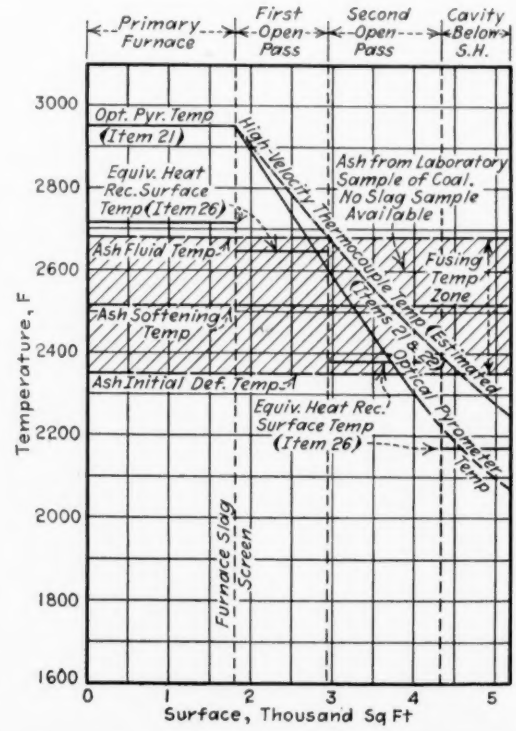


FIG. 10 GAS-ASH TEMPERATURE GRAPH FOR UNIT SHOWN IN FIG. 8, AT HIGH RATING

the superheater. There is no slag problem in the upper tubes of the boiler bank nor in the superheater, which checks with the temperatures and the slag characteristics.

This method of study is somewhat analogous to using Seger cones in ceramic work as a measure of furnace temperatures as related to the results desired in burning ware. The ash and slag supply a basis of comparison of effective results and enable performance results to be studied with greater assurance as to their accuracy and applicability to other designs.

The equivalent heat-receiving-surface temperature of the boiler generating tubes of 2225 F looks high, but it is to be remembered that it is a calculated temperature. However, these tubes are never thoroughly cleaned, for even the best of cleaning leaves a residual coating of ash which may be several hundred degrees above the tube temperature. An increase in ash or slag coating may raise the new surface temperature surprisingly high. While the equivalent black-body-surface temperature is not real and an emissivity of less than unity will lower it, the actual surface temperatures cannot be much lower and be in line with the behavior of the ash as observed at the various locations.

These equivalent black-body-surface temperatures will of course not be uniform throughout the secondary furnace and tube bank, but it is thought best for the time being to calculate them on the simple unity basis and plot them as horizontal lines until further studies can be completed. For the present it is thought this equivalent surface temperature will be useful in comparing results between different conditions such as methods of cleaning, effect of different kinds of coal, methods of firing, adjustment of excess air, and so on.

It should be noted that these data in Fig. 3 from the furnace of Fig. 2 are obtained when operating steadily at 600,000 lb of steam per hr and, for comparison with other graphs in this paper, the rate of liberation is 74,000 Btu per cu ft per hr in the primary furnace alone and 31,800 in the primary and secondary furnace combined.

The total input to the furnace is 833,000,000 Btu per hr, or 65,400 Btu per hr available per sq ft of total heat-receiving surface in the entire furnace and boiler generating-tube bank. The actual absorption in the primary furnace is 78,000 Btu per sq ft of wall surface per hr, while in the secondary furnace it is 30,500, and in the tube bank 15,600 Btu per sq ft of tube surface per hr.

The gas-ash temperature graph, Fig. 6, was obtained from a furnace, as shown in Fig. 5, operating at its full continuous-load condition. The ash is somewhat variable and the averages are shown in Fig. 6. The slag from this ash has a relatively high fluid temperature which, coupled with a tendency to form a sticky adhering formation at lower temperatures than indicated by its initial-deformation temperature, makes a troublesome cleaning and operating problem.

None of the rates of heat absorption in Figs. 5 and 6 are as high as the corresponding rates obtained from the furnace of Figs. 2 and 3, even though the rates of heat liberation are about 20 per cent higher and the heat input per square foot of total furnace-wall and boiler-tube surfaces (item 20) is about 10 per cent higher. This result is now believed to be largely due to the higher fluid temperature of the slag, which is about 2750 F as compared with 2550 F for Fig. 3. The nature of the slag structure, especially on the boiler tube banks, is also significant, as the same means for deslagging are much less effective in Figs. 5 and 6.

Another more direct comparison is between Figs. 6 and 7, where both have the identical cross section of furnace, but the latter is burning a different coal with lower-fusing ash than Fig. 6. The furnace of Fig. 7 is narrower in almost direct proportion to its output at full load, at which rate the data were obtained.

The observed behavior of the slag throughout the furnace and boiler of Fig. 5 with the different coals of Figs. 6 and 7 is consistent with the equivalent heat-receiving-surface temperatures.

The boiler of Fig. 8 has a rated continuous output of 300,000 lb of steam per hr. The test of Fig. 9 was run at 225,000 lb of steam per hr, or 75 per cent of its full-load conditions, with one kind of coal, and the test of Fig. 10 at 342,000 lb of steam per hr, or 14 per cent above full load with a different coal. Unfortunately, slag samples were not obtained from the furnace during the latter run, but the nature of the ash with low iron content would presumably produce a slag with little change in its fusing range from that of the laboratory determination from the coal ash, which is plotted.

The operation of this type of furnace, with two open passes and a small cavity between the last pass and the superheater, with no boiler tube bank whatever, has been very satisfactory.

The slag is easily tapped continuously from the primary furnace with ash of 2800 F fluid temperature and at ratings below 70 per cent of normal. The slag in the first open pass flows slowly back to the primary furnace, and this pass needs no cleaning or attention, which is also true of the primary furnace.

The second open pass accumulates some slag, but is more or less self-cleaning due to the down flow of gases and the gravity action on the accumulation which, with the coal being used, is near its initial-deformation temperature.

Some cleaning of slag from the screen tubes at the top of the first open pass is required. This is being done with a retractable-nozzle deslagger.

Single-jet retractable blowers are also used occasionally at the entrance to the superheater and economizer by-pass, to clear the slight amount of sponge ash which accumulates there.

The behavior of ash within a furnace is of such importance that a few comments regarding it should be kept in mind while discussing the subjects of temperature and rates of heat absorption.

In burning pulverized coal from modern direct-fired systems with turbulent burners in hot furnaces and the proper excess air, all but a small residue of the coarser coal is burned completely in the furnace and only residual ash particles remain. A microscopic examination of such fly ash discloses tiny fused black, brown, gray, white, and transparent spheres, together with crystals of unfused ash.

There are also some carbon or coke particles from the small residue of coarser coal present in the fly ash. A microscopic study of these is even more interesting and significant in connection with slag formation. Such a piece of coke has on its surface a number of beads of ash, many of which are fused into multicolored spheres and are the residue of ash in the coke which has already been burned from the surface of the particle. They are held to the coke by the adherence to small particles of ash which are only partly exposed at the surface of the coke. Many of these spheres and particles of carbon-free ash are continually falling free from the coke, hence the explanation for fly ash being so much smaller in size than if each particle of coal retained all of its ash as a single particle. The most significant result of this particle of coke with its nodules of ash is that its continued burning keeps its temperature above that of the carrying gas and the ash, being kept hotter, is more likely to stick to wall or tubes, holding the carbon where it burns still more rapidly due to the scrubbing action of the gases. As the combustion is completed, the residual ash is largely retained at a relatively higher temperature and is added to the accumulation of slag.

This is one explanation of the generally accepted fact that coarse coal and improper combustion are conducive to slag accumulations. The slag in boiler tube banks from fuel beds is largely carried up in the form of burning particles of coke rather than of free ash itself.

While it is well recognized that ash in coal is a complex sub-

stance and individual constituents of it vary in fusing temperature from as low as 1800 F to as high as 4000 F, it is treated as a mixture; in the laboratory, ash residue from a slowly burned out coal sample is formed into a cone-shaped briquet and its fusing-temperature range is determined in a muffle furnace and reported as (1) initial-deformation temperature, (2) softening temperature, and (3) fluid temperature. The initial deformation is that at which the cone starts to distort and is really indicative of when the lowest-fusing-temperature particles begin to soften and cement the higher-fusing particles. The softening temperature is that at which the cone has softened enough to turn the apex downward, or to bend the axis about 90 deg from its initial vertical position. This particular temperature has been referred to in the past as the fusing temperature and has been generally used as the important temperature in measuring the behavior of the ash and comparing the relative values of coal. When coal is to be burned in fuel beds this temperature is of value for comparisons, but it is now believed to be of little significance in large pulverized-coal-fired furnaces. The fluid or liquid temperature is important in connection with slag-tap furnaces and for a knowledge of the behavior of ash in water-cooled furnaces operating at temperatures above the fluid or liquid temperature of the ash, as most of them do at their normal and maximum ratings. In this paper "fusing temperature" is used as an inclusive generic term covering all three of the specific temperatures within the band just described.

Within a furnace or boiler unit, the ash collecting at any point may not be a true average of all that which is in the coal. Dense, hard-grinding particles, such as sulphide from pyrites, are more likely to be retained on the floor of a furnace. High-fusing-temperature particles are more likely to pass on through the entire furnace and boiler unit as fly ash, as they themselves are not sticky but they may be retained to some degree by adhering to sticky ash already retained on the surfaces. The particles which are molten have a double chance of being retained within the furnace, due to their sticking to cold surfaces, and surely to hot sticky surfaces.

Comparison of Methods for Measuring Furnace Temperatures

(*a*) The high-velocity thermocouple previously mentioned is the most accurate practical method now known for measuring gas temperatures.

(*b*) Certain forms of optical pyrometers (disappearing-filament type) are reasonably accurate for the highest-temperature zones of furnaces, where uniform temperatures prevail.

(*c*) Bare thermocouples have not been found particularly useful for gas-temperature measurement.

(*d*) Radiation pyrometers and other forms of elements sensitive to heat and light are being carefully studied, as it is desirable to know more than merely gas temperatures.

(*e*) It is desirable to know the temperature of slag particles floating in the gas stream, especially as they approach tube banks. Such slag particles are undoubtedly hotter than the carrying gases leaving a burner, if still within view of the hotter zones of the furnace; and cooler than the carrying gases when approaching cooler surfaces, especially when shielded from the radiant heat of the hotter zones. Perhaps a bare thermocouple of the proper size more nearly approaches the true temperature of the slag particles until some better method can be devised.

(*f*) It is desirable to know the temperature of floating particles of coke, especially just as they strike an arresting surface, also as they continue to burn when so arrested. No work has yet been done in this direction.

(*g*) A most important factor that should be known is the heat reflected and radiated from the wall surface. If this can be determined, it may give a more direct method than we now have of measuring the rate of heat absorption at the wall.

Summary

1 The fusing-temperature determinations on ash from a sample of coal, as now burned out in the laboratory according to A.S.T.M. standard methods, are not accurately indicative of the corresponding temperatures of the slag as formed in furnaces from the same coal. Many other data beside those shown here indicate much greater deviations. Usually the slag accumulated in the hotter zones near the burners and on the floor, or in the slag-tap section, is lower in all points than that determined from the coal sample by the standard method. There is a general tendency for the slag and ash collected beyond the hot zone to be higher in all significant temperatures.

2 Some ash or slag adherence takes place on all furnace walls, boiler, superheater, and other surfaces. So long as it adheres at all it tends to become cumulative. Dry nonsticky ash may maintain a close equilibrium by falling off due to gravity or being carried on by the normal gas velocity.

Sponge ash or even vitrified slag may fall off in pieces intermittently by its own weight, especially if the rate of combustion is changed from time to time, and the surface to which it adheres is not holding it too tenaciously. Continuous high-rating operation is usually conducive to continued accumulation of sponge ash or plastic ash in the zones where the ash is within the sticky-temperature range. This zone in the furnace should be as small as possible. It may be desirable to clean it by hand lancing or some deslagging means if the need for uniform conditions warrants.

The parts of the furnace where the temperature is maintained above the fluid temperature of the ash which deposits there, is kept at a more uniform condition than any other part because of the continued melting and running away, leaving a base of plastic slag. The lower the fluid temperature of the slag the greater the heat absorption in a given furnace with a given rate of heat input, and also the lower the gas temperature leaving such zone.

3 Ash and slag having a wide spread between the initial-deformation point and the fluid temperature are more likely to be troublesome than those with a smaller temperature range and a sharper melting point. With a wider spread like that in Fig. 6 from 2100 to 2730 F the fly ash must pass through a wide temperature range, and therefore a large surface exposure, before it is cooled from the liquid phase where it is easily handled, to the dry nonsticky stage. The coal of Fig. 9 covers even a wider spread from 2060 to 2850 F. In some furnace arrangements this might be extremely difficult to handle, although in the furnace of Fig. 8 no difficulty has been encountered.

4 A valuable method of comparing furnace-operating results has been discovered consisting of (1) taking accurate gas temperatures throughout a furnace, (2) calculating from them the rates of heat absorption in different zones, (3) calculating an equivalent heat-receiving-surface temperature for each zone, (4) taking samples of ash and slag from different portions of the furnace and determining the fusing-temperature range from such samples, (5) plotting these data as a gas-ash temperature graph against heat-absorbing surface, (6) comparing the data as shown on such graph with observed behavior of ash and slag in the furnace, and (7) comparing different furnaces, different coals, and methods of operation through the use of such graphs.

5 The equilibrium conditions with molten-slag-covered surfaces is a subject deserving further study. The data presented here were obtained under what were thought to be equilibrium conditions, brought about by long periods of operation at uniform load. However, only further continued observations will determine when equilibrium has actually been obtained.

Others who may follow this line of study should keep this in mind. For instance, if a unit that has been operating a long time at high rating is suddenly reduced to low rating, the reduced furnace temperature may cause a complete stoppage of flowing slag over the furnace walls and floor. Continued accumulation of ash will increase the thickness of slag until the furnace temperature is increased by the insulating effect, and fluid conditions are again restored. The ash accumulates more slowly at the lower rates of combustion. When the rate of heat input is again increased there will be a period of higher temperature until the surplus slag film is melted away.

6 The studies presented herein, indicate that the nature of ash in the coal being burned, has such an important bearing upon the operating results from furnaces, that proper selection of coal must be made for many, if not all, furnaces. It is believed that further studies along the lines herein set forth will enable furnaces to be designed which will permit wider latitude in the selection of fuel.

7 The superheater bank, usually consisting of relatively closely spaced small tubes, must be kept clean. This means that the fly ash should be below its sticky temperature, yet the gases should be as hot as possible for high superheat. The velocity and temperature of gases should be as uniform as possible to all parts of the entering area of the superheater for the protection of the tubes. Some cleaning device is needed, as a gradual accumulation of sponge ash or dust is inevitable. It should be kept uniformly clean, because if certain areas accumulate sponge ash and direct the flow of gases to limited portions of the superheater the higher gas-mass flow may quickly overheat such portions of the superheater tubes.

8 Colored moving pictures of furnace interiors have been helpful in studying ash and slag formation on heat-absorbing surfaces, with a view to obtaining better methods of removal and preventing the formation. By taking such pictures at high speed and from points close to the furnace doors, many phenomena can be seen in much more detail than from personal observation. Time and speed measurements can also be taken.

9 In conclusion, it should be remembered that this paper includes data from only four installations. The results were so illuminating that it was deemed to be to the benefit of professional progress to publish the data and method of comparison now, so that others active in this work can utilize what has been reported. Criticism and suggestions will be appreciated as an aid in developing and applying this method in studies which will surely be pursued further in the future.

Acknowledgment

Acknowledgment and appreciation are given to R. Shellenberger, Dr. H. F. Mullikin, and E. D. Grimison, associates of the author, for their diligent and constructive assistance in this work; without their help and cooperation the data here presented would not have been available.

Discussion

J. M. Drabelle.[4] This paper is particularly valuable to operating men who have in their charge large boilers and furnaces operating at high ratings.

For the last year the Iowa Electric Light and Power Company has been experimenting with the motion-picture camera and Kodachrome film for obtaining photographic records for use in studying stoker and furnace operation. Experience seems to indicate that colored motion-picture film is a valuable tool for obtaining a clear, concise, and readily examined picture of furnace operation. It should be borne in mind that, being in color, relative color values are not destroyed as they are when a furnace is observed through conventional blue-glass screens.

The equipment which we have been using is an Eastman Ciné Kodak equipped with an F-1.9 lens. To protect the camera from the intense heat of the furnace Aklo glass made by the Corning Glass Works, Corning, N. Y., has formed the shield through which the photographs have been taken. This glass has the peculiar property of transmitting light but not transmitting heat waves. Two screens of Aklo glass are used in series held loosely in a suitable frame. The lens of the camera is placed immediately back of the second sheet of glass. Up to the present time, no attempt has been made to use the conventional exposure meter. The diaphragm of the camera is stopped wide open. The resulting photographs have shown slag phenomena and burning phenomena very clearly, particularly on underfeed and chain-grate stokers. Formation of clinkers as well as the stratification in burning gas streams from the fuel bed are also depicted. To those having furnace trouble, this is a possible means of studying conditions within the furnace. Once the trouble is determined, a suitable remedy may be applied.

H. D. Harkins[5] and A. C. Flinn.[6] One of the most interesting points brought out in the paper is the conclusion that, for the same heat release, the transfer coefficient would be higher with coals having low-fusion-temperature ash due to the fact that the low-fusion ash forms a thinner, more fluid slag which offers less resistance to the flow of heat to the tubes. From this it would follow that low-fusion-ash coals would require a smaller amount of surface in the primary furnace or that, given similar conditions, a higher transfer rate would obtain in a slag-tap furnace, where the temperature must be such as to keep the ash fluid, than in a dry-bottom furnace. This indicates the possibility of building smaller and, consequently, cheaper furnaces for what have been considered the more difficult (low-fusion-ash) coals. Heretofore larger and more costly furnaces have been built for them.

The comparison between the furnaces, Fig. 2 and Fig. 5, shows about 19 per cent greater transfer rate in the primary furnace with the coal of lower ash-fusion temperature. However, it does not appear to have been proved conclusively that ash fusion is the direct cause or that other effects may not have contributed to the difference in heating rates.

For instance, Sherman[7] at Battelle Memorial Institute, in working on pulverized-coal flames from four types of bituminous coal, found variations in the emissivity coefficients and also in the initial flame temperatures. The emissivity coefficients varied as much as 30 per cent. Due to the fact that the coals having the highest emissivity also had the highest initial temperature, the heat radiated varied even more than this amount.

Since these data on radiation from the flames of several coals indicated a difference greater than that ascribed here to the difference in slag fluidity, the question arises as to whether emissivity or other similar effects may not have accounted for some of the difference in transfer rates?

E. A. Hitchcock.[8] It seems only a short time ago—about

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[5] Industrial Engineer, E. I. du Pont de Nemours & Co., Wilmington, Del. Mem. A.S.M.E.

[6] Fuel Engineer, E. I. du Pont de Nemours & Co., Wilmington, Del.

[7] "Burning Characteristics of Pulverized Coals and the Radiation From Their Flames," by R. A. Sherman, Trans. A.S.M.E., vol. 56, 1934, paper FSP-56-6, pp. 401–410.

[8] Dean Emeritus, College of Engineering, The Ohio State University, Columbus, O. Mem. A.S.M.E.

thirty years—that I was with a group of engineering students in a new pumping station of the city of Cleveland. How we admired that pumping equipment, which was the "last word." And the boiler-room equipment! I can still see the highly polished brass feed lines and those shining gages. In one respect we could hardly believe our eyes for those gages stood at the unbelievable pressure of 180 lb per sq in. My conservatism relative to future developments led me to exclaim: "That is the limit!" What an unwise statement! It certainly cured me of making prophecies applying to the fields of engineering and science.

Nearly twenty years ago I was dining with Benjamin G. Lamme, who was for many years chief engineer of the Westinghouse Electric & Manufacturing Company. At that time the largest turbogenerators built were about 50,000 hp. I felt the limit had been reached but ventured to ask Mr. Lamme what in his opinion would be the limit in size of such a machine. His immediate reply was: "The limit is the customer's pocketbook." I little thought at that time I would see a verification of Lamme's statement, but I did—a 225,000-hp machine going through the Westinghouse shops and later in operation in the Richmond Street Station of the Philadelphia Company. Now I assume, the end is not yet.

The most startling advancement, as I see it, has been on the steam-generating side. This progress has not been easy sailing by any means. One very important phase is now being presented by the author. When he became acquainted in a practical way with the burning of bituminous coals, boilers having steam capacities of 7500 lb per hr at 150 lb per sq in. pressure were about the upper limit. Where are we now? One million pounds of steam per hour at 1500 lb per sq in. pressure or more and 330 F superheat. An increase in capacity of over 130 times the equipment of thirty-five or forty years ago, to say nothing of the increases in pressure and temperature.

It was my privilege, about thirty-eight years ago, to be associated with the author on various kinds of fuel and combustion tests. For about four years we "bled and died" together, he occupying the important position of "go-between" from coal pile to boiler furnace, usually of the Dutch-oven type, while I kept the records and fulfilled sundry other little jobs which required very little brawn and, I guess, a small amount of mentality.

To generate the steam by the shovel method was no bed of roses. It was no punching of buttons and watching of recording and indicating gages. The slicing and cleaning of fires, and the pulling of clinkers was a man-size job; at times an asbestos suit would have been acceptable. The progress that has been made in this particular direction is as great as from the labor of the galley slaves to the duties of the boiler-room crew of an oil-burning ocean liner.

It was, I am sure, this early experience with many kinds of coal, producing ash and slag of various characteristics, which convinced the author that here was one of the most difficult and trying problems in the use of fuel. Many were the times, after a few hours running, that the grate bars became so clogged with fused ash that a shutdown was necessary! The importance of considering this slagging factor in testing coals became so apparent that all clinker above a certain size was sorted from the total refuse and so recorded. My records show that this factor was first determined by us in April, 1899. This early experience was, I believe, the beginning of the author's interest in ash and slag, considered as among the most deterrent factors in the economical generation of steam. As a result, there soon followed determinations of the fusing temperature of ash. The importance of this factor is recognized today.

It was in November, 1911, that the author read a paper before the Ohio Society of Mechanical, Electrical, and Steam Engineers. His statements and conclusions in that paper[9] were based upon the results of twenty-eight complete heat-balance boiler trials with fuels, having ash-fusing temperatures running from 2210 to 2700 F, and also upon a valuable tabulation of thirty-two fusing temperatures of fuel ash, ranging from that of anthracite at 3150 F to bituminous at 2140 F. This tabulation included also the percentage of ash, and sulphur, and in several cases the iron oxide in the ash.

My purpose in calling attention to this early investigation work is to bring about a consciousness that slag was a recognized problem nearly forty years ago. It was controlled partly by brute force and awkwardness. It was the "imp" in the coal pile. Most of us have had experience enough to know that Mother Nature seldom surrenders her treasures without some tough work somewhere by someone. That coal imp is still with us, although our methods of handling him have changed greatly.

We are, I am sure, pleased that the author has come forward with suggested methods for comparing furnace operation. Support for his recommendations is the results obtained upon three types of furnace settings. In these investigations a most unusual and important feature was the application of colored moving pictures in studying furnace activity. They were obtained with considerable hazard to the observer. It is to be expected that, as a result of these investigations and those to follow, there will be advancements in furnace designs and reduced operation costs. However, I believe a marked increase in heat efficiency, even with the best practice of today is not to be expected, for it is not there to get.

Ten days ago I was at the Glenlyn plant of the Appalachian Power Company and there learned from M. P. Lawrence, superintendent, that their latest installation, a powdered-coal furnace with wet bottom, had a record of nine months of 24-hr continuous operation, with Pocahontas coal, generating approximately 350,000 lb of steam per hr and giving an efficiency of between 85 and 86 per cent. I understand a later installation at the company's Logan plant, having a capacity of 1,000,000 lb per hr at 1425 lb per sq in. pressure, dry bottom, is giving about the same efficiency with a bituminous coal of much higher ash content than the Pocahontas.

The experience of M. P. Lawrence with powdered coal at the Glenlyn plant has been so satisfactory that he agrees completely with those who believe the use of powdered fuel in large generating units is destined to become general. It is conceded, however, that at present the ash characteristics are important deciding factors.

For example in the Glenlyn plant is a chain-grate equipment 24 ft wide by 26 ft long, upon which bone coal with from 15 to 25 per cent ash works most satisfactorily. That coal condition would probably make combustion by the powdered-coal road difficult and impractical. As I see it, from my forty-five year point of view, the ideal combustion of coal is not reached yet; there are a few factors still to be licked. The author of this paper has handed to us, on asbestos gloves, some hot nuts to crack.

P. B. METCALF.[10] Too many statments have been made in the past by various people that pulverized-coal firing is a panacea for troubles encountered with any and all types of coal, in other methods of firing. It is gratifying to have the author's frank statement: "The studies presented herein, indicate that the

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[9] "Fusing Temperature of Coal Ash and Its Relation to Rate of Combustion," by E. G. Bailey, *Journal* of the Ohio Society of Mechanical, Electrical, and Steam Engineers, November, 1911, pp. 53–67.

[10] Steam Engineer, New England Power Service Company, Boston, Mass.

nature of ash in the coal being burned, has such an important bearing upon the operating results from furnaces, that proper selection of coal must be made for many, if not all, furnaces."

The statement is made early in the paper: "If mechanical cleaning is applied to this area, and it is kept reasonably clean, the furnace may be too cold for stable ignition and efficient combustion, especially at lower rates." This conclusion appears to be at variance with other authorities on the subject, notably with a statement made by Henry Kreisinger,[11] who states: "Beyond the temperature required for quick ignition of the fuel, nothing is gained by going to higher temperature. In fact, there are experimental indications that high temperatures retard the combustion."

In view of this seeming conflict of conclusions on the subject, it would be helpful if the author could give further information toward clarifying the issue.

The explanation of why coarse coal and improper combustion are conducive to slag accumulations is most interesting and further emphasizes the need for fine pulverization with an absolute minimum of coarse particles.

J. F. Muir.[12] This paper, illustrated by motion pictures has the distinction, within the writer's experience, of being the first presentation of its kind dealing with the effects and significance of ash transformations as they occur in the boiler furnace.

The following comments are made as a member of the A.S.M.E. and not as an employee of any company.

One of the many points of importance brought out by the author and one that deserves repetition and emphasis is the fact that, in the design of the high-capacity boiler, it is not sufficient, as has been done heretofore, to take a single temperature value for the change in ash from and to its solid state in the combustion process. It is pointed out that, in setting up design fundamentals, it is necessary to give consideration to and make allowances for first, the initial-deformation temperature; second, the softening or plastic temperature, which has been commonly referred to as the fusion temperature; third, the fluid temperature which is the one that must be maintained in the slag-tap bottom. The author includes all of these three specific temperatures under the category of "fusion temperature," which classification incidentally might be referred to as the "fusion-temperature range," since it involves more than one temperature. While the application of this fusion classification in combination with the gas-ash-temperature graph to the design of the combustion system is quite complicated, nevertheless, these are definite advances in the development of design fundamentals having practical possibilities.

Operators of high-capacity units have recognized the significance of these three temperature classifications and have learned that combustion temperature, when properly concentrated to give complete combustion in the furnace, will, of course, liquefy any kind of ash. To avoid slagging of boiler, superheater, or economizer convection surfaces the gas-entering temperatures should, if anything, be below the softening or middle-temperature classification. Furthermore, to meet the operator's idea of slag prevention, gas flow should be uniform in all sections of the assembly and the velocity of the gas, entering the convection sections, should be relatively low to get away from adhesive impact of slag against tubes. The operator has also learned that for rapid and complete combustion, fineness of pulverization and high turbulence over the entire combustion zone are essentials.

It has been mentioned that slag in the furnace is inevitable, but it might be added that slagging is intensified to a great extent in units designed for high capacity, high-steam temperatures and high rates of heat liberation. Operating experience with units fired with pulverized coal during the last two years indicates that the greatest barrier to holding the boiler in service for extended periods is the deposit and accumulation of slag or ash while in a semiliquid or a plastic state in the convection sections, particularly those having narrow gas passages between rows of tubes.

The elimination of these troubles and to meet the more or less ideal conditions which have just been described might call for an appreciable increase in tube surfaces over that provided in many installations. The first cost would also be higher; but if an increase in the availability of the unit can be accomplished, the higher first cost might readily be warranted and approved. High availability, it should be emphasized, is the operating factor that is necessary to establish economic justification for the installation and for the money invested in the project.

It may be pertinent to mention that in some of the recent units the time of travel through the furnace, from the burner to the boiler-tube-bank inlet, is about 1 sec. During that extremely short interval the volatile and carbon in the coal are consumed while the ash liberated has been transformed from solid to liquid and back again to a solid, all of which process seems almost incredible in the space of time available. Under these conditions, it is not surprising that more than 50 per cent of the ash in various stages of plasticity passes through the assembly, requiring higher-cost collector equipment to prevent undue release of ash to the atmosphere.

One of the most serious troubles in the combustion system of the high-pressure and high-temperature units and the one, which possibly causes the most interruptions to service, is the stopping up of the gas passages in the convection-type superheater with slag and ash. Steam blowers in many cases have failed to dislodge accumulations, while hand lancing in many cases necessitates shutting down the boiler. Because of the inaccessibility of the superheater passages, the manual process is a slow, awkward, and expensive operation.

To superheat steam above 900 F, with superheaters of the convection type, at the entrance to the superheater section, requires a gas temperature close to 2000 F. With an ash-fusion-temperature range of, say, 1900 to 2300 F, it is next to impossible to cool the ash down to a more or less dry state until the ash particles have advanced a considerable distance into the superheater gas passages. Now, if the gas passages between the superheater tubes were large and the gas velocity relatively low or, say, in the order of 50 ft per sec, it is possible that the superheater would remain free from slag choking for long periods. However, superheaters have not been made with wide spacing; in fact, passages between tubes are actually less than 1 in. and gas velocities up to 100 ft per sec are quite common. These conditions may be satisfactory for ash of high-fusion temperature, but with ash fusion in the 1900 to 2300 F range and, particularly for gas carrying in suspension over 50 per cent of the ash content in the fuel, slag stoppages are quite likely to occur. It should be pointed out that these conditions refer particularly to boilers of the inclined straight-tube type with counterflow convection superheaters.

Why have close spacing and high gas velocity been adopted? As the author points out, the superheater for these high steam temperatures must absorb more than 25 per cent of the total heat produced in the furnace, which is an extraordinary job for any kind of a superheater. With this great amount of heat absorption, the unit must necessarily be compact to fit into restricted

[11] "Combustion of Pulverized Coal," by Henry Kreisinger, Trans. A.S.M.E., vol. 60, May, 1938, paper FSP-60-8, p. 289.

[12] Power Engineer, American Water Works and Electric Company, New York, N. Y. Mem. A.S.M.E.

accommodations. To hold the first cost at an acceptable and competitive point, small tubes and close spacings are desirable to obtain maximum rates of heat transfer.

With possible frequency of interruptions in service, caused by stoppages in superheaters of the closely spaced type, purchasers would, of course, be interested in developments which will minimize these interruptions to the greatest possible extent. With this end in view, the first consideration might be to avoid, if possible, this excessive concentration of closely spaced tubes and the second to lower the temperatures of the gas and the amount of ash in suspension entering the superheater surfaces.

If the superheater could be divided into two parts, transferring the final-temperature section to panels in the furnace where the superheating would be done by radiant heat, the remaining convection element would then operate at lower gas temperatures and would be called upon to absorb, say, about 15 per cent instead of 25 per cent of total heat produced in the furnace. Why not substitute sections of steam-cooled walls for water-cooled walls in the furnace, as has already been done in a few installations? It might be necessary to locate these steam-cooled walls in the upper section of the furnace to avoid exposing surfaces of this type to the extreme temperatures of the combustion zone.

Without doubt a combination superheater of this kind would be a great deal more expensive than a straight convection unit. Whether the results, as regards increased availability of the boiler, would be realized and whether the purchaser would feel justified in paying a much higher price is problematical. No doubt these things would take a long time to settle. Be that as it may, the fact remains that the convection-type superheater, in its present arrangement and design, restricts the availability of the modern boiler fired with pulverized coal. Operators can well afford to pay handsomely for an improvement in these operating conditions and they look hopefully to the future for an appreciable reduction in the loss of output and the high costs incident to removal of slag from superheater passages. Meantime the redesign and improvements in the arrangement of tubes in the convection elements of the boiler by the manufacturers may in the near future produce higher availability and an economical solution of slagging in the gas passages of these elements.

Availability and the production of steam-generating units for high pressures and temperatures that will deliver specified rated capacities without interruption for periods of 6 or 12 months or longer must be the objective. The central-station company cannot afford to add spare boiler capacity as was the usual practice ten years ago. There is no serious thought that slagging conditions, as they occur in the high-pressure and high-temperature boilers, will be instrumental in forcing a return to the almost forgotten spare-boiler installation. The results of the author's investigation and research constitute a definite advance in the development of units to cope properly with the slagging problem and should contribute greatly to improvements in boiler design and construction, in order to meet the operators' expectations as regards availability.

E. B. Powell.[13] This paper records an important step in the pathway toward scientific design of the steam-generating unit and one which brings attainment very markedly nearer. The research of Armacost, Gordon, Hottel, Mullikin, Schmidt, Uchida and Tanabe, and Wohlenberg, to mention some of the most recent investigators, was necessary in developing a sound theoretical basis for correlation and interpretation of data and in improving tools and technique. The work which the author describes represents, in the writer's opinion, the most comprehensive factual analysis of the practical aspects of heat transfer in the combustion and immediately related zones of the modern steam-generating unit that has so far appeared. Yet, as the author himself points out, this is but an initial step along a very promising pathway. I am sure we all hope that he may continue as his paper suggests. It would be of much assistance to those others who follow if he would supplement his paper by a specific statement of the methods which he employed in gas-temperature measurement. Precision in such measurement requires complete freedom from interference of surroundings and is particularly important when different surface arrangements are under analysis. The steam-power industry is again indebted to the author for his pioneering work.

[13] Consulting Engineer, Stone & Webster Engineering Coporation, Boston, Mass. Mem. A.S.M.E.

H. P. Reid.[14] The writer is particularly interested in the methods which were used by the author to measure gas temperatures at various locations in the boiler.

The difficulty in obtaining gas temperatures that are representative and usable in studying boiler and furnace operations is fully appreciated. For several years the writer has made use of the high-velocity thermocouple for research and test purposes and would seriously consider application of such thermocouples for continuous records, if a stable and satisfactory design could be obtained.

Table 2 contains representative data illustrating errors that may occur by the use of the protected thermocouple. These data were obtained from recent tests on the performance of a rotary cement kiln and the waste-heat boiler attached to it. Temperatures of the gases leaving the kiln were taken by both protected or shielded thermocouples and also by high-velocity thermocouples. The figures given are average for a number of readings taken during the test period.

TABLE 2 DATA ILLUSTRATING POSSIBLE ERRORS FROM USING PROTECTED THERMOCOUPLE

	Gas temperatures F leaving kiln			Boiler efficiencies, per cent, calculated from	
Test no.	By protected thermocouple	By high-velocity thermocouple	Temperature difference F	Protected thermocouple readings	High-velocity thermocouple readings
273	1465	1590	125	67	62
274	1540	1862	322	73	59

In the tests, Table 2, the waste-heat boilers were operating considerably below rating. This explains the low boiler efficiencies. It is believed they are representative for this particular type of waste-heat boiler installation.

Bernhard Schroeder.[15] The results of tests made on three different installations are of much value but insufficient as a basis for definite conclusions. No doubt more extensive investigations will be undertaken by many operating companies. In this way the problem can be much more rapidly solved for various kinds of coal.

Knowing how difficult it is to obtain reliable data on which to base conclusions, one is inclined to wonder what is the degree of accuracy of the data presented in the paper? For instance, when powdered fuel is blown into a furnace, the various-sized particles of fuel immediately start to distill off the volatile gases. If sufficient oxygen is present, the gases will be burned; if not, they will not be completely consumed. With the amount of excess air ordinarily present in a boiler furnace, such as given in the paper, namely, 14 to 25 per cent, it is not difficult to consider the furnace being filled with gases of almost any composition, together with solid particles of carbon. All of these can only be

[14] Operating Engineer, Universal Atlas Cement Co., Chicago, Ill. Mem. A.S.M.E.

[15] President, Sargent & Lundy, Inc., Chicago, Ill. Mem. A.S.M.E.

burned when coming into contact with the required oxygen. It would appear quite possible for the burning gases to pass through the primary and perhaps through the secondary furnace before finding the necessary oxygen to complete their combustion.

On looking over the author's calculations, it is evident that the combustion is assumed to be completed before the gases leave the primary furnace. If they are not, the calculations are all affected to some extent. Therefore, it is necessary that the completeness of the combustion should be established. In future work complete analyses of the gases should be made to determine this point.

The gases leaving the boiler may show 15 to 20 per cent excess air, but in actual boiler operation the proportions of excess air, as between the various burners, may be quite different. The distribution of coal in the burner itself often leaves much to be desired and, unless the turbulence is very great, a satisfactory fuel and air mixture is not obtained until the gases have entered the boiler tubes or even the superheater.

Until there is some radical change in the art of burning coal, slag in some form will always be present. If it can be determined where the slag shall be deposited, by suitably proportioning the heating surfaces and the gas temperatures, then adequate means can be provided for its removal with a minimum of trouble and expense.

V. J. Skoglund.[16] The data given in this paper are useful as a guide in the analysis of heat transfer by radiation in furnace cavities. It would be ideal if point-by-point heat-transfer rates (Btu per hr per sq ft) and temperatures could be calculated from fuel and air operating conditions for any type of furnace cavity, using only the fundamental laws of thermodynamics, heat transfer, and fluid mechanics. To do this is as yet impossible but, starting with simplifying assumptions based on experimental data, rational analysis may some day make a useful contribution to the solution of practical furnace-radiation problems. At present the furnace-radiation analyst is handicapped by the lack of reliable data on conditions within furnace cavities.

An unsolved analytical problem of practical value is the calculation of point-by-point heat-transfer rates (Btu per hr per sq ft) for specified temperatures and emissivities of gases and suspended particles which vary from point to point within a cavity. The following additional data, taken simultaneously with gas temperatures and the corresponding over-all furnace conditions, are necessary before undertaking the development of a solution for the foregoing problem:

1 Intensity of emission or total emissivity of the mass of gases and suspended particles within the cavity for all possible directions at different points on its surface; there is at present no practical way of measuring this and the first step would be the development of such a method.

2 Measured wall temperatures over all of the surface; in the ash and slag problem as discussed by the author, measured wall temperatures are needed to establish the significance of those deduced from fusion properties of ash and slag samples.

3 Approximate thickness and mean conductivity of the slag on absorbing surfaces; the thickness may still vary with time after a long period at a uniform load.

The author makes the statement, "(*g*) A most important factor that should be known is the heat reflected and radiated from the wall surface." Although a rational solution of the above unsolved analytical problem might be laborious, it would be very valuable inasmuch as it would include the determination of that factor as well as surface temperatures.

The author's method of calculating an "equivalent heat-receiving surface temperature" is very simple but, even though the Stefan-Boltzmann radiation law is used, the method should not be considered rational since the use of an average gas temperature and an emissivity of 1.00 are not consistent with the actual mechanism of radiation. The method is of value only if results can be correlated for a large number of different furnaces and coals. This may require the use of empirical values of emissivity differing in different zones instead of the value 1.00.

The paper includes the statement, "(*e*) It is desirable to know the temperature of slag particles floating in the gas stream, especially as they approach tube banks." Attention is called to the mathematical results of Wohlenberg and Wise,[17] which are a qualitative indication of Δt, the difference between gas and particle temperatures, for points not near cold surfaces. Although their analysis is for very much idealized furnace conditions, after studying their paper, the writer believes the statement in the summary applies to average values in actual furnaces, "Hence for any reasonable length of gas travel in the cavity the ash-particle temperature is substantially that of the gases except for regions close to cold surface." Where flow conditions are unsteady, as is usually the case in actual furnaces, instantaneous values of Δt might be considerable for particles accelerated relative to the gas stream in regions where the temperature gradient in the gas is high.

V. V. Soldatoff.[18] Although the author states that consideration of the efficiency of steam-generating units is not the object of the paper, it is of great interest to the writer to bring out this point when speaking of boiler furnaces. For the combustion of pulverized coal must be provided (*a*) time, (*b*) temperature, and (*c*) excess of air. These last two items are well taken care of in primary furnaces but not the item of time. One way to extend the time, in the writer's opinion, would be to place a furnace slag screen of full capacity in the proper place, so that the sticky particles of ash could be retained there, fused with the fluxes provided. The mass could slide down to the bottom of the furnace.

As long as one function of the primary furnace is the smelting of coal ash, more should be known about it. Besides the chemical analysis, its mineralogical structures should be known. For example, uncombined iron ore in the presence of coal or coke will supply oxygen for burning off the carbon. This would be carried to the upper parts of the boiler, spreading a deposit of dust there. This would not happen if the amount of flux necessary to fuse the ash could be blown in with the pulverized coal and thus complete the slag formation on the furnace slag screen. Bearing in mind that the initial temperature of gases is too high for direct application, the fluxing of ash will be justified.

The author points out that the temperatures of gases entering the generating tubes of the boilers, Figs. 2, 5, 7, and the second pass of the boiler Fig. 8, vary from 2500 to 2670 F. This proves that convection heat emission can be applied to the boilers if the vital portion of tubes be protected by a permanent covering, e.g., "porcelain enamel," and forced circulation of water applied to them. Porcelain enamel, which is a thin layer of refractory, would allow placing the boiler generating tubes immediately after the slag-screen tubes, giving a narrow space for possible dust collection. This would produce an insulating effect, but only for two or three rows of tubes.

[16] Instructor in Mechanical Engineering, Columbia University. Jun. A.S.M.E.

[17] "The Distribution of Energy in the Pulverized-Coal Furnace," by W. J. Wohlenberg and D. E. Wise, Trans. A.S.M.E., vol. 60, 1938, paper FSP-60-19, pp. 531–547.

[18] Consulting Metallurgical Engineer, New York, N. Y.

Convection heating of water-tube boilers is not new. The same principle applies in Steamotive and Velox boilers but without any protection of their water tubes. This method should produce rapid steam generation with considerable economy of fuel and a decrease in the structural cost of the power station.

C. H. SPARKS.[19] The author's researches stress an important point; they illustrate conclusively that space requirements, in the larger units at any rate, cannot be met by drastic alterations to the system of generating the steam. The problem is essentially a furnace problem and the fact remains that the furnace will have to be designed upon principles which remain unaltered irrespective of whether natural or forced circulation is employed.

The author implies that dry-bottom furnaces are suitable for units up to 200000 lb evaporation per hr, but that for units larger than this slag-tap furnaces should be used. The interpretation to be placed on this statement is that the slag-tap furnace cannot be built as economically as the hopper-bottom furnace for evaporations of 200000 lb and under. In practice however, boiler units having hopper-bottom furnaces have been successfully built for evaporations as high as 450000 lb per hr. The heat release at the maximum continuous rating of such boiler units seldom exceeds 25000 Btu per cu ft per hr. Rates of heat release much in excess of this figure are not generally acceptable for the reason that the gases and entrained dust reach the boiler heating surface at a temperature above fusion point. In slag-tap furnaces in order to insure that the ash can be readily tapped, a high rate of heat release is maintained in the primary furnace and this normally results in a boiler approximately two thirds as wide as that which would be accommodated over an equivalent hopper-bottom furnace. The slag-tap furnace can be justified if it can be shown that, at a given load, a lower gas temperature entering the tube bank results, than that which can be attained when burning the same fuel in a hopper-bottom furnace designed for the same evaporation. Severe problems have been encountered in keeping boiler units clean and although deslagging has been successfully accomplished with some of the later designs of water sprays the problem of deslagging is indeed formidable.

The author seems to be of the opinion that the trouble of slagging is accentuated in a high-pressure boiler plant. There may be a number of reasons for this, one of which is that to justify the expense of high pressure, a high continuous load is usually carried, which gives the slag slight chance of breaking away naturally. Another reason may be that the steam temperature is high. This makes it necessary to reduce the amount of boiler-heating surface in front of the superheater so that the slag eventually forms on the small-diameter tubes of the close-pitched superheater surface. It is not improbable that engineers in the near future will endeavor to exploit the savings to be made by the application of high steam temperatures to low and moderate pressures. When this time arrives, somewhat similar slagging conditions on boilers operating at lower pressures may have to be faced.

E. H. TENNEY.[20] Looking back over the varied experiences of the last fifteen years, the ash and slag factor alone has in one installation or another determined such major elements of boiler performance as (*a*) the upper limit of output, (*b*) continuity of service, (*c*) furnace efficiency as limited by excess air, (*d*) the practical and economic value of a specific fuel, and (*e*) the level of operating costs. This paper is an important contribution to the practical analysis of this problem and as such is of interest to designers and operators.

The author suggests that dry-ash furnaces of large size may call for less turbulent burners. If by this is meant high-velocity burners, intended to maintain furnace turbulence, the writer agrees. A truly turbulent burner is one designed to create an intimate mixture of coal with the air needed for combustion at or near the burner. Such treatment has proved essential for the successful use of low-fusion-ash Mid-West coals where high furnace capacities are to be developed. It results in a more rapid heat liberation within a smaller furnace volume. This takes place with a clearer flame than is the case with the streamline burner. The increased time element contributes to the cooling of the ash to a dry condition. The clear flame permits unimpeded radiation to the furnace walls and results in a lower furnace temperature than is the case with "clouded" or so-called "insulated" burning. When furnace temperatures are permitted to reach 2800 F, cooling the ash to 2000 F is difficult. Recourse must be had, as the author points out, to the questionable use of water sprays on high-temperature boiler surfaces for slag removal in the boiler passages.

As he has suggested, it is important to explore the furnace to determine temperatures at various stages of combustion to make certain that the ash is cooled sufficiently before striking boiler surfaces or superheaters. Several years ago the writer made a series of observations on furnace temperatures, comparing turbulent and streamline firing. It was demonstrated that for low-fusing-temperature ash coal, proper mixing of the coal and air led to more rapid combustion and a relatively low-temperature furnace. This eliminated the necessity for creating high temperatures in one part of the furnace (for ignition or melting the ash) to be followed by the troublesome necessity of cooling the gases before reaching the boiler proper.

W. J. WOHLENBERG.[21] The author is to be congratulated on having presented effective visual evidence, by means of motion pictures, concerning some of the phenomena which occur in the pulverized-coal-fired furnace cavity. It was noted, among other things, that, under most circumstances, by far the larger percentage of fuel particles ignited and burned out very soon after entering the cavity. Such evidence confirms the results of calculation based on theoretical analysis of these conditions which are found in reference.[17] Thus it demonstrates, among other things, that the radiant mean positions of burning particles is not far removed from the point at which the fuel particles enter the cavity, when most of the air supplied for combustion enters with the fuel.

The author's method of arriving at surface temperatures is interesting. Investigation of this indicates that the results for the secondary furnace and boiler generating tube zones are not affected greatly by such variations of emissivity of the contents of the cavity which are likely to occur. Thus variation of actual emissivity values, from the assumed value of 1.0 to, say 0.5, changes the results for surface temperatures in these zones by only about 200 F.

However, for the temperature conditions which exist in the primary zone the results of calculation are extremely sensitive to emissivity values. A change here from the assumed value of unity to 0.5 changes the calculated results 2000 to 2500 F. It is essential therefore in this case that accurate data concerning the emissivity be available. The mere assumption that it is close to unity does not appear to be sufficient to justify any conclusions concerning surface temperature values arrived at on the basis of the assumption.

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[21] Professor of Mechanical Engineering, Yale University, New Haven, Conn. Mem. A.S.M.E.

As shown in reference,[17] ash particles in suspension contributed very considerably to the radiating power, i.e., to the emissivity of the contents of the cavity. This is true even on the basis of a single ash particle for each coal particle. The author implies

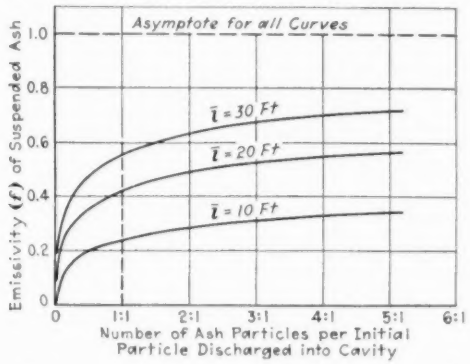


FIG. 11 INFLUENCE OF RATIO OF NUMBER OF ASH PARTICLES, FORMED PER INITIAL PARTICLE FIRED, ON EMISSIVITY OF CLOUD IN SO FAR AS PARTICLES ALONE ARE CONCERNED

(Ash particle surface A per lb fuel fired is 85 sq ft for abscissa ratio = 1:1, gas temperature T_g = 2500 F.A.)

that, on the average, there may be several ash particles per coal particle. If this is the case then radiation due to the ash particles is even more important than the results of reference[17] indicate in this respect. The author apparently based such implications on an examination of coarse particles. But by far the larger number of particles in a pulverized-coal sample are very small; in fact, so small that it is easily conceivable that many of them contain no ash, while others may be entirely ash.

In order to indicate the possible magnitude of effects of the above nature, curves shown in Fig. 11 were plotted. These are based on an application of Fig. 11, of reference.[17] It is assumed that each ash particle is an opaque sphere with a radiationally black surface. The radiant mean thickness of the cloud is $\bar{l}$. Differences of ordinate values, corresponding to differences in abscissa values, thus indicate the maximum amounts by which the emissivity of the cloud may be affected, because of variation in the number of ash particles formed per particle in the initial sample. The real differences would be less than those shown because, as the abscissa ratios increase, the ash particles become so small as to become more and more transparent to radiation. It is thus obvious that, until more is known about this state of affairs, the assumed ratio 1:1 used as a basis in reference[17] is probably safer than any other assumption which might be made in this respect.

Under (*e*) the author makes statements concerning the relation between temperatures of slag particles and other parts of the cavity. In this connection the reader is again referred to certain results in reference.[17] There it is conclusively shown that, thermal conduction between gases and ash particles is so powerful, the average temperature of the ash-particle surface must be very close to the gas temperature. This is true even when each fuel particle yields but a single ash particle. If, as the author implies, there are considerably more ash than fuel particles the difference between the temperatures of gases and ash-particle surface would be even less. The point is this: Thermal conduction between gases and particles has a far greater influence on the temperature of the particle surface than has radiation, if the particles are very small. The smaller the particles the greater is the relative influence of thermal conduction. Hence, the author's statements under (*e*), while probably quite true for the large particles, have less and less weight as the particles decrease in size. For the latter, the temperature will vary with that of the gases. If this varies due to local conditions, then the temperature of the small particle will follow substantially the variations of the gas temperature.

AUTHOR'S CLOSURE

Since the preparation of this paper a great many additional tests have been made, some on the same boilers and with the same kind of coal, others on additional boilers with different kinds of coal, sometimes varying the rating, the excess air, and other conditions. These later data definitely confirm the value of this method of measuring gas temperatures and using for comparison the calculated equivalent heat-receiving-surface temperature. They have shown that the rates of heat absorption in the primary furnaces of the slag-tap type vary closely with the adiabatic temperature and the actual slag-fluid temperature according to the Stefan-Boltzmann law.

Actual fluid temperatures of slag formed in a furnace may differ from those determined from the ash in the coal by the usual A.S.T.M. method due to segregation within the furnace and also due to the varying degrees of reducing and oxidizing influence on the iron in the ash in different parts of the furnace.

High rates of combustion seem to be beneficial from the standpoint of completeness of combustion, high rate of heat absorption, and large recovery of ash collected and tapped from a furnace.

The foregoing comments answer many of the questions raised in the discussion. Messrs. Harkins and Flinn properly bring out the importance of the radiating temperature as well as the receiving-surface temperature. Coal with low total-moisture content burned with high-temperature air and little excess air produces high furnace temperatures and correspondingly high rates of heat absorption. The temperature of the slag on the heat-receiving surface is an additional but independent factor.

Mr. Muir's comments are appropriate, and it is toward a more economical design of all parts of the furnace, boiler, and superheater that this method of testing is being actively continued.

The remarks of Messrs. Sparks and Tenney relate to the features of dry-ash removal. One point should not be overlooked, viz.: As furnaces become larger the relation between furnace volume and heat-absorbing surface changes materially. The Btu available per sq ft of heat-absorbing surface must be considered more than the Btu per cu ft of furnace volume. Long flame and delayed combustion usually give a lower peak temperature than does concentrated turbulent combustion, but as Mr. Tenney says, less slag trouble may result from properly arranged turbulent burners. This point should be further investigated with special reference to excess air and the fusing-temperature range of the ash as actually deposited, in comparison with that accredited to the ash in the coal by the A.S.T.M. method.

Professor Wohlenberg's discussion is much appreciated, and the data presented in the paper are inadequate to clarify some of the points which he has questioned. It is felt certain that additional and more thorough test data from the field will be helpful in learning more about furnace radiation. There is one point about the ash particles that certainly can be clarified by further field data. The author feels sure that there are very few, if any, particles of coal with no ash.

Dean Hitchcock's 38-year milestone makes one apologetic for not having accomplished more in that period of time. It now seems that the greatest hindrance in solving the slag problems of today arises from the fact that we learned too well the characteristics of slag in *fuel beds* and are only now beginning to realize that, when coal is burned in pulverized form, we must revise our methods of determining the ash-fusing temperature and approach the problem from a very different angle. We have

continued to obtain our laboratory data in a reducing atmosphere, and have then deposited the ash on furnace and boiler surfaces under varying conditions, often highly oxidizing with several hundred degrees difference between the two. We are well on the way toward making up for lost time, with a certainty that much more valuable progress is in sight.

Pulverized Coal-Tar Pitch as a Fuel

By E. W. CLARKE,[1] PITTSBURGH, PA.

This paper describes the method of adapting pulverizing equipment to burn fuel pitch, which is the hard residue remaining after the distillation of oils from coal tar when the maximum quantity of creosote has been removed. Early attempts to use this hard product as a fuel were beset with difficulties. However, a practical system was developed by the author in 1922 for pulverizing this fuel in a unit pulverizer and blowing it directly into the combustion space of a boiler through a nozzle designed specifically for its use. Since that time hard pitch has attained wide use as an economical and efficient fuel. The methods of firing fuel pitch are described by the author and specific examples of the results obtained are cited.

THE PRINCIPAL PRODUCTS of coal-tar distillation in the United States of America are road tar, creosote oil, and pitch. Not more than 2 per cent of the tar produced is used in making synthetic resins, dyes, and chemicals. The creosote-oil fraction varies from 15 to 60 per cent of the tar, depending upon the grade of pitch produced and the specifications of the wood-preserving industry in which the creosote is utilized. About 45 per cent of the creosote oil used in treating timber is consumed by the plants that treat railroad cross ties. Piling takes about 5 per cent, paving blocks 1 per cent, and construction timber 7 per cent of the creosote derived.

In 1908 about 55,000,000 gal of creosote were used for timber treatment. This steadily increased yearly to 108,000,000 gal in 1913, of which amount nearly two thirds were imported from Europe. When the outbreak of the World War reduced these importations, the American Tar Products Company, now the Tar and Chemical Division of the Koppers Company, was organized by the American Creosoting Company to supply its creosote requirements and to sell the surplus production in the American market. A ready market awaited the creosote but the opposite was true of the pitch, which is the residue left in the stills after distilling off the creosote from the tar.

This pitch is a liquid residue as produced in the stills and either flows by gravity or is pumped from them. At atmospheric temperatures it varies in character, depending upon the amount of oil distilled off from the tar, from a viscous, semisolid material to pitch hard enough for grinding. The point for stopping distillation is determined by the consistency of the pitch desired or by the specifications for the creosote, depending upon which is of the highest market value. Pitch comprises from 40 to 80 per cent of the tar, depending on the character of the tar and the hardness of the pitch.

The uses of distilled tars or pitches include saturant for roofing felts, protective coatings for iron and steel pipes, binder for aggregate in road construction by both hot and cold processes, surface treatment of roads by hot or cold applications, filler in block pavements, waterproofing walls below grade, built-up roofing, membrane waterproofing, saturating fibrous materials, binder in briquetting fuel, electric carbons, sealing and insulating compounds, core compounds, foundry facings, clay pigeons, and pitch coke.

Since the primary product of the newly organized American Tar Products Company was creosote oil, the bulk of the tar was distilled to give the highest creosote-oil yield, thus producing a hard pitch for which the demand was limited. This resulted in the need for a market to consume the hard pitch produced.

Early Experiments in Burning Pitch

Various attempts were made to use hard pitch as a special fuel. Pitch was mixed with coke breeze or noncoking coals to supply the cementing properties necessary for satisfactory coking. Pitch was mixed with coal and with ashes and used as fuel. The hard pitch had a softening point of 285 to 320 F. Attempts were made to liquefy it by heating and firing it with oil burners. However, all parts of the system had to be held at a temperature of about 500 F to maintain the pitch in a liquid condition. The suspended carbonaceous material, usually about 10 to 20 per cent, clogged up the oil burners and caused the abandonment of these experiments. To lessen these difficulties the liquid hard pitch was cut back with oil tar. At the Chicago plant of one tar distiller liquid hard pitch having a softening point of 280 to 300 F was cut back to a softening point of 145 F with oil tar and used for fuel under all its boilers and tar stills. The oil tar was obtained from gas companies operating water-gas sets. At the Philadelphia plant of the same company the boilers and tar stills were fired with a mixture of one part crushed pitch, having a softening point of 300 F, and two parts of run-of-mine coal burned on hand-fired grates. At the St. Louis plant of another tar distiller, a mixture of 40 per cent crushed hard pitch and 60 per cent coal screenings or coke breeze was used as fuel on Murphy stokers to heat boilers and tar stills.

The nuisance and added cost of these various methods of blending and mixing solid or liquid hard pitch with other fuels restricted such operations to the plants of the tar distillers and precluded the possibility of pitch making any headway in the open market as a fuel.

In 1922 the author took up the task of developing a better way to utilize hard pitch as a special fuel. At that time the opinion was generally held in the industry that the possibilities of burning pitch in pulverized form had been thoroughly investigated and that the difficulties were practically insurmountable. Prior to this, hard pitch had been pulverized for years with impact pulverizers with air separation and sold as the main constituent of core compound, which suggested the possibility of burning it as fuel, after running it through coal pulverizers. No difficulty was experienced in pulverizing it with the same type of machines as those used for core pitch. However, the heat radiated from the furnace caused the pulverized pitch to soften and to cake in the burners, soon plugging them shut.

Investigation of the patent literature showed that R. P. Perry and E. F. Stewart had been granted recent patents for air-cooled and water-cooled burners for pitch. Arrangements were made to borrow a small pulverizer and start experiments. The first step was to make burners such as those described in the Perry and Stewart patent and to conduct experiments with them. It was found that they had some tendency to clog when exposed to any considerable degree of radiant heat from the furnace. This suggested the idea that, if the burner could be located outside the furnace and the pulverized pitch could be discharged from it into

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NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

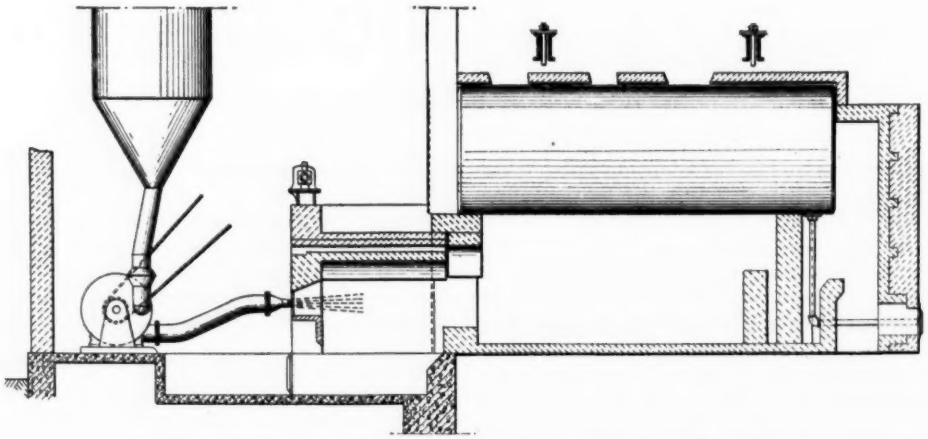


FIG. 1 PULVERIZING APPARATUS DESIGNED FOR BURNING FUEL PITCH

the furnace through an opening of sufficient size and shape so that the pitch would not impinge on any heated surfaces before being burned in suspension, the difficulties might be overcome. This was tried and found to be successful and a patent[2] was granted to the author on February 28, 1928, covering the apparatus and process. Fig. 1 shows the arrangement of the apparatus. The plants of the American Tar Products Company were then equipped to burn pulverized pitch instead of coal.

One thing that the hard pitch lacked was a name to identify it as a new type of fuel. The name "hard pitch," by which it was known to the industry, bore no relation to its use as fuel. Various designations were suggested, e.g., "burning pitch" or "furnace pitch." An excellent name was suggested by the oil industry's nomenclature and hard pitch was henceforth called "fuel pitch." Within a comparatively short time, fuel pitch became a stable article of commerce. The principal characteristics of fuel pitch are given in Table 1.

TABLE 1 CHARACTERISTICS OF FUEL PITCH

Proximate analysis	
Volatile matter, per cent	53.80
Fixed carbon, per cent	46.13
Sulphur, per cent	0.55
Ash, per cent	0.07
Ultimate analysis	
Carbon, per cent	93.71
Hydrogen, per cent	4.12
Nitrogen, per cent	0.98
Oxygen, per cent	0.65
Sulphur, per cent	0.44
Water, per cent	0.12
Ash, per cent	0.10

Softening point, 285 to 320 F
Calorific value per lb = 16,000 Btu
Weight by bulk = 50 lb per cu ft
Grindability, 150 Hardgrove

Fuel pitch can be pulverized in standard types of impact coal pulverizers with air separation. It requires only about one half the power that must be furnished for pulverizing coal and the wear on the pulverizers is negligible. The high volatile content of the pitch causes it to burn with a flame more like that of gas than pulverized coal, enabling the pitch to be ignited without preheating the furnace. Another characteristic is that dust explosions and puffs, which must be so carefully guarded against when burning pulverized coal, are entirely absent with fuel pitch.

Since pulverized pitch cannot be subjected to pressure without binding together, it cannot be used in central pulverizing plants where bins and screw conveyers are provided for the pulverized-fuel distribution. The type of equipment suitable is the unit pulverizer in which the pitch is pulverized and blown directly into the furnace. No preliminary drying is necessary since the moisture content of the pitch is usually less than 0.2 per cent. For the purpose of drying coal in unit pulverizers, it is customary to introduce heated air, but with pitch this is not only unnecessary but contrary to the nature of the material, since the heated air would soften the pitch in the mill and render it inoperative.

In proportioning combustion chambers, the same rules apply to pitch that determine the size for pulverized coal. Since the ash in pitch is about 0.10 per cent there is no difficulty with slag or ash removal. Furnaces fired with fuel pitch can be operated for months without shutting down for cleaning purposes.

MARKETS AND PRODUCTION METHODS

Because of the purity of the fuel, it can be used to replace gas and oil for chemical and metallurgical processes. Its low ash and sulphur content are especially desirable in the manufacture of highly alkaline materials by processes in which the combustion gases come in contact with the materials being produced; for example, high-purity chemical lime and sodium silicates.

A leading Midwestern manufacturer of lime found that the purity of the product from his rotary kilns was improved by firing them with fuel pitch instead of coal, due to the elimination of the ash from the coal, which acted as a source of contamination to the burned lime. Fuel pitch has been used in this plant for about thirteen years. Malleable-iron furnaces which had been fired with coal produced a stronger product when fuel pitch was substituted and this has proved to be another attractive market outlet. A furnace producing sodium silicate was successfully operated with fuel pitch which replaced oil at a saving in fuel cost. It is obvious that the tar distiller can demand premium prices if the use of pitch as fuel results in improved industrial products.

Fuel pitch is now produced either in flake form made on water-cooled drums and steel belts, or in crushed form processed by natural cooling in shallow open-topped steel tanks or bays built on the ground. From these it is reclaimed either by hand labor or mechanical means and then crushed to about $^3/_4$-in. size. From this point on to the pulverizers it is handled by the same type of conveying machinery and storage bins as used for coal. Neither crushed nor flaked pitch will solidify or bind together in bins or conveying systems, which facilitates its handling and makes it possible to ship it by trucks, cars, barges, or steamers without difficulty.

The heating values of fuel oil and fuel pitch are about 20,000 Btu per lb and 16,000 Btu per lb, respectively. On this basis,

[2] U. S. Patent No. 1,660,831.

the comparable values per ton for fuel pitch for various prices of fuel oil are as follows:

Fuel oil, cents per gallon	3	4	5	6	7
Fuel pitch, dollars per ton	6.85	9.12	11.36	14.71	16.00

Typical Fuel-Pitch Installation

The pioneer work with fuel pitch was done at a Wisconsin plant of the American Tar Products Company. After successfully demonstrating that pitch could be burned as a fuel commercially and equipping the boilers and stills at this tar-distilling plant for the purpose, a part of the surplus fuel pitch was sold for boiler fuel to a neighboring company. The boiler-room equipment at this plant included six 402-hp, 200-lb per sq in. pressure Stirling boilers, fitted with Foster superheaters which provided 100 F superheat.

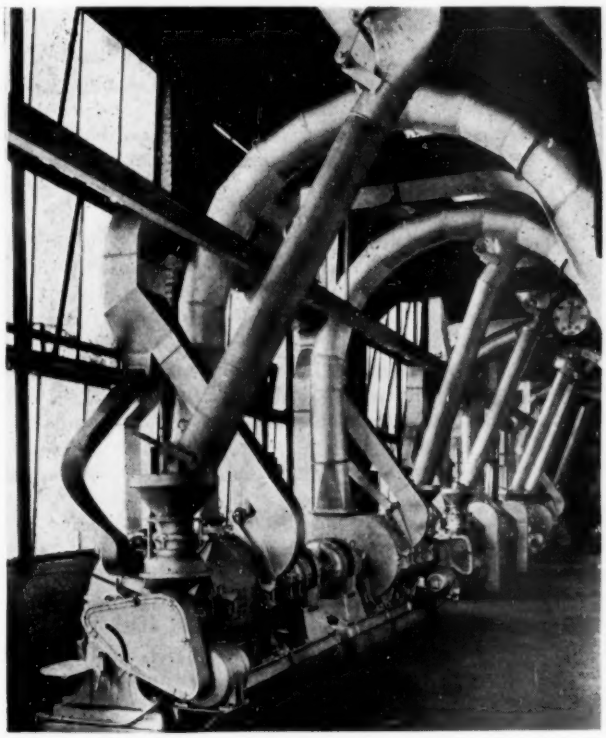


Fig. 2 A Typical Unit-Pulverizer Installation for Burning Fuel Pitch

The boilers were originally fired with stokers using coal as fuel. In February, 1926, the company purchased a unit pulverizer for one of the boilers and after removing the stoker remodeled the furnace by adding a small Dutch oven and lining the ashpit with firebrick.

After the first pulverizer had been installed and its operation compared with stoker firing, the savings were found to be of sufficient magnitude to warrant the installation of additional pulverizers. A second order for a unit pulverizer was placed five months after the first one was installed. In less than six months after the second pulverizer was ordered, a third machine was purchased so that three of the boilers were fired by unit pulverizers. The equipment permitted the use of either fuel pitch or coal as fuel. The pulverizers carried the boiler rating up to 200 per cent without difficulty.

The fuel pitch supplied to this plant at the outset had a softening point between 260 and 280 F, which is lower than that of the fuel pitch produced today. It had to be pulverized in machines where high friction losses were not prevalent and without the use of preheated air. Variable-speed control was used to drive the feed mechanism of the pulverizers in order to assure maximum flexibility. One pulverizer has handled 5500 tons of fuel pitch without the necessity of replacing any of the pulverizing elements. At the end of five months' operation, the second pulverizer had handled more than 2700 tons of fuel pitch without the casing of the machine having been opened.

Under average conditions the furnace temperature was about 2700 F; the CO_2, 14 per cent; the furnace draft about —0.10 in. water gage; and the stack temperature about 540 F at 175 per cent of boiler rating. The ash residue was cleaned out of the furnaces about once in every three months. The fuel pitch was very easy to ignite and a clear stack was readily maintained.

It will be realized from the foregoing that the maintenance costs on the pulverizers were most satisfyingly low. From a psychological standpoint, it is an interesting fact that the boiler-room operators preferred to burn pitch in the pulverizers rather than coal on the stokers.

The evaporation rate was 11.2 lb of water per lb of fuel pitch as fired. The fuel cost per 1000 lb of steam generated has been shown by carefully conducted test to be more than 23 per cent less than with coal fired on stokers. The pulverizers ran for hours at a time without attention. The additional capacity, supplied by the use of fuel pitch in the pulverizers, made possible the elimination from regular service of one of the boilers, even under the maximum demands for steam.

Some Observations on the Yield Point of Low-Carbon Steel

By JOSEPH WINLOCK[1] and RALPH W. E. LEITER,[2] PHILADELPHIA, PA.

A study has been made of the yield-point phenomena in low-carbon steel with the view of determining the mechanism of the transition of the metal from the elastic to the plastic state. Unlike copper and aluminum, low-carbon steel does not yield in tension until an excessively high elastic resistance has been built up. This high elastic resistance is the upper yield point. The upper yield point is believed to be of fundamental importance in the Lüders' line phenomena, and experimental evidence is given which supports this view.

THE peculiar behavior of low-carbon steel at the yield point has long been known and has been the subject of many researches both in this country and abroad. Of special importance has been the work of Nádai (1),[3] Siebel and Pomp (2), Sachs and Fiek (3), Rawdon (4), Kenyon and Burns (5), Gensamer (6), Gensamer and Mehl (7), Davenport and Bain (8), Fell (9), Deutler (10), Brinkman (11), and MacGregor (12).

The question of what is occurring during this singular behavior is not only of interest from a theoretical point of view, but is of great practical importance for at least two reasons: (*a*) The highly localized plastic flow, which occurs during the deep drawing of light-gage steel, gives rise to surface markings which seriously mar its appearance. If the forces causing the deformation are primarily tensile forces, these surface markings occur as depressions in the surface. Whereas, if the forces causing the deformation are primarily compressive, irregular lines of elevations occur. These surface markings are known as Lüders' lines, Hartmann lines, the Piobert effect, and, in the shop, as "stretcher strains." A photograph showing surface markings as depressions is shown in Fig. 1. And (*b*) a clear understanding of the significance and value of the yield point is, it would seem, essential to engineers engaged in the designing of structures.

The purpose of this paper is not an attempt to describe what causes the steel to behave in this peculiar manner, but an attempt to describe what the authors believe is taking place in the steel during its transition from the elastic to the plastic state. Its purpose, also, is to make some observations which will, it is hoped, serve to create a greater interest in this important field.

Deformation of Aluminum

In order to describe more clearly the phenomena which are occurring at the yield point of low-carbon steel, it will help if a description is first given of what occurs when aluminum is elastically and plastically deformed in tension.

There are, of course, two methods of deforming a metal in tension: (*a*) by actually hanging an increasing amount of dead weight on the metal; or (*b*) by deforming it at some given rate or rates by means of oil-pressure cylinders, screws, or the like. With the first method, the load is always present and obviously no drop in load can occur at any time. With the second method, the registered load is the resistance of the metal to the deformation being imposed upon it. The second method is the one most widely used, and is the one referred to throughout this paper.

At the instant deformation begins, the aluminum begins to offer a resistance to deformation and for a certain distance the load-deformation curve rises as a substantially straight line (13). So long as the curve remains substantially straight, the deformation is substantially elastic. But, when the deformation becomes so great that the resistance of the metal to elastic deformation is overcome, plastic deformation takes place. This is shown in Fig. 2. It is to be noted, too, that the metal exhibits a continually increasing resistance to deformation until the maximum load is reached; and also that the curve is smooth.

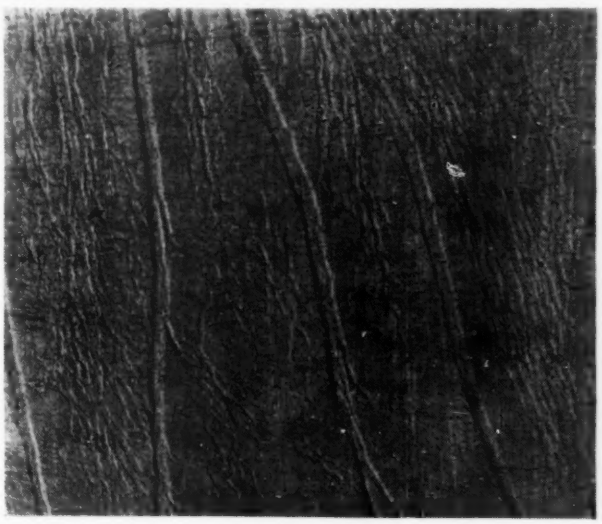


Fig. 1 Lüders' Lines or Stretcher Strains Shown as Surface Depressions

It should be further noted particularly that as the change from the substantially elastic state to the substantially plastic state takes place, the metal immediately offers a greater resistance to deformation, i.e., a higher load is recorded. In this case, the elongation is substantially uniform until the maximum load is reached.

Deformation of Low-Carbon Steel

When low-carbon steel is deformed, the metal appears to behave elastically in much the same manner, but when plastic deformation occurs there is a lowering of the resistance to deformation of the specimen and the registered load is lower. In testing parlance, this phenomenon is known as "the drop of the

[1] Chief Metallurgist, Edward G. Budd Manufacturing Company.

[2] Assistant Metallurgist, Edward G. Budd Manufacturing Company.

[3] Numbers in parentheses refer to the Bibliography.

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Discussion of this paper was closed January 10, 1939, and is published herewith directly following the paper.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

beam." This is shown by the solid line in Fig. 3. Experiments have shown that substantial plastic deformation does not occur in a uniform manner as it does with aluminum, but occurs first at some point or points and, as the deformation proceeds, spreads until all of the metal has been so affected. The elongation required to produce complete plastic deformation varies with the speed of deformation, but under ordinary conditions is some 5 per cent.

Attempts have been made to analyze this behavior by mathematical equations designed to establish conditions which show when heterogeneous or homogeneous flow can take place. Attempts have also been made to explain the phenomenon by postulating that, unlike aluminum, a sudden weakening occurs in the metal, or, more scientifically, the metal temporarily exhibits a low rate of work strengthening.

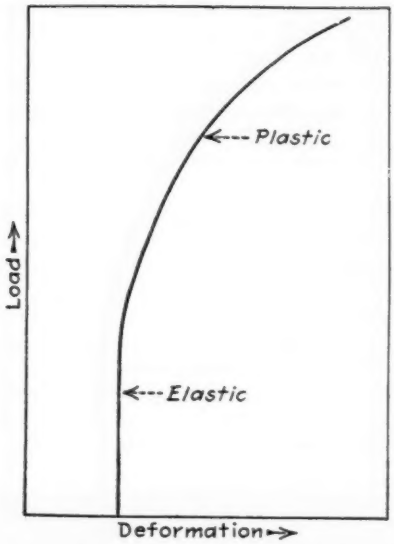


FIG. 2 LOAD-DEFORMATION CURVE FOR ALUMINUM

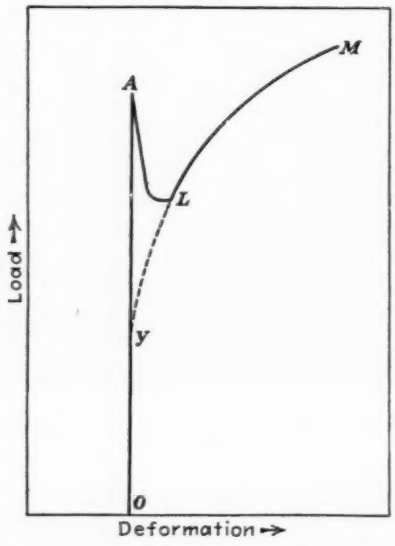


FIG. 3 LOAD-DEFORMATION CURVE FOR LOW-CARBON STEEL

None of these attempts is in accordance with the observations made in the authors' experiments. Mathematical equations cannot be established for this portion of the load-deformation diagram on the basis of homogeneous or heterogeneous flow because neither uniform nor nonuniform plastic flow is occurring! How can either of these be taking place when there are large portions of the metal which are not flowing at all? The authors believe also that there is not a sudden weakening but, indeed, a sudden strengthening of the metal undergoing plastic deformation.

It appears that, for steel to behave like aluminum, substantial plastic deformation should take place at the load Y in Fig. 3 and the load-deformation curve should be the curve $OYLM$. For some unexplained reason, however, the load Y is "overshot" and substantial plastic flow does not commence until the load A has been reached. This point has long been known as the "upper" yield point. Plastic deformation does not occur throughout the specimen at once, however, but first at one point. Therefore, at the instant plastic deformation commences, the plastically deformed metal finds itself overloaded, so to speak, with a resistance to further plastic deformation of only the load Y. The further behavior of this first increment of metal is dependent entirely upon its reaction to the precarious and unstable situation in which it finds itself. Because of the high load, it must deform plastically at a high speed, and it does so. The amount of metal forced to a great speed of deformation is governed by the amount of elasticity available in the other parts of the specimen and of the testing machine; and it is for these reasons that the downward path of the load-deformation curve veers away from the portion of the curve denoting elastic deformation.

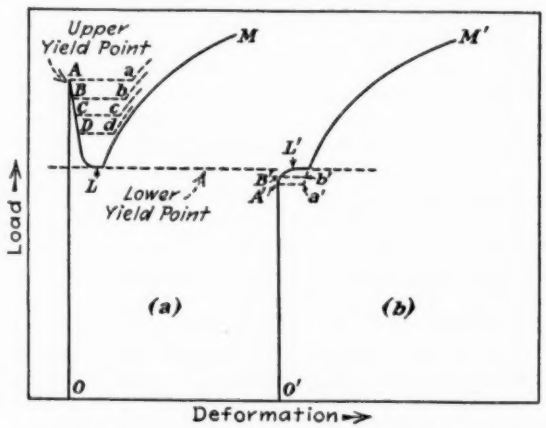


FIG. 4 LOAD-DEFORMATION CURVES WITH AND WITHOUT UPPER YIELD POINTS

The following is a more detailed explanation of what the authors believe occurs: When the load A, Fig. 4a, is reached, the first increment of metal deforms plastically at the high load which it has built up for itself while in the elastic state. And, since it now has the resistance to further plastic deformation of only the load Y of Fig. 3, it does what it is forced to do, namely, travel at a high speed along the dotted line A-a. At the next instant two other small increments, one on either side of the first, deform plastically at the load B an amount equal to B-b. The load on the specimen drops—first, because the metal has only the resistance to further plastic deformation of the load Y; second, because there is a concentration of stress at either end of the local constriction; third, because there is now present some metal which has been cold-worked, i.e., plastically deformed (the effect of this will be described in more detail later); and fourth because the increment's speed of travel is greater than the head speed of the testing machine.

The total load registered by the specimen never drops to the load Y but only to the load L, known as the "lower" yield point,

because a balance is reached between (*a*) the increase in resistance of the metal undergoing plastic deformation, i.e., work strengthening; (*b*) the increase in resistance of the metal because of the high speed of travel; and (*c*) the presence of metal in other parts of the gage length which has only been elastically deformed, and which must first be stressed to a value corresponding to the upper yield point before plastic deformation can occur in it too.

The extremely important point to emphasize is that, at the instant substantial plastic deformation is initiated, it is not necessary to have present the load which the metal has built up for itself while in the elastic state in order for further plastic deformation to occur. As opposed to this, the usual view, it seems, leads to as much information of the inherent properties of the metal as would be obtained by hanging a million-pound weight on a steel bar having a diameter of a quarter of an inch. After fracture, the bar would be considered (with a reduction in area of, say, 40 per cent) to have broken at a stress of the absurd figure of some 30,000,000 lb per sq in. This, of course, is true, but certainly no such stress as this was needed to cause the metal to fracture.

It has previously been considered that uniform elongation began when the load-deformation curve commenced to rise again at the termination of the yield-point elongation. But, if the authors' views are correct, uniform elongation does not begin until a load is reached at which all parts of the specimen have deformed the same amount that the first increment has deformed. In other words, the cross-sectional areas of the specimen taken when the end of the yield-point elongation has been reached should gradually increase from the smallest cross-sectional area of the metal first to deform plastically, up to the largest cross-sectional area of the metal last to deform plastically. And, furthermore, these areas should correspond to the elongations at the points *a*, *b*, *c*, *d*, and *L* of Fig. 4*a*. These points should not, of course, lie on the curve *LM* because of the greater speed at which these increments are deformed, and because of the short time these high loads existed.

Test on Steel Specimen

The following experiment was made in order to substantiate this. A highly polished and carefully annealed tensile specimen of low-carbon steel was prepared. The specimen was approximately 0.037 in. thick, had a uniform section 2.25 in. long and 0.5 in. wide, and the gage length was 2 in. The edges of the specimen were carefully smoothed and partially rounded to avoid any concentration of stress. It was then carefully aligned in a tension-testing machine (14) especially designed to give a constant head speed. The specimen was then pulled at the extremely low head speed of 0.0046 in. per min until the yield-point elongation had been completed. As is well known, Lüders' lines always start at the fillets of a test piece free from surface imperfections, notches, or the like, because of the inevitable concentration of stress at these points. For the purposes of this experiment, however, it was necessary to cause the Lüders' line to start near the middle of the gage length. Accordingly, two extremely small prick-punch marks were carefully made before the test on either side of the specimen at this point. These marks were, however, so small that an upper yield point was not removed from the load-deformation diagram.

During the test and while the load was dropping, pencil marks were made on the specimen at the boundaries of the spreading Lüders' line and at the same time were placed on the charted load-deformation curve. The specimen was then removed from the machine and careful measurements were made with point micrometers. After this, the specimen was replaced in the testing machine and, at the same head speed, was pulled a considerable distance beyond the yield-point elongation. The curve obtained is shown in Fig. 5.

TABLE 1

Location see Fig. 5	Width, in.	Thickness, in.	Area, sq in.	Change in area	Change in area, per cent
Original	0.4927	0.03740	0.01845	...	...
A	0.4885	0.03570	0.01743	0.00102	5.53
B	0.4890	0.03625	0.01775	0.00070	3.79
C	0.4890	0.03655	0.01788	0.00057	3.09
D	0.4890	0.03660	0.01790	0.00055	2.98
E	0.4880	0.03670	0.01791	0.00054	2.925
L	0.4880	0.03670	0.01791	0.00054	2.925

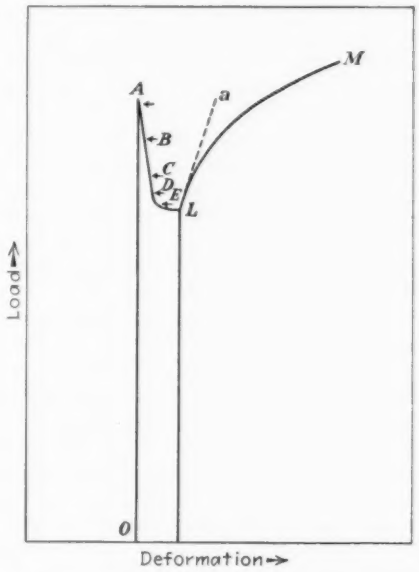


Fig. 5 Load-Deformation Curve for Data of Table 1

The measurements obtained are given in Table 1 and from these data the dotted line *L-a* in Fig. 5 was drawn. The small arrows indicate the loads which correspond to each set of measurements. The reason why this line does not coincide with the actual load-deformation curve has already been mentioned, viz., the greater speed at which these increments were deformed. These experimental data, it seems to the authors, definitely substantiate their views. These findings can be further supported by what is believed to be sound deduction of the existence of an upper yield point, and of the vital importance of this point in Lüders' line phenomena.

It is necessary, first, to describe a few experimental observations. As has been mentioned, the first substantial plastic deformation to occur is extremely local in character. That is, regions of only elastically deformed metal and regions of plastically deformed metal are present at the same time. This has been conclusively shown by placing extensometers on the elastically deformed regions and, after removing the load, noting that they return to zero. Careful visual observation of highly polished specimens show, also, no traces of plastic deformation in these regions.

It has also been conclusively shown by others and more recently by the present authors (14) that the horizontal portion of the load-deformation curve can be raised or lowered to an extraordinary degree by changing the speed of deformation. In spite of this, the same phenomena take place in exactly the same manner. The only difference is that the depth of the Lüders' lines is greater the higher the speed of deformation.

Now, since there are, during the stretching of a tensile bar, some regions which are only elastically deformed, these regions are not substantially in motion; that is, the elongation is taking

place only at the boundaries of the Lüders' line. And, since the horizontal portion of the curve can with changes in speed of deformation occur at almost any load level, an extremely pertinent observation, it seems, can be made: How is it possible for the only elastically deformed stationary regions to support the high loads produced by the deforming boundaries of their plastically deformed neighbors when these same elastically deformed regions would have yielded at a lower load had the speed of deformation been lower? In other words, how can the elastically deformed regions support, during a test carried out at a high speed, a load which is far greater than that which they could support if the test were carried out at a low speed? The absurdity of such a situation appears to be obvious. The only elastically deformed regions, therefore, must be able to withstand, in order to conform to this otherwise anomalous situation, a much higher load than that at which plastic deformation is occurring at their boundaries. If they could not, they too would yield at the same time. This higher load-carrying capacity of the elastically deformed regions results, the authors believe, is the phenomenon known as the upper yield point.

A further observation concerning the upper yield point follows: The load-deformation curve *OYALM* in Fig. 3 can always be obtained in annealed low-carbon steel of small or moderate grain size in either cylindrical or flat specimens if (*a*) the edges of the flat specimen are carefully rounded by polishing; if (*b*) the fillets on the specimens are large; if (*c*) great care is exercised in properly aligning the specimen in the testing machine; if (*d*) the surface is free from pits or scratches and other surface imperfections; and if (*e*) the specimen is free from any cold-worked material which might be present as a result of the machining operation, some local abrasion, or bending.

If, however, any or all of these precautions are not observed, an upper yield point is not obtained on the load-deformation curve. A localized spreading plastic deformation occurs, nevertheless, in exactly the same manner, but the resistance of the metal to the initiation of plastic deformation is lower, i.e., the registered loads are lower, and the loads at which the increments are deformed are lower. See Fig. 4*b*, lines *A'-a'* and *B'-b'*. And, as would be expected, the local constriction is less deep. Although the registered loads are lower, it is believed that the stresses are exactly the same as when an upper yield point is obtained. It is important to note from this figure two significant facts: (*a*) the horizontal portion of the curve, i.e., the load *L* is the same for a given steel and a given speed of deformation whether or not an upper yield point is obtained; and (*b*) the elastic limit does not coincide with the yield point as it does in Fig. 4*a*, but occurs at a lower load.

The Rate of Work Strengthening and the Stress-Strain Curve

The phrase "rate of work strengthening" is usually defined as: the slope of the stress-strain curve. The authors think that this definition is not a desirable one because work strengthening can only occur after plastic deformation has taken place. Indeed, it is plastic deformation which creates work strengthening. With this definition, however, the origin of the work-strengthening curve must be taken as the stress existing at the highest load which the steel can offer while in the elastic state. That is, the origin of the work-strengthening curve must be chosen as a stress existing when the metal is not in the plastic condition. According to this definition, then, the stress-strain curve through and somewhat beyond the yield-point elongation would be the curve *OYAPM* in Fig. 6. From this, it would be necessary to conclude that because the slope of the portion of the curve *AP* is low, the rate of work strengthening is low. It would seem that this definition leads to an erroneous conclusion.

The authors believe that the elastic condition must be totally divorced from the plastic condition in order to arrive at the proper rate of work strengthening. So long as there are regions of both conditions present at the same time, the true behavior of the metal in the plastic condition is obscured. As the authors see it, the influence of the upper yield point included in the definition should be removed. And the definition of the rate of work strengthening should be: The rate of increase of only the stress actually necessary to cause further plastic deformation in metal which has already been plastically deformed. To clarify, in Fig. 6 the rate of increase of stress actually necessary to cause an increment to deform plastically through the yield-point elongation is not the curve *AP* but the curve *YP*. That is, the stresses actually needed to cause the metal to deform through the yield-point elongation are the stresses S_1, S_2, S_3, ... The curve showing the rate of work strengthening then becomes the curve *YPM*. And it can be seen that the slope of the curve is greater during the yield-point elongation than at any other time. With this observation in mind, there is no necessity for postulating a negative rate of work strengthening (6) when the steel is tested in compression (in which no reduction of area is possible).

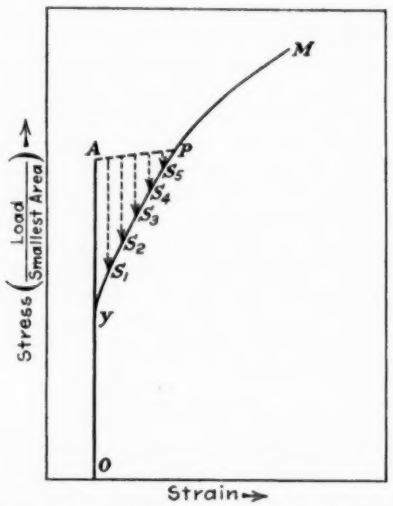


Fig. 6 Stress Strain in Relation to Work Strengthening

Other observations also indicate that this is correct. Hardness readings taken at the bottom of Lüders' lines formed at different speeds of deformation show the hardness curve to be rising in a normal manner. Of great importance, too, is the fact that if the steel is cold-worked slightly before testing in either tension or compression, all traces of the peculiar yield-point phenomenon completely disappear. The steel then behaves in exactly the same manner as aluminum, as shown by the curve *OYLM* in Fig. 3. That is, the load-deformation curve of cold-worked low-carbon steel is smooth (15), and substantial plastic deformation occurs at a much lower load.

Practical application of this has been made in the deep-drawing industry and special machines have been developed (16) to cold-work the steel a small amount before the deep-drawing operation is performed. In this condition, the steel does not show Lüders' lines (stretcher strains) after deep drawing and, because the deformation takes place uniformly, the deep-drawing properties are considerably improved.

The problem of what stress should be chosen for the yield point and the significance of the yield point raise questions which have not been given the attention they deserve. Experiments show that if an upper yield point is observed, the propor-

tional limit, the elastic limit, and the yield point occur at the same stress. On the other hand, if no upper yield point is observed, the elastic limit and the proportional limit occur at a considerably lower load, and the yield point, i.e., the horizontal portion of the curve, occurs at a slightly higher load than this.

What value, then, for yield point is important for engineering purposes: the stress at the elusive upper yield point (the stress at the drop of the beam); or, the lower yield point which, in ordinary testing procedure, is seldom noted? Should the horizontal portion of the curve be taken as the yield point when no upper yield point occurs? Is the stress at the load Y in Fig. 3 related to any other property of the steel, such as its fatigue strength? Of what significance to engineers is the upper yield point? Can it be obtained reliably? Is the absence of an observed upper yield point indicative of severe internal stress? (Locally cold-worked carefully prepared smooth specimens show no upper yield point.) Will a steel be thought to be of inferior quality because of a low proportional and elastic limit due only to failure to obtain an upper yield point? And, finally, what is the cause of the upper yield point anyway, i.e., the extremely high elastic strength of steel? These questions urgently need answers.

The authors wish to acknowledge the friendly and helpful advice and criticism of Dr. A. Nádai whose important work in this field is well known; and they wish to thank their colleague Dr. R. Eksergian for his interest and encouragement.

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Discussion

E. A. Davis.[4] In the latter part of this paper, the authors object to using the slope of the first portion of the stress-strain diagram, line A-P in Fig. 6, as a definition of the "rate of work strengthening." This objection, the writer believes, is entirely justified, for the rate of work strengthening should be a measure of the manner in which the stress is increased due to the effect of strain alone. If the material is such that the stress under which it yields is affected by changes in the speed of deformation, then these influences must be taken into consideration before the rate of work strengthening can be determined. Ordinarily, this can be avoided by stretching the specimen at a constant strain rate. In a like manner, if the properties of the material change with time (e.g., age hardening, annealing, etc.), the stress under which it yields will also change. This influence must be taken into account in measuring the rate of work strengthening. This may be a negligible factor in short-time tests, but it becomes important in evaluating the results of long-time tests.

In the case of mild steel or wherever localized yielding occurs, it is practically impossible to test any given fiber or element at a constant strain rate. For example, the speed of the movable head on the testing machine can be adjusted so as to give 5 per cent over-all strain in 5 min, yet each element may strain 5 per cent in less than 1 sec, and very little can be said about the uniformity of the rate of strain during this interval. If it were possible to select some infinitesimal gage length so that the strain inside this length would be uniform and, if this could be tested at a constant strain rate, then the slope of this stress-strain diagram should furnish a true "rate of work strengthening." This curve can be made to fit the ordinary curve in the region near M in Fig. 6 by choosing the strain rate the same as that in the ordinary test in this region. In the part of the diagram usually known as the yield-point elongation, the curve for the constant strain-rate test will be below the line A-P because of the slower strain rate. This curve should correspond in some way to the curve $OYPM$ shown in Fig. 6. Just what the shape of this curve should be does not seem obvious to the writer, and he would like to ask the authors if they will further explain their reason for drawing the curve between points Y and P as they have.

A. V. de Forest.[5] This paper again draws attention to the great difficulty of interpreting the yield-point phenomenon both from the point of view of the metallurgist and of the engineer. One reason for this is that, due to the limitations of our present test machines and method of recording the stress-strain diagram, there is no clear distinction between the idiosyncrasy of the machine and the properties of the metal under test.

In this work and in many previous papers, attention has been drawn to the upper yield point and the conditions necessary to measure its value. This is found to depend greatly on freedom from stress concentrations due to shape of the test specimen, preparation of its surface, and particularly on eccentricity of loading. It is never measured at all under commercial testing conditions where special precautions are not taken. The value of the lower yield point is similarly never measured because of imitations in the speed of response, both of the weighing mechanisms of the testing machine and the strain-measuring equipment. Furthermore, both points are sensitive to test speed and to temperature. It is, therefore, high time that the whole problem be reexamined and if possible, an agreement reached as to how this point shall be defined and measured for engineering

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purposes; how it is to be interpreted by engineers as a criterion of strength, and by metallurgists as a particular mode of deformation exhibited only under special conditions by special materials.

J. J. KANTER.[6] The authors showed in a previous paper[7] that the upper yield point of low-carbon steel sheet is influenced to an amazing degree by changing the rate of straining. They demonstrated a consistent rise in upper yield point with increase in straining rate for sheet steel about 0.036 in. thick of approximately 0.05 per cent carbon content prepared to five different grain sizes.

The fact that the upper yield point increases with increase in rate of straining suggests that the effective stress distribution at the yield point becomes increasingly homogeneous on a microscopic scale as the stress is applied. At stresses up to the upper yield point it is a fair approximation to say that the mode of deformation is essentially elastic and of uniform distribution. Beyond this point plastic deformation proceeds heterogeneously as the authors have demonstrated in Fig. 1. As stress increases the elastically deformed structure becomes increasingly less stable. Its existence as such becomes a transient condition. As the straining rate is increased, "snapshots" may be caught of this elastically deformed condition at somewhat higher loads, just prior to the plastic breakdown, which propagates at a rate governed to some degree by temperature.

Suppose the elastic straining were caused to proceed at a greater rate than that at which the propagation of plastic breakdown proceeds. Under this condition the microscopically homogeneous elastic-stress distribution might be preserved up to the rupture strength of the steel. Such a condition might well obtain in high-velocity tension impact tests, such as made by H. C. Mann[8] at the Watertown Arsenal, in which 0.35 per cent carbon steel exhibited a transition from plastic deformation of high energy absorption to brittle fracture of low energy absorption at impact velocities of about 30 ft per sec per in. of gage length. At striking velocities in excess of 30 ft per sec, it may well be that the upper yield point of the steel may be pressed to the point where it approaches the elastic cohesive limit of the material.

Consideration of the mathematical possibilities of dealing with the effect of straining velocities on yield point suggests at once the methods of statistical mechanics. We are dealing with a system, the elements of which are individually capable of undergoing an elastic type of deformation up to a cohesive limit. The energy necessary to reach this limit is, after a fashion, a measure of the individual yield point of the element involved. The upper yield point, which is measured for a particular aggregate of these elements subjected to a certain rate of straining, is a manifestation of a statistical average of states of strain. Due to the instability of the elastic state of deformation, once the stress has reached the point of instability, it is to be expected that but a fraction of the stressed elements simultaneously sustain the elastic strain necessary to attain the true upper yield point. The faster the rate of strain, the larger will be the proportion of elements simultaneously capable of sustaining the full elastic-cohesive effort without relaxation into slip. The kinetic oscillations of the particles comprising the steel conceivably contribute to the elastic instability which brings about yielding. Under rapid straining rates it is to be supposed that the kinetic oscillations are given a lesser opportunity to influence the elastic displacement and thus the unyielded steel is given a greater chance to persist at higher stresses.

If it be granted that the probability of kinetic disturbance underlies the inception of yielding in steel, it is to be inferred that the Maxwell-Boltzmann distribution law might be applied to correlating upper yield points with the straining rates at which they occur. In this law the chance of a number of eligible elements, out of a large number, being involved in an event is shown to be exponentially proportional to a quantity repre-

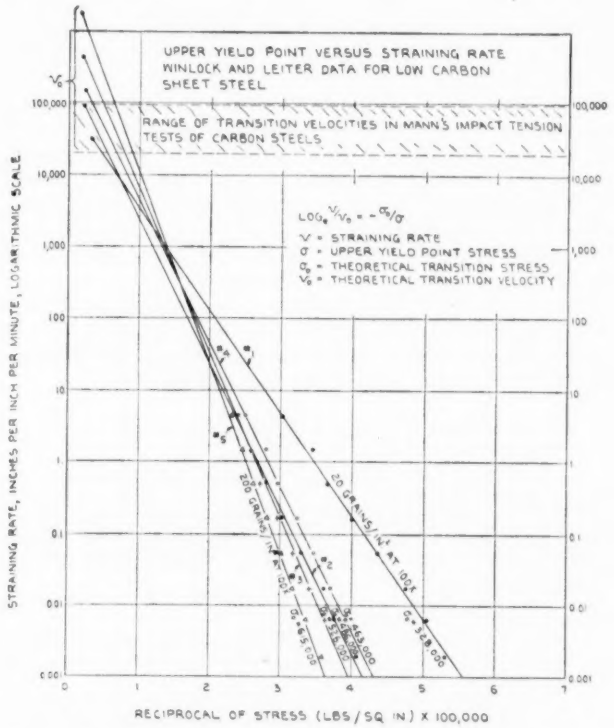


FIG. 7 CLOSE AGREEMENT BETWEEN STRAINING VELOCITY ESTIMATES AND RANGE OF TRANSITION VELOCITIES FROM MANN'S VARIABLE-VELOCITY IMPACT TESTS ON CARBON STEELS

senting a ratio between the actual energy required to produce the event and the average energy available. In the case at hand, the event with which we are dealing is elastic deformation to the full cohesive effort without degradation to slip. The probability that is expressed by the exponential function should involve the proportionate occurrence of such an event. The exponential quantity will be that ratio between the observed upper yield point and the true elastic-cohesive effort at which the elements of the steel reach a brittle rupture.

The testing of this theory upon actual data suggests employing a semilogarithmic plot of log straining rate as a function of the reciprocal of the upper yield stress. A straight-line alignment upon such a chart fulfills the requirements of the Maxwell-Boltzmann distribution law. The straining rate V is, in effect, a measure of the total number of events which occur during a time interval, while the upper yield stress σ is the average of all conditions of stress acting. The true elastic-cohesive effort of which the steel elements are capable will be a constant, σ_0, and associated with yielding under this full effort will be a straining-rate constant, V_0. The equation relating upper yield point and straining rate is thus

$$V/V_0 = e^{-\sigma_0/\sigma}$$

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[7] "Some Factors Affecting the Plastic Deformation of Sheet and Strip Steel and Their Relation to the Deep-Drawing Properties," by J. Winlock and R. W. E. Leiter, Trans. of the American Society of Metals, vol. 25, Mar., 1937, pp. 163–185.

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The five sets of data for effect of straining rate on yield point which the authors have published are strikingly correlated by this expression. The range of values for the constant σ_0 is in accord with similar values arrived at through other considerations. Values for this same sort of constant may be estimated from the breaking strength of cold-worked wires. The measurements made by E. A. Davis[9] on the effect of loading rate upon upper yield point, together with the Winlock and Leiter results on effect of straining rate, afford another means of estimating the magnitude of constant σ_0. The ratio between loading rate and straining rate is a value dimensionally equivalent to σ_0. Computations of this ratio from the results given for materials of similar physical characteristics, at equivalent yield points, give values for σ_0 of the same magnitude as determined from the slopes of the log straining rate versus reciprocal yield-stress plot.

If the value of σ_0 is accepted as representative of a transition-point yield stress, the straining velocity, V_0, associated with the point may also be estimated. The values of V_0 so computed are in surprisingly good agreement with the range of transition velocities found in Mann's variable-velocity impact tests on carbon steels at Watertown Arsenal as charted on the semilog diagram. See Fig. 7.

These observations suggest that additional work should be done to correlate the effects of straining rates on yield points over wider ranges of velocity than have been reported upon. The semilog chart shows a considerable gap of straining velocities between those explored and those corresponding to the range of impact velocities which have been studied. It would be a matter of considerable theoretical as well as practical significance if the Maxwell-Boltzmann distribution equation could be demonstrated to fit the yield-stress–straining-rate data over the entire range from tensile-testing speeds to impact velocities.

Authors' Closure

The authors wish to thank Mr. Davis, Dr. de Forest, and Mr. Kanter for their very helpful discussions.

[9] "The Effect of Speed of Stretching and Rate of Loading on the Yielding of Mild Steel," by E. A. Davis, *Journal of Applied Mechanics*, Trans. A.S.M.E., vol. 60, Dec., 1938, p. A-137.

The reasons for choosing *YP* (a continuation of the curve *PM*, Fig. 6), as the curve indicating the rate of work strengthening at the beginning of plastic deformation are as follows: (*a*) The curve *OYPM* is the stress-strain curve of metals, such as aluminum and copper, which show no Lüder's-line phenomena. (*b*) Hardness readings taken at the bottom of Lüder's lines, formed at a different speed of deformation, show the hardness curve to be rising in a normal manner. (*c*) If the steel is cold-worked very slightly before testing, the stress-strain curve has the shape shown by the curve *OYPM*. (*d*) The slower the speed at which the test is carried out, the shorter the yield-point elongation and the lower the load at which the horizontal portion of the load-deformation curve occurs. (There is, however, a yield-point elongation at even the lowest speed of deformation.) At decreasing testing speeds, a continuation of the curve *LM*, Fig. 3, approaches the elastic portion of the curve. Even though its position is lowered, its general shape is not changed.

The authors agree with Dr. de Forest that the upper and lower yield points are sensitive to test speed and to temperature, but do not believe that the ability to obtain the lower yield point accurately is limited because of the speed of response, etc., of the testing machine, when ordinary testing machines and ordinary testing methods are used. (It is, of course, difficult with a beam-type machine to obtain accurately the load to which the resistance of the metal drops unless the yield point is of considerable magnitude, but with a hydraulic machine this value can be obtained readily.) The rigidity of the testing machine, etc., affects the position of the downward course of the load-deformation curve, as the authors pointed out in their paper, but the ordinary testing machine does not affect the value of the lower yield point *if a constant load is obtained during the yield-point elongation*.

The authors are especially indebted to Mr. Kanter for his carefully prepared and constructive discussion. It would be well to emphasize, however, that no increase in the magnitude of the elastic forces due to speed alone can take place. The raising of the upper yield point by speed must then be due either (*a*) to the load-deformation curve not being elastic up to the upper yield point, or (*b*) to some sort of locking of the crystalline grains which is affected by the speed of test.

Five Years' Progress of Oil and Gas Power

By HOWARD E. DEGLER,[1] AUSTIN, TEXAS

The last "Five Years' Progress of Oil and Gas Power" has been primarily that of constantly lowering the engine weight, decreasing the bulk, increasing the flexibility, and lowering the installed cost per horsepower. With these worth-while improvements also came better performance, lower fuel consumption, and a more quietly operating engine. These gradual changes and improvements have broadened the field of application to such an extent that, from the first development, running into hundreds of pounds per horsepower and turning over at from 100 to 150 rpm, engines have been reduced in weight and increased in speed to an extent that now offers possibilities for universal application. Undoubtedly, the Diesel engine is recognized today as one of the most economical and reliable prime movers known, and is fast taking its place in applications for which it seemed unsuited a few years ago.

THE LAST five years, 1933 to 1938, have been devoted to the development of many new lines of lightweight high-speed engines and the improvement of the established heavy-duty types of oil and gas engines. The total installed horsepower has been doubled during this period and the field of application of oil and gas power has been extended enormously. The best performance of modern Diesel engines will go as low as 0.35 lb of fuel per bhp-hr; in fact, one foreign 5500-hp engine uses only 0.329 lb per bhp-hr with 18,000-Btu fuel, corresponding to a thermal efficiency of 41 per cent. Several Diesel aircraft engines have shown the remarkable economy of 0.32 lb at full load and cruising speed.

Diesel-Type-Size Standardization

The Diesel engine has been subject to progressive refinement and to continued increases in size. The largest Diesel engine in the world was installed in 1933 in the Copenhagen, Denmark, electric-light plant. This unit is of the solid-injection two-stroke-cycle double-acting type; each of the eight cylinders has a 33-in. bore and a 59-in. stroke; and at 115 rpm the engine delivers 22,500 bhp. One manufacturer has offered to build a 40,000-hp engine; another engine of 80,000 hp has been proposed for installation on a ship. America's largest units are the 24 × 36-in. 7000-bhp eight-cylinder two-cycle double-acting HOR engines at Vernon, California, and Salt River, Arizona. Larger capacity per unit has been sacrificed by other builders in favor of the greater simplicity of construction of single-acting engines that results from freedom from stuffing boxes and crossheads, and greater simplicity of piston cooling and of valve-operating mechanism.

Above 1000-bhp and below 4000-bhp capacity, two-cycle engines are favored almost three to one; and above 4000 bhp this type is used exclusively. At present there are no four-cycle double-acting engines, but of the large two-cycle units about half are double-acting. Considering only capacities within the range of both types it is clearly a choice between the simplicity and accessibility of the single-acting engine and the smaller space requirements and smaller number of cylinders for a given power of the double-acting type. All large Diesel engines have vertical cylinders. In sizes between 300 and 1000 bhp, the use of the two- or four-cycle engine is a matter of choice; many builders make both types.

Marked advances have been made in standardization, not only in the general design of the Diesel engine, which has been toward simplification of lines and centralization of control, but also in the design of many of its component parts. Rotative speeds range from 2000 rpm for small high-speed engines to from 120 to 150 rpm for large heavy-duty Diesel engines, and piston speeds from 1500 fpm in small engines to 1000 fpm in large ones.

Gas-Engines and Gas-Burning Diesels

The gas engine has found new ways to make the most of the numerous pipe-line extensions bringing gas fuel to many parts of the country. Once principally used in steel mills, gas plants, and oil fields, gas-powered units now find acceptance in the general power field. The modern gas engine like its oil-engine counterpart is generally a vertical multicylinder unit operating at higher speeds. It is versatile, burning many gases including sewage-sludge gas and butane. The largest vertical gas engine is a 2200-bhp unit at Carlsbad, New Mexico, with eight 23 × 28-in. cylinders. It operates on the four-stroke cycle at 240 rpm. The effect of the present economic trend upon oil- and gas-engine design is shown by the introduction of an increased number of convertible Diesel-gas engines capable of operating on either liquid or gaseous fuels. The gas-injection two-cycle engine with improved scavenging features is giving as good fuel economy as the four-cycle types without the loss of any of its inherent advantages of simplicity, compactness, low first cost, and low maintenance.

One of the outstanding Diesel-engine developments made in recent years (1936) is a dual-fuel engine of the two-stroke cycle type (fundamentally air-injection with port scavenging) that will burn either oil or gas. The advantages of this engine are appreciated when it is understood that the unit can actually burn either fuel without making any major changes in the engine, and further, that either fuel is burned on the Diesel two-stroke cycle. With an eight-cylinder 1500-bhp dual-fuel Diesel engine installed in the municipal power plant at Lubbock, Texas, comparison of the cost of producing power with oil or gaseous fuel shows the advantage to be greatly in favor of the latter. With natural gas at 13.5 cents per 1000 cu ft, the average gas-fuel cost was $0.00172 per kwhr, while with oil as used by other Diesel engines in this same plant costing 3.1 cents per gallon, the cost was $0.0032 per kwhr. This amounts to a reduction of 46.2 per cent for the gas-burning Diesel engine.

Heavy-Duty Diesel Progress

Considerable progress has been made in the development and application of alloy materials and welding, and in precision manufacturing methods. Controlled pressure lubrication, more careful consideration of fuel-injection methods and combustion, and better scavenging have also contributed much to the perfection of modern engines. Diesel-engine installations in communities have been silenced and isolated to prevent noise and vibration. Adequate intake and exhaust silencers are available, as well as engine mountings using cork, helical springs, and the like. In general, more stress has been put on better filtration of

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Note: Statements and opinions advanced in papers are to be understood as individual statements of their authors, and not those of the Society.

fuel oil, lubricating oil, and air before entering the engine. The near future will undoubtedly see the general application of the exhaust-gas turbosupercharging of four-cycle Diesel engines.

While there has been a trend in recent years toward lightweight high-speed engines, some manufacturers have not departed from the heavy-duty type of Diesel engine with which they have been so successful. Undoubtedly, they are not convinced that these newer trends will give the kind of performance which the more conservative designs are giving and which many users of Diesel equipment expect. Many of the Diesel engines now produced in the United States are being built under foreign patents, although American ideas of simplicity and accessibility are constantly modifying the designs. The greater use of large Diesel engines is one of the trends that is noticeable in this field; but America, because of conditions more favorable to the use of coal and natural gas, lags behind Europe in its adoption of the larger sizes. Weights per horsepower range from 25 to 300 lb with an average of 80 lb.

Other design trends, based upon unprecedented progress during this five-year period, indicate higher speeds, further weight reduction, greater unit output, supercharging, greater use of double-acting cylinders, more compactness, greater use of small engines, more gas-Diesel units, incorporation of accessories in the engine enclosure, the use of welding for frames and other parts, more complete enclosure, simplified control, higher mean effective pressure, increased use of normally wasted engine heat, combination of Diesel engines with existing steam plants to improve heat balance, wider application as auxiliary power to main generating units, also the starting, controlling, and stopping of Diesel units automatically in accordance with load requirements or with a minimum amount of personal supervision.

Some authorities doom the four-cycle engine, because the two-cycle type has no valves, has smaller cylinders, requires less floor space, weighs less, has lower first cost, and needs less maintenance. Others think it is not doomed, claiming that two-cycle units present greater cooling problems, are harder to lubricate, have lower mean effective pressures, require excellent scavenging equipment, frequently require supercharging, and present some cylinder-wear problems.

Each Diesel-engine manufacturer seems to have a different idea of the grade of fuel oil required for his product. Hence, ever-increasing demands have been made upon the petroleum industry to supply various grades of fuel oil for the many types of Diesel engines. During the last five years several engineering societies (The American Society of Mechanical Engineers, American Society for Testing Materials, and the Society of Automotive Engineers) have undertaken for the first time to prepare specifications for standard Diesel fuel oils for the mutual advantage of engine designers, manufacturers, and operators, and the petroleum industry. A tentative classification covering five grades of Diesel fuel was issued by the American Society for Testing Materials in September, 1936. Purchase of fuel according to this classification with resultant criticisms and comments should aid in developing a permanent series of specifications for Diesel fuel oil. The diversity of opinions regarding specification limits and the relative importance of the various items indicate the need for improved and continued cooperative work between engine builders, engine users, and oil refiners. In any case, for the good of the industry, the engine owner should be spared the trouble of purchasing fuel to too-rigid specifications.

Until the last five years, development of speed-regulating equipment for Diesel and other internal-combustion engines had lagged behind that of governors for the other two principal prime movers, steam turbines, and water turbines. The recent application of the hydraulic governor to Diesel and gas engines has resulted in close speed regulation and constant stability over the entire speed range. A hydraulic governor does not depend for its power on centrifugal force or inertia acting on a large mass, hence the size of the unit can be considerably reduced as compared to an equivalent mechanical governor for the same engine.

Smaller Diesels for Wider Applications

Many authorities see the greatest future of the Diesel engine in motive power such as that for portable industrial service, long-haul truck and bus transportation, lightweight high-speed trains, and other uses wherever its compactness and efficiency are effective. In marine work, the Diesel engine means low-cost operation and no fire hazard; in airplanes, greater dependability, reduced fuel cost, longer cruising range, and reduced fire hazard. But the automobile presents a different problem. Car owners in the United States today pay for easy starting, quick acceleration, surging speed, and smokeless exhaust but not fuel economy. Diesel engines have not proved as satisfactory for these desired characteristics as the present-day gasoline engine, but medium-compression spark-ignition oil engines approximating the Diesel principle may be used in the near future and make possible the wider adoption of Diesel engines for automobiles in this country.

Some of the long-established Diesel manufacturers devoted the last five years to the development of completely new lines of high-speed engines and to the improvement of their existing heavy-duty units. These companies and other manufacturers are producing high-speed Diesel engines that compare favorably with gasoline engines in power, speed, size, weight, ease of starting, performance, and reliability. Many of these units have been used in tractor, truck, and portable construction equipment; in 1937 over 80 per cent of the total number of engines manufactured had a capacity of less than 100 bhp. As compared with gasoline engines, the possibilities are evident when the following specifications of a typical high-speed Diesel engine are considered: Four cylinders, $3^3/_4 \times 4^1/_2$ in.; piston displacement, 199 cu in.; rating, 62 bhp at 2600 rpm; weight, 12 lb per bhp; and electric starting.

Not since the inception of railway travel has any mode of transportation aroused the public interest and captivated the imagination of railroad men so much as the first Burlington Diesel-powered *Zephyr*, the lightweight passenger train which on May 26, 1934, made its historic long-distance nonstop run from Denver to Chicago in 13 hr 5 min at an average speed of 77.6 mph.

Two developments which have been of great significance to the career of the high-speed Diesel engine, and have perhaps made that career possible, are the use of improved bearings of new alloys and the advance in the refining and compounding of lubricating oil. Not long ago all bearing metal was of babbitt, very satisfactory in its way for slower-speed heavier engines. The introduction of bearing metals containing silver and higher percentages of lead has permitted great increases in allowable unit bearing pressure on main, crank, and piston-pin bearings. Comparable results have been obtained in a different way by the use of "needle bearings," or roller bearings with rollers of very small diameter. Engine maintenance has been simplified by the use of what are termed "precision bearings," namely, thin steel bearing shells lined with a very thin layer of bearing metal. Such bearings require no fitting and when worn are thrown away and replaced.

A few years ago Diesel-engine lubricating oil was of the uncompounded mineral type, of either paraffinic or asphaltic base. Now there are addition agents available for the promotion of a number of desirable qualities. Some of these addition agents increase oiliness under normal conditions; others promote film

strength in order to preserve lubrication under abnormal conditions. Ring sticking, not long ago the cause of much trouble with high-speed units, has been materially decreased by the use of other additives. Unfortunately some of these additives are corrosive to some of the newer bearing alloys, but petroleum technologists are constantly at work on the problem and a solution is probably only a matter of time.

Auxiliary Combustion Chambers

With increasing speeds the principal design problem of any Diesel engine is in its injection and combustion system, and this has been the subject of much study and experimentation for the purpose of obtaining clean combustion and low rates of fuel consumption. Experience has shown that a high degree of turbulence in the combustion chamber just prior to fuel injection is necessary to obtain the desired results; however it was found that application of the methods employed for low-speed engines was not practicable in the case of high-speed units. Hence a new departure seemed necessary and accordingly various forms of auxiliary combustion chamber have been evolved to produce this turbulence. This chamber is generally located in the cylinder head and is completely water-cooled.

The auxiliary combustion chamber is in no sense a precombustion chamber, but contains all of the clearance volume, except that necessary immediately over the piston in order to secure sufficient mechanical clearance. Gratifying results have been obtained with this type of combustion chamber in that it has led to cleaner combustion and higher mean effective pressures. This design removes the spray valve from the center of the cylinder head, thereby permitting the use of one intake valve and one exhaust valve of large size instead of two of each, as is necessary with the centrally located spray valve if restriction of air flow is to be avoided. The location of the spray valve at the side of the cylinder head makes it accessible and shortens the length of the fuel line from the injection pump.

A departure from conservative ideas is represented by the development of the two-cycle high-speed opposed-piston Diesel engine. The compactness of the engine and its efficient scavenging will continue to be contributing factors in the wider application of these engines, especially for submarine and aircraft purposes.

Standardization of Engine Parts

An interesting feature of modern Diesel engines, especially high-speed types, is the increased use of parts made by specialty manufacturers. This applies not only to injection equipment and governors but also to bearings, valves, and, in the smaller engines, to pistons and cylinder liners. This makes possible the furnishing of parts made of superior materials at considerable savings in cost. In these smaller engines, drop-forged connecting rods are used. Their use effects a large saving in the weight of reciprocating parts and makes possible the safe operation of these engines at much higher speeds than could be attained with rods having ends of the marine type.

Trends in High-Speed Engines and Aircraft Engines

The future high-speed industrial Diesel engine will probably develop up to 150 bhp per cylinder, weigh from 10 to 15 lb per bhp, run at speeds up to 3000 rpm with piston speeds of 2000 fpm, and occupy less than $^1/_2$ cu ft per bhp. Present choice between four-cycle and two-cycle engines appears to favor the former. The argument that, because the four-stroke cycle has become the generally adopted system for high-speed gasoline engines, it must also be better for high-speed Diesel engines, is without value. Inasmuch as the simple two-cycle Diesel engine offers great possibilities with a high specific power output for a relatively small floor space, designers will consider more seriously ways and means of improving scavenging methods and lowering the fuel consumption of this type of engine. Design problems to be considered include: Higher compression ratios to insure a high temperature; increased speed of injection and good turbulence of the incoming fuel charge; a combustion chamber shaped to give rapid heating of the fuel spray; a long free fuel spray; cooling of the injection nozzle; use of the automatic or unit-injector type of fuel nozzle; and the all-important problem of scavenging.

The high-speed Diesel engine possesses many inherent qualities that render its application to aircraft purposes not only attractive but also desirable from the points of view of safety and economy. The German Junkers line of liquid-cooled opposed-piston Diesel engines has done more than all other engines put together to popularize the Diesel engine for aviation. Their first flight was made in 1929, but since that time development and successful application have been rapid. These engines are used extensively on the fast twin-engined (600 to 800 bhp each) ten-passenger airliners in daily service in Europe and on the flying boats which carry mail across the South Atlantic. It was with a Junkers four-engined flying boat, that the world's long-distance seaplane record was broken early in 1938 in a nonstop flight of over 5000 miles from England to Brazil. The weight of these engines is about $1^3/_4$ lb per bhp with Prestone (liquid) cooling.

At the present time the Junkers organization has its 1200-bhp engine on test and is proceeding with the construction of a 2000-bhp Diesel which may be ready for its tests in 1939. The 1200-bhp unit will be suitable for transatlantic and long-range aircraft, and in this respect will be the equivalent of our 1500-bhp gasoline engines because of smaller frontal area and lower fuel consumption. Paul H. Wilkinson, one of America's foremost Diesel aircraft-engine authorities states, "The 2000-hp engine will be far ahead of anything we have, or will have, in the United States. With a diameter of 39 in. and a weight of less than 1 lb per bhp, this powerful liquid-cooled two-cycle engine will be hard to beat. When its crankshafts are turning over at 3000 rpm there will be 144,000 power impulses per min from the 48 pistons of this square-type 24-cylinder engine—approximating the smooth power flow of the turbine."

With the exception of the work carried out by Guiberson in the air-cooled radial-engine field, aircraft Diesel development in the United States has not been supported by the government. The Guiberson Company has been fortunate in being able to continue this development, but its engines of 340 bhp are still on the restricted list. A modification of this engine is a 265-bhp unit recently adopted by the Ordnance Department to power medium-size tanks, for which service this type of engine has many decided advantages over gasoline engines. Other present-day American designers who have developed Diesel aircraft engines of considerable promise are Deschamps, Godfrey, and Lawrance. It is unfortunate that the government is not actively and financially supporting these developments which are so vital to our national defense and also to the safer and more economical operation of our commercial air lines.

Discussion

Oliver F. Allen.[2] Professor Degler has given a fair and constructive résumé of Diesel progress in this country during the last five years. In condensing such a report into the space and time available, an author must understress certain phases of the development which others might have elaborated upon.

[2] Consulting Engineer, New York, N. Y. Mem. A.S.M.E.

Without in any way criticizing his excellent paper, the writer wishes to call attention to some aspects of Diesel development which may justify a little more attention.

Considering developments in this country, in England, and on the European continent, it seems to the writer that the only outstanding progress has been made in land transportation.

Five years ago a few Diesels had been installed experimentally in busses and trucks in the United States. Only one or two Diesel manufacturers were then giving serious consideration to automotive applications. But at the Motor Truck Show, Port Authority Building, New York, last month, seven makes of commercial Diesels were shown for truck and bus service in sizes from about 100 hp up to 300 hp including both four- and two-cycle designs. There have been installed within the last few months, or are now on order, between 300 and 400 of the bus and truck engines of one make shown there.

In England, from a few trial installations five years ago, the use of Diesels in passenger vehicles has had such rapid growth that 5656 compression-ignition vehicles were registered in England at the end of October, 1937. This was an increase of over 2000 compared with a year before.

For many years, several hundred switching locomotives powered by Diesel engines have been in use here and abroad and Diesel-powered passenger trains were in regular service in Europe and in experimental service in this country prior to 1933 but, as the author points out, the long-distance runs of the light-weight passenger trains in 1934 inaugurated a development in passenger rail transportation by Diesel engines which has been growing by leaps and bounds ever since. This development is not limited to the United States although this country is the leader in long-distance high-speed Diesel passenger trains, some of which regularly reach schedule speeds exceeding 100 mph and go thousands of miles without changing locomotives.

There are now in service a considerable number of units of about 900 bhp at about 750 rpm of both the two- and four-cycle types. However, these weigh from $1^1/_2$ to 2 times as much per horsepower as the truck and bus engines. They are assembled to make many 1800-hp and some 3600-hp locomotives, and the designs of more powerful ones are already developed both here and in Europe.

Furthermore, in England there are already at least three Diesel-engine models which are now available for private automobiles.

Since 1933, land transportation by Diesel power has emerged from the experimental stage into a major industry.

The writer heartily agrees with what the author has said about the cleanliness and ease of operation of the gasoline passenger automobile as compared with the Diesel and, in this country where the cost of fuel is not a great factor, it is difficult to see how the Diesel pleasure vehicle can advance very rapidly. In Europe where the fuel cost is a much more important factor, there will be a more rapid development.

The development of double-acting Diesel engines has not been unqualifiedly successful and there is still considerable hesitation about using them except for special applications. On the one hand, the simplicity and other advantages of the single-acting type influence many users to restrict Diesel plants to such unit sizes as are available in single-acting engines; and on the other hand, recent improvements in small high-pressure high-temperature steam installations tend to decrease both the size of the unit and the size of station in which the steam plant is generally more attractive than the Diesel plant.

The author is perhaps too conservative in his reference to rotative and piston speeds. At least one European manufacturer offers four-cycle single-acting engines of over 2000 bhp at 250 rpm and 1180 fpm piston speed for regular power plants. Small light-weight engines are now designed for piston speeds up to 2000 fpm.

In his reference to gas engines, the author referred incidentally to the use of sewage gas in sewage treatment works. The number of these installations is increasing and their size often reaches well over a thousand horsepower in a single plant.

As is pointed out in the paper, more attention is being paid in this country to the supercharging of four-cycle engines than ever before. Attention is also being given to the supercharging of two-cycle engines. After all, the question of supercharging versus using a larger engine is primarily economic. If the combination of first cost and operating cost for a given application is less with the supercharged engine than with the conventional engine, the user will naturally take the supercharged combination, but if there is no economic gain he will prefer the simpler conventional design.

Closely affiliated with supercharging, as the term is ordinarily used, is the problem of better scavenging and higher volumetric efficiency with resultant higher mean effective pressures in conventional engines both two- and four-cycle. These are important aspects of employing higher pressures for supplying air to engine cylinders which have frequently been overlooked.

In referring to the two-cycle engines, it should not be overlooked that there are successful engines of this type both here and abroad using a combination of ports and poppet valves in the heads; in some designs the scavenging air is admitted through poppet valves and in others the exhaust is through the poppet valves.

As to fuel, there has been a tendency on the part of manufacturers to produce engines designed to use fuel having almost inflexible characteristics and therefore compelling the user to buy always a particular fuel for which the engine is designed. The continued growth of the Diesel-engine business would be facilitated by a policy on the part of designers and manufacturers of producing engines of greater flexibility in the matter of fuel requirements.

In the matter of improvements in speed control, it should be pointed out that there are Diesel central stations now in regular operation supplying thousands of secondary clocks which keep time accurately, and that both control equipment and Diesel engines are now available which will give the uniformity both in voltage and frequency needed for the best laboratories and equal to that supplied by the big steam stations.

The paper refers to auxiliary combustion chambers which are quite the fad now for high-speed engines. Precombustion chambers also continue to be used. In spite of the extensive use of some of these special designs, the plain open combustion chamber such as is used in the Junkers engine continues to be favored by many designers of good engines.

The standardization of injection equipment has been carried forward successfully by a few of the leading manufacturers as to their own product but, in view of the wide variety of pump designs, the many methods of controlling the metering and injection, as well as the wide divergence in nozzles and injection timing and pressures, it can hardly be said that injection equipment as a whole has been standardized.

The writer has felt for a long time that the life of the light-weight high-speed engine would be less than that of the old heavy-weight conventional engine, but some figures that have come to his attention during the last several months have made him doubtful. Now it rather looks as if the higher-speed light-weight unit, due to better design, better utilization of materials, and other characteristics, will have a maintenance cost per unit of power rather less than that of the old conventional engine.

In the matter of fuel economy, there seems to be very little

difference between the two- and the four-cycle engines of equally good design. For instance, the Junkers engines have a fuel economy quite equal to that of any conventional four-cycle engine.

PAUL H. WILKINSON.[3] The writer would like to add a few supplementary comments to the excellent paper of Professor Degler, which it is hoped will be of value in substantiating that section dealing with the Diesel aircraft engine. The author refers to the use of the Diesel in daily service in Europe, and to its flight across the ocean. Data which have recently been received from Deutsche Lufthansa, the only major international air line in the world using Diesel-engined aircraft, confirms his statements and should prove interesting inasmuch as they represent the result of several years' practical experience with the engine under varying conditions of air-line operation.

One interesting set of figures gives the mileage flown with Diesel-engined aircraft on its air lines. Starting with 1634 miles in 1931, there has been a substantial increase. In 1937, a distance of 1,166,023 miles was flown with the Diesel in regular scheduled operation. For 1938, the mileage should show a still greater increase inasmuch as they have further expanded their activities with this type of power plant.

With regard to the number of hours flown, Deutsche Lufthansa's figures show that up to the end of 1937 a total of 46,767 hr had been flown with aircraft equipped with Junkers Jumo 204 and Jumo 205 Diesels. The number of these engines in active service at the end of 1937 was 48, apart from engines in reserve. The fuel consumption obtained was 0.35 lb per hp per hr under regular air-line operating conditions.

While at first the overhaul period for the Diesel aircraft engines was every 200 hr, Deutsche Lufthansa states that this period has now been increased to 350 hr which compares favorably with the 500-hr period recently attained with the gasoline engine. It must be remembered that the Junkers Jumo Diesel cruises at approximately 80 per cent of its total power output compared with about 60 per cent power output for the gasoline engine, which accounts for most of the difference between their overhaul periods.

According to the latest information on the subject, Deutsche Lufthansa has found the Diesel to be satisfactory for short-range, medium-range, and long-range flights. With regard to short-range flights, approximately 28,000 miles a week were flown with the Diesel in 1938 over air routes in Europe, none of which exceeded 425 miles in length. On some of these routes stops were less than 50 miles apart which effectively discounts assertions that the Diesel cannot be used to advantage on any but long-range flights. The equipment used for the most part comprised standard Junkers Ju-86 ten-passenger air liners powered with two 600-hp water-cooled Junkers Jumo 205-C Diesels.

Flights that are now considered to be in the long-range category for gasoline-engined aircraft, are within the medium-range category for Diesel-engined aircraft. A typical example is the weekly air-mail service operated by Deutsche Lufthansa between Bathurst in British Gambia on the west coast of Africa and Natal in Brazil. Since 1936 this ocean flight of 1890 miles has been flown with Diesel-engined aircraft. In 1938 four twin-engined Dornier Do-18 flying boats powered with 600-hp Junkers Jumo 205-C Diesels with a pay-load capacity of 800 lb have been carrying the mail across the South Atlantic on regular schedule.

Numerous long-range flights with Diesel-engined aircraft have been carried out by Deutsche Lufthansa across the North Atlantic. This has given people in the United States an opportunity to see the Diesel in regular scheduled service. In the course of these transatlantic flights they completed 48 nonstop crossings over the 2400-mile route between New York and the Azores, a 100 per cent record which far exceeded the achievements of the gasoline-engined airplanes of Pan American Airways, Imperial Airways, and Air France in this field of endeavor.

The total mileage flown by Deutsche Lufthansa's Diesel-engined mail planes in the course of their survey flights across the North Atlantic amounted to 152,360 miles compared with 27,720 miles for Imperial Airways, 24,370 miles for Pan American Airways, and 8600 miles for Air France over the transatlantic route. The fastest eastbound trip of one of the four-engined Blohm & Voss Ha139 seaplanes with its four 600-hp Junkers 205-C Diesels was made in 11 hr 53 min at a speed of 202 mph. Westbound, the fastest trip took 13 hr 40 min at a speed of 176 mph. These seaplanes carry a crew of four and have a pay-load capacity of 1210 lb between New York and the Azores.

For transatlantic operations in 1939, Deutsche Lufthansa is taking delivery of a number of four-engined Dornier Do-26 flying boats powered with 600-hp Junkers Jumo 205-C Diesels. It is their intention to fly these airplanes nonstop between New York and Lisbon, covering the 3400 miles between these points in about 17½ hours. A crew of four and a pay load of 2500 lb, consisting of five or six passengers and 1000 lb of mail, can be carried on these flying boats which will take off from the water as is customary for passenger craft. The Dornier Do-26 flying boat cruises at 193 mph, and has a maximum flight range of 5600 miles.

This information received from one of the World's leading air lines relative to the performance of Diesel-engined aircraft, should be sufficient proof that the aircraft Diesel can do all that is claimed for it. In view of the tremendous strides being made with the engine abroad, it is to be hoped that the United States government will not continue to jeopardize our progress in aviation by "restricting" low-powered Diesel aircraft engines such as the Guiberson, or by withholding financial aid from those who are competent to build large engines suitable for long-range commercial and military aircraft.

EDGAR J. KATES.[4] The author is to be congratulated upon his success in handling the difficult job that was assigned to him. In the writer's opinion, two additional items of information deserve to be recorded.

First, in the matter of fuel-oil standardization, the A.S.M.E. Oil and Gas Power Division is entitled to recognition as having originated the movement the recent progress of which the author cites. In June, 1928, G. H. Michler of the Standard Oil Company of New Jersey read a paper on Diesel fuel-oil specifications at a meeting of the Oil and Gas Power Division held at The Pennsylvania State College. The discussion of this paper resulted in the formation of a subcommittee of the Oil and Gas Power Division. The committee functioned actively and one year later, again at an Oil and Gas Power Division meeting at the same place, Wiley H. Butler presented a progress report of the A.S.M.E. Research Committee on Diesel Fuel Oil Specifications (the committee into which the original committee had been transformed). The cooperative society work now in progress is a direct outgrowth of that start made by this Division.

The second matter which the writer wishes to mention is in connection with aircraft Diesel engines. Without attempting to detract in any way from the success and commercial progress that has been made by the Junkers engine, one should not forget that the Packard Company in this country designed and built a Diesel aircraft engine by means of which the world's first long-distance Diesel airplane flight was made in May, 1929. The author mentions the present-day American designers who have

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[4] Consulting Engineer, New York, N. Y. Mem. A.S.M.E.

developed Diesel aircraft engines of considerable promise. This would be an appropriate time also to pay tribute to the memory of Captain Woolson of the Packard Company who was responsible for the development of that first American aircraft engine and who died in the cause before his work was done.

H. W. BUKER.[5] With reference to the statement about "piston speeds of from 1500 fpm in small engines to 1000 fpm in large engines," the writer believes that 1200 fpm would be more nearly correct for the piston speeds of large engines. For example, the 22,500-hp engine in Copenhagen has a piston speed of 1130 fpm and an outside limit of 1200 fpm on big engines is not out of the way at the present time.

L. N. ROWLEY, JR.[6] With a paper as authoritative and comprehensive as this, it is difficult either to correct statements or to amplify them. However there are one or two small points that might bear further discussion.

First, with reference to the data on two- and four-cycle engines: The author states that "In sizes between 300 and 1000 bhp, the use of the two- or four-cycle engine is a matter of choice; many builders make both types." This might be construed to mean that in this size range, both types are more or less equally used. However, an analysis of annual tabulations of stationary Diesel-engine installations for 1936, 1937, and 1938 shows that the four-stroke cycle is used about twice as often as the two-stroke, while below 300 bhp the ratio is even greater. This is not meant as a recommendation for either type; it is merely a statement of fact based on present practice.

Mention of the combination of Diesel engines with existing steam plants is extremely interesting. Of course this combination may be equally effective in a newly designed plant. In either case the Diesel is a welcome aid to the power-plant designer faced with the problem of matching prime movers to power and heating or process loads. By adding to the number of possible combinations, it makes it easier to meet specific cases effectively and several Diesel-steam plants are demonstrating this fact at the present time. On the other hand, such a combination means a heavier investment in many cases and this added cost must be compared with possible operating economies before coming to a decision.

As a general thing more attention is given today to the accessory equipment that goes to make up a complete Diesel plant; technical developments in this field have kept pace with engine improvements. When accessory equipment is overlooked or given inadequate consideration, it is probably because the internal-combustion engine is considered as practically a complete power plant in itself. Present indications are that this kind of thinking is passing as it becomes generally recognized that careful selection of accessory equipment pays in operating efficiency and freedom from troubles.

Outstanding perhaps, are improvements in jacket-water cooling equipment. Cooling-tower size has been decreased for a given capacity and special designs for indoor or other close-quarter installations have been developed. The evaporative condenser, second cousin to the cooling tower and widely used in air-conditioning practice, has been applied to jacket-water cooling, with compactness and low first cost listed among its advantages.

Indicative of a trend away from past hit-or-miss methods of operation is the increasing use of instruments, particularly pyrometers.

Possibly stimulated by the efforts of the Oil and Gas Power Division to accumulate reliable data on performance and costs, plant operation is becoming more systematic. Maintenance is scheduled and maintenance needs and costs are carefully studied. More operating engineers are keeping comprehensive records of both cost and performance and these records are used as a guide to purchasing, operation, and maintenance.

M. P. CLEGHORN.[7] The author deserves commendation for the excellent paper he has presented. It gives a splendid review of the advances in this field in the last few years. It would seem, however, that in a discussion of this type mention might well be made of some of the places where advance has not been made as it should have been.

Much has been said regarding lower fuel consumption and better operating characteristics, but except for one brief sentence no mention was made regarding engine improvements for burning the low-grade fuels. As a matter of fact, engines are demanding better fuels which in turn mean more expensive fuels and this reacts against the present advantage of the Diesel engine in the small power plant.

Every time a new advance is made in the quality of a fuel, the engine manufacturer steps up the performance of his engines to take advantage of this new fuel, even though the fuel cost is greater. The cry for power-plant-Diesel performance to approach that of the automotive Diesel has led the engine builders to ignore the caution they should consider against still further narrowing the field of possible fuels.

The mere fact that fuels of high quality are becoming available generally for vehicle engines makes the engine purchaser feel that there will always be an adequate supply of such fuel and that the price should remain stable. But little account is taken of the probability that the demand for high-grade fuels will be greater for transportation needs and may overshadow the demand for the low-cost fuels for power plants.

The chemistry of fuels has already reached the point where practically all the low-grade stocks can be transformed into high-grade fuel, with a further curtailment of the supply of heavier grades. This can only result in an increased price charged for the lower-grade fuel. Further use of gas oils in furnaces and for household heating brings this fuel into a favorable sellers' market with a continuous trend toward higher prices. Any improvement therefore in the ability of the power-plant Diesels to handle low-grade fuels will relieve the pressure for high-grade fuels and slow up the gain in price.

At the present time high-grade Diesel fuel for trucks and busses is selling at retail in the Middle West for from 11.5 to 12.1 cents per gallon. Since the two main advantages of the Diesel engine are lower fuel consumption and lower price per gallon, the manufacturers of these engines may find themselves in an embarrassing position unless they can utilize satisfactorily the low-grade lower-priced fuels.

In view of these trends it would seem that the time is ripe to urge further development work on engines which will use fuel oils below the level of furnace oils, and to urge caution in choosing and installing power-plant engines having fuel requirements approaching those of the automotive Diesel. The rise in price of Diesel fuel oils, predicted for many years, apparently is facing us now.

O. D. TREIBER.[8] The summary of progress for the last five years in oil and gas power has a significant phase which seems to justify emphasis. This is the advent of the high-speed Diesel engine into American manufacturing. It is significant because American production methods with attending reduction in unit

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[6] *Power*, New York, N. Y.

[7] Professor, Head of Mechanical Engineering Department, Iowa State College, Ames, Iowa. Mem. A.S.M.E.

[8] Hercules Motors Corporation, Canton, Ohio. Mem. A.S.M.E.

costs open vast fields of applications heretofore impossible or impractical because of cost, weight, and size. Large-quantity low-unit-cost production requires a substantial investment in tooling together with carefully organized operations for quick and accurate duplication of parts and their assembly. However it makes economically possible the use of these engines in automotive equipment, tractors, and a myriad of industrial applications where the owners can enjoy the inherent low operating cost of a Diesel engine.

Low first cost is made possible not only by production methods but also by ratings at higher speeds which allow of reductions in weight and size to values which are comparable with gasoline engines. In fact the company with which the writer is associated has a line of four models and eight sizes all of which are dimensioned for interchangeable mountings with gasoline engines of corresponding sizes and horsepower.

Some idea of the significance of this important contribution of a "new tool" to the power-using world may be had from some comparisons. About fifteen years ago Diesel engines were sold for marine and stationary purposes at prices about a hundred times those of high-production gasoline engines stripped of accessories. Ten years ago Diesel prices were down to from 50 to 60 times gasoline-engine prices. The differential would buy gasoline for the life of many installations. Now with high-production tooling, prices of high-speed Diesels have been reduced to from a fifth to a tenth of the last low figure and to from 5 to 10 times those of gasoline engines.

Consequently, high-speed Diesel engines are now available for a great many applications not economically possible in the past, such as in trucks, tractors, motorboats (both pleasure and commercial), rail cars, industrial locomotives, power shovels and all kinds of excavating machinery, farm machinery, pumping plants, and electrical generating units. Even multiunit powerhouses of large aggregate capacity are feasible and practical.

It might be argued that high-speed engines will not last as long as the slow-speed units. This is granted (the higher the speed the shorter the life) but with the present knowledge of metallurgy and the mechanics of internal-combustion engines, their life is being lengthened sufficiently, it is believed, to warrant the use of high-speed Diesel engines even where requirements for continuity of power are most exacting.

The advantages are not only a great reduction in original investment but low costs of repairs and replacements. Furthermore the time required for overhauling and rebuilding is obviously much less on either a basis of unit power or engine unit than on large slow-speed engines.

There is no intention herein to depreciate the importance of large Diesel engines. They will continue to fulfill a requirement distinctly outstanding. But it is desired to call attention to the importance of the high-speed Diesel engine as a new power tool.

E. S. Dennison.[9] The writer would like to amplify the portion of the paper in which the author referred to the advantages that have accrued from the use of auxiliary combustion chambers. With small engines using the four-stroke cycle, the use of such chambers has made it possible to raise the mean effective pressure for rating the engine from about 80 lb, as it was some years ago with an open combustion chamber, to between 105 and 110 lb, now an almost equally conservative figure. It has also enabled the cylinder pressure in the high-speed engine to be brought down in one case at hand to a little under 700 psi whereas with open combustion chambers that pressure is often 1000 psi. It has enabled the pressure in fuel-injection systems to be brought down from between 6000 and 8000 psi for direct injection, to pressures from 4000 down to 2000 psi with an auxiliary combustion chamber. Furthermore it has made the timing requirements much less rigorous than they were, so much so that it is often possible to dispense with any adjustment of timing to correlate with speed.

Now in contrast to all that the auxiliary combustion chamber has done for the small high-speed four-cycle engine, it is well to question what it has done for the small two-cycle high-speed engine. In so far as the writer knows, auxiliary chambers are not now and have not been successfully applied to the latter and an interesting question today is whether they can be. As is well known, there is the unfortunate tendency of exposed metal parts in a two-cycle combustion chamber to burn away, and whether that will prevent the use of auxiliary chambers in these engines is an open question. The writer is a little pessimistic on that point.

Now if the two-cycle engine must use an open combustion chamber with high cylinder pressure, high injection pressure, and exacting requirements as to timing, the four-cycle type is left with a decided edge from the maintenance standpoint. As long as materials are used that are responsive at all to heat or to pressure and that have any tendency to wear, such wear is going to be more effective in the two- than in the four-cycle engine. Consequently the writer believes that although the small two-cycle engine may have an advantage as to weight perhaps and is about equal in economy, nevertheless, from the operator's standpoint the four-cycle will continue to have an advantage. These comparisons do not apply at all in the case of large engines where both types use an open combustion chamber.

Author's Closure

The author wishes to thank each of the gentlemen who have by their discussions contributed further information of interest to make the paper more complete, more interesting, and better historical material for future use.

He was glad to have Mr. Dennison's defense of the four-cycle high-speed Diesel engine with an auxiliary combustion chamber in comparison with the two-cycle engine and direct injection, and also to have Mr. Rowley's data showing that the relative use of the four-cycle and the two-cycle engine in that particular class was a two-to-one ratio rather than an even break.

Concerning preliminary papers on Diesel fuel-oil specifications and some of the related factors that engineers and engine users had been talking about over a period of years in this so-called preparatory fuel work that was being done, the author still believes that there was nothing definite accomplished along the lines of fuel standardization before the year 1933. However, the Oil and Gas Power Division of the A.S.M.E. does deserve much credit for having sponsored papers on this subject and contributing valuable suggestions during the preliminary years of uncertain Diesel-fuel requirements.

The discussion of Mr. Kates concerning the Packard engine was noteworthy, but the author is not personally familiar with developments previous to 1933. It might be said that the Packard was the forerunner of the Guiberson engine because the latter is also a radial aircraft type, but with many new features. Captain L. M. Woolson[10] indeed deserves considerable credit for his Packard Diesel aircraft-engine development and the author is glad that his name was mentioned in this meeting.

It is interesting to note that each of the discussers of this paper said something about fuels and their combustion. The author would like to reiterate in closing that these are the two important factors that designers, manufacturers, and users will continue to write and talk about in years to come, the two most important considerations in the progress of oil and gas power.

[9] Engineer in charge of Diesel-engine development, Electric Boat Company, Groton, Conn. Mem. A.S.M.E.

[10] "The Packard Diesel Aircraft-engine," by L. M. Woolson, *S.A.E. Journal*, April, 1930, and September, 1930.

Attack on Steel in High-Capacity Boilers as a Result of Overheating Due to Steam Blanketing

By EVERETT P. PARTRIDGE[1] AND R. E. HALL,[2] PITTSBURGH, PA.

In recent years a number of failures have occurred in certain portions of boilers operating in the pressure range near 1400 psi. Many of these failures have occurred under conditions such that oxygen corrosion or scale could not be considered as the cause. Characteristically the tubes have shown internal attack along the top, generally with definite grooving and the production of black magnetic iron oxide.

The fundamental cause of this type of failure is steam blanketing. In a tube which is only slightly inclined to the horizontal, the steam inevitably tends to rise to the top and move along it. This introduces a considerable resistance to the flow of heat from the tube wall to the fluid, and causes the top of the tube to become more or less seriously overheated. Boiler water of normal alkalinity passing through the tube and touching the overheated surface is concentrated to an excessive degree, with resultant attack on the steel. As has been shown by Berl, the rate of oxidation of steel by water at high boiler temperatures is greatly accelerated by increase in concentration of sodium hydroxide.

Condensed case histories of occurrences of this type of failure in a number of high-pressure boilers are presented. These histories include failures in ash screens in dry-bottom furnaces, floor tubes in slag-bottom furnaces, and slag-screen and top-row tubes in straight-tube cross-drum boilers.

From certain of the case histories it is evident that the dissolution of the steel along the top of steam-blanketed tubes may be stopped or greatly retarded by eliminating caustic alkalinity from the boiler water. This, however, does not affect the fundamental cause, and may merely postpone trouble, as indicated by other case histories in which steam blanketing has resulted not in dissolution of the steel, but in serious cracking.

It is concluded that the best chance of obviating the difficulties which may result from steam blanketing is so to design the surfaces exposed to high rates of heat input that steam blanketing cannot occur, or where slightly inclined tubes must be used, to protect them so that the rate of heat input will be sufficiently low to avoid excessive overheating of the metal.

IN THE LAST few years, a number of new boilers in the 1200 to 1400-psi range have lost tubes located in positions inclined at only a small angle to the horizontal. These failures have occurred in the majority of cases under conditions which precluded ordinary scale formation or oxygen corrosion, and have been characterized by a tendency for the tubes either to waste away internally along the top or to crack in this region, without excessive deformation.

This type of failure has been attributed to a variety of causes, but no systematic correlation of the information available from plant experience has yet appeared. It is the purpose of this paper to present such a correlation of a number of typical case histories in the hope that the problem will thereby be clearly defined, so that appropriate steps may be taken to meet it.

A paper such as the present one would have been impossible without the cooperation of the engineers and executives of the various companies, who in many cases have released data and information hitherto unavailable for publication, even when, as in a few cases, they disagreed with the interpretation made by the writers. The factual statements are, it is hoped, beyond argument, although the discussion based upon them is freely offered for criticism.

Certain concepts essential to a proper appreciation of the details in the case histories are introduced first, and after the experiences at the individual plants have been presented, a comprehensive and consistent interpretation of the observed facts is attempted.

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NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

FUNDAMENTAL CONCEPTS

When heat is transmitted to a vertical or highly inclined tube in a boiler at rates ranging up to 100,000 Btu per sq ft per hr characteristic of contemporary design, no difficulty is ordinarily encountered. The steam bubbles tend to separate immediately from the tube wall and the highly turbulent flow of steam and water up through the tube maintains a high surface conductance of heat from the metal to the boiling water. No actual data covering the high-pressure range are known to the writers, but at atmospheric pressure the surface conductance for boiling water may vary from 1000 to as high as 4800 Btu per sq ft per hr per deg F. It is probable that, at the higher pressures also, the resistance to the flow of heat from the steel tube to the water is of the same small order of magnitude as the resistance of the steel wall of the tube itself. The temperature of the inside of the wall accordingly does not run much above that of the water in the tube, perhaps exceeding it by 100 F under the most severe conditions.

Film Boiling

At extremely high rates of heat input in laboratory apparatus, it is possible to produce on the heat-transfer surface a continuous layer of vapor which acts as an insulator and accordingly increases the temperature drop from the metal to the water. The experiments of Sauer, et al. (1)[3] at atmospheric pressure show, however, that the maximum rate of heat transfer corresponding to the

[3] Numbers in parentheses refer to the Bibliography.

transition from ordinary to film boiling for water at an iron surface was 413,000 Btu per sq ft per hr. McAdams[4] has suggested that film boiling is not likely to occur in an actual boiler even in the portions of the heat-absorbing surface exposed to the most severe radiation.

Steam Blanketing

Even though film boiling may not constitute a problem in steam generation, the insulating effect of a blanket of steam must be considered in the case of tubes which are inclined at only a slight angle to the horizontal, such as the ash screens in dry-bottom furnaces and the floor tubes in slag-bottom furnaces of contemporary design. In such tubes, the heat transfer is downward to the top of the tube, while all of the steam generated in it inevitably tends to rise and move along its ceiling.

Even with moderately high heat input, the steel at any given point is probably wetted frequently by intermittent splashing of water against the top, and its temperature is thereby kept comparatively low. High rates of radiation to a tube may, however, produce an almost continuous steam blanket, with a resultant great increase in temperature. When water does strike against the overheated metal, a sudden drop in temperature would be anticipated.

Considering tubes in various positions, from the horizontal to the vertical, it is obvious that increase in the inclination will tend to make less probable the development of a nearly continuous steam blanket. From another viewpoint, the heat input necessary to produce an overheating of 500 F in the wall of a tube will vary progressively from some lower value for a horizontal tube up to the probably much higher value necessary to establish film boiling in a vertical tube.

Lack of data for high boiler temperatures and pressures precludes more than a rough estimate of the temperature conditions set up in a steam-blanketed tube. However, the highest value measured by Poensgen (2) for the surface conductance from steam at 100 psi and 412 F to the wall of a 3.75-in. pipe at 388 F was 70.5 Btu per sq ft per hr per deg F. If an admittedly long guess is made that the surface conductance might be 150 for heat transfer from an ash-screen tube to steam at 1400 psi and 590 F flowing along its top, then a heat input to the top of the tube of 75,000 Btu per sq ft per hr would raise the temperature of the inner wall 500 F above that of the steam in equilibrium with water in the tube.

The difference in temperature distribution resulting from a surface conductance of 150 to steam in a blanketed horizontal tube on one hand and of 1500 to boiling water in a vertical tube on the other hand is shown graphically in Fig. 1. The temperatures indicated, even though obtained by simplified calculation from assumed values, are probably of the correct order of magnitude.

Change in Microstructure Due to Overheating

If the top of a tube be actually overheated as suggested in Fig. 1 for a sufficiently long period of time, a definite change will be produced in the microstructure of the steel; the iron carbide originally present in the customary lamellar grains of pearlite will form globular particles of cementite.[5] Table 1, from the work of Bailey (3) shows how the time necessary to develop this spheroidized structure increases with decrease in the temperature to which the steel is subjected. The lowest temperature at which spheroidization may develop over a long time is unknown. On the basis of creep tests, White, Clark, and Wilson (4) state that spheroidization begins to be noticeable

[4] Private communication from W. H. McAdams.

[5] The change in structure is well illustrated in Fig. 5.

TABLE 1 TIME REQUIRED TO SPHEROIDIZE ANNEALED LOW-CARBON STEEL (3)

Temperature, C	Temperature, F	Time, hr
600	1112	200
625	1157	75
650	1202	20.5
675	1247	7.5
700	1292	3

in plain carbon steel after about 4000 hr at 1000 F when the stress is sufficiently great. A spheroidized region in a boiler tube may therefore be regarded as evidence that the metal temperature has exceeded this level.

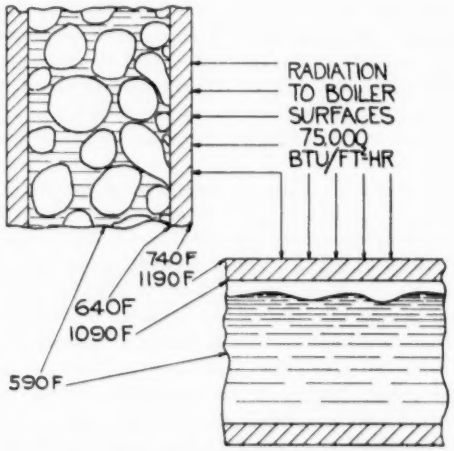


FIG. 1 COMPARISON OF TEMPERATURE CONDITIONS IN A VERTICAL AND IN A STEAM-BLANKETED HORIZONTAL TUBE

(Surface conductances of 1500 Btu per sq ft per hr per deg F from steel to boiling water and of 150 from steel to steam have been assumed.)

Excessive Local Concentration of Boiler Water

While steam blanketing tends directly toward overheating, indirectly it produces an equally serious effect. Normally, the concentration of dissolved substances in a boiler water will be low, but if this water is thrown intermittently against the excessively hot ceiling of a steam-blanketed tube, local concentration to any imaginable degree is possible.

Very soluble substances may be deposited as solids either on the top or at the water line on the sides of such tubes, decreasing correspondingly the concentration of these substances in the main part of the boiler water. The "hiding out" of phosphate and other soluble salts when boilers are operated at high ratings, followed by the reappearance of these substances in solution when the steam output is lowered has been observed in a number of boilers placed in service in recent years. The most potentially serious result of localized excessive evaporation is the high concentration of sodium hydroxide developed from a boiler water containing only 50 or 100 ppm of this substance.

Effect of Sodium Hydroxide on Oxidation of Steel by Water

What a concentrated solution of sodium hydroxide at 590 F will do to steel has been shown by Berl and van Taack (5) whose data are reproduced in Fig. 2. The vertical scale represents the extent to which carefully sized steel powder was oxidized when agitated for 7.5 hr in contact with solutions of sodium hydroxide of the concentrations shown along the horizontal scale. A minimum in the attack at 0.7 g per liter or 700 parts of NaOH per million parts of water is followed by a progressive increase at higher concentrations.

That water, even when free of dissolved oxygen is a powerful

oxidizing agent with respect to steel is not always recognized, because as soon as the reaction

$$3Fe + 4H_2O \rightarrow Fe_3O_4 + 4H_2$$

commences, it is slowed down to an almost infinitesimal rate by the protective coating of Fe_3O_4 which is one of the products. Because the role of dissolved oxygen in helping to destroy the protective effect of this oxide coating has been recognized, mechanical deaeration has been so perfected in recent years that water containing only 0.005 ml per liter may be fed to boilers, and the use of chemical scavengers such as sodium sulphite and ferrous hydroxide has been developed to remove even this last trace. Every pound of water in a boiler, however, contains 0.89 lb of combined oxygen which is available to oxidize the steel surfaces whenever and wherever the water can penetrate the thin barrier of oxide.

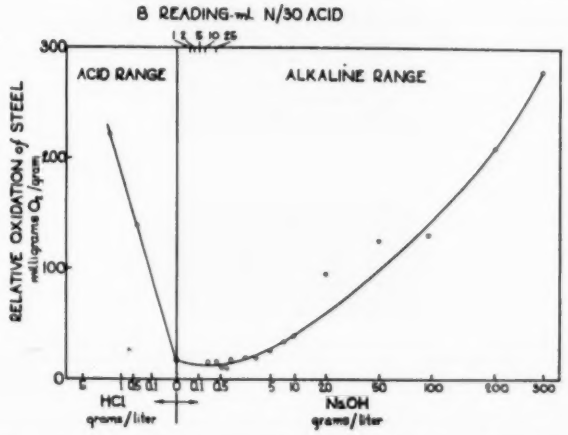


FIG. 2 EFFECT OF HYDROCHLORIC ACID AND OF SODIUM HYDROXIDE ON THE OXIDATION OF STEEL BY REACTION WITH WATER AT 590 F (FROM BERL AND VAN TAACK.)

(In each test five grams of sized steel powder was agitated in contact with solution in a bomb for 7.5 hr, and the extent of the reaction was measured by the degree of oxidation of the steel. Concentrations are plotted on a cube-root scale in order to show adequately the data over the wide range covered.)

The retarding effect of low concentrations and the accelerating effect of high concentrations of sodium hydroxide on the reaction of water with steel at high temperatures is believed to be due largely to the effect of this substance on the protective oxide, which may be regarded as a membrane full of minute pores through which water is always diffusing to the metal beneath to continue the reaction. A dilute solution of sodium hydroxide apparently helps to develop an oxide film which is less permeable, while a more concentrated solution tends to open it up, and a very concentrated one actually destroys it by forming soluble iron complexes.

Hydrogen Brittleness

Hydrogen, the second product of the reaction of water with steel, when produced in the atomic state at the surface of the metal, diffuses readily into it, causing a decided loss in ductility without decreasing the tensile strength. When high concentrations of sodium hydroxide produce accelerated attack of water on steel, it is therefore reasonable to expect that the steel may become more brittle, in the same manner that it does when pickled in acid (6).

Cracking as a Result of Thermally Induced Stresses

If the ceiling of a tube is repeatedly first overheated, because of the development of a steam blanket, and then suddenly quenched by water striking the surface, the reversed stresses set up in the steel may well lead to cracking of the stress-corrosion type.

Imperfect Circulation of Boiler Water

Steam blanketing may develop not only in nearly horizontal tubes receiving heat by direct radiation from the furnace, but also in tubes which, by reason of the sometimes unpredictable circulation in boilers, carry an inadequate flow of water. In straight-tube cross-drum boilers, for example, it is not uncommon to find the top-row tubes steam-blanketed near the uptake header, with definite evidence of a water line or even rope scale on the sides of the tubes.

CASE HISTORIES

From the preceding, it is evident that steam blanketing of tubes may lead not only to overheating, but also to accelerated attack on the steel due to the excessive concentration of normally alkaline boiler water, or to cracking by stress corrosion. These difficulties are not new. However, few case histories have been reported as completely as those of a 450-psi cross-drum boiler by Hanson (7), of 45-atm cross-drum boilers at Merseburg by Ziegler (8), and of the first 100-atm bent-tube boilers at Mannheim by Marguerre (9). A number of specific cases in which the authors believe that steam blanketing has constituted a problem will therefore be presented.

SLAG-SCREEN TUBES IN A STRAIGHT-TUBE CROSS-DRUM BOILER

South Amboy Station

The South Amboy Station of Jersey Central Power and Light Company comprises three 1400-psi straight-tube cross-drum boilers which went into service successively between Oct. 1, 1930 and Feb. 14, 1931. In these boilers, which are 40 tubes wide, the even-numbered tubes of the bottom row were originally bent downward, as indicated in Fig. 3, to form a slag screen, and the slope of the short, nearly horizontal section was 1.5 deg. Failure of these screen tubes along the top of the nearly horizontal section near the bend upward has occurred at intervals throughout the history of the station, while failure of the alternating straight tubes in the bottom row has been infrequent.

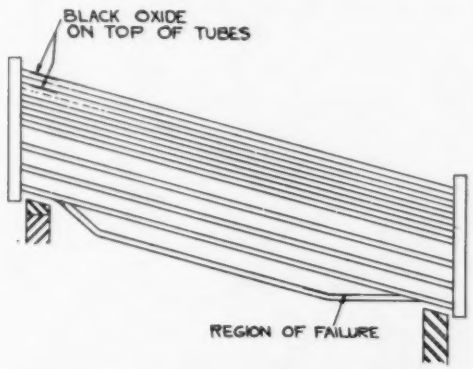


FIG. 3 ELEVATION OF GENERATING TUBES AT SOUTH AMBOY STATION

History of Failures. Interpretation of the early records is somewhat uncertain, because both oxygen corrosion and scale formation may have contributed to tube failures. When the plant started up and for some time thereafter, the pH of the boiler water was low and the oxygen content of the feed rose intermittently to 0.1 ml per liter, while considerable amounts of raw well water were introduced as make-up. After slightly less than two years of service, it was necessary to replace the en-

tire bottom row, both straight and drop-leg screen tubes, in boiler 1 toward the end of July, 1932, following individual failure of six tubes during the spring of that year.

Sporadic failure of screen tubes during the period from 1933 to 1936, always along the top of the nearly horizontal section, led to the decision to increase the slope of this section from 1.5 to 3 deg. On Apr. 25, 1936, all of the drop-leg tubes in boiler 1 were replaced with tubes rebent to give this increased slope. The 3-deg tubes were then installed in boiler 3, five on June 18, one on July 6, and the remaining fourteen on Aug. 23, 1936; and in boiler 2, two on June 9 and one on Oct. 26, 1936, and the entire twenty tubes on March 20, 1937.

In spite of the increased slope, replacement of tubes continued in the same discouraging manner, as indicated in Table 2. On

TABLE 2 TUBE REPLACEMENTS SUBSEQUENT TO INSTALLATION OF COMPLETE NEW SLAG SCREENS WITH 3-DEG SLOPE IN NEARLY HORIZONTAL SECTION

Date	Boiler	Tube	Time in service, months
8/13/37	3	38	11.7
1/26/38	3	16	17.0
		24	17.0
3/28/38	1	6	23.0
		8	23.0
4/26/38	1	4	24.0
7/1/38	3	16	5.0
		18	22.2
7/15/38	1	6	3.7
		8	3.7
8/25/38	1	4	4.0

July 1, 1938, tubes with a slope of 6 deg along the nearly horizontal section were installed as Nos. 16 and 18 in boiler 3. Shortly thereafter, however, it was decided to eliminate the drop-leg tubes altogether, and to replace them by straight tubes. This change was made in boiler 3 on Aug. 5, 1938.

Evidence of Overheating. One factor influencing the decision to abandon the slag screen was a study of tubes 6 and 8 removed from boiler 1 on March 28, 1938. Tube 6, which had developed a leak, and tube 8 which had not cracked but which was in a state of incipient failure, both showed typical attack along the ceiling of the horizontal section near the bend upward. The tube wall

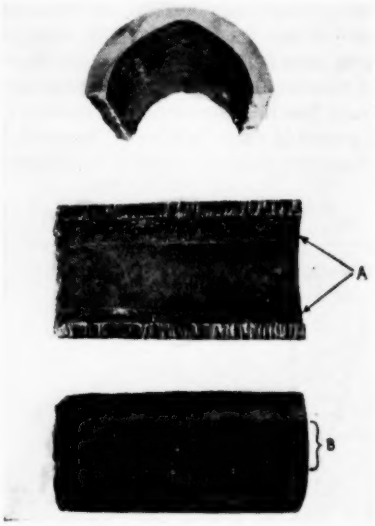


FIG. 4 APPEARANCE OF TOP OF NEARLY HORIZONTAL SECTION OF SCREEN TUBE FROM A 1400-LB STRAIGHT-TUBE CROSS-DRUM BOILER

(*A*, Water lines and corrosion grooves along ceiling of tube; *B*, red band on external surface of top half of tube.)

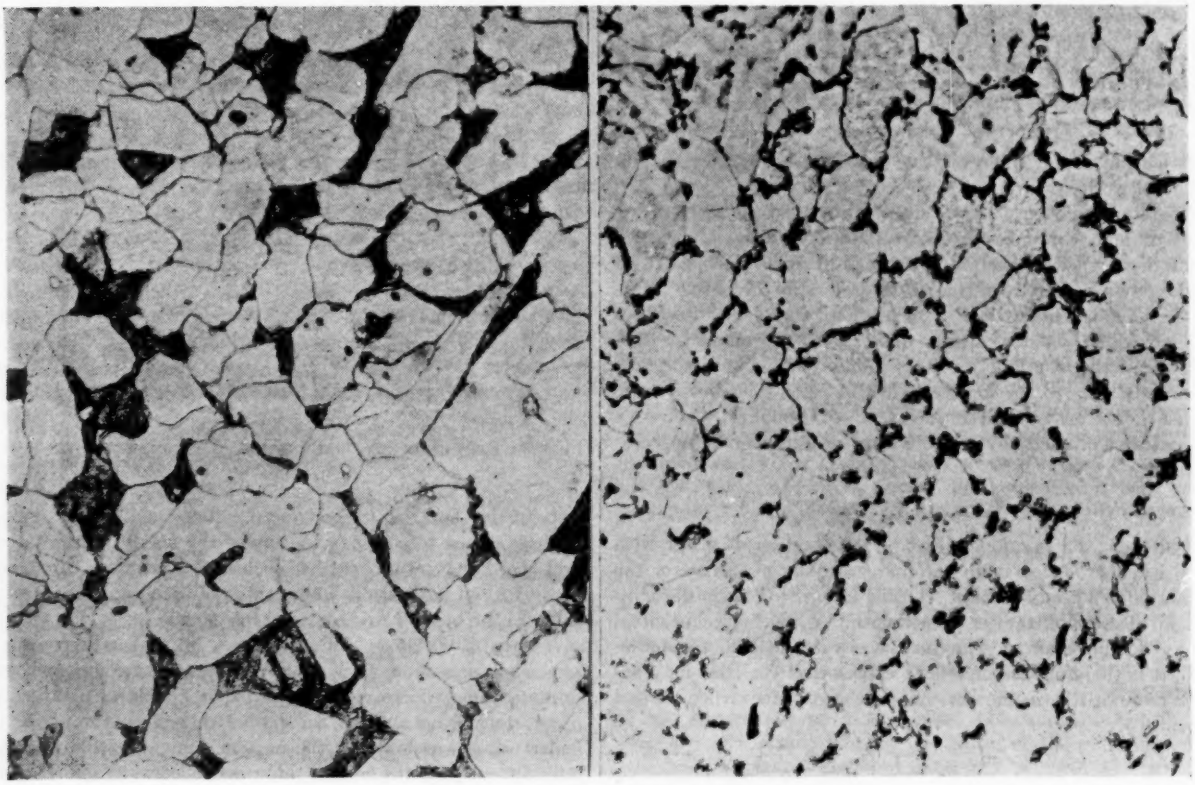


FIG. 5 MICROSTRUCTURE (×500) OF SPECIMENS OF TUBE

(Left: From side of tube, showing normal appearance. Right: From top of tube, showing severe overheating.)

had been thinned down as indicated in the cross section of Fig. 4, and at each side of the band of attack, which was covered by a thin relatively smooth coating of black magnetic iron oxide, there were narrow sharp grooves. The rest of the internal surface showed the typical appearance of old scale in the process of disintegration.

That the tops of the horizontal sections had been overheated was suggested by an external band of red oxide about 3 in. wide, which tapered off along the upturned portion of the tube. Conclusive evidence to this effect was obtained by metallographic examination of specimens cut respectively from the side and from the top of tube 8 at a point in the horizontal section 6 in. from the upward bend of the tube. While a microstructure typical of a seamless tube was observed in the side wall, extreme breakdown of pearlite and the formation of rounded particles of cementite characterized the top of the tube, as shown in Fig. 5.

Boiler-Water Alkalinity. An attempted correlation between the monthly average values for alkalinity and pH of the boiler water on one hand and replacement of screen tubes on the other

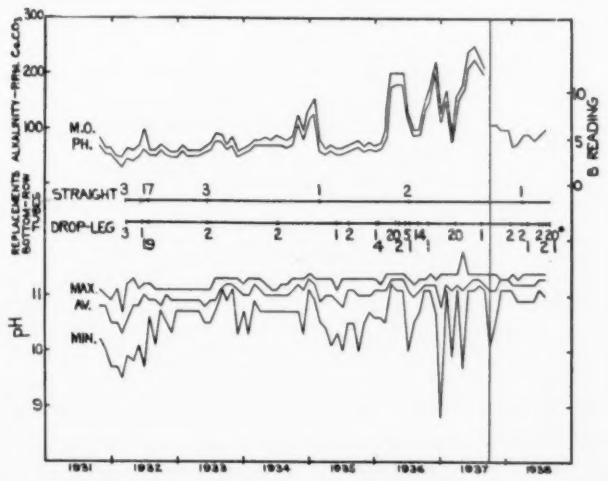


FIG. 6 RECORD OF SCREEN-TUBE REPLACEMENTS AND OF MONTHLY AVERAGE VALUES FOR ALKALINITY AND pH OF BOILER WATER

(Alkalinities are expressed as determined by titration to end-points of phenolphthalein (PH.) and methyl orange (M.O.), and by titration to end-point of phenolphthalein in the presence of an excess of barium chloride (B reading). While the results of the first two methods of titration of a boiler water do not allow calculation of the concentration of sodium hydroxide, multiplication of the B reading by 13.3 gives the approximate value for NaOH in ppm.)

in Fig. 6 suggests that the considerable increase in alkalinity of the boiler water early in 1936 may have accentuated the attack along the ceiling of the screen tubes and necessitated the immediate replacement of tubes which had been losing metal slowly during the preceding years. The maintenance of a moderate and carefully controlled alkalinity subsequent to Oct., 1937, did not, however, obviate the continued failure of the new screen tubes installed in all three boilers during 1936 and 1937.

That the replacement of the drop-leg tubes by straight tubes is likely to solve the problem is indicated in Fig. 6 by the fact that, while more than eighty of the former had been replaced since 1932 and prior to August, 1938, only seven of the straight tubes alternating with them in the bottom row had failed during this period, and most of these failures were due to external cutting by water and steam from leaking drop-leg tubes.

Attack in Upper-Row Tubes. While up to the present it has constituted a minor problem in comparison with the failure of screen tubes, some attack has also occurred, as indicated in Fig. 3, along the top internal surfaces toward the uptake header in the two upper rows of tubes in the boilers at South Amboy. During an inspection of boiler 3 in August, 1938, it was estimated that not more than 0.02 in. of metal had been removed. The attack apparently was more pronounced in the row next to the top than in the top row, but showed in all tubes in both rows. Only one tube in these rows has been replaced during the history of the plant, tube 16 of the top row of boiler 3 on Sept. 11, 1935, but it is apparent that the same condition exists here, though to a less serious degree, as in other straight-tube cross-drum boilers discussed in this paper.

TOP-ROW TUBES IN A STRAIGHT-TUBE CROSS-DRUM BOILER

San Antonio Station B

On Aug. 28, 1931, two top-row tubes failed in the 1250-psi boiler at Station B of San Antonio Public Service Company (10). This straight-tube cross-drum boiler had been operated intermittently for only about seven months, with B reading[6] held generally between 1 and 4, and phosphate between 20 and 60 ppm of PO_4, although occasional higher values were observed.

Failure occurred along the top of the tubes, near the uptake header. In addition to the two actual failures in the top row, many tubes in the upper four rows had been damaged, as attested by the considerable quantities of magnetic iron oxide in the top half of each tube. The attack was localized toward the right side of the tube bank; the iron oxide was thickest in the top-row tubes of headers 24, 25, and 26, and thinned out in 27 and from 23 back to 13, with practically none in 12. The tubes in the second row were in about the same condition as those in the first. In general, the oxide seemed to be thickest about 3 or 4 ft back from the uptake header, thinning out progressively toward the header.

In addition to the magnetic iron oxide still in place along the tops of the tubes, large quantities of this substance in the form of powder were found lying in the central portions of the waterwall

[6] B reading is the number of ml of $N/30$ acid required to titrate to the phenolphthalein end-point 100 ml of a boiler-water sample to which an excess of barium chloride has been added to precipitate buffer salts such as carbonate and phosphate.

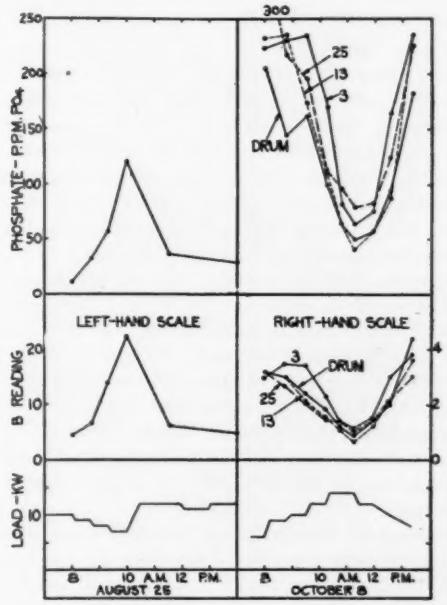


FIG. 7 INVERSE VARIATION OF BOILER-WATER CONCENTRATION WITH LOAD AT SAN ANTONIO STATION B

(Multiplication of the B reading by 13.3 gives the approximate value for NaOH in ppm.)

headers, apparently carried there and dropped by the circulation of the water.

"Hiding Out" of Soluble Compounds. For some time previous to the failure, the tendency of phosphate and alkalinity to disappear from the boiler water had been noted. Tests on Aug. 25, three days before the tubes let go, indicated that, when no addition of conditioning chemicals was made, the concentrations in the boiler water tended to vary inversely with the load. Although it did not seem credible at the time, the data, reproduced in Fig. 7, now may be interpreted as definite evidence of excessive local evaporation to substantial dryness in some portion of the boiler during operation at the higher loads, followed by return of the deposited substances to solution during subsequent operation at lower load. This hiding out of dissolved substances was observed repeatedly during later tests when samples were taken not only at the original sampling point in downtake header 3, but also in downtake headers 13 and 25, and from a perforated sampling line running the length of the drum. The data for Oct. 8 in Fig. 7 show how the concentrations changed in the same general manner with load at all four sampling points. As the load was raised progressively, both the alkalinity and the phosphate decreased; when the load was dropped they again increased.

Correlation Between Tube-Wall Temperature and Oxidation. Loss of dissolved substances from the boiler water during the test on Oct. 8 did not produce any consistent corresponding increase in the temperature of the top-row tubes as measured by thermocouples. However, during the period from Oct. 1, when the boiler was first returned to operation following the tube failures, until Oct. 16, there was a gradual increase in the tube-wall temperatures, with the highest values developing in the region where the heaviest formation of oxide had previously been observed. Inspection at this time showed that the oxide was developing as before, and that the largest amounts were in the tubes of header 23. Correlation of the temperature measurements with the observed condition of the tubes showed that the deposit of black oxide had accumulated only in those tubes which had exceeded 800 F during the operating period. Measurement of the internal diameter revealed that there had been little actual loss of metal from the tubes.

Changes in Baffling and Circulation. Experimentation with deflection baffles in the downtake headers to direct more of the flow into the upper rows, and with direct introduction of feed into the tops of the downtake nipples showed less promise during brief test runs than did a rearrangement of the baffles. The original monolithic baffle, resting on the top-row tubes and extending from the downtake headers nearly halfway along the tubes, was removed, and an alloy-steel baffle of similar extent was placed just under the superheater. At the same time a short baffle was installed, extending back from the front headers between the third and fourth rows from the top. The effect of these changes was to divert the upward flow of gases through the tube bank toward the rear, thus decreasing materially the heat input to the upper ends of the top three rows of tubes and increasing the heat input to the lower ends. During a run from Jan. 5 to 14, 1932, after this change had been made, the highest temperature observed in a tube wall at a load of 18,000 kw was 821 F, and the temperatures averaged approximately 200 F lower than during the previous run.

Extension of the short baffle 2 ft farther from the uptake header to still further deflect gas flow away from the upper ends of the tubes in the first three rows; installation of small restriction baffles in all downtake headers between rows 5 and 6 from the top; and replacement of the feed trough by a 4-in. pipe along the drum with 1-in. nipples extending downward into the tops of the downtake tubes solved the overheating problem early in March. At a load of 19,200 kw the maximum tube temperature was 613 F.

Recurrence of Trouble. An interesting postscript to this case was written into the record in 1938. For more than 6 years, from March, 1932, until July 22, 1938, the boiler had operated with no signs of any further difficulty with overheating of tubes. On the latter date the top-row tube of header 27 ruptured 6 in. from the uptake end. Inspection revealed black oxide deposits in this and adjacent tubes. At the end of the next operating period, from Aug. 5 to 20, the replacement tube had developed a coating of black oxide on the top quarter of its circumference extending over about 10 in. from 3 to 13 in. in from the end of the tube and increasing in thickness from the edges to a maximum of 0.25 in. at the center.

The reason for the reappearance of this old trouble was found in the removal of the end superheater tube immediately above the affected region. This tube had developed a leak and had been cut out and the header openings plugged during September, 1937, but all except the bottom two loops were left in place. Because the dead tube was found to be burning away, it was removed during the annual inspection in March, 1938, and the open space partially closed off by a vertical alloy-steel baffle running upward from the front edge of the inclined baffle under the superheater. Later, on May 25, the empty space toward the front of the boiler from this vertical baffle was partially filled with tile. Apparently, however, heat transmission to the top-row tubes was appreciably greater than when the dead superheater tube had been in place.

During the latter part of August the vertical metal baffle was removed, and the space formerly occupied by the bottom loop of the superheater was completely filled in with tile. When operation from Aug. 27 to Sept. 19 revealed continued though greatly decreased attack, a small protecting baffle was installed covering the upper ends of the top-row tubes in headers 25 to 27 for a distance of 27 in. from the headers. This effectively prevented serious overheating; the highest temperature observed was 720 F. While other changes may be made to reduce tube-wall temperature still further, the problem no longer seems serious.

Inclined Tubes in Generating Sections

Springdale Station

Failure in tubes in the generating section has occurred in the three 1300-psi boilers at Springdale Station of West Penn Power Company. These boilers, numbered 2, 4, and 6 respectively, are of the cross-drum sectional-header type with two-stage furnaces, and are rated to produce 500,000 lb of steam per hr.

The generating tubes are illustrated in elevation in Fig. 8. The first failure occurred in boiler 4 on Sept. 30, 1937, after about three weeks of service, when the top generating tube in header 17 ruptured along the top about 2 ft from the uptake end. During the subsequent 15 months, similar attack necessitated the replacement of 5 more top-row generating tubes and 2 division-wall tubes in boiler 4, 2 top-row and 1 division-wall tubes in

TABLE 3 COMPOSITION OF DEPOSIT FROM REGION OF FAILURE ALONG BOTTOM OF CORED TOP-ROW TUBE AT SPRINGDALE STATION

	Per cent
Sulphur trioxide (SO_3)	Trace
Carbon dioxide (CO_2)	0.0
Phosphorus pentoxide (P_2O_5)	1.7
Silica (SiO_2)	2.2
Iron oxide (Fe_3O_4)	82.9
Aluminum oxide (Al_2O_3)	Trace
Calcium oxide (CaO)	1.4
Magnesium oxide (MgO)	0.4
Copper (Cu)	8.9
Net ignition loss[a]	2.2

[a] Calculated from actual ignition gain of 3.5 per cent resulting from oxidation of Fe_3O_4 to Fe_2O_3 and of Cu to CuO.

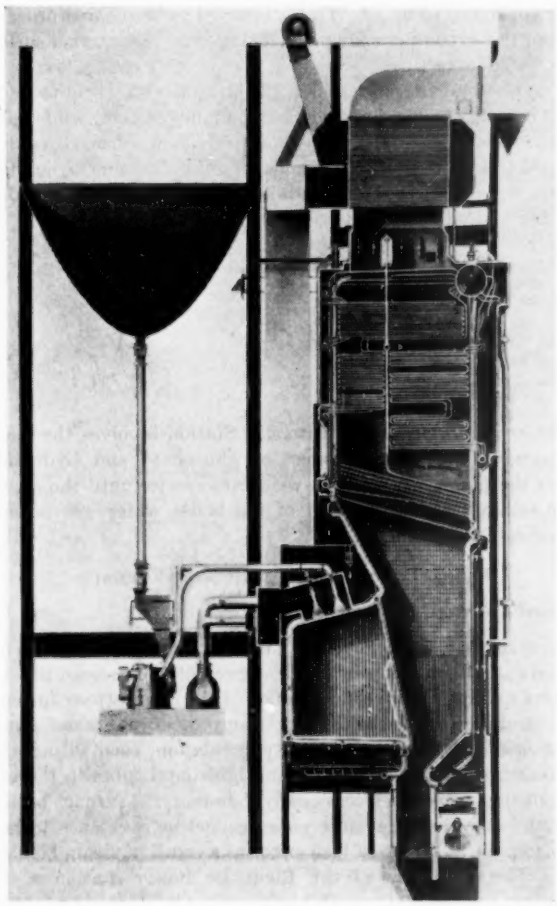


FIG. 8 CROSS SECTION OF 1300-LB BOILER AT SPRINGDALE STATION

boiler 6, and 1 screen tube in boiler 2. The type of failure is indicated in Fig. 9, top.

In the meantime, cores extending from the uptake headers to within about 18 in. of the downtake headers were installed in the top row of generating tubes in boiler 6 in an effort to obviate steam blanketing. This, after about 15 months' service, led to failure of these tubes along the bottom about 3 ft from the downtake end where a deposit of the composition shown in Table 3 accumulated, plugging the lower half of the annular space between the core and the tube wall. This deposit, almost entirely magnetic iron oxide and metallic copper, was partially loose material which had settled out of the water. The cross section of Fig. 9, bottom, shows the harder mass developed in place by oxidation of the tube wall. This was so hard and so firmly bonded to the metal that it was not loosened by turbining prior to removal of the tube.

Altogether, 12 of the 28 tubes of the top row have been replaced in boiler 6 as a result of this type of attack along the bottom toward the downtake end. The full-length cores have been removed, and other means of obviating steam blanketing are being tested.

The water conditions in the boilers at Springdale are indicated in Table 4.

TABLE 4 RANGE OF BOILER-WATER CONDITIONS IN 1300-PSI BOILERS AT SPRINGDALE STATION

Boiler	B Reading, ml			Phosphate, ppm PO_4		
	Maximum	minimum	Average	Maximum	minimum	Average
2	6.3	3.9	4.6	71	13	50
4	12.0	0.3	5.0	141	6	49
6	15.3	0.9	5.5	205	13	76

NOTE: Multiplication of the B reading by 13.3 gives the approximate concentration of NaOH in ppm.

Waterside Station

In the 1400-psi boilers which went into service at Waterside No. 2 Station in 1938, the roof of the furnace proper is formed by a straight-tube section, as indicated in Fig. 10. This section, 41 tubes wide, comprises 20 banks 3 tubes high, alternating with 21 banks 4 tubes high.

Boilers 4A and 4B had been in service several months when boiler 5A was started up about the middle of July, 1938. On Aug. 12, the top tube in the sixth header from the east end of 5A ruptured, and adjacent tubes were found slightly swollen. A month later, on Sept. 14, a second failure occurred, this time in the top tube in the twelfth header from the east end. Both of these headers are 4 tubes high.

Samples of the first tube failure showed little or no loss in wall thickness and little accumulation of oxide on the internal surface, but the steel was badly cracked both longitudinally and transversely over a considerable area, as illustrated in Fig. 11. In the case of the second failure, the wall thickness had been considerably reduced and a heavy coating of magnetic iron oxide was present, while the failure was rather localized, as shown in Fig.

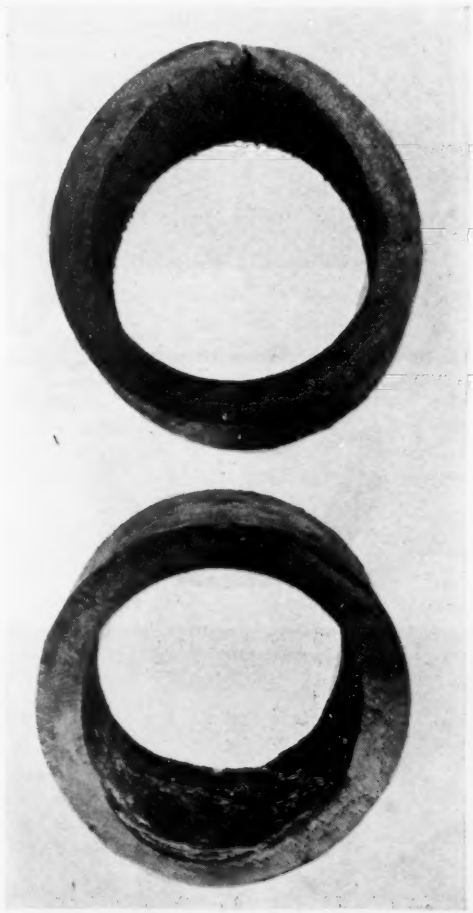


FIG. 9 CROSS SECTIONS OF FAILED TUBES FROM 1300-LB BOILERS AT SPRINGDALE STATION

(Top: Tube cracked along top as a result of steam blanketing. Bottom: Tube oxidized along bottom as a result of accumulation of sediment between core and tube wall.)

12. In both cases the damage was limited to the bottom half of the tube near the uptake header. Other tubes removed from the boiler, including one thought to be relatively undamaged, showed fine cracks in an early stage of development.

Microscopic examination of specimens from the damaged areas by F. E. Foster of the Research Bureau of the Consolidated Edison Company revealed that the cracks were intergranular, as is apparent in Fig. 13. This is believed to be the first observation of this type of cracking developing from the internal surface of a boiler tube.

The photomicrographs of Fig. 13 also show the steel to be almost completely decarburized near the inner surface, while spheroidized cementite, indicative of excessive overheating, is evident beyond this decarburized zone. In other specimens, various stages in the disappearance of the original lamellar pearlite could be observed.

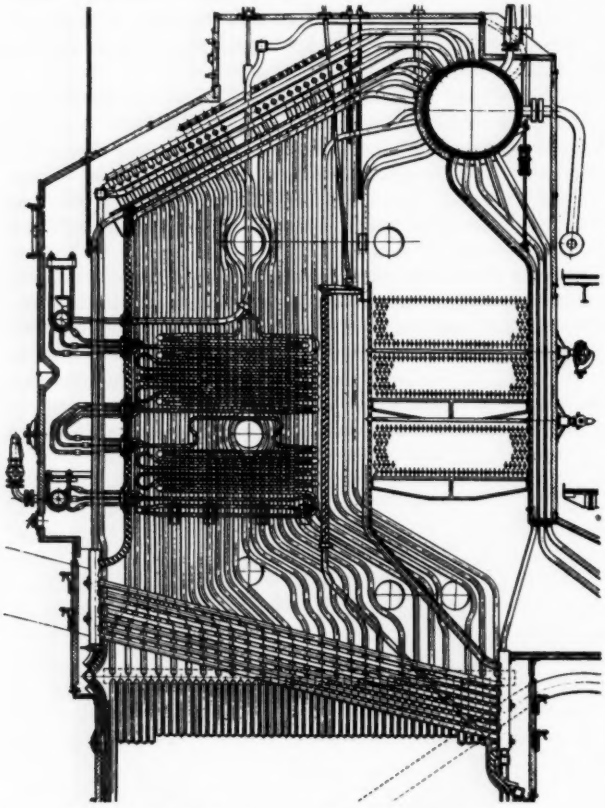


FIG. 10 CROSS SECTION OF HIGH-PRESSURE BOILERS
(Units Nos. 4 and 5, Waterside No. 2 Station.)

TABLE 5 RANGE OF BOILER-WATER CONDITIONS IN BOILER 5A AT WATERSIDE STATION JULY 19 TO SEPT. 14, 1938

	Concentration, ppm		
	OH	PO_4	pH
Maximum	88	160	11.4
Minimum	2	15	10.8
Average	29	51	11.2

Boiler-water control at Waterside Station involves the maintenance of low concentrations of phosphate and hydroxide. From the time that boiler 5A went into service until the second tube failure, the composition of the boiler water was normal, as indicated in Table 5.

FLOOR TUBES IN A SLAG-BOTTOM FURNACE

Rivesville Station

As stated by Caldwell (11), "The corrosion sometimes associated with overheating or deficient circulation in screen tubes is absent generally in the floor tubes of the slag-bottom furnace. The insulating nature of ordinary slag protects the water-cooled floor against overheating or faulty circulation, even with highly concentrated heat-release rates and horizontal tubes in the bottom of the furnace." Occasionally, however, the furnace bottom and the slag on it may interpose insufficient resistance to heat flow to prevent failure of floor tubes as a result of steam blanketing. The experience of the Rivesville Power Station of the Monongahela West Penn Public Service Company is a case in point.

The floor of the primary furnace in this 1300-psi straight-tube boiler contains 28 tubes about 18 ft long, nearly horizontal, with a slope of only about 1.5 deg. These tubes, which were studded over the top half, were originally covered with about 1 in. of

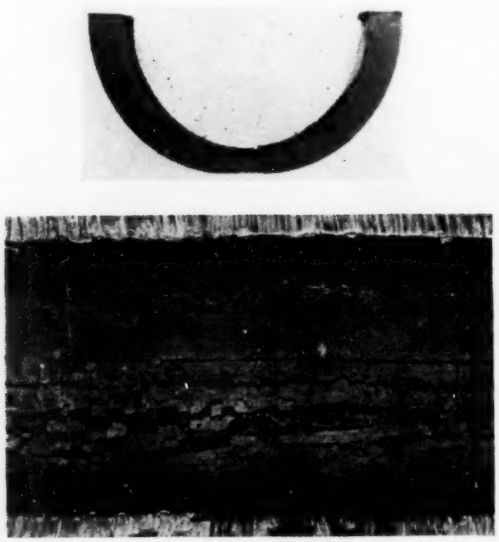


FIG. 11 CROSS SECTION AND INTERNAL APPEARANCE OF FIRST FAILURE OF A TOP-ROW TUBE AT WATERSIDE STATION

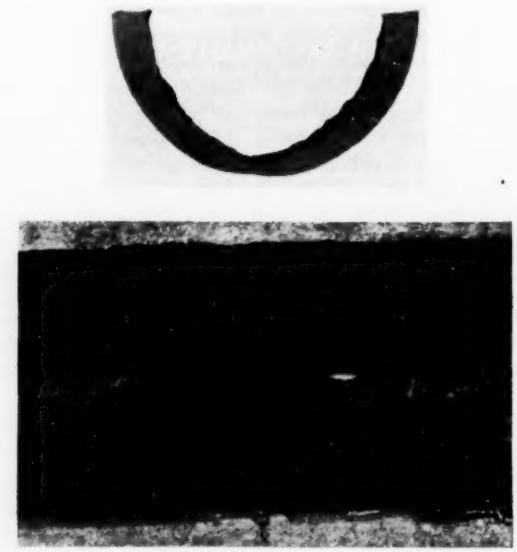


FIG. 12 CROSS SECTION AND INTERNAL APPEARANCE OF SECOND FAILURE OF A TOP-ROW TUBE AT WATERSIDE STATION

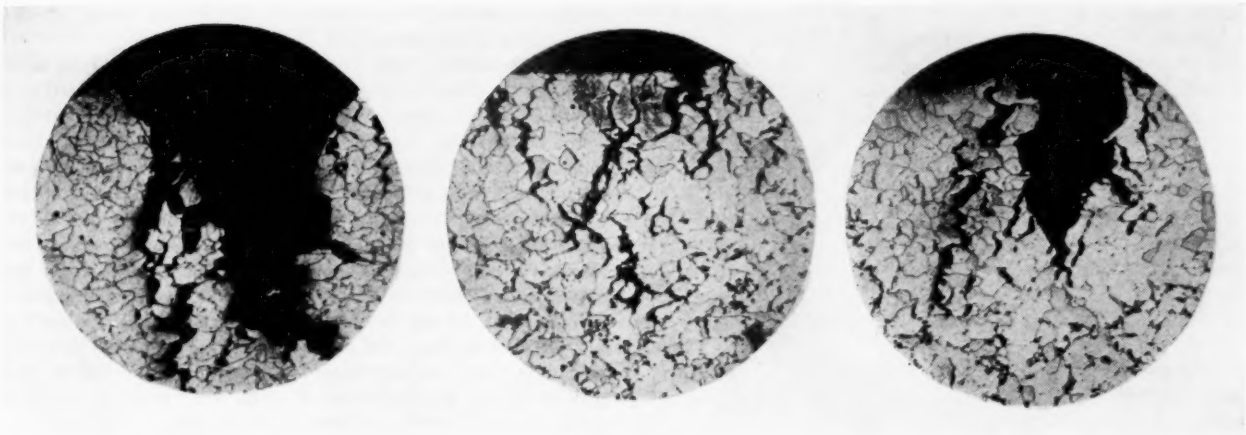


FIG. 13 INTERGRANULAR CRACKS DEVELOPED FROM INTERNAL SURFACE OF CRACKED TOP-ROW TUBES AT WATERSIDE STATION

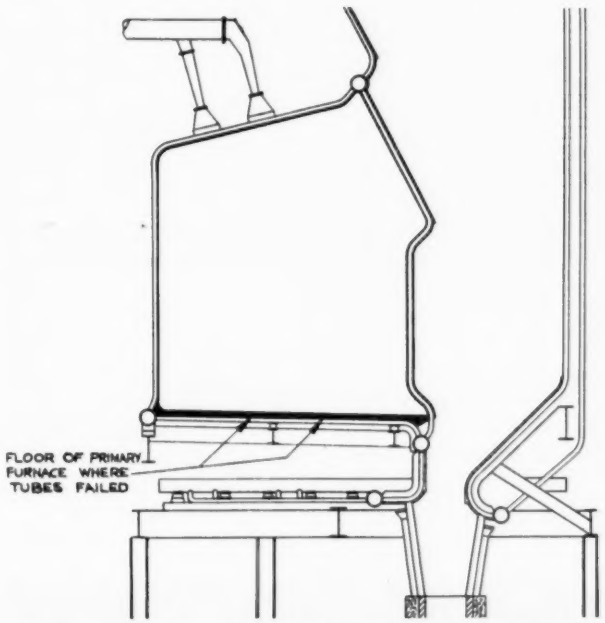


FIG. 14 CROSS SECTION OF 1300-LB BOILER AT RIVESVILLE STATION SHOWING FLOOR TUBES

chrome ore. Their location is indicated in the simplified cross section of Fig. 14.

The boiler went into service on Oct. 29, 1937. On Dec. 27, 1937, floor tubes 10 and 16 failed along the top, followed by tubes 14 and 18 on Dec. 30. Inspection revealed that the chrome-ore refractory had been worn away in the center of the floor, directly under the burners, exposing the ends of the studs, which were badly burned, some having lost as much as half the original metal. The center 20 tubes were then replaced, leaving only 4 of the original tubes at each side of the floor, the thickness of the layer of chrome ore was increased to 4 in., and provision was made to retain about 1 in. of molten slag on top of the chrome ore.

All of the 20 tubes which were removed had been attacked along the top, most of them for about two thirds of their length. Each tube showed sharp lines of demarcation along the sides, below which the steel was undamaged, but above which there were many deep pits. The actual failures were small holes in the tops of the tubes where the wall had been thinned until it yielded under the internal pressure.

Lying in the tubes and in the header into which they discharged was a large amount of loose black magnetic iron oxide, which was easily washed out.

The alkalinity and phosphate in the boiler water during the 2 months from the time the boiler was placed in service until the failures occurred were normal, as indicated in Table 6. During the year 1938 following the changes in the furnace floor, the alkalinity averaged somewhat higher than during the time prior to the failures, and the phosphate remained at about the same level, but no signs of further attack on the floor tubes were observed. Apparently, the considerable increase in the thermal insulation above the tubes so reduced the heat input to them as to minimize overheating and excessive localized concentration of the boiler water, even though the tubes, as a result of their nearly horizontal position, may still tend to be steam-blanketed.

TABLE 6 SUMMARY OF BOILER-WATER CONDITIONS IN 1300-PSI BOILER AT RIVESVILLE STATION PRIOR TO FAILURE OF FLOOR TUBES

	B Reading ml	Phosphate, ppm PO_4
Maximum	8.5	100+
Minimum	1.4	20
Average	4.9	70

One adverse result of decreasing the heat input to the floor tubes has been an increased temperature of the gases entering the tube banks, with a resultant tendency to choke up the boiler with slag, which at times makes it impossible to maintain superheat.

ASH SCREEN IN A DRY-BOTTOM FURNACE

Port Washington Station

The experience of Port Washington Station of Wisconsin Electric Power Company[7] with failure of screen tubes along the top has already been described in some detail (12) but will be reviewed here, together with supplementary information, because of the decided similarity between this case and the others described in this paper.

This boiler went into service in December, 1935. In October, 1936, the first tube failure in the ash screen was replaced. How serious a condition existed was realized in December. Four outages were necessary on Dec. 4, 17, and 26, 1936, and Jan. 3, 1937, to replace screen tubes which were found to have been grooved internally along the top. Newly installed tubes were thinned down to the point of leaking during 5 days of operation from Dec. 21 to 25. When a failure suddenly occurred on Jan. 12, it was decided to replace all questionable screen tubes.

[7] Formerly The Milwaukee Electric Railway and Light Company.



FIG. 15 CROSS SECTION OF SCREEN TUBE FROM PORT WASHINGTON SHOWING GROOVING ALONG TOP

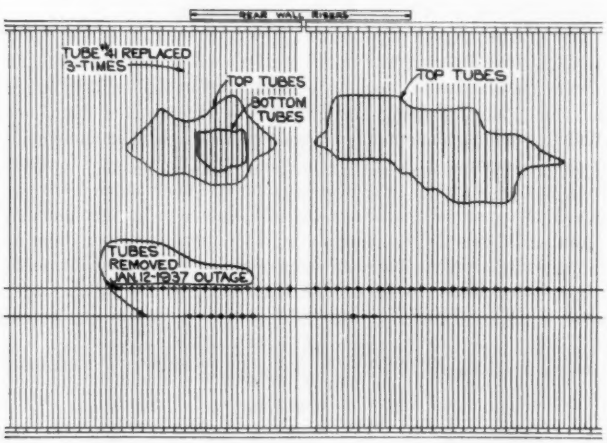


FIG. 16 AREAS OF ATTACK IN ASH SCREEN AT PORT WASHINGTON STATION

The typical appearance of the grooving of the screen tubes is indicated by the cross section in Fig. 15. Examination of the 41 top-row and 10 bottom-row tubes removed immediately after the outage on Jan. 12 demonstrated that the attack had been limited to two areas of the water screen about one third of the distance from the rear to the front headers, as shown in the plan sketch in Fig. 16. In the smaller left-hand area, 5 bottom-row as well as 14 top-row tubes had been damaged; in the larger right-hand area, the trouble was apparently confined to the top row, as the 3 bottom-row tubes removed showed no thinning along the top.

The replacement tubes were bent to give an increase of 0.5 deg on the average slope of 5 deg for the original design. Due to the natural sag of the tubes, the actual slope in the zone where trouble occurred was probably about half the average value.

Boiler-Water Conditions. In the search for some means of obviating further trouble, attention was concentrated on adjustment of the composition of the boiler water to maintain a protective coating on the tube surfaces, using the concentration of hydrogen in the steam as a measure of the rate at which the steel was being attacked. After the controlled formation of calcium-sulphate scale, on one hand, and the use of increased amounts of phosphate, on the other, had failed to limit the apparent rate of corrosion, it was observed that the lowest rates of hydrogen evolution frequently coincided with the lowest values for the alkalinity of the boiler water.

From December, 1935, when the boiler went into regular service, until the attack on the screen tubes first was recognized as a serious problem in December, 1936, the phenolphthalein alkalinity of the boiler water, expressed as ppm of NaOH, had been maintained generally between 20 and 30, and the phosphate concentration, expressed as ppm of Na_2HPO_4, was similarly controlled within the general limits of 50 and 70, as indicated in Fig. 17. After replacement of tubes during a week-end outage, the alkalinity was increased to an average value of 70 ppm for the period from Dec. 21 to 26. When the next week-end inspection revealed that, during the week, a new tube had been thinned to the point of leaking, the alkalinity was again raised and was held at a comparatively high level, ranging from 130 to 220 ppm of NaOH, until the failure of a tube on Jan. 12, 1937, led to the general replacement of tubes in the screen.

When the boiler was again placed in operation on Jan. 25, the alkalinity and phosphate were varied experimentally, while the rate of hydrogen evolution was watched as an indication of what

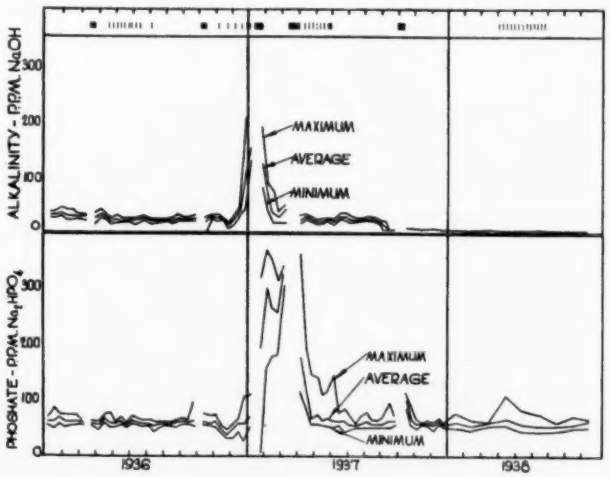


FIG. 17 RECORD OF ALKALINITY AND PHOSPHATE CONCENTRATION IN BOILER WATER AT PORT WASHINGTON

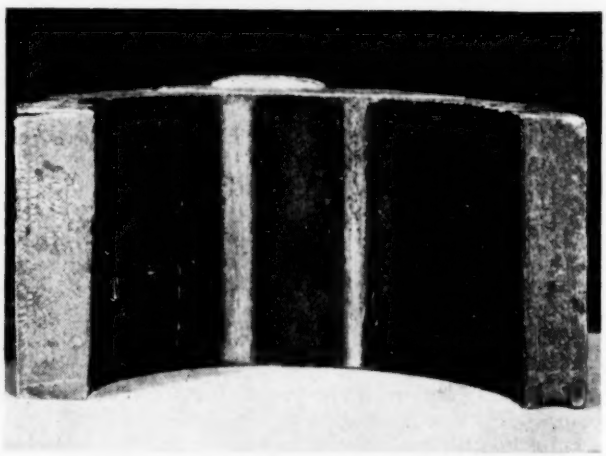


FIG. 18 APPEARANCE OF INTERNAL SURFACE OF TOP HALF OF SCREEN TUBE FROM PORT WASHINGTON AFTER OPERATION WITH LOW ALKALINITY IN BOILER WATER

(Central dark band is a thin continuous coating of magnetic iron oxide. Gray bands are exposed edges of thin layer of iron phosphate which covers remainder of surface, but is in turn covered by and intermixed with black magnetic iron oxide.)

was happening in the screen tubes. Alkalinity was progressively decreased until by the middle of February it was back in the range held during the major portion of 1936. In the meantime, phosphate concentrations were greatly increased, and held at a high level until the boiler was taken out of service in the middle of March. Specimen tubes removed at this time demonstrated that no appreciable attack had taken place since the January outage. Subsequently, the phosphate concentration was progressively lowered. For somewhat more than four months, from the latter part of April to the first part of September, both the alkalinity and the phosphate were controlled at the levels initially maintained during the first year of operation. A decision to reduce the alkalinity still further was put into practice about the middle of September, 1937, and since that time it has been held at an average value of 5 ppm of NaOH.

Removal of two more specimen tubes from the ash screen during the October, 1937, inspection confirmed the conclusion drawn

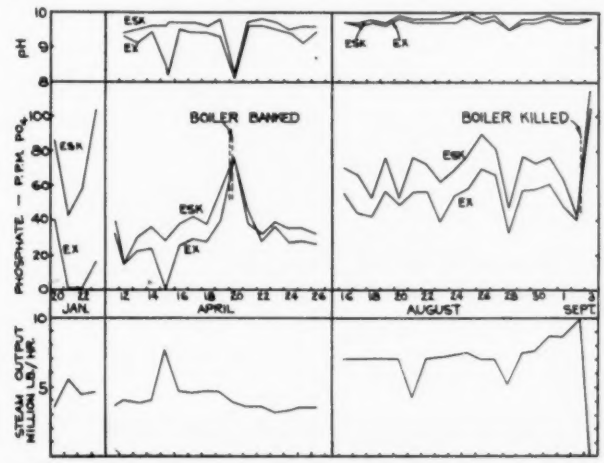


FIG. 19 COMPARISON OF TENDENCY OF SCREEN TUBES OF OLD AND OF NEW DESIGN TO STEAM-BLANKET

(Sample EX from special sampling line at top of tube of old design delivered substantially pure steam on Jan. 21 and 22 and on Apr. 15; during period from Aug. 16 to Sept. 3, no such steam blanketing was indicated by similar samples from line in tube of new design. Samples ESK came from skimmer in the east 48-in. drum.)

from the hydrogen measurements during the preceding few months that practically no attack had occurred. When four new tubes were installed in place of those removed previously and at this time for examination, the rate of hydrogen evolution went up, however, to 3 times its base value for about a week, indicating some reaction of the fresh metal surfaces with the boiler water. Pressure and load were decreased 20 per cent and the phosphate concentration was increased until the hydrogen evolution again came back to its base value corresponding to the loss of about 2 oz of iron per day from the entire unit, including the superheater.

Study of Internal Surfaces Under the Microscope. To ascertain the extent to which conditions at the internal surface of an ash-screen tube during the later period of freedom from attack differed from those during the period when failures were experienced, the polarizing microscope was applied to a study of the specimens illustrated respectively in Figs. 15 and 18. It was found that two substances were present on each of the two specimens, first the anticipated black magnetic oxide Fe_3O_4 and second, a crystalline substance similar to the natural mineral ludlamite,[8] which is regarded as a ferrous hydroxyphosphate.

[8] The crystalline substance present on the surface of the tubes was transparent, faintly blue-green, birefringent with minimum and maximum indexes of 1.655 and 1.700, and showed a definite cleavage apparently rectangular or nearly so.

Drewry has previously reported partial analyses (12) indicating that an iron phosphate was present on the internal surface of a tube removed during the later period of trouble-free operation. The microscopic study of the specimen shown in Fig. 18 has shown that the iron phosphate, which is readily dislodged from the tube in minute fragments comprising many crystals, is present over the entire internal surface of the specimen, with the exception of a dark band, 0.5 in. wide, along the top of the tube, where the steel is covered only by a thin, uniform film of iron oxide. The iron-phosphate coating is particularly evident in two gray bands, readily visible in Fig. 18 at either side of the central band. Elsewhere, the iron phosphate is partially covered by and intermixed with minute particles of Fe_3O_4, which give a black appearance to the surface.

That some slight attack had taken place along the central band at the top of the tube was evident even on visual examination. Under the microscope the surface appeared as smoothly rounded hummocks and valleys. The depth of the attack was, however, only of the order of 0.001 in.

Only a small amount of iron phosphate was found during micro-

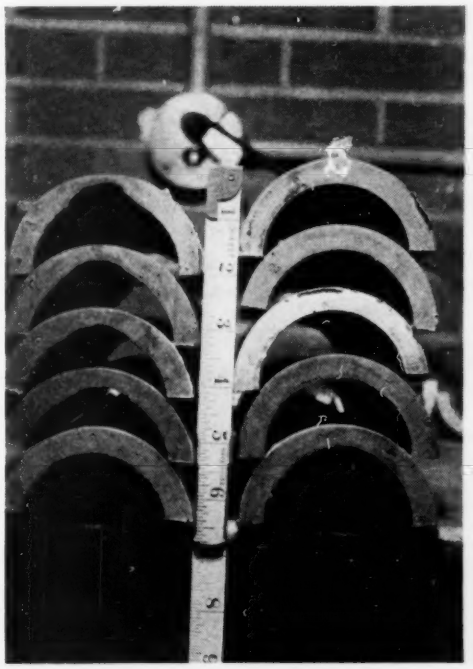


FIG. 20 CROSS SECTIONS OF ASH-SCREEN TUBE FROM LOGAN STATION SHOWING EXTENT OF GROOVING AT SUCCESSIVE 1-FOOT INTERVALS FROM SIDE-WALL HEADER

(Tube 4 SE, failed Jan. 31, 1938. Cross section at upper right is 1 ft from header, each succeeding cross section 1 ft farther away.)



FIG. 21 INTERNAL ATTACK ON TOP OF ASH-SCREEN TUBE AT LOGAN STATION DURING OPERATION WITH NORMAL BOILER-WATER ALKALINITY

(Section 6 to 7 ft from side-wall header of original tube 4 SE, failed Jan. 31, 1938.)

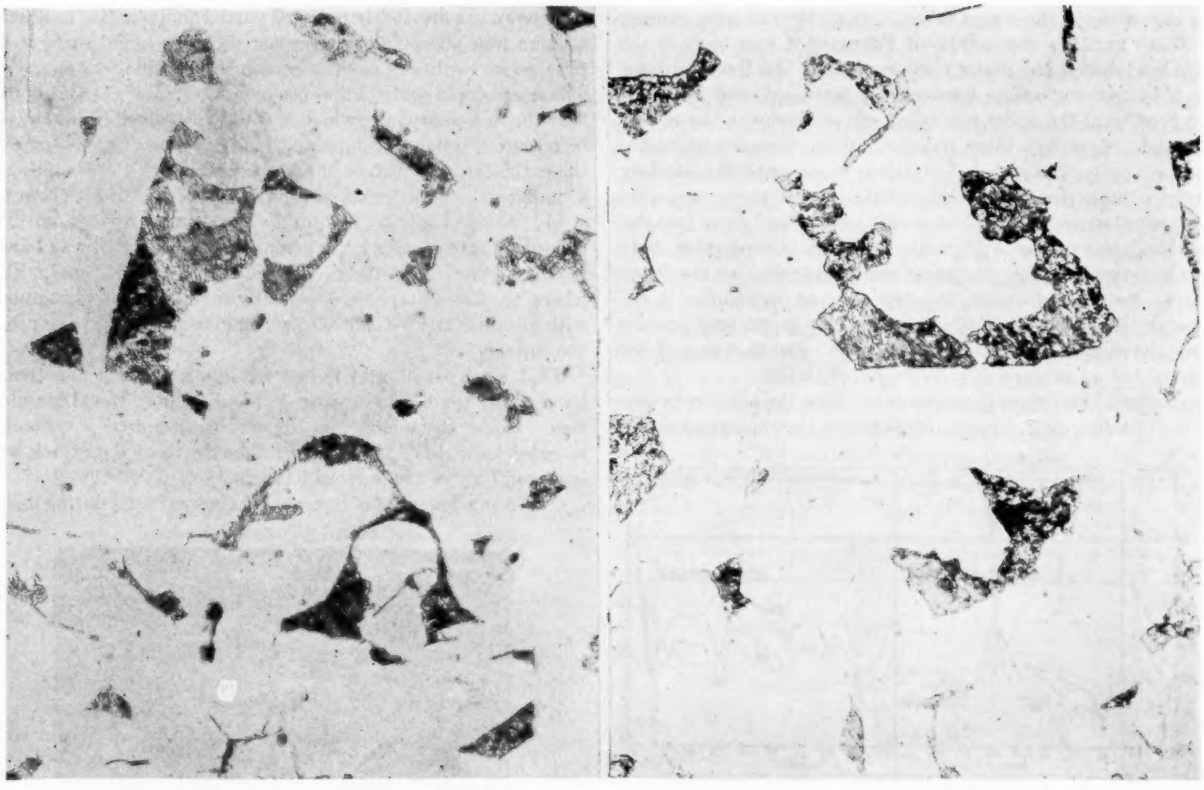


FIG. 22 STRUCTURE (×500) OF SIDE, LEFT, AND TOP, RIGHT, OF TUBE 4 SE FROM LOGAN STATION, 9 IN. FROM POINT OF FRACTURE
(Temperature at top of tube not sufficiently high to induce spheroidization of pearlite.)

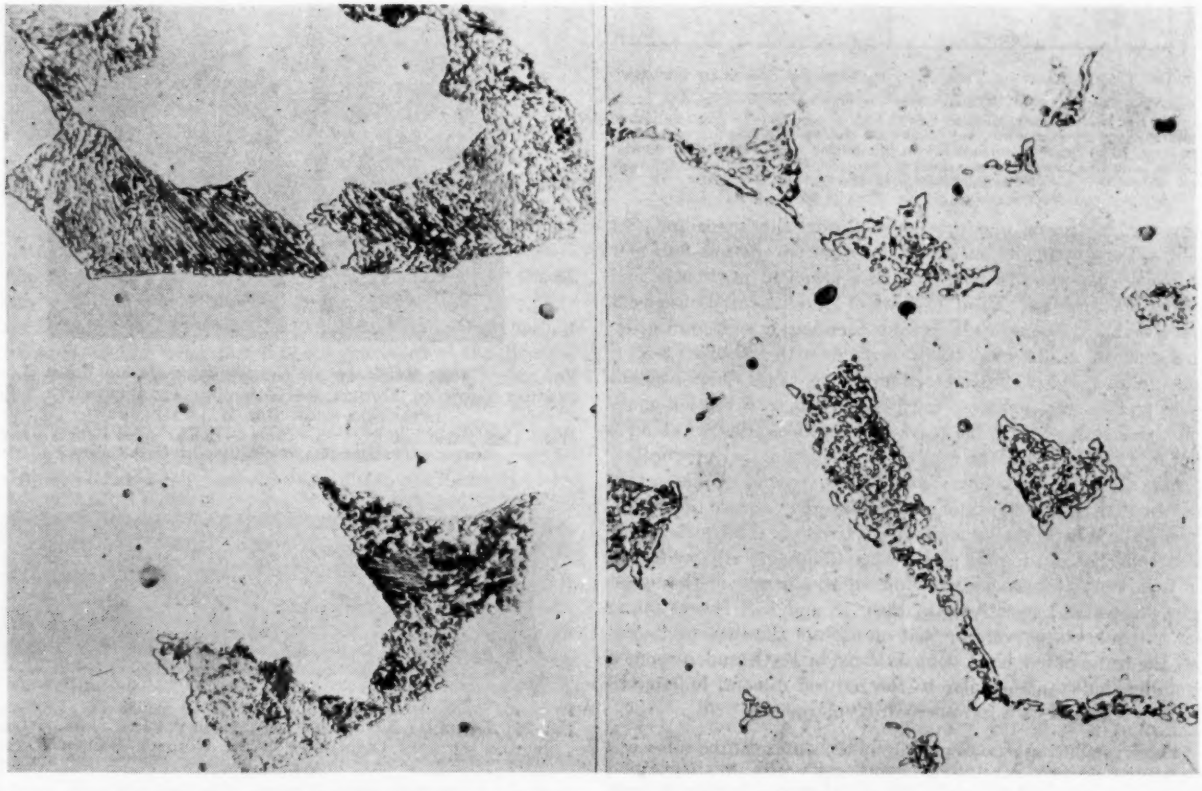


FIG. 23 STRUCTURE OF TOP OF TUBE 4 SE FROM LOGAN STATION. RIGHT, AT FRACTURE; LEFT, AT POINT 9 IN. FROM FRACTURE ×1000
(Spheroidization of pearlite definitely indicates excessive overheating in region where fracture developed.)

scopic examination of the specimen illustrated in Fig. 15, the internal surface of which was coated chiefly with black magnetic iron oxide. At the top of the tube where severe attack had occurred, this coating was thin and continuous, resembling that at the top of the practically undamaged specimen.

Logan Station

During October, 1937, a 1325-psi boiler went into operation at Logan Station of Appalachian Electric Power Company. This boiler, designed to produce 1,000,000 lb of steam per hr at a total temperature of 925 F, was operated intermittently at low loads during the fall of 1937. During a trial run from Dec. 5 to 8, inclusive, the output was raised to 725,000 lb per hr for a few hours, but averaged only 540,000. During the next operating period, from Dec. 15 to 25, the maximum and average outputs were, respectively, 550,000 and 440,000 lb per hr. Similar operation commencing Dec. 28, 1937, was terminated Jan. 7, 1938, by the development of a leak in a top-row screen tube, No. 2 from the southeast corner.

Study of Early Failures. The failure occurred about 6 ft from the side-wall header on the top of a portion of the tube that had been depressed approximately 7 in. by falling slag. Examination of this tube after removal showed that it had been attacked internally along a strip from 0.5 to 1.2 in. wide on the top of the tube. This region of attack extended over the portion of the tube between 4 and 9 ft from the side-wall header.

Inspection of the other top-row screen tubes after removal of handhole caps on the headers revealed similar corrosion, quite severe in the first six or eight tubes in each corner, but slight in the central portion of the furnace. To determine whether the tops of the tubes were steam-blanketed, a special sampling line consisting of a stainless-steel tube with an internal diameter of 0.09 in. was installed in the tube 17 NE, at the middle of the furnace. The end of the sampling line was turned upward and held in position close to the top of the tube at a point 6 ft from the side-wall header.

When the boiler was placed in operation on Jan. 20, comparison of the samples removed from the special sampling line EX, with those from the skimmer in the east 48-in. upper drum ESK, gave definite evidence of steam blanketing. As shown in the first section of Fig. 19, the phosphate concentrations of the EX samples from Jan. 20 to 23, inclusive, were always low compared to those of the ESK samples, and in two cases, substantially pure steam was issuing from the special sampling line. On Jan. 23 the run was terminated by the failure of a second top-row screen tube, 5 SE, along the top in the same manner as 2SE had failed previously.

An attempt to measure tube-wall temperatures by means of chromel-alumel thermocouples was made during the next test period, beginning Jan. 28. While the renewed tube 5 SE, which had been covered with protecting blocks, showed a temperature of only about 600 F during the brief life of the couples, the next tube 4 SE which had been left uncovered, simultaneously averaged 1250 F on the top and 640 F on the bottom, when the steam ouput of the boiler was 450,000 lb per hr. That the overheating indicated by the thermocouple was not fictitious was proved by the failure of this tube on Jan. 31 and by its appearance after removal and sectioning, illustrated in Fig. 20. Beginning at the upper right, the cross sections represent the condition of the top half of the tube at successive 1-ft intervals from the side-wall header. The grooving apparently began between 4 and 5 ft from the header, was deepest at about 6 ft, and continued to between 9 and 10 ft.

The appearance of the internal surface of the section from 6 to 7 ft from the side-wall header is shown in Fig. 21. The actual crack is about 3 in. from the 6-ft (left-hand) end. Specimens cut



FIG. 24 SECTION NEAR EXTERNAL SURFACE OF CRACKED ASH-SCREEN TUBE FROM LOGAN STATION, SHOWING PENETRATION OF OXIDATION ALONG GRAIN BOUNDARIES, ×1000
(Note also accumulation of globular iron carbide at grain boundaries.)

for metallographic examination from the side and from the top of the 7-ft end showed no significant difference in structure, as may be seen in Fig. 22, indicating that 9 in. from the point of failure the tube had not been overheated sufficiently to produce spheroidization. A specimen cut from the region of the fracture was, however, definitely spheroidized, as illustrated in Fig. 23, while the fine cracks near the main fracture at the external surface showed oxidation working inward along grain boundaries, as in Fig. 24.

Attempts to Obviate Attack. Since the most serious attack was evident in the screen tubes at the corners of the furnace, an attempt was made to improve circulation by removing the five top-row tubes in each corner, plugging tube holes 1, 2, 4, and 5, replacing each tube 3 with a new tube, and inserting baffles in the side-wall headers to isolate these sections from the central portions. In each of these corner sections there then were left a total of five tubes, one in the top row and four in the bottom, where previously there had been nine tubes. It was hoped that this change would nearly double water velocities through the remaining tubes.

While these changes were being made, the profile of the attack along the top of each tube on the south side was measured by a special calipering device, and series of thermocouples were peened into the tops of tubes 3 and 6 in each corner and tubes 17 NE

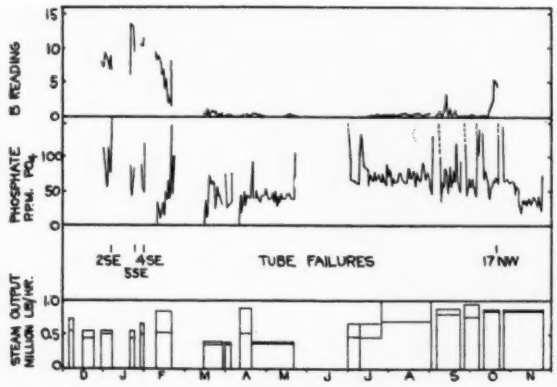


FIG. 25 RECORD OF LOAD, BOILER-WATER CONDITIONS AND FAILURES OF ASH-SCREEN TUBES AT LOGAN STATION

(Upper line for load indicates maximum, lower line average for each operating period. Multiplication of the B reading by 13.3 gives the approximate value for NaOH in ppm.)

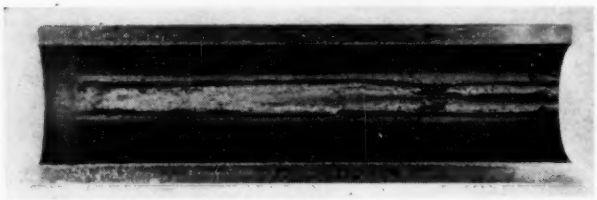


FIG. 26 INTERNAL APPEARANCE OF TOP OF ASH-SCREEN TUBE AT LOGAN STATION AFTER TWO MONTHS OF OPERATION WITH LOW BOILER-WATER ALKALINITY

(Replacement tube 6 NE, in service Mar. 14 to May 21, 1938. White deposit is iron phosphate.)

and SW at the center of the screen. When the boiler went into service on Feb. 9, however, the thermocouples failed within from 17 to 28 hr, before the steam output had exceeded 600,000 lb per hr. Although the maximum temperatures recorded in each tube during this brief period at the start ranged only from 640 to 730 F, appreciable attack occurred before the boiler was taken off the line on Feb. 21. On calipering, it was found that all tubes had lost some metal and that tubes 3 and 6 in both the SE and SW corners had suffered the most. The maximum decrease in wall thickness, approximately 0.1 in., had occurred in tube 3 SE about 3 ft from the side-wall header.

With the efforts to improve conditions by increasing velocity of flow apparently unsuccessful, two alternative procedures were urged: First, modification of boiler-water conditions in an attempt to stop the attack by chemical means, and second, modification of the design of the ash screen to eliminate or minimize steam blanketing along the top of the tubes. Since replacing the ash screen would involve rather considerable expense, and since the type of attack in question had apparently been obviated at Port Washington by decreasing the alkalinity of the boiler water (12), it was decided to maintain "Port Washington conditions" during the next operating period.

Operation With Low Alkalinity. As indicated in Fig. 25, the alkalinity was maintained at a low level not exceeding a B reading of 1 from March 14 to 28; in the succeeding periods from March 31 to April 4 and from April 11 to May 21 this maximum limit was further reduced to 0.25. During these periods the steam output did not exceed 500,000 lb per hr except on Apr. 14 and 15; a maximum of 880,000 was attained on the latter date. This relatively high rating again produced definite evidence of steam blanketing from the EX sampling line, at that time located in tube 3 NE. As shown in Fig. 19 the sample from this line on April 15 contained practically no phosphate and had a pH of only 8.2, while the samples taken simultaneously from the skimmer in the east 48-in. drum showed little change from the preceding day.

After this brief trial at higher ratings, the steam output was dropped to 460,000 and then to about 350,000 lb per hr for the rest of the period of operation, which terminated May 21. Measurements revealed little further loss of metal, but whether this was due more to low alkalinity or to low rating could not well be decided. The black bands previously observed along the tops of the tubes were still present, but were bordered and partially covered by a white deposit, evident in Fig. 26. This shows a portion of replacement tube 6 NE which had been installed new just before operation under Port Washington conditions was commenced on Mar. 14. To the eye, all of the surface except the band at the top was uniformly reddish brown. Examination under the polarizing microscope revealed that the surface of the black band was the familiar magnetic iron oxide, that the white deposit was iron phosphate similar to that previously observed in a tube from Port Washington, and that the reddish brown coating was chiefly finely divided metallic copper. Both the iron phosphate and the metallic copper could be readily rubbed off with a cloth, leaving the fundamental tightly bonded surface coating of Fe_3O_4 on the steel.

Change in Design of Ash-Screen Tubes. Before the boiler was again placed in service, the burners were swung out from the walls to decrease the diameter of the vortex in the furnace. At the same time, the entire east half of the ash screen was replaced, using tubes 3 instead of 3.5 in. in diameter, bent to give an increase in average slope from about 7.5 to about 12.5 deg. The change in slope is indicated in Fig 27. Replacement tubes of the original design were installed in 6 NW and 3, 6, 7, and 14 SW to allow direct comparison of the effect of tube design during the next operating period.

During the two months from June 30 to Sept. 2, water conditions were held closely the same in the east and west halves of the boiler, again with the low limit of alkalinity set by Port Washington. Toward the end of this period the steam output was progressively increased from a level of 700,000 to a maximum of 1,000,000 lb per hr, the rated capacity, just before the boiler was killed.

During this test period the EX sampling line was still located in tube 3 NE, but this was now of the new design. The representative data for Aug. 16 to Sept. 3 reproduced in Fig. 19 indi-

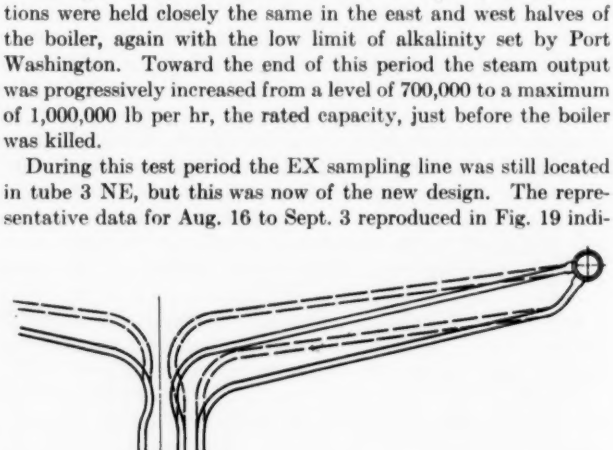


FIG. 27 CHANGE IN DESIGN OF ASH-SCREEN TUBES AT LOGAN STATION

(Broken lines: Original tubes, 3.5-in. diam, 7.5-deg slope. Solid lines: New tubes, 3-in. diam, 12.5-deg slope.)

cate a consistent dilution of the sample by the steam in the top of the tube, but not even at the maximum rating was there steam blanketing such as that shown for the old design of screen tube on Jan. 21 and 22 and on Apr. 15. This promise of better performance of the new tubes was substantiated when the boiler was inspected on Sept. 4. While all of the tubes of the old design in the west end showed the characteristic black band along the top, frequently edged and spotted with the white deposit of iron phosphate, the tubes of the new design in the east end were practically unmarked.

That the marks in the tubes in the west end were not merely carried over from previous operation was proved by the appearance of the replacement tubes of the old design. Thus 6 and 7 SW, new at the beginning of the test period, could not be distinguished from 8 SW, which had been in the boiler since it was erected.

While the appearance of the tubes of the old design attested continued steam blanketing, the actual attack during the two months under Port Washington conditions had been slight relative to that previously developed during the shorter early periods of operation with more alkaline boiler water. Reflecting the shift in the angle of the burners, tubes 8 to 10 SW lost more metal than those toward the corner or toward the center of the screen, and the maximum decrease of 0.05 in. in wall thickness occurred in tube 10 SW, 6 ft from the side-wall header.

Of the tubes of the new design in the east half of the screen, 12 SE was the only one to show a dark band along the top. This band, about 0.5 in. wide at its mid-point, extended from about 3 to about 6 ft from the side-wall header. The corresponding tube on the other side, 12 NE, showed a few discontinuous spots along the top. Measurement indicated an apparent maximum loss of 0.03 in. in wall thickness at one point in 12 SE and no loss greater than the uncertainty in measurement in 12 NE.

Effect of Increased Alkalinity on Tubes of New Design. The evidence now clearly indicated that the attack on the old design of tube had been greatly retarded by decreasing the alkalinity of the boiler water but that the fundamental cause of the attack, steam blanketing of the tops of the tubes, had been almost completely eliminated by the change in design. The next question was whether the new design would actually stand up when boiler-water alkalinity was again raised to the original operating level.

During the shutdown from Sept. 2 to 6, the deflector plates in the secondary-air passages of the upper burners were adjusted. Contrary to expectations, the change caused the flame to sweep down through the screen tubes during the period from Sept. 6 to 23. The plates were then changed back, restoring normal flame conditions during the operating period from Sept. 26 to Oct. 7. While the alkalinity of the boiler water was allowed to reach a B reading of 3 on one day during these two periods, it was not until shortly after the boiler again went on the line on Oct. 10, producing steam at a rate of about 840,000 lb per hr, that the alkalinity was progressively raised as indicated in Fig. 25. A B reading of 5.3, corresponding to a nominal concentration of 70 ppm of NaOH, was reached on Oct. 19 and held the following day. On Oct. 21 tube 17 NW, of the old design, located at the center of the furnace, developed a leak along the top which necessitated taking the boiler off the line.

Measurement of the screen tubes showed that while those of the old design in the NW quarter had lost quite generally about 0.15 in. of metal, with less severe attack indicated in the SW quarter, there had been no measurable attack on the tubes of the new design in either the NE or SE quarters. Complete replacement of the tubes in the west end with those of the new design was the result. This was carried out during a shutdown from Nov. 24 to Dec. 6, 1938. Further operation will be watched with no little interest.

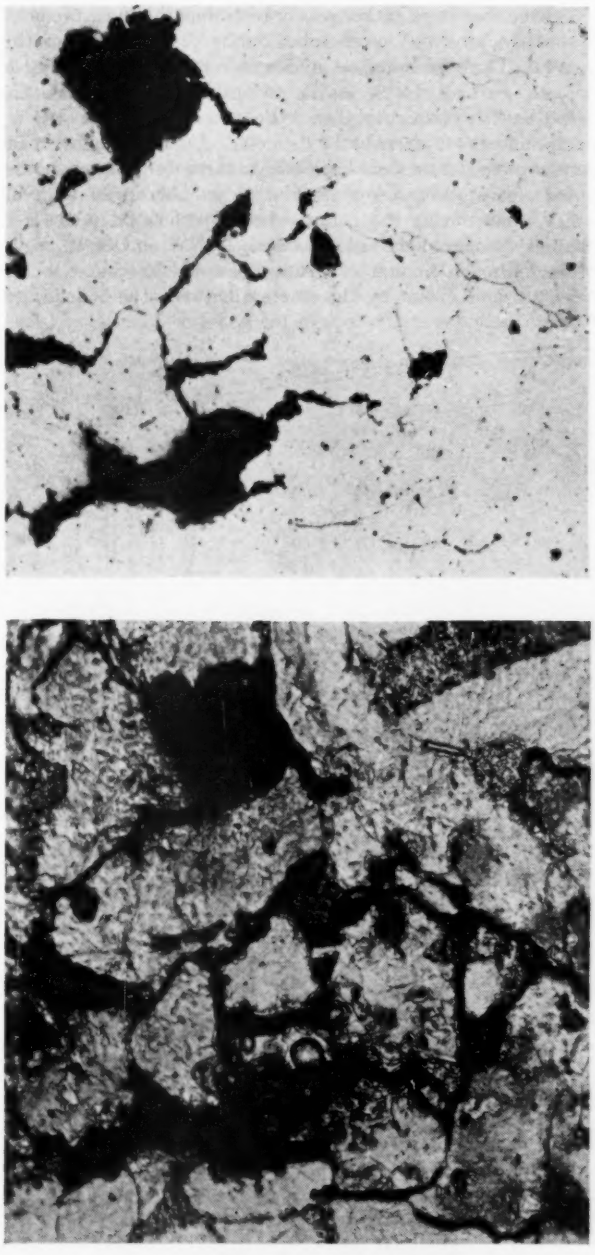


FIG. 28 STRUCTURE OF TOP OF TUBE 17 NW FROM LOGAN STATION, ×500

(Top: Unetched. Bottom: Etched with 4 per cent nital.)

Metallographic Examination of Failed Tube. Under the microscope, specimens from the area of failure in tube 17 NW revealed not only a structure indicative of excessive overheating, but also intergranular cracks developing from the internal surface. As may be seen from Fig. 28, the finer cracks in the unetched condition appeared only as faint lines but etching developed these into relatively broad black bands which seemed to follow the boundaries of the original grains.

Hydrogen Evolution as a Warning Signal. Theoretically the hydrogen content of the steam from a boiler should be a measure of the amount of boiler metal oxidized by reaction with the water. Actually the hydrogen measurements made on the steam from each end of the boiler at Logan Station daily since March 15,

1938 have correlated rather poorly with the other measurements of the attack on the screen tubes during the various operating periods. Thus the apparent hydrogen content was observed to increase markedly with steam output during one operating period at low alkalinity, but little attack was indicated by measurements of internal tube diameter. Likewise the hydrogen content of the steam went higher for a longer time during another period of operation at low alkalinity when there was negligible attack, than during the period when experimental increase in alkalinity produced the failure of tube 17 NW on Oct. 21.

The hydrogen content of the steam would, of course, be affected to some extent by the reaction between the boiler water

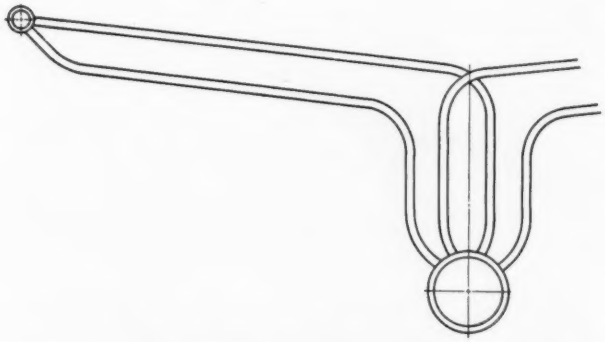


FIG. 29 ELEVATION OF ASH-SCREEN TUBES IN BOILER 5 AT ROUGE PLANT
(Tube diameter, 3 in.; slope, 7 deg.)

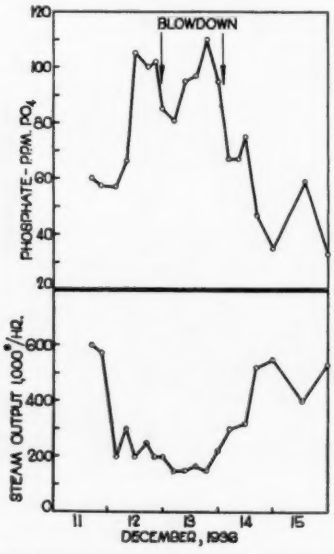


FIG. 30 INVERSE RELATION BETWEEN PHOSPHATE CONCENTRATION AND STEAM OUTPUT DUE TO "HIDING OUT"

and any excess of the ferrous sulphate used as an oxygen scavenger. However, 1 lb of iron combined in the form of ferrous sulphate would produce only 61 liters of hydrogen, as against 244 liters from 1 lb of metallic iron.[9]

[9] These values are based upon the equations:

$$3FeSO_4 + 6NaOH \rightarrow 3FeO \cdot H_2O + 3Na_2SO_4$$
$$3FeO \cdot H_2O \rightarrow Fe_3O_4 + 2H_2O + H_2$$
$$3Fe + 4H_2O \rightarrow Fe_3O_4 + 4H_2$$

The reactions of iron and of ferrous hydroxide with water to form Fe_3O_4 and H_2 have been observed experimentally by E. Berl and F. van Taack, *Archiv. für Wärmewirtschaft*, vol. 9, 1928, pp. 165–169.

Ford Motor Company

Power Plant No. 1 of the Rouge plant of the Ford Motor Company contains three 1350-psi dry-bottom boilers, which have experienced no difficulty with internal attack along the tops of the ash-screen tubes. Boilers 1 and 3, each with a rated output of 700,000 lb of steam per hr, have been in operation since 1931, and boiler 5, rated at 800,000 continuous, 900,000 for 2 hr, and actually operated on test at 1,040,000 lb per hr, went into service July 15, 1936.

Boiler 5 is in many respects similar to the Logan boiler. However, the screen tubes are 3 in., while the original tubes at Logan were 3.5 in. in diameter. The ash screen, which has a slope of approximately 7 deg, is shown in elevation in Fig. 29.

Initially the superheat on boiler 5 tended to run from 30 to 40 F too low and adjustment of the burners upward was necessary to attain 900 F. No such adjustment was necessary on the earlier boilers 1 and 3.

Boiler-Water Conditions. In boiler 5, as well as in the older high-pressure boilers at the Rouge plant, the water conditioning has always been carefully controlled. Only infrequently does the boiler water show concentrations which are outside of the control limits of from 10 to 20 ppm of OH and from 60 to 80 ppm of PO_4.

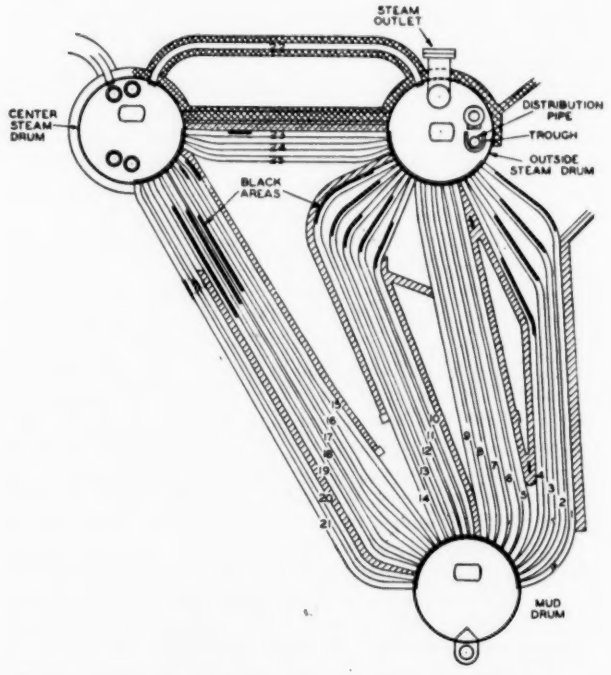


FIG. 31 LOCATION OF BLACK AREAS IN TUBES OF NO. 2 BOILER AT BEACON STREET
(Tubes 1 to 16 were inspected with a mirror. The lower ends of tubes 10 to 14 could not be seen because of bends. Tubes 17, 18, and 19 were removed and opened for examination.)

Although inspection has consistently shown all three high-pressure boilers to be free of attack in the ash-screen tubes, the boiler-water analyses show definite evidence of hiding out of phosphate during normal operation followed by its return to solution on reduction of load during the night or over a week end. The results of a test on boiler 5 are shown in Fig. 30.

This case has been included as representative of the considerable number of boilers in which no serious difficulty has yet been encountered, even though steam blanketing apparently is present.

Generating Tubes in a Bent-Tube Boiler

Beacon Street Heating Plant

The cracking of tubes in the double-set bent-tube boilers in the Beacon Street Heating Plant of The Detroit Edison Company has been reported previously (13), but is discussed here with some additional information. When the first two of these boilers went into service in September, 1926, they were the largest in the world with respect to heating surface. It was soon found, however, that the desired output of 435,000 lb of steam per hr from each boiler could not be reached without excessive carry-over. After trial of mechanical separators had failed to produce sufficiently dry steam, it was found by experiment that lowering the working water level achieved the desired result. Accordingly, during the summer of 1928, the water columns on each boiler were moved down 13 in. from their original location at the center line of the 54-in. drums.

For nearly eight years the boilers operated without difficulty. During inspections black areas were noted on the upper internal surfaces of many of the tubes, but little importance was attached to this condition until a No. 19 tube in boiler 2 developed a leak in February, 1936. When this was removed and sectioned, a thin, velvety-black coating of magnetic iron oxide was found on the high side, extending over a distance from 3 to 7 ft downward from the center drum, and in this area were many circumferential cracks. The tube wall had not been thinned, nor was it bulged.

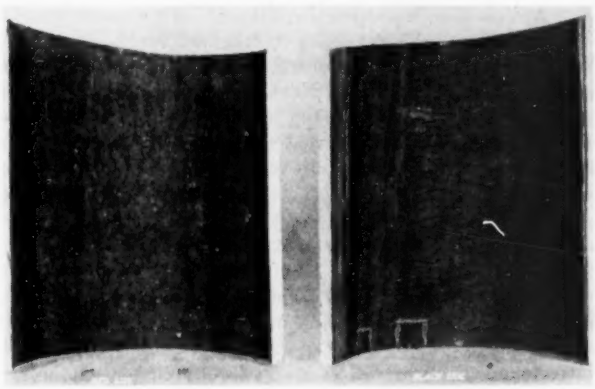


Fig. 32 Cracking as a Result of Steam Blanketing in a Bent-Tube Boiler at Beacon Street Heating Plant

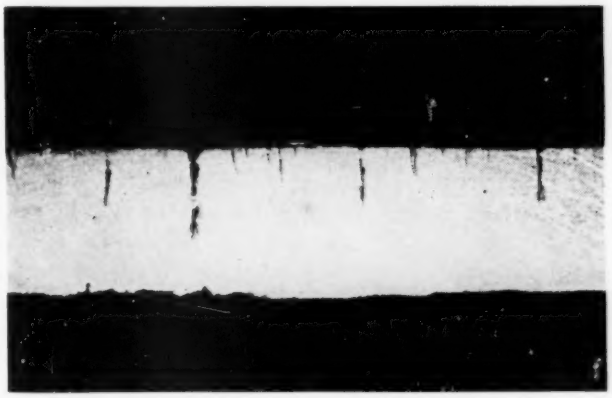


Fig. 33 Cracks Developed From Internal Surface of Boiler Tube at Beacon Street Heating Plant



Fig. 34 Decarburized Region Adjacent to Crack, ×500

However, along this portion of its length the tube was warped about 3 in. out of line, due to an apparent shortening of the black side.

Subsequent detailed examination revealed black areas as indicated in Fig. 31, in every case on the high side of the tube and in most of the tubes across the boiler. The No. 21 and No. 20 tubes, in front of the first baffle and closest to the furnace, were notably unaffected. The No. 19 tubes, however, not only showed well-defined black areas, but the majority were cracked in the same manner as the first tube removed. Fig. 32 shows the typical appearance of the undamaged front half and badly cracked rear half of one of these tubes.

Inspection revealed that boiler 1, and also boilers 4 and 5 installed respectively in 1929 and 1927, all showed black areas similar in appearance and location to those in boiler 2, but no cracks were apparent on visual examination of specimen tubes. A rough bending test caused the development of obvious cracks in the black areas but not in the normal areas of tubes from boiler 1. No cracks were revealed by this test in specimens from boilers 4 and 5.

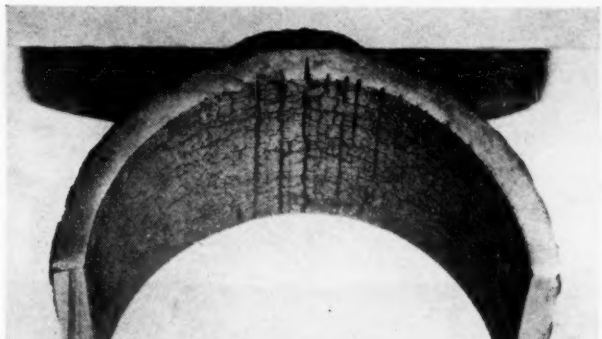


FIG. 35 LONGITUDINAL AND TRANSVERSE CRACKS IN TOP OF STEAM-CIRCULATING TUBE FROM 600-LB BOILER AT PIEDMONT PLANT

The typical character of the cracks is illustrated in Fig. 33. Microscopic examination showed them to be transcrystalline and revealed that the tube metal had been locally overheated. It was noted, moreover, that the inside surfaces of the cracks and the inside surface of the tube adjacent to the cracks were almost completely decarburized, as may be seen in Fig. 34.

Further evidence concerning the cause of the cracking was supplied by operating tests with thermocouples attached to tubes in the areas where the black-oxide coating had developed. It was found that, at the water level which had been employed, areas on tubes 15, 16, 17, 18, and 19 remained continuously at temperatures of from 600 to 1200 F, but that when the water level in the drum was raised, normal temperatures of from 360 to 460 F resulted. Phenomenally rapid and extreme changes in temperature were observed. Thus, on many occasions, raising or lowering the water level in the gage glass only $^1/_2$ in. produced a change of nearly 800 F in less than 1 min at some points on the high side of the tubes.

Overheating due to steam blanketing was recognized as the immediate cause of the difficulty. From the temperature studies, it was decided that the water level must be raised 9 in. toward the original level to establish adequate circulation through the tubes which had been affected. This again raised the problem of wet steam, which was solved by installing special baffles in the center steam drum and adding 184 new steam circulators between the center and the outside drums in each boiler.

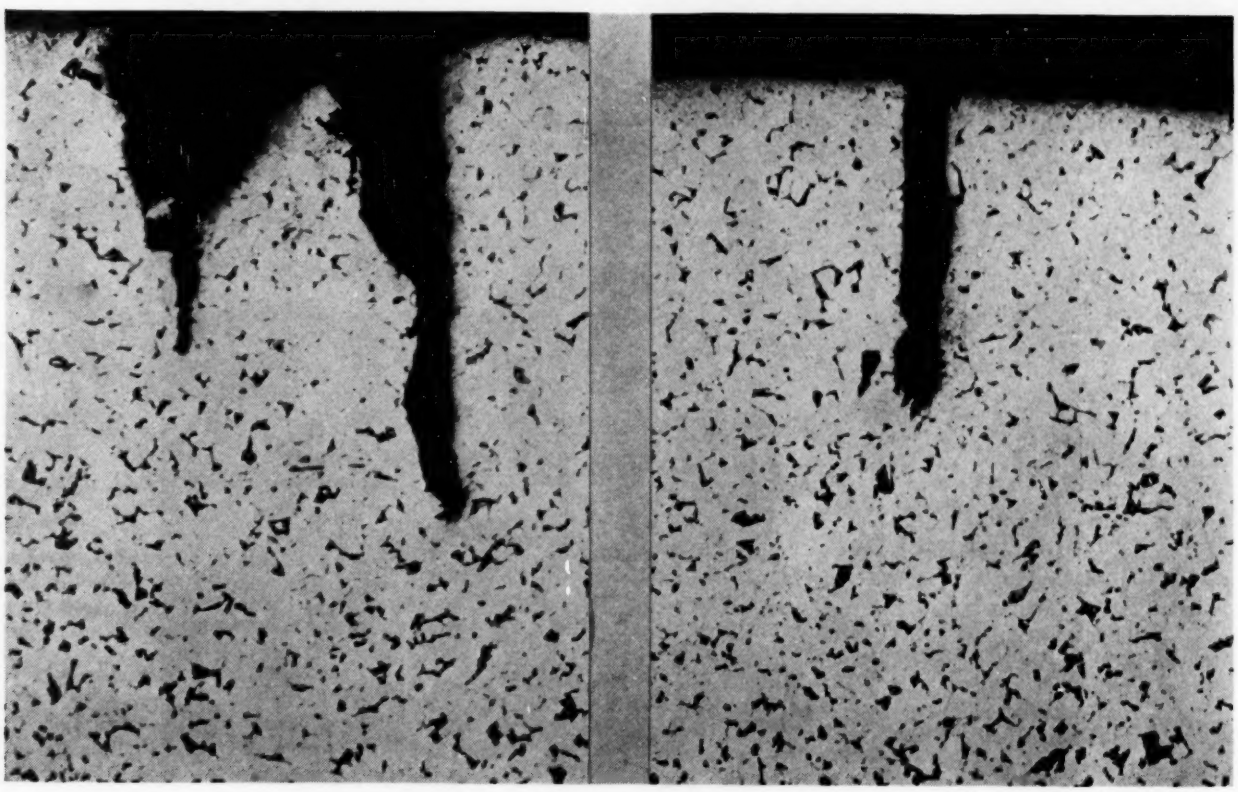


FIG. 36 CHARACTER OF CRACKS IN TOP OF STEAM-CIRCULATING TUBE

Cracking of Steam Circulators in a Bent-Tube Boiler

West Virginia Pulp and Paper Company

Boilers 22 and 23 in the Piedmont Plant of the West Virginia Pulp and Paper Company are 600-psi three-drum bent-tube units, which went into service respectively in December, 1934, and May, 1938. Cracking of steam-circulating tubes in each of these boilers has posed an interesting problem.

The first cracking in boiler 22 was observed in the spring of 1937, and all of the trouble developed in the second row of tubes from the top, at or close to the bend adjacent to the steam drum. Failure occurred first in the third and fourth tubes from one side and then in the fourth and fifth tubes from the other side. Efforts were made to continue these in service by welding up the first cracks observed but when further cracking led to the removal of tubes it was discovered that the internal surfaces were literally disintegrated. Fig. 35 shows the network of longitudinal and transverse cracks on the upper half of one of the tubes after pickling in acid to dissolve the thin coating of calcium phosphate present when it was removed from the boiler.

Under the microscope the tube wall had the typical appearance shown in Fig. 36, suggesting that the trouble was due to stress corrosion. The steel showed no signs of spheroidization.

By segregating the discharge from the wall tubes into the intermediate drum so that it was forced to pass out through the top row of circulators to the steam drum, conditions in the second row of circulators in boiler 22 were materially improved. This arrangement was accordingly installed in boiler 23. This did not prevent the development of the same type of cracking, which first became evident during the fall of 1938. In boiler 23, however, the failures occurred in the third instead of the second row from the top, and black oxide was evident on the ceilings of the tubes in the five lower rows.

A number of changes have been made to increase the ratio of water to steam in the circulating tubes. The third row of tubes, in which the cracks had developed, was removed and the holes were blanked; discharge from the upper waterwalls was divided between the top two rows; discharge from the lower waterwalls formerly entering the mud drum was routed into tubes delivering directly to the steam drum; and the baffling in the gas passage through the vertical tubes between the steam and mud drums was changed in an effort to increase the rate of downward flow of water through the tubes. While further modifications may be necessary for a final solution of the problem, recent operation indicates that the condition is much improved if not entirely cured.

The Relation Between Low pH, Pitting, and Stress Corrosion

Operation with little or no caustic alkalinity has proved effective in minimizing the attack of excessively concentrated boiler water on the tops of steam-blanketed tubes, as indicated in the preceding discussions of Port Washington and Logan Stations. While it is known that boilers fed with condensate containing consistently little dissolved oxygen may operate for years at a relatively low pH level with no difficulty from pitting or stress corrosion, nevertheless the experiences of a number of plants suggest that low alkalinity may contribute to these types of attack. As examples, the histories of Colfax Station of Duquesne Light Company and of Edgar Station of Boston Edison Company may be cited.

Pitting and Stress Corrosion at Colfax Station

At the time the first unit of seven 265-psi straight-tube cross-drum boilers was installed in Colfax Station of Duquesne Light Company in 1920, operation with pure distilled water was considered the ultimate answer to the then almost universal problem of boiler scale. With the exception of a few weeks during 1924, when soda ash was fed to the head tanks, and of some tests with sodium silicate on individual boilers, the plant was operated without chemical treatment until November, 1929. In the meantime a second unit of seven boilers had been added in 1922, a third unit of five in 1924 and 1925, and a fourth unit of three in 1927. While the design changed in detail from unit to unit, the general arrangement of the heat-absorbing surface is indicated in Fig. 37, which shows a boiler of the third unit. Boilers of the first two units were stoker-fired, while those of the third and fourth units were fired with pulverized coal.

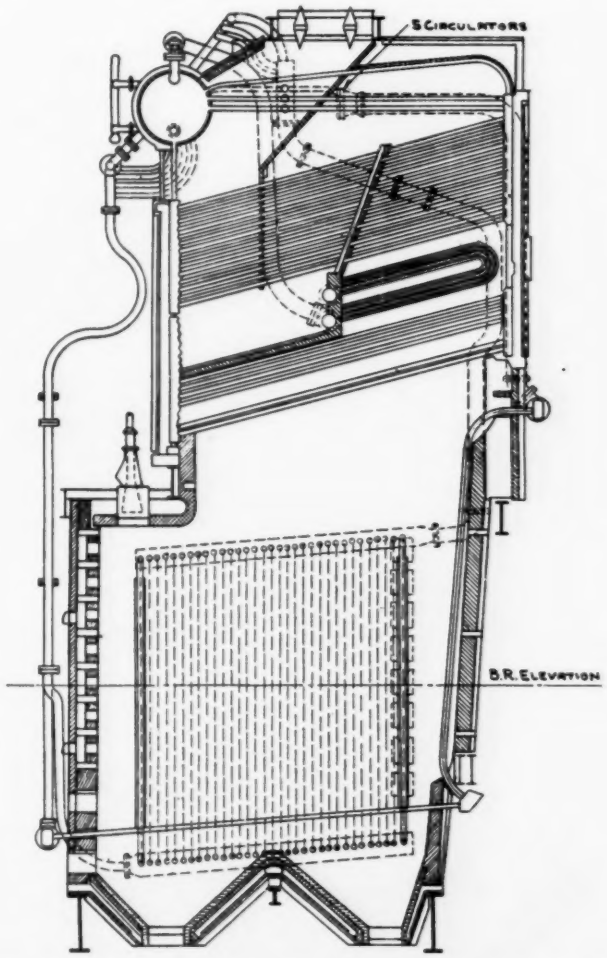


Fig. 37 Cross Section of Third-Unit Boiler at Colfax Station

Efforts to Reduce Oxygen Content of Feedwater. According to the information available when Colfax Station was designed, serious corrosion was not anticipated if the oxygen content of the feedwater was kept below 0.5 ml per liter. In retrospect, it is not surprising that this amount of oxygen introduced into boiler water at a low pH should have caused noticeable corrosion during the early operation of the plant. This led to the practice of covering the surfaces of the drums with a special paint. In 1923 and 1924 pitting of the tubes in the boilers of the first unit was recognized as a problem, and oxygen as a primary factor. Painting of the generating tubes, started late in 1925, proved ineffective.

A small deaerator installed to treat the 2 per cent make-up from the evaporators and auxiliary supplies went into service



FIG. 38*a* EXTERNAL VIEWS OF CRACKED NIPPLE

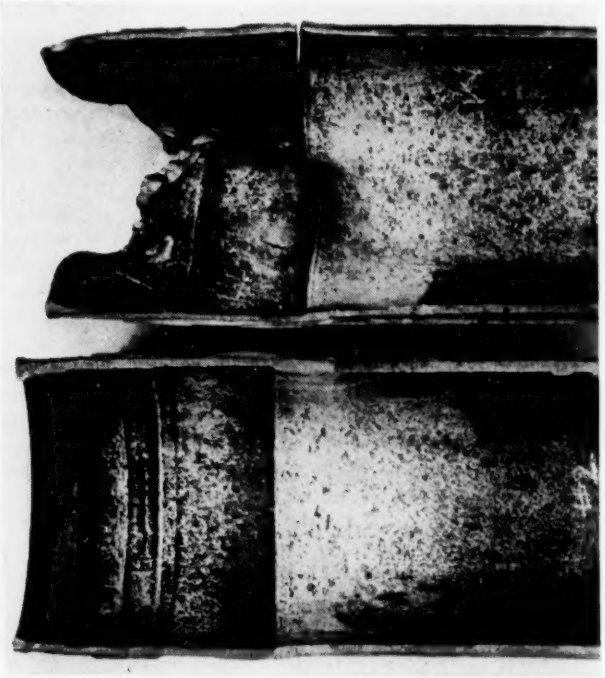


FIG. 38*b* INTERNAL VIEWS OF CRACKED NIPPLE

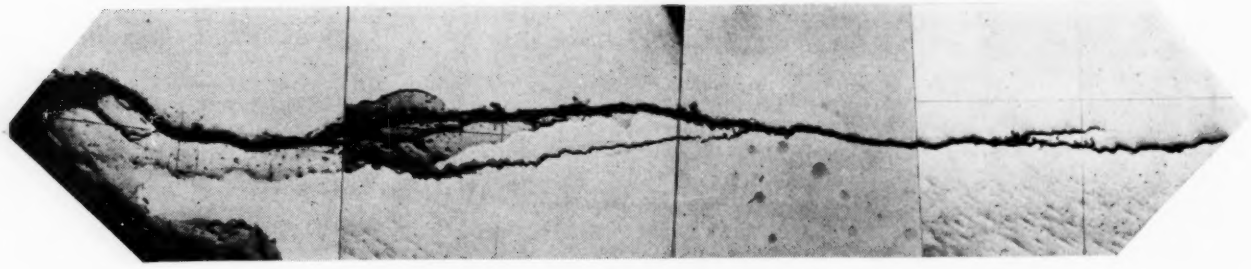


FIG. 40 TYPICAL CRACK IN HEADER, ×20

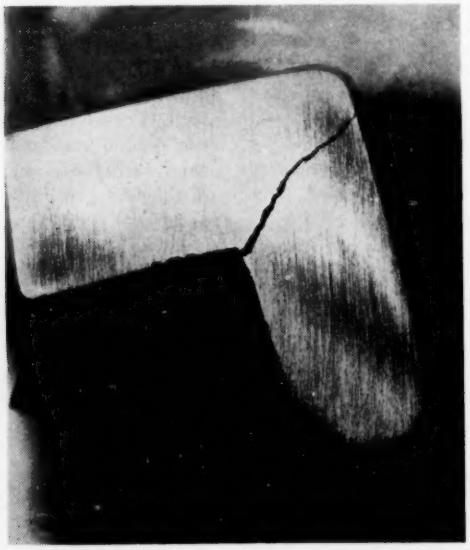


FIG. 39 STRESS-CORROSION CRACK IN CORNER OF HEADER

in April, 1924, and in September, 1925, closed heaters were substituted for the barometric condensers which had served as feedwater heaters, while possible sources of air leakage into the system were closely checked. As a result, the oxygen content of the feed was reduced from 0.5 to a maximum of 0.2 ml per liter (14) during full-load operation of the first and second units. A second deaerator was installed in 1927 with the fourth unit of boilers. In 1930 the small deaerator installed in 1924 was replaced by a larger one to deaerate surge return to the condensate system.

Cracking Due to Stress Corrosion. Not only did pitting continue in spite of these efforts, but nipples in the downtake headers and the headers themselves began to crack with increasing frequency as a result of what would now be called stress corrosion, which may have been developing for years. Figs. 38 and 39 illustrate typical cracking in a nipple and a header, respectively. The character of the cracks in the headers is shown by the photomicrograph of Fig. 40. All cracks developed from the internal surface.

Beginning in 1928, as a precautionary measure, all of the downtake nipples on the boilers at Colfax Station, except those in the newly installed fourth unit, were replaced with particular care to minimize deformation during rolling in.

In the boilers of the first and second units, which were equipped with umbrella-type feed nozzles in the drums, the cracking had occurred chiefly in the nipples in the downtake headers immediately below these nozzles. Boilers in the third and fourth units, equipped with perforated feed pipes extending the length of the drum below the water level, developed cracking of the nipples more at random. It was found however that the oxygen content of the boiler water was lower in the first and second units, where the entering feedwater was partially deaerated by spraying through the steam space,[10] than in the other units. The combined advantages of deaeration and distribution were secured in the boilers of the first two units by substitution of troughs for the feed nozzles during the period from May, 1929, to November, 1930.

Chemical Conditioning of Boiler Water. It should be remembered that during all of these efforts to reduce the oxygen content of the feedwater and to distribute the feed, the water in the boilers contained only what dissolved substances might be introduced by evaporator carry-over or condenser leakage, so that the pH inevitably remained low, ranging between 7 and 9. In November, 1929, however, after some tests on a single boiler, the feed of trisodium phosphate to all boilers was started for the combined purposes of preventing the formation of a thin but dense scale on the heat-transfer surfaces, and of minimizing corrosion. While control on the basis of maintaining from 25 to 50 ppm of PO_4 in the boiler water did eliminate scale, the average pH of the water in the boilers was increased only to about 8.8, and corrosion and cracking continued, although to a decreased extent.

In June, 1932, the trisodium phosphate was supplemented by caustic soda to maintain between 17 and 34 ppm of OH in the boiler water. Installation of steam seals on the surge tanks the following March reduced the dissolved oxygen in the feedwater to the range from 0.02 to 0.05 ml per liter, as determined by the standard Winkler procedure, and the feed of sodium sulphite was commenced in May, 1933. No failures due to cracking have occurred since 1933.

At the time the feed of sodium sulphite was started, corrosion was occurring primarily at the uptake ends of the tubes in the lower deck and chiefly on the lower half of the tubes. In the stoker-fired boilers of the first two units, the treatment with sodium sulphite substantially stopped this corrosion which, however, persisted in the pulverized-fuel boilers operated at somewhat higher ratings. It was found, as a result of a series of tests begun during the latter part of 1935, that ferrous hydroxide effectively inhibited this type of corrosion. As a result, it has been progressively substituted for sodium sulphite on all pulverized-fuel boilers. The reason for the improved results with ferrous hydroxide in these boilers is not known. During the last two years, there has been practically no indication of active corrosion at Colfax Station, although occasional failures of previously pitted tubes have occurred.

Summary of Replacements Due to Corrosion. Condensed in Table 7 is a record of replacements of generating tubes, steam circulators, and headers on account of internal corrosion. Many of the tube replacements were made before actual failure, particularly those in the third unit in 1930 and 1932. Not included in the table are a considerable number of tubes removed primarily for observation in the early years of the station, and the complete replacement of all downtake nipples in the first three units, which was started in 1928.

A number of interesting features are apparent in Table 7. Thus 10 out of 11 cases of header cracking occurred in the first unit, with the remaining one in the third unit. Cracking of nipples was distributed chiefly between the first and third units, with 16 in the former, 14 in the latter, and only 4 in the second unit.

The third unit has undergone by far the most extensive replace-

[10] The oxygen survey given in (14) indicated only from 0.04 to 0.06 ml of oxygen per liter in the boiler water at various locations when the entering feedwater contained 0.2, and the saturated steam 0.24 ml per liter.

TABLE 7 TUBE AND HEADER REPLACEMENTS AT COLFAX STATION DUE TO INTERNAL CORROSION

Unit	Boiler	1920	1921	1922	1923	1924	1925	1926	1927	1928	1929	1930	1931	1932	1933	1934	1935	1936	1937	1920 - 1937 Total
1	1 2 3 4 5 6 8	(12/8) (12/8) (12/22)	(2/9) (2/7) (1/4) (2/20)	1	1	1 3	1		1 1	1 2 2 3	14 6 2 1	4	4 1 1	3 1 1	1 2 2	1	2			16 19 1 8 0 2 0 1
Total				1	1	4	1	1	2	0	0	1	23	4	4	1	0	1	5	0	1	2	1	0	1	2				68
2	7 9 10 11 12 13 14			(10/18) (11/18) (11/26) (12/2)	(4/18) (2/27)	(2/18)					1 2 1 3	17 1	5 1	1	2 9 1	1	1		10 1	51 0 0 4 1 0 1
Total											4	17	4	0	1	0	13	0	0	1	0	0	0	1	11				42
3	15 16 18 20 22					(8/17) (10/7)		1		4	1 2 4	1	47 40 48 47 47	1	69 26 107	5 20	7 2 41 5 1	4 40 3	2 2	1 4 2 1	2	2 1	10 51 90 175 15	47 48 51 69 47	1 0 0 0 0	1 4 0 0 2
Total													227	1	202	25	57	72	2	8	4	3	347	255	1	14
4	17 19 21									(12/11) (12/17) (10/31)	1	1			5		1 1			1			7 2 0 12	0 1 0 4	0 0 0 0	0 0 0 0
Total											1	1	0	0	5		2	1	1				21	5	0	0
Total for all four units				2	1	4	1	1	1	2	10	1	2	1	10	84	13	4	31	231	6	1	1	282	29	2	65	1	55	2	9	15	2	663	272	11	84

G - Generating tubes.
S - Steam circulators.
F - Headers.
N - Nipples.

Dates in parentheses indicate when respective boilers were placed in service.

ment of tubes due to corrosion, with 367 out of a total of 465 generating tubes, and 262 out of a total of 273 steam circulators. The general replacement of the latter in 1930 in all the boilers of the third unit may have been due chiefly to overheating produced by secondary combustion in the upper part of the furnace adjacent to these necessarily steam-blanketed tubes. This view is substantiated by analyses which show that the top halves of the tubes were coated primarily with iron oxide, while the deposit on the lower halves comprised typical scale-forming constituents.

The experience at Colfax Station might be summarized briefly by the statement that, after reduction of dissolved oxygen in the feedwater and maintenance of higher alkalinities and oxygen-scavenging chemicals in the boiler water, pitting and stress corrosion have apparently been stopped.

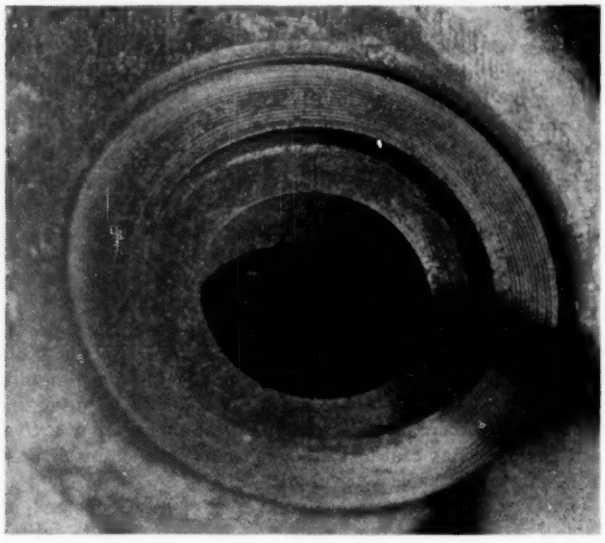


FIG. 41 CRACKS IN WATER COLUMN AT LOWER CONNECTION, BOILER 6, EDGAR STATION

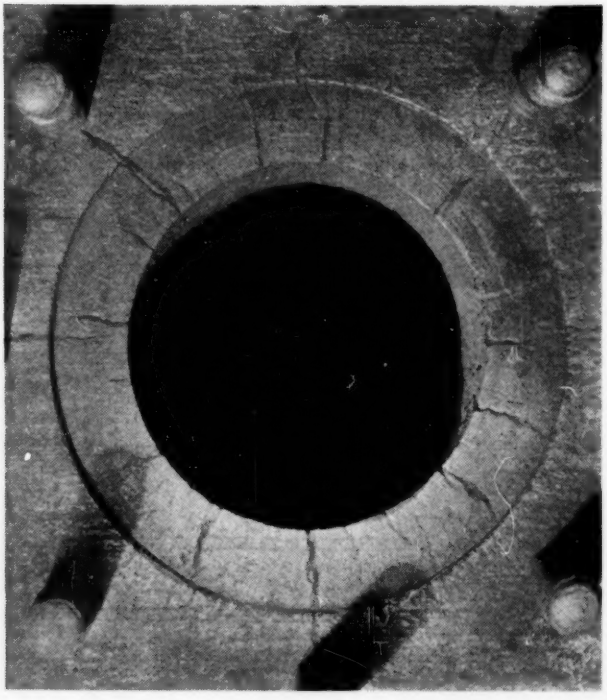


FIG. 42 CRACKS IN DRUM AT FEED-LINE CONNECTION, BOILER 5, EDGAR STATION

STRESS CORROSION AT EDGAR STATION

In Edgar Station of Boston Edison Company there are five high-pressure boilers. No. 3, installed in 1925 to operate at 1200 psi, was one of the pioneer units in this pressure range. Boilers 5 to 8, designed for 1400 psi, went into service successively from 1927 to 1929.

Late in 1935, it was found that the water columns on boiler 6 were cracked at the bottom companion joint, as indicated in Fig. 41. The same condition was then observed on boiler 5. In no case, however, were any cracks found in the corresponding joints at the upper end of the water columns.

The following year, cracks were discovered in boiler 3 in the cutouts for the feedwater connections, where the drum itself formed part of the feed line. Later inspections showed similar

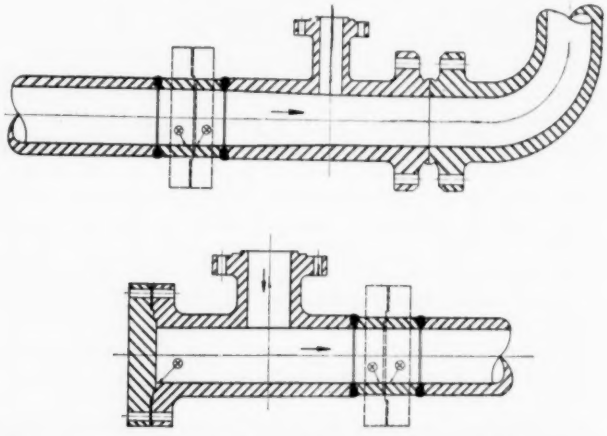


FIG. 43 LOCATION OF CRACKS AT BOTTOM OF FLANGED CONNECTIONS AT ENDS OF SUPERHEATER INLET AND OUTLET HEADERS, BOILER 5, EDGAR STATION

cracking in boiler 5, both in the drum at the feedwater connections and in the flanged joints of the superheater headers, and in boiler 6, in the Van Stone laps of the supply and riser tubes to the waterwalls.

The most serious condition was that in the drum of boiler 5. Fig. 42 shows how the cracks had developed to the stud circle on the inside surface. This boiler was removed from service in February, 1938, for rebuilding. The details of the inspection made at this time and of the changes made to obviate further cracking have been given by White (15).

Both repeated stress and contact with water at a low pH seemed to be important factors in the development of the cracks in the various locations. The feedwater to the drum, comprising condensate and evaporated make-up, was substantially neutral, with a pH between 6.9 and 7.2, and a temperature between 250 and 350 F. As a result of the intermittent introduction of feedwater by the automatic regulator, the temperature in the wall of the hole in the drum must have varied repeatedly between this range and the temperature of the water in the drum, about 585 F, presumably setting up localized thermal stresses in the steel.

On both the inlet and the outlet headers of the superheaters, tees are connected by flanged companion joints, as indicated in Fig. 43. Cracks were found in these joints at the bottom only, where a definite water line showed that condensate had col-

lected at various intervals. No cracks were found in the side outlets of the tees, which were in a position to drain readily, or in the Sargol joint on the outlet fitting.

In the water columns, normal condensation kept the bottom connections in contact with water which showed pH values as low as 7.3 when samples were withdrawn slowly to avoid flow of water from the drum back into the column.

The only cracking where water at a higher pH may have been in contact with steel was in the Van Stone laps on the supply and riser tubes to the waterwalls of boiler 6. Since the supply tubes take off from the drum, it might be expected that boiler water with a pH of 11 or higher would pass through them. Short-circuiting of feedwater down the supply tubes is, however, possible. No means are available for checking the actual pH of the water in this circuit.

The concentration of dissolved oxygen which may have contributed to the stress corrosion is not known. Although mechanical deaeration eliminated oxygen from the feedwater to the point where it could not be detected in the standard Winkler test, some trouble with pitting was encountered prior to the adoption of sodium sulphite as a chemical scavenger during September, 1932.

Interpretation of Evidence

By piecing together evidence from the various case histories a reasonably complete picture of steam blanketing and its effects may be sketched. The most direct proof of the development of a steam blanket in a slightly inclined tube is afforded by the experiments at Logan in which substantially pure steam issued from a special sampling line located with its intake at the top of one of the original ash-screen tubes, Fig. 19. The overheating which must result from high heat input to a blanketed tube was observed by means of thermocouples at San Antonio and at Beacon Street, and was indicated by the microstructure of the steel at South Amboy (Fig. 5) and at Logan (Fig. 23). The hiding out measured at San Antonio (Fig. 7) and Rouge (Fig. 30) may reasonably be ascribed to localized evaporation on the ceiling or at the water line along the sides of steam-blanketed tubes.[11]

Grooving and Cracking of Tubes

Grooving such as occurred at South Amboy (Fig. 4), Port Washington (Fig. 15), and Logan (Fig. 20) is believed by the authors to develop where a relatively constant condition of steam blanketing causes excessive localized concentration of a normally alkaline boiler water along the ceiling of a highly overheated tube. Cracking such as was experienced at Beacon Street (Fig. 32) and at Piedmont (Fig. 35), on the other hand, is attributed to repeated quenching of overheated steel by intermittent contact with boiler water.

It is interesting to note that grooving occurred in the high-pressure boilers at Mannheim, Germany, in 1929 (16). The published photographs show tube walls thinned in the same manner as those illustrated in the present paper.

An early case of cracking of upper-row tubes near the uptake headers occurred in 1925 at Hell Gate Station of the United Electric Light and Power Company. The study of tube temperatures and the application of cores to prevent overheating has been briefly described elsewhere (17).

Effect of Boiler-Water Composition

Maintenance of low controlled alkalinity characteristic of the best contemporary operation did not prevent failures of steam-blanketed tubes at South Amboy, San Antonio, Springdale, Waterside, Rivesville, Port Washington, Logan, Beacon Street, or Piedmont. Increasing the concentration of sodium hydroxide aggravated the trouble at South Amboy (Fig. 6) and at Port Washington (Fig. 17), while substantial elimination of this substance from the boiler water at Port Washington and at Logan minimized grooving (Figs. 17 and 25).

On the other hand, Rouge is representative of many plants which have experienced no difficulty when operating with normal low concentrations of sodium hydroxide in the boiler water, while stress-corrosion cracking has apparently been promoted by the absence of this constituent at certain localities in the boilers at Edgar and during the earlier operation at Colfax.

Formation of an iron phosphate, which takes place only to an insignificant extent in boilers operated with definite though low concentrations of sodium hydroxide, apparently is promoted by elimination of this caustic alkalinity. Both at Port Washington (Fig. 18) and at Logan (Fig. 26) considerable amounts of a white iron phosphate were found admixed with black Fe_3O_4 on the tube surfaces and exposed along the steam-blanketed strips on the tube ceilings. The authors believe this to be merely a casual result, with no particular significance as far as prevention of attack is concerned.

Effect of Tube Slope

The influence of tube slope is shown most clearly by the fact that the drop-leg tubes at South Amboy failed frequently, and always along the top of the nearly horizontal section, while the straight inclined tubes with which they alternated were practically free from trouble except as they were cut externally by leaks from the drop-leg tubes. At Logan the increase in slope of the ash-screen tubes was accompanied by a decrease in diameter; to what extent the resulting improvement may have been due to possible increase in velocity of flow can scarcely be guessed, but the writers believe that this effect was less important than the increased slope.

Effect of Circulation

So little is known about the rate at which water actually circulates through individual tubes that little can be done to evaluate the influence of this factor. The attempt at Logan to double the velocity in the tubes at the corners of the ash screen failed to produce any marked improvement, and the modifications with respect to flow at San Antonio apparently had no great effect.

In straight-tube sectional-header boilers in general, the circulation appears to be downward through the upper row or rows of tubes (7, 8). In most cases, any damage from the resultant steam blanketing is confined to the tops of the tubes near the uptake header, as at San Antonio, and at Springdale before insertion of cores. The location of the failures on the bottom of the tubes at Waterside is difficult to explain, unless it is assumed that at times little water was passing through them. The attack on the bottom of the cored tubes at Springdale suggests the same condition.

With respect to ash-screen tubes, even if the rate of flow were greatly increased, as by forced circulation, steam blanketing might still occur. In this connection, the authors have had the privilege of examining a specimen of studded tube from a forced-circulation once-through boiler which showed pits on the internal surface precisely under the studs on the portion of the wall toward the combustion chamber. Apparently the studs had served as channels for localized high heat input to the tube wall.

Effect of Rate of Heat Input

Heat absorption sufficient to develop steam blanketing in a specific tube may not actually cause any serious damage to the steel. Progressive increase in heat transmission must, however, increase tube temperature and localized concentration of boiler

[11] Hiding out is also indicated in the control reports of a number of other boilers of contemporary design.

water, thus accelerating attack. Unfortunately, there is little basis for determining the maximum rate at which heat may be safely absorbed by a steam-blanketed tube.

The experimentation at San Antonio revealed that as long as the temperature of the tube wall did not exceed 800 F, oxidation was slow. The practical solution of the problem at this plant, it will be recalled, was baffling of the gas flow to shift the zone of heat absorption away from the upper ends of the tubes.

Swinging the burners out slightly from the walls at Logan moved the region of most severe attack from the sides toward the center of the ash screen. Adjustments of this sort may have played a critical part in determining whether other plants would or would not encounter trouble as a result of steam blanketing. For example, the shift of the burners upward on boiler 5 at Rouge in order to achieve the desired superheat may have helped to obviate the failure of ash-screen tubes.

The three 1300-psi boilers at Lakeside Station of Wisconsin Electric Power Company represent the pioneer application of ash screens in the United States. What the actual heat transmission to the tubes in these screens may be is not known, but the fact that no failures have occurred in spite of probable steam blanketing suggests a lower input to these tubes than to those in more recent installations of similar type. It is interesting to note from the discussion by Anderson (18) that some development of black oxide took place at Lakeside at particular points in both the boiler and fin tubes where the heat transfer was the highest.

Stopping the failure of floor tubes at Rivesville by increasing the thickness of the floor insulation is perhaps the most direct example of the effect of rate of heat input on the damage to steam-blanketed tubes.

Intergranular Cracking

While in the tubes at Beacon Street and at Piedmont which were presumably subjected to repeated quenching, the cracks were characteristic of stress corrosion (Figs. 33 and 36), the intergranular cracks observed first at Waterside (Fig. 13) and later at Logan (Fig. 28) suggest the possibility of embrittlement by sodium hydroxide or by hydrogen. However, intergranular cracks produced in low-carbon steel during long-time creep tests at 1000 F in air by White, Clark, and Wilson (19) and cracks developed in Armco iron at 1100 F in air as well as in hydrogen by Thielemann and Parker (20) are similar to those in Fig. 13. The latter authors attributed the intergranular character of the cracks in their experiments, which also included low-carbon and alloy steels, primarily to long-continued load above the equicohesive temperature, and expressed the belief that intergranular cracking preceded intergranular oxidation rather than resulting from it. The similarity of prolonged overheating of a tube in a boiler to a creep test under low stress is obvious. Whether or not intergranular corrosion by the boiler water, analogous to that which sometimes develops in riveted seams, was a factor in the formation of the cracks at Waterside and at Logan, excessively high temperature seems to have been the fundamental cause.

Significance of Metallic Copper

Metallic copper present in deposits of magnetic iron oxide has sometimes been regarded as the principal cause of the process which produced the oxide (21). The authors believe, however, that the copper is merely an incidental product of the fundamental reaction of steel with boiler water, and if supplied to the boiler by attack on condensers, valves, pumps, or other auxiliary equipment, will be found chiefly wherever stress, steam blanketing, or other influence accelerates reaction between the water and the steel.

Black Oxide and Hydrogen as Warning Signals

Steam blanketing leaves its distinctive mark in a tube before the conditions produced by it become dangerous. This will usually appear as a dark band along the top of a tube, sometimes dull and sooty, sometimes scintillating with the light reflected from the faces of small but well-formed crystals of Fe_3O_4. The presence of such a band or area does not necessarily mean immediate danger, but it should be considered as a warning signal. When loose, powdery Fe_3O_4, which may be admixed with metallic copper, is found in any appreciable amount either adhering to tube surfaces or lying in tubes or headers, the condition should be regarded as serious.

Hydrogen in steam has been used as a measure of the attack on tubes under superheating conditions by Fellows (22) and more recently by Potter, Solberg, and Hawkins (23). As a measure of the over-all attack in the boiler it proved useful at Port Washington but yielded rather unreliable results at Logan. While the technique of accurate determination of the hydrogen content of a representative sample of steam is not simple, this type of measurement applied to saturated steam should tell whether or not any serious oxidation of steel by boiler water is taking place.

Definition of the Problem

Difficulties caused by steam blanketing in various types of boilers of four different manufacturers in nine plants have been described in this paper. The intention has not been to raise a great alarm, but to bring what seems to be a general problem clearly into focus, so that it may be discussed and solved to the advantage of both the manufacturer and the purchaser of contemporary high-capacity boilers.

How can the difficulties resulting from steam-blanketing of slightly inclined tubes in high-pressure boilers best be prevented? One answer would be to eliminate the steam blanketing by eliminating the slightly inclined tubes; another, to minimize steam blanketing by controlling heat input or improving circulation; and still a third, to modify the composition of the boiler water so that the rate of attack, even on overheated steel, would be insignificant.

A boiler without slightly inclined tubes is a problem for the designer. Where a boiler is already in service, extensive changes may be justifiable neither from the mechanical nor from the economic viewpoint. It may be possible, however, to redistribute heat absorption or improve circulation sufficiently to minimize the damage from steam blanketing, even though the latter may still persist.

To reduce the heat input over any appreciable fraction of the heat-absorbing surface of a boiler apparently runs counter to the current trend toward smaller furnace volume and higher heat release. The question may well be asked, however, whether a small increment in the initial cost of a boiler guaranteed for a specific output of steam might not save both the manufacturer and the purchaser several times this amount in money and in the money equivalent of the time during which the boiler is unavailable for service due to necessary modifications.

Over against the factor of design, which fundamentally determines whether steam blanketing will or will not occur in a boiler, and if so, to what extent the blanketed areas will be heated above normal temperature levels, must be set the factor of boiler-water conditions. By eliminating sodium hydroxide from a boiler water and operating only with the alkalinity contributed by phosphate, the wasting away of metal in steam-blanketed tubes may be greatly reduced, although attack will still continue at a slow rate. Control of the pH of the boiler water can scarcely, however, be expected to reduce the temperature of the overheated tube wall which, if quenched repeatedly by water striking it, may ultimately crack. Operation with zero caustic alka-

linity seems less a solution to the problem of steam blanketing than an expedient which, in competent hands, will allow operation until the necessary fundamental changes can be made.

The writers believe that the real solution of the difficulties described in this paper lies in the prevention of steam blanketing in tubes subjected to high heat input, or conversely, limitation of the heat input to tubes which are likely to develop a steam blanket. Once this is accomplished, the question of what alkalinity should be maintained in the boiler water is no longer conditioned by what must be done to save overheated steel from attack by excessive local concentration of the boiler water, but instead by what will give maximum protection to the boiler as a whole.

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17 Statement of The United Electric Light and Power Company, Edison Electric Institute Publication C4, July, 1935, p. 6.

18 "Operating Experiences With 1300 Pounds Steam Pressure," by J. Anderson, *Journal of the Institute of Fuels*, vol. 1, 1927, pp. 131–149; see particularly pp. 141–142.

19 "Influence of Time at 1000 F on the Characteristics of Carbon Steel," by A. E. White, C. L. Clark, and R. L. Wilson, Proceedings, American Society for Testing Materials, vol. 36, 1936, part 2, pp. 139–156, Figs. 7 and 8.

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Discussion

J. B. Romer.[12] The authors have presented considerable data that they have gathered on the question of high-temperature corrosion of steel by steam and the effect of boiler-water conditioning, particularly along the lines reported by M. K. Drewry (12) on this general problem.

Their paper may be summed up to the effect, that overheated tubes, irrespective of angle, in contact with steam corroded severely and that with the possible exception of a special treatment involving excess ferrous iron, there is no known feedwater treatment which adequately takes care of this situation.

The writer is in agreement with this summation and under these conditions the answer must lie in adequately taking care of this situation through the medium of design.

[12] The Babcock & Wilcox Co., Barberton, Ohio.

The microstructures as presented by the authors indicate that the nonoxidized metal, at the point of corrosion, had been at temperatures well above 1000 F and quoting from the authors:

> Correlation of the temperature measurements with the observed condition of the tubes showed that the deposit of black oxide had accumulated only in those tubes which had exceeded 800 F during the operating period. Measurement of the internal diameter revealed that there had been little actual loss of metal from the tubes.

These two items place the oxidation limit, of steel in steam, at or above 1000 F.

Straub in his paper[13] indicates that in laboratory tests the limit may be as low as 750 F—this when certain chemistry is adhered to. That control of the chemistry controls the extent of the oxidation is not only extremely interesting but it is also valuable information.

We should not lose sight of the effect of sodium sulphate. Straub shows that a decided decrease in the hydrogen generated and in the amount of corrosion observed follows as the result of an increased content of sodium sulphate.

This is quite significant and it is also unexpected that this particular problem should show up the advantage of the old caustic-soda antidote, sodium sulphate.

While the writer agrees that the answer to the formation of black magnetic oxide of iron in certain tubes at temperatures above about 950 to 1000 F is not one of proper water treatment, he wishes to point out that the feedwater chemist cannot dodge his two major problems:

1 To produce scale-free water at maximum operating temperatures by such methods that will

2 Produce water which will not damage the material of the boiler.

He should not raise the bugaboo that "lack of circulation is the cause of certain types of scale."

While there may be evidence indicating or pointing in the direction that certain types of scale form under these conditions of "dryness" there are other equally if not more definite indications that this dryness is the result of the formation of a type of sludge the porosity of which is insufficient to permit water to reach and cool the tube wall adequately, and further that this sludge formation may and does occur in the presence of ample circulation.

The second job, namely, to produce water that will not damage the material of the boiler generally involves controlling the pH. Experiences over a long period of time indicate that water having a low pH is excessively reactive toward iron and steel and unless this reactivity can be satisfied without using up or depleting the excess soluble ferrous ions then the boiler metal may suffer from some form of corrosion or corrosion fatigue.

In connection with the hiding out of soluble solids the writer would like to inquire as to a condition frequently encountered in modern boiler-water-conditioning practice. In many instances it is well known that the feedwater contains both calcium and magnesium apparently as carbonates or bicarbonates. Within the boiler these are treated with phosphate for the purpose of changing the calcium salts from a form of relatively low solubility to a more stable material supposedly of the sludge type. The blowdown from such units, even when the concentration of soluble salts is normal, is often crystal clear. Under these circumstances, what becomes of the sludge which is being formed in the boiler?

Another question which is quite pertinent to this problem is: Can dry areas be produced by water conditioning? In a number of instances sludge has accumulated in areas as shown in Fig. 1 and resulted in starvation of certain sections, thereby producing dry areas.

Among the case histories quoted by the authors are four with which the writer is familiar, namely, South Amboy, Rivesville, Edgar, and Colfax. At the present time it is considered that the first two of these have been satisfactorily taken care of by suitable changes in design. The third has been taken care of by changes in design which simultaneously involve changes in pH and the fourth case has been taken care of by changes in the pH alone.

The authors raise the question as to the possible secondary dangers as the result of maintaining low pH and point out the experiences at both Colfax and Edgar in this connection. Although the latter case may be of the stress-corrosion type, it is well known that low pH lowers the stress which may be applied to a pressure vessel to a marked degree, that many failures of boiler metal parts in the past have been directly due to low pH of the boiler waters, and that proper correcting of this value has eliminated the trouble.

Authors' Closure

That the reader does not always get out of a paper what the writer tried to put into it is illustrated by the remarks of Mr. Romer. His discussion implies that tube slope is not a factor in steam blanketing, and that oxidation of the ceiling of a steam-blanketed tube may possibly be avoided by maintaining excess ferrous iron in the boiler water. Without wishing to seem dogmatic, the authors again call attention to the improvement in conditions at South Amboy and at Logan following an increase in tube slope, and to the fact that the use of ferrous hydroxide at Colfax minimized only ordinary corrosion due to dissolved oxygen, rather than the oxidation due to steam blanketing, which was the chief concern of the paper.

Professor Straub and Mr. Nelson, in their paper[13] presented on the same program as that of the authors, concluded from laboratory studies of steam-blanketing that:

1 "This type of corrosion occurs only in partially dry areas.

2 "If the metal temperature is below 750 F, sodium hydroxide appears to be the active constituent in the boiler water causing the corrosion.

3 "If the metal temperature is below 750 F, the corrosion may be controlled by modifying the boiler water as follows: (*a*) reducing the causticity to zero, and (*b*) adding other salts, such as sulphate, silicate, etc.

4 "If the metal temperature is above 750 F, corrosion takes place in the absence of sodium hydroxide and chemical treatments effective at the lower temperature are no longer protective."

Unless one is willing to grant or to guarantee that the ceiling of a steam-blanketed tube will fail to exceed 750 F or some temperature not far above this, there is little basis either in the laboratory tests of Straub and Nelson or the operating experiences cited in the present paper for assuming that ferrous hydroxide or sodium sulphate or anything else added to the boiler water would prevent damage to the steel.

As the writers noted in the second sentence of the paper, neither the insulating effect of scale nor the acceleration of ordinary corrosion by dissolved oxygen are factors in the type of damage which is the chief subject of the paper. In most of the cases described, the amount of sludge found on the boiler surfaces was literally microscopic, and in no case was it sufficient to be a factor in the major problem. The authors hope, therefore, that, without evading any responsibility, they may postpone to some more appropriate forum a response to Mr. Romer's comments and questions relative to scale and sludge.

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[13] "Corrosion in Partially Dry Steam Generating Tubes," by F. G. Straub and E. E. Nelson, *Mechanical Engineering*, vol. 61, 1939, pp. 199–202.

Deflection of Helical Springs Under Transverse Loadings

By W. E. BURDICK,[1] F. S. CHAPLIN,[2] AND W. L. SHEPPARD[3]

This paper presents an analytical approach to the problem of the transverse deflection of a helical closely coiled spring under the action of a force perpendicular to its axis and an end moment. Supporting test data are included to substantiate the formulas within established limits. Expressions for the stresses under this loading in combination with an axial force are derived for the critical points on the spring.

THE USE of helical closely coiled spring suspensions for vibrating masses has not only necessitated a careful consideration of the characteristics of the spring under normal axial loading but in addition, where a thrust acts in a direction perpendicular to the axis of the helix, equal regard should be given to the properties of the coil under transverse loadings. This condition is evident in the case of those reciprocating mechanisms where an unbalanced force lies in the horizontal plane; also, it exists in certain types of vehicular suspensions in which a helical spring is utilized to take lateral or fore-and-aft reactions in addition to gravity loadings. The latter application is best exemplified in certain recently constructed types of railway-car trucks incorporating this particular capacity of a helical spring to cushion loads in more than one plane.

It is the authors' intention to present herein an analytical approach to the problem of a helical spring with one end resting on a rigid foundation and the other subjected to a transverse loading alone or in combination with an end moment. The investigation includes a derivation of the relationship for the resulting transverse deflection at the free end and attendant stresses under such a condition. Comparison will be made with actual test results made with a wide variety of spring sizes and loadings to demonstrate the validity of the expression for deflection and establish the approximate range of application. The stress derivations are dependent on the ability of the stock itself to follow Hooke's law of proportionality for a material in bending and torsion, with appropriate factors for stress concentration due to curvature and may be accepted with no more reservations than are accorded the present approved method of investigation for the axial loading condition.

The only specific information available on this subject which the authors were able to find was a paper by Marty.[4] In this case, the investigation was mainly concerned with a uniformly distributed transverse load over the length of the spring as occurs under centrifugal loading. This author expressed an effective flexural and torsional rigidity of a helical spring analogous to a beam as a function of certain of its dimensions, and stated the relationship for stress and deflection but without derivations or supporting test data. Timoshenko[5] has evaluated the equivalent column characteristics of a helical spring as an axially loaded member for purposes of determining its stability.

Nomenclature

The nomenclature used in this paper is as follows:

- h = height of helix
- ϕ = slope of end coil
- D = diameter of helix
- R = radius of helix
- d = bar diameter
- P = axial loading
- L = transverse loading
- M_0 = end moment on coil
- n = effective number of coils = $\left(\frac{\text{solid spring height}}{d} - 1\right)$
- f = pitch of helix = (h/n)
- θ = angular coordinate of any point on the helix
- M_t = torsional moment on any cross section of the bar
- J = polar moment of inertia of this section
- τ_t = shear stress on this section due to M_t
- M_x = bending moment on this section about the axis perpendicular to the axis of the helix
- I_x = moment of inertia about this axis
- σ_x = bending stress on this section due to M_x
- M_y = bending moment on this section about the axis parallel to the axis of the helix
- I_y = moment of inertia about this axis
- σ_y = bending stress on this section due to M_y
- S_x = shear load on this section perpendicular to axis of helix
- S_y = shear load on this section parallel to axis of helix
- Q = axial load on this section
- τ = shear stress
- E = modulus of elasticity in tension and compression
- G = modulus of elasticity in torsion
- Δ = vertical deflection of coil
- δ = transverse deflection of coil
- V = total strain energy
- C_v = vertical spring constant
- C_l = transverse spring constant

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Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[4] "On the Calculation of Helical Springs Loaded Transversely" (French), by R. Marty, Comptes Rendus, vol. 192, July 11, 1932, p. 105.

[5] "Theory of Elastic Stability," by S. Timoshenko, McGraw-Hill Book Company, Inc., New York, N. Y., 1936, pp. 165–167.

Deflection

Any analysis covering the exact deflection of a helical spring necessarily involves consideration of that portion of the deflection introduced by the end coils. Such an analysis introduces complex quantities, and for simplification only that portion of the helix will be considered which involves the full bar diameter.

Consider a helical spring with initially parallel ends supported on a rigid foundation, as shown in Fig. 1*a*. Assume the helix to extend along the center line of the bar from the first full diameter of the bar on one end to the corresponding point on the other end. This gives a helix height equal to the full distance

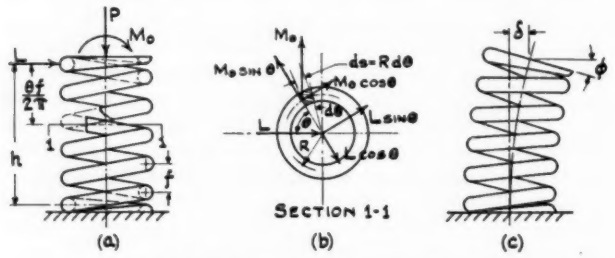


Fig. 1 Helical Spring With Initially Parallel End Supported on a Rigid Foundation

between coil ends less one bar diameter. Therefore, the number of turns in the helix n will be one less than the solid spring height divided by the bar diameter. Assume the spring to be initially loaded axially with P and consider the effect of superimposing a lateral load L and a moment M_0. Let L intersect the helix axis and lie in a horizontal plane containing the highest point on the helix. Let the transverse end couple M_0 have its vector perpendicular to the plane containing the helix axis and L. Since L is independent of P, only the deflections due to L and M_0 will be considered. The transverse deflection resulting from the application of P will be considered in the Appendix. The helix is developed by the continuous function $R\theta$ and the pitch f. The origin of θ will be taken at its coincidence with L.

The general equation for the strain energy[6] is expressed as

$$V = \frac{1}{2}\int_0^s \left(\frac{M_x^2}{EI_x} + \frac{M_y^2}{EI_y} + \frac{M_t^2}{GJ} + \frac{KS_y^2}{AG} + \frac{KS_x^2}{AG} + \frac{Q^2}{AE}\right) ds \quad \ldots\ldots [1]$$

This form of strain-energy equation neglects the effect due to curvature, but is sufficiently accurate for a first approximation.

In order to maintain the linear relationships required by the Castigliano theorem the influence of the couple $P\delta$ due to deflection is also disregarded. Since the strain energy of direct shear and compression is relatively small, only that portion of the energy equation dealing with bending and torsion will be considered. The slope and deflection may be expressed by

$$\delta = \frac{\partial V}{\partial L} \quad \text{and} \quad \phi = \frac{\partial V}{\partial M_0}$$

Hence, from Equation [1]

$$\delta = \int_0^s \left(\frac{M_x}{EI_x}\frac{\partial M_x}{\partial L} + \frac{M_y}{EI_y}\frac{\partial M_y}{\partial L} + \frac{M_t}{GJ}\frac{\partial M_t}{\partial L}\right) ds \quad \ldots. [2]$$

$$\phi = \int_0^s \left(\frac{M_x}{EI_x}\frac{\partial M_x}{\partial M_0} + \frac{M_y}{EI_y}\frac{\partial M_y}{\partial M_0} + \frac{M_t}{GJ}\frac{\partial M_t}{\partial M_0}\right) ds \ldots\ldots [3]$$

[6] "Strength of Materials," by S. Timoshenko, D. Van Nostrand Co., Inc., New York, N. Y., 1930, pp. 318–336.

For closely coiled springs, the pitch angle is small, and with sufficient accuracy, the reactions due to L and M_0 on any section with coordinate θ may be designated as follows:

1 A bending moment M_x in the vertical plane

$$M_x = L\frac{\theta f}{2\pi}\sin\theta + M_0\sin\theta \ldots\ldots [4a]$$

2 A bending moment M_y in the horizontal plane

$$M_y = LR\sin\theta \ldots\ldots [4b]$$

3 A torsional moment M_t

$$M_t = L\frac{\theta f}{2\pi}\cos\theta + M_0\cos\theta \ldots\ldots [4c]$$

In addition, there are the following reactions which have been disregarded as influencing deflections:

A direct horizontal shear

$$S_x = L\cos\theta \ldots\ldots [4d]$$

A direct compressive load

$$Q = L\sin\theta \ldots\ldots [4e]$$

The partial derivatives of Equations [4*a*], [4*b*], and [4*c*] with respect to L and M_0 are

$$\frac{\partial M_x}{\partial L} = \frac{\theta f}{2\pi}\sin\theta; \quad \frac{\partial M_y}{\partial L} = R\sin\theta; \quad \frac{\partial M_t}{\partial L} = \frac{\theta f}{2\pi}\cos\theta;$$

$$\frac{\partial M_x}{\partial M_0} = \sin\theta; \quad \frac{\partial M_y}{\partial M_0} = 0; \quad \frac{\partial M_t}{\partial M_0} = \cos\theta$$

Since $ds = Rd\theta$, Equations [2] and [3], with n = number of turns and $s = 2\pi nR$, become

$$\delta = \int_0^{2\pi n}\left(\frac{Lf^2}{4\pi^2 EI_x}\theta^2\sin^2\theta + \frac{M_0 f}{2\pi EI_x}\theta\sin^2\theta + \frac{LR^2}{EI_y}\sin^2\theta + \frac{Lf^2}{4\pi^2 GJ}\theta^2\cos^2\theta + \frac{M_0 f}{2\pi GJ}\theta\cos^2\theta\right) Rd\theta \ldots\ldots [4]$$

$$\phi = \int_0^{2\pi n}\left(\frac{Lf}{2\pi EI_x}\theta\sin^2\theta + \frac{M_0}{EI_x}\sin^2\theta + \frac{Lf}{2\pi GJ}\theta\cos^2\theta + \frac{M_0}{GJ}\cos^2\theta\right) Rd\theta \ldots\ldots [5]$$

The integration of Equations [4] and [5] with simplifying approximations, is shown in the appendix.

Hence with $f = (h/n)$

$$\delta = Rh\pi n\left(\frac{Lh}{3} + \frac{M_0}{2}\right)\left(\frac{1}{EI_x} + \frac{1}{GJ}\right) + \frac{LR^3\pi n}{EI_y} \ldots. [6]$$

$$\phi = R\pi n\left(\frac{Lh}{2} + M_0\right)\left(\frac{1}{EI_x} + \frac{1}{GJ}\right) \ldots\ldots [7]$$

Expressing M_0 in terms of ϕ and L from Equation [7], and substituting in Equation [6], the deflection is

$$\delta = \frac{LRh^2\pi n}{12}\left(\frac{1}{GJ} + \frac{1}{EI_x}\right) + \frac{LR^3\pi n}{EI_y} + \frac{\phi h}{2} \ldots\ldots [8]$$

In the case where the ends of the spring remain parallel after deflection ($\phi = 0$), and noting that from symmetry $M_0 = -Lh/2$, both Equations [6] and [8] become

$$\delta = \frac{LRh^2\pi n}{12}\left(\frac{1}{GJ} + \frac{1}{EI_x}\right) + \frac{LR^3\pi n}{EI_y} \ldots\ldots [9]$$

For a round bar steel spring with $E = 29{,}500{,}000$ lb per sq in., and $G = 11{,}500{,}000$ lb per sq in., the general Equations [6] and [8] for the lateral deflection become

$$\delta = \frac{Dn}{10^6 d^4}(0.825h^2L + 1.238M_0h + 0.271D^2L) \quad [10]$$

and

$$\delta = \frac{LDn}{10^6 d^4}(0.271D^2 + 0.206h^2) + \frac{\phi h}{2} \quad [11]$$

and for a spring whose ends remain parallel after deflection

$$\delta = \frac{LDn}{10^6 d^4}(0.271D^2 + 0.206h^2) \quad [12]$$

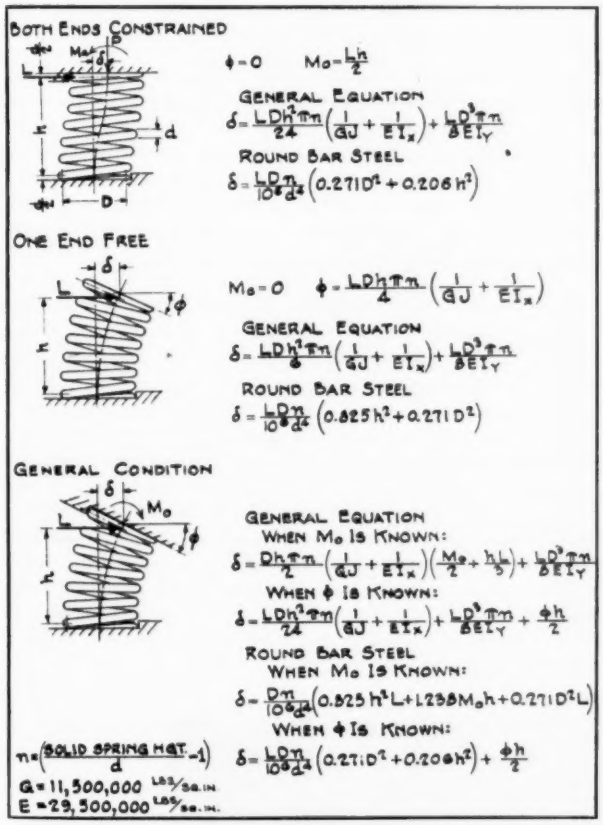


FIG. 2 TRANSVERSE-DEFLECTION FORMULAS FOR CLOSELY COILED HELICAL SPRINGS

The transverse-deflection formulas are shown with illustrative diagrams in Fig. 2.

STRESSES

The maximum stresses imposed by the loading described previously will now be discussed. As in the case of the deflections, the small influence of the helix angle has been neglected. The maximum shear stress under axial loading, including Wahl's[7] concentration factor, is given by the formula

$$\tau = \frac{16PR}{\pi d^3}\left(\frac{4c-1}{4c-4} + \frac{0.615}{c}\right) \quad [13]$$

where $c = D/d$.

[7] "Stresses in Heavy Closely Coiled Helical Springs," by A. M. Wahl, Trans. A.S.M.E., vol. 51, 1929, paper APM-51-17, pp. 185–200.

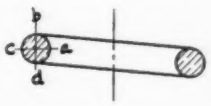


FIG. 3

This stress occurs on the inside of the coil as shown at a in Fig. 3, and is constant for all values of θ. The portion $\frac{16PR}{\pi d^3}\left(\frac{0.615}{c}\right)$ is due to direct shear P and will not be affected by the transverse loading. The remainder, $\frac{16PR}{\pi d^3}\left(\frac{4c-1}{4c-4}\right)$ is due to the torque PR and will be increased by any augment in the torque due to L and M_0. The expression for torque, that is Equation [4c], with the addition of PR is

$$M_t = \left(\frac{L\theta f}{2\pi} + M_0\right)\cos\theta + PR \quad [14]$$

It is obviously a maximum when the expression $[(L\theta f/2\pi) + M_0]$ is a maximum and $\cos\theta = \pm 1$. Also $[(L\theta f/2\pi) + M_0]$ will be maximum when θ is either 0 or a maximum, depending on the relative values and signs of L and M_0.

Thus, the maximum shear stress will occur on the inside of the coil in the plane containing L and P, shown in Fig. 1. Its magnitude will be

$$\tau = \left[\left(\frac{L\theta f}{2\pi} + M_0\right)_{\max} + PR\right]\frac{16}{\pi d^3}\left(\frac{4c-1}{4c-4}\right) + \frac{16PR}{\pi d^3}\left(\frac{0.615}{c}\right) \quad [15]$$

Disregarding the direct horizontal shear and compressive load due to L given by Equations [4d] and [4e], there remain the bending stresses due to M_x and M_y. An examination of the expressions for these moments shows every term to contain $\sin\theta$ and they are accordingly zero for the points already investigated. The moments M_x and M_y attain their maximum values when $\sin\theta = \pm 1$, i.e., at that section of the coil intersected by a plane which includes the axis of the helix and lies at right angles to (the direction of) L.

Thus, $M_x = [(L\theta f/2\pi) + M_0]\sin\theta$ and reaches a maximum value a quarter turn from the maximum for M_t. The moment $M_y = LR\sin\theta$ and is likewise a maximum twice every turn. Where M_x and M_y are maximum, M_t has the value of PR.

The maximum stress due to M_x occurs at the extreme fibers of the bar section at points b and d shown in Fig. 3. At these points the torsional and bending stresses are not increased by any stress concentration.

Therefore, at b the stresses will be

$$\tau_t = \frac{16PR}{\pi d^3} \quad [16]$$

$$\sigma_x = \frac{32[(L\theta f/2\pi) + M_0]}{\pi d^3} \quad [17]$$

The maximum principal stress

$$\sigma_{\max} = \frac{16}{\pi d^3}\left[\left(\frac{L\theta f}{2\pi} + M_0\right) + \sqrt{(PR)^2 + \left(\frac{L\theta f}{2\pi} + M_0\right)^2}\right] \quad [18]$$

and the maximum shear stress

$$\tau_{\max} = \frac{16}{\pi d^3}\sqrt{(PR)^2 + \left(\frac{L\theta f}{2\pi} + M_0\right)^2} \quad [19]$$

The maximum stress due to M_y occurs at the inside of the bar at a shown in Fig. 3, where it combines with a torsional shear.

At a, the stresses will be

$$\tau = \frac{16PRk}{\pi d^3} \quad [20]$$

where $k = \left(\frac{4c-1}{4c-4} + \frac{0.615}{c}\right)$, and

$$\sigma_y = \frac{32LRq}{\pi d^3} \quad [21]$$

From the curved-beam theory, $q = (4c-1)/(4c-4)$. The maximum principal stress

$$\sigma_{max} = \frac{16}{\pi d^3}\left[LRq + \sqrt{(PRk)^2 + (LRq)^2}\right] \quad [22]$$

and the maximum shear stress

$$\tau_{max} = \frac{16}{\pi d^3}\sqrt{(PRk)^2 + (LRq)^2} \quad [23]$$

In general it will be found for springs of normal proportions, that the maximum shear stress will be more critical than the maximum direct stress. Therefore, the critical stress will occur on the inside of the coil where M_t is a maximum at which point it will have the value

$$\tau = \left[\left(\frac{L\theta f}{2\pi} + M_0\right)_{max} + PR\right]\frac{16}{\pi d^3}\left(\frac{4c-1}{4c-4}\right) + \frac{16PR}{\pi d^3}\left(\frac{0.615}{c}\right) \quad [15]$$

It will be noted that the maximum moment occurs on the end coil. Obviously, if this coil were tapered, the maximum moment would occur on a reduced section and some provision may be necessary for its protection.

Discussion of Other Contributing Factors

As previously mentioned, Timoshenko[6] has discussed the stability of helical springs under axial loading. It is obvious that the addition of a transverse load will decrease the axial critical load. A complete investigation and discussion of stability was considered to be beyond the scope of this paper.

TABLE 1 CHARACTERISTICS OF SPRINGS TESTED

Spring no.	Mean diameter D in.	Bar diameter (d) in.	Free height, in.	Solid height, in.	Axial capacity, lb	$\frac{D}{d}$
1	5.812	1.25	9.219	6.594	10200	4.65
2	3.672	0.734	9.406	6.156	3700	5.00
3	6.726	1.241	15.250	8.72	12200	5.43
4	3.875	0.750	14.656	8.81	4050	5.16
5	0.566	0.123	0.798	0.553	139	4.60
6	0.569	0.123	1.211	0.841	121	4.63
7	0.572	0.123	1.664	1.108	121	4.65
8	0.752	0.123	1.313	0.715	98	6.12
9	0.752	0.123	2.078	1.102	98	6.12
10	0.752	0.123	2.801	1.455	96	6.12

The precise equation, including the effect of the helix angle, expressing a relationship for transverse deflection is shown in the Appendix. Numerical calculations on two springs, whose helix angle was approximately 5 deg, indicated an error of less than 0.5 per cent.

Description of Test Procedure

In order to demonstrate the validity of the foregoing formulas, tests were conducted on several sets of helical springs to determine the transverse-load-deflection rates. While the original tests were conducted on large springs of the railway type, the procedure was cumbersome due to the magnitude of the vertical loads required. Accordingly, to facilitate experimental work, subsequent tests were conducted on small springs, accurately coiled and with the ends ground. These springs were furnished by the Railway Steel Spring Company.

The pertinent characteristics of all springs tested for lateral deflection properties in conjunction with this paper are shown in Table 1. It will be noted that the experimental springs covered a wide range of ratios of height to diameter, and the spring indexes varied from 4.5 to 6. The experimental apparatus was essentially the same for both the large and small coils, employing in each case four identical helical springs with ground ends seated at the four corners of a rectangular base plate and surmounted by a similar plate on which the vertical load was placed. The assembled coils were then loaded transversely by a turnbuckle and tension rod in series with a load-measuring device lying in a horizontal plane and affixed, one end to a rigid foundation and the other to some point on the top plate according to whether pure translation or combined translation and rotation was to be applied to the top plate.



Fig. 4 Apparatus Used for Testing High-Capacity Springs

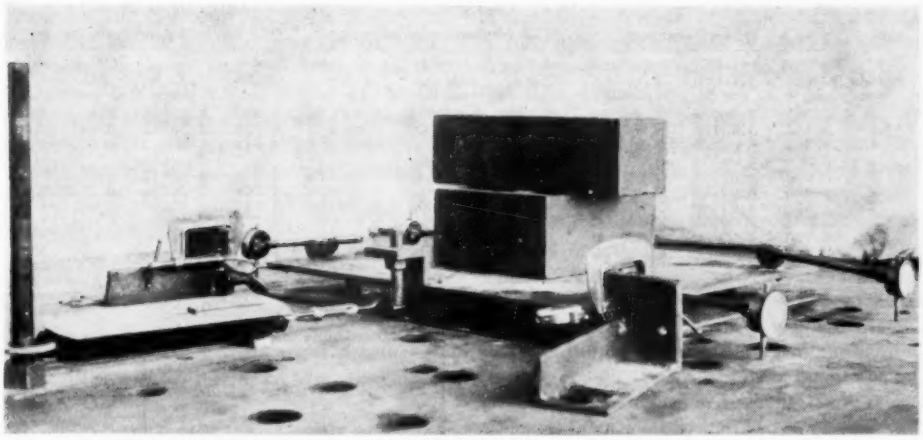


FIG. 5 APPARATUS FOR TESTING SMALL SPRINGS

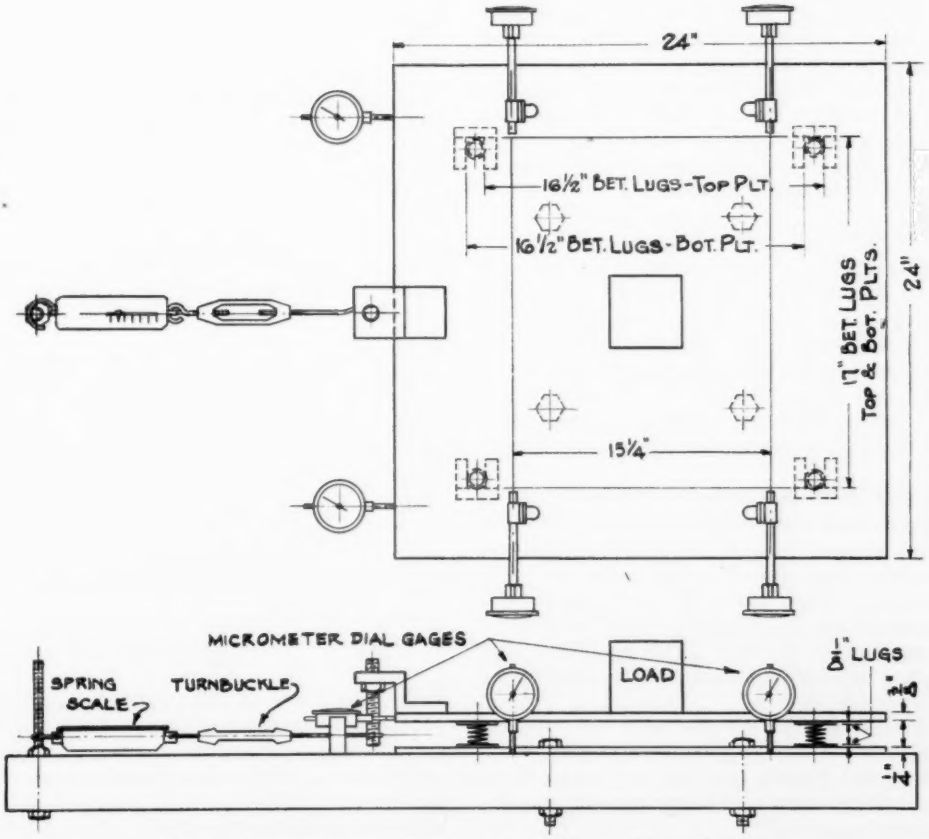


FIG. 6 ARRANGEMENT OF APPARATUS FOR TESTING SMALL SPRINGS

In the case of the large testing apparatus, which is shown in Fig. 4 the springs were retained at both extremities by set bolts tapped in an annular ring welded to each plate around the spring seat and abutting the end coil at the quarter-circle points. A slotted lug was provided at the midpoint of the underside of the top plate, which afforded an opportunity for varying the height of transverse-load application.

As shown in Figs. 5 and 6, the small springs were retained in their position by cleats of the same thickness as the bar diameter of the spring; the cleats were welded to the plates on such a side of the coil as to prevent horizontal movement of the ends under transverse loading. An auxiliary cleat was welded at right angles to this, thus forming an angle which thereby nested the end coil on two points of its circumference. An adjustable threaded rod affixed to the front edge of the top plate, in this case, provided a means of varying the height of transverse load application.

On both apparatus the amount of transverse load was determined by mounting a calibrated ring gage or accurate spring scale, according to the magnitude of the loads, in tandem with the turnbuckle. Micrometer dials or, wherever necessary, surface gages, were used to observe all necessary deflections. Lateral deflections were taken on either side of the center line of the

TABLE 2 COMPARISON OF ACTUAL AND CALCULATED TRANSVERSE DEFLECTIONS

	25 per cent axial capacity			50 per cent axial capacity			75 per cent axial capacity		
Spring no.	Working height, in.	Actual δ/L, in. per lb	Calculated δ/L, in. per lb	Working height, in.	Actual δ/L, in. per lb	Calculated δ/L in. per lb	Working height, in.	Actual δ/L, in. per lb	Calculated δ/L, in. per lb
1	7.301	0.00028	0.000214	6.679	0.000213	0.000196	6.039	0.00018	0.000176
2	7.860	0.00175	0.001560	7.047	0.001570	0.001310	6.234	0.00145	0.00111
3	12.384	0.00118	0.000827	10.760	0.001000	0.000704	9.141	0.00088	0.000562
4	13.076	0.01610	0.005650	....	Unstable	.....	...	Unstable	.....
5	0.612	0.00162	0.001425	0.554	0.001340	0.001305	0.512	0.00117	0.001227
6	0.934	0.00495	0.004160	0.884	0.004350	0.003600	0.764	0.00415	0.003020
7	1.378	0.01480	0.009550	1.226	0.014800	0.007960	1.143	0.01540	0.007160
8	1.048	0.00615	0.006000	0.902	0.005250	0.005080	0.771	0.00400	0.004370
9	1.720	0.03150	0.020000	1.480	0.026700	0.015870	1.262	0.02450	0.012630
10	2.347	0.11200	0.045700	....	Unstable	.....	...	Unstable	.....

top plate to insure an average value of this reading, while vertical deflections were taken at the four corners of the assembly to determine the change in spring heights between the front and rear sets of springs under rotational loading.

All springs were tested for transverse deflections under gravity loadings of about 25, 50, and 75 per cent of their axial capacity, except in those few cases where the instability of the coil prevented it. Springs Nos. 1, 2, and 3 were transversely loaded at the level of the top plate. It was then recognized that the slope ϕ contributed an appreciable increment to the deflection. Therefore, springs Nos. 4 to 10, inclusive, were loaded at the height of the point of contraflexure of the springs for the purpose of eliminating the end slope. Sufficient increments of transverse load were applied to establish the true rate of transverse deflection, although for obvious reasons the loading was not pursued to the point of instability of the spring. Upon release of the lateral load, no appreciable lag was observed.

Discussion of Test Results

Examination of the results, as shown in Table 2 indicates good agreement between the actual and the calculated values for a majority of the springs tested. However, a discrepancy exists in the case of certain springs. The approximations occurring in the formula are discussed elsewhere in this paper, together with their effect on the exact evaluation of δ. It will suffice at this point to consider wherein the actual conditions of testing depart from the original assumptions, and to what degree they influence the results obtained.

In order to simulate the general method of a helical-spring installation involving no rigid attachment of the end coils to their base, the testing arrangement made no provision for a constraining moment other than that offered by the loaded plate. In addition, the transverse loading was applied at whatever point of tangency happened to be established between the end coil and the restraining cleat (or set bolt). This procedure, while duplicating practical application, upset certain premises on which the theory was established.

(*a*) The end moment M_0 acted not only upon that portion of the spring contained in the helix proper, but also over the diminishing section of the tapered end coil, thus producing a finite deflection for this portion of the spring which was disregarded in the general equation.

(*b*) Likewise, the transverse loading L, unless the spring possessed exactly n plus $1/2$ coils (where n is a whole number) acted at a point within the tapered region of the end coil, and thereby added its component of transverse deflection in excess of the calculated value.

The magnitude of the error in the calculated value as the axial loading progressed from 25 to 75 per cent capacity was increased or decreased according to which of two effects was prevalent. (1) When the spring was well within the limits of stability under load, the decrease in M_0 as the working height diminished meant a corresponding decrease in its distorting effect on the end coil as just described in paragraph (*a*). (2) If the spring approached an unstable condition as the gravity load increased,

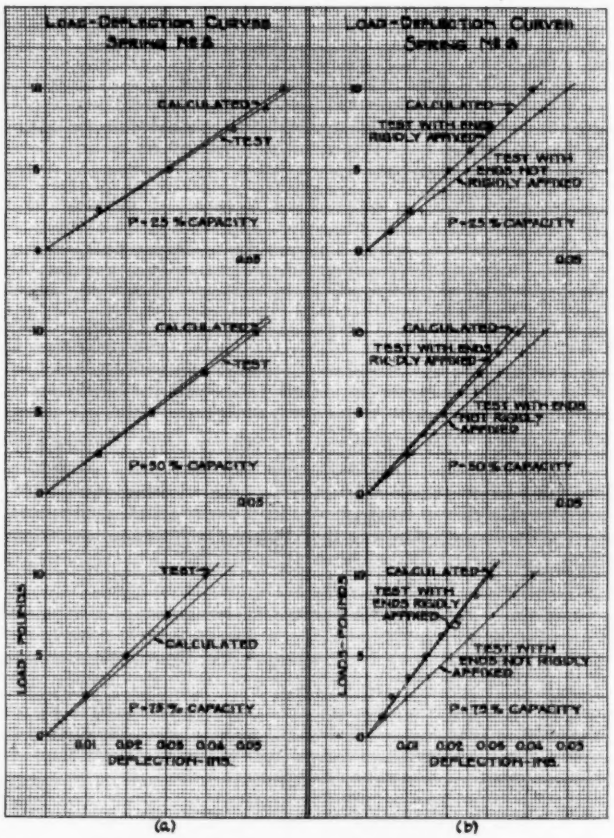


Fig. 7 Transverse-Deflection Characteristics of Two Representative Springs

a given transverse loading produced a greater deflection than would occur for the same transverse loading in combination with a smaller axial-load application.

The actual transverse-deflection characteristics of a representative spring (i.e., spring no. 8) is shown in Fig. 7*a* with the calculated curve for the extent of the test. Similar characteristics for spring No. 6 is shown in Fig. 7*b*. This latter spring was subsequently affixed rigidly to steel base plates at its extreme coil faces and almost perfect agreement with the calculated values was obtained. Examination of the actual plotted points in Figs. 7*a* and 7*b* indicates that there is no measurable deviation from the straight line. This apparently indicates that the omission of the influence of the couple $P\delta$ is justified.

Conclusions

A plot of the ratio of actual to calculated deflection versus the ratio of working height to the mean diameter D, on the basis of the actual tests, is shown in Fig. 8.

As previously noted, the springs were chosen to give a wide

variation of the latter ratio in order to establish the range of application of these formulas. It appears from the results of these tests that the equations may be used with reasonable accuracy when this ratio is less than 1.5. For the majority of engineering applications of compression springs subjected to transverse loadings, this ratio will not be exceeded. With springs in excess of this ratio there is the possible danger of approaching the limit of stability.

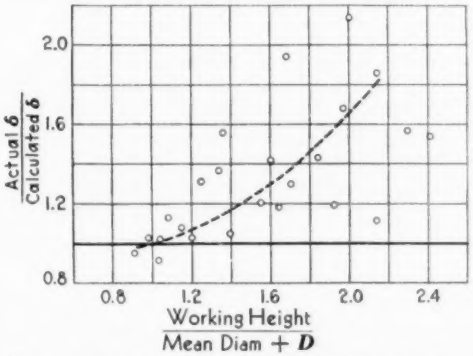


FIG. 8 RATIO OF ACTUAL TRANSVERSE DEFLECTION TO CALCULATED TRANSVERSE DEFLECTION VERSUS THE RATIO OF WORKING HEIGHT TO MEAN DIAMETER OF SPRINGS

The extent to which the instability and end-coil conditions weaken the validity of the expressions, can only be qualitatively appraised at this time.

Since the scope of this paper was to establish the expression for lateral deflection within the stable limits and did not consider the influence of the end coil, it is hoped that this research effort will furnish the impetus for further investigation into the subject.

Appendix

The integrals involved in Equations [4] and [5] are as follows:

$$\int_0^{2\pi n} \theta^2 \sin^2\theta d\theta = \left[\frac{\theta^3}{6} - \frac{\theta^2}{4}\sin 2\theta - \frac{\theta}{4}\cos 2\theta + \frac{1}{8}\sin 2\theta\right]_0^{2\pi n} \approx \frac{4\pi^3 n^3}{3} \quad [24a]$$

$$\int_0^{2\pi n} \theta \sin^2\theta d\theta = \left[\frac{\theta^2}{4} - \frac{\theta}{4}\sin 2\theta - \frac{1}{8}\cos 2\theta\right]_0^{2\pi n} \approx \pi^2 n^2 \quad [24b]$$

$$\int_0^{2\pi n} \sin^2\theta d\theta = \left[\frac{\theta}{2} - \frac{1}{4}\sin 2\theta\right]_0^{2\pi n} \approx \pi n \quad [24c]$$

$$\int_0^{2\pi n} \theta^2 \cos^2\theta d\theta = \left[\frac{\theta^3}{6} + \frac{\theta^2}{4}\sin 2\theta + \frac{\theta}{4}\cos 2\theta - \frac{1}{8}\sin 2\theta\right]_0^{2\pi n} \approx \frac{4\pi^3 n^3}{3} \quad [24d]$$

$$\int_0^{2\pi n} \theta \cos^2\theta d\theta = \left[\frac{\theta^2}{4} + \frac{\theta}{4}\sin 2\theta + \frac{1}{8}\cos 2\theta\right]_0^{2\pi n} \approx \pi^2 n^2 \quad [24e]$$

$$\int_0^{2\pi n} \cos^2\theta d\theta = \left[\frac{\theta}{2} + \frac{1}{4}\sin 2\theta\right]_0^{2\pi n} \approx \pi n \quad [24f]$$

In evaluating the foregoing integrals, only the first terms in the brackets have been retained. It will be seen that the balance of the terms diminish rapidly from the first and the evaluation of their maxima was found to affect the calculated deflection less than 1 per cent with n between 3 and 4. With larger values of n the error decreases.

The foregoing analysis deals only with the deflection and slope, resulting from the application of a lateral force and end couple, to a spring initially loaded with an axial force. It is to be noted that the axial force itself may produce a lateral deflection and slope, and by the reciprocal theorem the lateral force and moment may produce a vertical deflection. The following investigation will disclose this effect.

Referring to Fig. 1, let P, only, act on the spring, and apply a dummy load $L = 0$. The only coupling between the two, neglecting the helix angle, lies in the moment M_t, or

$$M_t = PR + \frac{L\theta f}{2\pi}\cos\theta \quad \text{and} \quad \frac{dM_t}{dL} = \frac{\theta f}{2\pi}\cos\theta$$

Hence

$$\delta_P = \int_0^{2\pi n} \frac{PR^2 f}{2\pi GJ}\theta\cos\theta d\theta$$

$$\delta_P = \frac{PR^2 h}{2\pi GJn}\left[2\pi n \sin 2\pi n + \cos 2\pi n - 1\right] \quad [25]$$

It will be seen that when n is a whole number the value of the bracket (and hence the deflection) is zero. At other values of n the deflection is expressed as follows:

n = whole number plus:	δ_p equals:
1/4 turn	$\frac{PR^2h}{GJ}\left(1 - \frac{1}{2\pi n}\right)$
1/2 turn	$-\frac{PR^2h}{GJ}\left(\frac{1}{\pi n}\right)$
3/4 turn	$-\frac{PR^2h}{GJ}\left(1 + \frac{1}{2\pi n}\right)$

To determine the end slope induced by P, the dummy moment $M_0 = 0$ may be applied, from which it will be found that

$$\phi_{Pe} = \frac{PR^2}{GJ}\sin 2\pi n \quad [26]$$

Similarly the vertical deflection induced by the application of L and M_0 will be found to be

$$\Delta_L = \frac{LR^2 h}{2\pi GJn}\left[2\pi n \sin 2\pi n + \cos 2\pi n - 1\right] \quad [27]$$

$$\Delta_{M_0} = \frac{M_0 R^2}{GJ}\sin 2\pi n \quad [28]$$

An evaluation of these reciprocal relationships involves an end-coil analysis to disclose the total effective number of turns so that the equations of the form $(An \sin Bn)$ and $(An \cos Bn)$ may be determined accurately. This is also true for a precise evaluation of the sinusoidal terms in Equations [24a] to [24f], inclusive. Such an analysis lies beyond the scope of this paper.

It is sufficient, however, to point out that with multiple-spring suspensions, it is possible to dispose the various coils so that their lateral deflection components may counteract.

The point has been raised regarding the lateral deflection under the action of axial loading, inasmuch as it indicates a field of profitable investigation in its bearing on the stable limits of spring loading.

The derivation of the transverse deflection in the body of the paper disregarded the helix angle. Including the effect of the helix angle [$\alpha = \tan^{-1}(f/\pi d)$] gives the following moments:

$$M_x = L\frac{\theta f}{2\pi}\sin\theta + M_0\sin\theta$$

$$M_y = \left(L\frac{\theta f}{2\pi}\cos\theta + M_0\cos\theta\right)\sin\alpha + LR\sin\theta\cos\alpha$$

and

$$M_t = \left(L\frac{\theta f}{2\pi}\cos\theta + M_0\cos\theta\right)\cos\alpha - LR\sin\theta\sin\alpha$$

Also

$$ds = \frac{Rd\theta}{\cos\alpha}$$

By rewriting Equations [4] and [5] and proceeding as in the previous case, the more exact expression for Equation [9] becomes

$$\delta = \frac{LRh^2\pi n}{12}\left(\frac{\cos\alpha}{GJ} + \frac{1}{EI_x\cos\alpha}\right) + \frac{LR^3\pi n}{EI_y}\cos\alpha - \frac{LR^2h}{2}\left(\frac{1}{GJ} + \frac{1}{EI_y}\right)\sin\alpha \quad [29]$$

for the deflection of a spring whose ends remain parallel.

ACKNOWLEDGMENTS

The authors acknowledge the cooperation of the following men:

R. Ebenbach, of the E. G. Budd Manufacturing Company, who assisted in conducting the tests.

E. G. Hallquist, vice-president of General Steel Castings Corporation, who placed the facilities of this plant at the authors' disposal for conducting the tests on the large springs.

E. Schmidt, of the E. G. Budd Manufacturing Company, who made possible the tests on the small springs. The latter were furnished through the kindness of T. R. Weber, Railway Steel Spring Division, American Locomotive Company.

The authors are grateful to R. Eksergian, of the E. G. Budd Manufacturing Company, for his personal assistance and contributions in the preparation of this paper.

Discussion

E. T. P. NEUBAUER.[8] In this paper valuable data are presented on the problem of spring mountings for the isolation of machinery vibration. In order effectively to isolate the vibration of heavy machinery it is necessary to have the critical or resonance frequency of all the various degrees of freedom well below the operating frequency of the machine. This requires a knowledge of the transverse deflection of springs as well as the longitudinal.

A. M. WAHL.[9] The authors have contributed interesting test data which, in conjunction with the analytical work, should be of considerable interest to spring designers.

The writer notes that in deriving the formula for lateral deflection, as mentioned in the paper, the authors have neglected the effect on the axial load. This would be in such a direction as to increase the deflection above the calculated value using the formulas given. Since most of the test deflections were higher than the calculated (in some cases twice as high), it is suggested that a part at least of the discrepancy between calculated and test deflections may be accounted for by the fact that this effect was neglected.

A formula to take into account the increased lateral deflection due to presence of an axial load on a spring laterally loaded (with both ends constrained) has been derived by E. Rausch.[10] This lateral deflection is

$$\delta_{max} = \alpha\delta_n$$

where δ_n = nominal deflection, figured by neglecting the axial load (this is given by Equation [12] of the paper, which agrees with the formula derived previously by Rausch).

The term α is a factor greater than unity which takes into account the increased deflection due to the axial load

$$\alpha = \frac{1 - \dfrac{\frac{\delta_1}{h}}{\left(2.6 + \frac{\delta_1}{h}\right)\left(1 + 0.77\frac{h^2}{D^2}\right)}}{1 - \dfrac{\delta_1}{2.6\,h}\left(1 + K\dfrac{h^2}{D^2} - \dfrac{\frac{\delta_1}{h}}{2.6 + \frac{\delta_1}{h}}\right)} \quad [30]$$

where

$$K = \frac{0.83 + 0.77\frac{h^2}{D^2}}{1 + 0.77\frac{h^2}{D^2}} \times 0.93$$

where

δ_1 = deflection under the axial load
h = working height of spring
D = mean diameter

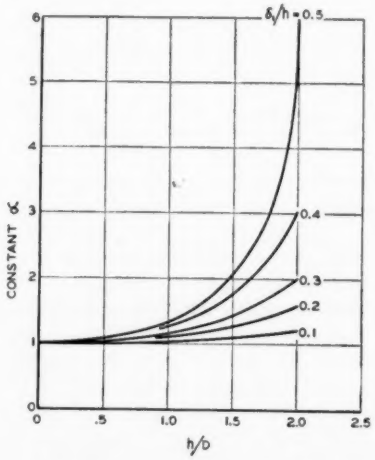


FIG. 9 CURVE FOR FINDING α

Curves given by Rausch for the application of this formula indicate that, where the ratio of working height h to mean diameter D is between 1 and 2, the factor α may be as high as 3,

[8] Machine Designer, York Ice Machinery Corp., York, Pa. Jun. A.S.M.E.

[9] Research Engineer, Westinghouse Electric & Manufacturing Company, East Pittsburgh, Pa. Mem. A.S.M.E.

[10] "Die Steifigkeit von Schraubenfedern senkrecht zur Federachse," by E. Rausch, *Zeit. V.D.I.*, vol. 78, Aug. 11, 1934, p. 964.

where the axial deflection δ_1 due to the axial load is around 40 per cent of the working height. This effect decreases as the axial load decreases. The constant, α, is plotted in Fig. 9 (given by Rausch) for various ratios of δ_1/h (for ease in calculation). Of course, as indicated by the authors, the question of stability must also be considered.

It would be of interest if this formula could be applied to the springs tested by the authors to see whether or not better agreement could be had between test and calculated results.

Authors' Closure

The authors wish to thank Mr. Wahl for calling their attention to the paper by Professor Rausch, which was overlooked in preparation of this paper. Professor Rausch's original abstract[11] is quite complete and the authors are gratified to note that his formula based on both ends constrained parallel agrees with Equation [12], although obtained through a different mathematical approach.

Equation [30] for the correction factor α was given by Professor Rausch in collaboration with Dr. A. Röver and appeared in a supplement[10] to the original paper. However, there was no proof or derivation given and the authors were therefore unable to check it. For this reason the authors thought it advisable to derive a correction factor which was based on the properties of a column.

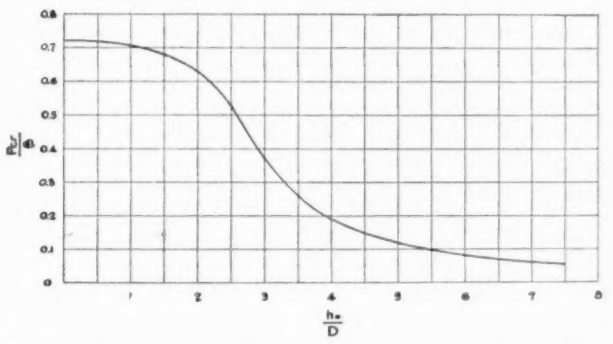


Fig. 10 Curve Showing Ratio of Critical Load to Compressive Rigidity Versus Ratio of Free-Spring Height to Mean Diameter

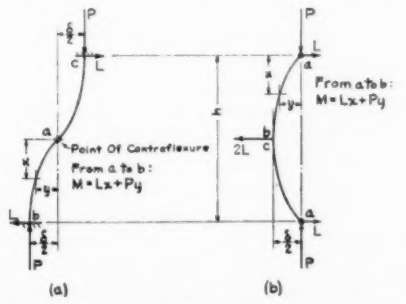


Fig. 11 Showing Similarity Between Column With Ends Constrained Parallel and Pin-Ended Column

It is true that the axial load influences the transverse deflection of the spring. The magnitude of this influence depends on the ratio of the imposed axial load to the critical load at which buckling of the spring occurs (without regard to transverse forces).

The buckling resistance of helical springs has been reviewed by Dr. Timoshenko.[5] For pin-ended connections the critical axial load may be obtained from the curve shown in Fig. 10 which is reproduced from Timoshenko's review,[5] using the values of G and E given previously in this paper. The compressive rigidity β may be found from

$$\beta = \frac{GIh_0}{\pi n R^3} = C_v h_0 \quad [31]$$

where h_0 = free spring height of helix.

From Fig. 11 it will be noted that the deflection curve of a column with parallel ends free to move laterally satisfies the conditions for a pin-ended column, since the bending moment acting on each half of the column (a) is duplicated on each half of the curve (b). It will be recognized that the curve, Fig. 11(a) represents the axis of the deflected spring.

The deflection for a pin-ended column, Fig. 11(b), transversely loaded at the center line has been derived by Dr. Timoshenko[12] and is expressed as

$$\delta = \delta_0 \lambda \quad [32]$$

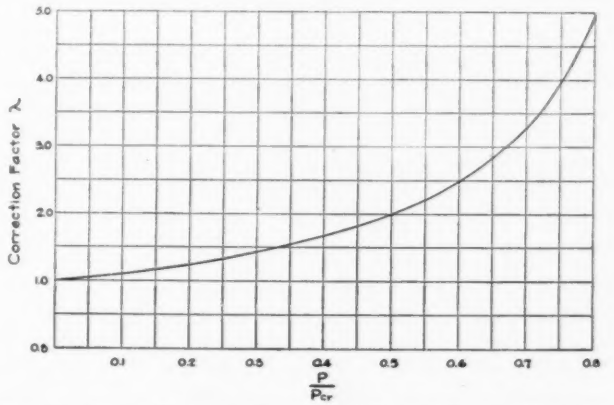


Fig. 12 Curve Showing Correction Factor Versus Ratio of Axial Load to Critical Load

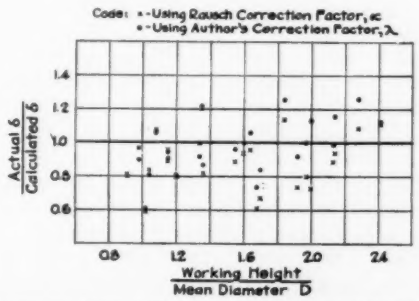


Fig. 13 Ratio of Actual Transverse Deflection to Corrected Calculated Transverse Deflection Versus Ratio of Working Height to Mean Height of Springs

where

$\lambda = \dfrac{3(\tan u - u)}{u^3}$ and δ_0 = deflection due to the transverse load acting alone.

$$u = \frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}} \quad [33]$$

Fig. 12 illustrates the correction factor λ expressed as a function of $\dfrac{P}{P_{cr}}$. To find the correction factor it is only necessary to deter-

[11] "Die Steifigkeit von Schraubenfedern senkrecht zur Federachse," by E. Rausch, *Zeit. V.D.I.*, vol. 78, March 24, 1934, p. 388.

[12] "Theory of Elastic Stability," by S. Timoshenko, McGraw-Hill Book Company, Inc., New York, N. Y., 1936, p. 4.

mine the critical load P_{cr} from Fig. 10, from which the ratio $\frac{P}{P_{cr}}$ may be obtained. The correction factor may then be determined directly from Fig. 12.

Examination of Equation [9] shows that the first term represents the portion of the deflection due to the action of bending moments, while the second term represents the portion due to shear forces alone. Since only the former is affected by the column action, Equations [9] and [12] may be written with the correction factor λ as

$$\delta = \lambda \frac{LRh^2\pi n}{12}\left(\frac{1}{GJ} + \frac{1}{EI_x}\right) + \frac{LR^2\pi n}{EIy} \quad \ldots\ldots [9a]$$

$$\delta = \frac{LDn}{10^6\, d^4}(0.271D^2 + 0.206h^2\lambda) \quad \ldots\ldots [12a]$$

The authors have recalculated the deflection ratios of the experimental springs based both on the Rausch correction factor and on the factor λ obtained as just outlined. Fig. 13 illustrates the results plotted similarly to Fig. 8. It will be noted that a much better agreement is obtained when the correction factors are applied; the majority of the calculating values agreeing within 20 per cent of the actual.

A treatment of the correction factors applied to the general case of a spring the ends of which do not remain parallel is too complex to be treated here.

Studies in Boundary Lubrication

Variables Influencing the Coefficient of Static Friction Between Clean and Lubricated Metal Surfaces

By W. E. CAMPBELL,[1] SUMMIT, N. J.

A thorough survey has been made of the literature on static-friction measurement. It is shown that there are large discrepancies among the results of different investigators and it is concluded that the differences are largely due to lack of control of experimental technique. A complete bibliography is presented.

An experimental study of several of the more important variables influencing static friction is described. The variables studied are the effect of load, vibration, and cleanliness of the surfaces. An inclined-plane and a horizontal-plate apparatus are used. It is concluded that the method of cleaning the surfaces is the factor which has the greatest influence upon the results and it is shown that invisible films which may be deposited upon the surfaces during the cleaning process can exert large effects upon static-friction measurements.

A method of cleaning is described which involves polishing the surfaces with moist abrasive. This method yields high and reproducible values for static friction between unlubricated surfaces. It is shown that sulphide and oxide films on steel, copper, and brass produce a lowering in static friction and that the amount of lowering depends upon the thickness of the film.

A comparison is made between the static and the kinetic values obtained on several bearing metals lubricated with typical oils. It is shown that the static values are in general considerably higher than the kinetic values, and it is concluded that the differences which are observed are due in large part to viscosity effects in the kinetic measurements.

An analysis of static-friction measurements on typical lubricating oils leads to the conclusion that moisture and corrosion effects are responsible for many of the phenomena observed. It is concluded that static-friction measurements do not lend themselves to routine testing of oils, but that they can be used to considerable advantage for studying single variables influencing boundary lubrication.

DURING the last few years increasing attention has been given to a study of the factors which govern lubrication in the region where loads and speeds are such that the film of oil separating the rubbing surfaces is no longer continuous. Under these conditions the coefficient of friction for constant operating conditions is no longer a simple linear function of the viscosity (1).[2] The composite effect of the new variables entering into the problem when operation takes place under incomplete film or boundary conditions has been assigned the vague term "oiliness," but no really satisfactory definition has been proposed, nor has much attempt been made to study the separate variables in a quantitative manner. A comprehensive review of the results and findings of investigators of oiliness is given in a paper by Herschel (2), which includes descriptions of testing machines commonly employed. More recently a complete discussion of the subject of boundary lubrication, containing a list of the various factors which may contribute to oiliness, has been given by Hersey (3). In general the important factors may be summarized as follows:

1 The adsorptive force operating at the metal-liquid interface, which may vary with the nature of the metal and of the liquid.

2 The structure of the molecules adsorbed at the interface.

3 Solid films formed on the surface during operation, such as adsorbed graphite films, or oxide, sulphide, and chloride films formed by chemical action between lubricant and metal.

4 The structure of the bearing surfaces.

5 The increase, with increasing pressure, of the viscosity of the lubricant.

A large number of oiliness investigations have been carried out using kinetic-friction measurements, and have been concerned primarily with the development of a machine for routine rating of oils. Kinetic measurements, though they have the advantage that they may be made under conditions closely approximating those obtaining in service (28), yield results complicated by viscosity effects (26). The static-friction method, on the other hand, has the advantage of giving results which are independent of viscosity (4, 5, 10*b*). It involves the use of simple apparatus and lends itself much better to the study of very small quantities of pure compounds under controlled atmospheric conditions.

The present investigation was originally undertaken with the object of studying the effect of molecular structure on boundary lubrication, and the static-friction method appeared to be the most suitable choice for such a study. Difficulties were encountered in the early stages in reproducing the measurements of earlier workers, and reference to the literature indicated considerable discrepancies among the data of different observers using similar methods. It was therefore found necessary to make a detailed study of the variables entering into static-friction measurements, which will be described in the experimental section of this paper.

Rayleigh (6) appears to have been the first to use static-friction measurements for studying boundary lubrication effects. He estimated that the thickness of the films between the surfaces was 1000A or less, and observed that lubrication by these films could not be treated hydrodynamically. Shortly afterward Langmuir (7) showed that static friction between glass surfaces could be lowered from a value between 1.1 and 1.7 to 0.10 by a monomolecular film of a fatty acid transferred to the glass from a water surface. Deeley (4) and Hyde (8) were able to show marked differences between lubricating oils which correlated with their efficiencies as judged by practical experience, and by the Lanchester worm-gear testing machine. Hardy and his collaborators (10) have carried out studies of a high degree of accuracy on a large number of pure organic compounds, and have obtained ex-

[1] Bell Telephone Laboratories.

[2] Numbers in parentheses refer to the Bibliography.

Contributed by the Special Research Committee on Lubrication and presented at the Semi-Annual Meeting of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS, held in Detroit, Mich., May 17–21, 1937.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

TABLE 1 VARIABILITY OF RESULTS OBTAINED BY DIFFERENT OBSERVERS FOR THE COEFFICIENT OF STATIC FRICTION BETWEEN METAL SURFACES

Observer	Brass-brass, μ	Steel-steel, μ
Rennie (14)	0.175	0.146
Jacob (15)	0.07	..
Hardy (10)	..	0.82
Tomlinson (16)	0.63	0.41
Deeley (4)	..	0.17

ceedingly reproducible results which indicate relations of astonishing simplicity between coefficient of friction and molecular weight for members of a given homologous series. Woog (11), Wilharm (12), Wilson and Barnard (5), and more recently Rhodes and his coworkers (13), have demonstrated the value of static-friction measurements between lubricated surfaces for the analysis of oiliness phenomena.

It is to be expected that the measurement of a property such as static friction, which is controlled largely by interfacial forces, would be profoundly influenced by very small variations in the state of the surfaces. A study of the literature bears this out, and indicates that the attainment of reproducible and significant static-friction measurements requires the development of an elaborate and time-consuming experimental technique. There is considerable difference of opinion, however, regarding the extent of control necessary for any given experimental variable, such as preparation of surface, control of temperature, and purity of atmosphere.

Some idea of the spread of comparable data in the literature can be seen from Table 1, in which are given results for dry static friction between similar metal combinations obtained by several independent workers.

The lack of agreement is probably due in large measure to the presence of different amounts of grease or other films on the surfaces. That Deeley's surfaces were contaminated is indicated by the fact that the friction rose when the surfaces were breathed upon, an effect of moisture on friction which is characteristic of greasy surfaces, but which Hardy (10*a*), and the present author have found to be absent for clean surfaces. Deeley also gives figures showing a continuous drop in friction coefficient with load for clean plane surfaces, which is contrary to the law of solid friction, but which is in agreement with results obtained by Hardy and Bircumshaw (10*h*) for thin invisible films of lubricants on steel.

In view of the variability of the results for dry friction, it is not surprising that those for lubricated surfaces are in poor agreement. Coordination of results of different investigators is further complicated by the fact that most of the work reported covers a wide variety of loads, and is concerned for the most part with commercial lubricating oils of indefinite compositions from widely different sources.

Only one group of investigators (10) has made a thorough study of the variables entering into measurements of static friction. In their early papers it is claimed that reproducible values can be obtained only by using highly purified chemical compounds and by carrying out the measurements in a very pure dry atmosphere. Woog (11), Deeley (4), and Hyde (8), however, have published data indicating a high degree of precision on surfaces lubricated with commercial lubricating oils and exposed to laboratory atmospheric conditions.

Woog (11) lays stress on the sensitivity of measurements to vibration and to minute irregularities in the surfaces. Wilharm (12) also found elaborate precautions necessary to minimize vibration effects, but other investigators (5, 17) appear to have obtained reproducible values in the presence of considerable vibration from a driving motor. There is fairly general agreement regarding the necessity for highly polished surfaces, but there is also evidence that the coefficient of friction is independent of the roughness of the surfaces (10*a*).

No work has been done on the possible effect of invisible films of oxide and sulphide which may be present on the surfaces before application of lubricant, although it is well known that such films form during and immediately after the polishing process (18), and Dow (19) has shown that oxide films on brass produce a friction-lowering effect.

There is some difference of opinion regarding the effect of load. Deeley (20), in describing the technique to be employed in using his machine, assumes that the coefficient of friction is independent of the load, but other workers (17*a*, 17*b*), using the same machine, have published data indicating a decrease in the static coefficient of friction up to a limiting load of from 50 to 100 psi, when it becomes constant. Other publications (10*h*, 21) confirm this latter finding and indicate that the coefficient of friction is constant with increasing load for a spherical slider on a plane surface. Hardy and Bircumshaw (10*h*) explain the friction results they observe for small loads and plane surfaces, where the friction is considerably higher than it is above the limiting load, as being due to orientation of molecules by attractive forces at the metal-liquid interface. They consider that these orientation effects extend to a distance of the order of 7μ from the surfaces. The work of Bulkley (22) and of Bastow and Bowden (23) indicates that the length of such oriented chains cannot exceed 0.01μ.

The effect of temperature on the static coefficient of friction appears to be rather complicated. The majority of observers assume no temperature effect over the range of temperatures from 20 to 35 C. This assumption has been clearly demonstrated for pure organic lubricants (10*d*), but seems less certain for commercial lubricating oils in the light of more recent work (10*j*, 10*k*, 13*b*), which shows that the coefficient of static friction may remain constant, decrease, or increase according to the nature of the adsorbable components in the oil.

Hardy (10*g*) has found that if μ_{a-a} and μ_{b-b} are the coefficients of dry friction for metal a against metal a and metal b against metal b, respectively, then the coefficient of static friction between metal a and metal b will be $(\mu_{a-a} + \mu_{b-b})/2$ and that the same law holds true for lubricated surfaces. Tomlinson (16) has confirmed this law, which Hardy calls the mean-value rule, for clean metal surfaces. He finds that it follows by considering friction to be the resultant of the attractive and repulsive forces between atoms, since the former is considerably greater than the latter and a less rapidly decreasing function of the distance between atoms. Similar views are expressed by Shaw (24). However, the data of other observers (8, 13*a*) on static friction between lubricated surfaces show little confirming evidence of such a rule,[3] and it is very uncertain whether it can be generally applied.

Hardy (10*h*) has also noted that large changes in friction may take place with time of contact between lubricated surfaces. The time during which an equilibrium value is approached he calls the latent period, which is always one of falling friction at loads above the limiting load. The length of the latent period is dependent on the molecular structure of the lubricant employed, being large for long-chain molecules having polar groups, such as fatty acids and alcohols, and negligibly small for nonpolar molecules such as straight-chain saturated hydrocarbons. This latent period is considered to be the time of orientation of molecules in the adsorbed layer.

Many of the measurements recorded in the literature have been made with the Deeley machine (20), in which the friction is measured between a stationary three-legged slider and a disk which is started slowly rotating from a position of rest so that both static and kinetic measurements can be made with the same apparatus. Although it is not always clear how the readings are

[3] Rhodes and Allen (13*a*), for example, find that the coefficient of static friction is the same for surfaces of steel, brass, or chromium, lubricated with the same material.

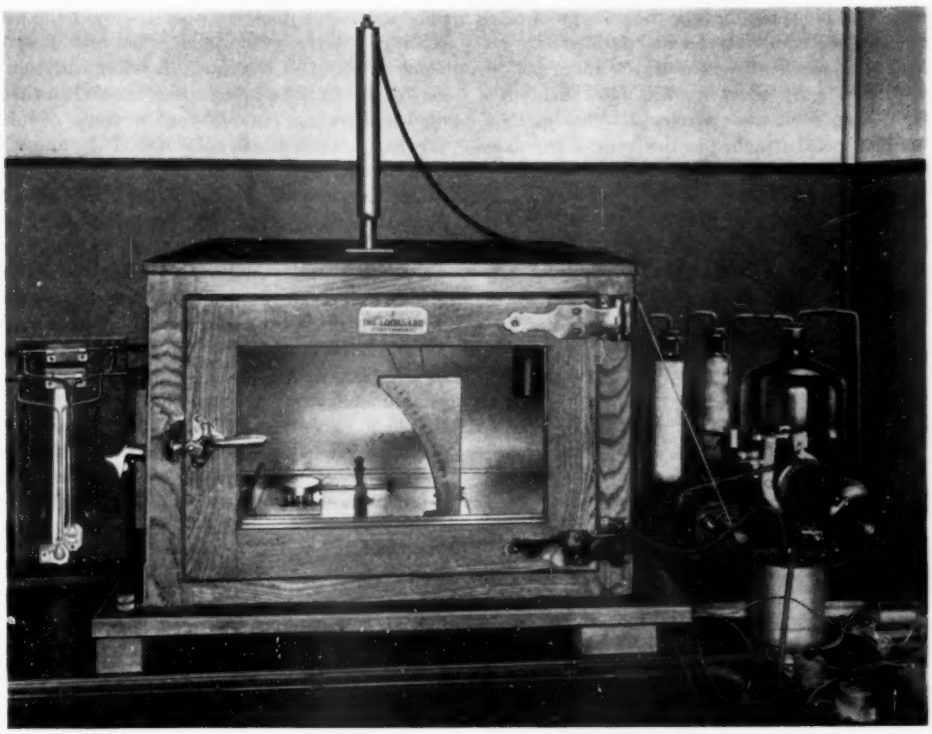


FIG. 1 INCLINED-PLANE APPARATUS

taken, it is likely that most of the values obtained with this machine are kinetic values. Deeley assumes that the static and kinetic coefficients of friction between lubricated surfaces are equal under boundary conditions, and recommends taking the latter value as being more reproducible, and Dover and Helmers (17*c*), using Herschel's modification of Deeley's machine (25), state that their kinetic measurements are independent of viscosity. On the other hand, the results of McKee (1) for a journal-bearing machine, and Bridgeman (26, 27) for a modified Herschel machine, indicate that the coefficient of friction is a function of viscosity practically down to zero speed, and that single measurements of kinetic friction at a given speed cannot be used as a direct measure of oiliness.

On only one point is there fairly complete agreement, and that is that static friction between lubricated surfaces in the boundary region takes place between thin adsorbed films of oriented molecules, although there is considerable difference of opinion, as has already been noted, regarding the thickness of these films. It is also fairly well established that long-chain molecules having a polar group at one end of the chain will be good boundary lubricants, but it is certain that high adsorptive power alone is not a measure of oiliness (5, 10*b*, 11) since substances like water or glycerin, though strongly adsorbed on metals, definitely produce no lowering effect on the static friction between metal surfaces.

EXPERIMENTAL PROCEDURE

Two forms of apparatus, differing mainly in the method of application of a tangential force to a sliding member, were used. The first apparatus, an inclined-plane machine which is essentially the same as that used by Wilson and Barnard (5), is shown in Fig. 1. A polished metal plate, upon which was placed a three-legged slider, rested against a stop on the plane. The plane was tilted by a motor operating through a reducing gear, and when slip occurred between slider and plate, a switch operating a solenoid brake on the motor was thrown, so that the movement of the plane was sharply halted. The fraction of the right angle through which the plane had moved was read to three significant figures by means of a vernier attachment, and the coefficient of friction calculated from this angle. The rate of raising the plane for most of the measurements was 12 cm per min measured at the moving end. This rate was chosen after it had been shown that the static coefficient of friction between lubricated steel surfaces was independent of the rate of raising the plane over the range of speeds from 5 to 20 cm per min. The apparatus was contained inside a metal-lined box through which a current of air, cleaned and dried by passage through sulphuric acid, soda lime, dehydrite, cotton, and glass wool, was passed at a rate of 500 liters per hr. Under these conditions a volume of air equal to that of the box was passed every 15 min and the relative humidity was maintained between 0.1 and 1.0 per cent. The slider was manipulated between readings through a small side door at the left of the box. The box and contents were insulated from vibration by means of layers of sponge rubber and tissue paper. For some of the work slight movement of the slider brought it against a contact attached to the stop on the base plate, closing a relay circuit which operated the solenoid brake on the motor. This method was not always convenient, particularly in certain cases when slip was very slow, and it was found that equally accurate results could be obtained by observing visually the opening of a gap between the back of the slider and a brass disk attached to the base plate.

The second form of apparatus, a horizontal-plate apparatus, which is similar to that used by Hardy, is shown in Fig. 2. The plate carrying the slider was placed upon a brass leveling platform, and the tractive force was applied tangentially by means of a light thread attached to a corner of the slider and passing over a pulley. The load was applied by running water into a light glass bucket attached to the other end of the thread. The plate was prevented from slipping by a step at one end of the leveling platform. The assembly was mounted inside a glass desiccator of 10

liters capacity, bored with holes to admit of the passage of clean dry air during the course of a run. The air was purified as previously described and was passed at a rate of 100 liters per hr. Slip was generally detected by observing the movement of a galvanometer needle when a circuit was closed by contact between a polished strip of metal attached to the front of the slider and a point ground on the end of the feeler of a depth gage set in the wall of the chamber. Successive readings were taken by

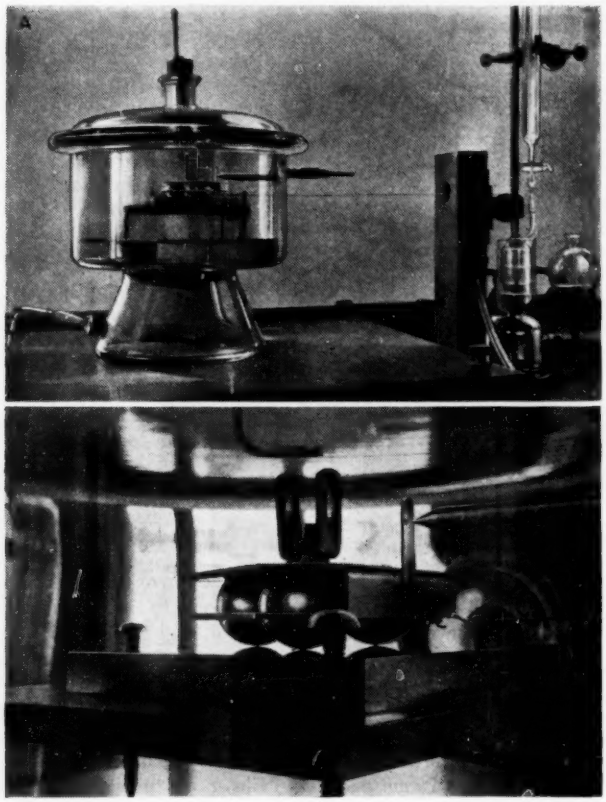


FIG. 2 HORIZONTAL-PLATE APPARATUS

drawing the feeler back 0.010 in. In some cases, however, when experimental conditions were such as to make the electrical contact uncertain, the slip was observed by watching the closing of the gap between the point of the feeler and its shadow on the polished metal strip.

The slider shown in Fig. 1 was the type used in the earlier experiments. The pegs were of speculum metal or steel, and the radius of curvature of their polished ends was 1 in. The mass was 370 g. Sliders of the type shown in Fig. 2 B were generally used. They consisted of three ball bearings at the apexes of a triangle of 2-in. sides, held between two triangular steel plates. By loosening a screw, the balls could be turned so that each of a series of readings could be made on an unscratched surface of the ball if desired. The sliders were of two sizes, fitted with balls of 1 in. and $^1/_2$ in. diam, respectively. Their masses varied from 290 to 300 and 125 to 135 g, respectively. The steel balls were hardened-tool-steel ball bearings and were obtained from the Atlas Ball Company, Philadelphia. The plates were of hardened steel. Two most frequently used were toolmaker's flats obtained from Pratt and Whitney. Others were cut to $3^1/_2 \times 3^1/_2 \times {}^1/_2$ in. from ground flat stock and were hardened, lapped, and polished.

The most important factor to be controlled in order to obtain significant results is the cleanliness of the surfaces. Fatty films only a molecule thick not only can lower the friction between unlubricated surfaces enormously but can lower considerably the value obtained on subsequent flooding with lubricant. The work carried out in these laboratories has shown that the only method of cleaning which removes the last traces of lubricant used in the previous test consists in polishing off the film with moist abrasive on a metallurgical lap.

The data in Table 2 indicate the large effects which an invisible film can produce on the friction between dry and lubricated surfaces. Before each run the surfaces, after solvent treatment, were cleaned by polishing with wet broadcloth sprinkled with magnesium oxide used for metallographic polishing. For the first four runs water was removed from the surfaces, after polishing and scrubbing under water, by rinsing with alcohol and drying in air. For runs 5 to 9 the water was removed by means of a clean linen cloth. Application of a test, to be described later, of the cleanliness of the surfaces, which consisted in the observation of their tendency to be wetted with water, showed that the alcohol-rinsed surfaces were clean, whereas those dried by wiping with the cloth were covered with an invisible contaminating grease film.

TABLE 2 EFFECT OF CLEANING ON COEFFICIENT OF FRICTION BETWEEN SPECULUM METAL AND STEEL

Treatment of surfaces	Run no.	Lubricant	μ	Per cent average deviation from mean
Polished under water, rinsed with alcohol, dried in air	1	None	0.45	3.6
	2	Acto oil	0.28	9.0
	3	S.A.E.-10 automobile oil	0.21	5.9
	4	Acto oil + 1 per cent oleic acid	0.16	3.9
Polished under water, dried with clean cloth	5	None	0.13	11.2
	6	None	0.08	8.4
	7	Acto oil	0.11	5.0
	8	S.A.E.-10 automobile oil	0.10	..
	9	Acto oil + 1 per cent oleic acid	0.16	6.8

It will be observed that the coefficient of friction between the surfaces rinsed with alcohol and dried in air after abrasion (run 1) is more than four times greater than that obtained when the surfaces were dried with a cloth (runs 5 and 6). The effect of the film deposited by the cloth is almost as great for the lubricated surfaces; the results for the two mineral oils are considerably higher when they lubricate the clean steel surfaces (runs 2 and 3) than when they lubricate the cloth-treated surfaces (runs 7 and 8). It will also be noted that the difference between the white Acto oil and the automobile oil is masked by the presence of a contaminating film, and that the effect of addition of oleic acid to the white oil is actually reversed (runs 4 and 9).

The values of the coefficient of friction are the means of ten determinations. It will be observed that the spread of the readings averaged is not lowered by improving the cleaning technique. This was rather disappointing since many published results (4, 10*c*, 11) indicated that static-friction determinations reproducible to three or four significant figures were obtainable.

Although the cleaning method giving the higher values in Table 2 gave more promising results than had been obtained hitherto, the reproducibility of dry values was still rather poor (μ = 0.4 to 0.6 for speculum on steel). After testing various cleaning methods, the following was adopted: The surfaces were first scrubbed with cotton soaked in benzene and acetone, and were then polished on wet broadcloth sprinkled with levigated alumina; the hands of the operator were covered with welt cotton gloves. When the surfaces appeared to be clean they were scrubbed with cotton under running tap water, washed with redistilled ethyl alcohol, and dried by resting the slider or plate on the lip of a glass beaker, cleaned by flaming, set under a bell jar through which was passed a current of clean dry air. The

sliders were harder to clean than the plates, particularly those having balls held between two steel plates, because contaminating material became lodged in crevices and became deposited on the lower surfaces by the alcohol rinse. These sliders were therefore frequently dismantled, the parts cleaned with solvents, reassembled, and extracted with ether in a large Soxhlet-type extractor before the final cleaning by abrasion. All handling of sliders and plates, except during the actual polishing, was done with crucible tongs which had been cleaned by heating to redness. Alcohol was found to be the best solvent to use for removing the water after polishing. In general, absolute alcohol as received left the surfaces clean, but it was found safer to use redistilled alcohol, which was stored in carefully cleaned Pyrex bottles. Other solvents invariably left a film of grease on the surfaces unless they were freshly distilled.

The plates were tested for freedom from grease by wetting with water and draining in a vertical position. When the surface is clean the water will remain in a continuous film until it has become so thin by drainage and evaporation as to show interference colors at the upper edge. A straight line of interference colors will then proceed down the plate. Any drying at the vertical edges should also be preceded by the appearance of interference colors.

It is rather difficult to tell when the surface of a ball is clean by the wetting test, particularly if the radius of curvature is small. The following method has been found the most satisfactory: After rinsing in redistilled alcohol and drying in air, a drop of water is placed on one ball. If the ball is clean, it will wet all over and will be somewhat duller in luster than the two unwetted balls. After about twenty seconds, a circular dry spot, fringed with interference colors, appears near the upper part of the curved surface and spreads fairly rapidly over the ball. If any contamination is present, the water will flash back immediately to the boundary between the ball and the base of the slider, leaving it dry all over.

No method of solvent cleaning alone, such as has been frequently used (5, 13, 20), was found to be satisfactory. Water invariably drew into droplets on steel surfaces cleaned in this manner and the dry coefficient of friction ran between 0.2 and 0.3, for a given slider-plate combination, whereas for surfaces cleaned as previously described, the dry coefficient ran between 0.75 and 0.80.

The necessity for a cleaning technique which provides surfaces free from organic contamination cannot be too strongly emphasized. The work carried out in these laboratories indicates that failure to use adequate cleaning methods is the most important single factor contributing to the lack of uniformity of published results on static friction. Hardy stressed the need for clean metal surfaces in static-friction work as early as 1919. The results of the present work are in full agreement with his findings on this point, and the friction coefficient for steel on steel checks closely his value for steel surfaces cleaned by a similar technique (10*j*).

The plates and sliders were dried for about half an hour after rinsing with alcohol, and were then transferred to the chamber for measurement. Before flooding the surfaces with oil, a few dry-friction measurements were always made as an additional check on the cleanliness of the surfaces. In Table 3 a series of typical results for the coefficient of friction between steel surfaces cleaned in this manner is given. Sliders 6 and 7 were fitted with 1-in-diam balls, and slider 9 with $^1/_2$-in-diam balls. Plates 1 and 5 were toolmaker's flats and plates 6 and 7 were made from ground flat stock. Each of the values given in the table is the mean of at least five readings. The first ten values were obtained using the horizontal-plate apparatus and are picked at random from more than one hundred obtained on surfaces in equilibrium with dry air (relative humidity less than 0.1 per cent) over a 2-year period. Determinations 11 to 14 were obtained using the inclined-plane apparatus in an atmosphere of approximately 1 per cent relative humidity, and determinations 14 to 18 were obtained using the horizontal-plate apparatus in an atmosphere of 70 per cent humidity.

TABLE 3 COEFFICIENT OF STATIC-FRICTION MEASUREMENTS BETWEEN STEEL SURFACES

Determination no.	Normal load, grams	Slider no.	Plate no.	μ	Per cent average deviation from mean
1	295	6	5	0.75	1.4
2	295	6	5	0.82	2.3
3	295	6	6	0.80	1.9
4	295	6	6	0.77	3.4
5	295	6	7	0.75	1.1
6	295	7	5	0.79	3.0
7	295	7	6	0.80	1.3
8	1700	9	5	0.74	2.0
9	133	9	6	0.80	4.1
10	133	9	6	0.82	2.5
11	230	6	1	0.79	1.1
12	470	7	1	0.84	1.7
13	450	7	5	0.80	1.8
14	104	9	5	0.80	1.8
15	295	6	6	0.78	2.0
16	295	6	7	0.74	2.1
17	295	7	6	0.77	3.0
18	295	7	7	0.76	2.5

It will be noticed that over the range of loads tested, the law of friction is obeyed, and that, as would be expected (10*a*), the coefficient of friction is independent of the humidity of the surrounding air.

The grand average of several hundred values obtained over a period of four years for a large number of slider-plate combinations is 0.78, with a maximum deviation of 0.06. For a given slider-plate combination, the maximum deviation is generally of the order of 0.03. This spread is not as low as would be expected from the literature (4, 10*c*, 11), but no higher reproducibility was obtainable, no matter what care was taken.

It had been assumed that the results were independent of temperature. In case temperature effects could account for fluctuations, runs were made at constant temperature with no improvement in reproducibility. It is possible that the extent and nature of irregularities in the surface may be the limiting factor, although no significant difference was observable between the spread of the results obtained on the toolmaker's flat which was optically plane, and of those on the square plates which were flat to only 0.001 in.

The method used for insulating against vibration reduced general building vibration fairly efficiently, but for measurements on the inclined-plane apparatus, an appreciable amount of vibration was transmitted from the motor through the string. In order to determine the effect of this vibration on the reproducibility and on the mean value of the friction readings, runs were made in which readings were taken alternately using the motor lift and a hydraulic lift mounted on top of the chamber. In all, ten readings for each method of lifting the plane were made in a given run, and the means are given (runs 1 and 2 of Table 4). For runs 3 and 4 the chamber and contents were mounted on a vibration-free mounting suspended by steel springs from the ceiling and the hydraulic lift was used.

TABLE 4 EFFECT OF VIBRATION ON COEFFICIENT OF STATIC FRICTION BETWEEN LUBRICATED STEEL SURFACES

Run no.	Surfaces	Method of lifting plane	μ	Per cent average deviation from mean
1	Steel, Acto oil, steel	Motor	0.233	6.5
		Hydraulic	0.262	5.5
2	Steel, oleic acid, steel	Motor	0.102	3.2
		Hydraulic	0.126	5.9
3	Steel, Acto oil, steel	Hydraulic	0.273	10.5
4	Steel, Acto oil, steel	Hydraulic	0.255	6.3

These results indicate that vibration from the motor exercises a small lowering effect on the coefficient of friction. This lowering,

TABLE 5 FRICTION BETWEEN SULPHIDED COPPER SURFACES FOR VARYING FILM THICKNESS

Run no.	Color of film	Equivalent air-film thickness cm × 10^{-5}	μ
1	No film	..	1.17
1	Slightly darkened	1.0	1.07
1	Rose I	1.2	0.91
2	Blue I	1.6	0.88
2	Yellow II	2.8	0.77
2	Purple II	3.9	0.78
2	Blue II	4.3	0.73
2	Blue green II	4.9	0.73
3	Light blue II	4.6	0.78

however, is scarcely greater than experimental error and is negligible for practical purposes. It is also clear from Table 3 that vibration effects were not responsible for the lack of reproducibility in earlier values—in fact the highest spread was found in run 3, which was carried out under practically vibrationless conditions.

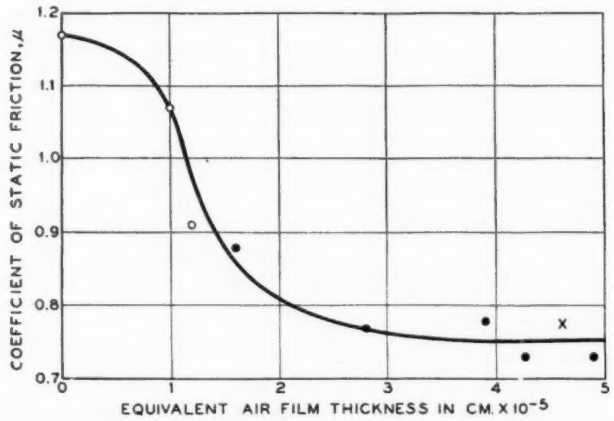


FIG. 3 RELATION BETWEEN COEFFICIENT OF STATIC FRICTION AND THICKNESS OF TARNISH FILM ON COPPER

In the early work there had been indications that the presence of thin tarnish films, such as oxide and sulphide, formed on the surfaces while drying in laboratory air, might be responsible for irregularities in both dry- and lubricated-friction results. Since also the formation of corrosion films plays an important part in the functioning of extreme-pressure lubricants, a series of experiments was carried out to determine the effect of oxide and sulphide films of various thickness on the static friction between steel, brass, and copper surfaces.

The sulphide films were made by immersion of slider and plate for varying times in 0.02 per cent sodium-sulphide solution, and the oxide films were made by heating in air at temperatures from 100 to 500 C.

A preliminary survey indicated that in general the very thin invisible or barely visible films produced little or no effect, but that the friction dropped sharply when a certain limiting thickness of film was reached. For example, the invisible oxide film formed on steel by a 5-min immersion in hot chromic acid and on brass by heating for 1 hr in air at 110 C gave a friction lowering only slightly greater than experimental error, whereas thicker films produced a reduction in friction of the order of 50 per cent. Some results obtained for sulphide films on copper over a wide range of thickness, which illustrate this point clearly, are given in Table 5 and plotted in Fig. 3. The procedure for each run was to dip slider and plate in the sodium-sulphide solution until a given uniform tarnish color was obtained. They were then rinsed with water and alcohol, dried, and used for a friction determination. After testing for cleanliness by the water-wetting test, they were redipped in the sodium-sulphide solution and the friction redetermined. The results given are the means of ten readings. The thickness of the films was not measured directly. It has been shown by Constable (29), however, that the equivalent air-film thickness, which is the thickness of an air film between glass plates giving the same color as the tarnish film under consideration, is directly proportional to the measured thickness. Air-film thicknesses are therefore given in the third column of Table 5 and provide a relative measure of the thickness of the film.

It will be observed that at the pressures used in these experiments (mass of slider = 297 g, diameter of balls = 1 in.)[4] no pronounced drop in friction takes place until the film is thick enough to show well-defined interference colors. The friction then drops sharply to a relatively constant value for thicker films. Similar results were obtained for oxide on copper. The film produced by the action of sodium sulphide on steel appears to be an exception to this behavior in that a sharp drop in friction was measured for an invisible film in this case.

Table 6 gives results for the friction between surfaces of steel, brass, and copper coated by a variety of solid films of thickness large enough that little or no breakdown took place. The unlubricated surfaces reacted clean to the water-wetting test after each set of ten readings, the means of which are given in the table. The results recorded in this table as well as those of Table 5 were obtained using the horizontal-plate apparatus.

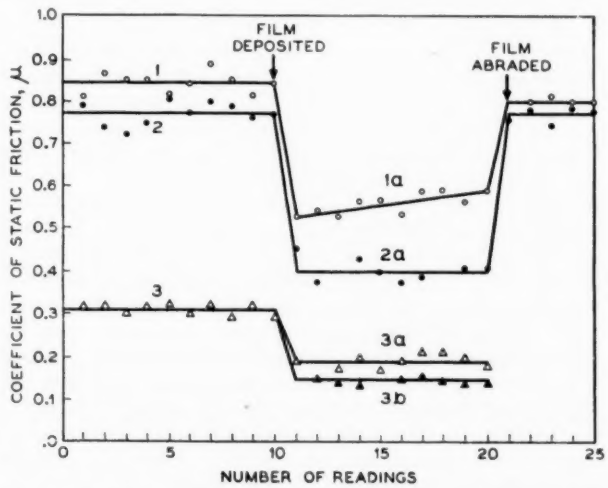


FIG. 4 EFFECT OF SULPHIDE FILMS ON BRASS AND STEEL ON THE COEFFICIENT OF STATIC FRICTION

1	Brass	1a	Sulphided brass
2	Steel	2a	Sulphided steel
3	Steel, Acto oil	3a	Sulphided steel, Acto oil
		3b	Oxidized Steel, Acto oil

TABLE 6 FRICTION-REDUCING EFFECT OF SOLID FILMS ON STEEL, BRASS, AND COPPER

Metal combination	Treatment of surfaces	Static coefficient of friction, μ — Untreated surfaces	Treated surfaces
Steel-steel	Oxide	0.78	0.27
Steel-steel	Sulphide	0.78	0.39
Brass-brass	Sulphide	0.88	0.57
Copper-copper	Sulphide	1.21	0.74
Copper-copper	Oxide	1.21	0.76
Steel-steel	Oleic acid	0.78	0.11
Steel-graphite	Oleic acid	0.21	0.09
Steel-steel	Acto oil	0.78	0.32
Steel-steel	Oxide, Acto oil	0.78	0.19
Steel-steel	Sulphide, Acto oil	0.78	0.16

It will be observed that a substantial drop in friction results from the presence of the solid films.

[4] Assuming Hertzian surfaces the average pressure is of the order of 1.5×10^4 psi.

The values for the lubricated solid films are lower than those for the metals lubricated by the same oil, but the reduction is not as large nor as consistent as that which would follow from application of the empirical rules given by Hardy and his coworkers.

Some of the results of Table 6 are given in more detail in Fig. 4. The figures plotted are those obtained for individual readings and a good idea of the reproducibility of the results in the tables may be gained from a study of the curves. The upward trend in the values for sulphided brass indicates some breakdown of the film on the balls; similar but considerably steeper curves were obtained by Dow (19) for friction as a function of the number of slips for thin layers of oxide on brass. He interprets the rise in friction as due to the abrasive action of thin oxide films. His results, however, are all considerably lower than those obtained in the present work, and it is believed that they may be interpreted as being due to the removal of an oxide film contaminated with a thin layer of fatty material.

The last series of readings for curves 1 and 2 in Fig. 4 were taken after the slider had been rubbed back and forth on the plate to produce three areas of clean but roughened metal surface of approximately 1 sq cm. A spot of clean metal approximately 0.04 sq cm in area was formed on each ball. The readings were taken on these abraded areas. The results are in good agreement with the values obtained for polished surfaces and offer additional evidence that the coefficient of friction is independent of the area of contact and of the roughness of the surfaces (10) for solid friction.

In order to find out to what extent the static-friction method could be used to differentiate among a variety of oils and in order to gain some idea as to how the static and kinetic methods of friction testing would compare, studies were made on a series of commercial lubricants of widely differing chemical properties and viscosities on several different bearing metals, after these same combinations had been measured by the Herschel disk friction machine. The static-friction measurements on the bearing alloys were made using the inclined-plane apparatus, and the kinetic measurements were made by Winslow Herschel in the friction and lubrication section of the U. S. Bureau of Standards. After the kinetic measurements were completed, the $^1/_2$-in. balls were removed from the slider and fitted into an aluminum frame so that they were no longer free to rotate, giving a slider mass of 75 g. The plates were the same disks that had been used for the kinetic-friction measurements. The results for steel were obtained using the slider with 1-in. steel balls described earlier, on the toolmaker's flat, using the horizontal-plate apparatus. The surfaces were completely flooded with oil after cleaning, and the plate was rotated through an angle of about 20 deg after each reading. The slider was pressed in place and agitated gently on the plate before a measurement, which was taken 5 min later. The plane was lifted using the motor.

The composition of the alloys was as follows:

	Per cent		Per cent		Per cent
A.S.T.M. No. 1	tin 91	copper	4.5	antimony	4.5
A.S.T.M. No. 8	lead 80	tin	5	antimony	15
A.S.T.M. No. 1	lead 83	tin	2	antimony	15
Frary metal	lead 96.9	tin	0.18	barium	1.71
	calcium 0.70	mercury	0.33	zinc	0.11

The following oils were tested:

	Viscosity, poises at 30 C
Atlantic spindle oil	0.33
Liberty aero oil	8.92
Castor oil	4.73
Lard oil	0.73
Atlantic spindle oil + 2 per cent oleic acid	0.34

The results of the tests are given in Table 7, and show the means of ten readings in the case of the static measurements. These results are represented in Fig. 5, and are reproducible to ±0.01.

TABLE 7 COMPARISON OF THE COEFFICIENT OF STATIC FRICTION BETWEEN STEEL AND VARIOUS LUBRICATED MATERIALS WITH THE COEFFICIENT OF KINETIC FRICTION BETWEEN THE SAME SURFACES

μ_s = Coefficient of static friction (inclined-plane apparatus)
μ_k = Coefficient of kinetic friction (Herschel oiliness machine)

	A.S.T.M. No. 1		A.S.T.M. No. 8		A.S.T.M. No. 10		Frary metal		Steel
Oil	μ_s	μ_k	μ_s	μ_k	μ_s	μ_k	μ_s	μ_k	μ_s
Atlantic spindle	0.23	0.16	0.17	0.14	0.25	0.13	0.19	0.12	0.23
Liberty aero	0.22	0.10	0.23	0.09	0.27	0.085	0.22	0.095	0.22
Castor	0.15	0.06	0.11	0.065	0.12	0.06	0.15	0.06	0.15
Lard	0.08	0.11	0.09	0.07	0.10	0.055	0.09	0.05	0.11
Atlantic spindle + 2 per cent oleic	0.085	..	0.08	..	0.11	..	0.05	..	0.17

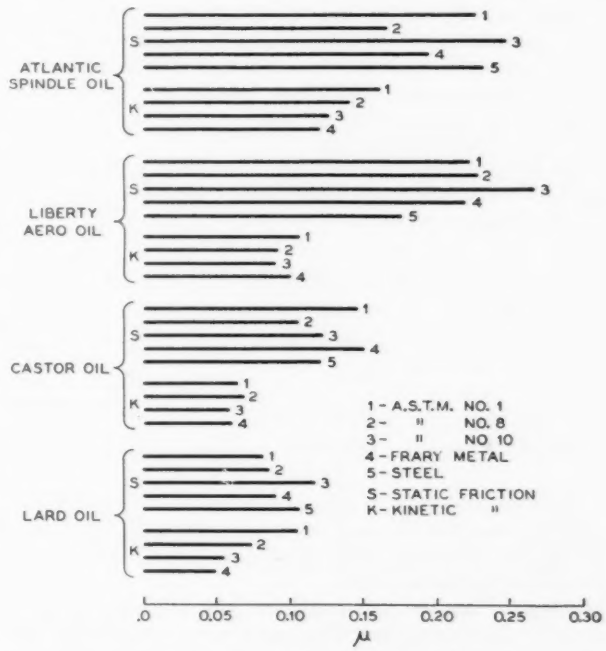


FIG. 5 REPRESENTATION OF COMPARATIVE DATA ON STATIC AND KINETIC FRICTION FOR SEVERAL OILS AND BEARING MATERIALS

It will be noticed that the kinetic values are in general considerably lower than the static values and that for the two mineral oils kinetic values for the low-viscosity Atlantic spindle are higher than those for the high-viscosity Liberty aero, whereas the static values for these two oils are practically the same. These differences are believed to indicate that the kinetic values are influenced by viscosity effects. With one exception, the kinetic values for the two fatty oils are similar. Here, it is believed, corrosion effects enter as an additional complicating factor.

The kinetic values show some tendency to rate the bearing materials in a given order. It would be expected that some similar effect of the bearing material would be apparent in the static values (10*g*, 16), but none is present. Here again corrosion effects may be in part responsible, but it is clear from other measurements made in these laboratories on a variety of materials, under experimental conditions closely approximating those of Hardy, that the quantitative relations describing the effect of the solid surface given by him do not hold generally.

In many of the runs a pronounced downward trend of values, somewhat reminiscent of the latent period mentioned earlier in this paper, was noticed. This latent period (from 1 to 2 hr) was considerably longer than that described by Hardy and Bircumshaw (10*h*), and was noticed for the relatively nonpolar mineral

as well as for the fatty oils, although it tended to be shorter in the latter case. In general the procedure adopted was to continue taking readings until a steady value for two successive means of ten was reached. It was found that the steady value could be reached immediately by rubbing in the lubricant with a linen pad extracted free from friction-lowering impurities, and then rubbing off all but a thin smear of oil. Some evidence was found that this rubbing-in technique gave more reproducible values than the standard method just described, and in one or two cases, where fatty oils were concerned (e.g., lard oil on A.S.T.M. No. 1), a difference greater than experimental error was found between the value obtained by the standard method and the rubbing-in value. This might be due to the presence of a corrosion layer at the interface in the case of the standard value, but the two methods were not investigated in sufficient detail to draw any definite conclusion. It is believed that the downward trend was due in part to the presence of an adsorbed moisture film at the metal-liquid interface. Moisture effects on static-friction measurements have been investigated in considerable detail and will be described in a later paper.

For certain of the runs in Table 7 the effect of an increased load was determined for alternate readings after the steady stage had been reached. The results are given in Table 8.

TABLE 8 EFFECT OF LOAD ON THE COEFFICIENT OF STATIC FRICTION BETWEEN LUBRICATED METAL SURFACES

Lubricant	Metal combination	Load, gr	μ
Atlantic spindle oil	Steel—A.S.T.M. No. 8	75	0.168
		175	0.176
Lard oil	Steel—A.S.T.M. No. 8	75	0.085
		175	0.094
Castor oil	Steel—A.S.T.M. No. 8	75	0.103
		175	0.110
Liberty aero oil	Steel-steel	292	0.22
		717	0.23

These results indicate that under the conditions of test there is a slight increase in the static coefficient with load. The effect, however, is not greater than experimental error and was not investigated in further detail.

Although clear-cut differentiation was obtained among a series of oils, it is clear that the static-friction method is not suitable as a routine test for oiliness. At best it can only give information regarding the state of surfaces at the beginning of motion, and the elaborate precautions required to obtain consistent results provide a further limitation on its use. However, when proper precautions are observed, it provides a useful supplement to other oiliness-testing methods, and can be used to good effect as a research tool for fundamental investigations of the factors operating at the solid-liquid interface in boundary lubrication.

The writer wishes to express his thanks to E. A. Thurber and to H. V. Wadlow for their assistance in obtaining many of the experimental results.

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Stresses in Helical Compression Springs—Present Status of the Problem

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Since Dr. Röver, in 1913, first formulated the modern theory of the stress distribution in a helical spring, including the effect of bar curvature, pitch angle, direct-shear stress, etc., a number of investigators have treated the problem. Wahl, Göhner, Adams, Vogt, Perkins, and Keysor have all presented interesting analyses. Most of these vary somewhat in the detailed method of treatment, but they all arrive at substantially the same result, in terms of the maximum fiber stress in the section. Wahl, in a notable series of experiments, demonstrated that these maximum stresses postulated by the theory are actually present in the spring.

Thus it would appear that we have a satisfactory working theory. But practical spring men feel that there must be something wrong; the theory, when applied to certain standard spring designs with very satisfactory service records, predict stresses which seem far too high to be compatible with good endurance in service. This feeling is supported by the results of a few endurance tests, which indicate rather clearly that the maximum stresses predicted by the theory, though undoubtedly present, do not correctly index the actual endurance.

In the author's opinion, all the evidence points to the conclusion that the problem of design stresses in helical springs is not one for the mathematician, but must be worked out in the laboratories, by actual experiment.

IT WAS as long ago as 1913 that Röver published an article (1)[2] in which was announced, for the first time, a first approximation to the modern theory of the stress in helical springs. Up to that time the Reuleaux formula (2) had been the universal standard; it assumed that the only stress present was pure torsion. Röver's theory recognized not only the torsional stress, but also the direct-shear stress, the bending stress due to the pitch angle, and the variation in torsional stress from point to point of the section, due to the curvature of the bar.

Röver developed his theory in three steps. He first worked out an approximate correction factor for the maximum value of the stress augment due to the bar curvature, plus the direct-shear stress; the pitch angle was taken as zero. He then amplified this correction factor to include the bending stress due to the pitch angle. Both of these formulas are approximate, and both are based on the assumption that the center of torsional rotation coincides with the geometrical axis of the bar. This is incorrect, as the shear moments over the section would not be in statical equilibrium. The center of torsion is displaced a small amount toward the spring axis.

Röver's third step was to develop a more exact but very complicated solution, based on elastic theory, for a particular value of the spring index. This development is therefore of value only as a check on his approximate formula. The latter gives a result differing by only 1 per cent from the more exact solution, for the numerical case examined.

Considering the importance of Röver's theory, it is surprising that it attracted so little attention. It seems to have been practically unnoticed for 15 years, when Wahl made his experiments at the Westinghouse laboratories (3).

This research of Wahl's was magnificently simple in conception and execution. He mounted a half coil of a spring in a testing machine, in such a way that the stress distribution accurately simulated that in the body coils of a complete helical spring, and measured the strains at the innermost helical element (the locus of maximum stress); then reduced these strains to corresponding shear stresses. He also took measurements at the outermost element (locus of minimum stress) and checked these with measurements on the outermost element of complete springs. Due to space limitations, it was impossible to check the complete springs at the innermost element.

Supplementing his experimental work, Wahl announced a revised stress formula. It takes into account the stress differentials due to bar curvature and to direct shear, but not the pitch angle. The correction for bar curvature recognizes the displacement of the center of torsion from the geometrical center of the bar, in accordance with curved-bar theory. Therefore Wahl's formula is presumably more accurate than Röver's first approximation. As to the experimental results, Wahl did not report any numerical figures; however, he did show a graphical plot of his results, together with the theoretical curve for the test springs based on his formula. He ran the tests on two specimens, and as the theoretical stress was a few per cent greater than the experimental result on one, and a few per cent less on the other, we can consider that he got substantial agreement.

Another interesting contribution to the subject was that of Adams (4), who developed an extremely complicated set of equations for the torsional stress. However, it is difficult to appraise accurately the value of Adams' work. The author himself states that "the results derived in the.....analysis are rigorous." But the analysis is not given in full; in an editorial footnote it is stated that the integrations are very difficult and lengthy, and cannot be reproduced, but that they have been examined and checked. Adams gives his final result (a correction factor for the stress augment due to bar curvature) in three separate equations. Then he arbitrarily inserts in the equation another correction factor, representing the effect of direct-shear and bending stresses, and he names certain values for this factor, for various index values. He says he has "shown (this) elsewhere." His formula for displacement of the axis of torsion gives about the same numerical value as the form used by Wahl, for an index of 4. His final correction factors agree closely with Wahl's for very small and rather large index values; however, for intermediate values there is a difference of several per cent. Thus Adams was

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[1] Manager, Bureau of Statistics, Crucible Steel Company of America.

[2] Numbers in parentheses refer to the Bibliography at the end of the paper.

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NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

TABLE 1 FORMULAS USED FOR COMPARING STRESSES IN HELICAL COMPRESSION SPRINGS

Author	Formula	Stress Components included	Value of coefficient for $c = 4$
Reuleaux	$\frac{8PM}{\pi d^3}$	A	1.000
Röver (1)	$\frac{8PM}{\pi d^3}\left\{\frac{c}{c-1}+\frac{1}{4c}\right\}$	A-B-C	1.396
Röver (2)	$\frac{8PM}{\pi d^3}\left\{\frac{7}{13}\left[\frac{c}{c-\cos^2\phi}+\frac{\sin^2\phi}{4c}\right]\sin\phi + \sqrt{\left[\frac{c}{c-\cos^2\phi}+\frac{1+\sin^2\phi}{4c}\right]^2\cos^2\phi+\left[\frac{c}{c-\cos^2\phi}+\frac{\sin^2\phi}{4c}\right]^2\sin^2\phi}\right\}$	A-B-C-D-E	1.450
Modification of (2)	$\frac{8PM}{\pi d^3}\left\{\frac{7}{13}\left[\frac{c}{c-\cos^2\phi}+\frac{\sin^2\phi}{4c}\right]\sin\phi+\sqrt{\left[\frac{c}{c-\cos^2\phi}+\frac{1}{4c}\right]^2+\left[\frac{\sin\phi\cos\phi}{4c}\right]^2}\right\}$	A-B-C-D-E	1.450
Wahl	$\frac{8PM}{\pi d^3}\left\{\frac{4c-1}{4c-4}+\frac{0.615}{c}\right\}$	A-B-C	1.404
Adams	$\frac{8PM}{\pi d^3}\left\{\frac{d^3}{16K}\cdot\frac{d-2x}{M-d}\right\}C_1$ where $x=\frac{d}{4}\left\{2c-\frac{1}{c-\sqrt{c^2-1}}\right\}$ $K=\frac{d^2}{12}\left\{\frac{d(c^2+1)}{\sqrt{c^2-1}}+6x\right\}-\frac{M}{6}\left\{M^2\left[\frac{c^2}{\sqrt{c^2-1}}-1\right]+6x[M-x]\left[1-\sqrt{1-1/c^2}\right]\right\}$ C_1 is a factor to allow for effect of combined stresses.	Doubtful	abt. 1.44
Göhner (1) for S_T	$\frac{8PM}{\pi d^3}\left\{\frac{c}{c-\cos^2\phi}+\frac{\cos^2\phi}{4c}+\frac{\cos^4\phi}{16c^2}+\left[\frac{0.62\cos^2\phi}{c}+\frac{0.53\cos^4\phi}{c^2-c\cos^2\phi}\right]\tan^2\phi\right\}\cos\phi$	A-B-C	1.324
Göhner (1) for S_B	$\frac{16PM}{\pi d^3}\left\{1+\frac{0.87\cos^2\phi}{c}+\frac{0.64\cos^4\phi}{c^2-c\cos^2\phi}+\frac{1}{4c}\right\}\sin\phi$	D-E	1.324
Göhner (2) for S_T	$\frac{8PM}{\pi d^3}\left\{\frac{c}{c-\cos^2\phi}+\frac{\cos^2\phi}{4c}-\frac{\cos^4\phi}{8c^2}\right\}\cos\phi$	A-B-C	
Göhner (3) for S_T	$\frac{8PM}{\pi d^3}\left\{\frac{c}{c-\cos^2\phi}+\frac{\cos^2\phi}{4c}\right\}\cos\phi$	A-B-C	
Göhner (4) for S_T	$\frac{8PM}{\pi d^3}\left\{1+\frac{5}{4c}+\frac{7}{8c^2}+\frac{1}{c^3}\right\}\cos\phi$	A-B-C	1.326
Göhner (4) for S_B	$\frac{16PM}{\pi d^3}\left\{1+\frac{1.12}{c}+\frac{0.64}{c^2}\right\}\sin\phi$	D-E	1.326
Vogt	(for Compression Springs) $\frac{8PM}{\pi d^3}\left\{\frac{0.9875c}{c-1}\right\}$	A-B-C-D-E	1.317
Vogt	(for Extension Springs) $\frac{8PM}{\pi d^3}\left\{\frac{1.0875c}{c-1}\right\}$	A-B-C-D-E	1.450
Perkins for S_T	$\frac{8PM}{\pi d^3}\left\{\left[1-\frac{\sin^2\phi}{2}\right]\left[\frac{2c+1.1\cos^2\phi}{2c-1.4\cos^2\phi}+\frac{0.3\cos^4\phi}{c^2(c-\cos^2\phi)}\right]-\left[\frac{\sin^2\phi}{4}\right]\left[\frac{2c-\cos^2\phi}{c-\cos^2\phi}\right]\frac{1}{\cos\phi}\right\}$	A-B-C	1.320
Perkins for S_B	$\frac{8PM}{\pi d^3}\left\{\frac{\sin^2\phi}{2c}+\frac{2c^2-\cos^4\phi}{c(c-\cos^2\phi)}\right\}\sin\phi$	D-E-F	1.320

SYMBOL NOTATION FOR STRESS COMPONENTS INCLUDED.

A- Pure Torsional Stress. B- Correction for Bar Curvature. C- Direct Shear Stress. D- Bending Stress. E- Direct Tension or Compression. F- Secondary Shear Stress.

able to check Wahl's experimental results to about 1.25 per cent; Wahl's test springs were of index 3.

The next contribution we find is that of Göhner. In 1930 and 1931 Göhner published three articles developing the pure theory of the stress distribution in helical springs. In March, 1932, he published a fourth article (5) in which he announced a series of simplified formulas based on his earlier work and in convenient form for practical spring calculations. The stress formula is stated in four different forms, each successive form being a further simplification.

In the year following Göhner's work, Vogt (6) made some ingenious simplifications of the equations developed by Röver as his second step; as explained before, this step includes the corrections for bar curvature, direct shear, and pitch angle. There-

fore Vogt's equations, like Röver's, involve the error of taking the center of torsion at the geometrical axis of the bar.

Vogt makes the interesting suggestion that the stress equation be stated in terms of tensional stress instead of torsional, and in his own paper his equations are so stated. Probably something could be said on both sides of this question, but a discussion of the subject does not come within the scope of the present paper.

At the December, 1933, meeting of the A.S.M.E., Professor H. C. Perkins of Cornell University presented a paper (7) in which he developed a very interesting set of equations. Perkins followed the method used by Röver in his accurate solution for a particular index value; but, he generalized the loading on the spring to cover a combination of an axial force and an axial couple, and he generalized his solution to cover any numerical value of index.

In the following year, Professor Perkins gave us a second paper, in which he gave stress equations for the case of an eccentrically loaded helical spring.

It is necessary to mention one other name. At the A.S.M.E. meeting in December, 1934, Keysor (8) gave us a discussion of the problem from a new standpoint. All the previous investigators had discussed a condition in which the stress distribution was the same in every section of the bar. Keysor, in considering the problem of the number of inactive coils, decided that any assumptions as to axial loading were not justified, and investigated the general case of a helical spring of standard end conformation compressed between two parallel plates. He found that the effective loading was in general nonaxial, the eccentricity varying with the spring index, the number of total coils, and the number of fractional coils. For certain combinations of these variables the eccentricity becomes zero.

While the Bibliography given with this paper is by no means complete, it probably covers the more important contributions to the subject. Now it will be interesting to look at the various formulas that have been mentioned, and compare the results they give.

In Table 1 are listed the formulas, their authors, the stress components accounted for by each, and the numerical value of the correction factor from each formula for an index value of 4. For ready comparison, all the formulas are reduced to a standard form, consisting of the Reuleaux formula times a correction factor, and a common notation is employed. The latter is

S_b = maximum tension or compression stress (bending plus direct tension or compression)
S_t = maximum shear stress (torsion plus direct shear)
S_s = maximum combined stress, in terms of shear stress
d = diameter of spring bar
M = mean diameter of helix
P = applied load, compressing the spring
c = spring index = M/d
ϕ = pitch angle.

In order to calculate the numerical values of the correction factors, it is necessary, where the formulas involve the pitch angle ϕ, to assign a particular value therefor. As the fiber stress is a maximum when the spring is completely compressed or solid, and this solid stress is usually the starting point for design calculations, it seems logical to use as typical the value of the pitch angle when the spring is solid. This value is

$$\phi = \tan^{-1} d/\pi M = \tan^{-1} 1/\pi c \quad \ldots\ldots [1]$$

Listed first in Table 1 is the Reuleaux formula, which probably needs no introduction to engineers. It is the well-known

$$S = \frac{8PM}{\pi d^3} \quad \ldots\ldots [2]$$

in which only pure torsional stress is considered.

Next come the several Röver formulas. It is easy to see how Röver arrived at his first and simplest form. Neglecting the pitch angle and the displacement of the axis of torsion, he assumed that the stress augment at any point on the periphery of the bar varied inversely as its distance from the cylindrical axis; its maximum value, at the innermost point of the periphery, was

$$S = S_0 \frac{M/2}{(M-d)/2} \quad \ldots\ldots [3]$$

where S_0 is the mean stress as given by the Reuleaux formula. The fractional term obviously reduces to the first term in the Röver equation $c/(c-1)$. For the direct shear at the innermost point on the cross section, Röver wrote $(P/2)/(\pi d^2/4)$, which, when the term $8PM/\pi d^3$ is factored out, becomes the second term in his correction factor, that is, $^1/_4c$.

Röver then modifies his formula to give effect to the influence of the pitch angle. Space will not permit a detailed description of the method used. The two formulas developed, for torsion-shear and bending-tension stresses, respectively, are

$$S_t = \frac{8PM}{\pi d^3}\left(\frac{c}{c-\cos^2\phi} + \frac{1+\sin^2\phi}{4c}\right)\cos\phi \quad \ldots\ldots [4]$$

$$S_b = \frac{16PM}{\pi d^3}\left(\frac{c}{c-\cos^2\phi} + \frac{\sin^2\phi}{4c}\right)\sin\phi \quad \ldots\ldots [5]$$

Röver then combines these in the classic form (taking Poisson's ratio as $m = 3^1/_3$)

$$S_s = 0.35S_b + 0.65\sqrt{(S_b^2 + 4S_t^2)} \quad \ldots\ldots [6]$$

Substituting the two values, and rewriting in terms of torsion-shear stress, we obtain the third Röver formula given in Table 1.

This formula can be rigidly transformed into the fourth form. The latter can be further simplified very greatly, with only very minor losses in accuracy. However, a discussion of these does not lie within the scope of this paper.

Reference to the last column in Table 1 shows that for index 4 the value of the correction coefficient has been increased about 4 per cent by including the effect of the pitch angle.

Going now to Wahl's formula, this is quite widely known and used, and a full description of the derivation is readily available; therefore, only a brief statement is necessary here. Wahl used for the displacement of the axis of rotation the well-known approximation $d^2/8M$. He then found that the Röver factor for torsional stress augment, that is, $c/(c-1)$, must be multiplied by another factor

$$\frac{(d/2) - (d^2/8M)}{d/2}$$

and becomes

$$\left(\frac{c}{c-1}\right)\left[\frac{(d/2)-(d^2/8M)}{d/2}\right] = \left(\frac{c}{c-1}\right)\left(1-\frac{1}{4c}\right) = \frac{4c-1}{4c-4} \quad \ldots\ldots [7]$$

For the direct-shear stress at the innermost point of the cross section, Wahl used the expression $1.23P/(\pi d^2/4)$, and his complete correction factor becomes

$$\frac{4c-1}{4c-4} + \frac{0.615}{c}$$

It may be noted that the first term of Wahl's correction factor yields considerably lower numerical values than Röver's and the second term gives higher values. The two values for the complete expressions, as shown in Table 1 differ by less than 1 per cent for $c = 4$.

Wahl made no allowance for the pitch angle, stating that "it can be shown that a change in slope from zero to 10 deg does not change the maximum shear stress more than a few per cent."

It is impossible to discuss Adams' formulas in detail for the reasons already given. A glance at Table 1 will reveal their complex character. As one of the terms in the final equation is an arbitrary constant, it was necessary to read off the representative numerical value of the correction factor, for index 4, from Adams' graphic chart. As the latter was too small to serve this purpose properly, the tabulated value is only approximate.

Göhner's equations are very interesting. In their simplified forms they are quite easy to manipulate, and calculations for several index values show that for "solid" pitch angles the error in these simplified forms is trifling. This is illustrated by the two values given in Table 1.

Next we come to Vogt's equations. In his simplification of the Röver formulas, Vogt wrote the equation for combined stress

$$S_c = \pm 0.36 S_b + 0.64 \sqrt{(S_b^2 + 4 S_t^2)} \quad [8]$$

with the double sign for the first term, and he said that the plus sign applied to extension springs and the minus sign to compression springs.

This procedure seems to be quite correct. In a compression spring, the bending component acts to bend the bar inward (in the direction of the curvature), and the resultant stress at the innermost fiber of the section is compressive. Also the $P \sin \phi$ component of the direct load is obviously a compressive force. According to the authorities, for combined torsion and compression stresses the equation must be written with a minus sign for the first term.

Röver ignored this distinction; in combining his own formulas he wrote the result as in Table 1. Therefore, while Vogt's extension-spring formula agrees with Röver's, his compression-spring formula gives a value about 10 per cent lower. Apparently Adams missed this point also. Göhner did not give a combined form for his own equations; in working out the values given in Table 1 Vogt's practice was followed.

Perkins' equations are quite interesting, as their accuracy is probably of a very high order. As already stated, Perkins followed Röver's method, except that he generalized it to cover any value of the index, instead of one particular value. He found that a "secondary shear stress" accompanies the bending, opposing the stress in torsion. However, its magnitude is very small.

The final equations for torsion-shear and bending-tension stresses are each stated in several successive steps, like those of Adams; however, Perkins' equations can be combined quite readily into the forms given in Table 1. Consolidation of these two into a single equation for the combined stress would yield a rather complicated expression, which is therefore omitted; for an index value of 4 the combined stress coefficient is 1.320, substantially the same as given by the Göhner and Vogt formulas.

Perkins' equations gave effect to an axial couple as well as an axial force; but in order to make his formulas comparative with the others in Table 1, the terms involving the axial couple have been omitted.

Finally, we have Keysor's paper, dealing primarily with stress augments due to the eccentricity of the effective loading. Keysor's complete analysis is not available at the present writing, and in any event there would be no comparison between his formula and those already dealt with. Keysor used Adams' correction coefficient for bar curvature, direct shear, etc., inserting a symbol in his formula to represent them.

But it should be carefully noted that Keysor's analysis predicts a stress augment, due to eccentricity of loading alone, of as much as 28 per cent in certain designs (springs of very few total coils), and that in the outside coil of the standard A.A.R. freight-car bolster spring this augment is about 8 per cent, superimposed on the stress augments already discussed. Some experimental tests have been made, and a more elaborate research is now in progress, the results from which so far seem to confirm Keysor's theory.

From the comparisons given in Table 1, we might reasonably conclude that we have a very satisfactory theory covering our stress problem. Now let us look for a moment at the practical aspect. In the ten years since Wahl published the results of his research, these results have become quite generally known to spring engineers and designers. While they are coming more and more into use, many practical spring men still regard them with a certain amount of skepticism. It is necessary to inquire into the reasons for this attitude.

What these practical men tell us is, briefly, that our correction factors, when applied to certain widely used spring designs, indicate stresses that look rather absurd. Consider, as an example, the outer coil unit of the standard A.A.R. freight-car bolster spring. Literally millions of these springs are riding American freight cars today. The maximum fiber stress at solid height, by the Reuleaux formula, is about 92,000 lb per sq in. The correction coefficient from Wahl's formula, less about $4\frac{1}{4}$ per cent for the effect of the pitch angle, is about 1.4, making the maximum stress 129,000 lb per sq in. If Keysor's theory be correct, we must now add to this about 8 per cent for load eccentricity, and we arrive at a stress of about 140,000 lb per sq in. in torsion. This is well above the usual yield strength of the material.

It is true that these springs rarely or never go solid in service, but they will frequently come close to it. Yet, if made under reasonably good manufacturing conditions, they will give reasonably good service. There are other spring designs in common use, in which stress conditions are more extreme.

The A.S.M.E. Special Research Committee on Mechanical Springs felt that this practical argument merited some investigation. Here we must distinguish carefully between the intrinsic correctness of the Röver-Wahl theory, and its practical application to spring design. There is no doubt that the maximum stresses predicted by the theory are actually present in the spring; Wahl's experimental work proved that beyond any reasonable doubt. The real question is: Are these the critical stresses which index the life of the spring in service, or in an endurance test? Only the test itself can furnish the answer to this question.

Accordingly the Committee arranged, in connection with its extensive fatigue-test program at Wright Field, to have two groups of springs made up from the same rolling and heat of steel, and otherwise alike in every respect except as to the spring index and related dimensions; one group was of index 5 and the other of index 3. The answer we want is a formula for maximum fiber stress, which, when applied to the results of the fatigue tests, will indicate the same endurance limit for both groups.

The test results are shown graphically in Fig. 1. Datum points for the individual runs are plotted in duplicate, with stresses calculated from the Reuleaux formula and alternatively with the Wahl correction factor. Probable S–N curves are plotted for each of the groups, likewise in duplicate. These curves were calculated from the test data by statistical methods, based on the assumption that the S–N curve for ferrous metals is of the form

$$(N - a)(S - b) = k \quad [9]$$

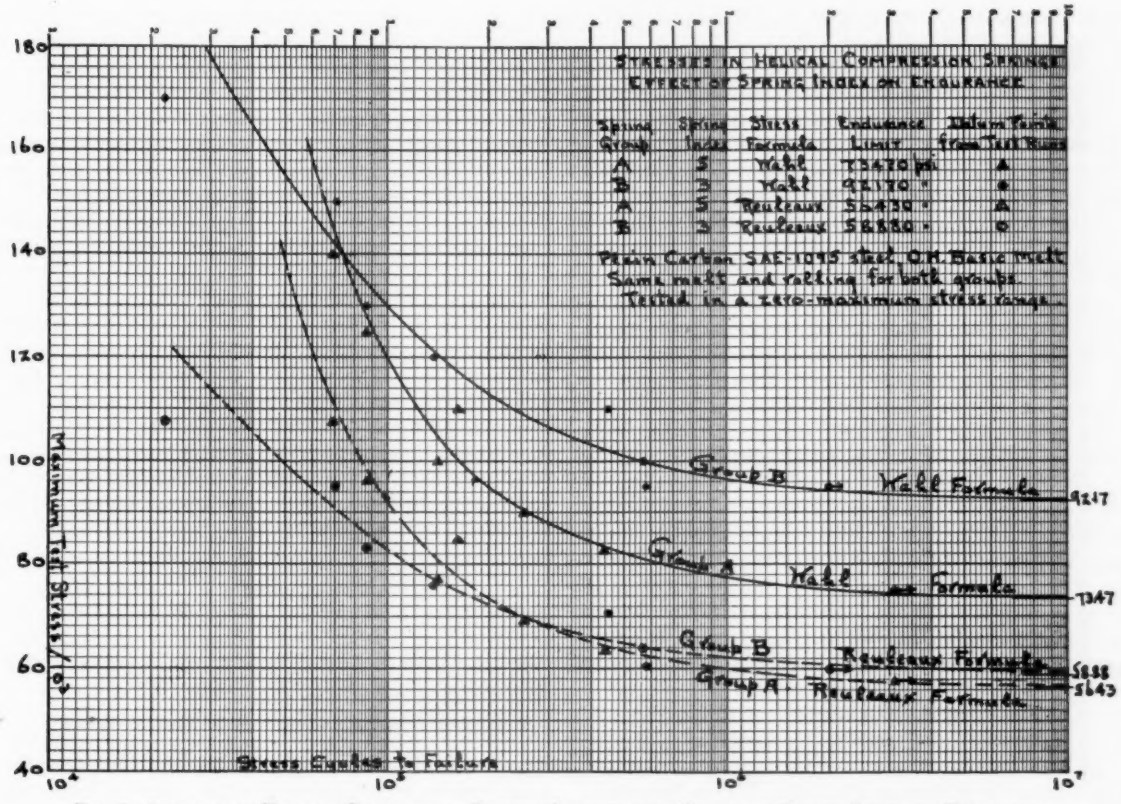


FIG. 1 STRESSES IN HELICAL COMPRESSION SPRINGS SHOWING THE EFFECT OF SPRING INDEX ON ENDURANCE

and that the value of b can be interpreted as the endurance limit. The correlation between the test data and the curves as plotted is quite good for both groups.

A study of Fig. 1 discloses a rather puzzling state of affairs. The two Wahl curves intersect at a rather high stress. Springs in this and higher stress regions are given a quite severe "cold setting" in manufacture, and a possible interpretation of the results might be that under these conditions the springs are very sensitive to local stress augments, and perhaps that they are being subjected to augments not provided for in the Wahl correction factor, such as eccentricity of loading and distortion occasioned by the cold setting; there are several possibilities. As the stress is reduced the two Wahl graphs diverge rapidly, and give predicted endurance limits of 92,170 lb per sq in. for group B (index 3) and 73,470 lb per sq in. for group A (index 5).

For the Reuleaux curves the situation is reversed. They are far apart at high stresses, converge rapidly as the stress is reduced, and the endurance limits are 58,880 and 56,430 lb per sq in., respectively. The apparent conclusion is that for moderately stressed springs local stress augments have little or no effect on endurance. This does not seem quite reasonable, but we can find nothing wrong with the test procedure and data. Further tests along the same lines are projected.

We then got F. P. Zimmerli interested in the problem, and he ran some tests on cold-wound wire springs of different index values. The results are given in Table 2, with an attempt at an analysis thereof.

TABLE 2 TEST RESULTS ON COLD-WOUND WIRE SPRINGS

Spring	Index	Wahl correction factor	Endurance limit, Wahl formula	Endurance limit, Reuleaux	Revised correction factor	Revised endurance limit
A	3.5	1.476	100×10^3	67.75×10^3	1.262	85.50×10^3
B	4.55	1.346	94×10^3	69.84×10^3	1.190	83.11×10^3
C	7.00	1.213	93×10^3	76.67×10^3	1.117	85.64×10^3
D	9.10	1.161	91×10^3	78.38×10^3	1.089	85.36×10^3
E	11.90	1.121	90×10^3	80.29×10^3	1.067	85.67×10^3

The revised correction factors in Table 2 are coefficients, the decimal portions of which have been reduced by a fixed percentage from those of the corresponding Wahl factors. The fixed percentage was established by cut and try, the object being to arrive at a set of revised endurance limits for the various springs which would show a minimum systematic deviation from a mean. The percentage arrived at was 45 per cent. So we might state, as a hypothesis for discussion, that the effective maximum fiber stress in a helical spring is obtained by using a correction factor, the decimal portion of which is 55 per cent of the decimal portion of the Wahl factor.

Mr. Wahl calls this 55 per cent the "sensitivity index," and he attacks the problem along a line of argument, quite different from the foregoing, in a paper[3] being presented coincidentally with this one. He has given a good deal of time and thought to the subject, and what he has to say will be more pertinent than any theory the present writer is able to offer at the moment.

But at least it seems obvious that our theory of stress in helical compression springs, despite the great amount of work that has been done on it and the very consistent results of the mathematicians, will have to be drastically modified to check with what we find in the experimental laboratory, and in actual spring service.

Here the author pauses, somewhat perplexed even to suggest a promising line of attack. Consider for a moment the intricate stress pattern in a helical spring. It includes:

Pure torsional stress.
Direct shear stress.
Bending stress.
Direct tensile or compressive stress.

[3] "Analysis of Effect of Wire Curvature on Allowable Stresses in Helical Springs," by A. M. Wahl, *Journal of Applied Mechanics*, Trans. A.S.M.E., vol. 61, 1939, p. A-25.

A stress increment or decrement, nonsymmetrical with the bar axis, due to curvature of the bar.

A stress increment or decrement, nonsymmetrical with respect to the cylindrical axis, due to eccentricity of the effective loading.

In springs which have been cold-set, entrapped stresses, of which little is known.

Possibly some kind of stress augment about two turns from the tip of the bar. Endurance-test failures show a tendency to localize in this region. This augment may be related to the load eccentricity.

At the Committee meeting last December, Dr. A. W. deForest remarked to the author that in view of the complex stresses present in a helical spring, it seemed hopeless to expect to solve them. Perhaps the best answer is that we can reasonably expect to find a definite consistency in the stress pattern. The proportions of a helical spring can only vary in a very limited number of ways, therefore, our stress pattern should only vary in a correspondingly limited number of ways. This means that while we may never attain to full understanding of the absolute stresses present, we should be able to find out a great deal about the relative stresses.

Stated in this form, the problem is no longer one for the mathematicians; it becomes definitely the problem of the research engineer. The function of this paper is to describe the problem, rather than to suggest solutions. However, the author has for some time been convinced that we must look primarily to the experimental laboratory for our code of design for helical springs.

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Discussion

A. M. WAHL.[4] From Table 1 of the author's paper, it would appear that a value of the coefficient of 1.326 (for a spring of index 4) would be obtained by using Göhner's formula, while by using the writer's formula a coefficient of 1.404 is found, the difference between these values being about 6 per cent. However, it should be mentioned that this difference is based on the use of the maximum-strain theory in combining the shear-stress component S_t with the bending stress S_b, the use of this theory being originally suggested by Röver (1)[5] and later used by Vogt (6). If the stress components S_b and S_t had been combined according to the shear-energy (von Mises-Hencky) theory, this difference would have been only about $1^1/_2$ per cent. There are considerable fatigue test data on spring steels which tend to support the shear-energy theory rather than the maximum-strain theory. Moore and Kommers[6] report a ratio between the endurance limit in reversed torsion and that in reversed bending of 0.52 for a 1.2 per cent carbon steel, heat-treated; for a 0.93 per cent carbon steel, heat-treated, a value of 0.53; and a value of 0.50 and 0.56 for quenched and drawn chrome-nickel steels, the average for a large number of alloy steels being 0.58. Also Ludwik[7] reports a ratio of 0.575 between the torsion and bending endurance limits for a chrome-nickel steel. These values are all quite close to the value of 0.577 predicted on the basis of the shear-energy theory, and are considerably different from the value of 0.77 predicted on the basis of the maximum-strain theory. Dr. Koerber, director of the K. W. Institute for Eisenforschung in Germany, in a lecture before the Fifth International Congress for Applied Mechanics also stated that the shear-energy theory appears to yield the best results when compared with the results of fatigue tests on steels made at their laboratories. For these reasons, there seems to be at present a distinct trend toward the use of this theory by engineers interested in strength of materials.

Using the shear-energy theory as a basis, it may be shown that the equivalent stress is given by

$$S_s = S_t \sqrt{1 + \frac{S_b^2}{3S_t^2}} \qquad [10]$$

where S_b and S_t have the same meanings as given in the author's paper and S_s is the equivalent torsion stress.

Using this formula, taking a pitch angle $\phi = \tan^{-1} \frac{1}{\pi c}$ as was done by the author, and calculating S_b and S_t from Göhner's Equations [3] and [4] given in Table 1 of the author's paper, the value of the coefficient for $c = 4$ becomes 1.384. This differs by about 1.5 per cent with the writer's value of 1.404, obtained by neglecting the pitch angle.

Another theory which is quite widely used in industry is the maximum-shear theory, according to which the ratio between the torsion and bending endurance limits should be 0.50 (which is somewhat below the average values obtained by tests). On the basis of this theory the equivalent stress is

$$S_s = S_t \sqrt{1 + \frac{S_b^2}{4S_t^2}} \qquad [11]$$

Using Göhner's Equation [4] of Table 1 as before for calculating S_b and S_t, a coefficient of 1.383 is found for a spring of index 4, which is practically the same as that obtained previously by using the shear-energy theory.

These examples show that if either the maximum-shear or shear-energy theories is taken as a basis, the simple formula previously proposed by the writer, in which the pitch angle was neglected, will give results for most practical springs within less than 2 per cent, as compared with those obtained by Göhner's equations, the derivation of which is based on the more exact methods of the theory of elasticity. For these reasons, it is the writer's opinion that for the present at least the simpler formula may be used with sufficient accuracy for practical work.

Since, at present, no theory for combining the various stress components present in a helical spring can be regarded as definitely established experimentally, it would seem logical to compare individually the stress components figured from the different formulas. The most important component in helical springs is the shear stress due to the axial load. Using Göhner's formula

[4] Research Laboratories, Mechanics Division, Westinghouse Electric & Manufacturing Company, East Pittsburgh, Pa. Mem. A.S.M.E.

[5] Numbers in parentheses refer to the Bibliography in the author's paper.

[6] "Fatigue of Metals," by H. F. Moore and J. B. Kommers, McGraw-Hill Book Co., Inc., New York, 1927, p. 147.

[7] "Kerb und Korrosionsdauerfestigkeit," by P. Ludwik, *Metallwirtschaft*, vol. 10, no. 37, September, 1931, pp. 705–710.

[4] of Table 1, for $c = 4$ and $\phi = \tan^{-1}\frac{1}{\pi c}$, the stress S_s becomes $\frac{8PM}{\pi d^3} \times 1.379$. Compared with the writer's value of 1.404 this is a difference of less than 2 per cent. Even smaller differences will be found for larger indexes. As far as the bending stresses S_b are concerned, there is little question about these since they may be figured by well-known theory. The formulas given by Göhner in Table 1 for S_b may be shown to be in close agreement with the results calculated by ordinary curved-bar theory,[8] taking the moment equal to $\frac{PM}{2}$ sin ϕ and adding on the direct stress due to the compression component of the axial load.

These examples indicate that, if elastic conditions are assumed, the formulas available will yield stresses in helical springs which are very close to the actual values. However, in most cases, when compressing the spring solid (as is usually the case during manufacture), the peak stress as given by the formula exceeds the proportional limit or even the yield point in torsion. In such cases yielding occurs and the actual maximum stress is less than the calculated. When the load is released, a residual stress of opposite sign is set up, so that when the load comes on again the peak stress is reduced by the amount of this residual stress.

Referring to the freight-car bolster springs cited by the author as an example, the stress of 140,000 lb per sq in., mentioned would not actually exist when the spring is compressed solid. Instead, yielding of the material would take place at localized regions near the inside of the coil so that the peak stress would not appreciably exceed the yield point. Then too, in usual service as mentioned by the author, the springs are subjected to relatively few cycles of the solid stress, although they may be subject to a great many cycles of a smaller stress. In the writer's opinion there is a definite need for fatigue-test data for cases where there are relatively few cycles of a maximum stress, but in between these cycles a relatively large number of cycles of a moderate stress takes place. When such data are available, the performance of these freight-car springs mentioned may be explained on a rational basis.

The results of the tests carried out on the springs at Wright Field seem rather surprising, since they indicate that neither the stress augment due to curvature nor to direct shear has any effect on the endurance limit. This result may be due to a combination of several factors, including: (*a*) The eccentricity of loading in the two sets of springs of different indexes may have been different and this may tend to mask the effect of bar curvature; (*b*) the material may not be fully sensitive to stress-concentration effects; (*c*) the effect of heat-treatment may be different in the springs of smaller index. In this connection, mention should be made that the sensitivity effect of 55 per cent discussed by the author is not the same as the sensitivity index of that used in the writer's concurrent paper (3).[9] In this paper "sensitivity index" is defined as

$$q = \frac{K_f - 1}{K_c - 1} \quad [12]$$

where K_f = actual fatigue-strength reduction factor and K_c = theoretical stress-concentration factor due only to curvature. The definition given by Equation [12] is in line with that used by research workers on fatigue of metals in this country and in Germany. Using the methods developed in paper (3), the results of Zimmerli's tests (mentioned by the author) may be explained, using a sensitivity index of 1 (defined in this manner). This means that the full sensitivity of the material to stress-concentration effects due to curvature are assumed in figuring the variable component of the stress, the static component being calculated by neglecting stress-concentration effects due to curvature.

The writer fully agrees with the author that a great many more laboratory tests must be carried out before a satisfactory code of design for helical springs will be available. However, it is his belief that analytical work carried out in conjunction with the fatigue tests will be of value in correlating and explaining the experimental results.

Author's Closure

The author notes with pleasure Dr. Wahl's extensive discussion. The point made by Dr. Wahl in connection with the use of the maximum-strain theory is probably well taken. Had the author the task of preparing this paper again, he would probably refrain from any attempt to combine the bending- and shearing-stress components into a single formula, except perhaps in the cases of Röver and Vogt, who themselves suggested combined forms. Apparently the correct theory for combined stresses is still a matter of controversy. In any event, it is of no great importance as related to the subject under discussion. Comparison among the various stress formulas can just as well be made on the basis of shear-stress components only. Dr. Wahl seems to feel the same way about it.

Coming now to the discussion of actual test results, it is the author's belief that Dr. Wahl's argument rather begs the question. The statement made in the paper was not that the stress of 140,000 lb per sq in. was actually present in the standard A.A.R. bolster spring, but that this was the stress predicted by the formulas. Obviously, if this stress is beyond the elastic range of the material, plastic deformation will occur, and the figured stress will not be attained. The author does not believe that Dr. Wahl will contend for a moment that such a condition would not have a detrimental effect on the life of the spring. Indeed, there is very satisfactory evidence from the Wright Field tests that the spring endurance is affected to a marked extent by the amount of "cold setting" in initial tests.

The situation with regard to this problem urgently calls for further experimental work. Assuming that the tests at Wright Field can be continued (regarding which there is some doubt), plans are now being made to run some more tests on the effect of spring index on endurance, in the near future.

[8] "Strength of Materials—Part II," by Stephen Timoshenko, D. Van Nostrand Co., New York, 1930, p. 427.

[9] The effect of sensitivity is treated in the author's bibliographical reference (3).

Discussion

Chlorination of Condenser Cooling Water[1]

H. K. Aldinger.[2] The author's treatise on chlorination of condenser cooling water presents a very interesting and valuable engineering problem. While this article confines itself principally to the chlorination of circulating water for steam condensers, it should be equally interesting to anyone using process water for other purposes, such as bleacheries, chemical plants, food-product plants, and breweries.

In discussing the author's treatise, the writer takes the liberty to depart somewhat from the main issue, that is, chlorination of cooling water for condensers, because he believes that the slime control in water is of equal importance for any other industrial purpose.

Most operating engineers of power and other plants have to deal more or less with slime and scale-forming conditions in such units as condenser tubes, boilers, air washers, tanks, and pipes. In the case of condenser tubes, as the author states, the economic losses due to slime and scale formation are more pronounced in low-temperature heat exchangers than in high-temperature exchangers.

While chlorination of circulating water in large plants may be of decided advantage, it is doubtful whether it would pay for itself in small plants where a condenser can be cleaned easily over week-end shutdowns, unless the slime and scale formation becomes excessive and retards considerably the heat transfer. Such a condition would occur mostly in the spring and summer months, and where the loss in vacuum would amount to 1 in. or more; however, the warmer circulating water in summertime in itself would reduce the vacuum somewhat. Of course, slime conditions in condenser tubes exist mostly with such plants where they take cooling water from rivers, lakes, or open ponds. It hardly occurs where the cooling water is recirculated through a cooling tower and the make-up water is obtained either from deep-well or city-water supply, as is the case with many small plants.

From the writer's observations, slime conditions are caused by algae in the water. Algae in turn are caused only when vegetable matter or pollen is deposited on or into the water, and have sufficient time to disintegrate under the influence of air and sunlight; they are, of course, most annoying in spring and summer months. Pure cellulose, such as wood and cotton fibers, are the best propagators for the development of algae. However, pollen from grass and other plants furnish their share.

These algae troubles are not nearly as bad in condensers as they are where open water from such sources as ponds and reservoirs is used for processing work, as for instance in bleacheries, food-product plants, and chemical plants. Even filtering will not entirely eliminate these algae, and their propagation is very rapid. The writer's studies have been made with water used mostly for finishing cotton products (bleachery); however, it makes very little difference whether reservoir water is used in bleacheries or in a condenser, slime conditions will occur as long as it is infested with algae. While the writer does not claim to have found a cure for such slime formation, he made some experiments for its prevention by exterminating algae with chemicals other than chlorine.

Of course, in using river, lake, or pond water, the algae are carried in with the water to the plant. It would not be practical to cover a lake or pond in order to prevent vegetable matters or any cellulose from entering the water. On the other hand, a reservoir or tank could be covered, provided that algae are not brought in from outside sources with the water entering the reservoir.

The writer does not intend to discuss the histology of cells. There are many books written on it, and a short and comprehensive treatise on algae can be found in Van Nostrand's Scientific Encyclopedia.[3]

Algae are generally referred to in industrial plants as reservoir or pond scum or waterbloom. They can be found in varied forms in stagnant and putrid water, especially in large reservoirs, ditches, ponds, tanks, and open wells where much vegetation exists and around cotton mills where flying lint and organic dust can easily settle on the water. Algae are classified under the group of Thallophyta.

In the case of algae, the pigment enclosing the nucleus contains chlorophyll, a green coloring matter, while fungi do not. Algae are divided into four classes, when classified by their color, but we are interested here only in the first and second group, that is blue-green algae (Cyanophyceae) and green algae (chlorophyceae). The other two groups occur mostly in sea water. The chlorophyll is what gives the algae its characteristic green or blue color.

While the individual cell of the protoplast of an algae is complex in structure, it can be considered generally to consist of three principal parts: (1) the nucleus; (2) the cytosome or cell body, which envelops the nucleus and is composed of cytoplasm; and (3) the outer cell wall or shell surrounding the cytoplasm. It is the cytoplasm of the unit cell which, when aggregated in large masses, becomes a soft slimy substance of considerable thickness and size and causes all the trouble in condenser and process work.

Figs. 1*D*, 1*B*, and 1*C* show this clearly. Figs. 2 and 3 show water infested with algae, while Fig. 4 shows the same water filtered.

The writer has developed, by photosynthesis, some large specimens of cytosome as shown in the test tubes in Figs. 1*A*, 1*B*, and 1*C*.

The process by which the cytosome develops and grows is still a hypothetical one as far as the writer knows. However, he assumes that under the influence of light, water, and chlorophyll, a great amount of CO_2 is formed in some way, probably by decay. This CO_2 is evidently taken up again by the cells, and in some complex way is transformed into glucose and starch, probably under the influence of some form of amino acids. Of course, this is purely the writer's conclusion, and is based on the starchy character of the cytosome.

A quite similar process can be observed in the fermentation of wine or in making vinegar from apples where a similar slimy skin of considerable thickness is formed and it is from this process the writer got his idea. In the case of either algae or vinegar there is no cellulose present in the cytoplasm. (The cell wall surround-

[1] Published as paper FSP-60-16, by R. B. Martin, in the August, 1938, issue of the A.S.M.E. Transactions, pp. 475–483.

[2] Mechanical Engineer, Fulton Bag & Cotton Mills, Atlanta, Ga. Mem. A.S.M.E.

[3] "Van Nostrand's Scientific Encyclopedia," D. Van Nostrand Company, Inc., New York, N. Y., 1938.

ing the cytosome or cytoplasm consists of cellulose, but the cytoplasm itself does not contain any cellulose.)

It is not the algae as such which gives all the trouble in condensers or other apparatus, but rather it is the slimy mass or scum inherent with algae. In bleacheries, for instance, the algae itself may cause thousands of yards of goods to be damaged by staining the goods in some way or another, mostly green or brown.

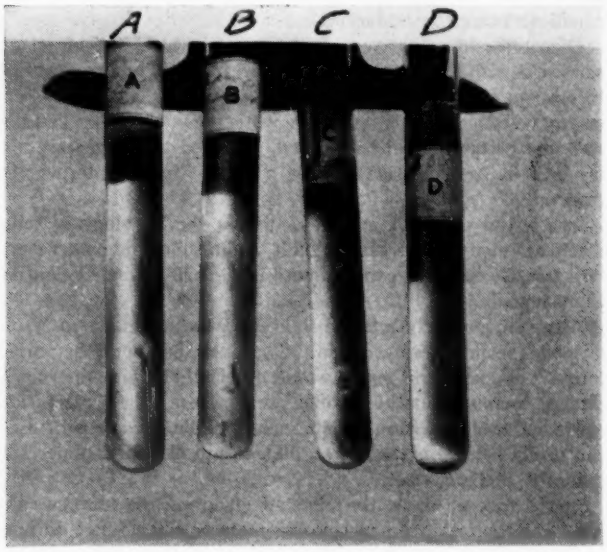


FIG. 1 ALGAE BEFORE AND AFTER TREATING WITH HYPOCHLORITE (*A*, *B*, and *C* show algae developed by photosynthesis; *D* is a sample of the same water treated with hypochlorite, and shows the algae dead at the bottom of the tube.)

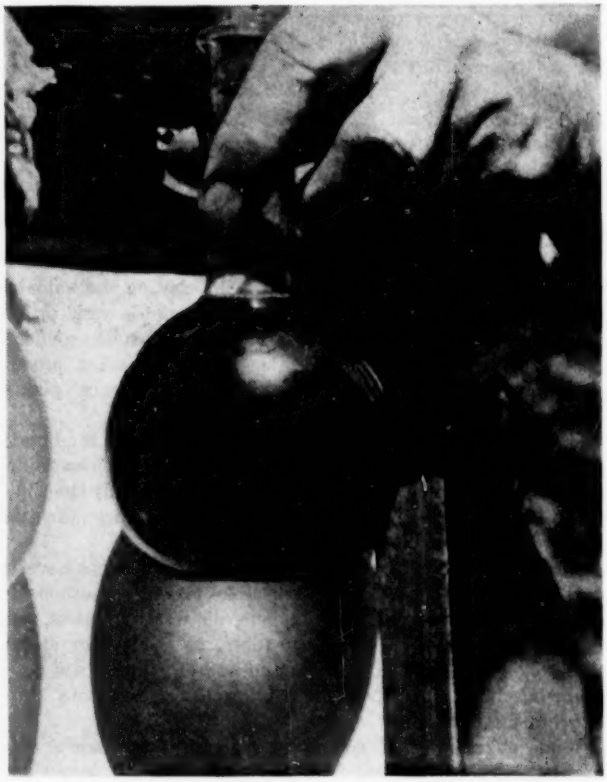


FIG. 3 ANOTHER WATER SAMPLE INFESTED WITH ALGAE

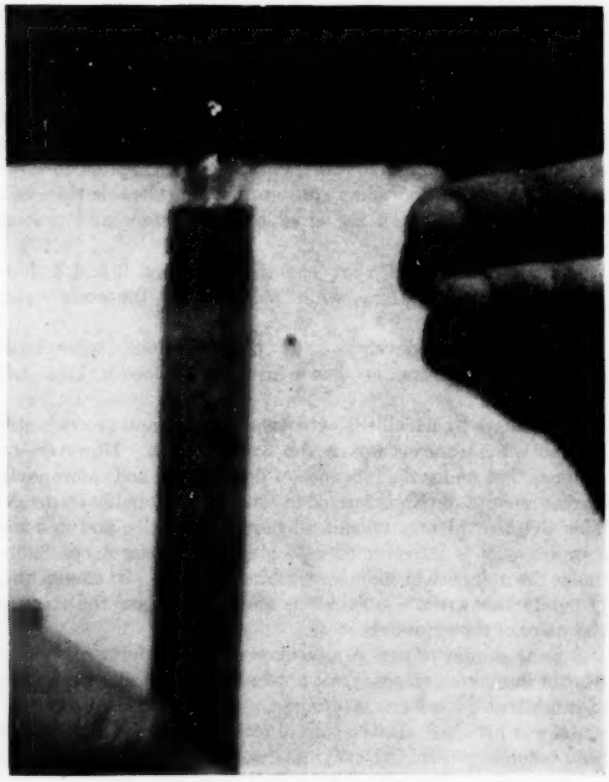


FIG. 2 WATER INFESTED WITH ALGAE



FIG. 4 THE WATER, SHOWN IN FIG. 3, AFTER FILTERING

Such staining is called "possum tack," and is caused when the cloth lays in kiers or bins for some length of time. Small puddles of water containing some algae form between the layers of goods. The water naturally drains in the bin, and the cloth, acting as a filter, holds the algae back. Thus the algae are deposited on the wet cloth, and form the characteristic streaks and stains.

With the common gravity or pressure filter, it is not quite possible to free the water of all algae unless the water is coagulated before and possibly after filtering. And even if the water were thoroughly free from algae, the fuzz and lint from the cloth, or the cloth itself, would only too soon create new algae if allowed to stand for any length of time in the bins, unless the cloth has a high residual chlorine content after bleaching which, however, is not desirable.

The writer conducted some tests with samples of bleached cotton cloth. The first sample was immersed in common clear well water. Algae developed in six days. The second sample was immersed in reservoir water already containing algae and was then exposed to sunlight. In two days the water and cloth sample were green and alive with floating cells, and had an oily and fishy smell. The writer immersed the third sample in city water; this sample held out 10 days before showing traces of algae. The fourth sample was immersed in city water and a 10 per cent hypochlorite solution; it was 20 days before algae developed in this sample.

There is no question but that chlorine or chlorides will inhibit or retard the formation of algae for some length of time and also kill algae already in the water, provided it is supplied continuously to the water in allowable and effective quantities. But as soon as this supply is stopped, only for a short time, the algae will appear again under favorable conditions, even within a few hours.

In inland industries, we have to deal mostly with the blue-green or green algae (waterbloom), which are the ones held together by a gelatinous sheath. The writer has no experience with the treatment of water with direct chlorine gas, but only with hypochlorite derived from the solution of chloride of lime. By calculation, there is about 39 per cent of chlorine present, but only 28.3 per cent is active with the oxygen radical, or approximately 11 grains per 1 lb of water. The writer experimented with a 4-Twaddell solution, but the results were rather disappointing. Figs. 1*A*, 1*B*, 1*C*, and 1*D* show such a treatment with the hypochlorite and while it arrested the further growth of algae, it destroyed neither the algae nor the gelatinous film. On the contrary, it seems to fix the film. Furthermore, the writer found that hypochlorite did not destroy the free swimming cells or gametes, and amoeba are always present where there are algae.

Like in all stagnant water loaded with algae, other organisms of the higher order can be found sooner or later. The writer does not know their names or origin, but supposes that these are carried from the air into the water, and probably are attached to vegetable matters or cotton fibers. Some may be mosquito larvae, which we call wigglers.

The wigglers, which are brownish red in color and develop to about $^3/_8$ in. in length, seem to be the chief enemy of algae. They seem to be especially fond of algae and incessantly eat a great mass of it. Unfortunately, a yellowish brown amoeba attacks and kills the wigglers.

One could conclude that if enough wigglers were present in the water they would destroy the algae and all that would be necessary, before using the water, would be to strain off the wigglers. The writer has not experimented along that line but found that hypochlorite has very little effect on these bugs unless used for a long time and in a strong solution which would eventually also kill the algae.

Somewhat disappointing as it was to use hypochlorite for condensers, air washers, and process water, the writer experimented with some other methods for directly eliminating the algae in the water, since filters let as much as 1 per cent in volume of the algae through; however, a strong solution of hypochlorite eventually arrested the growth of the algae, as shown in Fig. 1*D* where it can be seen the water is fairly clear and the algae dead at the bottom, it took nearly a full-strength solution of hypochlorite. Of course, neither in condensers nor in a bleachery can such a strong solution of hypochlorite be used without affecting the metal.

The writer first experimented with coagulation. Fig. 5*A* shows filtered water, containing some algae, which was treated with alum.

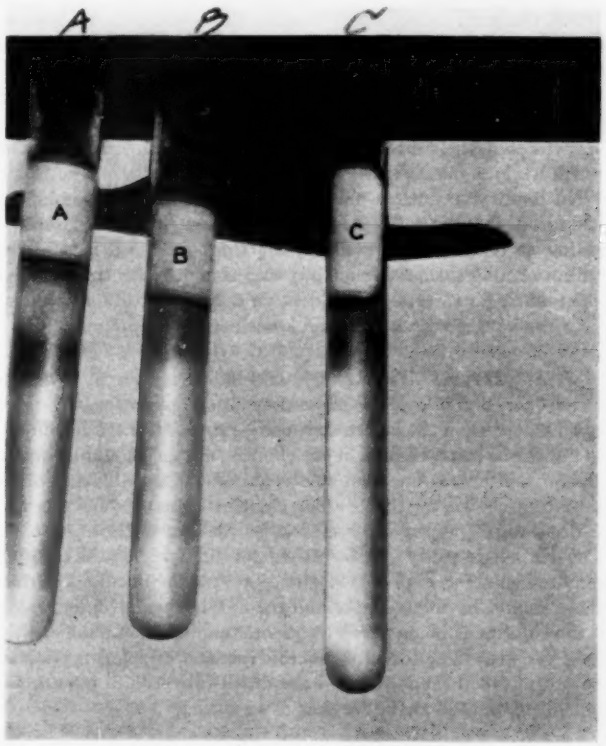


FIG. 5 FILTERED WATER, CONTAINING SOME ALGAE, TREATED WITH (*A*) ALUM, (*B*) ALUM WITH BICARBONATE OF SODA ADDED, AND (*C*) TRISODIUM PHOSPHATE AND HYPOCHLORITE

Fig. 5*B* shows the same water except bicarbonate of soda was added to the alum; the precipitate can be seen on the bottom of the test tube. The water was cleared perfectly, although it took some time.

Fig. 5*C* shows a precipitate made of trisodium phosphate (Na_3PO_4) and hypochlorite. It is this solution to which the writer would like to call the reader's attention, since it worked remarkably well and has many advantages. So far the writer tested this $Na_3PO_4 \times Ca(CLO)_2$ solution only in test tubes, but believes he has found a cheap, quick, and very effective coagulant that not only precipitates and actually kills the algae quickly, but also is noncorrosive to metal; furthermore, it precipitates all the hardness-producing salts, especially in well water, and actually softens the water. In other words, this combination of trisodium phosphate and hypochlorite not only is a strong coagulant but also a disinfectant and water softener.

Cloth samples in such treated water did not show any algae formation after 30 days of exposure. The strength of this test solution was 1 per cent Na_3PO_4 and 2 per cent $Ca(CLO)_2$ by volume.

The writer also found that this solution killed all other bacteria and insects in the water at least three times quicker than hypochlorite alone. There is only one drawback to that solution and that is that it intensified the smell of the dead algae; however, this could probably be overcome by aeration.

Of course, Na_3PO_4 has been and still is used for treatment of boiler feedwater, but the writer is not aware that it has been recommended as a coagulant and disinfectant for water in general and especially for algae extermination.

The cost is nominal—about $2^1/_2$ cents per lb for the trisodium phosphate and about 2 cents per lb for the chloride of lime. According to the strength of such a solution required, it would amount only to a few cents per 1000 gal of water.

But the main value, and the point the writer is coming to, is that it absolutely prevents any and all slime formation of the algae; also, it is practically noncorrosive and protects iron instead of attacking it. This last statement is based on the limited experimentation conducted by the writer. However, the writer believes that the prevention of slime formation alone would be worth while to justify further experimentation.

No doubt the author may already know of these materials, and has probably tried them; however, the writer would like the author to relate his experience along this line. But in case the author should not have made any experiments with the material mentioned, he certainly should be in a better position to do so than the writer, with his limited knowledge and apparatus.

W. A. Carter.[4] The writer would like to inquire if, as an aid to the operation of chlorine slime-prevention apparatus, any more rapid means than the laboratory death-rate studies can be offered to indicate when a change in the chlorine dosage should be made. There is often an unseasonable requirement for chlorine other than that shown by the test for chlorine demand and it is not realized until there is a partial loss of vacuum in the condenser. The death-rate study may be satisfactory in studying the chlorine requirements for a proposed slime-prevention installation, but there should be some quicker means of continually determining if the chlorine is being dosed frequently enough or for sufficiently long periods. The tests for chlorine demand or chlorine residual do not afford this answer, and the death-rate studies require too much time as well as the services of an expert.

Alex D. Bailey.[5] The writer would like to carry the history for this subject back beyond that which the author mentions. His attention was called to it by the paper on the treatment of water for condensing purposes with chlorine by L. L. Robinson,[6] who gave the origin of the idea and reports on tests conducted at the Hackney Electricity Works in England. About the same time a paper on the same subject was presented by H. W. Coulson,[7] who also reported on results at the Hackney Electricity Works.

As a result of this work, the engineers of the Commonwealth Edison Company, Chicago, took the matter up with representatives of Wallace & Tiernan Products and with the Electro Bleaching Gas Company. The representatives contacted did not know of the work done in England, but an agreement was made between the three organizations to conduct tests on the condenser of unit No. 3 at Northwest Station of the Commonwealth Edison Company. Twelve tubes were segregated in each of the two passes of the condenser and the water supplying these tubes was chlorinated. The test was started January 27, 1924, and continued until April 18, 1924. Although the results were quite satisfactory, it was decided to run a second test similar to the first, but using new tubes. This test was started April 20, 1924, and continued to June 21, 1924, and at that time the tubes were removed, split open, and comparisons were made with tubes using untreated water.

As a result of these tests, it was decided to treat all the water for this condenser, injecting the chlorine into the suction of the circulating pump. This test was started September 8, 1924, and ran until the end of October, the chlorine dosages varying from 0.25 to 0.45 ppm. In these tests, chlorine was fed continuously and the fact was established that the growth of slime could be prohibited, although it was not satisfactorily proved that it could be destroyed after it had formed. Because of the large amount of chlorine required and the high cost, which was then from 8 to 10 cents per lb, it was decided that the improvement would not warrant the expense of a permanent installation and its operation.

In the Transactions of the National Electric Light Association[8] there is a description of these tests, which is a part of the report of the Condenser Subcommittee of the Prime Movers Committee.

Shortly after the Crawford Station of the Commonwealth Edison Company was started, trouble was experienced with the sliming of the generator air coolers on unit No. 3, and chlorinating equipment was installed. This consisted of two chlorinators supplied by Wallace & Tiernan, one of which was of the vacuum type, intermittent feed; the other was of the old-style continuous feed. This equipment went into service August 16, 1926. The intermittent machine, probably the first of its type, proved unsatisfactory. It was subsequently rebuilt and put in service January 31, 1927, after which tests were continued until August of that year.

About this time, Dr. Enslow of the Chlorine Institute was contacted and he became quite interested in the results. On June 4, 1928, he came to Chicago and a meeting was held with representatives of Wallace & Tiernan, Electro Bleaching Gas, and the Commonwealth Edison Company. At this meeting it was agreed that the chlorination of all the circulating water for a large condenser should again be attempted using the intermittent process as this would reduce the amount of chlorine required. In the meantime the cost of chlorine had been considerably reduced. A very careful record of the performance of condensers for No. 1 unit at Crawford Station using raw circulating water was maintained for the balance of 1928, and arrangements were made to install intermittent chlorinating equipment on these condensers to be ready for service about the beginning of 1929. Those tests were successful and led to the permanent installation of chlorinating equipment on additional units.

M. S. Maurer.[9] Several parts of the author's paper express ideas which seem contrary to the writer's experience.

In the minimal chlorine death-rate studies, it appears that saline suspensions were used for the bacteria. These studies would seem to be of little practical value as they do not show the effect of the presence of organic matter. This is of major importance, for when the chlorine is added to a circulating water, it reacts with the organic matter first before acting on the bacteria.

It was shown by the studies in connection with the use of chloramines at Williamsburgh, that unless the organisms were

[4] Technical Engineer of Power Plants, The Detroit Edison Company, Detroit, Mich. Mem. A.S.M.E.

[5] Chief Operating Engineer, Commonwealth Edison Company, Chicago, Ill. Fellow A.S.M.E.

[6] "Chemical Treatment of Condensing Water," *Electrical Times*, June 22, 1922, p. 607.

[7] "A New Use for Chlorine," by H. W. Coulson, a paper delivered before a meeting of The Institution of Mechanical Engineers and the Chemical Engineering Group of the Society of Chemical Industry and later published in *Chemical Trade Journal* and *Chemical Engineer*.

[8] Vol. 83, 1926, p. 1009.

[9] Williamsburgh Power Plant Corporation, Brooklyn, N. Y.

freshly isolated and suspended in water taken from the circulating system, little if anything was accomplished by the laboratory work. It is well known that changes in the organic matter caused by bacterial growth will take place which affect the chlorine demand and therefore the germicidal efficiency. Thus, the time elapsing between taking the samples and running the tests is of great importance.

With regard to the ideas expressed by the author as to the formation of chloramines by the reaction of chlorine with ammonia occurring naturally in water, it appears to be generally recognized that chloramines are not formed from natural ammonia. We have examined a number of waters on the eastern seaboard in which the natural-ammonia content as determined by both direct Nesslerization and distillation was more than sufficient to form chloramines and in every instance, when an ammonium salt was added to the water, the chlorine demand was lowered. This is direct proof that the natural ammonia in these cases, did not form chloramines. It is true that chlorine will act more rapidly, in water free of organic matter, than chloramines. This property is lost, however, when the water contains sufficient organic matter to support the growth of bacteria in large numbers, as is the case in slime-forming waters. In this instance, chlorine reacts more rapidly with the organic matter than with the bacteria and a smaller amount of chlorine remains available for bactericidal action than where chloramines are used.

The author states that the formation of chloramines is of advantage in waters of abnormally high organic chlorine demand as it retards the combination of chlorine with the organic matter, thus requiring a lower quantity of chlorine to obtain a given residual. If this be true of high-organic-demand water then the same advantage must hold true in waters which have any organic demand whatever. As previously stated, all slime-forming waters have an organic content, and actual practice demonstrates that chloramines do have this definite advantage over straight chlorine in slime-forming waters.

In cases where the natural ammonia is high, unless an ammonium compound is added or in other words, a true chloramine treatment used, chlorine is at a decided disadvantage as the chlorine demand will probably be so high as to make a straight-chlorine treatment uneconomical.

The large generating station in the East referred to is the Williamsburgh plant of The Brooklyn Manhattan Transit Company. The author is entirely wrong in his statement as to the water conditions at this plant. The natural ammonia content is not low as stated, but high, and would be more than sufficient to form chloramines if they could be formed from natural ammonia. During the period when the author's company was trying out a straight-chlorine treatment at Williamsburgh, it was impossible to obtain slime control. However, by the addition of ammonium sulphate to this same water, the chlorine demand has been lowered and satisfactory slime control obtained. It would be interesting to know what reason can be ascribed as to why it is necessary to add ammonium salts at this plant, to get a chloramine treatment when the natural ammonia content is high.

In the cases cited in the Middle West and a small plant in the East, where the natural ammonia content is so high that excessive quantities of chlorine would be necessary to satisfy the demand due to the formation of chloramines, the writer is quite sure that the author is mistaken in the statement that chloramines would be formed, because if they were it would be unnecessary to add sufficient chlorine to combine with all the ammonia and only enough would need be introduced to form chloramines in amounts to give the desired germicidal action.

In waters containing natural ammonia there is usually an organic demand. It would seem the cases mentioned should be particularly adapted to the use of chloramines and were an ammonium compound to be added to any of these waters, the chlorine demand would drop and a more efficient bactericidal treatment obtained than if sufficient chlorine were added to satisfy the demand and give a germicidal residual.

With regard to the question of intermittent treatment, as interpreted in the past by the author's company, which necessitates complicated and expensive program equipment, it has been demonstrated by four large companies here in the East, namely, The Brooklyn Manhattan Transit Company, The Brooklyn Edison, The Interboro Rapid Transit, and the Public Service of New Jersey, that a half-hour dosage, once in 24 hr, is decidedly superior to short frequently repeated dosages. This is in accord with general bactericidal practice, the reasons for which are well known and it seems to be doubtful whether in any case the short frequent dosage would be preferable.

The mechanism of using a bactericidal treatment for slime control with the condenser in service is to use the circulating water as a carrier for the germicide which is to act on the organic growths on the tube surfaces. In the case of chlorine or allied chlorine compounds, the character of the water influences the amount of germicide available to kill the slime-forming organisms on the tubes. This germicidal residual is ordinarily determined with orthotolidin, and the orthotolidin is added immediately when the sample is taken. This immediate residual is not necessarily the germicidal residual available to kill the bacteria. At least 5 or preferably 10 minutes should elapse before the orthotolidin is added. The reason for this is that chlorine reacts first with the organic matter before reacting with the bacteria. This also applies to the organic matter in the layer of slime as well as the organic matter in the water. The sequence of events is the chlorine or chlorine compound first reacts with the organic matter in the water, then with any bacteria in the water and any residual remaining will then act first on the organic matter in the slime on the tube and last on the organisms in the slime layer. This is why the immediate residual has little significance and also one of the reasons for the half-hour dosage.

However, the most important thing in which we are interested is the slime layer on the tubes. This slime is very high in organic matter compared to the water from which it originates, and, as the author states, chloramines are superior to chlorine in the presence of high organic demand; it follows, due to this fact that chloramines will be advantageous over chlorine in acting on the actual layer of slime. This difference in action on organic matter results from chloramines being a much weaker oxidizing agent than chlorine.

Any water exhibiting an organic chlorine demand and which will form slime can be regarded as polluted. It is stated in the paper that chloramines are much slower in their action in slime control than chlorine, yet it seems to be generally recognized that in polluted water this is not so and the two are approximately equal in speed of killing in the presence of the organic matter, which is always present in slime-forming circulating water. It is difficult to reconcile some of the ideas the author expresses about chloramines with what has been published by his company on this same subject. For example, in Wallace and Tiernan's[10] Technical Publication No. 11 it is pointed out that the germicidal value of chloramine is almost three times that of chlorine. Further advantages of chloramine over chlorine are shown in Wallace and Tiernan's[10] Research Publication No. 327 where it is also pointed out that the use of chloramine is more specific than straight chlorine and that Wallace and Tiernan[10] found chloramines a great improvement over straight chlorine when used on slime-forming bacteria; also in acting on slime-

[10] Wallace and Tiernan Products, Inc., Belleville, N. J.

forming bacteria, chloramines require only one fourth the dose that chlorine required for the same length of contact and that, for the same dosage as chlorine, chloramines required only one fourth the period of contact.

This same publication also points out that the difference between chlorine and chloramine is that chlorine is a strong oxidizing agent and chloramine a very weak oxidizing agent; therefore, when bacteria are to be killed in the presence of a chlorine demand, the use of chloramines results in a larger proportion being available as a germicide than is the case with straight chlorine.

Further advantages of chloramine over chlorine are also shown in Wallace and Tiernan's[10] Research Publication No. 305.

The use of chloramines for the elimination of slime, at Williamsburgh and several other places, has indicated that they have the advantage over chlorine as stated in the Wallace and Tiernan[10] publications and that the ideas expressed by the author do not seem to be in accord with the results obtained.

Author's Closure

Mr. Aldinger has presented interesting details of a particular water treatment, in the handling of a type of problem common in the process industries. In the paper presented in December, 1937, the author was dealing with but one phase of the broad subject of water treatment, only that involved in the chlorination of very large volumes of cooling-water flow, as contrasted with the relatively smaller volumes of water that require sterilization in the recirculated spray pond or cooling-water systems, or in industrial-process water plants.

As was pointed out in the opening remarks of the paper, the present-day industrial uses of chlorination are founded on the accomplishments of chlorine in the field of drinking-water sterilization. An extensive review of the literature, including very old and more recent United States and foreign patents, discloses innumerable applications of a wide variety of chemicals and of special mechanical devices, having as objectives a method of bacteriological treatment that would prove to be reliable under any one of the many conditions common to the water-treatment field. These many alternate methods have not advanced into the realm of practical application, since in each case one or more important features have been found lacking, that have made chlorination so universally utilized at the present time. These features include availability of the chemical in convenient form, the extremely low cost now characteristic of chlorine, the development and advance in design of reliable proportioning and control apparatus, and the simple tests available for checking satisfactory chlorinator operation.

At the present time, in addition to the practically uniform use of chlorine in sterilization of drinking-water supplies, chlorination is also in general use in the smaller cooling-water plants, and in industrial-process water treatments. These smaller applications involve continuous rather than intermittent use of either chlorine or chloramines.

It has been the author's experience that slime growths are equally as troublesome, if not more troublesome in the smaller power and process water plants, and represent proportionately greater economic losses. It is true however that less frequently do these losses reach the same magnitude.

Wide experience is valuable in deciding upon the best and most economical method of accomplishing the desired result, without the risk of adverse reactions in the water-treatment plant, or in subsequent process steps, and in deciding whether continuous or intermittent applications of chlorine or of chloramines can best be utilized. The selection of type of treatment is dependent upon the nature of the contaminating organisms, the physical details of the treatment plant and the water distribution, and upon the nature of the process in which these treated waters are to be used.

Mr. Carter has inquired for a more rapid means by which it may be determined that the slime-prevention treatment is under proper control. Coincidentally with presentation of the original paper in December, 1937, experiments were undertaken and developments are now well under way, which involve electrometric means of analysis of water and slime-forming tendencies. It is proposed to report these developments more fully at a later date.

Mr. Bailey has called attention to an error on the part of the author, in failing through oversight to include in the paper presented in December, 1937, details of the Chicago experiments, a report of which had appeared in 1926 in the Transactions of the National Electric Light Association. This omission was corrected in an expanded version of the paper, which has received widespread distribution in reprint form. The author welcomes this opportunity to make a further apology.

Mr. Maurer's discussion is based on limited experience, and it is readily understandable that conclusions such as expressed might be reached without having a sufficiently broad background to permit of judging the relative merits of chlorine and of chloramines for a specific application. These conclusions fail to take into consideration basic differences in results obtained by these sterilization methods, as they may be made use of in drinking-water treatments, in the general industrial-process and paper-mill water-treatment field, as against the more specific applications in the condenser-cooling-water treatment.

Careful reading and unclouded interpretation of the original paper will serve to answer the questions raised. As experience is extended, the observations made in the paper will be confirmed, and a better understanding will be obtained of the various factors which must be taken into consideration in deciding in what types of applications the continuous or intermittent uses of chlorine or chloramines may prove to be most advantageous.

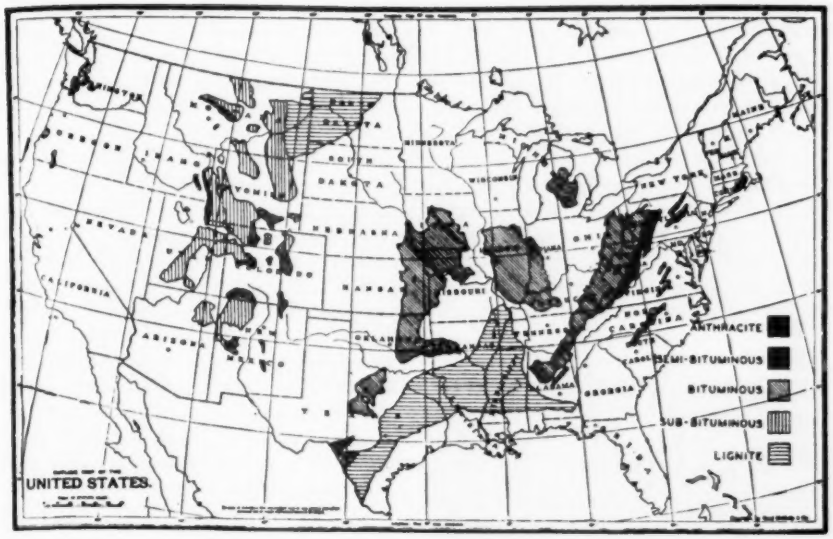


FIG. 1 COAL FIELDS OF THE UNITED STATES

Lignite—a Grate-Fired Fuel of the Future in the Southwest

BY CARL J. ECKHARDT, JR.,[1] AUSTIN, TEXAS

The paper describes the development of methods and apparatus for the practical use of Texas lignite as a grate-fired fuel. After presenting information on the extent and character of the deposits, and the constituents and characteristics of the fuel, the author describes early attempts at using it on hand-fired grates and follows the development through the overfeed-stoker stage to the present type of traveling-grate stoker using forced-draft and special furnace construction. A comparative table of heat balances indicates the improvement made. The author then discusses operating experience on boiler units with capacities up to 75,000 lb of steam per hr which give efficiencies as high as 76 per cent.

A résumé is also given of information on the firing of pulverized lignite, and increasing use of this fuel is predicted not only for grate and pulverized firing but also as a source for liquid and gaseous fuels.

THE FUEL reserves of the Southwest are divided by their constitution and characteristics into two distinct categories. One group is composed of the liquid and gaseous fuels, which can be used effectively at points far removed from their source as well as within the producing areas. The second group is composed of the solid fuels. A considerable portion of the Southwest's solid-fuel reserve possesses characteristics which limit its use to regions in the proximity of the mines.

The status of such states as Louisiana, Arkansas, Oklahoma, and Texas, with regard to the production of fuels of the first class, is well established because this section's distinction as a fuel-producing area is characterized by its prolific production of natural gas and petroleum products. The Southwest has produced 58 per cent of the natural gas consumed by the nation and 63.8 per cent of its crude oil for the last decade. The solid fuels of the area under consideration are as a general rule not superior to comparable solid fuels in other sections of the country. The region can, moreover, claim no great distinction for the magnitude of the production of this type of fuel at the present time. The failure to exploit its low-rank solid-fuel reserve to a greater extent must be attributed to the traditional regard of an area, within which fuel abounds, for fuels of a high rank and its disregard for fuels of the lower rank.

The prolific production of natural gas and petroleum products of the region is not being occasioned by the industrial development of the Southwest alone. On the contrary a great portion of these admirable fuels is being consumed in other sections of the country. Although the region is making a liberal contribution to the solution of problems confronting consumers of fuels in other sections of the country, it has made a comparatively small effort to develop processes for the utilization of vast amounts of potential energy which by their very nature are destined to be consumed in the area in which they are found. For the foregoing reason the low-rank solid fuels, which abound in this section of the United States, have assumed no position of importance in current industrial programs.

The Southwest produces a host of raw products of a vital order in prodigious quantities. Because artificial barriers have made necessary the processing of such products in other sections of the

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[1] Professor of Mechanical Engineering, Superintendent of Utilities, University of Texas. Mem. A.S.M.E.

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NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

country, the industrial development of the area has been held in abeyance. In the interim both fuels and raw materials in great abundance are being transported to other sections. Inasmuch as the deterrents are not warranted by natural circumstances, the Southwest will, when it liberates itself from these deleterious influences, resort to a much greater extent to the use of its low-rank solid fuels. Then such fuels as lignite, which must be consumed in the producing area, will assume a position of vital importance because industrial enterprises may be located directly upon and near the lignite beds where the use of this fuel will be prompted by economic expediency. The wisdom of achieving the fulfillment of such a plan is sound beyond any question of doubt. The time of its actual fulfillment remains a matter of conjecture.



FIG. 2 VIEW OF LIGNITE BED

More than 23,000,000,000 tons of lignite are deposited in Texas alone. The term lignite is generally used to denote "those coals which are distinctly brown and either woody or claylike in their appearance." In rank this material lies between peat and subbituminous coal. Although the biochemical action of the accumulation period of the formation of this fuel was quite complete, the absence of great thrust pressures in the course of the coalification period limited both dehydration and devolatilization processes to such an extent that it has remained a coal of low rank.

The depth of the lignite beds now being exploited varies from 15 to 160 ft. In many instances such factors as shallow overburdens, subterranean watercourses underlying the lignite beds, and the almost complete absence of rock make strip-mining methods expedient. At single mines as much as 65,000,000 tons of this material may be mined by such methods at depths ranging from 15 to 45 ft. For depths greater than 60 feet shaft-mining methods are generally employed. Such lignite beds, which are lenticular in character, vary from a few inches to as much as 16 ft in thickness.

The lignite which is now being mined possesses a claylike rather than a woody appearance and it has as its principal identifying characteristic a comparatively high moisture content. The magnitude of this moisture content "as mined" approximates 35 per cent. Its principal accompanying constituents are: Carbon, 40 per cent; hydrogen, 3 per cent; sulphur, 0.7 per cent; and ash, 9 per cent, all expressed on an "as mined" basis. Its heating value on the same basis varies from 7000 to 7500 Btu per lb.

Despite frequent reports to the contrary, lignite can be successfully burned as a grate-fired fuel. In the past many efforts have been made to burn this material upon grates of various types and designs. The reports of the results achieved have quite generally indicated that little, if any, success attended these efforts. Unfortunate indeed is the fact that almost without exception the fuel rather than the designing agencies was maligned because of such failures. Practically all unfortunate experiences of this kind should have been attributed to misapplication of equipment and improprieties in design. Instead such failures were believed to be due to the high moisture content of the fuel, to its comminution while in storage, to its disintegration while burning, and to its other burning characteristics.

The University of Texas in seeking the fulfillment of one of its fundamental purposes undertook, therefore, the task of making an investigation of the problems involved in the utilization of this fuel. Inasmuch as commercial agencies having means of loading larger units were in a better position to conduct research work related to the use of lignite in a pulverized form, the investigation has thus far been limited to grate-firing problems. The information which follows has as its background the investigations conducted at The University of Texas.

The water losses in the case of such lignitic fuels are by no means as great as they are quite generally supposed to be. Although the moisture content approximates 35 per cent, the actual heat loss due to moisture in the fuel varies from 5 to 6 per cent in the case of appropriately designed units. The heat loss due to the formation of water from the burning of the hydrogen in such units varies from 4 to 5 per cent and the heat loss due to moisture in the air approximates 0.3 per cent. The foregoing information indicates that water losses in the case of lignite render only approximately 10.5 per cent of its heating potential ineffectual. Although this aggregate loss is materially higher than that experienced in the case of higher-rank solid fuels and it is somewhat higher than that of fuel oil, the actual water losses in

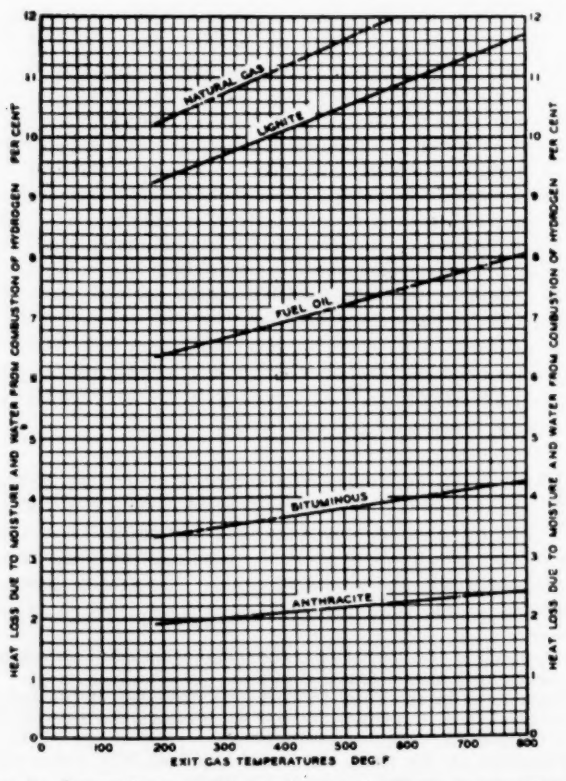


FIG. 3 CHART SHOWING WATER LOSSES INVOLVED IN THE USE OF TYPICAL LIQUID, GASEOUS, AND SOLID FUELS FOR A GIVEN ENTERING AIR TEMPERATURE

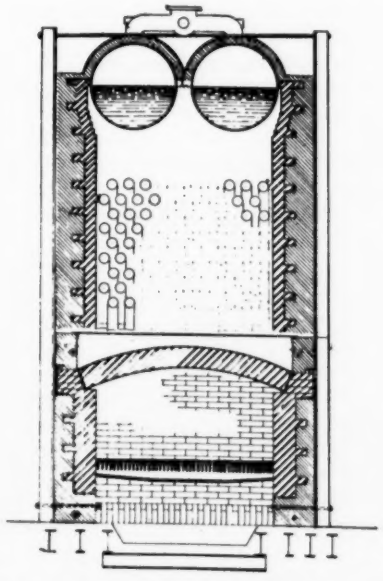


FIG. 4 VIEW OF SIMPLE LIGNITE-BURNING STEAM-GENERATING SYSTEM

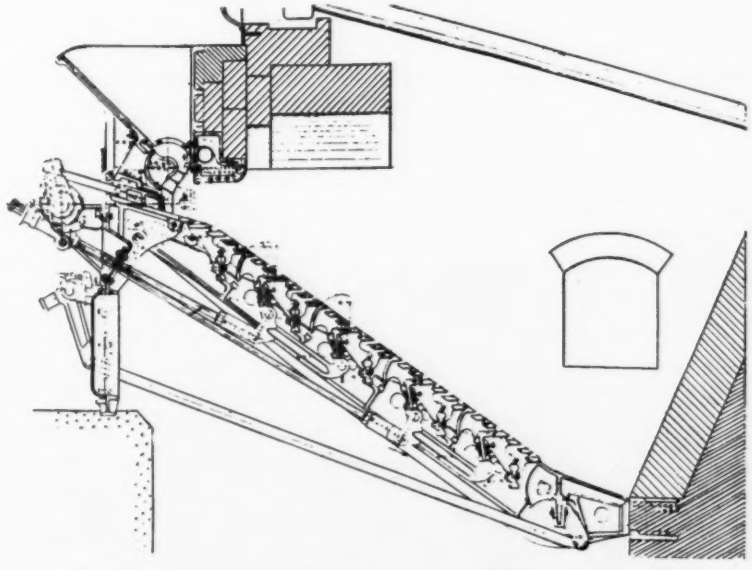


FIG. 5 VIEW OF LIGNITE-BURNING STEAM-GENERATING SYSTEM INVOLVING THE USE OF FRONT-OVERFEED TYPE OF STOKER

the case of lignite are not greater than comparable losses in the case of such an admirable fuel as natural gas.

The greatest opportunity for the reduction of water losses lies in dehydrating the lignite prior to its introduction into the furnace and in reducing exit gas temperatures to an economical minimum. Although satisfactory dehydration processes have been developed, at the prevailing fuel prices in the region under consideration, they are not commercially feasible. This fuel has, as a matter of fact, suffered its greatest disadvantages in the province of carbon and dry-gas losses. For that reason, in the consideration which follows, particular emphasis will be placed upon heat losses due to unconsumed gaseous combustible, unconsumed carbon in refuse, and dry chimney gases.

Original efforts made to burn lignite were made with a boiler, furnace, and grate possessed of greatest simplicity. Inasmuch as the lignite possessed a high moisture content, the ignition of this fuel was assumed to present a serious problem. Extended-front-type furnaces were employed in order that ignition arches might be used to effect the ignition.

Natural draft made mandatory the use of grates having liberally sized openings for the passage of air. The furnaces were arranged with such limited proportions that boiler heating surfaces were placed in the immediate proximity of the fuel bed itself. The difficulties actually experienced in operating such units might naturally have been expected by the designing agencies.

Improper control of the air supply resulted in the introduction of excessive quantities of air and in excessively high exit gas temperatures. The gases entered the first pass of the boiler in a stratified condition with zones of high oxygen and low carbon-dioxide content, and adjacent zones of low oxygen and high carbon-monoxide content. The use of a unidirectional forward flow of the gaseous products of combustion precluded the possibility of securing any degree of turbulence in furnaces which were too small to afford a reasonable period of time for the completion of the combustion process. The use of large air openings in the grates made unavoidable deleterious sifting losses.

Later efforts to eliminate desultory sifting losses employed the use of both chain- and traveling-grate stokers under natural-draft conditions. The arches continued to be used at the front of the furnace, no turbulence was provided, and furnaces were made entirely too small. Such units were frequently found to be unsatisfactory because of their inability to carry heavy loads and their incapability of responding appropriately to highly variable loads. The deficiencies of such units might well have been attributed to the failure of designing agencies to make due provision for those elements which are known to be absolute requisites for the completeness of the combustion process.

FRONT-OVERFEED STOKER

These experiences prompted the use of a different type of fuel-burning system which might be expected to eliminate serious sifting losses experienced in the case of the earliest units and at the same time to eliminate the restrictions of rapid responses to variations in load. For this purpose a front-overfeed type of stoker was next applied. The use of a grate surface which was inclined from the front to the rear of the furnace made possible the use of air openings placed in a horizontal rather than vertical plane. Natural draft was again used, but the size of the air openings was increased and the front arch was made materially shorter. Operating experience soon demonstrated the fact that in the use of this equipment the solution of old problems was accompanied by the introduction of new ones.

The introduction of air to the fuel bed in a horizontal plane to a free-burning fuel reposing upon a rather steep inclination, together with the oscillation of the grate bars, caused avalanching difficulties at moderate and comparatively high loads. The agitation of the fuel bed caused clinkering difficulties. Fractional loads could not be carried satisfactorily because of the absence of an appropriate control of the air supplied for combustion. Despite these deficiencies an improvement in performance over earlier units was effected. It is by no means improbable that by modifying the design of this system the magnitude of the deficiencies and difficulties experienced might have been rendered less potent.

TRAVELING-GRATE STOKERS

As the necessity for the design of larger units arose, the use of mechanical means for providing the air required for combustion

and for removing the products of combustion became imperative. By this circumstance the objectionable influence in the form of resistance to flow of primary air through numerous small air openings in the grate surface was overcome. In order that this free-burning lignite might be fired by a fuel-burning mechanism which provided a continuous undisturbed fuel bed from which the refuse disposition might be accomplished without cyclic fuel-bed disturbances, traveling-grate stokers were next applied. The multiplicity of the compartments through which the passage of the primary air to the various sections of the grate surface could be controlled not only afforded an opportunity for the reduction of dry-gas losses but also made feasible the use of perheated air.

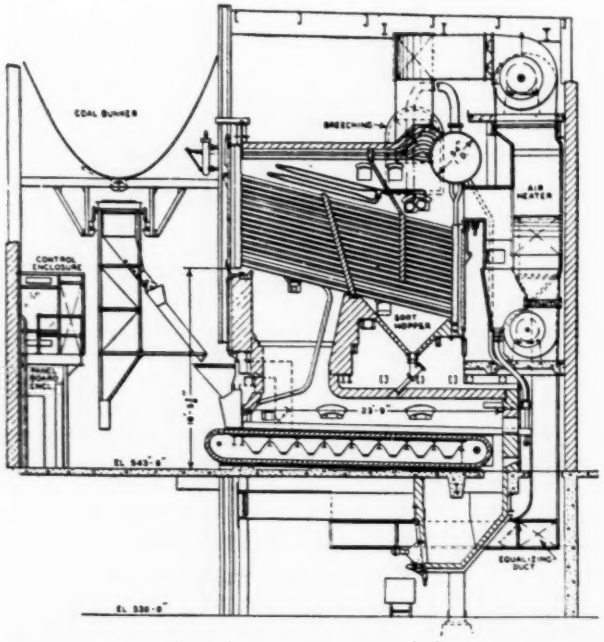


FIG. 6 VIEW OF LIGNITE-BURNING STEAM-GENERATING SYSTEM INVOLVING THE USE OF TRAVELING-GRATE STOKER AND MECHANICAL DRAFT

The principal departure from early designs in the case of the latest units consists of the almost complete abandonment of the front arch and the introduction of the use of a long low-hanging rear arch. By virtue of the fact that the ignition of lignite can in reality be accomplished with consummate ease, the elimination of the front arch involved no sacrifice in this respect and it materially reduced stratification tendencies.

With this type of design ignition is not only initiated, but its stable continuity is assured by the passage of hot gases from the rear of the furnace toward the front, and by the deposition of fuel particles partially burned in suspension as they are carried forward by the gas stream.

The reversal of the direction of flow of the gas stream reduced ashpit losses previously encountered with units having unidirectional backward flow of the products of combustion. The furnace arrangement is, moreover, such that the velocity of the lean gases flowing forward over the fuel bed is sufficient to create a desirable turbulence at the front of the furnace at the point at which the rich gases arise from the fuel just introduced. Inasmuch as the rear arch covers approximately 60 per cent of the grate surface, when the load on the unit is increased the amount of air admitted through the grate surface under the arch is likewise increased. A corresponding increase in the velocity of the forward-flowing gases intensifies the violence of the scrubbing action by which the rich and lean gases are mixed.

For high capacities air injection over the fuel bed is accomplished through the front furnace wall. For the reason just given, the turbulence just below the throat of the furnace is decreased as the capacity at which the unit is operated is reduced. The use of air admitted at the front of the furnace over the fuel bed is advisable at very light as well as very heavy loads. Just below the throat of the furnace the forward flow of the lean gases, the upward flow of the rich gases, and the backward flow of the secondary air admitted through the front furnace wall produce a scrubbing gaseous cylindroid. The height of the heat-absorbing surfaces above this cylindroid is sufficient to afford time for the completion of the combustion process. The heavier fuel particles entrained in the gas stream are thrown out onto the fuel bed and the length of travel of the lighter fuel particles is sufficient to assure the complete combustion of their combustible constituents before they are cooled to a temperature at which combustion is no longer supported. The carbon loss into the system beyond the furnace is for that reason of a negligible order. Table 1 shows heat balances for the three types of firing described.

PERFORMANCE

Five years of operating experience with units capable of generating from 30,000 to 75,000 lb of steam per hr has demonstrated that such lignite-burning steam-generating systems can be operated with remarkably high availability factors. Units of this design can be operated at the maximum capacities for

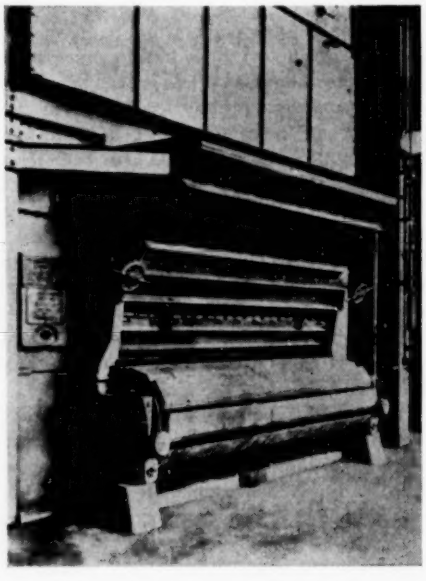


FIG. 7 VIEW OF TRAVELING-GRATE STOKER SHOWING MECHANISM BY MEANS OF WHICH FUEL-BED THICKNESS CAN BE VARIED

which they are designed with efficiencies as high as 75 per cent. Repetitive tests have demonstrated that under optimum conditions efficiencies as high as 76 per cent can be achieved. Although the air supplied for combustion is preheated to temperatures of from 250 to 375 F, in the 5-year period mentioned not a single stoker part has been burned out. In a like manner little difficulty has been experienced with solid refractory walls or with arches of refractory tile. Although from 1 to 12 per cent of the ash passes into the combustion loop, little difficulty has been experienced due to this circumstance. Frequent lancing of the narrow passages of the air preheaters has been found to be

TABLE 1 HEAT BALANCES FOR LIGNITE-BURNING STEAM-GENERATING SYSTEMS

	Stationary hand-fired grates		Front-overfeed stoker, natural draft		Traveling-grate stoker, mechanical draft	
	Btu per lb	Per cent	Btu per lb	Per cent	Btu per lb	Per cent
Calorific value of fuel as received	7066	100.0	7205	100.0	7475	100.0
Absorbed by water and steam	3715	52.6	4683	65.0	5681	76.0
Loss due to moisture in lignite	457	6.5	433	6.0	419	5.6
Loss due to water from hydrogen	301	4.2	295	4.1	302	4.0
Loss due to moisture in air	..	..	29	0.4	15	0.2
Loss due to dry chimney gases	..	..	1117	15.5	577	7.8
Loss due to unburned gases	..	..	0	0	0	0
Loss due to unburned carbon in ash	422	6.0	324	4.5	181	2.4
Unaccounted-for losses	2171	30.7	324	4.5	300	4.0

expedient. The use of side and scroll liners in induced-draft fans has eliminated difficulties which might otherwise have been experienced due to the abrasive action of the fly-ash particles which pass through the fan in the absence of a fly-ash precipitator.

Although it has been stated that, "Large quantities of unburned lignite drop through the grate surface even though the openings are very small," investigations conducted indicate that such statements may not be made with regard to appropriately designed systems. The actual amount of solid material passing through the grates of traveling-grate stokers amounts to approximately 0.1 per cent of the fuel fired. Inasmuch as a considerable portion of the material which sifts through the grates burns in the compartments and imparts its heat to the air passing to the grate surface, the actual heat loss is considerably less than 0.1 per cent of the heating potential of the lignite. It may, therefore, be neglected entirely without producing any appreciable difference in the performance as it is customarily computed.

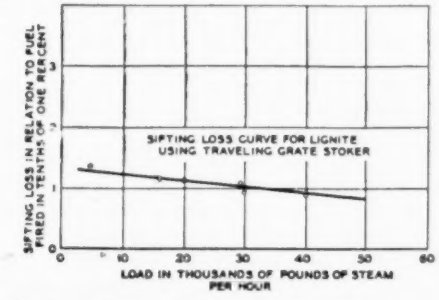


FIG. 8 CHART SHOWING AMOUNT OF LIGNITE AND ASH WHICH SIFTS THROUGH GRATE SURFACE OF APPROPRIATELY DESIGNED SYSTEMS

The foregoing information indicates that a factor which has frequently been regarded as possessing prohibitive importance can in reality be reduced to absolute insignificance.

As the combustion rate of such steam-generating systems is varied, both the amount of fuel introduced and the thickness of the fuel bed may be varied in a like manner. At the same time the amount of air supplied and its point of introduction may be changed in a corresponding manner throughout a comparatively wide range of operating conditions. Because of the speed and accuracy with which these variable factors can be

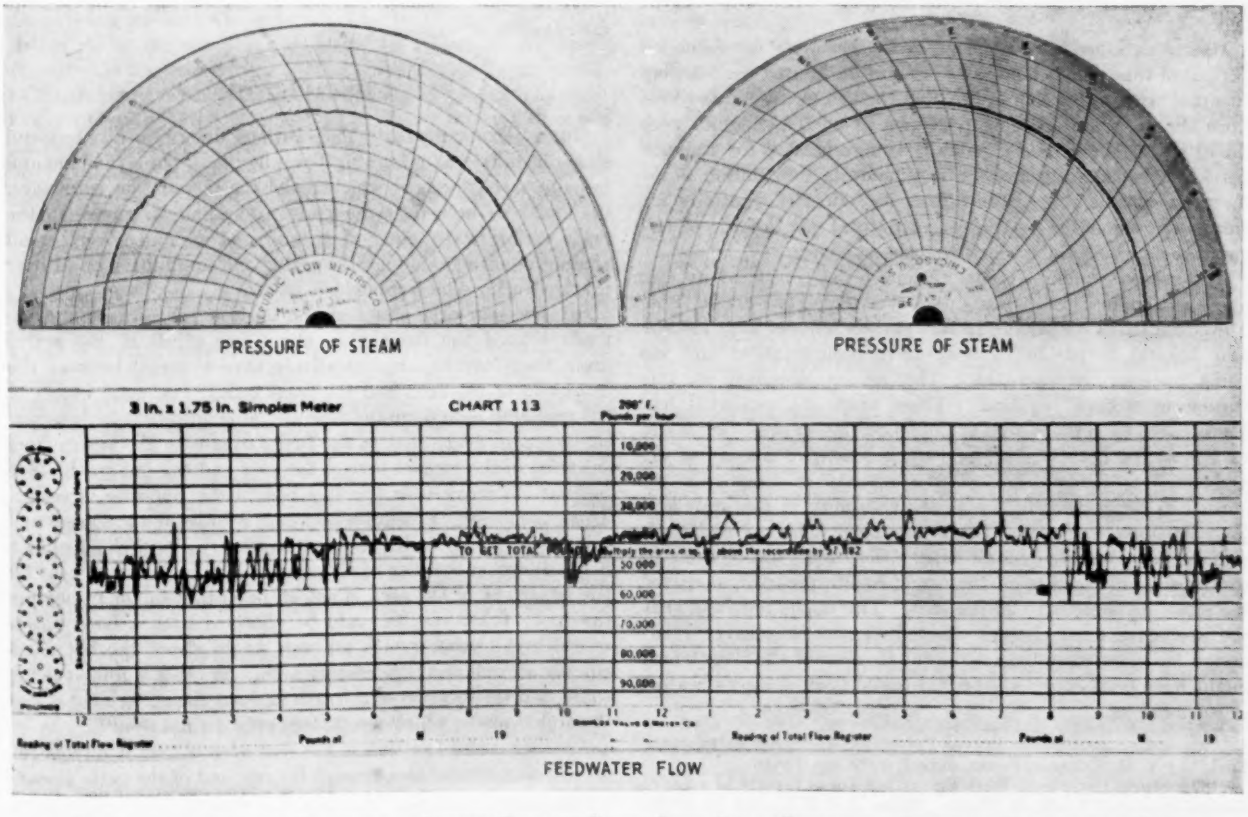


FIG. 9 CHART SHOWING RESPONSE OF LIGNITE-BURNING GRATE-FIRED STEAM-GENERATING SYSTEM TO VARIATIONS IN LOAD

controlled, there can no longer be any justification for the belief that lignite is an unsatisfactory grate-fired fuel because steam-generating systems, deriving their energy from this source, cannot be operated at high capacities and are incapable of responding to highly variable loads.

It has been said of lignite that, "With improper draft conditions holes will be formed in the fuel bed which will cause bad distribution of the air and poor combustion." This statement is true not of lignite alone but of all other solid fuels as well. There can be no justification for such a statement with regard to properly designed grate surfaces. In the case of the traveling-grate stokers the magnitude and arrangement of the air openings can be carefully fixed in the selection of the grate keys. In burning lignite on such fuel-burning equipment no difficulty of the nature mentioned need be experienced at all.

Pulverized Firing

In the foregoing part of the paper only grate-fired units of small and moderate sizes have been discussed. The purpose of the discussion has been to indicate that lignite can be satisfactorily burned on a grate surface. It should not be inferred that lignite can only be used to advantage in small units or as a grate-fired fuel. This paper has been limited to a discussion of grate-fired installations only because reliable information has already been published with regard to the use of this fuel in pulverized form. A brief review of two such papers will call to the reader's attention the source of such information. Early efforts to burn Texas lignite in pulverized form on a commercial scale were made at Comal Station of San Antonio Public Service Company where three steam-generating systems each capable of generating approximately 225,000 lb of steam per hr were installed. The reports[2] indicate that satisfactory results were obtained. These systems involved the use of forced- and induced-draft fans and recuperative air preheaters of the plate type.

Operating efficiencies as high as 79 per cent for extended periods of time and as high as 81 per cent for limited periods were reported when using lignite predried to such an extent as to reduce the moisture content to 29 per cent. These units were found to be reliable and capable of being operated for 4-month periods between inspections. Inasmuch as this station is removed a considerable distance from its source of lignite, it has since been converted to the use of natural gas because of circumstances which did not involve fuel cost alone.

The greatest amount of lignite now being consumed by any single station in Texas is being used at Trinidad Station of Texas Power & Light Company. In that station units of both 185,000 and 325,000 lb per hr capacity have been installed and are being successfully operated. The results obtained at this station have been reported.[3] These units are somewhat less complicated than those used at Comal Station in that they resort to the use of natural draft and do not involve the use of air preheaters or economizers. This simplicity in design was regarded as being contributive to the reliability of the units and their low operating cost. The performance achieved with such units involves yearly over-all boiler efficiencies ranging from 76 to 77 per cent based upon the use of lignite predried to reduce the moisture content to 26 per cent. The report indicates that the saving in fixed charges effected by the simplicity of the design has been more than sufficient to offset the savings which might have been expected from the installation of air preheaters or economizers.

[2] "Operating Experience Proves Pulverized Lignite a Satisfactory Fuel," by V. H. Braunig, *Power*, July 2, 1929, pp. 13–16.

[3] "Operating Experience With Pulverized Texas Lignite in a Large Central Station," by N. G. Hardy, Trans. A.S.M.E., vol. 58, 1936, paper FSP-58-4, pp. 267–275.

Fundamentally the use of lignite does not involve materially greater complications than other fuels of a solid nature. That systems of pulverized-fuel firing can be applied effectively in the case of large steam-generating systems has been definitely demonstrated. In the case of small units, the application of stokers seems advisable. Such stoker-fired units are capable of being operated in an entirely satisfactory manner. In the case of small units, the improvements in performance made possible by pulverization are generally more than offset by the preparation costs and the additional fixed charges resulting from a higher initial capital investment for preparation equipment. It is not impossible that, when the use of lignite becomes widespread, effective low-cost systems for pulverizing and firing lignite in small units may be developed.

Just cause for alarm does not exist in the Southwest because of any pessimistic outlook regarding either the exhaustion of its tremendous sources of potential energy, the classes of fuel previously mentioned, or for that matter any serious diminution of its bountiful natural resources. Cause for alarm does, however, exist because of the ruthless methods by which amounts of the more admirable fuels, almost beyond human comprehension, are being extravagantly wasted and because of the utter disregard of the area for its fuels of a lower order. In order to extend the benefits derived from the use of the more admirable fuels as far into the future as possible, definite steps must be taken to reduce wasteful practices which involve the better grades of fuel and to increase the amount of the lower-grade fuels consumed. The period of sufficiency of both liquid and gaseous fuels is assumed to be materially shorter than that of solid fuels. Approximately half of the nation's solid-fuel reserve consists of subbituminous coal and lignite. As the period approaches, in which these low-rank fuels will begin to assume a serious importance, lignite will not only be used as a grate- and pulverized-fired fuel, but it will, no doubt, likewise be used in the manufacture of liquid and gaseous fuels.

Discussion

Henry Kreisinger.[4] The author describes a successful stoker installation for burning Texas lignite. Three fundamental principles are involved which make the installation successful: (*a*) long rear arch over the grate; (*b*) small air spaces in the grate giving nearly equal distribution of air and a very small amount of siftings through the grate; and (*c*) addition of air over the fire for burning combustible gases.

(*a*) The rear arch starts at the back of the furnace and extends toward the front over about two thirds of the active grate area, leaving a comparatively narrow throat between the front end of the arch and the front wall. The main object of the rear arch is to force the products of combustion to flow over the incoming fuel; that is, the fuel moves with the stoker from the front of the furnace toward the rear and the hot products of combustion move over the fuel bed in the opposite direction. Thus, the incoming fresh fuel is swept over by the hot gases and is brought quickly to ignition temperature. The arch acts like a baffle directing the flow of the gases. The radiation of heat by the refractory of the arch is not an important factor in heating the fuel. Good results could be obtained with a bare water-cooled arch, because such an arch would direct the hot gases equally as well as the refractory arch. In other words, it is the shape and the location of the arch and not the material of which the arch is made, which accomplishes the desired results.

Another useful function of the rear arch is that it directs the air entering, around and through the rear end of the grate toward

[4] Engineer in Charge, Research and Development, Combustion Engineering Company, Inc., New York, N. Y. Mem. A.S.M.E.

the front part of the furnace, where it mixes with the combustible gases rising from the fuel bed. These gases generally contain no free oxygen and need air for their combustion.

(*b*) The small air spaces in the grate offer a high resistance to the flow of air as compared with the resistance of the fuel bed. Inasmuch as the air spaces are equally distributed over the grate area, the air distribution is nearly uniform even though the condition of the fuel bed may vary. In other words, the flow of air is determined mainly by the resistance of the grate which is constant and only to a small extent by the resistance of the fuel bed which is small and may vary.

(*c*) The gases rising from the front part of the active fuel bed contain a large percentage of combustible and very little or no free oxygen. Air is, therefore, supplied under pressure through the front wall at the level of the throat. This air owing to its high velocity bores into the stream of combustible gases and produces both rapid mixing and quick combustion of the gases. It is necessary that there be sufficient combustion space above the throat available for the mixing and for the combustion of the gases.

E. A. Wood.[5] Texas lignite undoubtedly has wide possibilities of usefulness. Strange as it may seem, The University of Texas uses lignite in its power plant and, because of the fine research work that has been done by the author, it saves money for the University. Up to the present time, neither oil nor natural gas has been able to compete successfully with lignite at The University of Texas. This is true in spite of the fact that there is no railroad spur at the University's power plant. The lignite has to be hauled by rail from Bastrop, Tex., a distance of 47 miles to Austin; thence from Austin to the University by truck, a distance of 2 miles.

Because of the Texas Planning Board's interest in the utilization of the state's natural resources by industry and because of the author's research and practical demonstration of the utility of this particular resource, the board has become very much interested in the possibilities of the future uses for Texas lignite. There are more than 23,000,000,000 tons of this fuel available in Texas. Consequently, this fuel should be of interest to many industries. The fact that it has not been of much interest in the past is probably largely due to the reasons given by the author, namely, too much moisture, poor combustion, ability to slack, and because it dropped through the grate.

It would appear from the paper that these difficulties have now been overcome and that Texas lignite should enjoy a much better reputation as a fuel in the future than it has in the past. Of course, its usefulness will be limited by present freight rates within a certain radius, which will equal the economical haul. Fortunately, however, the lignite beds of Texas lie close to the most heavily populated sections of the state and not too distant from the Gulf Coast, where great ports have already been developed. Among others should be mentioned Houston, Port Arthur, Beaumont, Galveston, Texas City (location of the Carbon and Carbide Company's new plant), Corpus Christi, Port Isabel, and Brownsville. Water transportation, either by barge on the Intracoastal Waterway, the Mississippi and its tributaries, or by ocean-going vessels is therefore readily available for those industries seeking a cheap and abundant fuel for manufacturing purposes.

In addition to The University of Texas, the Austin Street Railway Company in Austin, a cotton-oil mill in Austin, the Morton Salt Company at Grand Saline, the Round Rock Lime Company at Round Rock and others use lignite as fuel. At Marshall, the Darco Corporation is using lignite to make "Darco" an activated-carbon product used as a filtrant.

[5] Director, The Texas Planning Board, Austin, Tex.

Dr. E. P. Schoch, director of the Bureau of Industrial Chemistry of The University of Texas, reports tha the "has perfected a simple method for removing all the moisture from lignite, while keeping the material essentially in its original lump form and filming it throughout with a total of about 2 per cent of petroleum oil to make it waterproof. The process, mechanically, is practically identical with the procedure for treating wet timbers with creosote oil under pressure. Of course, it employs petroleum oil instead of creosote oil. This process has just been announced from the laboratory, and active steps for its commercialization are under way in Canada."

W. R. Woolrich.[6] It is fortunate for the Gulf-Coast area that its future industrial-fuel demand can be filled by a generous reserve of lignite. In much of the area in which lignite is available, the rainfall is very limited and uncertain and the rivers fluctuate widely. Thus for industrial power the rivers cannot be considered potential prime power sources to take the place of oil and gas when these fuels become less abundant. Lignite will be the future fuel for many of these areas.

The author's contribution to engineering research, in developing efficient and economical methods of burning lignite on chain-grate stokers, will have more and more historical significance as the existing fuel reserves of other more desirable forms of fuel become exhausted.

It is gratifying that the developed method eliminates the need of drying the fuel before firing under power-plant boilers. Lignite can be successfully dried for domestic use but the cost is considerable.

The Bureau of Industrial Chemistry of The University of Texas has developed a method of removing the excess moisture which seems most promising. At a relatively low cost, a very satisfactory domestic fuel can be provided by this new method. However, while natural gas and oil are quite generally available in most of the Southwest, no great expansion of lignite usage in domestic stoves and heaters is anticipated.

About 25,000 tons of lignite are burned annually in the University's plant under Professor Eckhardt's direction. This supplies the fuel needs for power, heat, and light to The University of Texas campus buildings on a 24-hour-per-day basis. Interruptions to service are practically unknown at the University power plant.

Author's Closure

The investigative work, which has heretofore been done at The University of Texas in the case of the utilization of lignite as a grate-fired fuel, has in its entirety been of a preliminary order. It has had as its objective the dissemination of information calculated to dispel the most serious erroneous beliefs which have been formulated with regard to this fuel. It is hoped that in the ultimate a sufficient amount of authoritative information may be made available by which the use of this fuel will be demonstrated to be economically expedient to a far greater extent than it now is. The abundance of very admirable fuels in the State of Texas and its adjoining states and the absence of a specific industrial-development program have given little, if any, impetus to research work calculated to improve the effectiveness with which solid-fuel deposits of the lower rank might be utilized.

The State of Texas is at this time producing forty per cent of the nation's petroleum, more than thirty-three per cent of its supply of natural gas, eighty-five per cent of its sulphur, and in prodigious quantities many other minerals by far too numerous to mention here. The value of its mineral products exceeds that of any other state in the country. Many valuable mineral de-

[6] Dean of Engineering, University of Texas. Mem. A.S.M.E.

posits are not now being exploited at all. Of the minerals produced, by far the greater portion is processed in other sections of the nation. This state produces almost twice as much cotton as any other state in the Union as well as the preponderant portion of the nation's wool and mohair. From a standpoint of its agricultural and ranching products it has assumed a position of great importance. Despite these and many other accomplishments it has continued to remain a poor region.

It is obvious that despite the fact that its natural resources have been exploited at a rapid rate, the region has not profited from transactions by which its raw materials and potential energy in the form of fuel have both been transported to other sections of the nation. The establishment of industrial enterprises for the utilization of Texas' forests in the paper industry and the establishment of initial factories for processing chemical deposits as well as cotton and wool gives evidence of the fact that American business has become conscious of the attractive possibilities offered by this section of the country for the conduct of industrial enterprises. Virtually all natural circumstances promote this consciousness. It does not seem unreasonable to hope that all deterrent artificial barriers will have been broken down in the not-distant future.

When the industrial development of this region is seriously undertaken then the utilization of fuels which can only be utilized gainfully in the region under consideration will not only be indicated, but will likewise be prompted by economic expediency. The fact that such fuels as lignite offer no attractive possibilities in regions far removed from their source should not be inferred to indicate that they cannot be expected to assume a position of importance in the producing areas. The research work must obviously be done now in order that the use of this fuel may be made as purposeful as possible.

By-Product Fuels in the Steel Industry

By E. G. FOX[1] and W. B. CLEMMITT,[2] CHICAGO, ILL.

A normal annual supply of by-product gases, having a heat content equivalent to about 32,000,000 tons of coal, results from the production of metallurgical coke and from the smelting of pig iron. This paper deals comprehensively with the uses within the steel industry of coke-oven gas, blast-furnace gas, and mixtures of the two gases, which constitute the by-product fuels. After outlining the characteristics and combustion properties of each, the authors give in some detail their applications to the various heating requirements of steel production. The process of mixing the two gases to obtain the maximum heat value is explained thoroughly. The fact is stressed that by-product fuels be utilized in processes where the greatest economic value will result. The method of arriving at the potential values of these gases in relation to that of tar, fuel oil, and coal is fully expounded, several examples of cost analyses being cited to illustrate the procedure.

APPROXIMATELY 37,000,000 tons of pig iron were produced in the United States in 1937. To supply metallurgical coke for this operation, it was necessary to coke about 54,000,000 tons of coal. The total gases resulting as by-products from these operations had a heat content equivalent to about 32,000,000 tons of coal. If the screened-out coke breeze and the tar be added, the total of potential by-product fuels, resulting from steel-plant operations, amounts to some 35,000,000 tons coal equivalent. This is equal to about 7.5 per cent of the national coal consumption. The effective use of these by-product fuels is obviously a matter of some import.

The principal by-product fuels of the steel industry, with an indication of their relative heat content in a balanced plant, are as follows:

Blast-furnace gas, per cent	55.0
Coke-oven gas, per cent	35.0
Coke breeze, per cent	6.5
Tar, per cent	3.5

Coke-Oven Gas. Coke-oven gas contains about 25 per cent of the heat value in the coal from which it is produced. Derived by releasing the volatile content of coal by heat in the absence of air, it comprises principally hydrogen and hydrocarbons and is low in carbon monoxide, carbon dioxide, and nitrogen. After being exhausted from the ovens, it is treated for removal of by-products. Extensive cleaning operations are essential to by-product recovery. The resulting product is a relatively clean gas of about 520 Btu per cu ft delivered substantially saturated with water vapor, at atmospheric temperature and gas-holder pressure.

While there is a wide range in the quantity and the heating value of the gas delivered from an individual oven during the coking operation, dependent upon the time elapsed since charging, the product from a battery of ovens which are charged in sequence, is very uniform. Changes in the coal mixture may affect both the gas quantity and analysis. Changes in the coke-production rate, effected by varying the coking time, affect the rate of gas production correspondingly. Such changes are infrequent. Operation is continuous 24 hr per day and 7 days per week.

Coke-oven gas is delivered either from the holder or from the supply line to boosters, which increase the pressure for distribution. For use at the bulk-consuming units in the steelworks, the gas should be delivered at a pressure of at least 5 lb per sq in. Small consumers, such as normalizing furnaces, commonly require pressures from 5 to 10 lb per sq in. It is customary practice to boost the gas for these uses in local compressors to a pressure between 15 lb and 20 lb per sq in.

Coke-oven gas burns with a transparent but rather white flame, which tends to be somewhat buoyant. It requires theoretically about 5 cu ft of air per cu ft of gas for perfect combustion. Very little excess air is required in practice, due to the high ratio of air to gas and the consequent intimacy of mixing. The flame is characteristically short. The gas will hold ignition temperature under less favorable conditions than most industrial gases, because of the high flame temperature and rapid rate of ignition. Coke-oven gas is not commonly preheated, as any substantial preheat (in excess of 500 F) causes the hydrocarbons to break down, resulting in deposition of carbon in regenerator or recuperator chambers.

Blast-Furnace Gas. Blast-furnace gas contains about one half of the heat value in the coke charged into the blast furnace. The analysis, heat value, and top temperature of blast-furnace gas differ according to the kind of iron or ferrous alloy being produced. The heat value usually ranges between 85 and 100 Btu per cu ft. For a given operation, the analysis is not entirely uniform, a range of plus-or-minus 5 per cent in heat value being typical. Blast-furnace operations are subject to delays ranging from five minutes to an hour or more, incident to tuyère changes and other operating emergencies. During such periods, gas production is discontinued. The rate of blowing may be varied over a wide range to suit production to demand. About once in three years a furnace must be blown out for relining and repairs which consume about two to three months. Since most plants have but a few blast furnaces, the shutting down of one furnace deducts substantially from the production of gas. The trend toward larger blast furnaces accentuates this condition. In general, it may be said that the supply of blast-furnace gas is subject to much wider variations than the supply of coke-oven gas.

Due to its bulk, it is not ordinarily considered feasible to store large quantities of blast-furnace gas. A 10,000,000-cu ft gas holder contains only about 2 hr gas output of one 1000-ton blast furnace. Consequently, it is necessary to adjust the consumption of blast-furnace gas to the supply.

Blast-furnace gas leaves the top of the furnace at a temperature between 250 and 700 F and at a pressure between 15 in. and 60 in. of water. It has a dew point of about 120 F corresponding to a moisture content of about 30 grains per cu ft, this being contained in the blast and picked up from the burden which may be wetted to restrict top temperature and dust pickup. It is generally cleaned in wet stationary towers to about 0.25 grain per cu ft, the gas leaving the towers saturated and carrying considerable entrained moisture. Improved towers, with cyclone dust-and-moisture eliminators, deliver saturated gas of about 0.1 grain dust per cu ft. Further cleaning may take place in rotary disintegrators or in electrical precipitators. By these means the gas may be cleaned to less than 0.01 grain dust per cu ft.

Gas which is to be used to underfire coke ovens, and that in-

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Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors and not those of the Society.

tended for gas-engine use, is generally cleaned to 0.01 grain per cu ft or better. Gas which is to be used at open hearths and heating furnaces should not contain in excess of 0.05 grain per cu ft.

Blast-furnace stoves can be designed with large checker openings (3.5 to 4 in.) to use gas having 0.25 grain dust per cu ft. Such stoves have efficiencies on the order of 80 per cent. Stoves having somewhat smaller checker openings (2.5 to 3 in.) can use gas having 0.15 grain dust per cu ft. Such stoves are about 85 per cent efficient. Stoves with small checker openings (2.0 to 2.5 in.) may be built to give 90 per cent efficiency. Such stoves require gas containing not more than 0.03 grain dust per cu ft.

Blast-furnace gas is delivered from wet-cleaning operations at a pressure as low as 1 in. or as high as 20 in., depending upon the nature of the gas-cleaning equipment and on the pressure required to effect distribution. The pressure at which the gas leaves the blast furnace may be sufficient to pass it through the several cleaning operations, transport it through distributing mains, and deliver it to consumers at a pressure of 2 to 6 in. water necessary for its utilization. In such event, the pressure is supplied by the relatively efficient blast-furnace blowing apparatus. In other instances, a lower furnace top pressure is desired and it is necessary to boost the gas pressure for distribution to about 25 in. Due to its low heating value and low pressure transport, blast-furnace gas requires relatively large mains for distribution.

Process Consumers of By-Product Fuels

Melting, refining, heating, and heat-treating are major steps in the production of steel. The cost of fuel for these operations is about 15 per cent of the total cost of conversion processes. The steel plant affords a large market for its own by-product fuels.

Coke Ovens

For many years by-product ovens in steel plants have been underfired with coke-oven gas. Most of the ovens constructed during recent years are adapted for underfiring with blast-furnace gas. The operation is one involving relatively low temperatures, for which blast-furnace gas is well-suited (coke pushed at 1750 to 2200 F). Both the blast-furnace gas and the combustion air are preheated. In order to prevent the clogging of flues, very clean blast-furnace gas is required, containing not to exceed 0.01 grain of dust per cu ft. The coking process is entirely continuous and there are no interruptions or substantial variations in gas use. Reliability of gas supply is essential, as stoppage of underfiring affects the quality of the coke, and expansion and contraction of refractories may lead to substantial injury to the oven structure. It is possible to change over from blast-furnace-gas underfiring to coke-oven-gas underfiring in an hour or less, but several hours may be required to adjust operations satisfactorily to the change. Therefore, frequent change-overs are undesirable. Usually no provision is made for firing of auxiliary fuels, but ovens are given first preference as to gas supply. Precautions may be taken, such as providing gas-holder storage.

Blast-Furnace Stoves

The moderate temperatures which are required for blast-furnace stoves (air-blast temperature 1000 to 1500 F) can be obtained from blast-furnace gas without preheat of either gas or air. Great reliability of gas supply is not essential. If a blast furnace is not being blown, stove heating is unnecessary. The refractories in blast-furnace stoves can store considerable heat. The storage capacity of the stoves is utilized by permitting the gas input to stoves to vary with fluctuations of gas production and use, proper combustion being maintained by automatic burner control. If the stoves are equipped with adequate pressure burners which place the combustion chamber of the stove under pressure, it is often possible to burn gas at a maximum rate equal to twice the average rate. In this manner the stoves act as a flywheel, tending to absorb some of the fluctuations in blast-furnace-gas supply.

The storage capacity of the blast-furnace stove permits of reduction in firing in the event of a temporary gas deficiency. This can be made up subsequently by increased firing. It is possible to reduce the blast temperature and the consequent sustained requirement of gas for firing the stoves. However, such action should be anticipated by about fourteen hours to permit compensation by charging additional coke to the blast furnace, the time element being that required for such coke to descend to the tuyère zone in the furnace.

Blast-furnace stoves have always been fired by blast-furnace gas exclusively. No auxiliary fuel is provided. Because of special conditions existing in certain plants which desire to release blast-furnace gas for use elsewhere, projects have been made for burning oil and natural gas at the stoves.

Open Hearths

The open-hearth department is usually the largest consumer of process heat in a steel plant. At the same time, it requires the highest temperatures. It is a batch process involving melting and refining operations. The total time of a heat will range from 8 to 16 hr. During the melting period, the heat input may be increased substantially above the average rate; during refining operations only enough heat may be applied to maintain temperatures, restricting the input to as low as 15 per cent of the average rate. Fig. 1 shows how the fuel input and the heat release in an open-hearth furnace vary through the heat.

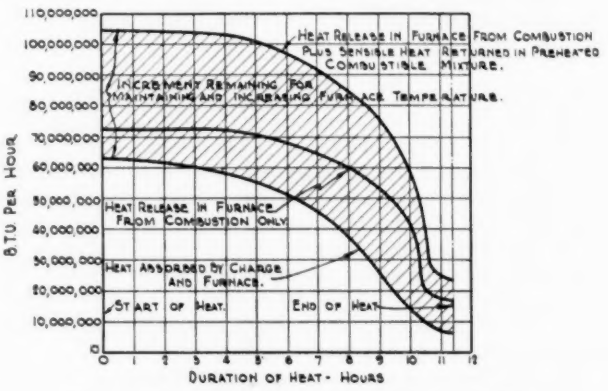


Fig. 1 Heat Release and Absorption for One Open-Hearth Heat

Coke-oven gas has been successfully applied to open-hearth furnaces. The gas burns with a transparent flame which is lacking in radiating power. Consequently, greater dependence must be placed on heat transfer by convection. Gas velocity, gas distribution, and turbulence play a role.

To avoid difficulties associated with transparent flame, it is customary to burn tar or oil in combination with coke-oven gas. The flame is directed close to the surface of the bath where its local luminosity expedites heat transfer to the near-by bath more effectively than to the more remote furnace roof. A combination is satisfactory in which 10 to 25 per cent of the heat input is in the tar or oil, the balance being in the coke-oven gas. However, in plants where the coke-oven-gas supply is limited, more of the heat input may be in liquid form.

Mixed coke-oven and blast-furnace gas is being successfully and increasingly applied to open-hearth furnaces. This mixture burns with a rather transparent, bluish flame. In at least one instance in the United States, the gas mixture is being preheated

to a temperature high enough to cause some cracking of hydrocarbons, a luminous flame resulting. The mixture tends to a lower theoretical flame temperature than straight coke-oven gas, and consequent lower furnace temperature. Absence of potential excessive heat head encourages even heat distribution and minimizes local overheating of refractories. Some variation in the composition of the mixture is permissible.

The fuel requirements of a single open-hearth furnace vary. There may be as many as fifteen furnaces in one shop, and some plants have several shops. In such plants the effect of diversity is to cause a substantially uniform fuel demand. Open hearths commonly operate continuously throughout the week.

Reliability of fuel supply is considered essential, although no serious damage will result if the fuel supply should fail for an hour or so. Most furnaces burning mixed gas are equipped to burn oil or tar in the event of failure or inadequacy of gas supply. Hence the open hearth may be considered as a large and steady consumer for which mixed blast-furnace and coke-oven gas is well suited. It is somewhat adaptable and accommodating in the event of variation or interruption of gas supply.

Soaking Pits and Heating Furnaces

Soaking pits and heating furnaces differ strikingly from open hearths in thermal requirements, in that they involve a lower range of temperature, such as 2200 to 2500 F as contrasted with the 2900 to 3250-F range required in the open hearth.

In soaking pits and heating furnaces, the function is to elevate the steel to rolling temperature, evenly distributed throughout the mass. The question of heat-penetration rate arises. A somewhat higher rate of heat penetration may be permissible during the earlier stages of heating, a limited rate of penetration during the later stage being desirable to obtain uniformity of temperature. The rate of penetration to cold steel is limited by possible damage to the steel resulting from wide temperature differences between surface and interior. The permissible heating rate is influenced by the quality of steel being heated.

Soaking pits and heating furnaces may be fired with coke-oven gas, mixed gas, or straight blast-furnace gas. The use of the latter two fuels is relatively recent. Coke-oven gas has long been applied. It is clean and convenient, but tends to be harsh if combustion air is preheated. Blast-furnace gas gives excellent results at soaking pits, where a moderate heat input is required for slow heating, with most of the heat transfer by convection. Straight blast-furnace gas has been used in continuous-heating furnaces, but the cost of such furnaces is rather high. Separately heated regenerators, for preheating the combustion air to a high temperature, are necessary. Mixed blast-furnace and coke-oven gas makes an excellent fuel for reheating furnaces. The mild, distributed heating obtained with mixed gas is particularly advantageous in furnaces heating thin sections As compared with producer gas, the cleanliness of mixed gas renders it more adaptable to use of automatic combustion controls. A mixture of about 33 per cent coke-oven gas and 67 per cent blast-furnace gas, having a heat value of about 230 Btu per cu ft, or any richer mixture, can be employed. It is not usual practice to vary the mixture at individual furnaces, but the mixture supplied to a common mill line, feeding a group of heating furnaces, may be changed without undue inconvenience. This possibility permits some flexibility in adjusting the supply of the two component gases to the demand.

Individual soaking pits and heating furnaces are relatively small consumers so that variations in their demands are readily absorbed by diversity in any extensive installation. Since most mills discontinue operations during week ends, the fuel requirements of these furnaces reduce nearly to zero over a period of about 20 hr each Sunday.

Annealing and Heat-Treating Furnaces

Annealing, normalizing, and heat-treating operations ordinarily involve temperatures on the order of 1200 to 1700 F. Slow, uniform heating is desirable. Preheat of combustion air is not ordinarily practiced. For such duty, blast-furnace gas or a lean mixture of coke-oven and blast-furnace gases, having inherently moderate flame-temperature characteristics, is ideal. The fact that such furnaces are generally small consumers and are often rather remotely located may cause the cost of distribution of blast-furnace gas to them to be prohibitive. On the other hand, annealing furnaces of the radiant-tube type require the use of straight coke-oven gas or a rich mixed gas.

Firing Boilers With Blast-Furnace Gas

It has long been customary to burn a major portion of the blast-furnace gas under boilers. This practice has been due in part to the usual proximity of the boiler plant, the ease and low cost of application, and the readiness with which supplementary fuels can be provided at boilers. It is common practice to burn blast-furnace gas in combination with powdered coal or fuel oil, or to provide for the exclusive burning of one of these supplementary fuels in the event of a deficiency of blast-furnace gas. When burned in combination, the supplementary fuel feed may be governed automatically as required to meet steam demands and blast-furnace-gas availability.

Blast-furnace gas is an eminently satisfactory fuel for boilers if these are adapted to its use. Because of the low flame temperature and consequently low furnace temperature, it is desirable that the combustion air be preheated. This is particularly essential when blast-furnace gas is used in a steam generator with waterwalls. Because of the relatively large volume of products of combustion (1.4 times coal), ample furnace volume and adequate provision for convection-heat transfer are necessary. When burning cold blast-furnace gas, care must be exercised not to reduce the rate of firing so low as to permit the furnace to cool below ignition temperature.

Blast-furnace gas may be fired to boilers as received from the dust catcher at a temperature of about 300 F, with the moisture content in superheated condition and carrying about 3 grains of dust per cu ft (standard), or it may be fired in cold saturated condition after wet cleaning. The hot dirty gas may produce about 5 per cent more steam, a factor of considerable import. The avoidance of gas-cleaning costs is an additional saving. The amount of dust carried into the furnace by hot, dirty, blast-furnace gas, is comparable with the ash in pulverized coal. This dust in the gas may have a salutary influence on combustion in that the hot dry dust particles glow and assist ignition. On the other hand, hot gas may require a larger gas main, which may be brick-lined to retain the sensible heat in the gas. More or less trouble may be experienced in keeping such a main clean, particularly if the amount of gas transported and the consequent gas velocity vary over a wide range.

In a few instances, blast-furnace gas is cleaned in electrical precipitators of dry type, retaining most or all of its sensible heat. A cleanliness such as 0.25 grain dust per cu ft is attained. Such gas is well-suited for boilers and blast-furnace stoves having large checkers.

If washed gas is fired under boilers, it is preferable that a cleanliness in the order of 0.15 grain dust per cu ft or less be attained. It is particularly desirable that the gas carry a minimum of entrained moisture. Wet dirty gas carrying perhaps 0.3 grain of dust and considerable entrained moisture can be utilized, but much trouble is experienced at the burners and in the gas lines due to sludge, and boiler tubes become encrusted. The cold wet dust tends to extinguish the gas and interferes with ignition at low ratings. Furthermore, entrained water is an immediate

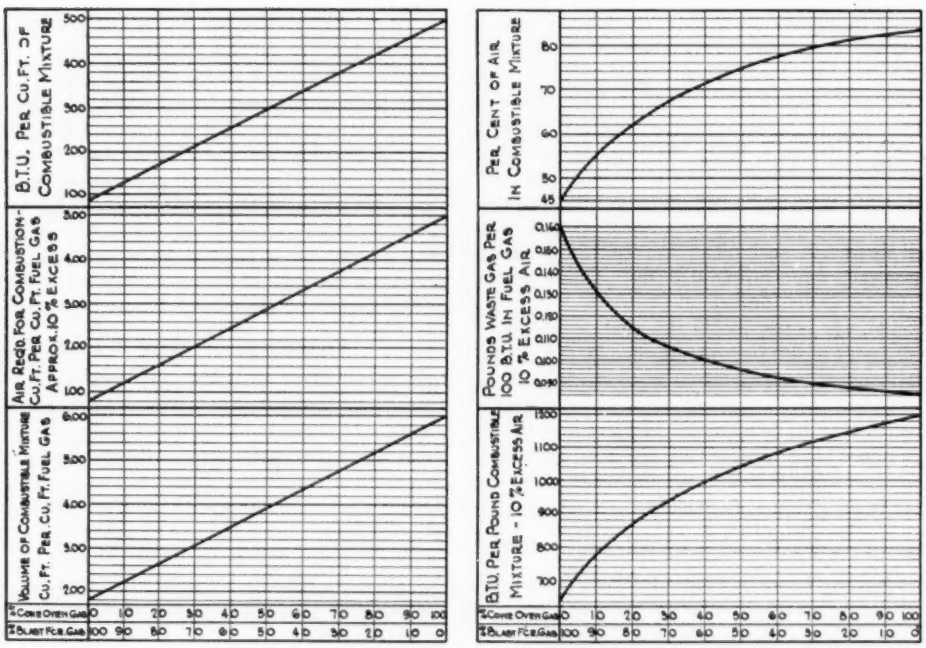


Fig. 2 Characteristics of Coke-Oven and Blast-Furnace-Gas Mixtures

and drastic debit against the net heat value of the gas, since it must be evaporated and superheated, with the absorption of almost 1/4 Btu per grain of water.

A choice between hot dirty gas and cool washed gas for boilers must depend on local conditions. If only a minor variable fraction of the gas is to be burned under boilers, it is most feasible to utilize the gas in the condition of its preparation for other uses. If, however, a major portion of the gas is to be burned under boilers regularly, the economies and advantages of hot gas should receive some consideration.

Coke-oven gas is seldom fired under boilers as it has been considered too valuable for other uses. However, it may prove desirable to burn some coke-oven gas under boilers on week ends. If coke-oven gas is substituted for blast-furnace gas in a boiler arranged for burning the latter, provision may be necessary to prevent overcooling of the lesser volume of combustion products in the air preheater or economizer, as pitting may result, particularly in view of the large amount of water vapor in the combustion products of coke-oven gas.

Gas Engines

Formerly a substantial portion of the blast-furnace gas was sent to gas engines driving blast-furnace blowers or electric generators. With improved practice in steam plants, the large space requirements, the heavy investment and maintenance charges associated with gas engines, are difficult to justify. The inflexibility of this use and the inability to substitute other fuels are serious disadvantages. Gas engines must be spared by steam-driven units. The advantages which accrue to the burning of the last variable increment of blast-furnace gas under boilers, supplemented by other fuel, do not accrue to gas-engine operations; in fact, gas engines are consumers which demand first consideration in the matter of regularity, reliability, cleanliness, and uniformity of supply.

Fuel Mixtures

As previously mentioned, coke-oven gas is frequently used in combination with tar or oil at open-hearth furnaces. Coke-oven gas has been added successfully to hot producer gas. There would appear to be a limited demand for such a combination, except as a means for disposing of an excess of coke-oven gas. Coke-oven gas has also been used in combination with still gas and the mixture has been used in combination with oil.

Principles of Mixed Gas Application

In the selection and application of fuels for steel-plant furnaces, it is desirable that two important considerations be met in order that the applications may be most effective and also most efficient. First, it is desirable that furnace-temperature conditions be developed which are well-suited to the process to be performed. Second, it is desirable that fuel consumption be minimized through the reduction of stack losses.

The latter consideration commonly requires that the heat in stack gases be used to preheat the combustion air, or both the air and fuel gas. The combustion products leave an open-hearth furnace at about 2700 F; they may leave a soaking pit or a side-door heating furnace at 2000 F; they often leave continuous-heating furnaces at temperatures up to 1000 F. For fuel economy, it is important that a substantial part of the heat contained in the products of combustion at these temperatures be recovered.

If part of the heat in the products of combustion is utilized to preheat the combustion air and/or the fuel gas, the effect of such preheat is to raise the temperature developed in the furnace by the fuel being used. An amount of preheat, which may be desirable from a fuel-economy standpoint, may not result in the most advantageous furnace temperatures and consequent heating performance.

The availability of two gaseous by-product fuels of widely differing characteristics offers opportunity, in the steel plant, to adapt the fuel to the needs of the application through the selection of one fuel or the other, or through use of mixtures of the gases in most advantageous proportions. The advantages which accrue through the use of mixed gas are being increasingly recognized.

The curves in Fig. 2 show pertinent data concerning mixtures of blast-furnace and coke-oven gas and their combustion charac-

teristics. The volume of combustible mixture per 100 Btu in the fuel, for blast-furnace gas, is about 1.66 times that corresponding to coke-oven gas. The weight of waste gases, per 100 Btu in the fuel, is 1.9 times greater for blast-furnace gas than for coke-oven gas. Because of the greater volume and weight of the products of combustion, recovery of waste heat from these products is somewhat more significant as related to the leaner gas mixtures, particularly those containing less than about 30 per cent coke-oven gas.

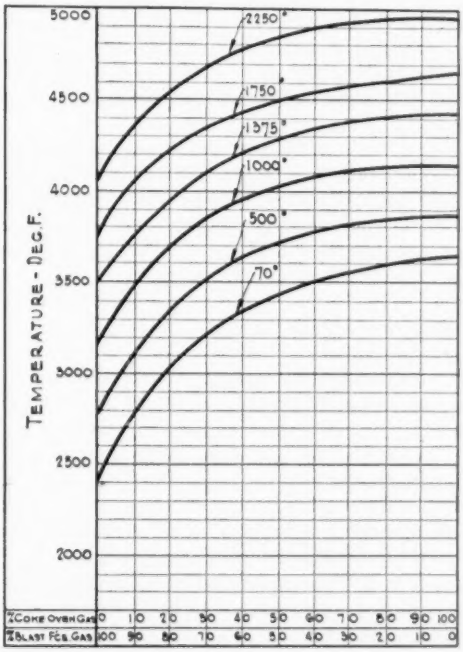


FIG. 3 THEORETICAL FLAME TEMPERATURES; MIXTURE OF BLAST-FURNACE AND COKE-OVEN GASES
(Combustible mixtures preheated to various temperatures; 10 per cent excess air.)

The amount of fuel and heat which can be handled by a given furnace is limited by speed of combustion, gas velocity, and draft considerations on the one hand, and by ability to deliver heat to the charge on the other. The lesser volume and weight of combustion products per 100 Btu in the fuel contributes to greater permissible heat release in a given furnace with coke-oven gas than with blast-furnace gas. The shorter coke-oven-gas flame is another favorable factor. Furthermore, the higher effective flame-temperature characteristic of the richer fuel increases the temperature difference or gradient between flame and charge, thereby favoring both radiant and convection-heat transfer. The richer fuels enjoy a distinct advantage with respect to rapid heating of cold steel.

It should be noted, in this connection, that preheat contributes to greater total heat release in a given furnace without encroaching on the limitations incident to heat release resulting from combustion. Fig. 1 gives an indication of the contribution to heat release made by preheat.

The greater relative volume of combustion products is advantageous to the leaner fuels, particularly with respect to moderate heat release when the temperature of the charge is high and the fuel feed is restricted. The larger relative volume of products may fill the furnace volume better, thus improving heat distribution; it may aid convection-heat transfer, thus minimizing the temperature difference necessary between the charge and the flame. The longer flame also favors uniformity of temperature through the furnace. The leaner fuels enjoy a distinct advantage as to quality of heating.

The curves, Fig. 3, show the influence of gas mixture and preheat on flame temperature. The lowermost curve shows theoretical flame temperatures corresponding to various mixtures of blast-furnace and coke-oven gas when burned cold in combination with cold combustion air. The next higher curve shows the effect of preheating the air or gas or both to such extent that the combustible mixture has a temperature of 500 F. Other curves show the effect of higher amounts of preheat.

It is impossible to attain the theoretical temperatures shown in these curves. The nearest approach to them can occur in an empty furnace or one in which the charge is fully heated. Under such conditions, all the heat released in the furnace is required to compensate for furnace losses and to maintain the furnace temperature. The maximum actual mean furnace temperature approximates 85 per cent of the theoretical flame temperature, depending partly on furnace design, exposure for radiation, and insulation.

When a cold charge is introduced into a furnace, it absorbs part of the heat released, thereby restricting the portion of the heat which is available for maintaining furnace temperature. The result is the drawing down of the mean furnace temperature, perhaps to about 50 per cent of the theoretical flame temperature.

As the charge becomes heated, the gradient between the furnace temperature and the charge tends to decrease. The charge therefore absorbs a smaller portion of the heat release. The furnace temperature increases. An increasing portion of the heat release per unit of fuel is then available and is required to maintain the higher heat level. The furnace temperature attained at any moment is determined by the portion of the heat release which is available to elevate the furnace temperature. If heating were continued indefinitely, the condition would eventually be attained under which the charge would absorb no further heat and "empty-furnace" conditions would apply.

Under typical working conditions, a portion of the heat release is absorbed by the charge and the mean furnace temperature approximates 75 per cent of the theoretical flame temperature.

Through application of these approximate ratios or percentages to the theoretical flame temperatures shown in Fig. 3, the following deductions may be made. These are merely explanatory of established practices.

1 An increase in preheat results in an increase in furnace temperature, which accelerates the rate of heating. An increase of 750 F in preheat may elevate the furnace temperature an amount, such as 200 F.

2 The richer fuels enable favorable heating rates (indicated by the disparity between theoretical flame temperature and mean furnace temperature) to be maintained at higher furnace temperatures.

3 A cold charge will draw down furnace temperatures further with a lean fuel than with a rich fuel; consequently heating is more rapid with the richer fuel. If a very lean gas is used with low preheat, furnace temperatures may be drawn down by a cold charge and cool furnace so low as to jeopardize ignition of the gas.

4 The use of a richer fuel permits the attainment of a given furnace temperature, with a given proportion of the heat release available for the charge, at a lesser preheat.

5 A temperature of about 3250 F cannot be attained, even with empty-furnace conditions, through use of a mixture containing less than 30 per cent coke-oven gas with 1000 F preheat. If 1750-F preheat is employed, a fair rate of heating may be attained. With this amount of preheat, a furnace temperature of 3250 F could hardly be attained with straight blast-furnace gas. All the heat input would be required merely to attain such a heat level.

6 With the richer fuels, unless the charge does absorb a substantial portion of the heat input, furnace temperatures will rise far beyond limits which the refractories can stand. This demonstrates the need of careful adjustment of fuel quantity to restrict furnace temperatures when rich fuels are combined with high preheat. If there is lack of uniformity of heat distribution or of absorption by the charge, use of a rich fuel with high preheat may easily lead to dangerous flame and furnace temperatures in localized areas. If less preheat is employed, the danger of excessive local temperature is reduced.

7 A rich gas mixture in combination with high preheat may be used to advantage to bring the charge up to melting temperature but, if a lean mixture is employed near the end of the heat,

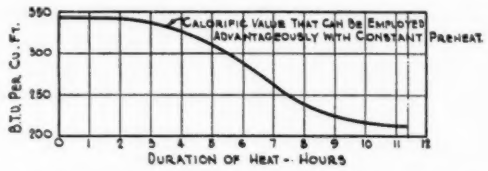


FIG. 4 POSSIBILITY OF VARYING CALORIFIC VALUE OF FUEL

when it is desired to deliver little heat to the charge, the danger of attaining damaging furnace temperatures through failure to restrict the fuel feed quickly enough, or through uneven distribution and local heating, is minimized. The manner in which this practice is applied is indicated in Fig. 4. This figure may well be viewed in relation to Fig. 1. In actual practice, the Btu content of the gas mixture is adjusted in steps, as the heat progresses, rather than along a smooth curve as shown in Fig. 4.

8 If mixed gas, containing about 80 per cent coke-oven gas by volume (420 Btu per cu ft) be used, it is possible to attain a furnace temperature of 3250 F with a good rate of heat input into the charge, if the combustible mixture is preheated to 1750 F. Such a mixture might be used advantageously during the initial stages of the heat. Since about 80 per cent of this mixture is air, preheat of the air only to about 2150 F gives an equivalent result. If this amount of air preheat is applied during the later stages of a heat to a gas containing 20 per cent coke-oven gas (170 Btu per cu ft), the combustible mixture will have a temperature of about 1300 F. With such a mixture, a furnace-temperature level of 3000 to 3250 F could be maintained only provided there is little or no heat input into the charge. Thus a mixture containing about 20 per cent coke-oven gas is a minimum which can be used to hold temperature in an open hearth where only the air is preheated to a maximum feasible temperature of about 2150 F.

9 As applied to heating furnaces requiring a heat level of about 2500 F, blast-furnace gas must be given high preheat if a fast rate of heating is to be carried up to rolling temperatures. A heating furnace, using blast-furnace gas fuel with 1000-F preheat of the combustible mixture, might be sluggish as rolling temperatures are approached.

10 If rich gas mixtures are used with high preheat, the furnace charge may be heated at high rates up to and well beyond desired rolling temperatures. Hence such combinations tend to be rash, causing uneven heating or washing of the steel. For heating furnaces which will employ high preheat, the use of lean fuel mixtures is indicated. Where but little preheat is to be employed, richer mixtures become desirable. If no preheat is to be employed, only the richest fuels will be adequate.

It should be recognized that the curves indicate general conditions applying when the gases are burned with about 10 per cent excess air at the furnace. Variations from this condition affect the situation. An increase in excess air not only increases the weight of waste gases, but it lowers the flame temperature substantially. The curves do not take into consideration the effect of chemical reactions on open-hearth-furnace temperatures. In general the curves represent conditions which are attainable with favorable design, construction, and practice.

PREHEAT AND RECOVERY OF STACK LOSSES

The amount of preheat which may be most advantageous from the viewpoint of furnace operation, is not a fixed quantity. When a charge of cool steel is being heated or melted, a high degree of preheat could be utilized without danger of attaining excessive furnace temperatures. On the other hand, when little heat is being delivered to the charge, a limited amount of preheat would be desirable, particularly with rich fuels, in order to limit the theoretical flame temperature and the consequent furnace temperature.

No practical method has been developed for varying preheat. It is prevailing practice to provide fixed preheat facilities, resulting in fairly uniform preheat. For a given fuel, it is desirable that the amount of preheat be such that a moderate rate of heat delivery to the charge can take place when the charge approaches or attains its maximum desired temperature.

As a general indication of relations between furnace temperatures, preheat, and recovery of heat in the products of combustion, the following examples are cited:

1 With fuel mixtures containing more than 60 per cent coke-oven gas, advantageous furnace conditions in a 2500-F furnace are obtained with no preheat and consequently no waste-heat recovery. With straight blast-furnace gas, burned at a furnace equipped with regenerators or recuperators, a recovery of about 55 per cent of the sensible heat in the combustion products between 2000 and 800 F is indicated to be adequate to the attainment of good furnace temperatures.

While it is true that heating furnaces burning coke-oven gas are often equipped with regenerators or recuperators, such provision is primarily directed at the recovery of waste heat. Some increase in theoretical flame temperature inevitably results, with a consequent tendency to furnace temperatures higher than necessary. Preheat tends to shorten the flame and is thus inimical to heat distribution resulting from slower combustion.

2 In an open hearth, using straight coke-oven gas, advantageous furnace conditions are obtained with a heat recovery of about 80 per cent of the heat in the waste gases contained in a 2700- to 1400-F range, or about 45 per cent of a 2700- to 400-F range. With a mixture containing about 25 per cent coke-oven gas (200 Btu per cu ft), a heat recovery of about 80 per cent of a 2700- to 800-F range is necessary from a furnace-operating standpoint. This would be a full practical recovery for this range in view of inevitable heat losses in the regenerators due to radiation and possible infiltration.

In general, the leaner mixtures afford greater possibilities for complete waste-heat recovery without attendant furnace conditions of a character which require close regulation of fuel input and uniform distribution of combustion and heat absorption in order to avoid excessive furnace temperatures. This statement does not apply to cases where the waste heat recovered is not returned to the furnace, i.e., to the cases where waste-heat boilers are used.

Improved types of recuperators which are now available, are factors favorable to the more extensive application of lean gas mixtures for heating-furnace duty.

QUANTITATIVE RELATIONSHIPS

The selection and mode of application of by-product fuels in the steel plant is naturally influenced by the availability of those fuels. Individual plants differ widely, dependent primarily upon the relative coke-, pig-iron-, and steel-making capacities;

the kind of coal, ore, and iron; whether all steel is by the open-hearth process; the amount of scrap used; the extent of rolling operations, etc. Existing pipe lines, furnaces, regenerators, and other equipment influence the practice adopted in by-product gas utilization. It may be enlightening, however, to consider the relations of gas supply and possible demand for a rather typical new plant where such restrictions were nonexistent. Table 1 is taken from a project for the Dnieprostal Steel Works in the Ukraine, U.S.S.R. The initial plant as projected, included four blast furnaces with coke ovens adequate to supply these furnaces. Most of the steel-melting capacity employed the open-hearth process, although some electric furnaces were included. A more or less typical complement of mills was involved. Electric power was to be received from the Dnieprostroy hydroelectric development near by. Steam turboblowers were installed to serve the blast furnaces.

TABLE 1 PROCESS-HEAT REQUIREMENTS IN A STEEL PLANT IN RELATION TO BY-PRODUCT-GAS SUPPLY

	Heat required				
Department	Total heat C-O[a] and B-F gases, per cent	Heat in B-F gas, per cent	Heat in C-O gas, per cent	All process heat, steam included, per cent	Process heat, excluding steam for blowers and coke ovens, per cent
Coke ovens (ovens)	17.4	27.2	48.5	16.2	19.0
Process steam	5.1	7.9	14.1	4.7	...
Blast furnace (stoves and losses)	18.1	28.2	50.5	16.8	19.6
Turboblowers	10.5	16.0	28.5	9.8	...
Open hearth	28.1	43.8	78.5	26.1	30.5
Soaking pits	8.2	12.7	22.6	7.6	8.9
Heating furnaces	13.8	21.5	38.3	12.8	15.0
Annealing and miscellaneous furnaces	6.4	10.0	17.9	6.0	7.0
	107.6	167.3	298.9	100.0	100.0
Deficit	7.6	67.3	198.9		

[a] C-O signifies coke oven; B-F signifies blast furnace.

While 64 per cent of the heat content of the by-product fuels at the plant indicated in Table 1 is in the blast-furnace gas, this represents 91 per cent of the gas volume. Nine per cent of the volume is coke-oven gas, containing 36 per cent of the total heat. If the two gases were mixed in toto, a lean mixture would result having a heating value of about 125 Btu per cu ft. Such a fuel would not be well-adapted for use at the open hearths and heating furnaces, for which a richer fuel is desirable. Moreover, if all the gas were mixed at one point, it would not be possible to select for each consuming plant the kind of gas or the mixture best suited to its requirements. Nor would it be possible to vary the mixture at the open hearth with the progress of the heat. It is more rational to distribute the gases separately and to use portions of the blast-furnace gas without mixing, thereby obtaining a richer mixture for the balance.

It is entirely logical to allocate straight blast-furnace gas for use at coke ovens and stoves. This gas is well-adapted to these operations. The layout of the plant in question is favorable to easy distribution of blast-furnace gas to these consumers. Moreover, these consumers require gas seven days a week, therefore introducing no week-end storage problem.

After thus allocating 55 per cent of the blast-furnace gas to stoves and coke ovens, the remaining 45 per cent might be mixed with the coke-oven gas. The resulting mixture would contain about 82 per cent blast-furnace gas and 18 per cent coke-oven gas, the heat value being about 165 Btu per cu ft. This is still too lean for open-hearth and heating-furnace use.

An additional 12.7 per cent of the blast-furnace gas may be used advantageously at the soaking pits. If about 15 per cent of the blast-furnace gas be allocated to annealing furnaces and to boilers to produce steam for coke ovens and blast-furnace use, a total of about 84 per cent of the blast-furnace gas will have been utilized. The remaining 16 per cent, mixed with all the coke-oven gas, gives a mixture of about 63 per cent blast-furnace gas and 37 per cent coke-oven gas, with a heat content of about 240 Btu per cu ft. Such a gas can be employed at open hearths and heating furnaces if provision is made therefor.

The foregoing analysis shows in a general way how the by-product fuels available in a plant may be allocated to process-consuming units. The relations given in Table 1 apply to capacity operations. If the plant as a whole, or any departments thereof modifies its production rates, the relations change. It is of course necessary to provide for any combination of conditions which may arise. If auxiliary fuels are required, investments are involved which must be included in the economic picture.

Economic Factors Involved in Allocation of Gases

It has been indicated that there are several definite potential consumers of blast-furnace and coke-oven gases in steel plants. Some practical considerations influencing the allocation of these gases have been mentioned. It is further of consequence that these gases be used where they will realize the greatest economy. Stated in another manner, it may be said that the gases should be sold to consuming plants which can pay the most for them. Table 2 gives some conception of the merit of various applications and indicates the factors involved.

In the case of oil, the cost of fuel plus necessary storage, heating and pumping, plus maintenance and fixed charges on storage and handling equipment, brings the cost to about $0.247 per million Btu. In a plant which must fire oil or tar, the value of the tar is substantially the same as that of oil, unless tar can be sold at a premium.

The cost of producer gas is indicated to be substantially lower, per million Btu, than oil. The cost figures for mixed gas are

TABLE 2 EVALUATION OF BY-PRODUCT GASES[a]

Elements of cost per million Btu in fuel	Oil or tar	Coal for producer gas	Coke-oven gas and oil, 50% mix	Mixed gas; coke-oven and blast-furnace, 230 Btu per cu ft
Fuel cost	$0.2400	$0.148	$0.1200	....
Preparation	0.0020	0.040	0.0017	$0.0016
Transportation	0.0020	...	0.0043	0.0070
Maintenance	0.0015	0.005	0.0017	0.0010
Fixed charges	0.0020	0.015	0.0038	0.0055
Total cost	$0.2475	$0.208 (coal)	$0.1315	$0.0151
Gas evaluation				
At open hearth:				
Millions of Btu per ton ingots	4.800	5.700 (coal)	4.800	4.500
Fuel cost per ton ingots	$1.190	$1.180	$0.630	$0.068
Bid price for coke-oven gas in gas-oil mixture:				
Per ton ingots	0.560	0.550	...	...
Per million Btu	0.234	0.229	...	...
Bid price for mixed gas:				
Per ton ingots	1.122	1.112	...	...
Per million Btu	0.250	0.248	...	...
At continuous heating furnaces:				
Millions of Btu per ton heated	1.400	2.000	...	1.200
Fuel cost per ton heated	$0.346	$0.416	...	$0.018
Bid price for mixed gas:				
Per ton heated	0.328	0.398	...	...
Per million Btu	0.273	0.332	...	...
At soaking pits:				
Millions of Btu per ton ingots	1.200	1.600	...	1.000
Fuel cost per ton ingots	$0.298	$0.333	...	$0.015
Bid price for mixed gas:				
Per ton ingots	0.283	0.318	...	...
Per million Btu	0.283	0.318	...	...

[a] Approximate or typical costs assumed:
Steam coal, cost ..... $3.50 per ton, delivered
Gas coal, cost ..... $4.00 per ton, delivered
Fuel oil, cost ..... 3½ cents per gallon
Electric power, cost ..... 8 mills per kilowatthour
Fixed charges ..... 15 per cent

Transportation cost based on 1200 ft of gas pipe line between respective gas sources and consuming department; length of lines might be doubled without material change.

based on a mixture having 230 Btu per cu ft. The charges include only the costs of preparation and delivery of the coke-oven and blast-furnace-gas components. In the case of a mixture of coke-oven gas and oil, the fuel cost of the oil only and all handling costs for both fuels are included. Fixed charges on all fuel preparation and transportation requirements are included.

In comparing these fuels for use at an open hearth, cognizance must be taken of the differences in fuel input required. The use of producer gas introduces substantial losses in producers. Mixed gas shows the highest economy for reasons already indicated. Fuel-consumption figures will vary in individual cases. The relations shown in Table 2 are typical, indicating that, under the conditions assumed, oil fuel costs $1.19 per ton ingots. If half the oil is replaced by coke-oven gas, the fuel cost per ton ingots is $0.63. The fuel value of the coke-oven gas for this application is the difference, i.e., $0.56 per ton ingots. The requirement of coke-oven gas is 2.4 million Btu per ton ingots. The value of the gas is $1.19 — 0.63/2.4 = $0.234 per million Btu.

Similarly, if coke-oven gas and oil were used in lieu of producer gas, the value of the coke-oven gas would be $1.18 — 0.63/2.4 = $0.229 per million Btu. This figure may be discounted if the producers already exist, so that their fixed charges continue whether or not the replacement fuel is used.

If mixed blast-furnace and coke-oven gas is used in lieu of oil, the value of the mixed gas is $1.19 — 0.068/4.5 = $0.25 per million Btu.

If mixed blast-furnace and coke-oven gas is used in lieu of producer gas, the value of the mixed gas is $1.18 — 0.068/4.5 = $0.202 per million Btu.

At the heating furnaces similar conditions exist. If mixed gas is used, somewhat lower fuel consumption may be expected. The figures tabulated show that, if mixed gas is used in lieu of oil, the value of such gas is about $0.273 per million Btu. If used in lieu of producer gas, the value of the mixed gas is about $0.332 per million Btu.

Similarly at the soaking pits, it is indicated that mixed blast-furnace and coke-oven gas has a value of about $0.283 to $0.318 per million Btu, under the conditions assumed. Since such a mixture can be replaced by straight blast-furnace gas with equivalent performance, blast-furnace gas has a value on this order for this application.

The values of mixed gas, as determined, refer to the component of coke-oven gas at the ovens before boosting the pressure for transmission, and the component of blast-furnace gas at the blast-furnace plant prior to cleaning and boosting.

These figures indicate that, under the conditions assumed for the example, heating furnaces and soaking pits can bid a higher price for mixed gas than can open hearths. This result is in conformity with general experience, but local considerations may affect the situation substantially. The cost of supplying mixed gas to scattered mill furnaces is likely to be higher than the cost of delivery to a bulk consumer such as the open hearth. Heating furnaces with regenerators or recuperators suitable for mixed gas may also involve additional first cost.

These figures for valuation of mixed gases for open-hearth and heating-furnace use are not inclusive of savings in refractory costs which may occur through use of milder fuels. Nor do they include such important factors as improved quality of steel resulting from better heating, decreased scale loss or elimination of the necessity for removal of suphur from coke-oven gas, which may be possible if it is diluted with blast-furnace gas. Other local or intangible factors may have a bearing.

Blast-furnace gas, burned under boilers, replaces coal which costs about $0.16 per million Btu, including preparation costs. If the gas is burned in cold saturated condition, the boiler efficiency is about 4 points lower with the gas than with coal. The gas thus has a replacement value of about $0.152. From this must be deducted $0.015 for preparation and transport, giving a net comparative value of the untreated gas of about $0.14. If the gas is burned hot, a substantial increase in heat value results from retention of the sensible heat and cleaning costs are avoided. On this basis the gas has a value of about $0.15 per million Btu.

If blast-furnace gas replaces coke-oven gas for underfiring coke ovens, it attains a value equal to that of coke-oven gas, less the rather nominal cost of preparing and transporting the blast-furnace gas for this use.

It has been rather customary to evaluate coke-oven gas according to the price which it will bring if sold to a utility. In some cases such an evaluation may not be entirely justified. If producer gas or oil is used in plant processes, the price which such consumers can bid for the gas for replacement purposes is a more realistic measure of value. Furthermore, it should be noted that coke-oven gas acquires a substantial increment of excess value if, through mixture with blast-furnace gas, it enables the latter to acquire a value such as $0.27 per million Btu for process use, as compared with $0.15 per million Btu for boiler use.

The foregoing examples bring out the fact that in many plants blast-furnace gas is the major by-product fuel. The problem is to use that lean fuel in processes where it has a higher value than when used under boilers. This can be accomplished if available coke-oven gas is directed to enrichment of a mixture rather than to separate application. Rather lean mixtures can be used if preheating is adequate. The use of mixed gas offers advantageous heating performance simultaneously with improved economy.

The Characteristics of Atmospheric-Type Burners When Used With Natural Gas

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In this paper it is determined that the design of atmospheric-type gas burners depends upon the stability of the flame produced, which is related to the speed of flame propagation in the combustible mixture. Experiments made upon single-port gas burners have served to show that the effective velocity of flame propagation is greater than the velocity of flame propagation in still mixtures. It has been found that natural gas can be burned at a rate of approximately 30 cu ft per hr per sq in. of port area. The authors summarize the matter of hydraulic design of inspirators in the form of an equation, the validity of which is demonstrated by experiment. This equation suffices to determine the mixture ratio when the gas rate and density are known.

FOR SEVERAL YEARS investigations have been under way at the University of California on the problem of designing atmospheric-type gas burners. This work was initiated by the Pacific Coast Gas Association Fellows and has lately been carried on by other graduate students and by Works Progress Administration research workers.

The particular design problem involved is that of the atmospheric- or Bunsen-type gas burner. This type burner may be regarded as consisting of two units, i.e., the inspirator and mixing tube, and the burner head or exit port. The inspirator-and-mixing-tube assembly acts as a jet pump, utilizing the kinetic energy of the gas jet to entrain the primary-air supply and force the mixture of gas and air through the burner-head passages and ports. The burner head serves to distribute the combustible mixture as desired within the furnace or space to be heated and also to form a barrier between the flame and the fresh mixture being delivered. Although the inspirator, mixing tube, and burner head are often cast in one piece they are treated separately in this paper, because the same design of inspirator and mixing tube may be used with any one of a variety of burner heads. Such a burner is shown schematically in Fig. 1. From a study of this diagram, it is evident that the combustible gas entrains some air as it leaves the jet and mixes with the primary air in the mixing tube. The mixture flows through the burner ports, beyond which the flame occurs. Additional or secondary air for completing the combustion is drawn from the space around the burner.

The design of this type of burner consists in determining (*a*) individual and total port areas; (*b*) gas-orifice diameter; (*c*) inspirator proportions and dimensions. The procedure in this paper will be to indicate the performance characteristics of the two elements, thereby placing limits on the design.

Performance of the Burner Head

The rate of heat generation possible with a burner head depends upon the stability of the flame. This in turn is a function of the primary mixture ratio, the velocity of the mixture through the ports, and the flame speed in the primary mixture. The range of stable operation of a burner head is limited by the phenomena of flash back and blowoff. A further limit to the useful range is imposed by the yellow-tipped or sooty flame. Flash back occurs when the flame speed is greater than the mixture speed through the ports so that the flame flashes back through the port openings. Blowoff occurs when the flame speed in the mixture is less than the mixture speed through the ports, thus causing the flame to blow away from the ports. The yellow-tipped or sooty flame occurs when the total air supply from both primary and secondary sources is insufficient to burn the gas along the center line of the burner port.

The useful limits of operation of a burner head are graphically presented in Fig. 2, which indicates the results of several investigations (1, 2)[3] using natural gas in single-port burners. The equipment is shown in its final form in Fig. 3. It consists of a sharp-tipped burner tube and the metering equipment necessary to determine the air-to-fuel ratio as well as the velocity of the mixture through the ports. No inspirator was used, as such equipment does not lend itself readily to accurate metering. In Fig. 3, it will be observed that air and fuel are separately metered, mixed, and then passed into the burner tube. A by-pass outlet is used so that various mixture velocities through the burner tube

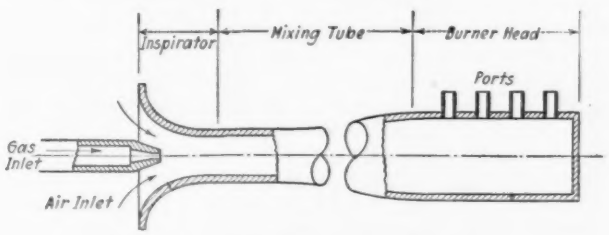


Fig. 1 Atmospheric-Type Gas Burner

may be obtained for the same total flow of mixture. This type of regulation was adopted in order to simplify the maintenance of a constant-mixture ratio. The quantity of mixture passing through the burner is metered as indicated.

The curves, Fig. 2, are the results of arbitrary definitions of the unstable phenomena. Blowoff and flash back are not critical since, under some conditions, they may be induced by slight disturbances of apparently stable flames. The definitions used are as follows:

(*a*) Blowoff is said to occur when the flame is in the act of blowing off almost continuously and when it leaves the burner within 10 to 15 sec after ignition.

(*b*) Flash back is said to occur when the flame moves slowly back into the burner port within 5 sec after being ignited.

The shape of the flash-back curve is similar to that of the flame-speed curve for still mixtures, Fig. 4. Both curves terminate at the two limits of inflammability of the primary mixture and have

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Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[3] Numbers in parentheses refer to the Bibliography at the end of the paper.

maximum values near the 100 per cent theoretical air point. That the two curves should be similar in shape is logical as will appear after some reflection. The maximum mixture speed at which flash back occurs is greater than the velocity of flame in still mixtures.

The shape of the blowoff curve may be explained in terms of flame speeds. Blowoff is an edge phenomenon, i.e., the part of the flame first to exhibit instability is that just at the burner tip. At this point secondary air enters the flame, thereby diluting the mixture and changing the flame speeds. For a rich mixture, with

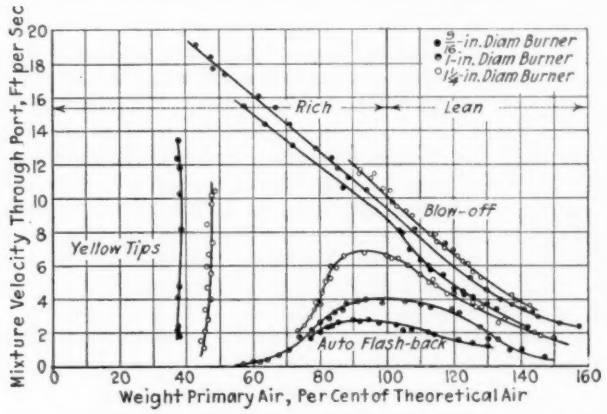


FIG. 2 PERFORMANCE CURVES FOR SINGLE-PORT BURNERS

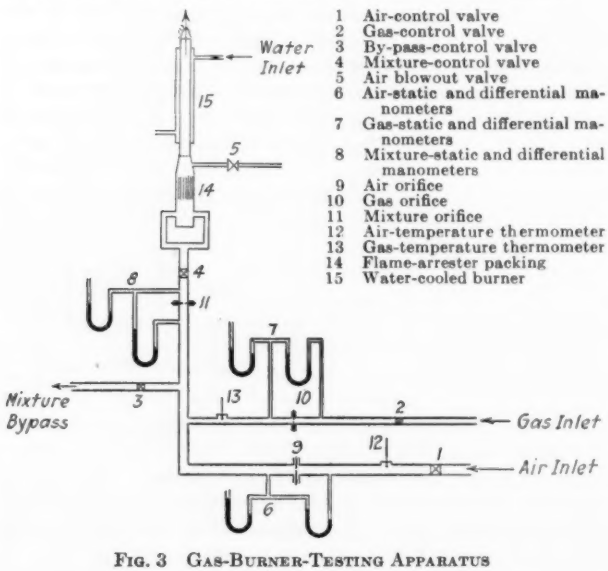


FIG. 3 GAS-BURNER-TESTING APPARATUS

a yellow-tipped flame, the effect of air entrainment is to increase the flame speed in the outer part of the flame. If the primary mixture is too rich to be inflammable, the flame speed in the mixture is zero and considerable air must be entrained to produce a flame speed great enough to give rise to blowoff. The amount of air entrained increases with the mixture speed through the ports. For primary mixtures, within the limits of inflammability, less air must be entrained for blowoff. Hence the blowoff curve should logically have the general shape shown. The yellow-tipped-flame curve should be nearly vertical, i.e., independent of mixture speed. This follows from the fact that the diffusion of secondary air toward the axis of the jet is a function chiefly of jet size, not of jet speed.

Inspection of Fig. 2 shows that the area of the region of stable operation increases as the port diameter is decreased. The boundary of the yellow tip moves toward the richer mixtures, while the flash-back curve descends more than does the blowoff curve. Studies (1, 3, 5) made with multiport burners show that the results found with these large-diameter single-port burners may be applied directly to the design of stove-top and heating-furnace burners. The primary mixture for natural gas should contain from 40 to 50 per cent of theoretical air and should issue from the ports with a speed between 0 and 8 ft per sec. Thus the design of the burner from the standpoint of port area requires a minimum of 1 sq in. of port area for each 30 cu ft per hr of natural gas burned.

The effect of secondary air upon the performance curves is shown in Fig. 5, the curves there plotted referring to the stability of the inner cone. The data were determined using the same burner and metering equipment as for the open flames. Secondary air was excluded from the burner tip by a flame separator as shown. This separator consisted of a glass tube, larger than the burner tip, placed coaxially with it and sealed to the tip at some distance from the outlet edge. This separator was similar to that ascribed to Smithells (6) and accomplished the division of the flame into two parts; the inner cone burning at the normal burner tip and the outer cone at the end of the separator tube where secondary air was available. The flash-back curve under

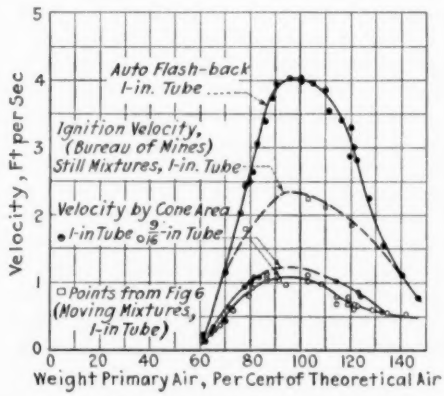


FIG. 4 IGNITION-VELOCITY CURVES FOR NATURAL GAS

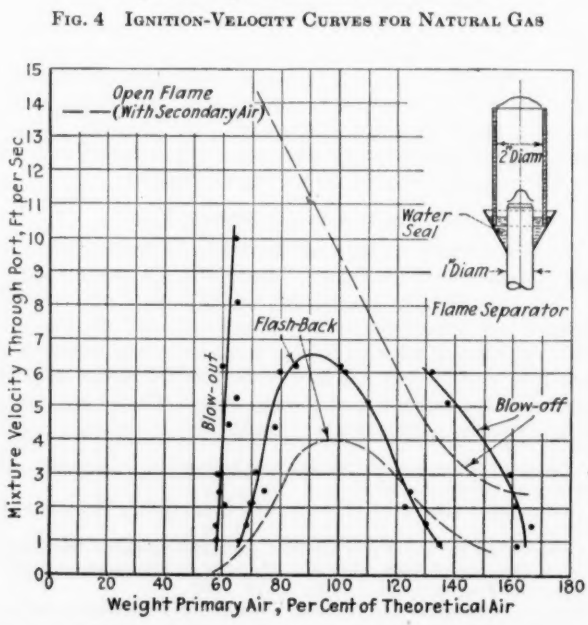


FIG. 5 PERFORMANCE OF 1-IN. BURNER EXCLUDING SECONDARY AIR

these conditions is seen to exceed that under normal conditions. The difference shown is not significant as it was possible to cause flash back at higher speeds in the open flame by disturbing the air about the flame. The blowoff curves for the two conditions differ widely in two parts. First, this curve for rich mixtures doubles back to terminate with the flash-back curve. This follows from the fact that without secondary air the flame cannot be maintained outside the limits of inflammability. Second, the blowoff curve for no secondary air lies definitely above that for the open flame in the lean-mixtures region. This fact is regarded as indicating that appreciable dilution of the mixture near the base of the inner cone does occur in the open flame. This conclusion assumes that the effect of dilution by the burned gases entrained by the primary mixture is negligible.

Extrapolation of Results to Other Gases

All of the work so far completed at the University of California has been done using Kettleman Hills natural gas, the average analysis of which is 76 per cent CH_4 and 24 per cent C_2H_6. If the performance curves for the same burner head using some other fuel mixture are to be predicted, it is necessary to relate them to the more fundamental characteristics of the fuel. For this reason the following properties of the fuel-air mixtures were studied: (*a*) limits of inflammability, (*b*) speed of flame propagation.

The limits of inflammability were obtained from extrapolation of the flash-back curves. As is shown in the appendix, these values agree well with the computed values, the comparison being 55 per cent and 190 per cent measured, as against 56.3 per cent and 186 per cent computed.

The determination of the speed of flame propagation presents a more serious problem. The several experimental methods for measuring this quantity include:

(*a*) Measurement of the speed of uniform motion of a flame through a stationary mixture confined in a long tube.

(*b*) Measurement of the average rate of flame-front propagation in a steadily burning Bunsen flame.

(*c*) Measurement of the rate of expansion of soap bubbles filled with combustible gas following spark ignition at the center of the bubble.

(*d*) Measurement of the speed with which the flame moves back through the burner tube following the occurrence of flash-back.

The flame speeds obtained by the several methods do not agree well, since the conditions are quite different. For instance, combustion for type (*a*) is nearly constant volume; that for types (*b*) and (*c*) nearly constant pressure; while that for type (*d*) is between these two other classes. Again, types (*b*) and (*c*) differ, since in the Bunsen flame secondary air is present at the flame front, thereby causing a variation of mixture ratio in the flame front, whereas the soap bubble is practically impervious to the surrounding air.

Types (*b*) and (*d*) were selected as the methods most nearly corresponding to the combustion of gas in the atmospheric-type burner. Consider first type (*b*): This method involves the simultaneous determination of the surface area of the inner flame cone (flame front) and the volume rate of flow of the particular primary air-gas mixture through this flame front. The cone area was determined from tracings of the inner-cone outline made on pieces of cellophane held against the ground glass of an enlarging camera which was focused on the steadily burning flame. This scheme assumes the inner cone to be symmetrical, a condition closely approximated in many of the tests. The flame-front area is determined from the tracings by dividing the cone area into several conical frustums and summing the resulting areas. This method gives results comparable to those of others (9) and utilizes the available facilities to the best advantage. The flame speeds so obtained are plotted in Fig. 4. Also plotted in Fig. 4 are the flash-back curve and a flame-speed curve obtained at the Bureau of Mines (10) for a methane-ethane mixture of nearly the same consistency as natural gas. This latter curve shows the speed with which the flame front moved through a gas-air mixture confined in a closed-end, 1-in-diam tube. Comparison of the two flame-speed curves shows agreement in the locations of zero and peak values. The magnitudes of flame speed as obtained by the cone-area method are definitely less than those obtained for still mixtures. Since, for rich mixtures, the entrainment of air increases the flame speed, it is difficult to account for the experimental difference.

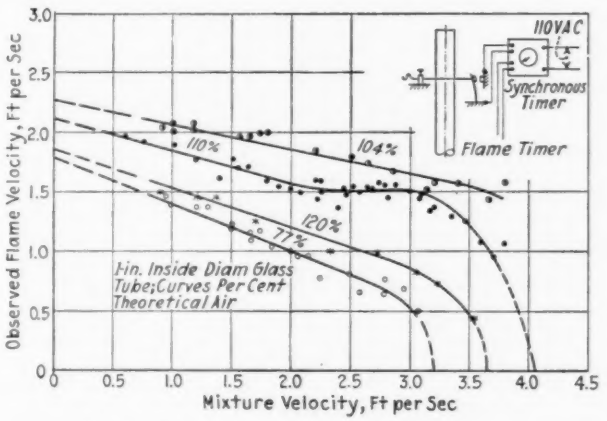


Fig. 6 Flame Velocities in Moving Mixtures

The second series of experiments was undertaken to determine the effect of motion of the mixture upon the flame speed. The flame speeds measured were determined for a flame under flash-back conditions. A long glass tube, shown in Fig. 6, was substituted for the single-port burner tip in the apparatus shown in Fig. 3. A steady flow of mixture was produced in the tube at a rate less than that for flash back. The mixture was ignited by a high-tension-spark discharge. After ignition the flame moved back along the tube at an essentially uniform speed. To check the uniformity of this speed electrical gaps were placed at 6-in. intervals along the tube. The movement of the flame past any gap tended to increase the conductivity of the gap, thereby permitting an electrical potential to drive a slight current through the grid circuit of a vacuum tube. This current was amplified and used to operate a chronograph pen. The results of this series showed uniformity of speed. Difficulty encountered with amplifiers led to abandonment of this scheme for the determinations of speed.

A second method, visual in nature, was next used for checking. In this arrangement a leather belt was mounted on two pulleys so that its length was adjacent to and parallel with the glass tube. White marks were painted across the face of the belt. In operation the belt was driven at constant speed, causing the white marks to move downward along the tube. By running the belt at several different constant speeds for one given mixture ratio and gas rate, a speed could be found and measured for which the belt and flame appeared to keep "in step."

The apparatus finally developed for observing flame speeds is shown in Fig. 6. It consists of a glass tube with two pairs of holes drilled through it. The two holes of each pair are diametrically opposite each other and the two pairs are 30 in. apart. A slender cotton thread was stretched across each pair of holes, serving to hold open a contactor. Passage of the flame would burn through the thread, thereby closing the control circuit of a synchronous timer.

The results of these tests are shown in Fig. 6. The coordinates of the curves are speed of the flame relative to the tube (as measured), and speed of the mixture. The absolute flame speed was considered unobtainable due to the fact that the mixture velocity varies from point to point along the diameter of the tube, whereas the flame front appears to move as a unit. Extrapolation of the curves to zero-mixture speed should yield values in agreement with the still-mixtures data. Points so obtained are plotted in Fig. 4, and are seen to lie close to the still-mixtures curve. It was hoped that extrapolation of the curves in Fig. 7,

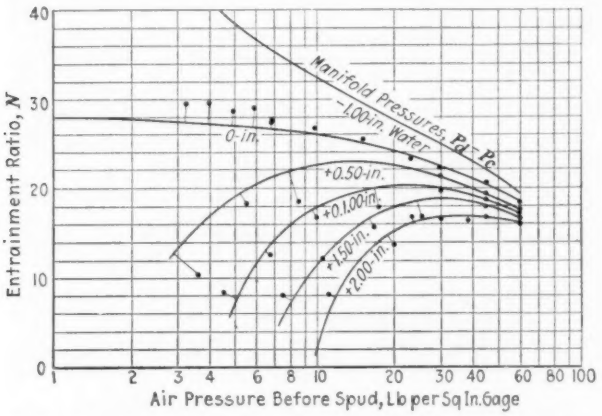


FIG. 7 PERFORMANCE CURVES FOR A VENTURI-TYPE ATMOSPHERIC INSPIRATOR
(Air operated; No. 48 spud; 1 1/8-in. throat.)

to the zero observed flame speed, would yield the flash-back velocity. As is seen from the chart, no dependable extrapolation can be made because of the dispersion of the points. The points obtained are of low accuracy as there was instability of motion. Also, it was found that, if forced into the tube entrance, the flame would move down into the tube at mixture velocities greater than that for flash back. It thus appears that stability of the open flame, from the standpoint of flash back, is increased by the secondary air. The flame shapes observed visually while the flame front was moving are similar to those in the literature (9). In most cases the flame front remained constant in shape during the passage of the flame throughout the tube. Since the flow was in most cases viscous, requiring a large change in velocity along any tube diameter, the maintenance of this constant flame front would require a variation of flame speed along the diameter of the tube. This conclusion is in accordance with the measurements.

Applying these findings to the performance curves of the burner head, a few generalizations may be made. First, the flash-back curve will be some distance above the curve for the flame speed in still mixtures. Since the latter curve is available for many gases, it will naturally form the basis for design. The velocities for flash back are roughly the squares of the flame speeds in the still mixtures for natural gas. If the performance curve for the burner head with one gas is known, the curves for any other gas may be estimated in terms of the flame-speed curves for still mixtures.

PERFORMANCE CURVES FOR THE INSPIRATOR AND MIXING TUBE

The proportions to be used in shaping the inspirator and mixing chamber must be such as to reduce hydraulic friction to a minimum and at the same time provide for sufficient mixing. These problems have been previously investigated (4, 11) and are adequately covered in the literature. The question as to how a given assembly will perform, when various gases are used and when wide ranges of pressure are applied, has not previously been fully treated. Equation [1] has been derived for this purpose and has been demonstrated to hold for one inspirator. The inspirator tested had the following dimensions:

Gas orifice, No. 48 drill size with 30-deg total angle of approach.

Throat, 1 1/8 in. diam × 6 in. long, beginning 1 1/2 in. from orifice outlet.

Expanding section, 12 in. long, expanding to 2 1/2-in. diam from 1 1/8 in. throat.

This unit was tested using air as the driving fluid. Constant back pressures were maintained at the outlet end. The driving-fluid rate was measured with a calibrated orifice and the mixture-discharge rate with a pitot tube. The results of the tests are shown in Fig. 7. The equation developed has been used to compute the solid curve in Fig. 7. This equation is

$$P_d - P_e = \frac{V_i W_i}{g A_t} + \frac{W_i^2 v_2}{g A_s}\left[\frac{1}{A_t} - \frac{1}{A_s}\right] N^2 - \frac{W_i^2}{g A_t^2}(1+N)(v_1 + N v_2)\left[1 - \frac{e}{2} + \frac{e}{2}\left(\frac{A_t}{A_d}\right)^2\right] \quad \ldots [1]$$

P_d = back pressure at burner outlet
P_e = pressure of air to be entrained just before entering induction tube
V_i = velocity of inducing fluid in jet
v_1 = specific volume of inducing fluid in jet
v_2 = specific volume of air to be entrained just before mixing
A_s = area of air passage just before mixing
A_t = throat area
W_i = weight rate of inducing fluid
N = mixture ratio in $\frac{\text{lb air}}{\text{lb gas (or inducing fluid)}}$
e = efficiency with which kinetic energy of mixture is converted into pressure head

The derivation (12) of this equation is based upon the usual energy equation for hydraulics. The flow may be regarded as incompressible when the driving fluid is first dealt with after leaving the orifice.

Inspection of the curves shows satisfactory agreement between the experimental and computed values. The curve applicable to the stove-top or heating-furnace burner is the one with zero manifold pressure, the efficiency being lowered to include the losses in burner-head passages and through the burner-head ports. This curve shows a constant mixture ratio at low-speed pressures, as has been found by previous investigators (12). At high gas pressures, the mixture ratio decreases, i.e., the mixture becomes richer in fuel. This latter fact probably accounts for the improved operation of certain burners when the gas-supply pressure is changed from low to high.

Applying this information to the design of atmospheric-type burners, it is seen that the equation derived may be applied to a particular case with confidence. The performance of the burner as a whole, from wide-open to closed gas cock, will produce a constant mixture ratio for gas-supply pressures less than 1.5 lb per sq in. gage. Changes in the fuel used will result in predictable changes in the air-fuel mixture produced. The adjustable air shutter on most burners will cover a large range of the fuels encountered.

ACKNOWLEDGMENTS

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Appendix

Limits of Inflammability of Natural Gas

The method used to determine the limits of inflammability of natural gas is that of Le Chatelier as outlined by Jones (7). The formula is

$$L = \frac{100}{\sum \frac{p}{n}}$$

where L = per cent gas in limit mixture
p = per cent by volume of gaseous constituent on air-free and inert-free basis
n = per cent by volume of same constituent at limit of inflammability with air when it alone is present as fuel

The limits of inflammability of single gases in air are obtained from experimental curve (7, 8).

$$L_1 = \frac{100}{\frac{76}{5.24} + \frac{24}{3.22}} = 4.55 \text{ per cent gas in air}$$

$$L_2 = \frac{100}{\frac{76}{14.02} + \frac{24}{12.45}} = 13.6 \text{ per cent gas in air}$$

Expressed in terms of per cent theoretical air $L_1 = 186$; $L_2 = 56.3$.

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Burning Pulp-Mill Waste From Sulphite Mill

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After first outlining the problem of stream pollution, as applied to sulphite pulp mills, this paper describes the recovery of lignin from digester waste and the elimination of stream pollution by the Howard process. This process produces a lignin fuel which can be and is burned in power-plant furnaces for the production of steam. Mechanical problems, such as the development of a filter press and various methods of burning the fuel so produced, are explained.

THIS PAPER relates to the use of industrial wastes as boiler fuel and specifically concerns some developments recently carried on under the author's supervision in burning, as a boiler fuel, a lignin organic product recovered from waste sulphite liquor at the Marathon Paper Mills Company, Rothschild, Wis., by the Howard process of treating such liquors.

Waste sulphite liquor results from the making of paper pulp by the acid-sulphite process. As drained from the pulp in the blowpits, these liquors are reddish brown in color and contain the organic matter dissolved from the wood in the pulping process. This dissolved organic matter represents the noncellulose constituents of the wood which are in part lignin substance and in part carbohydrate or sugar substances derived from the hemicellulose components of the wood. The yield of air-dried pulp is 45 to 50 per cent of the air-dried weight of the original wood, hence the amount of dissolved organic matter carried in the waste sulphite liquor is approximately 1 ton of such dissolved organic matter for each ton of pulp produced.

The annual production of sulphite pulp in the United States is around 1,400,000 tons and at least an equivalent tonnage of dissolved organic matter is carried in the waste liquors from such pulp production. This large tonnage of organic matter, in such waste liquors which now are almost entirely discharged into sewer outlets at the pulp mills, invites utilization to recover products of inherent value. It has become more and more necessary at least to find economical means of disposal for such liquors to avoid pollution of lakes and streams into which they are discharged.

Only limited amounts of such liquors are now being utilized in the manufacture of commercial products; such as, tanning materials, core binders, road-dressing materials, etc. Various proposals and attempts have been made to effect disposal of these liquors to avoid stream pollution; e.g., aeration, ponding, and concentration by evaporation to a fuel product. None of these has thus far afforded a satisfactory solution to this disposal problem and a discussion of their possibilities is outside the scope of this paper, save briefly to mention the evaporation of these liquors to a fuel product. In such procedures the liquors, either with or without neutralization, are evaporated in multiple-effect evaporators to a concentration of around 50 per cent solids to give a liquid product which can be burned as a boiler fuel. This procedure is handicapped by several inherent difficulties and has not afforded an economical disposal method. The liquors as draining from the pulp in the blowpits are necessarily mixed with wash water, hence it is not practical, with existing standard equipment, to collect more than about 60 per cent of the total liquor in sufficiently concentrated form to permit economical evaporation, even with multiple effects. The liquors are acid and, even after neutralizing, corrosive gases are evolved during evaporation; hence expensive acid-resisting evaporation equipment is necessary. Incrustation of evaporating surfaces is encountered and the high viscosity of the concentrated liquor results in low heat-transfer rates and the necessity of forced circulation in the evaporators.

On the other hand the Howard process is a patented fractional-precipitation treatment of these waste liquors which does not involve evaporation. This permits treating more dilute liquors than can be economically evaporated and allows processing a gallonage of liquor sufficient to carry 90 to 95 per cent of the total organic matter dissolved in the pulping process, the remaining amount being carried in wash liquors too dilute to warrant processing. This process has been under study and development at the Marathon Paper Mills Company, for the last ten years, under the direction of Guy C. Howard, an experienced chemical engineer who originated the process. A commercial plant is now in operation for treating all of the waste sulphite liquor at this pulp mill by this precipitation process. The objectives in the development of this process have been to find a disposal method for these liquors, which could be used at our own and other pulp mills, to avoid stream pollution through discharging such liquors into waterways, and also to develop means of utilizing the lignin and other constituents of these liquors as chemical raw materials for the production of various products of commercial value.

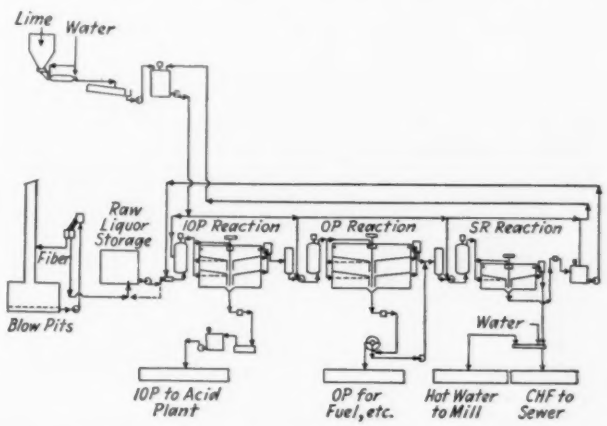


FIG. 1 FLOW DIAGRAM OF HOWARD PROCESS FOR TREATING WASTE SULPHITE LIQUORS

The Howard process is a three-stage precipitation treatment of the waste sulphite liquor which, as shown in the flow diagram, Fig. 1, may be briefly described as follows: The hot waste liquors are collected from the blowpits into raw-liquor storage tanks for equalization in concentration and then pumped through three reaction tanks in series, Fig. 2, in each of which a lime reagent in controlled amounts is added to the liquor. After each reaction tank, the liquor passes through a settling tank of the Dorr type, Fig. 3, for removal of a precipitated product. From the first settling tank an inorganic sludge product, containing

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NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

calcium sulphite, is removed as a slurry and pumped to the acid plant for making fresh cooking acid, thereby affording a credit to the process for its lime and sulphur values. The precipitate settled out in the second settling tank is an organic product which is removed as a sludge and filtered on a rotary vacuum filter of the Oliver type, Fig. 4, to give a filter cake which is the lignin fuel product with which this paper is particularly concerned. The precipitate in the third settling tank is a mixture of precipitated organic matter and unconsumed lime reagent. This mixed precipitate is removed as a sludge which is returned to the process as the lime reagent added to the liquors in the first reaction tank. The overflow liquor from the third settling tank is the process effluent which is passed through a liquor-to-water heat exchanger for recovery of its heat into wash waters used in the pulp mill and thence is discharged to the sewer. From 3500 to 4000 gal of waste liquor is processed per ton of pulp and this gallonage carries 90 per cent or more of the organic matter dissolved from the wood in the pulping process. The gallonage of the process effluent is approximately that of the waste sulphite liquor fed to the process but the original raw liquor has been materially altered by the hot-alkaline treatment to which it has been subjected by this processing. About half of its organic-matter content has been precipitated and removed; the sugars

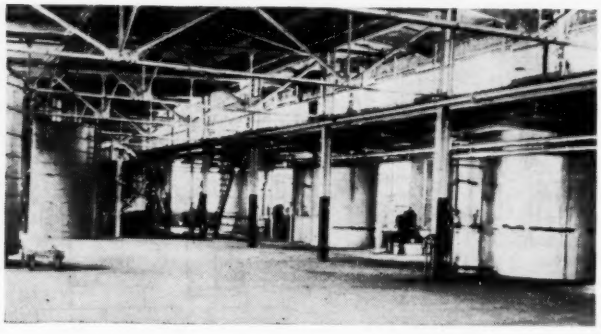


FIG. 2 REACTION TANKS FOR EQUALIZING RAW LIQUOR IN CONCENTRATION

in the raw liquor have been broken down to unfermentable derivatives; the original acid raw liquor has been made alkaline with lime; and a material portion of the sulphur content has been removed. As a result, the biochemical oxygen demand (BOD) of the process effluent, which is the criterion of its stream-pollution characteristics, has been reduced 80 to 90 per cent in comparison with the BOD of the untreated liquor. The process effluent therefore represents a greatly improved liquor from the standpoint of stream pollution and, under most pulp-mill conditions, can be safely discharged into sewer outlets.

The process thus affords a disposal means for handling such waste liquors which is self-contained in that its recovered products can be used by the pulp mill for making fresh cooking acid and as a boiler fuel, without the necessity of outside marketing of products. Further, the process is applicable to practically all sulphite pulp mills as an economical method of avoiding stream pollution.

The production of this lignin fuel product will range from 1000 to 1250 lb dry weight per ton of pulp produced, depending on the kind of wood used and details of the cooking operation. This means the recovery of 50 to 65 tons per day of such fuel product from the waste sulphite liquor, resulting from a daily production of 100 tons of pulp.

Some of the lignin filter cake, coming from the rotary vacuum filter at Marathon, is used as a chemical raw material for the production of special lignin products, but a considerable tonnage of it remains to be used as a boiler fuel. At installations of the process in other pulp mills, practically all of this filter cake will have to be utilized as a boiler fuel because of its large tonnage and insufficient market for special products made from it. Since the economics of the process depends largely on the fuel value of this filter cake as a credit to the process, we were confronted with the necessity of finding the most economical method of utilizing it to yield maximum heat value.

The wet filter cake is a light chocolate-colored colloidal organic material in flocculated condition which dries to a friable black granular product. It is essentially a basic calcium salt of lignin-sulphonic acid together with some nonligneous organic impurities carried in the mother liquor of the wet cake and is

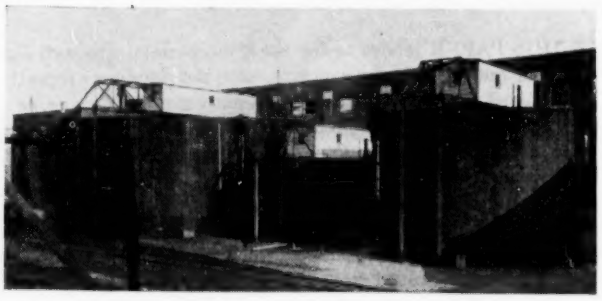


FIG. 3 DORR TYPE SETTLING TANKS

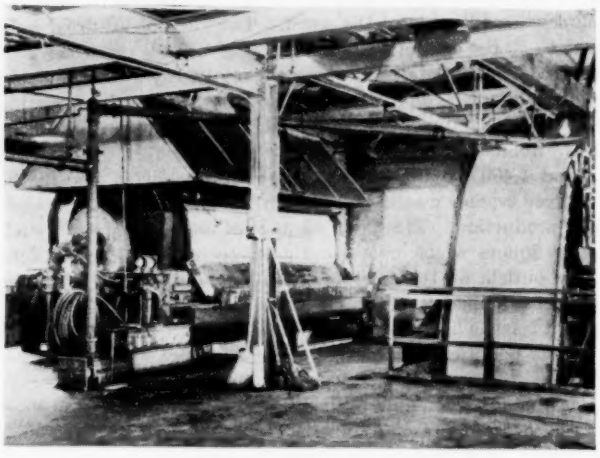


FIG. 4 ROTARY VACUUM FILTER, PUMP TO PRESS, AND TOP OF FILTER PRESS

alkaline with caustic lime. Its ash content by ignition is about 18 per cent. It has a calorific value of 8000 to 8500 Btu per lb dry basis. At its normal alkalinity of about pH 11, it filters readily to a cake containing 68 to 70 per cent moisture. An 8-ft diameter × 12-ft face rotary vacuum filter, Fig. 4, is sufficient to handle 65 tons dry weight of this cake per day. Various endeavors with the present filter have been made to reduce the moisture content of the cake but without success, because of its colloidal character and tendency to shrink and check on the filter drum. A cotton-duck filtering medium is used and the filtrate is clear. At this moisture content the cake discharges cleanly in flake form and appears quite dry but, on mixing, it readily puddles to a semifluid paste which can be pumped by a screw pump.

The material leaving the vacuum filter is now ready to be made into chemical products or fuel. The moisture content, however, is too high to burn for the production of useful heat.

Early experiments conducted with small amounts of dried material, led us to the belief that we could burn it usefully, if we could reduce the moisture content to 50 per cent. These experiments also indicated that, if the moisture content were reduced to 58 per cent or less, the material would no longer puddle. One of our greatest problems was to find the most economical method whereby the moisture could be reduced to 50 per cent and, at the same time, 65 to 70 tons dry weight could be handled per 24-hr day. This latter tonnage requirement and the nonfibrous colloidal character of the filter cake eliminated any press of standard design.

The methods given consideration were:

1 Partially or wholly drying the filter cake in fuel-fired direct-heat driers of the rotary or other type. This is practical to do but is expensive because of the large moisture content of a 70 per cent cake, with a consequent high fuel requirement to evaporate this moisture.

2 Partially or wholly drying the cake in a steam-heated tubular rotary drier, such as is in common use for drying brewery products. Economical drying can be accomplished by this means, provided the drier can be located alongside the boiler in which the fuel product is to be burned, thereby permitting the return of the high-pressure condensate to the boiler; saving the sensible heat in the fuel product as it discharges from the drier; and recovering heat from the vapor discharge of the drier. We have dried a considerable tonnage of wet filter cake in this type of drier satisfactorily.

3 Partially or wholly drying the cake with boiler-stack or other waste-heat gases. This is theoretically an economical procedure since no charge is necessary for the waste-heat gases. However, the available heat for drying in stack gases at 300 to 400 F, as produced from modern boiler plants, is rather low, consequently, large volumes of such gases would be required even partially to remove the high water content of a 70 per cent cake.

4 Removing water by pressing the cake mechanically. This would seem the most economical method of reducing the moisture content of this fuel cake, provided it could be accomplished. We therefore concentrated our efforts on this procedure.

Laboratory pressing tests indicated that the moisture content of this filter cake could be reduced to 50 per cent moisture on pressing at 200 lb per sq in. with 1 min retention under pressure for a 1-in. cake. At 70 per cent the cake contains 2.33 parts of water to 1 part dry solids and at 50 per cent contains 1 part water to 1 part dry solids, hence this relatively low pressure had removed over half the water content of the filter cake. Higher pressures failed to reduce the moisture content materially below 50 per cent, from which it was concluded that only the free water had been removed and that the moisture content of the cake could not be reduced lower than 50 per cent by mechanical pressing. At 50 per cent moisture the cake crumbles readily and can be burned alone or mixed with coal.

After much investigation and experimental work, we decided to make use of the principle in a press that had been designed by R. M. Thompson. We felt that, as constructed by him, it was too complicated and cumbersome for our needs. Incidentally, this is the only major piece of equipment required by the process which we were unable to find on the market.

In principle this press, Fig. 5, is a vertical, wedge-shaped chamber in which the material is pressed in its downward travel between two belts approaching one another. The belts are endless and tractor-like in construction and are made by attaching grid sections, Fig. 6, to three roller sprocket chains. The grid sections are 48 in. long, which constitutes the width of the belt, and are 6 in. wide in the direction of travel, with perforated

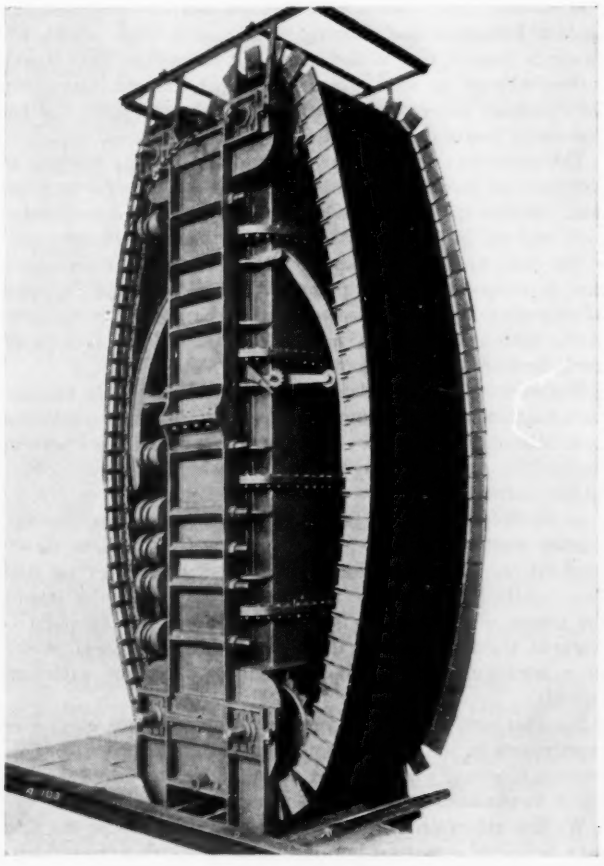


FIG. 5 FILTER PRESS

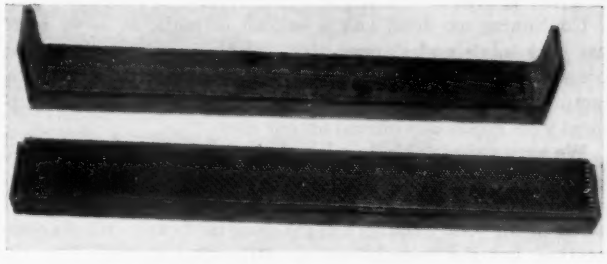


FIG. 6 MALE AND FEMALE GRIDS OF PRESS

metal faces. One of these belts is supported on a three-sprocket shaft at the top of a vertical supporting member and travels vertically downward to a drive sprocket at the base of this supporting member, thence returning to the top sprocket. The rollers in the belt chain run on tracks in this vertical supporting member.

The other chain is supported and driven in a similar manner, save that the top sprocket is offset somewhat to give an inclined travel to the belt downward, and the lower drive sprocket is carried in a floating bearing to move against spring tensions with increased clearance between the belts at the bottom, i.e., with increased thickness of cake.

The belts are driven by a $7^1/_2$-hp motor through reduction gears and a flexible coupling on the floating shaft in a manner similar to the roll drive used in steel mills.

The grid sections on one belt have flanged edges which overlap the grids on the opposite belt to give edge closure for the compres-

sion chamber and rubber astragals are used to give tightness against leakage. The filtering mediums through which the liquor is pressed out of the cake are two endless, loose-woven cotton belts 48 in. wide, which travel with the grid belts downward through the compression chamber and are supported on the perforated face of the grid belt.

The press was designed for a maximum working pressure of 250 lb per sq in. on the material and for a belt travel of 12 ft per min. At this speed the press would handle 70 tons dry weight of such cake per 24 hr at 1 in. thickness for the pressed cake.

The cake, as it comes from the filter at 68 to 70 per cent moisture, is pumped by a screw pump to the feed hopper of the press. After passage through the compression chamber, it is delivered at the bottom of the press in a cake 48 in. wide by about 1 in. thick, having a moisture content of 50 to 55 per cent.

Numerous mechanical defects which developed in the machine have been corrected, but nothing has occurred which would lead us to believe that the theory is wrong. The machine is operating successfully and producing a filter cake which averages close to 50 per cent moisture.

In our investigations of reducing the moisture content, we also studied waste-heat driers of several types, as well as direct-fired driers. Two obstacles prevented our going very far with these methods. First, the fact that for our particular layout, the source of waste heat is some distance from the point of origin of the fuel; second, the vacuum filter press cake reverts to a semiliquid condition and handling would be extremely difficult.

Simultaneously with our press development, we carried on experiments in the boiler room using both filter cake from the vacuum filter and material that we had dried to various degrees, which we obtained from our pilot plant.

We first attempted to burn a mixture of wood refuse and filter cake in one of our wood-burning furnaces which have ordinary Dutch ovens with flat grates and undergrate blowers—no result. Next we tried filter cake and various mixtures of coal—again no satisfactory result.

Continuing we dried half a carload of material to 85 to 90 per cent solids and shipped this, together with a quantity of press cake from the Oliver filter to the plant of a stoker-manufacturing company, the facilities of whose experimental equipment were generously offered for our use.

We were successful in burning the dried material in pulverized form in a rotary acid-sludge burner, and on a forced-draft chain-grate stoker. We tried to burn 30 per cent solids and 70 per cent water in the rotary burner and got along fairly well with the combustion, but the material was so sticky that it plugged up the feeder. We did find that, if the material were dried to about 50 per cent solids, we could get it through the feeder and handle it in the burner.

The combustion characteristics of this lignin fuel product are that, since it apparently contains no low-temperature volatiles, it is sluggish and slow to ignite in comparison with bituminous coal but that, at and above some critical decomposition temperature, the lignin material gasifies directly and completely into what are probably carbon monoxide and hydrogen, which burn readily above the fuel bed.

We also found in these experiments that the ash consisted of small white particles, which upon analysis, proved to be largely quicklime in sufficient quantities to produce a theoretical credit (if use could be made of it) in the acid plant at the beginning of the cooking process. However, it was contaminated with other calcium salts which must be kept out of the cooking acid.

After the foregoing experiments, we made a number of attempts to burn this material on our own underfeed stokers, together with coal. The principal result of these experiments was to find out that, when we had approximately one-fourth lignin and three-fourths coal, severe slagging occurred.

Investigation was also made with an inclined-grate Dutch-oven furnace for wood burning. If the material were wet, it would stick at the top of the grate and, if it were dry enough not to stick, it rolled to the bottom and burned so unevenly as to be unsatisfactory.

Mixtures of lignin material with fuel oil and with pulverized coal were also tried. The former experiment resulted in burning the oil and leaving the press cake in the furnace. No results were obtained with the coal mixture.

In the meantime, in order to improve our boiler room both as to capacity and efficiency, we had installed several spreader-type stokers. We then commenced experimenting with lignin fuel on these stokers and found, after a number of attempts, that, so long as the fuel was in the neighborhood of 50 per cent solids, we could use a mixture of approximately 15 per cent coal and 85 per cent lignin, operating continuously at from one to one-half times boiler rating. Some investigators of the waste-liquor problem have called attention to the possibility of damage to the boiler from corrosion. We are glad to say, however, that, after two years of burning this material, most of it in one boiler, we have found no indication of trouble arising from this source. This is apparently due to the alkaline character of the fuel ash.

Knowing that some mills, which would become interested in this process, would be equipped with coal pulverizers and, having learned by experiment that the material could be readily handled in the dry state in a pulverizer, we have gone to considerable length in attempting to burn the lignin by this method. It has been tried in five different plants with three different types of pulverizers. These tests indicate that, when it is only 50 per cent dry, it cannot be handled at all in a mill with close clearances but can be handled in an open-type mill. In a mill of this type, the capacity is reduced and therefore the horsepower per ton becomes high as compared to coal. For this reason we have, for the time being, dropped further experiments with pulverizers although it has not been decided to abandon them entirely.

So far as operating troubles go, it was stated previously that we have had no boiler-steel corrosion. Undoubtedly, if the furnace lining were acid, it would flux rapidly with the ash. Our firebrick comes from the Missouri field and has given the usual service life.

Due to the high lime content of the ash from this fuel, we have had to be more than ordinarily watchful of dust leaks from breechings and flues, because it is unpleasant to get any of this dust in one's nostrils. Again, we remove ashes with a steam vacuum system and have found it necessary to keep the ash dry to prevent the formation of mortar in the ash tank.

The final question to answer with a product of this nature is: At what efficiency can it be burned? To answer this question we engaged the services of Paul Pool, consulting combustion engineer for the M. A. Hanna Company, Cleveland, to conduct a boiler test with this fuel.

In order to eliminate any question as to how long a boiler could be operated with this material, it was decided to make the length of the test 36 hr. As no facilities for weighing the feedwater were available, a recently calibrated Bailey steam-flow–air-flow meter was used to give the weight of steam generated. Some difficulties were experienced in determining flue-gas analyses, due to the fact that the chemicals in the Orsat were rapidly rendered ineffective. Work is being done in our laboratory to determine the reason for this. Due to the uncertainty in the flue-gas analysis, the heat balance calculated is probably incorrect. However, the fact remains that, in order to have evaporated enough water to generate an average of 185 per cent rating with the

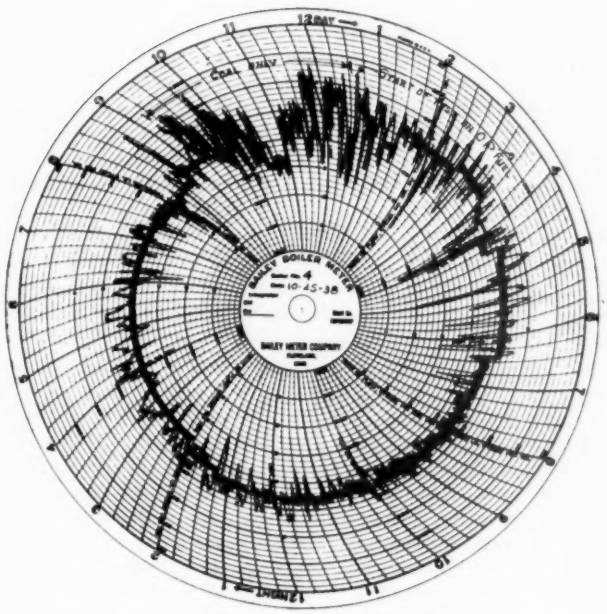


FIG. 7 CHART FROM STEAM-FLOW AIR-FLOW METER

weighed fuel supplied the stoker, the efficiency must have been quite close to that obtained. Expressed another way, we produced an average of 26,039 lb of steam per hr from a 386-hp Stirling boiler, equipped with a stoker having 96 sq ft of active area and a furnace volume of 1003 cu ft.

When operating at the rating just mentioned we were using all of the fuel which the recovery plant was then producing. Indications were that this rating could have been increased had there been more of the fuel available. At the beginning of the test, we used a fuel mixture of 15 per cent coal and 85 per cent lignin by weight as received. This was later changed to approximately 8 per cent coal and 92 per cent lignin, at which mixture the test was continued to the end with a much better steam chart. Fig. 7 is a chart from the steam-flow–air-flow meter covering the first portion of the test. The first 3 hr of this chart show the performance with coal only. The next 3 hr show a mixture of 15 per cent coal and 85 per cent lignin. The next $2^1/_2$ hr show the period during which we were gradually reducing the coal to 8 per cent, and the remainder, the performance with 8 per cent coal and 92 per cent lignin at which mixture the test was completed.

It should be noted that in Table 1, under proximate analysis, no figures are given for fixed carbon or volatile matter. The reason for this is that, so far as we have been able to determine, there is no fixed carbon in this fuel. Carbon is undoubtedly present

TABLE 1 ANALYSIS OF LIGNIN FUEL AND RESULTING ASH

	As received	Dry
Proximate analysis:		
Moisture, per cent	45.97	..
Ash, per cent	10.40	19.23
Heat value, Btu	4378	8103
Ultimate analysis:		
Carbon, per cent		46.68
Hydrogen, per cent		4.11
Nitrogen, per cent		.53
Oxygen, per cent		23.28
Sulphur, per cent		4.12
Ash, per cent		21.28
Composition of ash:		
Caustic CaO, per cent	26.43	
Caustic MgO, per cent	2.01	
$CaCO_3$, per cent	12.80	
CaS, per cent	15.50	
$CaSO_4$, per cent	24.40	
Fe_2O_3, per cent	5.54	
Al_2O_3, per cent	2.07	
Carbon, per cent	12.05	
Total accounted for, per cent	100.80	

TABLE 2 ECONOMIC RESULTS OF BURNING LIGNIN FUEL AND COAL

Actual evaporation, lb steam per lb fuel as fired	3.64
Factor of evaporation	1.045
Equivalent evaporation per lb dry fuel, lb steam per lb dry fuel	6.48
Efficiency of boiler, furnace, and grates, per cent	64.35
Efficiency of boiler, furnace, and economizer, per cent	67.77

but, as mentioned previously in this paper, we believe that, when the fuel reaches some distillation temperature, it breaks down into several gases, probably the most prominent being carbon monoxide and hydrogen.

The boiler on which this test was conducted has no provision for preheated air. We have, however, come to the conclusion that preheated air should be tried and believe it to be within the realm of possibility that, with preheated air, we could reduce the amount of coal required still further, perhaps even discontinue it entirely.

Another point under investigation, at the time this paper is being written, is that of furnace volume. We believe it to be quite possible that part of our difficulty with the Orsat equipment is due to some of the gases generated from this fuel being mechanically carried through the setting before they have had a chance to burn.

It will be noted that an efficiency of 64.35 per cent is quoted for the lignin fuel as burned. This means that we have recovered 64 per cent of the available heat in the solids leaving the digesters which were precipitated; however, as stated previously all of the organic matter has not been precipitated, so that the heat recovery of the total solids discharged from the digester, is lower than the figure just mentioned. Just how much, we do not know because we do not know what sort of fuel this remaining organic matter would make.

In closing, the author wishes to acknowledge assistance in the preparation of this paper from Guy C. Howard, inventor of the process, and George H. Urban, who is in charge of the boiler room.

Bagasse Furnaces

By E. W. KERR,[1] BATON ROUGE, LA.

This paper deals with the design of furnaces for burning bagasse, a woody product resulting from the removal of sugar juice from cane. Its calorific value when dry is about 8300 Btu, which is reduced considerably because of the moisture content averaging about 50 per cent. Because of the high moisture content large combustion-chamber volume is essential and the net boiler efficiency is limited. However, in a properly designed and balanced sugar factory, the heat from the bagasse will generate the necessary steam for operating it although some additional fuel (usually oil) is required during mill stops. Practically all makes and types of boilers are used for burning bagasse, water-tube boilers predominating. The author classifies the types of bagasse furnaces as grate, hearth, and combination grate and hearth. Examples of each are described and tests covering a wide range of furnace types over a long period of time are cited. The best installations range from 55 to 60 per cent boiler and furnace efficiencies; average practice about 50 per cent. A furnace developed recently by the late G. P. Ward has given an efficiency of 66 per cent.

BAGASSE or megass, as the English call it, is the woody matter remaining after the juice has been removed from sugar cane by mills. It is composed principally of fiber and moisture together with small quantities of sugar and other solids including dirt. The amount of sucrose in bagasse depends upon the number of crushings, the maceration and the pressure applied, especially on the last crushing. For example, a nine-roller mill (three crushings) will give bagasse with, say, 2.75 per cent sucrose, whereas with four crushings the sucrose in bagasse may amount to 2.25 per cent or less.

The amount of moisture in bagasse is approximately the same for different numbers of crushings, provided the pressure at the last crushing is the same in each case. The moisture varies somewhat also with different varieties of cane. The bagasse from hard-fiber cane is apt to have a lower moisture content than that from soft-fiber cane. One of the Louisiana-cane varieties not only has low fiber, with a correspondingly small weight of bagasse, but also the fiber is soft and retains more moisture. The result is that this bagasse is difficult to burn, causing the firemen to conclude erroneously that it has a lower heating value per unit weight of fiber.

In making calculations of the heating value of bagasse, it is customary to assume a moisture content of roughly 50 per cent, although modern mills produce bagasse of lower moisture than this, in fact quite commonly 45 to 48 per cent. It has been noticed that windrowed cane, if left in the fields for a considerable length of time, results in bagasse that is soft, high in moisture and difficult to burn. In addition to the fact that the net value is less as the moisture increases, excessively moist bagasse is hard to burn because it lies heavily on the grate or hearth, with the consequent difficulty of supplying sufficient air for combustion.

The amount of ash in bagasse (principally from dirt) depends upon the cleanness of the cane, which in turn depends largely upon the methods used for handling it in the fields. In Cuba and in other tropical countries, the cane is usually very clean, being loaded mostly by hand. In Louisiana, where much cane is handled mechanically, dirt may be picked up with the cane, especially in muddy fields. The amount of ash and for that matter sucrose has considerable effect upon the tendency to form clinker in the furnace. In relation to combustion space, this is of importance also, since it determines the time interval necessary between cleanings of the combustion chamber, including the space back of the bridge wall. In some cases, the combustion chamber behind the bridge wall is made very high in order that large amounts of ash may accumulate, before the remaining space is choked sufficiently to necessitate cleaning. In fact, one of the principal problems in designing a good bagasse furnace is to eliminate or reduce the amount of fly ash passing over the bridge wall into and underneath the boiler.

The woody fiber is, of course, cellulose. Ultimate chemical analyses made on bagasse from a Cuban factory gave the following composition:

	Per cent
Hydrogen	5.8
Carbon	46.31
Oxygen	45.82
Nitrogen	0 26
Ash	1.81

Calorific tests of bagasse by means of calorimeters have been made by various investigators, including some by the author. The values vary all the way from about 8000 Btu to 8600 Btu, the average being about 8300 Btu per lb of dry bagasse. With a 50 per cent moisture content, the gross calorific value of 1 lb of bagasse (moist) entering the furnaces will be approximately one-half this or, say 4000 Btu.

When the cellulose burns, hydrogen is formed, which in turn ignites and forms water. Therefore, the heat of combustion of the bagasse is not only required to evaporate free moisture, and raise it to stack temperature but also an added quantity resulting from hydrogen combustion. The net result is a reduction in the heat available for boiler absorption to about 2300 Btu.

Bagasse as it comes from the mill is light, weighing roughly 10 lb per cu ft in a pile. It is relatively easy to burn with sufficient air properly applied. Due to the high moisture content, a large combustion space, surrounded by linings of high-temperature refractory, is required for efficient combustion. The gases of combustion being so greatly diluted with steam (from the moisture), proper turbulence is especially necessary to insure against stratification and incomplete combustion. It is a simple matter to burn bagasse with low excess air in either grate or hearth furnaces of proper proportions. The author has found CO_2 as high as 18 per cent with negligible amounts of CO; in fact, 16 per cent CO_2 is common for good results. Due to the large combustion space, the heat release per cubic foot of furnace volume is relatively small. Actually, the best bagasse-furnace designs of recent years have a heat release per cubic foot of furnace volume in the order of 20,000 Btu per hr. This is very low as compared with the heat release in modern furnaces burning such fuels as coal, oil, and natural gas at high ratings, where a release as high as 100,000 Btu per cu ft of furnace volume per hr is attained.

The weight of bagasse per ton of cane varies with the fiber content which approximates 10 to 15 per cent. In Louisiana

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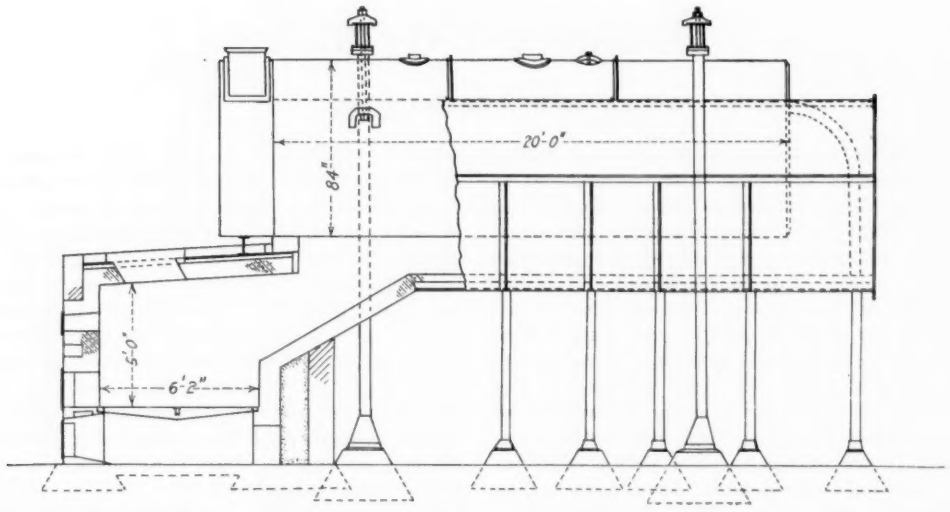


FIG. 1 HORIZONTAL RETURN-TUBULAR 250-HP BOILER AT VERMILLION SUGAR FACTORY WITH INSULATION AND STEEL CASING

where 13 per cent fiber in cane is a good figure, about 550 lb of bagasse (moist) is produced per ton of cane. For a factory of 1000 tons daily capacity, the bagasse would develop roughly 1500 boiler hp. In a properly designed and balanced sugar factory, the heat from the bagasse will produce the necessary steam for operating it. However some extra fuel (usually fuel oil) is nearly always necessary during mill stops, including weekly shutdowns for cleaning boilers, evaporators, etc., even when bagasse storage is provided. Provisions for burning the oil must be made either in the bagasse-burning boilers or in separate boilers.

Incidentally, one of the problems of the sugar-factory designer is to shorten the length of shutdowns for cleaning. However, up to the present time, the average factory has to be shut down periodically to clean the evaporators and other equipment in the fabricating department, whether or not it is necessary on account of the boilers. It is difficult to burn oil efficiently in the same combustion chamber with bagasse. Air supply through checker-work, even when located in the combustion chamber (other than the bagasse chamber), is also unsatisfactory due to fly ash and cinders. The tendency, therefore, is toward the use of steam- or mechanical-atomizing burners with registers, located in the side or rear walls or in some cases in a tunnel, for admitting air around the burners. It has been rather customary to install steam-atomizing oil burners through a 4-in-diameter hole in the front wall of the Dutch oven, rather high and to one side, in order not to spray the oil on the bagasse. However this does not permit a sufficient air supply; in fact, any considerable oil fire at this location is destructive of the furnace lining.

Boilers, in which bagasse fuel is fired comprise practically all types and makes. Most of them, when properly furnaced and operated, give good results. However, with a tendency toward large factories and higher ratings, the watertube type has gained in popularity. Large boiler units have proportionately less radiation loss. Operating at high ratings, a lower initial cost results.

The author was connected for many years with a large company in Cuba which had a total of over 100,000 boiler hp in its factories, 40 per cent of which was in straight-tube, watertube boilers (principally Babcock & Wilcox). The remaining 60 per cent was in horizontal-return-tubular boilers most of them 8 ft in diameter. In general, it was true of these factories that the average fuel-oil consumption for the watertube and horizontal-return-tubular boilers did not differ materially.

The steam requirements and oil consumption of sugar factories are affected largely by the design of the factory, including such factors as: Insulation of piping; balance in the production and utilization of exhaust steam; efficiency of heating and evaporating equipment (number of bodies, etc.). Nevertheless, the efficiency of the boiler plant is the most important factor in the matter of extra fuel consumption. In other words, if maximum efficiency is attained in the combustion of the bagasse and the conversion of its heat into steam, there will be slight need for extra fuel.

Reference has already been made to the fact that the bagasse from a 1000-ton factory should develop around 1500 boiler hp. This load, if developed with oil as fuel, would require some 14,000 gal of fuel oil per 24 hr. This indicates the large fuel requirement involved and emphasizes the importance of providing suitable furnaces and operating them at maximum efficiency. Originally, the oil consumption in the several factories in Cuba, with which the author was connected, averaged between 3 and 5 gal per ton, but, after several years, it was reduced to about 0.15 gal. Much of this reduction was effected through changes made in the design and in the operation of the boilers and the furnaces.

Coming now to the design of bagasse furnaces, the designer has to provide for the following: (*a*) Combustion volume sufficient for complete combustion of the moist bagasse at the desired rating; (*b*) a reasonable initial cost of boiler and furnace, keeping in mind that the average sugar factory in Louisiana operates only 2 to 3 months per year and in the tropics 3 to 5 months; (*c*) a reasonable radiation loss from walls of furnace and boiler setting; and (*d*) provision for cleaning and removing ash and cinders. Evidently the brick surface for a given horsepower rating is relatively high for a bagasse boiler, the furnace of which extends far in front of the boiler. Also increased combustion volume is obtained at the expense of increased radiation loss and vice versa.

Various means have been used for reducing the radiation loss. Among these occurs the use of a steel casing as shown in Fig. 1, a lining of asbestos being applied between the brick and the steel casing. Many of the installations in Cuba of the general type shown in Figs. 2 and 3 have insulation brick $4^1/_2$ in. in thickness in the side and front walls of the furnace and in the side walls of the combustion chamber. Insulating brick, usually 7 in. thick, are also applied under the floors of the furnace and combustion chamber. In some cases, a sheet-steel deck, as in Fig. 3, is installed, forming a closed space above the suspended flat roof of the furnace into which the forced-draft fan discharges. The slightly heated air is conducted from this space by means of ducts

to the air box and tuyères, thus eliminating radiation loss from the roof.

In order to minimize labor and loss of time for ash and clinker removal, the furnace design should permit removing the ash and clinker from the grate or hearth rather than from beneath the boiler behind the bridge wall. The former can be done during operation, whereas the latter may require taking the unit out of service. The solution of this problem has become increasingly important and difficult with the higher ratings of recent years.

Bagasse furnaces may be classified under three heads, namely, grate, hearth, or a combination of grate and hearth. The majority of grates for bagasse are flat, the bagasse being dropped on the center and forming a conical pile. However, in some countries the grates, made in steps, are inclined at a sharp angle, the bagasse being fed to the upper and front end as it burns. Flat grates are constructed either of stationary or hand-operated shaking bars, the latter being common in Louisiana. Flat grates are shown in Figs. 1, 2, and 4. Fig. 5 shows an inclined-grate furnace of recent design in which hand-operated pushers and dump grates at the lower end are used to facilitate cleaning and clinkering. The majority of grate furnaces are operated with natural draft although some have undergrate forced draft with centrifugal fans or steam-jet blowers, there being many of the latter in old Louisiana installations. Under-

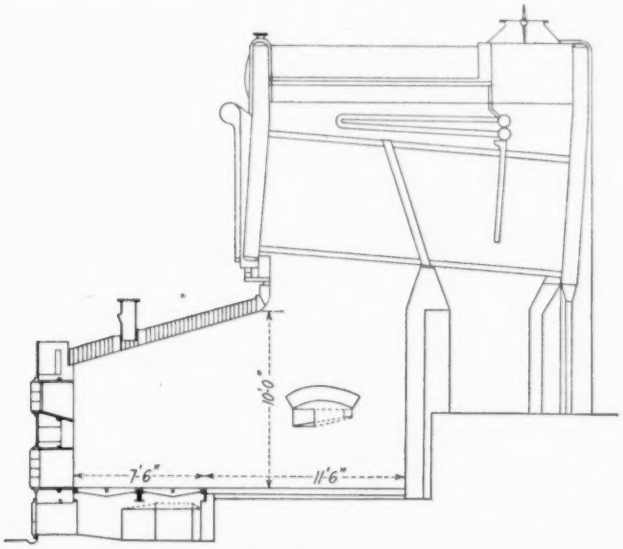


FIG. 2 GILCHRIST BAGASSE FURNACE FOR A LARGE WATERTUBE BOILER AT CENTRAL STEWART, CUBA

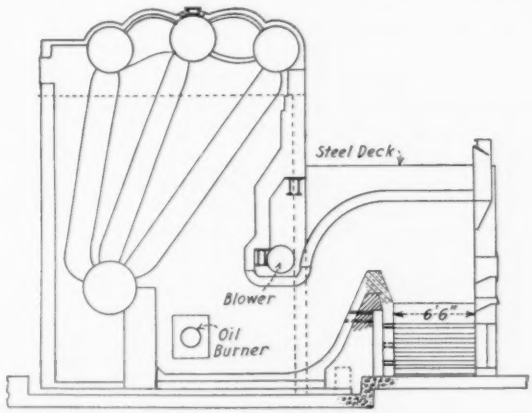


FIG. 3 BAGASSE FURNACE WITH SUSPENDED ARCH; THIS TYPE WIDELY USED IN RECENT YEARS

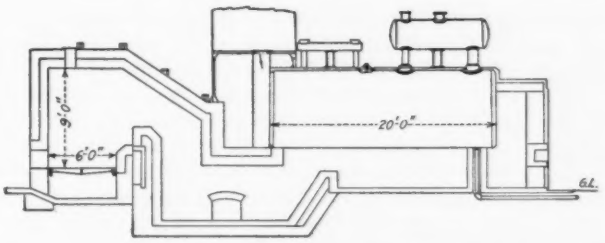


FIG. 4 HORIZONTAL-RETURN-TUBULAR 250-HP BOILER AT ADELINE FACTORY

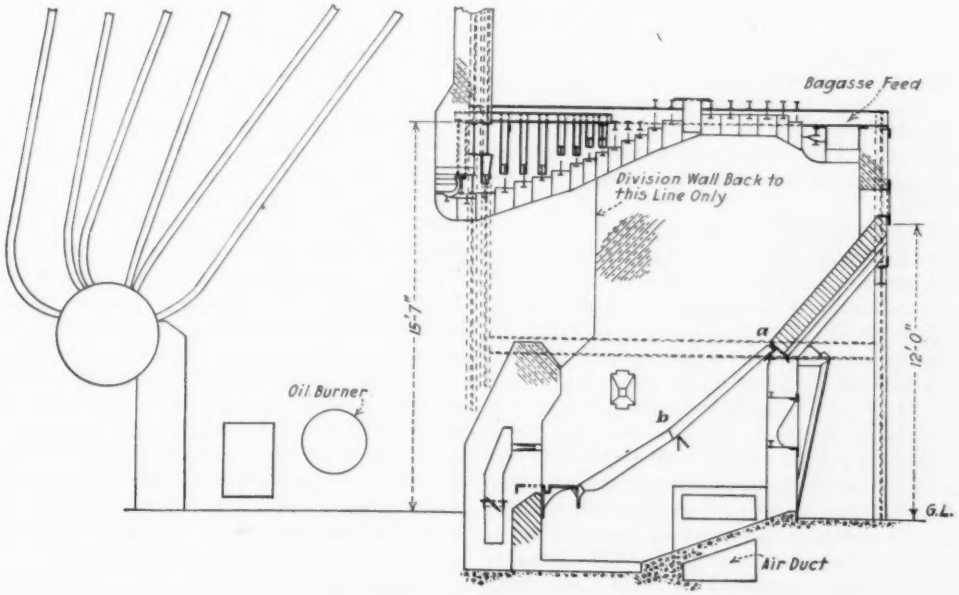


FIG. 5 INCLINED-GRATE FURNACE

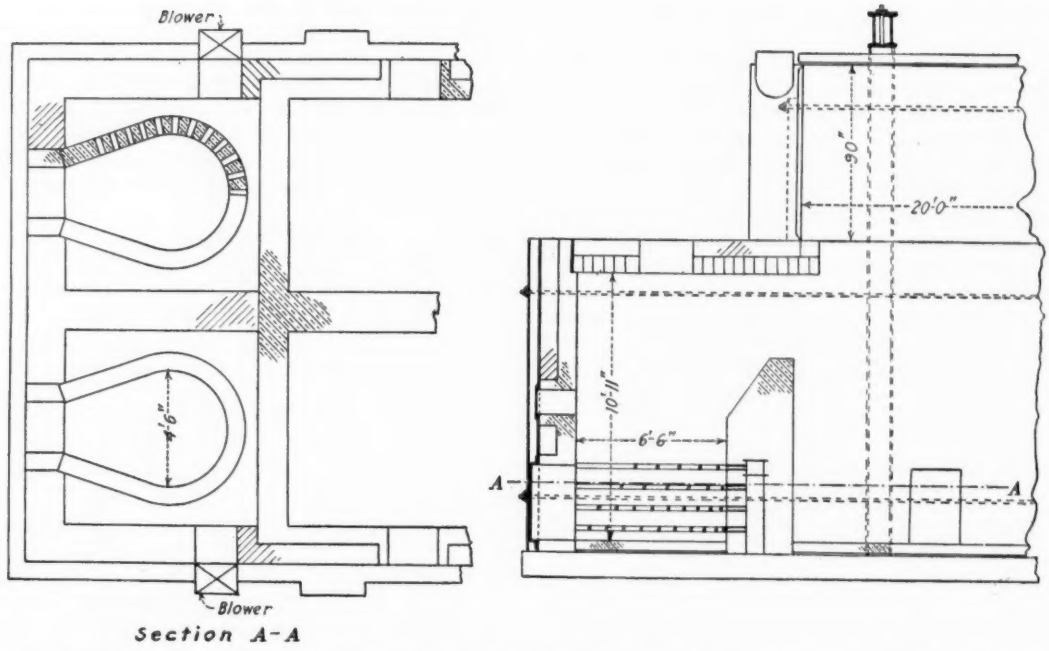


FIG. 6 HEARTH (HORSESHOE) FURNACE IN CONNECTION WITH TWO HORIZONTAL RETURN-TUBULAR 250-HP BOILERS; NO PROVISION IS MADE FOR BURNING OIL FUEL

grate blowers if forced cause carry-over of ash and cinders, even up through the stack, in some cases. The steam-jet blowers consume considerable steam which cannot be recovered.

In hearth furnaces, Fig. 6, the bagasse is dropped on a hearth of "horseshoe" of rectangular shape at ground level, air being supplied through cast-iron tuyères from an air chamber surrounding the hearth to which air is delivered by a blower. In more modern installations, the blower is of the impeller type set in the furnace walls. The hearth furnace affords maximum opportunity for control of capacity and air supply, and is especially suited for large boilers. In older designs, a single-hearth horseshoe was used for as much as 1200 bhp, frequently supplied by two 600-hp boilers. However, today the tendency is to subdivide the hearth into two or more sections for a single boiler of the same or smaller size, in order to facilitate cleaning during the run, also to promote increased ratings. In recent years, a considerable number of hearth furnaces has been installed in connection with horizontal-return-tubular boilers of 150 hp or less, with satisfactory results. This method of introducing air seems especially advantageous in the combustion of bagasse from certain varieties of cane.

Another feature in modern furnaces, especially of large size, is the use of suspended flat arches instead of sprung arches of circular shape to form the furnace roof. In the older and smaller sizes sprung arches were used and were satisfactory and cheap. However, where the spread exceeds 7 ft or so, the upkeep caused by the thrust and high furnace temperatures justifies the suspended flat type of arch which has come into common use and proved very satisfactory. Fig. 7 shows a furnace with sprung arches for a Babcock & Wilcox boiler of 859 hp. It will be noted that there are three arches, namely, a single arch of wide spread over the hearths, a single arch of wide spread over the bridge wall, and twin arches of narrow spread between the bridge wall and the boiler. The Cuba Cane Sugar Corporation at one time had a great many arches of this type which were expensive to maintain, in fact requiring rebuilding operations at regular intervals. Boilers of similar size installed later were provided with suspended arches, with very satisfactory results. One particular arch, of circular contour as shown in Fig. 3, was found very satisfactory, the author being associated with H. Clyde Gregory and the late G. P. Ward in its development.

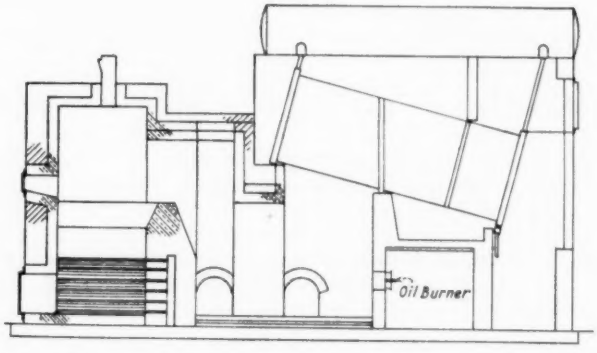


FIG. 7 BAGASSE FURNACE WITH SPRUNG ARCHES; THE UPKEEP OF THIS TYPE IS EXPENSIVE DUE TO THRUST, ETC.

Many boiler tests have been made by various investigators, the data from which are available. The author, while connected with the Louisiana State University, made fifteen boiler tests in 1909. In these tests, the weight of bagasse fed to the boiler was determined by weighing the cane supplied to the mill and subtracting the weight of juice which was measured in clarifier tanks. In this series, the average boiler efficiency was 46.3 per cent with an average rating of 125 per cent. The boilers were mostly 150 hp, or smaller, of the horizontal-return-tubular type. The furnaces were of the Dutch-oven type similar to Fig. 1, but without the steel casing or insulation. The efficiencies though low were representative of the time. Great accuracy was not possible under prevailing conditions. Furthermore, the bagasse moisture content was high, the mills being inefficient.

In 1910 the author conducted a series of tests on a 100-hp Stirling boiler with a Dutch-oven furnace, using partially dried ba-

gasse. The bagasse and the water were weighed. The average moisture of the material leaving the drier and being fed to the furnace was 46.2 per cent. The average efficiency of boiler and furnace in thirteen tests was 31.8 per cent, with an average of 80 per cent rating.

During 1914 and 1915 the author made thirteen boiler tests with bagasse as fuel on horizontal-return-tubular boilers at the Adeline, Vermillion, and Angola factories in Louisiana. In these tests the bagasse and the feedwater were weighed. In general these tests were made under better conditions and on better boilers and furnaces than the foregoing tests. The complete results of these tests, as well as those of 1909 and 1910, have been published.[1]

The tests at the Adeline factory were made on the unit shown in Fig. 4, which was designed by the Honolulu Iron Works, the boiler being rated at 250 hp. The efficiencies in the two tests were 52.8 per cent and 53.0 per cent, with ratings of 151.8 per cent and 102.7 per cent, respectively. The Vermillion factory tests were made on the unit shown in Fig. 1, which is a steel-encased boiler, rated at 250 hp, with asbestos lining, the furnace roof being flat. The boiler efficiencies were 60.4 per cent and 55.6 per cent, with ratings of 109.8 per cent and 105.9 per cent, respectively.

The tests at the Angola factory were made on a horizontal-return-tubular boiler, rated at 150 hp, having a furnace practically identical with that shown in Fig. 1, except that the setting was entirely of brick without the steel casing and with a sprung furnace arch. With ratings of 61.8, 103.3, and 125 per cent, the boiler and furnace efficiencies were 39.8, 47.7, and 50.1 per cent, respectively.

[1] "Report of Furnace Tests Using Bagasse as Fuel," Bulletin No. 160, Louisiana State Agricultural Experiment Station, October, 1916.

In 1922, the author conducted two tests on a Gilchrist furnace under a 1025-hp, four-pass Edgemoor boiler at Central Stewart in Cuba. The furnace was of the type shown in Fig. 2, although the boiler had four passes which gave a very low flue temperature. The efficiencies obtained were 61.32 per cent at 108.7 per cent rating and 60 per cent at 134.4 per cent rating. For purposes of comparison, a test also was made on a third boiler of the same size and make with a hearth, horseshoe-type furnace which, with a rating of only 88.7 per cent, showed an efficiency of 53.8 per cent. The efficiency would undoubtedly have been greater with a heavier load.

Special features in the Gilchrist furnace design are of interest, particularly the upward sloping roof and the wide floor extending back from the grate. In the Stewart furnace, the total distance from the front of the grate to the front of the bridge wall was some 18 ft. In order to clean the furnace, tools 20 feet long were required. Furthermore, the design necessitated a 20-ft firing pit in front. The construction cost as well as the operating inconvenience contributed to its discontinuance after five or six installations had been made, although the efficiencies were good. Another very reliable test on a furnace of the type shown in Fig. 2 was made at Central Florida, Cuba, by Professor Jesse J. Eames of the Massachusetts Institute of Technology in 1923. In this case, the boiler was an Edgemoor, 655-hp unit with three passes, which gave an efficiency of 56.8 per cent at 129.2 per cent of rating.

Fig. 3 shows a type of furnace which has probably been more widely adopted in recent years than any other design. The arch, which is of suspended construction and curved from front to rear, was developed by Messrs. Gregory and Ward of the Babcock & Wilcox Company in collaboration with the author, the first one

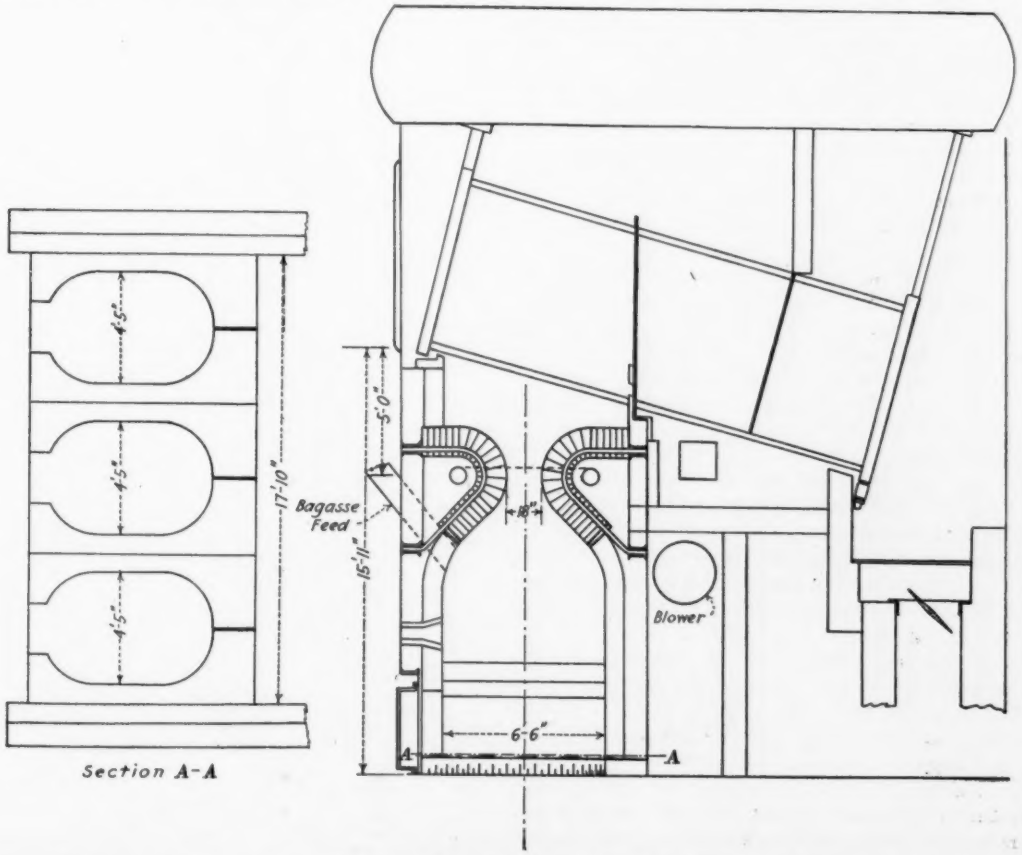


Fig. 8 Single-Pass Bagasse Furnace; a New Type With Reduced Radiation Loss; Also Less Time Lost for Cleaning

being installed at Central Perseverancia, Cuba, in one of the Cuba Sugar Cane Corporation factories. Since that time it has been adopted in many countries.

Fig. 7 shows a type of roof which, as already mentioned, was expensive to keep in repair and which the curved arch was designed to replace; the principal advantage of the curved arch being the reduced cost of up-keep and possibly somewhat less draft loss due to smoother flow.

In 1923 and 1924, a series of some sixteen tests was conducted at Central Baragua, Cuba, by Dr. N. E. Lewis, G. P. Ward, and the author on an 859-hp, Babcock & Wilcox, three-pass boiler in which the bagasse and water were weighed, all conditions being such as to make the results very reliable. The 1923 tests were made on the furnace shown in Fig. 7, which had a single horseshoe, whereas the 1924 tests were made with two horseshoes, each of somewhat smaller area. The ratings varied from 89.7 per cent to 155.4 per cent with efficiencies of 60.0 per cent to 54.3 per cent.

The latest and most revolutionary design of bagasse furnace, developed by the late G. P. Ward and known as the Ward furnace or single-pass furnace, is illustrated in Fig. 8, which shows a sectional view of a furnace under a straight-tube boiler, as well as a plan view through the furnace which has three hearths. The special features of this furnace are (1) the bottle shape narrowing to 18 in. at the neck; (2) cast-iron horseshoes through which air is admitted to the hearth by means of slots $^1/_2$ in. wide, and (3) the large air-insulated wall area. It will be noted that with this type of boiler the furnace is entirely underneath the first pass of the boiler and does not extend in front, thus eliminating a large area subject to radiation. The furnace can be and is used in connection with Stirling-type boilers. In this case, however, there is an extension of about 7 ft 6 in. in front of the boiler from the center of the front drum. Therefore, the reduction in radiation for Stirling boilers is not as great as for straight-tube boilers. The use of cast-iron wind boxes permits greater hearth area and lower air velocities through the tuyères. This, together with the high temperature at the neck, eliminates to a great extent the discharge of fly ash into the boiler.

Some twenty-two of these single-pass furnaces have been installed, one of them at Little Texas factory in Louisiana. They have given good satisfaction. Due to the large hearth area, etc., it is possible to operate this type of furnace at high ratings.

A test of a furnace of this type with a 450-hp watertube boiler was made by Mr. Ward at Central Preston in 1936. Complete data from the test are given in Table 1. The efficiency was 66 per cent with a developed rating of 165 per cent. Another test was made at Central Punta Alegre in which the developed rating was 165 per cent. Comparing this result with the Baragua tests, it will be noted that, with a rating of 155 per cent in two tests, the average efficiency was 55 per cent, which gives evidence of the increased efficiency with the single-pass furnace. Doubtless the greater part of this increase in efficiency is due to reduced radiation loss, although the elimination of fly-ash accumulation in the rear of the boiler is an important feature. It is claimed that even higher ratings, probably 200 per cent, are feasible with this furnace, which would give a heat release of some 60,000 Btu per cu ft of furnace volume per hr.

Fig. 5 illustrates a furnace developed in recent years which has been adopted extensively. The author was responsible for installing a furnace of this type under a 900-hp boiler at Central Moron, the first installation of its kind for bagasse. It will be noted that the bagasse is dropped onto an inclined refractory drying apron, sliding to cast-iron stepped grates, *a* to *b*, thence to plain grates having hand-operated pushers between them. Finally there is an ash dump, adjacent to the bridge wall, which is operated from the front. The furnace illustrated is actually composed of three furnaces with dividing walls between, the idea being to facilitate even feed and permit cleaning on the run. Air is admitted under the grate and through tuyères in the bridge wall. The arch is of the suspended type. The unit shown in Fig. 5 is a 750-hp Stirling boiler installed in a Louisiana factory.

A test was made on each of two 881-hp Casey & Hedges straight-tube boilers (watertube) with furnaces of this type at Reserve factory by Professor Wm. Whipple. A composite of the more significant data in these two tests is given in Table 2.

TABLE 1 BAGASSE-FUEL TEST AT CENTRAL PRESTON ON AN EXPERIMENTAL SINGLE-PASS, HEARTH-TYPE FURNACE

	March 29,
Date of test	1936
Duration, hr	8
Rated capacity, boiler hp	450
Steam pressure, lb gage	130.5
Draft at damper, in. water	0.8
Air pressure, in. water	0.8
Flue temperature, F	485
Air temperature, delivered, F	140
Developed hp	741
Rating developed, per cent	165
Evaporation from and at 212 F per lb bagasse, lb	2.937
Moisture in bagasse, per cent	48
Estimated heat value per lb dry bagasse	8300
Boiler and furnace efficiency, per cent	66
CO_2, per cent	16.45
O, per cent	3.75
CO, per cent	0.16
Excess air, per cent	25
Probable heat distribution:	
Heat absorbed by boiler, per cent	66.00
Loss due to H_2O from hydrogen, per cent Loss due to air moisture, per cent Loss due to H_2O from bagasse, per cent	23.26
Loss due to dry chimney gases, per cent	8.16
Loss due to radiation and unaccounted for, per cent	2.58

Test conducted by G. P. Ward

TABLE 2 TESTS ON TWO 881-HP CASEY & HEDGES WATER-TUBE BOILERS WITH INCLINED-GRATE FURNACES, USING BAGASSE FOR FUEL

	Dec. 13,
Date of test	1932
Duration of test, hr	24
Heating surface (two boilers), sq ft	17,620
Steam pressure, lb	135
Temperature flue gases, F	505
Rated capacity, boiler hp	1762
Developed hp	2940
Rating developed, per cent	167
Evaporation per lb of bagasse as fired, lb	2.64
CO_2, per cent	14.8
O, per cent	4.59
CO, per cent	0.0025
Bagasse analysis:	
Moisture, per cent	45.54
Sucrose, per cent	2.55
Evaporation per lb dry bagasse, from and at 212 F, lb	4.85
Calculated air excess, per cent	28.0
Heat absorbed by boilers (efficiency) per cent	58.8

TABLE 3 BAGASSE-FUEL TESTS AT CENTRAL MERCEDITA PUERTO RICO; GREGORY-WARD FURNACE WITH AIR HEATER

(Stirling watertube boiler, Class XIII, No. 36)

	Test No. 1 June 16, 1935	Test No. 2 June 17, 1935
Rated capacity, boiler hp	1290	1290
Type of bagasse furnace	Twin-hearth	Twin-hearth
Floor area hearth, total sq ft	70	70
Duration of test, hr	8	9
Steam pressure, lb gage	180	170
Superheat, F	125	120
Boiler hp, developed	2725	2684
Rating developed, per cent	211	208
Bagasse per hr per sq ft of hearth, lb	408	405
Evaporation from and at 212 F on bagasse as fired, lb	3.29	3.27
Moisture in bagasse as fired, per cent	45.6	45.18
Evaporation from and at 212 F on dry bagasse, lb	6.05	5.97
CO_2, per cent	15.0	14.3
Gas temperature leaving boiler, F	552	536
Gas temperature leaving air heater, F	348	340
Air temperature entering air heater, F	94 plus	99 plus
Air temperature entering furnaces, F	429	427
Furnace temperature in combustion chamber, F	2354	2330
Draft at boiler exit, in. water	2.07	2.02
Draft loss through air heater, in. water	1.13	1.08
Draft loss through boiler and furnace, in. water	1.52	1.40
Draft loss through boiler, furnace, and air heater, in. water	2.65	2.48
Btu per lb dry fuel	8270	8240

TABLE 4 HEAT BALANCE; BOILER TESTS WITH BAGASSE FUEL

	Stewart		Baragua					Mercedita	Preston	Reserve
	Grate	Hearth	Hearth					Hearth	Ward	Inclined grate
Furnace type										
Rated capacity, bhp	1025	1025	859	859	859	859	859	1290	450	880
Rating developed, per cent	134	89	85	98	119	155	131	211	165	167
CO_2, per cent	12.2	10.4	14.0	12.9	14.3	14.8	14.6	15.0	16.4	14.8
Heat Balance:										
Absorbed by boiler, per cent	60.0	53.8	59.7	58.0	55.6	54.3	58.0	71.0	66.0	58.8
Loss due to free moisture, per cent	13.3	13.4	14.4	14.9	16.3	16.4	15.1	11.9	...	...
Loss due to formed moisture, per cent	8.1	7.8	8.5	8.6	7.8	7.9	7.7	7.7	...	...
Loss in dry gases, per cent	8.6	10.0	9.9	8.8	8.7	9.0	9.1	5.9	...	...
Loss due to incomplete combustion, per cent	0.0	0.2	0.24	0.04	1.2	0.96	0.10	1.2	...	...
Loss due to radiation and unaccounted for, per cent	10.0	14.8	7.3	9.7	10.5	11.5	9.2	2.34	...	...

The efficiency of boiler and furnace was 58.8 per cent at a developed rating of 167 per cent. It will be noted that there is much cast iron in this furnace. The efficiencies in these tests were based on an average calorific value of 7980 Btu per lb, dry basis, which figure was the average of calorimeter tests made by two different laboratories. The calorific value for the five Baragua tests averaged 8200 Btu per lb dry basis.

The first hearth furnaces were designed and installed by Frederick Cook, who used heaters of cast-iron construction, employing flue-gas heat for raising the temperature of the air on its way to the furnace. In 1893 he ran a test on his "hot blast" furnace at Ingenio "Dos Hermanos," Cuba. The furnace was placed between two 640-hp Babcock & Wilcox boilers opposite the first pass. The gases divided, passing over the boiler heating surfaces in the usual way, then through individual tubular air heaters, thence to a common chimney. In this test he had to deal with bagasse of 58 per cent moisture. The test lasted 8 hours, one horsepower was developed with 13.25 lb of bagasse, the boiler operating at 76 per cent of rating. With a calorific value of 8000 Btu per lb of dry matter, this would be equivalent to an over-all efficiency of 57.5 per cent which for that time was exceptional.

In 1935 Mr. Ward conducted two tests on a 1290-hp Stirling boiler, equipped with a furnace of the general type shown in Fig. 3, but having a superheater and a vertical tubular air heater of modern design at the rear, together with forced- and induced-draft fans. Data covering these tests are given in Table 3. It will be noted that the efficiency of the boiler, air heater, and superheater was upward of 70 per cent at ratings of more than 200 per cent. In one test the gas temperature was reduced from 552 to 348 F by the air heater and 536 to 340 F in the other. The draft loss through the boiler and air heater was 2.65 in. water and 2.48 in. water, respectively. As far as the author knows these efficiencies are the highest ever obtained with bagasse fuel.

To complete the essential information concerning results obtained with bagasse fuel, Table 4 gives heat-balance data from the tests conducted at Centrals, Stewart, Baragua, Mercedita, Preston, and at Reserve factory.

The Effect of Preparation on Ash Fusibility as Revealed by a Study of Selected Illinois Coals

By L. C. McCABE[1] and O. W. REES,[2] URBANA, ILL.

The authors discuss the effect of sizing, and of sizing and specific-gravity separation on the softening temperature of the ash from ten representative coals mined in Illinois. The ash-softening characteristics of the sized coals fall into four groups. With the possible exception of two mines, there appears to be no grouping of ash-softening characteristics into specific-gravity fractions. Great differences in softening temperature were obtained in the ash of sized coals of narrow specific-gravity range. In general, the combining of ash of widely different fusion characteristics is to lower the high softening temperatures and to raise low softening temperatures of the individual portions.

STUDIES of the characteristics of the ash of coal have of recent years become increasingly common, such investigations being stimulated by the hope that they may provide criteria for establishing the particular suitability of the coal for specific uses. A variety of investigations have been made, but reference is necessary to only a few of these as examples. Ash-fusibility tests are being made in increasing numbers as part of the ordinary commercial analysis. Uncertainty as to the significance of these values has, in turn, stimulated investigations such as those of Nicholls, Selvig, and Ricketts (1)[3] into the relationship of fusibility determinations and clinkering tendencies. The relationship of ash composition to ash fusibility is also a problem that has received much attention recently (2). Some attention has been given to the bearing of the nature of the mineral components of the coal upon ash fusion (3). These and other investigations are providing a basis for a better understanding of the complicated relationship between ash composition and ash fusibility, and of ash fusibility to clinkering tendencies.

The investigation, the results of which are described in the present paper, was concerned with the effect of preparation upon the softening temperature of coal ash rather than with the relationship of ash composition to fusibility.

Few records of similar investigations have come to the attention of the authors. Ball (4) studied the amount and character of mineral matter of No. 6 bed coal from Franklin County, Illinois. Estep et al. (5) studied the effect of mixing coals on ash-softening characteristics of the mixture. Yancey and Fraser (6), in a report on some coal-washing studies, referred briefly to the effect of washing on ash fusibility. Selvig et al. (7), in discussing the relationship of ash fusibility to clinker formation commented on the effect of washing upon the fusibility and the clinkering characteristics of the coal. The only generalization of particular present interest that can be drawn from the last three papers referred to is that ash-fusion characteristics vary depending on the extent to which washing eliminates certain mineral substances. Furthermore, according to these authors, elimination of pyrite usually tends to raise the temperature of fusibility of the washed coal as compared with that of the raw coal.

This paper dealing with the effect of preparation on ash fusibility of coal screenings from Illinois mines is a part of an investigation of washability characteristics, size range, and petrographic and chemical nature of coal screenings, initiated by the Illinois Geological Survey in 1935. Ten mines were sampled so that each of the commercially important coal beds as well as the different producing districts in the state was represented, Table 1 and Fig. 1.

TABLE 1 LOCATION, COAL BEDS WORKED, THICKNESS OF BEDS, MINING METHODS, AND TONNAGES OF MINES SAMPLED

Mine	County	Coal bed number	Average thickness, ft	Average thickness, in.	Mining methods	Daily average, tons
A	Henry	1	4	1	Room-and-pillar, coal shot from solid, hand loading	450
B	Woodford	2	2	9	Long-wall, hand mining, hand loading	425
C	Peoria	5	4	2	Room-and-pillar, machine mining, hand loading	3000
D	Vermilion	Grape Creek	5	0	Room-and-pillar, machine mining, hand loading	3000
E	Sangamon	5 Springfield	5	9	Room-and-pillar, coal shot from solid, hand loading	1500
F	Christian	6	7	6	Room-and-pillar, coal shot from solid, hand loading	700
G	St. Clair	6	7	0	Room-and-pillar, machine mining, machine loading	1300
H	Marion	6	6	4	Room-and-pillar, machine mining, hand loading	1700
I	Williamson	6	10	0	Room-and-pillar, machine mining, mechanical loading	4000
J	Saline	5 Harrisburg	5	3	Room-and-pillar, machine mining, hand loading	2000

Increments of from 12 to 15 lb were taken from the loading chute at each mine throughout a day's run. One quarter of the gross sample of from 1000 to 1500 lb was sized and used in float-and-sink tests. Water solutions of zinc chloride were used in the float-and-sink tests of coal from $^3/_4$ to $^3/_8$ in. and larger, and organic solutions, carbon tetrachloride, benzene, and bromoform mixtures, in the sizes smaller than $^3/_8$ in. Results and details of procedure of the sizing and washability tests (9) have been published and will not be repeated here.

From the float-and-sink fractions, samples were taken which have furnished a fund of data on the ash-fusion characteristics and ash composition of these coals. Only the data pertaining to the effect of sizing and heavy-liquid separation on ash-fusion temperatures will be considered here; ash composition and its relation to ash-fusion characteristics will be treated at length in another publication.

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[3] Numbers in parentheses refer to the Bibliography

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FIG. 1 MAP OF ILLINOIS SHOWING LOCATION OF MINES WHERE SCREENINGS WERE SAMPLED

The screenings sample from each mine was sized as follows: $1^1/_4$ to $^3/_4$ in., $^3/_4$ to $^3/_8$ in., $^3/_8$ in. to 10 mesh, 10 to 48 mesh, and minus 48 mesh. Round-hole screens were used in sizing at $^3/_8$ in. and above and Tyler standard sieves for sizing below $^3/_8$ in.

Each size was separated by heavy liquids of 1.30, 1.35, 1.40, 1.50, and 1.70 specific gravities into the following fractions:

1.30 specific gravity, float
1.30 to 1.35 specific gravity, float
1.35 to 1.40 specific gravity, float
1.40 to 1.50 specific gravity, float
1.50 to 1.70 specific gravity, float
1.70 specific gravity, sink

The ash and sulphur values for the coals were obtained according to A.S.T.M. procedures (10). The ash-fusion data were obtained according to A.S.T.M. specifications (11) in a Barrett ash-fusion furnace. The ash analyses were made in accordance with procedures outlined by Hillebrand and Lundell (12) and Washington (13) for the analysis of silicate rocks.

Initial-deformation, softening, and fluid temperatures of the ash together with percentages of ash, sulphur, and weight of the gravity fractions are given elsewhere (14). Figs. 2 to 5 show the relationship between coal size and ash-softening temperature.

Plotting the ash-softening temperatures of the individual

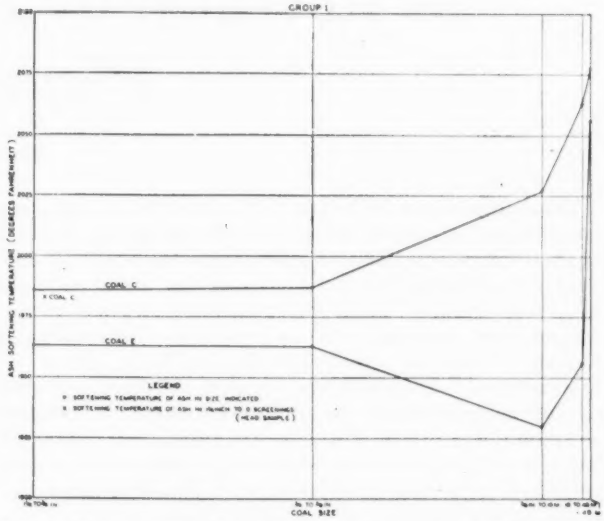


FIG. 2 RELATIONSHIP BETWEEN COAL SIZE AND ASH-SOFTENING TEMPERATURE, MINES C AND E

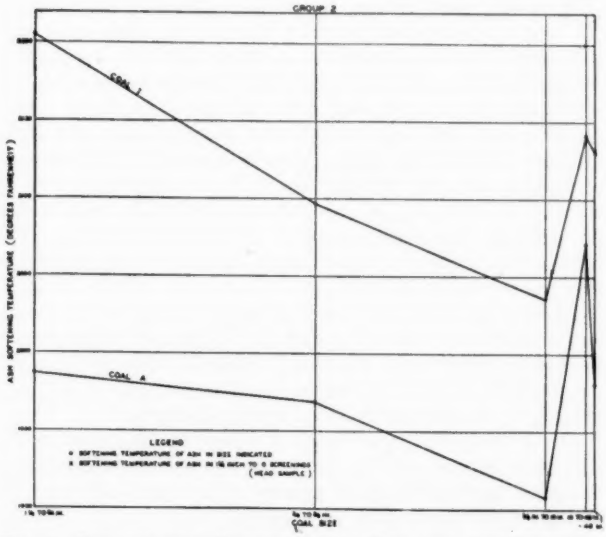


FIG. 3 RELATIONSHIP BETWEEN COAL SIZE AND ASH-SOFTENING TEMPERATURE, MINES I AND A

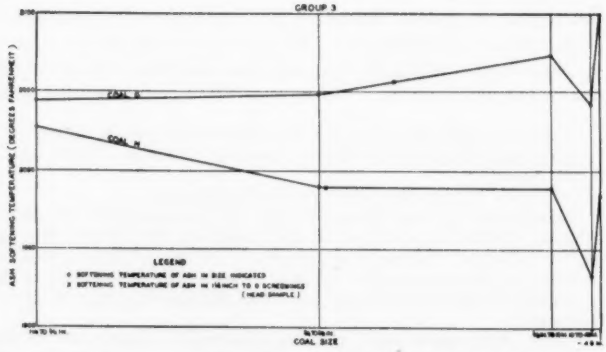


FIG. 4 RELATIONSHIP BETWEEN COAL SIZE AND ASH-SOFTENING TEMPERATURE, MINES G AND H

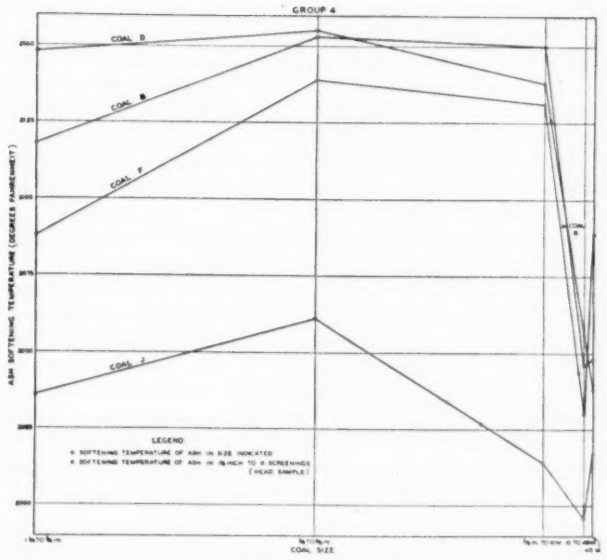


FIG. 5 RELATIONSHIP BETWEEN COAL SIZE AND ASH-SOFTENING TEMPERATURE, MINES B, D, F, AND J

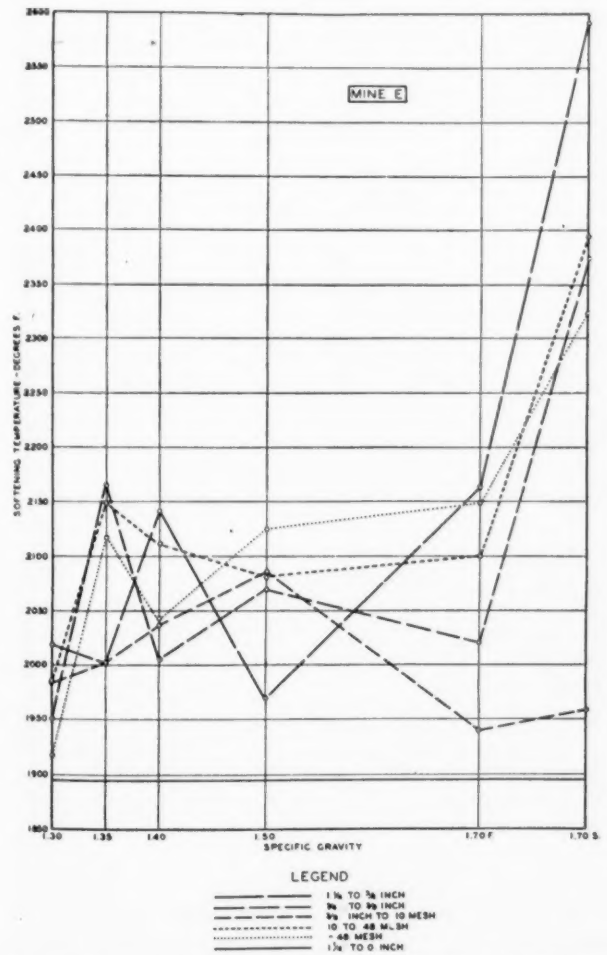


FIG. 6 RELATIONSHIP BETWEEN NARROW-RANGE GRAVITY AND ASH-SOFTENING TEMPERATURE, MINE E

sizes aids in comparing these values for the different sizes in the same mine and the curves formed by connecting the points make it possible to place the mines in characteristic groups.

The curves of ash-softening temperatures for the ten mines may be divided into four groups each containing two or more similar curves. In group 1 which includes coals from mines C and E (Fig. 2) the highest ash-softening temperature is in the minus 48-mesh size. The curves of group 2, representing coals I and A (Fig. 3) exhibit their lowest softening temperature in the $^3/_8$-in. to 10-mesh size, followed by an increase of from 100 to 150 F in the 10- to 48-mesh size with a reversal to a lower softening temperature for the minus 48-mesh dust. The group 3 curves, based on coals G and H (Fig. 4), show rather uniform ash-softening temperatures for all sizes except the 10- to 48-mesh size which is consistently lowest. The curves of group 4 representing coals B, D, F, and J (Fig. 5), show the highest softening temperatures to be in the $^3/_4$- to $^3/_8$-in. size. Three of these curves show the lowest ash-softening temperature in the 10- to 48-mesh size with a slightly higher softening temperature in the ash from the minus 48-mesh coal. In coal B no such reversal is shown.

While the number of mines represented is too small to be conclusive, with one exception there appears to be a relationship between the geographical location of the mines and the groupings indicated. The mines of group 1 are in No. 5 coal in the western part of the coal basin (Fig. 1). Mines of group 3 operate in No. 6 coal in the Belleville district of southwestern Illinois. Group 4 includes mines working the Grape Creek No. 2, No. 5, and No. 6 coals but all operating near the center or to the east of the center of the coal basin. Group 2 embraces two widely separated mines, mine A operating in No. 1 coal in northern Illinois and mine I producing from No. 6 coal in Williamson County.

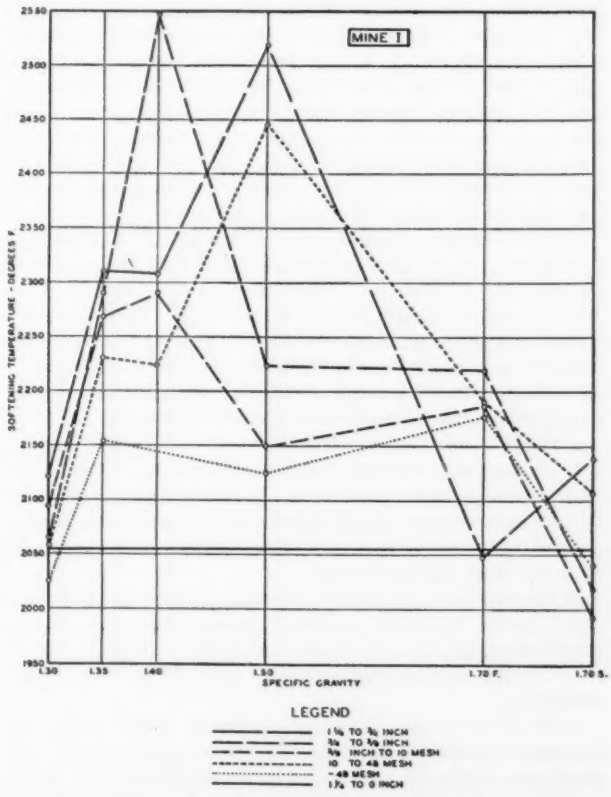


FIG. 7 RELATIONSHIP BETWEEN NARROW-RANGE GRAVITY AND ASH-SOFTENING TEMPERATURE, MINE I

Coals E and I were selected for an exhaustive study of ash composition in relation to ash-fusion characteristics because of the extremes of sulphur content. The sulphur content is 5.3 per cent for mine E 1¼ to 0 screenings and 1.9 per cent in screenings from mine I.

To investigate further the effect of preparation on ash fusion, samples of each of the five sizes of coals from mines E and I were separated by heavy solutions into fractions, 1.30, 1.35, 1.40, 1.50, and 1.70 float, and 1.70 sink. Chemical analyses of the ashes of the 1¼ to 0 samples of these coals appear in Table 2, while ash-fusion determinations as well as percentages of ash, sulphur, and weight of the specific-gravity fractions are given elsewhere (14).

TABLE 2 ASH ANALYSES OF SCREENING SAMPLES (1¼ IN. TO 0) FROM MINES E AND I

Ash constituents	Mine E, per cent	Mine I, per cent
SiO_2	40.15	46.94
TiO_2	0.77	0.93
Al_2O_3	11.81	20.19
Fe_2O_3	26.20	18.82
MgO	0.88	1.24
CaO	8.37	4.62
Na_2O	0.92	0.50
K_2O	1.35	2.12
P_2O_5	0.00	0.00
Total sulphur as SO_3	9.09	4.69
Total	99.54	100.05
Ash in coal (dry basis).....	13.8	10.4
Sulphur in coal (dry basis)...	5.3	1.9

Inspection of Table 2 shows the coal from mine E to be higher in ash and sulphur than that from mine I. The composition of the two ashes varies also; the SiO_2 and Al_2O_3 are lower in coal ash E while the Fe_2O_3, CaO, and SO_3 are considerably higher than in ash I.

These closely sized fractions of narrow specific-gravity limits cover a wide range of ash-softening temperatures but the curves (Figs. 6 and 7) show the ash-softening temperatures to be quite different for the two mines. In general the lowest ash-softening temperatures for mine E are in the fractions of low specific gravity and with the exception of the ⅜ in. to 10 mesh, the highest softening temperatures are in the minus 48-mesh size. The mine I curves show low ash-softening temperatures in both the low- and high-specific-gravity fractions and high ash-softening temperatures at intermediate specific gravities.

The ash-softening temperature of the mine I ¾- to ⅜-in. 1.40-specific-gravity fraction is 2547 F (14) and of the 1.70-sink fraction 2019 F, a range of 528 F between the maximum and minimum softening temperature in the size. The difference in softening temperature between the 1.50 (2518 F) and the 1.70 (2048 F) float in the 1¼- to ¾-in. size is 470 F. Mine E 1¼- to ¾-in. coal shows a range of 622 F in the ash-softening temperatures of the 1.50-specific-gravity-float fraction (1971 F) and the 1.70-sink fraction (2593 F).

It is noteworthy that the head sample for mine I has a lower ash-softening temperature than do most of the sizes prepared from it. All ash-softening temperatures of float and sink fractions derived from mine E head sample are higher than the softening temperature of the head sample.

It appears from the data for these two coals that sizing and close gravity separation produce products the ash-softening temperatures of which are usually higher than that of the original coal. It also appears that each coal possesses individual characteristics.

In Figs. 8 to 11 are presented ash-fusion data for sizes and cumulative gravity samples studied. The composite samples were made up according to percentages by weight obtained by gravity separations of each size. This procedure makes it possible to observe the effect on softening temperature of adding increments of high-ash coal to lower-ash coal or to determine the effect on softening temperature of washing at any given specific gravity.

It has been shown that the ash-softening temperatures of four of the high-specific-gravity fractions of mine E (Fig. 6) are considerably above the softening temperatures of the ash of the fractions of low specific gravity. It is apparent from an examination of the increment curves for this mine (Fig. 9) that the addition of increments of high-softening-temperature ash does not always raise the softening temperature of the composite; in most instances the effect is one of lowering the softening temperature of the whole. In the curve showing individual softening temperatures of the ¾- to ⅜-in. size (Fig. 6) fractions having specific gravities above 1.30 have higher softening temperatures than does the fraction floating at 1.30 specific gravity, yet when these fractions are combined in the proportion in which they occur in the coal, the effect of each added increment is to lower the softening temperature. The minus 48-mesh cumulative curve (Fig. 9) roughly follows the curve of individual softening temperatures (Fig. 6) in the first five increments but on the addition of the 1.70-sink fraction, which has a softening temperature 175 F

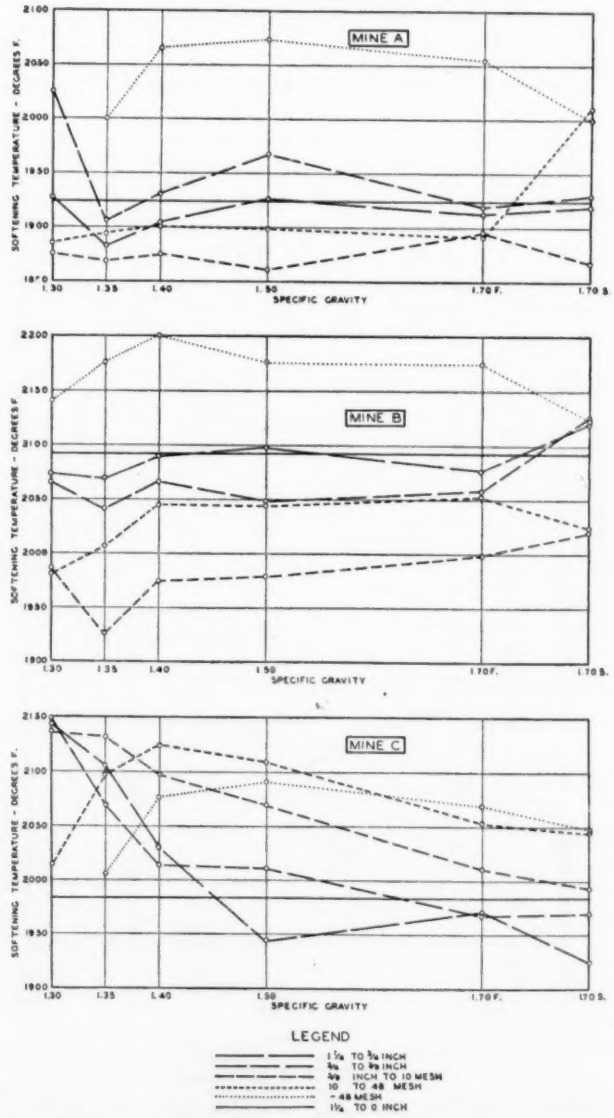


FIG. 8 RELATIONSHIP BETWEEN CUMULATIVE GRAVITY AND ASH-SOFTENING TEMPERATURE, MINES A, B, AND C

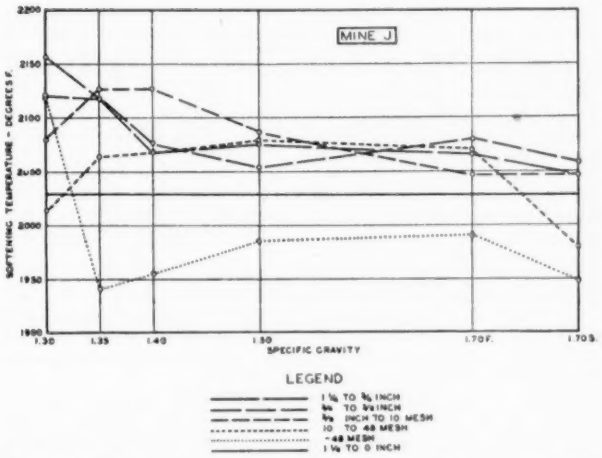


FIG. 11 RELATIONSHIP BETWEEN CUMULATIVE GRAVITY AND ASH-SOFTENING TEMPERATURE, MINE J

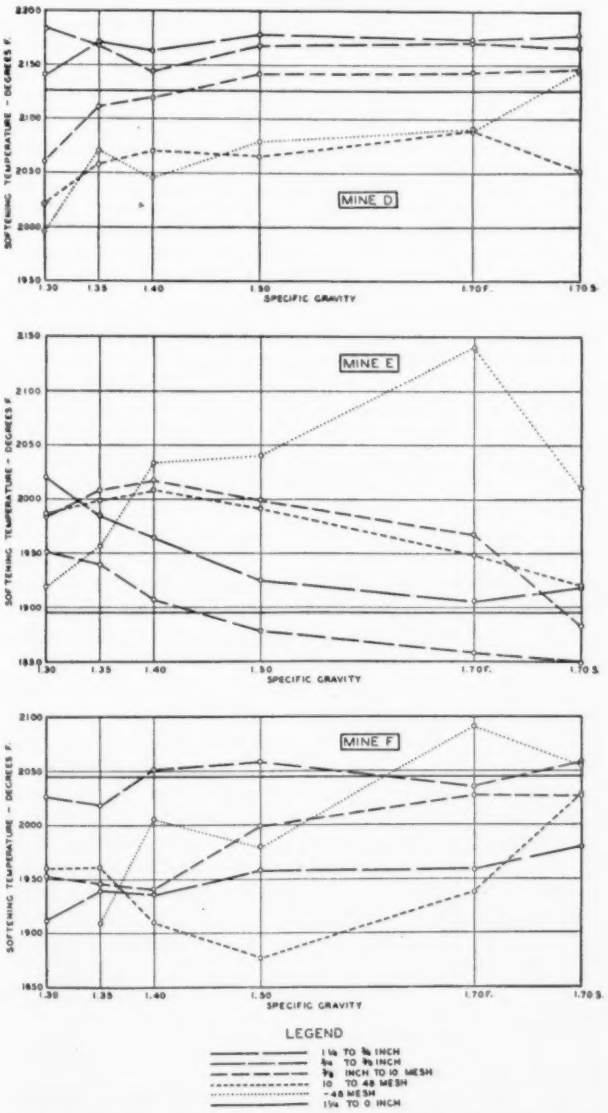


FIG. 9 RELATIONSHIP BETWEEN CUMULATIVE GRAVITY AND ASH-SOFTENING TEMPERATURE, MINES D, E, AND F

higher than that of the nearest increment (14), the softening temperature of the composite is lowered 129 F.

While the cumulative curves for mine I (Fig. 10) conform in a general way to the curves of softening temperature for individual fractions (Fig. 7) a modifying effect on both high and low points is apparent.

At 1.50 specific gravity (the washing gravity of most Illinois coals) the majority of sizes at all mines except B and F have higher ash-softening temperatures than the $1^1/_4$- to 0-in. head sample (Figs. 8, 9, 10, 11). However, it does not follow that, if these sizes were combined to form washed screenings, the fusion temperature would be higher than in the unwashed screenings. It might be higher or it might be lower.

The cumulative ash-softening curves for mines G and H (Fig. 10) are similar in shape and the curves for the various sizes occupy the same relative positions. These relations are not apparent for the coals of the other eight mines (Figs. 8, 9, 10, 11).

While sulphur values for all samples have been listed (14), no

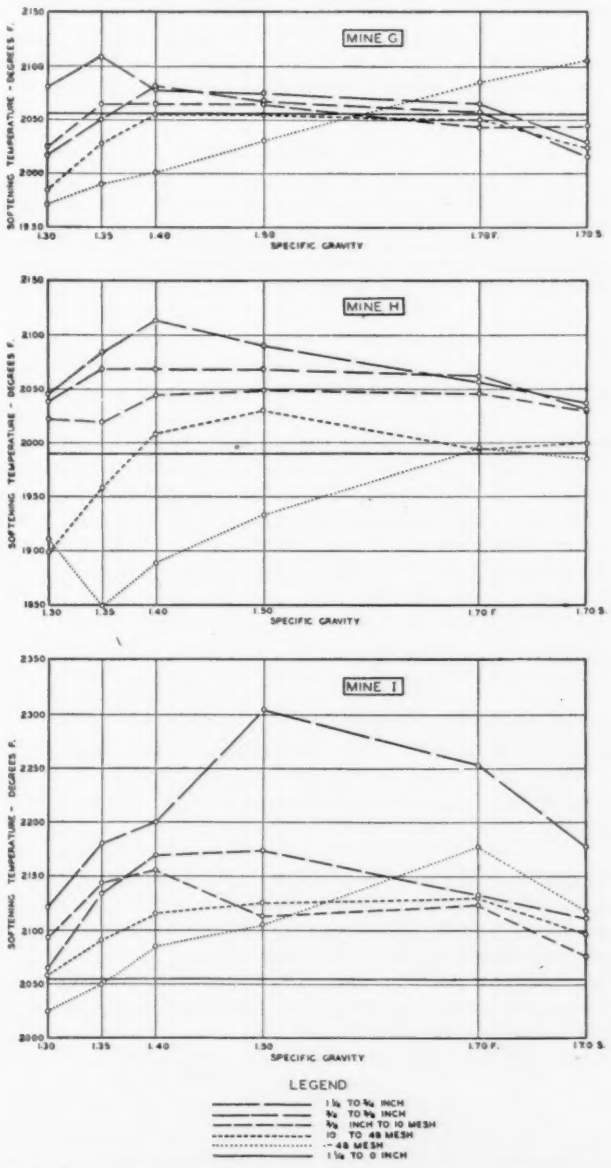


FIG. 10 RELATIONSHIP BETWEEN CUMULATIVE GRAVITY AND ASH-SOFTENING TEMPERATURE, MINES G, H, AND I

definite relationship of these values with ash-softening temperatures is apparent. Undoubtedly any such relationship depends on the nature and quantity of other minerals in the coal. These relationships will be discussed in a later publication.

Conclusions

Examination of the data presented in this paper leads to the following general conclusions:

1 On the basis of sizing alone the ten coals studied may be divided into four groups within which the ash-fusion relationships are similar.

2 With the possible exception of coals from mines G and H, the ash-fusion relationships of which are similar, no groupings similar to those derived from sizing appear possible on the basis of cumulative specific-gravity separations.

3 The greatest difference in ash-softening temperature was obtained in sized coals of narrow specific-gravity ranges. This may explain clinkering difficulties experienced with stoker coals when changes are made in sizing or washing procedure.

4 The effect, in general, of combining portions of narrow specific-gravity range is to lower high softening temperatures and to raise low softening temperatures of these component portions.

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Burning Various Types of Oil-Refinery Fuels

By A. L. WILSON,[1] LINDEN, N. J.

The paper describes the various types of waste by-product fuels which are produced in an oil refinery and the burning of them in boiler furnaces. Composition and characteristics of these by-products are given together with the methods of handling and utilization. Burner design and the use of steam for atomization are considered and attention is called to the fact that combination burners have been developed to burn simultaneously five different fuels in liquid, gaseous, and solid form.

OIL REFINING requires the utilization of relatively large volumes of fuels, among which are nonmerchantable by-product fuels, as well as high-grade merchantable fuels. It is the purpose of this paper to deal mainly with the burning of these nonmerchantable fuels as economically as possible.

Industries that utilize by-product fuels were compelled to design and develop suitable burners and furnaces to meet their individual requirements. This problem is particularly difficult because, due to the changing heat content of these fuels, it is impossible to set up performance charts that would indicate their true efficiencies.

Each installation must be considered as a special problem and, therefore, no attempt is made in the scope of this paper to set up a code that will be a cure-all for the designing and operating of a fuel-burning system by other refineries.

A new industry, while in its early stages, is concerned chiefly with the efficient conduct of the operation which yields its finished product. The substances which are produced at the same time, and are known as by-products, receive increasing attention when competition or other economic circumstances make it necessary to reduce the cost of the manufactured article, or when the amount of these by-products has risen to such proportions that better means for their disposal must be found. Under such circumstances, the attention of the industry is concentrated on these by-products, to provide a more suitable means for their disposal.

In the following discussion, oil-refinery fuels will be classified as follows: (1) Merchantable fuels, such as gas, coke, and fuel oils, (2) sludges, (3) tars, and (4) tank cleanings.

In this classification, the first group can be considered as high-grade, merchantable fuels, which serve as the basis for the evaluation of all other refinery fuels. This group of fuels is burned only after all by-product or nonmerchantable fuels are disposed of. In refineries where coke production is in large volumes, the greater percentage of this coke is a merchantable fuel, demanding a premium, particularly because of its adaptability to metallurgical and other uses requiring a relatively pure carbon. The amount of coke which cannot be considered merchantable, and which may be classed as a waste fuel for burning within the refinery, is small in volume, representing at most a negligible percentage of the total fuel burned. The remaining fuels listed can be considered as by-product or waste fuels, and the rest of the paper will deal mostly with these fuels.

Sludges are by-product materials from a purification process which uses sulphuric acid to remove tarry or gummy constituents from oil. Compared to fuel oil, some sludges are slow-burning and require high ignition temperatures to produce a reasonable heat release. The sludges vary extremely in characteristics, depending upon the initial crude stock, method of treating, and requirements of the finished product. Some sludges are readily blended with various tars to form a smooth-working fuel. Some have a heating value as high as 17,500 Btu per lb while in others it is as low as 9000 Btu on the as-fired basis. This variation is largely due to varying moisture content.

Due to the higher percentage of sulphuric oxide and water vapors in the products from the combustion of sludges, superheater supports, gas by-pass dampers, and economizer tubes deteriorate rapidly unless the exit-gas temperature is kept above the dew point. Exit-gas temperature is not always a true indication of metal temperatures, particularly in the economizer section of the boiler where the actual shell temperature of the tube is considerably lower than the average exit-gas temperature. As is the case when using high-sulphur coals, problems still exist in connection with economizers and preheaters where high efficiencies are desired when sludges are burned.

"Tar" is not exactly defined in refinery terminology, and is a type of sludge from acid refining operations, or may consist of distillation residues. Its combustion characteristics are quite similar to those of sludges.

Nonmerchantable coke is usually a hard nonfriable substance containing a small quantity of volatile matter and varying percentages of sulphur and ash. If the small nonmerchantable portion of this coke is to be burned as pulverized fuel, it must be powdered to a high degree of fineness. If petroleum coke is not finely pulverized the small particles pass out of the furnace partly burned, and may be deposited in the soot hopper or duct of the stack, or may pass through the stack to the atmosphere. The characteristics of the merchantable portion of any coke which is produced are such that it burns more freely and in a manner comparable with pulverized coal.

Small amounts of light oils or tars of light gravity, which are a by-product of some treating operations, average about 19,000 Btu per lb, and may be successfully burned in a separate burner system. Great care, however, must be exercised in handling these fuels, as they usually have a low flash point and high volatility.

Tank cleanings, or "bottoms," consist of a considerable portion of oil of varying gravity and volatility, of dirt, water, salts, asphaltic and waxy semisolid materials, and often of iron scale. The heat value of this material will vary greatly, depending upon the proportion of its various constituents. Tank cleanings may be disposed of in liquid form through a grinding operation, which will be discussed later, or may be cooked up with fluxing oil, by means of which treatment, water, salts, and noncombustible solids are settled out and rejected as such.

The grades and amounts of these various by-product fuels depend upon the crude run and the type of processing, but are seldom available in sufficient quantities to meet the entire fuel requirement of the refinery. The changing character of the sludge which constitutes about 90 per cent of the liquid residuals burned,

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NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

is indicated in Table 1 which shows two typical laboratory determinations.

TABLE 1 SHOWING VARYING CHARACTER OF SLUDGE

Gravity	16.1 API	12.3 Bé heavy
Specific gravity	0.9587	1.092
Weight, lb per gal	7.984	9.105
Sulphur, per cent	4.03	7.43
Heat content, Btu per lb	17,219	13,555

While these analyses are given as typical, the predominating characteristic of the sludges is extreme variability. The moisture or weak acid content of some sludges burned varies through a range up to 50 per cent and on rare occasions goes even higher. Such variations in moisture content cause a wide variation in the consistency of the sludge and require special handling equipment and operating procedures.

Sludges may be classed in two groups: Those from treating gasoline and kerosene, and those from treating heating oils and other heavy petroleum products.

Sludges From Treating Gasoline and Kerosene

Light-oil sludges are produced in two ways: (1) The acid oil, lighter than acid tar, settles out from sludge in the kettle or storage tank before processing. (2) The sludge is pumped into a kettle containing hot water where reaction takes place, resulting in a three-layer separation of acid oil, acid tar, and weak acid. The acid tar, the largest percentage in this separation, has the highest acid content, and is more readily handled and atomized than the heating-oil sludge. It is the most corrosive of all the liquid residuals, due to the amount of water entrained during the separation of the tar from the weak acid.

Sludges From Heating-Oil Refining

These are distinguished from those of gasoline by their thick viscous consistency. Also they are not separated into two layers by the addition of water, as the separated acid is too high in impurities. This sludge is obtained from the purification of heating oil, and is pumped directly to the sludge storage tanks, from which it is pumped to the burners. It requires no agitation, fluxing, or heating for handling. It is not as corrosive as the lighter sludges, either because of the undiluted acid or the inhibiting action of the sludge itself.

Paraffine Sludge

This sludge is a by-product from paraffine-wax treating operations. In many cases this sludge causes considerable difficulty, as it becomes entirely solidified at atmospheric temperature. A large eastern refinery, however, has developed equipment for burning it that has proved satisfactory. The sludge is kept in agitation by steam jets and fluxed with a heavy viscous oil at the treating plant. It is then pumped to the boilerhouse into lead-lined storage tanks, which are equipped with closed brass heating coils located about six feet above the bottom of the tank. Agitation is effected by means of a mechanical mixer located below the heating coil. The storage tank is also provided with a ring of steam jets in the bottom that serves the dual purpose of heating and agitating the sludge. This sludge is burned in a separate fuel system similar to that employed for fuel oil. Burners, using steam for atomizing are employed successfully and the sludge is supplied to the burners at a temperature of from 200 to 220 F. The temperature required depends upon the characteristics of the particular batch of sludge burned.

Preparation Technique

The efficient burning of various sludges constitutes a problem that must be developed by each refinery to meet its particular conditions. Successful utilization of by-product fuels in one refinery is no assurance that it will be equally successful in another, even under similar conditions, as each separate method of processing produces its own variety of sludge. Since various operations produce different types of sludge, no definite rules can be given for its treatment. Frequently it may be processed as a common fuel without troublesome coke deposits. As to whether it is properly processed or agitated, only the experience of trained operators can determine.

Waste Fuel Ground Up as "Colloidal" Fuel

Additional waste fuels consist of all the wastes that have any combustible characteristics. These residues consist of gummy coke, heavy tank bottoms, muck, asphaltic residues, and slop, i.e., plastic solids or semisolid materials which are neither truly solids nor liquids, and cannot be pumped. Collected and dumped into a pit, they may be ground or processed by the wet-pan type of grinder. The product of this operation is a frothy liquid containing appreciable quantities of carbon particles up to $^1/_8$ in. or more in diameter. The quantities of these wastes are generally not sufficient at any one time to warrant continuous operation of separate burning equipment for their individual disposal. In many cases the cost of disposing of these wastes in any other manner is prohibitive.

Sludge Handling and Storage Equipment

The storage and handling of sludges and tars present no difficult problem when the usually accepted standards are maintained. Lead-lined tanks are used for receiving and storing the sludge. These tanks usually are equipped with steam jets to provide constant agitation and proper consistency for pumping and burning, although many sludges are burned without additional heating or agitation. With all fuel-burning equipment, constant flow through the supply and return lines must be maintained in order to prevent stoppage of the system. It is frequently necessary, when shutting down a sludge-burning system, to have the line thoroughly blown out with steam to purge it completely of coke particles which tend to solidify and plug the line. Otherwise the line may have to be dismantled for cleaning. Receiving and storage tanks may be equipped with an acid-fume arrester, if necessary, to collect and dissolve any objectionable acid fumes that may originate through blowing or agitating operations.

Pipes and cocks used for handling the acid sludges are made of brass. This material contains about 85 per cent copper and 15 per cent zinc and usually can be considered as satisfactory.

The pumping quality of sludge is good in most cases and offers no particularly difficult problem in winter or summer. Reciprocating pumps of the type in general use for handling liquid fuels are utilized and in most cases are satisfactory. Pump-valve construction is of various types, being spring-actuated or of the plain rectangular-block type, depending considerably upon the extent of repairs necessary for maintenance. Pistons, piston rods, and lines usually are of acid-resisting bronze of sufficient hardness to avoid excessive wear.

Fuel-Burning Equipment

When faced with burning a variety of liquids, semiliquids, and solid by-product fuels, a novel combustion problem presents itself. The furnace design usually follows the accepted standards for high rates of combustion and steam generation and serves as an efficient plant for the disposal of by-product fuels as well. See Fig. 1.

It is generally desirable to have a combined-type burner, Fig. 2, which is capable of burning a combination of gaseous, liquid, or pulverized solid fuels without changing burner parts, or which permits all of these fuels to be burned simultaneously.

One refinery operation has one of its three large boilers, each of

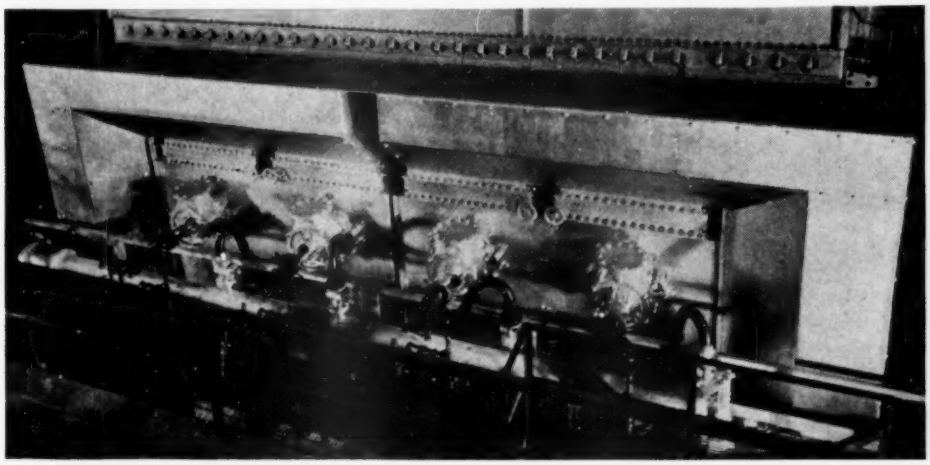


FIG. 1 UPPER PANEL OF FOUR COMBINATION BURNERS FOR BURNING PULVERIZED COKE, FUEL OIL, SLUDGE, AND GAS
(Another set of four is located below the platform, as shown in Fig. 3.)

which is designed to produce a maximum of 225,000 lb of steam per hr at 650 psi and 740 F, equipped to burn five separate fuels simultaneously, i.e., fuel oil, refinery gas, petroleum coke, acid sludge, and colloidal fuel. The other two boilers are equipped to burn refinery gas, bunker fuel oil, and sludge at the same time. Another boiler plant, consisting of four boilers, each with a capacity of 50,000 lb of steam per hr at 380 psi and 650 F, has twelve burners to each boiler. Seven of the burners may burn either fuel oil or sludge; the remaining five are for refinery gas. This boilerhouse generally is referred to as the waste-fuel disposal plant and burns waste fuels entirely when the supply is sufficient to carry the full load. Fuel oil is used only to keep settings warm when sludge is not available.

All boilers on which sludge is used as fuel are equipped with waterwalls and economizers. Induced and forced draft are used at the large plant, while induced draft is used at the smaller plant.

A full complement of indicating, recording, and integrating meters is advisable on high-pressure steam-generating units burning refinery by-products and sludges as fuels, as efficient burning of such fuels reduces the amount of commercial fuels required.

Combustion controls are usually operated manually due to the constantly changing heat content and varying amounts of the fuels.

Indicators are essential to detect instantly the presence of smoke. Smoke indicators used in conjunction with duplex gas recorders have been invaluable in securing maximum efficiency from the various fuels.

Due to the various fuels burned, a multiplicity of piping and valves introduces rather complex operating features, Fig. 3. Steam is employed in most cases to purge supply and return lines, headers, and burner connections, when it is necessary to change burners.

In the larger plant, four headers cross the front of each of the boiler burner panels—one for fuel oil, one for acid fuel, one for gas, with the fourth supplying steam for atomization and for purging lines. When it is necessary to purge an entire sludge system, steam or air is usually used in the parts of the system to be purged.

The large boiler equipped to burn pulverized coke has eight of the modern turbulent horizontal-type burners, and is fired from two levels of four burners each. Two ball mills, each having a coal capacity of 15,000 lb per hr, or sufficient to carry the boiler alone on full rating, comprise the pulverizer equipment. The exhauster of each mill has four ducts leading to one firing level so that one mill may be used at a time.

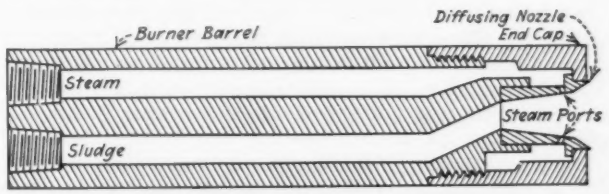


FIG. 2 TYPICAL SLUDGE BURNER USING STEAM FOR ATOMIZING

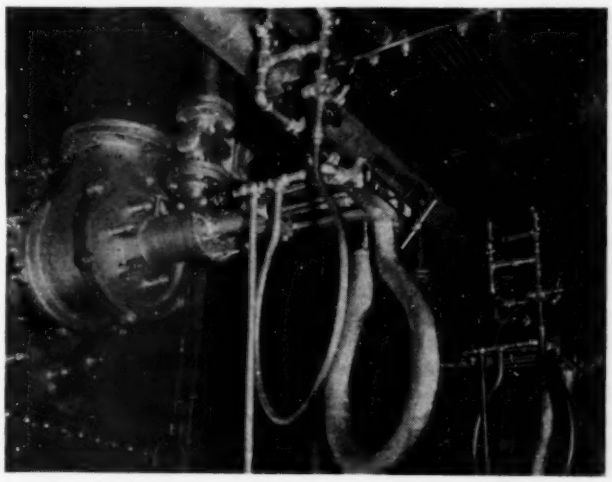


FIG. 3 COMBINATION BURNERS FOR PULVERIZED COKE, FUEL OIL, SLUDGE, AND GAS
(The large blank-covered top connection at the left is for pulverized fuel. The next vertical connection is from the gas header to the control cock. Steam, sludge, and fuel-oil headers are shown supported from the platform.

BURNER DESIGN

In firing various sludges, the most important factor is good atomization to reduce the sludge to a fine mist for its complete, combustion. Of the various approved methods of atomization, steam has generally been used for the liquid sludges, which often are of high specific gravity and contain a considerable amount of solid matter, carbonaceous or otherwise, in noncolloidal suspension. Due to these conditions, there has been much development in designing burners that would be successful in burning the various kinds and grades of refinery liquid sludges without deteriorating too rapidly.

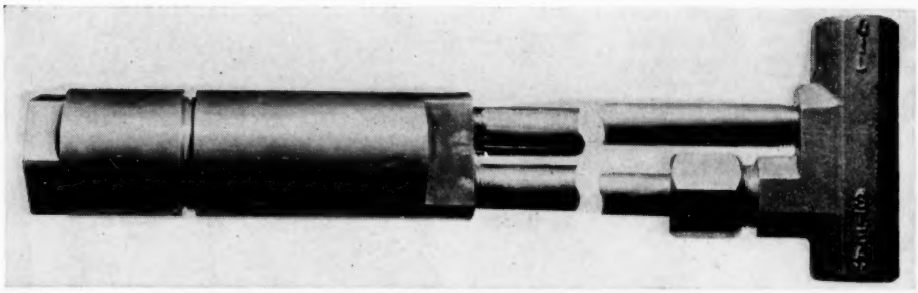


FIG. 4 BURNER FOR FUEL OIL AND SLUDGE

Factors that usually determine the selection of a burner are: (1) Simplicity of design; (2) low cost of installation; (3) thorough atomization of cold sludges; (4) low pump pressure; (5) flexibility and efficiencies on swinging loads; (6) capability of bringing a cold boiler on the line at short notice; (7) ability to burn extremely heavy oils and sludges; and (8) resistance to erosion and corrosion.

A burner, Fig. 4, in one refinery has been designed to burn successfully the different grades of sludge at a pressure of 50 psi and a temperature of 110 F. Effort was made at first to heat the sludge to lower its viscosity, but considerable plugging of the lines and burners resulted and it was finally decided to burn the sludges cold.

The burner in design conforms to the outside-mixing type. The assembly consists of a steam-and-sludge manifold arrangement on one end connected by about six feet of $^1/_2$-in. brass pipe to the burner at the other end. The burner is fastened to the ignition tube of the air register with a swinging yoke clamp for quick attachment. The same clamp attaches the sludge and steam-line connections to the burner manifold obviously avoiding many separate pipe connections. A by-pass valve is provided between the steam and sludge passages to blow the burner assembly clean before changing the burner or to remove solid particles in the burner which may temporarily shut off the sludge supply while in service.

The sludge and steam are admitted under pressure through separate valves and pipe arrangements into the burner tip before atomization takes place. The diffusing nozzle for atomization is located in the tip of the burner. This nozzle has a number of steam ports placed tangentially at the end of the nozzle in relation to the sludge stream and depends upon atomization off the burner tip. This arrangement avoids excessive burner wear. The burner tips are made of alloy metal that will withstand the strong erosive action as well as the corrosive attack of the by-product fuels.

The sludge is forced under pressure through the atomizer, past the steam ports which gives it a high velocity of rotation, breaking it up under the action of centrifugal force. The sludge leaving the burner tip is acted upon by three forces: Gravity, centrifugal force due to the rotating effect of the steam on the oil stream, and translation along the axes of the atomizer. In this manner the sludge is broken up in a fine fog-like mist which can be readily mixed with the secondary air admitted around the burner for complete ignition. The flame is a turbulent rotating cone that does not touch the rear wall tubes.

Regardless of the type of burner employed, satisfactory results cannot be obtained unless the furnace of the boiler is suited for the burning of by-product fuels. The design of the furnace must approach that suitable for good fuel oil by allowing sufficient furnace volume for maximum boiler requirements. The gas must be kept highly heated and afforded time and space in which to burn completely before reaching boiler heating surfaces.

Some important advantages of this type of burner are as follows:

1 Pulsation in the furnace is avoided by allowing sufficient mixture of air and sludge vapors for combustion.

2 There is no objectionable smoke to foul the boiler tubes with a carbon deposit and cause a serious loss of capacity and efficiency.

3 The admission of excess air to the furnace is avoided.

4 The required burning conditions, consisting of good atomization and burning in suspension, are obtained.

5 Low sludge temperatures and pressures are sufficient.

6 The plugging of burners, due to large sludge particles reaching the burner tips, is avoided.

The success of atomization with steam in many cases has been remarkable in that it has eliminated many of the objectionable features encountered in preparation and handling the heavy by-product fuels before they could be burned. The largest percentage of these by-product fuels is burned cold as it is received at the sludge storage tanks and is not fluxed with fuel oil to lower its viscosity.

Burning sludges cold has many advantages. It is difficult to obtain a metal with sufficient strength, corrosion resistance, and thermal transfer efficiency to heat the sludge in a closed heater. Agitating and heating by means of steam jets is necessarily accompanied by excessive steam costs and increase of moisture content in the sludge. Agitating in some cases causes the coke, held in a semicolloidal suspension in the sludge, to drop out in the tank, or in the lines and burners, thereby clogging them. When coke particles are precipitated through agitation and heating, they will not be absorbed again by the fluid even though boiled in heavy acid. This frequently makes it necessary to take the tanks out of service for cleaning and repairs to the heating and agitating system.

OPERATING RESULTS

In the refinery operation previously mentioned, the coke-pulverizing equipment is of standard make as applied to pulverizing coal. It has not been in service long enough to provide any definite data on wear or maintenance costs. The small portion of nonmerchantable coke, as it comes to the boilerhouse, has a high carbon content with a varying ash content as shown in Table 2.

TABLE 2 ANALYSES OF COKE

	Sample no. 1	Sample no. 2	
Oil (separately determined)	7.4	8.3	
Sulphur (separately determined)	3.24	2.4	
Volatile matter	12.61	14.12	Per cent by weight
Carbon	78.2	80.27	Per cent by weight
Moisture	0.77	1.21	Per cent by weight
Ash	8.42	4.4	Per cent by weight
Hardgrove scale	38	56	
Btu per lb	14,500	14,972	

The presence of part of the volatile matter may be explained by the surface oil which accumulates after the coke bed is formed.

In grinding, this oil naturally spreads over the fine particles of pulverized material and must be consumed by surface burning when injected into the furnace. While the surface burning is in process, the coke particle is traveling to cooler parts of the setting (see Fig. 5) where combustion is slowed up. In many cases, sufficient time has not been allowed to consume the particle of carbon itself, permitting it to pass through the furnace only partially burned.



FIG. 5 NEW BOILER UNDER CONSTRUCTION

(View looking vertically up from dry-slag hopper. This boiler will be fired by eight modern turbulent horizontal burners, capable of burning liquids, semiliquids, and solid by-product fuels. Capacity, 225,000 lb of steam per hr at 650 psi and 740 F; heating surface of waterwalls and boiler, 22,500 sq ft; economizer surface, 14,100 sq ft.)

Due to the mineral-ash content present in petroleum oils and removed by means of the acid treatment, which is finally deposited in the sludge, considerable slagging of an objectionable nature is often experienced on the boiler settings while burning these sludges. This slag, in many cases, adheres to the firebox row of screen tubes, the burner-panel water tubes, and particularly to the flat bottoms of the furnaces, when they are employed as sludge-burning units, thus lowering the boiler rating and efficiency to a considerable degree.

UTILITY AND HEAT VALUES

While each of these residues entails its own problem of disposal, much might be said about their utility value, as their true value when burned in a furnace cannot be determined on the basis of calorimeter indications alone.

It is difficult to allocate operating expenses, such as maintenance and repairs, to their proper location on steam-generating units that utilize varying proportions of these fuels. Burning by-product fuels may interfere with efficient evaporation from the heat-transfer surfaces, with a corresponding drop in over-all boiler efficiency and rating since the heating surfaces may be coated with a deposit of slag which is not present when standard fuels are burned.

It is practically impossible to secure a reliable heat-unit balance or true boiler efficiency on sludges that are burned as waste fuel. Due to slagging and fouling of the settings, maintenance and repair costs should be apportioned to the various grades of by-product fuels burned. This is accomplished best by establishing a reasonable operating efficiency for the basic fuel, and debiting or crediting any loss or gain against the waste fuels or the department that produces the by-product material. This method of accounting appears to be a fair means of arriving at or near the utility value of sludges burned, since any efficiency loss or excessive repair or maintenance cost will thus be properly charged.

Discussion

T. GARDNER.[2] The corrosion problem, resulting from the burning of sludges and coals of high-sulphur content, referred to by the author, is one of economic importance, in that boilers and auxiliary equipment attacked by active corrosion may fail and require replacement after only a few years of service.

Such corrosion, evidently, is activated by the accumulated deposits on the metal exposed to the flue gases and by the attendant moisture conditions. These deposits, which are hydroscopic, can become actively corrosive when the metal temperature at the point of corrosion is below the dew point of the surrounding gases. In practice, this condition frequently occurs in units while operating at low loads, while banked, or during outages. Even at higher loads, air-heater corrosion may occur when deposits on the gas side of the tube reduce the heat-transfer rate and lower the metal temperature.

In cases where active corrosion is anticipated, the practice of the writer's company is to advise the customer of the probable deterioration of the tubes, due to this cause, and to suggest that either tubes of heavier gage be used or that alternative designs be employed, to prolong the life of the equipment. One or the other of these suggestions is usually accepted.

G. F. GAST.[3] In general the writer's experience in the burning of nonmerchantable combustibles is in agreement with that of the author. However, the operation of the refinery with which the writer is connected is not comparable with that of the Bayway Refinery, is so far as the fuel situation is concerned. The only nonmerchantable fuels utilized at the Bayonne refinery are creek slop and high-viscosity bottoms. The proportion of water in the creek slop is not constant at all times. This difficulty is overcome, when the moisture content is too high, by burning good fuel along with the slop mixture.

The burning of high-viscosity bottoms is very simple, since the only question involved is that of heating the oil to the proper temperature in order to obtain the correct viscosity at the burner.

J. A. HAYES.[4] The introduction of the type of bunker-C fuel oil, as now produced by the principal petroleum-refining companies has presented a new problem both to users of bunker-C fuel oil and to oil-burning-equipment manufacturers. Cracked fuel oils require improved oil-burning equipment to prevent carbonization of the atomizers, as well as better methods of air introduction to permit faster and cleaner combustion. Present-day fuel oils are largely blends of residuum left in the still and a light product such as virgin gas oil, obtained from the early stages of the distillation, or a cracked gas oil, obtained from the pressure-stilling operation. Generally this blend contains solids in suspension. As the percentage of solids increases, the colloidal type of fuel is reached which requires the use of preheated air for rapid ignition and combustion.

Proceeding beyond the colloidal type of fuel we encounter the class of nonmerchantable fuels treated by the author. It is not

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[2] The Babcock & Wilcox Company, New York, N. Y.

[3] Power Engineer, Tide Water Associated Oil Company, Bayonne, N. J. Mem. A.S.M.E.

[4] Todd Combustion Equipment, Inc., New York, N. Y. Member A.S.M.E.

impossible nor, in fact unlikely, that fuels now considered nonmerchantable will be offered in the future for industrial use. Since the demand or need of the overhead product from the distillation of fuel oil is such as to necessitate obtaining the maximum yield, the residuum may be of almost solid consistency at ordinary temperatures. It may not be economical to "cut back" with lighter fractions to produce the fuel oils now available. In that event, fuel oils approaching the nonmerchantable types discussed in this paper may have to be utilized.

Common difficulties met in the use of bunker-C fuel oil may be mentioned as follows:

(*a*) Tendency toward instability in storage; (*b*) similar tendencies if too highly heated before reaching the burners; (*c*) atomizer deposits; (*d*) stack solids and ash and slag formation. These difficulties are aggravated as the grade of fuel oil becomes poorer. The nonmerchantable oil-refinery fuels, mentioned in this paper, demand a storage and preparation technique entirely unfamiliar to the ordinary plant operator. It is of comforting interest to note that the conventional boiler and furnace design is satisfactory for the combustion of exceedingly poor grades of fuel. The changes necessary in the fuel-oil-burning equipment can be determined only by actual test. Fuel-oil-burning equipment has kept in step with the quality of fuel oil. There is every reason to expect that, should the occasion arise, efficient methods of burning the available fuel oils will be developed.

The author has pioneered in the field of by-product fuels. His methods for obtaining the maximum heat recovery from these fuels will no doubt be of great assistance to operators and fuel-oil-burning-equipment manufacturers.

Author's Closure

In closing, the author wishes to express appreciation for the interest shown by the commentators. Comment and criticism in subsequent meetings would, no doubt, be of further assistance to engineers faced with the disposal of waste fuels under discussion.

The petroleum industry in particular is continually striving to reduce production cost, therefore, utilization of all by-products is becoming more and more important.

Mr. Hayes's comments are especially valuable in showing the possible relation between industrial fuels of today and tomorrow. The study is of economic importance and should bring forth further research in methods of handling, preparation, and burning of heavier fuels.

The many plants that pioneered in burning waste fuels have contributed a vast amount of information, if this were not true, modern installations of waste-fuel-burning plants would be far behind their present state of development.

It is felt that there is still much to be achieved in the future development of equipment for handling and burning waste fuels, and that such progress can only be attained by further experimentation in the field where these fuels are available.

The Analogy Between Fluid Friction and Heat Transfer

By TH. VON KÁRMÁN,[1] PASADENA, CALIF.

The results of the modern turbulence research on velocity distribution near solid walls are applied to extend Reynolds' analogy between fluid friction and heat transfer to liquids. Discrepancies between the experimental results and the theory of Taylor and Prandtl are explained. A new theoretical formula which fits the experimental results more satisfactorily is suggested and compared with the empirical equations of Dittus and Boelter.

IN the year 1874 Osborne Reynolds published a remarkable paper (1)[2] entitled "On the Extent and Action of the Heating Surface for Steam Boilers." In this paper Reynolds suggested that momentum and heat in a fluid are transferred in the same way. He concluded that in geometrically similar systems a simple proportionality relation must exist between fluid friction and heat transfer. Suppose, for example, a flow parallel to the x-axis in which the mean velocity of the fluid is a function of the y-coordinate only. It is known from Reynolds' previous investigations that the shear stress τ occurring in an arbitrary plane perpendicular to the y-axis is equal to the total momentum transferred by the molecular and turbulent exchange and is given by

$$\tau = \mu \frac{du}{dy} - \rho\overline{v'u'} \quad \ldots\ldots\ldots\ldots\ldots\ldots [1]$$

where u is the mean velocity, u' and v' are the velocity fluctuations parallel to the x- and y-axes, μ denotes the coefficient of the molecular viscosity, and ρ the density of the fluid. If the temperature θ of the fluid varies with the y-coordinate, the heat flow in the y-direction, i.e., the amount of heat transferred by unit area across the same plane, can be expressed in a similar way. It consists of two parts: The molecular heat conduction and the turbulent heat transfer due to the fluctuations of velocity and temperature. The first contribution is equal to $-k\, d\theta/dy$, where k is the coefficient of the molecular heat conduction (thermal conductivity), and θ the mean temperature, which is supposed to be a function of the y-coordinate only. The second part is equal to the mean value of the product $c\rho v'\theta'$ where c is the specific heat per unit mass of the fluid (in the case of a compressible fluid the specific heat for constant pressure), and θ' is the fluctuation of the temperature. Because of the temperature gradient in the y-direction, a correlation exists between θ' and v'; therefore the mean value $c\rho\overline{v'\theta'}$ is different from zero and gives the amount of heat transferred by the turbulent fluctuations.

Hence, the total heat flow is given by the equation

$$q = -k\,(d\theta/dy) + c\rho\overline{v'\theta'} \quad \ldots\ldots\ldots\ldots\ldots\ldots [2]$$

The second term in Equation [1] is called the contribution of the eddy viscosity, the second term in Equation [2] the contribution of the eddy conductivity. The analogy suggested by Reynolds amounts to the following statement: If the mean value $\overline{v'u'}$ is expressed in the form $\epsilon\, du/dy$, the mean value of $\overline{v'\theta'}$ is equal to $\epsilon\, d\theta/dy$, where ϵ has the same numerical value in the two expressions, and is called the coefficient of the turbulent exchange.

Introducing the kinematic viscosity $\nu = \mu/\rho$ and the thermometric conductivity $\kappa = k/c\rho$ one obtains the following fundamental equations

$$\left.\begin{aligned} \tau/\rho &= (\nu + \epsilon)du/dy \\ q/c\rho &= -(\kappa + \epsilon)d\theta/dy \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots [3]$$

It follows from Equations [3] that direct proportionality between shearing stress and heat transfer can be expected

(a) if ν and κ are negligible compared to ϵ,
(b) if ν and κ are numerically equal.

In the case of turbulent flow in pipes or around submerged bodies the condition (a) is generally satisfied with the exception of a relatively narrow range near solid walls. Consider two planes $y = y_1$ and $y = y_2$ in the turbulent region, and assume first that τ and q are independent of y. Then it follows by integration from Equations [3]

$$(u_2 - u_1)/\tau = c(\theta_1 - \theta_2)/q \quad \ldots\ldots\ldots\ldots\ldots\ldots [4]$$

This is the analytical expression of Reynolds' analogy. It is evident that the same relation prevails if τ and q vary with y according to the same law. In most cases, for example in a flow in a channel or a pipe, the law of variation of τ and q is not exactly identical. Between parallel walls or in a circular pipe τ varies linearly with the distance from the wall, whereas the variation of q is determined by the balance between the heat carried by the fluid in the flow direction and the heat flow across the stream, and therefore depends on the velocity distribution. However, the influence of the deviation between the distribution of τ and q is not very significant, and Equation [4] can be generally applied in the fully developed turbulent region.

Condition (b) is approximately satisfied by gases, since the kinematic viscosity and the thermometric conductivity of gases are of the same order of magnitude. However, for liquids ν is very much larger than κ. The ratio $\sigma = \nu/\kappa$ is for some liquids as high as $\sigma = 200$. Hence, Reynolds' analogy cannot be applied directly to the heat transfer between solids and liquids. The present paper is concerned mainly with this case. For completeness the case $\sigma = 1$ will also be discussed. However, in the next section the coefficients of friction and heat transfer will be defined first.

Coefficients of Friction and Heat Transfer

The quantity τ/ρ which occurs in the first Equation [3] has the dimension of the square of a velocity. Hence $\tau/(\rho/2)U^2$, where U is a velocity of reference of the physical setup, is a dimensionless quantity. In general, the ratio $\tau/(\rho/2)U^2$ is used as the coefficient of friction; it will be denoted by the symbol C_f. The choice of the reference velocity is more or less arbitrary. For

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[1] Director of Guggenheim Aeronautical Laboratory, California Institute of Technology. Mem. A.S.M.E.

[2] Numbers in parentheses refer to the Bibliography.

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Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

flow through a pipe the velocity at the center or the average velocity over the cross section may be chosen. In the case of a body submerged in a fluid, U is suitably taken equal to the relative velocity between the solid and the undisturbed region of the fluid.

The quantity $q/c\rho$ which occurs in the second Equation [3] has the dimension of velocity times temperature. Hence, if Θ denotes a reference temperature, $q/c\rho U\Theta$ is a dimensionless quantity. The reference temperature Θ is in general the temperature difference between two suitably chosen points of the system, or between temperatures averaged over certain regions. The ratio $C_H = q/c\rho U\Theta$ is a dimensionless quantity and will be called in this paper the *heat-transfer number*.

Introducing the coefficients C_f and C_H the shearing stress and the heat flow appear in the form

$$\left.\begin{aligned} \tau &= (C_f/2)\rho U^2 \\ q &= C_H \rho c U \Theta \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots [5]$$

It is seen that C_f is the ratio between two stresses, namely, the shearing stress τ and the dynamic pressure $U^2\rho/2$. The heat-transfer number C_H is the ratio between two heat flows: the heat flow q across the stream and the heat carried per unit cross-sectional area by a fluid of temperature Θ and velocity U, which is obviously equal to $\rho c U\Theta$. Since the shearing stress is equal to the momentum transferred across the stream, and the quantity ρU^2 can be considered as the momentum carried by the fluid in the direction of the stream, the concept of the coefficients $C_f/2$ and C_H appears analogous.

Another conception of the heat-transfer number is, however, commonly used in engineering practice. In the case of molecular heat transfer the heat flow is equal to $q = -k\, d\theta/dy$ (cf. Equation [2]) or if the distribution of the temperature is linear between two surfaces at the distance d, and $\Theta = \theta_1 - \theta_2$

$$q = k\Theta/d \ldots\ldots\ldots\ldots\ldots\ldots [6]$$

Now in the general case of molecular and turbulent heat transfer one can write the second Equation [3] in the analogous form

$$q = k'\Theta/d \ldots\ldots\ldots\ldots\ldots\ldots [7]$$

where $k' = c\rho(\kappa + \epsilon)$ can be interpreted as an increased value of the coefficient of the thermal conductivity k and d is a suitably chosen characteristic length of the physical setup. Then the ratio k'/k is a dimensionless quantity which expresses the rate of increase of the heat transfer due to turbulence. This ratio is sometimes called the *Nusselt heat-transfer number* and denoted by the symbol Nu. The equations

$$q = C_H \rho c U \Theta \ldots\ldots\ldots\ldots\ldots\ldots [8a]$$

and

$$q = Nu\, k\Theta/d \ldots\ldots\ldots\ldots\ldots\ldots [8b]$$

are two equivalent expressions for the heat flow, based on two different conceptions of the transfer mechanism.

The film coefficient or heat-transfer coefficient h generally used in practical calculations is equal to the heat flow per unit time and per unit surface per unit temperature difference between solid and fluid.

Hence, from Equations [8a] and [8b]

$$h = q/\Theta = C_H \rho c U = Nu\, k/d \ldots\ldots\ldots\ldots [9]$$

It is seen that the use of the Nusselt number is somewhat simpler for calculation, but the number C_H has the great advantage that it is directly linked to the coefficient of friction. The relation between the two numbers is the following

$$Nu = C_H \frac{\rho c U d}{k} = C_H \frac{\rho U d}{\mu} \frac{\mu c}{k} \ldots\ldots\ldots [10]$$

Now the expression Ud/ν is known as the Reynolds number Re, the ratio $\mu c/k = \nu/k = \sigma$ which already was mentioned in the introduction is called Prandtl's number, and the reciprocal of the latter is sometimes called Stanton's number. From Equation [10] one obtains

$$Nu = C_H Re\, \sigma \ldots\ldots\ldots\ldots\ldots [11]$$

The dimensionless number $\rho c U d/k = Ud/\kappa$ is called Peclet's number Pe; obviously $Pe = Re\, \sigma$. Hence, if Re and σ are given, Peclet's number is not an independent parameter.

The heat-transfer numbers C_H or Nu are functions of two of the three parameters Re, Pe, and σ. In this paper Re and σ are used as independent parameters.

Reynolds' Analogy for $\sigma = 1$

For an ideal gas both the kinematic viscosity ν and the thermometric conductivity κ are proportional to the product of the mean free path and the mean velocity of the molecules of the gas. According to the kinetic theory of gases the ratio ν/κ depends only on the number of degrees of freedom. Eucken (2) has suggested an empirical relation between σ and the ratio $\gamma = c_p/c_v$ of the specific heats of the gas

$$\sigma = 4\gamma/(9\gamma - 5) \ldots\ldots\ldots\ldots\ldots [12]$$

According to this equation $\sigma = 0.66$ for monatomic and $\sigma = 0.74$ for diatomic gases. Since σ is not very much different from unity the original Reynolds analogy can be applied to obtain an approximate value of the heat-transfer number between a solid and moving air.

For $\sigma = 1$, Equations [3] become, by replacing κ by ν

$$\left.\begin{aligned} \tau/\rho &= (\nu + \epsilon)du/dy \\ q/c\rho &= -(\nu + \epsilon)d\theta/dy \end{aligned}\right\} \ldots\ldots\ldots\ldots [13]$$

Assume first that τ and q are constant. Then if u_0 and θ_0 denote the velocity and temperature at the wall, and u and θ correspond to an arbitrary point in the fluid, it follows from Equations [13] by integration, that

$$\left.\begin{aligned} u - u_0 &= \frac{\tau}{\rho}\int_0^y \frac{dy}{\nu + \epsilon} \\ \theta - \theta_0 &= -\frac{q}{c\rho}\int_0^y \frac{dy}{\nu + \epsilon} \end{aligned}\right\} \ldots\ldots\ldots\ldots [14]$$

Hence, if velocity and temperature are measured relative to the wall, their laws of distribution are similar. This will be approximately correct also if q and τ are not constant, but vary approximately in a similar way. It follows that if U is the mean velocity of the fluid relative to the wall, and Θ is the difference between the mean temperature of the fluid and the temperature of the wall

$$U/(\tau/\rho) = -c\Theta/q \ldots\ldots\ldots\ldots\ldots [15]$$

or (cf. Equation [5])

$$C_f/2 = C_H \ldots\ldots\ldots\ldots\ldots [16]$$

This relation implies important conclusions:

(*a*) If the friction τ is proportional to the nth power of the velocity U, the heat transfer q is proportional to the power $(n - 1)$.

(*b*) Roughness increases the friction and heat transfer in the same ratio.

It must be emphasized that Equation [16] has an approxima-

tive character, first because of the assumption $\sigma = 1$, and second, because of the deviation in the laws of variation of the shearing stress and the heat flow over the region involved.

Extension of Reynolds' Analogy to Liquids. Approximations of G. I. Taylor and L. Prandtl

It was mentioned that for most liquids Prandtl's number σ is large compared to unity. Hence, Reynolds' analogy can only be applied directly to the fully developed turbulent region in which the kinematic viscosity ν can be neglected compared to the eddy viscosity ϵ. G. I. Taylor (3) and L. Prandtl (4) extended Reynolds' analogy to the case of liquids flowing through a pipe by subdividing the fluid into two regions: The turbulent region in which ν is negligible, and a laminar region in which ϵ can be neglected compared to ν.

If the thickness of the laminar layer is δ, the velocity at the wall is equal to zero, the temperature at the wall is Θ_0, and the velocity and temperature at the boundary between the laminar layer and the turbulent region are denoted by U_δ and Θ_δ, respectively, it follows from Equations [3] that

$$\left.\begin{aligned} U_\delta &= \frac{\tau}{\rho}\int_0^\delta \frac{dy}{\nu} \\ \Theta_0 - \Theta_\delta &= \frac{q}{c\rho}\int_0^\delta \sigma\frac{dy}{\nu} \end{aligned}\right\} \quad [17]$$

and therefore

$$\sigma\frac{\rho U_\delta}{\tau} = \frac{c\rho(\Theta_0 - \Theta_\delta)}{q} \quad [18]$$

Reynolds' analogy applied to the turbulent region gives the relation

$$\rho(U - U_\delta)/\tau = c\rho(\Theta_\delta - \Theta)/q \quad [19]$$

where U and Θ are the mean values of the velocity and temperature taken over the cross section of the pipe. Eliminating Θ_δ one obtains

$$\frac{c\rho}{q}(\Theta_0 - \Theta) = \frac{\rho U}{\tau} + (\sigma - 1)\frac{\rho U_\delta}{\tau} \quad [20]$$

or introducing $C_f = 2\tau/\rho U^2$ and $C_H = q/c\rho(\Theta_0 - \Theta)U$

$$\frac{1}{C_H} = \frac{2}{C_f}\left[1 + \frac{U_\delta}{U}(\sigma - 1)\right] \quad [21]$$

This result was obtained by G. I. Taylor in 1919. Nine years later L. Prandtl further developed the theory by computing the ratio U_δ/U by means of the semiempirical formulas which at that time were used for the expression of the velocity distribution. Prandtl found that for the case of the flow in a smooth pipe U_δ/U is proportional to $Re^{-1/8}$, where Re is Reynolds' number. Then Equation [21] gives the following relation for the heat-transfer number C_H (β is a universal constant)

$$C_H = \frac{C_f}{2}\frac{1}{1 + \beta Re^{-1/8}(\sigma - 1)} \quad [22]$$

In general, it can be assumed that $U_\delta/(\tau/\rho)^{1/2}$ is a universal constant for smooth surfaces. On the other hand, $U/(\tau/\rho)^{1/2} = (2/C_f)^{1/2}$; therefore $U_\delta/U = \text{const}\sqrt{C_f}$ and from Equation [21] it follows that

$$1/C_H = (2/C_f) + \text{const}\,(2/C_f)^{1/2}(\sigma - 1) \quad [23]$$

The experimental evidence has shown that Equation [22] or Equation [23] holds only for relatively small values of $(\sigma - 1)$, perhaps up to $\sigma = 2$; for larger values of σ there is an increasing discrepancy between the experimental results and Equation [23]. To obtain better results the transition between the laminar and turbulent region must be taken into account.

Extension of Reynolds' Analogy to Liquids. The Author's Analysis

Assume that in the region near the wall, i.e., for $y < \delta$ neither ν nor ϵ can be considered small, i.e., the molecular and eddy viscosity might be of the same order, however, δ is so small that τ can be considered constant for $y \leqq \delta$. This condition is in general well satisfied. Then from Equations [3]

$$\left.\begin{aligned} U_\delta &= \frac{\tau}{\rho}\int_0^\delta \frac{dy}{\nu + \epsilon} \\ \Theta_0 - \Theta_\delta &= \frac{q}{c\rho}\int_0^\delta \frac{dy}{(\nu/\sigma) + \epsilon} \end{aligned}\right\} \quad [24]$$

It is known from theoretical and experimental results of turbulence research (cf., for example, the excellent treatise (5) of Bakhmeteff) that in the vicinity of a smooth wall the ratio $u/(\tau/\rho)^{1/2}$ is a universal function of the dimensionless distance parameter $y^* = (\tau/\rho)^{1/2}y/\nu$, or denoting that universal function by $f(y^*)$

$$u/(\tau/\rho)^{1/2} = f(y^*) \quad [25]$$

It follows that

$$\frac{du}{dy} = \left(\frac{\tau}{\rho}\right)^{1/2} f'(y^*)\frac{(\tau/\rho)^{1/2}}{\nu}$$

and

$$\frac{\tau}{\rho du/dy} = \nu + \epsilon = \frac{\nu}{f'(y^*)} \quad [26]$$

In other words, the ratio ϵ/ν is a function of y^* only. Substituting

$$\epsilon = \nu\left(\frac{1}{f'(y^*)} - 1\right) \quad [27]$$

and introducing y^* as variable into Equations [24], one obtains

$$U_\delta = \left(\frac{\tau}{\rho}\right)^{1/2}\int_0^{y_\delta^*} f'(y^*)dy^* \quad [28]$$

and

$$\Theta_0 - \Theta_\delta = \frac{q}{c\rho(\tau/\rho)^{1/2}}\int_0^{y_\delta^*}\frac{dy^*}{\frac{1}{\sigma} - 1 + \frac{1}{f'(y^*)}} \quad [29]$$

The value of y_δ^*, i.e., the value of y^* which corresponds to the boundary of the fully turbulent region might be somewhat arbitrary, but is certainly the same for geometrically similar flows. Hence, the definite integral in Equation [28] represents a numerical constant A, and the definite integral in Equation [29] represents a function of σ only, say $B(\sigma)$. Then from Equations [28] and [29]

$$\frac{c\rho(\Theta_0 - \Theta_\delta)}{q} - \frac{\rho U_\delta}{\tau} = \left(\frac{\rho}{\tau}\right)^{1/2}[B(\sigma) - A] \quad [30]$$

Reynolds' analogy applied to the turbulent core yields the relation

$$\frac{\rho(U - U_\delta)}{\tau} - \frac{c\rho(\Theta_\delta - \Theta)}{q} = 0 \quad [19]$$

By subtraction of the two equations, one obtains

$$\frac{c\rho(\Theta_0 - \Theta)}{q} - \frac{\rho U}{\tau} = \left(\frac{\rho}{\tau}\right)^{1/2}[B(\sigma) - A]$$

or

$$\frac{1}{C_H} = \frac{2}{C_f} + \left(\frac{2}{C_f}\right)^{1/2}[B(\sigma) - A] \quad [31]$$

and

$$C_H = \frac{C_f}{2}\frac{1}{1 + (C_f/2)^{1/2}[B(\sigma) - A]} \quad [32]$$

In order to determine the function $B(\sigma) - A = g(\sigma)$ Niku-

radse's experimental results can be used. In Fig. 51 (p. 79) of Bakhmeteff's treatise (5), $u/(\tau/\rho)^{1/2}$ is represented as function of y^* in a semilogarithmic diagram. There are no experimental values for $y^* < 10$. However, for small values of y^* it can be assumed that the velocity distribution is linear and $u = (\tau/\mu)\, y$ or $u/(\tau/\rho)^{1/2} = y^*$. For $y^* > 30$ the logarithmic law which is considered a characteristic distribution law for the fully turbulent region is fairly well satisfied. Hence, $y_\delta^* = 30$ is chosen. The points between $y^* = 10$ and $y^* = 30$ can be approximated by a straight line which joins the curve $u/(\tau/\rho)^{1/2} = y^*$ tangentially at $y^* = 5$. The equation of this straight line is

$$u/(\tau/\rho)^{1/2} = 5[1 + \log_e(y^*/5)] \quad \ldots\ldots\ldots\ldots [33]$$

Substituting $f'(y^*) = 1$ for $y^* < 5$ and $f'(y^*) = 5/y^*$ for $5 < y^* < 30$ and carrying out the integrations, one obtains $A = 5(1 + \log_e 6)$ and $B = 5[\sigma + \log_e(1 + 5\sigma)]$. With these values Equation [31] becomes

$$\frac{1}{C_H} = \frac{2}{C_f} + 5\left(\frac{2}{C_f}\right)^{1/2}\left\{\sigma - 1 + \log_e\left[1 + \frac{5}{6}(\sigma - 1)\right]\right\}. \quad [34]$$

It is seen that

(*a*) for $\sigma = 1$, $1/C_H = 2/C_f$, according to Reynolds' analogy,

(*b*) for small values of $\sigma - 1$, by expansion of the logarithmic term

$$1/C_H \cong (2/C_f) + \text{const}\ (2/C_f)^{1/2}(\sigma - 1)$$

in accordance with the theory of Taylor and Prandtl.

Fig. 1 shows schematically the approximate velocity distribution used for the calculation. Fig. 2 shows the values of the exchange coefficient resulting from this assumed velocity distribution. The assumptions corresponding to the Taylor-Prandtl theory are indicated in both figures by dash-dot lines. The improvement introduced into the theory consists essentially of the elimination of the abrupt change of the exchange coefficient. There remains a discontinuity at the boundary between the transition layer and the fully turbulent region. However, it is believed that this discontinuity has no large effect on the result, since in the calculations (cf. Equations [24]) only the reciprocal values of the quantities $\nu + \epsilon$ and $\kappa + \epsilon$ appear. Therefore, the accuracy depends chiefly on the correctness of the assumptions in the region where these quantities are small.

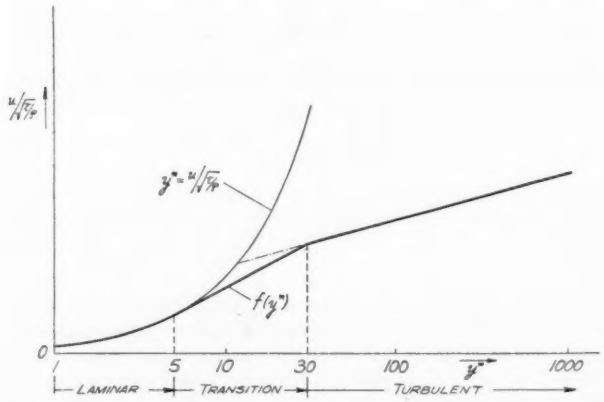


FIG. 1 VELOCITY DISTRIBUTION IN THE VICINITY OF A SOLID WALL

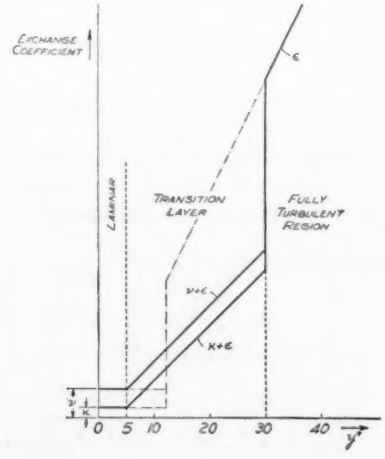


FIG. 2 EXCHANGE COEFFICIENTS IN THE VICINITY OF A SOLID WALL

COMPARISON BETWEEN THEORY AND EXPERIMENT

The theoretically computed values of the heat-transfer number can be compared with experimental data on the amount of heat transferred in smooth-walled pipes between the walls and the fluid at different rates of flow. However, an exact comparison is difficult for the following reasons:

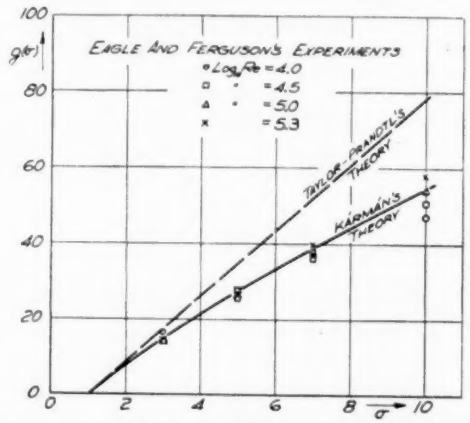


FIG. 3 COMPARISON BETWEEN THEORY AND THE EXPERIMENTS OF EAGLE AND FERGUSON

(*a*) Some experimenters use relatively short pipes, whereas for the application of the theory the pipe should have a long entry section, so that the velocity and temperature distributions can reach their final shapes.

(*b*) The physical properties vary with the temperature and, therefore, the heat-transfer coefficients depend on the rate of heat flow and are especially different for the case of heating and cooling of the fluid by the walls.

(*c*) Due to the variation of the physical properties over the cross section of the pipe, it is uncertain which values of the coefficients of viscosity and thermal conductivity are to be used for the evaluation of the transfer numbers.

(*d*) The value of the transfer numbers depends on the interpretation of the mean temperature, namely, whether the temperature is simply averaged over the cross-sectional area or is weighted according to the flow of matter passing through the elements of the cross section.

The difficulties mentioned under (*a*), (*b*), and (*c*) can be properly eliminated only by a careful extrapolation of the measured experimental results for an infinitesimal rate of heat flow, i.e., infinitesimal temperature difference between pipe and fluid. Such an extrapolation has been carried out by the British researchers, Eagle and Ferguson (6), for the heat exchange between smooth-walled pipes and water at different temperatures. Both Prandtl's and the author's formula can be reduced to the form

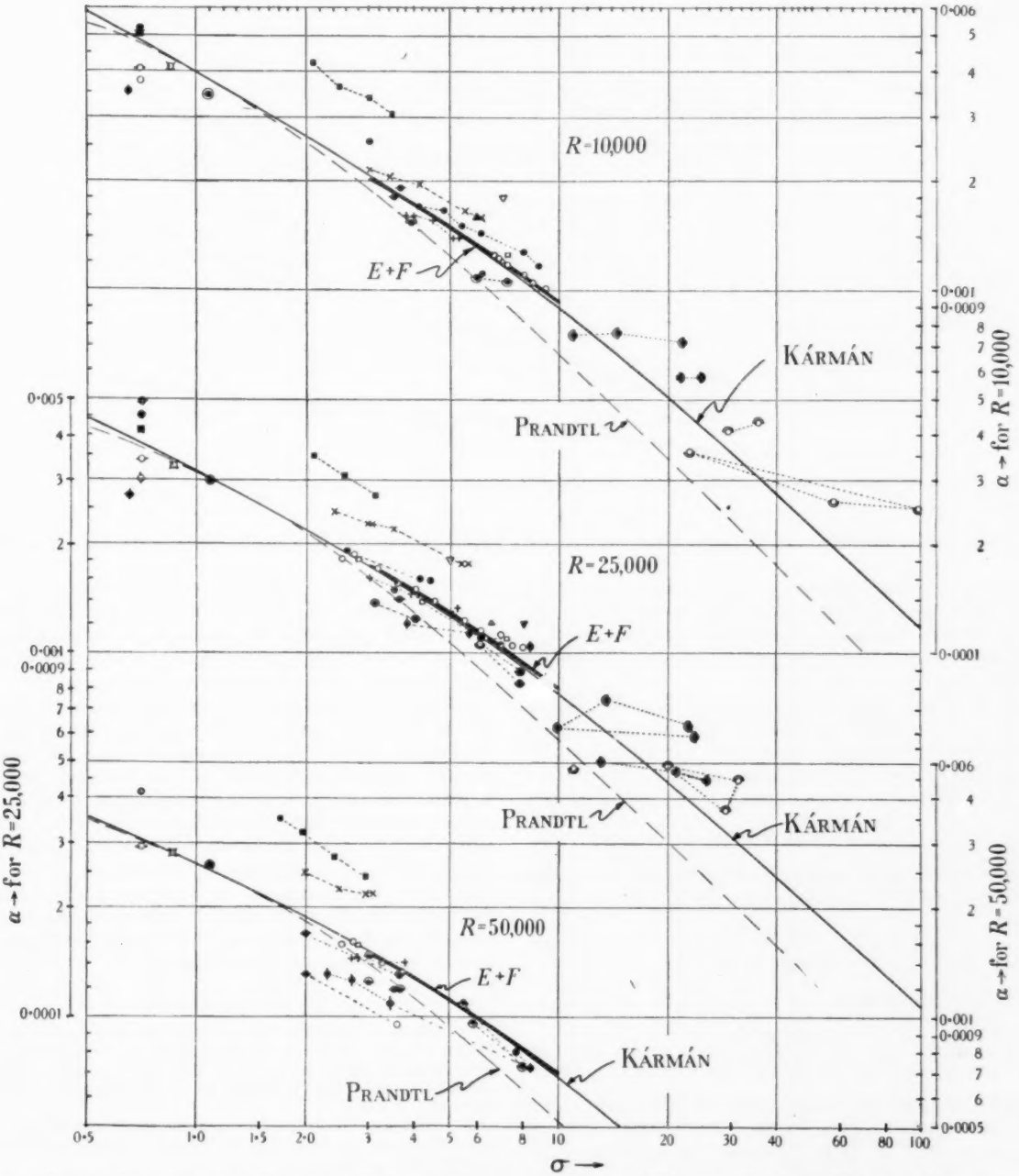


FIG. 4 HEAT-TRANSFER NUMBER IN PIPES FOR DIFFERENT REYNOLDS' NUMBERS COMPARED WITH THE THEORY
(α is equivalent to the symbol C_H used in this paper.)

$$\frac{1}{C_H} - \frac{2}{C_f} = \left(\frac{C_f}{2}\right)^{1/2} g(\sigma) \quad \ldots\ldots\ldots\ldots [35]$$

Prandtl uses for $C_f/2$ the approximation $C_f/2 = 0.04\ Re^{-1/4}$. With this value his theory leads to the expression

$$g(\sigma) = 8.7(\sigma - 1) \quad \ldots\ldots\ldots\ldots [36]$$

whereas according to the author's analysis

$$g(\sigma) = 5\{(\sigma - 1) + \log_e[1 + (5/6)(\sigma - 1)]\} \quad \ldots\ldots [37]$$

Fig. 3 shows the comparison of the two expressions [36] and [37], and the values of $g(\sigma)$ computed from the experiments of Eagle and Ferguson for $\log_{10} Re = 4$, 4.5, 5, and 5.3 for the range $3 < \sigma < 10$. It is seen that the accordance between the author's theory and the experimental results is very satisfactory. The results are not corrected for the error caused by the different interpretation of the mean temperature, item (*d*). G. I. Taylor made an estimate of this error for the case $\sigma = 1$, $Re = 10{,}000$ and found that it amounts to 3 or 4 per cent. However, for large values of σ the temperature distribution over the cross section becomes more and more uniform, so that the error is likely to be smaller than this value.

A great number of experiments have been carried out on heat exchange between pipes and different kinds of oils, petroleum, benzene, and so on. For these fluids σ varies from about 10 to 200. The results of these experiments show an extraordinary

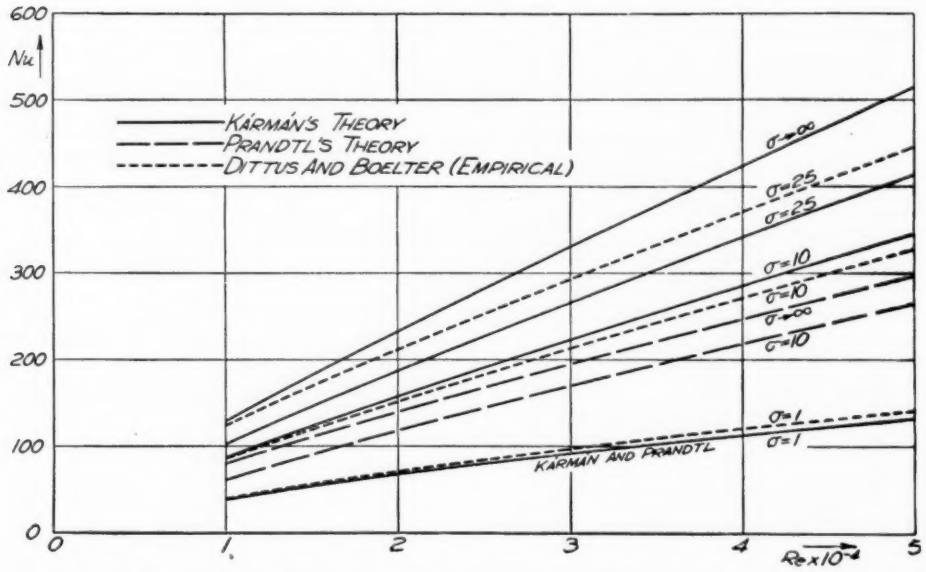


FIG. 5 COMPARISON BETWEEN THE THEORY OF PRANDTL AND THE AUTHOR WITH THE EMPIRICAL EQUATION OF DITTUS AND BOELTER

scatter, due to the great difficulties of experimentation and interpretation of the results. Fig. 4 shows the comparison between the heat-exchange numbers C_H computed from the results of different experimenters by H. Lorenz (7) and the values predicted by Prandtl's and the author's formula.

Dittus and Boelter (8) obtained two simple empirical formulas for the Nusselt number by averaging the scattered results of many experimenters: one equation for the case of the fluid's being cooled, the other for the fluid heated by the walls. The average of these two cases can be expressed approximately by the equation

$$Nu = 0.0254\, Re^{0.8}\sigma^{0.35} \quad \ldots\ldots\ldots\ldots\ldots [38]$$

On the other hand Prandtl's theory gives for the Nusselt number with $C_f/2 = 0.04\, Re^{-1/4}$ the expression

$$Nu = \frac{0.04\, Re^{3/4}\sigma}{1 + 1.74\, Re^{-1/4}(\sigma - 1)} \quad \ldots\ldots\ldots\ldots [39]$$

and the theory of the author

$$Nu = \frac{0.04\, Re^{3/4}\sigma}{1 + Re^{-1/8}\{\sigma - 1 + \log_e[1 + (5/6)(\sigma - 1)]\}} \quad [40]$$

The values corresponding to expressions [38], [39], and [40] are compared in Fig. 5 for the range $10{,}000 < Re < 50{,}000$. For very large values of σ Prandtl's theory gives the limiting form

$$Nu = 0.023\, Re^{7/8} \quad \ldots\ldots\ldots\ldots\ldots [41]$$

and the author's theory

$$Nu = 0.04\, Re^{7/8} \quad \ldots\ldots\ldots\ldots\ldots [42]$$

If the empirical formula of Dittus and Boelter is considered as representative for the averaged experimental values, it is seen that Prandtl's values are entirely too low, the author's values are very exact for $\sigma \leqq 10$, and give a fair approximation up to $\sigma = 25$. For larger values of σ the theory does not follow the increase of Nu with increasing σ. If σ is very large, there is a considerable drop of temperature in the laminar layer, and it is hard to expect that the hydrodynamic theory developed for constant viscosity can represent correctly the conditions. Moreover, the heat exchange in this case is determined chiefly by the thickness of the laminar layer, and, therefore, a small deviation in the function $f(y^*)$ used for the velocity distribution influences the results much more than in the case of moderate values of σ. Since the experimental values show an extraordinary scatter, more experimental research is necessary to furnish reliable data so that the theory can be extended to liquids with high viscosity.

It appears from the results of this paper that the fundamental idea of Reynolds' analogy can be applied with success to the problem of friction and heat transfer between fluid and solid. However, it must be mentioned that recent experiments make it doubtful whether the same idea is applicable to exchange phenomena between fluids flowing with different velocities. It seems that in this case the exchange of heat occurs at a relatively higher rate than the exchange of momentum. At the present time the number of experiments is still too small to draw final conclusions. It is believed that the exchange mechanism of friction and heat transfer for the case "fluid versus fluid" is one of the most important topics for future research.

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Mechanical Purification of Steam Within the Boiler Drum

By M. D. BAKER,[1] SPRINGDALE, PA.

The paper is a report of tests run on new high-pressure high-temperature boiler installations to determine a baffle arrangement for the steam drums that would prevent objectionable carry-over during normal operation.

The arrangements of three types of baffle are illustrated and the results are given in tables and graphs as conductivity measurements on condensed steam samples taken from eight different points in the drum.

The cyclone-separator baffle finally developed will deliver steam containing solids approximating 0.5 ppm with boiler-water concentrations up to 1600 ppm, and will permit a large range of load change and water-level variation.

THE OBJECT of this paper is to present a description and discussion of the development work that was required to produce satisfactory steam purity under all normal operating conditions on the 1400-psi boilers recently installed at the Springdale Station of the West Penn Power Company.

The new boilers consist of three Babcock & Wilcox cross-drum sectional-header boilers equipped with return-bend economizers and superheaters within the boiler setting, and two-stage furnaces. A cross section of the boiler is shown in Fig. 1. The inside diameter of the boiler drum is 54 in. and the length is 30 ft 1 in. Fourteen 4-in. tubes connect the boiler drum with the superheater inlet header. The boilers were guaranteed to deliver a maximum steam output of 500,000 lb per hr with a boiler-water concentration of 1275 ppm. Under these conditions, they were guaranteed to deliver steam containing not over 1 ppm total solids. This is equivalent to a conductivity of about 1.5 micromhos.

In order for a steam purifier in the drum to be satisfactory from an operating standpoint it should deliver high-quality steam and prevent priming with reasonably rapid changes in rating, with reasonable variations of the drum water level above normal, and with the allowable boiler-water concentration sufficiently high so that the control of concentration is not a major operating factor requiring constant vigilance.

The production of steam free from impurities and entrained boiler water has always been a problem of major importance in central stations. With concentration and other conditions equal, the height of foam produced upon the water surface in the boiler drum becomes greater as the pressure increases. This is caused principally by the fact that the ratio of the specific weight of water to the specific weight of the steam decreases as the pressure increases. In the older boilers at the Springdale Station, which operate at 340 psi, this ratio is 69 to 1 whereas in the new boilers described here, which operate at 1350 psi, it is about 13 to 1. As this ratio decreases, the rate at which the foam disintegrates becomes slower thereby producing a greater height of foam for equilibrium, if other conditions remain the same. The weight of this foam is so small that it has little effect on the height of the water in the gage glass; in one case in a boiler operating at 1300 psi with a boiler-water concentration of 350 ppm, it was found that the foam level in the drum was more than 20 in. above the water level shown in the gage glass. It is evident, therefore, that although the water level in the gage glass may be showing two or three inches below the center line of the drum the so-called steam space in the drum may be actually filled up with foam which, of course, would produce severe carry-over.

Test Procedure

Before describing the three types of drum baffles which were tried and the results obtained from them, a description of the method used for determining the steam purity will be given.

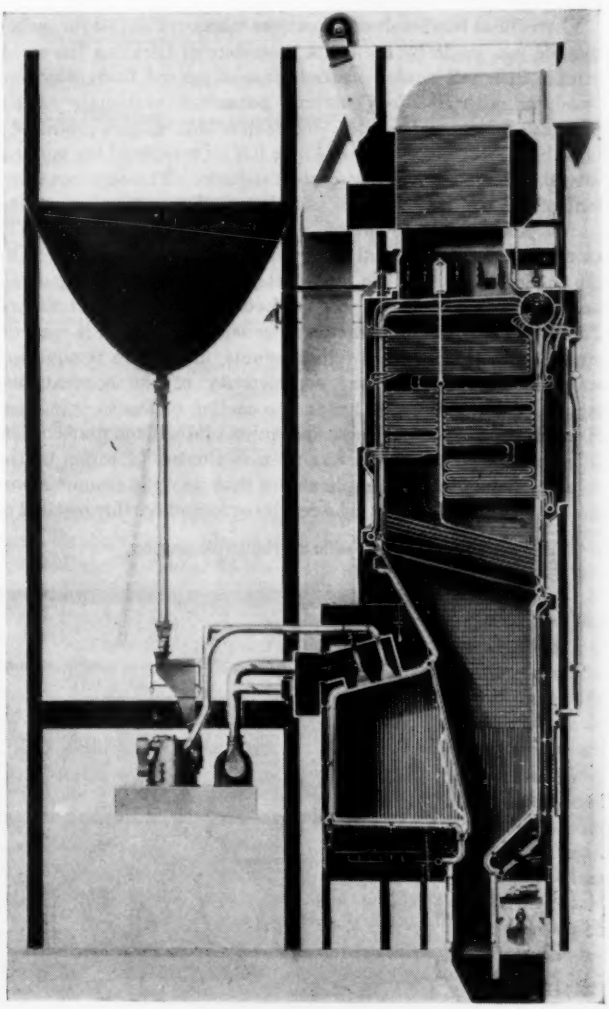


Fig. 1 Cross Section of Boiler

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Contributed by the Power Division and presented at the Spring Meeting of The American Society of Mechanical Engineers, New Orleans, La., February 23–25, 1939.

Discussion of this paper closed March 25, 1939, and is published herewith directly following the paper.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

There are two methods of determining steam quality in general use, namely, the conductivity method and the throttling-calorimeter method. The calorimeter method has been found inadequate where low moisture contents are being measured and where it is important to measure the entrained solids directly. The conductivity method of steam-purity determination is now used almost universally for this class of work. In its application a sample of the steam is condensed, sometimes deaerated to remove dissolved gases, and then the conductivity of the condensate is

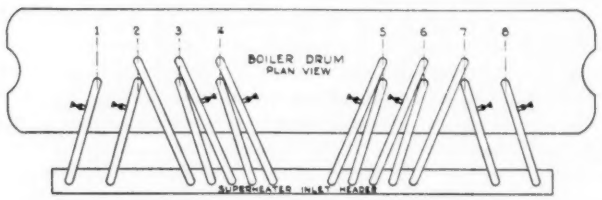


FIG. 2 SAMPLING POINTS FOR SATURATED STEAM

determined by means of a sensitive bridge and standardized conductivity cell. Rummel (1),[2] Hecht and McKinney (2), and Powell, Bacon, Jr., McChesney, and Henry (3) have fully outlined and described this method of determining steam purity, and the principles and the methods advocated by them were used in the tests reported in this paper.

Corrections for dissolved gases were made. The test for carbon dioxide was made by the usual procedure of titrating the steam sample in a tall narrow-necked glass-stoppered flask, with one two-hundredth (0.005 N) normal potassium carbonate to the phenolphthalein end point. One drop of this reagent produced a faint but positive color change in a 100 ml sample of the condensate showing an absence of carbon dioxide. The ammonia content of the condensed steam sample was determined by nesslerization and color comparison with a Hellige comparator. The ammonia content was not determined on all samples, especially the earlier ones. On the later tests, the following method was used to check the comparator method of determining ammonia. The condensed steam sample was boiled slowly until it was approximately one fifth its original volume, in order to remove any ammonia present, and the conductivity of the concentrated sample was measured. During the cooling of the sample great care was needed to prevent readsorption of dissolved gases. The difference in conductivity was then evaluated in terms of the volume of the original sample and in that way the ammonia correction was determined. The results obtained by this method of

[2] Numbers in parentheses refer to the Bibliography.

TABLE 1 TESTS TO DETERMINE THE EFFECT OF THROTTLING THE SAMPLE VALVE

	Locations 3	4	5	
	Micromhos per cm			
Test A				
	1.49	1.51	1.41	
	1.56	1.51	1.41	
	1.49	1.42	1.34	All valves throttled
	1.48	1.48	1.35	
	1.30	1.31	1.20	
	1.32	1.32	1.20	
	1.31	1.30	1.22	
	1.32	1.30	1.28	No. 4 valve open, excess steam vented to atmosphere. Nos. 3 and 5 throttled
	1.21	1.24	1.16	
	1.32	1.30	1.22	
	1.29	1.31	1.22	
	1.23	1.25	1.15	
	1.20	1.25	1.15	
	1.25	1.24	1.17	All valves throttled
Test B				
	1.23	1.29	1.20	
	1.30	1.29	1.21	
	1.27	1.26	1.20	
	1.29	1.22	1.18	All valves throttled
	1.25	1.21	1.15	
	1.24	1.22	1.16	
	1.20	1.17	1.15	
	1.19	1.19	1.13	
	1.26	1.22	1.18	
	1.24	1.24	1.17	All valves open, excess above condensing limit vented to atmosphere
	1.25	1.24	1.15	
	1.21	1.19	1.14	
	1.24	1.25	1.18	
	1.29	1.23	1.18	
	1.29	1.24	1.20	All valves throttled
	1.29	1.25	1.19	

determining the effect of ammonia on the conductivity were in close conformity with the conductivity-correction curve obtained by Rummel in 1934.[3]

Samples of the steam delivered from the boiler drum to the superheater were obtained by installing standard A.S.M.E. sampling nozzles in the saturated-steam tubes connecting the boiler drum to the superheater inlet header. As stated before, there are 14 of these tubes on each boiler arranged in pairs, except for the end ones, as shown in Fig. 2. It was originally thought that the two end and the two middle points, locations 1, 4, 5, and 8, would give a representative value of the steam purity, but it was found during the testing of the first baffle design that the center sampling nozzles showed higher conductivity values than the end ones, hence additional sampling nozzles were installed at locations 2, 3, 6, and 7. After the first six tests, samples were obtained at all eight locations on all subsequent tests. The steam from each sampling nozzle was delivered to its individual water-

[3] "Effect of Ammonia Content on Conductivity of Steam Condensate." Copies of this curve have been distributed by The Babcock & Wilcox Company, New York, N. Y.

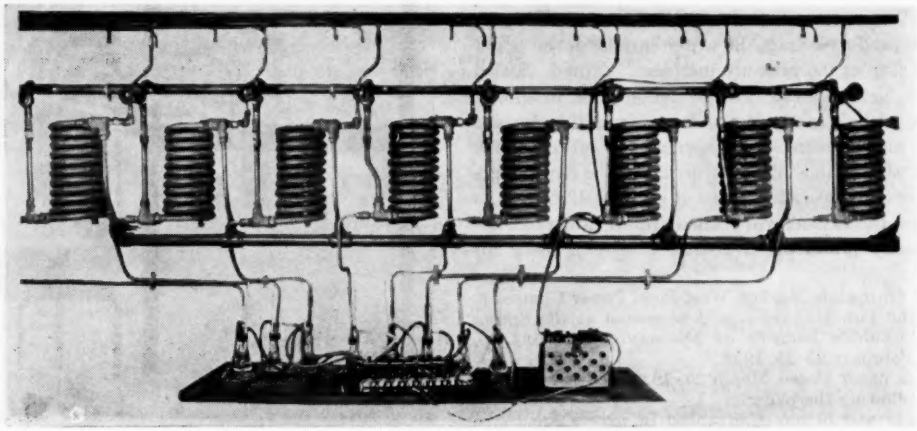


FIG. 3 CONDUCTIVITY APPARATUS AND STEAM CONDENSERS

cooled condenser and the condensate passed directly to its individual conductivity cell. Deaeration of this condensate was considered unnecessary because only occasional small amounts of ammonia were found, and all tests showed a complete absence of carbon dioxide. The individual condensers were placed at a convenient central location and connected by suitable piping to their corresponding sampling nozzles. Fig. 3 shows the arrangement of condensers with the cells, switches, and bridge in place ready for testing.

The condensers were not of sufficient capacity to condense the entire delivery capacity of the sampling nozzles, so throttling was necessary. This was done at the nozzle valve rather than at the condenser, since the sampling piping was not designed to withstand full boiler pressure. Tests were made to determine whether this throttling was detrimental to securing a representative sample. The sampling nozzle valves were throttled until the condensate temperature was reduced to 90 F with all available cooling water flowing through the condensers and the conductivity was determined. Readings were then taken with the sampling-nozzle valves wide open but with the additional steam above the condenser capacity escaping to the atmosphere. No appreciable difference in conductivity was noted, so it was concluded that the throttled samples were representative of the steam delivered by the boiler at the point of sampling. Table 1 presents data taken during this test.

Riser-Extension Baffle

The first steam-drum baffle installed is illustrated in Fig. 4.

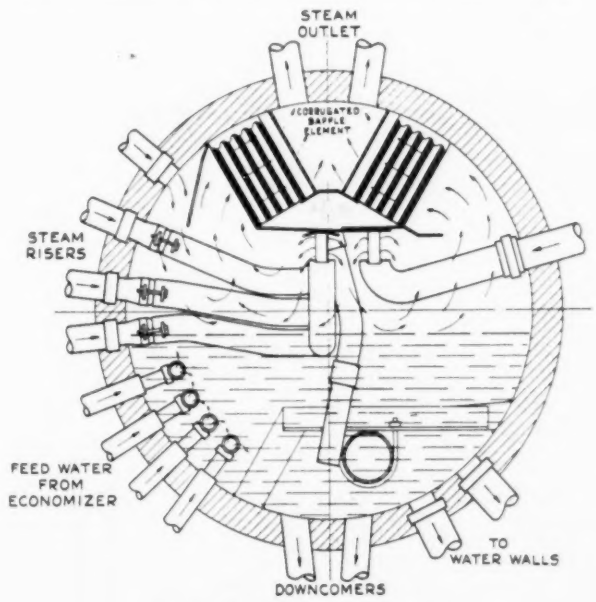


Fig. 4 Riser-Extension Baffle

The risers from the uptake headers of the boiler section and the waterwall headers were extended inside the drum to the vertical center line, and were so arranged that the steam and water were discharged from them vertically upward against deflector plates. These plates deflected the entrained water downward toward the water surface in the drum and the steam passed upward through the corrugated baffle elements located in the upper portion of the drum. The theory behind this design was to discharge all of the steam and the circulating water above the water level in the drum, and prevent the water and steam from impinging directly on the water surface at the discharge velocity. The corrugated steel plates were so arranged that the free-flow area for the steam was a large percentage of the total entrance area to the element, and they were so spaced that all the steam had to come in contact with some portion of the metal surface, thereby removing the moisture and returning it below the water level in the drum through the gravity drains. These corrugated baffle elements were retained and made a part of all three of the steam-drum

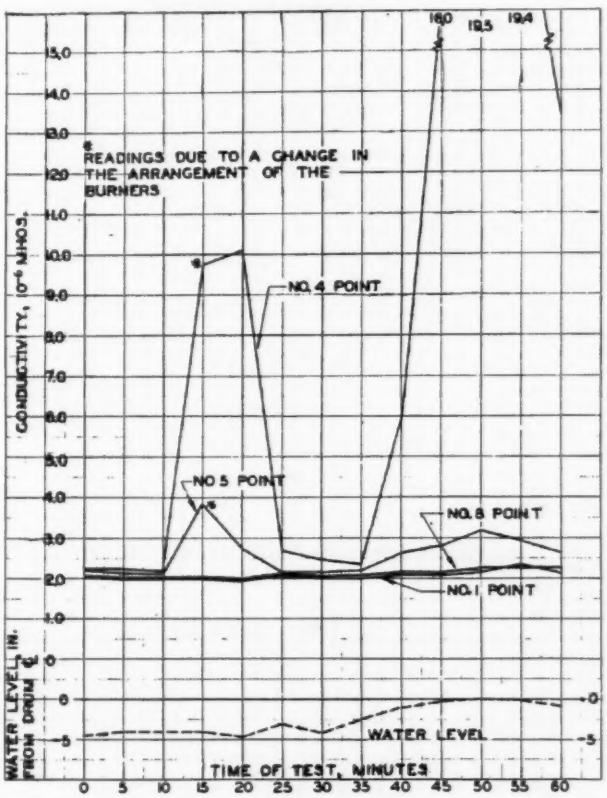


Fig. 5 Steam Quality With Riser-Extension Baffle

Feedwater, 253 F

Steam	Boiler-water analysis		Steam analysis
870–912 F, 1230–1325 psi, 270,000 lb per hr average	Hydroxide alkalinity, Concentration,	68 ppm 536 ppm	CO_2, 0.00 ppm NH_3, 0.00 ppm

baffles tried. The feedwater was discharged from the economizer tubes through tees so placed that the water surface would not be agitated and the metal in the drum would not receive a thermal shock.

The purity of steam delivered by this baffle is indicated by the data presented in Fig. 5. The first increase in carry-over occurred during a burner change but with other conditions remaining constant. The steam flow at this time was approximately one half of the maximum rating of the boiler and the boiler-water concentration was 536 ppm. As will be seen in Fig. 5, severe priming took place a short time later when the gage-glass water level was raised to approximately $2^1/_2$ in. below the center line of the drum. Table 2 presents the data obtained during seven tests conducted on two boilers equipped with this steam purifier. With low boiler-water concentrations and with the gage-glass water level remaining constant, the steam delivered through this baffle had a conductivity of from 2 to 3 micromhos. Increases of the water level caused either by the feedwater regulator or by sudden changes in rating, resulted in excessive carry-over. In some instances the moisture carry-over was sufficient to reduce the superheat as much as 100 F.

The data obtained proved quite conclusively that this design of

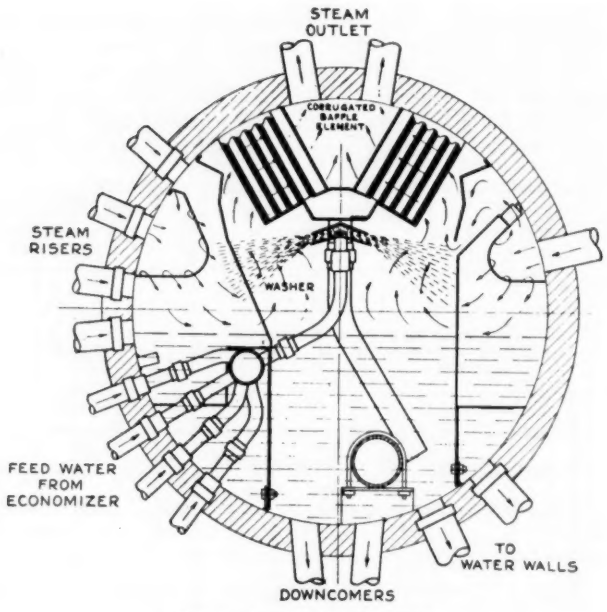


FIG. 6 IMPACT-BOX BAFFLE

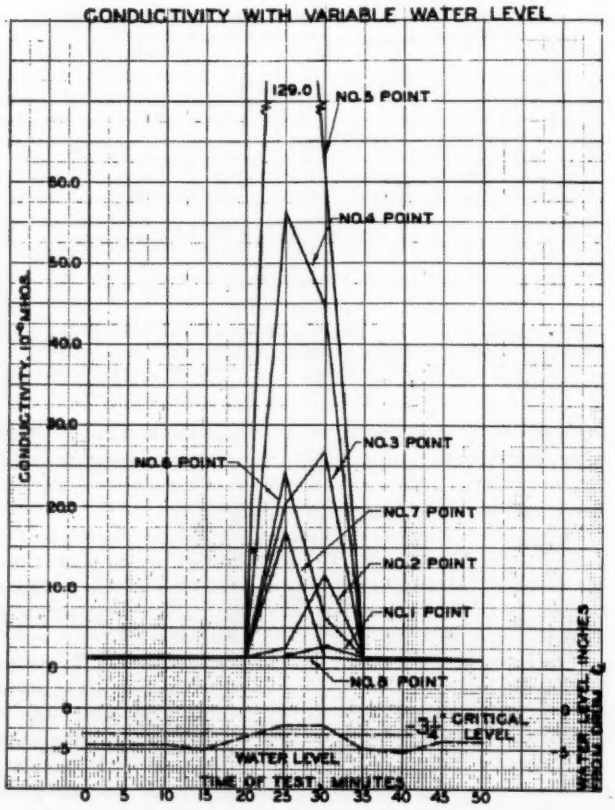


FIG. 7 STEAM QUALITY WITH IMPACT-BOX BAFFLE

Steam	Boiler-water analysis		Steam analysis
820–940 F, 1310 psi, 440,000 lb per hr average	Hydroxide alkalinity,	120 ppm	CO_2, 0.000 ppm
Feedwater, 330 F	Total solids,	517 ppm	NH_3, 0.024 ppm
	SO_4,	86 ppm	
	PO_4,	85 ppm	
	SiO_2,	7 ppm	
	Cl,	25 ppm	

TABLE 2 STEAM-QUALITY TESTS WITH RISER-EXTENSION BAFFLE
(Minimum and maximum values)

Test[a]	Test condition[b]	Readings per station[b]	Conductivity of condensed steam, micromhos per cm, Sample No. 1	2	3	4	5	6	7	8	Water level,[c] in.	Steam, deg F	Steam flow, 1000 lb per hr	Drum pressure, psi	Feed-water, deg F	Total solids, boiler water, ppm	k
6-1	Constant steam flow. Varying furnace condition	17	1.45			1.75	1.67			1.58	−5	880	350		247	316	1.54
			2.39			47.80	3.11			2.40	0	900	350				
6-2	Constant rating. Varying water level	17	1.56			1.79	2.27			1.34	−7	900	335	1200	270	328	2.01
			1.86			39.00	6.05			1.76	+4	920	340	1235			
6-3[d]	Low rating. Varying water level	13	1.96			2.19	2.11			1.99	−4.5	870	260	1230	253	536	1.18
			2.34			19.50	3.84			2.23	0	912	270	1325			
6-4	Low boiler-water concentration	37	1.91			2.24	2.17			1.97	−6	890	365	1200	240	40	1.48
			2.37			2.51	2.44			2.16	+2	950	390	1275	260		
6-5	Normal operation	23	1.66			2.17	1.91			1.65	−3.75	890	390	1260	235	360	1.48
			1.80			4.12	2.23			1.88	−2.75	922			249		
6-6	Effect of change in rating	22	2.77			3.62	3.06			2.83	−4.5	860	322	1220	240	276	2.08
			3.14			70.80	11.60			3.13	−2.75	940	415	1230	250		
4-1	Normal operation	13	1.65	1.87	2.14	1.76	2.34	2.16	1.22	1.47	−8	930	400	1170		440	1.88
			1.86	2.13	7.96	3.06	6.90	7.74	1.47	1.68	−7	970	430	1185			

k = Specific conductance of 1 ppm of dissolved solids in boiler water, micromhos per cm.

[a] The first number is the boiler number.

[b] Readings taken at start and every 5 min at each station.

[c] From center line of drum.

[d] Test shown in Fig. 5.

TABLE 3 STEAM-QUALITY TESTS WITH IMPACT-BOX BAFFLE
(Minimum and maximum values)

Test[a]	Test condition[b]	Readings per station[b]	Conductivity of condensed steam, micromhos per cm —Sample No.— 1	2	3	4	5	6	7	8	Water level,[c] in.	Steam, deg F	Steam flow, 1000 lb per hr	Drum pressure, psi	Feed-water, deg F	Total solids, boiler water, ppm	k
4-2	Low concentration. Normal operation	16	1.39	1.29	1.51	1.35	1.26	1.01	1.57	1.24	—7.75	900	380	1150	220	88	1.63
			1.70	1.67	1.83	1.63	1.51	1.15	1.98	1.51	—4	930	405	1165	230		
4-3	Water level above critical. Varying steam flow.	34	1.19	1.36	1.36	1.29	1.18	1.35	1.30		—8	880	250	1240	295	628	1.84
			5.87	16.86	53.60	63.50	89.50	19.90	1.91		—2.5	980	410	1270	300		
4-4	Water level above critical	17	1.47	1.61	1.61	1.57	1.29	1.61	1.42		—9	887	405	1250	270	460	1.89
			2.66	8.08	30.80	35.40	11.80	9.34	1.65		—2	919	420	1255	280		
4-5	Water level above critical	19	1.55	1.65	1.59	1.51	1.33	1.51	1.26		—8	885	410	1250	300	480	2.35
			3.29	9.86	23.20	42.20	37.25	11.22	1.40		—2.5	930	440	1255	310		
4-6	High concentration. Low water level	19	1.97	2.32	2.43		2.24	2.49	1.98		—6	873	290	1232	322	1150	1.84
			2.17	2.58	2.70		2.59	3.32	2.28		—5	919	326	1248	340		
4-7	High rating. High concentration	19	1.61	1.76	1.77	2.33	1.75	1.68	1.56	1.51	—6	860	435	1225	315	1185	1.84
			1.93	3.23	5.99	21.35	13.29	3.34	2.06	1.68	—5.5	915	455	1240	320		
4-8	High rating. Low concentration	19	1.43	1.59	1.66	1.78	1.51	1.52	1.33	1.14	—6	914	415	1200	310	442	1.58
			1.57	1.77	1.83	1.97	1.77	1.79	1.54	1.35	—5.25	936	430	1200			
4-9	Increasing load	19	1.63	1.64	1.73	1.77	1.74	1.71	1.56	1.48	—7	890	350	1100	315	736	1.49
			1.92	2.05	2.06	2.13	1.95	2.00	1.79	1.79	—5	935	425	1190	320		
4-10	Decreasing load	25	1.14	1.28	1.28	1.51	1.19	1.17	1.05	1.13	—7	805	200	1139	320	1052	1.60
			1.58	1.61	1.59	1.85	1.58	1.54	1.54	1.44	—5	955	440	1195	325		
4-11	Varying rating	19	1.53	1.67	1.81	1.94	1.72	1.75	1.52	1.35	—6.5	825	185	1135	315	391	1.60
			1.81	2.00	2.22	2.29	2.10	1.99	1.78	1.90	—4.7	950	410	1175	340		
4-12	High rating	25	1.80	2.05	2.07	2.03	2.03	2.08	1.67	1.52	—6	880	410	1250	315	410	1.50
			2.24	2.35	2.36	2.45	2.17	2.22	1.91	1.68	—5	955	520	1275			
4-13	Normal operation	25	1.83	2.02	2.04	2.06	1.92	2.01	1.69	1.47	—6	930	453	1270	320	375	1.50
			2.16	2.27	2.30	2.36	2.14	2.29	1.97	1.75	—5	940	469	1275			
4-14	350,000 lb per hr steam flow	25	2.08	2.38	2.40	2.36	2.17	2.38	2.00	1.74	—6	905	340	1245	315	327	1.50
			2.34	2.65	2.73	2.65	2.52	2.61	2.12	1.98	—5	950	370	1260			
4-15	250,000 lb per hr steam flow	25	1.80	2.07	2.21	2.10	1.95	1.96	1.61	1.44	—5.9	950	238	1215	330	324	1.50
			1.92	2.25	2.35	2.23	2.12	2.14	1.84	1.62	—5.4	922	250	1230	335		
4-16	450,000 lb per hr steam flow	34	1.78	2.03	2.07	2.16	1.88	1.92	1.61	1.39	—5.8	922	400	1250	330	312	1.50
			1.96	2.37	2.33	2.49	2.16	2.24	1.86	1.64	—5.0	955	455	1275	335		
4-17	High rating	17	1.98	2.08	2.07	2.12	2.06	2.14	2.00	1.85	—6.1	900	450	1285	330	550	1.50
			2.37	2.43	2.88	7.32	5.62	2.79	2.17	2.14	—5.4	949	510	1300	335		
4-18[d]	Varying water level	11	1.10	1.21	1.19	1.24	1.24	1.16	.99	1.04	—5.2	820	418	1305	330	517	1.88
			2.79	12.10	26.80	56.62	129.0	24.73	17.51	1.64	—2.0	940	510	1310	330		

k = Specific conductance of 1 ppm of dissolved solids in boiler water, micromhos per cm.
[a] First number is the boiler number.
[b] Readings taken at the start and every 5 min at each station.
[c] From center line of drum.
[d] Test of Fig. 7.

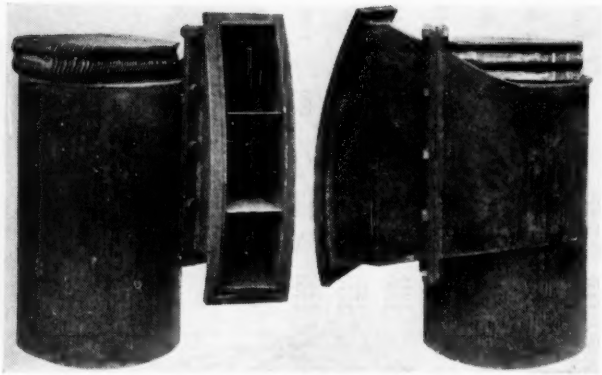


FIG. 8 CYCLONE-SEPARATOR BAFFLE

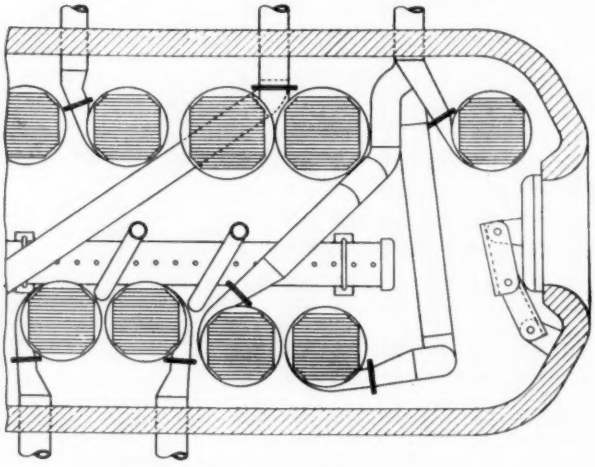


FIG. 9 CYCLONE-SEPARATOR BAFFLE
(Plan section in drum.)

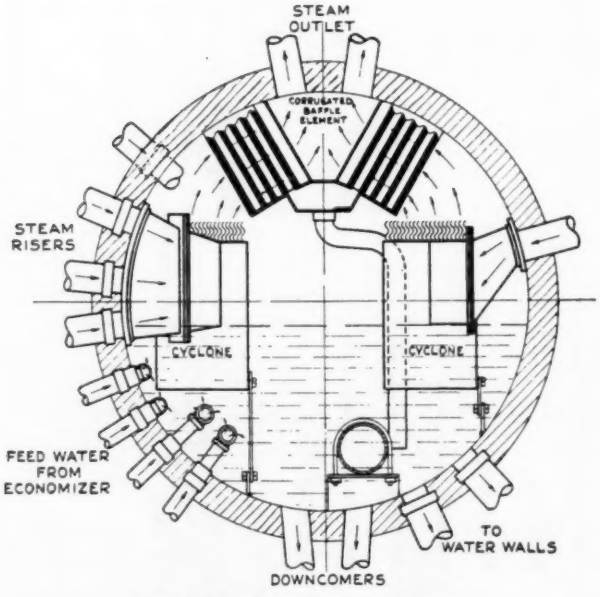


FIG. 10 CYCLONE-SEPARATOR BAFFLE
(End elevation in drum.)

steam-drum baffle would not meet the guarantee and imposed operating restrictions too stringent for satisfactory operation.

IMPACT-BOX BAFFLE

The second design of steam-drum baffle tried is shown in Fig. 6. Over each vertical row of riser tubes, there was placed a box with openings on either side above the drum center line and also at the bottom. The object was to dissipate the high velocity of the steam and water leaving the riser tubes and prevent impingement on the surface of the water. In addition to these boxes, the washing principle was also introduced by collecting the incoming feedwater into a manifold and spraying it into the upper part of the drum just below the scrubber baffle. Although this design was an improvement over the original design, it was not acceptable because a critical water level existed which was too low for satisfactory operation. As long as the water level in the gage glass was not above $3^1/_4$ in. below the center line of the drum, steam with a conductivity of 1.5 micromhos could be delivered, but with water levels $^1/_2$ in. higher than this, severe carry-over occurred; in some instances it was sufficient to increase the conductivity of the condensed steam to 129 micromhos and to reduce the steam temperature 120 F. Seventeen tests were made using this baffle and the significant data are shown in Table 3. The tests were made with boiler-water concentrations slightly above 1000 ppm. Fig. 7 illustrates the effect on the conductivity of the steam caused by raising the water level above the critical point.

CYCLONE-SEPARATOR BAFFLE

The third and final baffle design is shown in Figs. 8, 9, and 10.

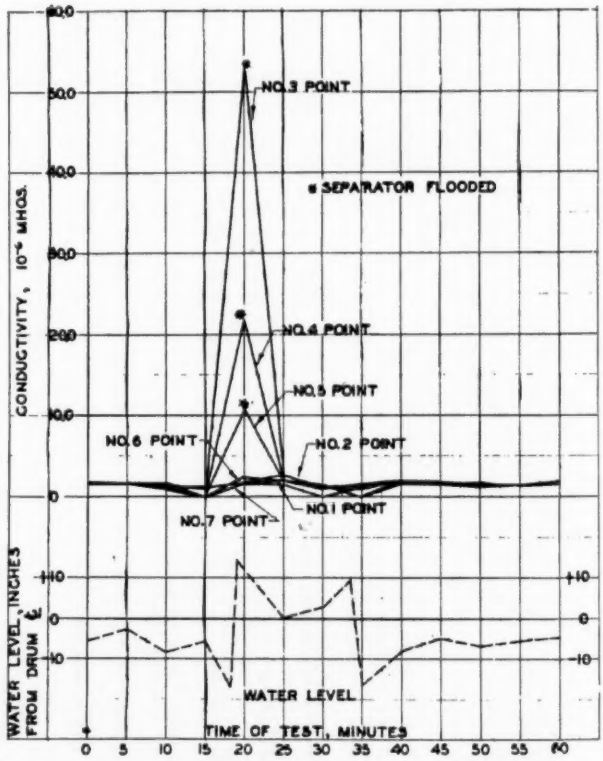


FIG. 11 STEAM QUALITY WITH CYCLONE-SEPARATOR BAFFLE AND VARIABLE WATER LEVEL

Steam	Boiler-water analysis		Steam analysis
842–960 F, 1135–1325 psi, 400,000 lb per hr average	Hydroxide alkalinity,	220 ppm	CO_2, 0.00 ppm
Feedwater, 315 F	Total solids,	1272 ppm	NH_3, 0.00 ppm
	SO_4,	210 ppm	
	PO_4,	287 ppm	
	SiO_2,	36 ppm	
	Cl,	45 ppm	
	Saponifiable,	15 ppm	

The essential difference between this design and those previously tried is that the steam and the circulating water delivered by the risers are discharged into the individual cyclones and the water and steam separated before being released into the main portion of the boiler drum. As will be noted from these figures, an individual cyclone is connected to a single riser or to a small group of risers. The steam and water enter the cyclone tangentially and the water flows downward and out of the cyclone at the bottom periphery of the cylinder, while the steam leaves through the opening at the top. The cyclones are placed vertically and are so located that the openings at the bottom are normally below the water level in the drum and the top openings are well above the usual water level. The small differential pressure and the centrifugal energy of the water within the cyclone are sufficient to keep a water level from forming within the cyclone, but are not great enough to break the seal and permit steam to leave the bottom of cyclone. In this way the steam is prevented from bubbling up through the water in the drum, which would cause foam. The steam leaves the top of the cyclone in such a way that it does not impinge upon the surface of the water in the drum, thereby preventing the formation of foam from this cause. It has been found that the actual water level in the drum is within from 1 to 2 in. of the water level indicated in the gage glass under all conditions. The corrugated elements placed on top and made a part of the cyclones act as a straightening medium to change the centrifugal motion of the steam to a linear one. The number and size of the cyclones were so determined that the velocity of the steam leaving the top of them is too low to carry an appreciable

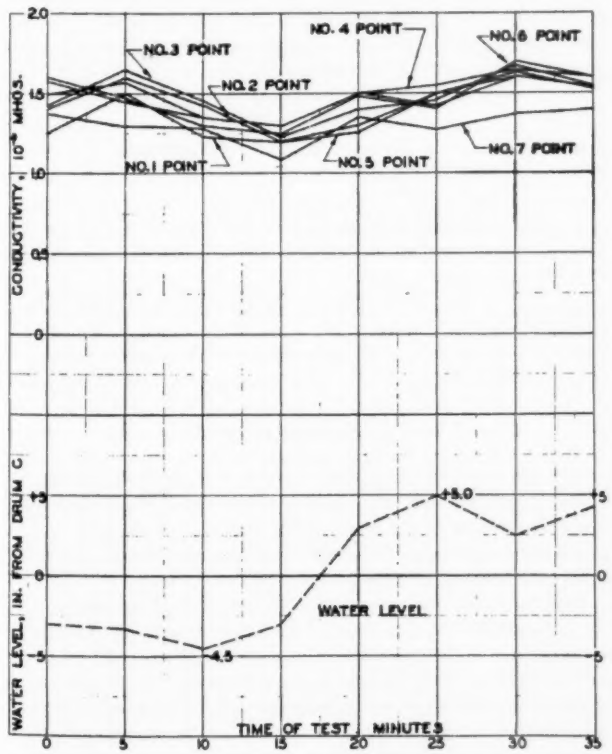


FIG. 12 STEAM QUALITY WITH CYCLONE-SEPARATOR BAFFLE AND HIGH RATING

Steam	Boiler-water analysis		Steam analysis
842–960 F, 1135–1325 psi, 500,000 lb per hr average	Hydroxide alkalinity,	208 ppm	CO_2, 0.00 ppm
Feedwater, 315 F	Total solids,	1215 ppm	NH_3, 0.00 ppm
	SO_4,	208 ppm	
	PO_4,	310 ppm	
	SiO_2,	35 ppm	
	Cl,	43 ppm	
	Saponifiable,	14 ppm	

TABLE 4 STEAM-QUALITY TESTS WITH CYCLONE-SEPARATOR BAFFLE
(Minimum and maximum values)

Test[a]	Test condition[b]	Readings per station[b]	Conductivity of condensed steam, micromhos per cm — Sample No. 1	2	3	4	5	6	7	8	Water level,[c] in.	Steam, deg F	Steam Flow 1000 lb per hr	Drum pressure, psi	Feedwater, deg F	Total solids, boiler water, ppm	k
6-8	Varying water level and steam flow	46	1.01	.97	1.03	1.04	.98	1.04	.93		−7	905	270	1235	320	239	1.50
			1.16	1.21	1.22	1.26	1.19	1.23	1.10		+3.2	975	400	1323			
6-9	Varying water level and steam flow. High ratings	44	1.55	1.70	1.75	1.65	1.61	1.77	1.56		−7	896	340	1210	322	983	1.50
			2.37	2.37	2.38	2.40	2.14	2.33	2.04		+4.5	984	520	1300	329		
6-10	Constant load. Varying water level	19	1.41	1.47	1.47	1.48	1.46	1.38	1.34		−5	925	365	1180	320	1573	1.50
			1.60	1.67	1.67	1.64	1.77	1.55	1.54		+3	959	372	1230			
6-12[d]	Cyclone separator flooded. High rating	37	1.20	1.20	1.23	1.14	1.29	.98	1.08		−17	840	360	1135	315	1272	1.50
			1.98	2.29	52.97	22.15	10.84	2.51	1.72		+14.5	960	540	1325		1215	
6-15	Varying rating	67	1.36	1.37	.99	.88	1.14	1.14	1.10		−3.0	898	290	1250	325	448	1.55
			1.81	1.73	1.77	1.77	1.66	1.76	1.60		−1.0	976	480	1320	340	518	
2-1[e]	Normal operation	19	.12	.68	.88	.71	.83	.78	.44	.60	−3.75	935	302	1300	322	673	1.86
			1.01	1.05	1.15	1.13	1.31	1.21	.81	.87	−3	950	340	1310			
2-2[e]	Normal operation[f]	13	.18	.18	.21	.14	.12	.17	.06	.14	−3.75	970	360	1270	301	485	1.82
			.50	.57	.69	.58	.44	.57	.43	.86	−2.75	980	390	1315			
2-3[e]	Normal operation. Low concentration	19	.57	.60	.66	.66	.61	.70	.49	.55	−3	940	280	1290	305	40	1.61
			1.09	.90	1.01	1.07	1.11	1.34	.95	.92	−2.5	955	375	1300			

k = Specific conductance of 1 ppm of dissolved solids in boiler water, micromhos per cm.
[a] First number is the boiler number.
[b] Readings taken at the start and every 5 min at each station.
[c] From center line of drum.
[d] Test of Figs. 11 and 12.
[e] Conductivity readings corrected for ammonia.
[f] Test of Fig. 13.

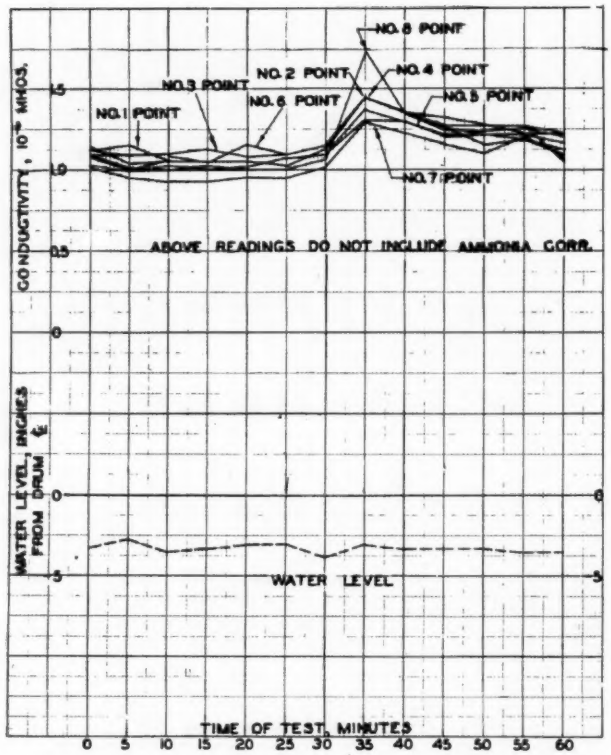


FIG. 13 STEAM QUALITY WITH CYCLONE-SEPARATOR BAFFLE UNDER CONSTANT CONDITIONS

Steam	Boiler-water analysis		Steam analysis
970–980 F, 1270–1315 psi, 375,000 lb per hr average	Hydroxide alkalinity,	72 ppm	CO_2, 0.00 ppm
Feedwater, 310 F	Total solids,	465 ppm	NH_3, 0.07 ppm
	SO_4,	46 ppm	
	PO_4,	130 ppm	
	SiO_2,	2 ppm	
	Cl,	14 ppm	

amount of moisture with the boiler delivering its maximum steam output. The corrugated elements used on the previous designs were retained in order to remove the final traces of moisture from the steam before leaving the drum.

This cyclone separator delivered steam having a conductivity of 1.5 micromhos until the water level in the gage glass was raised above the top of the cyclones. No appreciable effect on the amount of carry-over has been noted throughout a range of boiler-water concentrations up to 1573 ppm. Table 4 gives the data of five tests conducted on No. 6 boiler and three tests conducted on No. 2 boiler with this type of baffle. Corrections to the conductivity for dissolved ammonia were made only on the tests on No. 2 boiler.

The test data illustrated in Figs. 11 and 12 were obtained under abnormal conditions but present some interesting information. During these tests extreme changes in water level occurred; in one instance a 27-in. change in the gage-glass water level took place within one minute. The maximum gage-glass water level reached was $14^1/_2$ in. above the center line of the drum. After the cyclones had been flooded and the water level again lowered to their top, which is $6^1/_2$ in. above the center of the drum, the conductivity of the steam immediately decreased to 1.5 micromhos. The boiler-water concentration at the time this occurred was 1272 ppm. A high-rating test during which the water level was purposely varied was conducted. The results of this are shown in Fig. 12. With a steam flow averaging 500,000 lb per hr and a boiler-water concentration of 1215 ppm, the water level in the gage glass was raised as high as 5 in. above the center line of the drum and the average conductivity of the eight sampling points did not change from a value of 1.5 micromhos.

Fig. 13 presents data that were obtained on No. 2 boiler during normal operation. Under these conditions, after making corrections for pure water and for 0.07 ppm dissolved ammonia, the average conductivity of the steam during this test was 0.29 micromhos.

All of these tests have proved quite conclusively that the cyclone-separator baffle gives satisfactory results and permits a large operating leeway in rate of load change, in water-level variation, and in boiler-water concentration without jeopardizing satisfactory operation by increased carry-over or priming.

CONCLUSIONS

1 The importance of correct sampling and the use of a sufficient number of sampling points cannot be overemphasized. The variation in the conductance of the steam obtained from the eight sampling points, showed that the steam was not of uniform purity along the length of the drum at all times.

2 Determining of the dissolved-ammonia content in a condensed steam sample by nesslerization and comparison with a Hellige comparator, and correcting the conductivity of the steam sample by Rummel's correction curve were proved to be satisfactory methods of procedure.

3 The cyclone-separator baffle will deliver steam containing solids approximating 0.5 ppm with boiler-water concentrations up to 1600 ppm provided the drum water level is not raised above the top of the cyclones.

4 The cyclone-separator baffle permits a large operating leeway in rate of load change, in water-level variation, and in boiler-water concentration without jeopardizing satisfactory operation by increased carry-over or priming.

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3 "Design and Development of Apparatus for Measurement of Steam Quality by Electrical Conductivity Methods," by Sheppard T. Powell, Hilary E. Bacon, Jr., Irvin G. McChesney, and Frank Henry, Trans. American Institute of Chemical Engineers, vol. 33, 1937, pp. 116–138.

Discussion

MAX HECHT[4] AND D. S. MCKINNEY.[5] It appears that a highly efficient apparatus has been developed to accomplish a practically complete removal of the boiler water carried over in steam. It is gratifying to note that the author confirms the theory and principles which were presented by the writers[6] before the A.S.M.E. in 1930, as well as the independent work of J. K. Rummel.[7]

Inasmuch as it is obvious that the electrical conductivity testing procedure will replace the calorimeter method, and that there will be need for the preparation of a standard test code, it would be desirable for the author to present in his closure more complete detail of the testing procedure followed. The supplemental information desired should include the relative quantities of dissolved solids obtained by evaporation of both boiler water and steam to be used for comparison with the conductance of these

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[6] Referred to in the Bibliography of the paper (2).

[7] Referred to in the Bibliography of the paper (1).

two fluids; complete analyses of the boiler water; chemical treatment, if any, of the boiler feedwater; blowdown of the boiler water, continuous or intermittent, and what per cent of feed; point of sampling of boiler water; and finally, comparisons of the Rummel and Powell procedures, if these were made during the extensive test runs. The writers pointed out in their paper[6] the necessity of correcting the conductance of condensed steam for the conductance conferred by CO_2, NH_3, and H_2S. They also discussed in detail the CO_2 and suggested a similar procedure for other conducting gases. (The technique of correcting for the conductance of gases has been patented by the writers.)

It is assumed that the boiler-water concentration is the result of chemical treatment. Furthermore, in this treatment, if sodium sulphite were employed for oxygen cleanup, the author might advise whether or not tests on the condensed steam showed the presence of sulphite. The ratio of sulphite in steam to that in the boiler water might help decide the question as to whether sodium sulphite is entrained as such in the steam, or whether it is evolved as the gas SO_2. If SO_2 is produced, this has conductance characteristics and the total conductance of the condensed steam must be corrected.

In the Colfax tests reported in 1930, sulphite was not used. With no evidence of ammonia in the steam at that time, the conductance was corrected for pure water and CO_2. Prior to 1933, the same procedure was employed at another station where, in addition to CO_2, NH_3 was found in the condensed steam. The NH_3 was eliminated from the samples under test by boiling as mentioned in the present paper.

If the relationship of conductivity to concentration has been determined by test, it would be most desirable if the author would furnish details of the evaporative procedure. Does the author conclude that the conductivity is a measure of the soluble ions or has he secured data to show that the silica, iron, and aluminum oxides have conducting characteristics?

The author stresses, as have the writers previously, the necessity of exact sampling. In 1924 while conducting tests at Colfax, data were secured throughout the length of a drum, showing conclusively that different concentrations existed along the drum.[8] After the 1930 tests at Colfax and before starting test work at other stations, it was recognized that the available sampling facilities required improvement. The late E. G. Campbell of Pittsburgh designed and installed sampling nozzles (patented) for securing representative steam and boiler-water samples from various points along the length of the drum. The inlet nozzle of the sampler offers a minimum resistance to the flow of steam. The various nozzles of either the steam or water sampler are connected within the drum to a single pipe via an orifice plate and thence to a single opening in the drum, the steam or water discharging into a cooling coil. This arrangement overcomes the reluctance of boiler manufacturers to provide sampling openings in the equipment. In the past this reluctance has caused the operators much grief in their attempts to diagnose water and steam conditions both with regard to carry-over and water conditioning.

In the present paper, the author has made a rather elaborate sampling setup for securing steam samples, but it is not clear whether the same extensive setup was made to secure boiler-water samples. To secure the correct ratio of conductance of steam to boiler water, it seems essential that the dependence should not be placed on a sample of boiler water taken from a single point in the drum or from the blowdown line connection, for it will be recognized from reference to Fig. 104[8] that the boiler-water concentration varies across the drum.

[8] "Corrosion, Causes and Prevention," by F. N. Speller, second edition, McGraw-Hill Book Company, Inc., New York, N. Y., 1935, Fig. 104, p. 423.

With the acceptance of the conductivity method by the operators as a test for determining carry-over accurately, it would appear that continuous records of steam quality are desirable. With the development of sampling equipment, it is possible to record the average of numerous points across the steam space of the boiler drum for both steam and boiler water. It is also possible to record the conductivities, respectively, on a conductivity recorder and to make the necessary corrections for the gases having conductance characteristics.

W. H. Rowand.[9] Cyclone separators similar to those described by the author have been installed in 21 boilers of the writer's company and are performing to the complete satisfaction of the operators. In every case they are giving results which confirm those of the author.

Several of the 1400-lb per sq in. pressure installations have been tested up to 1700 ppm boiler-water concentration. Two units in the 500 to 700-lb pressure range have been tested over 3000 ppm. Up to the present time, in no case has a limit of boiler-water concentration been reached. During the tests, the concentrations were not carried higher because the values attained were well above those generally expected to be encountered during operation. It was decided not to run the risk of possible damage to the superheater or to the turbine from carry-over or to the boiler tubes from scale.

As pointed out by the author, tests show that the actual water level in the drum is within 1 to 2 in. of that in the gage glass. The writer's tests indicate that the cyclone separators will deliver high-purity steam and prevent priming until they are practically submerged below the water level. This means that the gage-glass water level may be carried 5 in. or 6 in. above the center line of the drum without increased carry-over or priming.

Referring to Mr. Gurney's question as to whether cyclone separators increase the circulation of the boiler, the writer has had no direct experience as yet in this connection on cross-drum, sectional-header boilers. However, circulation tests have been conducted on one open-pass boiler and on one radiant-type boiler which show that the separators increase the circulating flow more than 40 per cent. This result is due to the action of the cyclone separator which not only separates the water from the steam but also the steam from the circulating water, thus furnishing "solid" water for the downcomers and producing the optimum circulating head. It is safe to say that, when a boiler design is such that steam is recirculated down through supply tubes or pipes, the installation of cyclone separators will increase the circulation by preventing the recirculation of steam in the downcomers.

M. C. Schwartz.[10] Similar work to that of the author has been carried out at the Louisiana Station of the Gulf States Utilities Company.

The problem may be divided as follows: (1) Design of steam-purifying equipment to prevent priming or excessive carry-over, with variations in operating conditions as drum water level, boiler-water dissolved-solids concentration, and boiler load; (2) design or comparison of steam-purifying equipment to improve the quality of the steam being delivered.

The pursuit of a solution to such a problem depends upon methods of estimating steam quality or carry-over. At present the most useful method, and the one followed by the author, is the determination of the electrical conductivity of the condensed steam, correcting for the presence, if necessary, of any soluble gaseous substances, as carbon dioxide, or ammonia, which form

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[10] Research Chemist, Louisiana Station, Gulf States Utilities Company, Baton Rouge, La.

electrolytes in solution. Thus far to the writer's knowledge, no method of collecting the condensed steam is available which will secure an ammonia-free sample. The correction for ammonia is troublesome. Its magnitude is likely to be such, in some instances, as almost to equal the total electrical conductivity of the sample.

However, the change in electrical conductivity of the condensed-steam sample, which is collected during priming, is so great that, under these conditions, the corrections for carbon dioxide or ammonia are quite small. The measurement of the carry-over which takes place during normal "satisfactory" operation is, on the other hand, very difficult, since it is likely to be minute. The solution of this latter problem to a satisfactory degree of accuracy is yet to be attained. Because of its commercial importance it should be pursued.

Author's Closure

The chemicals used for conditioning the boiler feedwater at the Springdale Station are disodium phosphate, caustic soda, and sodium sulphite. Boiler-water samples are obtained from two locations on each boiler; the supply line to the recirculating pump, and the center of the top waterwall header in the front of the boiler. The recirculating-pump supply is taken from the downcomer supplying the furnace walls which in turn takes water at points throughout the length of the drum. Consequently, a sample from this location is representative of the water in the boiler drum.

The steam-quality guarantee specifies that the impurities in the steam, as determined by the conductivity method, will not exceed 1 ppm of dissolved solids. The following procedure was used to obtain the conductance of 1 ppm of dissolved solids in the steam. The conductance of condensed steam was first determined; boiler water, equivalent to 0.5 per cent by volume, was added and the conductance of the mixture determined. The difference in micromhos of the two readings, after applying gas corrections, was the increase caused by the dissolved solids present in the added boiler water. This difference was divided by the number of ppm increase in concentration and the result was the conductance of 1 ppm. The conductance of 1 ppm as determined from each of the two boiler-water samples never varied more than 0.05 micromhos, in any one test, which confirms the correctness of the boiler-water-sampling procedure. The concentration of the boiler water was determined by evaporation in platinum.

The quantity of treated boiler water handled by the recirculating pump conditions the feedwater to give full economizer protection. Sodium sulphite has been used as an oxygen scavenger of boiler feedwater by the West Penn Power Company at intervals since 1923. The excess sodium sulphite in the boiler water is maintained at between 5 and 10 ppm. A special test to determine whether or not the sodium sulphite decomposed in the boiler was made as follows: The total boiler-water concentration was 451 ppm and the sodium sulphite content 5 ppm. Sodium sulphite was added in two charges by means of a chemical bottle located in a by-pass on the recirculating-pump discharge. It required 5 min to empty the bottle; the elapsed time for feeding both charges was 45 min. After the second addition of chemical, the boiler-water concentration was 699 ppm and the sodium sulphite was 169 ppm. Conductivity readings taken at each of the eight sampling points at 5-min intervals, before, during, and for 2 hr after adding the chemicals, showed no increase, not even the increase which normally would be expected from the higher concentration of boiler water.

The amount of ammonia present in the steam was seldom in excess of 0.10 ppm and in most of the tests it was between 0.02 and 0.05 ppm. At first there appeared to be no correlation between the amount of ammonia found and the gas correction as determined but, after extreme care was taken in making the Nesslers solution and in avoiding ammonia contamination from the atmosphere during the determination, reproducible results were obtained.

Fundamental Relationships in the Design of Cooling Towers

By G. RUSSELL NANCE,[1] LOS ANGELES, CALIF.

This paper presents a few theoretical considerations encountered when an attempt is made to correlate actual tower performance with that which may be predicted from thermodynamic considerations alone. The results of observed tests are correlated by means of empirical equations and sufficient material has been presented to enable the reader to check the performance of different types of towers with a fair degree of accuracy. A discussion follows in which information relative to the merits of various types of cooling systems is presented and suggestions are made as to the most economical type of tower to install.

THE USES OF cooling towers in conjunction with industrial cooling problems of all types have become so general that engineers are becoming more familiar with their importance in both old and new fields of industry. The body of this article deals primarily with a few of the more practical considerations relative to the selection of a tower and the determination of its principal dimensions. It is the author's hope that sufficient material will be presented to enable the prospective purchaser to analyze carefully his specific problem and to enable him to make an intelligent decision regarding the engineering and economics of selecting a tower.

When water is cooled atmospherically, one of the two following conditions exists:

(*a*) $t_w > t_{db}$

(*b*) $t_w < t_{db}$

where t_w = tower-water temperature; t_{db} = dry-bulb air temperature

Under nearly all conditions the wet-bulb temperature of the air t_{wb} is less than t_{db}, and represents the lowest value which t_w can approach. When the temperature of the water is greater than the temperature of the ambient air, as in condition (*a*), cooling occurs both by the evaporation of some of the water being cooled and by a heat transfer through convection, conduction, and radiation from the water to the air. Heat exchange by any or all of the last three processes is usually spoken of as sensible-heat transfer. When the temperature of the water being cooled is less than that of the ambient air, as in condition (*b*), cooling takes place by evaporation only, since the sensible-heat transfer is from the air to the water. Water may be cooled under condition (*b*) because the amount of heat extracted by evaporation is much greater than that which is added by sensible-heat transfer.

In order to predict the performance which may be expected under various operating conditions, several methods have been proposed for aiding the engineer in the coordination of the initial and final water temperatures and the wet-bulb temperature. The problem resolves itself into a measurement of the so-called "cooling head," or the atmospheric cooling effect, under a given set of conditions.

An interesting method of coordination has been derived from the Ferrel psychrometric equation by means of which the total cooling head H relative to a datum condition is expressed as

$$H = (P_t - P_{wb}) + 0.0115\,(t_t - t_{wb}) \quad [1]$$

where t_t is the water temperature, F
t_{wb} is the wet-bulb temperature, F
P_t is the water-vapor pressure at condition t_t, in. Hg
P_{wb} is the water-vapor pressure at the wet-bulb temperature, in. Hg

Physically, this equation represents the sum of the vapor-pressure difference between any two conditions of water and wet-bulb temperatures, or the evaporation effect, and the temperature difference for the same condition, or the sensible-heat effect. It is also representative of the relative cooling proportions between the evaporative and sensible-cooling effects.[2]

The cooling performances of similar towers may be compared by a factor representing the cooling efficiency E defined as

$$E = [(S_i - S_f)/(S_i - S_{wb})]100 \quad [2]$$

in which $S = P - 0.0115t$, the subscripts i and f refer to the initial and final water temperatures, respectively, and wb refers to the wet-bulb temperature. Increasing values of E represent closer approaches of the final water temperature to the wet-bulb temperature. The following values of E are obtained for water cooled under the conditions shown:

Case	t_i, F	t_f, F	t_{wb}, F	E, per cent
A	100	80	70	73.5
B	100	80	60	60.4
C	90	70	60	73.2

Cases *A* and *C* are conditions in which the temperature differences and approaches are equal. The approach is defined as the difference between the final water temperature and the wet-bulb temperature. In case *B* the temperature drop is equal to that of either *A* or *C*, but the approach is twice as large. Under actual test conditions it has been found that conditions *A* and *C* are harder to obtain than *B*, which would indicate that higher efficiencies represent cooling conditions more difficult to obtain.

As the wet-bulb temperature decreases, the efficiency relative to a fixed temperature range also decreases with a corresponding increase in the ease of cooling. Under such conditions it would appear that the lower the wet-bulb temperature, the greater would be the quantity of water which could be cooled atmospherically through a given range, or the lower the final water temperature which could be obtained if a constant quantity of water were circulated. This is not strictly true, for in practice it appears that in spite of the larger cooling potentials attendant with lower wet-bulb temperatures there is a portion of this energy difference which is not available for cooling. Thus the efficiency concept must be somewhat modified for operation in the lower wet-bulb ranges.

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NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[2] "Water Cooling System Efficiency," by V. J. Azbe, *Mechanical Engineering*, vol. 46, 1924, p. 799.

Procedure in actual practice may be simplified by expressing the amount of cooling available under a given set of conditions as a function of the allowable water concentration per unit of deck (or tower) area which will produce these temperature conditions in actual tower performance. This empirical concentration factor may be expressed as a function of the cooling head after suitable modifications have been made to take into account physical characteristics of the tower and certain cooling discrepancies such as those noted in the preceding paragraph, although its use as a concentration factor is more convenient.

Fig. 1 illustrates a typical relation between concentration and temperature range. The curves represent constant approaches or values of $t_f - t_{wb}$. The differences between the initial and final water temperatures are expressed as ordinates, and values of the allowable water concentration C are shown as abscissas. These curves have been based on a 70-F wet-bulb temperature, but they may be referred to other wet-bulb temperatures by multiplying the values of C by K_{wb}, the values of which are shown in Fig. 2. A Fluor tower 12 ft wide and 35 ft high having 12 equally spaced horizontal decks was used in obtaining these curves.

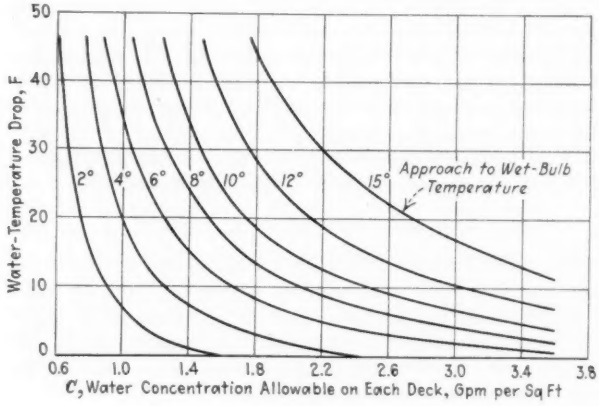


FIG. 1 COOLING-CAPACITY CURVES

(Values of water-concentration factor C plotted against drop in water temperature for various values of approach to wet-bulb temperature. Based on 12-deck tower, 12 ft wide × 35 ft high; wet-bulb temperature of 70 F; and wind velocity of 5 mph.)

An empirical method of determining the size of a tower is to consider the length L as

$$L = QDW/CA \quad \ldots\ldots\ldots\ldots\ldots\ldots [3]$$

in which Q = water circulated per unit of time, gpm
D = a design factor which varies with the physical characteristics of the towers
W = a factor which takes into account variations in wind velocity
C = water concentration, gpm per sq ft, Fig. 7
A = horizontally projected deck area corresponding to a unit of tower length, sq ft

A check on Equation [3] shows that it balances dimensionally since D and W are dimensionless factors.

The design factor D is used to correlate the relative effectiveness of towers of different widths, heights, and deck spacing. For a tower 12 ft wide and having a deck spacing of from $2^1/_2$ to 3 ft the factor varies with tower height approximately as shown in Fig. 3. Changes in width also affect the design factor, but not in any direct relation.

Fig. 4 shows the relation between tower length and wind velocity. It is at once apparent that it is quite important to select a wind velocity which is representative of the actual conditions existing at the proposed tower site. If too high a value is used, the tower will not be sufficiently large to cool the imposed heat load, while if the value is unnecessarily low, a tower larger and more expensive than actually warranted will be indicated.

For unobstructed locations and average conditions a 3-mph wind velocity is a safe figure to use, although Weather Bureau reports will usually show a 5-mph average. The values of C shown in Fig. 1 may be corrected to a 3-mph wind velocity by changing the abscissa scale by the ratio 1/1.25 as obtained from Fig. 4. The circumstances under which the plant is operating

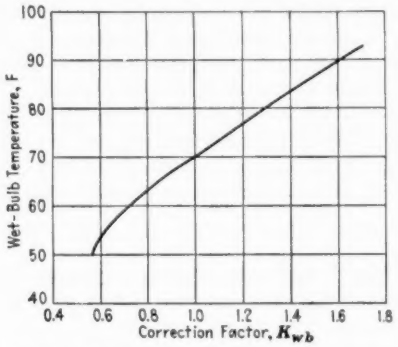


FIG. 2 WET-BULB CORRECTION FACTOR

(K_{wb} is used to correct the value of C in Fig. 1 to a wet-bulb temperature other than 70 F, $C_{wb} = C \times K_{wb}$.)

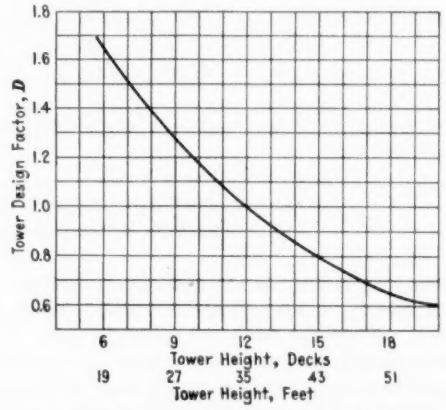


FIG. 3 DESIGN FACTOR, HEIGHT RELATION

(D is used to correct the values of C in Fig. 1 to tower heights other than 12 decks. Based on tower section 12 ft wide, having single-batten decks.)

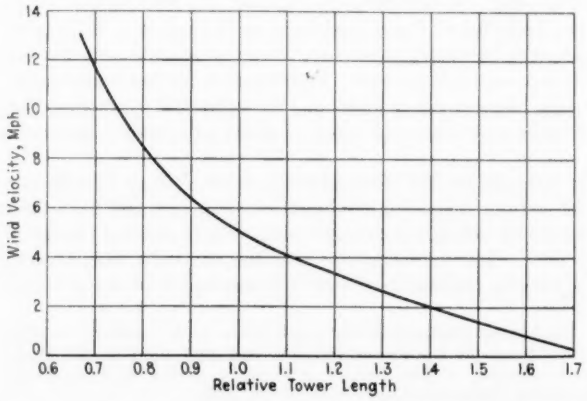


FIG. 4 VARIATION OF TOWER LENGTH WITH WIND VELOCITY

[Values of C in Fig. 1 can be corrected for other values of wind velocity by multiplying by (1/relative tower length).]

will in the last analysis decide whether an increase in tower size is justified by the savings resulting from a supply of lower-temperature water. Changes in wind velocity result in corresponding changes in the temperature range, providing the circulation remains constant.

The method of rating shown in Equation [3] is not only useful in determining the size of tower required for a given condition but also may be applied in other ways. Thus, if a tower of known size is available, the performance for a given water rate can be calculated. It is also useful in deciding whether a tower is meeting its guaranteed performance. It is very difficult to find the temperature ranges and wind velocities mentioned in tower specifications all occurring simultaneously. By this method suitable values are chosen for the atmospheric conditions, and the tower length, circulation, or temperature range can be checked against either of the other two variables. Solutions which satisfy the equation constitute a satisfactory check.

Fig. 5 has been included as a matter of interest to show the variation of temperature with height in a 20-section 12-deck tower. These data represent actual test conditions, and are indicative of the amount of heat removed from the water during various stages in the cooling cycle.

In specifying tower operating ranges, there should be borne in mind certain points, the application of which will result in more economical installations.

The size of a tower required to handle a given heat load varies with the performance required and care should be used in specifying the wet-bulb temperature, the approach of the final water

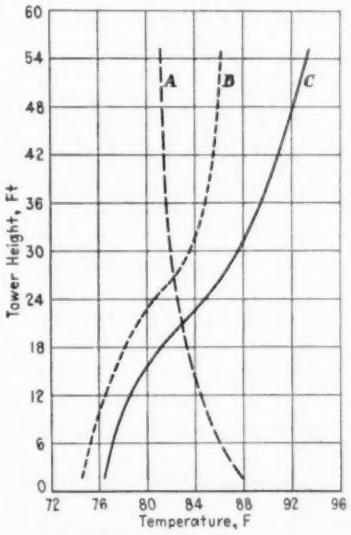


FIG. 5 TEMPERATURE RELATIONS BETWEEN AIR AND WATER AT DIFFERENT HEIGHTS IN A COOLING TOWER

(Curve *A*, temperature of entering air; curve *B*, temperature of leaving air; curve *C*, temperature of water in tower. Test on 12-deck vertical-louver (aerator) tower, 54 ft high. Wet-bulb temperature 73 F, wind velocity, 1.6 mph.)

temperature to the wet-bulb temperature, the quantity and temperature range of the water being circulated, and the wind velocity.

Plant operation is often dependent upon the maintenance of a fixed minimum water temperature, and care should be exercised in selecting the most economical vacuum to maintain, or temperature at which to operate equipment, if such flexibility is allowable.

It is generally preferable to base operation upon the worst atmospheric conditions experienced during the year in order that plant operation will not be impeded during the summer months when the wet-bulb temperature is the highest. This results in the selection of a larger tower than is necessary for operation during the winter months, but during this time the circulation may be decreased and a compensating economy effected.

Fig. 6 shows the relation between tower size and approach of the final temperature to a given wet-bulb temperature. It is apparent that as large an approach as is feasible should be used in order to keep the size of the tower within reasonable limits. In specifying operating conditions, an approach not less than 5 and preferably greater than 10 deg is recommended, depending upon the type of tower service.

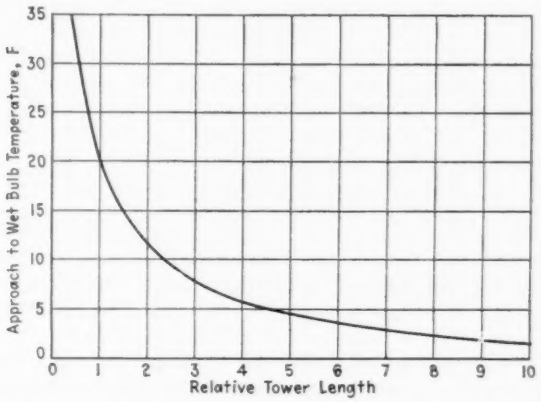


FIG. 6 VARIATION OF TOWER LENGTH WITH APPROACH TO WET-BULB TEMPERATURE

A calculation for a constant thermal load of 10,000,000 Btu per hr follows. Operating conditions make it undesirable to have more than a 25-deg or less than a 10-deg temperature drop.

The following selections could be made

$$\text{Gpm} = \frac{\text{Heat load, Btu per hr}}{\text{Temperature drop, F,} \times 8.33 \times 60}$$

Case	Solution	Gpm
1	10,000,000/25 × 8.33 × 60 =	800
2	10,000,000/20 × 8.33 × 60 =	1000
3	10,000,000/15 × 8.33 × 60 =	1333
4	10,000,000/10 × 8.33 × 60 =	2000

Table 1 shows the lengths of tower, 12 ft wide and 35 ft high, which would be required to handle these quantities, with the approach to the wet-bulb temperature and the wind velocity being constant.

TABLE 1

Case	Length, ft	t_i, F	t_f, F	Temp drop, F	t_{wb}, F	Approach, F	Projected area, sq ft	Wind velocity, mph
1	55.0	94	69	25	60	9	660	5
2	60.5	89	69	20	60	9	725	5
3	68.5	84	69	15	60	9	822	5
4	77.0	79	69	10	60	9	924	5

If the wet-bulb temperature were 65 F, the approach to the wet-bulb temperature would be decreased to 7 deg, making the final water temperature 72 F, and if the wet-bulb temperature were 70 F, the approach would be decreased to 6 deg, making the final water temperature 76 F. Fig. 7 further illustrates the relation between capacity, range, and tower size. These curves have been computed for conditions in which the wet-bulb temperature, approach of the final temperature to the wet-bulb temperature, and the wind velocity have been held constant. With the exception of the broken-line curve, these values have not been based on constant heat loads. In general, it may be

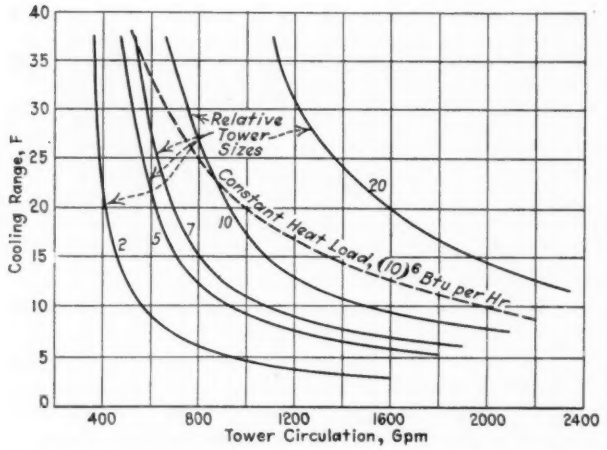


FIG. 7 COMPARATIVE TOWER SIZES RELATIVE TO WATER CIRCULATION AND COOLING RANGE
(Wet-bulb temperature, approach, and wind velocity constant.)

concluded that for the same heat load less tower is required for cooling a small quantity of water through a large temperature range than is required to cool a large quantity of water through a small temperature range.

In arriving at a final decision on a cooling system, it would be worth while to consider the relative economies of cooling ponds, spray ponds, and atmospheric and mechanical-draft towers.

Cooling ponds are generally economical to use only when a natural pond is available. If the required capacity is large, the initial cost will be high if any extensive excavation work is required, and since the cooling capacities of ponds are low per unit area, a more economical type of cooling system could be installed.

Spray ponds have a larger capacity per unit area than cooling ponds and frequently are used when a large amount of space is available and the temperature range through which the water is being cooled is small. As in the case of a cooling pond, the initial cost of a spray pond will usually exceed that of any type of cooling tower. In comparing a spray system with an atmospheric tower, it should be remembered that the pumping charges for the former will be about one half of those for a tower, and the prevailing power rates probably will determine which installation is the more suitable to install and operate over a given period of time. If a large temperature drop is required, respraying will be required, and the pumping charges will then be comparable to those of a tower. Modern mechanical-draft cooling towers have practically the same total pumping head as spray ponds when a low-head pressure-distributing system is used.

In the majority of cases an atmospheric tower will be an economical installation since its performance is dependable over a large range of conditions and the cost of maintenance is low. In selecting a site it will be necessary to place the tower broadside to the prevailing winds if the best performance is required.

As far as water cooling is concerned, tower height has little

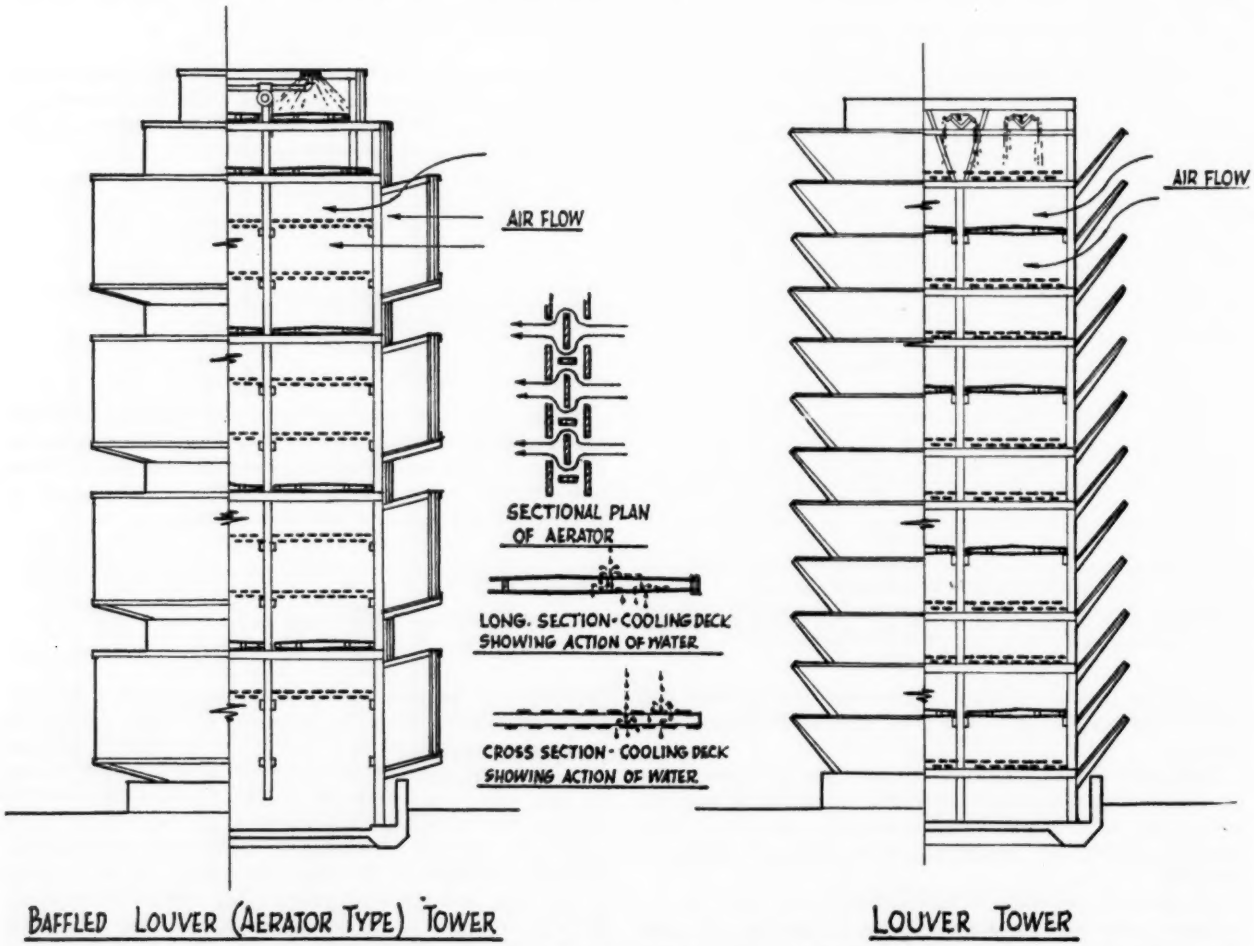


FIG. 8 SECTIONAL VIEWS OF TWO TYPES OF ATMOSPHERIC COOLING TOWERS

TABLE 2 COST COMPARISON OF ATMOSPHERIC COOLING TOWERS

No. of decks		12	15	18
Tower size, ft	length	108	84	66
	width	18	18	18
	height	35	43	51
Concrete basin, cu yd		32	29	26
Pumping horsepower		55.0	65.5	76.0
Cost tower and basin (approx)		$6100	$5825	$5450
Pumping cost[a]		$2.280	$2.715	$3.150
Depreciation, tower and basin[a]		$0.387	$0.369	$0.346
Total cost[a]		$2.667	$3.084	$3.496

a Cost per 1,000,000 gal of water pumped.

effect, since the length is correspondingly increased according to the curve of Fig. 3 for towers having a given capacity.

In selecting the economical size of tower to install, a check should be made of the initial cost of towers and basins in comparison with the costs of pumping water over the towers.

Table 2 outlines a suggested method of comparison. In this study atmospheric towers of various heights have been selected to cool 3000 gpm from an initial temperature of 120 F to a final temperature of 90 F based on a wet-bulb temperature of 73 F and a wind velocity of 3 mph. An arbitrary power rate of 1 cent per kwhr has been selected, and it is assumed that the initial cost of the tower and basin will be depreciated over a ten-year period.

The values are applicable only in this case, and will vary for different cooling ranges, tower prices, and power rates. The cost of the make-up water required to replace that lost by evaporation and windage and the cost of chemically treating this water have not been included in the total cost per million gallons circulated since they would be approximately the same for each tower. The property value of the tower site has not been included because of the difficulty in assigning a reliable value. The conclusion may be drawn that high towers are most economical in so far as the initial installation is concerned, but that low towers are more economical to use over a period of years.

If water is very expensive, it would be worth while to select a type of tower from which little water is blown out by the wind. A windage loss as small as 0.5 per cent of the water being circulated will often exceed in value the original price of the tower within a few years. The water lost by evaporation is of course the same for any tower cooling water under the same conditions.

Forced- and induced-draft towers have a greater unit concentration than atmospheric towers, and because of this find a great application under conditions in which space is limited. A positive cooling effect is assured by the use of fans which force air through the tower, and when the wind is not blowing this is often of paramount importance in plant operation.

In comparing mechanical-draft towers with atmospheric towers, a tabulation of the same general type as shown in Table 2 may be used. In order to obtain a satisfactory comparison, the atmospheric tower should be based on a wind velocity of from 0 to 2 mph, since it is assumed that if the wind velocity were greater, an atmospheric tower would be less expensive to install and operate, providing the space were available.

In this comparison, it will be necessary to take into consideration the power requirement of the fan as well as that of the pump. Generally, the pumping head on a mechanical-draft tower will be less than on an atmospheric tower, so that the over-all power consumption on the former will not be as large in comparison as it would appear to be on casual inspection. Another compensating factor is that the price of a mechanical-draft tower is usually less than that of an atmospheric tower for the same capacities and cooling conditions.

An installation should be selected which will give the least total annual cost including both operating cost and fixed charges, and only a careful study of all the factors involved will show whether an atmospheric tower or forced-draft tower should be installed.

A sectional view of a 12-deck atmospheric type of tower is shown in Fig. 8 in which the arrangement of the distribution system, decks, and louvers has been outlined as in a typical installation. The performance of the louver and baffled-louver tower is the same in so far as cooling properties are concerned. The baffled-louver (aerator) tower, however, has a much lower windage loss than the plain-louver tower.

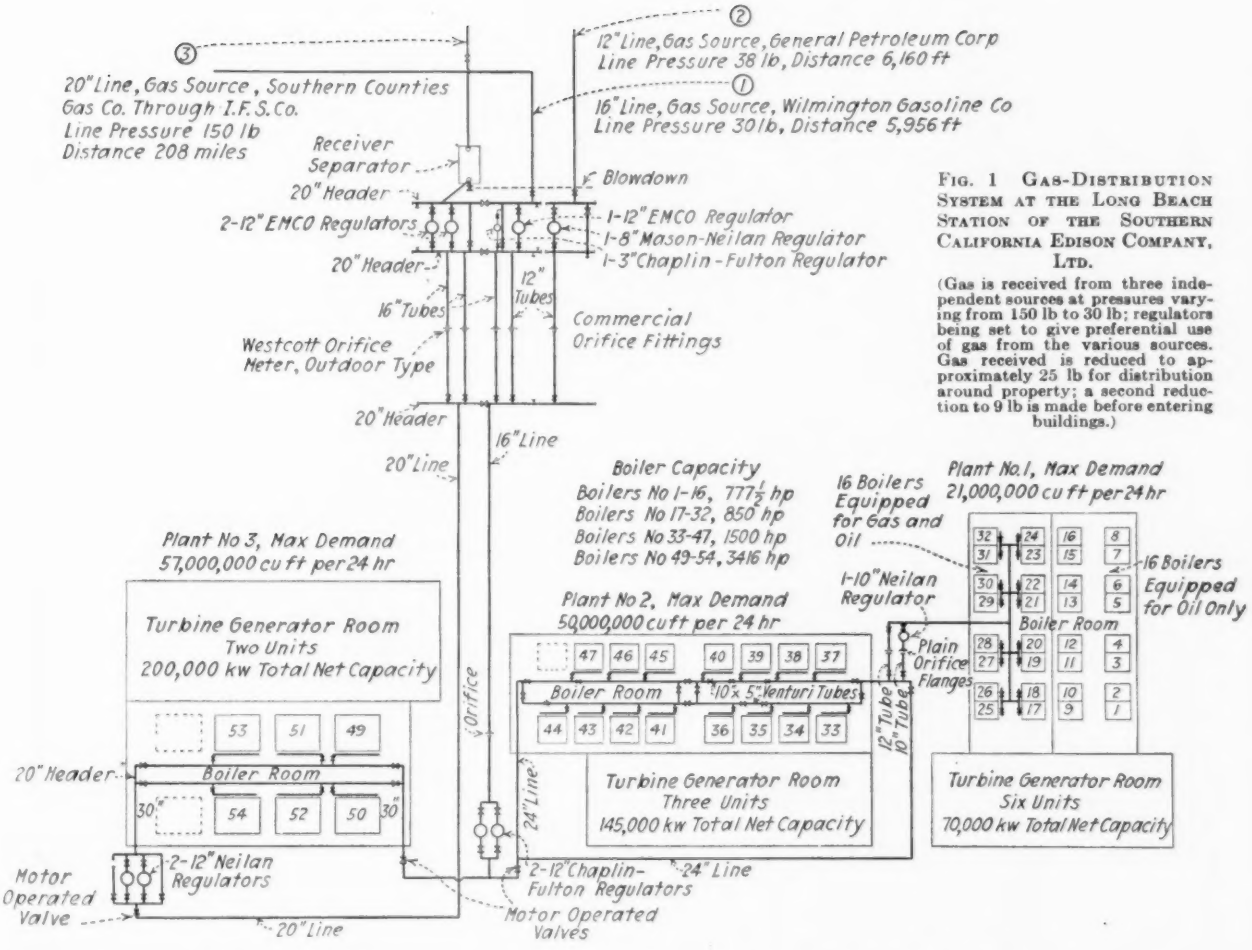


FIG. 1 GAS-DISTRIBUTION SYSTEM AT THE LONG BEACH STATION OF THE SOUTHERN CALIFORNIA EDISON COMPANY, LTD.

(Gas is received from three independent sources at pressures varying from 150 lb to 30 lb; regulators being set to give preferential use of gas from the various sources. Gas received is reduced to approximately 25 lb for distribution around property; a second reduction to 9 lb is made before entering buildings.)

Boiler-Plant Performance With Natural-Gas Firing

BY F. G. PHILO,[1] LONG BEACH, CALIF.

With its diversified operations, the Long Beach Steam Station of the Southern California Edison Company, Ltd., using natural gas as fuel has been required to solve many problems. The author presents the boiler-plant performance obtained with this fuel and outlines the methods and equipment adopted to give economic operation. A feature of this paper is the detailed instruction chart developed for the guidance of the boiler-plant operators. During the last fifteen years 120,000,000,000 cu ft of natural gas from three main sources have been burned in the installation. It is the author's opinion that a properly designed gas-piping system in the hands of competent operators probably presents fewer problems and no greater hazard than does the use of liquid or powdered fuels.

THE Southern California Edison Company, Limited is one of the world's largest consumers of natural gas as power-plant fuel. Beginning in 1922, considerable pioneer work was carried out at the Long Beach Steam Station and today 385,000-kw capacity at this station is equipped to utilize natural-gas fuel. The entire station capacity of 415,000 kw is equipped also for burning oil.

For many years base-load operation was customary. The economic depression of the last decade and the advent of Boulder Dam power made it highly desirable to operate as stand-by for emergency capacity. This change in basic function indicated certain alterations to be desirable in equipment and methods of operation, the most important being a method of burning minimum quantities of gas while being ready instantaneously to pick up and retain maximum station loads. Therefore, with its diversified operating requirements, the Long Beach Steam Station may be considered especially suited for study of the

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NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

TABLE 1 LONG BEACH STEAM STATION, PLANT NO. 3; PROCEDURE FOR BRINGING UP THREE BOILERS FROM COLD STAND-BY CONDITION WITH GAS FUEL

(Crew, four experienced operators; time, 3 to 3½ hr)

Head fireman	Assistant head fireman	Third operator (fireman or fan man)	Fourth operator (fuel-oil or gas man)
1 Received request to fire up for load from both the switchboard operator and watch engineer at least 3 hr before steam is needed for turbine.			
2 Check boiler-header gas pressure. Water level in boiler glass should be 2 to 4 in. Close following switches for combustion control on rear of boiler panel: (*a*) Fuel drive; (*b*) DC for combustion control; (*c*) Interrupter switch.	If boiler is filled to vent, open main boiler blowdown valves to blow down tank until water is down to firing level. Open cold-air doors.	Open stack dampers; open induced fan dampers; close forced-fan dampers; water to induced-fan bearing; close switch on induced-fan combustion-control box. Check oil level in fan and motor bearings. Check oil glass in combustion-control box.	Constantly watch gas pressure to maintain 18 to 30 lb intermediate pressure and 9 lb boiler-header gas pressure by making necessary regulator adjustments and using proper orifice to give desired reading on charts. Check chart for operation, time, setting, and ink.
3 Start induced fan—allow to run 10 sec to vent furnace. Set chronometer valve at 2 by pointer.	Open burner register on Nos. 3, 7, 13, and 17 burners. See that all gas plug valves are closed. Light torch and insert in burner front; open plug valve.	See that: Nonreturn valve stem is up; all inspection and explosion doors on boiler and preheater are closed; all superheater drains are open; high-pressure drip valve at superheater outlet is open; boiler-drum vent is open; water-glass, water-column, and feedwater regulator drains are closed; check all boiler blowdown valves.	
4 Open main gas valve to ½ lb at burner. Open main gas valve and adjust gas pressure with chronometer valve. Note: When furnace is very cold from cold stand-by service, it may be necessary to run induced fan until temperature is raised in furnace and stack.	Light one burner at a time with torch.		Watch gas pressures and regulators.
5 See that boiler steam header and feedwater header valves are open and that steam header lines are drained. Notify watch engineer of boiler-room condition and ask him to take care of steam receiver drains on pump floor. Open auxiliary steam valve to fuel-oil system.	Watch fires and water in boiler.	When steam starts to blow from boiler-drum vents, close drum-vent valves and superheater drains. The high-pressure drip valve is to be left open. See that steam-inlet valve to Fisher reducing valve in steam line to fuel-oil room is closed. Inspect high-pressure traps of this line in sub-basement to see that condensate is being taken care of.	Watch gas pressure and regulators.
6 When steam pressure is sufficient to lift the nonreturn valve, and send steam to receiver, close boiler-header drain and check with watch engineer for receiver draining.	Watch fires and water in boilers. Check charts and instruments for operation, time, setting, and ink.		
7 Watch fires and water in boilers. Note: Notify pumpman when water is needed.	Assisted by third and fourth operators blow all Bailey walls about 1 min each to start circulation and to prevent water hammer.	Help assistant head fireman.	Help assistant head fireman.

8 Gradually put in 20 fires per boiler with induced-draft fan only and fuel at 1/2 lb pressure.	Close cold-air doors. Help head fireman.	Open forced-draft-fan dampers; start motor in forced combustion-control box; close preheater-hopper drains; check fan and motor bearings for oil level, ring movement, and water flow from jackets. Help assistant head fireman.	Watch gas.
9 Watch fires and water in boilers.	At 75-lb and 300-lb steam pressure, check over boiler equipment for leaks and tighten up on boiler-drum manheads.		
10 Watch fires and water in boilers.	Blow all water-glass, water-column, and feed-water-regulator drains. Watch fires and water in boilers.		
11 Supervision. Note: After unit is rolling, the fires are to be regulated according to steam demand and the pressure increase required to be at 410 lb gage when unit goes on line.	Watch fires and water in boilers. Open oil-inlet and oil-return valves on oil headers at boiler front. Regulate oil pressure and temperature by equalizing oil circulation on boilers.	Routin nspection of fan floors.	Prepare one fuel-oil pump to circulate oil through one heater on No. 10 side and one heater on No. 11 side by: Open suction valve at oil-service tank to be used; set return oil valve in position to return oil to tank or oil-suction line of tank being used; open pump suction to tank to be used; open pump discharge valves; open oil inlet to heater; open oil outlet from heater; open oil inlet and outlet valves on oil heat exchangers; open division valves between No. 10 and No. 11 fuel-oil rooms; see that main strainer valves are in proper position; open fuel-oil header-line valves at pipe gallery; start pump by push button (if pump does not pick up the suction, start the booster pump and let it run 2 min and cut out). Check air pressure for temperature control; open steam valve at heater; open steam valve at Fisher reducing valves and set low-pressure side at 50 to 75 lb; check over heater traps and see that trap discharges are turned to heat exchangers; see that water from detector tank goes to sewer until there is no evidence of oil leaks; then turn condensate to distilled tank. Adjust oil temperature to about 220 F at heater for oil circulation and at about 250 F for oil-burning operation. For oil-burning operation maintain 220 lb pressure at pumps. This is done at present by regulating with the low-pressure relief valve at oil-pump discharge. Note: Temperature adjustment is made by turning pointer on controller to right or left as directed on dial. Steam pressure is regulated by adding or taking off weights at Fisher reducing valve.
Note: Standard operating conditions for boiler output of 250,000 lb per hr or more should be approximately as follows: Water level 6 to 10 in. in glass; gas-header pressure, 8 to 9 lb; burner-gas pressure, 1 to 3 1/2 lb; CO_2, 9 to 11 per cent; steam-flow, air-flow pens should be together; steam temperature, 775 F maximum; boiler drum pressure, 455 lb maximum. All pressures and temperatures will vary as boiler output varies and approximate standard conditions may be determined from curve charts made from tests of boiler No. 50.	Note: When picking up load from natural draft conditions, feedwater supply and the boiler control should be operated manually until boiler reaches at least 225,000 lb of steam per hr. Amount of air used for combustion should be determined from amount of gas-fuel input rather than from steam output or air-flow-steam-flow pen relation on boiler meter, until boiler and furnace temperatures become sufficiently high to insure proper combustion at established ratios.	Routine inspection.	

general problems of boiler-plant performance with natural-gas firing.

Gas is received at the station from three independent sources; one from Kettleman Hills approximately 208 miles distant; and two additional sources from the Wilmington field directly adjacent to the plant. Due to price and contractual obligations, it is necessary to establish and maintain a definite priority of use from these three sources.

The schedule is automatically carried out by means of differentially set gas-pressure regulators as shown in Fig. 1. Regulator No. 1 is set to maintain downstream pressure at 30 lb per sq in.; regulator No. 2 at 23 lb per sq in., and regulator No. 3 at 18 lb per sq in. Thus, when all gas is being taken from source No. 1, regulator No. 1 will be wide open. As the plant demand increases the pressure will be reduced to 23 lb per sq in., at which point

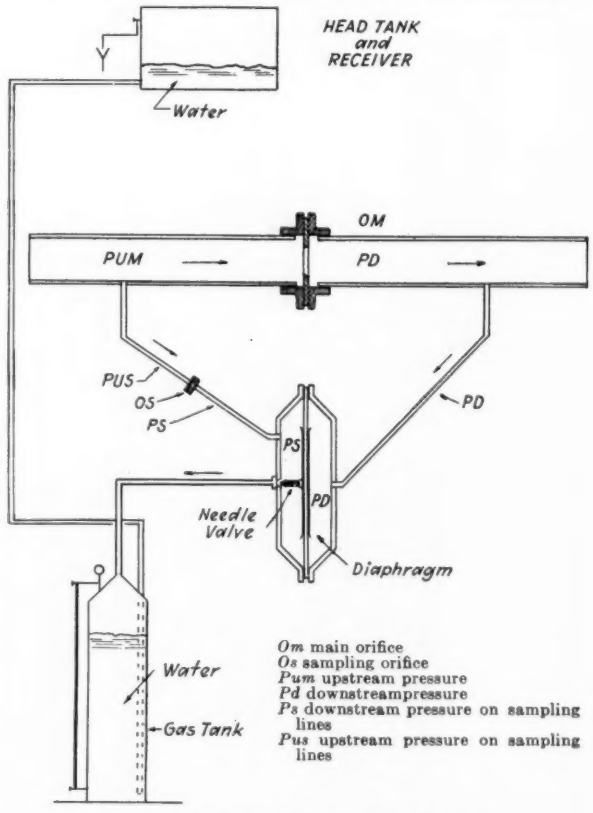


FIG. 2 THOMPSON PROPORTIONAL SAMPLER

(Sampler collects gas samples proportional to rate of use; samples used to determine specific-gravity and heating values which represent weighted average of fuel used.)

regulator No. 2 will open and gas will be taken from source No. 2. Similarly, when the full amount available from sources Nos. 1 and 2 is being used, the pressure will be reduced to 18 lb per sq in., and all additional requirements taken from source No. 3. These regulators are pilot-operated. With systematic care and maintenance they give accurate and dependable regulation. Before entering the boiler-room header, the pressure is further reduced to 9 lb per sq in. Each boiler is equipped with an individual header of its own branching off the main loop header below the boiler-room floor. Gas pressure in these individual boiler headers, Fig. 1, is regulated in plant No. 3 by chronometer valves which are operated automatically by a combustion-control system or manually if so desired. Plants Nos. 1 and 2 are controlled by manual operation only.

The standard procedure for firing these boilers is given in Table 1. This table is shown exactly as it appears in the "Station Standard Operating Methods" in order to illustrate the definite segregation of duties and the detailed instructions given. It has been found that detailed instructions of this kind tend to give a better understanding of operating problems to all concerned. They also, as illustrated, synchronize the duties of the various members of the operating personnel, each operation being done in its proper sequence, and each member of the crew having a thorough knowledge of what the others are doing. Careful preparation of such instructions and thorough drilling in their execution avoid confusion and enable the work to be done safely and with a minimum of supervision. The particulars of oil circulation for stand-by fuel are included, since it is felt that maintenance of a secondary fuel is a vital part of competent boiler operation with gas firing.

Natural gas is being purchased on a basis of heat content. This necessitates the measurement of the gas consumed in standard cubic feet and determination of the average Btu per standard cu ft. The gas volume measurement is carried out in accordance with the standard specifications and data based on results published in Gas Measurement Committee Report No. 2 of the American Gas Association. This report with its appendix gives a complete and authoritative summary of all the work in deriving the various factors involved and is universally accepted as standard throughout the gas industry. Consequently, although involving considerable detailed computations, the measurement of gas in standard cubic feet is clear-cut and well-defined. However, the translation into total Btu has presented some interesting complications. The station loading varies greatly during the day, and the heat content per cubic foot from the three independent sources is markedly different. In addition, the Btu per cu ft from any one source may fluctuate from hour to hour as much as 20 per cent.

In order to obtain the true weighted average Btu per cu ft for 24 hr of changing percentages of the different gases being used at widely different rates throughout the day, it has been necessary to build a proportional sampler. This sampler, based on a design by H. E. Thompson, is placed across an orifice through which the mixture of the three gases flows. It is constructed, as schematically shown in Fig. 2, to take a sample directly proportional to the flow through the main orifice.

The regulator is set to maintain *Ps* equal to *Pd*. Since the upstream pressures, *Pum* and *Pus*, are the same on the two orifices *Om* and *Os* and the downstream pressure on *Os* is maintained the same as *Pd*, the two orifices are subject to the same differential, and the flow of gas through orifice *Os* will be a constant small proportion of the flow through the main orifice *Om*. This small proportionate flow as it enters the sampler causes a displacement of the diaphragm which opens the needle valve and allows it to escape to the gas tank. This causes no change in pressure *Ps*, since the diaphragm is not restricted in its motion.

Samples are taken from the sampling tank by means of a rubber gas bag every 24 hr and run through the Thomas recording calorimeter, thus establishing the true average Btu per cu ft for the period. Proper care should be taken in this connection to include corrections for moisture in the fuel. Natural-gas calorimeters generally give Btu per cu ft of saturated gas. The gas as received and burned in the boilers is only partially saturated. The same sampling device is applied to the determination of true-average gravities for varying gas mixture. With these data and the steam pressure, steam and feedwater temperatures, and steam flow the station statistician is enabled to figure the true boiler performance and efficiency. Apparatus required for the determination of these quantities includes a recording calorimeter, recording thermometers, recording gravity balance, proportional samplers, orifice meters for gas, venturi meters for feedwater.

TABLE 2 ANALYSIS AND COMBUSTION CHARACTERISTICS OF A REPRESENTATIVE SAMPLE OF NATURAL-GAS FUEL

Constituent	Per cent by volume	Molecular weight	Weight, molecular fraction	Per cent by weight	Carbon: Per lb of constituent	Carbon: Per lb of fuel	Hydrogen: Per lb of constituent	Hydrogen: Per lb of fuel	N_2 per lb of fuel	O_2 per lb of fuel
CH_4	79.230	16.032	1270.22	67.16	0.7485	50.27	0.2515	16.89	..	..
C_2H_6	18.314	30.048	550.30	29.10	0.7987	23.24	0.2013	5.86	..	..
N_2	1.942	28.016	54.41	2.87	....	...	....	...	2.87	..
O_2	0.514	32.000	16.45	0.87	....	...	....	...	..	0.87
Total	100.00	....	1891.38	100.00	....	73.51	....	22.75	2.87	0.87

Constituent	Per cent by weight	Air requirements: Per lb constituent	Air requirements: Per lb of fuel	Theoretical combustion products: CO_2: Per lb constituent	CO_2: Per lb of fuel	Moisture: Per lb constituent	Moisture: Per lb of fuel	Nitrogen: Per lb constituent	Nitrogen: Per lb of fuel	Total
C	73.51	11.52	8.47	3.67	2.70	..	..	8.85	6.50	9.20
H_2	22.75	34.29	7.80	..	..	8.93	2.03	26.36	6.00	8.03
N_2	2.87	0.00	0.00	..	..	..	..	1.00	0.03	0.03
O_2	0.87	−4.32	−0.04	..	..	..	..	−3.32	−0.03	−0.03
Total	100.00	....	16.23	..	2.70	..	2.03	....	12.50	17.23

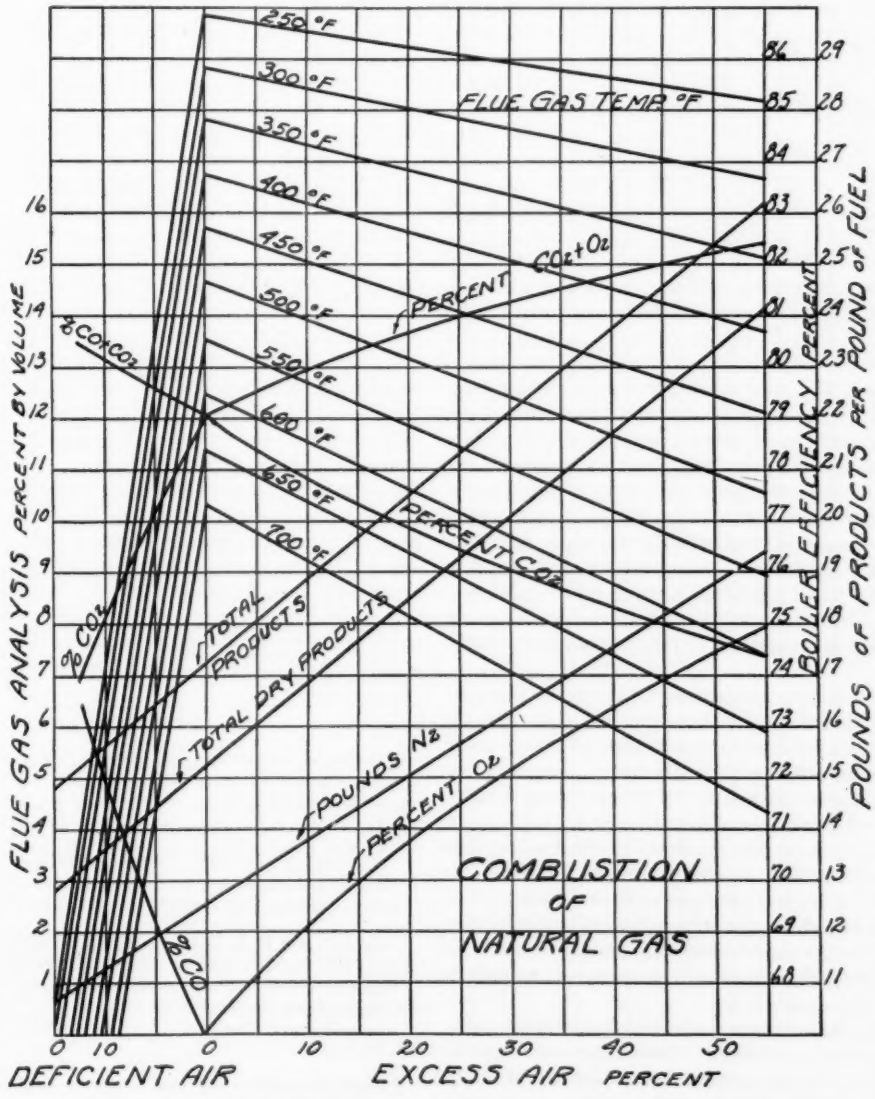


FIG. 3 COMBUSTION OF NATURAL GAS—PRODUCTS FORMED AND BOILER EFFICIENCY

(Analysis, per cent by volume: CH_4 = 79.230, C_2H_6 = 18.314, N_2 = 1.942, O_2 = 0.514; air temperature 70 F; higher heating value 22,458 Btu per lb, 1131 Btu per cu ft at 60 F and 14.696 psi; specific gravity 0.6584.)

In order accurately to evaluate the results thus obtained, it is necessary to give due consideration to the chemical constituents of the gas burned and their resultant combustion products and losses. Table 2 gives the analysis and combustion characteristics of a representative natural gas. Having these quantities and the specific heats of the various gases, it is possible to construct a graph such as Fig. 3, wherein the various known losses are subtracted from 100 per cent to give the theoretical heat-balance efficiencies for various values of excess air and flue-gas temperature. Such a set of curves permits an analysis of the relative importance of excess or deficient air and flue-gas temperature. It also establishes a standard with which reported efficiency may

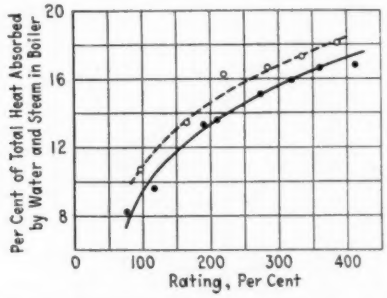


FIG. 4 COMPARISON OF TOTAL HEAT ABSORBED IN SUPERHEATER WHEN USING GAS AND OIL FUELS

be readily compared. Experience at Long Beach Steam Station has indicated actual efficiencies about 1 per cent lower than heat-balance efficiencies.

It should be noted in studying Fig. 3 that with the low flue-gas temperatures (300–350 F) that exist with the use of economizers and air preheaters that considerable increase in excess air has little effect on heat transfer efficiency. On the other hand, a deficiency of air causes losses to mount rapidly as shown on the left side of Fig. 3. Excess air losses however rise rather sharply when using the boiler alone without heat recovery equipment. Hence, although it is entirely possible with gas fuel to run with almost zero excess air and still attain complete combustion, it is considered good practice to operate with approximately 15 per cent excess air. This virtually obviates the possibility of slipping over into the deficient air range and CO in the flue gas with its resultant large losses.

Fig. 4 gives an interesting comparison of the percentages of total heat absorbed in the superheater when using gas and oil fuels. This boiler has a convection superheater. Consequently the heat absorbed is affected only by gas temperature and velocity passing the superheater tubes. With gas firing, at any rate of firing, there will be a larger volume of combustion products and, due to lower flame luminosity, relatively less heat absorption in the waterwalls and first bank of tubes than when firing fuel oil. Thus, flue-gas temperatures and velocities at the superheater will be higher with gas fuel and the quantity of heat absorbed in the superheater, as well as the superheated-steam temperature, will be higher. Obviously this relation will be reversed in boilers employing radiant-type superheaters.

Flexibility of performance and ease of control are two inherent advantages in the gas firing of boilers. These have been capitalized in the stand-by features of the plant No. 3 boilers at the Long Beach Steam Station. These 3416-hp boilers are operated with all twenty burners lighted, at outputs just sufficient to carry 1 per cent load on the main unit, and with the combustion-control mechanism in the "emergency" position. This emergency position requires manual starting of the forced- and induced-draft fans and selection of the "completely automatic" position on the control. The balance of bringing combustion up to full fire is entirely mechanical and requires 24 sec. Under test conditions, with two boilers being handled by each fireman, all boilers reached maximum fan speed and maximum fuel flow in 45 sec. The main turbogenerator units[2] accepted 85 per cent full load instantaneously, reaching full load in 5 min.

As shown in Table 1, the regular crew in plant No. 3 boiler room consists of only four men. These four men handle six boilers and related equipment furnishing steam to two 100,000-kw main units.

Operation of soot blowers is not required for gas fuel as burned in the stations described. However, soot blowers are installed and are used when burning oil.

Series 150 steel fittings, back-welded, are recommended for use in gas-piping systems.

The present installation is an outgrowth of a period in which gas from a source of very low line pressure, 15 lb per sq in., was available. In view of the present delivery of high-pressure gas only, future installations would be made delivering gas to the boiler room at not less than 25 lb per sq in., with consequent savings in size and cost of valves and fittings.

It is felt that orifices in the individual burner lines would allow the use of burners of ample gas-outlet areas and still restrict gas flow to a safe value in case of failure of local boiler gas-pressure controls. This control would be an individual reducing valve at each boiler, the setting for which would be regulated by the automatic-combustion-control system. Experience at Long Beach indicates the use of pilot-operated reducing valves of ample diaphragm area. The pilot mechanism may be located at a considerable distance from the reducing valve proper. It is highly desirable with parallel operation of reducing valves to have reducing-valve flow-pressure characteristics similar for all valves operating in conjunction with each other. These data are now available from the leading manufacturers of gas-control equipment. During the last fifteen years 120,000,000,000 cu ft of gas have been burned in the installation described above. A properly designed gas-piping system in the hands of competent operators probably presents fewer operating problems and no greater hazard than does the use of liquid or powdered fuels.

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[2] Manufacturer's rated load on these machines is 90 per cent of Long Beach Steam Station standard rating.

Power and Steam Plants for Oil-Refinery Service

By C. E. STEINBECK,[1] SAN FRANCISCO, CALIF.

This paper describes the general layout of the oil-company power plants, the various major items which will be used in their design, and the economy of the arrangement for both the Pacific Gas and Electric Company and the oil refineries.

THE DISCOVERY of oil in the lower San Joaquin Valley and in other sections of California approximately 40 years ago led to the establishment of oil refineries on the shores of upper San Francisco Bay. Some companies built pipe lines and others used ships and rail to transport the crude oil to the refineries, and the products of the refineries were shipped to all parts of the world by water and rail. As new and more efficient processes were developed and as demand for petroleum products increased, it became necessary to enlarge the refineries. This has required, from time to time, an increase in electricity for power and steam both for power and heat.

The increasing demand for steam and power at the refineries, together with the desire to increase operating efficiencies, led to a study of the question of whether to purchase these services or to build their own plants to supply them. It was found that the economies were in favor of purchased services and, hence, contracts which are similar for all companies, were made between the Pacific Gas and Electric Company and each of the following companies: Tide Water Associated Oil Company, Shell Oil Company, and Union Oil Company of California.

The Pacific Gas and Electric Company is constructing a steam-electric generating plant at the refinery of each company for the supply of steam and electric power. Since the steam plants are connected to the power company's transmission lines, no difficulty is presented due to unbalance in the electric and steam loads of the refineries. The quantities (for the three refineries) involved are approximately 20,000 kw maximum power demand, 120,000,000 kwhr per year, 875,000 lb per hr maximum steam demand, and 4 billion lb of steam per year. The power is delivered at 2300 volts and the steam at 160 psi with 50 F superheat.

The feedwater to the evaporators will be deaerated so that steam delivered to the refineries will be free of oxygen and CO_2. All water is furnished by the refineries with an allowance for blowdown depending upon the quality of the water.

A large natural-gas field has been developed a comparatively short distance from the sites of these plants and this supply, together with that from Kettleman Hills, will probably make natural gas the principal fuel.

Payment by the refineries for electric energy and steam will be made partly in cash and partly in fuel. This fuel may be fuel oil or refinery wastes such as sludges. The furnaces are suitable for the burning of petroleum coke as the use of this fuel is a future possibility.

Plant Design

Except for circulating-water supply, foundation conditions, and a few minor items, all plants will be the same. This will greatly reduce the engineering, construction, and operating costs. There are only a few miles separating the stations and as the Shell Oil plant at Martinez is centrally located between the Tidewater Associated plant at Avon and the Union plant at Oleum, the company's central office and plant shops will be located there.

Fig. 1 shows the general arrangement of the plant. Brick buildings will be erected to house the turbine, switch house, and control room. The boilers, evaporators, fans, water-softening equipment, and contact and deaerating heaters will be constructed for outdoor operation. The hot-air duct between the air heater and the burner front forms a roof over the firing aisle and by roofing over corresponding space between boilers, an enclosed firing aisle is obtained. The operating floors of the boiler firing aisle, the turbine operating platform, and that of the control room are all on the same level.

All panels necessary for operation of individual units, such as boilers, turbines, and pumps, will be located in the control room so that complete operation of the entire plant will be at one point. Automatic equipment will be installed wherever possible so that the operating personnel may be reduced to a minimum.

The design is such that changes which may be required by possible future conditions can be made. For example, should oil-refining processes change so that steam would no longer be required, a condensing unit would be installed to take the steam which is now extracted from the high-pressure unit for operation of the evaporators. On the other hand, should more steam be required, additional boiler, turbine, and evaporator capacity can be added.

Boilers

The nine radiant boilers to be installed, three at each station, are so called because, essentially, all the boiling or addition of latent heat is accomplished by the radiation of heat to the furnace walls. There is no conventional boiler convection bank, as the tubes between the furnace and the superheater are risers from a portion of the furnace waterwalls.

The units will have a maximum continuous capacity of 200,000 lb of steam per hr each, and a 24-hr overload capacity of 230,000 lb per hr. They are designed for a pressure of 1525 psi and a total steam temperature of 950 F at the maximum continuous steam flow.

The furnaces will be fired by natural gas, oil, and acid sludge through three combination burners arranged in a vertical row. Provision has been made for future firing with pulverized petroleum coke. Space has been allowed for pulverizers and piping to the burners which can be installed when necessary. The burners are so arranged that more than one of the fuels can be used simultaneously. For instance, when burning sludges, natural gas will be burned to support combustion and light the burners should some incombustibles be pumped to the burners.

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Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

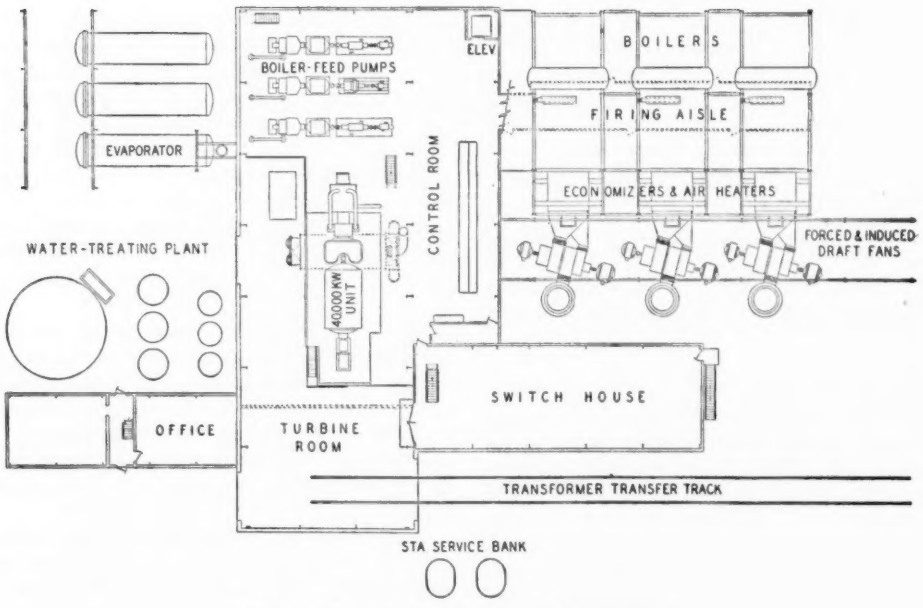


FIG. 1 GENERAL POWER-PLANT ARRANGEMENT

The furnaces are completely water-cooled and of stud-tube construction. The floors and the lower parts of the four walls are of fully studded tubes, and the upper portion of the furnaces of partially studded tubes. The fuels to be burned have so little ash that it was possible to design the furnaces with much simpler bottoms than is possible with coal-fired units for which special designs are necessary to provide for ash removal.

Each boiler has one forged steam-and-water drum, 60 in. ID, fitted with a steam scrubber to reduce the total solids carry-over in the steam to not over 1 ppm at all loads up to the designed capacity of the units, with a concentration in the boiler water of 1000 ppm

The boilers are designed to have just sufficient heating surface ahead of the superheaters to give them proper protection. The superheaters are pendant-type units and have sufficient surface to give 950 F total steam temperature at 100,000 lb output with gas fuel and 950 F at 150,000 lb with oil fuel. The metal in the superheater is grade C medium-carbon steel in the cooler section with special metal for the tubes and outlet header in the hotter portion to reduce the rate of oxidation and to provide the necessary creep strength for the pressure and temperatures involved. A gas by-pass of sufficient size to keep the superheat temperature down to 950 F at the top loads is provided along one side of the superheater. Automatic controls connected to dampers located in the gas stream behind the economizers will control the superheat temperature by causing gas to by-pass the superheater at the high loads. Quicker and safer starting up of the units will also be obtained by by-passing the superheaters to prevent overheating of the elements during the time no steam is being generated.

The by-passes around the superheaters are continued through the economizers. That section of the by-passes which parallels the superheater is formed by studded-tube water-cooled baffles. Economizer tubes with studs and plastic chrome ore form the baffles above the economizers.

The economizers are conventional horizontal water-tubular units, with gas flowing down across the tubes and the water entering at the bottom and flowing up through the tubes, and are designed so that they can steam at the high loads. The by-pass baffles cross all the economizer tubes so that the temperature of the water leaving all the tubes is as nearly uniform as possible. This insures good distribution.

Tubular air heaters have been located back of and below the economizers to permit washing the economizers, if necessary, and to provide better operating room between the air heaters and the burners.

These steam-generating units are designed to operate at 81.0 per cent efficiency and 9.0 in. draft loss through boiler, superheater, economizer, and air heater with natural-gas firing; 85.6 per cent efficiency and 12.8 in. with oil, and 88.3 per cent efficiency and 14 in. draft loss with pulverized petroleum coke at 200,000 lb of steam per hr at 1450 lb and 950 F total steam temperature at the superheater outlet. The efficiency given for petroleum coke does not include carbon loss. When the boiler is operating at a steaming rate of 200,000 lb per hr with gas fuel, the heat input per foot of furnace width is 23,000,000 Btu per hr. The equivalent release is 33,500 Btu per cu ft of furnace volume per hr. Fig. 2 shows the arrangement of boiler elements.

CONDENSER

The condensing equipment is designed to condense 300,000 lb of steam per hr with a heat surrender of 280,000,000 Btu per hr, giving a back pressure of 1.89 in. mercury abs when 37,500 gpm of water is circulated at 70 F, and 3.26 in. when inlet cooling water is 90 F.

The main condenser will have 18,000 sq ft of surface composed of $^{7}/_{8}$-in. OD, 18BWG tubes of 26 ft 0 in. over-all length. The water flow is single-pass and the water boxes and covers are divided so that half the tubes may be cleaned while the other half are in service. The tubes are roller-expanded at both ends. The tube sheet at the water-outlet end of the condenser is secured to the shell through a flexible-diaphragm expansion joint.

The tube-sheet layout of the condensers is of the double-folded type, each half an inverted U with a central exit lane to an air cooler, thus a short steam-flow path of low resistance is provided. Through the center of the entire length of the condenser shell is a V-shaped steam inlet passageway extending downward into the hot well and separated from the exit lanes by water seals between baffle plates. This arrangement permits accumulation

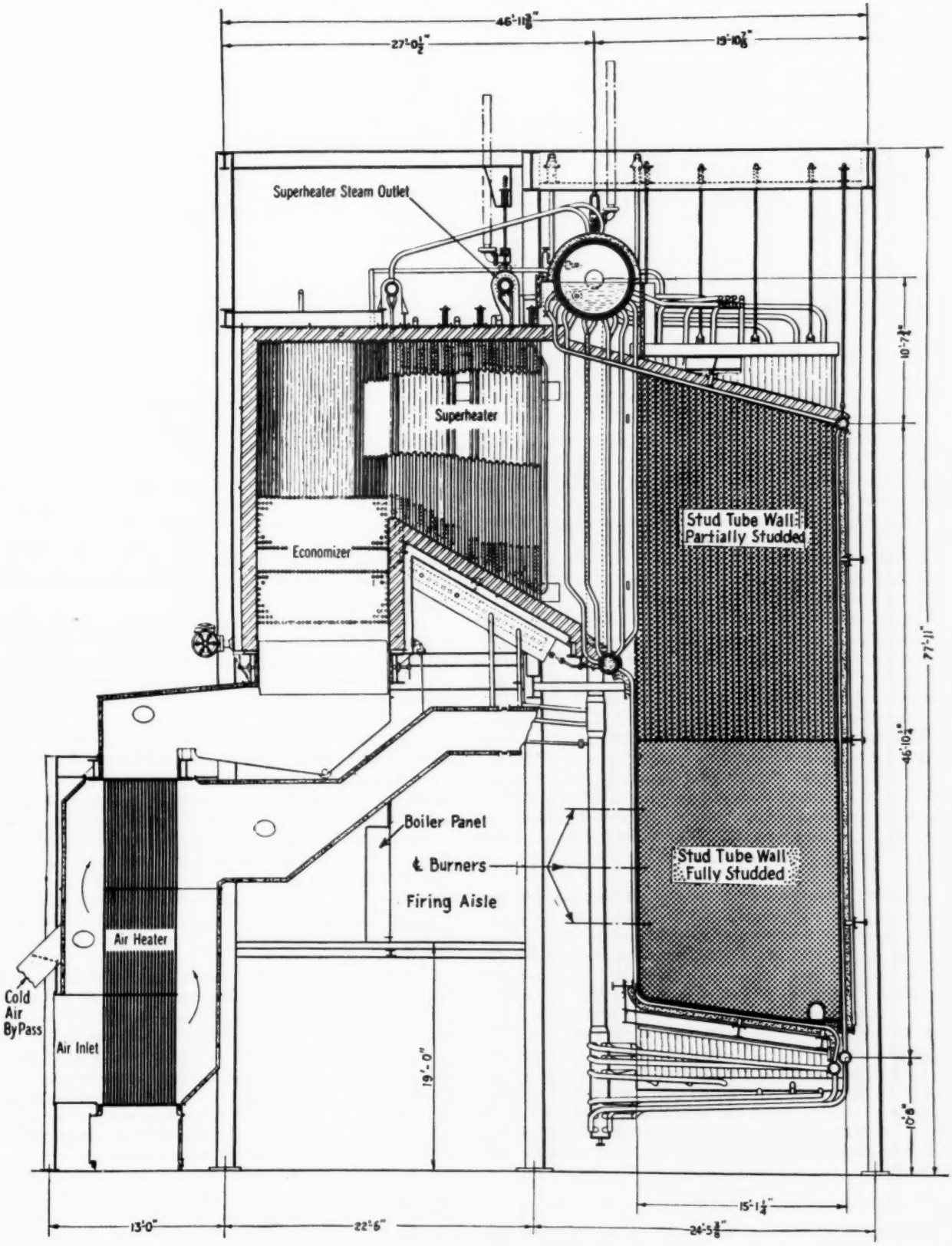


FIG. 2 BOILER ARRANGEMENT

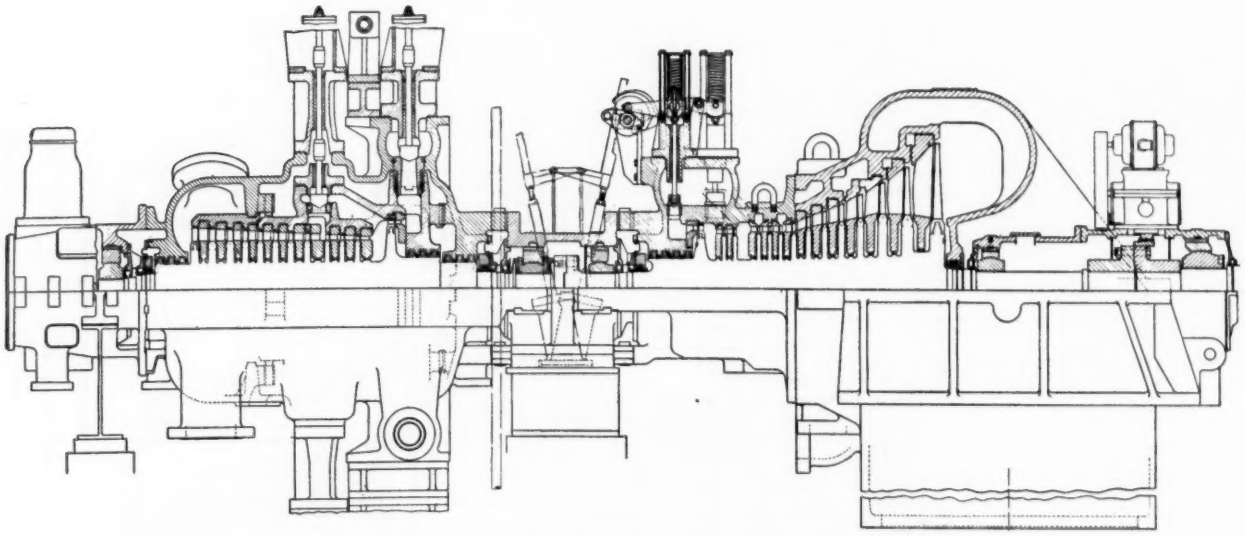


FIG. 3 TANDEM COMPOUND STEAM TURBINE

of pressure and temperature in the hot well for reheating of the condensate.

Under certain conditions, the unit will operate on stand-by service. At such times condensation of 15,000 lb of steam per hr and surrender of 15,000,000 Btu will serve to heat 300,000 lb of evaporator feedwater from 60 to 110 F at a pressure of 3 in. mercury abs. The heating of this water is to be effected in a four-pass heater having 1420 sq ft of surface made up of 242 tubes of same diameter and length as those in the main condenser. This heater will be located in the same shell but above the tubes of the main 18,000-sq-ft condenser. Between the heater and the main tube bank beneath, will be a diaphragm of shutter type at about 45 deg on each side of the condenser consisting of overlapping stainless-steel plates mounted upon shafts extending lengthwise of the shell parallel to but not coinciding with the center line of the plates. This will be counterbalanced to remain in closed position, separating the heater or stand-by section of the shell from the main or load-operation section of the shell during the usual stand-by operation until opened by predetermined pressure difference, thermostatic or load control, or combination of controls.

Air will be vented from the heater section to the main condenser section beneath. Condensate will be drained from the heater section through a conduit sealed deeply in the hot well at a point close to the hot-well pump suctions so that flashing may be avoided and the heat of condensate from the heater section at about 110 F conserved. Upon application of load, momentary rise in steam temperature within the heater to above 115 F will operate an automatic temperature control and cause the butterfly valves between the heater section and the main condenser section to open, thus placing the entire condenser in service. Liberal cutouts will be provided through the intermediate tube-supporting plates to permit free longitudinal distribution of the steam. A deaerating-type hot well of welded steel plate and of 300 cu ft volume is attached directly to the bottom of the condenser shell.

TURBINES

The turbine consists of a combination of high-pressure noncondensing 3600-rpm turbine, coupled to a single-flow 3600-rpm moderate-initial-pressure condensing machine. This turbine is arranged substantially as shown in cross section in Fig. 3. The inlet ends of the two machines are put together on either side of the middle bearing pedestal with the shafts solidly bolted together with forged-on couplings and the thrust bearing in between the two machines. The casings are anchored at the exhaust end of the condensing turbine nearest the generator, the middle pedestal slides guided by a center key, and the forward pedestal on the exhaust end of the high-pressure turbine is supported on a flexible cross member.

The governor is mounted on this forward pedestal and is driven by a worm gear on an extension of the exhaust end of the shaft of the high-pressure unit. The two hydraulic cylinders operating the inlet valves of the high-pressure turbine and the low-pressure turbine are both located and completely enclosed by the middle bearing standard between the two turbines, and the back-pressure governor and control mechanism are mounted on or alongside this pedestal.

The main exhaust of the high-pressure turbine is downward and the two auxiliary pipes leading into the inlet of the low-pressure turbine are from the upper half of the casing and over the top of the high-pressure unit down into either side of the nozzle chest of the low-pressure turbine. This arrangement simplifies the unit and greatly simplifies the foundation, provides ease of expansion in the piping, and results in less exhaust loss in the high-pressure unit, inasmuch as part of the steam is taken away directly from the upper half of the last-stage wheel and part of it from the lower half.

High-Pressure Unit. The high-pressure unit of this machine is of a double-shell internal-by-pass arrangement with the control-valve chests cast integral with the outer casing. This arrangement of the inner shell and outer shell is such that at full load, the pressure between the inner and outer shell is about 45 per cent of the steam pressure on the inside of the inner shell and the temperature about 75 F under the incoming temperature so that the outer shell is, therefore, subjected to quite low steam conditions and the inner shell which carries the high pressure on the inside is not subjected to a high pressure difference.

Primary control valves open until the full quantity of flow which can be passed by the machine in this condition is reached. The pressure drop through the first-stage nozzles is some 10 per cent and the temperature drop some 30 to 40 deg. After this load, additional flow can be passed, up to some 30 per cent, by opening the internal by-pass between the first-stage shell and the fourth-stage shell, lowering the first-stage shell pressure a sufficient amount to pass the additional flow, increasing the

pressure 25 per cent and increasing the temperature drop accordingly. In this way, initial boiler pressure and temperature never penetrate the machine further than the first-stage valve chest and nozzle parts, and the shells, wheels, and diaphragms are protected by the temperature drop incident to the flow through the first stage.

Low-Pressure Turbine. The low-pressure turbine is basically similar to the manufacturer's standard 20,000-kw 3600-rpm turbine. It is arranged with multivalve inlets for maximum economy, has a 20-in-long last-stage bucket shielded with stellite to prevent erosion, and is arranged in the low-pressure stages so as to extract heavier particles of moisture to improve efficiency

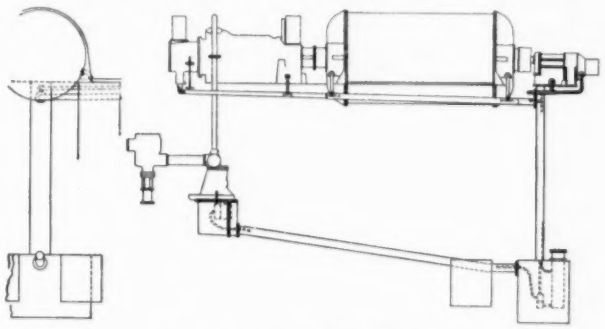


FIG. 4 TURBINE OILING SYSTEM

and reduce bucket and nozzle wear. The turning gear for the entire unit is incorporated on the exhaust-hood pedestal of this machine.

Oiling System. Fig. 4 shows the oiling system in schematic outline. The oil tank is located under the outboard-generator end of the turbine set and contains the coolers and the pumps. The high-pressure oil is conducted to the bearings and to the oil-operated hydraulic system by means of small pipes on the inside of the oil-drain returns from the corresponding bearings and hydraulic cylinders. In this way, no high-pressure piping is exposed and the fire hazard is reduced. The drive for the pumps is by a worm gear from an 1800-rpm shaft connected to the low-speed shaft of the reduction gear which drives an 1800-rpm exciter.

Governing. The governing of these extraction condensing turbines has been worked out, as was suggested by the engineers of the company in such a way as to permit controlling the high- and low-pressure turbines almost as though they were separate machines, i.e., the operation of the main controlling valves on the high-pressure section is controlled largely by the pressure in the exhaust of the high-pressure section with the back-pressure governor automatically reset by the pressure at the exhaust of the evaporators in such a way as to maintain the 180-lb pressure nearly constant. The controlling valves for the low-pressure section, on the other hand, are controlled by the speed governor and hence are responsive to the load requirements. In the event the load requirements are under the output of the high-pressure section when supplying the called-for steam, the high-pressure controlling valves are closed a corresponding amount. The governor of the low-pressure section is also so arranged as to permit holding these controlling valves open just sufficiently to supply cooling steam to the low-pressure turbine, to let no variation in the opening of these valves result from minor changes in frequency in the system, and yet to open the valves automatically to a predetermined limit in the event of a sudden reduction in frequency, i.e., a demand on the system for load from this unit. Various other safety factors are also being put into this governing system in order to insure that the combination will operate safely at all times.

Performance. The high-pressure turbine is designed for a maximum flow of 675,000 lb per hr and any part or all of this flow, with the exception of that required to cool the low-pressure turbine and the gland leakage, may be extracted. The operating back pressure is 240 lb abs. Under these conditions, the ouput is approximately 32,500 kw. The unit will also develop approximately 43,500 kw with zero extraction, i.e., with all of the flow passing to the low-pressure turbine but with this output there is a pressure rise in the exhaust of the high-pressure turbine above 240 lb abs. The unit will carry something in excess of 50,000 kw operating under certain extracting and condensing conditions.

Generator. The 50,000-kva generator will be hydrogen-cooled. There will be four water-cooled hydrogen coolers installed in the stator frame, two on each side. Oil seals will be provided and hydrogen-detraining apparatus will be provided for removal of the hydrogen from the oil. Hydrogen purity will be automatically maintained at 97 per cent.

Supervisory instruments will be installed either to indicate or

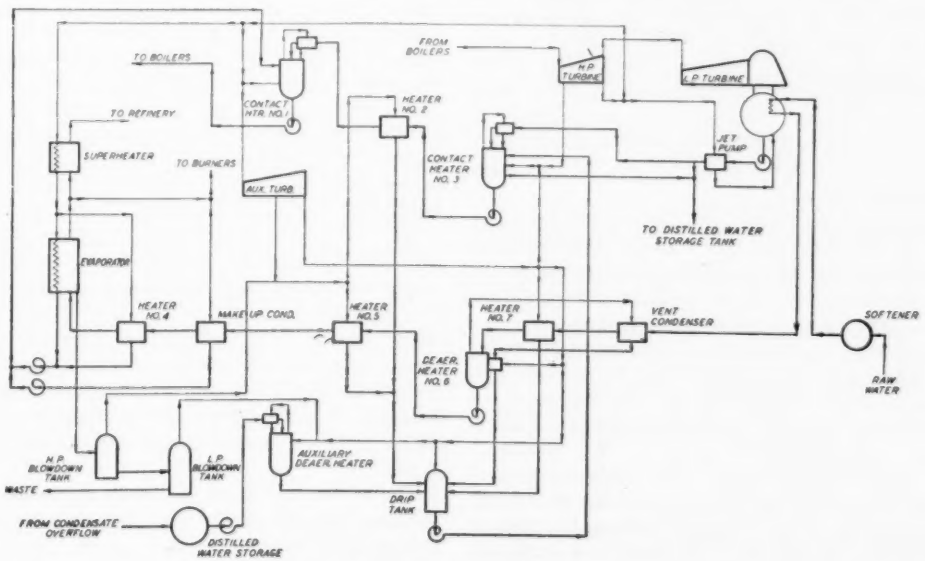


FIG. 5 FLOW DIAGRAM

record eccentricity, vibration, amplitude, expansion, and interference.

Flow Diagram

An examination of the flow diagram, Fig. 5, of the steam station will show that the high- and low-pressure steam cycles are kept separate. This was done in order to insure flexibility of operation and reliability of both the steam supply to the refineries and power output of the plant.

In the high-pressure steam cycle, the condensate after leaving the deaerating hot well of the condenser is pumped through the jet condenser to contact heater No. 3, through closed heater No. 2, and to contact heater No. 1. The pumps indicated at contact heaters Nos. 3 and 1 are the booster pumps and boiler-feed pumps. They are on the same shaft and are driven by an auxiliary turbine. Each auxiliary turbine has a capacity of 1675 kw and drives in addition to the pumps an induction generator of 1000 kw capacity. At light loads, the induction generator may be required to operate as a motor.

The water for the process-steam cycle is first treated by water softeners. At the Avon plant, the feedwater is of such a character that it first requires a lime pretreatment followed by a zeolite-and-acid treatment. At the Martinez and Oleum plants, a zeolite-and-acid treatment is all that will be required. After the water treatment, the process water passes through the feedwater-heating section of the condenser, heater No. 7, and then to deaerating heater No. 6. From heater No. 6, the evaporator-feed pump passes the water to the evaporators. The evaporator-feed pump and the evaporator-condensate pump are on the same shaft and are driven by a single motor.

The make-up condenser, as its name indicates, is used to furnish make-up to the high-pressure steam cycle. This will be regulated by throttling the steam supply to heater No. 5. The condensate from the make-up condenser will be pumped to contact heater No. 1. Provision is made so that, should this be more than the boilers require, condensate will be withdrawn at a point just beyond the jet condenser. An auxiliary deaerating heater No. 8 will be installed to deaerate the distilled water from storage, before it is returned to the high-pressure steam cycle.

Auxiliaries

Boiler-Feed and Evaporator-Feed Pumps. The boiler-feed pumps are divided into two units, a booster pump and the boiler-feed pump. These pumps are on the same shaft and driven by the same driver, as previously mentioned. The capacity of each booster pump is 350,000 lb per hr for a differential of 350 psi and that of the boiler-feed pump is 550,000 lb per hr for a differential head of 1500 psi. The maximum temperature of water to the booster pump will be 225 F and to the boiler-feed pump 400 F. The operating speed of the units is 3570 rpm and the power requirement 1870 hp.

Pumps from two manufacturers have been purchased. At Avon and Martinez, two horizontal split-case high-pressure units will be used. These are similar to those installed at station A, San Francisco. At these stations also will be one solid-case pump. At Oleum there will be three solid-case pumps.

The evaporator-feed pumps are likewise dual pumps driven by a single motor. There are three of these units for each station.

The main-evaporator-feed pumps will each be designed for a capacity of 300,000 lb of water per hr at a temperature of 225 F against a differential pressure of 225 psi gage. The maximum suction pressure will be 50 psi gage. The main-evaporator-condensate pumps will be designed for a capacity of 300,000 lb per hr at a temperature of 375 F against a differential head of 60 psi gage. The pump casings will be designed for a maximum suction pressure of 300 psi gage.

Evaporators. In all, ten horizontal evaporators were purchased for the three stations, four for Martinez and three for each of the other two stations. Each evaporator will have 7500 sq ft of heating surface and will produce 102,000 lb of steam per hr with a heat head of 17 F. The working pressures are 200 psi gage for the shells and 300 psi gage for the tubes. The evaporators are coil-type with welded steel shells, 9 ft diam × 40 ft long and tubes of arsenical copper.

Fans. A single combined forced- and induced-draft fan unit with both impellers mounted on the same shaft will be provided for each boiler. Each unit will be driven by either of two motors mounted on opposite ends, one of 400 hp, 900 rpm, and the other of 1000 hp 1200 rpm. Vane control is provided for each unit on both forced- and induced-draft sides. The forced-draft fan has a maximum capacity of 67,500 cu ft per min of 100 F air at 17.7 in. of water pressure and the induced-draft fan a maximum capacity of 134,000 cu ft per min of 430 F gas at 26.6 in. of water draft. The fans will be out of doors and at ground level.

Circulating Water. The condenser cooling-water supply at Avon will be pumped from a slough connected to upper San Francisco Bay. During the spring season when a large runoff occurs from the Sacramento and San Joaquin Rivers, the water will be fresh but at other times of the year the quality will vary from brackish to salt as the slough is subject to tide flow from the bay.

The Martinez station will be located on a hill some 100 ft above the level of the bay. As it would not be economical to pump bay water for cooling purposes, it has been decided to use a cooling tower, pumping only enough for cooling-tower make-up. This water will vary from brackish to salt.

The Oleum station will be located on the bay and there an ample supply of circulating water will be available. This water will also vary from brackish to salt.

Cooling Tower. A forced-draft cooling tower suitable for salt-water operation will be installed at Martinez. This will consist of 12 cells with a capacity to cool 37,500 gpm from 105 to 90 F with a wet-bulb temperature of 70 F.

Discussion

C. S. Smith.[2] The author describes a notable engineering and economic achievement resulting from the combined efforts of the respective managements of a power company and three oil refineries. His paper gives an interesting account of the advantages accruing on the side of the power and oil companies. From the viewpoint of the boiler manufacturer, the furnishing of nine essentially identical units presented an opportunity to install more efficient, high-pressure, high-temperature units than might have been feasible had the oil companies purchased independently and for their own specific needs. These units reflect the most recent progress in design and at the same time offer the advantages attendant upon a certain degree of standardization.

Of the various types of units available, the radiant boiler most satisfactorily and economically meets the customer's requirements in steam output, steam pressure, total steam temperature, and superheat control. At the same time it is capable of handling efficiently the multiplicity of fuels available. The total steam temperature of 950 F, which is among the highest in commercial use in this country, and superheat control down to one-half load, both of which were prerequisites, immediately eliminated from consideration many of the more conventional boiler types.

The author has briefly described the boiler and has shown a sectional side elevation. Comment on a few of the design details may be pertinent.

[2] The Babcock & Wilcox Company, New York, N. Y.

The various components of the boiler and furnace, such as the side walls, the two large outside downcomers, and inside risers while generally independent, are so integrally connected that the entire unit is suspended from overhead supports. As all pressure parts expand downward, the problem of providing for differential expansion seals and slip joints, is greatly simplified.

The water supply for the furnace walls and boiler is carried through two large vertical pipes leading one from each end of the steam and water drum, down the outside, to the bottom of the furnace waterwalls. Small tubes connect the lowermost portion of these downcomers to the various wall headers. The use of these outside supply tubes insures positive circulation and is simpler mechanically than their equivalent in small tubes.

The furnace wall surfaces and furnace volumes are designed especially for oil and gas fuel and are adequate for pulverized petroleum coke when the unit is operated at full capacity. This latter fuel necessitated a slight increase in the height of the furnaces, to insure proper combustion of the coke, maintain low carbon loss, and minimize the possibility of fouling the heating surfaces. Fully studded tube construction is used in the lower section of the furnace, where maximum protection is required and furnace temperatures must be relatively high to insure efficient combustion at all ratings with gas and oil. In the upper part of the furnace, where the service is relatively lighter and a higher heat-absorption rate is desirable, partly studded tubes are used.

The boilers will be equipped with the latest design of Babcock & Wilcox cyclone separators, a relatively new and important adjunct to modern high-duty boiler practice, and described in a paper by Baker.[3] The use of the cyclone separators, together with the steam scrubber, insures steam of the highest quality and purity known in commercial boiler operation. At the same time it permits wider fluctuations in water level without affecting steam quality, and promotes better circulation in the boiler by insuring that steam-free water enters the downcomers.

The nondrainable pendant superheater is entirely suitable for these units, as there will always be adequate time to bring a cold boiler to line pressure without overheating the pressure parts. Furthermore, there are no banking conditions that would cause condensation in the elements. Consequently, the desirable attributes of pendant superheaters, namely, simplicity of supports, accessibility of headers, and good steam distribution, are obtained.

The air heaters are provided with cold-air by-pass ducts from the forced-draft fan to the third air pass, to reduce the possibility of corrosion caused by condensation at the cooler ends of the tubes at reduced ratings. In other words, if the temperature of the cooler ends of the tubes were to fall below the dew point, moisture would collect, and this would form dilute sulphuric acid when high-sulphur fuels were burned.

Likewise, to prevent corrosion which might occur under these conditions where flue gases come in contact with cold steel, a portion of the casings for these units is made of a nonmetallic millboard that is resistant to such attack. These casings are also weatherproof. In conjunction with the arrangement of pressure parts, flues, and air heater, they render the boilers especially suitable for out-of-door service. By thus taking advantage of prevailing climatic conditions, it was possible to realize a substantial saving in the initial investment.

R. C. Vroom.[4] The author refers to the three combination burners to be used for firing each of the boilers. Each burner is designed for a heat liberation of 118,500,000 Btu per hr. With reference to the liquid fuels, the present method of atomization for sludge or oil will be steam although in the future the atomizers can be converted readily to mechanical atomizers of the wide-range type.

Fig. 6 shows a sectional elevation of the burners as they will be installed initially, without provision for handling pulverized fuels. Fig. 7 is a view taken from the furnace end of the burner as it will appear when assembled as a combined burner for liquid, gaseous, and pulverized fuels.

G. B. Warren.[5] The writer was privileged to work with the author and his associates in designing the turbine equipment to meet the requirements of these outstanding power plants and

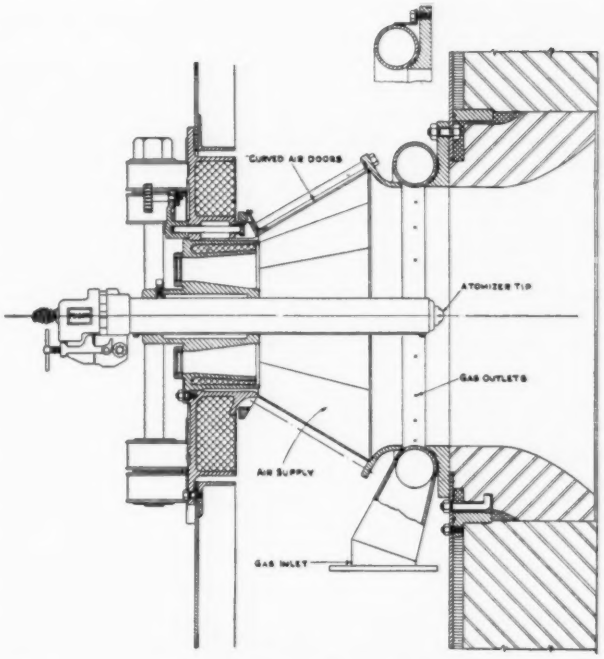


Fig. 6 Sectional Elevation of Burners Without Pulverized-Fuel-Handling Provisions

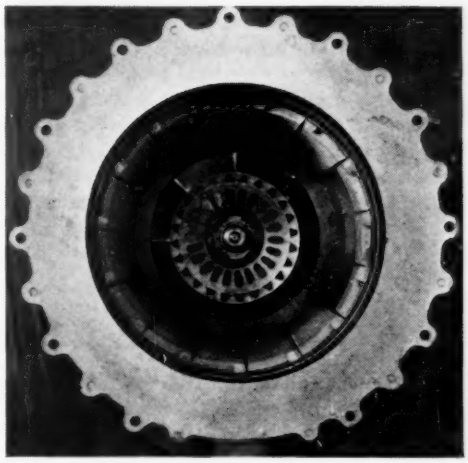


Fig. 7 Combined Burner for Liquid, Gaseous, and Pulverized Fuels

[3] "Mechanical Purification of Steam Within the Boiler Drum," by M. D. Baker, published in this issue of the Transactions, p. 711.

[4] Chief Engineer, Peabody Engineering Corporation, New York, N. Y. Mem. A.S.M.E.

[5] Designing Engineer, General Electric Company, Schenectady, N. Y. Mem. A.S.M.E.

is, therefore, familiar with the thoroughness and care with which every detail was developed by them.

In the writer's estimation, the success of these newly installed high-pressure, high-temperature power plants, together with the excellent record of the pioneer plants of this type, one of which was installed by the Pacific Gas and Electric Company, has created opportunities not heretofore possible in the construction of power plants for the purpose of furnishing low-grade heat and power as a joint undertaking. Condensing power plants of necessity waste the heat in the exhaust. Where such heat is needed and can be used, it offers the opportunity of obtaining large quantities of power at almost one hundred per cent thermal efficiency.

There is a keen realization of the difficulties which lie in the path of those who seek to apply these principles, because of the many conflicts in the demands of the industry for heat and the demands of the utility for power, together with the risks involved for the utility when its fortunes are tied up with an industry over which it has no control.

This matter was given very careful consideration in the installation under discussion. The risk to the utility was reduced to a minimum by the addition of the condensing element which permits operation of the complete set fully condensing at full power. Although this can be done only at a somewhat lower efficiency, yet, because of the high pressure and high temperature, the efficiency of the condensing set, when so operating, will compare very favorably with many quite modern condensing power plants now in operation.

Methods of Rotor-Unbalance Determination

By J. G. BAKER,[1] EVANSVILLE, WIS.

Rotor-unbalance determinations both in balancing machines and from vibration readings taken on the rotor during normal operation have in some cases been rather complicated and often subject to considerable error. This paper suggests methods intended to improve balancing methods in these cases.

In part I a more or less conventional method of calculating unbalance corrections from vibration readings is reviewed and the limitations and errors pointed out. Next, an extension of the method is developed which is in some cases simpler and inherently more accurate.

In part II a mathematical method is developed for determining unbalance corrections in cases where the locations of the corrections are completely prescribed in contradistinction to the ordinary case in which the axial position of the correction is prescribed but the angular position is to be determined. Also, a balancing machine is described which reads directly the unbalance corrections required at four preselected locations in a rigid rotor. This balancing machine is especially suited for the quick determination of unbalance corrections required in an internal-combustion-engine crankshaft.

In part III center-of-percussion balancing is described which is a method of taking vibration readings which can be interpreted directly in terms of unbalance. This method is applicable only under certain conditions which are noted.

PART I—CONVENTIONAL AND TRIAL-WEIGHT-GROUP METHODS

Introduction

FROM TIME to time methods have been described for determining unbalance corrections from vibration readings made with and without trial unbalance.[2,3,4] All of these methods are fundamentally the same and are based on the assumptions (1) that the system to be balanced is linear or nearly so and (2) that the unbalance remains constant except when deliberately altered by weights added or subtracted. With some systems one or both of these assumptions apply only roughly. The errors in the corrections determined from vibration readings in such cases are especially large if the vibrating system of the machine and the correction locations are selected, as is apt to happen, such that the determination involves the difference between large quantities. If this is the case the computed unbalance correction may be so inaccurate that the vibration of the machine is actually increased by it instead of being decreased.

Derivation of the Usual Formula

In Fig. 1 is shown a rotor supported in three bearings. The object of balancing is to eliminate vibration at each of these bearings. Correction locations are chosen at 1, 2, and 3. The three bearing pedestals on which vibrations are read will be designated as A, B, and C, respectively. The trial weights to be used in 1, 2, and 3 are w_1, w_2, and w_3, respectively. The unbalance component in 1 is W_{I}, that in 2 is W_{II}, and in 3 is W_{III}.[5] The vibration at pedestal A will be represented by the projection of the vector A, the vibration at the B pedestal will likewise correspond to the projection of the vector B, and that at C to the projection of the vector C. A subscript will be used to denote the cause of the vibration or increment in vibration. Thus, A_0 is a vector corresponding to the initial vibration at A; A_1 is a vector corresponding to the vibration at A due to the initial unbalance W_{I} only; and A_{01} is a vector corresponding to the vibration at A after adding the trial weight w_1 to the machine in its initial condition, that is, with W_{I}, W_{II}, W_{III}, and w_1 all present. A_1 is a vector corresponding to the increment in the vibration at A due to adding the trial weight w_1; thus

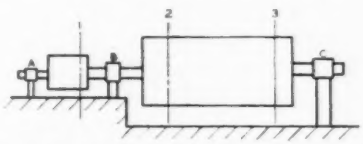


Fig. 1 Three-Bearing Rotor With Correction-Location Planes Indicated

$$A_1 = A_{01} - A_0$$

From the assumed linearity of the system

$$\left.\begin{aligned} A_0 &= A_{\mathrm{I}} + A_{\mathrm{II}} + A_{\mathrm{III}} \\ B_0 &= B_{\mathrm{I}} + B_{\mathrm{II}} + B_{\mathrm{III}} \\ C_0 &= C_{\mathrm{I}} + C_{\mathrm{II}} + C_{\mathrm{III}} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots [1]$$

Furthermore, A_{I} and A_1 are related in the same way as W_{I} and w_1 so that

$$A_{\mathrm{I}}/A_1 = W_{\mathrm{I}}/w_1 \quad \text{or} \quad A_{\mathrm{I}} = (W_{\mathrm{I}}/w_1)A_1 \ldots\ldots\ldots [2]$$

In which the product of two vectors is taken to mean a third vector, the angle of which is the sum of the angles of the two factors and the absolute magnitude of which is the product of the absolute magnitudes of the factors. Likewise the quotient of two vectors is taken as the difference between their angles and the quotient of their absolute quantities. Corresponding to Equation [2] A_{II} may be expressed as

$$A_{\mathrm{II}} = (W_{\mathrm{II}}/w_2)A_2 \ldots\ldots\ldots\ldots\ldots\ldots [3]$$

and

$$B_{\mathrm{I}} = (W_{\mathrm{I}}/w_1)B_1 \ldots\ldots\ldots\ldots\ldots\ldots [4]$$

[1] Mechanical Engineer, Westinghouse Research Laboratories, Hamilton Standard Propeller Division of United Aircraft Corporation, and Baker Manufacturing Company.

[2] "Turbine Vibration and Balancing," by T C. Rathbone, Trans. A.S.M.E., vol. 51, part 1, 1929, paper APM-51-23, pp. 267–284.

"Dynamic Balancing of Rotating Machinery in the Field," by E. L. Thearle, Trans. A.S.M.E., vol. 56, 1934, paper APM-56-19, pp. 745–753.

[3] "Balancing Rotating Machines in the Field," by F. C. Rushing and B. A. Rose, *Electric Journal*, vol. 34, November, 1937, pp. 441–444.

[4] "Balancing Rotors by Means of Electrical Networks," *Journal of The Franklin Institute*, August, 1936.

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Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[5] All terms appearing in the equations in this paper are vector quantities.

Using all the relations of the class of Equations [2], [3], and [4], Equation [1] may be rewritten

$$\left.\begin{aligned} A_0 &= \frac{W_{\mathrm{I}}}{w_1}A_1 + \frac{W_{\mathrm{II}}}{w_2}A_2 + \frac{W_{\mathrm{III}}}{w_3}A_3 \\ B_0 &= \frac{W_{\mathrm{I}}}{w_1}B_1 + \frac{W_{\mathrm{II}}}{w_2}B_2 + \frac{W_{\mathrm{III}}}{w_3}B_3 \\ C_0 &= \frac{W_{\mathrm{I}}}{w_1}C_1 + \frac{W_{\mathrm{II}}}{w_2}C_2 + \frac{W_{\mathrm{III}}}{w_3}C_3 \end{aligned}\right\} \quad \ldots\ldots [1a]$$

Since the angle of W_{I}/w_1 is the angle by which W_{I} exceeds w_1 and since its magnitude is the ratio of the absolute magnitude of W_{I} to that w_1 it is the result sought. It is the expression for the unbalance at 1 or the negative of the correction required at 1. Similarly, W_{II}/w_2 and W_{III}/w_3 are the results sought at 2 and 3, respectively. Equation [1a] may be solved for W_{I}/w_1, W_{II}/w_2, and W_{III}/w_3 by means of the rules of determinants thus if

$$\Delta = \begin{vmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{vmatrix} \quad \ldots\ldots [5]$$

Then

$$\frac{W_{\mathrm{I}}}{w_1} = \frac{\begin{vmatrix} A_0 & A_2 & A_3 \\ B_0 & B_2 & B_3 \\ C_0 & C_2 & C_3 \end{vmatrix}}{\Delta} \quad \ldots\ldots [6]$$

$$\frac{W_{\mathrm{II}}}{w_2} = \frac{\begin{vmatrix} A_1 & A_0 & A_3 \\ B_1 & B_0 & B_3 \\ C_1 & C_0 & C_3 \end{vmatrix}}{\Delta} \quad \ldots\ldots [7]$$

$$\frac{W_{\mathrm{III}}}{w_3} = \frac{\begin{vmatrix} A_1 & A_2 & A_0 \\ B_1 & B_2 & B_0 \\ C_1 & C_2 & C_0 \end{vmatrix}}{\Delta} \quad \ldots\ldots [8]$$

For a system of n locations at which it is desired to eliminate vibration, n correction locations are required. Corresponding to Equations [1a] there would be a system of n equations each with n right-hand members. Corresponding to the three formulas [6], [7], and [8], there would be n formulas each with the same nth order determinant for a denominator and each with its own nth order determinant for a numerator. Referring to formulas [6], [7], and [8], if the determinants are multiplied out the following equations are obtained

$$\frac{W_{\mathrm{I}}}{w_1} = \frac{A_0(B_2C_3 - B_3C_2) - B_0(A_2C_3 - A_3C_2) + C_0(A_2B_3 - A_3B_2)}{\Delta} \quad \ldots\ldots [9]$$

$$\frac{W_{\mathrm{II}}}{w_2} = \frac{-A_0(B_1C_3 - B_3C_1) + B_0(A_1C_3 - A_3C_1) - C_0(A_1B_3 - A_3B_1)}{\Delta} \quad \ldots\ldots [10]$$

$$\frac{W_{\mathrm{III}}}{w_3} = \frac{A_0(B_1C_2 - B_2C_1) - B_0(A_1C_2 - A_2C_1) + C_0(A_1B_2 - A_2B_1)}{\Delta} \quad \ldots\ldots [11]$$

$$\Delta = A_1(B_2C_3 - B_3C_2) - B_1(A_2C_3 - A_3C_2) + C_1(A_2B_3 - A_3B_2) \quad \ldots\ldots [5a]$$

In a similar way for a two-bearing case or in a case in which it is desired to reduce the vibration at only two bearings, the two equations corresponding to [9], [10], and [11] are

$$W_{\mathrm{I}}/w_1 = (A_0B_2 - B_0A_2)/\Delta_2 \quad \ldots\ldots [12]$$

$$W_{\mathrm{II}}/w_2 = (B_0A_1 - A_0B_1)/\Delta_2 \quad \ldots\ldots [13]$$

where

$$\Delta_2 = A_1B_2 - B_1A_2 \quad \ldots\ldots [14]$$

Criticism of Method

Any one of the formulas [9], [10], and [11] for the three-bearing case and [12] and [13] for the two-bearing case will yield results with large errors if the positive terms are about equal to the negative terms.

Also the algebraic or other work or the apparatus involved in evaluating these equations by machine means is large so that it appears desirable to seek another approach with the hope of reducing the complication and increasing the accuracy.

The ideal condition in so far as accuracy is concerned would be (1) to have a trial weight located such that only pedestal A would be affected by it, (2) to have another trial weight located such that only pedestal B would be affected by it, and (3) to have still another trial weight located such that only pedestal C would be affected by it. If this were possible Equations [9], [10], and [11] would become

$$\frac{W_{\mathrm{I}}}{w_1} = \frac{A_0}{A_1} \quad \ldots\ldots [15]$$

$$\frac{W_{\mathrm{II}}}{w_2} = \frac{B_0}{B_2} \quad \ldots\ldots [16]$$

$$\frac{W_{\mathrm{III}}}{w_3} = \frac{C_0}{C_3} \quad \ldots\ldots [17]$$

There appears no simple way to find the location for single trial weights to fulfill this ideal since information on a large number of locations for trial weights can apparently only be found by making a large number of trial runs. But there will be *groups* of trial weights located only in the planes already selected which will have the desired property of affecting only one pedestal for each group.

Trial-Weight Groups Which Affect the Vibration at Single Locations Only

Let the weights r_1, r_2, and r_3 inserted at 1, 2, and 3, respectively, be of such relative magnitude and position that the vibration increment at A due to these weights is A_r and the vibration increment at B and C is zero. This is the equivalent of saying that if the weights r_1, r_2, and r_3 only are present the vibration at A would be A_r and that at B and C would be zero. Thus in Equations [9], [10], and [11] if W_{I}, W_{II}, and W_{III} are replaced by r_1, r_2, and r_3 then A_0 must be replaced by A_r and B_0 and C_0 set equal to zero; or

$$r_1/w_1 = A_r(B_2C_3 - B_3C_2)/\Delta \quad \ldots\ldots [18]$$

$$r_2/w_2 = -A_r(B_1C_3 - B_3C_1)/\Delta \quad \ldots\ldots [19]$$

$$r_3/w_3 = A_r(B_1C_2 - B_2C_1)/\Delta \quad \ldots\ldots [20]$$

Since r_1, r_2, and r_3 are as a group to be again considered as a trial weight, only the relation between them is necessary so that Equations [19] and [20] are each divided by [18] to obtain

$$r_3w_1/r_1w_2 = (B_3C_1 - B_1C_3)/(B_2C_3 - B_3C_2) \quad \ldots\ldots [21]$$

and

$$r_3w_1/w_3r_1 = (B_1C_2 - B_2C_1)/(B_2C_3 - B_3C_2) \quad \ldots\ldots [22]$$

Similarly, if s_1, s_2, and s_3 are weights which together cause a vibration increment only at B and this vibration is B_s then from Equations [9], [10], and [11] as before

$$s_1/w_1 = -B_s(A_2C_3 - A_3C_2)/\Delta \quad [23]$$

$$s_2/w_2 = +B_s(A_1C_3 - A_3C_1)/\Delta \quad [24]$$

$$s_3/w_3 = -B_s(A_1C_2 - A_2C_1)/\Delta \quad [25]$$

$$s_2w_1/s_1w_2 = (A_1C_3 - A_3C_1)/(A_3C_2 - A_2C_3) \quad [26]$$

$$s_3w_1/s_1w_3 = (-A_1C_2 + A_2C_1)/(-A_2C_3 + A_3C_2) \quad [27]$$

Finally if t_1, t_2, and t_3 are weights which together cause a vibration increment C_t at C and no vibration increment elsewhere, then corresponding to Equations [21], [22], [26], and [27]

$$t_2w_1/t_1w_2 = (A_3B_1 - A_1B_3)/(A_2B_3 - A_3B_2) \quad [28]$$

$$t_3w_1/t_1w_3 = (A_1B_2 - A_2B_1)/(A_2B_3 - A_3B_2) \quad [29]$$

In the two-bearing case replacing W_I and W_{II} with r_1 and r_2, respectively, in Equations [12] and [13] and replacing A_0 with A_r and B_0 with zero in a way similar to that used in the three-bearing case

$$r_1/w_1 = A_rB_2/\Delta_2, \qquad r_2/w_2 = -A_rB_1/\Delta_2 \quad [30]\ [31]$$

and

$$r_2w_1/r_1w_2 = -B_1/B_2 \quad [32]$$

Again replacing W_I and W_{II} with s_1 and s_2,

$$s_2w_1/s_1w_2 = -A_1/A_2 \quad [33]$$

Equations [21], [22], [26], [27], [28], and [29] for the three-bearing case and [32] and [33] for the two-bearing case give the proper relation of the trial weights in a group with respect to each other so that with one group of weights the vibration at only one location is affected.

After the vector relations in the trial groups are determined, all other data are discarded and trial runs are made with each group of weights considered as a single trial weight. When the r group is inserted the measured increment in vibration at A will correspond to a vector which will be denoted by A_r'. If the correction required of the r group is $-R$ then R is related to r the same as A_0 is related to A_r' so that

$$R/r = A_0/A_r' \quad [34]$$

Similarly, if $-S$ and $-T$ are corrections required of the s and t groups then the following corresponding relations obtain:

$$S/s = B_0/B_s' \quad \text{and} \quad T/t = C_0/C_t' \quad [35]\ [36]$$

Equations [34] and [35] also apply to the two-bearing case.

Summary of Group Method

1 Runs are made in the usual way, i.e., vibration readings are taken on the machine initially and then after each trial weight is inserted.

2 For bearing pedestal A, the relation between trial weights in a group such that only pedestal A will be affected is calculated. This calculation is then repeated for each of the other bearing pedestals. (Equations [21] to [29], inclusive, for the three-bearing case and Equations [32] and [33] for the two-bearing case.)

3 New runs are made with each trial-weight group and the vibration measured. These runs should show the intended result of vibration increments at only one pedestal for a given group of trial weights.

4 On the basis of these measurements the required magnitude and phase of each group are calculated from Equations [34], [35], and [36].

Discussion of Group Method

Since the assumptions on which unbalance-correction determinations depend only crudely apply, the consideration of errors is an important aspect of the problem of balancing. In most cases the greater the complication of the calculation process used between experimental measurements, the greater will be the error introduced by the calculation process. It appears, therefore, important to keep the number of mathematical operations between tests a minimum. Note that in Equations [21], [22], [26], [27], [28], and [29] for the three-bearing case the number of products plus the number of quotients is twenty four and that the number of additions plus the number of subtractions is nine, whereas in the original equations, [9], [10], and [11], the number of products plus the number of quotients is fifty one and the number of additions plus the number of subtractions is twenty. Equations [32] and [33] for the two-bearing case contain only two quotients, whereas in the original equations, [12] and [13], the number of products plus the number of quotients is eight and the number of additions plus the number of subtractions is three.

The calculation of the trial-weight-group relations does not complete the calculation or the determination as the calculation to which it is compared does, but the calculation of the trial-weight-group relations is all of the calculation required between experimental measurements so that the comparison is fair as a rough indication of the errors introduced by the calculation.

In many cases due to errors, even if they are minimized, the balance is not satisfactory after the first set of computed corrections is introduced. In fact balancing is usually a series of reductions in vibration rather than the complete elimination of vibration with a single determination. If the trial-weight-group relations are found to experimentally check in the group trial, then the second series of corrections can be made very simply by the reapplication of Equations [34], [35], and [36] where A_0 is taken to correspond to the remnant vibration at A after the first set of corrections has been applied. B_0 and C_0 (in the three-bearing case only) are also taken to correspond to the remnant vibration after the addition of the first set of corrections. This is a simpler process than the repeat calculation with the conventional method. It has the further advantage that each group of correction weights has a simple physical significance so that it is easier to follow the correction process. In fact the group method reduces multiplane balancing to the equivalent of single-plane balancing.

It may be that there is enough similarity between many machines so that the group-relation calculations are necessary only in a few typical cases. At any rate, a given group can be checked experimentally by an operator who does not have sufficient training to make the calculation in the first place and even if the group does not show exactly the property of affecting one location at a time it will likely come closer to it than the use of a single correction weight at a time. If it does, the process of successively using the group will converge where the single-weight process may not.

PART II—BALANCING WITH SELECTED CORRECTION LOCATIONS

For some rotors, especially those of irregular shapes, the balancing problem is different from the usual rotor-balancing problem in that the corrections are conveniently made in completely selected locations instead of in the ordinary way in which transverse planes are selected, but the angular position of the correction is left for determination.

For the case of a two-bearing rotor, if completely selected locations for corrections are to be used, four will be required if either addition or subtraction of weight can be made at each

location. In the general two-bearing case the four correction locations may be in four different transverse planes and four different axial planes. In the case of multicrank internal-combustion-engine crankshafts the corrections are usually made in four different transverse planes but they can ordinarily be confined to two axial planes.

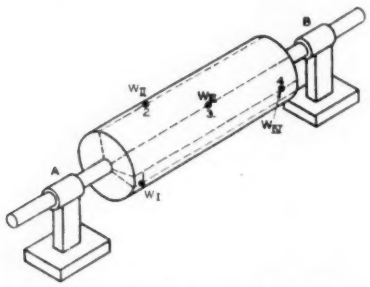


FIG. 2 DIAGRAM OF UNBALANCE COMPONENTS

To consider the general case first, refer to Fig. 2. The unbalance components at the general locations 1, 2, 3, and 4 are W_{I}, W_{II}, W_{III}, and W_{IV}, respectively.[6] The two bearing pedestals or other locations at which vibration is to be measured are A and B.

With a notation similar to that in part I,

$$\left.\begin{aligned} A_1(W_{\text{I}}/w_1) &= A_{\text{I}}; \quad A_2(W_{\text{II}}/w_2) = A_{\text{II}} \\ A_3(W_{\text{III}}/w_3) &= A_{\text{III}}; \quad A_4(W_{\text{IV}}/w_4) = A_{\text{IV}} \end{aligned}\right\} \quad [37]$$

$$A_0 = A_{\text{I}} + A_{\text{II}} + A_{\text{III}} + A_{\text{IV}} \quad [38]$$

$$\left.\begin{aligned} A_0 &= A_1\frac{W_{\text{I}}}{w_1} + A_2\frac{W_{\text{II}}}{w_2} + A_3\frac{W_{\text{III}}}{w_3} + A_4\frac{W_{\text{IV}}}{w_4} \\ B_0 &= B_1\frac{W_{\text{I}}}{w_1} + B_2\frac{W_{\text{II}}}{w_2} + B_3\frac{W_{\text{III}}}{w_3} + B_4\frac{W_{\text{IV}}}{w_4} \end{aligned}\right\} \quad [39]$$

where A_1 is the vector corresponding to the vibration at A due to the trial weight w_1 at location 1. A_{I} is the vector corresponding to the vibration at A due to the unbalance W_{I} only, which is completely known in position but unknown in amount. W_{I}/w_1 is a vector of zero angle and is the result sought. A_0 is the vector corresponding to the initially measured vibration, that is, with W_{I}, W_{II}, W_{III}, and W_{IV} present only. All the A's and B's are known in Equation [39]. Splitting the A and B vectors into their zero-degree and ninety-degree components and equating the zero-degree terms and the ninety-degree terms in Equations [39], four equations result

$$\left.\begin{aligned} \mathbf{A}_0 &= \mathbf{A}_1\frac{W_{\text{I}}}{w_1} + \mathbf{A}_2\frac{W_{\text{II}}}{w_2} + \mathbf{A}_3\frac{W_{\text{III}}}{w_3} + \mathbf{A}_4\frac{W_{\text{IV}}}{w_4} \\ \mathbf{a}_0 &= \mathbf{a}_1\frac{W_{\text{I}}}{w_1} + \mathbf{a}_2\frac{W_{\text{II}}}{w_2} + \mathbf{a}_3\frac{W_{\text{III}}}{w_3} + \mathbf{a}_4\frac{W_{\text{IV}}}{w_4} \\ \mathbf{B}_0 &= \mathbf{B}_1\frac{W_{\text{I}}}{w_1} + \mathbf{B}_2\frac{W_{\text{II}}}{w_2} + \mathbf{B}_3\frac{W_{\text{III}}}{w_3} + \mathbf{B}_4\frac{W_{\text{IV}}}{w_4} \\ \mathbf{b}_0 &= \mathbf{b}_1\frac{W_{\text{I}}}{w_1} + \mathbf{b}_2\frac{W_{\text{II}}}{w_2} + \mathbf{b}_3\frac{W_{\text{III}}}{w_3} + \mathbf{b}_4\frac{W_{\text{IV}}}{w_4} \end{aligned}\right\} \quad [40]$$

in which $\mathbf{A}$ and $\mathbf{B}$ indicate zero-degree terms and $\mathbf{a}$ and $\mathbf{b}$ indicate ninety-degree terms.

Solving Equations [40]

$$\frac{W_{\text{I}}}{w_1} = \frac{\begin{vmatrix} \mathbf{A}_0 & \mathbf{A}_2 & \mathbf{A}_3 & \mathbf{A}_4 \\ \mathbf{a}_0 & \mathbf{a}_2 & \mathbf{a}_3 & \mathbf{a}_4 \\ \mathbf{B}_0 & \mathbf{B}_2 & \mathbf{B}_3 & \mathbf{B}_4 \\ \mathbf{b}_0 & \mathbf{b}_2 & \mathbf{b}_3 & \mathbf{b}_4 \end{vmatrix}}{\Delta_3} \quad [41]$$

$$\frac{W_{\text{II}}}{w_2} = \frac{\begin{vmatrix} \mathbf{A}_1 & \mathbf{A}_0 & \mathbf{A}_3 & \mathbf{A}_4 \\ \mathbf{a}_1 & \mathbf{a}_0 & \mathbf{a}_3 & \mathbf{a}_4 \\ \mathbf{B}_1 & \mathbf{B}_0 & \mathbf{B}_3 & \mathbf{B}_4 \\ \mathbf{b}_1 & \mathbf{b}_0 & \mathbf{b}_3 & \mathbf{b}_4 \end{vmatrix}}{\Delta_3} \quad [42]$$

$$\frac{W_{\text{III}}}{w_3} = \frac{\begin{vmatrix} \mathbf{A}_1 & \mathbf{A}_2 & \mathbf{A}_0 & \mathbf{A}_3 \\ \mathbf{a}_1 & \mathbf{a}_2 & \mathbf{a}_0 & \mathbf{a}_3 \\ \mathbf{B}_1 & \mathbf{B}_2 & \mathbf{B}_0 & \mathbf{B}_3 \\ \mathbf{b}_1 & \mathbf{b}_2 & \mathbf{b}_0 & \mathbf{b}_3 \end{vmatrix}}{\Delta_3} \quad [43]$$

$$\frac{W_{\text{IV}}}{w_4} = \frac{\begin{vmatrix} \mathbf{A}_1 & \mathbf{A}_2 & \mathbf{A}_3 & \mathbf{A}_0 \\ \mathbf{a}_1 & \mathbf{a}_2 & \mathbf{a}_3 & \mathbf{a}_0 \\ \mathbf{B}_1 & \mathbf{B}_2 & \mathbf{B}_3 & \mathbf{B}_0 \\ \mathbf{b}_1 & \mathbf{b}_2 & \mathbf{b}_3 & \mathbf{b}_0 \end{vmatrix}}{\Delta_3} \quad [44]$$

$$\Delta_3 = \begin{vmatrix} \mathbf{A}_1 & \mathbf{A}_2 & \mathbf{A}_3 & \mathbf{A}_4 \\ \mathbf{a}_1 & \mathbf{a}_2 & \mathbf{a}_3 & \mathbf{a}_4 \\ \mathbf{B}_1 & \mathbf{B}_2 & \mathbf{B}_3 & \mathbf{B}_4 \\ \mathbf{b}_1 & \mathbf{b}_2 & \mathbf{b}_3 & \mathbf{b}_4 \end{vmatrix} \quad [45]$$

CRANKSHAFT BALANCING

The problem of balancing multicrank-engine crankshafts is a special case of the more general problem of balancing with selected correction locations just treated.[6] As illustrated in Fig. 3, two of the corrections can usually be placed in one axial plane or phase and two in another. This simplification makes the mathematical solution less involved than that of the more general case

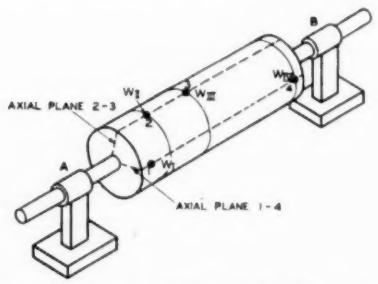


FIG. 3 DIAGRAM FOR CRANKSHAFT BALANCING

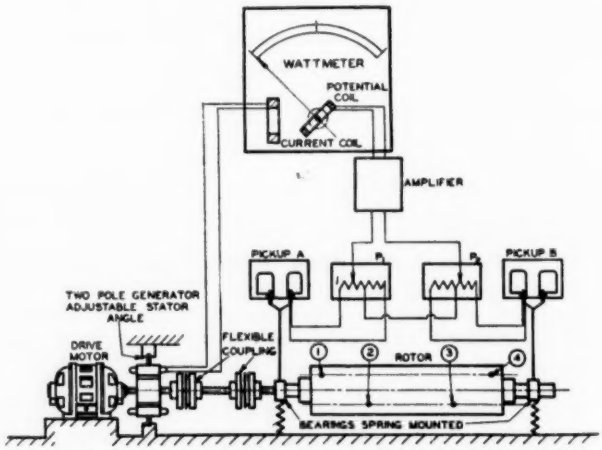


FIG. 4 ARRANGEMENT FOR READING THE UNBALANCE AT A GIVEN LOCATION

[6] If the rotor has different moments of inertia about the two principal inertia axes perpendicular to the rotational axis the analysis may not apply.

as given in Equations [41] to [45], inclusive, but still the mathematical solution is not practical since large numbers of shafts must be balanced. It appears, therefore, necessary to turn to machine means of obtaining the solution.

Description of Machine for Balancing. Fig. 4 shows a schematic diagram and Fig. 5 is a photograph of a commercial crankshaft-balancing machine. The circuit of Fig. 4 will indicate the unbalance component at one location at a time so that in the actual machine four such circuits are required with each setup for one unbalance location. Fig. 3 may be referred to to clarify the unbalance-correction position.

The rotor is mounted in very flexibly supported bearings so that the component of motion at one bearing due to one unbalance component will be either in phase or out of phase with the unbalance component. The motions of the bearings are used to actuate vibration pickups A and B so that voltages proportional to and in phase with the respective motions are produced. These voltages are divided by the potentiometers P_1 and P_2 and placed in series in such proportion that the resulting voltage is independent of any unbalance placed in one transverse plane. The settings of the potentiometers which will give a resultant change in voltage of zero for a weight added in the transverse plane of 4 will be called "setting 4," the settings of the potentiometers which will give zero resultant change in voltage for a weight added in the transverse plane of 3 will be called "setting 3," and so on. The resultant voltage in any case is amplified and placed on the potential coil of a wattmeter. The current coil of the wattmeter is energized by means of the two-pole generator turning with the rotor shaft.

The reading of the wattmeter, of course, depends on the phase relation between the current in the potential coil and the current in the current coil of the instrument. This phase relation may be adjusted by the manual rotation of the generator stator. In particular, the reading due to any unbalance in the axial plane

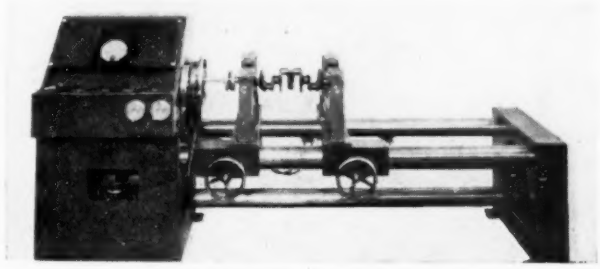


Fig. 5 Machine for Balancing Crankshafts

of 1 and 4 may be made zero by adjustment of the generator stator's angular position. If this adjustment is made and used in combination with a potentiometer setting 3, the deflection of the wattmeter must be due to unbalance at 2 only, so that calibration of this deflection is all that is necessary to make it a direct indicator of the unbalance correction required at 2 irrespective of what correction may be required at 1, 3, and 4.

Similarly if it is desired to read the unbalance at 1, the generator stator is adjusted so that unbalances in the plane of 2 and 3 do not register on the wattmeter and setting 4 on the potentiometers is used. The adjustments to read 3 and 4 correspond.

In short the indication of the unbalance component at a given location is obtained by using two elements. First, there is a measuring arrangement which can be adjusted so as not to indicate unbalances at a given phase.[7] This device eliminates from indication two of the four component unbalances. Second, a voltage-combining arrangement is used which eliminates the indication of a third unbalance component.[8] Thus the indication depends on only one of the four component unbalances.

It is of interest to note that the machine method corresponds to the mathematical method of solving a series of equations. The variables which are not sought are first eliminated and then the variable which is sought is evaluated.

PART III—CENTER-OF-PERCUSSION BALANCING[9]

For a rotor carried in a flexibly mounted frame, as in Fig. 6 or for the similar case of Fig. 7, a vibration reading taken at the proper location can be directly interpreted in terms of the unbalance corrections required in the planes 1 and 2.

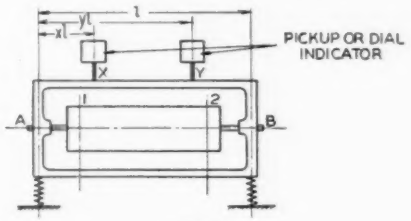


Fig. 6

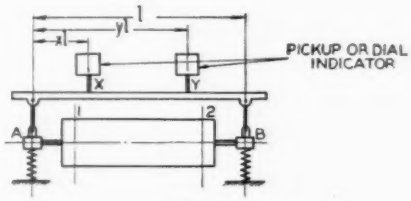


Fig. 7

Figs. 6 and 7 Diagrams for Center-of-Percussion Balancing

Let the motion perpendicular to the rotor axis at any point X along the frame (either Fig. 6 or Fig. 7) be X which is

$$X = A(1 - x) + Bx \quad [46]$$

where x is the ratio of the distance between X and A, to the distance l between A and B. Similarly the motion Y at another point will be

$$Y = A(1 - y) + By \quad [47]$$

where y is the ratio of the distance between Y and A, to the distance l.

The object is to choose x and y such that the vibration X_0 at X will be a measure of the unbalance component W_I at 1 and the vibration Y_0 at Y will be a measure of the unbalance W_{II} at 2.

Considering X and Y as stations similar to A and B and using the same subscript notation as in part I, an equation similar to [12] may be written

$$\frac{W_1}{w_1} = \frac{X_0Y_2 - Y_0X_2}{X_1Y_2 - Y_1X_2} = \frac{Y_0(X_2/Y_2) - X_0}{Y_1(X_2/Y_2) - X_1} \quad [48]$$

If X_2 could be made zero by proper selection of x then Equation [48] would become

$$W_1/w_1 = X_0/X_1 \quad [49]$$

[7] The wattmeter is not the only phase-sensitive device. Another example is a commutator running off the shaft of the rotor to be balanced.

[8] The voltage-combining arrangement may be replaced by a mechanical system.

[9] This method has been in use since about August 15, 1936, in Westinghouse service shops. It is so closely related to center-of-percussion theory that is doubtless very old, although no prior use has come to the author's attention.

Similarly, if Y_1 could be made zero

$$W_{II}/w_2 = Y_0/Y_2 \quad [50]$$

which are the results sought. Thus, the proper x and y might be obtained from Equations [46] and [57] by setting X_2 and Y_1 equal to zero thus:

$$0 = A_2(1 - x) + B_2x \quad [51]$$

$$0 = A_1(1 - y) + B_1y \quad [52]$$

or

$$x = A_2/(A_2 - B_2) \quad [53]$$

and

$$y = A_1/(A_1 - B_1) \quad [54]$$

Note that unless A_2 and B_2 are vectors of the same angle, and A_1 and B_1 are vectors of the same angle, x and y are not simple ratios as defined but vectors, so that, X_2 and Y_1 cannot be made zero and Equations [49] and [50] do not apply. However, with the condition of a flexible support imposed A_2 and B_2 will be vectors of the same angle provided the flexibility is high and the damping small since the rotor assembly would then be effectively free in space. Likewise, A_1 and B_1 will have the same angle and the method is applicable.

The point X as defined by Equation [53] is, of course, the node or center of oscillation for a force (of unbalance) at 2, and 2 is the center of percussion with respect to the point X. Y and 1 are similarly related. For simplicity w_1 and w_2 may be placed at zero angle, then X_0 and Y_0 will have the same angle as W_I and W_{II}, respectively.

Equations [49] and [50] may be used for the calibration of the readings.

Sometimes one of the calculated pickup locations may fall outside of the convenient range. This difficulty can be avoided either by (*a*) selecting a different correction combination, such as using an unbalance couple instead of single unbalance or by (*b*) using the two pickups in series instead of individually. In the latter case one pickup location may be selected and the other determined in a way similar to that described.

Tables of Moments and Deflections for a Rectangular Plate Fixed on All Edges and Carrying a Uniformly Distributed Load

By THOMAS H. EVANS,[1] CHARLOTTESVILLE, VA.

The problem of the clamped rectangular plate with a uniform load was solved by Hencky in 1913 and independently by Boobnoff in 1914. The former calculated values only for the case of a square plate. In 1937, Wojtaszak, using Hencky's method, calculated several other cases. In this paper the maximum moments and deflections for all side ratios between 1.0 and 2.0, by intervals of 0.1, are determined following Hencky's procedure. Growing out of the necessity for solving eight groups of simultaneous equations in twelve unknowns a short cut on the usual trial and error process of solution was developed.

THE PROBLEM of a rectangular plate clamped on four sides and carrying a uniformly distributed load is of great importance to the profession, but is one for which few numerical values for moment and deflection have been determined. For this reason the writer has calculated values of the maximum edge moments, center moments, and center deflections for various ratios b/a of the sides. Values omitted from the table, or otherwise noted, have been worked out by other investigators. Since the results at infinity are so little different from those at $b/a = 2.0$ the latter is taken as the upper limit.

The solution is obtained by combining the known solution for a simply supported plate under a uniform load with that of a plate bent by moments distributed along the pairs of opposite sides. It will be shown that by combining these two solutions we can finally satisfy all the requirements at the clamped edges.

The notation used in this paper is as follows:

- a = length of plate in x direction
- b = length of plate in y direction
- h = thickness of plate
- q = intensity of uniform load
- $D = \dfrac{Eh^3}{12(1-\nu^2)}$ = flexural rigidity of plate
- E = Young's modulus for the material
- ν = Poisson's ratio (taken equal to 0.3 in this paper)
- M = bending moment per unit length of plate
- w = deflection at any point
- $r = b/a$
- E_m and F_m are constants occurring in the expressions for moment and deflection
- m = 1, 3, 5, 7, ... ∞
- i = 1, 3, 5, 7, ... ∞
- n = 1, 3, 5, 7, ... ∞
- $\alpha = \dfrac{m\pi b}{2a}$, or deflection coefficient
- $\beta = \dfrac{m\pi a}{2b}$, or moment coefficient
- $K = -\dfrac{4qa^2}{\pi^3}$

[1] Department of Engineering, University of Virginia.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

For convenience in handling the relationships obtained, axes as shown in Fig. 1 are used, where the x- and y-axes are parallel to the short and long sides, respectively. In the case of a plate

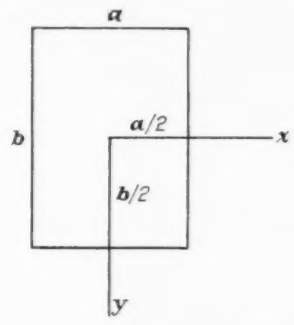


FIG. 1

with the two long edges simply supported and a moment M_y applied on the other two, the deflection is given by

$$w = -\frac{a^2}{2\pi^2 D}\sum_{m=1,3,5}^{\infty}\frac{E_m \cos\dfrac{m\pi x}{a}(-1)^{\frac{m-1}{2}}}{m^2\cosh\alpha_m}\left[\frac{m\pi y}{a}\sinh\frac{m\pi y}{a}-\alpha_m\tanh\alpha_m\cosh\frac{m\pi y}{a}\right] \quad \ldots\ldots [1]$$

It is assumed that $(M_y)_{y=\pm b/2} = \sum_{m=1,3}^{\infty} E_m \cos\dfrac{m\pi x}{a}(-1)^{\frac{m-1}{2}}$

The first derivatives of Equation [1] give

$$\left(\frac{\partial w}{\partial y}\right)_{y=b/2} = -\frac{a}{2\pi D}\sum_{m=1,3}^{\infty}\frac{E_m\cos\dfrac{m\pi x}{a}(-1)^{\frac{m-1}{2}}}{m}\left(\tanh\alpha_m+\frac{\alpha_m}{\cosh^2\alpha_m}\right) \quad \ldots\ldots [2]$$

and

$$\left(\frac{\partial w}{\partial x}\right)_{x=a/2} = \frac{a}{2\pi D}\sum_{m=1,3}^{\infty}\frac{E_m}{m\cosh\alpha_m}\left(\frac{m\pi y}{a}\sinh\frac{m\pi y}{a}-\alpha_m\tanh\alpha_m\cosh\frac{m\pi y}{a}\right)$$

$$= -\frac{1}{4D}\sum_{m=1,3}^{\infty}\frac{E_m}{\cosh^2\alpha_m}\left(b\sinh\alpha_m\cosh\frac{m\pi y}{a}-2y\cosh\alpha_m\sinh\frac{m\pi y}{a}\right) \quad \ldots\ldots [3]$$

To aid in making the last summation the quantity in parentheses is developed into a series, which gives

$$\left(\frac{\partial w}{\partial x}\right)_{x = a/2} = -\frac{4b^2}{\pi^2 aD} \sum_{m = 1, 3}^{\infty} \frac{E_m}{m^3} \sum_{i = 1, 3}^{\infty} \frac{i \cos \frac{i\pi y}{b} (-1)^{\frac{i-1}{2}}}{\left(\frac{i^2}{m^2} + \frac{b^2}{a^2}\right)^2} \quad \ldots\ldots [3a]$$

Now if moments are applied to the long edges, i.e., applied parallel to the x-axis, relationships can be worked out by similar reasoning. If $(M_x)_{x = \pm a/2}$

$$= \sum_{m = 1, 3}^{\infty} F_m \cos \frac{m\pi y}{b} (-1)^{\frac{m-1}{2}}$$

then

$$w = -\frac{b^2}{2\pi^2 D} \sum_{m = 1, 3}^{\infty} \frac{F_m \cos \frac{m\pi y}{b} (-1)^{\frac{m-1}{2}}}{m^2 \cosh \beta_m} \left(\frac{m\pi x}{b} \sinh \frac{m\pi x}{b} - \beta_m \tanh \beta_m \cosh \frac{m\pi x}{b}\right) \quad \ldots\ldots [4]$$

From which

$$\left(\frac{\partial w}{\partial x}\right)_{x = a/2} = -\frac{b}{2\pi D} \sum_{m = 1, 3}^{\infty} \frac{F_m \cos \frac{m\pi y}{b} (-1)^{\frac{m-1}{2}}}{m} \left(\tanh \beta_m + \frac{\beta_m}{\cosh^2 \beta_m}\right) \quad \ldots\ldots [5]$$

and

$$\left(\frac{\partial w}{\partial y}\right)_{y = b/2} = -\frac{4a^2}{\pi^2 bD} \sum_{m = 1, 3}^{\infty} \frac{F_m}{m^3} \sum_{i = 1, 3}^{\infty} \frac{i \cos \frac{i\pi x}{a} (-1)^{\frac{i-1}{2}}}{\left(\frac{i^2}{m^2} + \frac{a^2}{b^2}\right)^2} \quad \ldots\ldots [6]$$

The known solution for a uniformly loaded plate with simply supported edges is

$$w = \frac{4qa^4}{D\pi^5} \sum_{m = 1, 3}^{\infty} \left[\frac{1}{m^5} - \frac{(2 + \alpha_m \tanh \alpha_m)}{2m^5 \cosh \alpha_m} \cosh \frac{m\pi y}{a} + \frac{\frac{m\pi y}{a} \sinh \frac{m\pi y}{a}}{2m^5 \cosh \alpha_m}\right] \cos \frac{m\pi x}{a} (-1)^{\frac{m-1}{2}} \quad \ldots [7]$$

from which

$$\left(\frac{\partial w}{\partial y}\right)_{y = b/2} = \frac{2qa^3}{D\pi^4} \sum_{m = 1, 3}^{\infty} \frac{\cos \frac{m\pi x}{a} (-1)^{\frac{m-1}{2}}}{m^4} \left(\frac{\alpha_m}{\cosh^2 \alpha_m} - \tanh \alpha_m\right) \quad \ldots\ldots [8]$$

$$\left(\frac{\partial w}{\partial x}\right)_{x = a/2} = \frac{2qb^3}{D\pi^4} \sum_{m = 1, 3}^{\infty} \frac{(-1)^{\frac{m-1}{2}}}{m^4} \cos \frac{m\pi y}{b} \left(\frac{\beta_m}{\cosh^2 \beta_m} - \tanh \beta_m\right) \quad \ldots\ldots [9]$$

At $x = \pm a/2$ the summation of all $\partial w/\partial x$ terms should equal zero if the condition to be fulfilled is a fully clamped edge. Thus the sum of Equations [3a], [5], and [9] should equal zero. For like terms in these series the equality still holds; so if $m = i = n$, there results

$$\frac{F_n}{n}\left(\tanh \beta_n + \frac{\beta_n}{\cosh^2 \beta_n}\right) + \frac{8bn}{\pi a} \sum_{m = 1, 3}^{\infty} \frac{E_m}{m^3 \left(\frac{n^2}{m^2} + \frac{b^2}{a^2}\right)^2} = \frac{4qb^2}{\pi^3 n^4}\left(\frac{\beta_n}{\cosh^2 \beta_n} - \tanh \beta_n\right) \quad \ldots\ldots [10]$$

which provides one relationship for determining the constants $E_1, E_3, E_5, \ldots E_n$, and $F_1, F_3, F_5, \ldots F_n$. In the same manner another is found from the condition that $\partial w/\partial y$ must equal zero at the edges $y = \pm b/2$. That is

$$\frac{E_n}{n}\left(\tanh \alpha_n + \frac{\alpha_n}{\cosh^2 \alpha_n}\right) + \frac{8na}{\pi b} \sum_{m = 1, 3}^{\infty} \frac{F_m}{m^3 \left(\frac{n^2}{m^2} + \frac{a^2}{b^2}\right)^2} = \frac{4qa^2}{\pi^3 n^4}\left(\frac{\alpha_n}{\cosh^2 \alpha_n} - \tanh \alpha_n\right) \quad \ldots\ldots [11]$$

From Equations [10] and [11] as many terms and equations as thought necessary can be set up simultaneously. The form in which these equations for calculating the E and F coefficients were used is given in Equations [12] and [13]. For the values of r from 1.1 to 1.7 ten equations in ten unknowns were used; and for the remainder, twelve.

$$\left.\begin{aligned} &(n = 1)\ \frac{B_1 F_1 \pi}{1r} + \frac{8E_1}{(1 + r^2)^2} + \frac{24E_3}{(1 + 9r^2)^2} + \ldots \\ &\qquad + \frac{88E_{11}}{(1 + 121r^2)^2} = \frac{KB_1'\pi r}{1^4} \quad (a) \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ &(n = 11)\ \frac{B_{11}F_{11}\pi}{11r} + \frac{88E_1}{(121 + r^2)^2} + \ldots \\ &\qquad + \frac{968\,E_{11}}{(121 + 121r^2)^2} = \frac{KB_{11}'\pi r}{(11)^4} \quad (f) \end{aligned}\right\} \quad \ldots\ldots [12]$$

$$\left.\begin{aligned} &(n = 1)\ \frac{8F_1}{(1 + r^2)^2} + \frac{24F_3}{(9 + r^2)^2} + \ldots \\ &\qquad + \frac{88F_{11}}{(121 + r^2)^2} + \frac{E_1 A_1 \pi}{1r^3} = \frac{KA_1'\pi}{1^4 r^3} \quad (a) \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ &(n = 11)\ \frac{88F_1}{(1 + 121r^2)^2} + \ldots + \frac{968F_{11}}{(121 + 121r^2)^2} \\ &\qquad + \frac{E_{11}A_{11}\pi}{11r^3} = \frac{KA_{11}'\pi}{(11)^4 r^3} \quad (f) \end{aligned}\right\} \quad \ldots\ldots [13]$$

where $A_n = \tanh \alpha_n + \frac{\alpha_n}{\cosh^2 \alpha_n}$; $B_n = \tanh \beta_n + \frac{\beta_n}{\cosh^2 \beta_n}$

$$A_n' = \tanh \alpha_n - \frac{\alpha_n}{\cosh^2 \alpha_n}; \quad B_n' = \tanh \beta_n - \frac{\beta_n}{\cosh^2 \beta_n}$$

These equations could, of course, be solved by the trial-and-error method outlined by Wojtaszak.[2] Since, however, each

[2] "The Calculation of Maximum Deflection, Moment, and Shear for Uniformly Loaded Rectangular Plate With Clamped Edges," by I. A. Wojtaszak, JOURNAL OF APPLIED MECHANICS, Trans. A.S.M.E., vol. 59, December, 1937, p. A-173.

different ratio of b/a required the solution of one such group (making eight groups in all) a method involving many less trials was finally developed. As pointed out by Wojtaszak each group took at least six trials to obtain accuracy to the required number of significant figures; where in the following outlined method only two or three at the most are needed. As a means toward this end the relationships between E_1 and E_3, E_5, E_7, and E_9 were worked out, as well as those between F_1 and F_3, F_5, F_7, and F_9, for the three cases at hand, namely, $r = 1.0$, 1.5, and 2.0. With these values a set of smooth curves was plotted as shown in Fig. 2, where the abscissa represent the b/a ratios from 1.0 to 2.0. From these curves one can obtain reliable values of any E in terms of E_1, or of any F in terms of F_1, for other b/a ratios. Thus it is seen that by not ignoring *any* factors in making the first approximation for such a group of simultaneous equations the first values obtained for the E's and F's are close to their true values. In most cases it was found that accuracy to four significant figures could be had after only two trials.

As an example of the use of this particular technic the detailed steps for the case of $b/a = 1.9$ are included. It should also be noted that in the final form of the equations used the coefficients for any row of F's in Equation [13] are the same as for the corresponding column of E's in Equation [12]. After reducing all the coefficients to decimal fractions in these two equations for the case $r = b/a = 1.9$ the values shown in the Example were obtained. Then by using the curves of Fig. 2, Equations [12a] and [13a] could be reduced to two simultaneous equations in two unknowns, F_1 and E_1. After obtaining these values F_3 was found from Equation [12b], again using all the factors containing E. The relationships were solved in order until all the F's had been calculated; and these in turn were substituted into Equation [13] until all the E's had been determined. With these first approximate values of the E's a second trial was made on the F's by again feeding in to Equation [12]. The process was

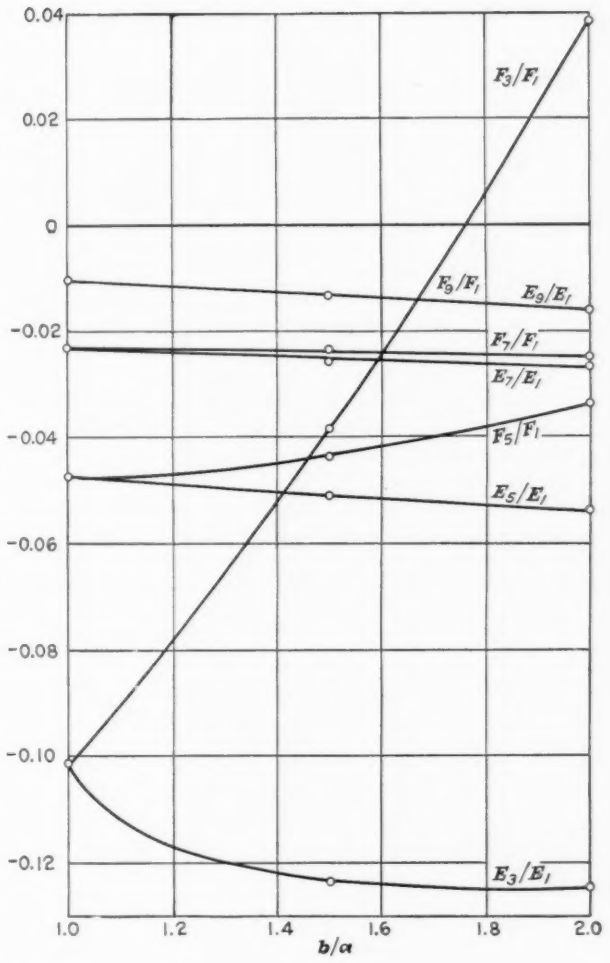


FIG. 2 (RIGHT) APPROXIMATE RELATIONS TO AID IN SOLVING THE EQUATIONS CONTAINING E AND F

EXAMPLE, $r = 1.9$

$$
\left.\begin{aligned}
1.85951\,F_1 + 0.37643\,E_1 + 0.02140\,E_3 + 0.00480\,E_5 + 0.00177\,E_7 + 0.00084\,E_9 + 0.00046\,E_{11} &= 1.38977\,K \quad (a)\\
0.58128\,F_3 + 0.15093\,E_1 + 0.04183\,E_3 + 0.01218\,E_5 + 0.00486\,E_7 + 0.00238\,E_9 + 0.00133\,E_{11} &= 0.06761\,K \quad (b)\\
0.33192\,F_5 + 0.04887\,E_1 + 0.03631\,E_3 + 0.01506\,E_5 + 0.00687\,E_7 + 0.00357\,E_9 + 0.00206\,E_{11} &= 0.00951\,K \quad (c)\\
0.23626\,F_7 + 0.02023\,E_1 + 0.02530\,E_3 + 0.01444\,E_5 + 0.00768\,E_7 + 0.00432\,E_9 + 0.00261\,E_{11} &= 0.00249\,K \quad (d)\\
0.18372\,F_9 + 0.01006\,E_1 + 0.01677\,E_3 + 0.01228\,E_5 + 0.00758\,E_7 + 0.00465\,E_9 + 0.00295\,E_{11} &= 0.00091\,K \quad (e)\\
0.15032\,F_{11} + 0.00567\,E_1 + 0.01121\,E_3 + 0.00986\,E_5 + 0.00683\,E_7 + 0.00463\,E_9 + 0.00311\,E_{11} &= 0.00037\,K \quad (f)
\end{aligned}\right\} \quad [12]
$$

$$
\left.\begin{aligned}
0.37643\,F_1 + 0.15093\,F_3 + 0.04887\,F_5 + 0.02023\,F_7 + 0.01006\,F_9 + 0.00567\,F_{11} + 0.46960\,E_1 &= 0.44178\,K \quad (a)\\
0.02140\,F_1 + 0.04183\,F_3 + 0.03631\,F_5 + 0.02530\,F_7 + 0.01677\,F_9 + 0.01121\,F_{11} + 0.15268\,E_3 &= 0.00566\,K \quad (b)\\
0.00480\,F_1 + 0.01218\,F_3 + 0.01506\,F_5 + 0.01444\,F_7 + 0.01228\,F_9 + 0.00986\,F_{11} + 0.09161\,E_5 &= 0.00073\,K \quad (c)\\
0.00177\,F_1 + 0.00486\,F_3 + 0.00687\,F_5 + 0.00768\,F_7 + 0.00758\,F_9 + 0.00683\,F_{11} + 0.06543\,E_7 &= 0.00019\,K \quad (d)\\
0.00084\,F_1 + 0.00238\,F_3 + 0.00357\,F_5 + 0.00432\,F_7 + 0.00465\,F_9 + 0.00463\,F_{11} + 0.05089\,E_9 &= 0.00007\,K \quad (e)\\
0.00046\,F_1 + 0.00133\,F_3 + 0.00206\,F_5 + 0.00261\,F_7 + 0.00295\,F_9 + 0.00311\,F_{11} + 0.04164\,E_{11} &= 0.00003\,K \quad (f)
\end{aligned}\right\} \quad [13]
$$

First Approximation.

From curves, Fig. 2.

$E_3 = -0.125\,E_1$; $E_5 = -0.052\,E_1$; $E_7 = -0.027\,E_1$; $E_9 = -0.016\,E_1$; $E_{11} = -0.008\,E_1$; hence $1.85951\,F_1 + 0.37344\,E_1 = 1.38977\,K$ ..... [12a]

$F_3 = +0.021\,F_1$; $F_5 = -0.035\,F_1$; $F_7 = -0.025\,F_1$; $F_9 = -0.016\,F_1$; $F_{11} = -0.008\,F_1$; hence $0.37717\,F_1 + 0.46960\,E_1 = 0.44178\,K$ ..... [13a]

Solving: $F_1 = 0.66586\,K$ $\quad E_1 = 0.40595\,K$

$$
\left.\begin{aligned}
F_3 &= (0.06761 - 0.05882)K/0.58128 = +0.00879K/0.58128 = +0.01512\,K \quad (b)\\
F_5 &= (0.00951 - 0.01757)K/0.33192 = -0.00806\,K/0.33192 = -0.02428\,K \quad (c)\\
F_7 &= (0.00249 - 0.00650)K/0.23626 = -0.00401\,K/0.23626 = -0.01697\,K \quad (d)\\
F_9 &= (0.00091 - 0.00285)K/0.18372 = -0.00194\,K/0.18372 = -0.01056\,K \quad (e)\\
F_{11} &= (0.00037 - 0.00142)K/0.15032 = -0.00105\,K/0.15032 = -0.00699\,K \quad (f)
\end{aligned}\right\} \quad [12]
$$

$$
\left.\begin{aligned}
E_1 &= (0.44178 - 0.25065 - 0.00228 + 0.00119 + 0.00034 + 0.00011 + 0.00004)K/0.46960 = +0.40573\,K \quad (a)\\
E_3 &= (0.00566 - 0.01425 - 0.00063 + 0.00088 + 0.00043 + 0.00018 + 0.00008)K/0.15268 = -0.05010\,K \quad (b)\\
E_5 &= (0.00073 - 0.00320 - 0.00018 + 0.00037 + 0.00025 + 0.00013 + 0.00007)K/0.09161 = -0.01998\,K \quad (c)\\
E_7 &= (0.00019 - 0.00118 - 0.00007 + 0.00017 + 0.00013 + 0.00008 + 0.00005)K/0.06543 = -0.00993\,K \quad (d)\\
E_9 &= (0.00007 - 0.00056 - 0.00004 + 0.00009 + 0.00006 + 0.00005 + 0.00003)K/0.05089 = -0.00590\,K \quad (e)\\
E_{11} &= (0.00003 - 0.00031 - 0.00002 + 0.00005 + 0.00004 + 0.00003 + 0.00002)K/0.04164 = -0.00384\,K \quad (f)
\end{aligned}\right\} \quad [13]
$$

Second Approximation.

$$
\left.\begin{aligned}
F_1 &= (1.38977 - 0.15273 + 0.00107 + 0.00010 + 0.00002 + 0.00000 + 0.00000)K/1.85951 = 0.66589\,K \quad (a)\\
F_3 &= (0.06761 - 0.06124 + 0.00210 + 0.00024 + 0.00005 + 0.00001 + 0.00001)K/0.58128 = 0.01510\,K \quad (b)\\
F_5 &= (0.00951 - 0.01983 + 0.00182 + 0.00030 + 0.00007 + 0.00002 + 0.00001)K/0.33192 = -0.02440\,K \quad (c)\\
F_7 &= (0.00249 - 0.00821 + 0.00127 + 0.00029 + 0.00008 + 0.00003 + 0.00001)K/0.23626 = -0.01710\,K \quad (d)\\
F_9 &= (0.00091 - 0.00408 + 0.00084 + 0.00025 + 0.00008 + 0.00003 + 0.00001)K/0.18372 = -0.01067\,K \quad (e)\\
F_{11} &= (0.00037 - 0.00230 + 0.00056 + 0.00020 + 0.00007 + 0.00003 + 0.00001)K/0.15032 = -0.00705\,K \quad (f)
\end{aligned}\right\} \quad [12]
$$

$$
\left.\begin{aligned}
E_1 &= (0.44178 - 0.25066 - 0.00228 + 0.00119 + 0.00035 + 0.00011 + 0.00004)K/0.46960 = 0.40573\,K \quad (a)\\
E_3 &= (0.00566 - 0.01425 - 0.00063 + 0.00089 + 0.00043 + 0.00018 + 0.00008)K/0.15268 = -0.05004\,K \quad (b)\\
E_5 &= (0.00073 - 0.00320 - 0.00018 + 0.00037 + 0.00025 + 0.00013 + 0.00007)K/0.09161 = -0.01998\,K \quad (c)\\
E_7 &= (0.00019 - 0.00118 - 0.00007 + 0.00017 + 0.00013 + 0.00008 + 0.00005)K/0.06543 = -0.00993\,K \quad (d)\\
E_9 &= (0.00007 - 0.00056 - 0.00004 + 0.00009 + 0.00006 + 0.00005 + 0.00003)K/0.05089 = -0.00590\,K \quad (e)\\
E_{11} &= (0.00003 - 0.00031 - 0.00002 + 0.00005 + 0.00004 + 0.00003 + 0.00002)K/0.04164 = -0.00384\,K \quad (f)
\end{aligned}\right\} \quad [13]
$$

TABLE 1 MOMENT AND DEFLECTION VALUES FOR A CLAMPED RECTANGULAR PLATE SUBJECTED TO A UNIFORM LOAD

b/a	β^*	β_1	β_2	β_3	α
1.0	—0.0513[a]	....	....	....	0.0138[a]
1.1	—0.0581	—0.0538	+0.0264	+0.0231	0.0164
1.2	—0.0639	—0.0554	0.0299	0.0228	0.0188
1.3	—0.0687	—0.0563	0.0327	0.0222	0.0209
1.4	—0.0726	—0.0568	0.0349	0.0212	0.0226
1.5	—0.0757[a]	....	....	....	0.0240[a]
1.6	—0.0780	—0.0571	0.0381	0.0193	0.0251
1.7	—0.0799	—0.0571	0.0392	0.0182	0.0260
1.8	—0.0812	—0.0571	0.0401	0.0174	0.0267
1.9	—0.0822	—0.0571	0.0407	0.0165	0.0272
2.0	—0.0829[a]	....	....	....	0.0277[a]

* Values of β obtained by estimating F beyond F_{11}.

At $x = a/2$, $y = 0$; $M_x = \beta q a^2$
At $x = 0$, $y = b/2$; $M_y = \beta_1 q a^2$
At $x = 0$, $y = 0$; $M_x = \beta_2 q a^2$
At $x = 0$, $y = 0$; $M_y = \beta_3 q a^2$
At $x = 0$, $y = 0$; $w = \alpha \frac{q a^4}{E h^3}$

[a] Values obtained by other investigators.

stopped when it became apparent that the variation in the fifth significant figure would be less than five, since accuracy to four significant figures was all that was desired.

After obtaining these constants the values of the moments were calculated by substituting in the expressions for M_x and M_y. Total maximum deflection was calculated by adding algebraically the results from Eq. [1], [4], and [7] when $x = 0$ and $y = 0$. The maximum moment values occur at the middle of the sides, and that on the longer side is the greater of the two. In order to get precise values of the moments it would be necessary to take n out to 21 in some cases (22 equations in 22 unknowns). Another possibility was to take it out to some value, such as 11 used in this study, and from the trend of the E's to E_{11}, and of the F's to F_{11}, to get as many E and F values as desired by extrapolation. These trends were definite in all the cases studied by the writer and it is expected that the moment values given in Table 1 are not in error by as much as 0.1 per cent, which is well within any practical limits. The maximum error involved by using only terms up to $n = 11$ was less than 0.8 per cent.

The writer wishes to express gratitude to Prof. Stephen Timoshenko for suggesting this work. The relationships used to start solution of this problem were developed during lectures given by Professor Timoshenko at the University of Michigan. It is understood that this general method of approach was first suggested by H. Hencky in 1913.

Friction Coefficients for the Compressible Flow of Steam

By JOSEPH H. KEENAN,[1] CAMBRIDGE, MASS.

The effect of friction on the flow of compressible fluids in pipes of uniform cross-sectional area was investigated analytically by Grashof[2] and Zeuner[3] who arrived at a relationship between velocity and friction coefficient for ideal gases. Stodola[4] showed that "the curves of Fanno permit a general graphical treatment for any law of friction." Frössel[5] presented the first extensive measurements of friction coefficients for air flow through a smooth tube with velocities above and below the velocity of sound. His measured coefficients for compressible flow were in excellent agreement at corresponding Reynolds' numbers with coefficients measured for incompressible flow. Egli[6] expressed in dimensionless form the equations for flow of an ideal gas through a channel, and used them to deduce friction coefficients from measurements of flow through channels varying in width from 0.0025 in. to 0.010 in.

In this paper are presented some experiments by Lang[7] on friction in a commercial 1-in. pipe for steam flow at velocities up to the velocity of sound, and some auxiliary experiments by Benson[8] on a similar pipe with liquid water. In an Appendix the analysis of flow of a compressible fluid in a passage of uniform cross section is restated without introducing difficult concepts, such as "work of friction" or "heat of friction." It is elaborated to include cases in which heat flow is appreciable.

Steam Test Apparatus

THE ARRANGEMENT of the test apparatus is shown in Fig. 1. Fluid was supplied to a standard 1-in. steel pipe whose inside diameter averaged 1.05 in. This pipe comprised three long pieces which were joined by couplings, and a short length at each end, which was joined to the longer pieces by unions. Care was exercised to make each pair of lengths concentric where they joined. The entrance to the test pipe was a rounded entrance brass nozzle, details of which are shown in Fig. 2. The nozzle was fed in turn by a 4-in. pipe. The pressure measurements, which largely determined the friction coefficients of Table III and Fig. 9, were made at taps labeled p_1 and p_4 in Fig. 1. The first of these is in the nozzle wall, shown in Fig. 2, and the second near the downstream end of the test pipe. Both holes were carefully drilled and cleaned. Auxiliary pressure measurements were made at taps in the couplings joining the longer pieces of the test pipe. These taps were less perfect than the others, despite the fact that the pipe lengths were butted together in the coupling. Nevertheless, nothing in the test results indicates that these measurements were unsatisfactory. Pressure taps were provided also in the 4-in. supply pipe, p_0, where the velocity was always negligible, and in the 4-in. discharge pipe leading to the condenser p_5.

From each pressure tap a short length of horizontal tubing led to the middle of a large reservoir. A copper tube connected the bottom of the reservoir with one side of a mercury manometer. Before tests were begun the connecting tubing and reservoir were filled with water from below so as to sweep all air out of the system. In computing the measured pressures, allowance was made for the unbalanced water column.

The initial pressure p_0 was measured with a calibrated Bourdon-tube gage. All other pressures were measured by means of differential mercury manometers or, if the pressure changes exceeded the range of the manometers, by calibrated Bourdon-tube gages. Locations of the gages and manometers are shown in Fig. 1.

A copper-constantan thermocouple, shown as T_0 in Fig. 1, with its hot junction in the middle of the 4-in. approach pipe and a few inches before the nozzle, measured the initial temperature of the steam.

In order to determine heat loss from the bare pipe, its surface temperature was measured by means of copper-constantan thermocouples at the positions indicated as T_a, T_b, T_c, and T_d in Fig. 1. The thermocouple wires were soldered into grooves in the pipe wall.

All thermocouples used in these tests were calibrated against the temperature of equilibrium of liquid and vapor phases of water up to 400 F.

When the pipe was lagged to reduce heat loss, the surface temperature of the lagging was measured roughly by means of mercury-in-glass thermometers. However, heat loss from the lagged pipe proved to be so small that it could be ignored in calculating the friction coefficient.

The steam for the tests came from the Institute power plant, where it was generated at 200 lb per sq in. abs and 100 F superheat, through a long tunnel before entering the laboratory. After reaching the laboratory it passed through a well-insulated pipe which was trapped at frequent intervals to remove condensate. Upon reaching the nozzle of the test pipe it retained a superheat anywhere from 1 to 60 F, the amount depending upon the steam demand for other purposes. Despite the low initial

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[1] Associate Professor of Mechanical Engineering, Massachusetts Institute of Technology. Mem. A.S.M.E.

[2] "Theoretische Maschinenlehre," by F. Grashof, L. Voss, Leipzig, 1875, pp. 593–597.

[3] "Technical Thermodynamics," by G. Zeuner, Leipzig, 1900, pp. 264–273.

[4] "Dampf- und Gas Turbinen," by A. Stodola, Julius Springer, Berlin, 1922, 5th edition, p. 50. Also: "Steam and Gas Turbines," by A. Stodola, McGraw-Hill Book Company, New York, N. Y., vol. 1, 1927, p. 61.

[5] "Strömung in glatten, geraden Rohren mit Über- und Unterschallgeschwindigkeit," by W. Frössel, *Forschung auf dem Gebiete Ingenieurwesens*, vol. 7, 1936, pp. 75–84.

[6] "The Leakage of Gases Through Narrow Channels," by A. Egli, *Journal of Applied Mechanics*, Trans. A.S.M.E., 1937, p. A-63.

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Discussion of this paper was accepted until January 10, 1939, for publication at a later date.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[7] Master's Thesis, by Hans J. Lang, Massachusetts Institute of Technology, Cambridge, Mass., 1936.

[8] Master's thesis, by John G. Benson, Massachusetts Institute of Technology, Cambridge, Mass., 1937.

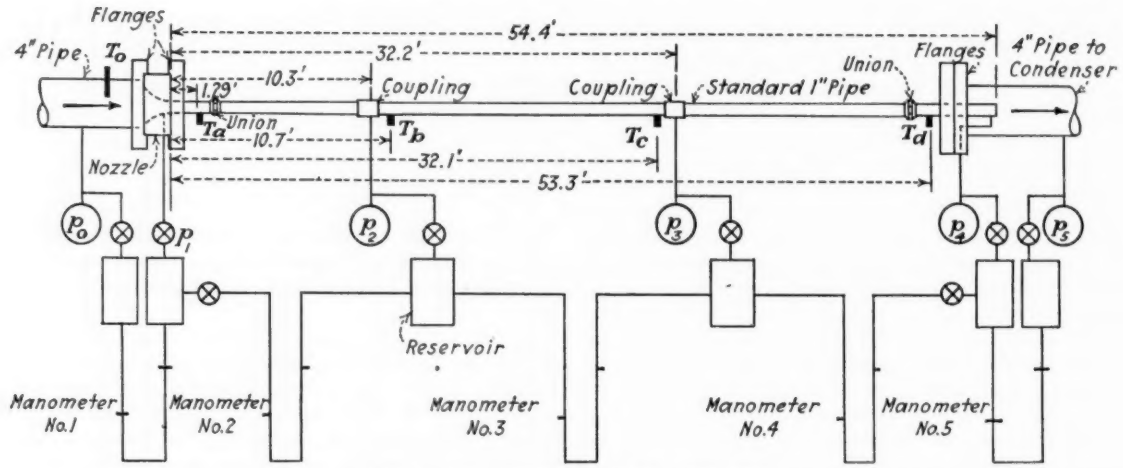


FIG. 1 ARRANGEMENT OF TEST APPARATUS

(The circles labeled P_0, p_2, etc., denote Bourdon-tube gages; T_0 denotes a thermocouple projecting into the stream; and T_a, T_b, T_c, T_d denote thermocouples soldered into grooves in the pipe wall.)

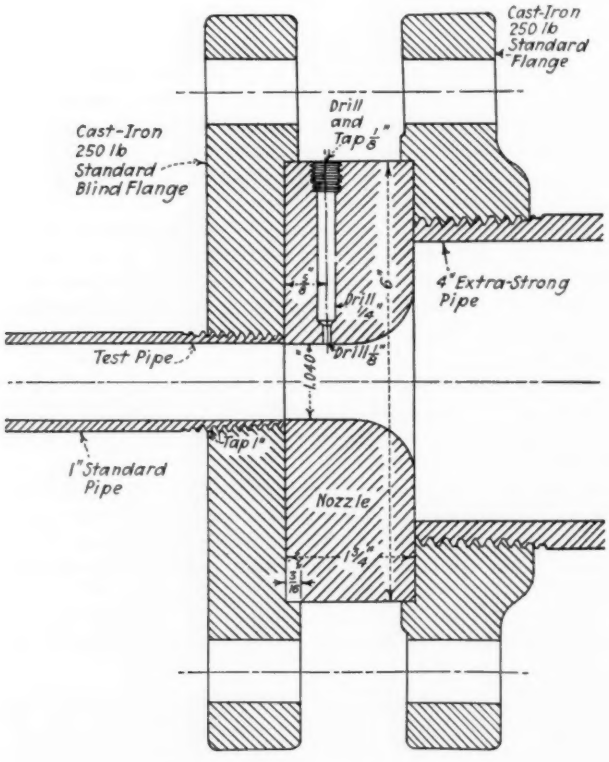


FIG. 2 INLET TO THE TEST PIPE

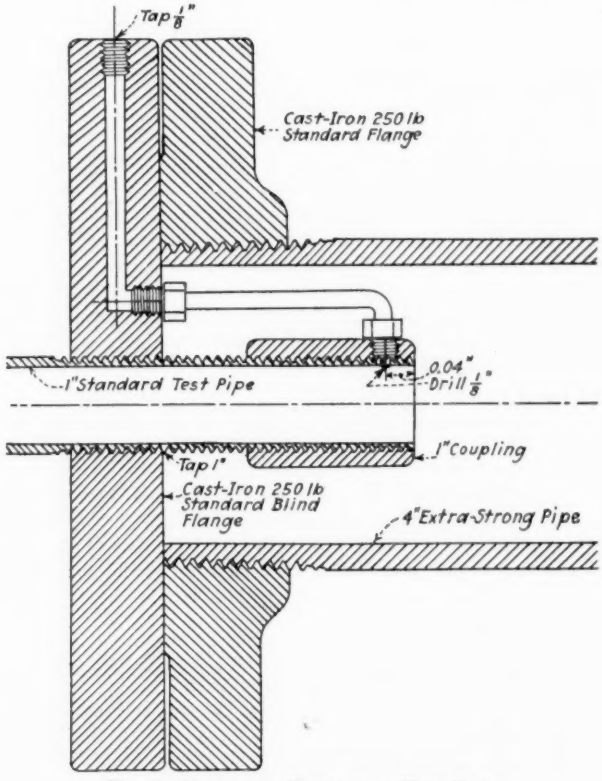


FIG. 3 DISCHARGE END OF THE TEST PIPE

(The final pressure tap is shown.)

superheat, which often was less than 20 F, the steam was usually slightly superheated where it left the nozzle and entered the test pipe.

Steam leaving the test pipe passed to a 4-in. pipe, as shown in Fig. 3, and then to a surface condenser. The condensate was weighed on calibrated platform scales.

WATER TEST APPARATUS

Essentially the same apparatus was used for the water tests as for the steam tests, but a few modifications were necessary. Since the water tests were made about eight months after the steam tests, the inside wall of the test pipe had become rough from corrosion. (In fact, some tests made on the corroded pipe gave friction coefficients about 15 per cent higher than those obtained from a new piece of pipe.) It was necessary, therefore, to install a new length of pipe in place of the middle length of the original test pipe. Because the pressure taps in the couplings were uncertain others were carefully made near each end of the new length. Only measurements made with these new taps were used in computing the friction coefficient.

Water from the city main was fed through a 2-in. pipe into a short length of 4-in. pipe preceding the nozzle. Straightening vanes in the 4-in. pipe insured against swirls which might have been introduced in some sharp bends in the preceding 2-in. pipe.

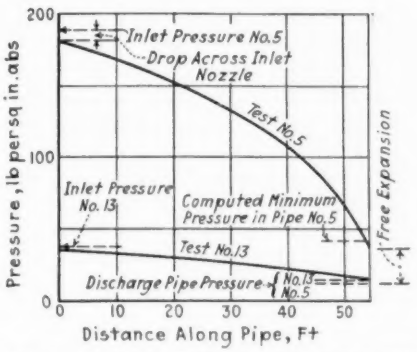


FIG. 4 PRESSURE DISTRIBUTION ALONG THE TEST PIPE

(The computed minimum pressure indicated for test 5 corresponds to single-phase expansion.)

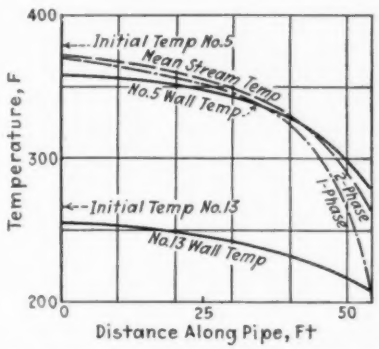


FIG. 5 TEMPERATURE DISTRIBUTION ALONG THE TEST PIPE

(The broken lines show computed values of the mean stream temperature.)

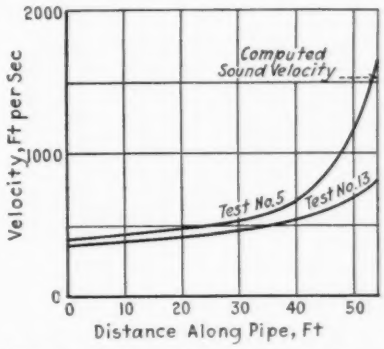


FIG. 6 VELOCITY DISTRIBUTION ALONG THE TEST PIPE

(The computed sound velocity indicated for test 5 corresponds to single-phase expansion.)

METHOD OF TESTING

In each test a control valve was set upstream from the apparatus to give the desired initial pressure or rate of flow. When a steady state was reached, readings were taken of all pressures and temperatures at definite intervals of time. The test was continued until adequate precision was attained in the measurement of flow.

RESULTS

The test results are presented in Tables 1, 2, and 3. The course of the pressure, temperature, and velocity along the length of the test pipe is shown in Figs. 4, 5, and 6, respectively, for two representative steam-flow tests, namely, tests 5 and 13. Test 5 is typical of seventeen tests in which the pressure in the exhaust pipe was below the pressure at the end of the test pipe and the rate of flow was therefore the maximum corresponding to the initial steam conditions. Test 13 is typical of the remaining six tests in which the steam left the test pipe at essentially the exhaust-pipe pressure, and the flow was therefore less than the maximum. The free expansion of the stream upon leaving the test pipe is indicated in Fig. 4 for test 5.

In Fig. 5 the outside wall temperature is compared with the mean steam temperature computed in two different ways: First, by assuming equilibrium between vapor and large water drops, for which the steam temperature would be the saturation temperature corresponding to the pressure; and second, by assuming that no liquid phase forms and that the vapor is in a supersaturated state. Evidence is presented in the Appendix that favors the second assumption. On either assumption the pipe-wall temperature is higher than the mean stream temperature for the last quarter of the length of the pipe. Since conduction of heat along the pipe must be small, it follows that near the pipe wall the fluid which has been slowed down by friction is at a temperature far above that of the main fluid stream.

It is shown in the Appendix that on the basis of certain assumptions it is possible to compute the lowest pressure and the highest velocity that can be attained in a pipe of uniform cross-sectional area, provided that the inlet velocity is less than the velocity of sound. The computed value of the lowest pressure (the pressure of maximum entropy) is shown for test 5 in Fig. 4, and the corresponding computed value of the highest velocity (the sound velocity) is shown in Fig. 6. It is evident that the expansion proceeds slightly beyond the computed limit. Frössel[8] found for air flow a deviation from the computed limit which is of the same sign and approximately the same magnitude as is indicated here for steam. He attributes the discrepancy to an assumption that underlies the computed value; namely, that the velocity is uniform across the pipe cross section.

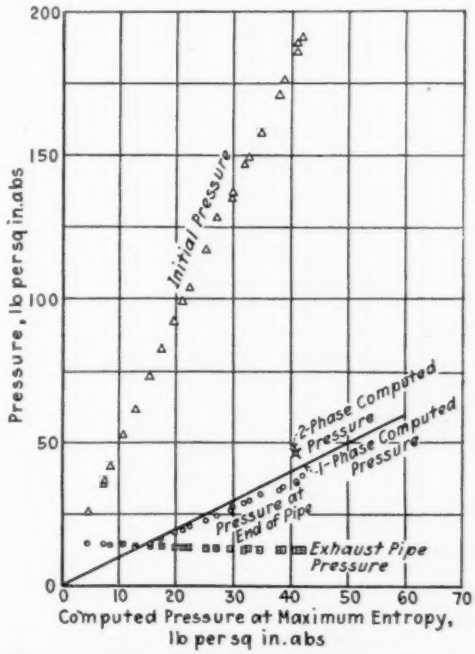


FIG. 7 RELATIONSHIP BETWEEN INLET PRESSURE, EXHAUST PRESSURE, AND PRESSURE AT THE END OF THE TEST PIPE

The relationship between the initial pressure, the final pressure in the test pipe and the exhaust-pipe pressure is shown in Fig. 7. The computed value of the minimum test-pipe pressure for single-phase expansion is shown by the solid line, which by virtue of the choice of abscissa is merely a line of equality between abscissa and ordinate. For comparison is shown a single computed value of the minimum pressure corresponding to two-phase equilibrium. The significance of this value is discussed in the Appendix.

In Fig. 8 the relationship between final test pipe pressure and flow is shown in dimensionless coordinates after the method employed by Frössel.[8] For comparison a part of the curve for frictionless flow is also shown. If continued it would reach $G/G_{crit} = 1$ at $p/p_0 = 0.542$.

Values of the friction coefficient, $R/{}^1/_{2\rho}V^2$, are given in Tables 1 and 3 and plotted against the mean Reynolds number[9] in Fig. 9.

For each of the tests with lagged pipe, Table 3 gives three friction coefficients, one for the first 32.2 ft of pipe length, one for the last 22.2 ft and one for the entire length. These values

[9] The method of computing the mean Reynolds number is described in the Appendix.

TABLE 1 RESULTS OF WATER TESTS

Test no.	Flow, lb per sec	Temperature, F	Pressure drop, lb per sq ft	Friction coefficient	Reynolds number $\times 10^{-4}$
1	8.168	39.5	2404.0	0.00522	11.350
2	7.017	39.5	1899.0	0.00572	9.740
3	5.760	39.9	1330.0	0.00595	8.000
4	1.640	40.4	128.2	0.00708	2.278
5	3.058	39.2	386.5	0.00638	4.250
6	4.586	39.0	904.0	0.00600	6.370
7	0.809	39.3	35.0	0.00870	1.124
8	1.225	39.5	81.9	0.00810	1.701

NOTE: Cross-sectional area of pipe = 0.006 sq ft, and pipe length = 21.5 ft.

cent. Four tests are shown for which the pipe was lagged with a 1 in. thickness of 85 per cent magnesia. The effect of heat loss on the coefficients computed for these tests was less than $^1/_2$ per cent and was therefore ignored. It can be seen from Fig. 9 that the agreement between the coefficients corresponding to lagged pipe and unlagged pipe is excellent.

The solid line in Fig. 9 is the relationship between friction coefficient and Reynolds' number proposed by Nickuradse.[10] It represents with high precision the experimental data on smooth-

TABLE 2 RESULTS OF STEAM TESTS

Test No.	Flow, lb per min	Initial pressure, lb per sq in.	Initial temperature, F	Exhaust-pipe pressure lb per sq in.	Test-pipe pressures, lb per sq in. 1	2	3	4	Test-pipe surface temp. F *a*	*b*	*c*	*d*	Surface temp. of lagging, F	Room temperature, F
1	36.04	117.0	355	13.0	112.0	102.0	80.0	23.0	334	332	321	285	...	85
2	28.67	92.0	327	13.5	88.0	80.0	63.0	18.6	310	308	298	263	...	88
3	32.31	104.0	339	13.5	100.0	91.0	72.0	20.9	320	318	308	272	...	82
4	41.56	135.0	358	12.8	129.0	120.0	95.0	26.8	337	336	325	285	...	84
5	58.75	189.0	379	12.8	181.0	168.0	129.0	36.7	359	356	341	283	...	85
6	58.07	186.0	387	12.8	178.0	165.0	126.0	37.0	364	362	351	307	...	85
7	43.03	137.0	353	12.8	131.0	122.0	96.0	27.7	334	332	322	281	...	87
8	46.88	149.0	359	12.7	143.0	133.0	102.0	30.0	340	338	328	283	...	88
9	54.22	176.0	372	12.7	169.0	153.0	122.0	34.9	352	350	339	295	...	85
10	53.50	171.0	389	12.8	164.0	152.0	118.0	33.8	366	364	353	308	...	74
11	49.55	158.0	372	12.7	151.0	141.0	109.0	32.1	351	349	338	296	...	75
12	45.40	147.0	368	12.7	141.0	130.0	101.0	29.2	345	343	333	292	...	73
13	11.20	36.9	266	15.0	35.5	32.5	25.8	14.8	255	253	241	211	...	79
14	6.70	25.5	272	15.0	24.7	23.1	19.5	14.9	251	247	232	209	...	80
15	21.97	73.2	351	14.5	70.4	63.0	48.0	14.7	327	326	314	280	...	93
16	24.50	82.6	365	13.8	79.5	72.0	55.0	16.1	340	339	330	294	...	89
17	18.28	61.6	351	14.5	59.3	54.0	43.0	13.5	329	327	315	280	...	87
18	12.75	41.5	271	14.7	39.8	36.4	28.6	14.4	261	260	248	213	...	78
19	15.96	52.6	285	15.0	50.5	46.1	36.0	14.5	280	279	268	235	...	77
20	59.60	191.0	379	13.1	183.0	170.0	130.0	38.6	366	362	348	314	142	87
21	38.70	128.0	347	13.0	123.0	111.0	85.0	24.7	332	331	324	296	136	86
22	30.43	99.3	329	13.6	95.3	88.0	68.0	19.5	318	317	310	285	131	82
23	10.50	35.7	285	14.6	34.4	31.6	25.1	14.4	270	272	269	256	120	79

NOTE: Cross-sectional area of the pipe = 0.006 sq ft.

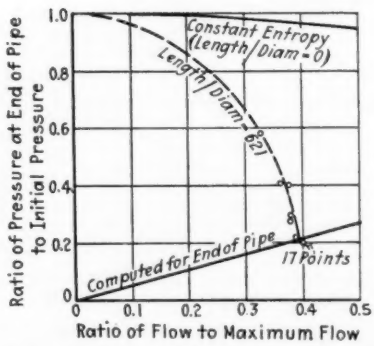


FIG. 8 RELATIONSHIP BETWEEN FLOW AND PRESSURE AT THE END OF THE TEST PIPE

(The computed minimum pressure at the end of the pipe for single-phase expansion is shown by the line sloping upward to the right. It meets the constant entropy curve at a pressure ratio of 0.542 and at a flow ratio of 1.0.)

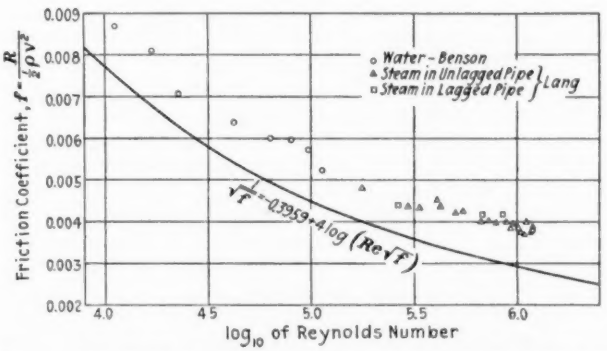


FIG. 9 MEASURED VALUES OF THE FRICTION COEFFICIENT FOR A 1-IN. COMMERCIAL PIPE

(The curve represents experimental data on smooth pipes.)

indicate that the friction coefficient is essentially independent of the velocity, provided that little change occurs in the Reynolds number. In Fig. 9 only the mean coefficient of friction for the entire pipe length is shown.

In the same figure are shown the friction coefficients obtained by Benson for water flow. The range of water pressure available did not permit tests with incompressible flow at as high Reynolds numbers as those for compressible flow. Nevertheless, the apparent continuity between the two sets of data offers evidence that the friction coefficient is substantially the same function of Reynolds' number for incompressible flow and for compressible flow up to the sound velocity.

Most of the steam tests were made with bare pipe, and for these an allowance was made for heat loss in the calculation of the friction coefficient. If no allowance had been made, the computed values of the coefficient would have been lower by 1 to 10 per

walled pipes up to a Reynolds number of 3.5×10^6. The experimental points of Fig. 9 for commercial 1-in. pipe show higher coefficients than are found for smooth pipes, but the character of the relationship between coefficient and Reynolds' number is much the same for the two cases. Because of the wide spread of values that have been published, no comparison is shown here with earlier data on commercial pipe. The present data are, however, consistent with various curves that have been selected on occasion to represent previously published values.[11]

The agreement between coefficients for compressible flow and incompressible flow shown in Fig. 9 is in accord with the results

[10] "Gesetzmässigkeiten der turbulenten Strömung in glatten Rohren," by J. Nickuradse, Forschungsheft No. 356, supplement of *Forschung auf dem Gebiete des Ingenieurwesens*, Berlin, September-October, 1932, p. 32.

[11] For example, see "Hydraulics," by G. E. Russel, Henry Holt & Co., New York, N. Y., 1934, fourth edition, wherein curve C on page 433 is in excellent agreement with the points of Fig. 9 of this paper.

TABLE 3 RESULTS OF STEAM TESTS

Test no.	Flow per unit area, lb per sq ft per sec	Distance from the nozzle throat, ft	Test-pipe pressure, lb per sq in.	Heat loss, Btu per lb	Specific volume, cu ft per lb	Velocity, fps	Friction coefficient		Mean Reynolds number $\times 10^{-6}$
							Steps[a]	Over-all	
		0	112.0	0.81	4.035	404	...		
1	100.11	32.2	80.0	5.01	5.533	554	...	0.00396	0.793
		54.4	23.0	7.51	16.380	1640	...		
		0	88.0	0.90	4.976	396	...		
2	79.64	32.2	63.0	5.41	6.800	542	...	0.00400	0.669
		54.4	18.6	8.04	19.770	1574	...		
		0	100.0	0.86	4.434	398	...		
3	89.75	32.2	72.0	5.22	6.029	541	...	0.00402	0.733
		54.4	20.9	7.67	17.790	1596	...		
		0	129.0	0.71	3.482	402	...		
4	115.44	32.2	95.0	4.47	4.642	536	...	0.00397	0.896
		54.4	26.8	6.57	14.060	1623	...		
		0	181.0	0.54	2.494	407	...		
5	163.19	32.2	129.0	3.50	3.439	561	...	0.00386	1.196
		54.4	36.7	5.17	10.250	1673	...		
		0	178.0	0.56	2.578	416	...		
6	161.31	32.2	126.0	3.70	3.576	576	...	0.00374	1.170
		54.4	37.0	5.47	10.370	1673	...		
		0	131.0	0.67	3.399	406	...		
7	119.53	32.2	96.0	4.24	4.553	544	...	0.00383	0.933
		54.4	27.7	6.25	13.500	1614	...		
		0	143.0	0.62	3.119	406	...		
8	130.22	32.2	102.0	4.02	4.290	558	...	0.00383	1.006
		54.4	30.0	5.92	12.470	1624	...		
		0	169.0	0.57	2.658	400	...		
9	150.61	32.2	122.0	3.72	3.619	545	...	0.00400	1.119
		54.4	34.9	5.49	10.820	1629	...		
		0	164.0	0.63	2.831	421	...		
10	148.61	32.2	118.0	4.08	3.862	574	...	0.00369	1.082
		54.4	33.8	6.02	11.430	1698	...		
		0	151.0	0.65	3.006	413	...		
11	137.64	32.2	109.0	4.07	4.088	562	...	0.00374	1.033
		54.4	32.1	6.00	11.880	1635	...		
		0	141.0	0.70	3.219	406	...		
12	126.11	32.2	101.0	4.30	4.409	556	...	0.00392	0.958
		54.4	29.2	6.32	13.020	1642	...		
		0	35.5	1.80	11.630	362	...		
13	31.11	32.2	25.8	9.48	15.510	482	...	0.00435	0.296
		54.4	14.8	13.80	25.910	806	...		
		0	24.7	3.05	17.030	317	...		
14	18.61	32.2	19.5	15.02	20.700	385	...	0.00480	0.178
		54.4	14.9	21.75	26.340	490	...		
		0	70.4	1.25	6.539	399	...		
15	61.03	32.2	48.0	7.91	9.319	569	...	0.00421	0.506
		54.4	14.7	11.65	26.140	1595	...		
		0	79.5	1.20	5.884	400	...		
16	68.06	32.2	55.0	7.73	8.274	563	...	0.00425	0.549
		54.4	16.1	11.40	24.100	1640	...		
		0	59.3	1.55	7.782	395	...		
17	50.78	32.2	43.0	9.61	10.410	528	...	0.00435	0.423
		54.4	13.5	14.15	29.040	1475	...		
		0	39.8	1.65	10.410	369	...		
18	35.42	32.2	28.6	8.77	14.060	498	...	0.00433	0.334
		54.4	14.4	12.75	26.430	936	...		
		0	50.5	1.44	8.325	369	...		
19	44.33	32.2	36.0	8.16	11.350	503	...	0.00452	0.408
		54.4	14.5	11.94	25.910	1148	...		
		0	183.0	0.54	2.464	408	...		
20	165.56	32.2	130.0	0.54	3.436	568	0.00378 ... 0.00381	0.00379	1.200
		54.4	38.6	0.54	9.950	1647	...		
		0	123.0	0.73	3.600	387	...		
21	107.50	32.2	85.0	0.73	5.160	554	0.00438 ... 0.00398	0.00422	0.851
		54.4	24.7	0.73	15.240	1638	...		
		0	95.3	0.89	4.587	388	...		
22	84.53	32.2	68.0	0.89	6.375	539	0.00405 ... 0.00426	0.00413	0.697
		54.4	19.5	0.89	19.110	1616	...		
		0	34.4	2.12	12.380	361	...		
23	29.17	32.2	25.1	2.12	16.850	491	0.00439 ... 0.00436	0.00438	0.265
		54.4	14.4	2.12	28.550	833	...		

[a] A systematic error is included in the friction coefficient by steps in those cases involving appreciable heat loss. The error is a result of an approximation to the Q-v relation which gives a satisfactory over-all coefficient, though it is inexact for the separate steps. Therefore, step values are given only for those tests in which heat loss was negligible.

of air-flow measurements made by Frössel.[5] In fact, the solid line of Fig. 9 is an excellent representation of a mean curve through all of Frössel's measurements both above and below the sound velocity.

Summary

It is found from tests with steam and liquid water on commercial 1-in. pipe that friction coefficients for compressible-fluid flow with velocities up to the velocity of sound and for incompressible-fluid flow are substantially the same function of the Reynolds number. The friction coefficient for commercial 1-in. pipe is higher for the same Reynolds number than that of smooth pipe by about 0.001.

The assumption that no moisture forms in a pipe if the steam

is initially slightly superheated is a satisfactory approximation to the truth. It seems to fit the observed phenomena better than the assumption of stable two-phase states.

The assumption that enthalpy is a linear function of the pressure-volume product facilitates analysis and computation, and is a satisfactory approximation to the true relationship for superheated and supersaturated vapor at low pressures.

The experimental results presented here on steam flow in commerical pipe are in qualitative agreement with those of Frössel[5] on air flow in smooth tubes.

Appendix

The Dynamic Equation for Flow in a Pipe of Constant Cross-Sectional Area

Consider an element of fluid which is bounded by two parallel planes transverse to the direction of flow and distant dx apart.

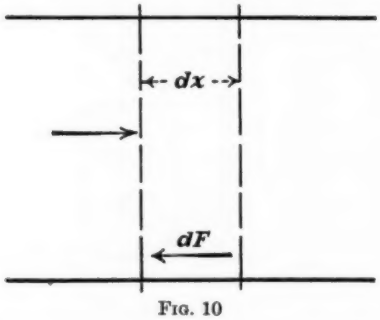


Fig. 10

The forces acting on this element may be classified as normal forces corresponding to hydrostatic pressures and shearing forces corresponding to wall friction. It can be shown that Newton's Second Law becomes for steady flow

$$-adp - dF = (w/g)dV, \qquad [1]$$

where a denotes the cross-sectional area of the passage, dp the increase in hydrostatic pressure of the fluid across distance dx, dF the wall friction force applied to the stream between the two planes, w the mass rate of flow, g the acceleration given to unit mass by unit force and dV the increase in the mean velocity of the stream across dx.

The wall-friction force dF may be expressed in terms of a friction coefficient which is commonly defined by the relation

$$f = R/{}^1\!/_2\rho V^2$$

where f denotes the friction coefficient, R the friction force per unit of wall surface, and ρ a mass density of the fluid which is otherwise $1/vg$. Then we may write

$$dF = R\pi D\,dx = fV^2\pi D dx/2vg$$

where D is the pipe diameter and dx is an element of length along the pipe. Substituting this expression for dF in Equation [1] dividing through by av and rearranging, we get

$$\frac{dp}{v} + \frac{G}{gv}dV + \frac{f}{2}\left(\frac{V}{v}\right)^2\frac{\pi D}{ag}dx = 0$$

where G is w/a. Since G for steady flow is constant along the length of the pipe and equal to V/v, the last equation may be written in the form

$$\frac{dp}{v} + \frac{G^2}{g}\frac{dv}{v} + \frac{2fG^2}{Dg}dx = 0 \qquad [2]$$

This is the dynamic equation of flow through a pipe. It may be used to determine the mean friction coefficient between two cross sections as follows:

Assume f to be constant between sections 1 and 2. Then Equation [2] integrates to the expression

$$\int_1^2 \frac{dp}{v} + \frac{G^2}{g}\ln\frac{v_2}{v_1} + \frac{2fG^2}{Dg}(x_2 - x_1) = 0 \qquad [3]$$

which may be solved for f. In an actual case f may be interpreted as the mean coefficient of friction. For a numerical solution it is necessary to know not only the dimensions of the pipe and the rate of fluid flow, but also the relationship between pressure and specific volume along the path of flow. In the incompressible case, which comprises the present tests with water, the specific volume is constant. Therefore, the first term becomes $(p_2 - p_1)/v$ and the second term vanishes. To get the pressure-volume relation for compressible flow we must turn to the first law of thermodynamics.

The Pressure-Volume Relationship

Let us first consider the adiabatic case, that is, the case in which heat flow to or from the fluid stream is negligible. Then from the first law of thermodynamics we know that for any section a along the pipe length the sum of the enthalpy and kinetic energy per unit mass of fluid crossing that section is constant and is equal to the enthalpy at a preceding section o where the cross-sectional area is very large and the kinetic energy is negligible: Thus

$$h + V^2/2g = h_o \qquad [4]$$

where h_o denotes the enthalpy at section o and the symbols without subscript denote quantities corresponding to section a. For V we may substitute Gv, so that Equation [4] becomes

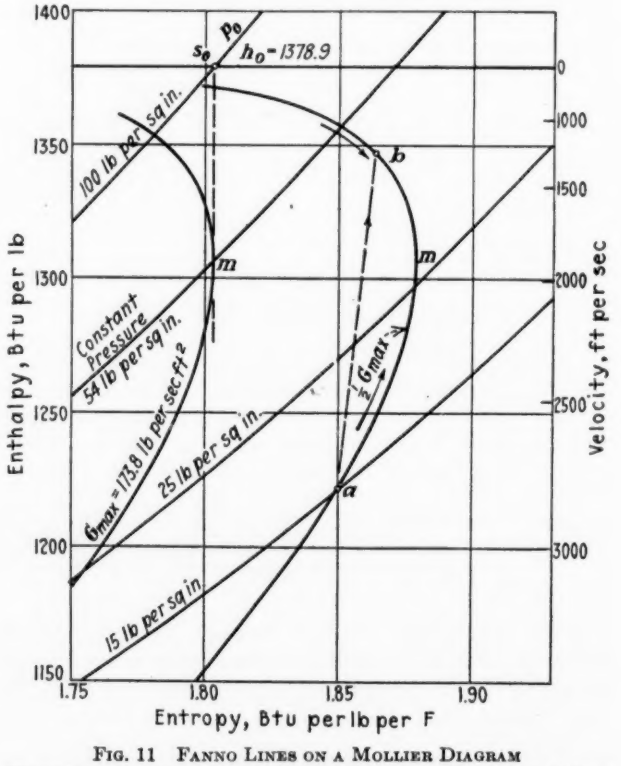


Fig. 11 Fanno Lines on a Mollier Diagram
(The velocity scale on the right holds good when the initial enthalpy is 1378.9 Btu per lb.)

$$h + G^2v^2/2g = h_o \quad \ldots\ldots [5]$$

where G is a constant in any steady-flow case for all sections having the same cross-sectional area. For a given state at section o, Equation [5] yields a series of relationships between h and v, each one of which corresponds to a value of G. In Fig. 11 two such relationships are drawn on an enthalpy-entropy diagram for one value of h_o. Stodola called them Fanno lines.[3]

Having determined by measurement the initial state o and the mass velocity G of a stream flowing through a pipe, we may determine by Equation [5] the h-v relationship and, with the steam tables as an aid, the p-v relationship corresponding. If two pressures, p_1 and p_2, at two sections 1 and 2 be now measured, it becomes possible to execute by graphic means or otherwise the integration indicated in the first term of Equation [3] and to compute the second term and the third term except for the single unknown f. Thus, the friction factor f is completely determined.

However, it would be convenient to have in place of the steam table an algebraic relationship between h, v, and p which represents the steam-table relations over the range in question with adequate precision and is yet simple in form. Such a relation is the equation

$$h = A + Bpv \quad \ldots\ldots [6]$$

in which A and B are constants. The constant A was chosen as 851.1 Btu per lb or 662,400 ft-lb per lb, and the constant B in its dimensionless form (corresponding to h in ft-lb per lb, p in lb per sq ft, and v in cu ft per lb) was chosen as 4.117. With these values, Equation [6] represents the relationship between h, p, and v for water vapor, superheated or supersaturated, to high precision for pressures under 100 lb per sq in.

Substituting Equation [6] into the Fanno-line Equation [5], we get

$$A + Bpv + \frac{G^2v^2}{2g} = h_o \quad \ldots\ldots [7]$$

which for given values of h_0 and G is a pure pressure-volume relation. Solving Equation [7] for p, differentiating and dividing through by v, we get for the first term of Equation [2]

$$\frac{dp}{v} = -\frac{G^2}{2gB}\frac{dv}{v} - \frac{h_o - A}{B}\frac{dv}{v^2}$$

Substituting this into Equation [3] and integrating between sections 1 and 2, we get

$$\frac{G^2}{g}\left(1 - \frac{1}{2B}\right)\ln\frac{v_2}{v_1} + \frac{h_o - A}{2B}\left(\frac{1}{v_2^2} - \frac{1}{v_1^2}\right) + \frac{2f\,G^2(x_2 - x_1)}{g\,D} = 0$$

or

$$= \frac{g\,D}{2G^2(x_2 - x_1)}\left[\frac{h_o - A}{2B}\left(\frac{1}{v_1^2} - \frac{1}{v_2^2}\right) - \frac{G^2}{g}\left(1 - \frac{1}{2B}\right)\ln\frac{v_2}{v_1}\right] \quad \ldots\ldots [8]$$

If measurements be made of the initial state, the rate of flow and the pressures at 1 and 2, the values of v_1 and v_2 can be found by solving the quadratic Equation [7]. Nothing further is needed to compute the friction factor from Equation [8].

Equation [8] is valid for any adiabatic case in which all states assumed by the steam are pure vapor states. However, in practically all of the tests reported here the Fanno line crosses the saturation line into the two-phase region before the discharge condition is reached. If moisture forms in large drops as soon as the saturation line is crossed, then Equation [6] does not hold good and Equation [8] is consequently invalid.

Evidence[12,13] has been accumulating recently that moisture does not form in a steadily flowing stream immediately upon passing the saturation line and almost never before reaching the end of the convergence in an expansion. Yellott and Holland[13] have shown experimentally that condensation begins in a nozzle only after the enthalpy has dropped some 60 Btu per lb below the saturation-line enthalpy at the same entropy. It has also been shown from Yellott's tests[14] that condensation does not immediately result in complete equilibrium with large drops of liquid.

Though the phenomena exhibited in Yellott's nozzles are not necessarily reproduced in flow through a pipe, it is reasonable to assume that within the supersaturation limit found by Yellott the state of steam in a pipe will more nearly approximate to a supersaturated vapor than to an equilibrium mixture of vapor and large drops. Since all states assumed by the steam in the present tests were within the supersaturation limit it was assumed that Equation [6] would hold good. Therefore, the friction coefficients presented here for the tests on insulated pipe (tests 20 to 23) were computed by means of Equation [8].

Friction With Heat Transfer

Enough heat transfer occurred from the unlagged pipe in tests 1 to 19 to justify an allowance for it in the computation of the friction coefficient. The dynamic Equation [2] is unaltered by heat transfer, because the Newtonian laws are independent of heat flow. The energy Equation [4], on the other hand, must be augmented by another term: Thus, for vapor states

$$A + Bpv + G^2v^2/2g = h_0 - Q \quad \ldots\ldots [9]$$

where Q is the heat transfer per unit mass of fluid between sections o and a. From this we find the first term of the dynamic equation to be

$$\frac{dp}{v} = -\frac{G^2}{2gB}\frac{dv}{v} - \frac{(h_o - A - Q)dv}{B \qquad v^2} - \frac{dQ}{Bv^2}$$

Substituting this in Equation [3], integrating and solving for f, we get

$$f = \frac{2ga}{G^2\pi D(x_2 - x_1)}\left[\frac{1}{B}\left(\int_1^2 \frac{dQ}{v^2} - \int_1^2 \frac{Q}{v^3}dv\right) - \frac{G^2}{g}\left(1 - \frac{1}{2B}\right)\ln\frac{v_2}{v_1} - \frac{h_o - A}{2B}\left(\frac{1}{v_2^2} - \frac{1}{v_1^2}\right)\right] \quad \ldots\ldots [10]$$

Values for v_1 and v_2 may be found from the measured quantities by solving the quadratic Equation [9].

Determination of heat transfer Q was based on the equations

$$dq = (H_c + H_r)\,\Delta t\,dA \quad \ldots\ldots [11][15]$$

$$H_c = 0.415(\Delta t/D)^{0.25} \quad \ldots\ldots [12][16]$$

and

$$H_r = 0.172e\,\frac{(T_1/100)^4 - (T_2/100)^4}{T_1 - T_2} \quad \ldots\ldots [13][17]$$

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[12] See "Steam and Gas Turbines," by A. Stodola, translated by Lowenstein, McGraw-Hill Book Company, New York, N. Y., 1927, vol. 1, pp. 117–122 for a review of earlier experiments.

[13] "The Condensation of Flowing Steam. Part I—Condensation in Diverging Nozzles," by J. I. Yellott and C. K. Holland, Trans. A.S.M.E., vol. 59, 1937, paper FSP-59-5, pp. 171–183. An adequate Bibliography covering recent work is included in this paper.

[14] Discussion by J. H. Keenan of the paper "Supersaturated Steam," by J. I. Yellott, Trans. A.S.M.E., vol. 56, 1934, pp. 427–430.

[15] "Heat Transmission," by W. H. McAdams, McGraw-Hill Book Company, New York, N. Y., 1933.

[16] Loc. cit. p. 243, Fig. 98.

[17] Ibid., p. 222.

where q denotes the time rate of heat transfer; H_c and H_r denote respectively the coefficients of heat transmission by conduction and by radiation, Btu ft^{-2} F^{-1} hr^{-1}; Δt denotes the temperature difference between pipe surface and room, F; dA an element of pipe surface area, sq ft; D the outside pipe diameter, in.; T_1 the pipe surface temperature, F abs; T_2 the room temperature, F abs; and e the emissivity of the surface. Various approximations were permissible in the calculation of Q because of the small influence of heat transfer on the value of f.

The heat transfer for the entire pipe length comprised the loss from the bare flanges and nozzle block, and the loss from the test pipe. The former was found from an estimated value of $(H_c + H_r)$ of 2.8 Btu ft^{-2} hr^{-1} F^{-1}, and the latter was computed from Equations [11], [12], and [13] using 0.86 for the value of e. In most calculations the integral of Equation [11] was replaced by the approximate equation

$$q = (H_c + H_r)_m \Delta t_m A$$

in which subscript m denotes a mean value and A is the total pipe surface area.

The heat transfer was calculated for two finite lengths; namely, that between the inlet section o and section 3, and that between section 3 and the end of the pipe. Next, it was assumed that along the test pipe the relationship between v and Q could be adequately represented by an equation of the form

$$1/v^2 = m + nQ \quad [14]$$

where m and n are constants. The constant n was chosen to be the mean of the chord slopes between sections 1 and 3 and 1 and 4 on a $[(1/v^2) - Q]$ chart, while m was chosen so that the equation gave the correct relation at section 1. With the aid of Equation [14] it was possible to evaluate with adequate precision the two integrals in Equation [10] involving Q. The value of f could then be determined.

The State of Maximum Entropy

A stream of fluid can expand in adiabatic flow in a passage of uniform cross section until the state of maximum entropy is reached. Any further expansion must occur outside the passage. Since heat transfer is least near the discharge end of the test pipe in the tests under discussion, it is reasonable to assume that the state at the minimum discharge pressure for any initial condition is a state of maximum entropy.

For an infinitesimal change in pressure at the state of maximum entropy, we may write

$$ds = 0$$

or, for a pure substance in the absence of gravity, electricity, and capillarity

$$ds = (du + pdv)/T = 0 \quad [15]$$

where s denotes entropy, u internal energy, and v volume, each per unit mass, and p and T denote pressure and absolute temperature, respectively. By definition, the differential of enthalpy is given by the equation

$$dh = du + pdv + vdp$$

so that Equation [15] becomes

$$(dh - vdp)/T = 0$$

Thus, we have

$$dh = vdp \quad [16]$$

at the point of maximum entropy.

For the vapor phase we have already assumed that the h-p-v relationship is

$$h = A + Bpv \quad [6]$$

so that for any infinitesimal change in state the differential of h is given by

$$dh = B(pdv + vdp) \quad [17]$$

Combining Equations [16] and [17], we get

$$\frac{dp}{p} = -\frac{B}{B-1}\frac{dv}{v} \quad [18][18]$$

The Fanno-line equation for adiabatic flow near the end of the pipe is

$$h + G^2v^2/2g = h_o - Q$$

where Q is the heat loss between the initial section o and any section near the end of the pipe. Differentiation gives the equation

$$dh + \frac{G^2}{g} vdv = 0 \quad [19]$$

provided that heat loss near the end of the pipe is negligible. For the state of maximum entropy we get by Equations [16] and [19]

$$vdp + \frac{G^2}{g} vdv = 0$$

which can be transformed with the aid of Equation [18] to

$$-\frac{v}{p}\frac{dp}{dv} = \frac{G^2}{g}\frac{v}{p} = \frac{B}{B-1} \quad [20][19]$$

Solving the latter equation for v and substituting this and the hpv relationship into the Fanno-line equation, we get

$$A - p_s^2 \frac{B^2}{1-B}\frac{g}{G^2} + \frac{G^2}{2g} p_s^2 \left(\frac{B}{1-B}\right)^2 \left(\frac{g}{G^2}\right)^2 = h_o - Q$$

in which p_s denotes the pressure of maximum entropy. Solving for p_s we get

$$p_s = \frac{(B-1)G}{B}\sqrt{\left[\frac{2(h_o - A - Q)}{g(2B-1)}\right]} \quad [21]$$

The computed values of the pressure of maximum entropy

---

[18] For a finite change in state at constant entropy this equation integrates to the usual relation

$$pv^k = \text{constant}$$

where k denotes $B/(B-1)$. Therefore, the assumed hpv relation comprises the commonly assumed isentropic relation. For the value of B chosen here, namely, 4.117, the corresponding value of k is 1.321, which is in accord with values indicated for superheated and supersaturated states near saturation at low pressures by Fig. 8 of "Thermodynamic Properties of Steam," by J. H. Keenan and F. G. Keyes, John Wiley & Sons, Inc., New York, N. Y.

[19] From this equation it is evident directly that the velocity at the point of maximum entropy is the velocity of sound. For upon substituting V/v for G and solving for V, we get

$$V = \sqrt{(gpvk)}$$

where k is the isentropic exponent. This is the expression for the velocity of sound in a fluid of pressure p and volume v. See, for example, "Hydrodynamics," by Lamb.[20]

[20] Hydrodynamics by Horace Lamb, Cambridge University Press, 1930, p. 453.

shown in Figs. 4, 7, and 8 are found from Equation [21].[21]

Comparison of the Adopted Method of Calculation With a Method Based on Two-Phase Equilibrium

Regardless of the validity of Equation [6], it is entirely possible to find a h-v relationship corresponding to any test for which the intitial enthalpy, the rate of flow and the heat transfer are known. The Fanno-line equation, namely

$$h + G^2v^2/2g = h_o - Q$$

may be solved for h by assuming a value for v and estimating a corresponding value for Q. From the hv combination thus found a corresponding value of the pressure may be obtained from the steam table. With this value of pressure a second approximation to Q is possible, since Q is known as a function of distance by calculation, and pressure is known as a function of the same variable by measurement.

The resulting pressure-volume relation may be used in solving the dynamic Equation [3] for the friction coefficient. It is necessary, of course, to use a graphic integration or other approximate method to evaluate the term $\int dp/v$.

When this method was applied to test 5, the computed value of the friction coefficient was 0.00382 as compared with 0.00386 by the method in which single-phase states were assumed.

From the h-v relation for the two-phase calculation it is possible to find an approximation to the corresponding state of maximum entropy. For test 5 the pressure at this state was 46 lb per sq in. as compared with a computed single-phase value of 41.1 lb per sq in. and a measured value of 36.7 lb per sq in. The computed value for two-phase equilibrium exceeds the measured value by 25 per cent, and the computed value for single-phase states exceeds the measured value by 12 per cent. Frössel[5] found for air flow, in which there was no question of the appearance of a second phase, an excess of the computed value over the measured value of about 10 per cent. The agreement between Frössel's result and the single-phase computation lends weight to the probability that the liquid phase plays a negligible part in these experiments.

[21] Equation [8] can be solved for the rate of flow in terms of the friction coefficient and other quantities: Thus

$$G = \sqrt{\left\{\frac{\frac{h_o - A}{2B}\left(\frac{1}{v_1^2} - \frac{1}{v_2^2}\right)}{\left[\frac{1-\frac{1}{2B}}{g}\ln\frac{v_2}{v_1} + \frac{f\pi D(x_2 - x_1)}{2g\,a}\right]}\right\}}$$

To obtain the expression for the maximum rate of flow per unit area corresponding to any initial enthalpy we must substitute for v_2 the value corresponding to p_s. For this a combination of Equations [20] and [21] gives

$$v_s = \sqrt{\left[\frac{2g}{G^2}\left(\frac{h_o - A}{2B - 1}\right)\right]}$$

When this substitution has been made, the only quantity remaining in the expression for the maximum G which cannot be readily obtained from the measurements is v_1.

Now if we assume the flow through the entrance nozzle to be frictionless, v_1 becomes a function of G and the initial state; thus

$$G = \sqrt{\left\{2g\,\frac{P_0}{v_0}\,\frac{k}{k-1}\left(\frac{v_0}{v_1}\right)^2\left[1 - \left(\frac{v_0}{v_1}\right)^{k-1}\right]\right\}}$$

The maximum flow through a given pipe for a prescribed initial state will be found at the intersection of the two curves of G against v_1 corresponding to the two expressions for G here given.

In an actual case, this process of solution becomes a method of successive approximations; for f, which was at first assumed, is dependent on the Reynolds number which is in turn dependent on G.

The Mean Reynolds Number

The friction coefficient computed from the steam tests is a mean friction coefficient. It can be seen by comparing Equations [2] and [3] that its definition in terms of point values of the friction coefficient f is

$$\bar{f} = \frac{\int_1^2 f\,dx}{x_2 - x_1}$$

where $\bar{f}$ denotes the computed value, hitherto denoted by f.

Assume that over the range of Reynolds' number covered by any one test the relationship between point values of the friction coefficient and the Reynolds number is given by an equation of the kind

$$f = a\mathrm{Re}^n$$

where Re denotes Reynolds number and a and n are constants. Then, we may write for the mean friction coefficient

$$\bar{f} = \frac{a\int_1^2 \mathrm{Re}^n\,dx}{x_2 - x_1}$$

which is identical with the point value of the friction coefficient corresponding to a value of Re given by

$$\left(\frac{\int_1^2 \mathrm{Re}^n\,dx}{x_2 - x_1}\right)^{1/n}$$

This may be called the mean Reynolds number.

Careful calculations of a few instances indicate that the mean Reynolds number can be approximated within 2 per cent by a simpler calculation. It consists of fitting a quadratic equation of the form

$$\mathrm{Re} = ax^2 + bx + c$$

to the Re-x relation at 0 ft, 32.2 ft, and 54.4 ft from the nozzle throat and then calculating for this quadratic the mean Reynolds number defined by the equation

$$\overline{\mathrm{Re}} = \int_1^2 \mathrm{Re}\,dx/(x_2 - x_1)$$

The values of mean Reynolds number given in Table 3 correspond to this equation with the integral and difference taken over the entire length of the test pipe.

Sample Computations

Water Test 1. From Equation [3]

$$f = \frac{Dg(p_1 - p_2)}{2vG^2(x_2 - x_1)}$$

From data given in Table 1, the following values are determined: $D = 1.05$ in. $= 0.0875$ ft; $p_1 - p_2 = 2404$ lb per sq ft; $v = 0.01602$ cu ft per lb; $G = 8.168/0.006$ lb per sec per sq ft; and $x_2 - x_1 = 21.5$ ft. Then, $f = 0.00530$.

Steam Test 5. By Equation [10]

$$f = \frac{2ga}{G^2\pi D(x_2 - x_1)}\left\{\frac{1}{B}\left[\int_1^2 \frac{dQ}{v^2} - \int_1^2 \frac{Q}{v^3}\,dv\right] - \frac{G^2}{g} \times \left(1 - \frac{1}{2B}\right)\ln\frac{v_2}{v_1} - \frac{h_o - A}{2B}\left(\frac{1}{v_2^2} - \frac{1}{v_1^2}\right)\right\}$$

Let us compute the over-all value of the mean friction coefficient. Referring to Tables 2 and 3, we get $a = 0.006$ sq ft; $G = 163.19$ lb per sec per sq ft; $D = 0.0875$ ft; $(x_2 - x_1) = 54.4$ ft; $B = 4.117$; and $A = 662{,}400$ Btu per lb.

Using a step-by-step integration of the heat-loss Equation

[11], we get the values given in Table 3 for the heat loss between the initial section and sections at distances 0, 32.2, and 54.4 ft downstream from the test-pipe entrance. Substituting these values into the Fanno-line equation

$$A + Bpv + G^2v^2/2g = h_o - Q$$

along with measured values of the pressure for each of the three stations, we get a quadratic which can be solved for v. The three resulting values are given in Table 3. The first and third may be substituted for v_1 and v_2, respectively, in the expression for f.

To evaluate the two integrals involving Q, we first choose values of the constants in the equation

$$1/v^2 = m + nQ$$

after converting the values for Q into ft-lb per lb. The value of n for the length 0 to 32.2 ft proves to be -3.3079×10^{-5}, while that for the remaining length is -4.1971×10^{-5}. Selecting the mean value, or -3.7525×10^{-5}, we get for m the value 0.17654. From the v-Q relation we get

$$\frac{1}{B}\int_1^2 \left[\frac{dQ}{v^2} - \frac{Q}{v^3}\,dv\right] = \left\{-\frac{3}{4n}\left[\left(\frac{1}{v_1^2}\right)^2 - \left(\frac{1}{v_2^2}\right)^2\right] + \frac{m}{2n} \times \left(\frac{1}{v_1^2} - \frac{1}{v_2^2}\right)\right\}\frac{1}{B} = (514.8 - 355.4)/4.117 = 38.72 \text{ lb}^2 \text{ ft}^{-5}$$

Other quantities appearing in the expression for f are

$$\frac{G^2}{g}\left(1 - \frac{1}{2B}\right)\ln\frac{v_2}{v_1} = 1028.04 \text{ lb}^2 \text{ ft}^{-5}$$

$$\frac{h_o - A}{2B}\left(\frac{1}{v_2^2} - \frac{1}{v_1^2}\right) = 4971.16 \text{ lb}^2 \text{ ft}^{-5}$$

$$\frac{2g\,a}{G^2\pi D(x_2 - x_1)} = 0.9694 \times 10^{-6} \text{ ft}^5 \text{ lb}^{-2}$$

Then $f = 0.9694 \times 10^{-6}\,(38.72 - 1028.04 + 4971.16) = 0.003860$

The Determination of the Expansion Forces in Piping by Model Test

By HAROLD W. SEMAR,[1] PHILADELPHIA, PA.

The paper describes apparatus for testing models of piping installations to determine the forces set up in the full-size installation due to expansion. The method consists in fixing one end of the model in a slide which can be moved a measured amount, and measuring the forces at the other end by means of a fixture which holds that end rigidly in place and permits measurement of six forces in three coordinate planes, which are the equivalent of forces in coordinate directions plus moments in coordinate planes. The slide of the movable end is placed so that its direction of motion coincides with the direction of the pipe expansion.

A complete example is given showing the steps in determining the end reactions of an inlet pipe for a steam turbine.

IN THE DESIGN of piping, consideration must be given to the forces and stresses set up by expansion of the pipe and by movement of the end supports. In modern central-station power plants where the main steam lines carry steam at 1200 pounds pressure and 925 F, this problem is especially important.

The design of the piping system, therefore, requires a means of predicting the expansion stresses and forces so that sufficient flexibility can be provided with a minimum of bends and extra pipe length. Methods of calculating piping have been published[2,3,4] and are in general use at the present time. When the pipe lies in a single plane the calculation can be readily made, but when the pipe is three-dimensional the calculation is long and tedious.

An accurate solution of complicated piping systems can be made, however, by tests of a scale model. This method is now in use by the Westinghouse Electric & Manufacturing Company in checking the forces exerted on large steam turbines by the inlet and exhaust piping.

When a pipe expands with an increase in temperature, the forces exerted by the ends of the pipe are the same as those produced by freeing one end of the pipe, allowing it to expand, and then restoring the free end to its original position. The expansion forces can thus be duplicated by a displacement of one end of the pipe equal to the expansion but in the opposite direction. The problem is then one of load deflection and can be exactly described by a scale model.

The model test is essentially a scale model of the full-size pipe made of steel tubing and rigidly clamped at both ends, with provision for moving one end without rotation and for measuring the forces at the other end.

Fig. 1 is a photograph of the apparatus set up for making a model test. The movable end of the model is bolted to a tool slide mounted so that its direction of motion coincides with the direction of the pipe expansion. The fixed end of the pipe is held in a frame that enables the forces which are acting on the pipe to be measured. This frame is the feature of the test arrangement.

The reaction of the end of the pipe can be defined by a force in each of the three coordinate directions and a moment in each of the three coordinate planes. The three moments are present because the end of the pipe is restrained from rotation in any direction. The six coordinate forces and moments are shown in Fig. 2. Each of the three moments can be replaced by a couple, one force of which acts in a coordinate direction. For instance, in Fig. 3 the moment M_{zy} is replaced by two forces in opposite directions, of magnitude M_{zy}/b, and a distance b apart, one of which is in the direction of P_x. By similarly replacing the other two moments by couples, the six equivalent forces of Fig. 4

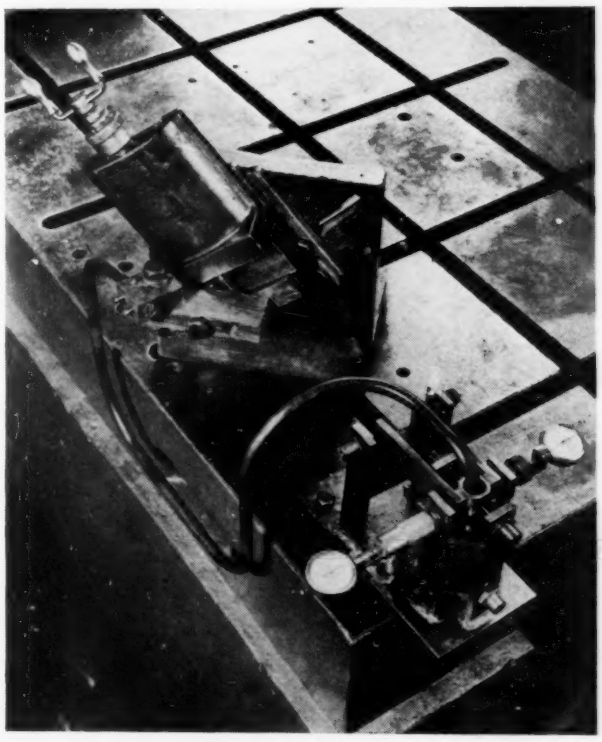


FIG. 1 ARRANGEMENT OF TEST APPARATUS

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[1] Mechanical Engineer, Westinghouse Electric and Manufacturing Company.

[2] "Design of Steam Piping to Care for Expansion," by W. H. Shipman, Trans. A.S.M.E., vol. 51, part 1, 1929, paper FSP-51-52, pp. 415–446.

[3] "Piping Handbook," by J. H. Walker and Sabin Crocker, McGraw Hill Book Co., Inc., New York, N.Y., 1930.

[4] "Stresses and Reactions in Expansion Pipe Bends, by A. M. Wahl, Trans. A.S.M.E., vol. 49-50, 1927-1928, paper FSP-50-49.

"Stresses in Three-Dimensional Pipe Bends," by William Hovgaard, Trans. A.S.M.E., vol. 57, 1935, paper FSP-57-12, pp. 401–415.

"End Reactions and Stresses in Three-Dimensional Pipe Lines," by G. B. Karelitz and J. H. Marchant, *Journal of Applied Mechanics*, Trans. A.S.M.E., vol. 59, 1937, pp. A-68 to A-75.

Contributed by the Applied Mechanics Division and presented at the Annual Meeting of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS, New York, N. Y., December 5–9, 1938.

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NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

result. The six coordinate forces and moments are readily converted to the six equivalent forces by the relations

$$\left.\begin{aligned} R_x &= P_x - \frac{M_{xy}}{b} + \frac{M_{zz}}{b} & R_{xy} &= \frac{M_{xy}}{b} \\ R_y &= P_y & R_{yz} &= \frac{M_{yz}}{b} \\ R_z &= P_z + \frac{M_{yz}}{b} & R_{zz} &= \frac{M_{zz}}{b} \end{aligned}\right\} \quad \ldots\ldots [1]$$

which are evident in Fig. 3.

Conversely, if the equivalent forces of Fig. 4 are known the coordinate forces can be found by

$$\left.\begin{aligned} P_x &= R_x + R_{xy} - R_{zz} & M_{xy} &= b\,R_{xy} \\ P_y &= R_y & M_{yz} &= b\,R_{yz} \\ P_z &= R_z - R_{yz} & M_{zz} &= b\,R_{zz} \end{aligned}\right\} \quad \ldots\ldots [2]$$

The bracket which holds the fixed end of the pipe is supported so that each of the six equivalent forces can be measured. This bracket is shown in Fig. 5. It is held in place by twelve pins or struts which act in the positive and negative directions of the six equivalent forces. In Fig. 5, for instance, the forces R_x and $-R_x$ represent the location of one pair of pins. These pins are supported as shown in Fig. 6. Each pin rests on a button which in turn rests on a shoulder of a sleeve that is screwed into a solid frame. The sleeves are adjusted to allow about 0.002 in. movement of the bracket so that one pin transmits the force to the bracket while the other is slack. Actually, the pin corresponding to $+R_y$ was omitted to allow clamping the model, and the model must be mounted so that R_y remains negative during the test.

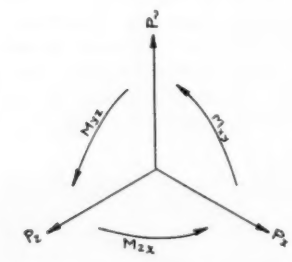


FIG. 2 COORDINATE FORCES

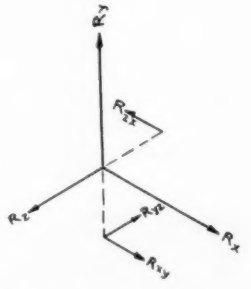


FIG. 4 EQUIVALENT FORCES

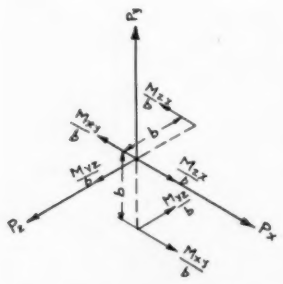


FIG. 3 MOMENTS REPLACED BY COUPLES

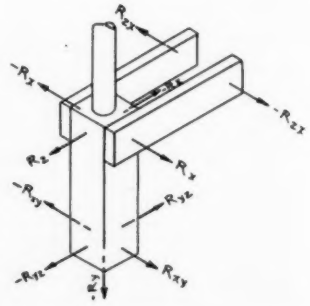


FIG. 5 BRACKET FOR HOLDING MODEL

The force transmitted to the bracket is then the force required to just lift the button from its seat. Measurement of this force is accomplished with the weighing instruments shown in Fig. 7. The instrument on the left is simply a dial indicator which bears on the button supporting the slack pin. It serves to show when the opposite button is moved, and to check the clearance between the pins. The instrument on the right contains a plunger and calibrated spring, and is screwed onto the sleeve supporting the tight pin. The plunger bears on the button with an increasing force as the instrument is screwed onto the sleeve, compressing the spring until the indicator on the opposite side shows that the button has been lifted. The indicator on the right measures the compression of the spring to determine the force acting. To measure $-R_y$, which has no pin opposite to show when the button is lifted, the weighing instrument is screwed over the sleeve in the same way. The point at which the button is just raised is determined by tapping the model. When the button lifts from its seat, the tapping is carried to the indicator of the weighing instrument. The six forces are noted for several positions of the movable end of the model. These are plotted against the linear movement of the pipe end, and the average slopes of these curves are taken as the model deflection constants, R_x, R_y, R_z, R_{xy}, R_{yz}, and R_{zz}, which are forces per unit deflection. The deflection is taken as positive in the direction opposite to that of the pipe expansion, and care must be taken of the signs of the model deflection constants.

If the model were exactly to scale and of the same material as the full-size pipe, the forces on the full-size pipe would simply equal the model forces divided by the scale of the model.

However, it is convenient to make the model only of standard sizes of steel tubing and to depart somewhat from a true scale model. The model forces can still be readily transferred to the full-size forces if the factor $\lambda = tR/r^2$ (where t = wall thickness, R = radius of pipe bend, and r = mean radius of pipe cross section), which is the index of the increased flexibility of the bends due to the flattening of the pipe section is kept the same. This can be done by selecting a standard size of steel tubing having approximately the same ratio of wall thickness to diameter and then taking a length scale to give the same value of λ. This condition will be met if the scale be taken as

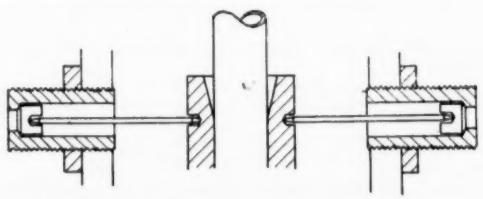


FIG. 6 METHOD OF SUPPORTING FIXED END OF MODEL

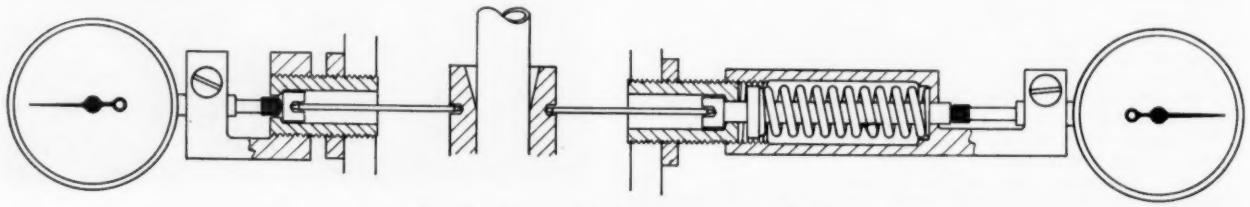


FIG. 7 MEASURING INSTRUMENTS IN PLACE

FULL SIZE PIPE

OUT. DIAM	= 12.75 in
INS. DIAM	= 10.125 in
WALL THICK	= 1.312 in
I_a	= 781.3 in^4
TEMP	= 925°F
E_a	= 20600000 lb/in^2

MODEL

OUT. DIAM.	= .625 in
INS. DIAM.	= .495 in
WALL THICK	= .065 in
I_m	= .00454 in^4
E_m	= 29000000 lb/in^2
SCALE	= 1/20

EXPANSION $= \Delta(T_2 - T_1)l = 7.75 \times 10^{-6}(925 - 70)l = .00662\,l$

	PIPE	TURBINE	TOTAL
Δx	.00662 × 138 = .91	40	1.31
Δy	.00662 × 60 = -.40	-1.98	-2.38
Δz	.00662 × 264 = 1.75	- .48	1.27

$e = \sqrt{1.31^2 + 2.38^2 + 1.27^2} = 2.99$

DIRECTION OF EXPANSION $= \tan^{-1}\frac{1.27}{1.31} = 44°10'$ WITH x AXIS

$= \tan^{-1}\sqrt{\frac{2.38}{1.27^2 + 1.31^2}} = 52°30'$ WITH xy

$K = \frac{E_a I_a}{E_m I_m} s^3 = \frac{20600000 \times 781.3}{29000000 \times .00454}\left(\frac{1}{20}\right)^3 = 15.3$

TEST DATA

DEFLECT.	R_x		R_y		R_z		R_{xy}		R_{yz}		R_{zx}	
	SPRING COMP.	FORCE										
0 in	+1.81 in	+31 LB	-.135 in	-.09 LB	+2.30 in	+56 LB	-1.96 in	-39 LB	+3.38 in	+107 LB	+1.58 in	+16 LB
100	+1.33	+0.8	-1.82	-3.2	+1.41	+1.2	-1.30	-0.6	+1.92	+3.7	+1.40	+1.2
300	-1.91	-3.6	-2.64	-7.2	-2.68	-7.4	+2.20	+5.0	-3.19	-9.8	+1.35	+0.9
400	-2.43	-6.2	-3.12	-9.5	-3.73	-12.5	+2.99	+8.9	-4.66	-17.0	+1.27	+0.8
		-23.6 lb/in		-21.5		-45.0		+31.8		-69.5		-2.1

MOMENT ARM $b = 5$ in

$P_x = (-23.6 + 31.8 + 2.1)\,15.3 \times 2.99 = +470$ LB

$P_y = -21.5 \times 15.3 \times 2.99 = -980$

$P_z = (-45.0 + 69.5)\,15.3 \times 2.99 = +1120$

$M_{xy} = 5 \times 20 \times 31.8 \times 15.3 \times 2.99 = +145000$ in LB

$M_{yz} = -5 \times 20 \times 69.5 \times 15.3 \times 2.99 = -318000$

$M_{zx} = -5 \times 20 \times 2.1 \times 15.3 \times 2.99 = -9600$

FIG. 8 EXAMPLE: DATA AND RESULTS

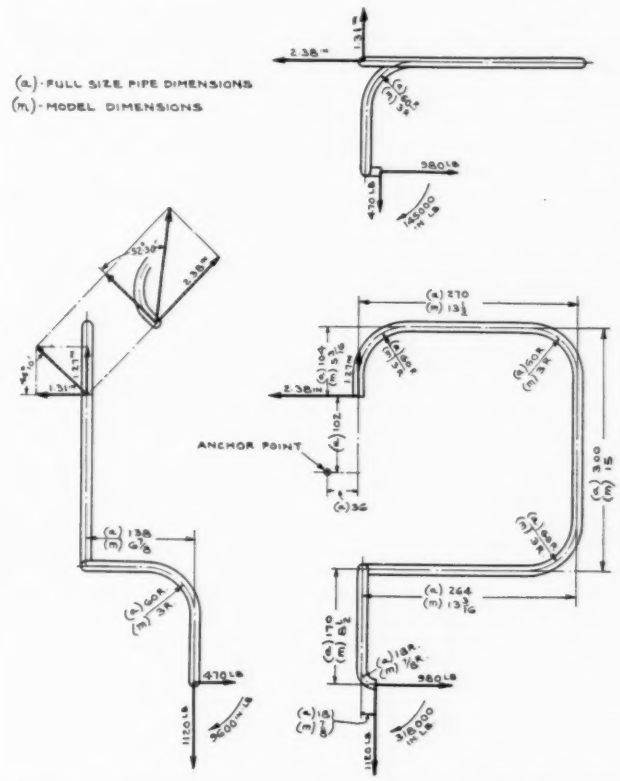


FIG. 9 EXAMPLE: LAYOUT OF PIPE SHOWING EXPANSIONS AND RESULTING FORCES

$$s = \frac{R_m}{R_a} = \left[\frac{r_m}{r_a}\right]^2 \frac{t_a}{t_m} \quad \ldots\ldots\ldots\ldots\ldots [3]$$

where subscripts m and a denote, respectively, the model and full-size-pipe dimensions.

Then, for the same deflection the forces in the full-size pipe and the model will be in the ratio

$$\frac{\frac{E_a I_a}{R_a^3}}{\frac{E_m I_m}{R_m^3}} = \frac{E_a I_a}{E_m I_m} s^2 = K \quad \ldots\ldots\ldots\ldots\ldots [4]$$

and the moments in the ratio

$$\frac{E_a I_a}{E_m I_m} s^2 = \frac{K}{s} \quad \ldots\ldots\ldots\ldots\ldots [5]$$

where

K = model-to-pipe conversion factor
E_a = modulus of elasticity of actual pipe
I_a = moment of inertia of actual pipe section
E_m = modulus of elasticity of model
I_m = moment of inertia of model section
s = scale of model.

The actual forces and moments exerted by the full-size pipe are then

$$\left.\begin{aligned} P_x &= (R_x + R_{xy} - R_{zx})Ke \\ P_y &= R_y Ke \\ P_z &= (R_z - R_{yz})Ke \\ M_{xy} &= (b/s)R_{xy}Ke \\ M_{yz} &= (b/s)R_{yz}Ke \\ M_{zx} &= (b/s)R_{zx}Ke \end{aligned}\right\} \quad \ldots\ldots\ldots [6]$$

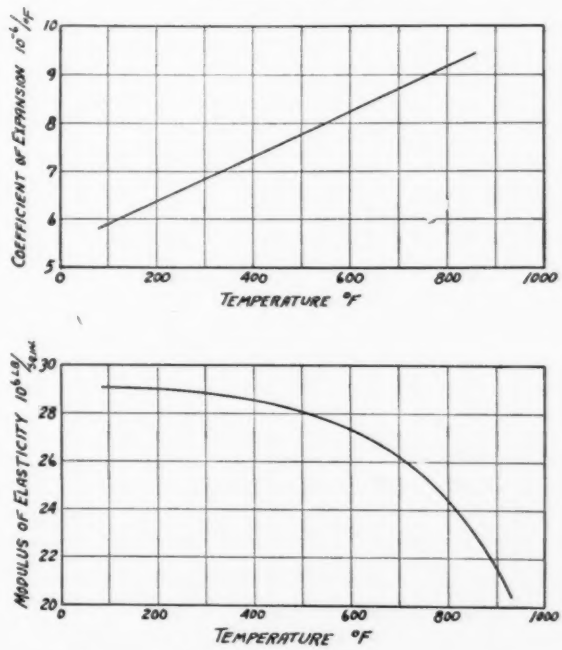


FIG. 10 MODULUS OF ELASTICITY AND COEFFICIENT OF EXPANSION FOR CARBON STEEL

where b is the moment arm of the model forces R_{xy}, R_{yz}, and R_{zx} as shown in Fig. 3 and e is the linear expansion or combination of expansion and end movement of the full-size pipe. It is the vector sum of the expansion and end movements in the coordinate directions.

A complete example is given in Figs. 8 and 9 which shows all the steps in determining the end reactions of an inlet pipe to a steam turbine. In this case, the movement of the end of the pipe due to expansion of the turbine is taken into account. No consideration is taken of cold springing of the pipe, since the forces are simply proportional to the expansion less the cold spring. Thus, if the pipe is cold-sprung 60 per cent of the expansion, the forces when the pipe is hot will be 40 per cent of those given.

As shown in Fig. 10 the modulus of elasticity of steel is affected by high temperatures and must be considered. Consequently, the forces when the pipe is cold will be 60 per cent of those given times the ratio of the moduli of elasticity cold and hot.

Branch systems are tested simply by using an additional slide for each additional branch.

By the method outlined, the expansion forces set up in steam piping can be reliably determined, with an error no greater than the experimental error in the measurement of the forces. Numerous and irregular bends and skew sections of pipe add only to the time required to construct the model and in no way complicate the test procedure.

Analysis of Effect of Wire Curvature on Allowable Stresses in Helical Springs

By A. M. WAHL,[1] EAST PITTSBURGH, PA.

The author suggests a method for determining the effect of wire or bar curvature on allowable stresses in helical springs, based on the endurance properties of the material. The method is based on the assumption, in agreement with available fatigue-test data, that stress-concentration effects produced by wire curvature may be neglected in calculating the static component of applied stress; also, it is assumed that the relation between the static- and variable-stress components at failure follows a linear law. On this basis, charts are given for determining maximum allowable stress as a function of spring index, ratio of minimum to maximum stress, and ratio between yield point and endurance limit. The effect of sensitivity of the material to stress concentration is taken into account by introducing a sensitivity-index q. Results calculated by means of the equations given show good agreement with results from tests of springs of different indexes. Some of the limitations of the method are discussed, and suggestions are made for future testing work.

AN ARTICLE[2] published in 1929 gave the results of strain measurements on helical springs, and indicated that the maximum stress in such springs may be much higher than that figured by using the conventional helical-spring formula. The difference is particularly pronounced for springs having small indexes, i.e., low ratios of coil diameter to wire diameter. This stress increase may be accounted for by (1) direct shear due to the axial load and (2) stress-concentration effects due to wire or bar curvature. On the basis of the strain measurements, reported in the 1929 paper,[2] an approximate formula was developed for calculating the maximum stress in such springs. The application of this formula involves the multiplication of the conventional formula for helical springs by a factor K which depends on the spring index.

The results which were computed by this theory were in close agreement for practical springs with an approximate formula given by Röver[3] in 1913, and also with a more exact formula developed later by Göhner who applied the theory of elasticity to the problem.[4] While it may be said that the problem of calculating the maximum stress in helical springs as affected by wire curvature has been solved[5] (assuming elastic conditions) the problem of predicting analytically the effect of these stresses on the actual fatigue strength of helical springs has received but little attention. There has, however, been a general opinion among spring engineers, based on experience, that the use of the factor K of Fig. 1 will result in too low values of the permissible working load. This view was confirmed to a certain extent by the results of a series of carefully made fatigue tests on small helical springs of different indexes carried out by Zimmerli[6] which showed that the limiting stress range in fatigue, when figured by using the K-factor, was higher for the springs of smaller indexes. Similar results were reported by Edgerton[7] in connection with the fatigue testing work being done on heavy helical springs by the A.S.M.E. Special Research Committee on Mechanical Springs. These tests will be discussed later.

An increase in the limiting stress range[8] with decrease in spring index may, in the author's opinion, be explained by the fact that

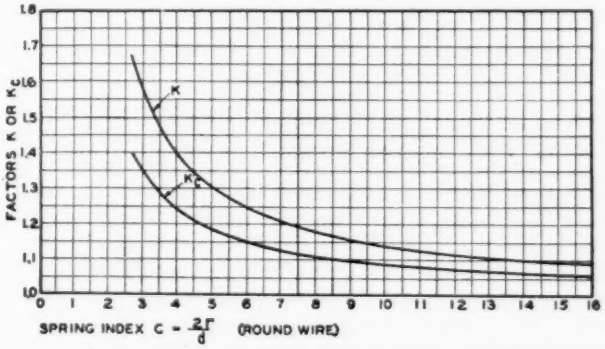


FIG. 1 CURVE FOR DETERMINING FACTORS K OR K_c FOR HELICAL TENSION OR COMPRESSION SPRINGS

(The factor K includes both the effects of curvature and direct shear; the factor K_c includes stress-concentration effects due principally to curvature.)

in helical springs the stresses as computed by the factor K are very much localized at the inside of the coil; for this reason the factor K may be considered to act in some degree as a stress-concentration factor. Fatigue tests[9] made in recent years on specimens having stress concentration indicate that the latter

[1] Research Engineer, Westinghouse Research Laboratories. Mem. A.S.M.E.

[2] "Stresses in Heavy Closely Coiled Helical Springs," by A. M. Wahl, Trans. A.S.M.E., vol. 51, part 1, 1929, paper APM-51-17, pp. 185–200.

[3] "Beanspruchung zylindrischer Schraubenfedern mit Kreisquerschnitt," by A. Röver, *Zeit. V.D.I.*, vol. 57, November, 1913, p. 1907.

[4] "Die Berechnung zylindrischer Schraubenfedern," by O. Göhner, *Zeit. V.D.I.*, vol. 76, March, 1932, p. 269.

[5] The present discussion will be limited to close-coiled springs, i.e., the effects of pitch angle are neglected. This is permissible for most practical springs.

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Discussion of this paper was accepted until January 10, 1939, for publication at a later date.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[6] "Relation of Wahl Correction Factor to Fatigue Tests on Helical Compression Springs," by F. P. Zimmerli, Trans. A.S.M.E., vol. 60, January, 1938, paper RP-60-2, pp. 43–44.

[7] "Abstract of Progress Report No. 3 on Heavy Helical Springs," by C. T. Edgerton, Trans. A.S.M.E., vol. 59, October, 1937, paper RP-59-8, pp. 609–616.

[8] It is assumed that all spring stresses are calculated by using the factor K of Fig. 1.

[9] See discussion by R. E. Peterson of the "Report of Research Committee on Fatigue of Metals," Proceedings of the American Society for Testing Materials, vol. 37, 1937, p. 162. Also: "General Considerations in Designing Mechanical Springs," by A. M. Wahl, *Machine Design*, vol. 10, January, 1938, pp. 30–35, and February, 1938, pp. 36–41.

may be neglected in computing the static component of the applied stress. In Fig. 2 are shown the results of some fatigue tests made on 0.7 per cent carbon-steel bars[10] in tension-compression. The full lines represent tests on bars without notches, the lower curve representing minimum and the upper curve maximum values of the limiting stress range. The dot-and-dash line represents the static or mean component of the applied stress, while the difference between this line and either the upper or lower curve represents the variable stress component. When the

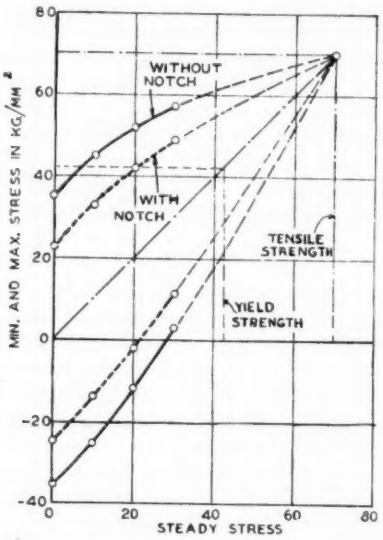


FIG. 2 NOTCH EFFECT IN FATIGUE STRESSING WITH INITIAL TENSION

(For 0.7 per cent carbon steel. According to tests by Schenck referred to in footnote 10.)

bars are notched, thus producing stress concentration, the endurance range is reduced as shown by the dashed lines on this figure, all stresses being figured in this case by neglecting the stress concentration due to the notch. It will be noted that the static component of stress as represented by the dot-and-dash line is practically unchanged by the presence of the notch; the effect of the notch is mainly to reduce the amount of variable stress which can be carried by the test piece, corresponding to some stress-concentration factor. Thus, in this case, it is seen that the static stress may be figured by neglecting stress-concentration effects while the latter must, however, be considered in calculating the variable stress.

Since practical helical springs are always under a combination of static and variable loading, if we neglect stress-concentration effects which are due to bar curvature the result is a higher permissible working stress (this is figured by using the K-factor of Fig. 1).

A further reason why the factor K cannot be directly applied in spring design lies in the fact that some materials, particularly in the smaller specimens, when tested in fatigue do not show the full stress-concentration effects predicted by theoretical methods (strain measurements, photoelastic tests, or mathematical analysis). This lack of sensitivity to stress concentration is a function of both wire or bar size and material,[11] the finer-grained high-strength alloy steels showing much higher sensitivity than the coarse-grained medium-carbon steels.

METHOD OF CALCULATION ASSUMING FULL SENSITIVITY TO STRESS CONCENTRATION

In accordance with the previous discussion, a method of calculation for helical springs is suggested. This method, which is similar to the general one proposed by Soderberg,[12] is based on the following assumptions: (1) Stress-concentration effects due principally to curvature in helical springs may be neglected in figuring the static component of the stress; (2) the relation between the values of static- and variable-stress components at failure follows a linear law; (3) the tensile and fatigue properties of the wire are the same in springs of different indexes, assuming the same size wire; (4) the effects of eccentricity of loading due to the end turns are neglected; (5) residual stresses produced by overstressing or surging the springs may be neglected. These factors will be more fully discussed later.

To determine the stress-concentration effect (due mainly to curvature) the stress multiplication factor K, shown in Fig. 1, is first split up into two parts, thus

$$K = K_s K_c \quad [1]$$

where K_s is a factor which takes into account the average stress produced by the direct shear due to the axial load, and K_c is a factor which takes into account the stress-concentration effect due mainly to curvature. Since the stresses due to direct shear are not localized, as is the case with those due to curvature, it is not believed that the factor K_s may be neglected in figuring the static component of stress. Assuming the shear stress due to the axial load as uniformly distributed over the cross section of the wire, the stress due to direct shear is[13]

$$s_s = \frac{16Pr}{\pi d^3}\frac{0.5}{c} \quad [2]$$

where the term $16Pr/\pi d^3$ is the stress figured by the conventional spring formula and c is the spring index $2r/d$. Thus

$$K_s = 1 + (0.5/c) \quad [3]$$

and from Equation [1]

$$K_c = \frac{K}{K_s} = \frac{K}{1 + (0.5/c)} \quad [4]$$

The factor K_s is shown in Fig. 1 as a function of the spring index c.

Referring to Fig. 3, the dashed line shows a typical experimental curve of failure for materials under a combination of static and variable stress. The ordinates represent values of variable stress which will just cause failure when superimposed on the static stresses shown by the abscissas. Assume that fatigue tests are made on a spring of large index ($c = \infty$) so that $K_c = 1$. Let s'_e denote the endurance limit in pulsating stress (zero to maximum) obtained on this spring. Then the point P on the diagram is determined, since for this case (pulsating stress) both the static- and variable-stress components are equal to $s'_e/2$. As an approximation, we replace the experimental curve by the straight line PA drawn to intersect the axis of abscissas at s_y, the yield point

[10] According to tests by Schenck; see "Federstähle" by Houdremont and Bennek, *Stahl und Eisen*, vol. 52, 1932, p. 660.

[11] For a discussion of "size effect" see the paper: "Model Testing as Applied to Strength of Materials," by R. E. Peterson, Trans. A.S.M.E., vol. 55, 1933, paper APM-55-11, pp. 79–85. Also: "Two and Three-Dimensional Cases of Stress Concentration, and Comparison With Fatigue Tests," by R. E. Peterson and A. M. Wahl, *Journal of Applied Mechanics*, Trans. A.S.M.E., vol. 58, March, 1936, p. A-15; and the discussion of this latter paper, *Journal of Applied Mechanics*, Trans. A.S.M.E., December, 1936, p. A-146.

[12] "Working Stresses," by C. R. Soderberg, Trans. A.S.M.E., vol. 55, 1933, paper APM-55-16, pp. 131–144. Also: "Working Stresses," by C. R. Soderberg, *Journal of Applied Mechanics*, Trans. A.S.M.E., vol. 57, September, 1935, p. A-106.

[13] All stresses considered in this analysis are torsion stresses.

of the material in torsion. This is done since in general no stress should exceed the yield point.[14]

To apply this diagram in actual design, assume that the spring is operating under a stress range from s_{min} to s_{max} in fatigue where these stresses are figured by using the factor K. Then the variable stress s_v is

$$s_v = \frac{s_{max} - s_{min}}{2} \quad [5]$$

and the static stress is

$$s_o = \frac{s_{max} + s_{min}}{2K_c} \quad [6]$$

The right side of the last equation is divided by K_c since this factor has already been used in figuring s_{max} and s_{min}; therefore, to neglect stress-concentration effects due to curvature, division

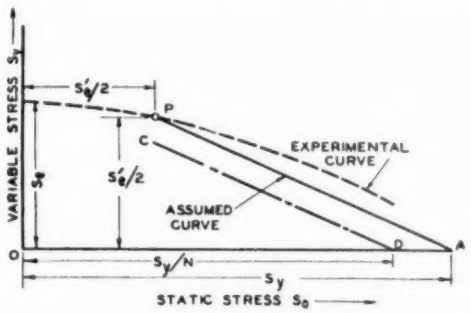


FIG. 3 APPLICATION OF STRAIGHT-LINE LAW TO HELICAL SPRINGS WHEN THE ENDURANCE LIMIT s'_e FOR PULSATING LOAD APPLICATIONS (ZERO TO MAXIMUM) AND THE YIELD STRESS s_y ARE KNOWN
(All stresses are torsion stresses.)

by K_c is necessary. The equation for the line PA, shown in Fig. 3, in terms of s_o and s_v is

$$\frac{s_o}{s_y} + \frac{s_v}{\frac{s'_e}{2}\left[\frac{s_y}{s_y - (s'_e/2)}\right]} = 1 \quad [7]$$

Substituting the values of s_v and s_o given by Equations [5] and [6] we find

$$s_{max} = \frac{2s_y}{\frac{1}{K_c}\left(1 + \frac{s_{min}}{s_{max}}\right) + \left(\frac{2s_y}{s'_e} - 1\right)\left(1 - \frac{s_{min}}{s_{max}}\right)} \quad [8]$$

This equation gives the maximum stress[15] s_{max} in terms of K_c, s_y, s_{min}/s_{max}, and s_y/s'_e and may be written

$$s_{max} = C_w s'_e \quad [9]$$

where C_w is a function of K_c, s_y, s_{min}/s_{max}, and s_y/s'_e. Thus

$$C_w = \frac{2s_y/s'_e}{\frac{1}{K_c}\left(1 + \frac{s_{min}}{s_{max}}\right) + \left(\frac{2s_y}{s'_e} - 1\right)\left(1 - \frac{s_{min}}{s_{max}}\right)} \quad [10]$$

Since K_c is a function of spring index c, values of C_w may be plotted in the form of charts for various spring indexes and various values of s_y/s'_e.

For design purposes we introduce a factor of safety N so that the working stress becomes

$$s_{w_{max}} = C_w(s'_e/N) \quad [11]$$

It may be shown that this amounts to assuming a curve CD, shown in Fig. 3, parallel to the line PA and intersecting the axis of abscissas at a distance s_y/N from O.

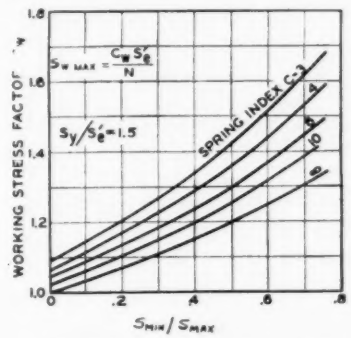


FIG. 4 CHART FOR DETERMINING THE WORKING-STRESS FACTOR C_w FOR $s_y/s'_e = 1.5$ AND $q = 1$ (FULL SENSITIVITY)

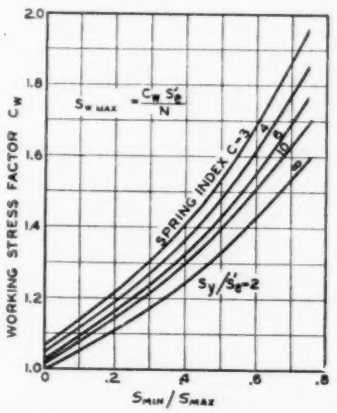


FIG. 5 CHART FOR DETERMINING THE WORKING-STRESS FACTOR C_w FOR $s_y/s'_e = 2$ AND $q = 1$ (FULL SENSITIVITY)

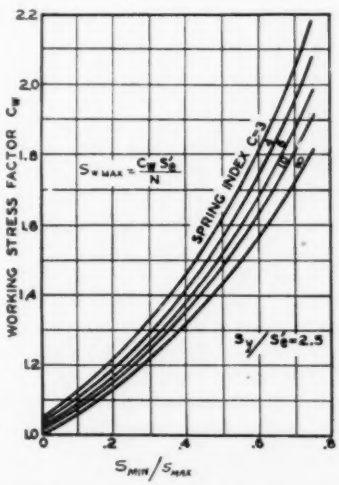


FIG. 6 CHART FOR DETERMINING THE WORKING-STRESS FACTOR C_w FOR $s_y/s'_e = 2.5$ AND $q = 1$ (FULL SENSITIVITY)

[14] The ultimate strength in torsion could be substituted for s_y if desirable. In some cases this would give results in closer agreement with tests, but the results obtained by using the yield point will, in general, be on the safe side.

[15] It is assumed that the variable component of stress is not greater than the static component, i.e., only stress conditions corresponding to the line PA in Fig. 3 are considered.

Charts showing the relation between C_w and s_{min}/s_{max} for various spring indexes at given values of s_y/s'_e are shown in Figs. 4, 5, and 6. Thus in Fig. 5 the ratio s_y/s'_e is taken equal to 2. These charts show clearly how the maximum permissible stress increases with increase in s_{min}/s_{max} and that this increase is greater for the springs of smaller index.

Method of Calculation Where Material Is Not Completely Sensitive to Stress Concentration

A similar procedure may be used for cases where the material is not completely sensitive to stress concentration. While fatigue tests on actual springs will be required to determine the actual sensitivity to stress concentration of the various materials made in various wire sizes, it may be expected that the sensitivity will be lower for the smaller wire sizes and for the coarser grained carbon steels, and higher for the larger sizes and the fine-grained alloy steels.

The sensitivity to stress concentration may be measured by the so-called "sensitivity index" q defined as[16]

$$q = \frac{K_f - 1}{K_c - 1} \quad [12]$$

where K_f is the fatigue strength reduction factor, i.e., the effective stress-concentration factor as determined by fatigue tests, and K_c is the theoretical stress-concentration factor as determined by strain measurements or mathematical analysis. For materials completely insensitive to stress-concentration, $K_f = 1$ and from Equation [12] $q = 0$, while for complete sensitivity $K_f = K_c$ and $q = 1$. Intermediate cases are represented by values of q between zero and unity.

To introduce the sensitivity index q in the expression for s_{max} in Equation [8], we note that in figuring the variable stress s_v we are to use the fatigue strength-reduction factor K_f rather than the theoretical stress-concentration factor K_c. But s_{max} and s_{min} have already been calculated by multiplying the nominal stress by the factor K which includes K_c. Hence, we must multiply the value given by Equation [5] by the ratio K_f/K_c. This gives

$$s_v = \left(\frac{s_{max} - s_{min}}{2}\right)\frac{K_f}{K_c} \quad [13]$$

But from Equation [12]

$$K_f = 1 + q(K_c - 1) \quad [14]$$

Substituting Equation [14] in Equation [13]

$$s_v = \left(\frac{s_{max} - s_{min}}{2}\right)\left(\frac{1 + q(K_c - 1)}{K_c}\right) \quad [15]$$

Using Equations [15] and [6] in Equation [7] we find

$$s_{max} = \frac{2s_y}{\frac{1}{K_c}\left(1 + \frac{s_{min}}{s_{max}}\right) + \left(\frac{2s_y}{s'_e} - 1\right)\left(1 - \frac{s_{min}}{s_{max}}\right)\left(\frac{1 + q(K_c - 1)}{K_c}\right)} \quad [16]$$

It is seen that where $q = 1$ (full sensitivity) Equation [16] reduces to Equation [8]. For $q = 0$ (no stress-concentration effect), the denominator of this equation is in effect divided by K_c, which means that stress-concentration effects due to curvature as represented by K_c are neglected entirely both for the static- and variable-stress components.

[16] See the second reference of footnote 11 for a discussion of "sensitivity index."

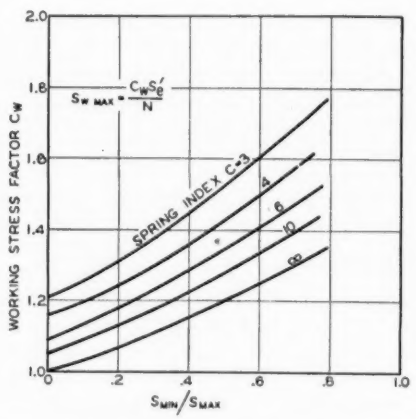


Fig. 7 Chart for Determining the Working-Stress Factor C_w for $s_y/s'_e = 1.5$ and $q = 1/2$

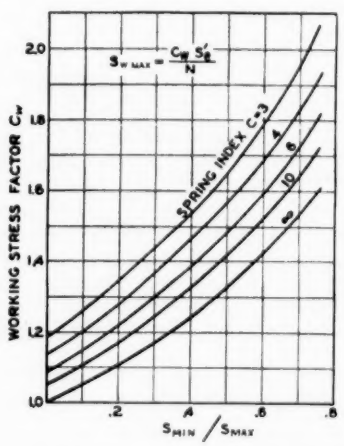


Fig. 8 Chart for Determining the Working-Stress Factor C_w for $s_y/s'_e = 2$ and $q = 1/2$

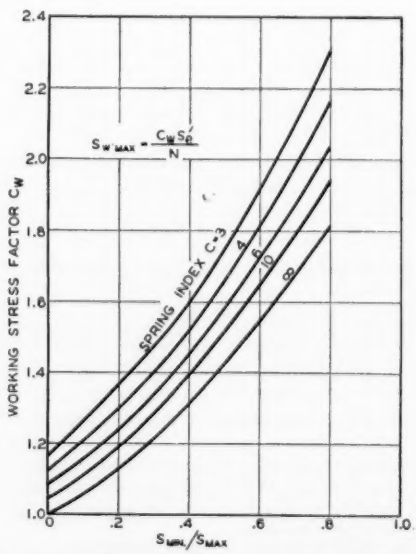


Fig. 9 Chart for Determining the Working-Stress Factor C_w for $s_y/s'_e = 2.5$ and $q = 1/2$

From Equation [16] we also obtain

$$C_w = \frac{2s_y/s'_e}{\frac{1}{K_c}\left(1+\frac{s_{min}}{s_{max}}\right)+\left(\frac{2s_y}{s'_e}-1\right)\left(1-\frac{s_{min}}{s_{max}}\right)\left(\frac{1+q(K_c-1)}{K_c}\right)} \quad \ldots . [17]$$

This also reduces to Equation [10] when $q = 1$.

Charts showing C_w as a function of spring index c for $q = 1/2$ and for s_y/s'_e equal to 1.5, 2.0, and 2.5 are given in Figs. 7, 8, and 9, respectively. These charts are given merely as examples to show the effect of a reduction in the sensitivity of the material, as measured by the index q, on the allowable stress.

Example of Application of Charts

To illustrate the application of the charts shown in Figs. 4, 5, and 6 in practical work, let us assume we have a spring of 1/4-in. wire diameter and 3/4-in. mean coil diameter, i.e., $c = 3$. Assume that fatigue tests on springs of large index and on the same size wire subjected to a pulsating load (zero to maximum) yield of value of endurance limit $s'_e = 60{,}000$ lb per sq in., while torsion tests show a yield point in torsion[17] of 120,000 lb per sq in. Assume further that the spring is under a fatigue range from s_{min} to s_{max}, where $s_{min} = 0.5\, s_{max}$. To be on the safe side, we assume full sensitivity of the material ($q = 1$). Since $s_y/s'_e = 2$, we use the chart shown in Fig. 5 and from this chart for $c = 3$, $s_{min}/s_{max} = 1/2$, we find $C_w = 1.53$. Thus, from Equation [9], we may on this basis expect fatigue failure at $s_{max} = C_w s'_e = 1.53 \times 60{,}000 = 92{,}000$ lb per sq in. Assuming a factor of safety $N = 1.5$, the working stress, from Equation [11], would be $s_{w_{max}} = C_w s'_e/N = 92{,}000/1.5 = 61{,}000$ lb per sq in., this stress being calculated by using the K-factor of Fig. 1. If the spring index were 10 instead of 3, the factor C_w would, from Fig. 5, be 1.39 and the working stress $s_{w_{max}} = (1.39 \times 60{,}000)/1.5 = 56{,}000$ lb per sq in., assuming a factor of safety $N = 1.5$.

To show the effect of a reduction in the q factor, let us assume that endurance tests on springs of smaller index in this particular wire size show a value of $q = 1/2$. Then, using the chart shown in Fig. 8, ($s_y/s'_e = 2$, $q = 1/2$) we find $C_w = 1.65$ for $c = 3$ and $s_{min}/s_{max} = 1/2$. In this case, the allowable working stress, using a factor of safety $N = 1.5$, would be $s_{w_{max}} = C_w s'_e/N = (1.65 \times 60{,}000)/1.5 = 66{,}000$ lb per sq in.

Discussion of Limitations of Method for Finding C_w

The method of design previously discussed is based on the assumption of a linear relation between the static- and variable-stress components necessary to cause fatigue failure, i.e., the line PA in Fig. 3 is assumed. Usually the experimental results will be somewhat above this line as indicated. Hence, if the value of the zero to maximum endurance limit s'_e were determined by actual tests on a spring of large index and having the wire size under consideration, it appears that if the line PA were used as a basis the results would be on the safe side. In this connection it is necessary to determine the value of s'_e by tests on the actual wire size used, since the endurance values may change considerably between small and large sizes. It has also been assumed that the line of failure PA tends to approach the yield point in torsion. In some cases it may be found that this line tends to approach the ultimate strength in torsion.[18] In such cases the ultimate strength s_u could be used instead of s_y in applying the charts and formulas. If this were done, however, a higher factor of safety should be used than would be the case otherwise.

In deriving the formulas for C_w, it has been assumed that the endurance properties of the material do not change between springs of small and large index, assuming a given wire size. Although this appears to be a reasonable assumption, there are cases where it may not be strictly true. For example, in cold-wound springs made from a given size of wire, the springs coiled to the smaller diameters will be subjected to the greatest amount of cold working; hence, such springs may possess somewhat different endurance properties than would otherwise be expected. If a stress-relieving treatment were given, this difference would possibly be slight. A similar result may be expected where springs are quenched after coiling, since the effect of the heat treatment may be different for springs of different indexes.

It is common practice, in the manufacture of compression springs, to compress the spring solid, thus giving it a permanent set. The effect of this is to introduce residual stresses of opposite sign so that when the spring is under the working load, the peak stress will be reduced by the amount of such residual stresses. For this reason the tendency will be for the fatigue strength to be increased because of the presence of these stresses. Because of the sharper curvature of the springs having smaller indexes, it will be easier to introduce such residual stresses in these springs, and for this reason it may happen that the fatigue strength of the springs of smaller index may be increased by such treatment to a relatively greater extent than is the case for springs of large index. Such effects may be taken into account by changes in the sensitivity index q when more test data are available.

In the present analysis, the effects of eccentricity of loading due to the presence of the end turns have been neglected. Some years ago the author had occasion to make a series of tests to determine eccentricity of loading in small helical springs in compression. It was found that such eccentricities may increase the maximum stress from 4 to 30 per cent, depending on the shape and form of the end turns, and the total number of turns; considerable scatter was evident in the results. For an accurate check of the effects of wire curvature on the endurance, it would seem desirable to make provision for applying central loads to springs under fatigue test in order to eliminate the disturbing effects due to eccentricity of loading which may tend to mask the effects due to wire curvature. In this connection, reference may be made to the torsional fatigue tests carried out on spring wires by Weibel.[19] These tests showed endurance limits in zero to maximum torsion of about 105,000 lb per sq in. compared with values around 65,000 lb per sq in. obtained on complete springs made from the same material by Zimmerli.[18] As mentioned by Weibel, part of this difference may be due to internal stress in the cold-wound spring and part may be due to the greater probability of the presence of serious flaws in the complete spring. It is the author's opinion, however, that eccentricity of loading of the complete spring may also play an important part in explaining this very considerable difference. Such eccentricity would lower the endurance limit of the complete spring in comparison to that which is obtained in simple torsion tests on straight specimens.

Comparison With Test Results

To the author's knowledge the most comprehensive series of tests yet made to check the effect of spring index on endurance strength of helical springs are those carried out by Zimmerli[6] who made a series of tests on springs of 0.148-in. diameter pre-

[17] If the yield point is not sharply defined it may, as an approximation, be taken as that point where the plastic strain is 0.2 per cent. See the paper: "Concerning the Yield Point in Tension," by J. M. Lessells, Proceedings of the American Society for Testing Materials, vol. 28, part 2, 1938, p. 387.

[18] "Permissible Stress Range in Small Helical Springs," by F. P. Zimmerli, Engineering Research Bulletin No. 26, University of Michigan, Ann Arbor, Mich.

[19] "The Correlation of Spring-Wire Bending and Torsion Fatigue Tests," by E. E. Weibel, Trans. A.S.M.E., vol. 57, November, 1935, paper RP-57-1, pp. 501–516.

tempered Swedish valve-spring wire with indexes c varying from 3.5 to 12. It will be of interest to compare these test results with those obtained by the application of the charts shown in Figs. 4, 5, and 6, which assume full sensitivity to stress concentration ($q = 1$). Values of the minimum and maximum points (s_{min} and s_{max}) of the limiting stress ranges as found by Zimmerli[6] for the various indexes are given in the second and third columns of Table 1. To determine the zero to maximum endurance limit s'_e, for a spring of large index, we use the test results for $c = 11.9$ as a basis, taking $s_{min} = 19{,}000$ lb per sq in., $s_{max} = 91{,}000$ lb per sq in., and $s_{min}/s_{max} = 0.21$. Assuming tentatively $s_y/s'_e = 1.5$, we find, from Equation [8], $s_{max} = 1.11\, s'_e$ for $s_{min}/s_{max} = 0.21$. Solving, $s'_e = 91{,}000/1.1 = 82{,}700$ lb per sq in. Since the yield point in torsion for this material will be around 120,000 lb per sq in. (or somewhat above the elastic limit in torsion) we may assume $s_y/s'_e = 1.5$ with sufficient accuracy and use the chart shown in Fig. 4. Using this value of s'_e and assuming the minimum values of the stress range (s_{min}) as given, the limiting values of maximum stress s_{max} were computed using the chart shown in Fig. 4 and Equation [9]. (Equation [8] may also be used for calculation.) The computed values of s_{max} thus found are given in the fifth column of Table 1. For comparison, values of the limiting stress range $s_{max} - s_{min}$ as found by Zimmerli,[6] and as determined by calculation, are given in the last two columns of Table 1.

TABLE 1

Spring Index c	Limiting stresses[a] from Zimmerli's fatigue tests[b]		Calculated values of limiting stresses (using Fig. 4)		Limiting range in stress $s_{max} - s_{min}$	
	s_{min}	s_{max}	s_{min}	s_{max}	By test (Zimmerli)[b]	Calculated (Fig. 4)
3.50	14000	100000	14000	95500	86000	81500
4.55	19000	94000	19000	96000	75000	77000
7.00	19000	93000	19000	93500	74000	74500
9.10	19000	90000	19000	92000	71000	73000
11.90	19000	91000	19000	91000	72000	72000

[a] These stresses figured using the K-factor.
[b] See footnote 6.

A comparison of the figures in the last two columns of Table 1 indicates that the test and calculated values of limiting stress range differ by only a few per cent. This offers some indication that the method of determining working stress by using Equation [8], which assumes full sensitivity to stress concentration ($q = 1$), will give results in fair agreement with actual fatigue tests, at least for some materials and wire sizes.

The tests by Edgerton mentioned previously[7] were made on two groups of springs coiled from $^3/_4$-in. diameter bar stock, one group of index 3 and the other of index 5. The endurance limits calculated by the use of the conventional formula were practically the same for the two groups of springs, while the endurance limits calculated by using the K-factor differed considerably. This would indicate that for these springs neither the bar curvature nor the direct shear stress have any effect on the endurance. As mentioned by Edgerton,[7] this is a most surprising conclusion, and further check tests are necessary. Since the tests by Zimmerli[6] discussed previously do show a definite effect on the endurance properties produced by wire curvature, even in small springs, it would seem that a more comprehensive series of tests should be made before drawing definite conclusions. In particular it would seem desirable to carry out a series of tests in which provision is made to obtain a centrally applied load on the spring in order to eliminate as much as possible the disturbing effects due to eccentricity of loading which, as mentioned previously, may tend to mask the effects produced by wire curvature.

Conclusions

The method described in this paper for determining the effect of bar or wire curvature on working stresses in helical springs under fatigue loading is suggested as a rational basis for the application of our present knowledge of the endurance properties of materials to this problem. While it is realized that the method may have to be modified in the light of future test results, it appears to yield reasonable results in certain cases when applied to existing fatigue-test data. Further fatigue tests wherein eccentricity of loading is eliminated would appear desirable in order to obtain a more accurate determination of the sensitivity index q for various wire sizes and spring materials. For design purposes, an additional factor to take eccentricity of loading into account may be introduced, but a discussion of this is beyond the scope of the present paper.

Acknowledgment

The author wishes to express his appreciation to L. W. Chubb and R. E. Peterson of the Westinghouse Research Laboratories for their interest and suggestions in connection with the preparation of this paper.

Discussion

Practical Aspects of Turbine Cylinder Joints[1]

GEORGE L. BASCOME.[2] The author's remarks relative to several factors which must be considered in the design of the longitudinal joint of a steam-turbine casing, subjected to high pressure and temperature, apply as well to a bolted flanged joint for steam piping.

Some years ago the writer was engaged in the testing of various types of flanged joints under consideration for a 1500-lb 900 F installation, for the power station of the Houston Lighting and Power Company. These tests were conducted jointly by the engineers of the utility and the fabricating companies responsible for the installation of the piping. At that time little was known concerning the behavior of the steel at high temperatures, so that in conducting these tests nothing was taken for granted, and careful micrometer measurements were made of bolts and flanges before and after each test. Thermocouple attachments were made at various locations about each joint, in order that temperature changes might be noted between the several parts.

At the beginning, it was felt that a type of joint, with ground faces, without a gasket would prove most satisfactory, but within a few hours the joint leaked. A close examination showed that due to the high temperature a slight warping of ground faces had occurred. This indicated that a metal-to-metal type of joint for this class of service should be thoroughly stress-relieved prior to finishing the surfaces. It is not recalled that the author made mention of this fact, but I presume that the turbine casings are stress-relieved prior to machining, in order to prevent distortion from heat under operating conditions.

In order to determine the best bolting material and a safe working stress, a crude but satisfactory relaxation-test method was employed. The bolts were stressed to 40,000 lb per sq in., between the faces of a cylindrical sleeve, or cage, carefully micrometered, and then submerged for a week in melted lead at 900 F. At the end of the period the increase in bolt length over the initial length was noted. This information determined the bolting steel best fitted for this service and also the permissible stress which might be employed. Micrometer measurements of the bolts during erection insured that proper unit pressure was provided to the joint faces.

The author stated that a net unit pressure between joint faces of the turbine casings of three times the internal pressure was necessary to maintain a tight joint under loaded conditions. The tests on pipe joints just referred to, indicated that the net pressure need be between two and three times the internal pressure when the joint was under pressure, including calculated bending moments. It is surprising that these early tests made with limited facilities should agree so closely with the author's findings.

It might be stated that the final design of the joint for this installation included a thin gasket to care for any slight warping of the joint faces, with a calculated gasket area, such as to provide the suitable net unit pressure with a safe working stress in the bolting material. It was stated that no leakage occurred during the first two years of service in any of the joints so designed and erected.

In conclusion it seems that if due consideration is given to all factors, a bolted joint can be designed to meet the most exacting service, and such information as was disclosed by the author is of great value to the profession.

C. RICHARD SODERBERG.[3] The relation between bolt pitch and flange dimensions which is given in the equation $t \geqq \sqrt{\left[d^2 + \left(\frac{P}{2}\right)^2\right]}$ appears somewhat arbitrary, although it is probably a safe working rule. The following presents a point of view which throws further light on this important question.

In a straight joint of the type shown in Fig. 3 of the paper, the flange may be represented as a double beam, each half of dimensions $b \times h$ held together by a series of concentrated forces P produced by the bolts. See Fig. 1 herewith. The material between the neutral axes of the beams is elastic under the influence

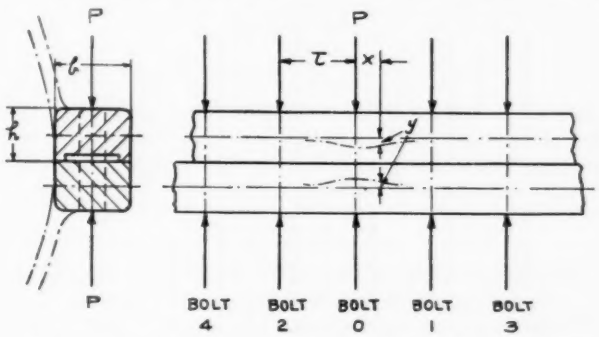


FIG. 1 STRAIGHT JOINT WITH CONCENTRATED BOLT LOADS

of the compression forces. Considering one of these beams, therefore, it may be regarded as supported on an elastic foundation, the modulus of which is

$$k = \frac{2Eb}{h} \text{ lb per sq in.} \quad [1]$$

The influence of relief and bolt holes will be neglected, although there are no serious difficulties in taking these into account, at least approximately. If the two beams deflect symmetrically toward each other a distance y, the resulting compression load per unit length is

$$q = ky \text{ lb per in.} \quad [2]$$

Thus, if the deflection of the beam can be expressed as a function of x, the resulting load distribution is also obtained.

The theory for a beam on an elastic foundation is well known,[4] and the results may be used to gain an insight into the present problem. Considering first the effect of a single force P in a long beam, the results of this theory give for the load distribution

$$q = q_0\varphi \quad [3]$$

[1] By C. B. Campbell. Published in the June, 1938, issue of the JOURNAL OF APPLIED MECHANICS, Trans. A.S.M.E., vol. 60, 1938, p. A-49.

[2] Valuation Engineer, State Corporation Commission, Richmond, Va. Mem. A.S.M.E.

[3] Department of Mechanical Engineering, Massachusetts Institute of Technology, Cambridge, Mass. Mem. A.S.M.E.

[4] "Applied Elasticity," by S. Timoshenko and J. M. Lessells, Westinghouse Technical Night School Press, E. Pittsburgh, Pa., p. 132, et seq.

where φ is a function of βx.

$$\varphi = e^{-\beta x} (\cos \beta x + \sin \beta x) \quad [4]$$

$$\beta = \sqrt[4]{\left(\frac{k}{4EI}\right)} = \frac{\sqrt[4]{6}}{h} = \frac{1.565}{h} \quad [5]$$

and

$$q_0 = \frac{Pk}{8\beta^3 EI} = \frac{\sqrt[4]{6}}{2}\frac{P}{h} = 0.783\frac{P}{h} \quad [6]$$

This distribution is shown in Fig. 2. At about $\beta x = 2.4$, the load becomes zero, and for larger values it is negative. Negative

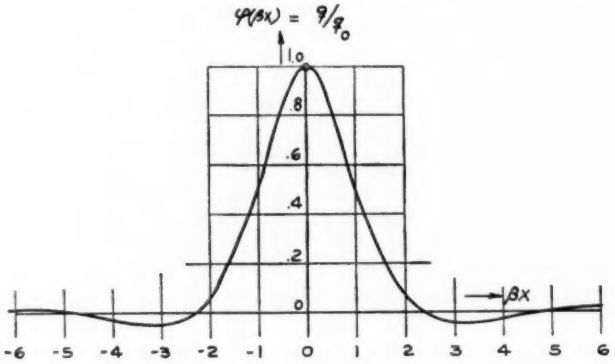


FIG. 2 LOAD DISTRIBUTION OF ONE BOLT

loads are excluded for practical reasons, but this is not serious since the case of a single bolt has only theoretical interest. The characteristic feature of the load distribution lies in the fact that it is a periodic function having a wave length

$$L = 2\pi/\beta \approx 4h \quad [7]$$

To obtain the load distribution for a series of bolts spaced at a pitch τ, it is necessary to add a series of such functions. The convergence is rapid for the pitches of practical interest, so that only a few terms are necessary. For convenience, it is assumed that the pitch is made a certain fraction ν of the wave length, so that

$$\tau = \nu L \quad [8]$$

Referring to Fig. 1, it is easily seen that the load distribution becomes

$$\begin{aligned} &\text{Bolt 0} \qquad \text{Bolts 1,3, ...} \qquad \text{Bolts 2,4, ...} \\ q = q_0 [\varphi(\beta x) &+ \varphi(2\pi\nu - \beta x) + \varphi(2\pi\nu + \beta x) \\ &+ \varphi(4\pi\nu - \beta x) + \varphi(4\pi\nu + \beta x) + \ldots.] \end{aligned} \quad [9]$$

where q_0 represents the maximum load due to a single bolt.

Fig. 3 shows the load distributions obtained in this manner for $\nu = \tau/L = \infty$, $^3/_4$, $^1/_2$, $^3/_8$, and $^1/_4$. Fig. 4 shows the ratio of q_{min}/q_{max} plotted as a function of ν.

These results must be regarded in the light of rough approximations of the real problem, but they permit the following conclusions:

1 The longitudinal load distribution is a sensitive function of the ratio of flange thickness h to bolt pitch τ.

2 The load midway between two bolts will fall to zero if the bolt pitch is greater than $(^3/_4)L = 3h$, which corresponds to

$$h \leq (^1/_3)\tau \quad [10]$$

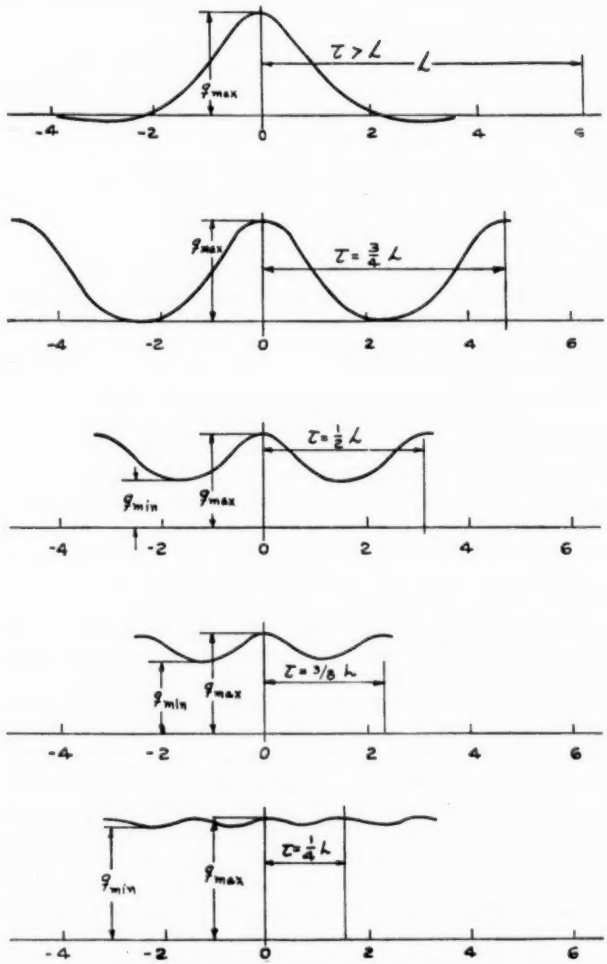


FIG. 3 LOAD DISTRIBUTION FOR VARIOUS VALUES OF $\nu = \tau/L$

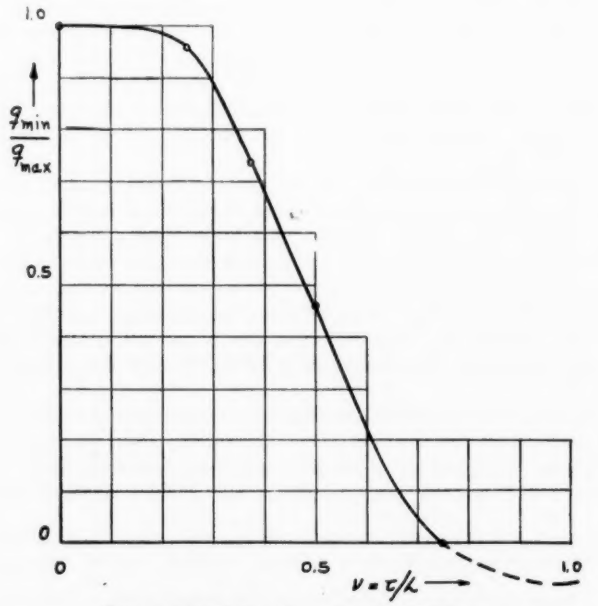


FIG. 4 RATIO q_{min}/q_{max} AS A FUNCTION OF ν

Flange thicknesses equal to or below this value must be regarded as definitely unsatisfactory.

3 In order to obtain approximately uniform load distribution along the flange, it is necessary that the bolt pitch be greater than $(1/4)L$, which corresponds to

$$h \geqq \tau \quad \ldots\ldots [11]$$

A satisfactory simple rule, therefore, is to make flange thickness and bolt pitch equal.

It is important to note that the width of the flange does not influence the longitudinal load distribution. This is not true, of course, if the flange is made so wide as to have appreciable deflection over the relieved portion. The design rules proposed by the author will satisfactorily guard against this.

Author's Closure

Mr. Bascome is correct in assuming that the turbine cylinder casings are stress-relieved prior to final machining.

As stated by Mr. Soderberg, the relation between bolt pitch and flange thickness $h \geqq \sqrt{\left[d^2 + \left(\frac{P}{2}\right)^2\right]}$ is arbitrary, and is purely a rule of thumb which results in rational and well proportioned design. Combined with Mr. Soderberg's conclusion that flange thickness should be equal to or greater than the bolt pitch, we find that the distance d, Fig. 3 of the original paper, should not exceed 0.9 × bolt pitch, an entirely rational result.

Stress Model of a Complete Airship Structure[1]

L. B. Tuckerman.[2] The evident care and foresight used by the authors cannot but impress anyone who appreciates the pitfalls which may lie on the path from test to structure. It would be difficult to find any essential property of their model which has been overlooked.

The discussion of the scales N is complete, and it is particularly gratifying to find second-order effects associated with buckling considered adequately. The model girder is ingenious, with its slotted tubes so placed that advantage is taken of the slot to reduce the torsional stiffness without affecting the flexural stiffness. Finally, the spacing of the tubes in the middle solves well the problem of controlling the axial stiffness.

A nice instance of the care exercised is shown in the method used in soldering the wires and allowing for the reduced tension in the bulkhead wires due to the installation of the shear wires. In speaking of the wires, it is the hope of the writer that the authors will publish a full description of their specially designed tensometer for measuring the tension in a taut wire. The writer has seen what he believes is this instrument or one like it, and it should be better known.

The many check tests, and especially those which lead to the results shown in Figs. 14, 15, and 16 inspire confidence. It is easy to say that the precautions taken throughout are fairly evident. So they are in many cases after they have been pointed out. Unfortunately, however, much testing is done in which adequate consideration is not given to what should be evident. It is a pleasure to find an experimental investigation carried out with the attention to important detail which the authors have shown.

[1] By L. H. Donnell, E. L. Shaw, and W. C. Potthoff. Published in the June, 1938, issue of the Journal of Applied Mechanics, Trans. A.S.M.E., vol. 60, 1938, p. A-67.

[2] Assistant Chief, Division of Mechanics and Sound, National Bureau of Standards, Washington, D. C.

An Improved Method for Calculating Free Vibrations in Systems of Several Degrees of Freedom[1]

Geometric Interpretations of the Method of Successive Approximations

M. A. Biot.[2] The method of successive approximations using matrix algebra is based on the same fundamental principles as what is generally known as Vianello's or Stodola's method. Starting from an arbitrary deflection of the system, the corresponding dynamic forces are evaluated assuming a given angular frequency, for instance, $\omega = 1$. A new set of deflections due to these dynamic forces is found, and with this new set, the process can be repeated. A certain number of such processes leads toward a convergent shape of the deflection, which is the shape of the fundamental mode, and the ratio of two successive deflections is equal to the square of the fundamental angular frequency.

It will be shown hereafter that a simple geometrical interpretation exists for this method.

Consider a system of two masses m_1 and m_2 connected to each other and to a fixed base B by springs, as shown in Fig. 1. It is assumed that there are only two degrees of freedom determined by the displacements x_1 and x_2 of the masses.

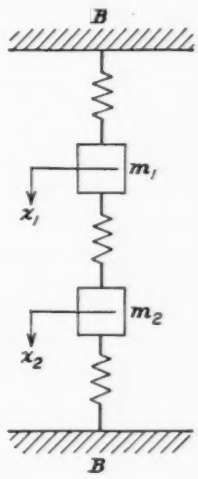


Fig. 1

Two loads P_1 and P_2 applied, respectively, to the masses m_1 m_2 produce deflections

$$\left.\begin{aligned} x_1 &= d_{11}P_1 + d_{12}P_2 \\ x_2 &= d_{21}P_2 + d_{22}P_2 \end{aligned}\right\} \quad \ldots\ldots [1]$$

when the d's are the influence numbers. According to Maxwell's reciprocity theorem, we have

$$d_{12} = d_{21} \quad \ldots\ldots [2]$$

The elastic potential energy under the loads P_1 P_2 is

$$W = \tfrac{1}{2}(x_1P_1 + x_2P_2) = \tfrac{1}{2}[d_{11}P_1^2 + 2d_{12}P_1P_2 + d_{22}P_2^2] \ldots [3]$$

Equations [1] of this discussion may be written as

$$\left.\begin{aligned} x_1 &= \partial W/\partial P_1 \\ x_2 &= \partial W/\partial P_2 \end{aligned}\right\} \quad \ldots\ldots [4]$$

For the sake of simplicity let us first assume that the two masses are both equal to unity. The problem of finding the natural vibrations of our system will be solved if we know values of P_1 and P_2 such that they are proportional to the displacements x_1 and x_2 produced by these forces, that is

$$P_1/x_1 = P_2/x_2 = \omega^2 \quad \ldots\ldots [5]$$

This follows from the fact that $\omega^2 x_1$ and $\omega^2 x_2$ are the inertia forces when the angular frequency is ω.

This problem is illustrated geometrically as follows:

Consider a coordinate system P_1 P_2. Equations [1] of this discussion define a linear transformation of the vector P of components P_1 P_2 into a vector X of components x_1 x_2. The vector X

[1] By Winston M. Dudley. Published in the June, 1938, issue of the Journal of Applied Mechanics, Trans. A.S.M.E., vol. 60, 1938, p. A-61.

[2] Professor, Department of Physics, Columbia University, New York, N. Y.

is a so-called linear vector function of P. The natural vibrations will correspond to those directions for which the vectors P and X are parallel.

Because of the symmetric character ($d_{12} = d_{21}$) of its coefficients, the transformation Equations [1] are associated with a certain conic

$$W = {}^1/_2[d_{11}P_1^2 + 2d_{12}P_1P_2 + d_{22}P_2^2] = \text{const}. \quad [6]$$

in the coordinate system $P_1\,P_2$. If we rotate the coordinate axes so as to make them coincide with the axes of the conic, the vector P is expressed by new components $\overline{P}_1\,\overline{P}_2$ and the equation of the conic becomes

$$W = {}^1/_2[D_{11}\overline{P}_1 + D_{22}\overline{P}_2^2] \quad [7]$$

Since the elastic potential energy is positive, D_{11} and D_{22} must be positive and the conic defined by Equation [6] must be an ellipse.

Equations [4] show that the linear vector function, expressed by

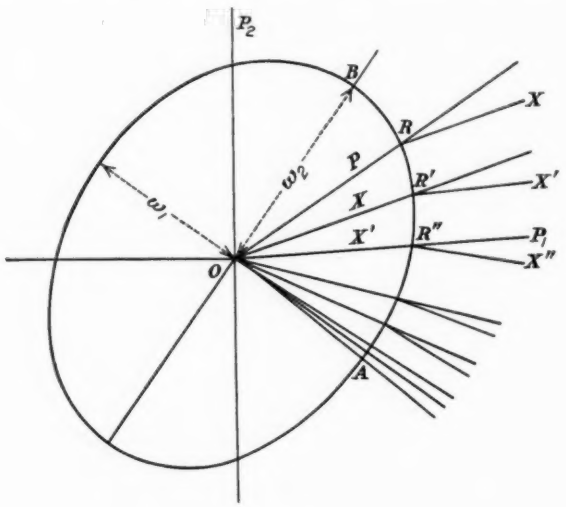


FIG. 2

Equations [1], is entirely defined by the scalar function W. They may be written in the form

$$X = \text{grad}\, W \quad [8]$$

They also show that when P is in the direction of a radius vector OR of the ellipse, then X is parallel with the normal to the ellipse at point R, as shown in Fig. 2.

Equation [5] will therefore be satisfied if we choose P in the direction of one of the axes of the ellipse. Hence, these directions determine the natural vibrations of our system.

The small axis corresponds to the lower frequency. This can be seen from Equations [7] and [8]. Along the axis $\overline{P}_1$ we have

$$\bar{x}_1 = \partial W/\partial \overline{P}_1 = D_{11}\overline{P}_1$$

According to Equation [5], $D_{11} = 1/\omega_1^2$ where ω_1 is the natural angular frequency corresponding to the axis $\overline{P}_1$. In the same way, if ω_2 is the angular frequency corresponding to the axis $\overline{P}_2$ we have

$$D_{22} = 1/\omega_2^2$$

The equation of our ellipse is

$$\frac{\overline{P}_1^2}{\omega_1^2} + \frac{\overline{P}_2^2}{\omega_1^2} = \text{const} \quad [9]$$

The axes are proportional to the corresponding natural frequencies.

The problem of finding the natural vibrations of our system is reduced to that of finding the direction of the axis of the ellipse defined by Equation [6].

The point which we finally want to stress is that the direction of the minor axis may be found by a method of successive approximations, which is fundamentally the same as the matrix method explained by the author in his paper,[1] and may also be considered as the geometrical illustration of what is generally known in the literature as Vianello's or Stodola's method.

Considering the ellipse defined by Equation [6] and shown in Fig. 2, we start with a vector P lying along the radius vector OR and draw a vector X in the direction of the normal to the conic at R. Now we take a radius vector OR' parallel with X and obtain by the same process a vector X'. Taking a radius-vector OR'' parallel with X' we find a new vector X'', and so on. It can be seen that the direction of these successive vectors will converge toward the direction of the smaller axis.

According to Equation [5], the ratio of the successive values of x will tend toward ω_1^2, the square of the fundamental angular frequency.

We have supposed the masses to be equal, but, as we shall see, this is not a restriction. Suppose the masses to be respectively m_1 and m_2. The transformation Equations [1] will be replaced by

$$\left.\begin{aligned} x_1' &= d_{11}m_1x_1 + d_{12}m_2x_2 \\ x_2' &= d_{21}m_1x_1 + d_{22}m_2x_2 \end{aligned}\right\} \quad [10]$$

The natural vibrations will be determined by a solution of this system such that

$$x_1'/x_1 = x_2'/x_2 = \omega^2 \quad [11]$$

We note that the coefficients of the transformation Equations [10] are not symmetric. However, they can be made symmetric by a simple change of variable. They may be written

$$\left.\begin{aligned} \xi_1' &= d_{11}m_1\xi_1 + d_{12}\sqrt{m_1m_2}\,\xi_2 \\ \xi_2' &= d_{21}\sqrt{m_2m_1}\,\xi_1 + d_{22}m_2\xi_2 \end{aligned}\right\} \quad [12]$$

with $x_1'\sqrt{m_1} = \xi_1'$; $x_1\sqrt{m_1} = \xi_1$; $x_2'\sqrt{m_2} = \xi_2'$; and $x_2\sqrt{m_2} = \xi_2$. The ellipse associated with the transformation is defined by the equation

$$d_{11}m_1\xi_1^2 + 2d_{12}\sqrt{m_1m_2}\,\xi_1\xi_2 + d_{22}m_2\xi_2^2 = \text{const}$$

The general case is hereby reduced to the one with equal unit masses, studied previously in this discussion.

Using the matrix notation, the transformation Equations [12] are written as

$$\begin{Bmatrix} \xi_1' \\ \xi_2' \end{Bmatrix} = \begin{Vmatrix} d_{11}m_1 & d_{12}\sqrt{m_1m_2} \\ d_{21}\sqrt{m_2m_1} & d_{22}m_2 \end{Vmatrix} \begin{Bmatrix} \xi_1 \\ \xi_2 \end{Bmatrix} \quad [12]'$$

For a system with three degrees of freedom, the transformation would be

$$\begin{Bmatrix} \xi_1' \\ \xi_2' \\ \xi_3' \end{Bmatrix} = \begin{Vmatrix} d_{11}m_1 & d_{12}\sqrt{m_1m_2} & d_{13}\sqrt{m_1m_3} \\ d_{21}\sqrt{m_2m_1} & d_{22}m_2 & d_{23}\sqrt{m_2m_3} \\ d_{31}\sqrt{m_3m_1} & d_{23}\sqrt{m_3m_2} & d_{33}m_3 \end{Vmatrix} \begin{Bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{Bmatrix} \quad [13]$$

with the variables $\xi_i' = x_i'\sqrt{m_i}$ and $\xi_i = x_i\sqrt{m_i}$.

The associated surface is here an ellipsoid with axis proportional to the three natural frequencies. The transformation Equations [13] define a linear vector function ξ' of ξ. Drawing a radius vector OR of the ellipsoid in the direction of ξ, the vector ξ' will be parallel with the normal to the ellipsoid at R. Applying the same transformation of direction to ξ', we obtain a new direc-

tion ξ'' and so on. It can be seen that by this process we travel along the ellipsoid and we will tend to reach the smallest axis, and hence, the fundamental mode of vibration of the system. However, if we start the process with a vector ξ lying in the plane of the medium axis and the biggest axis, the process will lead us to the medium axis or the first harmonic. Starting with a vector ξ having no component along the minor axis of the ellipsoid means that we start the process of successive transformations with a set of deflections from which the fundamental mode is completely eliminated.

In practice, however, it is never possible to eliminate exactly the fundamental mode, and the process will lead toward a direction which might become very close to that of the medium axis, but will finally converge toward the minor axis along a path *DEA* shown in Fig. 3. An improved method is to eliminate at every step the component along the minor axis. It is shown in the paper[1] how this can be done. The elimination of the component along the minor axis may be affected by expressing that the scalar product of the vector $\xi(\xi_1\xi_2\xi_3)$ and the vector $\alpha(\alpha_1\alpha_2\alpha_3)$ along the minor axis is zero.

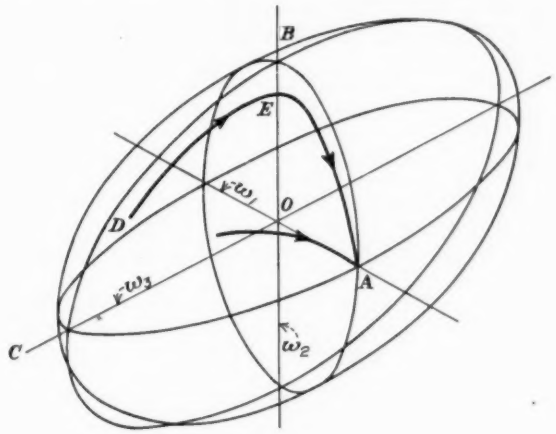


FIG. 3

$$\xi_1\alpha_1 + \xi_2\alpha_2 + \xi_3\alpha_3 = 0$$

or since $\alpha_i = \sqrt{m_i}\, a_i$, where a_i are the deflections of the fundamental mode

$$\Sigma m_i x_i a_i = 0 \ldots\ldots\ldots\ldots\ldots\ldots [14]$$

which is the expression [12] found by the author in his paper.[1] It has the same form as Equation [21] of this discussion in the theory of orthogonal functions. In the case of a system of n degrees of freedom the analogy will apply for an ellipsoid in n-dimensional space.

Relation With the Properties of Orthogonal Functions

A load $p(x)$ per unit length applied to a one-dimensional elastic system as a beam or a string produces a deflection

$$w(x) = \int_0^l g(x\xi)p(\xi)d\xi \ldots\ldots\ldots\ldots\ldots [15]$$

where $g(x\xi)$ is the influence number. We have

$$g(x_1\xi) = g(\xi_1 x)$$

The transformation Equations [1] correspond to the transformation Equation [15]. Introducing dynamic forces

$$p(x) = \omega^2\rho(x)\, w(x)$$

where $\rho(x)$ is the mass per unit length, we find the integral equation

$$w(x) = \omega^2 \int_0^l \rho(\xi)\, g(x\xi)\, w(\xi)\, d\xi \ldots\ldots\ldots [16]$$

which corresponds to the matrix Equation [3] of the paper,[1] $\{w\} = \omega^2[d][m][w]$.

The dynamical matrix $[d][m]$ corresponds to $g(x\xi)\rho(x)$ in the integral Equation [16] of this discussion. The mode of vibrations $w_i(x)$ and the corresponding frequencies ω_i are solutions of the integral Equation [16] of this discussion. The functions $w_i(x)$ are orthogonal, and satisfy the relation

$$\int_0^l \rho(x)\, w_i(x)\, w_j(x)\, dx = 0 \ldots\ldots\ldots\ldots [17]$$

where $i \neq j$. This corresponds to the relation given later in Equation [18] of this discussion. It may be considered as the mathematical formulation of the fact that the axis of the associated ellipsoid are orthogonal. In the example given in this discussion, for instance, if $\xi_1^0\ \xi_3^0\ \xi_3^0$ and $\xi_1'\ \xi_2'\ \xi_3'$ are the components of two of the axes of our ellipsoid, we have

$$\Sigma\xi_i^0\xi_i' = 0$$

or

$$\Sigma m_i x_i^0 x_i' = 0 \ldots\ldots\ldots\ldots\ldots [18]$$

An arbitrary deflection $w(x)$ may be developed into a series of orthogonal functions

$$w(x) = A_0w_0(x) + A_1w_1(x) + \ldots\ldots A_nw_n(x) \ldots\ldots [19]$$

The coefficients are determined by the expression

$$A_i = \frac{\int_0^l \rho(x)\, w_i(x)\, w(x)\, dx}{\int_0^l \rho(x)\, w_i^2(x)\, dx}$$

If we apply to $w(x)$ the transformation

$$w'(x) = \int_0^l \rho(\xi)\, g(x\xi)\, w(\xi)\, d\xi \ldots\ldots\ldots [20]$$

each function $w_i(x)$ in the series of Equation [19] is transformed into $w_i(x)/\omega_i^2$, and

$$w'(x) = \frac{A_0}{\omega_0^2} w_0(x) + \frac{A_i}{\omega_1^2} w_1(x) + \ldots\ldots\ldots\ldots \frac{A_n}{\omega_n^2} w_n(x)$$

Applying this transformation a sufficient number of times we find[2]

$$w^{(k)}(x) = \frac{1}{\omega_0^{2k}}\left[A_0w_0(x) + A_1\left(\frac{\omega_0}{\omega_1}\right)^{2k} w_1(x) + \ldots\ldots\right]$$

Hence the shape of $w^{(k)}(x)$ tends toward that of the fundamental mode $w_0(x)$ if $A_0 \neq 0$ and we also have

$$\frac{w^{(k)}(x)}{w^{(k+1)}(x)} \to \omega^2$$

If we want the deflection to converge toward the first harmonic, we must choose an initial deflection $w(x)$ such that $A_0 = 0$, or

$$\int_0^l \rho(x)\, w_0(x)\, w(x)\, dx = 0 \ldots\ldots\ldots\ldots [21]$$

The initial transformation Equation [20] corresponds to the linear transformation

$$\{w'\} = [d][m][w] \ldots\ldots\ldots\ldots\ldots [22]$$

and the condition given by Equation [21] corresponds to the relation expressed by Equation [14].

[2] "Mechanical Vibrations," by J. P. DenHartog, McGraw-Hill Book Company, New York, N. Y., 1934, p. 179.

The method of successive approximation was first suggested in 1898 without much justification by Vianello[4] in connection with the buckling problem of a beam and the equation

$$Dw(x) + Pw(x) = 0$$

where D denotes a differential operator. This is essentially the same method as that derived from the integral equation. In fact, the integral equation is equivalent to a very general class of differential equations of the type

$$Dw(x) + \omega^2\rho(x)\, w(x) = 0 \quad \ldots\ldots\ldots\ldots [23]$$

Such parallelism between the properties of systems with a finite number of degrees of freedom and continuous systems was to be expected since the integral equation can be considered as the limiting case of a system of linear equations with an infinite number of variables.

C. Concordia.[5] It may be pointed out that Mr. Dudley's methods, although developed only for a system of point masses, may be extended to systems having also rotary inertia. Then the mass matrix (i.e., the components of the mass tensor along the reference frame in question) in the last equation on page A-61 of his article will in the general case be the sum of a mass matrix having nonzero components only in the main diagonal and a moment-of-inertia matrix all of whose components may be nonzero. Such a general mass matrix is illustrated in Equations [11] and [17] of the article "The Use of Tensors in Mechanical Engineering Problems," *General Electric Review*, vol. 39, July, 1936, p. 335. The form having only main diagonal components can, of course, always be obtained by transforming either to normal system coordinates or to the principal axes of each rigid body of the system.

Author's Closure

Mathematically inclined readers should be helped to an understanding of the method of successive approximations by Professor Biot's geometrical interpretation of it. He also points out that when the number of degrees of freedom of a system is increased to infinity, we pass from a matric grouping of linear equations to a single integral equation. This may lead to additional practical applications of the latter subject.

Mr. Concordia suggests the applicability of matric methods to systems involving moments and products of inertia. It should perhaps be explained that a tensor is a matrix whose elements are differential operators.

Correspondence has also been received from Messrs. Collar and Duncan,[6] the originators of the method. It is stated that no special methods are required for the case of equal roots. Two sets of initial amplitudes will then give two different modes of the same frequency. Physical motions may then consist of any linear combination of these modes. A forthcoming book, "Elementary Matrices," by R. A. Frazer, W. J. Duncan, and A. R. Collar, Cambridge University Press, England, deals fairly exhaustively with the applications of matrices to dynamics and differential equations. These writers have independently found the property noted in Equation [10] of the author's paper, and give a shorter proof of it. They also bear out the statement at the end of part 3 of the paper, that high accuracy is necessary if several modes of a problem are to be found.

[4] "Graphische Untersuchung der Knickfestigkeit gerader Stäbe," by L. Vianello, *Zeit. V.D.I.*, vol. 42, July-December, 1898, pp. 1436–1448.

[5] Engineering Division, Central Station Department, General Electric Company, Schenectady, N. Y.

[6] See Bibliography of author's paper, references (1) and (6).

Heat Dissipation Through Annular Disk or Fin of Uniform Thickness[1]

Sam P. Soling.[2] This paper presents a good solution to the problem of the use of extended surfaces for heat transfer. The application of the results by the use of the base area of the fin and n, the "fin effectiveness," is rather unusual, not presenting as clear a picture of fin performance as might be obtained otherwise.

In many cases involving the use of finned surface, most of the external surface is extended surface and not primary surface. A factor to be applied to the usual heat-transfer equation involving surface coefficient, external surface area, and temperature difference is preferable, as it would never be greater than unity and would readily indicate the effectiveness of an extended surface. A copper fin might give a 90 per cent effective surface while, for an identical steel fin under the same conditions, the effectiveness would be but 60 per cent.

This factor, which may be designated by X, can be obtained by dividing n by the ratio of the fin surface area to the fin base area,

or
$$X = \frac{n}{\dfrac{R^2 - r_0^2}{2r_0 y}} = \frac{2\,r_0}{R^2 - r_0^2}\,\frac{\xi}{m}$$

In the example that the author has given, the fin effectiveness is 26.8, while the ratio of the fin surface to the fin base is 26.7, making X equal to 1.006. It is apparent that X should be slightly less than 1.0. The discrepancy is due to the difficulty of reading ξ from Fig. 2.

It appears to the writer, that it is more valuable to know that fins applied as in the author's example are almost completely effective, rather than that a multiplier of 26.8 should be used for the fin base area.

The factor X has been used for some time for finned surfaces with approximate values computed from the equation for bar fins,[3] and its use as outlined has proved valuable.

The author's Fig. 2 would be more useful if the two variables mR and mr_0 were extended to include higher values of surface coefficients, of approximately 300 Btu per sq ft per deg per hr.

Author's Closure

As Mr. Soling has pointed out there are two methods for expressing the heat transfer from a fin. The first scheme, which has been in use here and in Europe for some time, makes the comparison by means of the fin efficiency, Mr. Soling's quantity X, which is the ratio of the actual heat transferred by the fin to the heat which would be transferred if the whole of the fin were at the temperature of the base. The fin efficiency provides a good means of comparing two different fins but gives no information as to the actual heat-transfer without recourse to the fin area.

The second method using the fin effectiveness eliminates the necessity for calculating the fin area and at the same time provides a means of comparison although the measuring stick is slightly different from that of fin efficiency. The method given in the paper was developed specially to eliminate the step of finding the fin area in the calculation of the total heat-transfer from a finned tube but it is apparent that the adoption of either method of computation depends largely upon personal choice and any special conditions which may be involved.

[1] By William MacGregor Murray. Published in the June, 1938, issue of the Journal of Applied Mechanics, Trans. A.S.M.E., vol. 52, 1938, p. A-78.

[2] Assistant Research Engineer, York Ice Machinery Corporation, York, Pa. Jun. A.S.M.E.

[3] "Heat Transmission," by W. H. McAdams, McGraw-Hill Book Co., Inc., New York, N. Y., 1933.

Book Reviews

Men of Mathematics

MEN OF MATHEMATICS. By E. T. Bell, Professor of Mathematics at the California Institute of Technology. Simon and Schuster, New York, 1937. Cloth, 6¼ × 9¼ in., xv and 592 pp., illustrated, $5.

REVIEWED BY A. NÁDAI[1]

THE well-known author of this book, past-president of the Mathematical Association of America, has endeavored to acquaint the nonmathematical reader with the principal achievements and with the short life stories of a few men who have been the founders of "the Queen of the Sciences" from ancient to present times. The reader of this book will first find singular quotations gathered on three pages from many of the finest mathematical as well as nonmathematical thinkers expressing their ideas concerning the purposes for which mathematics has been developed. For Kronecker, "God made the integers, all the rest is the work of man;" for the Pythagoreans, "number rules the universe;" for Plato, "God ever geometrizes;" for the great Jacobi, "God ever arithmetizes;" but the truest statement of all of them was perhaps made by an analyst and founder of new functions, "a mathematician who is not also something of a poet will never be a complete mathematician" (K. Weierstrass).

The intentions of the author in this remarkable book may have been inspired by another celebrated book, namely, "Great Men," by W. Ostwald, published some thirty years ago. While in Plutarch's still so fascinating comparative biographies the ancient ages with their partially fabulous achievements are reflected, our rational times find or search the scales for a comparison in other properties of the great men. Ostwald selected his great men exclusively from among the physicists and chemists of the nineteenth century. Ostwald saw in the founders of the exact sciences two distinct types of men, which he somewhat associated with the different temperaments distinguished already by the ancient writers and philosophers: The quickly reacting "romanticists," among whom he believes can be found the great inspirers of youth and the founders of new schools, and the second type, the phlegmatic and precise "classicists." Davy, and Liebig, were for Ostwald romanticists, while Faraday and Helmholtz were classicists for him.

This new book by Bell may be characterized by his own words, "the gist of the story is in the lives and personalities of the creators of modern mathematics," no schemes being advanced for their classification other than the fundamentality and originality of their accomplishments. In contrast to other biographers, Bell attempts to show "that a mathematician can be as human as anybody else." According to him the mathematicians as a group have been "vigorous, alert, keenly interested in many things outside mathematics," statements which it is hoped will contribute much to reverse current ideas still perhaps prevailing among educated men for whom mathematicians have been concentrations of dryness, abstraction, and absent-mindedness. Bell quotes that during the eighteenth and early nineteenth centuries quite a number were soldiers, for the reason that mathematics and mechanics were not yet separated from each other nor from military science. We would like to add that perhaps quite a number were also outstanding engineers. The list comprising some thirty names starts with a few early Greeks (Zeno and Archimedes) and ends with Henri Poincaré and Georg Cantor (1918). It is known that many proofs of an early age in mathematics (the Sumerian, Babylonian, and Egyptian) exist. But these times were excluded, and leaving the Greeks, the list jumps to the names of Descartes, Fermat, Pascal, Newton, Leibnitz, the Bernoulli family, Euler, Lagrange and others up to Weierstrass, Riemann, and Hermite. The book contains 30 portraits; the frontispiece is Albrecht Duerer's "Melancholia" with the winged genius absorbed in contemplation over the unsolved puzzles of nature, a large crystal block resting beside, and a rainbow appearing above, the seated genius in the sky. The author has admirably understood how to paint with a few strokes the personalities he has selected. His essays do not tire the reader with scholarly details. He must have studied them and their works most carefully. He discloses, for example, quite a remarkable knowledge of prewar conditions and of life in academic circles in Europe and lifts the veil from circumstances under which appointments were made, which have subsequently influenced the lives of the great as well as the less great. One of the most thrilling life stories is perhaps that of the unfortunate Galois, "an all-time record in stupidity" of circumstances adverse to genius, combined with the absent-mindedness of influential contemporary authorities and ending at 21 years of age in a duel. Under "Master and Pupil" the author describes the personalities of the "father of modern analysis," of the unselfish, helpful, broad-minded Weierstrass and that of his brilliant pupil, Sonja Kovalevski. May one not be tempted to admit, when recalling his belief, that pure mathematics is perhaps the art of creation of entirely new forms for exact expression and of new harmonies between functional relations, that the boundaries between the arts and the exact sciences have, after all, been drawn too artificially and that a great worker in the purest of all sciences has much in common with a creator of music, or a sculptor of forms.

[1] Consulting Mechanical Engineer, Westinghouse Research Laboratories, East Pittsburgh, Pa. Mem. A.S.M.E.

The Scientist in Action

THE SCIENTIST IN ACTION. By William H. George, Royal Society Sorby Research Fellow, and Honorary Lecturer in Physics, University of Sheffield. Williams and Norgate Ltd., London, 1936. Cloth, 5½ × 8¼ in., 355 pp., 31 figs., 10s 6d.

REVIEWED BY C. RICHARD SODERBERG[2]

THIS book cannot be classified as easy or entertaining; it commends itself to the reader chiefly through the immense significance of the subject to the welfare of our civilization. The author's purpose is to classify the processes whereby the scientific worker achieves his results. To this end he analyzes in separate sections "The Scientific Outlook," "Getting Scientific Facts," and "Arranging Scientific Facts."

The author professes "an uncompromisingly nonphilosophical point of view" and treats scientific research simply as "a problem in human action." In the opinion of this reviewer, this limitation is to be regretted, and may be one of the reasons for a certain dryness and tediousness of presentation, which makes for hard reading. This is particularly true of the first two sections.

The last section, occupying practically one half of the book, is by far the most interesting and readable. Some of the quota-

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tions from scientific writers help to relieve the monotony. Edmund Gosse's "Father and Son" provides an illuminating example of the processes by which the six-year-old son applied the experimental technique to the efficacy of prayer. He felt a need for a large humming top, a need that impressed him as much more real than the conversion of the heathen, or the restitution of Jerusalem to the Jews. When the father forbade him to pray for things like expensive humming tops, he reached the fatal suspicion that the whole objection lay in the fact that the humming top was too expensive to buy. The famous sentence "I determined, however, to test the matter for myself" was related to the injunction against idolatry. This he subjected to an experimental test with completely negative results. His interpretation of this result left his belief in the existence and power of God intact; it merely convinced him that his father was not really acquainted with the divine practice of idolatry.

There is one important aspect of the human mind which for some reason has been given very little emphasis. This is the intuitive process, or the influence of unconscious behavior upon scientific work. Whatever the ultimate scientific explanation of the intuitive process, it cannot be denied that it has played an important rôle in the final formulation of many discoveries and inventions. This omission may be justified by the author's nonphilosophical point of view, but it has eliminated much that is valuable and pertinent. Planck's statement that "creative minds have felt a personal aversion from the idea of unfolding before the public gaze those delicate threads of thought out of which their productive hypotheses were woven, and the myriad other threads which failed to be interwoven into any final pattern" is interpreted as a reluctance of great scientists "to give in public detailed account of their failures." This appears somewhat unfair. Is it not equally true that the great minds usually have been unable to give a detailed account of the mental processes by which they reached their discoveries? All scientific discoveries have been preceded by long hours of concentrated conscious activities, out of which intuitive flashes were precipitated.

Experimental research, particularly in our industries, requires one setting which money alone cannot buy. This is a body of capable research workers animated by enthusiasm for their work and by zeal for opening up new perspectives in their field. No one has yet explained just what the premises are for this, but one of the essential elements is probably some one man with the gift of leadership. Once this condition has been realized, something very precious has been created, something which is easily lost if adequate moral and financial support is withheld. Few institutions are thus endowed more than once in their history. Industrial research furnishes many examples where this rare setting has been brought about without full realization of its potential value on the part of the financial leaders. Lack of support to an institution of this kind means unpredictable losses in scientific and technical advance; any amount of support to an institution lacking in this setting usually means wasted expenditures.

A better understanding of the mental processes by which great men arrive at their discoveries will probably not enable lesser minds to produce great thoughts. If the world at large could be persuaded to value the veneration for truth, and more particularly, the disdain for Absolute Truth, which has always been the prerogative of great minds in the world of science, some hope could be held out for the eventual solution of our social problems. While the author disclaims any such purpose he has, nevertheless, succeeded well in demonstrating that the most precious contribution of men of science to the world is just the ability to accept truth regardless of whether or not it coincides with preconceived notions.

Physics

RECOLLECTIONS AND REFLECTIONS. Sir J. J. Thomson, O.M., D.Sc., F.R.S., etc. Master of Trinity College, Cambridge. The Macmillan Company, New York, 1937. Cloth, 6 × 9¼ in., viii and 451 pp., illustrated, $4.

THE NEWER ALCHEMY. By Lord Rutherford, O.M., F.R.S., Cavendish Professor of Experimental Physics in the University of Cambridge. Cambridge University Press. The Macmillan Company, New York, 1936. Cloth, 5 × 7½ in., viii and 67 pp., xiii plates, $1.50

REVIEWED BY J. C. BOYCE[3]

FEW, if any, university professorships can claim as distinguished incumbents as the Cavendish Professorship of Experimental Physics in the University of Cambridge. The chair, created in 1871, was held by James Clerk Maxwell until his death in 1879. Following the five-year term of Lord Rayleigh, J. J. Thomson was elected in 1884 and retired in 1919. Lord Rutherford, the fourth Cavendish Professor, maintained until his untimely death on October 19, 1937, the high traditions established by his illustrious predecessors.

J. J. Thomson entered Cambridge as an undergraduate in Maxwell's time and has resided there ever since as research student, fellow, professor, and then as Master of Trinity College. "Recollections and Reflections" deals far more with the scientific and academic life of that whole period than it does with the personal achievements of its author. In that sense it is a very modest autobiography, and yet delightful personal glimpses are scattered through it. Most of the previous generation of British physicists were Thomson's colleagues or friends; many of the present generation were numbered among his students. In 1895 the University began to offer its degree for research to graduates of other universities. From then on a considerable number of students from abroad were to be found every year in the Cavendish Laboratory, coming largely but by no means entirely from the British Empire and from the United States. Apropos of this Thomson writes, "The advantage gained by our own students by their intercourse with men of widely different training and experience, of different points of view on political, social, and scientific questions, of very different temperaments, can, I think, hardly be overestimated."

Strictly scientific matters are reserved for the last quarter of the book. In the earlier portions glimpses are given of many scientists and men of affairs, Maxwell, Rayleigh, Stokes, Kelvin, Crookes, Arthur Balfour, Woodrow Wilson, to mention a few. Readers of this journal will be particularly interested in the account of Dr. Routh, who influenced so greatly the teaching of analytical statics and of rigid dynamics. Thomson made three visits to the United States, in 1896, 1903, and 1923. He comments on the development of American universities during that period as well as on aspects of student life here. Thomson looked into the matter of psychical research and devotes a brief chapter to his quite unconvincing (to him as well) experiences.

The strictly scientific portion of the book gives an elementary account of some of Thomson's researches, including those related to the conduction of electricity in gases and the discovery of the electron.

The title of the work is an accurate one. It is pleasant to note that Thomson's life has been full of interesting people and interesting problems. "I owe to the Laboratory not merely the opportunities it has given me for indulging my scientific tastes in a way that would not have been possible in any other place. I owe to it besides many valued friends, and memories of friendships which can no longer be more than memories."

Turning from these "Recollections and Reflections" we find in

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Rutherford's "The Newer Alchemy" a brief, nonmathematical and extremely lucid survey of the present state of knowledge of the nuclei of atoms and of transmutation. It may be read with interest by anyone having an elementary knowledge of physics and chemistry, yet to experts it speaks with the authority of the greatest master of the subject.

Reviewing the properties of radium and of the other radioactive substances, Rutherford describes the various elementary particles which take part in transmutation experiments—the alpha particle, proton, neutron, deuteron, and negative and positive electrons. Even the most severe critic must approve of the omission of that recondite particle, the neutrino, from a book of this scope. Methods of detecting individual particles are described, particularly that most spectacular one due to C. T. R. Wilson in which the track of the individual particle is made visible by water drops condensed on the ions formed along its path. Photographs of such tracks appear in illustration of several transmutations. In 1919 Rutherford found evidence that alpha particles directed against nitrogen could transmute a few of the nitrogen atoms into oxygen. Here then was the first true controlled transmutation, but a most expensive and inefficient alchemy it was, for only one alpha particle out of about 50,000 scored an effective hit. Further experiments with alpha particles produced transmutations among other light elements and led to the discovery of the neutron by Chadwick in 1932 and of artificial radioactivity by Irene Curie and F. Joliot in 1933.

The practice of this alchemy was limited by the available supply of radium as a source of alpha particles. Rutherford describes the electrical methods (including the cyclotron of E. O. Lawrence and the electrostatic generator of Van de Graaff) by which ions of hydrogen or of helium may be given more energy than the alpha particles from radium, and may be supplied far more copiously and conveniently. Rutherford describes some of the transmutations effected by these methods on the larger (but still tiny) scale. In conclusion he disposes of two familiar fallacies. Gold indeed may be produced by this alchemy, but from platinum rather than from lead (and at a cost far greater than the cost of the platinum). Large quantities of energy are produced by individual atomic encounters, but the occurrence of such successful hits is so rare among the many bombarding projectiles that "there is little hope of gaining useful energy from atoms by this process."

Science and Human Experience

SCIENCE AND HUMAN EXPERIENCE. By Herbert Dingle, Assistant Professor of Astrophysics, Imperial College of Science and Technology, South Kensington. First edition. The Macmillan Company, New York, 1932. Cloth, $5^1/_2 \times 8^3/_4$ in., 136 pp., $1.75.

REVIEWED BY J. ORMONDROYD[4]

NOTHING is more amusing and diverting than paradox. The popular writers on modern science have reveled in the paradoxical contrast between the pedestrian findings of classical physics and the more startling results of recent years. Jeans and Eddington instantly come to mind as the leaders of this activity. They have diverted the world so successfully that wealth and honors have been showered upon them. But it is no exaggeration to state, on sober reflection, that their attempts at explanation have ended in bewildering the reader. They have convinced their public that science in its modern form is completely unintelligible and apparently lacking in sense.

There have been inevitable reactions to this stressing of paradox and surprise. It is significant that the philosophical writings of the modern masters of physics—Planck, Bohr, Einstein, and Bertrand Russell—have always, during this period, been nearer to the common man's outlook. But their works, because they are more sober, come less to the attention of everyday readers.

Among the recent works which attempt to bring modern scientific activity within the understanding of the ordinary educated man, at least four should be mentioned. "Science and Human Experience" by Herbert Dingle, which appeared in 1932, "Through Science to Philosophy" by the same author published in 1937, "The Renaissance of Physics" by Karl Darrow, 1937, and "The Evolution of Physics" by Einstein and Infeld, 1938. All of these essays are healthy antidotes against the sensational mysticism so widespread in this field.

Our review concerns the oldest of these publications, "Science and Human Experience" by Herbert Dingle. In a certain sense Professor Dingle considers this older work superseded by his later book, "Through Science to Philosophy." But this book, which is longer and more abstruse than the older volume, still only reemphasizes the attempt at intelligibility started in the earlier, simpler discussion. Professor Dingle, a man widely versed in modern science, is not rated among the brilliant physicists and philosophers of our time. In fact, he identifies himself with the ordinary mortal who is disturbed and dissatisfied with the confused picture of modern science drawn in the great mass of current popular scientific literature. For these very reasons he possesses the qualities necessary to give an interpretation more satisfying to the intelligent layman.

Dingle defines science as "the recording, augmentation, and rational correlation of those elements of our experience which are actually or potentially common to all normal people." The entire content of "Science and Human Experience" consists in the development of this text. The source of all our usual confusion, according to Dingle, lies in the supposed contrast and interaction between "subject and object," between sensations and a real, external world. But "real, external world" and "object" have no meaning to Dingle. The true contrast is between experience (remembered sensation) and reason—both completely resident in the subject. The chief activity of science lies in the attempt of reason to correlate experience, that is, to bring the description of experiences to the simplest and most orderly form, based on the fewest possible elements. Anything which makes correlation possible is permissible, whether it take the form of mechanical models—which can be imagined to have possible physical counterparts—or purely mental concepts which have no prototype in familiar surroundings. The search for correlation is something entirely different from a search for "reality." If we insist on tying our understanding of modern science to some criterion of everyday "reality," we are doomed to confusion and intellectual despair. The train of successful scientific thought starts from observation—permits itself the widest latitude of abstraction and hypothesis—and ends up with an accurate description of the observation. This is the only reality we must demand of science. This is the reality Dingle demands.

The strangest thing about this defense of modern science is that it is exactly the defense which classical science needed. The apologist who wrote the preface to "The Revolution of the Heavenly Bodies" by Copernicus used the very same argument as long ago as 1543.

The correlating devices consist of abstraction and hypotheses. Hypotheses are classified by Dingle into three kinds: Hypotheses which can be and are checked (becoming thereby "facts"); hypotheses which cannot be checked since the conditions assumed in them do not exist within the reach of observation, such as the nebular hypothesis; and hypotheses which by their very nature can never be checked, such as the atomic hypothesis.

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These last Dingle christens "correlates." All three are admissible and useful in the business of science—which is the correlation of experience.

The theory of relativity represents the extremity of abstraction and the quantum theory is the extremity of hypothesis. "The apparent absurdity of Relativity is entirely a matter of limited experience, and that of the quantum theory is entirely a matter of limited thought. Relativity teaches us about phenomena because the notions it deals with are abstracted from our experience of phenomena; it surprises us because it reveals possibilities of experience which we never contemplated. The quantum theory surprises us because it reveals *impossibilities* of experience which we never contemplated; we thought we were justified in extending to atoms the abstractions discussed in relativity, and we find that we were not."

The meat of this work is contained in the first 102 pages. Chapters 10 and 11 discuss "Science and Criticism" (of Art) and "Science and Religion." They do not add anything worth-while to the viewpoint developed in the work.

It is indeed unfortunate that Dingle will never reach the audience that his more acrobatic competitors have charmed. A little bit of Jeans and Eddington should always be followed by large doses of Dingle. However, the poison is served in large doses and the antidote comes in a very small package.

Theory of Statically Indeterminate Structures

THEORY OF STATICALLY INDETERMINATE STRUCTURES. By Walter Maxwell Fife, Associate Professor, and John Benson Wilbur, Associate Professor, Department of Civil Engineering, Massachusetts Institute of Technology. First edition. McGraw-Hill Book Company, Inc., New York, 1937. Cloth, 6 × 9¼ in., ix and 248 pp., figs., $3.50.

REVIEWED BY J. M. GARRELTS[5]

THIS TREATISE possesses a superior technique in its clear and thorough development of principles of structural mechanics. The authors have stressed the importance of the so-called classic methods of analysis on the basis that they are essential to an understanding of structural action. Those familiar with the theory of statically indeterminate structures will generally agree with this viewpoint.

In the first chapter, fundamental assumptions involved and consequent limitations of the theory are carefully considered. Requirements for structural stability and the conditions for equilibrium of statically indeterminate structures are well presented. The equations for virtual work are evolved from a consideration of the virtual work of distortion produced during virtual deformation of an element under the action of given stress intensities. The development of the equations in a general form to include the effects of bending, direct stress, temperature differences, and shear is a splendid demonstration suggesting the power of the virtual-work principle as a tool in structural analysis. Application is made of the law of virtual work as an expedient for determining the displacement of any point in a distorted structure. The bar-chain method for computing deflections of the joints of a truss is clearly explained and a number of examples illustrate the principle. The moment-area theorems and the conjugate-beam method are adequately demonstrated.

The chapter on stress analysis for statically indeterminate structures is outstanding for its clearness of presentation. The law of virtual work and Castigliano's theorem are applied to the analysis of statically indeterminate trussed structures and redundant frames with moment-resisting joints. Simplifications resulting from the use of the elastic center are discussed. The development of the slope-deflection equations is concise and should be readily followed by the student. The necessary free-body diagrams for determining equations of equilibrium are included with the problems illustrating the application of these equations. The principle of moment distribution for determining the stresses in structures with moment-resisting joints is developed from slope-deflection equations.

Of particular interest is the chapter on influence lines for analyzing the structural effect of variable load conditions. The elastic curve and Maxwell's law are used to construct influence lines for deflections, stresses in redundant truss members, and redundant reactions. The conception of a statically indeterminate primary structure is discussed as a basis of mechanical methods of analysis, and its advantages in drawing influence lines are considered. In constructing influence lines for continuous beams of several spans, the fixed-point method illustrated is an aid in solving the three-moment equations.

In considering secondary stresses in trusses, approximate analyses for determining these secondary effects are presented and the correlation of primary and secondary stresses discussed. The methods of Manderla, Mohr semigraphic and moment distribution are treated. Under the heading of "Participating Stresses," the authors have properly called attention to a problem that may require consideration in a bridge design.

The solution of many illustrative problems in the text facilitates application of the theory to practical problems, while a number of problems for solution by the student are included following the text material.

This book warrants careful study by those interested in the theory of statically indeterminate structures. The clarity of development and presentation is stimulating and the thoroughness with which fundamental principles are demonstrated is gratifying.

[5] Assistant Professor of Civil Engineering, Columbia University, New York.

Statically Indeterminate Frameworks

STATICALLY INDETERMINATE FRAMEWORKS. Thomas F. Hickerson, Professor of Civil Engineering, University of North Carolina. The University of North Carolina Press, 1937. Cloth, 8½ × 11 in., xi and 205 pp., figs. and tables, $3.

REVIEWED BY JOHN B. WILBUR[6]

"STATICALLY Indeterminate Frameworks," is a revision of Professor Hickerson's previous volume, "Structural Frameworks." In addition to a complete revision of the former text, the new volume has been extended to deal with statically indeterminate frames.

The tremendous impact of "moment distribution" on the field of structural thought is clearly in evidence in Professor Hickerson's treatment of the subject. Starting with a consideration of restraint, continuity, and stiffness, the author discusses the slope-deflection theorem, and from this as a base, develops the method of moment distribution. His treatment of this method is unusually complete. Applications to structures of many types are considered, with special emphasis on building frames and rigid-frame bridges. Many tables are given which greatly reduce the labor involved in the procedures suggested.

As a treatment of new methods of applying basic theory, this book should be of value not only to students, but to designers as well. The writer is of the opinion, however, that if the volume were intended to be a complete treatment of statically indeterminate frameworks, more discussion of the so-called "classical"

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methods of stress analysis for statically indeterminate structures would have been in order. The method of moment distribution is essentially a technique of solution, rather than a method based on principles which are fundamentally new. It seems to the writer that this volume tends to accent the "how" of stress analysis, rather than the "why," and the question is raised in his mind as to whether the searching student of structures may not feel the need of further and more basic treatment of the subject.

Theory of Modern Steel Structures

THEORY OF MODERN STEEL STRUCTURES. By Linton E. Grinter, Ph.D., C.E., Professor of Structural Engineering, Agricultural and Mechanical College of Texas. The Macmillan Company, New York, 1936. Volume 1, fabrikoid, 6 × 9 in., xxv and 317 pp., illustrated, $4; Volume 2, xvii and 285 pp., illustrated, $4.50.

REVIEWED BY L. C. MAUGH[7]

IN MAKING a review of two volumes that treat of so comprehensive a subject as the theory of modern steel structures, the reviewer immediately encounters difficulty in discussing the relative importance of the selected subject matter and its arrangement in a limited amount of space. To make the discussion as definite as possible, the two volumes will be treated separately.

Volume 1 is restricted to the analysis of determinate structures although this classification is, in many instances, based more upon the assumptions that are made for purposes of analysis than upon the physical characteristics of the structures. The first chapter provides an interesting summary of the historical development of the various types of present steel structures, following which a well-illustrated and clearly described chapter on the fundamentals of stress analysis is given. Both graphical and algebraic methods of determining reactions and internal stresses are presented as well as shear and bending-moment diagrams for stationary loads.

The application of theory of stress analysis to mill buildings, highway and railway-truss bridges, plate girders, and tall steel buildings is presented in subsequent chapters. The material on truss bridges, although following the usual arrangement, is exceptionally complete and well illustrated. It is in the chapter on tall-building frames that the most noticeable differences from current textbooks are found. Here we find a discussion of structural arrangement of beams and columns, of analysis for wind stresses based on approximate methods, of the use of brackets and knee braces, and the advantages of various types of connections. These practical descriptions and illustrations should be helpful both to the beginner and to the more experienced designer. It seems that some discussion of space structures could well be placed in the first volume rather than the second. In general the subject matter of the entire first volume is so well illustrated by free-body diagrams and photographs that acquiring a working knowledge of the fundamental concepts of structural theory should be a pleasant task to any interested reader.

In the second volume, which is primarily concerned with the analysis of statically indeterminate structures, there are the same clear descriptions and excellent use of illustrations that characterize the first volume but the choice of subject matter is more open to question. After a general discussion of indeterminate structures, a chapter on the analysis of space structures such as towers, pedestals, and framed domes is given. Only space structures of a determinate type are considered. The next chapter considers the deflection of beams and trusses by various methods such as virtual work, Castigliano's theorem, area moments, and method of elastic weights. As the reader is expected to have had some of the mathematical development of these principles in courses in strength of materials, the text is largely confined to a statement of the fundamental concepts involved and their application to engineering problems. Although this description is undoubtedly sufficient for area moments and the method of elastic weights, the use of the strain-energy methods and the reciprocal theorem could well be given a more extensive treatment. Also, the slope-deflection method would be more useful if developed from the general case for members with variable cross section rather than for constant cross section only.

The analysis of continuous beams and trusses by various methods is then considered, following which a short description of movable, cantilever, arch, and suspension bridges is given. The subject of continuous frames receives considerable attention in the remaining portion of the second volume, principally by a study of the effect of the rotational and linear displacements of the joints. In this analysis, the use of the moment-distribution method in determining the effect of joint rotation, of restraining forces to control translation, together with the general rule of superposition, are all emphasized and illustrated. The subject of arches and rings is treated by both the neutral-point (elastic-center) and column analogy methods. The latter is accorded an unusually comprehensive treatment.

A closing chapter on the analysis of continuous frames by balancing angle changes, a method originally introduced by the author, is presented. Even though this discussion enables some consideration to be given to the effect of semirigid connections, a more extensive treatment of this problem might well be made. In general, a study of the many procedures that are discussed by the author for the analysis of continuous frames gives rise to a feeling that perhaps too many tools are placed before the reader before he is sufficiently familiar with the fundamental concepts. The final result will, of course, depend upon the experience of the reader.

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Principles of Powder Metallurgy

PRINCIPLES OF POWDER METALLURGY. By W. D. Jones, M.E., Ph.D. Lecturer in Metallurgy, Sir John Cass Technical Institute. Longmans, Green & Co., New York, 1937. Cloth, 5½ × 8½ in., xii and 199 pp., 73 figs., $5.

REVIEWED BY S. L. HOYT[8]

"POWDER Metallurgy" by W. D. Jones, "the first work in English on this subject," was opened by your reviewer with more than the usual interest in a new book. The field covered, though limited, has rich possibilities for a fine scientific treatment and interesting technical applications are in daily practice. Many workers in all parts of the world are engaged in this new branch of metallurgy and your reviewer was curious to learn how they were treated by their first "text" in English. It is with real gratification that he can report that the author has written his book with rare skill and fine judgment. This has obviously been a labor of love and we are fortunate that it was one of Dr. Jones's ability who set himself the task.

The scope of the subject matter is adequate. By chapters the book treats of principles of sintering and massive metals; principles of sintering and comminuted metals; physical properties of sintered metals; industrial applications; manufacture of hard metal alloys by sintering; manufacture, properties, and testing of metal powders, in all, 189 pages. Numerous illustrations and sketches are given of equipment and products. The author has gone to great pains in his discussion of the many points that arise

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and records the results of numerous scientific investigations for quantitative treatment. The general metallurgist will doubtless be surprised at the extent and content of the subject matter though it will be the specialist who will really profit by a careful reading. Even in this special field of metallurgy there are specialties and here we find them all carefully and thoroughly discussed.

In view of the excellence of this book, the reviewer hesitates to interject adverse criticism. However, it was only the discussion of tungsten that was found to be inadequate and below the standard of the book as a whole. In view of the large body of competent opinion in this particular field, it is perhaps unfortunate that care was not exercised to make this part one of the strongest. It is only fair to say that the author refers to the well-known text of Smithells for a discussion in English of the subject. Of a different character is the criticism that no attempt is made to acquaint the reader with the names and work of many of those who were responsible for the present position of powder metallurgy; instead the author utilizes sources which are best suited to scientific or technical discussions and they are generally of recent date. With a little additional effort the historical development could have been given. Finally the author appears not to be acquainted with the interesting copper-tungsten electrode material known as Elkonite.

This book is to be highly recommended and for all those engaged in the study or practice of powder metallurgy it rates first place as "must" reading.

Heat and Thermodynamics

HEAT AND THERMODYNAMICS. By Mark W. Zemansky. First edition. McGraw-Hill Book Company, Inc., New York, 1937. Cloth, 6 × 9¼ in., xii and 388 pp., 101 figs., $4.

REVIEWED BY JOSEPH H. KEENAN[9]

THERE IS little in the literature of thermodynamics that serves as a guide from first steps to the place where Gibbs begins. Professor Zemansky in his book "Heat and Thermodynamics" has made a creditable effort to strengthen our elementary literature. He does not attempt to cover all the thermodynamic analysis that one encounters in current literature; but he does cover the ground leading up to Gibbs.

Many engineers would not classify this book as engineering thermodynamics. There can be little argument, however, that all but a few of its 388 pages could be classed as thermodynamics the engineer should know.

From the preface we learn that "The first ten chapters deal with the fundamental ideas of heat, reversibility, entropy, etc., in a manner which strives at a compromise between rigor and simplicity;" and that "An attempt has been made to reduce the number of occasions in which the student is asked to refer back to a previous equation." Both these objectives have been attained to an unusual degree. The material is attractively presented and easy to read. Concerning kinetic theory and thermodynamics we read "The author believes that the study of molecular physics embodied in statistical mechanics and kinetic theory should not run parallel with but should follow a course in thermodynamics, in order that the power and limitations of the macroscopic point of view may be clearly understood and not confused with those of the microscopic point of view." Among his references he makes particular mention of the texts by Roberts, Planck, Lewis and Randall, and Kiefer and Stuart.

The book opens appropriately with a thoughtful and detailed discussion of temperature and its measurement. A chapter entitled "Work" includes the usual discussion of change of volume against a pressure, but is general enough to comprise work corresponding to change of length against a tension, change of area against a surface tension, change of charge against an electromotive force, and change of magnetization against a magnetic field.

The chapter entitled "Heat" adequately defines heat and describes its measurement. The 15-degree calorie is chosen and accounts given of several measurements of its mechanical and electrical equivalents, though the latest measurements by Osborne are not mentioned. It is unfortunate that the only calorie on which there is unanimity as to definition and magnitude, the international-steam-table calorie, is not mentioned. The rest of the chapter on heat is made up of a discussion of heat transfer and of the measurement of heat capacity.

The second law is adequately introduced and is stated essentially in the words of Planck. The chapter on "Reversibility and Irreversibility," which follows that on the "Second Law," is good. It is to be regretted, however, that the author chooses to follow the precedent of Clausius by introducing "the universe" into his reasoning. For example, he says, "a reversible process is one that is performed in such a way that, at the conclusion of the process, both the system and the local surroundings may be restored to their initial states, without producing any changes in the rest of the universe." Only an omnispective scientist or engineer could apply this definition as stated; but even he would doubtless find that for experimental purposes an isolated system is less cumbersome than the universe.

In the chapter on "Entropy" there is a good discussion of "energy that is rendered unavailable for work" in an irreversible process. Though this item was probably inspired by engineering texts, it is a comfort to find that the author has not accepted the usual treatment of unavailable energy which requires for its validity many interpretations and reservations which are almost never expressed.

Two chapters entitled "Applications of Thermodynamics" comprise a collection of illustrative cases that are not especially difficult. Most of them are of interest to the engineer. There are chapters of intermediate difficulty on "Change of Phase" and "Mixtures of Inert Ideal Gases." More advanced chapters deal with "Chemical Thermodynamics," "Ideal Gas Reactions," and "Heterogeneous Systems." In these the author follows the method of Gibbs in treating of equilibrium and equilibrium constants, the phase rule, the effect of pressure on vapor pressure, etc.

The illustrations are good—particularly those in the chapter on "Properties of Pure Substances." The problems which follow each chapter are the product of imagination and originality. Nowhere in the book is the diligence of the author more evident.

Despite the good quality of the work as a whole, a long list of defects, major and minor, might be prepared. The weakest member in the author's structure is doubtless the chapter on the "First Law." Surely it is meaningless to say "The principle of conservation of energy applied to a thermodynamic system and its surroundings is called the first law of thermodynamics" unless the term *energy* is defined. It is futile to expect the student to draw upon sciences previously studied for the definition, because none of them provides a general definition. In fact, no general definition of energy existed until the first law was enunciated.

The first section of this chapter is devoted to the subject of internal energy, which it defines quantitatively only by the remotest implication. The real definition appears in an equation at the end of the chapter. In view of that definition a casual statement at the bottom of the second page of the chapter is an adequate statement of the first law.

In a treatise as general as this one there is much to be gained by following Gibbs in permitting the term *internal energy* to in-

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clude kinetic energy and potential energy of all kinds. A loss of simplicity and a multiplication of terms results from the author's method of reserving the term *internal energy* to a system in the simplest kind of state.

We may be glad that a book has been written which is devoted to the task of developing a sound elementary exposition of thermodynamics, and which applies to that task an unusual amount of skill and imagination.

Statically Indeterminate Stresses

STATICALLY INDETERMINATE STRESSES. By John Ira Parcel, Professor of Structural Engineering, University of Minnesota, and George Alfred Maney, Professor of Structural Engineering, Northwestern University. Second revised edition. John Wiley & Sons, Inc., New York, 1936. Cloth, 6 × 9¼ in., xvi and 432 pp., 200 figs., $5.

REVIEWED BY L. C. MAUGH[10]

IN THIS revision of a text and reference book that has given valuable service during the last twelve years, the authors have naturally retained the essential features and basic principles of the first edition. Although several changes in the arrangement of the original text have been made, the essential difference is embodied in the addition of the moment-distribution method for the analysis of frames and of a chapter on suspension bridges.

A chapter on methods of calculating the deflection of structures provides the necessary tools for the solution of the various problems that are treated in the subsequent chapters. The Maxwell-Mohr or unit-load method, Castigliano's theorem, moment area, method of elastic weights, and Williot-Mohr diagrams are all explained and illustrated. The relative merits of the various methods are discussed, particularly the type of problems for which each is best adapted.

In the following chapters on the calculation of stresses in continuous beams and indeterminate frame structures, the use of the unit-load method, moment area, and method of elastic weights together with Maxwell's reciprocal theorem is given particular attention. The slope-deflection method has been made the basic method for the analysis of rigid-frame structures and for the calculation of secondary stresses in trusses. The generalized slope-deflection equations for members with variable cross section are considered and a procedure for determining the necessary coefficients is given. In the problem of calculating the coefficients, it is doubtful if the generalized solutions that are given are necessary, as the statement of the solution together with a numerical example should suffice. The moment-distribution method is also used for the analysis of rigid frames although in general the space devoted to the use of successive approximations seems somewhat brief. The physical principles of the problem are clearly defined, but the writer believes that a more comprehensive treatment of the slope-deflection concepts that are involved in the moment-distribution method is necessary for the average reader.

The analysis of the ordinary elastic arch rib and arch truss is fully described and illustrated by numerical examples. The treatment is limited to arches with two-hinge and fixed supports.

An important addition to the new volume is a chapter on suspension bridges. This treatment considers both the approximate and the deflection methods, and each has been made sufficiently complete to give the average reader an excellent picture of the numerical calculations that are necessary for the ordinary types of suspension bridges. The physical principles that are involved and the assumptions that are necessary for analytical purposes are clearly described.

In general, the new edition treats those principles and methods that are necessary for the solution of the average problem in indeterminate structures. The large scope of problems that is considered in this treatment may be too comprehensive for the average course in structural theory, but, nevertheless, it is the material that should be made available for the student.

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Elasticity, Plasticity, and Structure of Matter

ELASTICITY, PLASTICITY, AND STRUCTURE OF MATTER. By R. Houwink, Plastics Department, N. V. Philips Gloeilampenfabrieken, Eindhoven, Holland. With a chapter on the Plasticity of Crystals by W. G. Burgers, Natuurkundig Laboratorium, N. V. Philips Gloeilampenfabrieken, Eindhoven, Holland. Cambridge University Press Department, The Macmillan Company, New York, 1937. Cloth, 5½ × 8¾ in., xviii and 376 pp., 214 figs., 40 tables, $6.

REVIEWED BY A. NÁDAI[11]

THE AIM of this book may be characterized by some statements quoted from the preface according to which the purpose was to bring "the physicist, chemist, and the technologist into a closer contact with each other, so that they may understand one another more easily." According to the author "unity of thought" is seldom encountered in the treatment of the properties of matter mentioned in the title of the book and "one of the tasks the author has set himself is the attempt to trace such a unity as far as this is at present possible."

He adds, however, that "the results of the contemplated comparison between the observations on the flow of such different materials as crystalline substances, glass, resins, asphalts, textile fibres, rubber, clay, sulphur, and others and some general laws, which are assumed to hold for deformation, are often disappointing." He hopes that progress will allow a "deliberate synthesis of materials with defined elastic and plastic properties." The trends in the manufacturing processes of metallic and of many other artificial materials recently developed seem indeed to confirm the author's hopes. It is known that mineral oils, asphalts, rubbers, etc., have been considerably improved, so that their mechanical properties can, to the advantage of the consumer, be predicted and guaranteed in advance.

In the first five chapters elastic and plastic deformation and questions of a structural nature are discussed in a general manner. One chapter contributed by W. G. Burgers summarizes the views concerning the mechanism of plasticity in crystals. The following chapters are devoted to the flow of special materials: Glass, resins, asphalts, rubber, cellulose, clay suspensions, etc. Under the heading "the yield value" some views are reproduced about the existence of a "tension at which flow becomes observable practically under the usual conditions of test." Against the phenomenological introduction of a "practical yield value" for the stress at which the plastic deformations start, not much need be said, although the way in which this was done may not be entirely satisfactory for a group of the prospective readers of the book, namely for the "technologists," i.e., engineers better acquainted with the laws of the yielding of ductile metals. However, when the author continues to state that "the existence of a practical yield value will be accepted as a possibility and in such cases we shall speak of solids;" and "if no practical yield value can be observed, the material will be called a fluid" (pp. 14 and 15) engineers will shake their heads.

[11] Consulting Engineer, Westinghouse Research Laboratories, East Pittsburgh, Pa. Mem. A.S.M.E.

A polycrystalline, ductile, annealed metal (copper and aluminum) at room temperature which starts to yield at the smallest stress should then belong to the "fluids," and a soft asphalt, if it happens to have a small "yield value," to the group of "solids." Not much seems to be gained by definitions of this nature when they are based on such a comparatively weak foundation. Is not the existence or nonexistence of a "yield value" dependent on many circumstances subject to gradual change, and on the order of magnitude of the speed of deformation, etc.? Whether a "yield value" has a significance or not is also entirely dependent on the temperature at which the "solid" is tested. Apart from such considerations, it should perhaps be mentioned that the term "yield value" is not used by mechanical engineers. In testing of metals, ever since steel has been used, the terms "yield point" or "yield stress" have long been in daily usage.

The efforts of the author of the present book in endeavoring to reach a more precise description of the flow of solids and to give a certain uniformity of expression should be commended. It seems, however, to the reviewer that if this were the purpose of the book the needs of a large group of readers interested in the flow of materials, namely, those of the engineers, should have been considered more. Because of the enormous variety of substances which can flow it might have been better to have separated entirely substances into two or three main groups (for example, into the metallic, the soft amorphous materials, such as glass, resins, etc., and the plastic clays, mixtures of solids with water), and develop the laws and the mechanics of flow for each group separately.

As far as the flow of ductile metals is concerned, this has been thoroughly investigated by engineers during many decades and at the end of the nineteenth century a number of good treatises already existed in which many important effects, including that of the speed of plastic deformation, etc., were clearly attacked. For the flow and the elastic deformation of the metals, definitions were in existence which were, and still are, entirely adequate.

In some recent publications in this country, a few definitions have been suggested for long-established and clearly defined material properties, which seem to be inadequate and which are also superfluous. The terms in mind refer to the three fundamental modes of deformation, namely, to the elastic strains, the plastic and the viscous flow of matter. When these are introduced in mechanics the best that can be proposed is to consider these three types of deformation *as important ideal cases for the possible modes of deformation and as abstractions for three of the most important types of existing distortion.* They are described fully in treatises on mechanics of continuously distributed substances, in the theories of elasticity, of plasticity, and of viscous substances. Furthermore an essential element in the mechanical treatment (for engineers at least) is the fact that stress and strain must be considered in three dimensions, on account of the many practical applications. Compared to these general theories the restriction to axial "tension" or "compression" would also limit the field to a considerable extent if engineering applications were to be considered. For these and other reasons it is felt that it would have been preferable to separate substances into the afore-mentioned main groups, perhaps excluding entirely the ductile metals from a treatment in which so many chemical and other viewpoints are important for the understanding of the flow of soft materials. For certain types of flow which, by the way, have a great significance for solid metals, in the book, terms such as "Newtonian" and "non-Newtonian" fluids are used. These seem rather forced combinations of names and words which could certainly be replaced by simpler terms.

It is to be hoped that these remarks do not obscure the considerable merits of the author, who has brought together, in a comparatively limited space, much valuable and recent information on flow. The book will be of value to all those who wish to find in a short time a maximum of reliable information with regard to the behavior of a number of very different materials, crystalline or amorphous. Considerable attention was given to include also in the book various chemical and structural questions.

Soil Mechanics

THEORIE DER SETZUNG VON TONSCHICHTEN. By K. v. Terzaghi and O. K. Fröhlich. Franz Deuticke, Leipzig and Wien, 1936. $6^1/_2 \times 9^1/_2$ in., 168 pp., 100 figs., bound, 14.40 rm, paper, 12 rm.

ERDBAUMECHANIK UND BAUPRAXIS, EINE KLARSTELLUNG. By K. v. Terzaghi and O. K. Frölich. Franz Deuticke, 1937. $6^1/_2 \times 9^1/_2$ in., 33 pp., 2 figs., 2 tables, 2 rm.

REVIEWED BY A. NÁDAI[12]

THE FIRST-mentioned book is the second issue in a series of elementary treatises to be devoted to the science of soil mechanics. The first issue on pressure distributions in the ground by Fröhlich has already been published and a third one by Terzaghi on the theory of earth pressure is in preparation. The well-known authority on soil mechanics, who has been teaching at Massachusetts Institute of Technology for a time, published in the present book jointly with Dr. Fröhlich an introductory volume containing the theoretical foundations for the treatment of the subsidence of the ground under buildings erected on soft-clay layers. Both authors have carried out with their collaborators during the past years many experimental investigations on the settling of foundations which are continued now in Professor Terzaghi's quite remarkable new laboratory on soil mechanics at the Technical University in Wien (Austria).

This book has perhaps also been written to replace certain portions of Terzaghi's "Erdbaumechanik" through which his new theory of soil pressures became better known several years ago. The physical properties of soft clay are discussed and variables introduced which make the former ones susceptible to an analytical treatment.

This leads to Terzaghi's well-known "thermodynamic analogy" of soil mechanics according to which a certain problem of the flow of water through soft-clay layers leads to the same differential equation, on which the theory of heat conduction through solids is based (Fourier's equation or the equation of diffusion). A number of simple solutions about the time-pressure distributions in such layers of clay are given, resting on pervious or impervious layers, in which cases a vertical flow of water is assumed. In later chapters cases are added in which the ground water penetrates from one side into the porous layer and leaves it in a horizontal direction on the other side. In the third chapter a number of exact solutions of Fourier's equation are discussed; the fourth chapter contains practical or numerical applications of the theory. This brief introduction to a soil mechanics of wet-clay layers needs no further recommendation to American readers; those who are probably not yet sufficiently acquainted or familiar with these attempts to introduce an exact treatment when dealing with these phenomena will welcome this clearly written book.

The second joint short monograph mentioned is more of the character of a pamphlet. It refers to a controversy on soil mechanics. Because of the death of the author of several articles, which started the controversy some time ago, it suffices to point out here that this pamphlet contains a refutation of the viewpoints expressed in these other articles.

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Fluid Mechanics

AN INTRODUCTION TO FLUID MECHANICS. By Alex. H. Jameson, M.Sc. Emeritus Professor of Civil Engineering of London. Longmans, Green and Co., New York, 1937. Cloth, 5½ × 8¾ in., x and 239 pp., 116 figs., $2.50.

REVIEWED BY HUGH L. DRYDEN[13]

THIS book appears in response to the demand for a textbook for use in teaching fluid mechanics as a basic science of engineering. It deals with fundamentals, and numerous exercises for the student are included.

The author has chosen to omit all discussion of lifting surfaces, boundary layer, flow separation, turbulence except as it occurs in flow in pipes, lubrication, flow of compressible fluids except for a brief appendix, propellers, fans, and hydraulic machinery.

The book follows a carefully, thought-out logical plan; it first reviews the concept of pressure and the physical properties of fluids, treats the statics of fluids, derives Bernoulli's theorem from energy considerations, applies it to venturi meters and orifices, applies momentum and energy relations to impact of jets, and continues with the mechanics of fluids. Due consideration is given to dimensional reasoning and to model experiments. The author has succeeded admirably in his expressed aim of "illustrating the subject fully with diagrams, worked-out examples, and 'guided' exercises, and to keep it as free as possible from empirical formulae of tables of coefficients."

Two matters of terminology deserve comment. The author recognizes the common usage of the word "pressure" in the sense of total force and attempts to introduce the hyphenated expression "pressure-intensity" for the quantity usually designated pressure by most writers on fluid mechanics. The reviewer believes that it is preferable to teach prospective engineers to avoid the use of the word pressure in the sense of force and to reserve pressure for force per unit area. The author uses the words "boundary layer" for the region in a pipe which is usually designated "laminar sublayer."

The reviewer recognizes the difficulty of compressing the elements of an extensive subject within the allotted hours of a university course, but believes that the engineer should be introduced to the concept of pressure and velocity fields around objects immersed in fluids and within conduits and channels carrying fluids, and that some knowledge of the theory of lifting surfaces, of boundary layers, and of propellers, fans, and turbines is essential. Lubrication as such, and compressible fluids may perhaps be omitted. In view of the very concise manner in which the author has treated the subjects included, these additions could well be made in a subsequent edition without undue enlargement of the text.

Harmonic Analysis

RECHNERISCHE VERFAHREN ZUR HARMONISCHEN ANALYSE UND SYNTHESE. By A. Hussman. Julius Springer, Berlin, 1938. Cardboard, 8½ × 12½ in., 28 pages of text and 10 loose folding tables, 9.60 rm.

REVIEWED BY J. P. DEN HARTOG[14]

OF THE several methods for calculating the Fourier coefficients of a given periodic curve, the one devised by Runge is the simplest and most efficient. It consists of measuring a number of equidistant ordinates of the curve and performing a few simple addition and multiplication operations on them. Runge himself disclosed the general method and gave explicit tables for 12 and for 24 ordinates. The present book gives first a comprehensive discussion of various other methods of analysis, then goes into some detail on Runge's procedure, and gives explicit tables for 12, 24, 48, and 72 ordinates. With the latter 35 sine harmonics, 36 cosine harmonics, and the constant term of the Fourier series can be determined. The book is clearly written and the tables are easy to use.

The Theory and Performance of Axial-Flow Fans

THE THEORY AND PERFORMANCE OF AXIAL-FLOW FANS. By Curt Keller, Chief Engineer of Esher Wyss Laboratories, Zurich, Switzerland; adapted for the use of fan designers by Lionel S. Marks, Professor of Mechanical Engineering, Harvard University, with the assistance of John R. Weske, Assistant Professor of Aerodynamics, Case School of Applied Science. First edition. McGraw-Hill Book Company, Inc., New York, 1937. Cloth, 6 × 9 in., x and 140 pp., 36 tables, 112 figs., $4.

REVIEWED BY A. L. KIMBALL[15]

THIS IS the first book on axial-flow fans which has appeared in the English language wherein fan design is thoroughly treated from the standpoint of modern airfoil theory. The original of this book was in German by a well-known Swiss engineer.

The translators of this book are to be congratulated in making available information for which there has been a real need in this country.

It might indeed seem strange that modern airfoil theory was not sooner applied to fan design. The fact is that in some applications fans with blades of airfoil section have been disappointing. This was not the fault of the design but the result of misunderstanding as to how this type of fan should be applied.

A little consideration will make it clear that airplane propeller blades are designed for cases where the air flow through the blade area is almost entirely axial because of the high speed of motion of the plane itself. On the other hand, in most commercial fan applications the fan operates in air initially at rest which has to be greatly accelerated before it is delivered by the fan. As a result of this fact free-air propeller fans without guide vanes have to be designed to take account of the air inflow from all directions rather than for flow along the axis only. On the other hand if guide vanes are used so that the air is directed axially toward the fan blades a fine performance can be obtained from an airfoil type of fan. If the air is not properly directed in the front of the fan, however, its performance may be poor. It is for these reasons that no airfoil type of fan is really complete in itself alone. In fact, this volume discusses only cases where a fan is used with supplementary means of directing the air through the blade area.

Although the book contains 140 pages, it is made up of only four chapters.

In the first of these the basic theory is presented in which performance characteristics are given in terms of dimensionless constants.

The second chapter contains a study of fan losses in which it is shown how to design for maximum efficiency.

The third chapter presents the results of some wind-tunnel experimental studies of the performance of certain blade section profiles.

The fourth chapter gives the results which are of particular interest to most fan designers, i.e., actual performance data for four representative fans over a wide range of conditions, as determined from actual tests.

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[14] Associate Professor of Applied Mechanics, Harvard University, Cambridge, Mass. Mem. A.S.M.E.

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A few words on how to interpret some of these data may be of value to those not having time to thoroughly study this book.

The results of the tables on pages 109 to 122 which give data from actual tests are set forth graphically in Figs. 81 to 98. The constant ϕ represents the flow coefficient, i.e., the ratio of velocity through the blade area to blade-tip speed. The constant ψ represents the total static-pressure head as a fraction of fan-blade-tip velocity head. The efficiency, although spoken of as total efficiency, is what we would usually speak of as static efficiency. It is that which corresponds to ψ, being based on the actual over-all static pressure produced, including both turning vanes and diffusers. With these constants in mind these sets of curves can readily be used in design.

One point which deserves mention is the use of Δp_{tot} and Δp_{st}. The first of these is total pressure increase as defined in the list of symbols and from which the efficiency curves on pages 97 to 105 are derived. The second of these Δp_{st} as used in this volume refers sometimes to total theoretical stage head (see Equations 14, 20, and 22, for example), again it is sometimes an actual static-pressure drop which is represented (see Equation 12, also page 79). The definition in the list of symbols regards this quantity purely as the static drop across the fan blade which is not in general its most important usage in this volume.

There are a few typographical errors which might be mentioned:

On page 4, tenth line from the bottom, w_{u_2} should read c_{u_2}.

On page 5, third line from the bottom, dA should read dL.

In Fig. 19, 0.05 should be 0.5; furthermore, the curves of Figs. 18 and 19 should evidently be interchanged.

On page 47, "these" should read "there."

In Table 11, in the third row from the bottom, g_1 should read q_1.

Any serious student of the theory and application of axial-flow fans will find a careful reading of this book of great value.

Problems of the Magnetizing Curve

PROBLEME DER TECHNISCHEN MAGNETISIERUNGSKURVE. By R. Becker, Göttingen. Julius Springer, Berlin, 1938. Paper, 6 × 9 in., v and 172 pp., 102 figs., 16.50 rm.

REVIEWED BY F. BITTER[16]

THE DEVELOPMENT of the theory of ferromagnetism during the last ten years has not only explained many of the peculiarities of actual materials, but has stimulated research workers to examine still unexplained phenomena in greater detail, and to develop magnetic measurements as a tool for studying other metallurgical problems. In this field the German school has led the world, as is nicely demonstrated in this booklet, containing ten papers on vital new developments of interest to all serious students of metals. R. Becker, who organized the meeting at which these papers were first presented, and who did much to stimulate interest in the subject by his own original contributions, contributes the first paper, introductory in nature, describing in a general way today's generally accepted model of a ferromagnetic substance. There follow two papers by K. Sixtus and W. Döring on large Barkhausen discontinuities, which discuss the manner in which regions of reversed magnetization can form and propagate themselves along a wire.

M. Kersten, who collaborated with Becker in his earlier work and who has done much original work on the subject of the properties of mechanically deformed crystals, contributes an article on the interpretation of coercive force. J. L. Snoek continues with a discussion of the coercive force in Fe-Co-Mo and Fe-Ni-Al alloys, discussing in a most convincing way the importance of "inner demagnetization factors" due to the precipitation of nonmagnetic material in a ferromagnetic alloy. Then there are two papers on magnetic aftereffects and their importance in connection with a-c losses by G. Richter and H. Schulze. Some most striking effects in recrystallized carbonyl iron are described and correlated with primary creep. The book ends with two papers on precipitation hardening by G. Masing and W. Gerlach, and one on rates of transformation in steel by H. Lange, emphasizing the usefulness of magnetic methods of observation. These papers have none of that amateurish superficiality that characterizes many of the older papers in which magnetic methods are used in metallurgical investigations.

Work of this caliber, and perhaps even more thorough and elaborate investigations, are needed if magnetic researches are to contribute in an important way to our understanding of the nature of metals and alloys.

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Railway News

TRACTION NOUVELLE, January-February, 1936, first year, No. 1 and January-February, 1937, second year, No. 7. Published bimonthly by J. B. Bailliere & Fils, Paris. Price: 1 yr, France, 40 fr; Belgium, 50 fr; other countries, 60 fr.

REVIEWED BY A. I. LIPETZ[17]

AS IS known, the French railways have now in service a great number of Diesel-electric rail cars and trains; they had over 600 by the middle of the summer of 1937. A rail car committee of representatives of all French railways has started a new publication called *Traction Nouvelle*, published by J. B. Bailliere & Fils, 19, Rue Hautefeuille, Paris 6e, as a beautiful bimonthly magazine in an aluminum-finished paper cover. It started in January, 1936, and the first number contained an interesting article on characteristics of rail cars. "Les Coefficients Caracteristiques des Automotrices." In addition to the usual coefficient of resistances of rail cars on track, air resistance, efficiency of transmission to the wheels from the internal-combustion engine, a new coefficient called "nervousness," is introduced, which represents the following:

The accelerations and speeds of the car, or train, are graphically determined in accordance with the power and resistance curves, depending upon the profile of the line, by one of the usual methods of integration of the equation of motion. The results are plotted as velocities in function of time and distance and compared with actual velocity curves obtained after the car has been built and tested. It is usually observed that the ordinates, which represent actual velocities, have a certain relation to the theoretical ordinates, always in the same ratio, less than 1, for the same car. This is explained by the lag in action of the power of the car from the engine to the wheels. This ratio is called the coefficient of nervosity, or nervousness. It has been included in the list of car characteristics adopted by the Permanent Commission of the International Railway Congress Association.

In the January-February issue of 1937 (No. 7 of the *Traction Nouvelle*) appeared an interesting article on a Czechoslovakian rail car by Mr. Jan Leiner. This car is called the "Slovakian Arrow," and has two 165-hp gasoline engines with two electromechanical direct transmissions to the wheels, one engine and one transmission on each truck. The transmission, known under the name "Sousedik," is of the differential electric type, similar to the Entz or Thomas transmissions described in *Mechanical Engineering* for September, 1926, class B-1. This represents a revival of an old idea which is technically sound, and in small units may be even practical. So far on the Czechoslovakian

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car it has given good results. The car and transmission are fully described also in the Bulletin of the International Railway Congress Association (English edition, August, 1937, page 1787).

Buckling

KNICKUNG, KIPPUNG, BEULUNG. By Friedrich Hartmann, Professor an der Technischen Hochschule, Wien. Franz Deuticke, Leipzig and Wien, 1937. 7 × 10 in., vi and 201 pp., 143 figs., 18.40 rm.

REVIEWED BY A. NÁDAI[18]

THIS IS an enlarged introduction to the lectures on statics of wooden and steel constructions given by the author at the Technical University of Wien. It is a monograph devoted exclusively to the subject of the elastic stability of engineering constructions. The number of papers devoted to this part of the theory of elasticity has increased quite considerably during the last ten to twenty years. While in former times it seemed sufficient to treat the cases of the elastic buckling of straight bars and perhaps the conditions of instability of a few frameworks, according to the opinion of the author of the book a summary of the methods for solving stability problems and of the results of investigations of this kind is needed.

The book is written for use at university lectures and as a textbook for advanced students and engineers. It presents this difficult subject in a brief way. Of the eight chapters the following titles may be of interest: The buckling of straight bars with uniform and with variable cross sections, both in the elastic and the plastic range of strains, eccentric compression, buckling of frames and of bars with elastic connections, "Kippung" (lateral buckling of flat bars), buckling of flat plates.

Considerable effort is made in this book to develop the general equations and conditions of buckling. The theory of the inelastic buckling of a straight bar is treated in greater detail following the pioneer work by Engesser and von Kármán. In this chapter the author has added much original work of his own. He has for example, carefully discussed the relation between the shape of a given stress-strain curve for a structural steel and the corresponding shape of the curve for the critical load as a function of the slenderness ratio. While some recent work on this subject of American authors has not been discussed here (Westergaard, Young, and others), other new work, for example by Chwalla on buckling under eccentric loading is reported. An instructive long chapter is devoted to the general theory of the buckling of flat plates loaded in their plane, preceded by a careful derivation of the integral condition known as the "energy method."

Although this monograph contains many subjects which have been treated recently in Timoshenko's "Theory of Elastic Stability" (McGraw-Hill Book Co,. Inc., N. Y., 1936), the book having been carefully and critically written will be welcomed by readers who wish perhaps to study some of these difficult investigations in more detail, which in their originals cannot easily be reached. The book can well be recommended as an up-to-date record of methods dealing with all kinds of stability questions.

DIE KNICKFESTIGKEIT VON STÄBEN UND STABWERKEN. By Julius Ratzersdorfer. Julius Springer, Wien, 1936. Paper, 6 × 9¼ in., ix and 321 pp., 151 figs., 27 rm.

REVIEWED BY N. M. NEWMARK[19]

AS THE title indicates, this book is concerned only with the buckling of bars and of frameworks of bars. With the exception of his comprehensive chapter on the stability of plane and space trusses and frames, the author discusses many of the problems contained in chapters 1 to 4, pages 1 to 238, of Timoshenko's "Theory of Elastic Stability." The manner of treatment, the arrangement, and the point of view offer interesting comparisons to Timoshenko's work.

The book opens with a detailed discussion of the buckling of straight bars of uniform cross section under axial load. The author studies the effect of the various approximations that are customarily made in deriving the critical buckling load. For example, he derives the relation between deflection and load for a bar fixed at one end and loaded axially at the free end, assuming first that the deflections are small, and then removing this restriction. The approach here, as elsewhere in the book, is first by means of the customary approximations, followed by more exact approximations and in some cases by "exact" solutions (within the limitations of the classical "Theory of Elasticity"). The influence of the change in length of the bar is also discussed. These studies are made in order to lead to more precise concepts of the nature of buckling, and also to furnish quantitative estimates of the accuracy of the ordinary procedures.

Other end conditions and types of loading, the influence of the shearing stresses on the stability, and the behavior of bars of constant cross section made of material that does not follow Hooke's law are considered, and comparisons are made between the results of theory and experiment.

In the second chapter the author takes up a study of the deformation of bars subjected to both bending and direct stress. With a consideration of the effect of large deflections it is shown that the deflection does not actually become infinite when the critical load is reached. A treatment is given also of stresses beyond the elastic limit for combined flexure and axial load.

In the third chapter are considered problems of bars of varying cross section with concentrated end thrusts or with varying direct stress along the length of the axis. The cases treated are those in which the moment of inertia or the distributed axial load or both vary according to some power of the distance along the axis.

A brief discussion is given of approximate methods and methods of successive approximations. An illustration is made of Timoshenko's application of Ritz's method as an example of the use of the "energy method" in studying buckling. Several examples are given of the graphical procedure of successive approximations associated with the name of Vianello. It would have been desirable to extend this part of the work so as to present a single procedure capable of general application to problems of buckling of bars of variable cross section both in and beyond the elastic range.

The fourth chapter is concerned with bars having continuous elastic supports, generally with resistance proportional to displacement. Several end conditions are considered, and the case of a bar elastically supported over part of its length is treated.

In the fifth chapter is discussed the buckling of plane and space trusses and of plane frames. The author also treats "framed bars" or built-up columns and compares the theoretical results with test results. It would seem that for practical applications the effects of departure from Hooke's law should be taken into account in the buckling of frames. Also, in any discussion of built-up columns the problem of local buckling or wrinkling is of extreme importance, and such problems are not considered in the present work.

The sixth chapter covers the buckling of bars continuous over fixed or over elastic supports, but not carrying lateral loads. Consideration is given to the problem of lateral stiffening particularly in through bridge trusses without lateral bracing of the upper chord (the pony truss). The seventh chapter concerns the buckling of circular and parabolic arches in their own planes.

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[19] Research Assistant Professor of Civil Engineering, University of Illinois, Urbana, Ill.

Professor Ratzersdorfer has contributed a comprehensive and interesting work. The book contains the solution of a number of special problems of practical interest. However, one is led to wonder if the problem of buckling of bars is really as complicated as the length of this treatise (which is by no means an exception in this field) would indicate. There seems to be a necessity for some general approach that does not develop into an entirely new procedure for each special case that arises.

Springs

DIE FEDERN—IHRE GESTALTUNG UND BERECHNUNG. By Dipl.-Ing. Siegfried Gross and Dr.-Ing. Ernest Lehr. V.D.I. Verlag G.M.B.H., Berlin, 1938. Cardboard, 8 × 11½ in., 136 pp., 215 figs., 52 tables. 25 rm.

REVIEWED BY A. M. WAHL[20]

THIS BOOK, published under the auspices of the Springs Committee of the V.D.I., is primarily an attempt to bring together into convenient form for the spring designer the results of a great many research investigations carried out in the last few years, on the calculations of stresses and deflections in mechanical springs of various types. The introduction consists mainly of a brief discussion of the fundamentals of spring design and the properties of spring materials, together with illustrations of numerous practical applications, particularly in the railway and automotive fields. For convenience, the subject matter is divided into: Part 1, springs subjected principally to bending stress (leaf springs, spiral springs, etc.), part 2, springs subjected primarily to torsion stress (helical compression springs), and part 3, springs subjected to direct stress (ring springs).

Because of the importance of leaf springs in railway and automotive work, in part 1 the calculation of such springs is discussed in great detail. Among the particular cases treated are (1) leaf springs with large deflections, (2) effect of variable leaf thickness, (3) effect of interleaf friction, (4) oblique load application, and (5) effect on stress produced by varying the initial curvature of the different leaves. The results of numerous tests and comparison with theory are given. In this part the calculation of spiral and torsion springs is treated as well as that of Belleville and plate springs.

The calculation of helical tension and compression springs both by exact and approximate methods forms the principal subject of part 2. The more exact formulas derived by Göhner for calculating such springs are given, while the results obtained by these methods are compared with those obtained by strain measurements on helical springs of rectangular cross section. Besides the calculation of conical helical springs, the determination of buckling loads and the deflection of laterally loaded helical springs are also treated; in addition a section is devoted to the problem of vibration and surging of valve springs. In the concluding part the calculation of ring springs (as used in railway car buffers) is discussed, a comparison of analytical and experimental results also being included. For the convenience of the practicing engineer the formulas for each type of spring are summarized in convenient form in the final pages of each part of the book; in addition numerical examples throughout serve to illustrate the methods of calculation described.

Although in the reviewer's opinion considerably more data on spring materials and working stresses as used in practice could have been included with benefit, the information contained in the book should make it of considerable value as a reference work for those interested in spring design.

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Sound

SOUND. By Arthur Taber Jones, Professor of Physics, Smith College. D. Van Nostrand Company, Inc., New York, 1937. Cloth, 5½ × 8¾ in., xii and 450 pp., figs. 141, $3.50.

REVIEWED BY A. L. KIMBALL[21]

THIS BOOK is unique in that it is largely descriptive in character, in contrast to the usual distinctly analytical presentation of the subject. At the same time it is thoroughly scientific in that no phenomena are taken for granted but are viewed from every angle before conclusions are drawn. The book is delightfully written, in fact the author shows a capacity for clearly explaining physical phenomena so that they are apprehended by the reader with minimum effort.

The author's analysis does not go much further than to cover the case of the standard formula for frequency and the physical explanation of how this formula is applied, including also an interesting derivation from it of the frequency formula for a stretched string based on considerations of similarity. The more mathematical aspects of the subjects of elasticity and elastic vibrations are very little touched upon which most books on sound cover in some detail.

The author shows a broad interest in the general field of the physical phenomena of sound. For example, such matters as what causes the zones of audibility and inaudibility of sound from heavy guns, the question of under what conditions sand on a vibrating plate will become distributed on the antinodes rather than the nodes, the explanation of the ripples in a Kundt tube, the "wolf" note of a violin or cello, are interestingly discussed with details of the experiments which have been made to better understand these phenomena and the conclusions they show.

Among other things the author points out the little known fact that if the energy necessary to raise a blade of grass above the earth as it grows in the spring were all translated into sound, a continuous sound would result which would be clearly audible in a quiet room.

The explanation of the detailed features of thunder as produced by a lightning stroke is the best that the writer of this review has ever seen. The author clearly perceives that echo plays a minor part in the rolling character of thunder, but that its major cause is the extension of the source of this sound through space.

There is considerable material under vibration as applied to different kinds of musical instruments, including bells, which subject has been a specialty of the author in which he has done some fundamental work.

The history of our knowledge of sound phenomena has an appeal to the author in showing the progress of our understanding of them. The reference list is extensive, covering nearly four hundred separate items in the form of footnotes.

Although there is a chapter on technological applications, the subjects of noise prevention and the reproduction of sound by electrical means are not much developed.

There is a rather complete list of questions for possible use in classroom work which are intended as a test of the students' understanding of the subject as well as knowledge of facts presented. The reviewer notes 341 such questions in the text.

There are several short supplementary appendixes covering about ten pages in all at the end of the volume.

Although the book contains over 400 pages it can be covered in three or four evenings by an intelligent and interested reader, though he may not be versed in analytical methods. In fact one of the chief values of the book, aside from its use as a text, is this feature.

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Bearing Pressures and Cracks

Bearing Pressures Through a Slightly Waved Surface or Through a Nearly Flat Part of a Cylinder, and Related Problems of Cracks

By H. M. WESTERGAARD,[1] CAMBRIDGE, MASS.

The task is undertaken of determining the bearing pressures, and the stresses and deformations created by them, in some cases that differ from those considered by Hertz[2] in his classical study of contact. Thus two solids are examined which, before loading, are in contact along a row of evenly spaced lines in a horizontal plane, as indicated in Fig. 1(*a*). Between these lines the surfaces have a separation defined by a nearly flat cosine wave. A uniform pressure on top of the upper solid creates contact over an area consisting of a row of strips, reduces the separation of the solids between the strips, as suggested in Fig. 1(*b*), and creates contact pressures distributed as indicated in Fig. 1(*c*), with vertical rises in the diagram of pressure at the edges of the strips. At a greater load the width of the strip becomes equal to the wave length, and the contact is complete. At still greater loads the stresses increase as if the two solids were one. The procedure by which this problem is solved is demonstrated first by showing its easy application to some well-known cases, especially Hertz's problem of circular cylinders in contact.[2]

Further applications are to a noncircular cylinder resting on a solid with a flat top, with an initial separation of the surfaces varying as the fourth power of the distance from the initial line of contact; to partial contact of two surfaces which are initially plane, except that one of them has a ridge or several parallel ridges; and to some related problems in which two parts of the same body are partially separated by the forming of one or more cracks.

Notation

x, y	= rectangular coordinates, y vertical
r, θ	= corresponding polar coordinates
z	= $x + iy = re^{i\theta}$ = complex variable
Z	= function of z, Equation [1], defining the stresses by Equations [4] to [6]
$Z', \overline{Z}, \overline{\overline{Z}}$	= derivative and first and second integral of Z, Equations [2]
$\sigma_x, \sigma_y, \tau_{xy}$	= normal stresses and shearing stress in the directions of x and y
ξ, η	= displacements in the directions of x and y
η_0	= displacement η at $y = 0$
s	= initial separation of two surfaces
E, G, μ	= Young's modulus, modulus of elasticity in shear, and Poisson's ratio
F	= Airy's stress function
P	= force on slice parallel to the x,y-plane one unit thick, measurable in pounds per inch
p	= average pressure or tension, measurable in pounds per square inch
a, l	= horizontal distances on axis of x
c, c_1	= constants

Function of a Complex Variable Used as Stress Function

A stress function will be applied of a type which was introduced by Carothers[3] in 1920 and, evidently independently, by Nádai[4] in 1921. Both expressed the significant values in terms of harmonic functions, and both made use of the following fact: A harmonic function of x and y can be obtained as the real part ReZ or the imaginary part ImZ of an analytic function Z of the complex variable $z = x + iy$, with Z being written in the forms

$$Z = Z(z) = Z(x + iy) = \mathrm{Re}Z + i\mathrm{Im}Z \quad \ldots\ldots [1]$$

In the present applications it is expedient, as done by MacGregor,[5] to use the function Z itself as stress function.

The further functions Z', $\overline{Z}$, and $\overline{\overline{Z}}$ are the derivative and first and second integrals of Z, so that

$$Z' = \frac{dZ}{dz}, \quad Z = \frac{d\overline{Z}}{dz}, \quad \overline{Z} = \frac{d\overline{\overline{Z}}}{dz} \quad \ldots\ldots [2]$$

The properties of derivatives are noted

$$\frac{\partial \mathrm{Re}Z}{\partial x} = \frac{\partial \mathrm{Im}Z}{\partial y} = \mathrm{Re}Z', \quad \frac{\partial \mathrm{Im}Z}{\partial x} = -\frac{\partial \mathrm{Re}Z}{\partial y} = \mathrm{Im}Z' \ldots [3]$$

In a restricted but important group of cases the normal stresses and the shearing stress in the directions of x and y can be stated in the form

$$\sigma_x = \mathrm{Re}Z - y\mathrm{Im}Z' \quad \ldots\ldots [4]$$

$$\sigma_y = \mathrm{Re}Z + y\mathrm{Im}Z' \quad \ldots\ldots [5]$$

$$\tau_{xy} = -y\mathrm{Re}Z' \quad \ldots\ldots [6]$$

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[1] Gordon McKay Professor of Civil Engineering and Dean of the Graduate School of Engineering, Harvard University. Mem. A.S.M.E.

Presented by title at the Joint Meeting of The Applied Mechanics and Hydraulic Divisions of The American Society of Mechanical Engineers, Ithaca, N. Y., June 25–26, 1937.

Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y., and will be accepted until August 10, 1939, for publication at a later date. Discussion received after the closing date will be returned.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[2] Heinrich Hertz, Crelle's *Journal für die reine und angewandte Mathematik*, vol. 92, 1881, p. 156 (also in his Gesammelte Werke, vol. 1, 1895, p. 155). See, for example "Theory of Elasticity," by S. Timoshenko, McGraw-Hill Book Co., Inc., New York, N. Y., 1934, pp. 339–350.

[3] "Plane Strain: The Direct Determination of Stress," by S. D. Carothers, Proceedings of the Royal Society of London, series A, vol. 97, 1920, pp. 110–123, especially p. 119.

[4] "Über die Spannungsverteilung in einer durch eine Einzelkraft belasteten rechteckigen Platte," by A. Nádai, *Der Bauingenieur*, vol. 2, 1921, pp. 11–16, especially p. 12. Nádai applied the function to express curvatures and twists of elastic slabs. The curvatures and twists can be interpreted as stresses through Airy's stress function.

[5] "The Potential Function Method for the Solution of Two-Dimensional Stress Problems," by C. W. MacGregor, Trans. American Mathematical Society, vol. 38, no. 1, July, 1935, pp. 177–186.

By referring to Equations [3] it is observed that these stresses satisfy the two conditions of equilibrium of the form

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = 0 \quad \ldots\ldots [7]$$

The limitation of this type of solution appears in Equations [4] to [6], which require that

$$\sigma_x = \sigma_y \text{ and } \tau_{xy} = 0 \text{ at } y = 0 \quad \ldots\ldots [8]$$

With deformation in the direction perpendicular to the x, y-plane prevented, the displacements ξ and η in the directions of x and y are defined by the formulas

$$2G\xi = (1 - 2\mu)\,\mathrm{Re}\overline{Z} - y\mathrm{Im}Z \quad \ldots\ldots [9]$$

$$2G\eta = 2(1 - \mu)\,\mathrm{Im}\overline{Z} - y\mathrm{Re}Z \quad \ldots\ldots [10]$$

For, it is found that these displacements define the stresses in Equations [4] to [6] through Hooke's law, which can be stated in the form

$$\sigma_x = 2G\left[\frac{\partial \xi}{\partial x} + \frac{\mu}{1-2\mu}\left(\frac{\partial \xi}{\partial x} + \frac{\partial \eta}{\partial y}\right)\right] \text{ and } \tau_{xy} = G\left(\frac{\partial \xi}{\partial y} + \frac{\partial \eta}{\partial x}\right) \quad \ldots\ldots [11]$$

A useful observation from Equation [10] is that the value of η at $y = 0$ is

$$\eta_0 = \frac{1-\mu}{G}\,\mathrm{Im}\overline{Z} = \frac{2(1-\mu^2)}{E}\,\mathrm{Im}\overline{Z} \quad \ldots\ldots [12]$$

It is noted, furthermore, that the Airy function defining the stresses by the equations

$$\sigma_x = \frac{\partial^2 F}{\partial y^2}, \quad \sigma_y = \frac{\partial^2 F}{\partial x^2}, \quad \text{and} \quad \tau_{xy} = -\frac{\partial^2 F}{\partial x \partial y} \quad \ldots\ldots [13]$$

is

$$F = \mathrm{Re}\overline{Z} + y\mathrm{Im}\overline{Z} \quad \ldots\ldots [14]$$

In a slice parallel to the x, y-plane one unit thick the total vertical force transmitted between two points is the increase of the derivative

$$\frac{\partial F}{\partial x} = \mathrm{Re}\overline{Z} + y\mathrm{Im}Z \quad \ldots\ldots [15]$$

between the points. Similarly, the total horizontal force transmitted between two points is the increase of

$$\partial F/\partial y = y\mathrm{Re}Z \quad \ldots\ldots [16]$$

between the points.

Introductory Application to Boussinesq's Problem

The semi-infinite solid $y \geqq 0$, with y positive downward, is under consideration. The function

$$Z = P/(i\pi z) \quad \ldots\ldots [17]$$

gives

$$\overline{Z} = \frac{P}{i\pi}\log(z/c_1) = \frac{P}{i\pi}\left[\log(r/c) + i\left(\theta - \frac{\pi}{2}\right)\right] \quad \ldots [18]$$

that is

$$\mathrm{Re}\overline{Z} = P[\theta - (\pi/2)]/\pi \quad \ldots\ldots [19]$$

According to Equations [15] and [19], in a slice parallel to the xy-plane and one unit thick the total vertical force transmitted between $\theta = \pi$ and $\theta = 0$ is $-P$. It is concluded that Equation [17] represents the solution of Boussinesq's problem in two dimensions for a normal pressure P concentrated at $z = 0$. The familiar formulas for stresses and displacements are obtained readily by substituting from Equations [17] and [18] in Equations [4], [5], [6], [9], and [10].

Rows of Forces

Equation [17] suggests consideration of two modified functions

$$Z_1 = -i\frac{P}{l}\cot(\pi z/l) \text{ and } Z_2 = -i\frac{P}{l\sin(\pi z/l)} \quad \ldots [20]$$

Near $z = 0$ both approach Z in Equation [17]. Further inspection shows that Z_1 represents a row of equal pressures P at $z = 0, \pm l, \pm 2l, \ldots$, and Z_2 represents a row of pressures P at $z = 0, \pm 2l, \pm 4l, \ldots$ and a row of pulls P at $z = \pm l, \pm 3l, \pm 5l, \ldots$ on the solid $y \geqq 0$. When y becomes great, Z_1 converges toward $-P/l$, making the stresses in Equations [4] to [6] converge toward a uniform pressure P/l; while Z_2 converges toward zero, making the stresses converge toward zero, as they should under the self-balancing load.

Demonstration by Application to Hertz's Problem of Two Circular Cylinders in Contact

The solid $y \geqq 0$ is considered again. As stress function is chosen

$$Z = -\frac{2P}{\pi a^2}\left[\sqrt{(a^2 - z^2)} + iz\right] \quad \ldots\ldots [21]$$

$$\text{or} \quad Z = -\frac{2P}{\pi a^2}\left[\sqrt{(a^2 - x^2 + y^2 - i\,2xy)} + ix - y\right] \quad \ldots [22]$$

At $y = 0$ the shearing stress $\tau_{xy} = 0$, and the normal stresses, according to Equations [4] and [5], are both equal to $\mathrm{Re}Z$. Accordingly

$$\sigma_x = \sigma_y = 0 \text{ at } y = 0,\ x < -a \text{ or } x > a \quad \ldots\ldots [23]$$

$$\sigma_x = \sigma_y = -(2P/\pi a^2)\sqrt{(a^2 - x^2)} \quad \text{at } y = 0, \quad -a < x < a \quad \ldots\ldots [24]$$

Equations [23] and [24] show that the diagram of pressures on the surface $y = 0$ can be drawn as a half-ellipse between $x = -a$ and $x = a$; outside there is no load. The total pressure on the slice one unit thick is P.

When z becomes numerically great, with y remaining positive, one may write

$$\sqrt{(a^2 - z^2)} = -iz\sqrt{[1 - (a^2/z^2)]} = -iz\,(1 - a^2/2z^2 - \ldots) \quad \ldots [25]$$

Therefore, Z in Equation [21] converges toward Z in Equation [17], which represents Boussinesq's problem.

In the interval $-a < x < a$ at $y = 0$ Equations [12] and [22] give

$$\frac{d\eta_0}{dx} = \frac{2(1-\mu^2)}{E}\,\mathrm{Im}Z = -\frac{4(1-\mu^2)Px}{\pi E a^2} \quad \ldots\ldots [26]$$

that is, along the axis of x there is produced a constant concave curvature

$$\frac{1}{R} = -\frac{d^2\eta_0}{dx^2} = \frac{4(1-\mu^2)P}{\pi E a^2} \quad \ldots\ldots [27]$$

If instead of being initially flat along the axis of x the surface has an initial convex curvature equal to that in Equation [27], under the pressures defined by Equation [24] the surface will be flattened out and become plane in the interval $-a < x < a$; outside this interval it will be flattened out less.

It follows that if two parallel cylinders with radii R are pressed together by the load P per unit of length, the width $2a$ of the strip of contact will be defined by a in Equation [27], which agrees

with Hertz's classical solution. With a known, the contact pressures are defined by Equation [24] and the stresses and displacements in the surrounding region by Equations [21], [4], [5], [6], [9], and [10].

Bearing Pressure Through Slightly Waved Surface

Equation [21] suggests investigation of the stress function

$$Z = -\frac{2p\cos(\pi z/l)}{\sin^2(\pi a/l)}\left\{\sqrt{[\sin^2(\pi a/l) - \sin^2(\pi z/l)]} + i\sin(\pi z/l)\right\} \quad \ldots [28]$$

as applying to the solid $y \geqq 0$. It is assumed that $a < l/2$. By computing as in Equation [25], it is found that when y is positive and great compared with a, Equation [28] may be replaced by

$$Z = -ip\cot(\pi z/l) \quad \ldots [29]$$

According to the comments on Equations [20], Z in Equation [29] represents a row of pressures pl with spacing l at $y = 0$, and a uniform pressure p at great values of y.

At the surface $y = 0$ one finds in the interval $-a < x < a$

$$\sigma_x = \sigma_y = \mathrm{Re}Z = -\frac{2p\cos(\pi x/l)}{\sin^2(\pi a/l)}\sqrt{[\sin^2(\pi a/l) - \sin^2(\pi x/l)]} \quad \ldots [30]$$

and in the interval $a < x < l - a$

$$\sigma_x = \sigma_y = \mathrm{Re}Z = 0 \quad \ldots [31]$$

The function Z is periodic, and the period is l. The values are repeated in the similar intervals. The strips $nl - a < x < nl + a$ are loaded by pressures $-\sigma_y$ defined numerically by Equation [30]; the remaining strips are unloaded.

Within the loaded strips of the surface Equation [28] gives

$$\mathrm{Im}Z = -\frac{p\sin(2\pi x/l)}{\sin^2(\pi a/l)} \quad \ldots [32]$$

Over the whole surface $\mathrm{Im}Z$ is antisymmetrical with respect to the center lines $x = nl/2$ of the strips. By referring to Equation [12] it is then found that within the loaded strips the deflection of the surface can be stated as

$$\eta_0 = \frac{2(1-\mu^2)}{E}\mathrm{Im}\bar{Z} = \frac{(1-\mu^2)pl}{\pi E}\frac{[\cos(2\pi x/l) - 1]}{\sin^2(\pi a/l)} \quad \ldots [33]$$

with the integration constant being the same for all the loaded strips.

Assume now that instead of being initially flat the surface is slightly waved, having the equation

$$y_0 = \frac{c}{4}[1 - \cos(2\pi x/l)] \quad \ldots [34]$$

with

$$c = \frac{4(1-\mu^2)pl}{\pi E\sin^2(\pi a/l)} \quad \ldots [35]$$

Then under the pressures defined by Equation [30] the ordinates $y_0 + \eta_0$ of the deformed surface will be zero within the loaded strips. The loaded strips will be flattened out and be contained in a single plane. A further examination of $\mathrm{Im}Z$ as defined by Equation [28] shows that $y_0 + \eta_0$ will be positive between the loaded strips.

It is concluded that if another solid of the same material and shape is placed in contact with the one considered, so that the axis of x becomes an axis of symmetry, and if thereafter a uniform pressure p is produced at numerically large values of y, the contact pressures will be as defined by Equation [30]; the

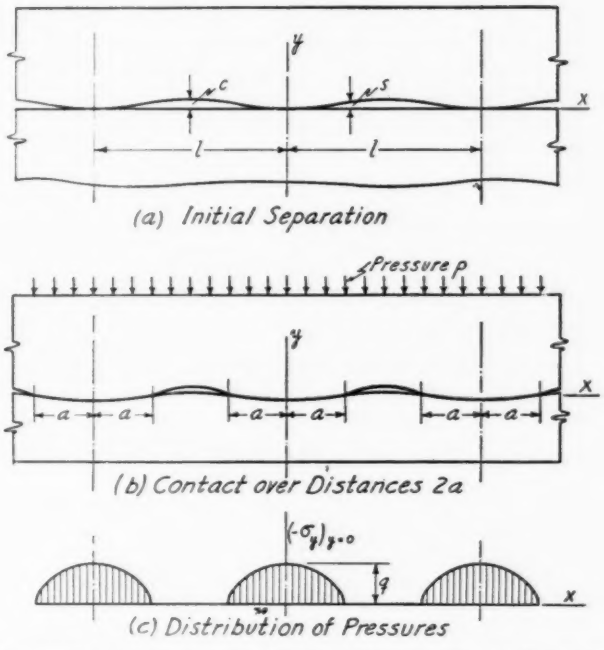


Fig. 1 Bearing Pressures Through Slightly Waved Surface

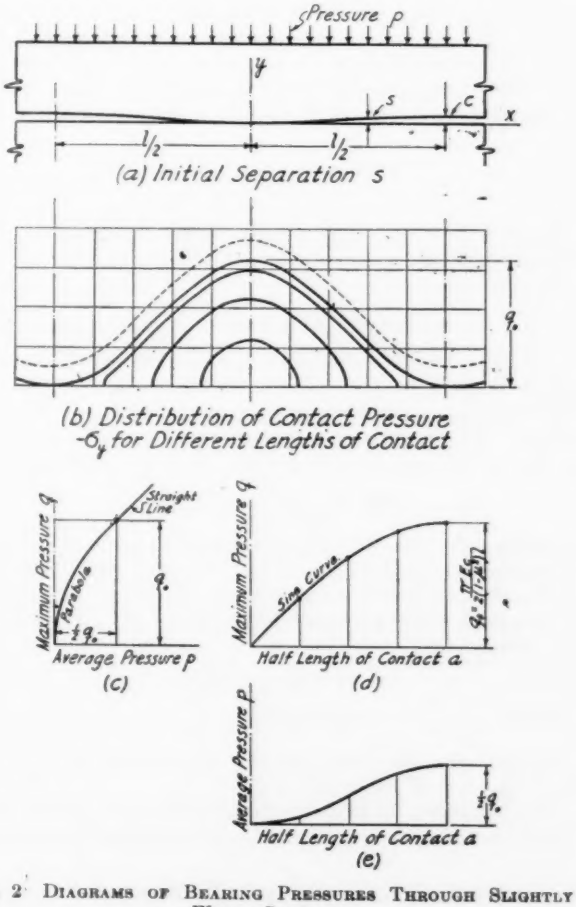


Fig. 2 Diagrams of Bearing Pressures Through Slightly Waved Surface

area of contact through which the pressures are transmitted will consist of the strips of width $2a$ defined by Equation [35]. Equations [34] and [35] are verified by Equation [27] when a is small compared with l.

It is noted that the initial separation of the two surfaces, before pressure is applied, is

$$s = (c/2)[1 - \cos(2\pi x/l)] = c \sin^2(\pi x/l), \quad s_{max} = c \ldots [36]$$

The conclusions that were drawn continue to apply if the two nearly flat surfaces in contact have a different shape, as long as the initial separation is defined by Equations [36].

Fig. 1 illustrates this case. Fig. 2 shows some results obtained from Equations [30] and [35].

Noncircular Cylinder With Nearly Flat Bottom

The function

$$Z = -\frac{8P}{3\pi a^4}\left[(z^2 + a^2/2)\sqrt{(a^2 - z^2)} + iz^3\right] \ldots [37]$$

applied to the solid $y \geqq 0$, is examined first for numerically great values of z. By writing

$$\sqrt{(a^2 - z^2)} = -iz\left(1 - \frac{a^2}{2z^2} - \frac{a^4}{8z^4} - \ldots\right) \ldots [38]$$

Z in Equation [37] is found to converge toward Z in Equation [17], which represents Boussinesq's problem. Again, at distances great compared with a the stresses are as in Boussinesq's problem, and the total load on the slice one unit wide is P.

At $y = 0$ only the interval $-a < x < a$ is loaded; the pressures are $-\mathrm{Re}Z$. At $x = 0$ the pressure is $8/3\pi$ times the average, that is, less than the average; the maximum pressure occurs at some distance from the center of the load. These pressures can be produced by contact of two solids. The required initial separation s is computed by considering the interval $-a < x < a$. One finds

$$s = -\frac{4(1-\mu^2)}{E}\,\mathrm{Im}\bar{Z} = \frac{8(1-\mu^2)P}{3\pi E a^4}\,x^4 \ldots [39]$$

The lower solid may have a flat top while the upper solid is a noncircular cylinder shaped at the bottom according to a parabola of fourth degree.

Flat Surfaces With One or More Ridges

Fig. 3(a) shows two solids with surfaces that are initially plane except for a single ridge on one of the surfaces at $x = 0$. Under the pressure p contact is missing in the intervals $-a < x < 0$ and $0 < x < a$. The same situation may be created by driving a plug in between the two surfaces. The stress function

$$Z = -p\sqrt{(1 - a^2/z^2)} \ldots [40]$$

represents this case, with the provision that a uniform horizontal tension, for example, $\sigma_x = p$ may be superposed. Fig. 3(b) shows the distribution of the pressures of contact. The force P at the ridge is found by stating Z near $z = 0$ for $y > 0$ in the two forms

$$Z = -ipa/z = P/(i\pi z) \ldots [41]$$

which gives

$$P = \pi pa \ldots [42]$$

The value of a will depend not only on p but also on the height and sharpness of the ridge.

Fig. 3(c) shows the related problem of a number of equal parallel ridges with spacing l. The corresponding stress function is

$$Z = -p\sqrt{\left[1 - \frac{\sin^2(\pi a/l)}{\sin^2(\pi z/l)}\right]} \ldots [43]$$

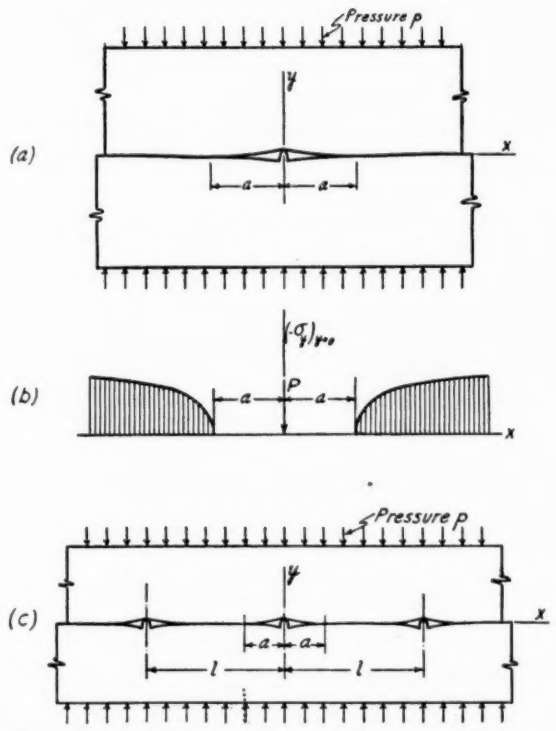


Fig. 3 Contact Between Initially Flat Surfaces With Ridges

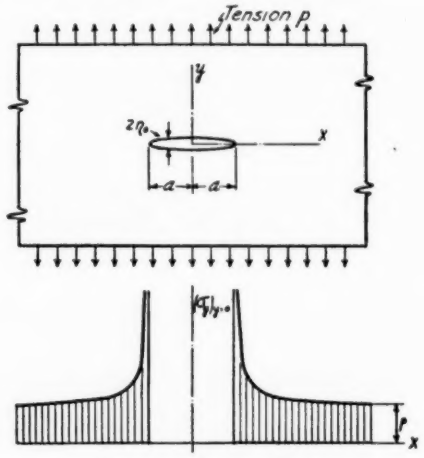


Fig. 4 Internal Crack

Internal Crack

Fig. 4 shows an internal crack which has opened from $z = -a$ to $z = a$ under the influence of an average tension p. The function

$$Z = p/\sqrt{[1 - (a^2/z^2)]} \ldots [44]$$

solves the problem. Z converges toward p when z becomes numerically great. At $y = 0$ one finds outside the crack the tension

$$\sigma_y = p/\sqrt{[1 - (a^2/x^2)]} \ldots [45]$$

and within the length of the crack the opening

$$2\eta_0 = \frac{4(1-\mu^2)}{E}\,\mathrm{Im}\bar{Z} = \frac{4(1-\mu^2)p}{E}\sqrt{(a^2 - x^2)} \ldots [46]$$

which shows the shape of the crack to be elliptic. The concentration of stress and the infinite slope $d\eta_0/dx$ at $x = \pm a$ are subject to the usual interpretation applicable to singularities. A uniform horizontal compressive stress p may be superposed without disturbing the remaining features of the solution.

Equation [44] suggests examination of the function

$$Z = p \Big/ \sqrt{\left[1 - \frac{\sin^2(\pi a/l)}{\sin^2(\pi z/l)}\right]} \quad \ldots\ldots\ldots\ldots [47]$$

At numerically great values of y this function converges toward p and defines a uniform tension p. At $y = 0$ the function accounts for a system of cracks, each of length $2a$, with centers at $x = 0, \pm l, \pm 2l, \ldots$.

The function

$$Z_1 = Z - p \quad \ldots\ldots\ldots\ldots\ldots\ldots [48]$$

with Z as in Equation [44] or [47], accounts for a crack or a system of cracks at $y = 0$, created by a liquid pressure p in the cracks as the only load.

Crack Opened by Wedge

Fig. 5(*a*) shows a crack opened by a wedge exerting pressures P. The stress functions

$$Z_1 = \frac{P}{\pi(a+z)}\sqrt{\frac{a}{z}} \quad \text{and} \quad Z_2 = -\frac{P}{\pi(a+z)}\sqrt{\frac{z}{a}} \quad \ldots [49][50]$$

represent two possible solutions, which require different loads at the outer boundary. Fig. 5(*b*) and (*c*), show the corresponding diagrams of stresses at $y = 0$. A change of the load on the outer boundary may bring about the change from Z_1 to Z_2, replacing the concentration of tension in Fig. 5(*b*) by the diagram of finite compressive stresses in Fig. 5(*c*). The form of the latter diagram near $x = 0$, with the vertical tangent at $x = 0$, should be considered as characteristic of brittle materials, such as concrete.[6]

[6] "Stresses at a Crack, Size of the Crack, and the Bending of Reinforced Concrete," by H. M. Westergaard, *Journal* American Concrete Inst., November-December, 1933, or, Proceedings, vol. 30, 1934, pp. 93–102. Contains an analysis of this feature of cracks.

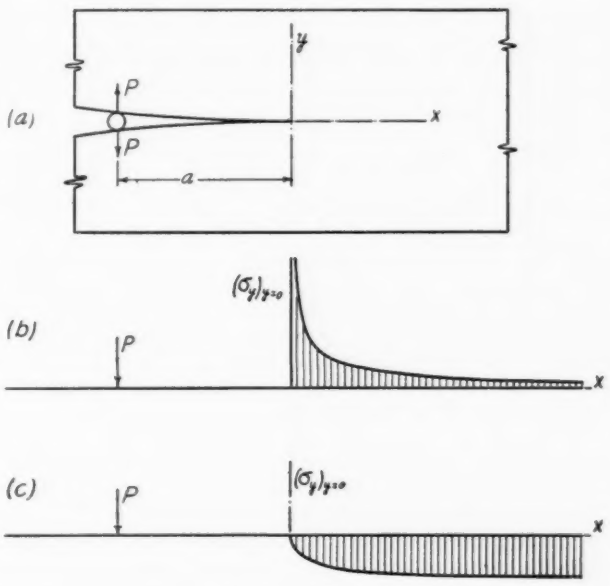


Fig. 5 Crack Opened by Wedge

An internal crack which has been opened between $z = -a$ and $z = a$ by a wedge exerting the pressure P at $z = 0$ is accounted for by the stress function

$$Z = Pa/[\pi z\sqrt{(z^2 - a^2)}] \quad \ldots\ldots\ldots\ldots [51]$$

This function shows concentration of tension at $z = \pm a$, and vanishing stresses at great distances from the crack. If an external pressure is superposed, of the magnitude p defined by Equation [42], Z in Equation [51] will be replaced by Z in Equation [40], and the concentration of tension is replaced by moderate compressive stresses.

Concluding Comment

It is easy to add further examples. Those that have been shown indicate a type of problem to which the method that was used lends itself.

The Forces Required for Rolling Steel Strip Under Tension

By A. NÁDAI,[1] EAST PITTSBURGH, PA.

In the continuous mills which have been installed during the last ten years, wide strips of steel are being rolled hot or cold. While the sheets are reduced under the lateral pressure of the rolls, tensile stresses are set up in the rolled strips. The influence of front or of back tension on the roll pressures is investigated and a theory developed for the rolling process considering tension.

The localized compression of wide sheets or strips under simultaneous application of tensile stresses is discussed under various conditions of surface friction, such as solid friction, uniform surface shearing stresses or a resistance to slipping proportional to the relative velocity of slip. Simultaneous pulling reduces considerably the pressures otherwise required for deforming metal through concentrated compression.

The theory of the rolling of wide sheets is discussed with particular reference to the influence of front and of back tension upon the rolling pressures and the results of such computations are shown in several examples. Treatment of the equations aims at a simplified use of certain variables and parameters which are needed in such computations.

I INTRODUCTION

IN VARIOUS large steel plants of the United States continuous hot- and cold-strip mills have been recently installed on a large scale. With these mills the art of making thin sheets and plates of steel has been changed and improved quite considerably, if not entirely revolutionized. Thin sheets of steel are now being rolled to a width of 70 in. or more in lengths of several hundred feet, with the same strip being engaged in 5 or 6 heavy rollstands. Improved designs of the driving and rolling equipment have made possible the production on a more economical basis of wide strips to supply the large-scale consumption of this kind of product in other industries. The advances in the manufacturing process of forming thin steel plates are: Increase of length and width of sheets, higher velocities of rolling (delivery speeds after the last stand of from 1200 to 1500 fpm or more are commonly used), the simultaneous pulling of sheets between stands, better uniformity and smoothness of surface, and an improved inner structure of the rolled products. The outstanding change perhaps was the application of large tensile stresses to the strips which, during the rolling operation, pull the sheets in the direction of rolling. This in turn brought with it the necessity of developing new means for controlling the tensile forces as well as influencing the rolling pressures required for the plastic forming of the sheets.

The ease of control of the torques and velocities of electric motors and motor generator sets which drive the mills, when compared with the driving machines of large mills used a generation ago, proved here invaluable. The electric drive made these developments possible. For example five or more consecutive pairs of heavy rolls are speeded up or slowed down in as many rollstands during the rolling of a long strip. At the same time ratios of the peripheral velocities of the pairs of working rolls must be maintained to hold constant the prescribed drafts (reductions in thickness of the sheet) to be taken in each of the consecutive stands.

For deforming metal between two rolls a torque is required which consists of two portions—one is the resultant of all the roll pressures p in the contact zone times its lever arm and the second portion is the difference between the resultants of the front and the rear tensions times the roll radius. One of the aims of this paper is to determine the pressure distribution under the rolls and thereby establish the position of the resultant of the roll pressures. Where single stands are used or special operations are to be carried out, present tendencies seem to be to increase back tension rather than front pull. One of the main reasons why utilization of more tension during rolling proved to be advantageous is evidently but another expression of the well-known facts about the plastic yielding of the ductile metals under combined stresses. If an element of a metal is deformed by compression, the stress can be diminished by applying tensile stresses at right angles to the compression direction, for example in a sheet, in the plane of the sheet (small additional tensile or compression stresses which may be present in the third direction which is mutually perpendicular to the principal compression and to the one in which the tensile stress was assumed, are of secondary importance) because yielding is controlled by the major of the principal-stress differences. Simultaneous pulling reduces the heavy rolling pressures otherwise required for deforming the metal and with them also the friction in the contact areas between the rolls and sheet. Thus wear and abrasion of the surface of the rolls are considerably lessened. This is an important factor in the maintenance costs of large mills. At the same time the uniformity of the structure of the metal sheets is improved. Large roll pressures mean massive frames, expensive bearings, power losses, and excessive wear. The power required for the pure plastic forming (compression) of the metal under the rolls increases if the frictional resistance becomes larger during the unavoidable forward and backward slipping of the moving particles of the metal along the contact area between the rolls and the compressed sheet. It is also of importance to note that the flow changes its character with increasing frictional surface resistance. The larger the friction is the more nonuniform the unit shearing strains must become in both surface layers of the sheet. Tension makes the flow more uniform through the entire cross section of a rolled sheet, and the structure is improved.

Recent developments in this field have been quite remarkable and practical questions related to the rolling of strips have been dealt with in several papers published in the last few years which cannot be quoted here in detail, but theoretical investigations on plastic flow which are adaptable to the rolling process and to details to be treated in this paper are comparatively scarce. In this connection reference to a paper prepared by Th. von

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Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y., and will be accepted until August 10, 1939, for publication at a later date. Discussion received after the closing date will be returned.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

Kármán[2] and to the extensive pioneer work by E. Siebel[3,6] done at the Steel Institute in Düsseldorf and that by W. Trinks[4] at Carnegie Institute of Technology in Pittsburgh should be made.[5] Some ideas which have been worked out in a revised way in what follows were the subjects of discussions the writer had several years ago with L. Prandtl in Göttingen and with E. Siebel. Concerning questions of rolling with tension the author is indebted for much valuable information to P. G. Lessmann in Pittsburgh with whom these details were frequently discussed recently.

An analysis of the forces required for rolling thin sheets has to consider the plastic compression of the elements of material passing under the rolls while at the same time considerable stresses act in the direction of rolling and probably minor stresses in the direction parallel to the roll axis. The frictional forces in the areas of contact have to be considered. The following factors seem to be of importance: (1) The effect of the speed of plastic deformation upon the yield stresses in the piece to be worked, particularly in hot rolling; (2) strain hardening in cold rolling; (3) the influence of the friction—extreme-pressure lubricants may be applied in the contact surfaces between the rolls and the steel sheet; (4) the elastic deflection of the heavily loaded backing-up rolls in bending; (5) "crowning" of the working rolls; (6) elastic flattening of these latter in the arc of contact; and (7) the elasticity of the rollstand. These questions can be treated only step by step and gradually. When rolling very thin sheets under heavy pressures, the radius of curvature of the rolls is increased in the area of contact so much that the original radius r of the undeformed working rolls is not defining the conditions with regard to reduction and draft. When the elasticity of various parts of the frame of the rollstands and of the rolls must be considered, the roll pressures, the tension in the strips, the curvatures of the cylinders, and the drafts will become functionally dependent on each other, particularly when very thin sheets are to be rolled.

In the present paper the plastic compressing of thin sheets will be considered first and the influence of various types of frictional resistance. The effect of front and back tension on the strips will be discussed but it will be assumed that there is no spreading of sheets in the direction parallel to the axis of the roll. The elastic flattening of the roll cylinders will be dealt with in a later paper.

II COMPRESSION OF A THIN SHEET OF MATERIAL

Some simple cases of compression will be mentioned first, for which it is easy to compute the distribution of the pressures and which may serve as typical examples of what might be expected under more complicated conditions. Friction of course is to be considered acting along the surface of plates and opposing the flow of the compressed solids.

(A) Parallel Flow in Viscous Layer in Compression

A material is supposed to fill the space between two parallel hard plates of rectangular shape. For a steady slow flow of such a solid following the law of a very viscous substance the forces can be assumed to be in equilibrium. It is assumed that the distance h between the plates is small and that the strains are infinitesimal. If the length of one pair of sides $2b$ of the rectangular compression plates is much larger than the length of the other pair $2a$, flow occurs in parallel planes, for which the x, y-plane may be chosen. (Fig. 1.) In other words, spreading of the material in the direction of the z-axis is excluded. The velocity components of flow are u and v parallel to the x- and y-axes, the coefficient of viscosity of the substance is η, and p is

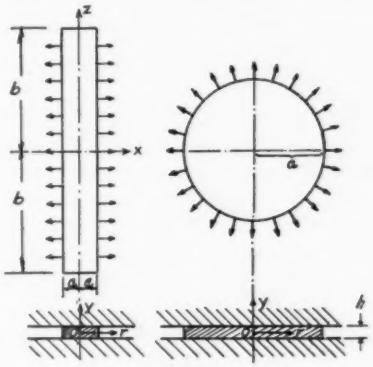
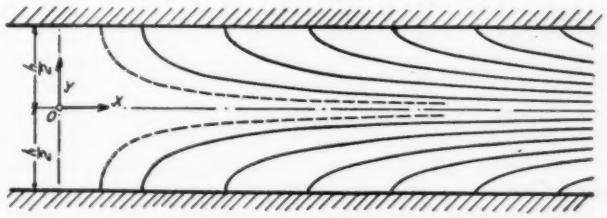


Fig. 1 Parallel Flow

Fig. 2 Radial Flow



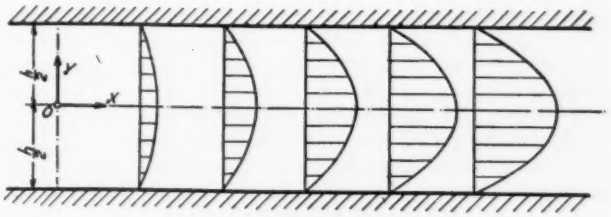


Fig. 3 Streamlines

Fig. 4 Velocities u for Parallel Flow

the pressure perpendicular to the middle plane $y = 0$ of the flat rectangular body. Suppose that the substance adheres to the compression plates. Then the boundary conditions are

$$y = \pm h/2, \quad u = 0, \quad v = \pm v_0 \ldots\ldots\ldots\ldots [1]$$

in which v_0 is the velocity with which each of the plates is moving toward the middle plane. The stream function

$$\psi = v_0 x(3h^2 y - 4y^3)/h^3 \ldots\ldots\ldots\ldots [2]$$

satisfies the imposed conditions and Equations [1] furnishing for the two velocity components of flow

[2] "Beitrag zur Theorie des Walzvorganges," by Th. von Kármán, *Zeitschrift für angewandte Mathematik und Mechanik*, vol. 5, 1925, p. 139.

[3] "Die Formgebung im bildsamen Zustande," by E. Siebel, Verlag Stahleisen Düsseldorf, 1932. The work by E. Siebel on the plastic forming of metals published in this monograph is now available also in an English translation by J. Hitchcock and published in a series of articles in *Steel*, starting in vol. 93, Oct. 16, 1933, p. 23.

A summary of Siebel's outstanding first actual measurements of the distributions of the rolling pressures is to be found in the reference.[6]

[4] "Roll Pass Design," vols. 1 and 2, second edition, The Penton Publishing Company, Cleveland, Ohio, 1933. A number of former and more recent papers on rolling were published by W. Trinks in *Blast Furnace and Steel Plant*. See "Roll Pressures in Hot Rolling," September, 1937; "The Mechanics of Cold Rolling," March, 1937; "Pressures and Roll Flattening in Cold Rolling," June, 1937.

[5] With reference to the forming of metals compare also: "Grundbegriffe der Mechanischen Technologie der Metalle," by G. Sachs, Akademische verlagsgesellschaft, Leipzig, 1925.

$$u = \frac{\partial \psi}{\partial y} = \frac{3v_0 x}{h}\left(1 - \frac{4y^2}{h^2}\right), \quad v = -\frac{\partial \psi}{\partial x} = -v_0\left(\frac{3y}{h} - \frac{4y^3}{h^3}\right) \quad \ldots\ldots\ldots [3]$$

The streamlines of this flow are shown in Fig. 3, the velocities u in Fig. 4. The pressure p which is expressed by the formula

$$p = 12\eta v_0(a^2 - x^2)/h^3 \ldots\ldots\ldots\ldots\ldots [4]$$

changes according to a parabola with the distance x from the origin. The maximum pressure

$$p_{max} = 12\eta v_0 a^2/h^3 \ldots\ldots\ldots\ldots\ldots [5]$$

is proportional to the square of the width ($2a$) of the compression plates and inversely proportional to the cube of the distance (h) between them. The total load is equal to

$$P = (2/3)(2a)(2b)p_{max} = 32\eta a^3 b v_0/h^3 \ldots\ldots\ldots [6]$$

(B) Radial Flow in Viscous Layer

If the compression plates are circular (Fig. 2) with a radius a the pressure at a distance r from the center (Fig. 5) is equal to

$$p = 6\eta v_0(a^2 - r^2)/h^3 \ldots\ldots\ldots\ldots\ldots [7]$$

The maximum pressure is $p_{max} = 6\eta v_0 a^2/h^3 \ldots\ldots\ldots\ldots\ldots [8]$

The load is $P = \pi a^2 p_{max}/2 = 3\eta\pi v_0 a^4/h^3 \ldots\ldots\ldots\ldots [9]$

It is noted that all stream lines of the flow (Fig. 3) start perpendicularly to the compression plates and that the maximum shearing stresses act along these latter. The shearing stresses τ in the interior of the substance are

$$\tau = -24\eta v_0 xy/h^3 \quad \text{and} \quad \tau = -12\eta v_0 ry/h^3 \ldots\ldots [10]\ [11]$$

respectively. These formulas may be useful for orientation.

(C) Plastic Solid in Compression

Suppose again, as in Fig. 1, that a solid is squeezed between two long parallel rectangular plates $2b \times 2a$ the distance h between which is small. Suppose that $2b$ is much larger than $2a$. Taking again one of the planes in which the material flows as the xy-plane (Fig. 6) the distribution of the pressures $p = f(x)$ can in a first approximation be found. It is assumed that plastic strains remain infinitesimal and that the three components of normal stress σ_x, σ_y, and σ_z can be taken as the principal stresses in every point of the solid. In other words, the influence of the shearing stress τ_{xy} on the condition of plasticity is neglected. The letter τ will be used for this stress later when the condition of equilibrium has to be considered. The influence of these shearing stresses τ in the bounding surfaces of the sheet $y = \pm h/2$ in which it appears as friction will therefore be taken into consideration. Under these assumptions the condition of plastic yielding is expressed by

$$(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 = 2\sigma_0^2 \ldots\ldots [12]$$

in which σ_0 is the yield stress in pure tension. Because the distance h between the compression plates is small

$$\sigma_y = -p \ldots\ldots\ldots\ldots\ldots [13]$$

throughout the solid, where p is the pressure exerted by the compression plates at a distance x from the origin, Fig. 6. The strain ϵ_z is equal to zero, that is, it is assumed that there is no "spreading" in the z-direction. Therefore

$$\sigma_z = (\sigma_x + \sigma_y)/2 = (\sigma_x - p)/2 \ldots\ldots\ldots [14]$$

The stress σ_z is determined after substituting Equations [13] and [14] in the equation of yielding [12], which gives $\sigma_x + p = \pm 2\sigma_0/\sqrt{3}$ and, taking the positive sign

$$\sigma_x + p = 2\sigma_0/\sqrt{3} = \text{const} \ldots\ldots\ldots\ldots [15]$$

This constant $2\sigma_0/\sqrt{3} = 1.155\sigma_0$ will be called the "constrained yield strength" for tension and will be designated as s_0. It is the tensile yield stress in a bar, which is allowed to contract freely in one of the lateral directions, while it is constrained not to deform in the other lateral direction (this latter is the direction,

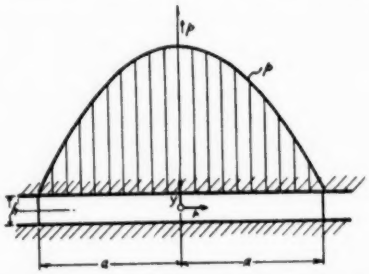
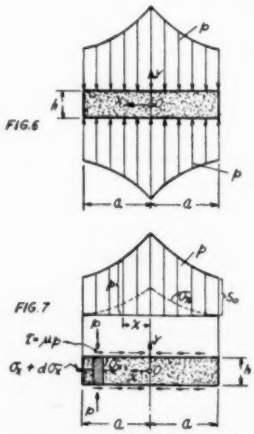


Fig. 5 Distribution of Pressure p in Radial Flow



Figs. 6 and 7 Plastic Solid in Compression

in which spreading in the sheet was excluded). It is seen from Equation [15] that for plastic yielding (in first approximation) in a wide sheet the sum of the normal stress σ_x and of the pressure p remains constant and equal to the constrained yield strength s_0:

$$\sigma_x + p = s_0 \ldots\ldots\ldots\ldots\ldots [16]$$

To this must now be added the condition for equilibrium of the horizontal forces deforming any small column of material of the substance under compression (Fig. 7), taking into consideration the frictional resistance or a shearing stress τ along the compression plates. This condition is

$$d\sigma_x/dx = -dp/dx = 2\tau/h \ldots\ldots\ldots\ldots [17]$$

Several kinds of frictional resistance may be considered:

(1) *Assume Dry Friction Between the Plastic Solid and the Plate.* Then the shearing stress $\tau = \mu p$ where μ is the coefficient of friction. Using Equation [17]

$$dp/dx = -2\mu p/h \ldots\ldots\ldots\ldots\ldots [18]$$

is obtained. When the edge $x = a$ is free of stress, $\sigma_x = 0$ for $x = a$ and therefore $p = s_0$ and the pressure p becomes when $o < x < a$

$$p = s_0 e^{2\mu(a-x)/h} \ldots\ldots\ldots\ldots\ldots [19]$$

If $-a < x < 0$ the negative sign before x in the exponent has to be changed to a plus sign. These two pressure curves are plotted in Fig. 7. When on the other hand the compressed sheet is simultaneously subjected to tensile forces at right angles to the direction of the compression it is easy to see that the pressures p must follow the same exponential law but that they are reduced. It is assumed that the compression plates remain parallel as they are approaching each other. If the compressed sheet is subjected to a tension equal to $\sigma_x = \sigma_1$ on the left side

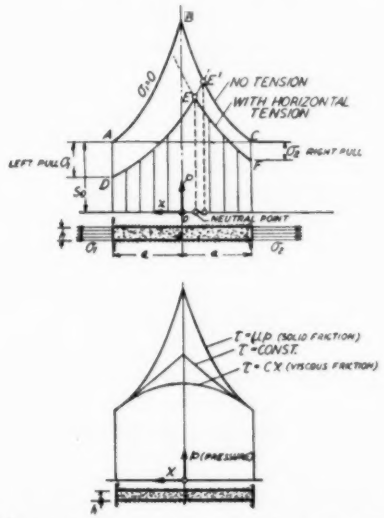


FIG. 8 PRESSURE CURVES FOR PLASTIC SOLID

FIG. 9 PRESSURE CURVES $p = f(x)$ FOR PLASTIC SOLID

of the compressed portion ($x = +a$) and a tension of $\sigma_x = \sigma_2$ on the right side (for $x = -a$) of it, two new pressure curves are obtained

$$\left.\begin{aligned} &\text{left curve,} \quad p = (s_0 - \sigma_1)e^{2\mu(a-x)/h} \\ &\text{right curve,} \quad p = (s_0 - \sigma_2)e^{2\mu(a+x)/h} \end{aligned}\right\} \quad \ldots\ldots\ldots [20]$$

as indicated in Fig. 8. Only the portions DE and EF of these are to be taken. Their point of intersection E defines a point in the compression plane, at which the frictional forces τ must change the direction in which they act. This point has been called the "neutral point" by Trinks in applications to rolling pressures. Its position depends on the tensions σ_x and one can see that it shifts away from the side on which more pull is applied.

Conditions with regard to the friction in the case of rolling may be very different under different conditions. Unfortunately not much is known about these conditions or even about the type of frictional forces which are active in the contact areas of heavily loaded working rolls. During hot rolling of billets when a scale covers these latter and hot water or steam may or may not be present in the contact areas, conditions may be very different from those during cold rolling, after the sheets have been cleaned by pickling in acid solutions. And again while large reductions in pickled strips are being taken cold and frequently in the presence of a lubricant which has been applied on the surface of the sheet, the friction may be very different from that which is active during a skin pass, where the reduction is quite small and no lubricant is used. It seems probable however, that neither a clear case of "solid" friction nor of an "oil film" showing simple fluid or viscous resistance can be present or representative of the conditions during cold rolling. The extremely high local pressures in the contact areas (of the order of 100,000 lb per sq in.) exclude the formation of the latter with certainty. However the assumption of dry friction, according to which $\tau = \mu p$ and the coefficient of friction μ should be independent of the pressure and of the relative velocity of slip between sheet and roll (an assumption which has been used exclusively in the theoretical work on the rolling process), seems also open to criticism. It leads to various conclusions which are not so well supported by tests.

The curves in Fig. 8 bring out a consequence of the assumption. The pressure curves $p = f(x)$ show a sharp peak, which is the more pronounced, the larger μ is assumed. The actual measurements of the pressures in the contact area of rolls by E. Siebel and W. Lueg[6] do not show such sharp breaks but more or less rounded maxima. (The finite dimensions of the piezoelectric pressure gages used by them would account only partially for this rounding off of the pressure curves near their maxima.)

Another consequence of the assumption $\tau = \mu p$ should also be mentioned. If $\tau = \mu p$ the stress τ must increase with the pressure p. A limit to this is however set by every material itself. If the pressure is sufficiently high due to the increase of the tangential resistance either the tensile strength or the plastic limit of the material must finally be reached. Let σ_0 be the yield stress in pure tension. The maximum shearing stress to which a plastic material can be subjected is $\tau_m = \sigma_0/\sqrt{3}$. The friction cannot become larger than τ_m. Hence a maximum pressure p_m must exist, at which the shearing stresses produced by dry friction just becomes equal to τ_m. This pressure is equal to

$$p_m = \sigma_0/(\mu\sqrt{3}) = s_0/2\mu \ldots\ldots\ldots\ldots [21]$$

It should be noted that in the preceding and following considerations the effect of strain hardening is not considered. When for example $\mu = 1/4$, this pressure would be equal to $2s_0$. Such and still larger pressures can be reached in the contact areas. At these high pressures the ordinary friction rule cannot be true. It will be shown later how some of these difficulties can be avoided.

From tests on solid friction it is known that if the minutest quantities of a fluid are present which is wetting the surface of a solid, the coefficient of friction is lowered. Some kind of extreme-pressure lubrication in the contact areas of the rolls and the sheets must be postulated. It is therefore perhaps of practical interest to compare one or two further cases for the friction other than the one ($\tau = \mu p$) which is most frequently assumed, for which pressure curves are most easily obtained, namely:

(2) *Assume τ = Const.* In this case the pressure gradient becomes constant and the pressure lines $p = f(x)$ are straight lines.

(3) *Assume Viscous Friction.* When on the other hand a frictional resistance is considered which is proportional to the relative velocity of slip between the sheet and the pressure plates (rolls) and when the flow is symmetrical to the point $x = 0$, the shearing stress τ may be taken as

$$\tau = Cx \ldots\ldots\ldots\ldots\ldots\ldots [22]$$

This leads to conditions somewhat similar to those which were described for the compression of a viscous layer in section II(A) and to a pressure distribution

$$p = s_0 + (C/h)(a^2 - x^2) \ldots\ldots\ldots\ldots [23]$$

leading to a maximum pressure $p_{max} = s_0 + (Ca^2/h)$. Fig. 9 shows the pressure curves for these three cases: (1) dry friction, $\tau = \mu p$; (2) τ = const; and (3) viscous friction, $\tau = cu$ (u is the relative tangential velocity of flow along the compression

[6] "Untersuchungen über die Spannungsverteilung im Walzspalt," by E. Siebel and W. Lueg, *Mitteilungen aus dem Kaiser-Wilhelm-Institut für Eisenforschung zu Düsseldorf*, vol. 15, 1933, Report No. 218, p. 1.

plates). It is assumed that the gradient of p is the same for $x = \pm a$.

(D) Distortion Due to Friction in Surface of Compressed Sheet

As mentioned in the introduction there is ample experimental evidence available that the outermost layers of a rolled or laterally compressed sheet can be considerably more distorted than the interior. The unit shearing strains γ_{xy} near the surface of the sheet can be considerably larger than in the central portions. An exact solution for a plane state of plastic strain by L. Prandtl[7] brings this out well and should therefore be recalled here. He found the following distribution of stress describing a flow of a material constrained between two parallel planes $y = \pm h/2$

$$\left.\begin{aligned} \sigma_x &= C + s_0\left[\frac{x}{h} + \sqrt{\left(1 - \frac{4y^2}{h^2}\right)}\right] \\ \sigma_y &= C + (s_0 x/h) \\ \tau &= -s_0 y/h \end{aligned}\right\} \quad [24]$$

A family of cycloids (Fig. 10) as slip lines corresponds to this system of stresses. When $y = h/2$

$$\sigma_y = -p = C + (s_0 x/h), \quad \tau = -s_0/2 = \text{const.} \quad [25]$$

The same conditions are obtained which were mentioned in case (2). The gradient of the pressure dp/dx and the shearing stress τ in the plane $y = h/2$ remain constant. A system of plastic displacement components ξ, η which is consistent with Equations [24] has been given by the writer

$$\left.\begin{aligned} \xi &= \lambda\left[x - x_0 + h\sqrt{\left(1 - \frac{4y^2}{h^2}\right)}\right] \\ \eta &= -\lambda y \end{aligned}\right\} \quad [26]$$

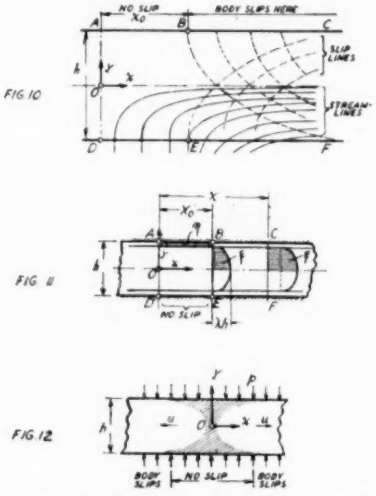
To it corresponds a unit shear

$$\gamma = \frac{\partial \xi}{\partial y} + \frac{\partial \eta}{\partial x} = -\frac{4\lambda y}{h\sqrt{\left(1 - \frac{4y^2}{h^2}\right)}} \quad [27]$$

These last equations show how a plastic layer must deform under the stress system of Equations [24]. Straight lines, which were formerly perpendicular to the compression plates are distorted into semiellipses (Fig. 11). The unit shear γ increases rapidly with y and even becomes infinite in the planes $y = \pm h/2$.

The behavior of the solution in the neighborhood of these two planes $y = \pm h/2$ is extremely interesting from a mathematical point of view. These two parallel planes are not only natural boundary surfaces of the plastic body, but along them two distinct portions of the mathematical solution branch off from each other.[8] At any rate these details show that conditions with regard to adhesion and slip along the planes $y = \pm h/2$ cannot be quite simple when a plastic solid is forced to move in a tangential direction along a hard plate under a heavy pressure.[9] It is seen from Equation [26] that $\xi = 0$ when $x = x_0$, $y = h/2$, and the solid adheres at this point to the compression plate.

When the flow is symmetrical to a vertical axis another point should be mentioned. The preceding solution fails near the axis of symmetry. L. Prandtl and Hilda Geiringer[10] have drawn plausible extensions of the slip lines characterizing the trend of the solution in the central region of the compressed layer, which has been marked in Fig. 12 by shading. For reasons



Figs. 10, 11, and 12 Retarding Influence of Surface Upon Flow

of symmetry in this case τ must vanish in the axis $x = 0$. Hence a certain region $-x_0 < x < x_0$ must exist in the compressed sample, in which

$$-s_0/2 < \tau < s_0/2 \quad [28]$$

The writer believes that in this region the plastic solid adheres to the surface of the compression plates. As τ increases it becomes just equal to $s_0/2$ at a point x_0 and here the plastic solid starts to slip. This has been indicated in Figs. 10 and 11. In truth γ cannot become infinite, because of the strain hardening of the metal, which was here entirely neglected.

III ROLLING PRESSURES IN STRIPS UNDER SIMULTANEOUS PULLING

With reference to Fig. 13 representing a section through the rolls and the sheet let r be the radius of the working rolls, h the variable thickness of the sheet in the contact region, varying between the values of h_1 and h_2, x the horizontal distance of point A

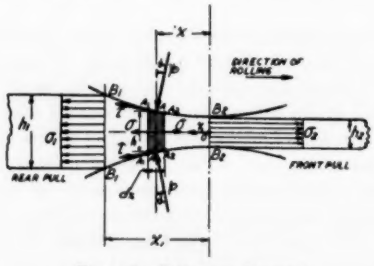


Fig. 13 Rolling Forces

in the sheet from the point O, x_1 the projected length of the contact arc B_1B_2, and α the angle between the normal to the rolls at point A and the vertical line passing through the roll centers.

[7] "Anwendungsbeispiele zu einem Henckyschen Satz über das plastische Gleichgewicht," by L. Prandtl, *Zeitschrift für angewandte Mathematik und Mechanik*, vol. 3, December, 1923,

[8] "Über die Gleit- und Verzweigungsflächen einiger Gleichgewichtszustände bildsamer Massen und die Nachspannungen bleibend verzerrter Körper," *Zeitschrift für Physik*, vol. 30, October, 1924, p. 106.

[9] Of interest in this regard is the paper, "The Distortion of Wires on Passing Through a Draw-Plate," by G. I. Taylor and H. Quinney, *Journal of the Institute of Metals*, vol. 49, 1932, p. 187. The authors have observed the distortion of wires after the drawing operation and their observations show this effect clearly.

[10] "Fondements Mathématiques de la Théorie des Corps Plastiques Isotropes," by Hilda Geiringer, Mémorial des Sciences Mathématiques, Paris, 1937, p. 76. Compare particularly her Fig. 15 on page 86 showing the flow lines.

Introducing for the reduction of thickness the notation

$$\lambda = (h_1 - h_2)/h_1 \quad [29]$$

and assuming small angles α for which

$$x/r = \sin\alpha = \alpha, \quad 1 - \cos\alpha = \alpha^2/2 = x^2/2r^2 \quad [30]$$

the variable thickness h can be taken as

$$h = h_2 + 2r(1 - \cos\alpha) = h_2 + r\alpha^2 = h_2 + (x^2/r) \quad [31]$$

and the length of the contact region becomes

$$x_1 = \sqrt{[r(h_1 - h_2)]} = \sqrt{(\lambda r h_1)} \quad [32]$$

Now let p be the surface pressure in lb per sq in. exerted at the points A by the roll on the deforming sheet, σ the mean value of the stress in the sheet in the rolling direction, τ the friction between rolls and sheet at the point A, and u the mean horizontal velocity with which the sheet moves when the cross section passes the point A at the distance x from O. The product of the mean velocity u of the sheet times the thickness h (volume of steel passing per second) is constant. Therefore

$$uh = u_1h_1 = u_2h_2 \quad [33]$$

where u_1 is the entering and u_2 the delivery velocity. From this it follows that the sheet must slip relatively to the roll with a velocity equal to $(u/\cos\alpha) - u_0$, where u_0 is the peripheral velocity of the rolls. Due to the smallness of the angles α this can also be taken equal to $u - u_0$. In the front portion of the contact zone where $u > u_0$ the sheet moves faster than the rolls (forward slip) and in the rear end of the pressure zone where $u < u_0$ the sheet moves slower than the rolls (backward slip).

Considering a small prism of material $A_1A_1A_2A_2$ (Fig. 13) of width dx and height h, the following forces act: On the top and bottom faces, the pressures $p\,dx$ normal to the surface of the rolls and the friction forces $\tau\,dx$ in a tangential direction (as long as $u > u_0$ these latter act on the sheet in the directions indicated in Fig. 13), and finally on the two side faces of the prism, the normal forces σh and $\sigma h + d(\sigma h)$ respectively. Equilibrium of the force components taken in the direction of rolling requires that

$$\frac{d}{dx}(h\sigma) + 2\tau + 2p\frac{x}{r} = 0 \quad [34]$$

Second, it is necessary to express that the sheet is deformed plastically. According to Equation 16 this is the case when

$$\sigma + p = s_0 \quad [34a]$$

where s_0 is the constrained yield strength as previously defined. Third a statement about the friction has to be made. Three different cases of friction will be considered again: (1) solid friction, $\tau = \pm\mu p$; (2) τ is constant; or (3) frictional resistance proportional to the relative velocity of slip between the sheet and the roll, if some kind of extreme-pressure lubricant be used. Before proceeding further the variables x and p are better transformed. Instead of the pressure p the ratio of p to the yield strength s_0 will be used as a new dependent variable $y = p/s_0$ and instead of the coordinate x (or the angle α) a new independent variable z defined by $x = \beta r z$, where the new dimensionless constant β is given by

$$\beta^2 = h_2/r = (1 - \lambda)h_1/r \quad [35]$$

This constant β depends on the ratio of the thickness to the radius of the roll. The variable thickness h which appears in Equation [34] can be expressed in terms of z using Equation [31] together with [35] which gives

$$h = h_2[1 + (x^2/rh_2)] = h_2(1 + z^2) \quad [36]$$

Taking Equations [35] and [36] and introducing

$$z = x/\beta r \text{ and } y = p/s_0 \quad [37]$$

Equation [34] is transformed to the following equation containing the new variables y and z

$$(1 + z^2)\frac{dy}{dz} - \left(\frac{2}{\beta s_0}\right)\tau = 2z \quad [38]$$

Of *this differential equation of the rolling problem*, which has figured in other forms in the papers by von Kármán, Siebel, Trinks, and Hitchcock, various applications will now be made. For this purpose the assumptions about the shearing stress τ expressing the resistance due to the surface friction of the rolls must be considered.

(1) Rolling Pressures With Solid Friction

Two branches of the pressure distribution have to be distinguished according to whether in the contact arc forward or backward slip occurs. In the first case $\tau = \mu p = \mu s_0 y$ and in the second case $\tau = -\mu p = -\mu s_0 y$. After writing for the constant $2\mu/\beta$

$$k = 2\mu/\beta = 2\mu\sqrt{[r/[(1 - \lambda)h_1]]} \quad [39]$$

Equation [38] becomes for forward slip

$$(1 + z^2)(dy/dz) - ky = 2z \quad [40]$$

This can be integrated after substituting instead of z a new variable v given by

$$z = \tan v, \quad dv = dz/(1 + z^2) \quad [41]$$

$$(dy/dv) - ky = 2\tan v \quad [42]$$

from which

$$y = e^{kv}(C + 2\int e^{-kv}\tan v\,dv) \quad [43]$$

is derived. In most cases this integral can be computed by replacing the term $\tan v$ by v and integrating.[11] This furnishes for the forward slip as the solution for the pressure curve

$$y = p/s_0 = C_2e^{kv} - 2(1 + kv)/k^2 \quad [44]$$

and for the backward slip

$$y = p/s_0 = C_1e^{-kv} - 2(1 - kv)/k^2 \quad [45]$$

After the integration constants C_1 and C_2 have been determined, rolling pressures can be computed from Equations [44] and [45]. It is noted that while the coordinate x varies between zero and $x_1 = \sqrt{(\lambda r h_1)}$ the variable z changes from the value zero to a value z_1 for which from Equations [32], [35], and [37]

$$z_1^2 = \frac{x_1^2}{\beta^2r^2} = \frac{\lambda h_1}{\beta^2 r} = \frac{\lambda}{1 - \lambda} \quad [46]$$

If for example the reduction in thickness is 50 per cent, $\lambda = 0.5$, $z_1 = 1$, and $0 < z < 1$. Usually $z_1 < 1$. The two integration constants C_1 and C_2 can be determined from the conditions at the ends of the contact zone so that the prescribed front and rear tensions σ_2 and σ_1 are reached there. From Equation [45] when

$$x = x_1; \quad z = z_1 = \sqrt{[\lambda/(1 - \lambda)]}, \quad \text{and } \sigma = \sigma_1 \quad [47]$$

and similarly from Equation [44] when

$$x = 0; \quad z = 0 \quad \text{and } \sigma = \sigma_2 \quad [48]$$

[11] The integrand could be expanded in a series writing $\tan v = v + v^3/3 + \ldots$ and the integration be carried out term by term. This series converges in the cases under consideration so rapidly that it is sufficient to take only the first term.

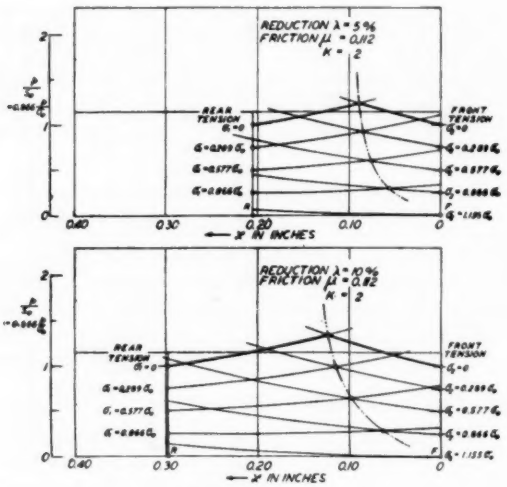
σ_1 is the specified back- and σ_2 the given front-tension stress in the rolled sheet. σ_1 acts at the entering and σ_2 at the delivery end of the pressure zone.

The rear and the front values of the pressure p_1 and p_2 are obtained from Equations [16] and [37] which also show that when using the conditions of Equations [47] and [48] we obtain when

$$z = z_1 = \sqrt{[\lambda/(1-\lambda)]}; \quad p_1 = s_0 - \sigma_1, \quad y_1 = 1 - (\sigma_1/s_0) \ldots [49]$$

and when $$z = 0; \quad p_2 = s_0 - \sigma_2, \quad y_2 = 1 - (\sigma_2/s_0) \ldots [50]$$

Conclusions. Summing up the results of the preceding analysis it is seen that the rolling pressures p can be computed as follows: Given are usually the sheet thickness h_1, the friction μ,



Figs. 14 and 15

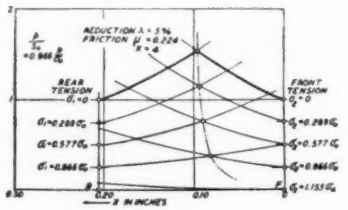


Fig. 18

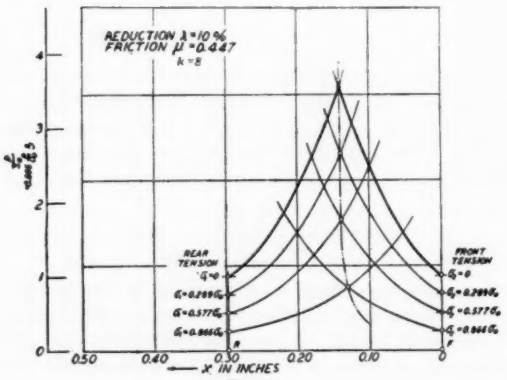
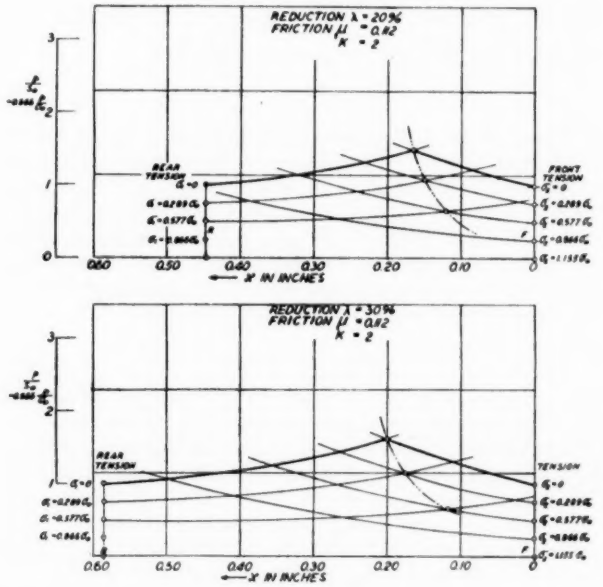
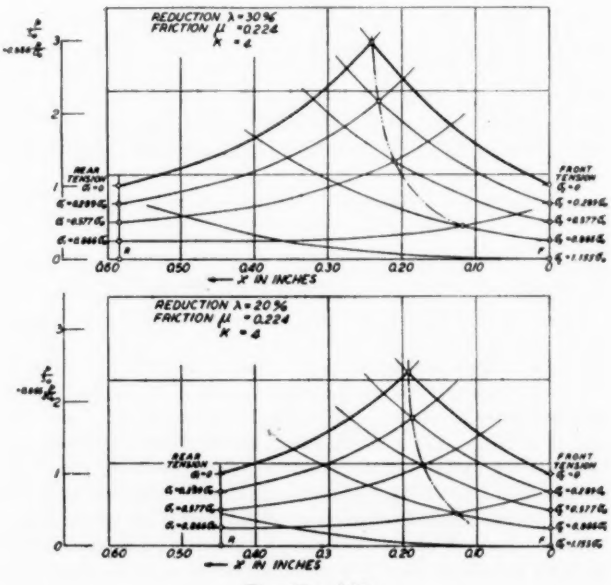


Fig. 21



Figs. 16 and 17



Figs. 19 and 20

FIGS. 14 TO 21 ROLLING PRESSURES FOR VARIOUS AMOUNTS OF FRONT AND REAR TENSIONS

NOTATION FOR FIGS. 14 TO 21

(See Fig. 13)

p = pressure on roll surface, lb per sq in.
σ_1 = rear tension on strip, lb per sq in.
σ_2 = front tension on strip, lb per sq in.
σ_0 = yield stress in tension, lb per sq in.
μ = coefficient of friction
λ = $[(h_1 - h_2)/h_1] \times 100$, per cent (reduction)
h_2 = thickness of rolled sheet = 0.1 in.
r = radius of rolls = 8 in.
k = $2\mu\sqrt{(r/h_2)}$
s_0 = constrained yield strength, lb per sq in.

Ordinates, $p/s_0 = 0.866\ p/\sigma_0$

the radius of the rolls r, and the reduction λ. From these are computed the three constants

$$\beta^2 = (1 - \lambda)h_1/r, \quad z_1^2 = \lambda/(1 - \lambda), \quad k = 2\mu/\beta \ldots [51]$$

If the yield strength s_0 is known and the back and front tensions σ_1 and σ_2 are given, the back pressure $p_1 = s_0 - \sigma_1$ and the front pressure $p_2 = s_0 - \sigma_2$. Thus also $y_1 = 1 - (\sigma_1/s_0)$ and $y_2 = 1 - (\sigma_2/s_0)$. For a given distance x the roll pressure p is finally found after computing the parameters v and v_1

$$\tan v = x/\beta r, \quad \tan v_1 = \sqrt{[\lambda/(1 - \lambda)]} \ldots\ldots [52]$$

and p from either

$$p = s_0 \left\{ \left[y_1 + \frac{2(1 - kv_1)}{k^2} \right] e^{k(v_1 - v)} - \frac{2(1 - kv)}{k^2} \right\} \ldots [53]$$

or

$$p = s_0 \left[\left(y_2 + \frac{2}{k^2} \right) e^{kv} - \frac{2(1 + kv)}{k^2} \right] \ldots\ldots [54]$$

The first of these last pressure curves refers to the entering (rear) side of the sheet and the latter curve to the delivery (front) side. The smaller of the two values of p thus found has to be taken. It is noted that Equation [53] is valid when $x_0 < x < x_1$ and Equation [54] when $0 < x < x_0$, in which x_0 is the coordinate of the "neutral point" at which the friction changes its direction. The value x_0 is defined by the intercept of curves [53] and [54] referring to the pressures in the rear and front portions of the pressure area respectively.

Using these expressions a few pressure distributions $p = f(x)$ have been computed numerically[12] and the results plotted in Figs. 14 to 21. Three cases have been chosen, assuming a small, medium, and large friction coefficient $\mu = 0.112$, 0.224, and 0.448 respectively. The following reductions of thickness were compared: $\lambda = 5$, 10, 20, and 30 per cent.

The delivery thickness $h_2 = (1 - \lambda)h_1$ of the sheet was $h_2 = 0.1$ in., the radius of the rolls was $r = 8$ in. The abscissas in the figures represent the coordinate x or the distance of a point from the center line of the rolls. The ordinates show the corresponding roll pressures in the contact zone. The ratios p/s_0 were plotted. The three cases of normal, medium, and large friction correspond to a value for the constant k of 2, 4, and 8 respectively. The direction of sheet travel in all pressure plots is from the left to the right. The top pressures shown with heavy lines correspond to the case when the rear and front tensions are zero. In each diagram additional pressure lines are indicated, corresponding to a rear or a front tension equal to σ_1 or $\sigma_2 = 0.289\,\sigma_0$, $0.577\,\sigma_0$, and $0.866\,\sigma_0$. Thus by interpolating between these curves, intermediate pressure lines may be constructed for other values of the front or rear tensions. The pressure hills are the steeper the larger the friction is and they become less steep with increasing front or rear tension. The neutral point shifts its position toward the delivery side or in the direction of sheet travel when both the front and the rear tension simultaneously increase in the same proportion. On the contrary the neutral point is shifted in an opposite direction when the front tension increases more than the rear tension. These conclusions may be useful for estimating the forward slip in the sheets passing under the rolls after a change in front or rear pull has occurred.

(2) Rolling Pressures When Surface Friction Is Assumed Constant

Let the surface friction be designated by τ (lb per sq in.). According to whether $z < z_0$ (forward slip) or $z > z_0$ (backward slip), $\tau = \tau_0$ or $\tau = -\tau_0$. After writing for the combination of constants

$$2\,\tau_0/\beta s_0 = k \ldots\ldots [55]$$

Equation [38] becomes

$$\frac{dy}{dz} = \pm \frac{k}{1 + z^2} + \frac{2z}{1 + z^2} \ldots\ldots [56]$$

This differential equation of rolling, integrated for the pressure, gives the function

$$y = p/s_0 = \pm k \tan^{-1} z + \ln(1 + z^2) + C \ldots\ldots [57]$$

The upper and lower signs are to be taken for forward and backward slip respectively. Assuming no front and no rear tension the integration constants become

$$C_1 = 1 \text{ and } C_2 = 1 + k \tan^{-1} z_1 - \ln(1 + z_1^2) \ldots [58]$$

These pressure curves are shown in Fig. 22 in an example in which it is assumed that the surface friction is one tenth of the constrained yield strength s_0, that the ratio of the roll radius to the sheet thickness h_1 is 100, and that a reduction of 50 per cent is taken. Thus $\tau = 0.1 s_0$, $r/h_1 = 100$, and $\lambda = 0.5$. This gives $\beta^2 = h_2/r = 0.00500$, $\beta = 0.0707$, $z_1 = 1$ and $k = 2.829$.

(3) Rolling Pressures When the Surface Friction Is Assumed Proportional to the Relative Velocity of Slip

It was mentioned that little is known about the friction when lubricated sheets are rolled. In cold reducing of very thin sheets palm oil is used as a lubricant. The viscosity of oils is known to decrease with temperature, but to increase with the pressure p. According to recent tests by R. B. Dow[13] increasing the pressure by 10,000 lb per sq in. increases the viscosity of mineral oils about ten times. Viscosity of oils η seems to increase with the pressure according to an exponential function of p. It is certain therefore that η in an oil layer or in the patches of oil which may be present in the areas carrying such high pressures as are known to exist during the rolling of very thin steel sheets, must be of quite another order of magnitude than at atmospheric pressure. Although observations are not available to substantiate the following assumption, which is rather doubtful in its present shape, it was thought that it might at least be interesting to inquire what will become of the pressure hill and what shape the pressure line $p = f(x)$ will take if the tangential surface forces be assumed proportional to the relative velocity of slipping in the contact area.[14] The constant which expresses this proportionality may contain η and the thickness δ of the oil layer. For η may be taken a mean value of the viscosity at the high rolling pressures. Whether a continuous oil layer can exist or whether the oil is only present in patches with a varying δ must be left an open question. Let u_0 be the peripheral velocity of the roll and u the variable velocity of translation with which the sheet moves in the contact zone. Since the variable thickness is

$$h = h_2 (1 + z^2) \ldots\ldots [58a]$$

the velocity is

$$u = u_2 h_2/h = u_2/(1 + z^2) \ldots\ldots [59]$$

and the velocity of the slip is equal to $u - u_0$. Since

[12] The author acknowledges the valuable assistance received in these computations by Mr. Frankenberg.

[13] "The Effects of Pressure and Temperature on the Viscosity of Lubricating Oils," by R. B. Dow, *Journal of Applied Physics*, vol. 8, 1937, p. 367.

[14] Also, this assumption cannot be based on experimental information. It is rather more probable that the law $\eta = \text{const}$ is not applicable at high pressures and in very thin films of oil but that $\tau = f(\partial\gamma/\partial t)$.

$$u_0 = u_2/(1 + z_0^2) \quad [59a]$$

where z_0 is the (unknown) value of z corresponding to the point at which the slip is zero, the friction law which was postulated can be expressed by the equation

$$\tau = \frac{\eta}{\delta}(u - u_0) = \frac{\eta u_2}{\delta}\left(\frac{1}{1+z^2} - \frac{1}{1+z_0^2}\right) \quad [60]$$

in which δ may be interpreted as a kind of mean thickness of the

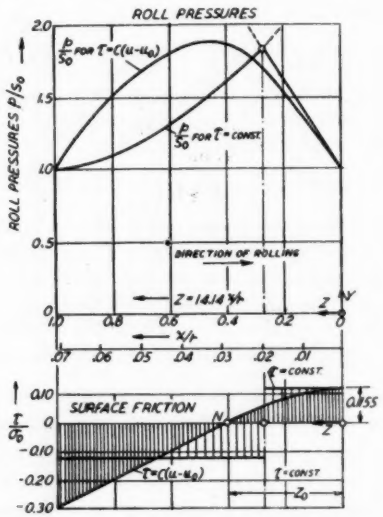


FIG. 22 DISTRIBUTION OF ROLLING PRESSURE AND SURFACE FRICTION CORRESPONDING TO TWO ASSUMPTIONS FOR FRICTION ($\lambda = 0.5$, no front or back tension, $r/h_1 = 100$, $r = 10$ in.)

oil film. After replacing $\eta u_2/\delta$ which has the dimension of a shearing stress by the symbol τ_0 and using

$$k = 2\tau_0/\beta s_0 \quad [60a]$$

the surface friction between roll and sheet can be written as

$$\tau = \tau_0\left(\frac{1}{1+z^2} - \frac{1}{1+z_0^2}\right) \quad [61]$$

and the differential equation of rolling [38] becomes

$$(1+z^2)\frac{dy}{dz} = k\left(\frac{1}{1+z^2} - \frac{1}{1+z_0^2}\right) + 2z \quad [62]$$

The ambiguous $\pm$ sign has now entirely disappeared from the equation and with it the need of distinguishing two distinct branches of pressure lines which complicated the former discussions. It is also noted that if $z = z_0$, $dy/dz \neq 0$, the neutral point does not coincide with the location of the maximum in the pressure hill as one would have perhaps guessed. After carrying out the integration of Equation [62] the following pressure distribution is found (when no tensions act)

$$y = \frac{p}{s_0} = 1 + \ln(1+z^2) + \frac{k}{2}\left[\frac{z}{1+z^2} - \left(\frac{1-z_0^2}{1+z_0^2}\right)\tan^{-1}z\right] \quad [63]$$

The variable z in this function is related to the distance x by $x = \beta r z$, where the constant $\beta^2 = h_2/r$. The pressure distribution given by Equation [63] satisfies the condition that for $z = 0$, $y = 1$. Similarly at the other end of the pressure zone, where $z = z_1$, y must also become equal to 1. This latter condition

$$z = z_1 = \sqrt{[\lambda/(1-\lambda)]}, \qquad y = 1 \quad [64]$$

really defines the location of the neutral point (the coordinates z_0 or x_0) and the forward slip in the sheet

$$\nu = (u_2 - u_0)/u_0 \quad [65]$$

As according to Equation [59a] $u_2 = u_0(1 + z_0^2)$, it is seen that the *forward slip* becomes equal to

$$\nu = z_0^2 \quad [66]$$

That the forward slip must increase when a lubricant is acting follows from Fig. 22 by comparing the two curves for τ corresponding to the two cases (2) and (3).

Fig. 22 shows the rolling pressures for this case in an example in which the same data were assumed as in the last example: $r = 10$ in., $h_1 = 0.10$ in., $h_2 = 0.050$ in., $\lambda = 0.5$, apparent friction coefficient at delivery point $\mu = 0.1$. Under these conditions $\beta = 0.0707$ and $z_1 = 1$. The length of the contact zone is $x_1 = \beta r z_1 = 0.707$ in. The distance x_0 for the neutral point is $x_0 = 0.283$ in. corresponding to $z_0 = 0.40$. Fig. 22 shows both the pressures p and the distribution of the surface frictions τ.

It is hoped that the various examples which have been worked out in detail will prove of some assistance to rolling-mill engineers until more experimental information becomes available with regard to the fairly complex conditions under which the thickness of metal sheets is reduced in mills.

ACKNOWLEDGMENTS

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Numerical and Graphical Method of Solving Two-Dimensional Stress Problems

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In developing his numerical and graphical method for solving two-dimensional stress problems, while the authors use a tension member with a semicircular notch as an example, the procedure actually has a wide range of application. Comparison of the calculated results and those obtained from photoelasticity shows close agreement. The maximum value of the stress in the plate used as an example obtained by this method is 3.25. This compares with a maximum of 3.20 obtained photoelastically by Wahl and Beeuwkes and a maximum of 3.05 obtained in the same manner by M. M. Frocht.

THE PURPOSE of this paper is to develop a method of solving two-dimensional stress problems. While the procedure is illustrated on a tension member with a semicircular notch, Fig. 1, it is very general in its applicability.

1 *Introduction.* A comparison of the calculated results with those obtained from photoelasticity showed very good agreement indeed. While the expense of carrying out a calculation by the present method compares favorably with that of photoelastic analysis, the method is still somewhat lengthy.

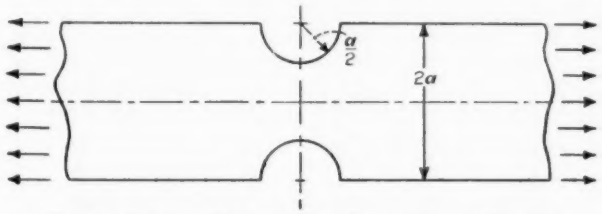


FIG. 1 TENSION MEMBER WITH SEMICIRCULAR NOTCHES

It will be recalled that plane-stress distributions may be expressed in terms of Airy's function F, which satisfies the differential equation

$$\nabla^4 F = (\nabla^2)^2 F = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)^2 F = \left(\frac{\partial^4}{\partial x^4} + 2\frac{\partial^4}{\partial x^2 \partial y^2} + \frac{\partial^4}{\partial y^4}\right) F = 0 \quad \ldots\ldots [1]$$

In terms of F the stresses are given by

$$X_x = \frac{\partial^2 F}{\partial y^2}, \quad X_y = -\frac{\partial^2 F}{\partial x \partial y}, \quad Y_y = \frac{\partial^2 F}{\partial x^2} \quad \ldots\ldots [2]$$

The problem of determining the stresses which arise in a simply connected region from the application of given tractions over the boundary C, leads to a solution F of Equation [1] with prescribed values of F and of $\partial F/\partial n$, the normal derivative of F, along C.

Equation [1] is also of interest in connection with *bending* or *transverse* deflections of thin plates, in which case it is satisfied by the normal deflection over regions free from normal load. The boundary conditions prescribing F and $\partial F/\partial n$ correspond to displacing the edge of the plate and forcing a proper slope.

2 *Finite Differences.* The method of solution makes use of finite-difference approximations. First and second derivatives, such as du/dx, $\frac{d^2u}{dx^2}$, are approximated, respectively, by the following quotients

$$\frac{du}{dx} = \frac{u(x+\delta) - u(x)}{\delta} \quad \ldots\ldots [3]$$

$$\frac{d^2u}{dx^2} = \frac{u(x+\delta) - 2u(x) + u(x-\delta)}{\delta^2} \quad \ldots\ldots [4]$$

whose limits, as δ approaches zero, yield the true derivatives. Likewise the Laplacian ∇^2 is replaced by

$$\nabla^2 F = \frac{F(x+\delta,y) + F(x-\delta,y) + F(x,y+\delta) + F(x,y-\delta) - 4F(x,y)}{\delta^2} = 4(F_m - F)/\delta^2 \quad \ldots\ldots [5]$$

where F_m is the mean of the values of F at the four points obtained by adding or subtracting it from x or y of the point in question, Fig. 2.

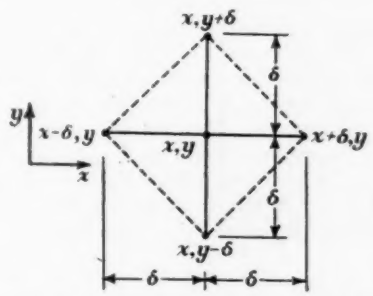


FIG. 2 REGION USED FOR FINITE-DIFFERENCE APPROXIMATION

Introducing a square lattice by cutting the region by means of the two sets of lines $x = x_i$, $y = y_j$ each a distance δ apart, and replacing Equation [1] by

$$s = \nabla^2 F, \ \nabla^2 s = 0 \quad \ldots\ldots [6]$$

causes departure from Equation [6] to

$$(F_{i+1,j} + F_{i-1,j} + F_{i,j+1} + F_{i,j-1}) - 4F_{i,j} = \delta^2 s_{i,j} \quad \ldots [7]$$

$$(s_{i+1,j} + s_{i-1,j} + s_{i,j+1} + s_{i,j-1}) - 4s_{i,j} = 0 \quad \ldots\ldots [7a]$$

where $F_{i,j} = F(x_i, y_j)$ and similarly for s_{ij}. In this way Equa-

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NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

tion [1] is replaced by a finite number of equations in a finite number of unknowns s_{ij}, F_{ij}.

If C is a rectangle of sides $m\delta$, $n\delta$, and if the boundary values of F and s are prescribed, then there are $2(m-1)(n-1)$ Equations [7] and [7a] for an equal number of variables. However, for other regions or boundary conditions complications arise at the boundaries. These could to some extent be overcome by

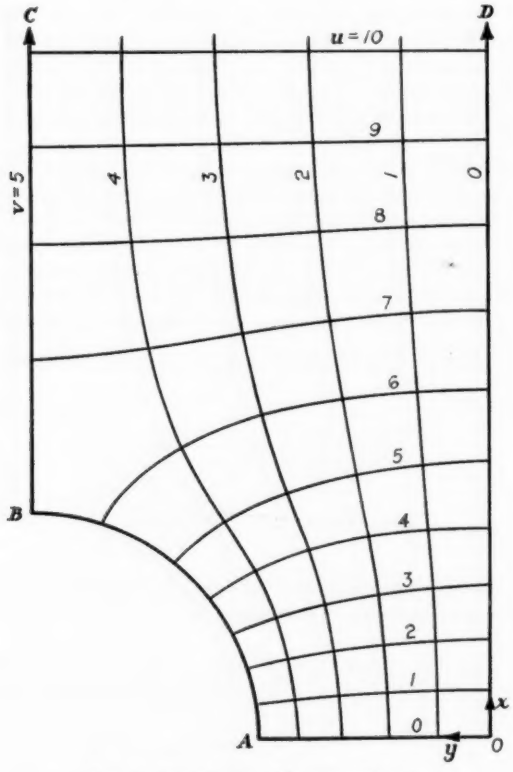


FIG. 3 CONFORMAL MAP, x, y-PLANE

using differences of values of s, F corresponding to unequal spacing of either x or y, but only at the expense of introducing much more complicated coefficients. This is avoided to a large extent by "mapping" the region inside C "conformally" on a rectangle, infinite strip, or some other simple region of another plane, say the u, v-plane. In general it is necessary to resort to a free-hand plot for this purpose. Thus Fig. 3 shows a conformal map of a quarter of the region of Fig. 1 on the corresponding strip of the u, v-plane. The boundary ABC of Fig. 3 has been "straightened out" into $v = 5$ in Fig. 4. It may be shown that in terms of the new variables Equation [6] is replaced by

$$\nabla^2_{u,v} F = s/r^2 \quad \ldots\ldots [8]$$

$$\nabla^2_{u,v} s = 0 \quad \ldots\ldots [8a]$$

where r is the ratio of element of length in the u, v-plane to the corresponding element of length in the x, y-plane

$$r = \frac{\text{side of (very small) square in the } u, v\text{-plane}}{\text{side of corresponding square in the } x, y\text{-plane}}.$$

The value of r is most conveniently found by plotting curves of x and y versus u, measuring the slopes $\partial x/\partial u$ and $\partial y/\partial u$, and utilizing the relation

$$r^2 = \frac{1}{(\partial x/\partial u)^2 + (\partial y/\partial u)^2} \quad \ldots\ldots [9]$$

Values obtained from Equation [9] should be checked by plotting x and y versus v and using

$$r^2 = \frac{1}{(\partial x/\partial v)^2 + (\partial y/\partial v)^2} \quad \ldots\ldots [10]$$

If the conformal plot is good the values obtained by Equations [9] and [10] will be essentially identical.

Equations [8] and [8a] can now be approximated by the difference equations

$$(F_{i+1,j} + F_{i-1,j} + F_{i,j+1} + F_{i,j-1}) - 4F_{i,j} = \delta^2 s_{i,j}/r^2_{i,j} \quad [11]$$

$$(s_{i+1,j} + s_{i-1,j} + s_{i,j+1} + s_{i,j-1}) - 4s_{i,j} = 0 \quad \ldots\ldots [11a]$$

At the boundaries, say at v = constant, no equation similar to Equations [11] and [11a] can be used, since this would involve values of F or s outside the region. Here $\nabla^2_{u,v} F = s/r^2$ is approximated by

$$\frac{s}{r^2} = \frac{\partial^2 F}{\partial u^2} + \frac{2}{\delta^2}\left[F_1 - F_0 + \delta\left(\frac{\partial F}{\partial v}\right)_0\right] \quad \ldots\ldots [12]$$

where subscript $_0$ indicates boundary values and subscript $_1$ indicates values at a distance δ from the boundary, while

$$\left(\frac{\partial F}{\partial v}\right)_0 = -\frac{1}{r}\left(\frac{\partial F}{\partial n}\right)_{x,y} \quad \ldots\ldots [13]$$

The last term of Equation [12] is obtained by using a second-degree polynomial through F_0, F_1, F_2; the preceding term of Equation [12], $\partial^2 F/\partial u^2$, is found best by plotting the boundary F against u, since the second difference method was not found sufficiently exact on the boundary.

3 *Solution by Successive Improvement.* To solve the resulting equations directly by determinants or elimination proved too difficult, since for 5×12 squares in the u, v-plane there are nearly 120 equations in 120 variables (counting the s- and F-values).

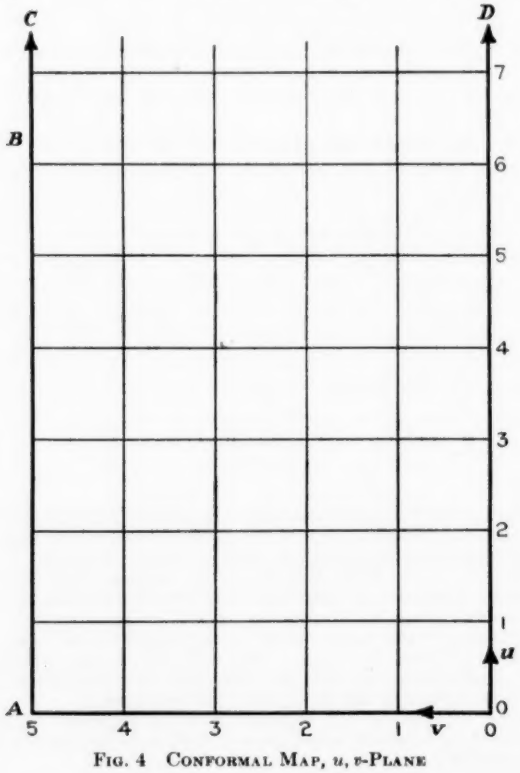


FIG. 4 CONFORMAL MAP, u, v-PLANE

Only in case of a square where there is symmetry about two perpendicular axes so that only one eighth of the region need be considered, is it at all advisable to use direct solution (1).[4] An indirect method was tried consisting in writing Equations [11] and [11a] in the form

$$s_{i,j} = \text{mean of four surrounding } s\text{-values} \ldots\ldots [14]$$

$$F_{i,j} = -\frac{\delta^2 s_{i,j}}{4r^2} + \text{mean of four surrounding } F\text{-values} \quad [14a]$$

starting with an intelligent guess at s and F, and improving it by substituting the guessed values on the right-hand side of Equations [14] and [14a] and using the resulting values of s, F as new values. These new, presumably improved, values were again substituted on the right side of Equations [14] and [14a], obtaining the next improvement or approximation, and so forth. A similar treatment is also given to Equation [12].

Methods similar to the foregoing have been reported by A. Thom and have been used by him successfully for the determination of the flow of ideal fluids (this reduces to the Laplace equation), for viscous fluids (2, 3) and for the problem of torsion of circular shafts of varying radius (4). In the case of the viscous fluid the differential equations involve repeated Laplacian as well as nonlinear combinations of lower derivatives. It was Thom's work, in fact, that suggested this method to the authors.

An attack was made on a plane-stress problem along these lines, but it proved unsuccessful. Even after repeating the successive improvements twelve and more times, there was no indication of convergence to a limiting set of values. Whether Thom's success lay in unreserved man power or whether it had to do with the nonlinearity of the problem—a feature which in general hampers classical methods of solution—the authors have never discovered. Perhaps the slowness of convergence has to do with the fact that the process of taking a mean of the four surrounding values reaches out only a distance δ, while its repetition reaches out over a square of larger size. One would expect that if convergence takes place at all, it would take place only after these squares have expanded over several times the size of the region, so that the "reflected wave fronts" have passed through each point several times. Now these wave fronts creep slowly, advancing only a step δ with each repetition of the process; thus it becomes apparent that many repetitions, indeed, are required before convergence is obtained.

The convergence was studied by considering solutions of $d^2s/dx^2 = 0$ (the Laplace equation in one dimension) and seeing how many repetitions were required to obtain convergence in simple cases. As many as 55 repetitions were required to secure 90 per cent accuracy. Various modifications were tried on the one-dimensional case. The results which proved successful were incorporated in the method finally adopted.

4 *Modifications Introduced to Secure Convergence.* One modification introduced in the method described in section 3 consisted in using *continuous corrections*. Instead of using all the *old* values of F in the right-hand members of Equations [14] and [14a] as many of the *corrected* values as have been obtained are used there. Thus the effect of a change in the boundary values is felt at once everywhere (though not to the full extent away from the boundary). The correcting process was begun at $u = 12$, $v = 3$, and from there proceeded inward to $v = 0$, and then inward along $u = 11, 10, \ldots\ldots 1, 0$, in turn.

The second modification introduced consisted in solving the Laplace Equation [8] for s exactly, the strip being a region for which such an exact solution in terms of boundary values is possible. For the strip

$$-\frac{h}{2} < v < \frac{h}{2}$$

if the boundary values of s are symmetric about $v = 0$, it may be shown that

$$s(u, v) = \frac{\cos\frac{\pi v}{h}}{h} \int_{-\infty}^{\infty} \frac{s(u') \cosh\frac{\pi}{h}(u' - u)}{\cosh^2\frac{\pi}{h}(u' - u) - \sin^2\frac{\pi}{h} v}\, du' \ldots [15]$$

where $s(u')$ represents boundary values of s. This integral was evaluated by the trapezoidal rule. Similarly, for a rectangle, the exact solution for s in terms of boundary values is possible, although the place of Equation [15] is taken by integrals involving elliptic functions. Thus the s-equations of Equation [14] were dispensed with completely, an exact solution for the s-values being given by Equation [15].

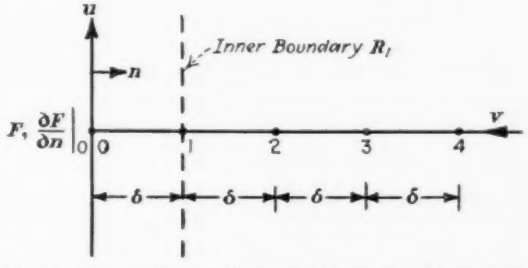


FIG. 5 POINTS USED IN APPLYING BOUNDARY CONDITIONS

The third modification of the original procedure consisted in eliminating the s-values along the boundary of the strip from the various stages of the calculations. The boundary values used for s in applying Equation [15] were taken to be those at points a distance δ from the true boundary, lying on two lines R_1 a distance δ from the true boundary R. Not until the final set of values of F and s was obtained over R_1 were the boundary values of s over R found. We shall refer to R_1 as the "inner boundary."

In addition to diminishing the number of s-values that had to be found at each stage of the successive approximations, the elimination of the boundary values is of advantage in that concentrations of stress at the boundary result there in rapid variations of s, so that the finite difference approximations are not sufficiently accurate. As is known, any irregularities or discontinuities in boundary values of a harmonic function are rapidly smoothed out away from the boundary.

Finally a change was introduced in connection with the manner of improving both F and s over the inner boundary R_1. Instead of using Equation [8] or its approximation Equation [14a] for improving F, it was used for improving s after the F-values had been improved. The new F-values along R_1 were then determined by passing a third-degree polynomial through the boundary value F_0 and having the given slope $\partial F/\partial v|_0$ there and through the two recently improved values F_2 and F_3 just inside the inner boundary R_1 Fig. 5. The value of F at R_1 as given by this polynomial turns out to be

$$F_1 = \frac{1}{18}\left(11F_0 + 9F_2 - 2F_3 - 6\left.\frac{\partial F}{\partial v}\right|_0 \delta\right) \ldots\ldots [16]$$

It is this method of improving the inner boundary values of F that forces the solution to fit both known boundary conditions. A third-degree polynomial was chosen rather than a second-degree, because of somewhat better accuracy, since it reaches further into the inside.

[4] Numbers in parentheses refer to the Bibliography at the end of the paper.

Thus modified, the procedure turned out to be completely successful. After the s- and F-values on and within R_1 had converged to proper limits, $\nabla^2 F$ was evaluated at the boundary R. Using the polynomial referred to in the foregoing, it is found that

$$\left.\frac{\partial^2 F}{\partial v^2}\right|_0 = \frac{1}{18}\left(-19F_0 + 27F_2 - 8F_3 + 30\left.\frac{\partial F}{\partial v}\right|_0 \delta\right) \ldots [17]$$

$$= 2F_0 - 5F_1 + 4F_2 - F_3 \ldots\ldots\ldots\ldots [18]$$

The component $\left.\frac{\partial^2 F}{\partial u^2}\right|_0$ of ∇^2 was evaluated graphically. The following checks should be made on the values thus determined. First, the boundary values of s must form a smooth curve when they are plotted against the tangential coordinate; second, each value must agree well with the value obtained by extrapolating the curves of the inner s-values to the boundary; third, the values must be such that the relation $\nabla^2 s = 0$ is satisfied.

5 *Summary of the Procedure Employed*. The procedure outlined may be summarized as follows:

a By means of a small square graphical plot make a conformal transformation from the original shape of Fig. 1 to a strip and determine r^2 at the corners of all the small squares.

b Calculate the values of F and its inner normal derivative, corresponding to all the corners of squares which fall on the boundary.

c Assume a set of F-values at all of the inner corners of squares. This should be as good a guess as possible.

d From the assumed F-values, calculate the s-values at all points on the inner boundary R_1 by means of the relation $\nabla^2_{u,v} F = s/r^2$, or rather its approximating Equation [14*a*].

e By means of Equation [15], determine the values of s at all points within R_1 so that $\nabla^2 s = 0$ is satisfied in this region.

f Correct all the values of F within the inner boundary by means of the numerical approximation to $\nabla^2_{u,v} F = s/r^2$, using continuous corrections such as described.

g Correct the F-values along R_1 by means of Equation [16].

h From the newly determined set of F-values, obtain another set of values of s along R_1.

i Repeat steps *a* to *h* until the F- and s-values converge.

j Determine graphically or numerically the actual boundary values of s in accordance with the relation $\nabla^2_{u,v} F = s/r^2$ and check these values by the methods which are described at the end of section 4.

6 *Results*. Table 1 gives the values of the free boundary stresses, while Fig. 6 shows a plot of these stresses for one quarter of the member itself.

The maximum value of the stress obtained by this method is 3.25. This compares with a maximum of 3.20 obtained photoelastically by Wahl and Beeuwkes (5), and a maximum of 3.05 obtained photoelastically by M. M. Frocht (6).

TABLE 1 VALUES OF FREE BOUNDARY STRESSES

u	0	1	2	3	4	5	6
s	3.25	3.15	3.00	2.80	2.15	1.05	0.10
u	7	8	9	10	11	12	
s	0.10	0.60	0.85	0.90	0.95	0.99	

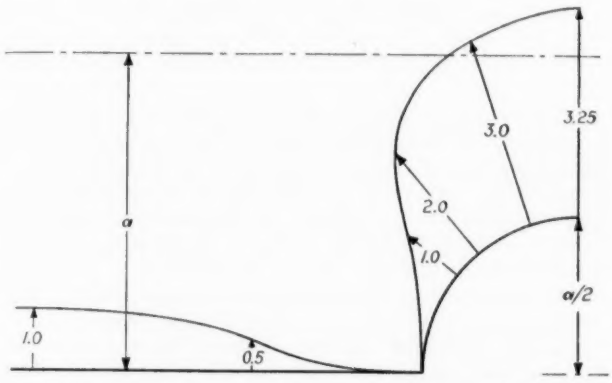


FIG. 6 FREE BOUNDARY STRESSES IN NOTCHED MEMBER

The method was also used in determining the stresses in a loaded T-head dovetail joint and in particular for investigating the effect of the size of the fillet upon the stress-concentration factor of these dovetails. In this case the dovetail region was conformally mapped upon a rectangle. The exact determination of inner values of the harmonic function s was not made by Equation [15] but by a somewhat similar expression involving elliptic functions. The final numerical results thus obtained were in excellent agreement with photoelastic results.

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A Method of Calculating Energy Losses During Impact

By C. ZENER[1] and H. FESHBACH[2]

When an elastic sphere collides with another perfectly elastic body, part of the initial kinetic energy is lost in starting elastic waves in the two bodies. The energy thus dissipated can be calculated by previous analytical methods only when it is a small fraction of the initial kinetic energy. This paper develops an approximate analytical method for this calculation which is applicable even when the greater part of the energy is dissipated. Thus in a particular case where the energy dissipated is 90 per cent of the original kinetic energy, an error of only 0.7 per cent is made. The new element in this analysis is the introduction of a "normalized" interaction force which is necessarily quite insensitive to one's ignorance of the exact interaction force.

The powerfulness of the method is illustrated by a complete survey of the problem of impact of spheres with beams fixed at each end. Graphs are constructed showing the variation of the coefficient of restitution with the length of the beam and the mass of the sphere. Impacts with multiple blows are excluded. It is found that when the mass of the sphere is kept constant, the coefficient of restitution has a minimum for a certain optimal beam length, and is independent of the beam length for values greater than twice the optimal length. The coefficient is a minimum when the period of the fundamental mode of vibration is approximately equal to 2.4 times the time of contact.

The method is also applied to the impact of spheres with large thin plates. The semiempirical formula of Raman is derived.

Although this analytical method cannot be applied to impacts with multiple blows, it nevertheless gives the conditions for such multiple blows.

Introduction

THE EXACT theory for the collision of two elastic bodies has been developed only for two extreme cases. In the first,[3] contact between the two bodies is established instantaneously and likewise ceases instantaneously. These conditions are satisfied in the longitudinal collision of two bars with ideally smooth plane surfaces. In this case, the two bodies are treated as a single elastic body after the establishment of contact. The process of collision is studied by merely following the propagation of a discontinuous elastic wave. In such collisions a large part of the original kinetic energy may be transformed into vibrational energy. In the second case,[4] the time required to establish maximum contact is very long compared to the time required for an elastic wave to traverse the length of the two bodies. This condition is satisfied in the collision of two spheres with small relative velocity of approach. In this case the strains are localized to the vicinity of the contact region. The force F which each body exerts on the other is then a function only of the distance of approach ξ after contact is first established. The variation of ξ with time is then determined by Newton's second law. The total kinetic energy of the two bodies is unaltered by such a collision.

All actual collisions are of an intermediate type. The time required for the establishment of maximum contact is finite, but only under special conditions is it long compared to the time required for an elastic wave to traverse both of the colliding bodies. The exact treatment of such a collision leads to an integrodifferential equation. An investigation of this equation has shown that under certain conditions a single impact consists of a series of distinctly separated blows.[5] The mere labor involved in solving this integrodifferential equation has, however, unfortunately prevented finding how the loss of kinetic energy depends upon the various parameters of a collision. There is thus a need for an approximate method by means of which the effect of the various collision parameters can be studied. Such a method is developed in this paper for the collision of spheres with other bodies. Its usefulness is demonstrated by a complete survey of the problem of the collision of a sphere with a beam fixed at both ends. Finally, by means of this method the solution is obtained for the collision of a sphere with large plates.

General Theory

The duration of an impact of a sphere with another body is so long, compared to the period of the fundamental mode of the sphere, that the elastic vibration of the sphere absorbs only a negligible fraction of the initial kinetic energy.[6] Thus it is only necessary to calculate the energy ΔE absorbed by the body which the sphere strikes. This energy is completely determined by the force $F(t)$ which the sphere exerts upon this body. This force has been calculated by Hertz for the particular case where the strains are localized to the immediate vicinity of the region of contact. It will be denoted by $F_H(t)$. In more general cases a gross error would be introduced by replacing the actual force by the Hertz force.

Fully realizing our ignorance of the force $F(t)$, we nevertheless proceed in the standard way to evaluate the energy absorbed by an elastic body acted upon by the force $F(t)$. In our final expression for ΔE, we shall then make a simple transformation

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[3] This case was investigated initially by F. Neuman and B. de Saint Venant. A thorough review of the literature is given in "Handbuch der Physik," Julius Springer, Berlin, 1928, vol. 6, chap. 7.

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Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y., and will be accepted until August 10, 1939, for publication at a later date. Discussion received after the closing date will be returned.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

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[6] "On the Production of Vibrations by Forces of Relatively Long Duration, With Application to the Theory of Collisions," by Lord Rayleigh, *Philosophical Magazine*, series 6, vol. 11, 1906, p. 283.

which renders ΔE exceedingly insensitive to errors in $F(t)$. The power of this transformation can be appreciated by comparing the calculated ΔE with the ΔE obtained by Timoshenko[5] by the numerical solution of the exact integrodifferential equation for a special case. In this example, the error in ΔE obtained by replacing $F(t)$ by $F_H(t)$ is reduced by this transformation from 160 per cent to less than 0.7 per cent, the rebound velocity of the sphere is obtained with an error of less than 3 per cent.

The differential equation for an elastic body acted upon by a force with volume density $\boldsymbol{f}(\boldsymbol{r}, t)$ is given by

$$(\partial^2/\partial t^2)\boldsymbol{U} = L\boldsymbol{U} + \rho^{-1}\boldsymbol{f} \quad \ldots\ldots\ldots\ldots\ldots [1]$$

Here $\boldsymbol{U}(\boldsymbol{r},t)$ is the vector displacement function, L a linear differential operator of the second order, and ρ is the mass density. We expand $\boldsymbol{U}$ in terms of the normalized eigenfunctions $\boldsymbol{U}_k(\boldsymbol{r})$ of the differential equation

$$(L + \omega_k^2)\,\boldsymbol{U}_k = 0$$

and of the boundary conditions appropriate to the problem. Upon substitution of this expansion

$$\boldsymbol{U}(\boldsymbol{r}, t) = \Sigma C_k(t)\,\boldsymbol{U}_k(\boldsymbol{r})$$

into Equation [1], multipling on the left by $\boldsymbol{U}_n$, and integrating over the entire solid, we obtain

$$[(\partial^2/\partial t^2) + \omega_n^2]C_n = \rho^{-1} f_n(t)$$

where $f_n(t) = \int \boldsymbol{f}\cdot\boldsymbol{U}_n d\boldsymbol{v}$. Before collision all the C's and their first time derivatives are zero. The solution of these equations subject to this boundary condition is

$$C_n(t) = (1/\omega_n\rho)\int_{-\infty}^{t} \sin\omega_n\,(t - t')f_n(t')dt' \quad \ldots\ldots [2]$$

The energy of elastic vibration is given by the equation

$$\Delta E = (\rho/2)\Sigma_n[(\partial C_n/\partial t)^2 + \omega_n^2 C_n^2] \quad \ldots\ldots\ldots\ldots [3]$$

The energy absorbed after the collision is now obtained by substituting Equation [2] into Equation [3], and setting $t = \infty$. First, however, we shall transform the function f_n. The force density $\boldsymbol{f}$ due to the impact is confined to a very small region. All the eigenfunctions $\boldsymbol{U}_k$ may be regarded as constant in this region except those with very high ω. But for these the corresponding C's after collision are negligibly small.[6] Hence we may write $f_n(t) = \delta_n F(t)$, where δ_n is equal to the component of $\boldsymbol{U}_n$ normal to the surface at the region of contact, and $F(t)$ is the magnitude of the total force, i.e., $F(t) = \left|\int \boldsymbol{f} d\boldsymbol{v}\right|$. We thus obtain for the energy absorbed after the collision

$$\Delta E = (1/2\rho)\Sigma_n\delta_n^2\left|\int_{-\infty}^{\infty} F(t)e^{i\omega_n t}\,dt\right|^2$$

We shall now endeavor to remove as far as possible errors resulting from our ignorance of the force $F(t)$. From Newton's third law, $-F$ is the force which acts upon the sphere. Hence $-\int F dt$ is equal to the change in momentum of the sphere. If m is the mass of the sphere, v its initial velocity, and ev its final velocity in the opposite direction, then the change of momentum is $-mv(1 + e)$, i.e., $\int F(t)dt = mv(1 + e)$. We may thus write $F(t)$ as $F(t) = mv(1 + e)\mathfrak{F}(t)$, where $\mathfrak{F}$ is the normalized force

$$\mathfrak{F}(t) = F(t)\Big/\int_{-\infty}^{\infty} F(t)dt.$$

Denoting by E the initial energy of the sphere $mv^2/2$ we obtain $\Delta E/E = (1 + e)^2 R$, where

$$R = m\rho^{-1}\Sigma_n\delta_n^2\left|\int_{-\infty}^{\infty}\mathfrak{F}(t)e^{i\omega_n t}\,dt\right|^2$$

Our difficulties have now largely disappeared. In the first place, we may eliminate the unknown coefficient of restitution e by means of the equation $\Delta E/E = 1 - e^2$, obtaining

$$\Delta E/E = 4R/(1 + R)^2 \quad \ldots\ldots\ldots\ldots\ldots [4]$$

Conversely, we may eliminate $\Delta E/E$, obtaining

$$e = (1 - R)/(1 + R) \quad \ldots\ldots\ldots\ldots\ldots [5]$$

In the second place, the quantity R is comparatively insensitive to errors in the force $F(t)$. This is so since the integrands of R contain explicitly only the function $\mathfrak{F}(t)$ which has been so defined that the integral $\int_{-\infty}^{\infty}\mathfrak{F}(t)dt$ is independent of the function $\mathfrak{F}(t)$. Only the shape of $\mathfrak{F}(t)$ is thus subject to error.

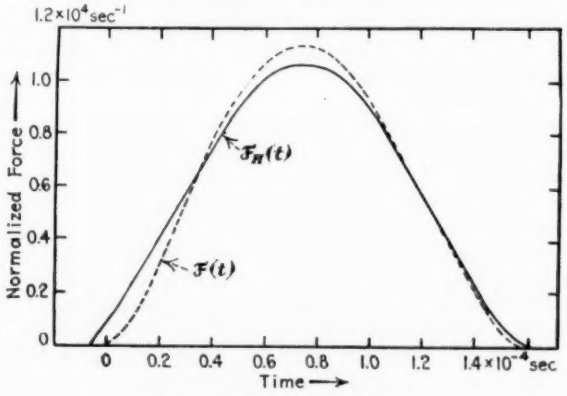


FIG. 1 NORMALIZED FORCES

(The broken curve is the normalized exact force for the particular example studied by Timoshenko. The full curve is the normalized Hertz force for the same example. In this example 90 per cent of the initial energy of the sphere is lost during collision.)

As evidence that the shape of $\mathfrak{F}(t)$ is little different from the shape of the normalized Hertz force

$$\mathfrak{F}_H(t) = F_H(t)\Big/\int_{-\infty}^{\infty} F_H(t)\,dt$$

we compare in Fig. 1 these two functions for the particular example for which $F(t)$ has been exactly calculated by Timoshenko.[5] Although in this particular example the sphere loses more than 90 per cent of its energy during collision, nevertheless $\mathfrak{F}_H(t)$ differs only slightly from $\mathfrak{F}(t)$. It is thus to be expected that only a very slight error will be made by substituting the first function for the second, excluding of course the case of multiple collisions. Such multiple collisions will occur whenever the calculated e is negative.

We have now the problem of evaluating the integrals in R, with $\mathfrak{F}(t)$ replaced by $\mathfrak{F}_H(t)$. This is most readily done by finding an analytical function which closely approximates $\mathfrak{F}_H(t)$. We have found the following function satisfactory

$$\begin{aligned} &[1 + \cos(2\pi t/T)]/T \quad \text{for} \quad -T/2 < t < T/2 \\ &0 \quad\qquad\qquad\quad \text{for} \quad |t| > T/2 \end{aligned}$$

in which T is adjusted so that the maximum of this function $2/T$ is equal to the maximum of $\mathfrak{F}_H(t)$. It may readily be shown that

TABLE 1 COMPARISON OF NORMALIZED HERTZ FORCE WITH ANALYTICAL APPROXIMATION $T = 1.0873\ T_H$

(The origin of time is taken as the instant of closest approach)

t/T_H	0	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.5438
$\mathfrak{F}_H(t) \times T_H$	1.839	1.802	1.694	1.523	1.303	1.051	0.785	0.525	0.291	0.104	0	0
$T^{-1}[1 + \cos(2\pi t/T)]T_H$	1.839	1.801	1.690	1.515	1.291	1.036	0.771	0.519	0.299	0.132	0.029	0

the maximum of $\mathfrak{F}_H(t)$ is $1.8395/T_H$, where T_H is the Hertz time of collision. Hence $T = 1.0873\ T_H$. The close agreement of this function with $\mathfrak{F}_H(t)$ is shown in Table 1.

Each integral in R is a function only of the one parameter $Q_n = \nu_n T$, $\nu_n = \omega_n/2\pi$, which is the number of oscillations of the nth mode in the time T. The percentage error in the integrals introduced by replacing $\mathfrak{F}_H(t)$ by this function is appreciable only for those integrals for which Q_n is large compared to unity. But these integrals are so small that their contribution to R is negligible. In this way we obtain

$$R = m\rho^{-1}\Sigma_n \delta_n^2 \left[\frac{\sin(\pi Q_n)}{\pi Q_n(Q_n^2 - 1)}\right]^2 \quad \ldots\ldots\ldots\ldots [6]$$

We have now obtained explicit formulas for $\Delta E/E$, Equation [4], and for e, Equation [5], in terms of a single parameter R. This parameter in turn is given by Equation [6] in terms of the displacement δ_n and frequency ν_n, of each normal mode of vibration, and in terms of the Hertz time of collision T_H. This time of collision may be calculated from the equation[3]

$$T_H = 10.97\left[\frac{(m')^2(E_1 + E_2)^2}{v r^2 E_1^2 E_2^2}\right]^{1/5} \quad \ldots\ldots\ldots\ldots [7]$$

where m' is the reduced mass of the two bodies. It is equal to the mass of the sphere alone if the struck body is supported at its boundaries. Also v is the initial velocity of the sphere; E_1 is the ratio of Young's modulus over $1 - \sigma_1^2$, where σ_1 is the Poisson ratio of the sphere; and E_2 refers to the corresponding quantity for the struck body. If the surface of the struck body is plane, r is the radius of the sphere. More generally, it refers to $r_1 r_2/(r_1 + r_2)$, where r_1 and r_2 are the radii of curvature of the two surfaces.

Application to Collisions of Spheres With Beams

In this section we apply the results of the previous paragraph to the collision of a sphere with a beam pivoted at both ends, with the impact being at the middle of the beam.

The normalized displacement functions for a beam pivoted at both ends are $U_n = (2/V)^{1/2}\sin(n\pi x/L)$, $n = 1, 2, 3\ldots$. Here V is the volume of the beam, L its length. The value of the displacement function at the point of contact, $x = L/2$, has been denoted by δ_n. In this case $\delta_n^2 = 2/V$ with n odd, and $\delta_n = 0$ with n even. The frequency ν_n of the nth mode is related to the frequency of the first by $\nu_n = n^2\nu_1$. Upon substituting these relations into Equation [6], we obtain

$$R = (2m/M)\sum_{n=0}^{\infty}\left\{\frac{\sin[(2n+1)^2\pi Q_1]}{(2n+1)^2\pi Q_1[(2n+1)^4 Q_1^2 - 1]}\right\}^2$$

Here m and M refer to the mass of the sphere and beam, respectively. As Q_1 approaches zero, the summation approaches infinity as $Q_1^{-1/2}$. It is thus convenient to write R in the form $R = (m/MQ_1^{1/2})G(Q_1)$, where

$$G(Q_1) = 2\,Q_1^{1/2}\sum_{n=0}^{\infty}\left\{\frac{\sin[(2n+1)^2\pi Q_1]}{(2n+1)^2\pi Q_1[(2n+1)^4 Q_1^2 - 1]}\right\}^2$$

Values of this function are given in Table 2.

TABLE 2

Q_1	$G(Q_1)$	Q_1	$G(Q_1)$	Q_1	$G(Q_1)$
0	0.840	0.25	0.922	1.0	0.500
0.02	0.840	0.30	0.976	1.1	0.378
0.04	0.840	0.35	1.008	1.2	0.278
0.06	0.840	0.40	1.028	1.3	0.205
0.08	0.846	0.45	1.032	1.4	0.123
0.10	0.832	0.50	1.018	1.5	0.078
0.12	0.814	0.55	1.000	1.6	0.038
0.14	0.803	0.60	0.972	1.7	0.010
0.16	0.810	0.7	0.880	1.8	0.002
0.18	0.824	0.8	0.766	1.9	0.000
0.20	0.850	0.9	0.632	...	...

We can now obtain at once $\Delta E/E$ and e for any set of collision parameters. Q_1 is given in terms of the fundamental frequency ν_1 of the beam, and the Hertz time of collision T_H by

$$Q_1 = 1.087\nu_1 T_H$$

or in terms of the mass, radius, and initial velocity of the sphere, m, r, and v, respectively, by

$$Q_1 = 11.93\,\nu_1\left[\frac{m^2(E_1 + E_2)^2}{rvE_1^2E_2^2}\right]^{1/5}$$

From Table 2 we now obtain the function $G(Q_1)$. Finally, by substituting R into Equations [4] and [5] we obtain the fractional energy loss $\Delta E/E$ and the coefficient of restitution e, respectively.

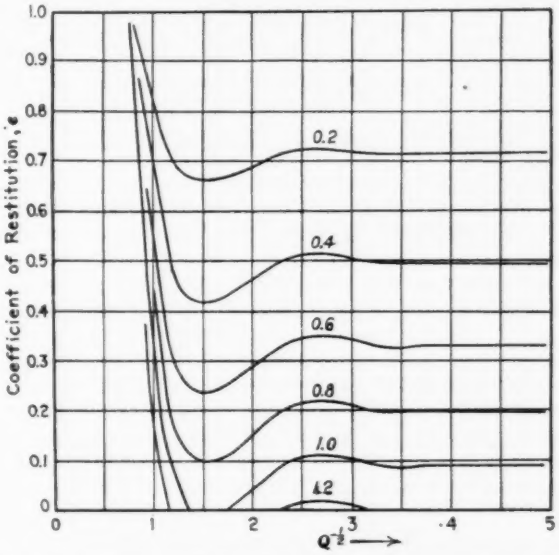


Fig. 2 Coefficient of Restitution of a Sphere Colliding With a Beam Pivoted at Both Ends
(The label on each curve refers to the ratio $m/MQ_1^{1/2}$.)

The fundamental frequency ν_1 and hence also Q_1, varies inversely as the square of the beam length. Hence the variation of e with beam length is best illustrated, as in Fig. 2, by plotting e as a function of $Q_1^{-1/2}$ for various values of the parameter $m/(MQ_1^{1/2})$, which is independent of the beam length. At $Q_1 = 0.45$, the value of e has a deep minimum corresponding to a fundamental period 2.4 times as long as the Hertz time of collision. The value of e is nearly independent of beam length for lengths greater than twice the optimal length. The variation of e with the mass of the sphere m is obtained by passing from one curve to another, with a slight change in the scale of the abscissa. This change of scale is due to the fact that $Q_1^{-1/2}$ is proportional to $m^{-1/5}$.

As a check upon our approximate method, we shall find the e and $\Delta E/E$ for that particular case for which Timoshenko[4] numerically solved the exact integrodifferential equation. In this example the necessary data are: $\nu_1 = 1000$, $T_H = (8/45) \times 10^{-3}$, $(m/M) = 0.273$. We obtain $Q_1 = 0.193$, and hence from Table 2 $G(Q_1) = 0.840$, $R = 0.521$, and finally $e = 0.314$. This calculated e is 3 per cent higher than Timoshenko's value 0.305. Our calculated fractional energy loss $\Delta E/E = 1 - e^2 = 0.901$ is thus 0.7 per cent lower than that obtained by Timoshenko. Timoshenko also examined a case in which the collision consisted of a series of blows. The parameters $Q_1^{-1/2}$ and $m/MQ_1^{1/2}$ for this case are 3.0 and 3.3, respectively. Reference to Fig. 2 shows that our calculated e is negative, and hence the collision must be multiple.

Application to Collisions of Spheres With Large Thin Plates

In this section we apply the general results of general theory to the collision of spheres with large plates. We shall take the plate to be circular with a radius R much larger than its thickness. Then the normalized displacement functions are given by

$$U_n = J_0(k_n r) \Big/ \left[4\pi h \int_0^R J_0^2(k_n r) r dr \right]^{1/2}, \quad n = 1, 2 \ldots\ldots$$

Corresponding to the condition that U_n vanish at the boundary, k_n is the nth root of $J_0(kR) = 0$. Corresponding to the condition that the point of contact be at the center of the plate, δ_n is given by the value of U_n at $r = 0$. For large values of R this gives $\delta_n^2 = k_n/4hR$. The assumption that R is large enables us to replace the summation in Equation [6] by an integration. Since consecutive k's differ by π/R, the summation sign is replaced by $(R/\pi)\int_0^\infty dk_n \ldots\ldots$ Upon using the following relation[7] between k_n and ν_n

$$k_n^2 = (2\pi/h)(3\rho/E_2)^{1/2} \nu_n$$

where E_2 is the same elastic constant as used in the last section, we obtain

$$R = \left(\frac{m}{4\pi h^2 T}\right)\left(\frac{3}{\rho E_2}\right)^{1/2} \int_0^\infty \left[\frac{\sin \pi Q}{\pi Q(Q^2 - 1)}\right]^2 dQ$$

$$= [3m/(16\pi h^2 T)]\,[3/(\rho E_2)]^{1/2}$$

In order to conform to the notation of Raman,[8] we replace T by $1.087\, T_H$, and introduce the symbol

$$k = \pi h^2 T_H (\rho E_2/3)^{1/2} = 19.90 h^2 (\rho E_2)^{1/2} \left\{ \frac{m^2(E_1 + E_2)^2}{v r E_1^2 E_2^2} \right\}^{1/5}$$

Upon substituting R into Equation [5], we obtain finally

$$e = (k - 0.542m)/(k + 0.542m)$$

This equation is identical to the equation which Raman[8] gave as fitting his experimental data, save that his numerical coefficient was 0.56 in place of 0.542. The accuracy of his data is not sufficient to distinguish between these two coefficients. Raman obtained his formula by choosing, ad hoc, that wave form in the plate which gave the best agreement with experiment. The experiments disagree with the theoretical formula for values of e less than about $1/_2$.

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Calculation of Stresses Within the Boundary of Photoelastic Models

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A method is proposed for the determination of the internal stresses in a two-dimensional system from data furnished by a photoelastic analysis. The method involves the numerical integration of the Laplace difference equation over a region with known boundary values by the iteration of a set of improvement formulas. The underlying theory is discussed briefly, reference being made to a more mathematical treatment appearing in another paper by the authors. New procedures for increasing the speed and accuracy of such computations are described and application is made to a typical photoelastic study. In this the complete system of internal stresses is computed, using the data obtained from the usual fringe photograph, without recourse to isoclinic lines, or other supplementary experimental measurements. The method is also applicable to other problems of potential theory which involve the vanishing of the Laplacian. This includes electric fields, steady-state heat conduction, shapes of membranes, and problems in hydrodynamics and gravitation.

THE METHODS which have been previously used for the determination of the internal stresses in a two-dimensional stress system from photoelastic data have been very laborious and in many cases inaccurate. These internal stresses have been obtained by graphical methods which make use of the isoclinic-line families,[3,4] methods depending on the measurement of thickness deformation,[5] the membrane analogy,[6] and direct interferometer methods. In this paper the authors will describe a procedure for obtaining these stresses by numerical integration directly from the boundary stresses. In this procedure no data are required beyond those given by the usual fringe photograph. The accuracy of the method is limited only by the accuracy with which the boundary stresses may be obtained from this photograph. No great skill is required in carrying out the computations. It is believed that the time and labor involved are much less than for other methods.

In the following discussion, the application of this method to a typical photoelastic problem will be described and a brief statement concerning the underlying theory will be made. However, most of the subject matter will be devoted to the technique of operation. A detailed treatment of the mathematical properties of the functions involved, together with data concerning rates of convergence, accuracy of results, etc., has been published in another paper by the authors.[6]

It is shown in any treatise on the theory of elasticity[7] that the sum of the principal stresses S_1 and S_2, in a two-dimensional stress system obeys the Laplace differential equation

$$\frac{\partial^2(S_1 + S_2)}{\partial x^2} + \frac{\partial^2(S_1 + S_2)}{\partial y^2} = 0$$

To solve this equation in a given region, the values of $(S_1 + S_2)$ along the boundary of the region are required. These are conveniently obtained from the fringe photograph at points which are not subject to external forces, since at a free boundary the principal stress directions are normal and tangential, the normal stress being zero. Hence on a free boundary, the sum of the principal stresses equals the difference between the principal stresses, which latter the fringe photograph gives directly. The stress at parts of the boundary where loads are applied cannot be evaluated in this way. In such cases boundary values suitable for use with Laplace's equation may be obtained by an approximate analytical treatment and the use of St. Venant's principle. Frequently for purposes of calculation, it is desirable to use an artificial boundary which avoids loaded parts of the actual boundary, obtaining approximate analytical values of the stresses along the artificial boundary. The same treatment of loaded boundaries is used in the membrane analogy.

The Liebmann Procedure

The Laplace equation can be solved analytically in only a few trivial cases since, in the general case of an irregular boundary, systems of separable coordinates cannot be found. However, an approximation to the solution may be obtained by solving the so-called Laplace difference equation in place of the differential equation.[8] In using the difference equation, proceed by laying down a square network on the region to be treated. Then consider the values of $(S_1 + S_2) = \psi$ only at the corners of the net, rather than at every point within the region. The difference equation requires that the value of ψ at any point in such a square net be the average of its values at the four nearest neighboring points. Let Fig. 1 represent a region in which a function ψ is to satisfy the Laplace difference equation and hence satisfy the differential equation approximately. It is assumed that the boundary values of ψ are known and may be written down at points where the net intersects the boundary. The requirement that the value of ψ at each interior point be the average of its four nearest neighbors gives a set of nine linear equations for the nine interior values of ψ. A typical equation of this set is

$$\psi_T = \frac{1}{4}(\psi_X + \psi_V + \psi_Z + \psi_K)$$

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[1] Mendenhall Laboratory of Physics and Engineering Experiment Station, The Ohio State University.

[2] Mendenhall Laboratory of Physics and Engineering Experiment Station, The Ohio State University.

[3] "Treatise on Photoelasticity," by E. G. Coker and L. N. G. Filon, University Press, Cambridge, 1931.

[4] "New Method of Deriving Stresses Graphically From Photo-Elastic Observations," by H. Neuber, Proceedings of the Royal Society of London, series A, vol. 141, 1933, pp. 314–324.

[5] "A Membrane Analogy Supplementing Photoelasticity," by J. G. McGivern and H. L. Supper, Trans. A.S.M.E., vol. 56, 1934, paper APM-56-9, pp. 601–605.

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Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y., and will be accepted until August 10, 1939, for publication at a later date. Discussion received after the closing date will be returned.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[6] "The Numerical Solution of Laplace's Equation," by G. H. Shortley and R. Weller, *Journal of Applied Physics*, vol. 9, 1938, pp. 344–348.

[7] "Theory of Elasticity," by S. Timoshenko, McGraw-Hill Book Company, 1934, New York, N. Y., chap. 1.

[8] "Partial Differential Equations of Mathematical Physics," by H. Bateman, University Press, Cambridge, England, 1932.

In general there are as many equations of this type as there are interior points of the net; too many equations to attempt to solve directly. Liebmann[9] has suggested a convenient iterative procedure for solving these equations for the unknown values of ψ at the interior points. At the beginning of this procedure entirely arbitrary values are assumed, based on whatever experience the individual may have had in such matters. The more nearly these assigned values approach the correct ones, the more rapid will be the convergence of the iterative procedure. However, any finite assigned values whatever will lead to the correct result.

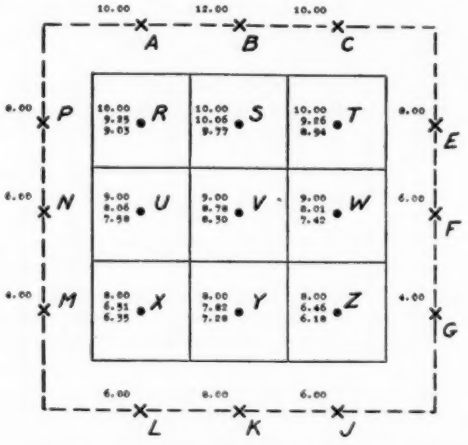


FIG. 1 SMALL NET ILLUSTRATING THE USE OF THE SIMPLE LIEBMANN IMPROVEMENT FORMULA

(Actual points of the net are shown as small black circles. The solid lines are so drawn as to form squares around each of the net points. This enables the writing down of all numbers relating to any particular point inside a single square, thus avoiding confusion as to which point is designated by any one number. Boundary points at which the function values are assumed to be known are indicated by crosses.)

The procedure is then to replace each value by the average of the four nearest neighbors, going over the set of points in some definite order, say alphabetically from R to Z in Fig. 1. The result of this operation is a new set of values for all points except those on the boundary. These values furnish a closer approximation to the true solution than the original arbitrary values. A second traverse over the net will yield a still better approximation, and so on. This should be repeated until the values reproduce themselves at each point to the required number of significant figures. Then it will be observed that the final values satisfy the difference equation. For example in Fig. 1, a column of numbers is associated with each point inside the net and a single number with each boundary point. Assume that the boundary values are given. The top number at each of the interior points is a value arbitrarily assigned. The second number in the column represents the value obtained by the first traverse over the set of points. In calculating the improved value at a given point, the previously altered values are used whenever they are available. Thus in the first traverse

$$9.25 = \frac{1}{4}(8 + 10 + 10 + 9) \qquad \text{(point } R\text{)}$$

$$10.06 = \frac{1}{4}(9.25 + 12 + 10 + 9) \qquad \text{(point } S\text{)}$$

$$8.01 = \frac{1}{4}(8.78 + 9.26 + 6 + 8) \qquad \text{(point } W\text{), etc.}$$

[9] "Die Angenaherte Ermittelung Harmonischer Funktionen und Konformer Abbildungen," by H. Liebmann, Sitzungberichte der Bayerischen Akademie der Wissenschaften zu München, Mathematische Physikalische Klasse, 1918, p. 385.

The third number at each point results from a second traverse. Further traverses would soon result in the appearance of stationary values, thus providing the solution of the Laplace equation within the given region to four significant figures.

The foregoing process is called the Liebmann procedure after its discoverer. A brief discussion together with a proof of convergence is given by Frank and von Mises.[10] It may be applied without modification to stress problems if desired, although the authors have developed a modified procedure which results in much more rapid convergence.

BOUNDARY POINTS

In the case of a point which is not equidistant from its four nearest neighbors, the simple improvement formula is not applicable. Fig. 2 shows such a point. In this case there are two

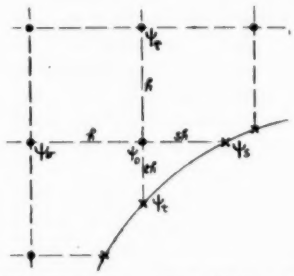


FIG. 2 TREATMENT OF A POINT NEAR A BOUNDARY, SEE EQUATION [1]

neighbors which lie at distances which are less than the net spacing h. The formula for the improved value of ψ_0 at this point is

$$\psi_0' = \frac{st}{s+t}\left[\frac{1}{1+t}\psi_r + \frac{1}{1+s}\psi_\sigma + \frac{1}{t(1+t)}\psi_t + \frac{1}{s(1+s)}\psi_s\right] = c_r\psi_r + c_\sigma\psi_\sigma + c_t\psi_t + c_s\psi_s \quad [1]$$

The values s and t here represent fractions of the normal net spacing. Note that $c_r + c_\sigma + c_t + c_s = 1$; this may be used as a check of these coefficients. In setting up the problem, the values of the coefficients c may be calculated once for all and written down on the net. In particular, since the two points at odd distances are almost always boundary points and hence do not change in value as the iteration is carried on, the product of the coefficients and values may be computed and written on the net. These points will be illustrated later in detail by treatment of a particular example (see Fig. 9).

A procedure has thus been described which may be used to solve problems in stress distribution, as follows:

1 Determine the stress values along the model boundary (or an artificial boundary) by the usual photoelastic method.

2 Lay down a square network on the model.

3 Evaluate by interpolation the points where the net intersects the boundary.

4 Apply the improvement formulas given until the values at each point reproduce themselves.

In applying this procedure to a practical problem, the fineness of the net which is to be employed must be considered, since the accuracy with which the solution of the difference equations agrees with that of the differential equation is increased by refining the net; but so is the amount of labor required to solve the difference equations. A criterion of accuracy is furnished by the use of two nets of different fineness over the same region, the second having one half the spacing of the first. If the values of ψ

[10] "Differential und Integral Gleichungen der Mechanik und Physik," by P. Frank and R. von Mises, Friedrick Vieweg and Son, Berlin, 1930, part 1, p. 735.

obtained from the two nets differ by less than three times the permissible error, the value from the finer net is satisfactory, since the error is approximately proportional to the square of the net spacing. A net with intervals one half as great as those of a previous net will converge to a value having an error with respect to the true continuous solution of Laplace's equation of about one fourth as much as that of the coarser net. It is advisable always to start by solving the difference equations with a very coarse net having, say ten to fifteen interior points. Then in successive steps cut the net spacing in half until the desired fineness is attained. The previous solutions of the coarser nets are desirable because, at each halving of the net constant, the convergence is about four times as slow. Hence it is economical of time to get as good a function as possible from a coarse net to use as the initial trial function for the finer net, in order to keep the error in the trial function as small as possible.

The use of the procedure described will result in a rather slow rate of convergence when a net of considerable fineness is employed. The means by which the convergence may be speeded up will be discussed later. The behavior of the nets and of how a consideration of this behavior leads to improvement in their treatment will now be discussed briefly.

Elementary Theory

The initial arbitrary values assigned to the interior points of a net may be considered as a set of numbers which constitutes the true solution of the difference equations plus an undetermined error at each interior point. It is clear that the use of an improvement formula does not change the true solution, but merely causes it to reproduce itself. The effect of the improvement operation is simply to reduce the amount of the error with each traverse over the net. The amount of reduction depends on the shape of the function representing the error. If the error is smooth and pillow-shaped, it will not sink very rapidly, but any roughness or jaggedness will disappear very quickly. It is possible to find certain error functions which, when improved, will be reduced to a constant fraction of themselves without being changed in shape. Such a function is called an eigenfunction of the improvement process. These functions, of course, have zero boundary values. The different eigenfunctions are denoted by $\phi^{(1)}, \phi^{(2)}, \phi^{(3)}, \ldots$ On improvement these go over to

$$\phi^{(1)\prime} = \lambda_1\phi^{(1)} \qquad \phi^{(2)\prime} = \lambda_2\phi^{(2)} \ldots$$

The λ values (eigenvalues) will all be less than unity, and one of them, say λ_1 will be the largest. It is now possible to make a sort of Fourier expansion of the actual error function ϕ in terms of the eigenfunctions, i.e.

$$\phi = a_1\phi^{(1)} + a_2\phi^{(2)} + a_3\phi^{(3)} + \ldots . \qquad \ldots\ldots\ldots (2)$$

On improvement this becomes

$$\phi' = a_1\lambda_1\phi^{(1)} + a_2\lambda_2\phi^{(2)} + a_3\lambda_3\phi^{(3)} + \ldots$$

After n improvements, the eigenfunctions will have been reduced by λ_k^n and soon all the functions of lower eigenvalue will disappear, leaving only a constant times $\phi^{(1)}$. The rate of convergence of the procedure depends upon the rate at which $\phi^{(1)}$ disappears, which in turn depends on λ_1. After the iteration has been carried on for a number of traverses, the error begins to be reduced at a uniform rate, each traverse reducing it by the same fraction λ_1.

Table 1 gives values of λ_1 for the ordinary Liebmann procedure taken over a square area with various numbers of intervals p, along the sides. This gives some idea as to the dependence of the rate of convergence upon the fineness of the net. For large p, $1 - \lambda_1 = \pi^2/p^2$.

For example, the error at any point in a square region with ten intervals on a side, that is, with 81 interior points, is reduced to about nine tenths of itself by each traverse after it has been improved a few times. At this rate it would take about 23 traverses

TABLE 1

p	λ_1
3	0.250
4	0.500
6	0.750
10	0.9045
18	0.9698

to reduce the error to one tenth of its initial value. For a net of eighteen intervals about 75 traverses would be necessary, etc. Later it will be seen, how, by extrapolation, the number of necessary traverses can be reduced to a fraction of these figures, even for the Liebmann procedure.

Symmetry

The treatment of a symmetrical model is much simpler than that for a nonsymmetrical one. If the original net is laid down so that a line of points lies along an axis of symmetry, these points may be improved in the usual way by assuming that values on opposite sides of the line are equal and are improved simultaneously. Thus, in Fig. 1 the line through B and K is a line of symmetry. The point at V is then improved by the following formula

$$4\psi'_V = \psi_S + 2\psi_U + \psi_Y$$

and only one half of the area need be treated. Each time R is improved, for example, T may be considered to have been improved at the same time. It is of course necessary that both the model contour and the loading system be symmetrical in order to have a symmetrical stress system. In the case of a fine net, the labor of calculation is roughly cut in half by each axis of symmetry. Lines of symmetry need not lie parallel to the principal directions of the net but they may equally well lie at 45 deg to it.

Multiple-Point Methods

The rate of convergence may be greatly increased by improving the points a block at a time rather than singly. For example consider Fig. 3, in which the internal points have been divided

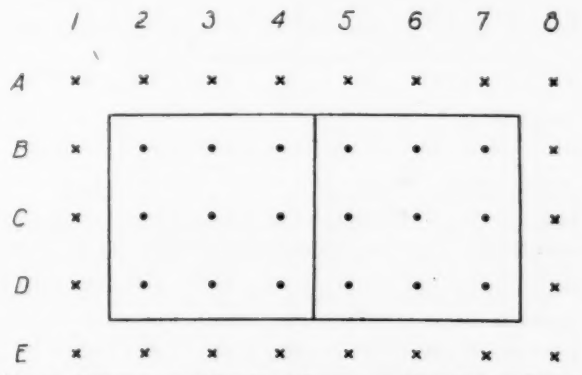


Fig. 3 Division of a Net Into Groups of Nine Points Each

into two square arrays of nine each. The borders of these groups are indicated by heavy lines. Consider the block for which $C3$ is the center point. A simultaneous solution of the set of nine equations, which express the requirement that the value at each of these points be the mean of its four neighbors, gives the following formula which may be used in calculating the improved value of the center point of the block directly from the boundary values

$$\psi'_{C3} = \frac{1}{16}\Big[2(\psi_{A3} + \psi_{C5} + \psi_{E3} + \psi_{C1}) + \psi_{A2} + \psi_{A4} + \psi_{B5} + \psi_{D5} + \psi_{E4} + \psi_{E2} + \psi_{D1} + \psi_{B1}\Big] \quad \ldots\ldots\ldots [3]$$

Having the center point, the four corner points may be improved by diagonal application of the single-point formula

$$\psi'_{B2} = \frac{1}{4}(\psi'_{C3} + \psi_{C1} + \psi_{A1} + \psi_{A3})$$

$$\psi'_{B4} = \frac{1}{4}(\psi'_{C3} + \psi_{C5} + \psi_{A5} + \psi_{A3})$$

$$\psi'_{D4} = \frac{1}{4}(\psi'_{C3} + \psi_{C5} + \psi_{E3} + \psi_{E5})$$

$$\psi'_{D2} = \frac{1}{4}(\psi'_{C3} + \psi_{C1} + \psi_{E1} + \psi_{E3})$$

In these formulas the improved value of ψ_{C3} is used. The five improved values now available may be used for the improvement of points $B3$, $C4$, $D3$, and $C2$ in the usual way, e.g.,

$$\psi'_{B3} = \frac{1}{4}(\psi'_{B2} + \psi_{A3} + \psi'_{B4} + \psi'_{C3})$$

This provides a procedure by means of which a network such as that of Fig. 3 may be improved much more rapidly than by the single-point treatment. The left block is improved as described and then the right block, improving first the center point $C6$ and subsequently the others. In the event that a nine block lies adjacent to a boundary or, if for any other reason the value of $A1$, say, is not available, the point $B2$ may be improved by the formula

$$\psi'_{B2} = \frac{1}{8}(2\psi'_{C3} + \psi_{A3} + \psi_{C1} + 2\psi_{A2} + 2\psi_{B1})$$

in place of the formula just given.

The value of λ_1 for this procedure will be less than that for the Liebmann process, indicating a more rapid rate of convergence. Table 2 gives values for the nine-block method of improvement. In this case, for large p, $1 - \lambda_1 = 3.5\pi^2/p^2$. For any value of p, the number of traverses, yielding a given error reduction with the nine-block procedure, is reduced over the single-point method by a factor of about 3.5, which results, in most practical problems, in a very satisfactory rate of convergence.

TABLE 2

p	λ_1
7	0.503
10	0.712
16	0.874
22	0.931

The use of four-point blocks is useful when small areas are to be treated which do not permit the placing of a nine block. A group

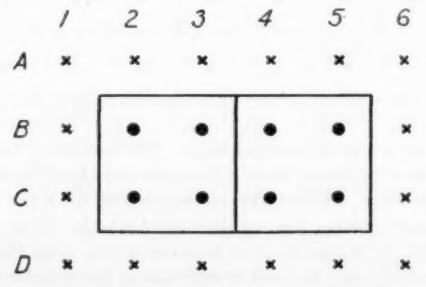


FIG. 4 DIVISION OF A NET INTO GROUPS OF FOUR POINTS EACH

of this nature is shown in Fig. 4. Assuming the boundary values to be given and arbitrary values assigned to interior points, improvement is made as follows

$$\psi'_{B2} = \frac{1}{24}\Big[7(\psi_{A2} + \psi_{B1}) + 2(\psi_{A3} + \psi_{C1} + \psi_{B4} + \psi_{D2}) + \psi_{D3} + \psi_{C4}\Big]$$

$$\psi'_{C3} = \frac{1}{4}(\psi'_{B2} + \psi_{B4} + \psi_{D4} + \psi_{D2})$$

$$\psi'_{B3} = \frac{1}{4}(\psi'_{B2} + \psi_{A3} + \psi_{B4} + \psi'_{C3})$$

$$\psi'_{C2} = \frac{1}{4}(\psi_{C1} + \psi'_{B2} + \psi'_{C3} + \psi_{D2})$$

The points are to be improved in the foregoing order, using improved values as soon as they are available in the succeeding formulas. The rate of convergence resulting from this procedure is greater than that for the Liebmann but less than that for the nine block. Table 3 gives values of λ_1.

TABLE 3

p	λ_1
5	0.347–0.444
9	0.753–0.779
13	0.878–0.886

Further reduction of the λ_1 values may be secured by the use of still larger blocks of points, but the complication of the formulas increases rapidly and the gain in reduction per traverse is more than offset by the labor involved. Experience of the authors indicates that the most satisfactory method for handling an area with any considerable number of interior points consists in fitting into it as many blocks of nine points as possible, then as many blocks of four as possible, and finally treating the remaining points by the Liebmann formula.

THE DIFFERENCE FUNCTION

A further saving in labor is achieved by carrying out operations on what will be designated the difference function, which represents the change in ψ with one traverse. Suppose one operation is carried out on a net such as Fig. 1, obtaining the first set of improved values for the interior points. Then let the initial net values be subtracted at each point from this set of improved values. These differences may now be treated with the same improvement formulas as the original set of values. Certain advantages are inherent in this treatment. In the first place, the number of significant figures is reduced in general, so that the labor of calculation is also reduced. The boundary values of the difference function vanish and hence give many zero terms in the improvement formulas. Finally, the successive traverses over this net soon result in the reduction of the value at each point by a factor λ_1 directly, so that succeeding values form a geometric series which approaches zero and which permits of convenient extrapolation. It will be seen that successive values, obtained on the difference net, represent amounts which must be added to the original net values in order to obtain the actual values of ψ. This represents an additional step which must be performed with this procedure, but the over-all labor is sufficiently reduced so that this is not objectionable. In practice it is logical to add all but the first value on the difference net to the first improved value of the original net. Thus, if $\psi, \psi', \psi'', \psi''', \psi^{(iv)}$ are the original and successive improved values at a given point on the original net and, if δ, δ', δ'', and δ''' are successive improved values on the difference net

$$\delta = \psi' - \psi$$

and, for example,

$$\psi^{(iv)} = \psi' + (\delta' + \delta'' + \delta''')$$

Since the type of series represented by successive values on the difference net is similar to Equation [2], it becomes a simple matter to extrapolate to a final value after all except the highest eigenfunction have disappeared. Suppose that after the difference net has been traversed k times, the values at each point become roughly a constant fraction of their immediate predecessors with each improvement, that is,

$$\delta^{(k+1)} = \lambda_1 \delta^{(k)}$$

Then successive values of the difference function may be represented as $\delta^{(k)}$, $\lambda_1\delta^{(k)}$, $\lambda_1^2\delta^{(k)}$, $\lambda_1^3\delta^{(k)}$, ... etc., and the sum which is to be added to the first improved value ψ' to get the final solution is

$$\begin{aligned}&\delta' + \delta'' + \ldots + \delta^{(k-1)} + \delta^{(k)} + \lambda_1\delta^{(k)} + \lambda_1^2\delta^{(k)} + \lambda_1^3\delta^{(k)} + \ldots . \\ &= \delta' + \delta'' + \ldots + \delta^{(k-1)} + \delta^{(k)}(1 + \lambda_1 + \lambda_1^2 + \lambda_1^3 + \lambda_1^4 + \ldots) \\ &= \delta' + \delta'' + \ldots + \delta^{(k-1)} + \delta^{(k)}/(1 - \lambda_1)\end{aligned}$$

Thus if a series of values at a point on the difference net reduces as follows: 100, 90, 81, 72.9, etc., it is seen that λ_1 is 0.9 and the series is

$$100\ (1 + 0.9 + \overline{0.9^2} + \overline{0.9^3} + \overline{0.9^4} + \ldots)$$

for which the sum is

$$\frac{100}{(1 - 0.9)} = \frac{100}{0.1} = 1000$$

By considering the last term of the series one may write the sum as

$$100 + 90 + 81 + \frac{72.9}{(1 - 0.9)}$$

which of course also turns out to be 1000.

By determining in this way the sum of the series of values which results from repeated traverses over the difference net, and adding this sum to the first improved value as described, an extrapolated value is obtained which will be very close to the true solution of the Laplace difference equation in the region under consideration. Then, for all but the smallest nets, the following procedure applies:

1 Determine the boundary values at intersections of the net with the boundary.

2 Assign arbitrary values to interior points of a coarse net, say, of ten to fifteen points, carrying an arbitrary significant figure beyond that given by the boundary data.

3 Apply the Liebmann procedure to approximate convergence.

4 Transfer the values obtained to a net of one half the net interval (four times as many interior points) carrying another arbitrary significant figure (two in all).

5 Divide the area into as many nine-point and four-point blocks as possible.

6 Improve the values once to obtain ψ'.

7 Subtract to obtain the difference function δ.

8 Improve the difference net *in the same order* until fairly smooth convergence is indicated.

9 Determine the limit of the series of improved values at each point.

10 Extrapolate to obtain the final ψ-value.

11 Improve this value once as a check on convergence.

12 If necessary calculate new differences and repeat the extrapolation, or transfer the values to a finer net and repeat Items 4 to 11, depending on the accuracy desired.

Application

As an illustration of the application of this method to an actual photoelastic problem, a compression piece has been chosen with symmetrical notches, Fig. 5. This is a shape which is quite familiar to most workers in the field of photoelasticity since it has been used frequently in the literature to illustrate both theoretical and practical points. In connection with this model, certain assumptions have been made. The stress in the shanks of the model at points removed from the notches is assumed to be substantially uniform and equal to P/A. This enables a consideration of the central section of the model, omitting any discussion of the points at which the load is applied. The sections through

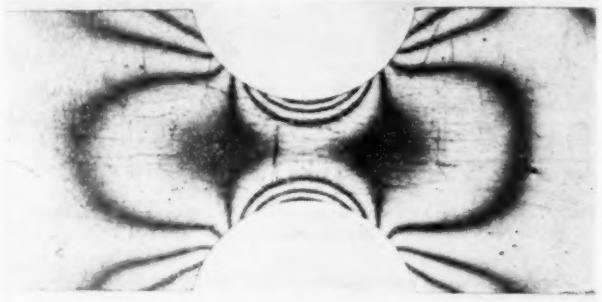


Fig. 5 Light-Field Fringe Photograph of Notched Compression Piece

(This shows the section of the model which was actually considered. The remaining portions of the shank were subjected to stress which was substantially uniform across the cross section. This permitted the use of the sections lying at the ends of the figure as artificial boundaries. The model was cut from clear bakelite, 1/4 in. thick, and was loaded in compression. Actual stresses in pounds per square inch may be taken as 342 times the fringe order. In this paper no actual stresses were computed, all data being given in equivalent fringe order.)

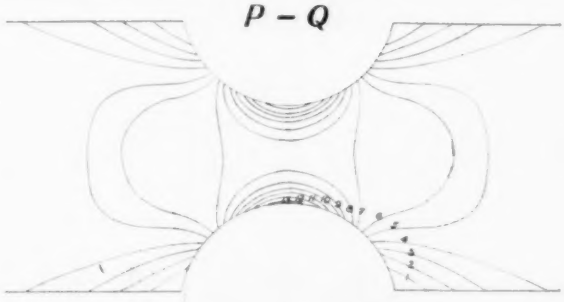


Fig. 6 Isochromatic Lines

(Obtained from Fig. 5 and a similar dark-field photograph. Center lines of the dark fringes were traced from each and superposed.)

the shank at the edges of Fig. 5 are taken as artificial boundaries along which the stress is constant and at which it may be taken as equal to the value at the boundary. The sum of the principal stresses is then known across this section. These lines constitute a part of the boundary of the model as it will be discussed. It further appears that there are two axes of symmetry in this shape, so the labor of solution will be reduced by a factor of about four by using these as outlined. In order to arrive at a more nearly representative solution, the fringe positions were averaged for the four quadrants of the model, the resulting average fringes being shown in Fig. 6. It may be of interest to note that these fringes were taken in two groups, one with polarizers crossed and the other with parallel polarizers. This gives, in effect, the half-order fringes and thus increases the precision which may be obtained from the data.

In order to evaluate the points at which the net intersected the boundaries of the model a curve was plotted as in Fig. 7. For the

shank of the model this is a plot in Cartesian coordinates and for the notch it is a polar diagram. Points were plotted where the fringes intersect the boundaries and a smooth curve drawn through the points. Values of fringe order were plotted instead of stress in pounds per square inch since the numbers are easier to handle. The boundary values of the net were taken from this curve.

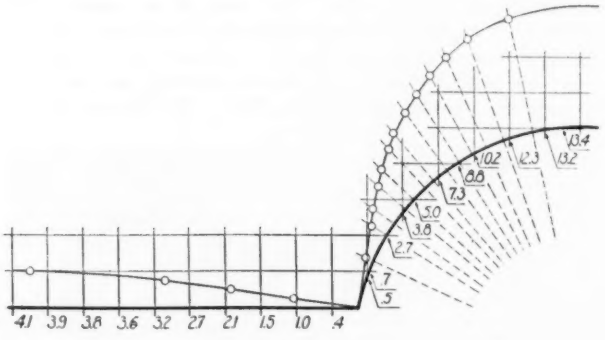


FIG. 7 INTERPOLATION CHART FOR DETERMINING STRESSES AT BOUNDARY POINTS

Three nets were calculated for this model in order to give an idea as to the factors which influence the choice of net fineness. Fig. 8 shows the intermediate net. The other two nets had respectively half the net interval and twice the net interval of this one. Reference will be made to these as the fine and the coarse nets. They were treated as follows: The coarse net was first calculated by the use of the Liebmann procedure, using entirely arbitrary values at the interior points. Final values from this net were transferred to the intermediate net at points which coincided. At other points the initial values for the intermediate net were computed by a diagonal application of the single-point formula and subsequent application of the ordinary single-point formula. Thus the point $D4$ was obtained as

$$\psi_{D4} = \frac{1}{4}(\psi_{C3} + \psi_{E3} + \psi_{E5} + \psi_{C5})$$

where $C3$, $E3$, $E5$, $C5$, etc., were obtained from the coarse net, and the point $D3$ as

$$\psi_{D3} = \frac{1}{4}(\psi_{C3} + \psi_{D2} + \psi_{E3} + \psi_{D4})$$

With each point in Fig. 8 are associated two columns of numbers. The top number in the left column is the original value assigned to the point by the coarse net. Although the boundary values were taken as significant to only two figures, three figures were carried in the interior of the coarse net and four in the other two nets. This was done to insure that the final result should contain no rounding-off errors and also to permit a closer observation of the improvement process. No decimal points are indicated on the net. In general it seems desirable to carry two more significant figures than the data justify and to discard them when the calculation is complete. Thus the calculation can be terminated when the extrapolation formula indicates that the error is less than 50 in the last place. The second number in the left column is the result of a single traverse over the net, using the appropriate improvement formulas. The order of going over the net is not of great importance, but a specific order must be retained throughout a problem in which the difference net is used. In this case the nine block was improved first; then points $A2$, $A3$, $A4$, $D5$, $D6$; the four block in the order $C5$, $B6$, $B5$, $C6$; then $A5$, $A6$, $C7$, $B7$, $A7$, $B8$, $A8$, $B9$, and $A9$. Points $D6$, $C7$, $B8$, and $B9$ were so placed that the boundary point formula was necessary. These were therefore treated as discussed under that heading. Thus point $D6$ was improved by the formula

$$\psi'_{D6} = 0.1448\psi_{D5} + 0.1188\psi_{C6} + 1243$$

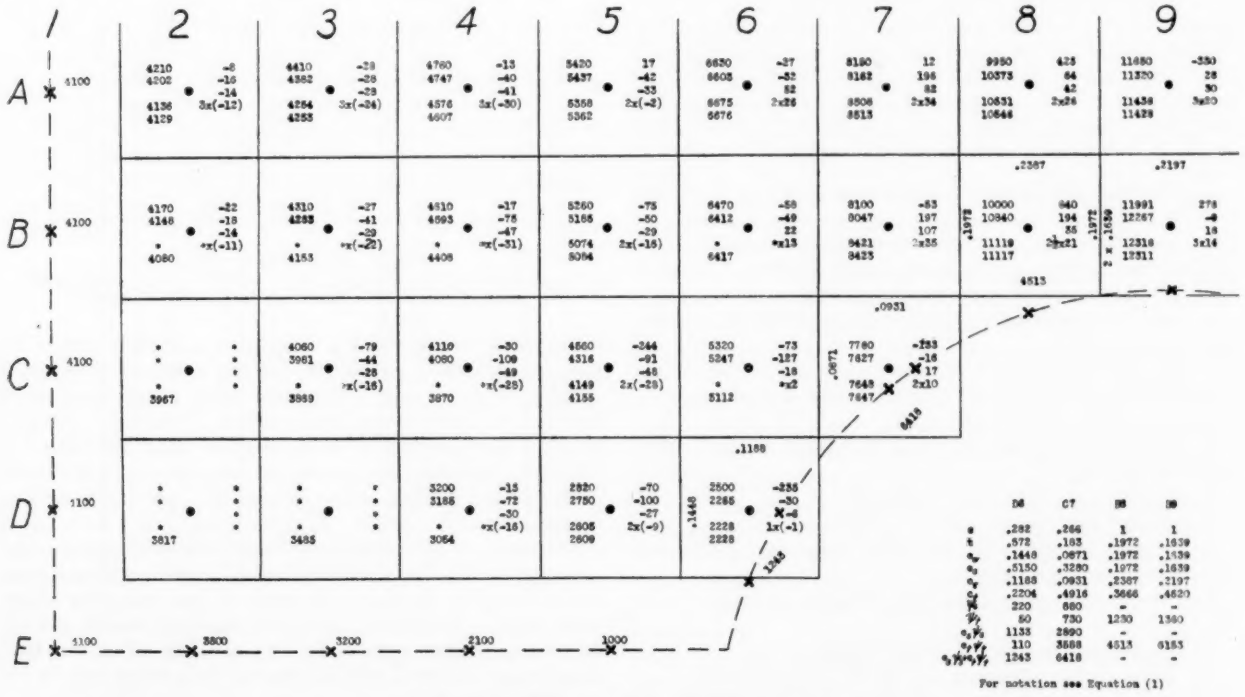


FIG. 8 SOLUTION OF THE INTERMEDIATE NET

(The table gives data for the calculation of the coefficients of the points near the boundary. These are written in the corresponding squares adjacent to the neighbors to which they apply. Values in the net marked * are not needed in subsequent calculations and hence were not computed.)

The first number in the right-hand column is the difference of the second and the first in the left column. This establishes the difference function with which to proceed in further iteration of the improvement formulas. The difference function was improved three times, giving the succeeding values in the right column. It will be observed that positive and negative areas tend to develop, each finally decreasing. If the iteration were prolonged sufficiently, the values would become one-signed and smoothly distributed. The authors did not wait for this, however, but proceeded to extrapolate after the third traverse over the difference net, since at this point the difference function had approximately a stationary node. Thus it evidently contained little of the first (nodeless) eigenfunction of the process but resembled more one of the other eigenfunctions. For this reason the error is decreasing more rapidly than would ordinarily be expected. The decrease seems to be by a factor of about $^2/_3$, so a multiplying factor $1/(1 - {}^2/_3) = 3$ has been used in the center of each pillow. Near the node where the decrease is more rapid smaller factors have been used. From observation of a converging series a reasonable guess is thus being attempted as to how far it is going. The choice of multiplying factors is influenced by a knowledge of the λ_1-value for an equivalent square area, by judgment acquired through experience, by the observed rate of decrease of particular points, etc. The exact amount by which each point is extrapolated is not so very important so long as the overall treatment is about correct. This is due to the rapidity with which a random error is ironed out by the subsequent application of the improvement formula.

The third value in the left column at each point is now obtained by adding the second, third, and fourth entries in the right-hand column to the second value on the left. Consider the point *B*5. The value 5260 was assigned from the previous coarse net, but in so far as treatment is concerned, it might have been entirely arbitrary. The value 5185 resulted from a single traverse over the net in the order indicated previously. The difference of 5185 and 5260 is —75, the top right value. Subsequently —50, —29, and —16 were obtained by traversing the difference net. The final multiplier for the last value was taken to be 2 as indicated by the considerations above. The sum of —50, —29, and 2 × (—16) and 5185 then gave 5074, the third value in the left column. This is the result of the extrapolation procedure. A final traverse is now made over the net, using extrapolated values, the result being the number 5084 at *B*5. This is considered to be the solution of the problem. Now drop the two significant figures which were added earlier and place the decimal point, giving the equivalent fringe value as 5.1 at this particular point. The degree to which it really represents a solution may be determined by the use of a finer net. Fig. 9 shows the arrangement of blocks in the fine net. Here the size permitted the use of six complete nine blocks, and three more which were split along an axis of symmetry.

The results of the fine net were obtained by a process identical with that employed on the intermediate net, except that after the first extrapolation was made, a second was also performed, in order to leave no doubt as to the correctness of the result. Table 4 shows the values obtained at various points from each of the three nets.

TABLE 4

Point	Coarse	Intermediate	Fine
*A*3	4.4	4.3	4.2
*A*5	5.4	5.3	5.3
*A*7	8.2	8.5	8.6
*A*9	11.7	11.4	11.3
*C*3	4.1	3.9	3.8
*C*5	4.6	4.1	4.1
*C*7	7.8	7.7	7.7

It will be observed that there is no significant change in going from the intermediate to the fine net. The coarse net, however, was somewhat farther from the final values than is desirable in this problem.

The results of the calculations were plotted as contours of constant $(S_1 + S_2)$, forming what are generally known as isopachic lines. These are shown in Fig. 10. When these are combined with the isochromatic lines, the several principal stresses can be separated. These separate values are given in Fig. 11. Thus the problem is completed.

Conclusions

Commenting upon what has been presented; with regard to

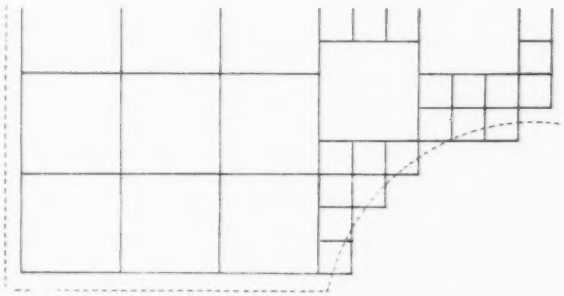


Fig. 9 Arrangement of Blocks in the Fine Net

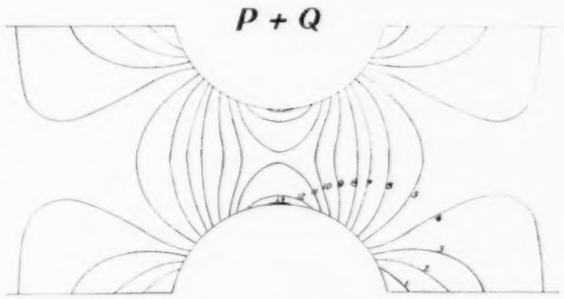


Fig. 10 Isopachic Lines of the Model

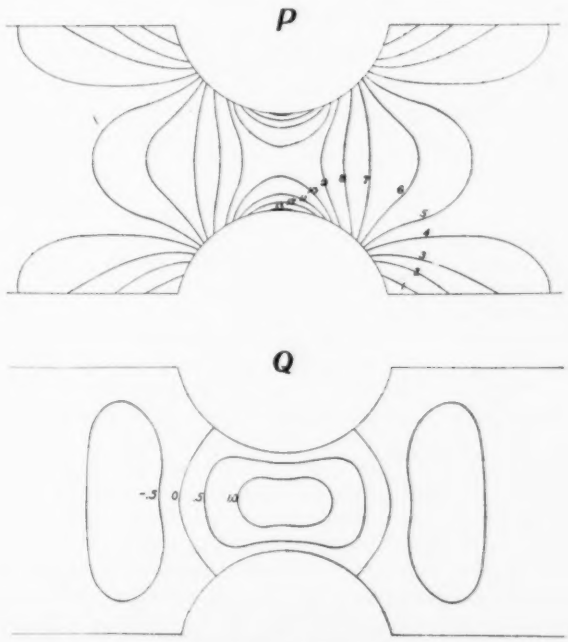


Fig. 11 Separate Principal Stresses

the photoelastic problem which was used as an illustration, it may be said that the result represents as nearly the correct values as it is possible to obtain with any method depending on the fringe pattern. No possible significant error can exist in the numerical treatment of the data. In the present instance the authors went to a great deal more trouble than was necessary in solving the problem. The intermediate-net solution was entirely adequate in so far as engineering accuracy was concerned.

As to the time consumed in this method, it will be of interest that the solution by the intermediate net, after the boundary values had been obtained, required approximately two hours. This is based upon two persons using a computing machine. The fine-net solution required about seven hours. The time necessary in working over the original coarse net was very short. It is estimated that the total amount of work included in this problem could be accomplished by one experienced person in a day. This represents a marked saving of time and energy over other methods which have been employed for this type of work.

It is evident that the method is a general one which may be applied with profit to many other fields, such as steady-state heat conduction, electric fields, membrane shapes, and perhaps certain problems in hydrodynamics and gravitation.

Creep, Elastic Hysteresis, and Damping in Bakelite Under Torsion

By HERBERT LEADERMAN,[1] CAMBRIDGE, MASS.

The creep in torsion of various forms of bakelite under constant load and the creep recovery on removal of load is found to follow closely the superposition principle or "memory law" of Boltzmann, if the previous maximum strain in a test is not exceeded. If the previous maximum strain is exceeded, an additional plastic flow takes place which is not, at any rate immediately, recoverable.

A function of the elapsed time called the "equivalent time" is introduced in order to analyze the creep and creep recovery when the loading history is complex.

A method is developed for calculating the stress-strain loop due to creep for step-by-step cyclic loading. Results of cyclic-loading tests reveal the presence of a true elastic-hysteresis loop in addition to the loop due to creep.

The specific damping capacity in torsion at a given stress, calculated from the measured creep and elastic-hysteresis loop, is shown to agree fairly well with the damping measured directly.

The Inelastic Phenomena

THE PROBLEM of the dissipation of mechanical energy under alternating stress, known as damping capacity, has in the past received much attention both in America and in Europe. It was originally hoped that damping capacity in metals might be an important factor in limiting the amplitude of resonant vibrations. Unfortunately, damping in metals appears to be too small to be of much use in this respect. In synthetic materials however damping can be large, and can therefore be more easily measured and studied. The following investigation was undertaken to elucidate the nature of damping and related phenomena in various commercial forms of bakelite. The conclusions drawn from this investigation are probably true for other organic materials as well.

It was originally thought that damping capacity as manifested by the decay of torsional oscillations of a pendulum was of the nature of a solid friction, analogous to fluid friction. This concept leads to the conclusion that the logarithmic decrement (the natural logarithm of the ratio of successive oscillations) is independent of stress but proportional to frequency. In practice, over the usual range of frequencies, this quantity is found to be a function of stress but independent of frequency. On the other hand, the assumption that damping is due to a departure from perfect elasticity is in agreement with experimental evidence. The study of damping is therefore the study of the small imperfections of elasticity within the elastic range.

Damping and Elastic Hysteresis. Damping is usually measured by a nondimensional quantity, the specific damping capacity ψ, which is the ratio of the energy dissipated in one oscillation to the strain energy present at the maximum amplitude of the oscillation

$$\psi = \Delta W/W \quad [1]$$

This is just twice the logarithmic decrement δ when this is small. We can thus measure the damping capacity of a specimen by the decay of torsional oscillations of a system in which the specimen forms the elastic member.

Careful measurements by Rowett (1)[2] of the stress-strain relationships of steel tubes under reversed torsion over a given stress range showed that this relationship was not exactly a straight line but was in fact a narrow closed loop. These loops of course correspond to a loss of energy on traversing a cycle. The specific damping capacity calculated from the static tests agreed closely with values obtained by measuring the decrement of free oscillations from the dynamic tests. The stress-strain relationship was a narrow closed loop, of the same shape and size, no matter how speedily or how slowly the stress-strain test was made. Such a stress-strain loop is usually referred to as an elastic-hysteresis loop. Since elastic hysteresis is a phenomenon independent of time, this can be a cause of damping capacity independent of frequency.

Primary and Secondary Creep. In amorphous materials such as glass, and also to a small extent in metals, the strain under constant stress varies with time. This phenomenon is known as "creep." In hard metals and in glass at room temperature the creep varies approximately as the logarithm of the time since the application of the load (2, 3). At higher temperatures there is a superimposed plastic flow in which the creep is proportional to time but not necessarily to stress. Under these conditions the deflection x varies with the time of application of load t according to the equation

$$x = b + a \log_{10} t + ct \quad [2]$$

where the term ct represents the plastic flow; this is of appreciable magnitude in metals, only at elevated temperatures. The creep rate

$$dx/dt = (a'/t) + c \quad [3]$$

at first decreases with time and then becomes approximately constant (4). Kanter (5) has referred to the first stage of creep where the logarithmic component is predominant as the "primary stage," and to the stage where the plastic flow is predominant as the "secondary stage." It is suggested as an extension of these definitions that the logarithmic component be called "primary creep" and the plastic-flow component "secondary creep."

The physical distinction between the two components is roughly that secondary creep is analogous to fluid flow, in that it represents a permanent distortion which is not recovered on removal of load. On the other hand, the deflection due to primary creep is gradually recovered after removal of load, as a negative creep or creep recovery takes place. The creep of bakelite at low stresses is very largely primary creep.

The Memory Law or Superposition Principle of Primary Creep. The law of primary creep and creep recovery was first enunciated

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Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y., and will be accepted until August 10, 1939, for publication at a later date. Discussion received after the closing date will be returned.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[2] Numbers in parentheses refer to the Bibliography.

by Boltzmann (6), and has been restated in other forms by Bennewitz (3) and Becker (7). The basic principle is that the primary creep of a specimen of material is a function not only of the last load applied, but also of the previous loading history. If for example a cylindrical specimen of bakelite is twisted first in one sense and then for a short time in the reverse sense, after the load is removed it recovers at first in the opposite direction to the latter twist. This recovery slows down, ceases, and then reverses. Fig. 1 shows the nature of creep recovery after such a

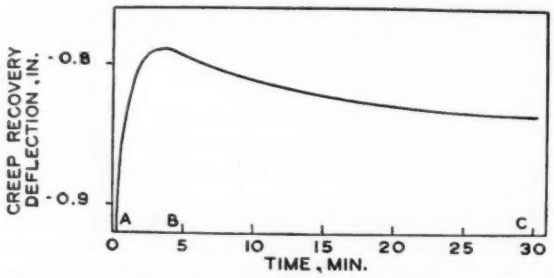


FIG. 1 CREEP RECOVERY AFTER TORSION LOADING IN OPPOSITE DIRECTIONS

(Angular twist in radians equals deflection divided by the optical lever arm of 148.4 in.)

loading history. This recovery can be explained by assuming that the specimen possesses a "memory," and that from A to B, Fig. 1, the memory of the more recent briefer action exceeded the memory of the earlier action of longer duration; from B to C the reverse is the case. The general theory of Boltzmann assumes that the rate of primary creep is a function of the entire loading history, and that the creep rate due to each augmentation of load is proportional to the increase in load and to a function of the time elapsed since the application of the load; the effect of each loading action is assumed to continue indefinitely. Removal of load is regarded as being equivalent to the addition of a negative load. If for example a load W be applied at time t_0, and if t be the current time, the creep rate at time t will be given by

$$dx/dt = Wa\varphi(t - t_0) \quad [4]$$

If the load be then removed at time t_1, the subsequent creep rate at time t will be given by

$$dx/dt = Wa\varphi(t - t_0) - Wa\varphi(t - t_1) \quad [5]$$

The assumption of logarithmic creep due to a single load is equivalent to assuming for the function $\varphi(t)$ a value $1/t$. Hence the creep rate for a load W applied at time t_0 and removed at time t_1 becomes

$$\frac{dx}{dt} = Wa\left[\frac{1}{t - t_0} - \frac{1}{t - t_1}\right] \quad [6]$$

This is negative, indicating that a creep recovery is taking place. The deflection x therefore varies with time t according to the equation

$$x = Wa \log\left(\frac{t - t_0}{t - t_1}\right) \quad [7]$$

If the load be reapplied at t_2, the subsequent deflection will be given by

$$x = Wb + Wa \log \frac{(t - t_0)(t - t_2)}{(t - t_1)} \quad [8]$$

where Wb represents the elastic deflection. If it be assumed that the creep rate due to a given load is proportional to that load, then in general, if the load be applied at times t' in increments W, he deflection at a subsequent time t will be given by

$$x = b\Sigma W + a\Sigma W \log(t - t') \quad [9]$$

Primary Creep and Damping. The creep due to a sinusoidal applied load will now be considered. Under certain assumptions (6, 7, 8) it can be shown that the deflection, due to the action of the "memory," is also sinusoidal but lags behind the deflection by a small angle ϕ; in other words the dynamic stress-strain curve due to the action of creep is a narrow ellipse, and hence primary creep obeying the memory law can be a cause of damping. The mathematical theory indicates that the specific damping capacity due to primary creep should be independent of frequency over a wide range of frequencies (7). The specific damping capacity due to creep is then

$$\psi = 4\pi\Phi = \frac{\pi^2 a}{Mb} = 4.28\,\frac{a}{b} \quad [10]$$

where $M = \log_e 10$, b is the elastic deflection per unit load, and a is the slope of the plot of the creep curve per unit load against logarithm (to base 10) of the equivalent time (in the absence of secondary creep). This formula has been used (8) to show that the directly measured damping for glass agrees closely with that computed from the creep measurements.

CREEP AND ELASTIC HYSTERESIS

Using cylindrical bakelite specimens in torsion, the author found that the measured damping was much greater than that computed from the creep using Equation [10]. It was considered that the discrepancy might be due to the presence, in addition to primary creep, of a true elastic hysteresis, of a statical nature independent of time. It was of course impossible to measure the area of the elastic-hysteresis loop directly by the method of Rowett, since the creep was of such magnitude as to interfere with the true elastic hysteresis, if the latter were present. Accordingly, static tests were made in which the load was applied in equal steps up to the maximum load; the load was then removed in steps of equal magnitude. In this manner stress cycles were traced. Increase or decrease of load took place in equal intervals of time; the reading corresponding to the twist of the specimen was made just before the load change. In this way stress-strain loops were obtained. It was now necessary to determine whether the width of loop at any load level could be accounted for entirely by creep. Accordingly the Boltzmann memory law was used to calculate the deflections due to creep for this particular loading pattern, and a separate creep test was made to determine the value of the creep constant a.

It was found, after applying the corrections due to creep deflection to each observation, that a finite width of loop remained, indicating that there was a true elastic-hysteresis loop. Repeating the step-by-step cyclic-loading test with different time intervals gave for the same stress range nearly the same value of hysteresis-loop area. The combined contributions to damping of primary creep (calculated from a creep test) and of elastic hysteresis (calculated from the deduced elastic-hysteresis loop) agreed reasonably well with the value obtained for the same stress range from the decrement of free oscillations of the specimen. It is to be concluded, for the four commercial forms of bakelite tested, that there exists a true elastic hysteresis as well as a primary creep, and that the damping is due to the action of both these inelastic phenomena.

THEORETICAL AND EXPERIMENTAL METHODS

The conclusion just stated was reached after a four-fold attack on the problem. First, methods were devised to test the memory law. Secondly, these methods were applied to four specimens of bakelite in torsion. Thirdly, the Boltzmann memory law was used to calculate the deflections due to creep in the step-by-step

cyclic-loading tests. Finally, cyclic-loading and decrement tests were made. From the former the damping due to creep and hysteresis was calculated, and from the latter the damping was obtained directly.

Materials Used. A complete set of tests was made on each of four types of bakelite material:

(*a*) Phenol-formaldehyde resin, white, transparent, and unfilled, as used for photoelasticity tests.

(*b*) Bakelite commercial molding material with wood-flour filler.

(*c*) Bakelite X 199 shock-resisting material. The filler consisted of fabric snippings impregnated with phenol-formaldehyde resin and molded. These three materials can be considered to have an isotropic structure.

(*d*) Bakelite cord material. The reinforcement consisted of closely packed cotton cords running parallel to the axis of the specimen.

Solid cylindrical specimens were tested in torsion, with the load applied by means of weights hung from the ends of a lever arm. For the creep and cyclic-loading tests, light, reflected onto a fixed vertical scale from a front-silvered mirror attached to the bar, was used to obtain readings of the torsional deflection. For the damping tests a high light reflected from a ball bearing was photographed on a moving strip of cinematograph film. The load was released by burning through a cotton thread, the lever arm acting as the inertia bar.

Tests of the Memory Law

As was previously shown, the deflection at a time t according to the memory law is equal to the elastic deflection together with a deflection proportional to the logarithm of a function of time dependent on the previous loading history; for example, if a load W be applied at time t_0, removed at time t_1, and reapplied at time t_2, the time function for the subsequent creep will be, from Equation [8]

$$(t - t_0)(t - t_2)/(t - t_1) \quad \ldots\ldots\ldots\ldots\ldots\ldots [11]$$

Let such a time function be called the "equivalent time." The concept of equivalent time will be shown to be a powerful tool in separating primary creep, secondary creep, and elastic deflection for any complex loading history.

For each material the following three tests were made.

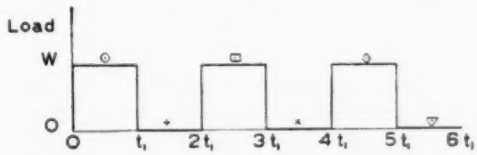


Fig. 2 Loading Diagram for Test A, Torsion Repeated-Loading Test

(The symbols on the diagram correspond with those in Fig. 5.)

Test A, Repeated Loading. Fig. 2 represents the loading diagram for this test. A given load W was applied for 10 min and the deflection read at intervals during this period. The load was then removed and the deflection read during a recovery period extending over an equal interval of time; the load was then reapplied. The unloading and reloading process was continued for a total of 60 min. It follows from the Boltzmann law that the equivalent times for the creep periods are

$$t, \quad \frac{t(t-2t_1)}{t-t_1}, \quad \frac{t(t-2t_1)(t-4t_1)}{(t-t_1)(t-3t_1)} \quad \ldots\ldots\ldots [12]$$

and for the recovery periods

$$\frac{t}{(t-t_1)}, \quad \frac{t(t-2t_1)}{(t-t_1)(t-3t_1)}, \quad \frac{t(t-2t_1)(t-4t_1)}{(t-t_1)(t-3t_1)(t-5t_1)} \ldots [13]$$

respectively, where t is the time from the beginning of the experiment, and t_1 is in this case equal to 10 min. In the absence of plastic flow, the deflections observed in the creep periods when plotted against the logarithm of the equivalent time should lie along one straight line of slope Wa, and the deflections in the recovery periods should lie along a parallel straight line.

Test B, Primary Creep and Stress. The purpose of this test was to prove that the primary creep was proportional to stress. Test A was carried out to say an elapsed time of $4t_1$ min, and then repeated after a while with half the load. See Fig. 3 (*a*) and (*b*). The slope of the creep-recovery line in the second test should be half of that in the first test.

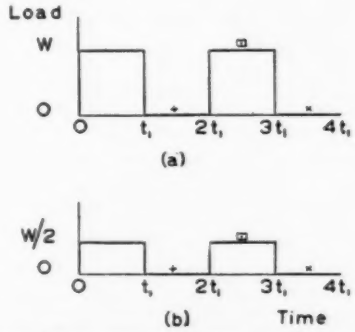


Fig. 3 Loading Diagrams for Test B, Primary Creep and Stress in Torsion

(The symbols correspond with those in Fig. 6.)

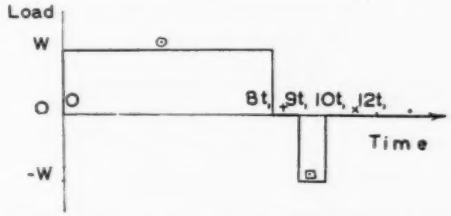


Fig. 4 Loading Diagram for Test C, Memory-Law Test, Torsion

(The symbols correspond with those in Fig. 7.)

Test C, Memory-Law Test. Fig. 4 represents the loading diagram for this test. The load was applied from time $t = 0$ to $t = 8t_1$, and then removed. The load was applied in the reverse sense from $t = 9t_1$ to $t = 10t_1$ and then finally removed. This form of test was so chosen that the memory law could be tested in the absence of any possible complications from plastic flow.

The equivalent time for the creep recovery from $t = 10t_1$ onward is then

$$t(t - 10t_1)/(t - 8t_1)(t - 9t_1) \quad \ldots\ldots\ldots\ldots [14]$$

This has a maximum at $t = 12t_1$, i.e., according to the memory law the creep recovery from $t = 10t_1$ to $t = 12t_1$ should be in the direction of the original creep, whereas from $t = 12t_1$ onward it should be in the reverse direction. Furthermore, the creep-recovery deflections from $t = 10t_1$ onward should all lie on a single straight line, when plotted against the appropriate value of the logarithm of the equivalent time.

Results of the Creep Tests

Test A. Fig. 5 shows the result of a test on cord material with the deflections in each of the six stages plotted against the logarithm of the equivalent time. The other specimens gave similar results. It is seen that in the first creep period the plot is not linear, indicating that some kind of plastic flow was taking place in this stage. The first recovery stage, when plotted against the

logarithm of the equivalent time, gives a straight line. The second creep stage gives a line which is straight and parallel to the creep-recovery line up to the termination of the first creep stage; for the rest of this period there is a further plastic flow. The second recovery stage gives a straight line parallel to but displaced from the first by an amount equal to this plastic flow. It is seen that the Boltzmann memory law gives correctly the nature of the creep recovery and also of the creep if the previous maximum strain is not exceeded. When stress-strain loops are traced therefore, it is safe to say that, within the stress range considered, after the first loop no plastic flow occurs and the creep and creep recovery are given completely by the Boltzmann law.

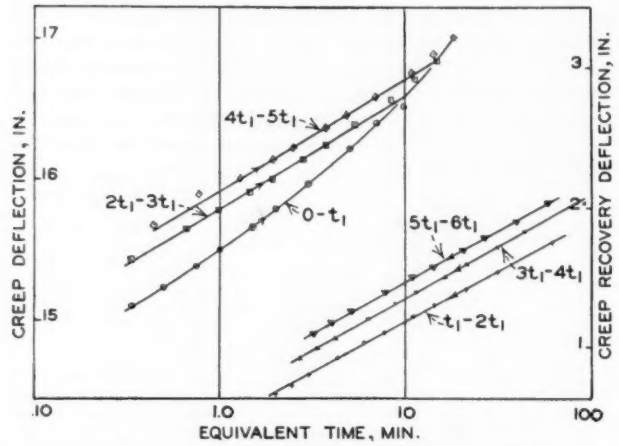


FIG. 5 CREEP AND CREEP RECOVERY, TEST A, CORD MATERIAL

(The left-hand scale refers to scale deflections under a torque load of 15 lb-in., and the right-hand scale to the recovery deflections. The angular twist in radians is obtained by dividing the deflection by the optical lever arm, 148.4 in. The symbols used for each creep and recovery stage correspond with those in Fig. 2. Time t_1 = 10 min.)

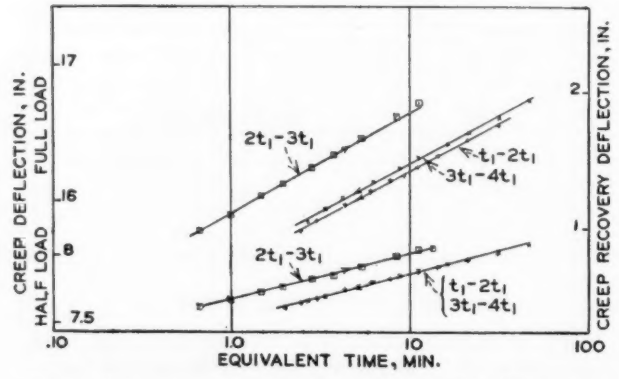


FIG. 6 CREEP AND CREEP RECOVERY, TEST B, CORD MATERIAL

(Tests at full load (15 lb-in.) and half load. In neither case is the first creep stage recorded. The symbols correspond with those in Fig. 3. Time t_1 = 10 min.)

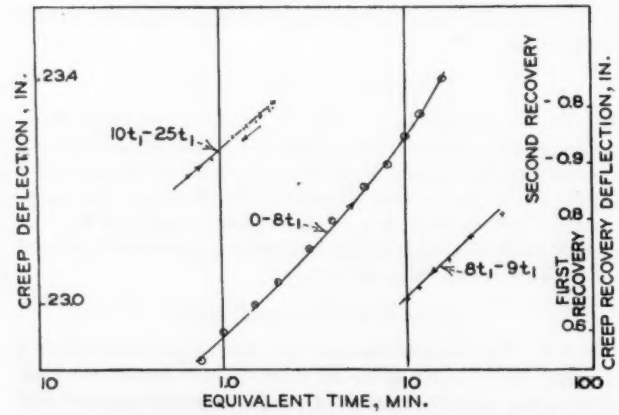


FIG. 7 CREEP AND CREEP RECOVERY, TEST C, BAKELITE RESIN

(The second creep stage is not recorded. The second recovery stage in this test is plotted in Fig. 1 against a linear time base. The symbols correspond with those in Fig. 4. Time t_1 = 2 min.)

Test B. Fig. 6 shows the first and second creep-recovery stages, and the second creep stage for cord material, with 10-min creep and recovery intervals as before. The lower set of lines refers to creep and recovery using one half the load used to obtain the upper set. It is seen that the slope of the upper lines is almost exactly twice that of the lower set, indicating that primary creep is proportional to the applied load for the cylindrical specimen. Therefore it follows the primary creep is proportional to stress, and hence the stress distribution across the cross section of the cylindrical specimen remains linear.

Test C. Fig. 7 shows the result of a test on transparent unfilled resin. The unit of time t_1 in this test was equal to 2 min. The creep recovery during the final recovery stage lies on a straight line when plotted against the logarithm of the equivalent time, and reverses in direction at $12t_1$ or 24 min, as predicted by the Boltzmann law. The creep recovery from $8t_1$ or 16 min to $9t_1$ or 18 min is also shown; this is a parallel straight line, displaced by an amount corresponding to the plastic flow in the intervening creep stage. By comparison the creep recovery from $10t_1$ or 20 min onward is plotted directly against time in Fig. 1.

It is to be concluded that for short-period tests at low stresses the primary creep and creep recovery obey the Boltzmann memory law, and that plastic flow is not a disturbing factor.

STRESS-STRAIN LOOP DUE TO CREEP

Let it be assumed that a cylindrical specimen is loaded in equal steps in reversed torsion, and that one quarter of the maximum load $\pm 4W$ is applied or removed in each step. Each loading increment or decrease takes place in equal intervals of time t_1, and the torsional deflection is observed just before the load change takes place. Due to the action of primary creep, stress-strain loops will be obtained. By means of the Boltzmann law it is possible to calculate the deflection due to creep for each observation. From the slope of the creep lines obtained from such a test as test A, it is then possible to evaluate these deflections. These deflections due to creep subtracted from the observed deflections would reveal whether, in addition to primary creep, elastic hysteresis were present and, if this were so, the area of the elastic-hysteresis loop.

In calculating the stress-strain loop due to creep, a theoretical difficulty arises. In the absence of plastic flow, the creep deflection due to a single load W is given by

$$x = W(b + a \log t) \quad \ldots\ldots\ldots\ldots\ldots\ldots [15]$$

This equation appears to be true for very small values of t, and consequently a value of "immediate elastic deflection" cannot be defined. If however Wb (the deflection at unit time) be called the "time-independent" part of the deflection and $Wa \log t$ the "time-dependent" part, then it is possible to calculate the time-dependent part for any given loading history. If this happens to be negative, it means that the deflection with the given loading history is less than that which would be obtained (of course in the absence of elastic hysteresis and plastic flow) if the same load were applied for unit time. The unit of time in all this work was one minute, though of course the results would not be affected if one second or one hour were used as the time unit.

In the experimental work, a radius arm of 7.5 in. was used; the maximum load of 2 lb was applied in four equal steps of $\frac{1}{2}$ lb

on a specimen $^5/_{16}$ in. in diameter corresponding to a maximum stress range of ±2520 psi. The effective length of the specimen was 2 in. The time intervals between load changes were in different tests $^1/_2$ min, 1 min, and 2 min. Two complete cycles were traced in each test, but the observations of the second cycle only were considered. In this way the effects of plastic flow were probably eliminated.

The procedure was to calculate the creep due to the previous loading history in terms of the current time t, reckoned from the beginning of the experiment, and then to substitute for t. The load W was applied at $t = 0$. The subsequent deflection is given by Equation [15]. Hence the deflection at t_1 is $W(b + a \log t_1)$. At this instant the load was augmented to $2W$. The subsequent deflection is given by

$$W(b + a \log t) + W[b + a \log(t - t_1)] = 2Wb + Wa \log t(t - t_1) \quad [16]$$

Hence deflection at time $2t_1$ is given by

$$2W(b + a \log t_1) + Wa \log 2 \quad [17]$$

At this instant the load was augmented to $3W$, and the subsequent deflection is

$$3Wb + Wa \log t(t - t_1)(t - 2t_1) \quad [18]$$

Deflection at $3t_1$ is therefore

$$3W(b + a \log t_1) + Wa \log 3! \quad [19]$$

Similarly, deflection at $4t_1$ is

$$4W(b + a \log t_1) + Wa \log 4! \quad [20]$$

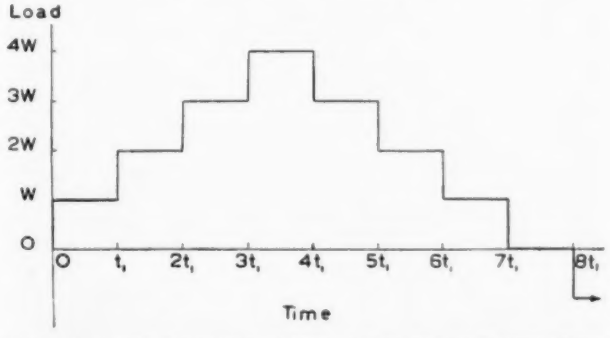


FIG. 8 LOADING DIAGRAM FOR STEP-BY-STEP CYCLIC-LOADING TESTS

Fig. 8 represents part of the loading diagram for the step-by-step cyclic-loading tests. From the diagram it is seen that the deflection from $5t_1$ to $6t_1$ is

$$2Wb + Wa \log \frac{t(t - t_1)(t - 2t_1)(t - 3t_1)}{(t - 4t_1)(t - 5t_1)} \quad [21]$$

Hence the deflection at $6t_1$ is

$$2W(b + a \log t_1) + Wa \log \frac{6!}{2!2!} \quad [22]$$

Similarly, the deflection at $8t_1$ is

$$Wa \log \frac{8!}{4!4!} \quad [23]$$

It is seen that the deflection at any given load station nW consists of a part $nW(b + a \log t_1)$ together with a part $Wa \log F$ where F is a factorial function dependent on the load history. It is seen further that a stress-strain loop due to the action of primary creep has appeared and that the width of this loop is independent of the loading interval; the effect of altering the time interval t_1 is to shear over the loop by an amount proportional to $\log t_1$. It is necessary therefore to consider only the progress of the factorial function F with loading history. At $t = 4t_1, 12t_1, 20t_1, \ldots\ldots$, a discontinuity of the loading process occurs, and hence F changes in nature. At $8t_1, 16t_1, 24t_1, \ldots\ldots$, there is no discontinuity and hence no change in shape in F. From this observation the value of F can be tabulated for the end of each quarter cycle for the first two cycles, as in Table 1.

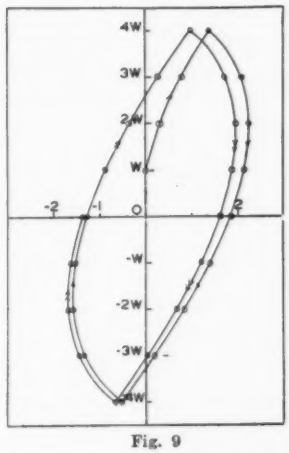


Fig. 9

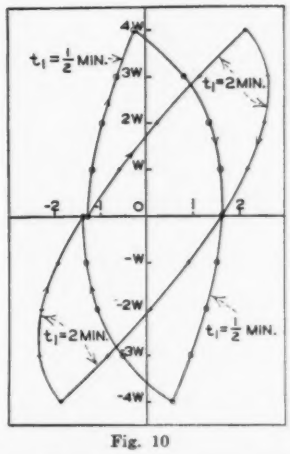


Fig. 10

FIG. 9 TIME-DEPENDENT DEFLECTIONS

(The diagram shows the calculated time-dependent deflections for the first two loading cycles when the loading interval t_1 (Fig. 8) is 1 min. The abscissas, when multiplied by Wa, the slope of the logarithmic-creep plot for load W, give the appropriate creep corrections.)

FIG. 10 TIME-DEPENDENT DEFLECTIONS, SECOND LOADING CYCLE

(The curves give the creep corrections when the loading intervals (Fig. 8) are $^1/_2$ min, and 2 min, respectively.)

TABLE 1 VALUES OF FACTORIAL FUNCTION F

First cycle		Second cycle	
Time	F	Time	F
$4t_1$	$4!$	$20t_1$	$\frac{20!\ 8!\ 8!}{16!\ 16!}$
$8t_1$	$\frac{8!}{4!\ 4!}$	$24t_1$	$\frac{24!\ 12!\ 12!}{20!\ 20!\ 4!\ 4!}$
$12t_1$	$\frac{12!}{8!\ 8!}$	$28t_1$	$\frac{28!\ 16!\ 16!}{24!\ 24!\ 8!\ 8!}$
$16t_1$	$\frac{16!\ 4!\ 4!}{12!\ 12!}$	$32t_1$	$\frac{32!\ 20!\ 20!\ 4!\ 4!}{28!\ 28!\ 12!\ 12!}$

The values of F for intermediate values of t_1 are obtained by using the next succeeding one listed as a model.

Fig. 9 shows the values of $\log F$ for the first two complete cycles. This multiplied by Wa is of course the time-dependent deflection when the loading interval is equal to 1 min. When the load interval is different it is necessary to add an amount $nWa \log t_1$ when the load is nW to obtain the time-dependent deflection. Fig. 10 shows the values of the time-dependent deflections in terms of Wa for the second cycle when $t_1 = ^1/_2$ min and $t_1 = 2$ min, respectively.

For each material, step-by-step cyclic-loading tests of two complete cycles were conducted. For the cord and fabric molding material, three, and for the other two specimens, two tests were made using different values of time interval. For any one material the tests were carried out on the same occasion, with short resting periods between each test; after this a creep test (such as test A) was carried out to determine a, the slope of the logarithmic plot corresponding to $^1/_2$-lb load on the torsion arm. The factor Wa multiplied by the appropriate value of time-dependent deflection recorded in Figs. 9 and 10 gave the actual

values of time-dependent deflections. These deflections when subtracted from the deflections observed in the second cycle of the tests gave the time-independent deflections. These latter may be regarded as the deflections in the absence of primary creep.

Results of Step-by-Step Cyclic-Loading Tests

In all cases after applying the corrections, elastic-hysteresis loops of appreciable magnitude were obtained. The loop areas in different tests on the same specimen showed slight differences; this was attributed to the low over-all accuracy of the tests and to the fact that in practice a small interval of time elapsed between the reading of the deflection and the change of load.

Table 2 gives the observed readings, and these readings corrected for primary creep for the cord material for $t_1 = 1/2$ min, 1 min, and 2 min, respectively; the observed and corrected total strain ranges are also given. In all cases a load of 2 lb on the lever arm corresponds to a torque of 15 lb-in. The observations are in inches of scale with an optical lever arm of 148.4 in. The second cycle only is recorded.

TABLE 2 STRESS-STRAIN LOOPS IN TORSION, SECOND CYCLE

Specimen, cord material, 5/16 in. diam, effective length, 2 in.

$t_1 = 1/2$ min load, lb[a]	Observed loop[b]			Loop corrected for creep[b]		
2		52.81			52.86	
1.5	48.44	49.11		48.56	48.95	
1	44.15	45.20		44.33	44.94	
0.5	39.91	41.16		40.13	40.86	
0	35.73	37.04	35.70	35.96	36.73	35.96
—0.5		32.82	31.53		32.54	31.78
—1		28.53	27.42		28.29	27.63
—1.5		24.07	23.39		23.89	23.48
—2		19.47			19.37	
$t_1 = 1$ min load, lb	Observed loop			Loop corrected for creep		
2		53.04			52.86	
1.5	48.59	49.27		48.54	48.94	
1	44.23	45.32		44.30	44.95	
0.5	39.93	41.24		40.10	40.88	
0	35.68	37.04	35.64	35.92	36.73	35.90
—0.5		32.79	31.41		32.56	31.72
—1		28.41	27.24		28.28	27.56
—1.5		23.89	23.15		23.88	23.42
—2		19.19			19.32	
$t_1 = 2$ min load, lb	Observed loop			Loop corrected for creep		
2		53.28			52.87	
1.5	48.75	49.47		48.53	48.97	
1	44.31	45.48		44.26	45.00	
0.5	39.94	41.34		40.05	40.93	
0	35.63	37.10	35.62	35.87	36.79	35.88
—0.5		32.77	31.32		32.54	31.68
—1		28.32	27.10		28.30	27.53
—1.5		23.73	22.94		23.89	23.38
—2		18.93			19.28	

Total strain ranges

Time interval, min	Observed loop[b]	Corrected loop[b]
1/2	33.34	33.49
1	33.85	33.54
2	34.35	33.59

[a] Applied on lever arm of 7.5 in.
[b] The observations are in inches of scale. The twist in radians is equal to the scale observation divided by the optical lever arm of 148.4 in.

TABLE 3 MEAN SPECIFIC DAMPING CAPACITY IN TORSION

Material		Computed specific damping capacity, per cent —Time interval, min— 1/2	1	2	Mean
Cord	Creep	22.9	22.9	22.9	
	Hysteresis	15.0	16.1	17.3	
	Total	37.9	39.0	40.2	39.0
Fabric molding	Creep	18.9	18.9	18.9	
	Hysteresis	13.4	15.0	16.0	
	Total	32.3	33.9	34.9	33.7
Wood-flour material	Creep	14.4	14.4	....	
	Hysteresis	18.9	18.9	....	
	Total	33.3	33.3	....	33.3
Bakelite resin	Creep		6.5	6.5	
	Hysteresis		3.5	3.9	
	Total		10.0	10.4	10.2

It is seen that the theory accounts for most of the discrepancy between the observed total strain ranges in the different tests, and this can be regarded as a partial check on the theory. It is also observed that the corrected observations are nearly the same in the three tests. Similar results were obtained in the tests on the other materials.

Computed Specific Damping Capacity. From the slope of the plot of creep against the logarithm of the equivalent time the contribution of primary creep to damping was calculated by means of Equation [10]. From the area of the hysteresis loop (obtained from the step-by-step observations corrected for creep) the contribution of elastic hysteresis to damping at a surface shear stress of ± 2520 psi was calculated by means of Equation [1]. Table 3 gives the computed mean specific damping capacity in torsion for circular cylinders of the four specimens when the surface shear stress is equal to 2520 psi.

Direct Determination of Damping

At the conclusion of the step-by-step cyclic-loading tests, the damping test for each material was carried out. The damping was determined by the decrement of free oscillations, using the loading lever as the inertia bar. Fig. 11 shows typical damping

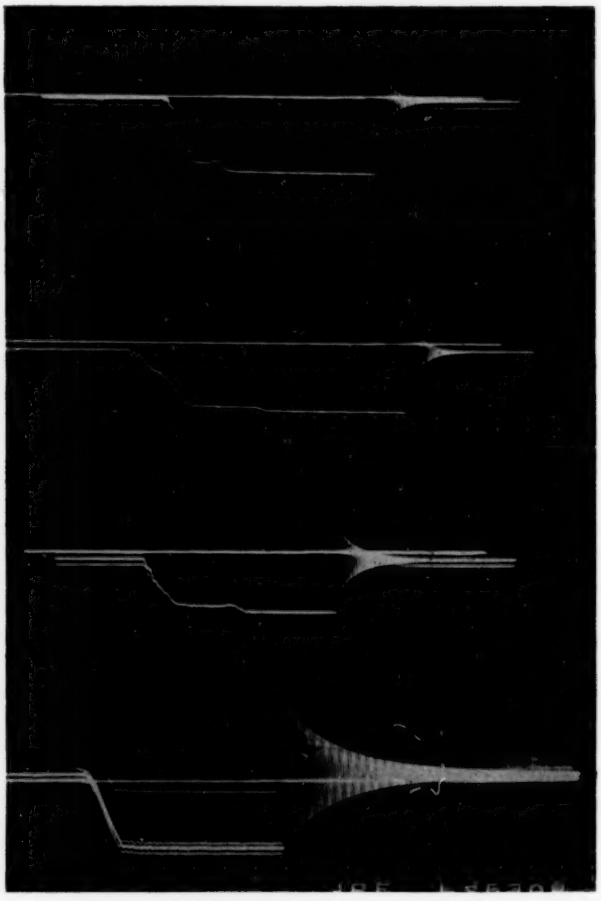


Fig. 11 Decrement of Free Torsional Oscillations
From top to bottom: Cord material
Fabric molding material
Wood-flour material
Bakelite resin, unfilled

records. From the base line of the record the deflection corresponding to the applied load was obtained, and hence the stress corresponding to any given deflection on the film record. The specific damping capacity was computed on the assumption that it was equal to twice the logarithmic decrement at any given stress. Though this assumption is true for small values of

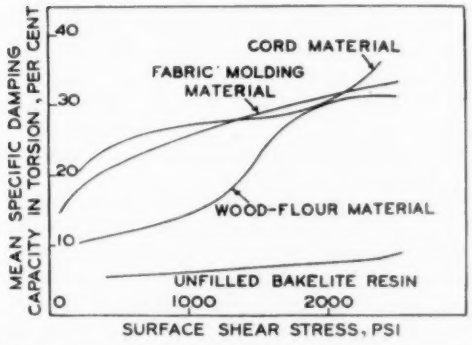


FIG. 12 SPECIFIC DAMPING CAPACITY AND STRESS

damping, it is liable to be in error for the higher values recorded. Fig. 12 shows the relation between the mean specific damping capacity in torsion ψ_m and surface shear stress for the four materials. In all cases the curves were very nearly reproducible. Table 4 shows the values obtained from the curves at a surface shear stress of 2520 psi, compared to the mean values obtained in the static tests.

TABLE 4 MEAN SPECIFIC DAMPING CAPACITY IN TORSION, PER CENT

Material	Cord	Fabric molding	Wood flour	Bakelite resin
Dynamic test	36.6	33.2	31.1	8.9
Static test (mean)	39.0	33.7	33.3	10.2

Though the agreement between the dynamic and the static tests is only fair, it is probable that a more carefully designed apparatus would give better agreement, especially if it were designed to measure, not the specific damping capacity, but the actual energy dissipated per cycle. Under these circumstances, the comparison of the static with the dynamic tests would involve no difficulties in interpretation of the data.

ACKNOWLEDGMENTS

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Design Data

It is important that the data contained in technical papers be made readily available to designing engineers. In order to satisfy these needs of industry, this section of the JOURNAL will include a concise presentation of data and information drawn chiefly from papers previously published by the Applied Mechanics Division of The American Society of Mechanical Engineers.

The first series dealt with the subject "Strength of Materials" and the present series will cover the subject "Vibration." The data have been prepared by a subcommittee of the Applied Mechanics Division (J. Ormondroyd, chairman), under the general guidance of S. Timoshenko.

The first article, which follows, is by J. Ormondroyd. This treats of the systems with one degree of freedom and gives a list of symbols and definitions which will be adopted in future articles of the series.

Vibration Problems

Systems of One Degree of Freedom and Basic Symbols and Definitions

Part I

By J. ORMONDROYD,[1] ANN ARBOR, MICH.

THE problem of vibration in machines and structures of engineering interest is merely a special problem in dynamics. If the motion of every particle in the system could be specified for every instant, then a great number of facts of technical interest could be deduced. For instance, the distribution of stresses in the system is known when the motions of all the particles are specified, and the possibilities of kinematic interference are indicated by the same data.

The number of independent coordinates needed to specify completely the configuration of any mechanical system at any instant gives the number of degrees of freedom of the system. From this it follows that if one wishes to describe accurately the motion of every particle in any system, the number of degrees of freedom is infinite. However, for practical purposes, it is possible to imagine systems approximately equivalent dynamically to the system under consideration which have a small number of degrees of freedom. The criteria used to determine how many degrees of freedom to ascribe to any system under discussion are practical in nature. For instance, some, or most, of the possible motions in the system may be so small that they are not of practical interest. Some or most of the motions of the particles in the system may be practically similar permitting such particles to be lumped into a single body which can be considered to be rigid. The range of frequencies of the disturbing forces or torques may be so small that only one, or at most a few, of the natural frequencies of the system can ever be involved in resonance. All groups of particles in the system which cannot possibly undergo significantly large distortions under the magnitude and frequencies of the applied forces or moments may be considered single bodies, thereby reducing the number of degrees of freedom necessary to consider. These practical considerations which permit the creation, in imagination, of systems of approximate dynamical similarity to the actual systems which concern the engineer lead to the concepts of lumped masses which are absolutely rigid—connected by members which are flexible but have no mass. It is with such systems that most of the design-data papers to be published will deal. These systems give first approximations to the actual motions. Usually the motions predicted by using such approximate systems are close enough to the actual motions to satisf y all practical demands.

In some systems a second approximation can be found by taking into account the mass which the elastic members obviously possess. This is necessary only when the flexible members have masses which are appreciable in magnitude in relationship to the masses of the parts of the system which are considered as rigid bodies.

Finally there are many systems of practical interest which have such simple shapes that they can be considered as systems possessing an infinite number of degrees of freedom. For these there are the conceptual and mathematical tools which make it possible to do so. The vibration of strings, simple beams, plates, shells, and infinite and finite bodies of gases and liquids all come in this class.

SYSTEMS OF ONE DEGREE OF FREEDOM

A surprisingly large number of practical vibration problems which arise in the mechanisms and structures designed by engineers can be treated with a sufficiently high degree of accuracy by imagining the actual system to consist of a single rigid body, the motion of which can be described by a single coordinate. In reality the simplest imaginable system always consists of two bodies—the body the motion of which is of interest and the immediate surroundings of this body. It is assumed that the surrounding body is so large and rigid that the forces acting on the first body which react on the surrounding body do not move or distort the surrounding body. The motions measured in the first body are measured relative to the fixed surrounding body.

The problem of treating such a simplified system is fourfold. The first part consists in deciding what part of the system is the rigid body and what parts are the flexible members. The second part consists in calculating the values of the dynamic specifications of the rigid body and flexible parts. The third part consists in writing the equations of motion of the equivalent system. Finally, the fourth part consists in solving the equations. The first two steps require judgment and experience which come with practice, that is, with the repeated process of assuming equivalent systems, predicting their motions and checking the pre-

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dictions against actual measurements on the real systems. The last two steps merely consist in applying procedures which were worked out long ago by physicists and mathematicians. The real work lies in the first two stages, while the last two stages are mere mechanical applications of recipes, so to speak.

The first two stages can never be avoided or changed, but the last two stages can be varied. For instance, there are many approximate methods, numerical and graphical in nature, which can be used in the place of the classical procedure previously outlined. Methods using the kinetic and potential energies of the system are frequently used in the place of the last two stages.

Before proceeding to any detailed useful results, a table of symbols which will be used in the subsequent papers is given. A list of definitions which are necessary in treating vibrations is also appended.

Nomenclature

Quantity	Symbols	Units
Force	P, X, Y, Z	lb
Moment	M, T, M_x, M_y, M_z	lb-in.
Amplitude (linear)	A, B, x_0, y_0, z_0	in.
Amplitude (torsional)	$A, B, \theta_0, \varphi_0$	radians, deg
Circular frequency	ω	radians per sec
Natural circular frequency	p	radians per sec
Frequency	f	cycles per sec
Natural frequency	f_c	cycles per sec
Period	T	sec per cycle
Cycles per second	$\sim$	cycles per sec
Wave length	λ	in.
Wave velocity	v	in. per sec
Linear displacement	x, y, z	in.
Linear velocity	$v, \dot{x}, \dot{y}, \dot{z}$	in. per sec
Linear acceleration	$a, \ddot{x}, \ddot{y}, \ddot{z}$	in. sec^{-2}
Torsional displacement	$\theta, \varphi, \alpha, \beta, \gamma$	radians
Torsional velocity	$\omega, \dot{\theta}, \dot{\varphi}, \dot{\alpha}, \dot{\beta}, \dot{\gamma}$	radians per sec
Torsional acceleration	$\alpha, \ddot{\theta}, \ddot{\varphi}, \ddot{\alpha}, \ddot{\beta}, \ddot{\gamma}$	radians sec^{-2}
Mass	m	lb sec^2 in.$^{-1}$
Mass density	ρ	lb sec^2 in.$^{-4}$
Weight per unit volume	γ	lb in.$^{-3}$
Spring constant (linear)	k	lb per in.
Spring constant (torsional)	k	lb-in. per radian
Mass moment of inertia	I	lb in. sec^2
Moment of inertia (geometrical)	I	in.^{4}
Polar moment of inertia (geometrical)	J	in.^{4}
Damping constant (linear)	c	lb sec in.$^{-1}$
Damping constant (torsional)	c	lb in. sec
Critical damping constant (linear)	c_c	lb sec in.$^{-1}$
Critical damping constant (torsional)	c_c	lb in. sec
Logarithmic decrement	δ	
Harmonic number	n	
Order number	m	cycles per rev
Radius of gyration	k	in.
Acceleration of gravity	g	in. sec^{-2}
Static deflection	δ_{st} or Δ	in.

Definitions

Note: In these definitions the motion of a particle has usually been specifically in mind. However, the definitions hold also for torsional motion if "angular rotation" is substituted for motion and "rotating body" is substituted for particle or point.

Simple Harmonic Motion. Any motion which can be described by an equation similar to $x = A \cos \omega t + B \sin \omega t$ is simple harmonic motion. The displacement x described by this equation is best imagined as being represented by the motion of the end

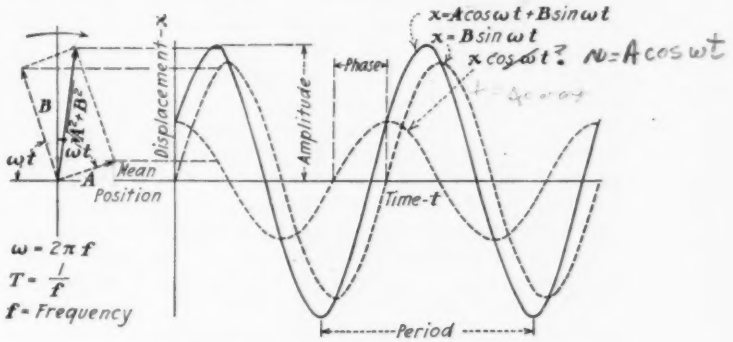


Fig. 1 Simple Harmonic Motion

of a rotating vector projected on the x-axis. The length of the vector is $(A^2 + B^2)^{1/2}$ and it is rotating with a constant angular velocity of ω radians per sec.

Cycle. Any simple harmonic motion repeats itself in every respect at regular and constant time intervals. This is true no matter where the time intervals are started. One complete progression of all the possible positions of the moving particle is called a cycle.

Period. The time interval taken to complete one cycle of motion is the period.

Frequency. The number of complete cycles of motion per unit of time is the frequency. This is the reciprocal of the period.

Amplitude. The extreme distance from the mean or middle position in any simple harmonic motion is the amplitude.

Range. Distance between positive and negative extreme positions—distance from crest to trough—is the range. Sometimes range is called *wave height* and *double amplitude.*

Phase. The phase of a simple harmonic motion at any instant is the fraction of the whole period which has elapsed since the moving point last passed through its mean or middle position in the positive direction. The phase of the motion at any time t, measured from the positive passage through the mean position, is t/T.

Phase Angle. Two simple harmonic motions having equal frequencies can be said to have a phase relationship if one wave leads the other in time. The lead is measured by the difference in time τ between the positive passages of both waves through their mean positions. The phase angle between the two waves is then $\omega\tau$.

Periodic Motion. The displacement x of a particle from a mean position is periodic if its magnitudes repeat themselves for values of time t differing by a constant. Simple harmonic motion is the simplest possible periodic motion.

Harmonics. All periodic motions which are not simple harmonic motions can be considered as combinations of simple harmonic motions having frequencies which are exact integer multiples of the fundamental cyclic frequency of the periodic motion. Such motion can be described by equations of the form

$$x = A_0 + \Sigma A_n \cos n\,\omega t + \Sigma B_n \sin n\,\omega t$$

Here ω is the angular velocity of the vector whose projection represents the wave of fundamental cyclic frequency, n is the harmonic number, $n\omega$ is the angular velocity of the vector whose projection represents the wave of the nth harmonic; n does not necessarily have to take on all of the integer values from 1 to ∞, nor do the values of n have to be contiguous integers.

Beats. When two simple harmonic motions with frequencies which are nearly equal combine, the resultant motion gives rise to the phenomenon of "beats." The resultant motion may or may not be strictly periodic depending on whether the frequencies are commensurable or incommensurable. No matter which of these cases exists, the envelope of the motion varies from a maximum to a minimum in a cycle which has a definite period $T = T_1T_2/(T_2 - T_1)$. The envelope of the motion is a periodic function with a frequency $f = f_2 - f_1$.

Damped Simple Harmonic Motion. This motion has all the characteristics of simple harmonic motion except that the amplitude is variable and is a continuous function of time. The amplitude may be continuously decreasing or increasing depending on whether the damping is positive or negative.

Logarithmic Decrement. The logarithm of the ratio of any two successive excursions from the mean position of any damped simple harmonic motion taken at a time interval equal to the period of the motion is the logarithmic decrement. Usually it is taken as the logarithm of the ratio of two successive maximum excursions from the mean position.

Aperiodic Motion. Motion which takes place in one direction only is aperiodic motion.

Vibration. Any periodic motion is a vibration. Frequently vibration is defined as a small periodic motion. But "small" used in this sense is a relative term. In a vibrating system a "small" relative motion exists between the various parts of the system when the motions have maximum excursions from their mean positions not exceeding a small percentage of the important linear dimensions of the system. The motion is also "small" when the proportional limits in tension, compression, or shear stresses are not exceeded in the elastic members of the system during the motion.

Free Vibration. Vibration which takes place when no external force is acting on the system. If the system is distorted from equilibrium configuration and then released, it will vibrate with free vibration. If any part of the system is struck a blow, the system will vibrate freely. Free vibration exists when the forces acting on the system arise solely from motion of the system itself. The frequencies of the free vibrations are fixed functions of the mass, elastic, and damping properties of the system itself. They are called natural frequencies.

Forced Vibration. Vibration which takes place under the excitation of external forces is forced vibration. External forces in any system are forces which have their reactions acting on bodies which are not parts of the system isolated for study.

Transient State. Whenever any changes of the mass or elastic properties in the system or in the magnitude or the frequency of the external exciting force on the system take place, a free vibration ensues which is superimposed on the existing state of motion. In all actual systems this free vibration gradually dies out because of damping. During the existence of the free vibrations having the natural frequencies of the system, the system is said to be in a "transient state" of motion.

Steady State. Forced vibration which takes place when neither the system itself nor the exciting forces are changing and after any transient states have died out or before they have started.

Natural Frequency. The frequencies of any free vibrations of a system that has been disturbed from its equilibrium position and then left undisturbed by any external force are called natural frequencies. For any particular system they have definite constant values.

Resonance. When a system is excited by a periodic external force which has a frequency equal to or nearly equal to a natural frequency of the system, the ensuing vibratory motion becomes relatively large even for small amplitudes of disturbing force. The system then is in a state of resonance.

Critical Speed. Any speed of operation of a machine at which the periodic disturbing forces inherent in the operation of the machine give rise to the largest or most dangerous vibratory responses in the parts of the machine. A critical speed exists when the frequency of the disturbing force is equal to or nearly equal to a natural frequency of the machine.

Wave Length. When regular trains of waves exist in some continuous medium, the shortest distance between contiguous wave peaks is called the wave length.

Wave Velocity. When waves move in some continuous medium (sound waves in air), the velocity of advance of any particular wave peak is the wave velocity.

Order. In many rotating machines the vibration frequencies encountered are simple multiples of the rotation frequency. The number of complete cycles of vibration per revolution of the machine is called the "order" of the vibration.

Major Critical Speed. A major torsional critical speed occurs in a multicylinder engine whenever the harmonic torques of the same frequency acting at the different cylinders are in phase. The harmonic torques with frequencies which are multiples of the firing frequency give rise to major critical speeds. Major critical speeds cannot be avoided or modified by changing the engine firing order.

Minor Critical Speed. A minor critical speed occurs in a multicylinder engine whenever the harmonic torques of the same frequency acting at the different cylinders are out of phase. The harmonic torques with frequencies which are not multiples of the firing frequency give rise to minor critical speeds. Alteration of the firing order can sometimes change the magnitude of motion in a minor critical speed.

"Major" and "minor" are somewhat misleading terms since they do not necessarily indicate the relative violence of motions in all cases. Minor critical speeds can be just as dangerous as major critical speeds.

Node. A point (line, plane, volume) in a vibrating system which has no vibrational motion. The node moves at the average velocity of the system and does not take part in the periodically changing motion.

Loop. A point (line, plane, volume) in a vibrating system which undergoes maximum motion. There may be several loops in the system. A loop is contrasted to a node. A loop moves with both the average velocity of the system and the vibratory velocity superimposed.

Mode of Motion. The mode of a vibration is characterized by the number of nodes which exist in the system at the various natural frequencies. For instance, the "two noded mode of motion" exists at resonance with that natural frequency which gives rise to two nodal points (lines, planes, volumes) in the system.

Discussion

The Effect of the Speed of Stretching and the Rate of Loading on Yielding of Mild Steel[1]

R. K. Bernhard.[2] This paper describes "the effect of the speed of stretching and the rate of loading on the yielding of mild steel," mainly from the point of view of the material itself. It might be of interest to treat the subject more dynamically,[3] in other words, discuss the change of equilibrium between specimen and testing machine or natural frequency of test machines and accuracy of indication in high-speed tests with respect to the determination of yield points.

In high-speed testing of essentially the same material, different results may be obtained depending upon whether the material is tested in a massive or in a light testing machine. Furthermore, the stress-strain diagrams of certain materials indicate peculiar forms near the yield point, which in some cases, may not be dependent upon the qualities of the tested material alone.

(1) *Static System.* Any testing machine, for example, with a hydraulic cylinder, may be considered as a system composed of masses and springs. Masses are the frame of the machine (the crosshead, etc.) while springs with a definite elastic constant represent the test specimen and the hydraulic medium. At any moment there must be equilibrium between the summation of the elastic forces of the springs and the load on the ram of the machine.

In the case of a testing machine having a low elastic constant, a very small decrease of load will result with the increase of strain; a high elastic constant, on the other hand, will result in a considerable decrease in load with increase of strain. When plastic flow ceases, equilibrium between the two elastic forces (specimen and pressure medium) may be restored and the point of equilibrium again rises as soon as the pressure pump starts to force new liquid into the cylinder. It must be borne in mind, however, that with decreasing tension in the specimen, the plastic flow will also cease.

Any rapid flow gradually changes into some kind of creeping flow before equilibrium is finally achieved. Hence, such drop in the load-deformation diagram, due to plastic flow, may be independent, in certain cases, of the quality of the tested materials, in high-speed testing.

(2) *Dynamic System.* The plastic flow or any pressure changes in the cylinder may take place very suddenly and the indicating device of the testing machine must follow these rapid changes immediately in both coordinates, load and deformation. It is well known that as soon as rapid changes come into consideration, the accuracy of any indicating device depends on the natural period of the whole vibrating system.

Any fluctuations during the test having a frequency of more than one tenth of the frequency of the machine will cause incorrect indications, assuming for simplicity that only vibrations of a sine form with a small damping factor come into consideration. This indicates also that different values for the upper and lower yield points, independent of the quality of the tested material and due to rapid changes of plastic flow near to these yield points, may be found in certain cases of high-speed testing.

(3) *Effect Upon Yield-Point Determinations.* The surplus in tension at the upper yield point is some kind of delayed phenomenon. Once plastic flow of the material in the specimen has started, the further shape of the diagram depends, at least to a certain amount, upon the character of the test machine. If the elastic constant of the test machine is low, any lower yield point will be suppressed, as the elongation of the test specimen produced by plastic flow means no decrease in tension for the testing machine itself. A very high elastic constant of the testing machine, however, will produce the well-known fluctuations (leaping effect).

A similar result will be produced if the test specimen is long and has a low elastic constant, thus causing a heavy drop in the diagram. Consequently, a very short and rigid specimen may indicate no lower yield point at all.

(4) *Conclusions.* The conclusion is accordingly reached that the upper and lower yield points are associated, also, with the characteristics of the testing machine and are to a certain degree independent of the quality of the material, and that the form of the specimen may be in *specific* cases of high-speed testing, secondary.

A. V. de Forest.[4] The method of test chosen by the author prevents any possibility of investigating the lower yield point but gives significant information as to the effect of rate of loading under relatively slow loads. Unfortunately, it appears that the upper yield point is largely determined by test conditions and is thus difficult to appraise as a true mechanical property of the material itself. The author has emphasized the lack of information as to the actual gage length over which the yield strain is taking place, a most important point. It is to be hoped that some method of measurement can be developed to clarify this point.

A reference is made to the stress-concentrating effect of the notch existing at the junction of the strained and unstrained metal. While there is a mechanical notch at this point, it is believed that the real notch effect is a metallurgical one rather than a mechanical one and that the flowing layer between the stressed and unstressed metal is the crux of the problem. The character of this layer approximates the out-moded Beilby layer of viscous fluid. The strength of the layer is greatly influenced by the rate of deformation, the temperature of the test piece which itself is a function of the rate of deformation, and hardens or sets at a rate greatly dependent on the particular quality of steel involved. In other words, the rate of strain aging is the most important part of the phenomenon.

One of the convolutions of curve *F*, Fig. 4, illustrates discontinuities in the stress-strain diagram which have been noticed in many other cases. It is believed to be important as showing that, while the singular yield point in steel is the one primarily of interest to engineers, any explanation of the yield-point phenomenon must also be capable of explaining diagrams in which a multiplicity of yield points are found throughout the flow portions of the diagram.

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[1] By E. A. Davis, published in the December, 1938, issue of the Journal of Applied Mechanics, Trans. A.S.M.E., vol. 5, 1938, p. A-137.

[2] Professor and Head of Department of Mechanics and Materials of Construction, The Pennsylvania State College, State College, Pa.

[3] "Einfluss der Federung der Zerreissmaschine auf das Spannungs-Dehnungs-Schaubild," by W. Späth, *Archiv für das Eisenhüttenwesen*, No. 6, December, 1935, pp. 277–283.

[4] Professor, Massachusetts Institute of Technology, Cambridge, Mass. Mem. A.S.M.E.

JOSEPH WINLOCK[5] AND R. W. E. LEITER.[6] The author lays great emphasis on the effect of a concentration of stress and a resulting high strain rate. It is agreed that this is of great importance, but not that he is justified in attributing the cause of the phenomena simply to a concentration of stress. Are there not similar stress concentrations in a tensile specimen of aluminum or copper? Yet no similar yield-point phenomena are manifested by these metals. Again, if a high concentration of stress is produced during a tension test of aluminum or copper by

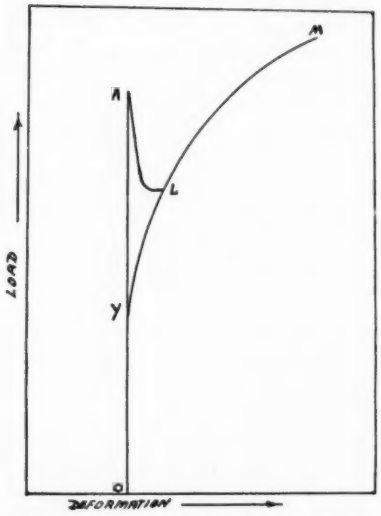


FIG. 1 LOAD-DEFORMATION CURVES FOR METALS WHICH DO AND DO NOT SHOW YIELD-POINT PHENOMENA

suddenly putting a notch in the specimen, a drop in the registered load occurs, but there is no yield-point elongation; that is, a localized elongation which widens until the whole gage length is affected does not occur under these circumstances.

The writers believe that the inordinately high elastic resistance of low-carbon steel, before plastic deformation is initiated, and the comparatively low resistance to plastic deformation, once this has started, are the causes of the phenomena; and that this conception is essential in order to obtain a true picture of what is occurring at the yield point and during the yield-point elongation. Such a conception appears to be lacking in the author's paper.

In order to make this point clear, the writers would like to refer to Fig. 1 of this discussion. The curve *OYLM*, Fig. 1, is the load-deformation curve of metals like aluminum and copper which do not show the yield-point phenomena. As will be noted, the curve is smooth even though there undoubtedly are concentrations of stress during the deformation. On the other hand, low-carbon steel does not yield at the load *Y*, but overshoots this load and yields only after the excessively high load *A* has been reached. (The difference in modulus of low-carbon steel and aluminum and copper is omitted for the sake of clearness). At the instant yielding occurs, the resistance of the metal is believed to be only the load *Y*. As a result of this, an extremely localized yielding occurs accompanied by a very high strain rate which produces a high concentration of stress in the metal immediately adjacent to the stretcher strain. The remaining portion of the load-deformation curve takes the shape shown by *ALM* simply as the result of the several conflicting conditions now existing: (1) The high elastic strength of those parts of the specimen not yet plastically deformed; (2) the low resistance of the metal at the instant plastic deformation occurs; and (3) the work strengthening of the plastically deformed metal.

As the author points out (in conclusion 6) the rate of work strengthening during the yield-point elongation is unknown, but, as the writers point out,[7] a continuation of the curve *CD* in Fig. 2 of his paper seems to us to be justified. The reasons for this conclusion are: (1) The slower the speed at which the test is carried out, the shorter the yield-point elongation and the lower the load at which the horizontal portion of the load-deformation occurs. (There is, however, a yield-point elongation at even the lowest possible speed of deformation.) At decreasing testing speeds, a continuation of the curve *CD* approaches the elastic portion of the curve, and even though its position is lowered, its general shape is not changed. (2) Hardness readings taken at the bottom of Lüders lines formed at different speeds show the hardness to be increasing in a normal manner and the shape of the curve is similar to that of *CD*. (3) An extremely slight amount of cold rolling removes the yield-point phenomena entirely and produces a condition of plasticity throughout the specimen. Test specimens taken from such material show a low yield point, i.e., the load *Y* in Fig. 1 in this discussion, and a load deformation curve like that of *OYLM*.

AUTHOR'S CLOSURE

As Professor Bernhard points out, the effect of the rigidity of the testing machine upon the resulting upper and lower yield points is of great importance. The work of Welter, Siebel, and others listed in references (6, 7, 8) of the paper shows this quite well. However, it seems evident that changing the elasticity factor of the testing machine can only change the rate of loading in the elastic range and the speed of deformation after yielding starts. The point, which the author would like to make, is that, when the speed of deformation and the stress concentrations are taken into account, the differences due to the elasticity factor of the testing machine tend to disappear.

In the constant-load-rate tests reported in the paper, the vibrations and inertia forces were small and hence were neglected. In the most rapid tests, the large moving weight dropped approximately two inches in one second during the yield-point elongation so that the acceleration was small compared to that of gravity.

Two statements of Professor Bernhard seem to conflict with each other: First, he says that, in a machine having a low elastic constant, a very small decrease in load will result from a sudden increase in strain while, in a machine having a high elastic constant, there will be a considerable drop in the load with a similar increase in strain. Later, he states that, if the test specimen is long so that it has a low elastic constant, there will be a heavy drop in the diagram. If the specimen is short and rigid there may be no indication of a lower yield point at all. The first statement seems logical and is in agreement with the results of the workers to which reference has already been made. If this is the case, however, it is difficult to understand how the latter statement can also be true. How can a low elastic constant in a test specimen cause a large drop in load at the yield point, when a low elastic constant in the testing machine tends to lessen this drop of load? Of course, if the properties of mild steel were such that a sudden increase in strain occurred simultaneously throughout the length of the test specimen, then a long bar would probably show a greater drop in load than a short bar. Actually, mild steel deforms locally and the elastic deflection of the remaining elastic portion of the long bar would have the same effect as elastic deflections in the testing machine. Therefore, a specimen with a

[5] Chief Metallurgist, Edward G. Budd Manufacturing Company, Philadelphia, Pa.

[6] Assistant Metallurgist, Edward G. Budd Manufacturing Company, Philadelphia, Pa.

[7] "Some Observations on the Yield-Point of Low-Carbon Steel," by Joseph Winlock and R. W. E. Leiter, presented at 1938 Annual Meeting AMERICAN SOCIETY OF MECHANICAL ENGINEERS.

low elastic constant should cause a smaller or, to be more exact, a less rapid drop in the load.

Professor de Forest brings out the fact that the time t may become an important variable in describing the properties of mild steel. This becomes necessary whenever changes, due to time alone, such as age hardening or annealing take place. Usually, when the effect of strain aging appears in the stress-strain diagram of mild steel, it shows up in that portion of the curve beyond the yield-point elongation. While the localized yielding is spreading along the length of the bar, the time that any given element of material is taking part in the yielding is rather short. Any aging, which might affect the yield point or yield-point elongation, would have to take place during the time that the wave of yielding was sweeping over the element in question. This time, although short, may still be important. Part of the yield-point phenomena may yet be explained in terms of strain aging.

With regard to the discussion by Messrs. Winlock and Leiter, concerning the cause of the yield-point phenomena, it may be well to recall that the intention of the paper was to discuss the influence of certain mechanical factors upon the yield point and in particular to show how they affect the stress-strain diagram. The reason behind the localized yielding in the material was not discussed and is not known so far as the author is concerned. It is granted that mild steel is weaker in the initial plastic region than in the elastic region, but the phenomena will probably not be fully understood until it is known what kind of a breakdown occurs which prohibits a gradual transition from the elastic to the plastic region.

In conclusion, the author would like to emphasize a few points on the ever-present question of whether the yield-point phenomena in mild steel are properties of the material or of the testing machine. Localized yielding and the apparent instability of the initial plastic flow definitely are properties of the material itself. This type of yielding occurs regardless of whether the load drops or increases. The shape of the standard recorded load-mean elongation curve (to be distinguished from the real stress-strain diagram) and the manner in which the load behaves after yielding starts are due largely to the characteristics of the testing machine, as has been shown by several experimenters. The reason why one material can accommodate itself to differently shaped stress-strain diagrams is that the actual stress in the yielding layers is not a function of strain alone but also depends upon the speed of deformation and possibly upon a time factor.

Steady Oscillations of Systems With Nonlinear and Unsymmetrical Elasticity[1]

R. K. Bernhard.[2] The amplitude-frequency curve as shown in Fig. 28(*a*) has been derived mathematically by the author and fills the gap between theory and practice in an ingenious way.

Curves with the same characteristic form have been obtained by actual measurements, indicating the presence of nonharmonic mechanical vibrations. Two examples of such curves may supplement the author's paper and emphasize the practical importance of analogous states of unstable equilibrium in the field of the structural engineer. Investigations have been made by the writer to determine the effect of unbalanced parts of counterweights of driver axles of steam locomotives passing bridges at critical speed.

[1] By Manfred Rauscher. Published in the December, 1938, issue of the Journal of Applied Mechanics, Trans. A.S.M.E., vol. 5, 1938, p. A-169.

[2] Professor and Head of Department of Mechanics and Materials of Construction, The Pennsylvania State College, State College, Pa.

(1) For a simple girder a series of resonance curves, Fig. 1, has been measured, showing the relation between discord (i.e., the ratio between exciting frequency and natural frequency of the girder) versus the corresponding excited amplitudes of the girder for varying magnitudes of the exciting forces. The lowest curve No. 1 is measured with only an exciting force of the unit 0.25, in order to produce a maximum amplitude within the range of a linear spring constant and a constant friction. Hence, this particular curve No. 1 still has a normal form. However, the next curve No. 2 shows a slight tilting effect, which appears to be substantially increased in curve No. 3 measured for an exciting force with the unit 1.0. The considerable increase in damping is shown by the wider gap at the top of curve No. 3. This phenomenon of increased damping effect is still more striking in

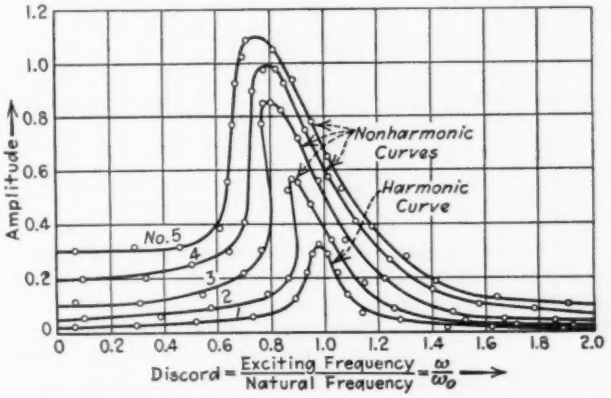


Fig. 1 Resonance Curves for Nonharmonic Vibrations (Amplitude versus discord for varying magnitudes of the exciting forces.)

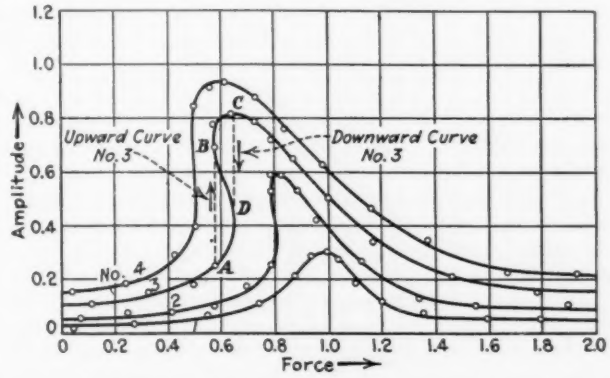


Fig. 2 Resonance Curves for Nonharmonic Vibrations (Amplitude versus force for constant exciting frequency.)

curve No. 4. Furthermore, the maximum amplitudes have increased only a relatively small amount, though the exciting force has double the size as shown in the increase of static amplitude from 0.1 to 0.2 (discord $\frac{\omega}{\omega_0} = 0$). The tilting of the three curves Nos. 1, 2, and 3 in the direction of lower frequencies is a characteristic sign of nonharmonic vibrations. Finally, in curve No. 5 this tilting effect gradually disappears, caused by the relatively larger values of the damping, which simultaneously prevents the building up of higher amplitudes. However, the broadening of the range of resonance, combined with the steep incline in the rising branch, produces an unsymmetrical shape, also, of curve No. 5.

(2) In Fig. 2, a series of similar resonance curves for a simple girder has been measured, showing, however, the relation be-

tween exciting force versus the amplitudes of the girder. Furthermore, a constant exciting frequency has been applied, imitating a constant speed of the passing loads (locomotives). The force = 0 means that only the elastic force of the vibrating girder is acting. All curves show analogous forms as obtained in Fig. 1. Again, a certain range of curves, Nos. 2 and 3, shows a pronounced tilting effect. With rising forces, an abrupt step-up from small to higher amplitudes takes place, indicating a state of unstable equilibrium (see curve No. 3, dotted line *A–B*); with decreasing forces, a similar abrupt step-down from high to low amplitudes takes place, however, at a point of larger forces (see curve No. 3, dotted line *C–D*) as compared with the step-up.

All these curves have been measured in applying the method of induced vibration, which has been described in previously published papers.[3]

Results. Some of the conclusions which can be drawn from these investigations may be summarized as below. The writer intends to give a more detailed report on this particular subject in a subsequent paper.

1 The tilting effect, as indicated in the amplitude-discord and amplitude-force resonance curves for vibrating structures shows obviously that nonharmonic vibrations are present in practical cases.

2 The nonharmonic vibrations cause an abrupt jump in the magnitude of the deflections of the structures, due to nonlinear elasticity and nonlinear damping.

3 A new structure, with still rigid riveted connections, shows a relatively small nonharmonic effect and hence, will vibrate smoothly; an older, worn-out structure, with more or less loosened connections, will produce the tilting effect, and hence, will endure higher dynamic impact stresses.

4 When computing the dynamic influence on bridges, which carry loads of passing steam locomotives with unbalanced parts of their counterweights, the classic theory of pure elasticity will yield only a relatively rough approximation.

R. P. Kroon.[4] Free oscillations of nonlinear systems without damping can be found by simple integration. The author proposes to take care of forced and damped vibrations by lumping the exciting forces and friction forces together with the spring force into a general restoring force. Therewith the problem is reduced to that of an artificial free vibration.

The exciting force being a function of time and the friction force depending on velocity, some way has to be found to transform these into functions of displacement like the spring force. To do this, it is proposed to assume for a first approximation that "the displacement curves of the free motions and that of the forced motion at the same amplitude are identical" and that "the frequency of the force is the same as that of the free motion."

These assumptions are obviously good for forced oscillations at resonance (where exciting and damping forces are small compared with elastic forces and inertia forces which govern the free vibration). How well they work in other cases is indeed remarkable. The writer cannot resist demonstrating this for the case of a simple linear undamped system.

In a linear system with one degree of freedom x, the elastic force can be written as

$$E = kx \quad [1]$$

and the free harmonic motion will be expressed as

$$x = x_0 \cos \sqrt{\frac{k}{m}}\, t \quad [2]$$

(k = spring constant, m = mass, t = time.)

For the forced vibration a periodic exciting force

$$F = \pm F_0 \cos \omega t \quad [3]$$

is now assumed. The two signs are to indicate that F can be either in or out of phase with the motion.

F_0 being given, the value of the impressed frequency ω is not prescribed at first. Rather, the problem becomes: What will be the value of ω at which an amplitude x_0 can be sustained?

According to the paper ω is first assumed to equal the natural frequency of the system. The exciting force is then

$$F = \pm F_0 \cos \sqrt{\frac{k}{m}}\, t \quad [4]$$

In view of Equation [2] this can be written

$$F = \pm F_0 \frac{x}{x_0} \quad [4']$$

The elastic force E and the impressed force F are now lumped into a "net restoring force" R

$$R = E - F = kx \pm F_0 \frac{x}{x_0} \quad [5]$$

This new restoring force will be made the elastic force in an imaginary "free vibration," which would be harmonic and have the form

$$x = x_0 \cos \sqrt{\frac{k \pm \dfrac{F_0}{x_0}}{m}}\, t \quad [6]$$

the circular frequency being

$$\omega = \sqrt{\frac{k \pm \dfrac{F_0}{x_0}}{m}} \quad [7]$$

This is already the expression for the frequency of the exciting force necessary to create a forced vibration of amplitude x_0. According to the paper Equation [6] should be used as a second approximation, instead of Equation [2] and should give the exciting force Equation [3] the frequency just found in Equation [7]. It is obvious that, from Equation [4'] on, the equations remain unaltered so the answer is final. The solution Equation [7] is, of course, the equivalent of the more familiar form

$$x_0 = \frac{F_0}{k - m\omega^2}$$

Author's Closure

Professor Bernhard, by offering data on the actual occurrence of nonharmonic vibrations in practice, contributes a welcome point in his comment. A particularly interesting feature of his curves is their behavior on the fringes of the unstable region to the left of the resonance peaks. Mr. Kroon's contribution adds clarity to the paper in another detail. To both commentators the author wishes to express his thanks.

[3] "Dynamic Tests by Means of Induced Vibrations," by R. K. Bernhard, American Society for Testing Materials, Proceedings of the 40th Annual Meeting, vol. 37, part 2, Technical Papers, p. 634. Also: "Theorie und Praxis der Schwingungsprüfmaschinen," by W. Späth, Julius Springer, Berlin, 1934.

[4] Experimental Division Engineer, Westinghouse Electric & Manufacturing Company, South Philadelphia, Pa. Mem. A.S.M.E.

Book Reviews

Vibration

EINFÜHRUNG IN DIE TECHNISCHE SCHWINGUNGSLEHRE. By Dr. Karl Klotter. Volume 1. Julius Springer, Berlin, 1938. Paper, 6¾ × 10¼ in., vii and 206 pp., 208 figs., 18 rm.

REVIEWED BY FRANK M. LEWIS[1]

THE FIRST of the three volumes of this work is devoted entirely to the study of the single-degree-of-freedom system. Subsequent volumes will respectively consider systems of several, but a finite number of, degrees of freedom and distributed systems.

To a large extent any new book on vibration must cover problems made familiar by many previous texts, and criticism can be made only of the manner of presentation. About half of the text is devoted to the classical problem of the free and forced vibration of the single-degree-of-freedom system and here the presentation must be considered excellent. Some forty pages treat the forced vibration of the linear system with a thoroughness which certainly exhausts the subject, and possibly the reader also.

Here free use is made of complex notation and the methods parallel those developed in the theory of alternating currents. Presentation of results is both by the familiar resonance curves and by a method used in electrical engineering in which families of curves represent the termini of the displacement or other vectors.

The remainder of the text largely considers systems which are nonlinear in their damping and spring constants and a considerable variety of problems is given. Systems with nonlinear and variable mass are not discussed. The treatment of forced vibration of systems with nonlinear springs is that of Den Hartog and Heiles, the recent exact relation of Rauscher evidently not having come to the author's attention before publication.

The case of accelerated vibration, i.e., passing through a critical, is discussed in abstract and the book concludes with a very brief discussion of self-sustained oscillation.

Fatigue of Metals

LA FATIGUE DES MÉTAUX. By R. Cazaud and L. Persoz; preface by A. Caquot. Dunod, Paris, 1937. Paper, 6½ × 10 in., viii and 190 pp., 121 figs., tables.

REVIEWED BY JOHN M. LESSELLS[2]

THIS SMALL book on fatigue of metals is of interest since while this subject has been intensely studied in other countries very few publications have come from French engineers. The purpose of the book is given in a note, sounded at the end of chapter one, commenting on the fact that certain phases of fatigue investigations have been badly coordinated, which has given rise, in certain cases, to contradictory results. An attempt is made in the book to present the results of the more important experiments, in review form, with a view to clarifying the existing data where doubt now exists.

The book contains eight chapters. The first is historical in character, and the following five chapters contain more or less well-known facts on the various theories of fatigue, testing machines used, and the results obtained. Chapter five contains data on the fatigue of nickel and nickel-molybdenum and other steels and is of interest because the fatigue strength is related to the microstructure. This is the first time that such a detailed effort has been made along these lines to correlate structure with fatigue strength. The subject of the fatigue of different kinds of jointed members, riveted, welded, brazed, is considered in chapter seven, while the last chapter is devoted to general conclusions. In the last chapter the data on allowable stresses, as contained in the published leaflets of the Verein deutscher Ingenieure, are discussed. In this regard it is to be regretted that the authors did not see fit to include and discuss the work of the Applied Mechanics Division on the subject as contained in the Design Data section of the JOURNAL OF APPLIED MECHANICS, September 1935.

The book is, however, of considerable interest and contains an excellent bibliography at the end of each chapter.

Fluid Mechanics for Hydraulic Engineers

FLUID MECHANICS FOR HYDRAULIC ENGINEERS. By Hunter Rouse, Assistant Professor of Fluid Mechanics, California Institute of Technology. First edition. Engineering Societies Monographs. McGraw-Hill Book Company, Inc., New York, 1938. Cloth, 6 × 9 in., xvi and 422 pp., 245 figs., and 6 tables, $5.

REVIEWED BY HUGH L. DRYDEN[3]

THE SUCCESS of modern fluid mechanics in providing an analytical approach to many engineering problems encountered in the design of aircraft has stimulated interest in fluid mechanics as a basic science and in its possible applications to other fields of engineering, in particular to hydraulics. Many American technical schools are experimenting with a basic course in fluid mechanics as a substitute for the traditional course in hydraulics, and a number of textbooks intended for use in this course have appeared recently. All encounter a fundamental difficulty which the authors have tried to meet in several ways.

In the present state of fluid mechanics, it is not possible to begin from fundamental principles and give a rational development of the theory of the varied phenomena encountered in nature. Rather one must recognize certain more or less typical phenomena, make simplifying assumptions which by rational analysis reproduce the phenomena. The selection of assumptions is based on the observed phenomena, and contradictory assumptions are used in different phenomena. Thus Bernouilli's theorem is of little use in studying the pressure changes in a closed conduit of constant cross section, since the kinetic energy is constant, and the fall in pressure corresponds to the loss of energy by friction. In flow in an open channel or in a conduit of variable cross section, Bernouilli's theorem may be very useful in estimating the much larger changes in static pressure, the friction being neglected. Such fickleness with regard to the applicability of a fundamental theorem is very confusing to the student. He has not only to become acquainted with certain groups of experimental data, and with certain basic principles of classical fluid mechanics, but also with many groups of often contradictory assumptions and the rational analysis of the con-

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sequences of the assumptions, and most difficult of all he must learn to recognize under what conditions a given group of assumptions is likely to yield a correct description of the behavior of the fluid.

Mr. Rouse meets this difficulty by assuming that the reader is already familiar, as a consequence of the traditional elementary course in hydraulics or otherwise, with the empirical facts of experience, i.e., with cavitation, secondary flow, general aspects of flow in closed conduits and open channels and over weirs, etc. His book is specifically addressed to graduate students and practicing engineers, offering a sound rational analysis of the observed phenomena.

The phenomena are classified in the main according to the property of the fluid of paramount importance. In part one, except for the last chapter, there is a selection from the classical treatment of the incompressible, nonviscous fluid of "all that is immediately useful or will have bearing upon later developments." For psychological reasons the term "ideal fluid" is not used, and the assumptions involved are not given prominence. There are practically no comparisons with experiment. The last chapter of this section very briefly describes the phenomena of compressibility of gases, cavitation of liquids, and flow separation as limiting the applicability of the "fundamental hydromechanics," in which density and weight are the only fluid properties considered.

Part two considers the mechanics of fluid resistance and occupies about one half the book. The discussion covers the Navier-Stokes equations, laminar flow, turbulence, boundary layer, flow around immersed bodies, in closed conduits, in open channels, and the transportation of sediment. The treatment is in general clear, accurate, and up-to-date. The author was evidently not familiar with the more recent work on the effect of initial turbulence on the drag of spheres and cylinders, and hence states that a particular curve for sphere and cylinder applies to "normal" conditions. This, however, is a relatively minor error in a book addressed to hydraulic engineers. Part three covers the mechanics of wave motion.

Throughout the book dimensional analysis holds a deservedly prominent place, and the discussion of the needed ingredients of judgment, common sense, and ingenuity in the use of this powerful method in the analysis of data and the planning of model experiments is especially to be commended. Unfortunately while these ingredients may be acquired or improved, they probably cannot be taught.

Vibration Problems in Engineering

VIBRATION PROBLEMS IN ENGINEERING. By S. Timoshenko, Professor of Theoretical and Applied Mechanics, Stanford University. Second edition. D. Van Nostrand Company, Inc., New York, 1937. Cloth 5¾ × 9 in., 462 pp., 229 figs., $5.50.

REVIEWED BY J. ORMONDROYD[4]

THERE existed in the early 1920's many treatises by physicists on the vibration of bodies producing sound, a complete theory of electric oscillations, a fairly large volume of scattered papers on mechanical vibrations and even several complete treatises on this subject in foreign languages. It is a somewhat lugubrious commentary on the state of mechanical-engineering education of that not too distant time when we consider that all this available material was almost a closed book to the profession as a whole. The number of mechanical engineers in the United States competent to solve any of the innumerable vibration problems in machinery existing at that time could easily be counted on the fingers of one man's hands.

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The times cried out for someone to transfer the existing state of knowledge concerning vibration to a level at which the mechanical engineer could comprehend it and use it. Timoshenko may be fairly said to have carried through this much-needed action when in 1928 he published his treatise on "Vibration Problems in Engineering." The first edition contained 346 pages of printed matter, 165 figures, and an appailing number of errata which slipped past in proofreading. This edition went through three printings in nine years and copies of it found their way into the libraries of a large number of practicing engineers. During these years several engineering schools introduced the subject into the curriculum, at least for advanced study, and the number of practicing engineers capable of handling vibration problems increased enormously.

When, in 1937, a second edition of this work appeared, the mechanical-engineering profession was prepared to welcome it and a younger generation of engineers existed which recognized its usefulness and possessed a background more suitable for understanding it. The second edition contains 462 pages of printed matter, 229 figures. It is comparatively free from the errata which marred the first edition. The material treated has been increased some 35 per cent. In the preface of the second edition the author outlines the principal additions as follows: "In the first chapter a discussion of forced vibration with damping not proportional to velocity is included, and an article on self-excited vibration. In the chapter on nonlinear systems an article on the method of successive approximations is added and it is shown how the method can be used in discussing free and forced vibrations of systems with nonlinear characteristics. The third chapter is made more complete by including in it a general discussion of the equation of vibratory motion of systems with variable spring characteristics. The fourth chapter, dealing with systems having several degrees of freedom, is also considerably enlarged by adding a general discussion of systems with viscous damping; an article on stability of motion with an application in studying the vibration of a governor of a steam engine; an article on whirling of a rotating shaft due to hysteresis; and an article on the theory of damping vibration absorbers. There are also several additions in the chapter on torsional and lateral vibrations of shafts."

These additions represent the results of important papers which have appeared in the technical literature since the first edition was printed. The citation of original papers on the subject in numerous footnotes forms a very valuable part of the treatise.

For those who are not familiar with the general contents of the volume it will be well to give a brief outline. The first chapter, containing well over 100 pages, discusses harmonic vibrations of systems having one degree of freedom. This chapter contains all the nomenclature, concepts, and mathematical methods which must be mastered by any engineer who expects to be reasonably effective in the field of vibrations. This chapter covers many problems of actual technical interest. The brief, qualitative discussion of self-excited vibrations at the end of this chapter will help to clarify many mysterious vibrations problems which confront engineers. The next two chapters discuss problems in systems of one degree of freedom which lead to nonlinear differential equations. Engineers who can handle the simple cases given in the first chapter will be especially pleased with the help and suggestion they receive in these two chapters in a field likely to be more baffling and elusive. Chapter 4 on systems with more than one degree of freedom opens with a clear, concise development of LaGrange's equations and includes, among many other good things, a discussion of stability of motion, a subject of supreme interest to all engineers dealing with control problems. Chapter 5 is a very practical chapter on tor-

sional and lateral vibrations of shafts carrying many rotors and supported in several bearings. This chapter is perhaps the most directly useful in the whole treatise. Chapter 6 deals with the vibration of continuous elastic bodies such as bars, membranes, and plates. This introduces the use of partial differential equations. Besides being useful for many practical problems, this chapter serves as an excellent introduction for those who wish to read intelligently volume 1 of Rayleigh's "Theory of Sound." All of the problems discussed in that volume—with the exception of the string, curved shells, and electrical oscillations—are treated in more understandable detail by Timoshenko in his chapter 6. The final chapter gives an exceedingly valuable description of all the common vibration-measuring devices—barring the strange omission of accelerometers and pressure indicators.

Readers of "Vibration Problems in Engineering" who are familiar with scientific history will realize that Timoshenko has done for the theory of mechanical vibrations what Rayleigh did for the theory of sound sixty years ago. They may also remember Rayleigh's quaint statement that "Without ears we should hardly care much more about vibrations than without eyes we should care about light." Timoshenko's treatise indicates that men, even if not blessed with ears, would still have many reasons for being interested in vibrations.

Timoshenko's "Vibration Problems in Engineering" bids fair to become a classic on the subject. It brings the general methods of the mathematical physicist down to the level of engineering comprehension by treating familiar problems in complete and particular detail.

Lubrication of Light Internal-Combustion Engines

LUBRICATION OF LIGHT INTERNAL-COMBUSTION ENGINES. By P. I. Orloff. Edited by N. R. Brilling, O.N.T. I., Leningrad, 1937. 462 pp., 336 figs.

REVIEWED BY GEO. B. KARELITZ[5]

THE AUTHOR says in his preface that he could not find in literature any treatise on the subject which he could use as a prototype. The best book in the field, to his knowledge, the German volume of E. Falz was unsatisfactory due to its "narrow practicality and absence of a theoretical background," and neglect of the vast experimental and theoretical works of American investigators. Mr. Orloff attempted to present a short encyclopedia of lubrication. He realized that this task was too difficult for one man.

The book is divided into seven chapters. In the first chapter the author presents a very able critical compilation of the existing knowledge on the behavior of the viscous oil film in a bearing. He uses Guembel's treatment of the subject but amplifies it by the recent work of the American and German authors. He gives therefore a rather complete picture of the hydrodynamical theory of lubrication in its present state. Whenever possible, he attempts to introduce practical coefficients which would bring together theory and practice into a closely knit fabric. It is evident from the treatment that the author had the opportunity to apply the theory and check it with practical designs, and thus acquired a critical approach to the curves and diagrams published by different authors. A section on rotating loads gives interesting considerations on the applications of the theory to bearings so loaded.

The material of Orloff's book is based on sources up to the year 1936. Some recent work which throws more light on lubrication phenomena is therefore not included, particularly that on internal-combustion engines such as the diagrams of Dennison for heavily loaded bearings, or the experiments of Bowden on surface temperatures of rubbing surfaces. However, together with the well-selected bibliography, this chapter is an interesting review of the hydrodynamic theory of lubrication as applied to bearing design.

The second chapter treats of design factors, running characteristics, and maintenance factors of a bearing. The problems met with are indicated and mentioned in a necessarily brief and general way.

The short third chapter mentions the tests used to determine the stability of lubricants and the means used to prevent oxidation and gumming of oils in internal-combustion engines. Chapter 4 gives a short survey of common bearing materials, based mostly on American practice. The compositions and mechanical properties of various babbitts and bronzes, and of several "oilless" bearing materials are listed.

Chapter 5 discusses rather briefly the application of ball, roller, and needle bearings to internal-combustion engines. All these chapters are of a descriptive character and serve only to call attention to important points which should be considered when designing a bearing.

Chapter 6 describes the general arrangement of the lubrication of the Kestrel motor of Rolls-Royce and of the Cyclone engine of the Wright Company. This is followed by a detailed description of the oil feed to main and connecting-rod bearings. The friction and lubrication of piston rings is mentioned just in passing. This short treatment is to be regretted since this is the most difficult phase of lubrication in internal-combustion engines, and since it appears from various reports that much experimental work is being done on this phase in the Russian research institutes. Auxiliary apparatus such as oil pumps, filters, packings are described.

Chapter 7 gives a short account of the theory of heat transfer and its application to the design of oil coolers for airplanes, with the bibliography of the subject.

The bibliography given in the appendix is a valuable feature of the book. It is well chosen and compiled. It compares with the bibliography given by M. D. Hersey in his recent book on lubrication. In general the book of Orloff could be well recommended to engineers if it were not written in Russian. It leaves the impression that the field remains open for a treatise which would cover the subject of lubrication and bearings in a way satisfactory to engineers.

Strength of Materials

STRENGTH OF MATERIALS. A First Course. By Norman C. Riggs, Professor of Mechanics, and Max M. Frocht, Associate Professor of Mechanics, Carnegie Institute of Technology. The Ronald Press Company, New York, 1938. Cloth, $5^3/_4 \times 8^1/_2$ in., xii and 432 pp., 454 illustrations, $3.75.

REVIEWED BY O. J. HORGER[6]

IT IS ALWAYS difficult to criticize a textbook prepared for a specific purpose and to meet the individual requirements of an author. So many books of this nature are being published that a book as a whole generally lacks in comparison with the outstanding books in their respective fields but contain valuable selections which may be considered as new contributions. In this latter respect the reviewer believes that the book under consideration excels in its introduction of the subject of photo-

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elasticity. The authors present a legitimate case for the use of photoelasticity by both the student and the practicing engineer in the rational study of stress analysis. Few investigators have developed the technique exemplified in the excellent fringe-photograph reproductions given in this book. The authors use their photoelastic examples to show the fallacy of determining stresses from the usual calculations. There may be some question as to whether the discussion of photoelasticity occupies a disproportionate amount of the book as a first course in strength of materials. This could be argued in many ways but the reviewer feels that one cannot learn too early or too much about stress concentration, especially since economic necessity is requiring the use of lower factors of safety and lighter weights.

This book includes a treatment of continuous beams by the Hardy Cross method of moment distribution and omits the fundamental but cumbersome analysis of determining deflections by double integration. In a first-course text it would seem apropos first to give the student the necessary fundamentals and then proceed to short-cut methods. It is realized that books have size limitations but generally speaking authors make little use of footnotes and discussions in small type to which the student may refer for detailed analysis of many problems too lengthy for consideration in any one book. The use of many references in any textbook immensely increases its value not only to the student in later life who is already familiar with its contents but also to the practicing engineer. Exceptionally few problems presented in everyday experience have their solution in the elementary treatment appearing in the text, but footnotes more often permit the reader to cover the literature in a more comprehensive manner. Neglecting this comment, the usual essentials in a first course in the strength of materials are presented in this book and the illustrations and problems are numerous and well-arranged.

One may take exception to the omission of reference to alloy steels in the discussion of the properties of materials appearing throughout the book with the exception of brief mention under the subject of fatigue. A section on impact, dividing materials into "ductile" and "tough," is questionable as many materials exist which possess both these properties and it would appear that the term "brittle" is commonly applied to the author's expression "tough." No mention is made of Izod and Charpy impact test values or fact that a transition velocity of impact is characteristic of many materials above which velocity the impact strength is considerably reduced. The statement "the stress which will sustain four or five millions of cycles differs but little from the endurance limit" is not true in cases of corrosion fatigue, some nonferrous materials, some forms of stress concentration, and so-called delayed fractures in hardened steels.

Thermodynamics

TECHNISCHE THERMODYNAMIK. Part 2. By Fr. Bosnjakovic. Theodor Steinkopff, Dresden and Leipzig, 1937. 294 pages with 243 figures and 24 pages of problems and their solution. Price 19 rm.; in addition a portfolio of large charts, Price 8 rm.

REVIEWED BY JOHN A. GOFF[7]

THIS book constitutes a unique contribution to the literature of thermodynamics by presenting in one volume a complete and coherent treatment of the properties of binary mixtures and of the processes in which they take part. A principal feature of the treatment is the use of what may be called fundamental coordinates expressing directly the content of the three fundamental laws which must be strictly observed in the analysis of any physical process. The law of conservation of matter is expressed by whichever of the coordinates, mol-fraction, weight fraction, or weight ratio, is most convenient to the problem at hand; the law of conservation of energy is expressed by the coordinate, enthalpy, since most processes contemplated are of the steady-flow type in which the energy convected with the fluid is its enthalpy; the second law of thermodynamics is expressed by the coordinate entropy. The book makes extensive use of graphical methods; the diagrams are numerous and show obvious signs of care in their preparation.

The first chapter is devoted to the liquefaction of gases with particular reference to the Linde and Claude cycles both of which are based upon the well known Joule-Thomson-effect phenomena. The extent of inherent irreversibilities in these cycles is brought into prominence by application of the second law and the various methods of reducing them are discussed.

The second chapter is devoted to the properties of air-water mixtures, commonly referred to as moist air, and to an analysis of the important processes of evaporation and condensation occurring in such mixtures. The Mollier, i-x, diagram is used throughout; this diagram is constructed with those coordinates which most directly express the fundamental laws governing the processes involved. In the introduction of this chapter, the author states his opinion that any attempt to investigate the departures from Dalton's law in the case of moist air is not only superfluous, but would pretend to a degree of accuracy which is not real because, in the air actually encountered in practice, there are always present impurities which affect its properties to an unknown extent. This statement, it is believed, not only violates the spirit of thermodynamic consistency in evidence throughout most of the book, but actually underestimates the practical importance of these departures. It is also surprising that the author should pretend to derive the so-called Lewis law which has long been known to be incorrect in the general case and to owe its approximate correctness in the case of moist air to fortuitous circumstances.

In the third chapter, the general properties of binary mixtures, with and without phase differences, are discussed with particular reference to the Merkel diagram in which specific enthalpy and weight-fraction, (alternatively mol-fraction) are used as coordinates. The integral and differential heats of dilution and solution, also heats of evaporation and of fusion are clearly explained. The next chapter is devoted to typical processes in which binary mixtures take part, such as evaporation, sublimation, absorption, distillation, rectification and separation. These are two of the most valuable chapters of the book. There follow discussions of the problems of material transport between liquid and vapor, and a short development of the theory of the absorption refrigeration cycle.

Following a chapter devoted to the general thermodynamic relations and conditions for equilbrium, there appear chapters on combustion of liquid and solid fuels, production of gas from solid fuels, equilibrium under chemical reactions in gas mixtures, and finally on Nernst's third law of thermodynamics.

The book carries with it a pamphlet containing 74 problems with solutions; also a portfolio containing Mollier diagrams and entropy-concentration diagrams for moist air, ammonia-water, ethylalcohol-water, and nitrogen-oxygen mixtures. In view of the increasing importance of the subjects treated, the fortunate choice of coordinates in terms of which the treatment is worked out, and the care with which illustrations and diagrams are prepared, the book will be found to have immense value to student and engineer alike.

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Analysis of Spherical Shells of Variable Wall Thickness[1]

By MERHYLE F. SPOTTS,[2] BALTIMORE, MD.

This article presents a solution to the problem of finding the forces and moments which occur in a spherical shell which is axisymmetrically loaded, when the variation in wall thickness is taken as a quadratic function of the coordinate of latitude.

The so-called Love-Meissner differential equations for the case of nonuniform wall thickness are derived herein. By appropriate substitutions these are reduced to one linear equation of the fourth order having constant coefficients. The solution to this equation when taken in the homogeneous form is first given. This homogeneous solution will be the general solution for all problems where the shell surface is free from external force, and the only loads on the shell consist of forces and moments applied at the boundaries.

When, however, the shell surface itself is loaded, a particular integral, which will depend upon the type of loading, must also be obtained for the nonhomogeneous equations.

The solution contained herein is illustrated by a numerical example for a shell of one boundary when acted upon by dead-load forces, and the results are plotted.

Historical

THE MODERN theory for shells having a wall thickness sufficiently great to resist bending may be said to date from Love (1)[3] who in 1892 gave the equilibrium equations for an element of the surface.

Differential equations for a spherical shell of constant wall thickness with axisymmetrical boundary loads were obtained by Reissner (2). These were solved by Blumenthal (3) by an asymptotic integration. Meissner (4) also obtained a solution in the form of a hypergeometric series.

Later Meissner (5) pointed out that the solution obtained for cylindrical shells could be applied to sufficiently thin spherical shells when the angle of opening is large. This is satisfactory in the region near the boundary since the stresses produced by bending are rapidly damped out. Geckeler (6) arrived at this same conclusion by taking the differential equations for a spherical shell and neglecting the terms of less importance.

In regard to the problem of shells of rotation of variable wall thickness, a solution has been presented by Meissner (7) for different variations in thickness and shape of meridian curve. An approximate solution for spherical shells of variable thickness has also been given by Geckeler (6).

Notation

a = radius of sphere
d = thickness of shell at pole or apex
D = $E\delta^3/12(1-\mu^2)$ modulus of flexural rigidity
E = Young's modulus
M_1 = unit bending moment with respect to φ
M_2 = unit bending moment with respect to θ
N_1 = unit meridional normal force
N_2 = unit hoop force
Q = unit meridional shearing force
r_0 = radius of latitude circle
v = meridional displacement
w = radial displacement
Y = unit external load parallel to meridian
Z = unit external load normal to surface
γ = weight per unit volume of shell
δ = thickness of shell
ϵ_1 = elongation along meridian
ϵ_2 = elongation along latitude circle
θ = coordinate of longitude
κ_1 = change of curvature with respect to φ
κ_2 = change of curvature with respect to θ
μ = Poisson's ratio
φ = coordinate, complement of latitude
χ = rotation of meridional tangent

Solution for the Case When the Shell Surface Is Free From External Forces

The usual elastic assumptions are made, namely, that the material is homogeneous, isotropic, and obeys Hooke's law, and that straight lines initially normal to the middle surface remain straight and normal to that surface after bending.

Fig. 1 shows an element cut from a sphere having a vertical axis, and a radius a. The coordinate φ of the element is measured from the axis in a downward direction, and the coordinate θ is measured from any fixed axial plane.

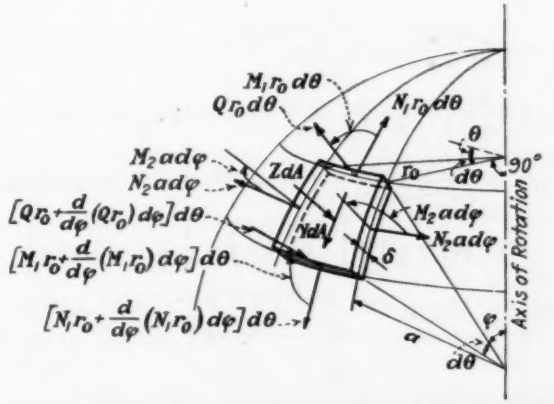


Fig. 1 Element of Spherical Surface With the Forces and Moments Which Produce Equilibrium

[1] Taken from a dissertation submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in the University of Michigan.

[2] Instructor, Department of Mechanical Engineering, Johns Hopkins University.

[3] Numbers in parentheses refer to the Bibliography.

Presented at the National Meeting of the Applied Mechanics Division of The American Society of Mechanical Engineers, New York, N. Y., June 14–15, 1939.

Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y., and will be accepted until October 10, 1939, for publication at a later date.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

The top and bottom edges of the element are formed by two conical surfaces inclined to the axis by the angles φ and $\varphi + d\varphi$. The sides are formed by two meridional sections an angle $d\theta$ apart.

The quantities N_1, N_2, and Q represent force per unit length of edge; M_1 and M_2 represent moment per unit length; and Y and Z represent force per unit area. Positive directions are as shown in Fig. 1. The element, together with all forces, moments, and external loads, is symmetrical with respect to the axis. Thus no other forces or moments than those shown in Fig. 1 need be considered. All forces and moments are thus functions of the coordinate φ only.

As shown in the Appendix it is possible to write three equations of equilibrium for the element together with four more equations expressing relationships between the deformations. These seven equations make it possible to solve for the seven unknowns N_1, N_2, Q, M_1, M_2, v, and w, where v is the displacement of the element along a meridian, positive downward, and w is the displacement along the radius, positive outward.

If the variation in wall thickness is now taken as

$$\delta = d(1 + \alpha\varphi)^2 \quad \ldots\ldots [1]$$

where d is the thickness at the pole, and α is an arbitrarily chosen constant, and the same approximations are made as in the case of constant thickness (10), these equations may be reduced to one linear equation of the fourth order having constant coefficients.

The values of the unknowns,[4] when the equation is taken in the homogeneous form, are found to be[5,6]

$$^\circ Q = [d^{1/2}/(\sin\varphi)^{1/2}]\{C_1(1+\alpha\varphi)^{(3/2)+k}\cos[k\log(1+\alpha\varphi) - \gamma_1] + C_2(1+\alpha\varphi)^{(3/2)-k}\cos[k\log(1+\alpha\varphi) - \gamma_2]\} \quad \ldots\ldots [2]$$

$$^\circ Q' = \frac{C_1 d^{1/2}}{(\sin\varphi)^{1/2}}(1+\alpha\varphi)^{(1/2)+k}\left\{-\alpha k\sin[k\log(1+\alpha\varphi) - \gamma_1] + \left[\alpha\left(\frac{3}{2}+k\right) - \frac{1}{2}(1+\alpha\varphi)\cot\varphi\right]\cos[k\log(1+\alpha\varphi) - \gamma_1]\right\} + \frac{C_2 d^{1/2}}{(\sin\varphi)^{1/2}}(1+\alpha\varphi)^{(1/2)-k}\left\{-\alpha k\sin[k\log(1+\alpha\varphi) - \gamma_2] + \left[\alpha\left(\frac{3}{2}-k\right) - \frac{1}{2}(1+\alpha\varphi)\cot\varphi\right]\cos[k\log(1+\alpha\varphi) - \gamma_2]\right\} \quad \ldots\ldots [3]$$

$$^\circ\chi = \frac{2\alpha^2 k^2}{Ed^{1/2}(\sin\varphi)^{1/2}}\{C_1(1+\alpha\varphi)^{-(5/2)+k}\sin[k\log(1+\alpha\varphi) - \gamma_1] - C_2(1+\alpha\varphi)^{-(5/2)-k}\sin[k\log(1+\alpha\varphi) - \gamma_2]\} \quad \ldots\ldots [4]$$

$$^\circ\chi' = \frac{2\alpha^2 k^2 C_1}{Ed^{1/2}(\sin\varphi)^{1/2}}(1+\alpha\varphi)^{-(7/2)+k}\left\{\alpha k\cos[k\log(1+\alpha\varphi) - \gamma_1] + \left\{\alpha\left(-\frac{5}{2}+k\right) - \frac{1}{2}(1+\alpha\varphi)\cot\varphi\right\}\sin[k\log(1+\alpha\varphi) - \gamma_1]\right\} + \frac{2\alpha^2 k^2 C_2}{Ed^{1/2}(\sin\varphi)^{1/2}}(1+\alpha\varphi)^{-(7/2)-k}\left\{-\alpha k\cos[k\log(1+\alpha\varphi) - \gamma_2] + \left[\alpha\left(\frac{5}{2}+k\right) + \frac{1}{2}(1+\alpha\varphi)\cot\varphi\right]\sin[k\log(1+\alpha\varphi) - \gamma_2]\right\} \quad \ldots\ldots [5]$$

$$^\circ N_1 = -^\circ Q\cot\varphi \quad \ldots\ldots [6]$$

$$^\circ N_2 = -^\circ Q' \quad \ldots\ldots [7]$$

[4] The index $^\circ( )$ indicates the homogeneous solution.

[5] Certain minor terms have been omitted in these results.

[6] Differentiation with respect to φ is represented by the prime $( )'$.

$$^\circ M_1 = (D/a)(^\circ\chi' + \mu\,^\circ\chi\cot\varphi) \quad \ldots\ldots [8]$$

$$^\circ M_2 = (D/a)(^\circ\chi\cot\varphi + \mu\,^\circ\chi') \quad \ldots\ldots [9]$$

where C_1, C_2, γ_1, and γ_2, are constants of integration which must be determined from the loading and boundary conditions;

$$k = (1/\alpha)(a/d)^{1/2}[3(1-\mu^2)]^{1/4} \quad \ldots\ldots [10]$$

μ = Poisson's ratio

χ = rotation due to deformation of the meridional tangent, or the normal to the shell surface, taken positive clockwise

$$D = E\delta^3/[12(1-\mu^2)] \quad \ldots\ldots [11]$$

modulus of flexural rigidity, a variable because of the presence of δ.

It may be observed from the foregoing formulas that the pole ($\varphi = 0$), is a singular point, but its local character does not affect the accuracy of the results at other points.

This solution applies to the general case of the shell of two boundaries. Constants C_1 and γ_1 are determined from conditions which prevail on the lower boundary, and C_2 and γ_2 from conditions on the upper boundary.

When the shell is closed, that is, extends all the way to the pole, $C_2 = \gamma_2 = 0$ and only the constants C_1 and γ_1 must be evaluated. A proof may be given for this based on consideration of the fact that for the closed shell the shearing force Q and the rotation of the meridional tangent χ must both be zero at the pole.

Particular Solution When the Load Is Distributed Over the Surface of the Shell

The particular integral for the nonhomogeneous differential equations will of course depend upon the type of loading to which the shell surface is subjected.

In general it is not possible to obtain a particular integral to the lengthy nonhomogeneous equations by means of any of the recognized methods, and resort must be made to approximations.

For this purpose for most types of loading it is satisfactory to use the membrane forces as the particular solutions for N_1 and N_2, and take Q as equal to zero.

Example

The foregoing solution will now be applied to a numerical example.

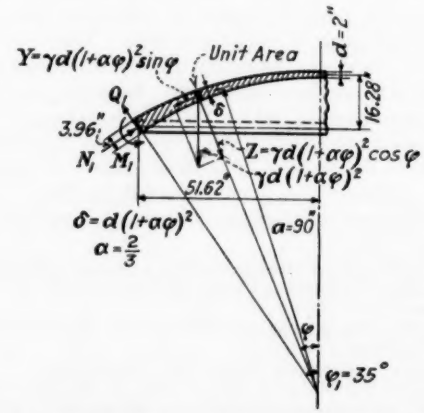


Fig. 2 Shell of Example Solved for Dead Load

Consider the closed shell of concrete with built-in boundary, as illustrated in Fig. 2, when the loading is taken as its own weight.

The numerical data for this shell are: $a = 90$ in. radius:

$\varphi_1 = 35$ deg, angle of opening; $d = 2$ in., thickness at pole; $\alpha = 2/3$; $E = 3{,}000{,}000$ psi; $\mu = 0.15$; $\gamma = 0.0868$ lb per cu in., weight; and from Equation [10], $k = 13.1676$.

Calculation of Constants of Integration

As shown in Fig. 2, the tangential and normal components of the surface load are

$$Y = \gamma d(1 + \alpha\varphi)^2 \sin \varphi \quad [12]$$

$$Z = \gamma d(1 + \alpha\varphi)^2 \cos \varphi \quad [13]$$

For a fixed-edge shell the conditions to be satisfied on the boundary are

$$v = 0, \quad w = 0, \quad \text{and} \quad \chi = 0 \quad [14][15][16]$$

As shown in the Appendix, the expression for w is

$$w = (a/E\delta)(N_2 - \mu N_1) - v \cot \varphi \quad [75]$$

This will be satisfied on the boundary where both v and w are zero if

$$N_2 - \mu N_1 = 0 \quad [17]$$

Since N_1 and N_2 denote the general solution for these forces this equation may be written

$$^{\circ}N_2 + {}^{DL}N_2 - \mu(^{\circ}N_1 + {}^{DL}N_1) = 0 \quad [18]$$

or

$$^{\circ}N_2 + {}^{m}N_2 - \mu(^{\circ}N_1 + {}^{m}N_1) = 0 \quad [19]$$

where the index $^{DL}(\ )$ indicates particular solution for dead load, and $^{m}(\ )$ indicates the membrane force.

Substituting the known expressions for the membrane forces for a sphere, and the values of the homogeneous solution from Equations [6] and [7] yields

$$-{}^{\circ}Q' + \frac{F(\varphi)}{\sin^2 \varphi} - Za - \mu\left[-{}^{\circ}Q \cot \varphi - \frac{F(\varphi)}{\sin^2 \varphi}\right] = 0 \quad [20]$$

where

$$F(\varphi) = a \int_0^{\varphi} (Y \sin \varphi + Z \cos \varphi) \sin \varphi d\varphi \quad [21]$$

For dead load as given by Equations [12] and [13] this becomes

$$F(\varphi) = a\gamma d[2a(1 + \alpha\varphi) \sin \varphi - (1 + \alpha\varphi)^2 \cos \varphi + 2a^2 \cos \varphi + 1 - 2a^2] \quad [22]$$

For the calculation of the constants, it will be convenient to let

$$\gamma_1 = k \log (1 + \alpha\varphi_1) + \psi_1 \quad [23]$$

where φ_1 is the angle of opening of the shell and ψ_1 is another constant.

Substituting for $^{\circ}Q$ and $^{\circ}Q'$ in Equation [20] and performing the numerical computations for $\varphi = \varphi_1 = 35$ deg gives

$$C_1(\sin \psi_1 + 0.96510 \cos \psi_1) = -0.00534 \quad [24]$$

Equation [16] may be written as

$$^{\circ}\chi + {}^{DL}\chi = 0 \quad [25]$$

In Equation [54] as derived in the Appendix the value of $^{DL}\chi$, if the particular solution for Q is neglected, is found to be

$$E\,{}^{DL}\chi = (1/\delta)\Phi(\varphi) \quad [26]$$

where $\Phi(\varphi)$ as given by Equation [55] becomes for dead load

$$\Phi(\varphi) = -(1 + \mu)\frac{\delta'}{\delta}\frac{F(\varphi)}{\sin^2 \varphi} + a\gamma d(2 + \mu)(1 + \alpha\varphi)^2 \sin \varphi \quad [27]$$

Substituting Equations [4] and [26] in [25] and performing the numerical work for $\varphi = \varphi_1 = 35$ deg yields

$$C_1 \sin \psi_1 = 0.001054 \quad [28]$$

Solving Equations [24] and [28] gives

$$C_1 = -0.006707 \quad [29]$$

$$\psi_1 = -(9°2'32'') = -0.15782 \text{ radians} \quad [30]$$

from which

$$\gamma_1 = 4.34067 \quad [30a]$$

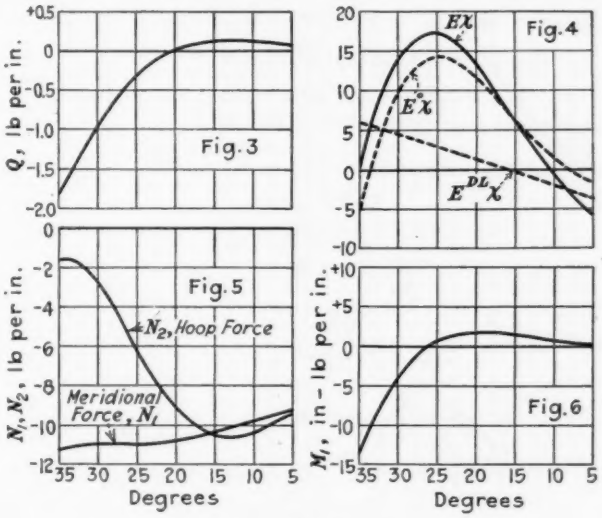


FIG. 3 SHEARING FORCE Q

FIG. 4 ROTATION OF MERIDIONAL TANGENT

FIG. 5 HOOP FORCE N_2

FIG. 6 MERIDIONAL BENDING MOMENT M_1

When the constants C_1 and γ_1 are determined, the solution as represented by Figs. 3, 4, 5, and 6 may be completed by substitution in the appropriate equations.

In computing the values of the bending moments it is necessary to have $^{DL}\chi'$. Much labor will be saved if, instead of differentiating Equation [26], the curve for $E\,{}^{DL}\chi$ as shown in Fig. 4 is first represented by an empirical polynomial of the second degree and then differentiated.

Appendix

(*a*) *Equilibrium Equations*

The equations of equilibrium of the element shown in Fig. 1 are

$$(Q \sin \varphi)' + N_1 \sin \varphi + N_2 \sin \varphi + Za \sin \varphi = 0 \quad [31]$$

$$(N_1 \sin \varphi)' - N_2 \cos \varphi - Q \sin \varphi + Ya \sin \varphi = 0 \quad [32]$$

$$(M_1 \sin \varphi)' - M_2 \cos \varphi - Qa \sin \varphi = 0 \quad [33]$$

Expressions for N_1 and N_2 may be obtained as follows: Multiply Equation [31] by cos φ and Equation [32] by sin φ and add.

$$(N_1 \sin \varphi)' \sin \varphi + N_1 \sin \varphi \cos \varphi + (Q \sin \varphi)' \cos \varphi - Q \sin^2 \varphi + a(Y \sin \varphi + Z \cos \varphi) \sin \varphi = 0$$

Integrating this equation gives

$$(N_1 \sin \varphi + Q \cos \varphi) \sin \varphi + F(\varphi) = 0 \quad \ldots\ldots [34]$$

where $F(\varphi)$ is as given by Equation [21].

From Equations [34] and [31] is obtained

$$N_1 = -Q \cot \varphi - F(\varphi)/\sin^2 \varphi \quad \ldots\ldots [35]$$

$$N_2 = -Q' + [F(\varphi)/\sin^2 \varphi] - Za \quad \ldots\ldots [36]$$

Equations [35] and [36] become Equations [6] and [7], respectively, when taken in the homogeneous form. On the other hand if the shell is a membrane and incapable of resisting shear and moment, the membrane forces N_1 and N_2 may be obtained from Equations [35] and [36] by setting Q and Q' equal to zero.

(*b*) *Relationships Involving Deformations*

Fig. 7*a* shows a vertical section through the element of Fig. 1 before and after deformation of the shell.

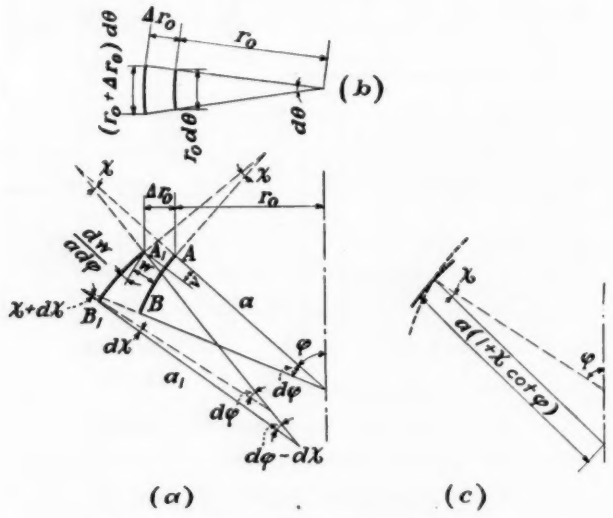


FIG. 7 DEFORMATION OF THE ELEMENT

[(*a*) Rotation of meridional tangent, also radius of curvature in direction of coordinate φ; (*b*) view of upper edge of element; (*c*) radius of curvature in direction of coordinate θ.]

A point originally at A on the upper edge of the element moves to A_1 undergoing the two displacements v and w.

Point B on the lower edge of the shell moves to B_1 after receiving displacements of $v + v'd\varphi$ and $w + w'd\varphi$.

The tangent to the meridian drawn at A has thus rotated counterclockwise through an angle v/a due to v, and clockwise through an angle w'/a because of the variation in w. The total rotation is thus

$$\chi = (w' - v)/a \quad \ldots\ldots [37]$$

It will be convenient to use χ as one of the unknowns rather than v and w separately.

Consider the curvature of the normal section of the surface whose plane contains the tangent to the meridian curve.

Normals drawn from the points B and B_1, Fig. 7*a*, will intersect at an angle $\chi + d\chi$. Normals from the points A_1 and B_1 will then intersect at an angle $d\varphi - d\chi$. The length of the arc AB then is

$$ad\varphi = a_1 (d\varphi - d\chi)$$

if a_1 be the radius of curvature after deformation, and if any change in length between AB and A_1B_1 be neglected.

The curvature for the deformed element then is

$$1/a_1 = (1/a)(1 - \chi')$$

and the change of curvature in the φ-direction is

$$\kappa_1 = \chi'/a = (w' - v)'/a^2 \quad \ldots\ldots [38]$$

Now consider the curvature of the section at right angles to φ-direction. Neglect the slight displacement of the element and regard only the effect of the rotation. The new radius of curvature extends from the element to the axis of the sphere, and as shown in Fig. 7*c*, has a length approximately equal to $a(1 + \chi \cot \varphi)$.

The change of curvature in the θ-direction is then found to be approximately

$$\kappa_2 = (\chi \cot \varphi)/a = [(w' - v)/a^2] \cot \varphi \quad \ldots\ldots [39]$$

Thus for the bending moments

$$M_1 = D(\kappa_1 + \mu\kappa_2) = (D/a)(\chi' + \mu\chi \cot \varphi)$$
$$= (D/a^2)[(w' - v)' + \mu(w' - v) \cot \varphi] \quad \ldots\ldots [40]$$

$$M_2 = D(\kappa_2 + \mu\kappa_1) = (D/a)(\chi \cot \varphi + \mu\chi')$$
$$= (D/a^2)[(w' - v) \cot \varphi + \mu(w' - v)'] \quad \ldots\ldots [41]$$

As shown in Fig. 7*a* the length of the arc A_1B_1 is given approximately by $(ad\varphi + v'd\varphi)[(a + w)/a]$. Neglecting terms of higher order the increase in the length of the element then is $(v' + w)d\varphi$ and the deformation is

$$\epsilon_1 = (v' + w)/a \quad \ldots\ldots [42]$$

In the θ-direction the new length of arc as shown in Fig. 7*b* is $(r_0 + \Delta r_0)d\theta$. The increase in length then is

$$\Delta r_0 d\theta = (v \cos \varphi + w \sin \varphi)d\theta$$

and the elongation is

$$\epsilon_2 = (1/a)(v \cot \varphi + w) \quad \ldots\ldots [43]$$

The edge forces are thus seen to be

$$N_1 = [E\delta/(1 - \mu^2)](\epsilon_1 + \mu\epsilon_2)$$
$$= [E\delta/a(1 - \mu^2)][v' + w + \mu(v \cot \varphi + w)] \quad \ldots\ldots [44]$$

$$N_2 = [E\delta/(1 - \mu^2)](\epsilon_2 + \mu\epsilon_1)$$
$$= [E\delta/a(1 - \mu^2)][v \cot \varphi + w + \mu(v' + w)] \quad \ldots\ldots [45]$$

The seven Equations [31], [32], [33], [40], [41], [44], and [45] make it possible to solve for the seven aforementioned unknowns contained therein. These equations will first be reduced to two equations in the unknowns Q and χ.

(*c*) *Differential Equations of the Shell*

Substituting Equations [40] and [41] into Equation [33] gives the first differential equation of the shell.

$$\chi'' + \chi'[\cot \varphi + 3(\delta'/\delta)] - \chi[\cot^2 \varphi + \mu - 3\mu(\delta'/\delta) \cot \varphi] = a^2Q/D \quad \ldots\ldots [46]$$

For the second equation proceed as follows. Combine Equations [35] and [36] as here indicated

$$N_2 - \mu N_1 = -Q' + \mu Q \cot \varphi + (1 + \mu)\frac{F(\varphi)}{\sin^2 \varphi} - Za \quad \ldots [47]$$

$$N_2' - \mu N_1' = -Q'' + \mu Q' \cot \varphi - \mu Q \csc^2 \varphi + \frac{1 + \mu}{\sin^2 \varphi} F'(\varphi) - \frac{2(1 + \mu) \cos \varphi F(\varphi)}{\sin^3 \varphi} - Z'a \quad \ldots [48]$$

$$N_1 - N_2 = Q' - Q \cot \varphi - \frac{2F(\varphi)}{\sin^2 \varphi} + Za \quad \ldots\ldots [49]$$

Combine Equations [44] and [45]

$$v' + w = (a/E\delta)(N_1 - \mu N_2) \quad [50]$$

$$v \cot \varphi + w = (a/E\delta)(N_2 - \mu N_1) \quad [51]$$

Subtract Equation [51] from [50]

$$v' - v \cot \varphi = (a/E\delta)(1 + \mu)(N_1 - N_2) \quad [52]$$

Differentiating Equation [51] and subtracting the product of Equation [52] times cot φ gives

$$w' - v = (a/E\delta)[-(\delta'/\delta)(N_2 - \mu N_1) + (N_2' - \mu N_1') - (1 + \mu)(N_1 - N_2)\cot\varphi] \quad [53]$$

Substituting Equations [47], [48], [49], and [37] into Equation [53] gives the second differential equation of the shell.

$$Q'' + Q'[\cot\varphi - (\delta'/\delta)] - Q[\cot^2\varphi - \mu - \mu(\delta'/\delta)\cot\varphi] = -E\delta\chi - (1+\mu)\frac{\delta'}{\delta}\frac{F(\varphi)}{\sin^2\varphi} + a[(1+\mu)Y + \frac{\delta'}{\delta}Z - Z'] = -E\delta\chi + \Phi(\varphi) \quad [54]$$

where

$$\Phi(\varphi) = -(1+\mu)\frac{\delta'}{\delta}\frac{F(\varphi)}{\sin^2\varphi} + a[(1+\mu)Y + \frac{\delta'}{\delta}Z - Z'] \quad [55]$$

Equations [46] and [54] are general, and will apply to any type of variation in δ, and to any type of external load.

(d) Solution of the Homogeneous Equations

Equation [46] is homogeneous in the variables χ and Q, and Equation [54] may be made thus by omitting the term $\Phi(\varphi)$ which involves the components Y and Z of the external load.

An approximate solution may be obtained to Equations [46] and [54] based on the assumption that the expressions which represent χ and Q are of such a type that the first two terms on the left of Equations [46] and [54] are of much more importance than the third term, and that this third term may therefore be neglected.

Equation [46] becomes then when the term in χ is omitted

$$\chi'' + \chi'[\cot\varphi + 3(\delta'/\delta)] = a^2Q/D \quad [56]$$

The term in χ' may be removed from this equation by the following substitution giving an equation in the new variable $\bar{\chi}$. Let

$$\chi = \bar{\chi}e^{-1/2\int(\cot\varphi + 3\delta'/\delta)d\varphi} = \bar{\chi}/(\delta^3\sin\varphi)^{1/2} \quad [57]$$

Substitute in Equation [56] and neglect terms in the variable itself.

$$\bar{\chi}''/(\delta^3\sin\varphi)^{1/2} = a^2Q/D \quad [58]$$

Omitting the term in Q, Equation [54] becomes

$$Q'' + Q'[\cot\varphi - (\delta'/\delta)] = -E\delta\chi \quad [59]$$

The term in Q' is removed by the substitution

$$Q = \bar{Q}e^{-1/2\int(\cot\varphi - \delta'/\delta)d\varphi} = \bar{Q}(\delta/\sin\varphi)^{1/2} \quad [60]$$

Substituting in Equation [59] and neglecting terms in the variable itself gives

$$\bar{Q}''(\delta/\sin\varphi)^{1/2} = -E\delta\chi \quad [61]$$

Equation [60] should now be substituted into [58] and Equation [57] into [61].

$$\bar{\chi}''\delta = [12a^2(1-\mu^2)/E]\bar{Q} \quad [62]$$

$$\bar{Q}''\delta = -E\bar{\chi} \quad [63]$$

Eliminating $\bar{\chi}$ between Equations [62] and [63] gives

$$\delta^2\bar{Q}^{IV} + 2\delta\delta'\bar{Q}''' + \delta\delta''\bar{Q}'' + 12a^2(1-\mu^2)\bar{Q} = 0 \quad [64]$$

Substituting the value of δ as given by Equation [1] gives

$$(1+\alpha\varphi)^4\bar{Q}^{IV} + 4\alpha(1+\alpha\varphi)^3\bar{Q}''' + 2\alpha^2(1+\alpha\varphi)^2\bar{Q}'' + 12(a^2/d^2)(1-\mu^2)\bar{Q} = 0 \quad [65]$$

This equation is reduced to one with constant coefficients by the following substitutions. Let

$$u = 1 + \alpha\varphi \quad [66]$$

$$u^4\frac{d^4\bar{Q}}{du^4} + 4u^3\frac{d^3\bar{Q}}{du^3} + 2u^2\frac{d^2\bar{Q}}{du^2} + 4k^4\bar{Q} = 0 \quad [67]$$

where k is given by Equation [10]. Now let

$$u = e^t \quad [68]$$

$$\frac{d^4\bar{Q}}{dt^4} - 2\frac{d^3\bar{Q}}{dt^3} + \frac{d^2\bar{Q}}{dt^2} + 4k^4\bar{Q} = 0 \quad [69]$$

The corresponding characteristic equation has roots of

$$m = (1/2)[1 \pm (1 \pm 8ik^2)^{1/2}] \quad [70]$$

but since $1 \ll 8k^2$ the term unity under the radical will be neglected giving values of m of

$$m = (1/2) \pm (1 \pm i)k \quad [71]$$

The homogeneous value of the shearing force is thus found to be

$$\begin{aligned} {}^cQ &= \frac{d^{1/2}(1+\alpha\varphi)}{(\sin\varphi)^{1/2}}\{A_1e^{[1/2+(1+i)k]t} + A_2e^{[1/2+(1-i)k]t} + A_3e^{[1/2-(1-i)k]t} + A_4e^{[1/2-(1+i)k]t}\} \\ &= \frac{d^{1/2}(1+\alpha\varphi)}{(\sin\varphi)^{1/2}}\{e^{(1/2+k)t}(A_5\cos kt + A_6\sin kt) + e^{(1/2-k)t}(A_7\cos kt + A_8\sin kt)\} \\ &= \frac{d^{1/2}(1+\alpha\varphi)}{(\sin\varphi)^{1/2}}[C_1e^{(1/2+k)t}\cos(kt-\gamma_1) + C_2e^{(1/2-k)t}\cos(kt-\gamma_2)] \end{aligned} \quad [72]$$

Upon substitution of $t = \log(1+\alpha\varphi)$ the value for the shearing force as given by Equation [2] is obtained. The rotation of the meridional tangent χ may be obtained by substitution into Equations [60] and [61].

Combining Equations [42] and [43] gives

$$v' - v\cot\varphi = a(\epsilon_1 - \epsilon_2) \quad [73]$$

The integral of this is

$$v = C\sin\varphi + a\sin\varphi\int\frac{\epsilon_1 - \epsilon_2}{\sin\varphi}d\varphi \quad [74]$$

where C is the constant of integration.

Substituting Equations [44] and [45] into Equation [43] yields

$$w = (a/E\delta)(N_2 - \mu N_1) - v\cot\varphi \quad [75]$$

Summary

An approximate method of treating spherical shells of variable thickness, similar to that developed for spherical shells of constant thickness (10), is developed in this paper.

It has been shown by Hetényi (10) that the method of approximation as used here for shells of variable thickness will, when applied to the case of constant wall thickness, give results which

are almost identical with those given by the exact solution of the differential equations. Very likely this same relative accuracy will hold when the wall thickness varies. In all cases substitution in the differential equations constitutes the final criterion.

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The Spring Clutch

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The mathematical theory is developed for the spring clutch which consists of two coaxial cylinders placed end to end and coupled torsionally by a coil spring fitted over them. Relations are derived whereby it is possible to design spring clutches in terms of the requirements and the constants of the spring material. Experimental verification of the relations is given. The theory of residual and active stresses as applied to the springs is discussed.

THE TYPE of spring clutch to be discussed here consists of two cylinders placed end to end, rotating on a common axis, and torsionally coupled by the friction between the cylinders and a coil spring fitted over the cylinders. Such clutches have been extensively used on telephone-dial governors to permit the dial to be wound up without dragging the governor and to couple the governor to the system with a negligible backlash for motion in the pulsing direction. A photograph of such a clutch, assembled and apart, from a telephone dial is shown in Fig. 1. In spite of widespread use there seems to be little theoretical discussion of this device in the literature.

It is obvious that if the driving drum be rotated in the direction to wind up the spring and decrease the diameter, the spring will grip the cylinders and will be capable of exerting more torque than it would in the direction of rotation which tends to unwind the spring. Equations are to be developed which will permit the calculation of these two torque values in terms of the physical dimensions and the material constants of the clutch.

Torque of Spring Clutch in the Free Direction

In Fig. 2 assume that the spring is fastened to the left-hand arbor in order that any slipping which may take place must occur on the driving drum on the right. Assume also that the spring is so formed that the inward radial force on the drum per unit length of the material is constant when no torque is applied.

Notation

- l = length along the line of contact of the spring on the arbor, measured from the free end to any point, in.
- μ = coefficient of friction between the spring and the arbor
- r_2 = radius of the arbor, in.
- R_2 = radius to the neutral bending axis of the spring when on the arbor, in.
- N = number of turns on the right-hand arbor
- P = compression in the spring wire at any point due to the applied torque; this is not the stress in the material but the resultant force acting across the entire cross section of the wire, lb
- f_0 = radial force of spring on arbor when no torque is applied, lb per in. of contact line.

As compression exists in the spring wire at any point when the arbor is turned to make the spring unwind, there will be a radial force subtracting from f_0 at every point. This subtracting force is P/r_2. The increase of compression in the wire along the length of the line of contact due to friction is

$$dP = \mu(f_0 - P/r_2)dl \quad [1]$$

which upon integration gives

$$l = -(r_2/\mu) \ln [f_0 - (P/r_2)]C$$

where C is a constant of integration equal to $1/f_0$ since $P = 0$ at $l = 0$. Hence

$$P = r_2 f_0(1 - e^{-\mu l/r_2}) \quad [2]$$

Since $l = 2\pi r_2 N$, $P = r_2 f_0(1 - e^{-2\pi N\mu})$ [3]

Since the torque is equal to Pr_2

$$T = r_2^2 f_0(1 - e^{-2\pi N\mu}) \text{ (in-lb)} \quad [4]$$

It will be observed that for any but fractional values of $N\mu$ the exponential term becomes very small and

$$T = r_2^2 f_0 \qquad (N\mu > 1) \quad [5]$$

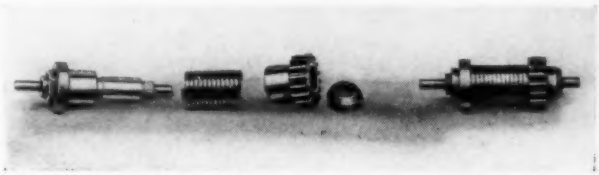


Fig. 1 Telephone-Dial Clutch

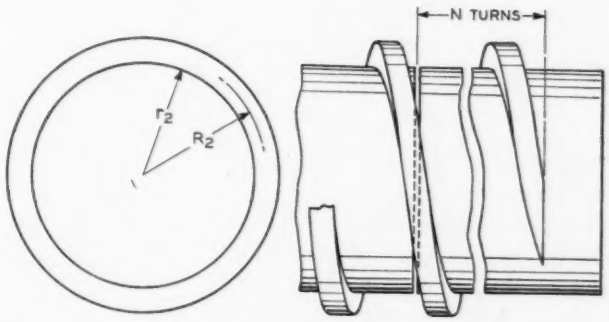


Fig. 2 Diagram of Spring Clutch

If $N\mu = 1.0$ this expression is in error by only 0.2 per cent. It can thus be seen that provided only that $N\mu$ does not become too small, variations in N or μ do not affect the torque exerted. The torque will depend only on the radius of the arbor and on the force f_0 which is controlled entirely by the dimensions and the elastic properties of the spring.

Torque of Spring Clutch in the Gripping Direction

If the torque is applied to the clutch in the direction to wind up the spring, instead of unwind it as in the previous case, the force P'/r_2 due to the tension P' in the spring wire adds to the inward force f_0 and the relation corresponding to Equation [1] is

$$dP' = \mu(f_0 + P'/r_2)dl \quad [6]$$

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[1] Member of the technical staff of the Bell Telephone Laboratories, Inc.

Presented at the National Meeting of the Applied Mechanics Division of The American Society of Mechanical Engineers, New York, N. Y., June 14–15, 1939.

Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y., and will be accepted until October 10, 1939, for publication at a later date. Discussion received after the closing date will be returned.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

From which by the same method as before

$$P' = r_2 f_0(e^{2\pi N\mu} - 1) \quad [7]$$

The corresponding torque is

$$T' = r_2^2 f_0(e^{2\pi N\mu} - 1) \text{ (in-lb)} \quad [8]$$

In this case the torque increases rapidly with an increase in either N or μ especially for large values of $N\mu$. The coefficient of friction is in general a rather variable factor. It is to be expected that the slipping torque will also be variable, but since a lower limit can in general be set for this coefficient it will always be possible to make the number of turns of the spring such as to give any desired lower limit of torque.

The Radial Force on the Arbor

One method of evaluating the force f_0 occurring in the torque relations depends on equating the potential energy of strain per unit length of the wire when on the arbor to the work done in expanding the spring from its free diameter to the diameter of the arbor.

Let

E = Young's modulus for the spring material, psi
I = the area moment of the wire section, in.^{4}
h = the radial thickness of the wire, in.
R_1 = free radius to the neutral axis, in.
r_1 = free inner radius of the spring, in.

Consider the portion of spring wire shown in Fig. 3 straightened out from the initial radius of curvature R_1 to the radius R_2. The

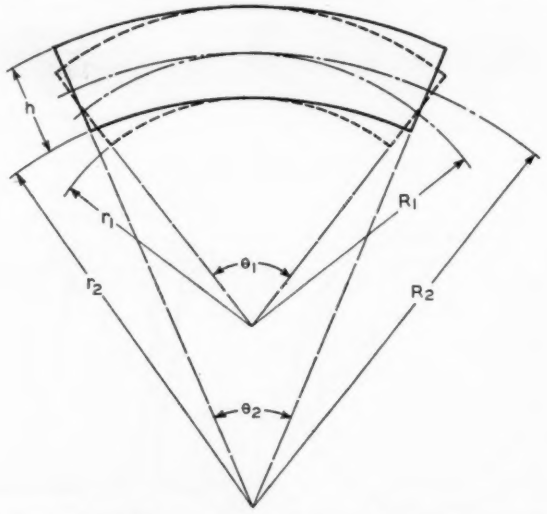


Fig. 3 Element of Spring in Initial and Expanded Condition

fibers above the neutral axis will be compressed while those below the neutral axis will be stretched. Let y be the distance of any given fiber from the neutral axis. The length of the undistorted fiber will be $L = (R_1 + y)\theta_1$ while after bending this same fiber will have the length $L' = (R_2 + y)\theta_2$. The strain or the change in length per unit length is

$$\frac{L - L'}{L} = 1 - \frac{(R_2 + y)\theta_2}{(R_1 + y)\theta_1} \quad [9]$$

Since along the neutral axis there is no change in length, $\theta_2 = L_0/R_2$ and $\theta_1 = L_0/R_1$. Substituting these values in Equation [9] gives

$$\text{Strain} = (y/R_2)\,(R_2 - R_1)/(R_1 + y) \quad [10]$$

The potential energy per unit volume in a material strained in tension or compression is

$$W/V = (E/2)(\text{Strain})^2 \quad [11]$$

Substituting the value of strain from Equation [10] in Equation [11] the energy density at any point of the deflected wire will be

$$\frac{W}{V} = \frac{E}{2}\left[\frac{y(R_2 - R_1)}{R_2(R_1 + y)}\right]^2 \quad [12]$$

Let b represent the width of the wire at the point y. Then for a wire symmetrical about the neutral axis the strain energy per unit length of the wire becomes

$$\frac{W}{l} = \int_{-h/2}^{h/2} \frac{E}{2}\left[\frac{y(R_2 - R_1)}{R_2(R_1 + y)}\right]^2 \frac{R_1 + y}{R_1}\, b\,dy \quad [13]$$

where the factor $(R_1 + y)/R_1$ represents the ratio of the length of the fiber at the point y to the length along the neutral axis. If all values of y, and hence also $h/2$, are small compared to R_1 there will be little error made in neglecting the y, which carries both positive and negative values, in the expression $R_1 + y$. Hence

$$\frac{W}{l} = \int_{-h/2}^{h/2} \frac{E}{2}\left(\frac{R_2 - R_1}{R_2 R_1}\right)^2 by^2 dy \quad [14]$$

The integral of by^2dy is the area moment I of the section and therefore

$$\frac{W}{l} = \frac{1}{2} EI \left(\frac{R_2 - R_1}{R_2 R_1}\right)^2 \quad [15]$$

This must be equal to the work done per unit length of the neutral axis, by the force per unit length $F(\Delta R)$ working through the distance ΔR where $\Delta R = R_2 - R_1$. That is

$$\int_0^{\Delta R} F(\Delta R)\, d\Delta R = \frac{1}{2} EI \left[\frac{\Delta R}{R_1(R_1 + \Delta R)}\right]^2 \quad [16]$$

Differentiating both sides with respect to ΔR gives $F(\Delta R)$ for the left-hand side, and after simplification

$$F(\Delta R) = \frac{EI}{R_1} \frac{\Delta R}{(R_1 + \Delta R)^3} \quad [17]$$

Substituting for ΔR its value $R_2 - R_1$ gives

$$F(\Delta R) = EI(R_2 - R_1)/R_1 R_2^3 \quad [18]$$

The equivalent force per unit length measured along the surface of the arbor must be larger than this value in the ratio of R_2/r_2 since the same total force is here distributed over a shorter length. This latter force is f_0, hence

$$f_0 = \frac{R_2}{r_2} F(\Delta R) = EI \frac{R_2 - R_1}{R_1 r_2 R_2^2} \text{ lb per in} \quad [19]$$

The value of f_0 calculated from this equation in terms of the constants of the spring material and the dimensions of the spring may be used in Equations [4] and [8] to calculate the free torque and the slipping torque of the clutch.

Experimental Check of the Free-Torque Relation

In order to check the validity of the relation for the free torque, Equation [4], and that for the radial force on the arbor, Equation [19], the free torque of a given spring on arbors of various diameters as well as the torque for different numbers of turns on the same arbor was measured. The spring of 0.0085 × 0.022-in. phosphor-bronze ribbon was attached to a short vertical shaft

suspended by a torsion fiber of measured torsional stiffness. The free end of the spring was placed over a vertical arbor capable of rotation. The arbor was rotated and the angle of twist of the torsion fiber was measured thus giving a measure of the slipping torque. The precision of the measurements of torque was about 0.5 per cent although the sensitivity to small changes was about 0.2 per cent. No measurable increase of torque occurred by increasing the number of turns on the arbor beyond six. This is to be expected if the coefficient of friction exceeds about 0.12.

For all succeeding measurements seven to eight turns were used. As a further check on the independence of the torque and the coefficient of friction for this number of turns a measurement was made before and after oiling the arbor and spring with a light lubricating oil. There was a decrease in torque of approximately 0.25 per cent.

The inside diameter of the spring as measured by a taper gage was 0.180 in. ± 0.001 in. The torque for arbors ranging in size from 0.182 to 0.193 in. was determined and is shown in Fig. 4. This curve extrapolated to zero torque gives, for an accurate measure of the inside diameter of the spring, 0.1794 in. Using this value and the quantity EI, determined by obtaining the resonant frequency of a straight short length of the ribbon, of which the spring was made, vibrating as a fixed free reed the radial force on the arbor as calculated by Equation [19] is shown by the dotted curve as a function of the arbor diameter $2r_2$. The points indicate values of f_0 obtained from the measured values of torque by the use of Equation [5]. The two sets of values agree within about 2 per cent.

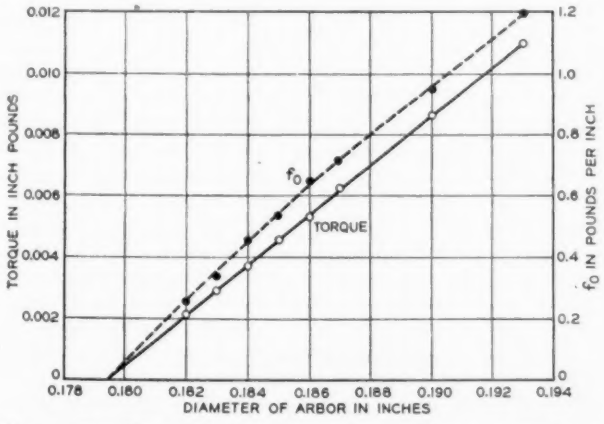


Fig. 4 The Free Torque and the Radial Force on the Arbor for a Phosphor-Bronze Spring on Arbors of Different Diameters

As a further check on the validity of the calculations under practical conditions, the free torque of a phosphor-bronze spring on a dial-governor arbor was measured for various numbers of turns of the spring engaging on the slipping arbor. The torque due to bearing friction alone with no spring in place was also measured and found to be approximately 0.001 in-lb. This constant value was subtracted from the other measured values of torque. The resulting values of free spring torque are plotted as a function of the number of turns in Fig. 5. The torque values calculated by Equations [4] and [19] and using the value of $\mu = 0.165$ obtained as described in the next section, are shown on the curve to indicate the agreement.

Experimental Check of the Holding-Torque Relation

It is to be expected that any relation for which the coefficient of friction is a controlling factor will be difficult to check accurately. Equation [8] for the slipping torque is of this type. It is possible however by taking a large number of measurements to establish the validity of the equation and then by determining limiting values for the coefficient of friction, to use this equation as a design relation, especially if only a minimum torque limit is set. Measurements of the slipping torque, as a function of the number of turns, were made on the same dial clutch that was used for checking the free torque.

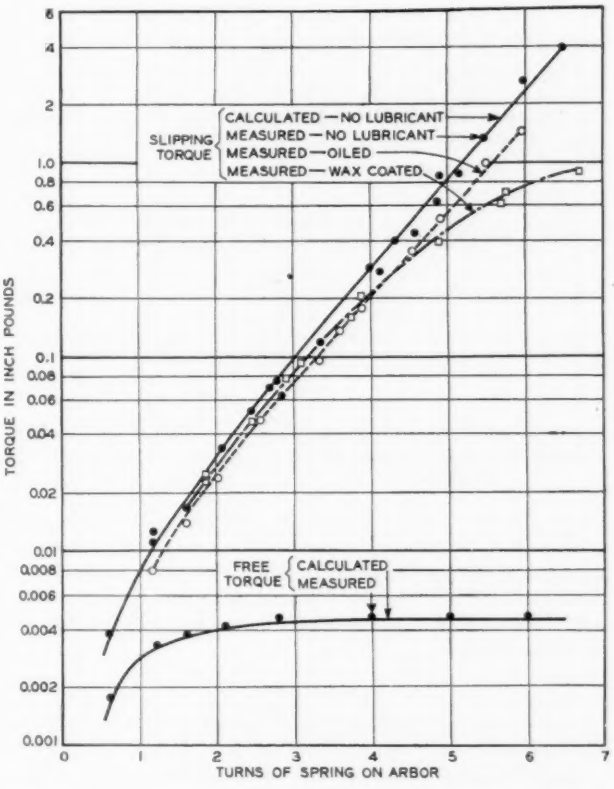


Fig. 5 Dependence of Torque on the Number of Turns of the Spring Engaging the Arbor

This slipping torque was not steady as was the case in the free direction but varied as much as ±20 per cent. An average value was taken in each case. An uncertainty also existed regarding the number of turns engaging the rotating arbor. Since the crossover from one arbor to the other requires practically one whole turn, slight differences in the arbor diameters may result in a gain or loss of almost half a turn. In addition to these factors there was an end effect due to the fact that the free end of the spring wire was cut off square rather than beveled but this factor although calculable was neglected in view of the other uncertainties. The results of these measurements are shown in Fig. 5.

From Equation [8] it can be seen that for large values of N the plot of T' versus N will be a straight line provided μ is independent of the force between the spring and the arbor. The slope of this straight line when multiplied by the proper constant, which can be shown to be 0.368, gives the coefficient of friction. For the experimental points shown in Fig. 5 this value of μ is 0.165. Using this value of μ in Equation [6] the calculated curve was plotted. Considering the uncertainties involved the calculated and measured curves are in good agreement.

The dotted curve of Fig. 5 shows the effect of lubricating the clutch with a light machine oil. This resulted in only a small

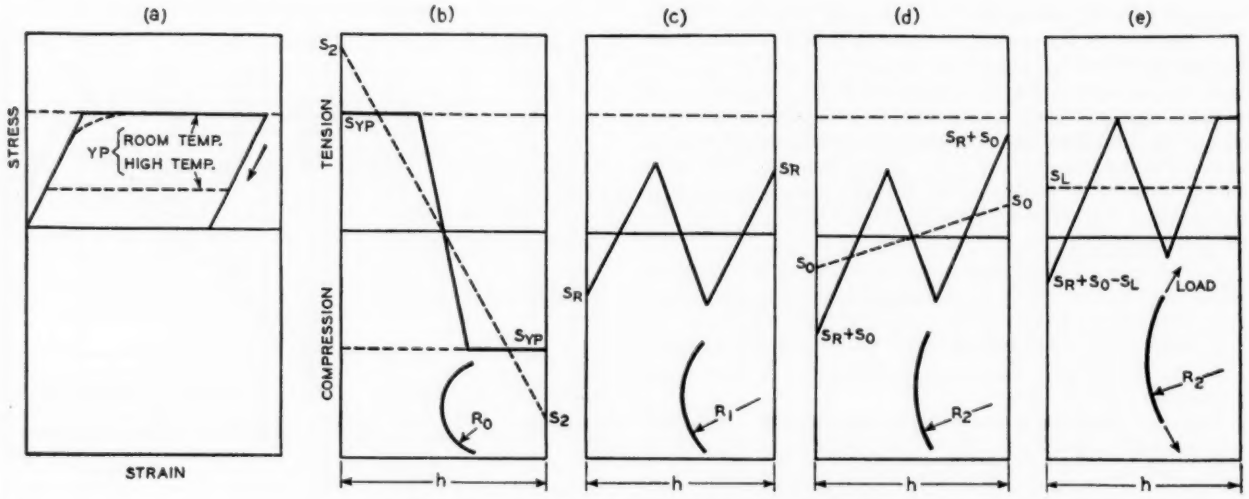


FIG. 6 IDEALIZED STRESS CONDITIONS IN CLUTCH SPRINGS

decrease of the coefficient of friction. The curve shown by the dashes illustrates the effect of lubricating a clutch with spermaceti. The coefficient was no longer a constant but decreased with an increase in load.

SPRING STRESSES

In determining the load that a spring clutch will withstand, first without stretching which will result in backlash, and second without breaking, initial as well as load stresses must be considered. The initial stresses are made up of the residual stresses due to forming the spring, plus the stresses due to expanding the spring to fit the arbor, that is from an inner radius r_1 to an inner radius r_2.

Of these limiting load values the easiest to calculate is the torque required to break the spring. This is given by the product of the radius of the spring and the breaking strength of the spring wire. Loads much smaller than this value would stretch some of the fibers of the spring, especially those in which a high initial stress already existed. As will be shown, such stretching will cause the radius to increase at those portions of the spring where the applied stresses are highest, that is, for those turns near the dividing line of the arbor.

Substituting for y, in Equation [10], the distances from the neutral axis to the extreme inner and outer fibers will give the strain in these fibers. Provided R_1 is considerably larger than $h/2$, half the thickness of the material, the distances to these extreme fibers becomes $h/2$ and the y in the denominator can be neglected in comparison to R_1. The maximum fiber stress due to placing the spring on the arbor is this value of strain multiplied by Young's modulus. Then

$$S_0 = \frac{h}{2R_2}\,\frac{R_2 - R_1}{R_1}E \qquad [20]$$

This stress is in the form of compression for the outer fibers and tension for the inner. Since a load on the clutch results in a tension in the spring, the stress given by Equation [20] must be added to the load tension stress to get the total stress on the inner fibers of the spring. This initial stress therefore reduces the load-carrying capacity of the clutch.

RESIDUAL SPRING STRESSES

When a straight wire is wound upon an arbor to form a coil spring the strain on the inner and outer fibers must exceed that corresponding to the yield-point and plastic-flow results. To simplify the discussion assume the idealized stress-strain curve shown by the heavy lines of Fig. 6(a). The stress distribution across any section of the wire while wound on the winding arbor will be as shown by the heavy lines of Fig. 6(b) where S_{YP} is the yield-point stress, the maximum stress that the material will sustain. The moment, across the section, required to produce this bending is

$$M = \int_{-h/2}^{h/2} Sbydy \qquad [21]$$

where b is the width of the wire at any point of y distance from the neutral axis and S is the stress at the same point. If the spring is released, it expands to a radius R_1 in which condition the external and the internal moments are both zero. It is now possible by applying the same moment as was given by Equation [21] to reduce the radius of curvature again to R_0 but without causing additional plastic flow. The added stress distribution produced by this second bending must therefore follow a straight line as shown by S_2-S_2, which together with the residual stresses in the relaxed condition (radius R_1) must equal the distribution S_{YP}-S_{YP} resulting from the original forming operation. Therefore the stress distribution in the relaxed condition (radius R_1) must be the difference between S_{YP}-S_{YP} and S_2-S_2 or as indicated in Fig. 6(c). The value of S_2 is of course so determined that the moment as specified by Equation [21] is the same for the dotted-line as for the solid-line stress distribution.

The value of $S_R = S_2$-S_{YP} is relatively easy to determine for rectangular and round wire, on the basis of the straight-line stress-strain characteristic if the bending has been sufficiently severe to have caused plastic flow almost to the neutral axis. The moment given by the actual stress distribution will then differ but little from that obtained by Equation [21] with S replaced by S_{YP}, a constant. The values of S corresponding to the S_2-S_2 distribution are given by

$$S = 2S_2y/h \qquad [22]$$

For a rectangular wire b is a constant and equating the moments corresponding to the two stress distributions gives

$$\int_{-h/2}^{h/2} S_{YP}bydy = \int_{-h/2}^{h/2} 2\,\frac{S_2}{h}\,by^2dy$$

from which $S_2 = (3/2)S_{YP}$ or

$$S_R = S_2 - S_{YP} = (1/2)S_{YP} \text{ (rectangular wire)} \qquad [23]$$

Similar integrations in the case of a round wire of radius $h/2$ for which $b = 2\sqrt{[(h/2)^2 - y^2]}$ gives

$$S_R = 0.7\ S_{YP} \text{ (round wire)} \quad [24]$$

These equations give the residual fiber stresses in the extreme inner and outer fibers under the assumed conditions. In any case where the stress-strain characteristic is known a correct value for the residual stress can be obtained by graphical integration. The values given by Equations [23] and [24] will however be fair approximations even in cases where the stress-strain characteristic is curved provided it does not exhibit strain hardening to a decided extent. Since a limited amount of plastic flow takes place for any stress above the proportional limit, which is generally far below the yield point, the residual stress may be sufficient to cause a small amount of creep. Any additional stresses will then cause permanent deformation of the spring.

The analysis will be continued on the basis of the idealized straight-line characteristic. Fig. 6(*d*) shows the result of placing the spring on the clutch arbor. The line S_0-S_0 shows the stresses added by this expansion where S_0 is given by Equation [20]. The sum of these stresses and those shown in Fig. 6(*c*) gives the total stresses as shown by the solid line of Fig. 6(*d*). If now a load be put on the clutch a uniform stress S_L will be added but this stress for even relatively light loads may be sufficient to cause the total stress on the inner fibers to exceed the yield point as is indicated in Fig. 6(*e*). The inner fibers are consequently stretched and when the load is released and the spring taken from the arbor it will be found that the center turns of the spring have expanded. Even with the spring on the arbor if the clutch is turned in the free direction it will be noticed that these center turns raise off the arbor. It was shown in the paragraphs on the clutch torque in the free direction that the torque did not increase appreciably after the first few turns. This can be explained by the fact that as soon as the outward radial force due to the compression along the wire is equal to the initial inward radial force of the spring on the arbor the friction on these turns vanishes. The value of the compression will be fixed by a relatively few end turns. Hence if the inward force of some of the center turns decreases due to their stretching this compression will be sufficient to expand the turns to clear the arbor. If S_L is still further increased the stretch will be sufficient to cause the diameter of the center turns to exceed the arbor diameter even when no torque is applied in the free direction.

Since the yield point of metals decreases at higher temperature it is possible to produce a spring having lower residual stresses by the proper heat-treatment. If the wire is wound on an arbor and then heated, additional plastic flow takes place since the maximum stress that can be sustained at the high temperature is that shown in Fig. 6(*a*) as the high-temperature yield point. If the spring is then cooled and released the expansion will not be as great as for the untreated spring. The residual stresses will again be given by Equation [23] or [24] where S_{YP} is taken as the lowest yield point reached in the temperature cycle. Fig. 7(*a*), (*b*), and (*c*) show the stresses in the heat-treated specimen corresponding to those shown in Fig. 6(*c*), (*d*), and (*e*) for the untreated spring. Fig. 7(*c*) shows that for the same load stress as for Fig. 6(*e*) no permanent deformation has taken place.

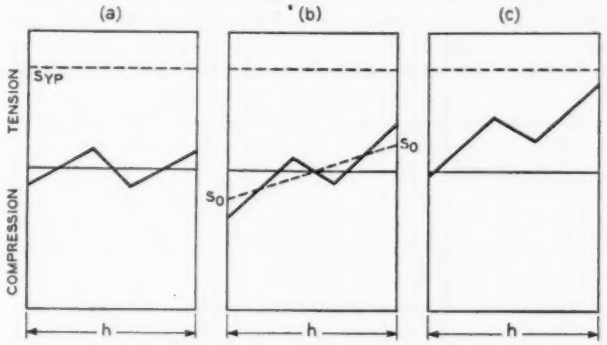


FIG. 7 STRESS CONDITIONS IN A STRAIN-ANNEALED CLUTCH SPRING

It is of course important that the strain-relieving temperature should not go high enough to lower permanently the strength of the material. This limit[2] for phosphor bronze is about 320 C.

TABLE 1 EFFECT OF HEAT-TREATMENT ON SPRINGS

Ribbons of 18-8 stainless steel, 0.0068 × 0.021 in., heat-treated on 0.1486-in. winding mandrels

Heat-treatment temp, C	Time, hr	Diam after release, in.	Residual stress, psi
25	..	0.228	111000
100	4	0.206	88000
200	4	0.188	66000
300	4	0.182	58000
400	4	0.175	47000
470	4	0.169	37000
500	4	0.166	33000
400	1/4	0.177	51000
400	1/2	0.176	49000
400	1	0.176	49000
400	2	0.175	47000
400	3	0.175	47000
400	4	0.175	47000

To determine the stress-temperature characteristics of 18-8 stainless steel[3] a number of springs of 0.0068 × 0.021-in. ribbon were wound on 0.1486-in. arbors and given various heat-treat-

[2] "Better Instrument Springs," by Robert W. Carson, Trans. A.I.E.E., vol. 52, September, 1933, p. 869.

[3] 8 per cent nickel, 18 per cent chromium.

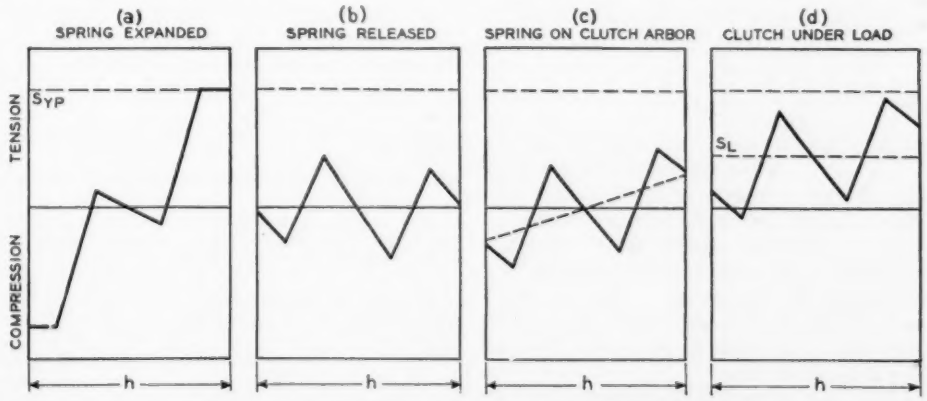


FIG. 8 STRESS CONDITIONS IN AN EXPANDED CLUTCH SPRING

ments. They were then cooled, released from the arbors, and measured for inside diameter. Table 1 gives the results of these measurements.

The residual stresses were calculated by noting from Equation [23] that $S_R = S_2/3$ and then obtaining S_2 from Equation [20] rewritten as

$$S_2 = \frac{h}{2R_1} \frac{R_1 - R_0}{R_0} E \quad \ldots\ldots\ldots\ldots [25]$$

Straight pieces of the stainless-steel ribbon were given the same series of heat-treatments as the springs. Young's modulus was determined for each of these samples and a bending test was also applied to determine whether the wire had been permanently annealed. No appreciable effect was noted. The proportional limit and the ultimate tensile strength of this ribbon at room temperature as measured on a tensile-testing machine were 41,600 and 252,000 psi, respectively. It is thus seen that except with the high-temperature anneals the residual stress alone exceeds the proportional limit and any additional stress will cause a permanent deformation.

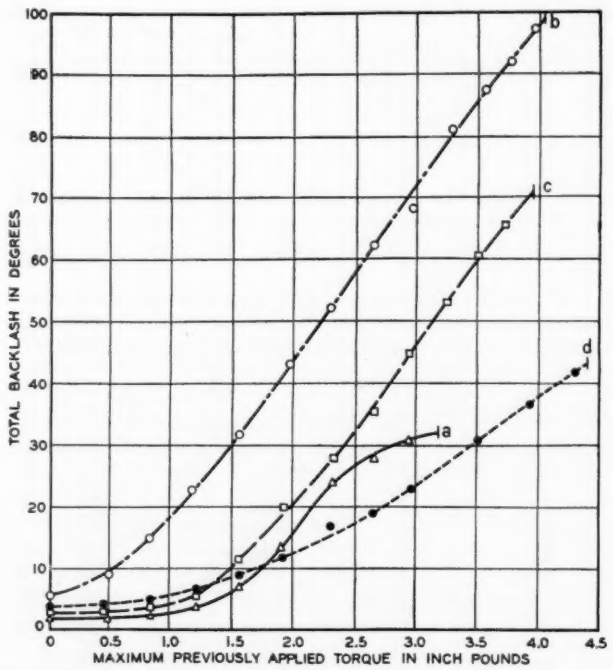


FIG. 9 EFFECT OF OVERLOAD ON CLUTCH BACKLASH. ARBOR DIAMETER, 0.190 IN.

(*a*) Phosphor-bronze-ribbon spring, 0.0085 × 0.022 in., heat-treated hr at 230 C, 0.1835 in. ID.
(*b*) 18-8 stainless-steel ribbon, 0.0068 × 0.021 in., wound on 0.127 in. mandrel and released, 0.183 in. ID.
(*c*) 18-8 stainless-steel ribbon, 0.0068 × 0.021 in., wound on 0.120-in. mandrel and released, 0.170 in. ID, mechanically expanded to 0.183 in. ID.
(*d*) 18-8 stainless-steel ribbon, 0.0068 × 0.021 in., wound on 0.157-in. mandrel, heated 4 hr at 470 C, cooled, and released, 0.182 in. ID.

To verify the validity of these arguments three springs of stainless-steel ribbon with different preliminary treatments and one of heat-treated phosphor bronze were tested for backlash as a function of previous loading. The backlash angle was measured from a point of slipping in the free direction to the point in the holding direction at which it would sustain a load of 0.05 in-lb. An initial load of 0.5 in-lb was then applied and removed and the backlash measured as before. This was repeated for various loads up to the breaking point of the spring. The results are shown in Fig. 9. In the case of the untreated stainless steel the backlash began to increase immediately. Its higher initial value was probably due to the unavoidable stressing occasioned by assembling the spring on the clutch arbor and to the 0.05-in-lb testing torque. The backlash of the other three samples remained constant to slightly above 0.5 in-lb and then began to rise. At low loads the phosphor bronze was better while at higher loads the heat-treated stainless steel had the advantage; the breaking load for the latter was also about 30 per cent higher.

CONCLUSION

The relations developed in the preceding sections are sufficient to determine uniquely the correct spring dimensions for a spring clutch of specified free and gripping torque provided the material constants and the cross-sectional shape (round or rectangular) of the wire as well as the length and diameter of the clutch arbor, are specified. Choice of values for the last two factors is based largely on permissible heating resulting from the slipping in the free direction. Furthermore, as would generally be the case, if only maximum values of the free torque and minimum values of the gripping torque are given a number of solutions can be obtained from which to choose the most convenient. By combining the relations derived, in a manner to permit step-by-step calculation, the design of spring clutches is reduced to a simple routine.

Boundary Friction in Bearings at Low Loads

By L. M. TICHVINSKY[1] and E. G. FISCHER,[2] EAST PITTSBURGH, PA.

The paper describes bearing tests performed in the region of semifluid or boundary lubrication. Bearings 2½ in. in diameter mounted in a rigid housing were loaded lightly and their performance studied in an attempt to correlate it with various physical properties of the bearing and journal materials. In this connection special consideration was given to journal surface finish and its measurement.

A CAREFULLY designed bearing performs well in the region of fluid lubrication. Such performance is characterized by a continuous viscous film possessing high load-carrying capacity. Due to accurate machining of the rigid journal and bearing also to correct selection of materials and lubricants the journal runs smoothly in the shell. It is separated from the latter by an oil film, the minimum thickness of which must be larger than the sum of the journal- and shell-surface irregularities (1).[3]

Such smooth bearing performance can be attained in bearings after they have gone through the short period of boundary or semifluid friction at the beginning of rotation before the continuous film is established. The same type of boundary friction must occur just before the full stop. These regions are indicated in Fig. 1 where the coefficient of friction f is plotted against the bearing characteristic number ZN/P. Z is the viscosity of the lubricant in centipoises, N = revolutions per minute, and P = bearing pressure on the projected area, pounds per square inch. The minimum oil-film thickness h_0 is also indicated, and is fully developed only in the region of fluid friction.

It is realized that a bearing performs under boundary conditions for only a very short interval. However, it can well be realized how detrimental such action might be due to wear of the sliding surfaces in the absence of a continuous oil film. Interesting experiments made in the National Physical Laboratory in England (2, 3) revealed that with sliding contact, high localized temperatures occur not only during dry-friction performance but also in the region of boundary lubrication. This high temperature undoubtedly accelerates local wiping and scoring of the bearing metal. In the boundary region the thin film separating the sliding surfaces has a thickness which is measured in terms of molecules. The unknown reactions between molecules of the fluid and surfaces determine the nature of the frictional resistance. Unfortunately little is known of the complex mechanism of boundary friction. Most experiments reveal that semifluid friction depends on a number of factors, such as adsorption, adhesion, and molecular orientation. Apparently in this type of friction adsorption plays a predominant role. The mechanism for lubrication becomes possible as a result of this physical property of the lubricating substance which causes it to adhere to the sliding surfaces. On two such surfaces the molecules of lubricant form continuous adsorption layers. A schematic representation of the adsorption layer made by Woog (4) often appears in the literature due to its simplicity and clearness. The boundary layers consist of molecules of lubricant that cling by their more active ends to the sliding surfaces, as shown in Fig. 2. The relative motion takes place on the sliding plane E, where the less active parts of the molecules meet and cause frictional resistance to decrease considerably. The magnitude of the forces which attract the active parts of molecules to the metal surfaces depends on the chemical properties of the lubricant and the type and finish of both the journal and the shell. For most of the organic compounds, to which all lubricants belong, the adhesion to such metals as steel, bronze, and babbitt is extremely high.

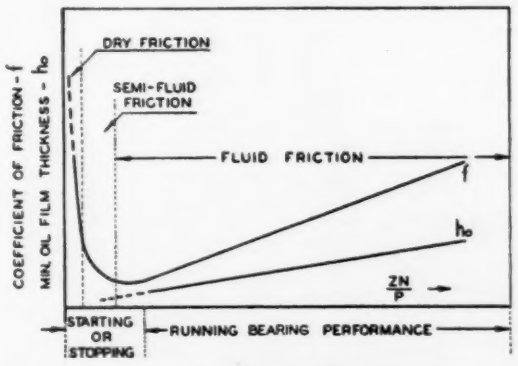


FIG. 1 JOURNAL-BEARING PERFORMANCE
(When starting or stopping, the bearing usually performs in the regions of dry and of semifluid friction.)

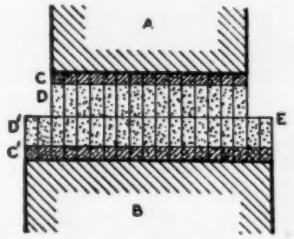


FIG. 2 SCHEMATIC REPRESENTATION OF ADSORPTION LAYERS
A and B, metal surfaces
CD and $C'D'$, molecular layers
E, sliding plane
C and C', active parts of molecules
D and D', less active parts of molecules.

Because of this some physicists consider the adsorption layers to be firmly attached to the metal.

The bearing-testing apparatus used in this investigation incorporates many features common to actual bearing operation. For example, the sliding surfaces are curved, actual bearing and journal materials are used, and the grinding scratches of the journal are parallel to the direction of sliding motion. The authors endeavor to answer the questions: Does every kind of journal material and grade of finish give equal performance with the same babbitt shell? Does the boundary oil film formed on various combinations of bearing and journal materials have the same strength and frictional resistance?

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[2] Mechanical Engineer, Westinghouse Electric & Manufacturing Co. Jun. A.S.M.E.

[3] Numbers in parentheses refer to the Bibliography.

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Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y., and will be accepted until October 10, 1939, for publication at a later date. Discussion received after the closing date will be returned.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

Test Setup

The tests were made on the bearing-testing machine shown in Fig. 3. The loading and torque-recording apparatus of the machine are shown schematically in Fig. 4. The bearings, as seen in Fig. 3, were mounted by means of steel bushings in a rigid housing made in two halves and bolted together. Interchangeable $2^1/_2$-in-diam journal sleeves, Fig. 5, were mounted on the shaft of the testing machine, Fig. 6. The shaft was mounted on two roller bearings and connected through a flexible coupling to a variable-speed motor, which provided a wide range of speed by means of armature and field voltage control.

Light bearing pressures up to 28 psi were applied by tightening the turnbuckle and the load was registered on the two spring scales P_L and P_R mounted parallel. An auxiliary weighing device was used to measure directly the friction torque in the

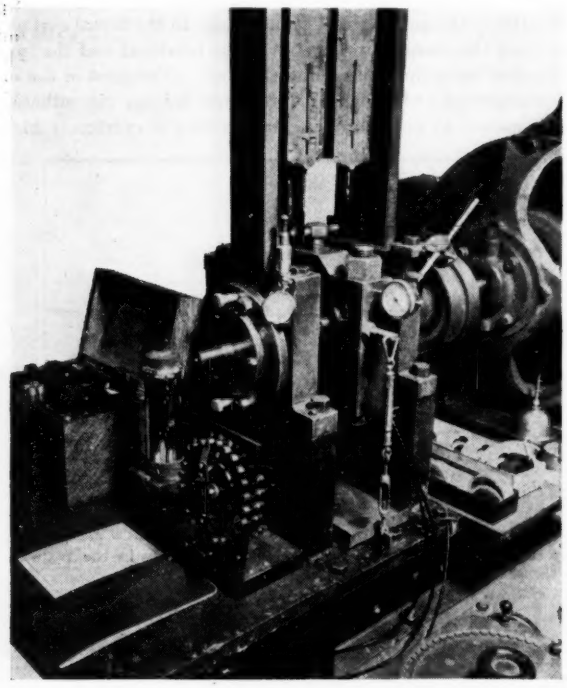


FIG. 3 MACHINE FOR TESTING BEARINGS

bearing. The zero position was previously set by the bubble level with the apparatus suspended free of the journal. When balanced, there was a negligible component of load opposing the bearing friction forces, and the weight of bearing housing and the friction pull were subtracted from the reading of the total suspended load.

Test Specimens

Journal sleeves were prepared from various shaft materials to the dimensions shown in Fig. 5. Measurements by micrometer caliper showed that all the 2.5-in-diam sleeves were ground within ±0.0005 in. A description of the journal materials used in the tests follows:

1 Nickel steel. Machined from a $2^3/_4$-in. round bar.

2 Free machining 18-8 steel. Machined from a 3-in. round bar.

3 Chromium steel, 12 per cent chromium, chromium-plated. Machined from a $2^3/_4$-in. round bar, ground to 2.495 in. OD, chromium-plated, above size, and reground to 2.500 in. OD.

4 Carbon steel sprayed with high-carbon steel. Machined

TABLE 1 PHYSICAL PROPERTIES OF JOURNAL-SLEEVE MATERIALS

Sleeve no.	Yield point, psi	Tensile strength, psi	Elongation, per cent	Reduction, per cent	Brinell hardness: Material	Brinell hardness: Tested sleeves[a]
1	76000	107500	23.5	57.3	...	217
2	46000	91500	63.5	69.9	170	180
3	92000	111000	23.0	65.4	235	275
4	56000	94600	26.0	48.0	...	213
5	56000	94600	26.0	48.0	...	82
6	116500	160000	25.0	44.9	321	330

[a] These data taken on the edge of the journal-sleeve surfaces.

TABLE 2 CHEMICAL COMPOSITIONS OF JOURNAL-SLEEVE MATERIALS

Sleeve no.	C	Mn	P	S	Si	Ni	Cr	Se
1	0.33	0.73	0.015	0.021	0.19	3.43	...	...
2	0.08	0.86	0.129	0.014	0.52	8.79	18.42	0.253
3	0.10	0.48	0.019	0.016	0.43	0.18	12.20	...
4	0.48	0.95	0.037	0.030	0.20	..	...	...
5	0.48	0.95	0.037	0.030	0.20	...	...	...
	C	Mn	Si	Cu	Ni	Fe	Al	Zn
6	0.22	0.45	0.30	29.20	64.78	1.38	2.70	0.95

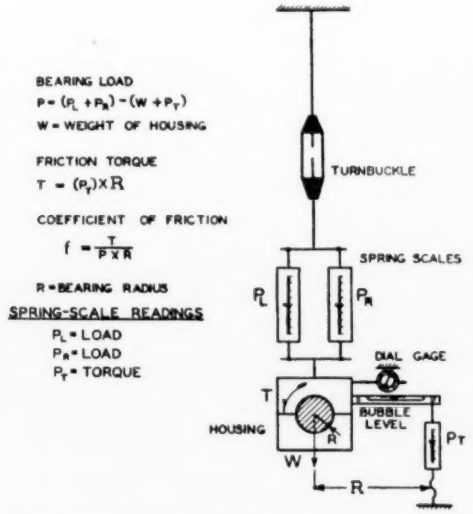


FIG. 4 SCHEMATIC DIAGRAM OF LOADING AND TORQUE APPARATUS

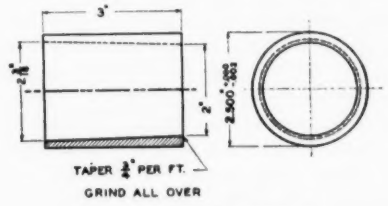


FIG. 5 JOURNAL SLEEVE

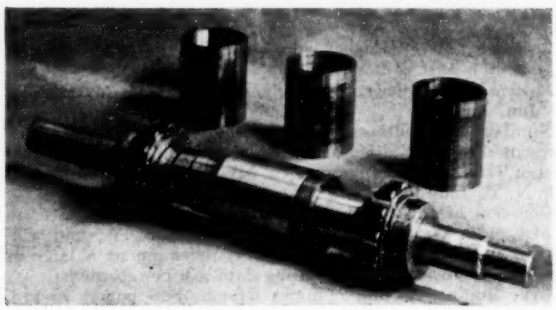


FIG. 6 JOURNAL SLEEVES AND SHAFT OF TESTING MACHINE

TABLE 3 MAXIMUM DEPTH OF GRINDING SCRATCHES

Sleeve no.	Material	Microinches: Before tests	Microinches: After tests	Difference
1	Nickel steel	89	80	− 9
2	Free-machining 18-8 steel	133	89	−44
3	Chromium steel, chromium-plated	44	25	−19
4	Carbon steel sprayed with high-carbon steel	22	100	+78
5	Carbon steel sprayed with 18-8 stainless steel	100	114	+41
6	Monel metal	36	36	0

from a $2^3/_4$-in. round bar to 2.480 in. OD, sprayed oversize with 1.2 per cent carbon steel, and ground to size.

5 Carbon steel sprayed with 18-8 stainless steel. Machined from a $2^3/_4$-in. round bar to 2.480 in. OD, sprayed oversize with low-carbon 18-8 stainless steel, and ground to size.

6 Monel metal. Machined from $2^5/_8$-in. round bar.

The physical properties and chemical compositions of these materials are given in Tables 1 and 2.

Photographs of the surface finish of these journal sleeves were made by the optical method[4] before and after tests so that changes in the profiles could be measured. In Figs. 7 and 8 the upper photographs show the surface profiles of the six new journals, the lower, the same surface profiles after tests. All photographs were made with a magnification of about 280×. Each division of the scale shown on the profile photographs corresponds to 0.0002 in. The roughness of the journals, measured as the maximum depth of grinding scratches, is given in Table 3.

[4] "Surface Finish," by Stewart Way, *Mechanical Engineering*, vol. 59, November, 1937, pp. 826–828.

TABLE 4 CHEMICAL COMPOSITION OF BEARING MATERIALS

Bearing no.	Material	Sn	Cu	Pb	Sb	Fe	Ag
1	Babbitt	85.00	5.00	0.17	9.63	Trace	...
2	Bronze no. 1	1.00	61.00	38.00	..	...	...
3	Bronze no. 2	9.00	72.00	19.00	..	...	...
4	Silver	...	...	...	..	...	100

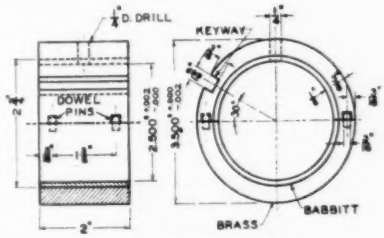
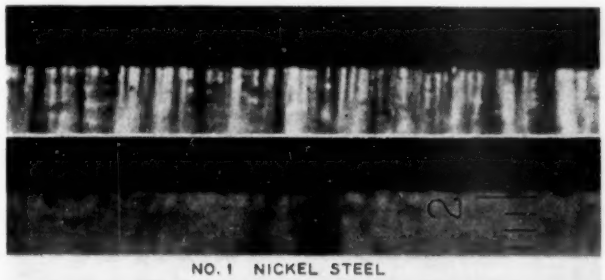
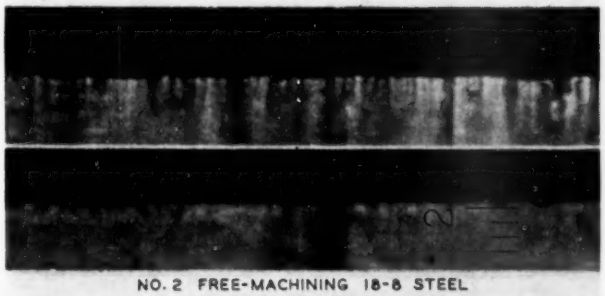


FIG. 9 BEARING BUSHING

The bearing bushings were prepared in accordance with Fig. 9. A small hole was drilled in the middle of the bottom (loaded) half of the bearing as close as possible to the bearing surface, and a thermocouple was inserted to measure the temperature. Two additional thermocouples were used to measure the rise of housing


NO. 1 NICKEL STEEL


NO. 2 FREE-MACHINING 18-8 STEEL

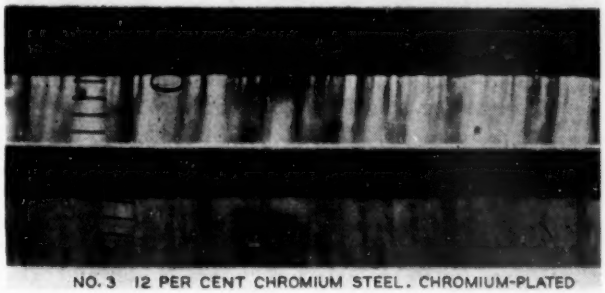
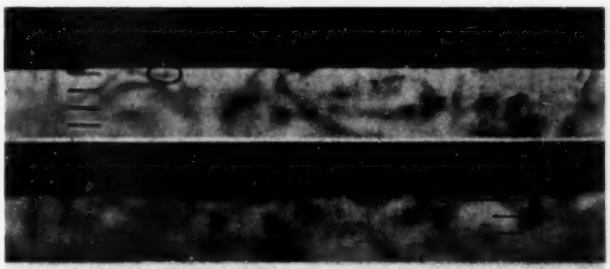
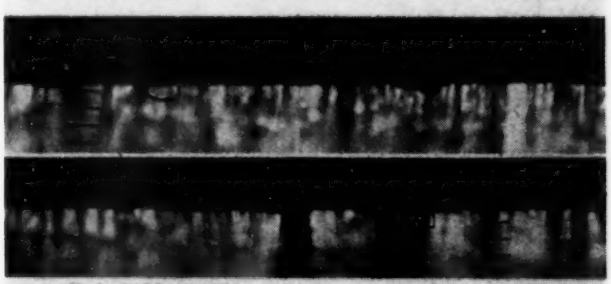

NO. 3 12 PER CENT CHROMIUM STEEL. CHROMIUM-PLATED

Fig. 7


NO. 4 CARBON STEEL SPRAYED WITH 1.20 PER CENT CARBON STEEL


NO. 5 CARBON STEEL SPRAYED WITH STAINLESS STEEL

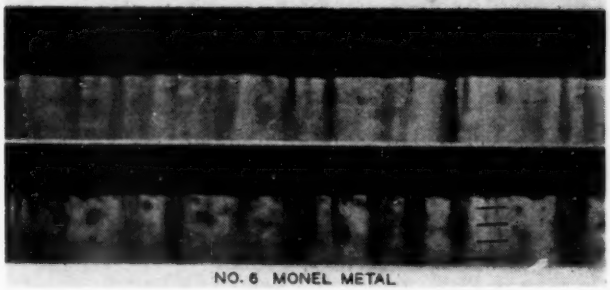

NO. 6 MONEL METAL

Fig. 8

FIGS. 7 AND 8 PHOTOMICROGRAPHS OF SURFACE FINISH OF JOURNAL SLEEVES
(The upper and lower views show the profiles before running-in and after testing. The scale divisions represent 0.0002 in.)

temperature above ambient. The hole shown in the upper half of the bearing permitted lubricating by means of a drop-feed oil cup.

The chemical composition of all bearings tested is shown in Table 4.

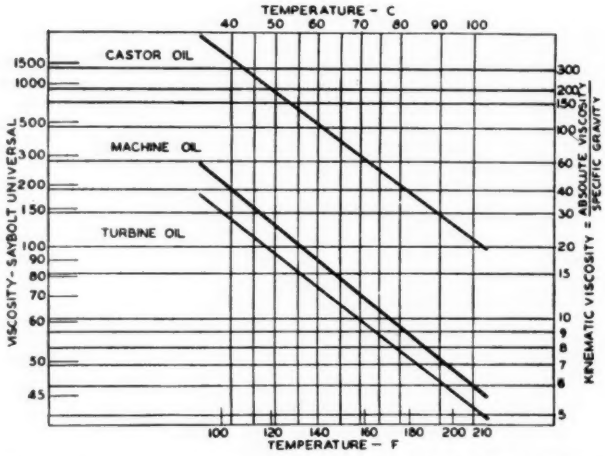


FIG. 10 VISCOSITY-TEMPERATURE CHART OF OILS USED DURING TESTS

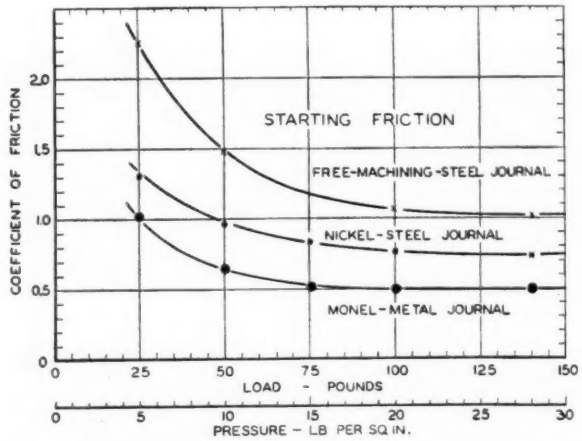


FIG. 11 PERFORMANCE OF THREE DIFFERENT JOURNALS IN A BEARING OF TIN-BASE BABBITT, STARTING FRICTION

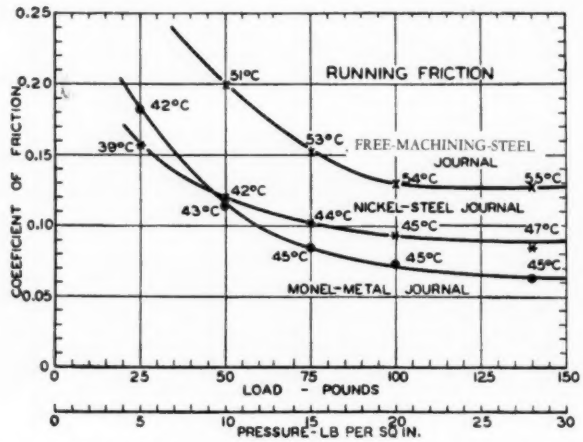


FIG. 12 PERFORMANCE OF THREE DIFFERENT JOURNALS IN A BEARING OF TIN-BASE BABBITT
(Running friction with boundary-layer lubrication.)

Properties of oils which were used during tests are shown in Fig. 10.

Method of Testing

The bearing machine was designed for investigating the condition of partial or boundary lubrication.

Before actual tests all bearings were satisfactorily run-in with oil (from 4 to 6 drops per minute of machine oil) inside of eight hours, during which time the load was increased to 28 psi at 600 rpm.

During the tests, torque measurements were made under starting and running conditions for various loads ranging from 25 to 140 lb (5 to 28 psi of projected area). Running-friction tests were made at 600 rpm (400 fpm) which allowed the maximum range of loading without excessive heating. The drop feed was

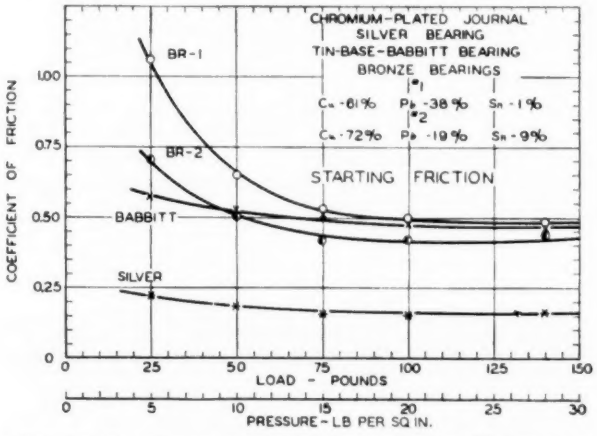


FIG. 13 PERFORMANCE OF DIFFERENT BEARING MATERIALS

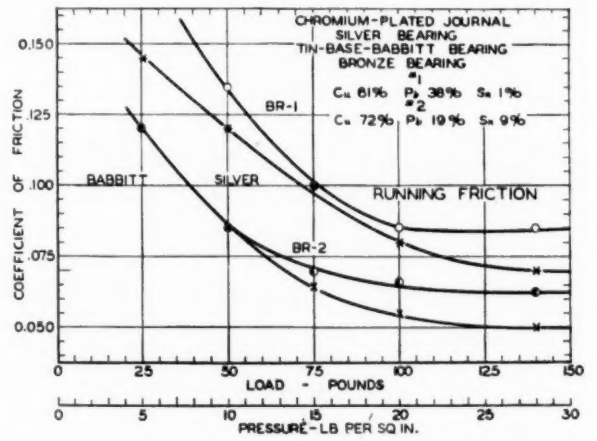


FIG. 14 PERFORMANCE OF DIFFERENT BEARING MATERIALS
(Running friction with boundary-layer lubrication.)

adjusted to provide enough oil for boundary lubrication but not enough to form a fluid film or to provide cooling.

Test Results

Figs. 11 and 12 show the values of the coefficient of friction, starting and running, respectively, for the three journal materials Nos. 1, 2, and 6. The temperatures shown next to each point on the curves for running friction indicate the final bearing-surface temperature for steady-state performance in still air. These journals were run in a tin-base-babbitt bearing. In both cases, starting and running, the free-machining steel showed the most friction, monel metal showed the least, and nickel steel was intermediate. These three materials have been arranged accord-

TABLE 5 COMPARISON OF JOURNAL MATERIALS

Sleeve no.	Material	Friction	Hardness, Brinell	Surface finish, microinches Before tests	After tests
6	Monel metal	Low	330	36	36
1	Nickel steel	Medium	217	89	80
2	Free-machining steel	High	180	133	89

ing to the values of friction, hardness, and roughness in Table 5.

It is of interest to note that the journal roughness, even at the low pressures used, decreased slightly during the running-in and actual test. The hardest material did not show any change in its surface qualities. The wear of the tin-base babbitt was not measured because of the light loads used. The visual examination did not show any wiping or scoring.

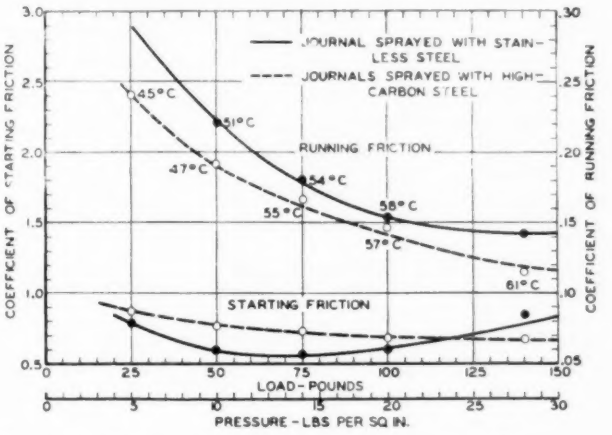


FIG. 15 PERFORMANCE OF SPRAYED JOURNALS IN A BEARING OF TIN-BASE BABBITT

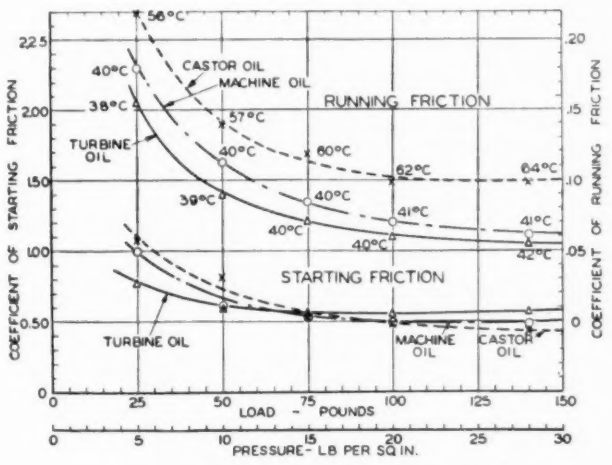


FIG. 16 EFFECT OF DIFFERENT LUBRICANTS
(Journal, monel metal; bearing, tin-base babbitt.)

Next, four different bearings were tested with one grade of journal material—No. 3, 12 per cent chromium steel, chromium-plated. As may be seen from Fig. 13 the silver bearing showed minimum value of starting friction while the three other materials performed at higher values. During running tests, Fig. 14, the babbitt and the high-tin high-copper bronze performed better than the silver or the high-lead bronze.

The performance of the two sprayed journals, Nos. 4 and 5, running in a tin-base-babbitt bearing is shown in Fig. 15. The starting and running friction are higher than for the low-friction materials shown in Figs. 11 and 12. As can be seen from Table 3, the originally high roughness of these two journals increased after tests.

The influence of lubricant is shown in Fig. 16. Under starting conditions the effect of the three oils is almost alike. It is of interest to point out the performance of the castor oil which shows a higher friction value at low pressure and a lower friction at higher pressures. During running conditions the friction values of the three oils are proportional to their respective viscosities.

Conclusion

Within the inherent limits of the experiment it was shown that various journal materials, similarly machined and ground, perform differently in a babbitt shell. A higher value of the coefficient of friction was associated with a relatively rougher journal finish having grinding scratches of the order of 100 microinches. Such finish in turn was associated with soft journal materials (180 Brinell).

The boundary oil film, formed between several grades of bearing materials and a chromium-plated journal, offered different frictional resistance to motion. In the combinations of different bearing materials and the same chromium-plated journal, babbitt proved to be superior to bronze or silver.

ACKNOWLEDGMENT

The authors are indebted to R. E. Peterson, manager, mechanics division, research laboratories and N. L. Mochel, metallurgical engineer, both of Westinghouse Electric & Manufacturing Co., who made these investigations possible. Dr. S. Way, of the research laboratories, helped in applying his method for the measurement of surface profiles.

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Clamped Rectangular Plates With a Central Concentrated Load

By DANA YOUNG,[1] STORRS, CONN.

A GENERAL method of solution for rectangular plates with clamped edges and any kind of loading has been developed by Professor S. P. Timoshenko (1).[2] The present paper gives the results of calculations using this method for the maximum deflection, moment, and edge shears for rectangular plates of various proportions with all four edges clamped and loaded by a single concentrated load at the center. Similar data for a clamped rectangular plate with a uniformly distributed load have been given by I. A. Wojtaszak and also T. H. Evans (2). A report of an experimental investigation of this problem with some analytical results has been given by R. G. Sturm and R. L. Moore (3).

The notations used are as follows:

- a = length of plate in the x-direction
- b = length of plate in the y-direction
- h = thickness of plate
- E = modulus of elasticity of the plate material
- μ = Poisson's ratio. In the numerical calculations this is assumed equal to 0.3
- $D = \dfrac{Eh^3}{12(1-\mu^2)}$ is the flexural rigidity of the plate
- P = central concentrated load
- M = bending moment per unit length of plate
- Q = shearing force per unit length of plate
- R = total vertical reaction along one edge of plate
- w = deflection of plate at any point
- $\alpha_m = \dfrac{m\pi b}{2a}$
- $\beta_m = \dfrac{m\pi a}{2b}$

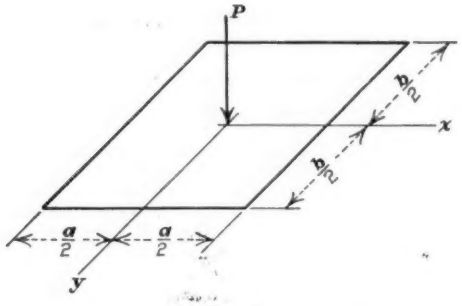


FIG. 1

The origin of coordinates is taken at the center of the plate as shown in Fig. 1. The general procedure as applied to this particular problem is as follows:

For a plate with no distributed lateral load, the deflection surface is governed by the differential equation

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = 0 \quad \ldots\ldots [1]$$

To solve this equation for the given problem, the known solution for a simply supported plate with a central concentrated load is combined with that for a simply supported plate with distributed bending moments applied along the edges. The applied edge moments are then so determined that the slope at the boundaries is zero; that is, take

$$w = w_1 + w_2 + w_3 \quad \ldots\ldots [2]$$

where

$$w_1 = \frac{Pa^2}{2\pi^3 D}\sum_{m=1,3,5\ldots}\frac{1}{m^3}\cos\frac{m\pi x}{a}\left[\left(\tanh\alpha_m - \frac{\alpha_m}{\cosh^2\alpha_m}\right)\cosh\frac{m\pi y}{a} - \sinh\frac{m\pi y}{a} - \tanh\alpha_m\frac{m\pi y}{a}\sinh\frac{m\pi y}{a} + \frac{m\pi y}{a}\cosh\frac{m\pi y}{a}\right] \quad \ldots\ldots [3]$$

is the solution for a simply supported rectangular plate with a central concentrated load (4). Also where

$$w_2 = -\frac{a^2}{2\pi^2 D}\sum_{m=1,3,5\ldots}A_m\frac{(-1)^{\frac{m-1}{2}}}{m^2\cosh\alpha_m}\cos\frac{m\pi x}{a}\left[\frac{m\pi y}{a}\sinh\frac{m\pi y}{a} - \alpha_m\tanh\alpha_m\cosh\frac{m\pi y}{a}\right] \quad \ldots\ldots [4]$$

and

$$w_3 = -\frac{b^2}{2\pi^2 D}\sum_{m=1,3,5\ldots}B_m\frac{(-1)^{\frac{m-1}{2}}}{m^2\cosh\beta_m}\cos\frac{m\pi y}{b}\left[\frac{m\pi x}{b}\sinh\frac{m\pi x}{b} - \beta_m\tanh\beta_m\cosh\frac{m\pi x}{b}\right] \quad \ldots\ldots [5]$$

are the deflections for a simply supported plate with moments applied along the pairs of edges $y = \pm\dfrac{b}{2}$ and $x = \pm\dfrac{a}{2}$, respectively (5). These applied edge moments are given by

$$(M_y)_{y=\pm b/2} = \sum_{m=1,3,5\ldots}(-1)^{\frac{m-1}{2}}A_m\cos\frac{m\pi x}{a} \quad \ldots [6]$$

$$(M_x)_{x=\pm a/2} = \sum_{m=1,3,5\ldots}(-1)^{\frac{m-1}{2}}B_m\cos\frac{m\pi y}{b} \quad \ldots [7]$$

in which the Fourier-series coefficients A_m and B_m are to be determined from the condition that the slope at the boundaries

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[2] Numbers in parentheses refer to the Bibliography at the end of the paper.

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Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y., and will be accepted until October 10, 1939, for publication at a later date. Discussion received after the closing date will be returned.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

TABLE 1 SYSTEM OF EQUATIONS [11]

$$
\begin{aligned}
1.1210\,A_1 + 0.73912\,B_1 &+ 0.12112\,B_3 + 0.03147\,B_5 + 0.01211\,B_7 + 0.00583\,B_9 + \ldots = -0.17006\,P\\
0.3334\,A_3 + 0.06774\,B_1 &+ 0.08213\,B_3 + 0.04581\,B_5 + 0.02407\,B_7 + 0.01346\,B_9 + \ldots = 0.001400\,P\\
0.2000\,A_5 + 0.01607\,B_1 &+ 0.03260\,B_3 + 0.02957\,B_5 + 0.02132\,B_7 + 0.01447\,B_9 + \ldots = -0.0000194\,P\\
0.1429\,A_7 + 0.00602\,B_1 &+ 0.01460\,B_3 + 0.01687\,B_5 + 0.01509\,B_7 + 0.01207\,B_9 + \ldots = 0.00000032\,P\\
0.1111\,A_9 + 0.00286\,B_1 &+ 0.00753\,B_3 + 0.00987\,B_5 + 0.01010\,B_7 + 0.00912\,B_9 + \ldots = 0
\end{aligned}
$$

TABLE 2 SYSTEM OF EQUATIONS [12]

$$
\begin{aligned}
1.1958\,B_1 + 0.51331\,A_1 &+ 0.04704\,A_3 + 0.01116\,A_5 + 0.00418\,A_7 + 0.00199\,A_9 + \ldots = -0.18125\,P\\
0.3351\,B_3 + 0.08411\,A_1 &+ 0.05703\,A_3 + 0.02264\,A_5 + 0.01014\,A_7 + 0.00523\,A_9 + \ldots = 0.005467\,P\\
0.2000\,B_5 + 0.02186\,A_1 &+ 0.03181\,A_3 + 0.02054\,A_5 + 0.01171\,A_7 + 0.00685\,A_9 + \ldots = -0.000239\,P\\
0.1429\,B_7 + 0.00841\,A_1 &+ 0.01672\,A_3 + 0.01481\,A_5 + 0.01048\,A_7 + 0.00702\,A_9 + \ldots = 0.0000125\,P\\
0.1111\,B_9 + 0.00405\,A_1 &+ 0.00934\,A_3 + 0.01005\,A_5 + 0.00838\,A_7 + 0.00634\,A_9 + \ldots = -0.00000071\,P
\end{aligned}
$$

TABLE 3 VALUES OF A_m AND B_m FOR VARIOUS RATIOS b/a

b/a	1.0	1.2	1.4	1.6	1.8	2.0
A_1	$-0.1025\,P$	$-0.0764\,P$	$-0.0524\,P$	$-0.0339\,P$	$-0.0206\,P$	$-0.0118\,P$
A_3	$0.0263\,P$	$0.0200\,P$	$0.0145\,P$	$0.0099\,P$	$0.0065\,P$	$0.0040\,P$
A_5	$0.0042\,P$	$0.0036\,P$	$0.0026\,P$	$0.0016\,P$	$0.00099\,P$	$0.00049\,P$
A_7	$0.0015\,P$	$0.0011\,P$	$0.00075\,P$	$0.00044\,P$	$0.00025\,P$	$0.000080\,P$
A_9	$0.00055\,P$	$0.00040\,P$	$0.00025\,P$	$0.00014\,P$	$0.00007\,P$	$0.000003\,P$
A_{11}	$0.00021\,P$					$-0.000017\,P$
A_{13}	$0.00006\,P$					$-0.000028\,P$
B_1	$-0.1025\,P$	$-0.1196\,P$	$-0.1246\,P$	$-0.1224\,P$	$-0.1168\,P$	$-0.1100\,P$
B_3	$0.0263\,P$	$0.0318\,P$	$0.0360\,P$	$0.0397\,P$	$0.0431\,P$	$0.0461\,P$
B_5	$0.0042\,P$	$0.0035\,P$	$0.0014\,P$	$-0.0015\,P$	$-0.0049\,P$	$-0.0083\,P$
B_7	$0.0015\,P$	$0.0018\,P$	$0.0018\,P$	$0.0019\,P$	$0.0021\,P$	$0.0027\,P$
B_9	$0.00055\,P$	$0.00066\,P$	$0.00061\,P$	$0.00045\,P$	$0.00019\,P$	$-0.00014\,P$
B_{11}	$0.00021\,P$					$0.00023\,P$
B_{13}	$0.00006\,P$					$0.00004\,P$

is zero. Thus taking $\left(\frac{\partial w}{\partial y}\right)_{y=b/2} = 0$ and expressing the terms containing $\frac{m\pi x}{b}$ $\sinh\frac{m\pi x}{b}$ and $\cosh\frac{m\pi x}{b}$ by a cosine series, we obtain

$$
\frac{Pa}{2\pi^2 D}\sum_{m=1,3,5\ldots}\frac{1}{m^2}\cos\frac{m\pi x}{a}\,\frac{\alpha_m\tanh\alpha_m}{\cosh\alpha_m} + \frac{a}{2\pi D}\sum_{m=1,3,5\ldots} A_m\,\frac{(-1)^{\frac{m-1}{2}}}{m}\cos\frac{m\pi x}{a}\left(\tanh\alpha_m + \frac{\alpha_m}{\cosh^2\alpha_m}\right) + \frac{4a^2}{\pi^2 bD}\sum_{m=1,3,5\ldots}\frac{B_m}{m^3}\sum_{i=1,3,5\ldots}\frac{i(-1)^{\frac{i-1}{2}}}{\left(\frac{a^2}{b^2}+\frac{i^2}{m^2}\right)^2}\cos\frac{i\pi x}{a} = 0 \ldots [8]
$$

This relation holds for any particular value of the cosine term, say $\cos\frac{k\pi x}{a}$. Therefore, the summation signs may be dropped and this cosine term canceled, thus obtaining

$$
A_k\frac{1}{k}\left[\tanh\alpha_k + \frac{\alpha_k}{\cosh^2\alpha_k}\right] + \frac{8ak}{\pi b}\sum_{m=1,3,5\ldots}\frac{B_m}{m^3}\frac{1}{\left(\frac{a^2}{b^2}+\frac{k^2}{m^2}\right)^2} = -P\,\frac{(-1)^{\frac{k-1}{2}}}{\pi k^2}\,\frac{\alpha_k\tanh\alpha_k}{\cosh\alpha_k} \ldots\ldots [9]
$$

Similarly, for zero slope at $x = \frac{a}{2}$ is obtained

$$
B_k\frac{1}{k}\left[\tanh\beta_k + \frac{\beta_k}{\cosh^2\beta_k}\right] + \frac{8bk}{\pi a}\sum_{m=1,3,5\ldots}\frac{A_m}{m^3}\frac{1}{\left(\frac{b^2}{a^2}+\frac{k^2}{m^2}\right)^2} = -P\,\frac{(-1)^{\frac{k-1}{2}}}{\pi k^2}\,\frac{\beta_k\tanh\beta_k}{\cosh\beta_k} \ldots\ldots [10]
$$

Equations [9] and [10] are used to determine the coefficients A_k and B_k. They form a double-infinite system of simultaneous equations which may be solved approximately by neglecting the higher terms. Thus for a plate with the proportions $b/a = 1.2$, from Equation [9] the system of Equations [11], given in Table 1, is obtained by assigning to k the values 1, 3, 5, ..., in turn. Similarly, from Equation [10] the system of Equations [12], given in Table 2, is derived.

Neglecting terms higher than those given, Equations [11] and [12] (Tables 1 and 2) may be solved by successive approximations. For the first approximation the terms to the right of the stepped line are neglected. For the second approximation, the values of A_3 to A_9 and B_5 to B_9 found from the first approximation are substituted in the terms to the right of the stepped line and the system solved again. This process is repeated as many times as necessary. Three or four approximations have been found sufficient for these calculations. Using this procedure, the coefficients for rectangular plates of several proportions were calculated. The results are given in Table 3.

The deflection at the center is then found from Equations [2], [3], [4], and [5] and may be reduced to the form

$$
(w)_{\substack{x=0\\y=0}} = \alpha\frac{Pa^2}{Eh^3} \ldots\ldots [13]
$$

The maximum edge moment is found from Equation [7] to be

$$
(M_x)_{\substack{x=\frac{a}{2}\\y=0}} = \sum_{m=1,3,5\ldots}(-1)^{\frac{m-1}{2}}B_m = -\beta P \ldots [14]
$$

in which the negative sign is used for a moment which bends the plate concave downward.

The maximum edge shear occurs at the center of the long side and is given by

$$
(Q_x)_{\substack{x=a/2\\y=0}} = -D\frac{\partial}{\partial x}\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right)\Bigg|_{\substack{x=a/2\\y=0}} = -\gamma\frac{P}{a} \ldots [15]
$$

in which the negative sign denotes an upward shear at this point.

The total vertical reactions along the edges are given by

$$R_x = \int_{-b/2}^{b/2} (Q_x)_{x=a/2}\, dy = -\delta_x P \ldots\ldots\ldots [16]$$

TABLE 4 VALUES OF DEFLECTION, MOMENT, SHEAR, AND REACTION COEFFICIENTS AS DEFINED IN EQUATIONS [13] TO [17]

b/a	α	β	γ	δ_x	δ_y
1.0	0.0611	0.1257	0.794	0.250	0.250
1.2	0.0706	0.1490	0.877	0.347	0.153
1.4	0.0755	0.1604	0.908	0.417	0.083
1.6	0.0777	0.1651	0.916	0.459	0.041
1.8	0.0786	0.1667	0.918	0.484	0.016
2.0	0.0788	0.1674	0.919	0.498	0.002

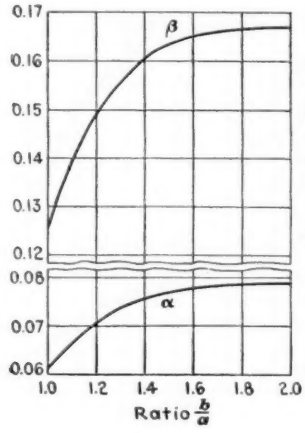


FIG. 2

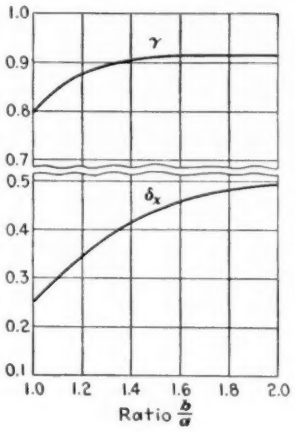


FIG. 3

$$R_y = \int_{-a/2}^{a/2} (Q_y)_{y=b/2}\, dx = -\delta_y P \ldots\ldots\ldots [17]$$

The values of the coefficients α, β, γ, δ_x, δ_y in Equations [13] to [17] were calculated and are given in Table 4 and Figs. 2 and 3. In the calculations for the coefficient α, the value of Poisson's ratio has been taken as $\mu = 0.3$.

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The Thick-Film Lubrication of Full Journal Bearings of Finite Width

By M. MUSKAT[1] and F. MORGAN,[2] PITTSBURGH, PA.

A discussion is given of the results of analytical and experimental studies of the thick-film lubrication of full journal bearings of finite width. The Reynolds differential equation has been solved by a successive approximation method for (*a*) flooded full journal bearings, (*b*) full journal bearings with central circumferential grooves, and (*c*) full journal bearings provided with point sources of lubricant. Calculations were made on the variation of the journal eccentricity, load-carrying capacity, and friction coefficients with the Sommerfeld dimensionless variable for the three cases and for different bearing widths. Direct experimental tests are described which confirmed the theoretical predictions for bearings fed with lubricant at point sources.

ALTHOUGH the basic physical principles underlying the problem of lubrication were clearly outlined by Osborne Reynolds some fifty-three years ago, there still remains much to be done in the development of a complete understanding of the actual operation of modern industrial equipment. From the empirical point of view there has been a remarkably close approach to what may be considered as ideal performance. However, there appears to be little question but that the improvements in the actual operation of machines subject to lubrication have far outdistanced the advances in the physical theory as to just why and how these improvements are achieved.

From the physical point of view as well as from the practical aspects of the problem, the subject of lubrication conveniently divides itself into that pertaining to the so-called "thick-film" conditions of lubrication and that usually described as "thin-film" lubrication. In the former, which refers to conditions where the film of lubricant is of a thickness great as compared to the heights of the surface irregularities of the sliding parts, the film may be assumed to be nothing else than a hydrodynamic viscous liquid of a special geometry. Under thin-film conditions of lubrication, where the term "thin" indicates that the film thickness becomes of the order of the height of the projections upon the metallic surfaces, the detailed characteristics of these surface irregularities and the nature of the metallic surfaces will play roles as important as, if not predominant over, those of the properties of the lubricant which normally characterize it as a viscous liquid. This latter part of the subject, though of greater importance from a practical point of view, is obviously more complex physically. Much excellent work on it has been described in the recent literature, but it has not yet been subjected to a comprehensive correlation and formulated into a unified physical and quantitative theory.

Thick-film lubrication, on the other hand, involves only the hydrodynamic behavior of the lubricant film, and is therefore much simpler from the physical point of view. Mathematically, however, it may still be considered as a formidable problem, and much work yet remains to be done in developing all the implications of what are otherwise rather simple physical postulates regarding this type of lubrication. As a preliminary approach to the more practical thin-film phases of the problem the authors have begun a study of the hydrodynamics of thick-film lubrication, which it was hoped might provide a desirable and helpful background in investigating later the more complex features of the thin-film region. Some of the preliminary developments in this study will be reported here.

In order to interpret correctly the implications of experimental data on thick-film lubrication with regard to the validity of the hydrodynamic theory, one must obviously have available the predictions of the latter under the experimental conditions being studied. In the case of journal bearings this means that solutions of the hydrodynamic equations must be derived for bearings of finite width, since all practical and experimental bearings will necessarily be of finite width. As the standard solutions previously available for journal-bearing lubrication referred to the infinitely wide bearing originally treated by Reynolds and Sommerfeld, it was necessary to develop first the corresponding analysis for the bearings of finite width.

This discussion will not enter into the mathematical details but will be confined essentially to the final results.[3] These results are the direct consequences of the solution of the Reynolds equation, which may be expressed in dimensionless form as

$$\frac{\partial}{\partial\theta}\left\{(1+\eta\cos\theta)^3\frac{\partial p}{\partial\theta}\right\}+(1+\eta\cos\theta)^3\frac{\partial^2 p}{\partial w^2}=-\beta\eta\sin\theta \quad [1]$$

where p is the pressure distribution in the film, θ the azimuthal angle in the direction of rotation, and η, w, and β are defined by

$$\eta=\frac{e}{c};\quad w=\frac{z}{r};\quad \beta=6\mu Ur/c^2 \quad [2]$$

Here e is the displacement between the axes of the journal and bearing, c is the radial clearance, z the axial coordinate, r the journal radius, μ the lubricant viscosity, and U the surface velocity of the journal, as indicated in Fig. 1. The film thickness is taken as

$$h=c+e\cos\theta \quad [3]$$

Equation [1] was solved by a successive approximation method in which all the physical quantities of interest were expanded in powers of the eccentricity η. That is, p was first expressed formally as

$$p(\theta, w)=\Sigma\eta^m p_m(\theta, w) \quad [4]$$

This was inserted in Equation [1] and after expanding the various

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NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[3] For the full mathematical developments of the solution the reader may refer to the papers by the authors in the *Journal of Applied Physics*, vol. 9, 1938, p. 393; vol. 10, p. 46, 1939. A number of literature references are cited in these papers. Still others are given in "Theory of Lubrication," by M. D. Hersey.

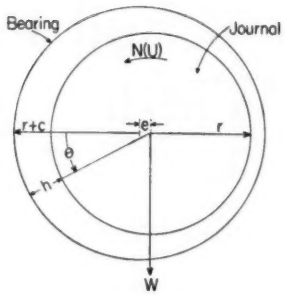


FIG. 1 CROSS SECTION OF A FULL JOURNAL BEARING

terms the coefficients of like powers of η were equated. The following series of Poisson equations for the $p_m(\theta, w)$ was thus obtained

$$\left.\begin{aligned}
\nabla^2 p_0 &= \frac{\partial^2 p_0}{\partial \theta^2} + \frac{\partial^2 p_0}{\partial w^2} = -\beta\eta \sin\theta \\
&\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
\nabla^2 p_m &= -3\left(\cos\theta \nabla^2 p_{m-1} - \sin\theta \frac{\partial p_{m-1}}{\partial \theta}\right) \\
&\quad -3\cos\theta\left(\cos\theta\nabla^2 p_{m-2} - 2\sin\theta\frac{\partial p_{m-2}}{\partial\theta}\right) \\
&\quad -\cos^2\theta\left(\cos\theta\nabla^2 p_{m-3} - 3\sin\theta\frac{\partial p_{m-3}}{\partial\theta}\right)
\end{aligned}\right\} \quad \ldots[5]$$

These equations were then solved successively in analytical form. For the solutions involving only β, corresponding to flooded bearings or bearings with circumferential grooves, the first five equations of the series were solved. For the solutions involving point sources of lubricant the solutions of only the first three equations were derived explicitly.

Once the pressure distribution was found, the load supported by the film was calculated by integrating the pressure components parallel and perpendicular to the polar axis, i.e., by the integrals

$$\begin{matrix} P_\parallel \\ P_\perp \end{matrix} \int_{-\bar{w}}^{\bar{w}} dw \int_0^{2\pi} p \begin{matrix} \cos\theta \\ \sin\theta \end{matrix} d\theta \quad \ldots\ldots\ldots\ldots[6]$$

where $\bar{w}$ represents the dimensionless half width of the bearing, i.e., the width divided by the diameter.

The frictional forces required to keep the journal in rotation or the bearing stationary were calculated by integrating the surface stresses in the film over the journal and bearing surfaces.

In the final application and interpretation of the analysis, particular attention has to be paid to the character of the source of lubricant with which the system is provided. Thus far the detailed calculations have been carried through for the three cases where: (1) The bearing is flooded and has no external lubricant source, (2) the bearing is provided with a central circumferential groove as a lubricant source, and (3) the lubricant is fed to the bearing through a small circular hole, which was represented mathematically as a point source, somewhere along the central circumference of the bearing. As the results differ markedly in these various cases they will be discussed separately.

FLOODED FULL JOURNAL BEARINGS

With regard to the pressure distribution itself it was found that in the case of flooded full journal bearings the pressure distribution over the semicylinder on one side of the line of centers was the exact negative reflection of that on the semicylinder on the other side of the line of centers. As an immediate consequence the resultant load which could be supported by the film is normal to the line of centers, so that when the external load is applied vertically the displacement of the journal in the bearing must lie along the horizontal. This situation is exactly the same as in the case of the infinitely wide bearing originally treated by Sommerfeld.

The variation in the position of the center of the journal may best be seen by plotting the value of the journal eccentricity η against the dimensionless Sommerfeld variable, $S = (r/c)^2\mu N/P$, where P is the load per unit projected bearing area and N is the speed in rps. The practical significance of this variable is that it may be used to specify roughly the thick- and thin-film

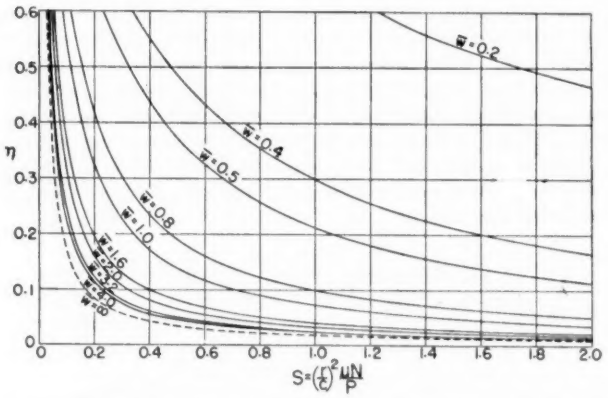


FIG. 2 THE VARIATION OF THE JOURNAL ECCENTRICITY η WITH THE SOMMERFELD VARIABLE S FOR FLOODED JOURNAL BEARINGS OF FINITE WIDTH

[$\bar{w}$ = (bearing width)/(bearing diameter); r = journal radius; c = radial clearance; μ = lubricant viscosity; N = journal speed, rps; P = load per unit projected bearing area.]

ranges of lubrication. Thus when $S > 0.05$, viscous or thick-film conditions will usually obtain, while values of $S < 0.05$ will as a rule correspond to partial or thin-film lubrication, although these limits are by no means rigidly applicable under all conditions. The numerical results for the flooded full journal bearing are shown in Fig. 2 for different bearing widths defined by the parameter $\bar{w}$ which, as already indicated, is equal to the actual bearing width divided by the bearing diameter. The dotted curve in Fig. 2 represents the values given by the Sommerfeld theory for the infinitely wide bearing.

It will be seen that the eccentricity assumed by a bearing of finite width for a given value of S is uniformly higher than that for the infinitely wide bearing. Moreover this eccentricity increases as the bearing width decreases. This is of course due to the fact that the dropping off of the pressure to atmospheric at the ends of the finite-width bearing leads to smaller average pressures in the bearings of short width, which in turn necessitate greater eccentricities and wedging effects to permit the support of loads of the same magnitude. This effect may be seen more clearly by computing the relative load-carrying capacities of the finite-width bearings and the infinitely wide bearing having the same journal eccentricity. Assuming equal values of r/c, μ, and N, the load-carrying capacity will clearly be proportional to the reciprocal of the Sommerfeld variable. The relative load-carrying capacities will therefore be given simply by S_0/S, where S_0 and S are given by the abscissas of the dotted and solid curves of Fig. 2. Such comparisons are shown graphically in Fig. 3 for different values of $\bar{w}$.

As Fig. 3 shows, the relative load-carrying capacities for fixed eccentricities of the finite-width flooded bearings are uniformly less than one. Moreover, the load-carrying capacity decreases

continuously with decreasing bearing width. Thus, for a bearing width equal to the bearing diameter ($\bar{w} = 1$), the load-carrying capacities for the same eccentricity will be of the order of from 25 to 30 per cent of those of the infinitely wide bearing, and for a bearing width equal to one half the diameter, the load-carrying capacities will fall to less than 10 per cent of the infinitely wide bearing values. These results may be contrasted with the values $2/3$ and $2/\pi$, for all values of η, obtained on the common assumptions that the axial pressure distributions fall in parabolic or trigonometric manner, respectively, from the Sommerfeld distribution along the central circumference.

While these results do show the importance of taking into account the "end effects" of the finite-width bearing and the associated dropping off of the pressure distribution to atmospheric at the ends, they do not necessarily imply corresponding differences in ultimate load-carrying capacities of the finite-width journal bearings. Rather they give only the relative values of the load-carrying capacity of the finite-width bearings as compared to those for the infinitely wide bearing for the same value of η, that is, *the same geometry*. All that would be necessary, to obtain in a finite-width bearing any predetermined load-carrying capacity corresponding to an infinitely wide bearing with fixed

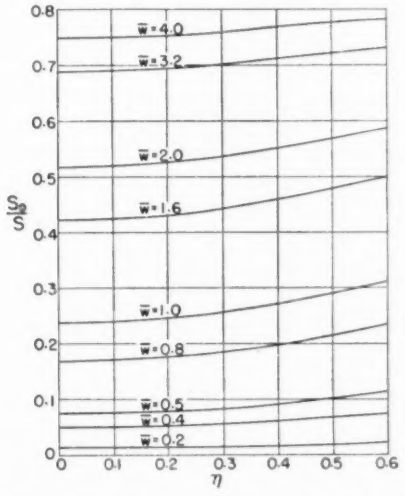


FIG. 3 RELATIVE LOAD-CARRYING CAPACITIES OF FLOODED JOURNAL BEARINGS OF FINITE WIDTH

[S_0/S = (load-carrying capacity of finite-width bearing)/(capacity of infinitely wide bearing of same eccentricity); $\bar{w}$ = (bearing width)/(bearing diameter).]

eccentricity, would be to increase the eccentricity of the former. In principle, it will be possible to obtain arbitrarily large load-carrying capacities with the finite-width bearing to the same extent that such would be possible with bearings of infinite width, although from a practical point of view, the higher eccentricities that would be required in the case of a finite-width bearing might lead to seizing or scoring of the bearing somewhat earlier than under the hypothetical case of the infinitely wide bearing.

The results of the calculation for the friction coefficient on the journal are shown in Fig. 4, in which the dotted curve again refers to the case of the infinitely wide bearing. It will be seen that in general the friction coefficients for the finite-width bearing are higher than those for the infinitely wide bearing, and that the difference increases as the bearing width decreases. Asymptotically, however, they all approach the Petroff line through the origin, of slope $2\pi^2$. The reason for the higher friction coefficients for the finite-width bearings obviously lies in the higher eccentricities previously noted for fixed values of S.

These results are valid for relatively large values of S, i.e., for those corresponding to values of the eccentricity not exceeding 0.6. To obtain the corresponding information in the thin-film

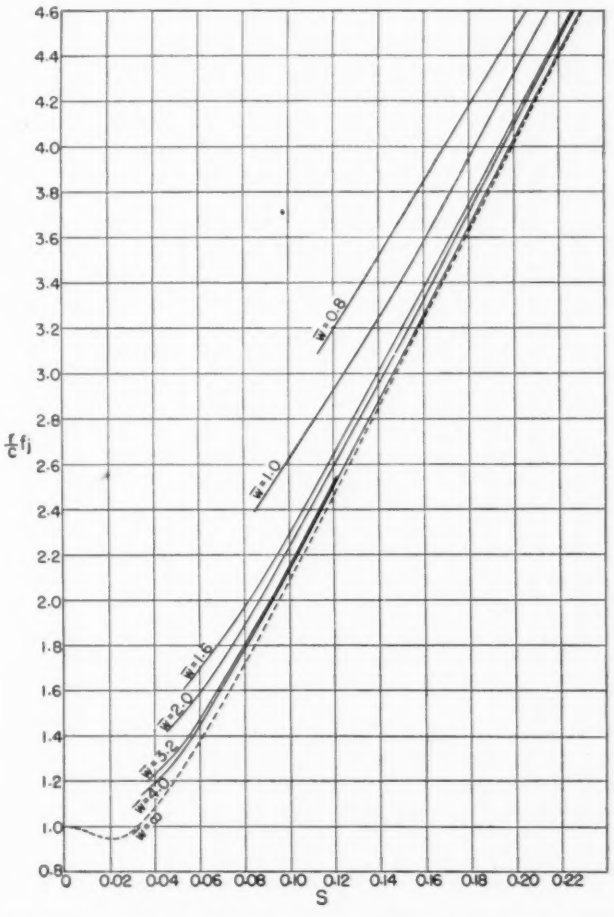


FIG. 4 FRICTION COEFFICIENTS ON THE JOURNAL fj VERSUS S FOR FLOODED BEARINGS OF FINITE WIDTH

[$\bar{w}$ = (bearing width)/(bearing diameter); c = radial clearance; r = journal radius. Dotted curve represents Sommerfeld curve for infinitely wide bearing.]

region or for values of the eccentricity approaching 1, different analytical procedures must be used.

FULL JOURNAL BEARINGS WITH CENTRAL CIRCUMFERENTIAL GROOVES

While at first thought the case of a full journal bearing with a central circumferential groove may appear to demand a considerably more complex analysis than that required for the flooded journal bearing, it turns out that the difference in the analytical solutions for the two cases consists merely in the almost trivial addition of a linear axially varying pressure term which takes into account the presence of the circumferential groove. For all that such a groove actually does is to split the original bearing into two end-fed bearings, with the fed ends adjacent to the groove. The pressure distributions in such end-fed bearings are identical with those in the flooded bearing with the same journal eccentricity, except for the variation in pressure parallel to the axis, which is induced by the end feed. However, this linear axial pressure term does not affect the load that can be supported by the film, nor does it affect the friction properties. The line of centers in the bearing with the circum-

ferential groove will therefore also remain horizontal if the load be vertical.

The only ultimate effect of the circumferential groove is to reduce the composite bearing with the groove to two flooded bearings of half the original bearing width. The detailed properties, such as the variation of the journal eccentricity with S or of the friction coefficient with S will be given by exactly the same numerical results already derived for the flooded bearings, provided only that the effective width of the flooded bearing be taken to be half of the true width of the composite bearing. In other words, for a bearing with a circumferential groove for which the over-all dimensionless width = 1, the curve of η versus S will be exactly the same as that shown in Fig. 2 for the case of a dimensionless width equal to $^1/_2$ or $\bar{w} = 0.25$. The same relationship will obtain with respect to the load-carrying capacities of the bearing as shown in Fig. 3, and the friction-coefficient curves as shown in Fig. 4. In short, a circumferential groove in a bearing cuts the effective bearing width in half—except for the actual width of the groove—with respect to its load-carrying power and friction properties. As will be seen from Figs. 2, 3, and 4, the magnitude of this effect is much larger than that which could be attributed to merely cutting the width of the composite bearing by the width of the groove, especially if the bearings are initially of relatively short width.

Because of the lower load-carrying capacities and higher friction coefficients of the flooded bearings of short width it follows that from a strictly hydrodynamic point of view a circumferential groove would impair the lubricating effectiveness of a journal bearing. However, there are two considerations pertinent to the practical use of journal bearings which may offset these hydrodynamic disadvantages of the circumferential groove.

The first is that the higher rate of lubricant flow through a bearing with such a groove will tend to cool the lubricant film and thus maintain a higher viscosity of the lubricant. This in turn will lead to higher load-supporting capacity of the film as compared to that in a flooded bearing where the film temperature may become high and the viscosity low. This effect may be especially important when the bearing is operated under thin-film conditions, when the fall in lubricant viscosity due to the temperature rise would tend to aggravate the tendency for metallic contact to develop and ultimately cause seizing or scoring of the bearing.

An even more valuable practical feature of the circumferential groove may result from the possibility of overcoming the negative pressures in the lubricant film by means of the added feed pressure applied at the groove. For in the theoretical hydrodynamic film the pressures over the bearing surface on one side of the line of centers of the journal and bearing are just as much below atmospheric as those on the other half are above atmospheric. If there is no external feed pressure applied to overcome these negative pressures the film may break and thus cut down the load-supporting power. As the feed pressures in the circumferential groove are uniformly distributed over the whole periphery of the bearing they will tend to overcome the negative pressures that would theoretically obtain in the flooded bearing, thus preventing the breaking of the film and permitting it to support the full value of the load as predicted by the hydrodynamic theory.

Full Journal Bearings Fed With Lubricant at Point Sources

In contrast to the case of the feed pressure as applied to a circumferential groove, a pressure applied at a point source dissipates itself rapidly in moving away from the location of the source to the distant parts of the film. Nevertheless such point sources of lubricant may exert significant reactions upon the behavior of the lubricated system. The exact effects depend upon the position of the lubricant source as well as upon its strength q_0, to which reference is made later. For the present purpose it will suffice to consider only the two cases where the source is set through the vertical line of application of the load, and either at the crown of the bearing or unloaded part of the film, or at the base of the bearing or the loaded part of the film. It is supposed that the actual load is applied to the journal.

When the source is at the crown of the bearing it is found that its general effect is to cause a displacement of the journal still greater than that merely corresponding to its rotational position. This in turn leads to higher journal eccentricities than for the case of the flooded bearing, correspondingly lower load-carrying capacities for fixed eccentricity, and higher values of the friction coefficient for fixed values of the Sommerfeld variable. As in the case of the flooded bearing the journal eccentricity, load-carrying capacity, and friction coefficient all depend upon the actual value of the bearing width, and moreover vary also with the strength of the lubricant source. Thus the relation between the friction coefficient and the Sommerfeld variable is not given by a universal curve applicable under all conditions with deviations implying the presence of oiliness effects, but rather the Sommerfeld variable appears to be but one parameter required for the complete specification of an actual bearing system of finite width.

When the lubricant source is set at the base or the loaded part of the bearing its effect is radically different. Here it tends to increase the load-carrying power of the film arising from the journal rotation and lessens the eccentricity required to support any given load. In fact if the source strength is made sufficiently high so as to attain a certain critical value the pressure distribution due to the lubricant source will suffice alone to carry the whole of the external load without any need for the supporting effect due to the journal rotation. Under such circumstances the journal becomes concentric with the bearing so that the load-supporting power of the film due to the rotation automatically vanishes. And when the source strength is increased still further the journal is raised beyond its central position, forcing the rotational effect to give a load-depressing component so as to counterbalance some of the load-supporting forces due to the lubricant source. The displacement of the journal center will no longer lie along the horizontal, but will move into any of the four quadrants, depending upon the actual strength of the lubricant source, its position, and the value of the Sommerfeld variable.

Perhaps the most striking effect of the lubricant source when placed at the base of the bearing is its ability to eliminate completely all thin-film effects normally encountered when the Sommerfeld variable becomes vanishingly small. For with the lubricant source set in the loaded part of the bearing the journal can be supported without coming into metallic contact with the bearing surface even though the rotation of the journal ceases. The resting position of the journal under such conditions will of course depend upon the strength of the source, and the ultimate separation between the journal and bearing surfaces will increase as the source strength increases. When the critical value previously mentioned is reached, the two will become concentric whether the journal is rotating or not.

With regard to the coefficient of friction, it will in general be decreased by a source set at the base of the bearing as long as the source strength is less than the critical value required to support the load completely. For higher values of source strength the friction coefficient begins to increase, and ultimately it may even exceed the values appropriate to flooded journal bearings.

Experimental Studies

The essential features of the theoretical predictions for journal-bearing systems fed with lubricant at point sources have been confirmed by direct experimental tests. These experiments were performed with a machine shown diagrammatically in Fig. 5. The Graham variable-speed transmission driving the journal provided a continuous variation of speeds from 0 to 680 rpm. The bearing was $9^3/_4$ in. wide and $3^1/_2$ in. in diameter. The ratio of the journal radius to the radial clearance was 1470. Two oils and diethylbutyl alcohol were used as the lubricants, the viscosities of which at 25 C varied from 51.1 to 3.2 centipoises. The load was varied from 61.7 to 215.4 lb. The film

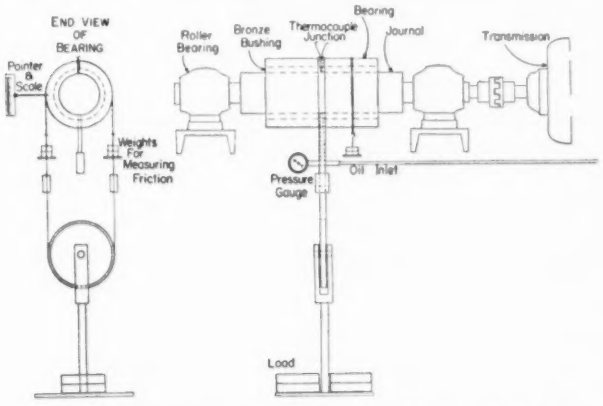


Fig. 5 Diagrammatic Representation of Machine for Measuring Journal-Bearing Friction

viscosity was determined by means of a thermocouple junction set flush with the bearing surface. The rate of flow of lubricant through the bearing was measured by collecting the leakage out of its ends in drains from which it flowed into a burette. Experiments were performed both with the source set at the crown of the bearing and at the base, and the torque was measured on the bearing.

A typical set of results is shown in Fig. 6, which was obtained with the source set in the loaded part of the bearing. The parameters q_0 indicate the values of the dimensionless flux strength appearing in the theoretical analysis as the quantity characterizing the strength of the lubricant source. It was calculated by means of the formula

$$q_0 = 3\mu Qr^2/\pi c^3 W \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots [7]$$

where Q was the actual rate of lubricant flow through the bearing and W the total load.

In contrast to friction-coefficient curves previously published in the literature, which, at high values of S, approach straight lines with positive intercepts on the friction axis, all the curves of Fig. 6 asymptotically approach the Petroff line of slope $2\pi^2$ passing through the origin, as theoretically predicted. Moreover, as the analysis requires, the curve for the critical source strength $q_0 = 0.091$ follows this Petroff line throughout its whole extent, since for this value of q_0 the source should alone be able to support the whole of the load and the journal should run concentrically in the bearing. The experimental data also verify the theoretical prediction that the friction coefficient on the bearing for a fixed value of the Sommerfeld variable uniformly increases as the source strength increases when the source is in the loaded

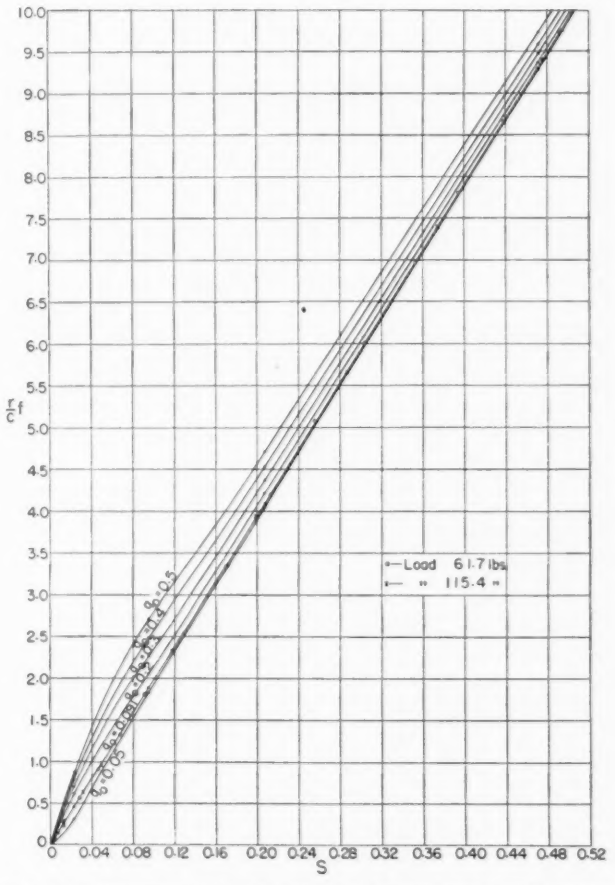


Fig. 6 Experimentally Determined Coefficients of Friction on the Bearing Versus S
(q_0 = dimensionless source strength.)

part of the bearing. Finally it will be seen that for this position of the lubricant source all the friction curves are directed so as to pass through the origin with vanishing value of the Sommerfeld variable, without encountering the slightest indication of the usual thin-film phenomena. And it may be added that, again in accordance with the theoretical prediction, the thin-film behavior does appear at low values of the Sommerfeld variable when the source is placed in the unloaded part of the bearing.

It would be misleading to claim that the theory as thus far developed agrees completely in quantitative detail with the experimental findings under all conditions. Deviations have been found at high values of the source strength and at very low values of the Sommerfeld variable. These discrepancies, however, do not at all necessarily imply a breakdown of the hydrodynamic picture, but rather may be reasonably correlated, both with the inherent approximations of the mathematical analysis and some of the unavoidable uncertainties and errors in the experimental determinations. But with respect to all the essential major features of the problem, the experiments do show that the Reynolds hydrodynamic equations provide a satisfactory physical explanation of journal-bearing lubrication obtaining under thick-film conditions.

The authors are indebted to Paul D. Foote, executive vice-president, Gulf Research & Development Company, for permission to publish this paper.

A Theory of Flexure for Beams With Nonparallel Extreme Fibers

By WILLIAM R. OSGOOD,[1] WASHINGTON, D. C.

A theory of flexure for beams with nonparallel extreme fibers is presented. The theory is shown to reduce to the ordinary theory of flexure as the angle between the extreme fibers approaches zero. Maximum fiber stresses computed by the theory and by the ordinary theory for a plate girder with nonparallel flanges show that for angles between the flanges up to twenty degrees the ordinary theory can never be in error more than about five per cent, but for larger angles the error in general increases rapidly with the angle. Finally an example is given of the application of the theory as an approximation to a beam with curved extreme fibers. In this case the theory was confirmed by the results of tests.

Introduction

THE ANALYSIS of the stresses in a beam with nonparallel extreme (top and bottom) fibers arises occasionally as a problem in structural design. The ordinary beam theory is commonly used in such an analysis and, when the extreme fibers are nearly parallel, this theory is good enough for practical purposes. It cannot but be seriously in error when the extreme fibers are far from parallel. An accurate theory exists for a very narrow or a very wide wedge with plane parallel ends loaded at its edge. This theory can be extended to a wedge with nonparallel but symmetrical ends, with a degree of approximation which should be comparable to that of the ordinary beam theory applied to an ordinary beam with nonparallel sides (in the plane of bending). It is of interest to compare extreme-fiber stresses obtained by this extended theory with those obtained by the ordinary beam theory applied to the wedge. Furthermore, recent tests[2] indicate that the theory of the wedge may be exploited to yield satisfactory results in cases where the extreme fibers are not only not parallel but not straight.

Equations of Equilibrium and Compatibility

When it is justifiable to assume a condition of plane stress in a disk of nonuniform thickness, the equations of equilibrium in polar coordinates become in the absence of body forces[3]

$$\left.\begin{aligned} \frac{\partial(\sigma_r t)}{\partial r} + \frac{1}{r}\frac{\partial(\tau_{r\theta} t)}{\partial \theta} + \frac{\sigma_r t - \sigma_\theta t}{r} &= 0 \\ \frac{1}{r}\frac{\partial(\sigma_\theta t)}{\partial \theta} + \frac{\partial(\tau_{r\theta} t)}{\partial r} + \frac{2(\tau_{r\theta} t)}{r} &= 0 \end{aligned}\right\} \quad \ldots\ldots\ldots [1]$$

where t is the thickness of the disk at any point, and the other notation is well known. The equation of compatibility remains unchanged[3]

$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} + \frac{1}{r}\frac{\partial}{\partial r}\right)(\sigma_r + \sigma_\theta) = 0 \quad \ldots\ldots\ldots [2]$$

The Theory of the Wedge

Consider a wedge, Fig. 1, having an arbitrary symmetrical circular-cylindrical section, as shown by arc A-A. The thickness t is assumed not to vary radially. If it does, the following analysis may be expected to be in error by an amount comparable to the error in the ordinary beam theory when applied to a similar straight beam of which symmetrically situated elements (fibers) of the sides are not parallel. Introduce a system of polar coordinates r, θ with the pole at the edge of the wedge and the

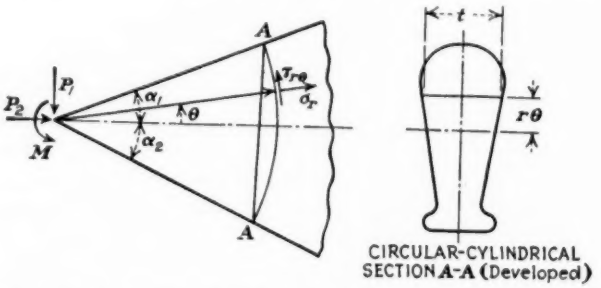


FIG. 1 SIDE ELEVATION AND SECTION OF WEDGE-SHAPED BEAM

polar axis in a temporarily arbitrary direction. Take the wedge to be loaded at the edge by a force P_1 perpendicular to the polar axis, a force P_2 along the polar axis, and a couple M, all positive as shown. The loads will be considered one at a time, and the stresses due to them may then be added by the principle of superposition. The equation of compatibility, Equation [2], is satisfied in each case.

The theory of the wedge with plane parallel ends[3] suggests that the radial stress σ_{r1} due to the force P_1 may be given by $\sigma_{r1} = (c_1/r)\sin\theta$, where c_1 is a constant, and that the tangential stress $\sigma_{\theta 1} = 0$ everywhere. The equations of equilibrium [1] together with the condition that the sides of the wedge are free from stress then require the shearing stress $\tau_{r\theta 1}$ to be everywhere zero. The solution for this case is therefore of the form

$$\sigma_{r1} = (c_1/r)\sin\theta, \quad \sigma_{\theta 1} = 0, \quad \text{and } \tau_{r\theta 1} = 0 \ldots [3]\,[4]\,[5]$$

and it only remains to locate the polar axis and to determine the constant c_1, which may be done with the aid of the conditions of equilibrium (forces parallel and perpendicular to the polar axis) of the portion of the wedge between the edge and the circular-cylindrical section A-A

$$\int_{-\alpha_2}^{\alpha_1} \sigma_{r1} t r \cos\theta \, d\theta = 0$$

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[1] U. S. Department of Commerce, National Bureau of Standards.

[2] "Strength of a Riveted Steel Rigid Frame Having a Curved Inner Flange," by Ambrose H. Stang, Martin Greenspan, and Wm. R. Osgood, National Bureau of Standards, *Journal of Research*, vol. 21, December, 1938, RP 1161.

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Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y., and will be accepted until October 10, 1939, for publication at a later date. Discussion received after the closing date will be returned.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[3] See for example "Theory of Elasticity," by S. Timoshenko, McGraw-Hill Book Company, New York and London, 1934, chap. 3.

and $$\int_{-\alpha_2}^{\alpha_1} \sigma_{r1} tr \sin\theta \, d\theta - P_1 = 0$$

that is, $$\int_{-\alpha_2}^{\alpha_1} t \sin 2\theta \, d\theta = 0 \quad \ldots\ldots [6]$$

and $$c_1 \int_{-\alpha_2}^{\alpha_1} t \sin^2\theta \, d\theta - P_1 = 0 \quad \ldots\ldots [7]$$

Since $\alpha_1 + \alpha_2$ together with t as a function of θ are supposed known, Equation [6] will give α_1 or α_2, thus locating the polar axis, and Equation [7] will then give c_1.

Just as in the case of the force P_1, the theory of the wedge with plane parallel ends[3] suggests that the radial stress σ_{r2} due to the force P_2 may be given by $\sigma_{r2} = (a_1/r)\cos\theta$, where a_1 is a constant, and that the tangential stress $\sigma_{\theta 2} = 0$ everywhere. Again as before, Equations [1] together with the condition that the sides of the wedge are free from stress require the shearing stress $\tau_{r\theta 2}$ to be everywhere zero. The solution for this case is therefore of the form

$$\sigma_{r2} = (a_1/r)\cos\theta, \quad \sigma_{\theta 2} = 0, \quad \tau_{r\theta 2} = 0 \ldots [8]\ [9]\ [10]$$

and it remains to locate the polar axis and to determine the constant a_1, which may be done in the same way as previously with the aid of the conditions of equilibrium (forces perpendicular and parallel to the polar axis) of the portion of the wedge between the edge and the circular-cylindrical section A-A.

$$\int_{-\alpha_2}^{\alpha_1} \sigma_{r2} tr \sin\theta \, d\theta = 0 \text{ and } \int_{-\alpha_2}^{\alpha_1} \sigma_{r2} tr \cos\theta \, d\theta + P_2 = 0$$

that is, $$\int_{-\alpha_2}^{\alpha_1} t \sin 2\theta \, d\theta = 0 \quad \ldots\ldots [11]$$

and $$a_1 \int_{-\alpha_2}^{\alpha_1} t \cos^2\theta \, d\theta + P_2 = 0 \quad \ldots\ldots [12]$$

Equation [11] is the same as Equation [6], and thus the direction of the polar axis is independent of the line of action of any force acting at the edge of the wedge. The constant a_1 can be obtained from Equation [12] as soon as the polar axis has been located from Equation [11].

The theory of the wedge with plane parallel ends[4] suggests that the radial stress σ_{r3} due to the couple M may be given by $\sigma_{r3} = -(4/r^2)d'_2 \sin 2\theta$, where d'_2 is a constant, and that the tangential stress $\sigma_{\theta 3} = 0$ everywhere. Equations [1] then give

$$\tau_{r\theta 3} = -\frac{4d'_2}{tr^2}\int t \sin 2\theta \, d\theta + \frac{a'_0}{tr^2}$$

or $$\tau_{r\theta 3} = \frac{4d'_2}{tr^2}\int_{+\theta}^{\alpha_1} t \sin 2\theta \, d\theta + \frac{c}{tr^2}$$

where a'_0 and c are constants. The condition that $\tau_{r\theta 3}$ must equal zero when $\theta = \alpha_1$ requires that $c = 0$, and the condition that $\tau_{r\theta 3}$ must equal zero when $\theta = -\alpha_2$ then requires that

$$\int_{-\alpha_2}^{\alpha_1} t \sin 2\theta \, d\theta = 0 \quad \ldots\ldots [13]$$

This equation is the same as Equations [6] and [11] and hence locates the polar axis in the same position as for a force at the edge of the wedge. Since, from Equation [13]

$$\int_{\theta}^{\alpha_1} t \sin 2\theta \, d\theta = \int_{\theta}^{-\alpha_2} t \sin 2\theta \, d\theta = \int_{\theta}^{\theta_e} t \sin 2\theta \, d\theta$$

where θ_e is either extreme value of θ, that is, either α_1 or $-\alpha_2$ (it is immaterial which), the solution for the wedge loaded by a couple at its edge is of the form

$$\sigma_{r3} = -(4/r^2)d'_2 \sin 2\theta \quad \ldots\ldots [14]$$

$$\sigma_{\theta 3} = 0 \quad \ldots\ldots [15]$$

$$\tau_{r\theta 3} = \frac{4d'_2}{tr^2}\int_{\theta}^{\theta_e} t \sin 2\theta \, d\theta \quad \ldots\ldots [16]$$

The constant d'_2 is most easily determined from the condition of equilibrium of moments. With respect to the edge of the wedge (Fig. 1)

$$\int_{-\alpha}^{\alpha_1} \tau_{r\theta 3} tr^2 d\theta + M = 0$$

that is, $$4d'_2 \int_{-\alpha_2}^{\alpha_1}\int_{\theta}^{\theta_e} t \sin 2\theta \, d\theta \, d\theta + M = 0$$

or, by transforming the integral by integration by parts[5]

$$4d'_2 \int_{-\alpha_2}^{\alpha_1} t\theta \sin 2\theta \, d\theta + M = 0 \quad \ldots\ldots [17]$$

When there is symmetry with respect to a plane bisecting the angle of the wedge, the problem is simplified somewhat: $\alpha_1 = \alpha_2 = \alpha$, and Equations [6], [11], and [13] (all the same equation) drop out.

The Ordinary Theory of Flexure as a Limiting Case of the Theory of the Wedge

The theory that has been presented reduces to the ordinary theory of flexure as α_1 and α_2 approach zero and r becomes infinite. Thus Equation [6] when multiplied by r^2 may be written for small values of θ

$$\int_{-\alpha_2}^{\alpha_1} t\theta r^2 \, d\theta = 0$$

With $z = r\theta$, $z_1 = r\alpha_1$, and $z_2 = r\alpha_2$, this equation becomes

$$\int_{-z_2}^{z_1} tz \, dz = 0$$

that is, the statical moment of the area of the section (now the same as the cross section) with respect to the polar axis (now the neutral surface) must be zero, which is the same thing as saying that the neutral axis passes through the centroid of the cross section. Equation [7], multiplied by r^3, may be written for small values of θ

$$c_1 \int_{-\alpha_2}^{\alpha_1} t\theta^2 r^3 d\theta - P_1 r^3 = 0$$

or $$c_1 \int_{-z_2}^{z_1} tz^2 \, dz - P_1 r^3 = 0$$

The integral in the last equation is the moment of inertia I of the area of the cross section with respect to the neutral axis, and therefore $c_1 = P_1 r^3/I$. Substitution of this value of c_1 in Equation [3] gives, with $z = r\theta$

$$\sigma_{r1} = P_1 rz/I$$

[3] "Plane Strain in a Wedge, With Applications to Masonry Dams," by S. D. Carothers, Proceedings of the Royal Society of Edinburgh, vol 33, 1912–1913, p. 292.

[4] By letting $u = \int_{\theta}^{\theta_e} t \sin 2\theta d\theta$, $dv = d\theta$, as indicated to the author by Dr. G. H. Keulegan.

the stress which would be obtained by the ordinary flexure formula.

Similarly, when θ is small, Equation [11] (same as Equation [6]) indicates that the line of action of P_2 passes through the centroid of the cross section, and Equation [12] when multiplied by r may be written

$$a_1 \int_{-\alpha_2}^{\alpha_1} tr\, d\theta + P_2 r = 0$$

or

$$a_1 \int_{-z_1}^{z_1} t\, dz + P_2 r = 0$$

The integral in the last equation is the area A of the cross section, and thus $a_1 = -P_2 r/A$. Substitution of this value of a_1 in Equation [8] gives for $\theta \approx 0$

$$\sigma_{r2} = -P_2/A$$

the familiar expression for simple tension or compression.

Again, in the case of the couple M Equation [13], being the same as Equations [6] and [11], locates the neutral axis as passing through the centroid of the cross section in the limit $\alpha_1 = 0$, $\alpha_2 = 0$, $r = \infty$. Equation [17] multiplied by r^3 may be written

$$8d'_2 \int_{-\alpha_2}^{\alpha_1} t\theta^2 r^3 d\theta + Mr^3 = 0$$

or

$$8d'_2 \int_{-z_2}^{z_1} tz^2 dz + Mr^3 = 0$$

Since the integral in the last equation is the moment of inertia I of the area of the cross section with respect to the neutral axis

$$d'_2 = -Mr^3/(8I)$$

Substitution of this value of d'_2 in Equations [14] and [16] gives for $\theta \approx 0$, with $z = r\theta$

$$\sigma_{r3} = Mz/I$$

and

$$\tau_{r\theta 3} = -\frac{M}{rIt} \int_z^{z_2} tz\, dz$$

The expression for σ_{r3} is again the ordinary flexure formula, and the expression for $\tau_{r\theta 3}$ is the one which would commonly be used in an ordinary beam subjected to a finite shearing force M/r at the cross section in question. If M is finite and $r = \infty$, $\tau_{r\theta 3} = 0$.

The Theory of the Wedge and the Ordinary Theory Applied to a Plate Girder With Nonparallel Flanges

Some idea of the magnitude of the error involved in using the ordinary theory instead of the theory of the wedge may be obtained by comparing the maximum fiber stresses computed by both theories in the case of a plate girder which is triangular (or trapezoidal) in elevation and symmetrical with respect to the line bisecting the angle between the flanges, $\alpha_1 = \alpha_2 = \alpha$. The cross-sectional area of each flange is assumed to be constant and concentrated at its centroid in order to simplify the computation.

The stress σ_{r1} along a circular-cylindrical section A-A in Fig. 1, due to a force P_1 at the intersection of the flanges (or the flanges extended) is obtained by determining the constant c_1 from Equation [7] and inserting its value in Equation [3]. Equation [7] becomes in the present case

$$2c_1 \frac{A_f}{r} \sin^2 \alpha + c_1 t \int_{-\alpha}^{\alpha} \sin^2 \theta\, d\theta - P_1 = 0$$

where A_f is the cross-sectional area of one flange and t is the thickness of the web. This equation solved for c_1 gives

$$c_1 = \frac{P_1 r}{tr(\alpha - \sin \alpha \cos \alpha) + 2A_f \sin^2 \alpha}$$

and this value of c_1 inserted in Equation [3] gives

$$\sigma_{r1} = \frac{P_1 \sin \theta}{tr\,(\alpha - \sin \alpha \cos \alpha) + 2\,A_f \sin^2 \alpha} \qquad [18]$$

which is a maximum, at A Fig. 1, when $\theta = \alpha$

$$\sigma_{r1\max} = \frac{P_1 \sin \alpha}{tr\,(\alpha - \sin \alpha \cos \alpha) + 2A_f \sin^2 \alpha}$$

According to the ordinary beam theory the maximum fiber stress f_1 on a cross section, A-A in Fig. 1, due to the force P_1 occurs at A and is

$$f_1 = \frac{P_1 r \cos \alpha \cdot r \sin \alpha}{(2/3)tr^3 \sin^3 \alpha + 2A_f \sec \alpha \cdot r^2 \sin^2 \alpha}$$

$$= \frac{P_1 \cos \alpha}{(2/3)tr \sin^2 \alpha + 2A_f \tan \alpha}$$

The ratio of f_1 to $\sigma_{r1\max}$ is

$$\frac{f_1}{\sigma_{r1\max}} = \frac{\cos^2 \alpha \left(\dfrac{\alpha - \sin \alpha \cos \alpha}{2\alpha \sin^2 \alpha} + \dfrac{A_f}{tr\alpha} \right)}{\dfrac{\sin \alpha \cos \alpha}{3\alpha} + \dfrac{A_f}{tr\alpha}}$$

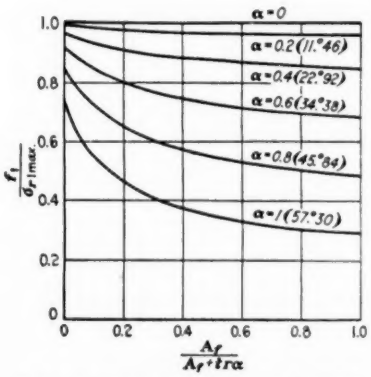


FIG. 2 CANTILEVER-LOADING CURVES

(Showing ratio of maximum fiber stress by the ordinary beam theory to maximum fiber stress by the wedge theory for all values of ratio of flange area to total circular-cylindrical-sectional area and for different values of the half angle of a wedge-shaped plate girder loaded as a cantilever.)

This ratio is shown in Fig. 2 for various values of α and for all possible values of the ratio $A_f/(A_f + tr\alpha)$ of the circular-cylindrical-sectional area of the flanges to the total circular-cylindrical-sectional area.

The stress σ_{r2} along a circular-cylindrical section A-A in Fig. 1, due to a force P_2 at the intersection of the flanges (or the flanges extended) is obtained by determining the constant a_1 from Equation [12] and inserting its value in Equation [8]. Equation [12] becomes in the present case

$$2a_1 \frac{A_f}{r} \cos^2 \alpha + a_1 t \int_{-\alpha}^{\alpha} \cos^2 \theta\, d\theta + P_2 = 0$$

from which

$$a_1 = -\frac{P_2 r}{tr(\alpha + \sin \alpha \cos \alpha) + 2A_f \cos^2 \alpha}$$

This value of a_1 inserted in Equation [8] gives

$$\sigma_{r2} = -\frac{P_2 \cos\theta}{tr(\alpha + \sin\alpha\cos\alpha) + 2A_f\cos^2\alpha} \quad \ldots\ldots [19]$$

which is a (numerical) maximum at the polar axis $\theta = 0$ so that

$$\sigma_{r2max} = -\frac{P_2}{tr(\alpha + \sin\alpha\cos\alpha) + 2A_f\cos^2\alpha}$$

By elementary methods the stress f_2 on a cross section through the same point (the intersection of the polar axis and the arc A-A in Fig. 1) due to the force P_2 would be computed as

$$f_2 = -\frac{P_2}{2tr\tan\alpha + 2A_f\sec\alpha}$$

The ratio of f_2 to σ_{r2max} is

$$\frac{f_2}{\sigma_{r2max}} = \frac{\cos^3\alpha\left(\dfrac{\alpha + \sin\alpha\cos\alpha}{2\alpha\cos^2\alpha} + \dfrac{A_f}{tr\alpha}\right)}{\dfrac{\sin\alpha}{\alpha} + \dfrac{A_f}{tr\alpha}}$$

This ratio is shown in Fig. 3 for various values of α and for all possible values of the ratio $A_f/(A_f + tr\alpha)$.

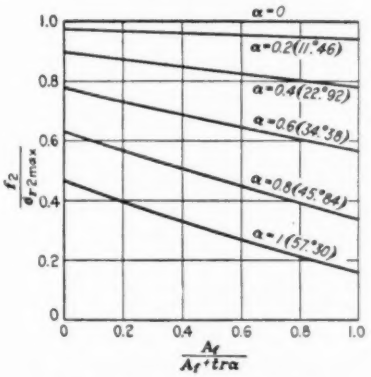


FIG. 3 AXIAL-LOADING CURVES

(Showing ratio of maximum stress by the ordinary prism theory to maximum stress by the wedge theory for all values of ratio of flange area to total circular-cylindrical-sectional area and for different values of the half angle of a wedge-shaped plate girder loaded axially.)

The stresses σ_{r3} and $\tau_{r\theta3}$ along a circular-cylindrical section, A-A in Fig. 1, due to a couple M at the intersection of the flanges (or the flanges extended) are obtained by determining the constant d'_2 from Equation [17] and inserting its value in Equations [14] and [16]. Equation [17] becomes in the present case

$$8d'_2\frac{A_f}{r}\alpha\sin 2\alpha + 4d'_2t\int_{-\alpha}^{\alpha}\theta\sin 2\theta\,d\theta + M = 0$$

from which

$$d'_2 = -\frac{Mr}{2[tr(\sin 2\alpha - 2\alpha\cos 2\alpha) + 4A_f\alpha\sin 2\alpha]}$$

This value of d'_2 inserted in Equation [14] gives

$$\sigma_{r3} = \frac{2M\sin 2\theta}{r[tr(\sin 2\alpha - 2\alpha\cos 2\alpha) + 4A_f\alpha\sin 2\alpha]} \quad \ldots [20]$$

When $\alpha \leqq \pi/4$, σ_{r3} is a maximum at A, Fig. 1, $\theta = \alpha$, and

$$\sigma_{r3max} = \frac{2M}{r[tr(1 - 2\alpha\cot 2\alpha) + 4A_f\alpha]}$$

If $\alpha \geqq \pi/4$, σ_{r3} is a maximum at $\theta = \pi/4$ and is given by setting $\sin 2\theta = 1$ in Equation [20].

According to the ordinary beam theory the maximum fiber stress f_3 on a cross section A-A, Fig. 1, due to a couple M occurs at A and is

$$f_3 = \frac{Mr\sin\alpha}{(2/3)tr^3\sin^3\alpha + 2A_f\sec\alpha\cdot r^2\sin^2\alpha} = \frac{M}{2r[(1/3)tr\sin^2\alpha + A_f\tan\alpha]}$$

The ratio of f_3 to σ_{r3max} for $\alpha \leqq \pi/4$ is

$$\frac{f_3}{\sigma_{r3max}} = \frac{\alpha\cot\alpha\left[\dfrac{1}{4\alpha^2}(1 - 2\alpha\cot 2\alpha) + \dfrac{A_f}{tr\alpha}\right]}{\dfrac{\sin\alpha\cos\alpha}{3\alpha} + \dfrac{A_f}{tr\alpha}}$$

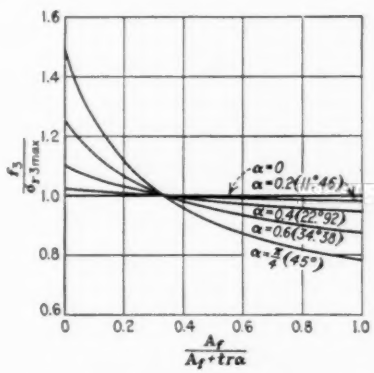


FIG. 4 COUPLE-LOADING CURVES

(Showing ratio of maximum stress by the ordinary beam theory to maximum stress by the wedge theory for all values of ratio of flange area to total circular-cylindrical-sectional area and for different values of the half angle of a wedge-shaped plate girder loaded by a couple at the edge end.)

This ratio is shown in Fig. 4 for various values of α and for all possible values of the ratio $A_f/(A_f + tr\alpha)$.

Inspection of Figs. 2, 3, and 4 shows that so long as the included angle of the wedge does not exceed 20 deg ($\alpha = 10$ deg) the ordinary beam theory will never be in error on the unsafe side by more than about 5 per cent nor on the safe side by more than about 2 per cent. For larger angles the error may increase rapidly with the angle, however.

In order to obtain $\tau_{r\theta3}$ in a useful form, it is necessary to evaluate the integral in Equation [16]. In the present case one obtains

$$\int_{\theta}^{\theta_e} t\sin 2\theta\,d\theta = \frac{A_f}{r}\sin 2\alpha + t\int_{\theta}^{\alpha}\sin 2\theta\,d\theta = (A_f/r)\sin 2\alpha - (t/2)(\cos 2\alpha - \cos 2\theta)$$

and insertion of this value of the integral and the previously found value of d'_2 in Equation [16] gives

$$\tau_{r\theta3} = -\frac{M[tr(\cos 2\theta - \cos 2\alpha) + 2A_f\sin 2\alpha]}{tr^2[tr(\sin 2\alpha - 2\alpha\cos 2\alpha) + 4A_f\alpha\sin 2\alpha]} \quad \ldots [21]$$

which is a (numerical) maximum at the polar axis $\theta = 0$ so that

$$\tau_{r\theta3max} = -\frac{M[tr(\csc 2\alpha - \cot 2\alpha) + 2A_f]}{tr^2[tr(1 - 2\alpha\cot 2\alpha) + 4A_f\alpha]}$$

In the case of pure bending by a couple M if the ordinary beam theory were used, the corresponding shearing stress would be zero.

Curved Extreme Fibers

The theory may be applied as an approximation to beams having curved extreme fibers. In such a case a wedge may be formed for analytical purposes by first drawing a tangent to the extreme fiber at any point and then swinging an arc through this point with center on the tangent and with such a radius that it intersects the opposite extreme fiber at right angles. The tangents from the center of this arc to the extreme fibers, together with the arc, form a wedge and the stresses along the arc (circular-cylindrical section) may be computed by the theory which has been presented. It is to be expected in the case of curved extreme fibers that the theory will give better values, the smaller the curvature.

An example of this kind will be given in which the theory was substantiated by actual determination of stresses from strains measured on a rigid-frame specimen with curved inner flange. This specimen was tested recently at the National Bureau of Standards.[2] One half of the specimen is shown in

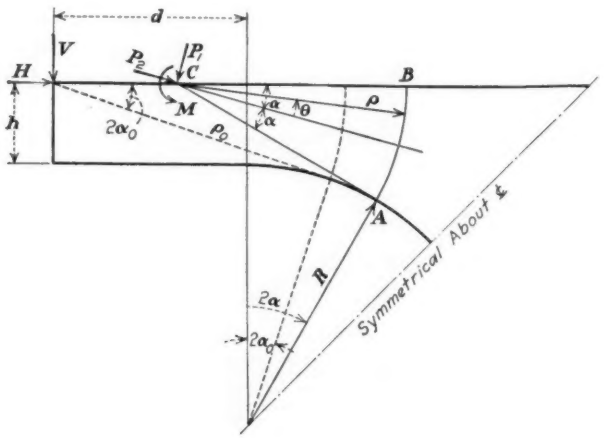


Fig. 5 Outline of One Half of a Rigid-Frame Specimen With Curved Inner Flange

outline in Fig. 5 and the entire specimen is shown in Fig. 6. The flange (indicated by heavy lines in Fig. 5) consisted of two $4 \times 4 \times {}^1/_2$-in. angles, and the web plate was ${}^3/_8$ in. thick. The distance h between the centroids of the cross-sectional areas of the flanges was 15.7 in., the distance d from the point of application of the load to the point of tangency of the curved part of the inner flange with the straight part was 49.85 in., and the radius R of the centroidal line of the curved inner flange was 61.15 in.

In order to determine the state of stress in the specimen, tangents were drawn from several points, such as A in Fig. 5, on the

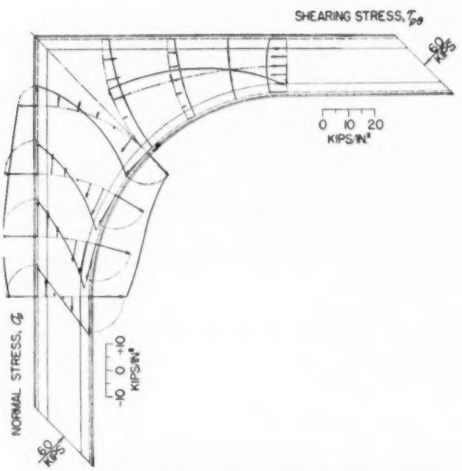


Fig. 6 Rigid-Frame Specimen With Curved Inner Flange
(One kip equals 1000 lb.)

curved inner flange intersecting the straight outer flange, as at C. With these points of intersection as centers, arcs such as AB were swung through the points of tangency. The stresses along these arcs of radius ρ and included angle 2α could then be determined from Equations [18], [19], [20], and [21] by inserting the values of $r = \rho$ and the proper values of P_1, P_2, and M. These quantities in terms of the components H and V of the applied load and the other known quantities are easily found to be

$$\rho = \frac{h + R(1 - \cos 2\alpha)}{\sin 2\alpha}$$

$$P_1 = V \cos \alpha - H \sin \alpha$$

$$P_2 = V \sin \alpha + H \cos \alpha$$

$$M = V\left[d - \frac{h \cos 2\alpha - R(1 - \cos 2\alpha)}{\sin 2\alpha}\right]$$

The dotted lines in Fig. 5 indicate the wedge (included angle $2\alpha_0$, radius ρ_0) for which $M = 0$. Along the dotted arc $2\rho_0\alpha_0$ the shearing stresses are everywhere zero, and at the extremities of this convenient arc the normal stresses are nearly a maximum. For a detailed discussion the reader is referred to the paper cited in footnote 2.

Fig. 6 shows by means of curves at various circular-cylindrical sections the theoretical stresses on these sections and also by means of arrows the stresses which were determined from measurements of strain under a 60,000-lb load applied as indicated. It is considered that the agreement is within the experimental error.

Design Data

It is important that the data contained in technical papers be made readily available to designing engineers. In order to satisfy these needs of industry, this section of the JOURNAL will include a concise presentation of data and information drawn chiefly from papers previously published by the Applied Mechanics Division of The American Society of Mechanical Engineers.

The first series dealt with the subject "Strength of Materials" and the present series will cover the subject "Vibration." The data have been prepared by a subcommittee of the Applied Mechanics Division (J. Ormondroyd, chairman), under the general guidance of S. Timoshenko.

The second article, which follows, is by J. Ormondroyd. This considers the dynamic specifications of a system and there is included a table of typical spring constants both linear and torsional.

Vibration Problems

PART II

BY J. ORMONDROYD,[1] ANN ARBOR, MICH.

THE major part of any vibration analysis lies in the calculation of the dynamic specifications of the simplified system which represents the machine or structure being studied. The dynamic specifications of a system are the numerical values of the masses, moments of inertia, spring constants, and damping constants which determine its dynamic behavior. These quantities are the coefficients which appear in the differential equations of motion of the system.

DYNAMIC SPECIFICATIONS

Mass.

The calculation of mass requires little comment except that the value of g used in the lb-in-sec system is 386 in./sec^2. In the lb-mass-ft-sec system the unit of mass is the slug; 1 slug weighs 32.2 lb, 1 lb mass weighs 1 lb. The unit of mass in the lb-in-sec system has no official name. For engineers it is always safest to forget the named units of mass and name the mass units after the units of w/g, that is, lb sec^2/in. (g = 386 in. sec^2) or lb sec^2/ft (g = 32.2 ft/sec^2).

For bodies of simple shape and uniform material

$$m = \gamma V/g \text{ lb sec}^2/\text{in.}$$

where γ = weight per cubic inch
V = volume in cubic inches
g = acceleration of gravity, in./sec^2

In many cases actual weights W can be measured, in which case

$$m = W/g \text{ lb sec}^2/\text{in.}$$

Moments of Inertia.

Methods of calculating moments of inertia are given in most treatises on mechanics and in all handbooks. A complete and understandable treatment of the analytical method of calculating moments of inertia is given by Timoshenko and Young (1).[2] Condensed treatments of the subject are given in Marks' Handbook (2) and Eshbach's Handbook (3).

Tables of mass moments of inertia for bodies of simple shapes are given by Timoshenko and Young (4), Marks (5), and Eshbach (6).

In American engineering practice three totally different quantities are designated by the single term "moment of inertia." In calculating the deflections of beams a geometrical moment of inertia of the beam cross-sectional area taken with respect to the neutral axis is used. This has *in.*^{4} units. For twisted beams a geometrical polar moment of inertia of the beam cross section taken with respect to the axis of twist is used. This also has *in.*^{4} units. The mass moment of inertia taken with respect to the axis of rotation is defined by the equation $I = \int r^2\, dm$ and has the units *lb in. sec*^{2}. All three are usually discussed together. The engineer must be careful to select the right kind for the problem he has at hand.

Although the geometrical moments of inertia must not be confused with the mass moments of inertia, a simple relationship does exist between the geometrical polar moment of inertia J of the cross section around the axis of rotation and I the mass moment of inertia around the same axis. For bodies of constant axial thickness

$$I = J \times \frac{\text{total mass}}{\text{cross-sectional area}} \text{ lb in. sec}^2$$

Some of the older handbooks define moment of inertia in Wr^2 (weight × length squared) units. This is sometimes called "flywheel effect." The flywheel effect divided by g (the acceleration of gravity) is the mass moment of inertia as used in these articles.

Moments of inertia can be measured if the body in question can be suspended as a pendulum and set into oscillation.

$$I = \frac{WlT^2}{4\pi^2} \text{ and } I_0 = Wl\left(\frac{T^2}{4\pi^2} - \frac{l}{g}\right)$$

where
W = weight, in pounds, found by weighing
l = shortest distance from axis of rotation to center of gravity, found by measurement
T = period of pendulum in seconds, measured experimentally

Spring Constants.

The spring constants of the various types of elastic members which occur in machines and structures can be calculated by

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[2] Numbers in parentheses refer to the Bibliography.

methods developed in the theory of elasticity. The sections of this theory which deal with the deflections and deformations of bodies and groups of connected bodies under the action of various types of loading can be used directly to indicate methods of spring-constant calculation.

Tables of linear and torsional deflections of beams under loads and twisting moments are given in all handbooks. These can be used to calculate the spring constants, since these are the load divided by the deflection. Fundamental methods for calculating deflections of beams and structures in bending are given by Timoshenko and Lessells (7) and by Timoshenko (8). Torsional-deflection calculations are discussed by Timoshenko and Lessells (9) and by Timoshenko (10).

Methods using *strain energy* are very important practically. These methods are discussed by Timoshenko and Lessells (11), Timoshenko (12), and Van den Broek (13).

Timoshenko and Lessells (14) give a very detailed discussion of the axial and bending deflection of helical springs with large pitch angle in which the energy method is used.

Graphical methods become necessary when beams with many concentrated loads, many changes in cross-sectional dimension, and irregularly distributed loadings are under investigation. This method is discussed by Timoshenko and Lessells (15).

For many torsional-deflection problems, Prandtl's Membrane Analogy gives the only easily attainable results. This method is outlined by Timoshenko and Lessells (16) and by Timoshenko (17).

Grammel (18) gives a method for treating helical springs as solid beams for compression, tension, bending, and twisting.

Tables which give deflections of beams under various loadings can often be used to calculate spring constants. Such tables for bending are given in Marks' Handbook (19) and Eshbach's Handbook (20); and for twisting, Marks' Handbook (21) and Eshbach's Handbook (22). Marks' Handbook (23) gives the deflection of many types of springs in a separate section.

On the last page of this contribution 24 typical spring constants are given. In reading the table, it should be kept in mind that the spring constants are defined as ratios. The linear constant $k_l = P/\Delta$; P = load or force causing the deflection, Δ = the deflection in the direction of the load causing it at the point of application of the load. The torsional constant $K_t = M/\varphi$; M = the twisting moment causing the angular distortion. It acts in a plane perpendicular to the axis of twist; φ = the angular deflection around the axis of twist in the plane of the twisting moment. The torsional constant in bending $K_{tb} = M/\varphi$; M = bending moment acting on the beam, φ = the angle between the tangents to the neutral axis of the beam in the plane of the bending moment before and after the moment is applied.

It should be remembered that in unsymmetrical structures or in symmetrical structures unsymmetrically loaded, other deformations beside the ones which appear in the spring constants can occur. The point of application of the load may deflect at right angles to the load. Rotations around the point of application of the load may also occur. In many simple problems these additional distortions may be ignored—in a more refined analysis they would have to be considered.

Damping Constants.

Damping or friction forces are always associated with the energy losses of the system. Whenever the macroscopic or molar motion of the system is transformed into microscopic or molecular motion in the bodies of the system itself, or in the medium which surrounds the body, damping exists. Most generally stated, damping forces exist when momentum is transferred from the vibrating system to the rest of the universe which is not included in the system isolated for study. Naturally transfer of molar momentum to other macroscopic bodies in the same system is not considered as damping.

Two general types of damping forces exist—friction forces and impactive forces. Friction forces occur between the surfaces of bodies moving relatively to each other. They are tangential to the surfaces at the points of action, they are functions of velocity and always act in the opposite direction to the velocity. Friction forces occur internally between the particles which form the body as well as on its outer surface. Impactive forces occur whenever the moving body sets the surrounding medium in motion, whether this medium be fluid or solid.

A complete discussion of the various types of damping forces is given by Stanton (24). A very general paper considering damping as specifically applying to vibrating systems was written by Kimball (25).

Coulomb Friction.

The kinetic or sliding Coulomb friction force is $F = \pm \mu N$, where μ is a coefficient of friction and N is the normal force exerted on the moving bodies by the surrounding bodies which touch it and to which it is moving relatively. Extensive references to this type of damping are given by Timoshenko (26). Tables of values for kinetic or sliding friction between several materials are given in Marks' Handbook (27) and Eshbach's Handbook (28). Free and forced vibration of systems of one degree of freedom on which Coulomb friction is acting are discussed by Timoshenko (29) and Den Hartog (30). Den Hartog (31) has also given a rigorous solution for this problem. Kimball (25) has discussed an energy method for dealing with this type of vibration damping and Jacobsen (32) has given a valuable approximate method for forced vibration with Coulomb damping.

Internal Friction in Solids.

No accurate and simple detailed description of internal hysteresis forces has yet been made. They are usually handled quantitatively by experiments which give the energy lost per cycle of distortion as a function of the maximum stress occurring during the cycle of motion. Kimball (25) has treated this subject. Kimball and Lovell (33) have described an ingenious experiment in which the internal hysteresis of materials in tension and compression can be measured. Rowett (34) and Föppl (35) have made investigations of hysteresis losses in metals in torsion.

Viscous Friction.

Viscous friction occurs at lubricated surfaces and on the surfaces of bodies moving slowly through gaseous and fluid media. The whole theory of viscous friction stems from the simple definition proposed originally by Newton and put into its modern form by Maxwell

$$F = -\eta A v/h$$

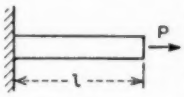
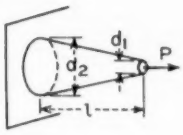
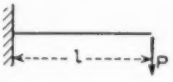
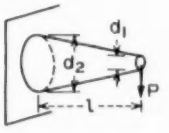
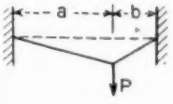
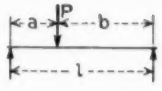
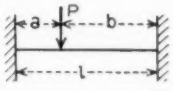
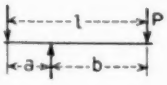
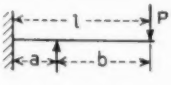
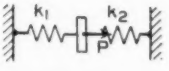
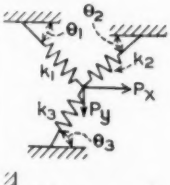
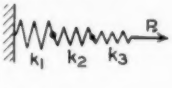
Here η is the coefficient of viscosity of the fluid involved, A is the area of the surface in contact with the fluid, h is the thickness of the lubricating fluid between the bodies in relative motion and v is the relative velocity of the two contiguous surfaces.

Viscous friction is discussed extensively in all treatises on hydrodynamics. Stanton (24) devotes a large section of his book to its discussion. This type of friction lends itself to convenient mathematical manipulation. For this reason nearly all theoretical investigations on vibration assume it to be the only friction existing. This is equivalent to assuming that all types of damping can be described by an equivalent viscous damping. Jacobsen (32) has indicated that this assumption is practically substantiated.

Impactive Damping.

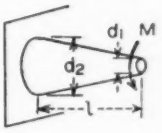
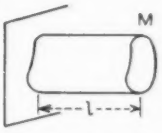
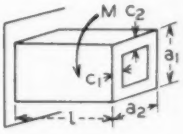
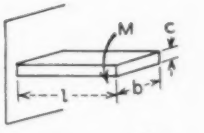
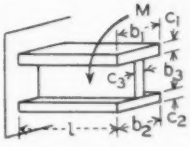
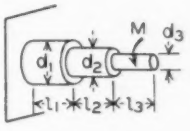
Impactive damping forces are described by simple equations

LINEAR SPRING CONSTANTS

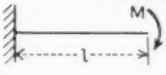
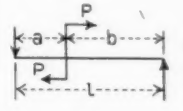
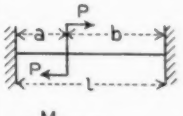
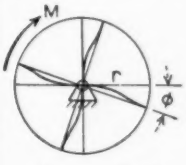
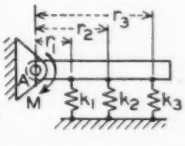
	DESCRIPTION	SPRING CONSTANT LB PER IN.
	BARS OF CONSTANT CROSS SECTION IN TENSION OR COMPRESSION E = MODULUS OF ELASTICITY, LB PER SQ IN. A = CROSS-SECTIONAL AREA, SQ IN. l = LENGTH	$\frac{EA}{l}$
	TRUNCATED CONE IN TENSION OR COMPRESSION	$\frac{\pi d_1 d_2}{4}\ \frac{E}{l}$
	CANTILEVER BEAM, CONSTANT SECTION, LOADED AT FREE END	$\frac{3EI}{l^3}$
	TAPERED CONE, AS CANTILEVER BEAM LOADED ON FREE END	$\frac{3E}{l^3}\ \frac{\pi d_2^4}{64}\ \frac{d_1}{d_2}$
	SMALL LATERAL DEFLECTIONS OF STRETCHED ELASTIC STRING T = ORIGINAL TENSION FORCE IN THE STRING	$T\ \frac{a+b}{ab}$
	SIMPLY SUPPORTED BEAM, CONSTANT CROSS SECTION	$\frac{3EI}{l^3}\ \frac{(a+b)^4}{a^2b^2}$
	CONSTANT CROSS SECTION, BEAM BUILT IN AT BOTH ENDS	$\frac{3EI}{l^3}\cdot\frac{l^2}{a^3b^3}$
	SPRINGBOARD BEAM, CONSTANT CROSS SECTION	$\frac{3EI}{l(l-a)^2}$
	SPRINGBOARD BEAM BUILT IN AT ONE END	$\frac{3EI}{l(l-a)^2\left(1-\frac{1}{4}\frac{a}{l}\right)}$
	SPRINGS IN PARALLEL	$k_1 + k_2$
	A GROUP OF STRETCHED SPRINGS ATTACHED TO ONE CENTER. θ_n IS THE SMALLEST ANGLE EACH SPRING MAKES WITH THE HORIZONTAL AT EQUILIBRIUM BEFORE LOADING	$k_x = \sum k_n \cos^2\theta_n$ $k_y = \sum k_n \sin^2\theta_n$
	SPRINGS IN SERIES	$k = \frac{1}{\frac{1}{k_1}+\frac{1}{k_2}+\frac{1}{k_3}}$
	FOR m SPRINGS IN SERIES	$k = \frac{1}{\sum_{n=1}^{n=m}\frac{1}{k_n}}$

TORSIONAL SPRING CONSTANTS

ROTATIONS CAUSED BY TWISTING MOMENTS

	DESCRIPTION	SPRING CONSTANT LB-IN. PER RADIAN
	TRUNCATED CONE G=MODULUS OF ELASTICITY IN SHEAR	$\frac{G}{32l}\ \frac{\pi(d_2-d_1)d_1^3 d_2^3}{d_2^3-d_1^3}$
	PRISM, CONSTANT CROSS SECTION OF SIMPLE OR IRREGULAR SHAPE. NO HOLES IN CROSS SECTION. A = AREA OF CROSS SECTION J = POLAR MOMENT OF INERTIA OF CROSS SECTION (ST. VENANT'S ELLIPTIC SECTION GOOD APPROXIMATION FOR ALL SECTIONS)	$\frac{G}{l}\ \frac{A^4}{4\pi^2 J}$
	RECTANGULAR HOLLOW TUBE	$\frac{G}{l}\ \frac{2}{\frac{1}{a_1 a_2^2 c_1}+\frac{1}{a_1^2 a_2 c_2}}$
	RECTANGULAR-CROSS-SECTION BAR	$\frac{G}{l}\ \frac{bc^3}{3}$
	I-BEAM	$\frac{G}{l}\ \frac{\sum b_n c_n^3}{3}$
	TORSIONAL SPRINGS IN SERIES	$k = \frac{\pi G}{32}\ \frac{1}{\sum_{n=1}^{n=m}\frac{l_n}{d_n^4}}$

ROTATIONS CAUSED BY BENDING MOMENTS

	CANTILEVER BEAM, CONSTANT CROSS SECTION	$\frac{EI}{l}$
	SIMPLY SUPPORTED BEAM, ROTATION AT POINT OF APPLICATION OF MOMENT	$\frac{3EI}{l}\ \frac{1}{1-\frac{3ab}{l^2}}$
	BEAM BUILT IN AT BOTH ENDS	$\frac{6EI}{l}\ \frac{(l+b)l^4}{9a^2b^3+abl^2(12b-3a)}$
	FLYWHEEL SPOKES, CONSTANT CROSS SECTION n = NUMBER OF SPOKES r = LENGTH OF SPOKES	$n\frac{4EI}{r}$
	EQUIVALENT TORSIONAL-SPRING CONSTANT OF LINEAR SPRINGS AROUND AXIS A	$\sum k_n r_n^2$

of the type $F = Cv^n$, where $n \geq 2$. In ordinary vibration problems perhaps only the case in which $n = 2$, velocity-squared damping, has any importance. This damping occurs when the vibrating bodies have high velocities in a fluid or gaseous medium. The term "high velocity" here has meaning only in terms of the Reynolds number which exists for the particular case. Wherever waves and eddies are created by the vibrating body in the medium surrounding it, damping proportional to v^2 can be assumed to exist. This probably holds true, at least approximately, even in the case where the vibration is limited to impact with solid bodies. Hort (36) discusses free vibrations with velocity-squared damping and Forchheimer (37) gives a more extensive treatment to the subject. Forced vibrations with velocity-squared damping can be treated by Jacobsen's (32) energy method.

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Discussion

Methods of Rotor-Unbalance Determination[1]

T. C. Rathbone.[2] Developments of methods for balancing turbine-generator installations under operating conditions became necessary, since the slow-speed dynamic-balancing machine correction at room temperature in the factory does not assure satisfactory operation.

The field-balancing methods are based on vibration observations at the bearings, which include determination of both the amplitude and the phase relation between vibratory motion and the angular rotor position, in other words, the "vibration vector." The change in the initial vectors brought about by the addition of a known unit of unbalance in a known position determines the "unit vectors" for all bearings, resulting from the unit of unbalance at the location applied.

With the assumption of linearity, these groups of unit vectors, defining the response of all bearings produced by unit unbalance at each corrector location, determine a fixed characteristic of the installation. Once evaluated, any unknown unbalance can then be corrected by canceling its vectors with appropriate adjustment of the unit vectors.

In the writer's paper,[3] a graphic solution by trial was described for the two-bearing case, also a rough method by inspection indicated for the four-bearing case. E. L. Thearle[4] described an analytical method for the final step, which yields a direct solution for the two-bearing case. F. C. Rushing and B. A. Rose[5] proposed another vectorial-operation method for the solution of the two-bearing case.

The author points out that the previous analytical methods are cumbersome and, under certain conditions, lead to large errors. This is particularly true, where the response at a distant bearing is large in relation to the effect at a bearing nearest the corrector application or, where differences between large quantities are handled. His experience has also been that frequently the desired correction is not obtained at once and additional trials are necessary for further reduction of the residual. He proposes an intermediate step using groups of trial weights, selected so that their effect is nil at all but the bearing selected for observation, and develops expressions involving vector quantities for this determination for both the two- and the three-bearing case. The residual or remnant errors then rapidly converge for elimination within the tolerance. The author's analytical and instrumentation contributions in balancing have been outstanding.

The writer has preferred the purely graphic handling of the vector solutions, as being simpler and more easily followed by the practical engineer. Since the original paper he also has extended the graphic method to take advantage of the reduction in graphic trials afforded by adopting combinations or groups of trial weights which affect only the selected bearing.

The method of using combinations of weights was suggested by the analogy with the Marsland process,[6] used with the Westinghouse dynamic-balancing machine, in which pairs of corrector weights are computed, which affect one end only without upsetting the balance at the other end.

The case of the two-bearing rotor is shown in Fig. 8, where A and B are the unit trial weights and the following numerical values are assumed:

$1A$ is the unit vibration vector at bearing No. 1 due to unit weight A
$2A$ is the unit vibration vector at bearing No. 2 due to unit weight A
$1B$ is the unit vibration vector at bearing No. 1 due to unit weight B
$2B$ is the unit vibration vector at bearing No. 2 due to unit weight B

These unit vectors have been determined by observations on the two bearings for the initial condition with unknown unbalance as found, and on the two trial runs with the trial weight first at A and then at B. The unit vectors are the closing vectors between the initial condition as found and the condition as altered by the application of the unit trial weights.

It is required to replace the unit trial weight A by a combination of A and a weight b at the other end, such that the vector at the No. 2 bearing is nil. Unit weight A produces a secondary

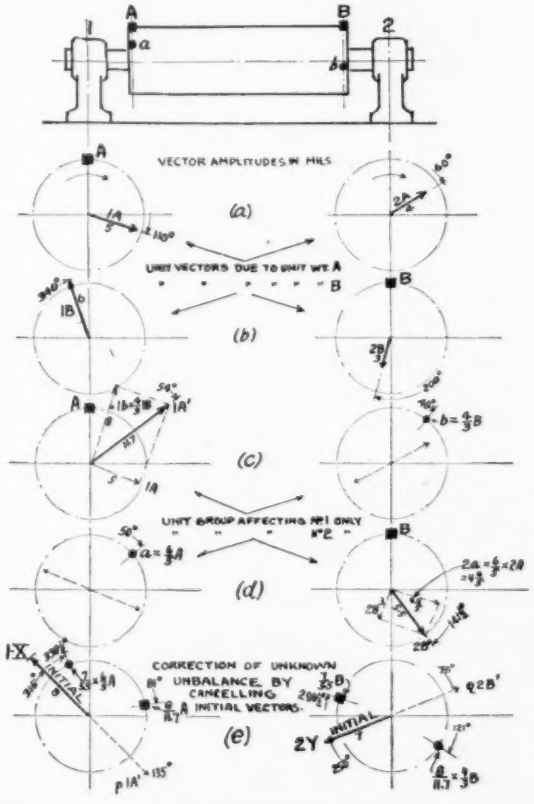


Fig. 1 Graphic Method Employing Groups of Trial Weights Applied to Two-Bearing Rotor

[1] Paper by J. G. Baker published in the March, 1939, issue of the Journal of Applied Mechanics, Trans. A.S.M.E., vol. 61, 1939, p. A-1.

[2] Chief Engineer, Turbine Division, The Fidelity and Casualty Company of New York.

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[6] U. S. Patent No. 1,591,853.

vector $2A$ at the No. 2 bearing. A weight b is selected whose magnitude is in the ratio of the magnitude of the vectors $2A$ and $2B$, and whose angular position is such that the vectors cancel. The amount and location is represented by b, Fig. 1(c).

However, the addition of unit b produces a secondary vector at the No. 1 bearing, in the same relation that B produces the vector $1B$. Call this secondary vector $1b$. The vectorial sum of $1A$ and $1b$ is $1A'$. Thus, for the unit trial combination of A and b, the unit vector is $1A'$ at the No. 1 bearing, and zero at the No. 2 bearing. By the same process, the combination of trial weight B and a produces a zero vector at the No. 1 bearing and unit vector $2B'$ at the No. 2 bearing, Fig. 1(d).

These single-unit vectors can now be applied directly to nullify the initial vectors due to the initial unbalance. Thus, if in Fig. 1(e) $1X$ and $2Y$ are the original unknown unbalance vectors, the adjusted-unit groups are found directly, whose vectors $p1A'$ and $q2B'$ are equal and opposite, canceling the originals. In practice, the two corrector weights at each end are combined into single weights for final application to the rotor.

The three and four-bearing installations are of course more difficult to solve graphically, than the two-bearing case, but the labor is still entirely graphical. In Fig. 2 the usual four-bearing set is illustrated, each rotor having two corrector planes, and ignoring for the time the corrector zones now commonly provided along the body of long, flexible rotors.

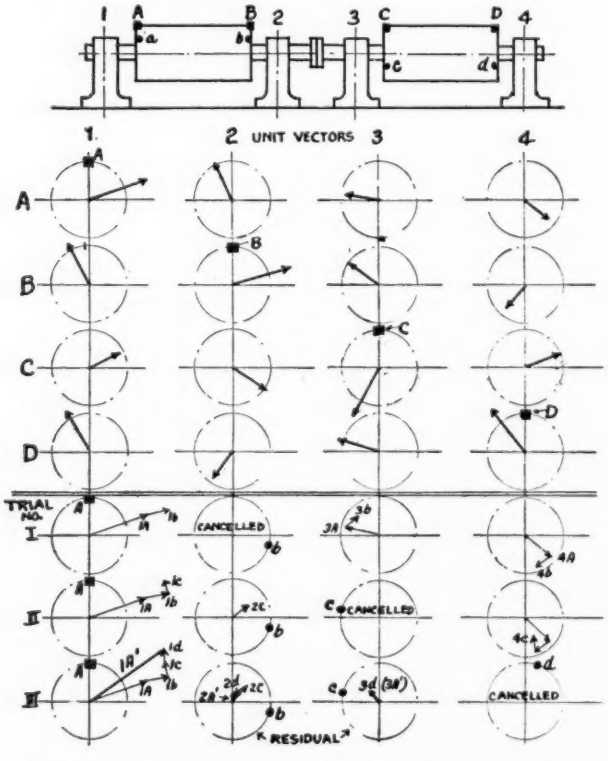


FIG. 2 UNIT WEIGHT GROUP TO AFFECT NO. 1 BEARING ONLY
(Residual reduced by continuing trial process; groups for other bearings found in similar manner.)

The four-bearing case involves sixteen vectors, evaluated by four trial runs with the unit weight in each of the locations. The direct application of these vectors for correction of the initial unbalance, even by the graphic process, would be difficult. However, if these data are reduced to unit groups affecting one bearing only, with zero vectors at all other bearings, the problem becomes simple, as the author points out.

In Fig. 2, the graphic trial process is indicated for determining the weight groups which will produce a "primary" vector at the No. 1 bearing, with zero or insignificant residual vectors at the other three bearings. The vectors at Nos. 2, 3, and 4 due to A are canceled in turn by appropriate adjustment of amount and angular position of units B, C, and D. This is accomplished with merely a protractor and simple arithmetic. A primary vector is left at No. 1, secondary residuals at Nos. 2 and 3, and zero at No. 4. If the residuals at Nos. 2 and 3 are still significant, the process is repeated until they are less than one third of the permissible tolerance. If the response from a weight is greater at a distant than at a nearer bearing, an inversion is necessary for convergence.

In like manner, unit groups are found which produce a primary vector at Nos. 2, 3, and 4, respectively, and zero at the other bearings. These unit groups can now be applied directly to cancel the initial unbalance vectors graphically. The four weights resulting at each location would in practice be combined into single corrector weights for actual application to the job. The most unfavorable sum of the residuals would still be within the tolerance.

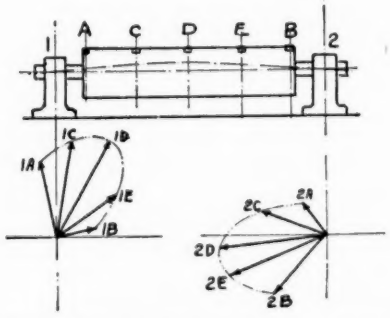


FIG. 3 TRIAL RUNS ON A FLEXIBLE ROTOR

The Vectorscope,[7] described by G. J. Dashefsky, developed to aid in the analysis of Diesel-engine balancing and torsional vibration, would seem to have application to the present problem. Four Vectorscopes, geared together with means for altering the vectors proportionately, would aid in determining the necessary trial group combinations.

The balancing of large rotors in service is not a mass-production procedure. There is usually ample time between runs for the graphic work which should greatly reduce the number of shutdowns.

During the meeting, the author pointed out some situations which introduced nonlinearity between cause and effect. One of the conditions, which brings about nonlinear vector response, arises when the trial-unbalance location is shifted axially on long flexible rotors. From the flexural consideration, a given unit of unbalance produces the greatest mass displacement and hence disturbance, when located midway between the bearings.

In Fig. 3, representing trial runs on a flexible rotor, the vibratory responses for a unit of unbalance at various axial locations would be expected to appear as indicated by the vectors. The vectors at Nos. 1 and 2 bearings are represented by $1A$ and $2A$, respectively, for the weight located at A, and so on. The magnitudes increase as the unit weight approaches the mid-position.

If this relationship were established by actual trial, it might be useful for computing the transfer of corrections from the ends to the middle body, except for the fact, which the author cited, that the particular mode of vibration may introduce a complication.

[7] "The Vectorscope," by G. J. Dashefsky, Trans. A.S.M.E., vol. 61, July, 1939, p. 403.

If the rotor and its elastic supporting structure are vibrating in the mode with the pedestals in phase, Fig. 4(*a*), the flexural relationship may hold. If a resonant condition is approached, in the "seesaw" mode with the pedestals moving in opposite phase, as indicated in Fig. 4(*b*) the unbalance at the node at the middle is not as effective as at the rotor ends.

The flexural condition might be tested by making a trial run with a unit weight at the center, as at *W*, Fig. 4 (*a*) and two equal weights of $^1/_2$ *W* at the ends, in the opposite direction. As these weights constitute a dynamically balanced system, the

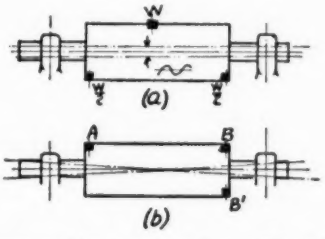


FIG. 4 (*a*) PEDESTALS VIBRATING IN PHASE WITH ROTOR AND SUPPORTING ELASTIC STRUCTURE; (*b*) PEDESTALS MOVING IN OPPOSITE PHASE

introduction of any disturbance would be a measure of the flexural effect.

The sensitivity to either mode of vibration might be tested by making a trial run with the weight first, at *A*, Fig. 4(*b*), then leaving *A* in, with a similar weight at *B*. From these runs, the unit vectors for the weights at *A* and *B* are obtained. Finally, a test run is made with *B* rotated 180 deg to *B'*, forming a couple with *A*. A comparison of the resulting vectors with the vectors to be expected by using the data from the first two runs should indicate directly any difference in sensitivity to the two modes of vibration.

It is understood of course that, in the foregoing discussion, a definite operating speed is assumed, and both the lateral and vertical components will undergo independent changes at other speeds. Rushing and Rose[8] have reported an extreme case where vibrations both at the operating speed and a lower critical speed were corrected only after taking into account the flexural effect with axial locations.

The Fundamentals of Three-Dimensional Photoelasticity[1]

M. M. FROCHT.[2] The fundamental questions in connection with the possible extension of the photoelastic method into the domain of three-dimensional problems are: First, Is the stress pattern which is obtained by slicing a model, which is stressed at a high temperature and annealed to room temperature, affected by the cutting process itself? and second, Does the interpretation of this stress pattern follow the same stress-optic law which governs two-dimensional photoelasticity?

Considerable work has been done in the photoelastic laboratory of the Carnegie Institute of Technology to determine the answers to these questions and it is a source of satisfaction to report that our findings completely corroborate the results obtained by Oppel, Hiltscher, and Kuske in Germany and by Hetényi in this country regarding the elastic behavior of bakelite at high temperatures, as well as the magnitude of the fringe value given by the author in the paper under discussion.

Our experiments, like those of the author, show that a frozen stress system is not affected by being cut in any manner whatsoever. Fig. 1 shows a frozen stress pattern of a bar with a circular hole in a field of pure tension, while Fig. 2 shows the stress pattern after two circular holes and a groove have been cut in the same model. Our work further shows that in a stress range of from 20 to 200 psi the behavior of bakelite at temperatures equal to or exceeding 110 C is completely elastic, that the optical sensitivity at 112 C is 26.3 times as great as at room temperature, and that it remains essentially constant above 110 C. Fig. 3 shows the variations in the fringe value in a series of twelve tests. It is seen that none of the results exceed the average by more than plus or minus 2 per cent. These results have been obtained from constant-load tests with tension models. No such consistency in the results is obtainable from constant-deformation tests, especially at low stresses.

Dr. Hetényi has made a significant contribution by bringing



FIG. 1 FROZEN OR FIXED STRESS PATTERN OF A BAR WITH A CIRCULAR HOLE IN TENSION
(Load, 5.04 lb; width 0.847 in.; thickness, 0.258 in.; fringe value, 12.22 psi tension.)



FIG. 2 SAME BAR AS IN FIG. 1 AFTER TWO HOLES AND A GROOVE HAVE BEEN CUT
(No disturbance of the stress pattern due to cutting.)

[1] By M. Hetényi. Published in the December, 1938, issue of the JOURNAL OF APPLIED MECHANICS, Trans. A.S.M.E., vol. 60, 1938, p. A-149.

[2] Associate Professor of Mechanics, Carnegie Institute of Technology, Pittsburgh, Pa. Mem. A.S.M.E.

into the field of photoelasticity the current theory of molecular structure of phenolic resins which throws much light on the puzzling phenomenon of frozen loadless stress patterns. One may perhaps question his conclusion that ". . . . . the observed birefringence is also due *entirely*[3] to the deformations of this element in the resin." If this statement is to be interpreted as meaning that only the permanently elastic network of the bakelite is responsible for the final optical effects a question would arise as to why the bulk of the material, which ultimately returns to its normal optical properties, makes no contribution to this birefringence? Does the theory assume that this material is strainless? If so, wouldn't that mean that a body maintains an elastic material in a state of strain without itself being strained? It would seem to the writer more reasonable to assume that the final stress pattern is the result of the combined effect of all the molecules in the model, and not only of the permanently elastic network.

[3] Italics by the writer.

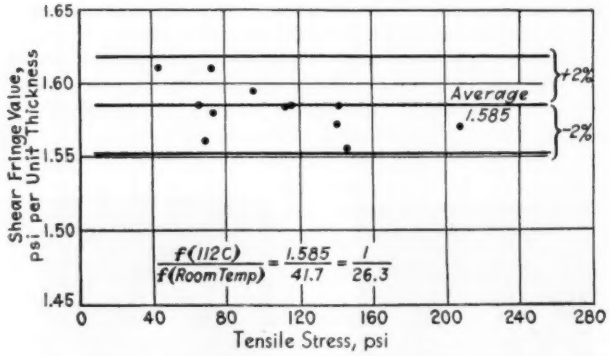


FIG. 3 SHEAR FRINGE VALUE OF BAKELITE-BT-69-893 AT 112 C WHEN STRESSES ARE FROZEN

Fig. 4 shows an attempt to use the relatively large strains produced at high temperatures as an aid in the determination of the sum of the principal stresses in two-dimensional problems. With small loads, variations of several thousandths of an inch can be obtained in models 1/4 in. thick and measured with a fair degree of accuracy with ordinary micrometers or dial gages. The curves shown in Fig. 4 represent the changes in the thickness of the model the stress pattern of which is shown in Fig. 2, in which the uniform stress was 23 psi. This method of procedure may prove of some value for first approximations in determining factors of stress concentration. It is extremely simple and necessitates no equipment other than an oven, micrometer, or dial gage, and model.

To return to three-dimensional problems the writer would like to make one suggestion which may simplify the equipment and procedure in the determination of the direction of the principal stresses as well as the difference of one pair at any point of a general section cut from a body in an arbitrary direction. If such a plate be placed in a transparent container filled with carbon disulphite and this container placed in the field of an ordinary photoelastic polariscope, then by gradually turning the submerged

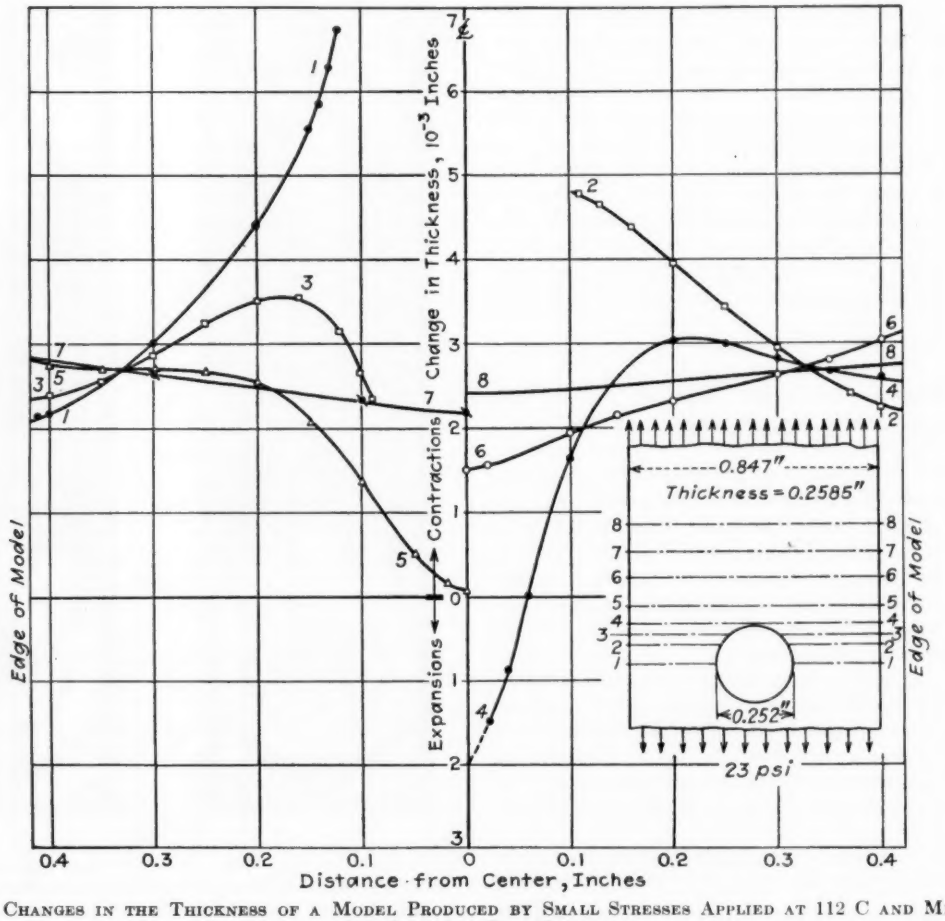


FIG. 4 CHANGES IN THE THICKNESS OF A MODEL PRODUCED BY SMALL STRESSES APPLIED AT 112 C AND MEASURED BY ORDINARY DIAL GAGES

section of the model about horizontal and vertical axes it should seem possible to determine the directions of the principal axes at every point as well as the difference of one pair of principal stresses. These directions should coincide with the positions which would give the clearest photoelastic image at the points in question.

Dr. Hetényi is to be congratulated for having made an important contribution in the extension of the science of photoelasticity to space problems.

RAYMOND D. MINDLIN[4] AND DANIEL C. DRUCKER.[5] Dr. Hetényi's discovery and explanation of the extraordinary properties of certain resins at elevated temperatures places the freezing procedure, recently developed for three-dimensional photoelasticity, on a sound physical basis. The optical properties of slices cut from the frozen model may now be accepted with confidence as indicating very closely an elastic state of stress.

When analyzing the fringes observed in a slice cut from a three-dimensional model, care must be taken to distinguish between their significance and the significance of the fringes obtained in the ordinary two-dimensionally stressed plate. In the latter case, the fringes are loci of points of constant principal-stress difference but, for the former case, this is usually not true. In this connection the author's remarks in footnote 20 are somewhat misleading.

Dr. Hetényi states that, in a bar subjected to torsion, the principal stresses are located in a plane making an angle of 45 deg with the axis of the bar. If this were true, the fringes observed in the 45-degree slice could be interpreted according to the usual two-dimensional methods. However, a more detailed examination of the stress distribution shows that at most points of the section none of the three principal stresses lies in this plane. Furthermore, the orientations of the principal stress directions with respect to the plane of the slice vary from point to point. Except for very special cases, slices cut from a three-dimensionally stressed model will have this characteristic. In order to correctly translate the fringe pattern into terms of stress, it is generally necessary to make measurements and calculations in addition to those made in two-dimensional work. Most important of these is the determination of the orientations of the optic axis in the artificial crystals produced by the stress.

For torsion, it can be shown that one of the optic axes at each point in the bar is parallel to the axis of the bar and the second optic axis is perpendicular to the first. The orientation of the first optic axis is thus known a priori while the orientation of the second may be calculated from data obtained by observations on the slice analogous to the isoclinic determinations of two-dimensional photoelasticity. With these two orientations known, the information obtained from the fringe pattern may be converted to values of principal stresses.

The section at 45 deg to the axis of the bar is not the only one on which observations can be made to yield the distribution of shearing stress over the cross section. It does show the greatest number of fringes, but any other section, even a normal one, gives sufficient data when viewed properly. However, for no section are the fringes contours of constant principal shear.

Knowing the stress distribution, one can predict the fringe pattern. For example, it can be shown that light at normal incidence to any section of a circular bar (under torsion) will produce circular fringes and that at oblique incidence the pattern becomes elliptical.

AUTHOR'S CLOSURE

It is gratifying to know that by using sufficiently high annealing temperature Professor Frocht obtained results coinciding with those of the writer. This fact itself, since the annealing time in the two cases was presumably different, seems to prove also that at elevated temperature the material is truly elastic and shows no creep which is so easily observable in room temperature tests.

The statement that ". . . . . the observed birefringence is also due *entirely* to the deformation of this element in the resin" was made for the high-temperature state and the author maintains the belief that for that state the statement holds accurately true. Doubt could exist only for the case when, after the annealing process, the load is removed from the specimen. Then, the reaction forces will be transmitted to the solidified fusible part whose stress or strain will then participate in the birefringence of the sample. But even this participation will be small, which is proved by the fact that by unloading the piece the number of fringes decreases only in the ratio of 25 to 26.

The author wishes to correct his statement made in the paper regarding the chemical composition of bakelite BT-61-893. This material is not phenolic resin, but is glycerine phthalic anhydride. Its molecular structure has however the same characteristics as the ones described in the paper for phenol formaldehyde resins and therefore the explanation given in the paper regarding the freezing process remains essentially the same.

The proposal to determine the sum of principal stresses from lateral measurements on the frozen sample is of practical value. The method has been used also by the author in determining stresses in rotating disks by the freezing procedure.[6]

The author appreciates the remarks of Messrs. Mindlin and Drucker regarding the corrections to be made when determining the shearing-stress distribution in the cross section of a bar subjected to torsion. When writing the paper he was already aware of the fact that such corrections are necessary but did not go into details since for several other reasons the method does not seem suitable for determining shearing-stress distribution in the cross section of a twisted bar. In such cases the main point of interest would be to obtain stress-concentration factors for reentrant corners of the cross section, but at those places, because of the oblique section and the finite thickness of the slices, the definition of the fringes is poor; therefore, the method cannot compete with the membrane-analogy test which seems to be eminently suitable for problems of this type. On the other hand, by the author's method one can solve problems of torsional stress distribution where the membrane analogy is unusable. Such a case was presented by him in Figs. 13 to 15 where also no correction of the nature indicated by Messrs. Mindlin and Drucker is necessary.

Friction Coefficients for the Compressible Flow of Steam[1]

J. I. YELLOTT.[2] The growing importance of superposition in power-plant practice has given rise to difficult problems in the low-pressure lines leading from the reducing valves. It is encouraging to learn that the familiar friction factors can be used with confidence.

During the last year, experiments similar to those reported by the author have been carried out at Stevens Institute of

[4,5] Department of Civil Engineering, Columbia University, New York, N. Y.

[6] "The Application of Hardening Resins in Three-Dimensional Photoelastic Studies," by M. Hetényi, *Journal of Applied Physics*, vol. 10, no. 5, May, 1939.

[1] Paper by Joseph H. Keenan, published in the March, 1939, issue of the JOURNAL OF APPLIED MECHANICS, Trans. A.S.M.E., vol. 61, p. A-11.

[2] Assistant Professor of Mechanical Engineering, Stevens Institute of Technology, Hoboken, N. J. Jun. A.S.M.E.

Technology, using a 36-ft length of standard 1-in. pipe. The principal difference between this apparatus and the author's lies in the use of a reducing valve directly ahead of the test section instead of a flow nozzle. The character of the flow patterns must be decidedly different in the two cases, but preliminary test results show surprisingly good agreement with those presented in the paper.

The author's assumption that no moisture forms in the pipe when the steam is initially superheated is probably correct. Experiments carried out by the writer on a nozzle with a very long throat indicate that supersaturation prevails under such conditions.

It should be noted that the friction coefficients presented in the paper are just one quarter of the usually quoted friction factors.[3] This discrepancy doubtless arises from the definition of the wall-friction force in terms of the friction coefficient.[4]

Students of this subject will be glad to know that a complete translation[5] of Frössel's paper is available.

Author's Closure

Professor Yellott's corroboration of the assumption that the liquid phase does not form in the pipe is most welcome.

Some doubts linger as to whether any one definition of a friction factor is "usually quoted" in the literature of fluid mechanics. However, no confusion need arise in this instance because the definition employed is clearly stated in the second paragraph of the Appendix.

The Calculation of Steam-Turbine Reheat Factors[1]

J. R. Finniecome.[2] Although much attention has been given in the past to the accurate determination of the reheat factor, still further new methods, formulas, or data on this subject are welcome. This problem is of vital importance to turbine designers.

In comparing the author's published formulas with those already in existence and used, it is found that his formula for the reheat factor for superheated steam, according to Equation [6], agrees with an already well-established one. No further comments are thus required. One has only to study H. M. Martin's classic publication, referred to in the author's Bibliography (9). At the time, this attracted general attention for it contained the first exhaustive mathematical analysis on reheat factors.

1 *Superheated Steam.* Although Martin's formulas and tables were published exactly twenty years ago, they are still in general use. His formula for the superheated region is accepted by other authorities. One is thus not surprised to find that the author, after applying the basic principles of thermodynamics, arrives ultimately at the same formula for the reheat factor for an infinite number of stages.

It is rather fortunate that the author has compared the graphical method of determining the reheat factor with the values obtained by Equation [6] and proved that these agree very closely as shown in his Table 1, the discrepancy being minute.

The graphical method has been applied successfully for many years and is to some extent used today. However, during the last decade the improvement in the stage efficiency, the increase in the number of stages, the higher pressures and temperatures have made it essential for the designer to determine the reheat

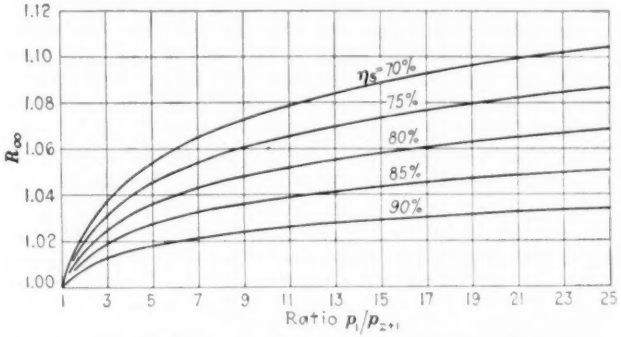


Fig. 1 Reheat Factors for Superheated Steam: Plotted as Function of Pressure Ratio
(Infinite number of stages.)

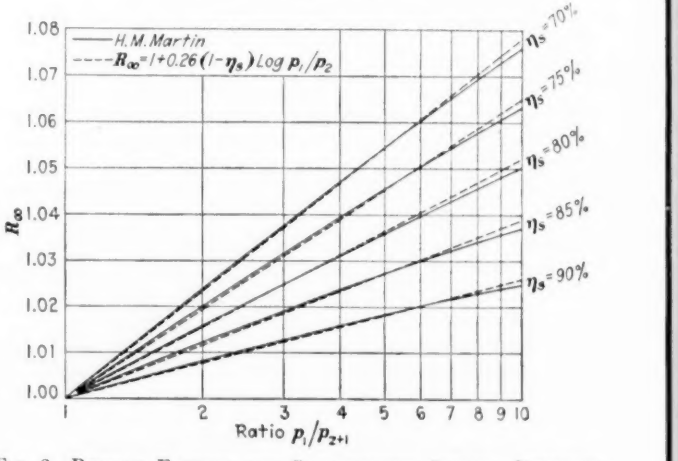


Fig. 2 Reheat Factors for Superheated Steam: Pressure Ratio Plotted on Logarithmic Scale
(Infinite number of stages; values of p_1/p_{z+1} up to 10.)

factor more accurately by calculating the values from a formula. An increase in stage efficiency gives a decrease in the value of the reheat factor, so that accuracy is of considerable importance.

The writer has plotted in Fig. 1 the reheat factors for an infinite number of stages for the superheated region as a function of the pressure ratio at the various stage efficiencies generally used in turbine practice today. Fig. 2 shows the same relationship with the pressure ratio plotted on a logarithmic scale for

[3] "The Flow of Fluids in Closed Conduits," by R. J. S. Pigott, *Mechanical Engineering*, vol. 55, August, 1933, p. 499.

[4] "Thermodynamics, Fluid Flow and Heat Transfer," by H. O. Croft, McGraw-Hill Book Company, New York, 1938, chap. 7.

[5] "Flow in Smooth Straight Pipes at Velocities Above and Below Sound Velocity," by W. Frössel, National Advisory Committee for Aeronautics, Washington, D. C., Technical Memorandum No. 84.

[1] By R. B. Smith. Published in the December, 1938, issue of the Journal of Applied Mechanics, Trans. A.S.M.E., vol. 60, 1938, p. A-156.

[2] Mechanical Engineer, 4 Rye Bank Road, Manchester 16, England.

TABLE 1 INFLUENCE OF NUMBER OF STAGES ON REHEAT FACTOR

1	2	3	4	5	6	7	8	9	10
		Author	Equation	Number of stages					
1	Pressure ratio	.....	....	2	2	20	2	4	20
2	Reheat gain (ρ_∞)	R. B. Smith	[6]		0.01533			0.03018	
3	Reheat gain (ρ_∞)	J. R. Finniecome	[7]		0.01581			0.03094	
4	Reheat gain (ρ)	R. B. Smith	[5a]	0.00745	0.01140	0.01455	0.0144	0.0225	0.02865
5	Reheat gain (ρ)	J. R. Finniecome	A. Zinzen	0.00796	0.012054	0.015102	0.0152	0.023	0.02944
6	Ratio ρ/ρ_∞	R. B. Smith	....	0.486	0.744	0.949	0.477	0.745	0.949
7	Ratio ρ/ρ_∞	J. R. Finniecome	....	0.5034	0.7622	0.9551	0.4913	0.7434	0.9516
8	$(z-1)/z$	.....	....	0.50	0.75	0.95	0.50	0.75	0.95

the values of p_1/p_{s+1} up to 10 and in Fig. 3 for values up to 50. These reheat factors were calculated from Martin's formula, which is equivalent to Equation [6] of the author's paper.

The writer has derived from these curves a simple formula which is represented by dotted straight lines through the origin; these lines approach as closely as possible to the accurate curve. It will be found that the complicated exponential formula by Martin can be simplified and thus we get

(*a*) For a pressure ratio 1 to 10

$$1 + \rho_\infty = 1 + 0.26(1 - \eta) \log_{10} (p_1/p_{s+1}) \quad \ldots\ldots [1]$$

(*b*) For a pressure ratio 1 to 50

$$1 + \rho_\infty = 1 + 0.248(1 - \eta) \log_{10} (p_1/p_{s+1}) \quad \ldots\ldots [2]$$

Usually the pressure ratio does not exceed 10, so that Equation [1] can be applied over the complete range. A closer examination of the full-line curves and the dotted lines of Figs. 2 and 3 indicates that the difference in the values for the reheat factors is very minute.

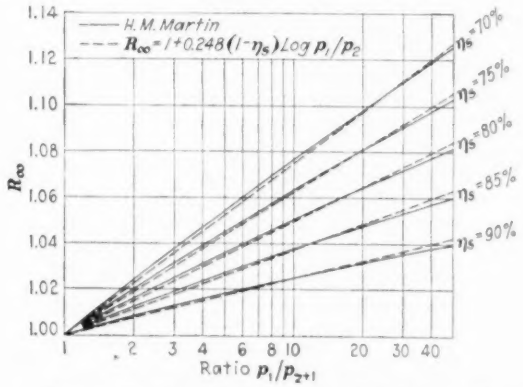


FIG. 3 REHEAT FACTORS FOR SUPERHEATED STEAM; PRESSURE RATIO PLOTTED ON LOGARITHMIC SCALE
(Infinite number of stages; values of p_1/p_{s+1} up to 50.)

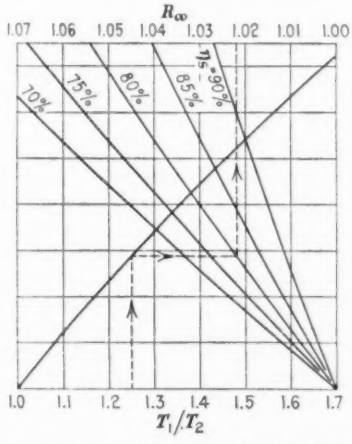


FIG. 4 REHEAT FACTORS OF SATURATED STEAM; PLOTTED AS RATIO OF INITIAL TO FINAL ABSOLUTE TEMPERATURE
(Author's Equation [10].)

2 *Saturated Steam.* For the saturated-steam region the author has derived, on applying the principles of thermodynamics, a very interesting relationship of the initial and final absolute temperatures to the reheat factor. This is represented by his Equation [10]. It will be found that the mathematical solution to this expression is too complicated for practical use, so the writer decided to solve this equation graphically. This is shown in Fig. 4, the basis of plotting being the ratio of the initial to the final absolute temperature. The author's Equation [10] can be modified as follows:

$$\frac{\rho_\infty}{(1-\eta)}\left(1 + \frac{\eta}{1-\eta} \times \frac{\rho_\infty}{2}\right) = \frac{T_1}{T_1 - T_{s+1}} \log_e \left[\frac{T_1}{T_{s+1}} - 1\right] \quad \ldots\ldots [3]$$

and by substituting $T_1/T_{s+1} = k_T$ we get

$$\frac{\rho_\infty}{1-\eta}\left(1 + \frac{\eta}{1-\eta} \times \frac{\rho_\infty}{2}\right) = \left[\frac{1}{1-k} \log_e k - 1\right] \quad \ldots [4]$$

So far as the writer is aware this is the first time the reheat factor for saturated steam has been plotted as a function of the ratio of the absolute temperatures. By following the arrows in Fig. 4, one is able to read off the reheat factor for a specified absolute-temperature ratio, based on the saturated temperature of the initial and final pressures.

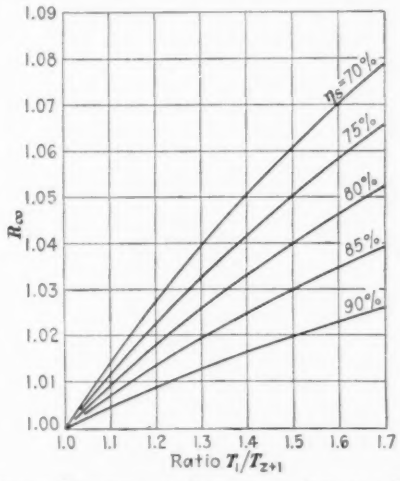


FIG. 5 REHEAT FACTORS OF SATURATED STEAM; PLOTTED AS FUNCTION OF ABSOLUTE TEMPERATURE
(Author's Equation [10].)

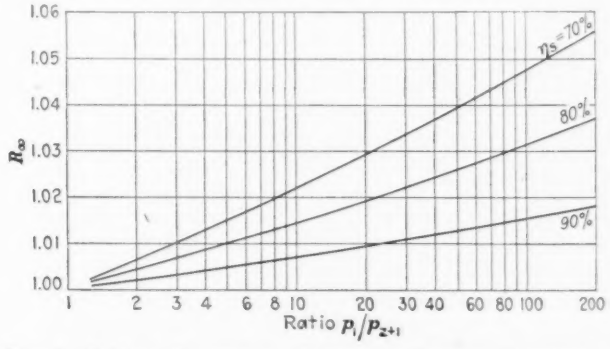


FIG. 6 REHEAT FACTORS OF SATURATED STEAM; PLOTTED AS FUNCTION OF PRESSURE RATIO
(Due to H. M. Martin.)

In Fig. 5 the reheat factor is simply plotted directly as a function of the absolute temperature without applying any intermediate curve. The values can thus be read off directly. The writer has checked the reheat factors obtained from the curves plotted in Fig. 5, derived from Equation [10] with those obtained by Martin, plotted in Fig. 6, as a function of the pressure ratio and he finds they agree.

In 1935, H. G. Yates expressed the reheat factor for the saturated region also as a function of the initial and final absolute temperatures. In substituting into his original formula the

symbols used in this paper we get for an infinite number of stages

$$1 + \rho_\infty = 1 + (1 - \eta)\frac{T_1 - T_{z+1}}{2T_{z+1}} \quad \ldots\ldots\ldots [5]$$

This equation can be written in this simple form

$$1 + \rho_\infty = 1 + (1 - \eta)\frac{1}{2}\left(\frac{T_1}{T_{z+1}} - 1\right) \quad \ldots\ldots\ldots [6]$$

This formula is graphically represented on the basis of the ratio of the absolute temperatures as shown in Fig. 7. It is found that the values derived from Yates's formula are much higher

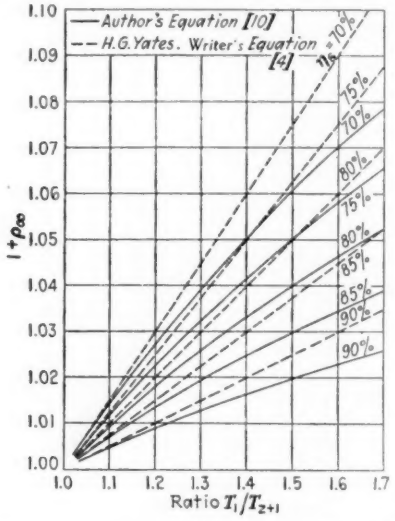


FIG. 7 REHEAT FACTORS OF SATURATED STEAM; PLOTTED ON BASIS OF RATIO OF ABSOLUTE TEMPERATURE (Author's Equation [10]; writer's Equation [7].)

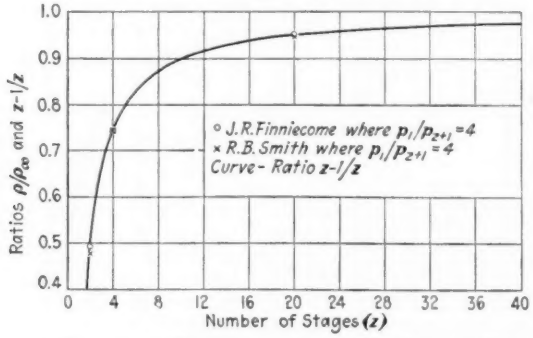


FIG. 8 EFFECT OF NUMBER OF STAGES ON REHEAT FACTOR

than the author's calculated from his Equation [10]. This comparison is shown in Fig. 7, where the dotted lines are Yates's values and the full-line curves are based on the author's Equation [10], which is more accurate.

3 *Effect of the Number of Stages on the Reheat Factor.* The foregoing discussion and curves have referred to the reheat factor for an infinite number of stages. The author gives a painstaking derivation of the correction factor to be applied to the reheat factor for an infinite number of stages, so as to obtain those at a specified number of stages. The now generally accepted formula is $(z - 1)/z$, which is very accurate indeed and the reheat gain is then calculated from

$$\rho = \frac{z - 1}{z}\rho_\infty$$

This has been used by the author in comparing the more accurate values determined analytically and expressed by Equation [5*a*] of this paper.

It may be worth while to mention here that A. Zinzen gives for this correction factor the following slightly simpler equation

$$\frac{1 + \rho}{1 + \rho_\infty} = \eta\left[\frac{1 - (p_1/p_{z+1})^{\frac{n-1}{n}\frac{1}{z}}}{1 - (p_1/p_{z+1})^{\frac{\eta}{z}\frac{n-1}{n}}}\right] \quad \ldots\ldots\ldots [7]$$

In Table 1 are given the values calculated by Equation [7] for the cases considered by the author. Fig. 8 shows these plotted on the basis of the number of stages.

E. L. ROBINSON.[3] This paper attempts to present in algebraic form a complicated property of steam. Although the algebra looks formidable when set in type the actual expressions, Equation [6] presented in Fig. 4 and Equation [10] presented in Fig. 6, are really comparatively simple expressions. At best, the expressions of formal algebra can only represent complicated physical properties to a certain degree of approximation. Under such circumstances, it has always been the preference of the writer for tabular or graphic presentation of the properties rather than approximate representation by formulas. However, it must be admitted that a formula is very convenient for computation and for assuring consistency of tabular values.

In following through the rather large number of closely approximate assumptions made by the author, it becomes increasingly difficulty to visualize what the outcome might be in final accuracy.

In this respect comparison with Professor Thatcher's results[4] is very valuable. Thus, if the author's Fig. 4 and Professor Thatcher's Fig. 7 are plotted on the same semilog sheet, the degree of agreement between the perfect gas formula and the step-by-step calculation becomes evident and the magnitude of the deviations is easily seen. It must, of course, be recognized that the adiabatic index for steam is subject to variation. The writer has carried out a number of sample calculations extending from the superheated region into the wet region and finds the results for the most part agree within a minor fraction of 1 per cent.

In the derivation of Equation [10] for the wet region, the assumption that temperature variation is proportional to enthalpy change is approximately the same as that made by the writer on the suggestion of Prof. J. H. Keenan in preparing the report,[5] referred to by the author. In the report the use of that assumption was confined to the superheated region entirely. The deviations from accuracy in the wet region were investigated and the corrections published in the report. The use of this assumption for expansions exceeding 100 Btu led to errors exceeding 0.1 Btu and, consequently, this assumption was rejected for greater expansions.

Direct integration of Equation [9] gives the following expression for reheat factors which appears to the writer quite as convenient for computation as the quadratic form given in Equation [10]

$$1 + \rho = \frac{e^{\eta B} - (1 - \eta)}{\eta e^{\eta B}}$$

---

[3] Turbine Engineering Department, General Electric Company, Schenectady, N. Y. Mem. A.S.M.E.

[4] "Reheat Factors for Expansions of Superheated and Wet Steam," by C. G. Thatcher, published in the December, 1938, issue of the JOURNAL OF APPLIED MECHANICS, Trans. A.S.M.E., vol. 60 1938, p. A-161.

[5] "Reheat Factors," by E. L. Robinson, *Mechanical Engineering*, vol. 50, February, 1928, pp. 155–156.

In this expression B is the function of the temperature ratio, which appears on the right-hand side of Equation [10], namely,

$$B = \left[\frac{T_1}{T_1 - T_{z+1}}\left(\log_e \frac{T_1}{T_{z+1}}\right) - 1\right] = \left(\frac{\theta \log_e \theta}{\theta - 1} - 1\right)$$

where $\theta = T_1/T_{z+1}$.

This alternative expression appears to agree closely with the values presented in the author's Fig. 6 and, of course, involves the same assumption as regards variation of enthalpy and temperature. It is to be noted that this expression gives the reheat factor in terms of the stage efficiency and the temperature ratio. For a definite efficiency of 0.80, as chosen for Fig. 6, a single curve suffices to define the result more simply than the family in Fig. 6. Thus, for the wet region:

Temperature ratio	= 1.05	1.10	1.15	1.20	1.25	1.30	1.35	1.40
Reheat gain, $\eta = 0.80$	= 0.0049,	0.0095,	0.0139,	0.0181,	0.0221,	0.0260,	0.0296,	0.0331

The author gives a more exact expression for the saturated region in Equation [12], of which the right-hand parenthesis may be used for the value of B in Equation [10]. A chart or table of "wet" reheat factors showing how Equation [12] differs from Equation [10] would be welcome.

C. G. THATCHER.[6] The writer wishes to express his appreciation of the author's fine work in approaching this subject from an analytical angle, and also for the excellent spirit of cooperation which he has exhibited at all times toward the writer and his work.

TABLE 1 COMPARISON OF REHEAT FACTOR VALUES

(In superheat region)

Constant stage efficiency or e_s	Pressure ratio $P_1 \div P_2$	Author's Equation [6] or Fig. 4	Thatcher's[4] Fig. 7
0.90	10	1.0250	1.0250
	5	1.0167	1.0177
	2	1.0067	1.0072
0.85	10	1.0367	1.0386
	5	1.0261	1.0272
	2	1.0108	1.0117
0.80	10	1.0487	1.0515
	5	1.0346	1.0353
	2	1.0153	1.0140
0.70	4	1.0456	1.0470
	3	1.0365	1.0365
	2	1.0230	1.0220
0.60	3	1.0492	1.0490
	2	1.0306	1.0300

TABLE 2 COMPARISON OF REHEAT FACTOR VALUES

[In wet region (nonmoisture correction) at constant stage efficiency of 0.80]

P_1	P_2	P_1/P_2	H_1	H_2	$H_1 - H_2$	Author's Fig. 6	Thatcher's Fig. 9
5	1.0	5	1131.0	1031.0	100.0	1.0102	1.0105
10	0.5	20	1143.5	962.0	181.5	1.0190	1.0191
10	1.0	10		999.0	144.5	1.0150	1.0152
10	2.0	5		1038.5	105.0	1.0108	1.0111
25	1.25	20	1160.5	970.0	190.5	1.0205	1.0200
25	2.5	10		1008.0	152.5	1.0162	1.0161
25	5.0	5		1050.0	110.5	1.0117	1.0116
50	2.5	20	1174.0	975.0	199.0	1.0218	1.0209
50	5.0	10		1015.5	158.5	1.0173	1.0167
50	10.0	5		1059.0	115.0	1.0125	1.0121
100	5.0	20	1187.0	981.0	206.0	1.0235	1.0217
100	10.0	10		1022.5	164.5	1.0187	1.0174
100	20.0	5		1067.5	119.5	1.0136	1.0126

It will be noted from Tables 1 and 2 of comparative values that in the superheat region the values check very closely. Some discrepancy may creep in through the author's assumption of a constant value of k in the equation PV^k = constant. This does vary some through the superheat region, as shown by Keenan and Keyes.[7] In the wet region there is a close agreement in the figures, although at higher initial pressures the author's values are somewhat higher than those of the writer.

In the author's Equation [12], he introduces the specific heat of the liquid with the assumption that it remains constant. This may possibly be the cause of the discrepancy, since at higher initial pressures the value of specific heat does increase beyond unity.

[6] Associate Professor of Mechanical Engineering, Swarthmore College, Swarthmore, Pa. Mem. A.S.M.E.

[7] "Thermodynamic Properties of Steam," by J. H. Keenan and F. G. Keyes, John Wiley & Sons, Inc., New York, N. Y., 1936.

AUTHOR'S CLOSURE

Since publication of the paper, it has been possible to reduce the mathematical relations which define the reheat factors in the superheat and the saturated regions to rapidly converging series suitable for slide-rule work. Equation [6] for the superheated reheat region may be written

$$1 + \rho_\infty = 1 + \frac{1-\eta}{2}\,\frac{k-1}{k}\log_e \frac{P_1}{P_{z+1}} - \left(\frac{k-1}{k}\right)^2 \frac{1}{12\eta}\{9 - \eta^5 - (\eta - 1)^2(\eta + 2)^3 - (1 + 2\eta - 2\eta^2)(\eta + 1)^3\}\left[\log_e \frac{P_1}{P_{z+1}}\right]^2 \quad \ldots\ldots\ldots [8]$$

The first three terms of Equation [8] yield results which are accurate to within 0.1 per cent for pressure ratios as high as 50. For the most common values, namely, $\frac{k-1}{k} = 0.225$ and $\eta = 0.80$, this expression becomes

$$1 + \rho_\infty = 1 + 0.225 \log_e \frac{P_1}{P_{z+1}} - 0.000492\left[\log_e \frac{P_1}{P_{z+1}}\right]^2 \quad \ldots [9]$$

For Equation [10] of the paper, representing approximately the saturated reheat factor, there can be written

$$\rho_\infty\left(1 + \frac{\eta}{1-\eta}\,\frac{\rho_\infty}{2}\right) = \frac{1-\eta}{2}\,\frac{T_1 - T_{z+1}}{T_{z+1}}\left\{1 - \frac{1}{3}\left(\frac{T_1 - T_{z+1}}{T_{z+1}}\right) + \frac{1}{6}\left(\frac{T_1 - T_{z+1}}{T_{z+1}}\right)^2 - \frac{1}{10}\left(\frac{T_1 - T_{z+1}}{T_{z+1}}\right)^3 + \frac{1}{15}\left(\frac{T_1 - T_{z+1}}{T_{z+1}}\right)^4\right\} \quad \ldots\ldots\ldots [10]$$

These relations may be of value to those working with the reheat problem.

Mr. Finniecome refers to the work of Martin in connection with the analysis of reheat factors. The author is familiar with Martin's classic study and it has not been his intention to claim any originality for the superheated reheat factor expressed by Equation [6] of the paper. However, this result is merely a simplification of the more complete relation expressed by Equation [5]. It is certainly in the derivation of this equation that the work of Martin and the author differs.

Martin's work, particularly in the superheated region, has been generally dismissed in this country as being too theoretical to give accurate results with such an imperfect vapor as steam. Consequently, there has been an overemphasis on the calculation of reheat by laborious summations on the steam charts. It is rather fortunate, in this connection, that the comparison of Table 1, of the paper, as well as the later more extended work of Professor Thatcher indicates such good agreement between analytical and practical computations.

In the saturated region, Martin's analysis leaves much to be desired. If one compares Mr. Finniecome's Fig. 6 with Fig. 6 of the paper, it is apparent that there are differences as great as 0.5 per cent. The work of Yates, which is graphical in source, leads to an expression which is a first-term approximation to the series expressed by Equation [10] of this closure. Only for small pressure ratios is it accurate.

The derivation of Zinzen, pertaining to the influence of the

number of stages on the reheat factor, and to which Mr. Finniecome refers, is based upon the assumption that the pressure ratio is uniform over each stage. Actually, this assumption is not even close to the truth on a turbine of normal design, although it must be admitted that it yields results, as Mr. Finniecome points out, which are in good agreement with the more exact expression represented by Equation [5] of the paper.

Mr. Robinson's remarks are always of interest, particularly so in this instance, in view of his past experience with the problem of reheat. He has succeeded in integrating directly the left side of Equation [9] of the paper, whereas the author was able to achieve this result only approximately.

The differences are small between the saturated reheat factor calculated by Equation [12] and by Equation [10], both of the paper. Actually the complicated relation represented by Equation [12] can be expressed as follows:

$$\rho_\infty \left(1 + \frac{\eta}{1-\eta}\,\frac{\rho_\infty}{2}\right) = (1-\eta)\left\{\frac{T_1 \log_e \dfrac{T_1}{T_{z+1}}}{T_1 - T_{z+1}} \times f(T) - 1\right\} \qquad \ldots\ldots [11]$$

where, for $c = 1.0$

$$f(T) = 1 - \frac{\dfrac{1}{12}\left(\dfrac{T_1 - T_{z+1}}{T_{z+1}}\right)^2\left\{1 - 1.5\left(\dfrac{T_1 - T_{z+1}}{T_{z+1}}\right) + 1.78\left(\dfrac{T_1 - T_{z+1}}{T_{z+1}}\right)^2 - 1.96\left(\dfrac{T_1 - T_{z+1}}{T_{z+1}}\right)^3\right\}}{S - \log_e \dfrac{T_1}{492} + \dfrac{1}{2}\left(\dfrac{T_1 - T_{z+1}}{T_{z+1}}\right)\left\{1 - \dfrac{2}{3}\left(\dfrac{T_1 - T_{z+1}}{T_{z+1}}\right)\right\}} \qquad \ldots\ldots [12]$$

These series when used in conjunction with Equation [10] of this closure are quite easily handled. In Table 3, herewith, are typical comparative values of saturated reheat factors computed from the analytical expression and also graphically by Professor Thatcher. The differences are small.

TABLE 3 SATURATED REHEAT FACTORS; INFINITE NUMBER OF 80 PER CENT EFFICIENT EXPANSIONS

Initial pressure, lb per sq in., abs	Back pressure, lb per sq in., abs	Entropy	Enthalpy drop, Btu per lb	Calculated reheat factor Eq. [10][a]	Calculated reheat factor Eq. [12][a]	Reheat factor from Thatcher's summation
100	50	1.60	54	1.0061	1.00605	1.0057
	20		120	1.0135	1.0132	1.0126
	10		165	1.0187	1.0182	1.0174
	5		206	1.0234	1.0226	1.0217
	3		235	1.0267	1.0257	1.0247
	2		256	1.0293	1.0283	1.0270
50	25.0	1.65	52	1.00566	1.0056	1.0055
	10.0		115	1.0126	1.0124	1.0121
	5.0		157	1.0174	1.0171	1.0165
	2.4		200	1.0222	1.0216	1.0210
	1.6		222	1.0247	1.0240	1.0234
	1.0		246	1.0275	1.0267	1.0259
25	12.0	1.70	53	1.00557	1.00555	1.0056
	5.0		109	1.0117	1.0116	1.0115
	2.4		153	1.0165	1.0162	1.0161
	1.2		191	1.0208	1.0203	1.0200
	0.8		213	1.0232	1.0226	1.0224
	0.5		236	1.0258	1.0252	1.0248
10	5.0	1.75	46	1.00484	1.00481	1.0048
	2.0		102	1.0108	1.0106	1.0107
	1.0		141	1.0150	1.0148	1.0148
	0.5		177	1.0190	1.0187	1.0186
	0.3		202	1.0218	1.0212	1.0212
	0.2		222	1.0240	1.0232	1.0234

[a] Reference is to equation in original paper.

Professor Thatcher's graphical analysis of the reheat problem is timely and his comparative tabulation between the analytical and graphical results substantiates the work that has been done. In regard to the saturated analysis—while it is true that the specific heat of the liquid is far from constant, it is also true that over the range of practical turbine design the variation when applied in Equation [12] of the paper is insignificant. This is further borne out by the comparison in Table 3 of this closure. Actually, the differences that do exist between the two methods of solution can be partly explained by the fact that Professor Thatcher presents his reheat factors as a function of the isentropic-enthalpy drop. It is easy to demonstrate that entropy is also a necessary variable.

Reheat Factors for Expansions of Superheated and Wet Steam[1]

E. M. Fernald.[2] The use of PV^k = constant for the isentropic, with a value of k pertaining to the region in which the process takes place, will yield values of the available energy (isentropic change of enthalpy) precise to 0.01 Btu without any difficulty, although the Keenan and Keyes tables[3] are precise only to 0.1 Btu. Calculation of available energy to 0.01 Btu as the difference of two enthalpies, each run out to 0.01 Btu in the course of interpolation, as the author does, seems less desirable.

The mention of perfect-gas laws by R. B. Smith,[4] in connection with the PV^k equation for isentropics, brings up a point in thermodynamics which appears to be widely misunderstood. It is true that a perfect gas, characterized by constancy of PV/T, does conform to that relation, k being then the ratio of specific heats C_p/C_v. It is not true, however, that conformity to a relation of the general form PV^k = constant for individual isentropics necessitates perfect-gas behavior to the extent that PV/T is constant over all states. The assumption is not needed. It is only necessary to make use of the convenient fact that isentropic expansion of superheated steam, wet steam, and other fluids, proceeds according to the relation PV^k = constant, at least near enough for engineering. The k is merely a constant, approximately, and has no direct relation to the ratio of specific heats. Indeed, for fluids approaching incompressibility, k approaches infinity (volume constant), whereas the ratio of specific heats does no such thing.

Proceeding on this basis, we get for the difference between internal energies, calculated as the work for expansion in a cylinder,

$$U_1 - U_2 = \int_2^1 Pd\,V/J = \frac{P_1V_1 - P_2V_2}{J(k-1)}, \text{ Btu}$$

For the difference between enthalpies, or the available energy, we get, from $H = U + PV/J$,

$$H_1 - H_2 = (U_1 + P_1V_1/J) - (U_2 + P_2V_2/J)$$

$$= \frac{P_1V_1}{J(k-1)} + \frac{P_1V_1}{J} - \frac{P_2V_2}{J(k-1)} - \frac{P_2V_2}{J}$$

$$= \frac{k}{J(k-1)}\,(P_1V_1 - P_2V_2), \text{ Btu}$$

[1] By C. G. Thatcher. Published in the December, 1938, issue of the Journal of Applied Mechanics, Trans. A.S.M.E., vol. 60, 1938, p. A-161.

[2] Professor, Department of Mechanical Engineering, Lafayette College, Easton, Pa. Mem. A.S.M.E.

[3] "Thermodynamic Properties of Steam," by J. H. Keenan and F. G. Keyes, John Wiley & Sons, Inc., New York, 1936.

[4] This is a combined discussion of the present paper and the paper, "The Calculation of Steam-Turbine Reheat Factors," by R. B. Smith, published in the December, 1938, issue of the Journal of Applied Mechanics, Trans. A.S.M.E., vol. 60, 1938, p. A-156.

This agrees with R. B. Smith's relations for superheated steam. Perfect gases thus need not be mentioned.

To prove that PV^k = constant can be used for individual isentropics where PV/T = constant could not possibly be used over all states, only wet steam need be considered. Over the common region of wet steam, k is near enough to constant for most engineering, whereas PV/T varies, of course, widely. It is of interest, however, that PV/T is about as constant for a particular isentropic of wet steam as k is. As PV^k = constant applies reasonably well to isentropic expansion of wet steam, the writer surmises that Mr. Smith, had he chosen, could have treated wet steam acceptably for engineering by the same method that he used for superheated steam.

J. R. Finniecome.[5] There has been a considerable controversy concerning the correct method to determine most accurately the value of the reheat factor. In the step-by-step calculation the author has succeeded in his endeavor to present a set of very interesting curves on the reheat factor for both the superheated and saturated region.

The object of the writer's contribution is to satisfy the turbine designer, who is particularly interested in the reheat factor, that the full-line curve in the author's Fig. 5, representing the average of the figures obtained from his analysis, corresponds actually to those determined by the following formula generally used for the superheat region

$$RF_\infty = \frac{1}{e_s} \cdot \frac{\left[1 - (p_2/p_1)^{e_s \frac{n-1}{n}}\right]}{\left[1 - (p_2/p_1)^{\frac{n-1}{n}}\right]}$$

where e_s = the constant stage efficiency
p_1 = the initial pressure
p_2 = the final pressure
n = 1.3 (superheated steam)
$\frac{n-1}{n}$ = 0.2308

The reheat factors for an infinite number of stages and for a constant stage efficiency of 85 per cent at various pressure ratios are calculated from the above formula and are given in Table 1.

TABLE 1 REHEAT FACTORS AT VARIOUS PRESSURE RATIOS

Pressure ratio p_1/p_2	Reheat factor RF_∞
2	1.0120
4	1.0238
6	1.030
8	1.0342
10	1.0371
20	1.0473
50	1.0602

When plotting these values in the author's Fig. 5, it is found that they lie on the full-line curves, indicating that Equation [1] agrees as expected with the step-by-step calculation method. On analyzing the reheat factors in Figs. 7 and 8, the writer finds that they also agree exceptionally well with Equation [1]. For comparison the writer gives in the accompanying Figs. 1 and 2 reheat factors based on Equation [1] for pressure ratios up to 48 and 50, respectively, and stage efficiencies varying from 70 to 90 per cent, generally used in turbine design.

Incidentally it may be mentioned that the author's Fig. 4 confirms the already well-known relationship of the reheat factor with the number of stages.

In addition the variation of the reheat factor in the saturated region with the adiabatic heat drop without and with wetness

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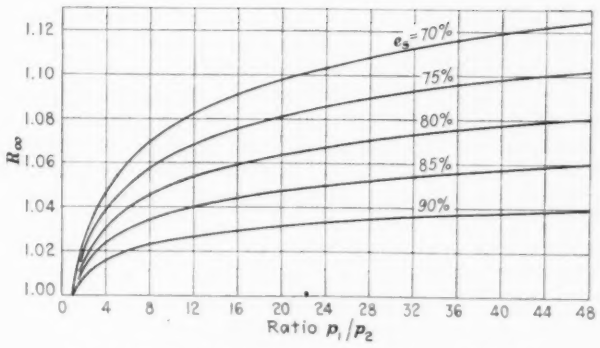


Fig. 1 Reheat Factors for Superheated Steam
(For an infinite number of stages.)

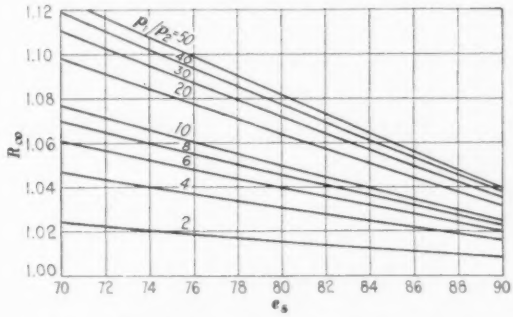


Fig. 2 Reheat Factors for Superheated Steam
(For an infinite number of stages; due to H. M. Martin.)

correction is the result of the step-by-step calculation method. These curves constitute a valuable contribution and indicate the advantage of this method of determining the reheat factors in the saturated region.

It is perhaps desirable to mention that the turbine designer would have welcomed from the author an empirical formula expressing the relationship of the reheat factor to the adiabatic heat drop and the constant stage efficiency, based on the curves of Figs. 9 and 12, respectively, to enable one to plot the reheat factor on a larger scale chart.

J. T. Rettaliata.[6] Reheat factors, based on the latest Keenan and Keyes[3] Mollier diagram, have been much needed for some time and the author has rendered a valuable service by supplying them. The existent uncertainties associated with the expansion of steam in turbines, as mentioned by the author, necessitate the use, in turbine design, of as much reliable data as can be obtained, in order to minimize the effect of these uncertainties. Hence the need for reheat factors directly applicable to the Mollier diagram in universal use in this country today.

The step-by-step method employed by the author is a tedious process but the accuracy of its results justifies its use. As stated in the paper, the enthalpy-entropy diagram is so constructed in the superheat region that, a movement in the direction of increasing entropy on the diagram, gives rise to two mutually compensating effects, tending to keep the reheat factor constant for any given pressure ratio. To justify his statement, the author determined reheat factors for a pressure ratio of 2 in various regions of the superheat field of the Mollier diagram and showed that they were essentially the same. The writer, in a mathematical treatment, wishes to present further veri-

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fication of the existence of a single value of the reheat factor under the aforementioned conditions.

For an infinite number of expansions, the reheat factor may be given as the ratio of area *ABCE* to area *ABDE*, in the *P-V* diagram, Fig. 3, where *BC* represents the actual expansion, from P_1 to P_2, with exponent n; and *BD* the isentropic expansion with exponent k, where k is the ratio of the specific heats at constant pressure and constant volume.

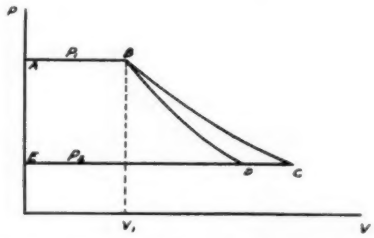


FIG. 3 ISENTROPIC AND POLYTROPIC EXPANSIONS ON A PRESSURE-VOLUME PLANE

Using the proper relations for the areas, the reheat factor may be expressed by

$$RF_\infty = \frac{\frac{n}{n-1} P_1V_1\left[1-\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}\right]}{\frac{k}{k-1} P_1V_1\left[1-\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}\right]} \quad \ldots\ldots [1]$$

If the specific heat at constant volume is assumed to remain sensibly constant, the value of n may be shown to be

$$n = \frac{1}{1-e_s\left(1-\frac{1}{k-1}\right)} \quad \ldots\ldots [2]$$

where e_s is the stage efficiency of the expansion.[7]

If Equation [2] is substituted in Equation [1] the latter becomes

$$RF_\infty = \frac{1}{e_s}\left[\frac{1-\left(\frac{P_2}{P_1}\right)^{e_s\left(\frac{k-1}{k}\right)}}{1-\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}}\right] \quad \ldots\ldots [3]$$

Using Equation [3], reheat factors were calculated for a stage efficiency of 0.75 and pressure ratios from 5 to 50, assuming an average value of 1.3 for the isentropic exponent in the superheated region on the Mollier diagram. A comparison of the calculated results and those obtained from the $e_s = 0.75$ curve of Fig. 7 in the paper is given in Table 2.

TABLE 2 COMPARISON OF REHEAT FACTORS FOR STAGE EFFICIENCY OF 0.75 AND PRESSURE RATIOS 5 TO 50

$\frac{P_1}{P_2}$	Reheat factor		Difference, per cent
	Calculated	Thatcher	
5	1.042	1.0450	0.287
10	1.064	1.0640	0
15	1.074	1.0740	0
20	1.081	1.0805	0.0462
25	1.085	1.0860	0.0921
30	1.090	1.0910	0.0917
35	1.094	1.0940	0
40	1.097	1.0970	0
45	1.100	1.1000	0
50	1.102	1.1015	0.0454

As can be seen from the last column of Table 2, the agreement between the calculated values and those given by the author is extremely good.

In reality the isentropic exponents for superheated steam do not remain constant but have values as given by Keenan and Keyes.[8] Using the writer's Equation [3] with a stage efficiency of 0.75 and random values of pressure ratio and entropy, and isentropic exponents applicable to the particular regions of the Mollier diagram, where the expansions are occurring, reheat factors as shown in Table 3 were obtained.

TABLE 3 REHEAT FACTORS FOR STAGE EFFICIENCY OF 0.75 AND RANDOM VALUES OF PRESSURE RATIO AND ENTROPY

$\frac{P_1}{P_2}$	Entropy	Isentropic exponent	Reheat factor		Difference, per cent
			Calculated	Thatcher	
5	1.7	1.2890	1.043	1.045	0.1914
5	1.8	1.274	1.046	1.045	0.0960
10	1.8	1.3050	1.064	1.064	0
10	1.9	1.2930	1.061	1.064	0.2820
10	2.0	1.2760	1.060	1.064	0.3760
20	1.8	1.2985	1.080	1.0805	0.0460
20	1.9	1.2850	1.078	1.0805	0.2310

Here, again, close agreement exists between the calculated and the author's values.

Although by no means exhaustive, this analysis indicates that the reheat factors for a given pressure ratio remain essentially constant in the superheat region regardless of the location of the expansion on the Mollier diagram.

E. L. ROBINSON.[9] This paper gives the essential information from which reheat factors can be determined for turbine expansions. The calculations appear to have been carried out carefully with a view to presenting the reheat factors for an infinite number of stages as a cardinal property of the working fluid, steam.

It would be advantageous if reheat factors for over-all expansions from the superheated to the wet region could be presented on a single diagram like the author's Fig. 14. He does not appear to consider this feasible, and it must be admitted that interpolation is not easy among crisscrossing curves, such as characterize Fig. 14.

On the other hand, there is great virtue in being able to see at a glance the difference between reheat factors for one type of expansion as compared with another. The scheme presented for getting over-all reheat factors toward the end of the paper requires a simple three-step calculation which is easily carried out, but is sufficiently long to prevent the seeing of a dozen reheat factors at once.

The two essential diagrams are Fig. 7 for the superheated region and Fig. 9 for the wet region. In many respects, it would be desirable to have Fig. 7 presented on a semilog plot like Fig. 5, since it is difficult to make accurate readings for small pressure ratios on Fig. 7.

The importance attached to moisture correction appears to the writer to be quite out of proportion, in view of the fact that other and compensating losses have been completely overlooked. To mention only the more obvious, as the expansion progresses in the low pressure stages of the turbine in a logarithmic manner, the specific volume increases rapidly. In consequence, nozzle and bucket lengths increase rapidly. End losses, rotation losses, and packing leakages are reduced to a small percentage of what they had been. These influences tend to improve the efficiency of the low pressure stages. The increasing moisture content tends to reduce the efficiency.

There is thus good reason for the use of a condition curve of uniform efficiency for the computation of reheat factors.

[7] "Steam Turbines," by W. J. Goudie, Longmans, Green and Company, London and New York, 1922, pp. 574–576.

[8] See p. 82 of footnote 3, on p. A-140 of this issue.

[9] Turbine Engineering Department, General Electric Company Schenectady, N. Y. Mem. A.S.M.E.

There has been some lack of clearness in technical writing about this matter of moisture loss. If two identical turbines are operated, one with wet steam and the other with dry steam, the one with wet steam will have its efficiency lowered approximately in proportion to the average moisture content. This is not the same as saying that each successive stage will have a lower efficiency in proportion to the moisture content.

Thus, the only basis for the use of Fig. 12 would be a design in which the actual stage efficiencies correspond with those assumed by the author in which, in spite of all the influences making for better performance in the low-pressure region, the net over-all result would be as presented.

H. G. YATES.[10] The author's step-by-step calculation of reheat factors for a large number of particular cases must have entailed much labor. The results will be of use to all engineers interested in this subject.

Referring to Equation [2] of the paper

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There is undeniably a certain amount of doubt and disagreement as to the effect on blading efficiency of 1 per cent of wetness. The author is no doubt correct in assigning part of this doubt to uncertainty in the choice of reference point. In addition the loss is not amenable to calculation or to exact measurement. It would not have been surprising if the various authorities differed even more than they do.

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AUTHOR'S CLOSURE

Professor Fernald points out the doubtful value of the second decimal place in calculating isentropic drops and reheats to 0.01

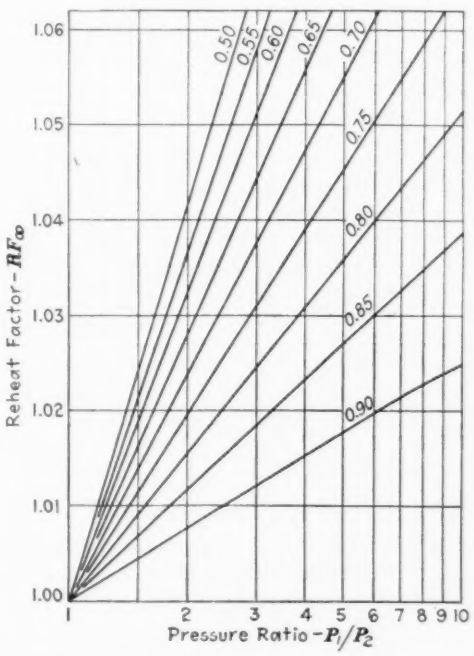


FIG. 4 REHEAT FACTORS IN SUPERHEAT REGION FOR VARIOUS STAGE EFFICIENCIES

[10] Rugby, England.

[11] "Calculations of Turbine Reheat Factor From the Geometry of the Heat Chart," by H. G. Yates, *The Engineer*, vol. 160, August 30, 1935, pp. 212–213.

fication of the existence of a single value of the reheat factor under the aforementioned conditions.

For an infinite number of expansions, the reheat factor may be given as the ratio of area *ABCE* to area *ABDE*, in the *P-V* diagram, Fig. 3, where *BC* represents the actual expansion, from P_1 to P_2, with exponent n; and *BD* the isentropic expansion with exponent k, where k is the ratio of the specific heats at constant pressure and constant volume.

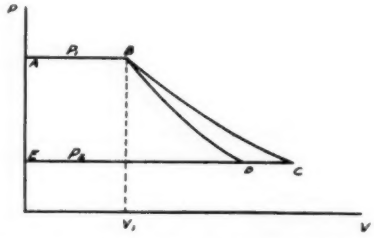


FIG. 3 ISENTROPIC AND POLYTROPIC EXPANSIONS ON A PRESSURE-VOLUME PLANE

Using the proper relations for the areas, the reheat factor may be expressed by

$$RF_\infty = \frac{\frac{n}{n-1} P_1V_1 \left[1 - \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}\right]}{\frac{k}{k-1} P_1V_1 \left[1 - \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}\right]} \quad \ldots\ldots\ldots [1]$$

If the specific heat at constant volume is assumed to remain sensibly constant, the value of n may be shown to be

$$n = \frac{1}{1 - e_s\left(1 - \frac{1}{k-1}\right)} \quad \ldots\ldots\ldots [2]$$

where e_s is the stage efficiency of the expansion.[7]

If Equation [2] is substituted in Equation [1] the latter becomes

$$RF_\infty = \frac{1}{e_s}\left[\frac{1 - \left(\frac{P_2}{P_1}\right)^{e_s\left(\frac{k-1}{k}\right)}}{1 - \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}}\right] \quad \ldots\ldots\ldots [3]$$

Using Equation [3], reheat factors were calculated for a stage efficiency of 0.75 and pressure ratios from 5 to 50, assuming an average value of 1.3 for the isentropic exponent in the superheated region on the Mollier diagram. A comparison of the calculated results and those obtained from the $e_s = 0.75$ curve of Fig. 7 in the paper is given in Table 2.

TABLE 2 COMPARISON OF REHEAT FACTORS FOR STAGE EFFICIENCY OF 0.75 AND PRESSURE RATIOS 5 TO 50

$\frac{P_1}{P_2}$	Reheat factor		Difference, per cent
	Calculated	Thatcher	
5	1.042	1.0450	0.287
10	1.064	1.0640	0
15	1.074	1.0740	0
20	1.081	1.0805	0.0462
25	1.085	1.0860	0.0921
30	1.090	1.0910	0.0917
35	1.094	1.0940	0
40	1.097	1.0970	0
45	1.100	1.1000	0
50	1.102	1.1015	0.0454

As can be seen from the last column of Table 2, the agreement between the calculated values and those given by the author is extremely good.

In reality the isentropic exponents for superheated steam do not remain constant but have values as given by Keenan and Keyes.[8] Using the writer's Equation [3] with a stage efficiency of 0.75 and random values of pressure ratio and entropy, and isentropic exponents applicable to the particular regions of the Mollier diagram, where the expansions are occurring, reheat factors as shown in Table 3 were obtained.

TABLE 3 REHEAT FACTORS FOR STAGE EFFICIENCY OF 0.75 AND RANDOM VALUES OF PRESSURE RATIO AND ENTROPY

$\frac{P_1}{P_2}$	Entropy	Isentropic exponent	Reheat factor		Difference, per cent
			Calculated	Thatcher	
5	1.7	1.2890	1.043	1.045	0.1914
5	1.8	1.274	1.046	1.045	0.0960
10	1.8	1.3050	1.064	1.064	0
10	1.9	1.2930	1.061	1.064	0.2820
10	2.0	1.2760	1.060	1.064	0.3760
20	1.8	1.2985	1.080	1.0805	0.0460
20	1.9	1.2850	1.078	1.0805	0.2310

Here, again, close agreement exists between the calculated and the author's values.

Although by no means exhaustive, this analysis indicates that the reheat factors for a given pressure ratio remain essentially constant in the superheat region regardless of the location of the expansion on the Mollier diagram.

E. L. ROBINSON.[9] This paper gives the essential information from which reheat factors can be determined for turbine expansions. The calculations appear to have been carried out carefully with a view to presenting the reheat factors for an infinite number of stages as a cardinal property of the working fluid, steam.

It would be advantageous if reheat factors for over-all expansions from the superheated to the wet region could be presented on a single diagram like the author's Fig. 14. He does not appear to consider this feasible, and it must be admitted that interpolation is not easy among crisscrossing curves, such as characterize Fig. 14.

On the other hand, there is great virtue in being able to see at a glance the difference between reheat factors for one type of expansion as compared with another. The scheme presented for getting over-all reheat factors toward the end of the paper requires a simple three-step calculation which is easily carried out, but is sufficiently long to prevent the seeing of a dozen reheat factors at once.

The two essential diagrams are Fig. 7 for the superheated region and Fig. 9 for the wet region. In many respects, it would be desirable to have Fig. 7 presented on a semilog plot like Fig. 5, since it is difficult to make accurate readings for small pressure ratios on Fig. 7.

The importance attached to moisture correction appears to the writer to be quite out of proportion, in view of the fact that other and compensating losses have been completely overlooked. To mention only the more obvious, as the expansion progresses in the low pressure stages of the turbine in a logarithmic manner, the specific volume increases rapidly. In consequence, nozzle and bucket lengths increase rapidly. End losses, rotation losses, and packing leakages are reduced to a small percentage of what they had been. These influences tend to improve the efficiency of the low pressure stages. The increasing moisture content tends to reduce the efficiency.

There is thus good reason for the use of a condition curve of uniform efficiency for the computation of reheat factors.

[7] "Steam Turbines," by W. J. Goudie, Longmans, Green and Company, London and New York, 1922, pp. 574–576.

[8] See p. 82 of footnote 3, on p. A-140 of this issue.

[9] Turbine Engineering Department, General Electric Company Schenectady, N. Y. Mem. A.S.M.E.

There has been some lack of clearness in technical writing about this matter of moisture loss. If two identical turbines are operated, one with wet steam and the other with dry steam, the one with wet steam will have its efficiency lowered approximately in proportion to the average moisture content. This is not the same as saying that each successive stage will have a lower efficiency in proportion to the moisture content.

Thus, the only basis for the use of Fig. 12 would be a design in which the actual stage efficiencies correspond with those assumed by the author in which, in spite of all the influences making for better performance in the low-pressure region, the net over-all result would be as presented.

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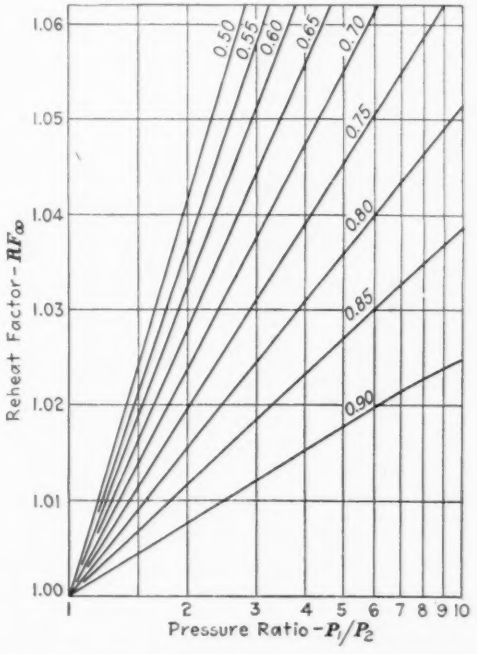


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Btu. He is probably correct in that our steam tables, in most cases, are not closer than 0.10 Btu. In the case of an isentropic-enthalpy drop of 100 Btu, where the constant stage efficiency is 0.80 and the reheat is 19.0 Btu, the reheat factor works out about 1.0125. If the reheat is 19.1, the reheat factor becomes 1.01125 while, if the reheat is 19.2, the reheat factor becomes 1.010. One-tenth Btu seems to make a great difference in the calculated result, especially when the isentropic drops are quite small. The use of the second decimal place is introduced in an effort to refine the work.

Mr. Finniecome suggests that an empirical formula, expressing the relationships shown in Figs. 9 and 12 of the paper, would be a welcome addition. The following expression, therefore, represents the relationship between reheat factor for an infinite number of stages, isentropic-enthalpy drop, and stage efficiency for expansion entirely within the wet region.

$$RF_\infty - 1 = 0.000525\ (I.E.D.)(1.00 - e_s)$$

This equation is based on Fig. 9. When a moisture correction is to be made, Fig. 12, add to the right-hand side of the equation

$$\frac{(I.E.D.)^{2.75}}{6 \times 10^8}$$

The equations used by Messrs. Finniecome, Rettaliata, and Ronald B. Smith to express the reheat factor for an infinite number of stages in the superheat region are really all the same equation. Apparently, they yield values which check very closely with those determined by the author in the superheat region.

As Mr. Robinson appropriately points out, it is apparently a difficult matter to present the entire range of reheat factors in a simple form such as that demonstrated in Fig. 14. The author hopes to be able to do something to simplify this problem in the near future.

In answer to Mr. Robinson's just criticism of Fig. 7 of the paper, the author herewith presents Fig. 4 which shows more accurately the values of RF_∞ in the superheat region for various constant stage efficiencies.

The stage efficiency does vary through the turbine as noted by Mr. Yates but, in a great many cases, not as much as he has indicated. If the stage efficiency at the high-pressure end is 0.78, increasing to somewhat better than 0.80, just above the saturation curve, finally falling off again in the wet region, an assumption of a constant stage efficiency of 0.80, or thereabouts, is probably justified. This eliminates the correction for moisture, for high density at the throttle end, and altogether makes the work simpler. If it is desired to approach more closely the true condition curve, the expansion may well be taken in three or four steps, using an appropriate constant stage efficiency for each of the steps.

Mathematical-Machine Determination of the Vibration of Accelerated Unbalanced Rotor[1]

By J. G. BAKER,[2] EVANSVILLE, WIS.

For an unbalanced rotor, amplitudes of vibration are given for different rates of constant angular acceleration and deceleration.[3] The setup used to obtain these results on the differential analyzer at the Massachusetts Institute of Technology is described briefly and shown diagrammatically.[4] The selection of spring constant, damping, and mass of suspended unit as a means of avoiding serious vibration in bringing a heavily unbalanced spring-mounted rotor up to speed is discussed in connection with the results.

SPRING mountings for rotors or reciprocating machinery are often designed to reduce vibration transmission to the foundation at full running speed without formal consideration of the process of getting up to speed.[5] This is permissible except in cases where the unbalance is large in comparison with other pertinent factors such as the mass of the mounted element and the acceleration with which it is brought up to speed. Where the unbalance is large the vibration of the spring-supported unit may increase rapidly enough to absorb all the energy supplied through torque to the rotor so that the machine cannot be brought to speed.

Of the number of schemes which have been used to avoid serious vibration during acceleration, perhaps the most obvious is that of adding damping. The addition of damping has the disadvantage that the vibratory force on the foundation at full speed is larger. Clamping of the spring-mounted element until the speed is well above that corresponding to the natural frequency and then unclamping has been done with good results. However, the complication of a clamping mechanism is usually objectionable.

It is the purpose of this study to consider the effects of spring stiffness, acceleration, damping, and mass of the spring-mounted element on the maximum amplitude of vibration attained during acceleration primarily to see to what extent the maximum amplitude can be reduced by the proper choice of the magnitudes of these factors.

In most actual cases of spring-mounted rotors the torque applied to the rotor by the source of power is roughly constant during most of the acceleration period, so that the angular acceleration would be also roughly constant were it not for the part of the torque which is absorbed by the vibration. Because of the building up of the vibration, a torque associated with the unbalance appears which is opposite in direction to the torque applied by the source of power and which therefore reduces the angular acceleration below that which would have taken place had there been no vibration. The equations of motion of a simple spring-mounted unbalanced rotor with a constant torque applied appear so difficult to solve or to handle in the differential analyzer in its present form that instead of constant applied torque, constant acceleration has been assumed.[6] Thus, the solution which has been arrived at does not show or take into account the reduction of acceleration due to the building up of vibration. Of course, if the vibration is small the difference between the constant-torque case and the constant-acceleration case is also small because the countertorque due to the vibration building up is negligible; hence, the analysis based on constant acceleration should be a good basis for design in the case of constant torque where the amplitude is limited to a small magnitude. In order to estimate when the error due to assuming constant acceleration is important, an approximate calculation of the torque required to maintain constant acceleration has been made. In those cases where the torque is practically constant, the assumption of constant acceleration is evidently justified.

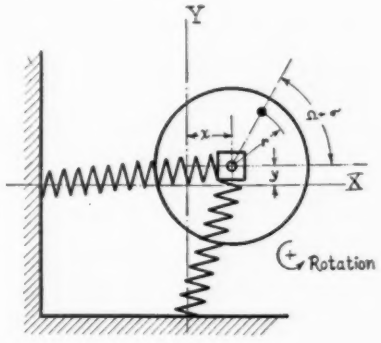


FIG. 1

DETAIL OF SYSTEM ANALYZED AND NOTATION

Fig. 1 shows a spring-mounted rotor movable in a plane perpendicular to its axis. The springs supporting the bearings on which the rotor is mounted are such that displacement of the rotor in

[1] The mathematical machine work was done on the differential analyzer at M.I.T. under the guidance of Prof. S. H. Caldwell with the assistance of J. L. C. Löf. The project as a whole was sponsored by and carried out in cooperation with the Westinghouse Electric & Manufacturing Company.

[2] Vice-President, Baker Mfg. Co., and consulting engineer, Westinghouse Elec. & Mfg. Co., and Hamilton Standard Propellers.

[3] A similar problem in which a vibrating system is disturbed by a constant amplitude force of changing frequency instead of an unbalance (with which the force amplitude varies approximately with the square of the frequency) has been solved exactly without the aid of a mathematical machine and was given in the article: "Vibration During Acceleration Through a Critical Speed," by F. M. Lewis, Trans. A.S.M.E., vol. 54, 1932, paper APM-54-24, p. 253.

[4] For a complete description see "The Differential Analyzer, a New Machine for Solving Differential Equations," by V. Bush, *Journal of The Franklin Institute*, vol. 212, October, 1931, p. 447.

[5] No distinction need be made between a spring-mounted rotor and a spring-mounted reciprocating machine in the discussion. It will be noted, however, that the vibration of a spring-mounted rotor absorbs twice as much energy from the rotating torque as is absorbed by a spring-mounted reciprocating machine, other factors being equal.

Presented at the Annual Meeting, Philadelphia, Pa., December 4–8, 1939, of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS.

Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y., and will be accepted until January 10, 1940, for publication at a later date. Discussion received after the closing date will be returned.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

[6] With the new differential analyzer now being constructed at Massachusetts Institute of Technology, it is believed that the solution of the case of constant applied torque can be obtained.

one direction does not produce spring forces perpendicular to the displacement. The spring constant and damping will be considered the same in both x- and y-directions. The means of applying damping is not shown.

In the following notation, the dimensions are indicated by M, L, and T for mass, length and time, respectively; M is also used for the mass of the rotor assembly less the unbalance. Let

M = mass of rotor assembly including mass of all parts moving with rotor except the unbalance $[M]$

m = mass of unbalance $[M]$

d = radius of unbalance $[L]$

K = spring constant in either x- or y-direction $\left[\frac{M}{T^2}\right]$

C = damping constant for either x- or y-direction $\left[\frac{M}{T}\right]$

Ω = angular displacement of rotor in radians from position at which the rotor velocity is zero (dimensionless)

ω = instantaneous angular velocity of rotor in radians per unit time $\left[\frac{1}{T}\right]$

ω_0 = instantaneous frequency of rotor lateral motion in radians per unit time $\left[\frac{1}{T}\right]$

t = elapsed time $[T]$

t_1 = time at which angular velocity is zero for deceleration $[T]$

σ = phase angle of unbalance in radians when rotor velocity is zero [dimensionless]

I = moment of inertia of rotor including the unbalance $[ML^2]$

v = displacement of unbalance m in x-direction $[L]$

h = acceleration of rotor in cycles per unit time per unit time $\left[\frac{1}{T^2}\right]$

T = torque required to produce specific acceleration $\left[\frac{L^2M}{T^2}\right]$

T_1 = $2\pi hI$ = torque required to increase angular momentum $\left[\frac{L^2M}{T^2}\right]$

T_2 = torque required to supply vibrational energies $\left[\frac{L^2M}{T^2}\right]$

e = $md/(M+m)$ = eccentricity of center of gravity of rotor $[L]$

E_s = energy stored in system due to vibration $\left[\frac{L^2M}{T^2}\right]$

E_d = energy dissipated by damping $\left[\frac{L^2M}{T^2}\right]$

E_r = $\frac{I\omega^2}{2}$ = energy stored due to rotation

N = $\frac{1}{2\pi}\sqrt{\left(\frac{K}{M+m}\right)}$ natural frequency in cycles per unit time $\left[\frac{1}{T}\right]$

x = displacement of rotor bearing, center to right, Fig. 1 $[L]$.

y = displacement of rotor bearing, center up, Fig. 1 $[L]$.

x_a = amplitude of x during acceleration $[L]$

x_d = amplitude of x during deceleration $[L]$

x_0 = amplitude of x in any case $[L]$

x_{ma} = maximum amplitude of x during acceleration $[L]$

x_{md} = maximum amplitude of x during deceleration $[L]$

x_s = amplitude calculated from constant-speed theory $[L]$

u = x/e; $u_a = x_a/e$; $u_d = x_d/e$; $u_{ma} = x_{ma}/e$; $u_0 = x_0/e$; $u_s = x_s/e$

u_{md} = x_{md}/e; u_m = either u_{md} or u_{ma} (all dimensionless)

q = N^2/h = number of undamped free vibrations there would be in the time interval between rotational-speed zero and rotational frequency equals the natural frequency (dimensionless); vice versa for deceleration

$q/2$ = number of revolutions the rotor makes in time interval between the time when the rotational velocity is zero and the time when the rotational frequency and t. natural frequency are equal [dimensionless]; vice versa for deceleration

γ = $C/\sqrt{K(m+M)}$ = damping constant (dimensionless)

$\frac{1}{\gamma}$ = amplitude of u that would be obtained were there no acceleration and the system was operated at resonance (dimensionless)

β = $\omega/2\pi N$ = ratio of the instantaneous rotational frequency to the natural frequency (dimensionless)

β_p = increment in β for one period of vibration as measured from differential-analyzer chart (dimensionless)

β_a = β for acceleration; $\beta_d = \beta$ for deceleration

β_d = $\frac{-(\tau-\tau_1)\sqrt{2}}{\sqrt{R}}$ (dimensionless)

t_1 = time at which $\beta_d = 0$

Substitutions made for algebraic simplification:

$F = \gamma\sqrt{(\pi q)}$; $\tau = t\sqrt{(\pi h)}$; $\tau_1 = t_1\sqrt{(\pi h)}$

τ_p = increment in τ for one period of vibration $\tau_p = \beta_p\sqrt{\pi q}$

$F = \gamma\sqrt{\pi q}$ for acceleration only $\tau = \beta_a\sqrt{(\pi q)}$

$R = 2\pi q$; for deceleration $(\tau - \tau_1) = -\beta_d\sqrt{(\pi q)}$

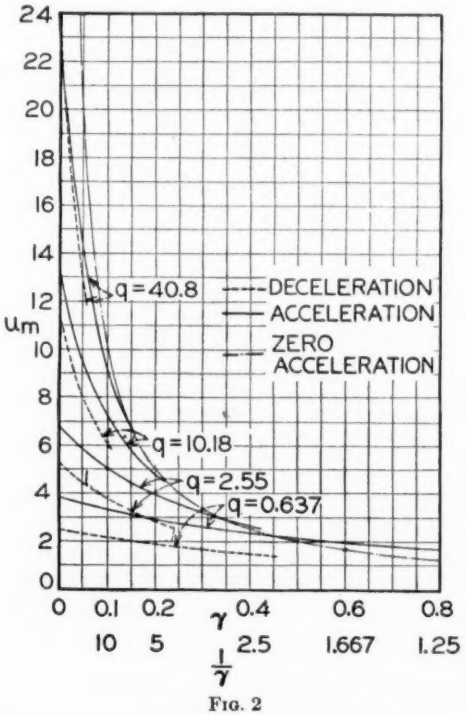


FIG. 2

RESULTS WITH APPLICATION TO SPRING MOUNTINGS

Fig. 2 shows the results as far as maxima are concerned. Figs. 3 and 4 were traced from the curves plotted automatically by the

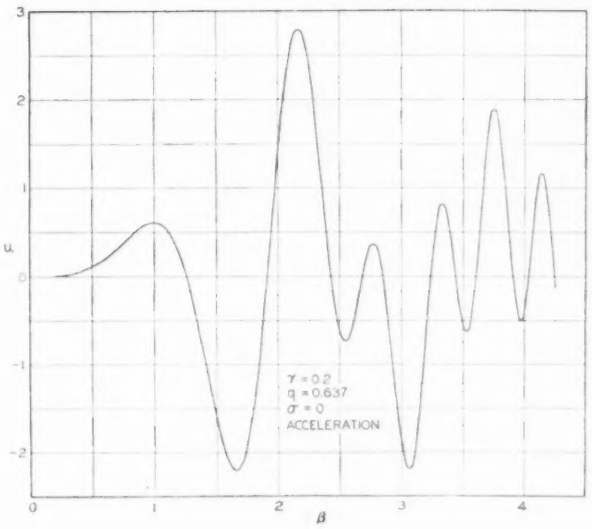


FIG. 3

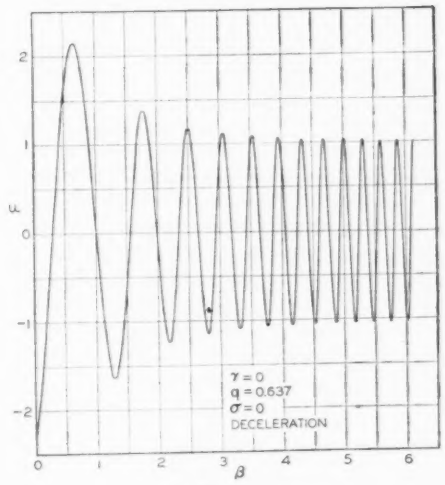


FIG. 4

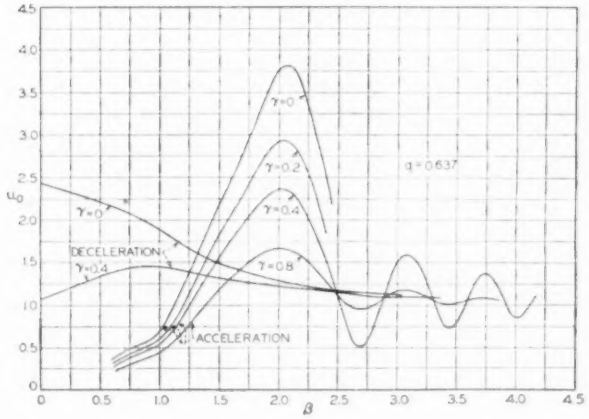


FIG. 5

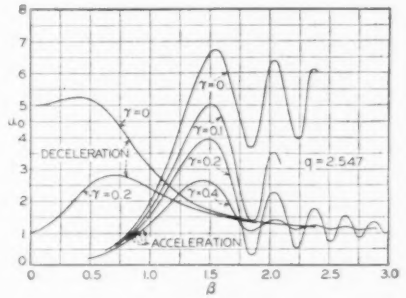


FIG. 6

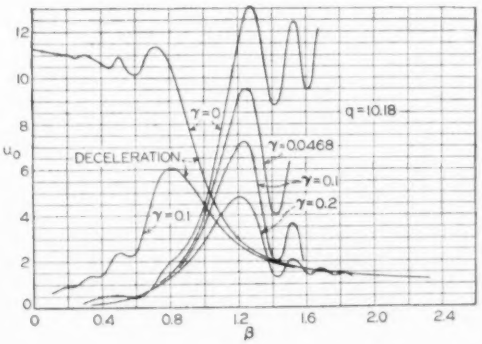


FIG. 7

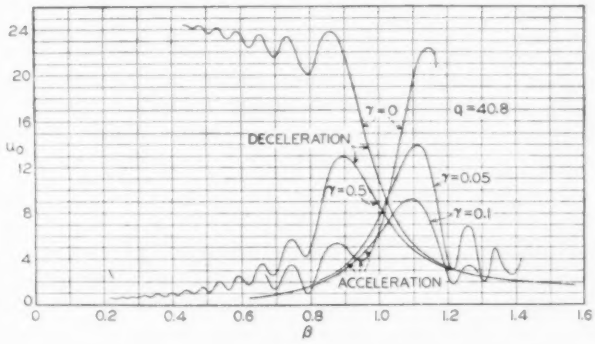


FIG. 8

differential analyzer. They show the displacement as a function of frequency ratio; (u as a function of β). Figs. 5, 6, 7, and 8 show the amplitudes u_0 as functions of frequency ratio β. For each low value of q several values of σ were used to obtain enough points to plot u_m curves.

Fig. 2 shows, not quite directly, the importance of choosing a low spring constant for a spring mounting, where it is desirable to minimize the maximum amplitude attained in coming up to speed with the mass and acceleration of a rotor given. To illustrate, consider two cases, both with the same total mass, $M + m$, and the same acceleration h but with case I with a lower spring constant K_1 than the spring constant K_2 for case II and compare the damping constants C_1 and C_2 in cases I and II, respectively, required to limit the maximum amplitude u_{ma} to the same magnitude in both cases. Let γ_1 and γ_2 be the dimensionless damping ratios for case I and case II, respectively, and let q_1 and q_2 be the respective magnitudes of q. Under these conditions and from the definitions of q and N

$$\frac{K_1}{K_2} = \frac{q_1}{q_2} \quad \text{[1.1]}$$

also from the definition of γ, and from Equation [1.1] and for no change in $M + m$

$$\frac{C_1}{C_2} = \frac{\gamma_1}{\gamma_2}\sqrt{\frac{q_1}{q_2}} = \frac{\gamma_1}{\gamma_2}\sqrt{\frac{K_1}{K_2}} \quad \text{[1.2]}$$

For a specific comparison, let u_{ma} be limited to 3. Take for case I $q_1 = 0.637$, then from Fig. 2, $\gamma_1 = 0.18$. For case II take $q_2 = 2.55$, then also from Fig. 2, $\gamma_2 = 0.33$. From Equation [1.1] $K_1/K_2 = 1/4$, and from Equation [1.2] $C_1/C_2 = 0.273$. In short, in reducing the spring constant to one quarter, the damping constant under the conditions considered can also be reduced to approximately one quarter for the same maximum amplitude of vibration. The amplitude of force on the foundation at speed will be reduced to about one quarter since u is approximately unity at full speed irrespective of the spring constant so long as the natural frequency is low compared to the full-speed frequency.

A comparison may also be made to show the effect of spring-constant reduction on amplitude with a given damping constant. Taking the same values of q_1, q_2, K_1, K_2 as before, then from Equation [1.2]

$$\gamma_1/\gamma_2 = 2 \quad [1.3]$$

For a maximum amplitude for the new case I, u_{ma1} of 3, $\gamma_1 = 0.18$ as before and from Equation [1.3], $\gamma_2 = 0.09$. From Fig. 2 with $q_2 = 2.55$ and $\gamma_2 = 0.09$, $u_{ma2} = 5.15$ or the ratio of the two maximum amplitudes is $u_{ma1}/u_{ma2} = 0.582$. The ratio of the maximum amplitudes of vibratory force will be $u_{ma1}K_1/u_{ma2}K_2$ or 0.146. That is, a reduction in spring constant to one quarter its original value, for the conditions under comparison, reduces the maximum force on the foundation to about one seventh. The frequency of motion at the occasion of the maximum force is also reduced so that the whole effect is made more than proportionally milder by a reduction in spring constant.

Discussion of Errors in the Mathematical-Machine Operation

In the setting up of the machine numerous checks were made of the setup. As a check of the results a comparison can be made between the u_a at frequencies high above the natural frequency and the amplitude as calculated from the ordinary constant-speed theory which u_a would be expected to approach except in the case of no damping where a sustained free vibration might be expected. In expressing the constant-speed solution for frequencies high above the natural-frequency damping, unless it is very large, need not be taken into account so that the amplitude u_s becomes simply

$$u_s = \frac{1}{(1/\beta^2) - 1} \quad [1.4]$$

Of the graphs which were run out far enough to compare with Equation [1.4], discrepancies of about 5 per cent were noted.

Erratic errors are apparently small in the differential analyzer for, in a few instances, parts of runs were repeated with the new line traced by the machine falling on top of the old. Another check was the observation of the accuracy with which the machine generated sine and cosine functions. Many cycles would often be generated with less than 1 per cent error as observed by changing amplitude. In the runs evolving many cycles, corrections were introduced at the shafts generating sine and cosine functions in order to minimize accumulated error. In one case ($q = 2.547$; $\gamma = 0$) two runs were made, one with these corrections and one without. At the maximum amplitude the run without the corrections gave 1.5 per cent higher value than with the corrections. Further along for β about 3 the difference was 4 per cent at the peaks; however, except at the peaks the curves were identical as far as could be detected.

In another check it was noted that the maxima, for runs differing only in the value of the phase σ used, all plotted on a single smooth curve; for large values of q only one value of σ was used so that this check could not be made. It is expected that the runs for the smaller q's are more accurate than the runs with larger q's

Derivation of Differential Equation for the Case of Acceleration

The displacement of m with respect to the rotor center will be $d \cos(\Omega + \sigma)$. The total displacement v of m in the x-direction will be

$$v = x + d \cos(\Omega + \sigma) \quad [1]$$

The lateral force to the right, of m on M will be

$$-m\frac{d^2v}{dt^2} = m\frac{d^2x}{dt^2} - md\frac{d^2}{dt^2}(\cos[\Omega + \sigma]) \quad [2]$$

Equilibrium of the disk mass then gives

$$(M + m)\frac{d^2x}{dt^2} + C\frac{dx}{dt} + Kx = -md\frac{d^2}{dt^2}(\cos[\Omega + \sigma]) \quad [3]$$

Carrying out the indicated differentiation in the right-hand member of Equation [3], Equation [3] becomes

$$(M + m)\frac{d^2x}{dt^2} + C\frac{dx}{dt} + Kx = md\frac{d^2\Omega}{dt^2}\sin(\Omega + \sigma) + md\left(\frac{d\Omega}{dt}\right)^2\cos(\Omega + \sigma) \quad [4]$$

Setting $\Omega = t^2\pi h$, which expresses a displacement of uniform acceleration, and introducing the dimensionless parameters defined previously, Equations [3] and [4] become Equations [5] and [6], respectively

$$\frac{1}{2}\frac{d^2u}{d\tau^2} + F\frac{du}{d\tau} + Ru = -\frac{1}{2}\frac{d^2}{d\tau^2}(\cos[\tau^2 + \sigma]) \quad [5]$$

$$\frac{1}{2}\frac{d^2u}{d\tau^2} + F\frac{du}{d\tau} + Ru = \sin(\tau^2 + \sigma) + 2\tau^2\cos(\tau^2 + \sigma) \quad [6]$$

For the purpose of machine solution Equation [5] was selected and put in the form

$$u = -\cos(\tau^2 + \sigma) - 2\int[R\int u d\tau + Fu]d\tau \quad [7]$$

Differential Analyzer Setup for Case of Acceleration[4]

The differential-analyzer mathematical machine is made up of a number of shafts which may be used to connect machine units of several types. One type of machine unit is the "adder," which is a differential gearing arranged so that one shaft rotates through an angle equal to the sum of the angles through which two other shafts rotate. Another type is the "gear train" by means of which one shaft is made to rotate a fixed fraction of the rotation of another shaft. Thus, the function of the gear train corresponds to multiplication by a constant. Another type of machine unit is the "integrator" which is a variable-ratio transmission between a driving shaft of angular displacement s and a driven shaft of angular displacement z. A third shaft of angular displacement w determines the transmission ratio which may be also denoted by w. Fig. 9 is the standard conventionalized diagram for an integrator with the shafts z, s, and w indicated.[4] For a small displacement of s equal to ds the displacement of z will be dz, so from the definition of w



Fig. 9

$$w = \frac{dz}{ds} \quad [8]$$

and

$$\int w ds = z \quad [9]$$

In the differential analyzer used z must be the driven shaft for

mechanical reasons. On this account z is considered the dependent variable.[7]

Fig. 10 illustrates the setup of two integrators corresponding to the differential Equation [10]. The dots in Fig. 10 indicate connections between perpendicular shafts. Thus

$$\frac{d^2\alpha}{ds^2} + \alpha = 0 \quad \ldots\ldots [10]$$

A solution of Equation [10] is

$$\alpha = \sin(s + \phi) \quad \ldots\ldots [11]$$

where ϕ is a boundary condition. Equation [10] may be written

$$\alpha = -\int\int \alpha\, ds ds \quad \ldots\ldots [12]$$

In Fig. 10, taking the displacements of shaft b and shaft c as s and α, respectively, then integrator III will cause shaft d to rotate an

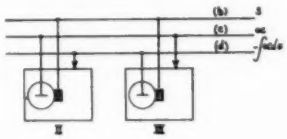


FIG. 10

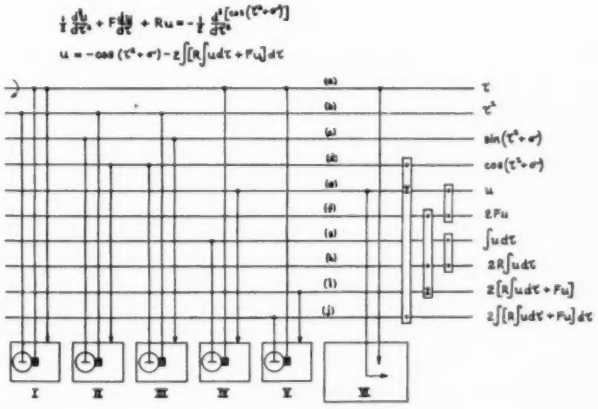


FIG. 11

amount $\int \alpha ds$. Because of the connection to integrator II through a reverse gear, shaft c will rotate $-\int\int \alpha\, ds ds$. Since c must rotate an amount equal to both α and $-\int\int \alpha\, ds ds$ Equation [12] is satisfied by the setup and the motion of c can be $\sin(s + \phi)$ (from Equation [11]) provided the starting conditions are properly selected. If the displacement of c is $\sin(s + \phi)$ then the displacement of d, which is $-\int \alpha ds$, must be $\cos(s + \phi)$.

Fig. 11 shows the setup for Equation [7]. To the displacement of the shaft a is assigned the letter τ. The integrator I performs the operation $\int \tau d\tau$ so that in conjunction with a 2:1 gear shaft b moves an amount τ^2. The arrangement of the integrators II and III and the shafts b, c, and d is, as has already been described, in connection with Fig. 9 (the letter and number indications are the same on the two figures) and Equations [10], [11], and [12]; hence, if s and ϕ are taken equal to τ^2 and σ, respectively, the displacements of c and d indicated on Fig. 11 are accounted for. The displacement of shaft e is taken as u. By means of a gear train of ratio $2F$ connecting e and f (indicated by a small rectangle with two dots) f moves an amount $2Fu$. The integrator IV because of its connections to a and e moves g an amount $\int u d\tau$. Shaft h is driven an amount $2R\int u d\tau$ through a gear train of ratio $2R$ connected to g. Shaft i moves $2[R\int u d\tau + Fu]$ because of the adder connecting f and h to it (indicated by a rectangle with dots at f and h, the displacements of which are to be added, and a summation sign at i the displacement of which is to be the sum of the displacements of f and h). Integrator V moves j $2\int [R\int u d\tau + Fu] d\tau$ due to its connection to i and a. Another adder causes the displacement e to be the sum of the motion of d and j. This addition is expressed by Equation [7] so that with starting conditions of the differential-analyzer machine corresponding to the boundary conditions of the problem rotation of e will be a measure of u which is the result sought. For the case of acceleration τ, u, and $du/d\tau$ are initially zero and σ may be assigned any value. These conditions fix the starting position of all shafts.

The shafts a and e are connected to the plotting mechanism VI. The curve appearing on VI thus has τ for an abscissa and u for an ordinate. The curves of Figs. 3 and 4 were traced directly from such curves. (The τ variable has been exchanged for β.)

Analysis and Differential-Analyzer Setup for the Case of Deceleration

The case of deceleration is taken as that in which the rotor is initially operating at a speed far above its natural frequency and decelerating at a uniform rate. It is assumed that no appreciable free vibration is present at this time. As with the case of acceleration, constant deceleration has been assumed instead of constant retarding torque.

Equations [3] and [4] still hold in the case of deceleration but Ω must no longer be set equal to $t^2\pi h$. If the rotor is rotating rapidly and slowing down, then if its velocity is negative

$$\frac{d^2\Omega}{dt^2} = 2\pi h \quad \ldots\ldots [13]$$

If $t = t_1$ when $\frac{d\Omega}{dt} = 0$ and $\Omega = 0$, then

$$\frac{d\Omega}{dt} = 2\pi h(t - t_1) \quad \ldots\ldots [14]$$

or

$$\Omega = \pi h(t - t_1)^2 \quad \ldots\ldots [15]$$

So that if in the equations for acceleration τ^2 is replaced by $\tau - \tau_1{}^2$ the equations for deceleration are obtained; thus, Equation [5] would be changed to

$$\frac{1}{2}\frac{d^2u}{d\tau^2} + F\frac{du}{d\tau} + Ru = -\frac{1}{2}\frac{d^2}{d\tau^2}(\cos[(\tau - \tau_1)^2 + \sigma]) \quad \ldots [16]$$

where τ_1 has a large value. Since the initial speed will be far above the natural frequency the ordinary constant-speed solution neglecting damping would be expected to give a good approximation of the initial amplitude. On this basis, Equation [16] may be approximated for frequencies high above the natural frequency as

$$\frac{1}{2}\frac{d^2u}{d\tau^2} + Ru = 2[\tau - \tau_1]^2 \cos([\tau - \tau_1]^2 + \sigma) + \sin([\tau - \tau_1]^2 + \sigma) \quad \ldots [17]$$

For τ_1 large compared to τ Equation [17] becomes

$$\frac{1}{2}\frac{d^2u}{d\tau^2} + Ru = 2(\tau_1{}^2 - 2\tau_1\tau)\cos(\tau_1{}^2 - 2\tau_1\tau + \sigma) \quad \ldots [18]$$

[7] In order to obtain accuracy of transmission ratio and to avoid large torques in the w shaft, no appreciable power can be supplied by the z shaft as it leaves the transmission. The necessary power is supplied to a z' shaft not directly connected to the z shaft. Just sufficient torque is supplied to the z' shaft to make it move with the z shaft. The torque supplied is controlled by minute relative displacement between the z and z' shafts. Because z and z' move practically together no distinction will be made between their displacements.

Furthermore the $-2\tau_1\tau$ may be neglected in the coefficient of the cosine term because it is small compared to τ_1^2. However, $-2\tau_1\tau$ may not be neglected under the cosine sign because here even though τ_1^2 is extremely large the value of the cosine function is markedly affected. Equation [18] may then be further simplified to

$$\frac{1}{2}\frac{d^2u}{d\tau^2} + Ru = 2\tau_1^2 \cos(\tau_1^2 - 2\tau_1\tau + \sigma) \quad [19]$$

The steady-state solution of Equation [19] is

$$u = 2\tau_1^2 \cos(\tau_1^2 - 2\tau_1\tau + \sigma) \quad [20]$$

$$\frac{du}{d\tau} = -\frac{4\tau_1^3 \sin(\tau_1^2 - 2\tau_1\tau + \sigma)}{R - 2\tau_1^2} \quad [21]$$

Equations [20] and [21] were used to calculate the boundary conditions for the deceleration runs. For simplicity the part of the cycle was chosen where

$$\frac{du}{d\tau} = 0 \quad [22]$$

and therefore where

$$u = \frac{2\tau_1^2}{R - 2\tau_1^2} \quad [23]$$

The differential analyzer set was the same as for acceleration with these boundary conditions and considering that the variable τ is replaced by the variable $\tau - \tau_1$, which starts out with a large negative value ($\tau = 0$). Fig. 4 is typical of the solutions obtained in this way. It will be noted that there is no perceptible free vibration transient at the highest β where the solution started. In cases including damping, slight free-vibration transients were recorded which might be expected since damping was neglected in estimating the boundary condition. However, the presence of damping removed the transient after a few cycles so that the neglect of damping in estimation of the boundary condition is justified.

Torque Required to Accelerate Rotor

The energy equation of the system may be used to calculate approximately the torque required to accelerate.

For a nonrotating mass equally sprung in both directions and vibrating in a circular path, one half of the stored energy is kinetic and one half is potential (elastic energy in springs). The total stored energy may therefore be considered twice the elastic energy or

$$E_s = Ke^2u^2 \quad [24]$$

The energy stored per unit time is

$$\frac{dE_s}{dt} = 2Ke^2u_0\frac{du_0}{dt} \quad [25]$$

The energy dissipated by damping is per unit of time

$$\frac{dE_d}{dt} = C\omega_0^2x_0^2 = C\omega_0^2u_0^2e^2 \quad [26]$$

If the vibrating mass is rotating and being accelerated, the energy due to rotation is $I\omega^2/2$ and the increase in rotational energy per unit time will be

$$\frac{dE_r}{dt} = I\omega\frac{d\omega}{dt} = I\omega 2\pi h \quad [27]$$

The only source of energy is the external torque so that equating the energy supplied per unit time to the total energy stored plus the energy dissipated

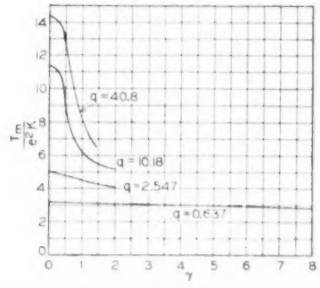


FIG. 12

$$T\omega = 2Ke^2u_0\frac{du_0}{dt} + C\omega_0^2u_0^2e^2 + I\omega 2\pi h \quad [28]$$

or

$$T_2 = \frac{C\omega_0^2u_0^2e^2}{\omega} + \frac{2Ke^2u_0}{\omega}\frac{du_0}{dt} \quad [29]$$

or introducing nondimensional notation

$$\frac{T_2}{e^2K} = \frac{u_0}{q\beta}\left[\frac{\gamma u_0}{\beta_p^2 q} + \frac{1}{\pi}\frac{du_0}{d\beta}\right] \quad [30]$$

Which is an approximate expression, because the rotor's path was assumed to be a circle, whereas it is actually a complicated spiral. Values of u_0, β_p, and $du_0/d\beta$ were obtained from the charts drawn by the differential analyzer and T_2/e^2K calculated from the approximate Equation [30]. The results are plotted in Fig. 12.

Some Applications of Photoelasticity in Turbine-Generator Design

By M. HETÉNYI,[1] EAST PITTSBURGH, PA.

This paper deals with three particular problems in turbine-generator design, namely (*a*) stress concentrations in transverse grooves of two-pole generator rotors, (*b*) centrifugal stresses in rotors, and (*c*) stress concentrations in T-heads (bolts and blade fastenings). The connection between these problems is only their mutual sphere of application and the fact that in each case the solution has been obtained by photoelasticity. The problems have been investigated as two-dimensional ones, the first and third being analyzed by the standard photoelastic technique, while in the second problem the fundamentally three-dimensional method of "freezing" the stresses in bakelite samples was applied.

Stress Concentration in Transverse Grooves of Two-Pole Generator Rotors

IN A two-pole generator the flexural rigidity of the rotor varies from a maximum value in the pole axis to a minimum value in the winding axis. This unequal rigidity in the two principal axes results in a variation in the static deflection of the rotor as it rotates. The variation in the deflection occurs at a frequency equal to twice the frequency of rotation and results under certain circumstances in an objectionable vibration of the rotor.

The most effective way of preventing the occurrence of this double-frequency vibration is to equalize the flexural rigidity of the rotor in the two principal planes, specifically to reduce the rigidity in the pole axis until it is essentially the same as the combined rigidity of the rotor teeth and field coils in the winding axis. The required reduction in rigidity can be obtained most effectively by cutting segment-shaped transverse grooves in the pole body in uniform intervals along the axis of the rotor as shown in Fig. 1. In order to obtain sufficient reduction of rigidity with a minimum removal of material, the grooves will have to be shaped as narrow slots, in consequence of which the radius R of the semicircular ends of the grooves becomes very small in comparison to other dimensions, such as the net diameter d of the rotor. Under such circumstances a considerable concentration of stress can be expected at the bottom of the grooves and, though the nominal stresses in the rotor body are always low, the safety of design required a determination of the exact stress conditions existing in such cases, specifically in the design shown in Fig. 1.

The problem has been considered as a two-dimensional one and bakelite plane models were prepared representing in $^1/_8$-in. scale the longitudinal middle section of the rotor through the pole axis and the center of the grooves. In order to establish the effect of the spacing of the grooves c on the amount of stress concentration, the tests were started with only one groove on each side of the sample, which corresponds to a $c = \infty$ spacing. Then the spacing $c = 2a$ (corresponding to the proportions of the actual rotor shown in Fig. 1) with five grooves on each side was investigated. Finally the spacing was reduced to $c = a$ when nine grooves were on each side of the plane model. The values $D/d = 1.31$ and $d/2R = 69.3$ were the same in each case, and all three samples were tested in pure bending. The photoelastic pattern obtained in the edge region of the sample with $c = 2a$ spacing of the grooves is shown in Fig. 2.

The maximum stress at the bottom of the grooves can be expressed as $\sigma_{max} = k\sigma_{nom}$, where $\sigma_{nom} = 6M/td^2$ is the nominal fiber stress in bending the net section of width d and thickness t, and k is the concentration factor. From the tests, the k values given in Table 1 were obtained.

TABLE 1 STRESS-CONCENTRATION FACTORS k OBTAINED FROM TESTS

Spacing $c = \infty$	Spacing $c = 2a$	Spacing $c = a$
$k = 6.25$	End groove, $k = 4.75$	End groove, $k = 4.25$
....	Middle groove, $k = 3.95$	Middle groove, $k = 3.20$

In the rotor design the concentration factors for the middle slots should be used because they are in the region of maximum

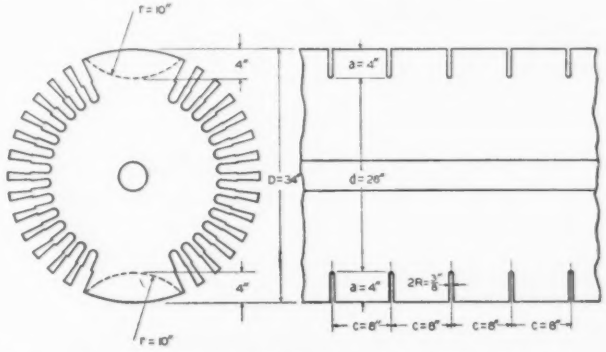


FIG. 1 TWO-POLE TURBINE-GENERATOR ROTOR WITH TRANSVERSE GROOVES IN THE POLE BODY

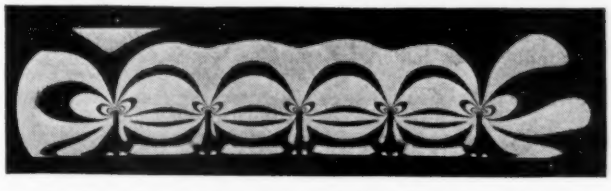


FIG. 2 PHOTOELASTIC PATTERN IN THE EDGE REGION OF A GROOVED SAMPLE IN BENDING

bending stress. Knowing the limiting cases ($c = 0, k = 1$) and ($c = \infty, k = 6.25$) and two other data between, the effect of spacing on the amount of stress concentration can be plotted as shown with the solid curve in Fig. 3. This curve gives the k factor for any middle groove, since the values $D/d = 1.31$ and $d/2R = 69.3$ are kept constant and only the spacing of the

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[1] Research Laboratories, Westinghouse Electric & Mfg. Co.

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Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y., and will be accepted until January 10, 1940, for publication at a later date. Discussion received after the closing date will be returned.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

grooves changes and shows clearly the stress-relieving effect as the grooves are spaced closer. In comparison with these values, the partial relief of stress which occurs at the end grooves is seen from the numerical data for the k values given in Table 1. In actual generator-rotor design the slot proportions are such that the k factor must always be close to the 3.95 value found for the actual case of an 81250-kva two-pole generator rotor. The ratio $d/2R = 69.3$ gives a bending-stress-concentration factor $k = 7.15$ which, since only one pair of grooves was considered, corresponds also to a $c = \infty$ spacing of grooves. Compared to our test results the k factors for infinitely deep grooves should approach the limiting theoretical value $k = 7.15$ for $c = \infty$ approximately in the manner shown by the dashed curve in Fig. 3.

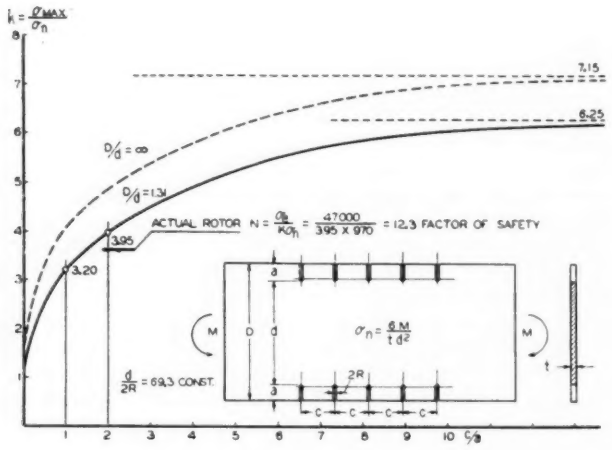


FIG. 3 STRESS-CONCENTRATION FACTOR k FOR TRANSVERSE GROOVES PLOTTED AS A FUNCTION OF THE SPACING OF THE GROOVES

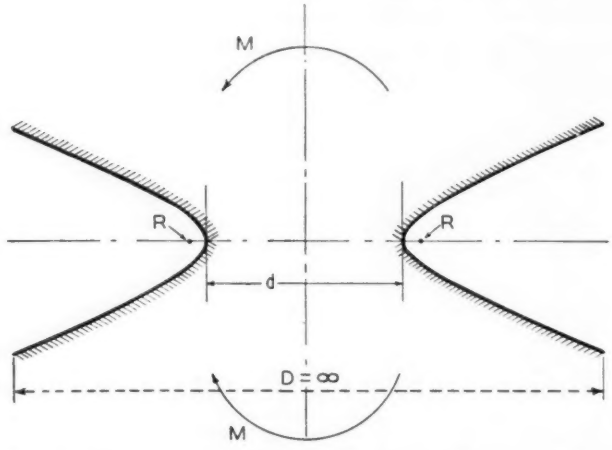


FIG. 4 HYPERBOLIC GROOVES WITH RADIUS OF CURVATURE R AT THE ROOTS, INVESTIGATED THEORETICALLY BY H. NEUBER (1)

nominal stress σ_n for this case is 970 lb per sq in. which multiplied by 3.95 gives a maximum stress of 3800 lb per sq in. The endurance limit of the material σ_e for this case is 47,000 lb per sq in. The factor of safety is $\frac{\sigma_e}{k\sigma_n} = \frac{47000}{3.95 \times 970} = 12.3$, which is well within the standards of accepted design practice and represents a condition of safety equal to or better than that of the regions near ventilation holes and other details encountered in rotor-design practice.

There is no theoretical solution for finite D/d values with which to compare the test results. The only solution of this kind is for the case when $D = \infty$, that is, for an infinitely deep slot of hyperbolic contour line with an R radius of curvature at the root; it was derived by Neuber (1).[2] His formula (2) for a

[2] Numbers in parentheses refer to the Bibliography at the end of the paper.

Under the same assumptions as in the plane problem in Fig. 4, Neuber has also obtained a theoretical solution for the corresponding three-dimensional case when the grooves form a surface of revolution. According to his formula (3) in simple bending, the value of maximum stress at the root of such circumferential grooves for $d/2R = 69.3$ and Poisson's ratio $\mu = 0.3$ is $\sigma_{max} = k\sigma_{nom}$, where $k = 6.42$ and $\sigma_{nom} = 32M/\pi d^3$ is the nominal fiber stress in bending of the net section of diameter d.

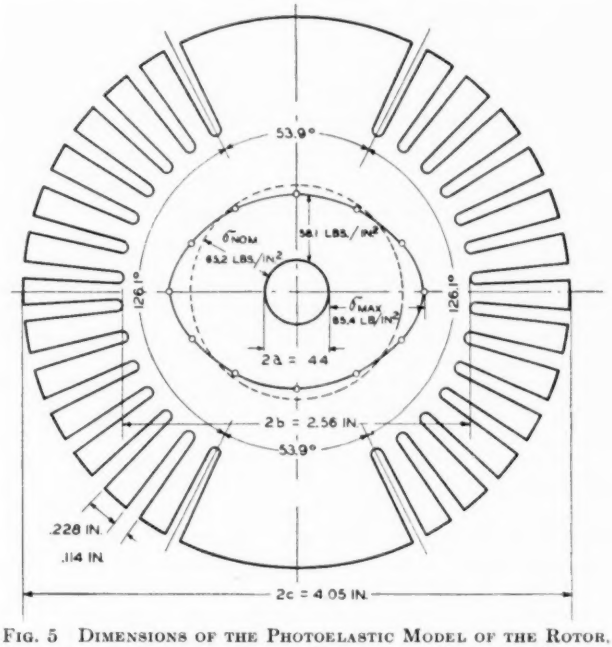


FIG. 5 DIMENSIONS OF THE PHOTOELASTIC MODEL OF THE ROTOR, SHOWING THE DISTRIBUTION OF CENTRIFUGAL STRESSES AROUND THE CORE

It is seen that the concentration factor for the three-dimensional case is about 10 per cent less than for the corresponding two-dimensional problem. On this basis it is justifiable to say that the slightly three-dimensional character of the segmental grooves in our problem, shown in Fig. 1, will cause not more than a few per cent deviation in the value of the factor k obtained from plane model tests.

CENTRIFUGAL STRESSES IN ROTORS

When a sample of heat-hardening resin, such as bakelite BT-61-893 commonly used in photoelastic tests, is annealed in a loaded condition, one obtains a specimen in which the elastic strains and birefringence produced by the loading at the annealing temperature are permanently preserved. This method of solidifying or "freezing" elastic-stress fields has been successfully applied in the investigation of three-dimensional stress problems (4) and can be used in every case where the loading of the model is maintained constant during the annealing process. The stress problem of a body rotating with constant velocity is merely a particular case of that of constant loading, which makes the freezing method directly applicable to problems of this kind. As has been shown in a previous publication by the author (5).

from solidified test pieces it is possible to determine the complete stress distribution, comprising the magnitude and direction of each of the principal stresses at any point of a rotating specimen.

The same method has been applied in the present case to determine the distribution of centrifugal stresses around the core of a two-pole generator rotor. The dimensions of the model, made of bakelite BT-61-893, are shown in Fig. 5. This model, 0.27 in. thick, was rotated in an electric oven at a speed of 3600 rpm at a temperature of 120 C for 1 hr and was then slowly cooled down in about 5 hr to room temperature while the speed of rotation was maintained the same. Since at the annealing temperature the modulus of elasticity of bakelite is greatly reduced and consequently the model undergoes noticeable deformations under the influence of centrifugal forces, particular precautions had to be taken in order to keep the sample in a concentric position while it was being rotated in the oven. For this purpose the rotor model was surrounded by a casing lined with rubber and rotated together with the specimen. Knowing from previous tests (6) the modulus of elasticity of the material at the annealing temperature (E = 1080 lb per sq in.), it was possible to predict the expansion of the model during the test and to provide a corresponding clearance in the casing around the sample. At the beginning of the test the specimen was held by friction at its sides but as the temperature was raised, due to the extension of the model and the reduction of its thickness, this friction on the sides was relieved and from this moment on at the latter part of the test the expanded specimen was guided in its central position by its casing along the circumference. This arrangement was simple and satisfactory and no contact pressure of detectable magnitude took place between the model and the casing.



FIG. 6 ISOCHROMATIC LINES OF CENTRIFUGAL STRESSES IN A GENERATOR-ROTOR MODEL

In the rotor model, the width of the teeth was chosen in such a way as to represent the combined weight of the rotor teeth and windings, the sum of which per unit volume is equivalent to two thirds of the specific weight of the rotor body. Making the model with such proportions, the distribution of centrifugal stresses around the core will be the same as in the actual generator rotor.

The photoelastic picture of the "frozen" model is shown in Fig. 6. By calibrating the material it was found that at the annealing temperature in green light one isochromatic line is equivalent to 12.37 lb per sq in. difference in principal stresses in the specimen 0.27 in. thick. Using this fringe value, the core stresses were obtained from the photoelastic pattern and the resulting stress diagram is shown in Fig. 5.

The experimental results can be conveniently compared to a constant nominal stress value easily obtainable by computation if the extra weight is disregarded in the solid poles of the rotor. Under such assumptions the core stresses can be calculated by considering the inner solid part of the rotor between the radii a and b subjected in addition to its own centrifugal loading to uniformly distributed radial tensile forces along its outer circumference equivalent to the inertia forces due to the combined weight of the rotor teeth and windings. Both of the foregoing cases can be computed by known formulas (7) and their sum, the nominal stress σ_{nom} at the core will be

$$\sigma_{nom} = \frac{3+\mu}{4}\rho\omega^2\left(b^2 + \frac{1-\mu}{3+\mu}a^2\right) + \alpha\frac{2b}{b^2-a^2}\rho\omega^2\int_b^c r^2dr \quad \ldots . [1]$$

where a, b, and c are radii, as shown in Fig. 5; μ is Poisson's ratio; ω is the angular velocity; ρ is the mass per unit volume of the material of the rotor; and α is the ratio of the combined weight of rotor teeth and windings per unit volume to the specific weight of solid rotor body. In the photoelastic test we had a = 0.220 in., b = 1.280 in., c = 2.025 in., μ = 0.5, ω = 376.8 sec^{-1}, ρ = 0.000128 lb-sec^2 in.$^{-4}$, and α = $^2/_3$ which, when substituted into Equation [1] give σ_{nom} = 24.40 + 40.80 = 65.20 lb per sq in. Referring the maximum core stress obtained in the test to this calculated nominal stress value, we obtain a stress-concentration factor k = 85.40/65.20 = 1.13 as a result of the extra weight in the poles which was not considered in the calculation.

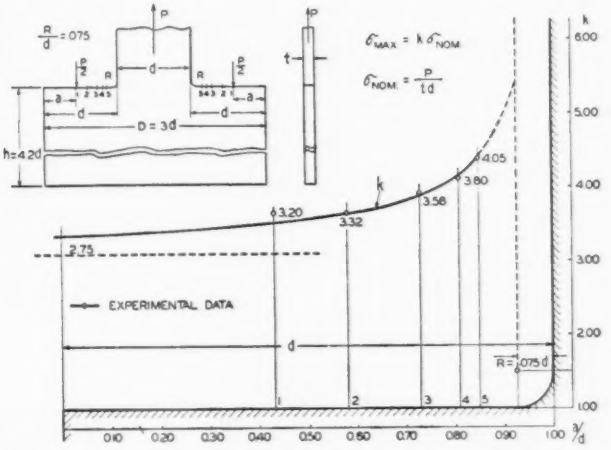


FIG. 7 EFFECT OF THE PROXIMITY OF SUPPORTS TO THE FILLET ON THE STRESS CONCENTRATION IN T-HEADS

The stress increase at the bore due to nonsymmetry of the rotor is thus shown to be negligible as judged from conventional design practice.

STRESS CONCENTRATION IN T-HEADS

T-heads have frequent application in machine design as bolt heads, turbine-blade roots, and various types of fastenings. Tests of such elements have revealed that the stress concentra-

tion at the fillets is usually much greater than what one could expect from a comparison with known results obtained with tension and bending specimens of similar fillet ratios. This fact pointed to the necessity of a more fundamental study of the stress conditions in T-heads which would establish the effect of proportions and other factors on the resulting stress concentration. A partial result of such a study will now be discussed.

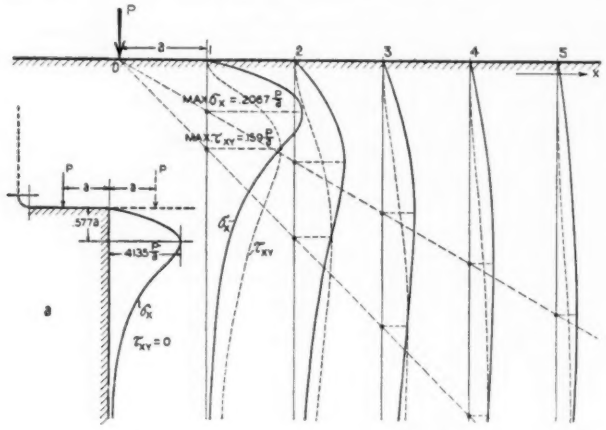


FIG. 8 STRESSES IN A SEMI-INFINITE PLATE DUE TO A CONCENTRATED FORCE

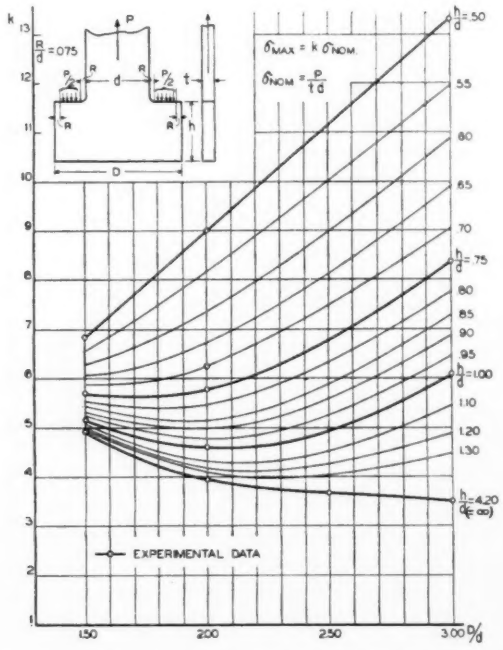


FIG. 9 STRESS-CONCENTRATION FACTOR FOR T-HEADS WITH A CONSTANT FILLET RATIO OF $R/d = 0.075$

Photoelastic tests have been made with T-head models supported along their flanges and subjected to an axial pull P in their center line. The ratio of the fillet R to the width of the mass d was kept constant $R/d = 0.075$ in these tests while other proportions of the head were varied. The maximum stress σ_{max} occurring in the fillet at the junction of the head and the neck can be conveniently expressed as a multiple of the nominal tensile stress in the neck section $\sigma_{nom} = P/td$. Thus $\sigma_{max} = k\sigma_{nom}$, where k is the stress-concentration factor. It was observed in the tests that there were three main factors which contribute to the increase of σ_{max}, namely (*a*) the proximity of the points of support to the fillets, (*b*) the proximity of the edges of the head to the points of support, and (*c*) the bending of the flanges of the head. The influence of these three factors will be discussed separately in the following.

The effect of the proximity of the points of support to the fillets is shown in Fig. 7. A model was tested with the ratios $D = 3d$, $R = 0.075d$ and with such large depth of the head $h = 4.20d$ that the bending of the flanges could be disregarded. This sample was supported at concentrated points successively in positions 1, 2, 3, 4, and 5, and in each case the amount of stress concentration at the fillet was determined. The result of this test series is shown in Fig. 7. It is seen that the concentration factor k rapidly increases as the points of support approach the fillets. It was not advisable to approach the fillets beyond the point No. 5 since the material underwent a considerable distortion in the neighborhood of the concentrated supports and if they were placed close to the fillets, the region of maximum stress could have been also in the yielding region leading to a false reading of σ_{max}. For this reason the value of k in the immediate neighborhood of the fillet was determined by extrapolation. The $k = 2.75$ value is for the case when the sample of $D/d = 3$ and $R/d = 0.075$ ratios is subjected to simple axial

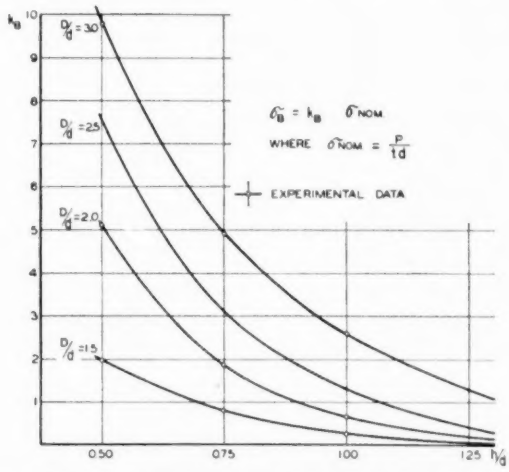


FIG. 10 EFFECT OF BENDING OF SHOULDERS, SEPARATED FROM FIG. 9

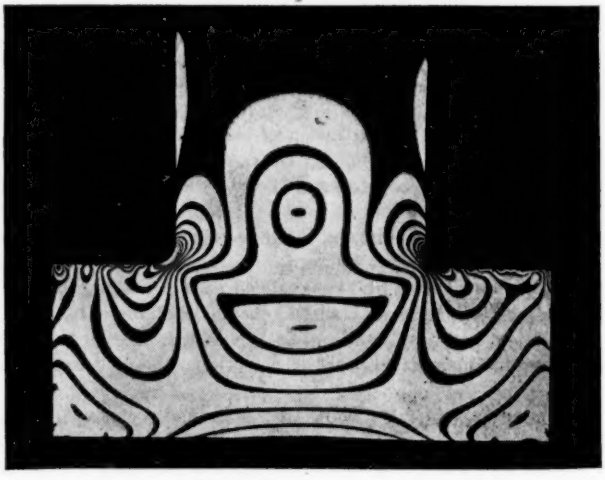


FIG. 11 PHOTOELASTIC PATTERN OF A T-HEAD SUBJECTED TO AN AXIAL PULL AND UNIFORMLY SUPPORTED ALONG THE FLANGES



FIG. 12 ISOCHROMATIC LINES AT THE FILLETS OF THE T-HEAD SHOWN IN FIG. 11

tension and it is believed that this value corresponds to the imaginary case when the same sample is supported at the flanges at points in infinity. It is seen that if the concentrated supports are exactly at the start of the fillets, the produced stress concentration might amount to about twice the value of that obtained from simple tension specimens of similar dimensions.

When the sample is supported at points Nos. 5 and the width of the head is reduced from $D = 3d$ to $D = 1.5d$, the concentration factor $k = 4.05$, shown in Fig. 7, will increase to a value of $k = 5.10$. This indicates the effect referred to previously as the second factor in the increase of stress concentration, namely, the proximity of the edges of the head to the points of support. A qualitative explanation of this effect can also be obtained from theoretical considerations. When a semi-infinite plate is subjected to a concentrated force P at its straight boundary, the distribution of the normal and shearing stress σ_x and τ_{xy} originated in the vertical sections at various distances from the point of application of the load is shown in Fig. 8. The action of cutting the plate through any of these vertical lines would be equivalent to superposing the negative value of the σ_x and τ_{xy} stresses along the same line which would create there a free unloaded vertical boundary. The τ_{xy} shearing stresses can be removed by applying another force P symmetrically on the other side of the vertical section, doubling thereby the compressive normal forces σ_x along the same line as shown in Fig. 8*a*. Thus the action of cutting can be considered as superposing σ_x tensile forces of this double intensity on the stress conditions existing in the case shown in Fig. 8*a*. It is seen from Fig. 8 that if a is small, that is, if the vertical section is close to the point of application of the load, then the σ_x stresses used for producing the free boundary at that place will be of considerable magnitude. Their maximum will be right under the surface so that they will be apt to increase substantially the tensile stresses in the fillet which is assumed to be not far away on the other side of the point of support shown in Fig. 8*a*. As a physical picture, the action of the σ_x tensile stresses can be also conceived as "shearing off" the flanges at an oblique angle sloping outward, which is easy to realize in the case when the points of support are close to the edges of the head.

The last factor, the bending of the flanges, was investigated in a series of tests, starting for every D/d value with a large depth of the head ($h = 4.20d$) and successively reducing the depth h, increasing thereby the bending stresses in the flanges. In these tests the support of the specimens was provided by uniformly distributed reaction forces extending to one fillet-radius distance toward the ends of each flange as shown in Fig. 9. It was necessary to adopt this type of support in order to eliminate one of the variables, the position of the reaction forces, and to reduce thereby the combinations to be investigated. The system of curves shown in Fig. 9 is the result of this test series for the particular value of fillet ratio $R/d = 0.075$. The bottom curve is obtained for T-heads of large depth ($h = 4.20d \rightarrow \infty$), where the bending action of the flanges was negligible, the successive curves above show the rise of k due to increasing bending while the top curve shows that for $h = 0.50d$ ratios the bending stresses are dominant. The heavy lined curves in Fig. 9 were based on test data shown by small circles. The intermediate curves were determined by graphical interpolation from the curves in Fig. 10 where the effect of bending is separated from the data of Fig. 9 and plotted as a function of h/d. Fig. 11 shows the photoelastic picture of one of the samples tested in the previously mentioned series. The proportions of this specimen were $D/d = 2.00$, $h/d = 0.70$, and $R/d = 0.075$; the concentration factor was found to be $k = 6.25$. An enlargement of the isochromatic lines at the right-side fillet of this sample is shown in Fig. 12.

ACKNOWLEDGMENTS

The author wishes to thank C. M. Laffoon and B. A. Rose of the generator engineering division and F. T. Hague and R. P. Kroon of the steam engineering division of the Westinghouse Electric & Manufacturing Co. for their interest and cooperation. He is also indebted to R. E. Peterson of the Westinghouse Research Laboratories for his valuable and helpful suggestions.

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A Two-Load Method of Determining the Average True Stress-Strain Curve in Tension

By C. W. MacGREGOR,[1] CAMBRIDGE, MASS.

The author describes a method, developed at the Massachusetts Institute of Technology, whereby the complete average true stress-strain curve in tension may be determined for a material from the beginning of yielding to fracture under ordinary testing speeds by the observation during the test of only two loads applied to a tapered specimen, namely, the maximum and fracture loads. Diameters at various positions along tapered specimens are measured before and after the test, and stress and reduction-of-area values computed from these observations.

IT WAS suggested recently[2] that instead of plotting stress versus strain in the usual manner for tensile tests of metals, that the average true stress $s = P/A$ be plotted as a function of the true reduction in area $q' = \log (A_o/A)$, where P, A, and A_o are the instantaneous load, the actual area, and the original area, respectively. The advantages of this method of procedure are fully discussed in the references given and will therefore not be repeated here. It may be mentioned, however, that the reduction in area q' thus defined has a better physical basis than the commonly used $q = (A_o - A)/A_o$, and that numerous tests have shown the s-q' curve becomes a straight line from the point corresponding to the maximum load to fracture, which simplifies the recording and interpretation of data. Further, assuming the volume remains constant during plastic deformation (which is true for many metals strained in this fashion to within about 1 per cent), the q' is the same as the true axial strain $\epsilon = \log (L/L_o)$, where L_o is a small original gage length and L is the new distance between gage points produced by strain. Hence, a further advantage of measuring the diameter and computing the true reduction in area q' is obvious since, during necking down, the average true strain in the axial direction may be determined easily in this manner, whereas by ordinary axial-strain measuring methods it would be difficult.

The Two-Load Method[3]

If a tensile-test piece of mild steel having a uniform diameter is stressed in tension, measurements of the diameter along the length of the bar during the test have shown that after the lower yield point is passed, the diameters decrease practically uniformly along the bar until the maximum load is reached. As the load starts to drop from this maximum value, a local necking develops until the bar finally fractures. The diameters of a certain uniform portion of the bar not immediately adjacent to the necked portion change only elastically as the load drops and, as this change is very small compared to the plastic deformation, the true stress corresponding to the maximum load could have been obtained by measuring this uniform diameter after the test and dividing the area of cross section obtained into the maximum load. In other words, with the exception of the necked-down region, the plastic deformation in the form of a reduced area on the fractured bar is produced by the maximum load acting and no further appreciable change in this is produced after the maximum load is applied.

It was then conceived that if a tapered bar were fractured in the same manner, the maximum load acting during the test would produce, with the exception of a small elastic change as the load drops off, the final reduction in area of each diameter of the tapered bar, excluding the necked-down region as before. It would then be possible to determine the s-q' curve (that is, the true stress versus reduction-in-area curve) or the true stress-strain curve (s-ϵ) for the region from the lower yield point to the maximum load point by merely noting the maximum load during the test and measuring the changes in diameter for a number of sections along the tapered bar after the test piece was fractured. This could be done by scribing a uniform spiral along the bar with a diamond tool as will be discussed later, measuring corresponding diameters before and after test, computing the true stress s from the final diameters and the maximum load, and $q' = \epsilon$ from the relation $\log (A_o/A)$.

For the region of the curve from the maximum-load point to fracture, that is, during necking down, the only point for which the diameter and the corresponding load can easily be determined from measurements after fracture is the last or fracture point. Since, however, it is known from previous tests[2] that the s-q' curve is a straight line from the maximum-load point to fracture, the previously mentioned method may be extended to the determination of the complete curve from the lower yield point to fracture by merely noting one extra load, namely, the fracture load. One difficulty which presents itself immediately, however, is that for a tapered specimen it is not clear which diameter corresponds to the maximum load. It was found that this may be overcome by plotting an s-q' curve from first yielding to fracture, where $s = P_{max}/A$, and A is determined from the various diameters along the bar after fracture. This curve possesses an inflection point at the position on the curve corresponding to the maximum load. Therefore, to obtain the remainder of the actual s-q' curve from the maximum-load point to fracture, it merely entails the drawing of a straight line from the point at fracture determined by the fracture load and area to the inflection point of the (P_{max}/A)-q' curve obtained previously

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[1] Associate Professor of Applied Mechanics, Department of Mechanical Engineering, Massachusetts Institute of Technology. Jun. A.S.M.E.

[2] "Relations Between Stress and Reduction in Area for Tensile Tests of Metals," by C. W. MacGregor, A.I.M.E. Metals Technology, Technical Publication No. 805, April, 1937. Also: "Differential Area Relations in the Plastic State for Uniaxial Stress," Stephen Timoshenko 60th Anniversary Volume, The Macmillan Company, New York, N. Y., 1939.

[3] The tapered test specimen was suggested by Prof. A. V. de Forest and the work was carried through with the assistance of P. R. Shepler.

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Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y., and will be accepted until January 10, 1940, for publication at a later date. Discussion received after the closing date will be returned.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

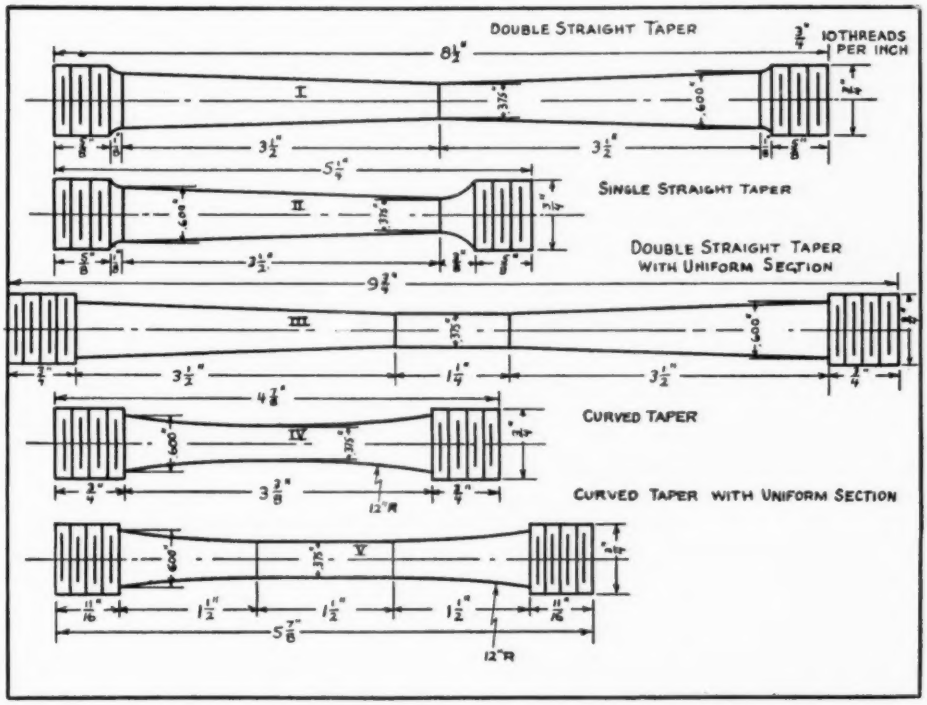


FIG. 1 STRAIGHT AND CURVED TAPERED SPECIMENS USED IN THE TENSILE TESTS

This latter point is not difficult to determine since the actual s-q' curve for a uniform specimen from the beginning of yielding to the maximum-load point will become tangent to the straight line connecting the maximum-load and fracture points.

An experimental program which was carried through to investigate the accuracy of this method will now be described.

EXPERIMENTAL RESULTS

A series of tapered tensile specimens (at least two of each) of types I, II, III, IV, and V shown in Fig. 1 were prepared from fully annealed S.A.E. 1112 steel. These were supplemented by a number of test pieces of the uniform diameters of 0.375 in. and 0.505 in., respectively. After machining, the tapered speci-

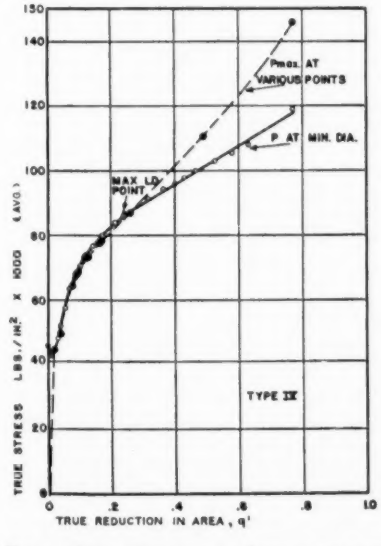


FIG. 2 COMPARISON OF s-q' CURVES DRAWN FROM (1) MEASUREMENTS TAKEN DURING TEST AT MINIMUM-DIAMETER SECTION (SOLID CURVE) AND (2) MAXIMUM LOAD AND FINAL AREAS AT VARIOUS SECTIONS (DASHED CURVE)

(Several specimens of each of the types shown in Fig. 1 were tested. These data are for specimen No. 5C of type IV.)

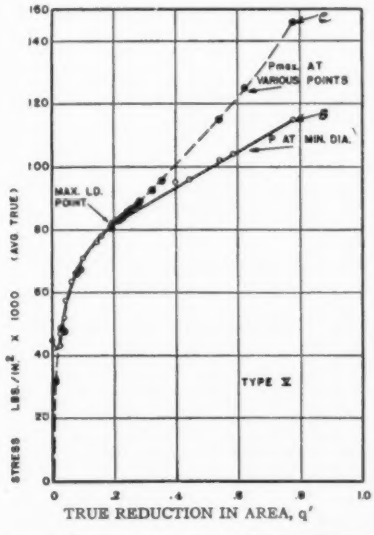


FIG. 3 COMPARISON OF s-q' CURVES DRAWN FROM (1) MEASUREMENTS TAKEN DURING TEST AT MINIMUM-DIAMETER SECTION (SOLID CURVE) AND (2) MAXIMUM LOAD AND FINAL AREAS AT VARIOUS SECTIONS (DASHED CURVE)

(These data are for specimen No. 6C of type V.)

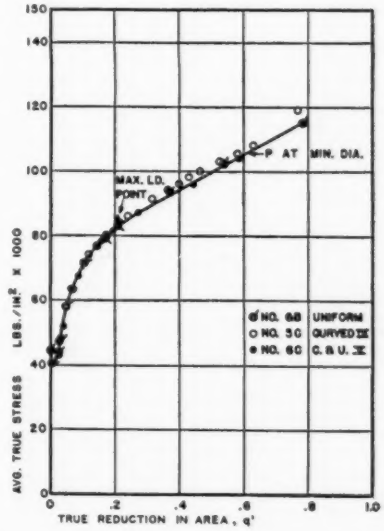


FIG. 4 COMPARISON OF s-q' CURVES OF FIGS. 2 AND 3, DETERMINED BY MEASUREMENTS AT THE MINIMUM DIAMETERS FOR TAPERED BARS, WITH THAT DETERMINED ON SPECIMEN NO. 6B, OF UNIFORM SECTION

mens were polished axially with No. 000 emery paper to remove all toolmarks and to produce scratches in the longitudinal direction only. They were then ruled in a lathe with a continuous spiral scratch produced by a diamond, the axial pitch of the spiral being 0.10 in. These fine rulings were clearly visible against the background of axial scratches produced by the No. 000 emery paper.

The tensile tests were carried out on a Baldwin-Southwark 30-ton hydraulic testing machine. Instead of measuring the axial strain, the reduction in area was determined during the test at the smallest diameter of the tapered specimens by means of a specially designed dial gage and clamp. Corresponding readings of load and minimum diameter were taken to fracture in order to compare these results with those received from the uniform bars.

Referring to Figs. 2 and 3, the solid curves following the points indicated by open circular rings represent the s-q' curves to fracture received from such measurements during the test as described previously on the tapered specimens of types IV and V, respectively. The dashed curves passing through the solid circles are the (P_{max}/A)-q' curve obtained from the maximum load and measurements of final areas along the axes of the tapered specimens after fracture. It will be noted that for type V the s-q' curves for both procedures practically coincide from the lower yield point to the maximum-load point after which they diverge. From Fig. 2 it will be seen that in the case of curved specimen of type IV, the material on either side of the section of minimum diameter produced a slight reinforcing effect. The curves, however, are rather close. It was found that types I and II had an appreciable reinforcing effect and consequently do not give results which compare favorably with the uniform bars. The results of these tests have therefore not been included in this discussion. Type III is difficult to machine and did not give as favorable results as type V. Fig. 4 shows a comparison of the s-q' curves of Figs. 2 and 3, determined by measurements at the minimum diameters for the tapered bars, with that determined on a uniform specimen. The agreement is quite close. It is believed that the specimens of type V are the best suited for this purpose.

Therefore, it will be seen by referring to Fig. 3 that the complete s-q' curve could have been constructed from measurements of the tapered specimen after fracture with the observation of the maximum and fracture loads only. According to this procedure, the dashed curve would first be constructed from measurements of the areas along the specimen after fracture, dividing these final areas into P_{max}, and plotting the curve $0C$ through the solid points so obtained. Point B is also known from the diameter at fracture and the fracture load. From point B then a line would be drawn to the inflection point of $0C$. This line would then represent this portion of the s-q' curve as checked by the measurements shown (the open rings of Fig. 3) on the section of minimum diameter. The only reasons for taking measurements of corresponding load and diameter at the minimum section during the test were to check the reinforcing effect, if any, and to check the described method of construction.

Application of the Method

From the point of view of time required to determine an s-q' or an s-ϵ curve for a given material, at normal temperatures and at ordinary testing speeds, there is little to recommend the two-load method using a tapered specimen as compared to a uniform test piece. More time is consumed in the preparation of a tapered test specimen than for a uniform test bar. If time spent were counted from the beginning of the test, the difference in time required by both methods would not be quite so marked, but probably still in favor of the uniform test bar.

There are, however, cases arising in the testing of materials where it is not practical or feasible to measure the change in diameter of the test bar during the test. Such examples as the determination of s-q' curves at temperatures above room temperature but not high enough to permit appreciable scaling, or the s-q' curve for impact at high velocities, may be mentioned. It is hoped that this method will prove useful in the future when applied to these problems.

Rubber Springs—Shear Loading

By J. F. DOWNIE SMITH,[1] PHILADELPHIA, PA.

The designer in industry should be in a position to predetermine by calculation the size and hardness of, and the deflection to be expected from, a rubber spring which is to serve a special purpose. Unfortunately the literature contains little which would be of help; and it is the author's intention in this paper to supply such information for rubber springs loaded in shear. Most of the designs ordinarily encountered in practice have been considered.

The agreement of theory and experiment is shown in all cases where data are available.

A chart is given which enables anyone to determine immediately the shear angle expected from a plane shear mounting of any size and rubber hardness within the limits ordinarily used for springs.

THE GROWING importance of rubber as a spring material warrants an investigation of the means available for calculating the load-deflection curves for rubber utilized in several possible ways. This present paper is restricted to shear loading of rubber.

The theory involved is developed and the agreement of theory and experimental data is shown graphically for each case studied, where such data are available.

Case 1. Plain Sandwiches

In Fig. 1 the two most common shear rubber springs are pictured. The theory involved is simple. Consider Fig. 1*a*. The modulus of elasticity in shear = G = stress/strain; and stress = W/A.

The definition of strain is set up in engineering literature for small angles either as (*a*) the tangent of the angle, or (*b*) the angle in radians. In the numerous samples tested by the author, and in those cases noted in published experiments, the latter definition gives the better agreement between theory and practice. For that reason it has been adopted in this paper.

Therefore the strain (in radians) = W/AG ..... [1]

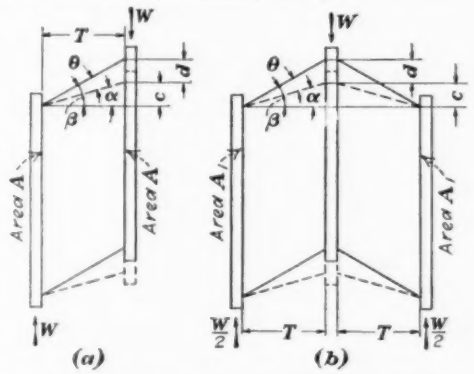


Fig. 1 Two Common Forms of Rubber Shear Springs

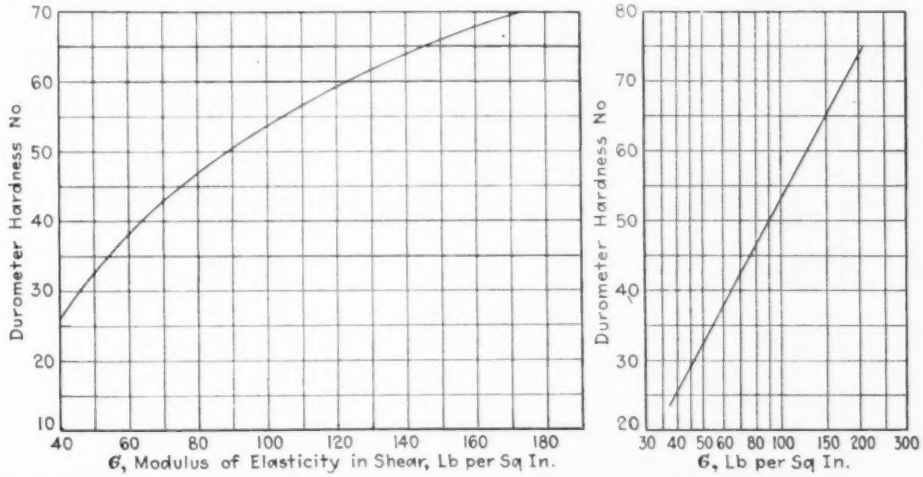


Fig. 2 Relation Between Modulus of Elasticity of Rubber in Shear and Durometer Hardness Number
(The equation of the curve is $G = 16.9e^{D/30.25} = 16.9e^{0.033D}$. The curve should not be extrapolated.)

The strain (in degrees) = $57.3\ W/AG$ ..... [2]

It is then a simple matter to find the deflection d when the original dimensions of the rubber are known. Thus

$$\tan(\theta) = \tan(\beta - \alpha) = (\tan\beta - \tan\alpha)/(1 + \tan\alpha\tan\beta)$$
$$= (d/T)/\{1 + (c/T)[(c + d)/T]\}$$

If θ is in degrees, then

$$\tan(57.3\ W/AG) = (d/T)/\{1 + (c/T)[(c + d)/T]\} \quad \ldots\ldots [3]$$

Ordinarily c/T is small and $(c + d)/T$ is in the order of magnitude of 1 or less. Thus a first degree of approximation can be obtained from

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Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y., and will be accepted until January 10, 1940, for publication at a later date. Discussion received after the closing date will be returned.

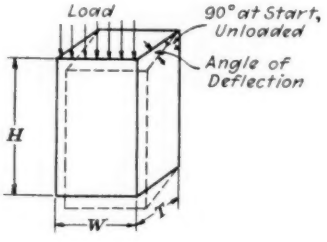
Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

$$d = T \tan (57.3\ W/AG) \quad [4]$$

If in Fig. 1, $2A_1 = A$, Equations [1] to [4] apply also to the spring of Fig. 1*b*.

From those equations it is possible to calculate the strain or the deflection provided that the physical dimensions of the rubber, the load, and the modulus of elasticity of the rubber in shear are known.

It is common practice to specify the hardness of rubber in terms of an arbitrary scale on a commercially available instrument known as a durometer. It is also possible to correlate the durometer hardness number with the modulus of elasticity in shear, as shown in Fig. 2. The agreement of theoretically derived values and experimental data justifies the conclusion that the durometer hardness number and the shear modulus of elasticity are related in the manner shown to within quite close limits.



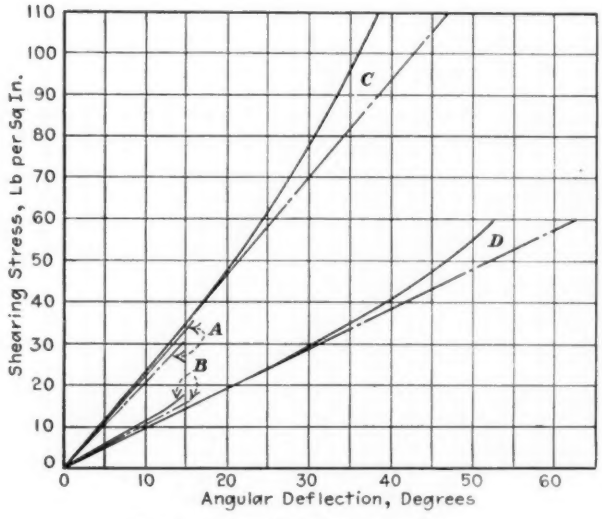


FIG. 3 SINGLE RUBBER SHEAR PADS

Curves	Dimensions, in. W	T	H	Shear area, sq in.	Durometer number
A	7.87	1.77	3.94	31	58
B	7.87	1.77	3.94	31	38
C	3	3	8	24	60
D	1.2	3/4	3	3.6	35

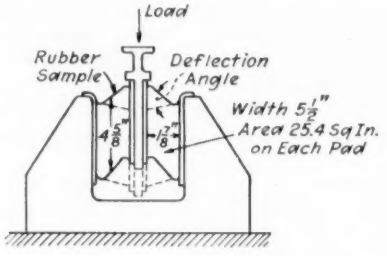
(Solid lines, experimental curves. Dot-and-dash lines, calculated.)

Unfortunately, it is difficult at present to get two durometers picked at random to agree perfectly, so that for each durometer a curve such as is shown in Fig. 2 is necessary for accurate work. On the other hand, rubber can be obtained from a rubber company with a known durometer hardness number and, from the company's own chart, the modulus of elasticity in shear can be obtained. If necessary, this modulus can be referred to the customer's chart and a durometer hardness number on his own scale obtained. It is regrettable that such a procedure should be necessary.

In Figs. 3 and 4 the close agreement of theory and experiment can be observed, for reasonable angles of deflection, with both single and double shear pads. Fig. 5 is presented to give immediately the angular deflection for any given load and rubber hardness.

Frequently it is necessary to design for greater deflection than can be obtained from a single pair of pads. A double pair is shown in Fig. 6 and a quadruple pair in Fig. 7.

Notice the reasonable check with theory even with the quad-



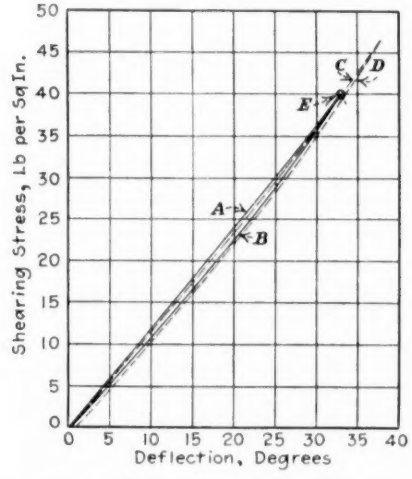


FIG. 4 DOUBLE RUBBER SHEAR PAD

(Room temperature, 83 F; rubber, 43 durometer. *A*, first loading; *B*, first unloading; *C*, second loading; *D*, second unloading; *E*, theoretical point.)

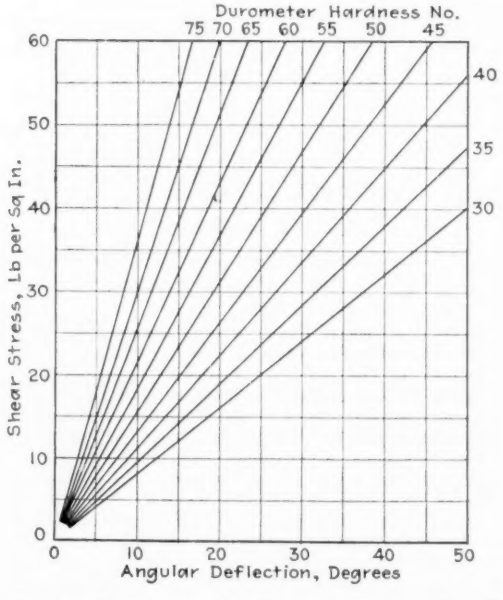


FIG. 5 LOAD-DEFLECTION CURVES FOR RUBBER SHEAR PADS

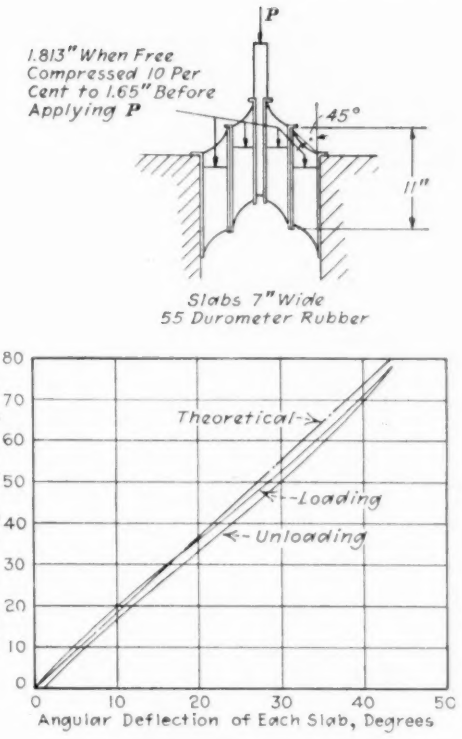


FIG. 6 COMPRESSION-SHEAR LOADING
(The angular deflections are based on the free thickness of the rubber.)

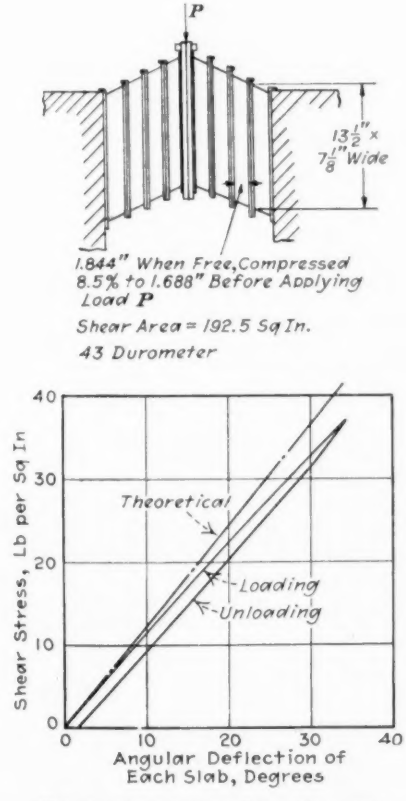


FIG. 7 COMPRESSION-SHEAR LOADING
(The angular deflections are based on the free thickness of the rubber.)

ruple pair, where the metal plates separating the rubber slabs were nowhere near parallel under full load. It would be expected that the greater the number of pads the greater would be the departure from the simple theory outlined. Even with this quadruple pair, however, the error was small.

In the design of such sandwiches it is essential to keep the height large compared with the thickness of the group for the sake of stability. An example of instability due to too small a height-thickness ratio is shown in Fig. 8.

It is customary to supply lateral compression to rubber slabs which are to be loaded in shear. Automatically one would inquire the effect of the compression on the loading curve. Fig. 9 shows that the effect is slight, so that if the angle of deflection is to be calculated the free thickness of the rubber should be used.

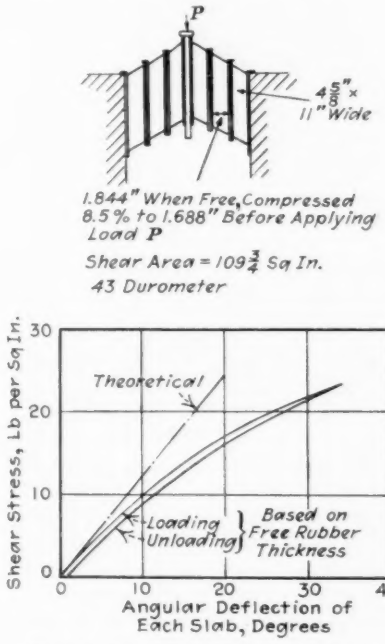


FIG. 8 COMPRESSION-SHEAR LOADING
(Showing instability due to too small a height-thickness ratio. Angular deflections are based on the free thickness of the rubber.)

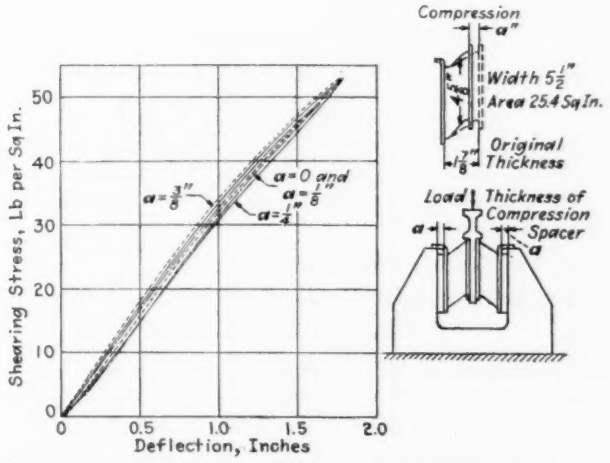


FIG. 9 EFFECT OF DIFFERENT AMOUNTS OF COMPRESSION ON THE LOADING CURVES
(Vertical shearing stress with different amounts of horizontal compression. Room temperature, 75 F; 43 durometer rubber.)

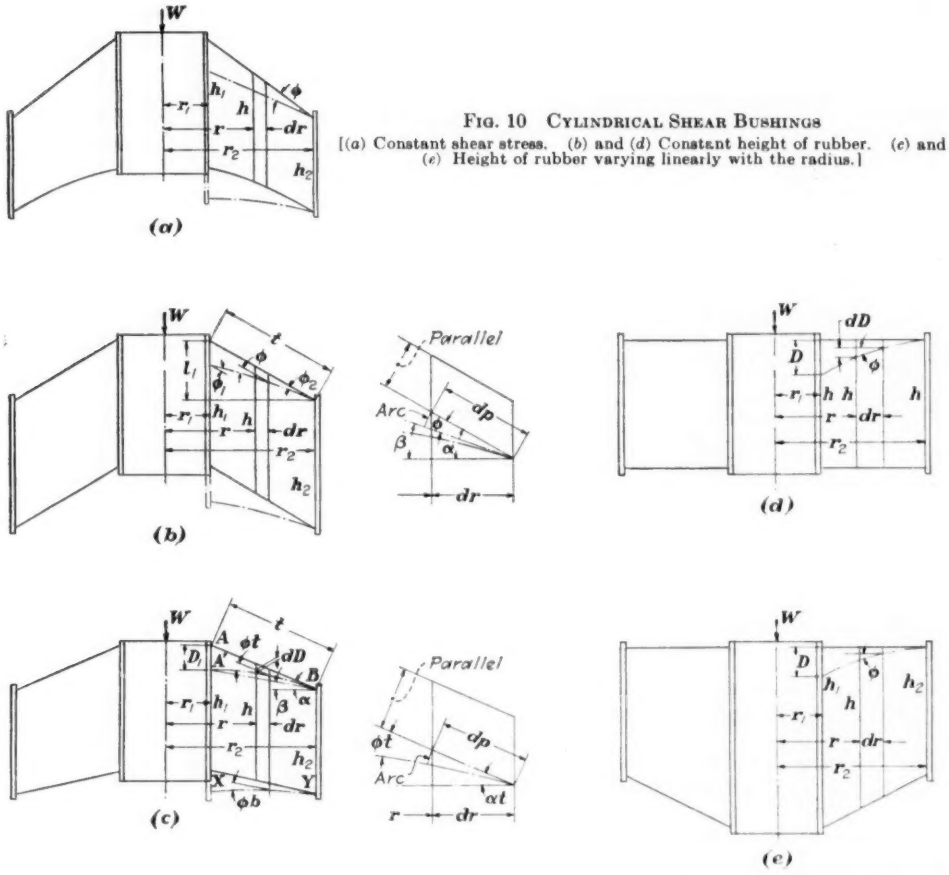


FIG. 10 CYLINDRICAL SHEAR BUSHINGS
[(*a*) Constant shear stress. (*b*) and (*d*) Constant height of rubber. (*c*) and (*e*) Height of rubber varying linearly with the radius.]

The test pieces described so far have all been rectangular. Would the same load-deflection curve be obtained for the same area and rubber hardness if another shape were used? Several different shapes have been tested experimentally and, so long as the thickness of the rubber is appreciably less than the shortest perpendicular length, no great departure from the preceding equations has been found.

CASE 2. CYLINDRICAL SHEAR BUSHINGS

(*a*) ***With Constant Shear Stress, Fig. 10a***

With constant stress, $S/G = \phi = \text{const}$ ..... [5]

$$S = W/2\pi rh = W/2\pi r_1 h_1 = W/2\pi r_2 h_2 \text{ .....} [6]$$

Or, for any load and stress, $rh = \text{const}$ ..... [7]

which is the equation of a hyperbola. Therefore the height should vary inversely as the radius, or $rh = r_1h_1 = r_2h_2$.

The deflection can be obtained from angle ϕ and the physical dimensions of the bushing.

(*b*) ***With Constant Height of Rubber, Fig. 10b***

Here, $h = h_1 = h_2$ and $S = G\phi$. If $W/2\pi h$ is represented by a, then $S_r = a/r$, $S_1 = a/r_1$, $S_2 = a/r_2$.

Therefore the stress varies inversely as the radius.
Now $\phi = S/G = a/rG = b/r$, where $b = a/G$. But $\phi = \text{arc}/dp$ and $dr/dp = \cos\alpha = (r_2 - r_1)/t = g$.

$$\therefore \phi = \text{arc}/dp = \text{arc}(g/dr) \text{ and arc} = (\phi/g)dr = (b/gr)dr$$

$$\therefore \Sigma \text{ arc} = \frac{b}{g}\int_{r_1}^{r_2}\frac{dr}{r} = \frac{b}{g}\log_e\left(\frac{r_2}{r_1}\right)$$

$$= \frac{W}{2\pi hgG}\log_e\left(\frac{r_2}{r_1}\right) = \frac{Wt}{2\pi hG}\frac{\log_e(r_2/r_1)}{(r_2 - r_1)} \text{ .....} [8]$$

Let the angle of movement be θ, then Σ arc $= \theta t$ approximately, and

$$\theta = \frac{W}{2\pi hG(r_2 - r_1)}\log_e\left(\frac{r_2}{r_1}\right) \text{ .....} [9]$$

Another method of solving this problem is to assume that the rubber of the original design of Fig. 10*b* would behave similarly to that shown in Fig. 10*d*.

In this case, $\phi = W/2\pi rhG = b/r$ as before. Roughly, $\tan\phi = dD/dr = \tan(b/r)$.

$$\therefore D = \int_{r_1}^{r_2}\tan\left(\frac{b}{r}\right)dr$$

Now b/r is always less than $\pi/2$.

$$\therefore D = \int_{r_1}^{r_2}\left[\left(\frac{b}{r}\right) + \frac{1}{3}\left(\frac{b}{r}\right)^3 + \frac{2}{15}\left(\frac{b}{r}\right)^5 + \frac{17}{315}\left(\frac{b}{r}\right)^7 + \dots\right]dr$$

$$= b\log_e\left(\frac{r_2}{r_1}\right) - \frac{b^3}{6}\left(\frac{1}{r_2^2} - \frac{1}{r_1^2}\right) - \frac{b^5}{30}\left(\frac{1}{r_2^4} - \frac{1}{r_1^4}\right) - \frac{17b^7}{1890}\left(\frac{1}{r_2^6} - \frac{1}{r_1^6}\right) + \dots$$

$$= b\log_e\left(\frac{r_2}{r_1}\right) + \frac{b^3}{6}\left(\frac{1}{r_1^2} - \frac{1}{r_2^2}\right) + \frac{b^5}{30}\left(\frac{1}{r_1^4} - \frac{1}{r_2^4}\right) + \frac{17b^7}{1890}\left(\frac{1}{r_1^6} - \frac{1}{r_2^6}\right) + \dots$$

$$= \frac{W}{2\pi hG}\log_e\left(\frac{r_2}{r_1}\right) + \frac{1}{6}\left(\frac{W}{2\pi hG}\right)^3\left(\frac{1}{r_1^2} - \frac{1}{r_2^2}\right) + \frac{1}{30}\left(\frac{W}{2\pi hG}\right)^5\left(\frac{1}{r_1^4} - \frac{1}{r_2^4}\right) + \frac{17}{1890}\left(\frac{W}{2\pi hG}\right)^7\left(\frac{1}{r_1^6} - \frac{1}{r_2^6}\right) + \ldots\ldots\ldots [10]$$

(c) With Height of Rubber Varying Linearly With the Radius, Fig. 10c

Here $(h - h_2)/(h_1 - h_2) = (r_2 - r)/(r_2 - r_1)$

$$\therefore h = h_2 + [(h_1 - h_2)/(r_2 - r_1)](r_2 - r) \ldots\ldots [11]$$

Let $a = h_2 + [(h_1 - h_2)/(r_2 - r_1)]r_2$ and $b = (h_1 - h_2)/(r_2 - r_1)$, then $h = a - br$ and

$$\text{strain} = \frac{\text{stress}}{G} = \frac{W}{2\pi rhG}\begin{cases} = \phi_t \text{ at top} \\ = \phi_b \text{ at bottom} \end{cases}$$

$\phi_t = \dfrac{W}{2\pi rG(a - br)} = \dfrac{k}{ar - br^2}$, where $k = \dfrac{W}{2\pi G}$. Now $\phi_t = \text{arc}/dp$, and $dr = dp\cos\alpha_t = dp(r_2 - r_1)/t$

$$\therefore \phi_t = \frac{\cos\alpha_t \text{ arc}}{dr} = \frac{(r_2 - r_1)\text{ arc}}{dr\ t} \ldots\ldots [12]$$

$$\therefore \text{arc} = \frac{\phi_t\ t\ dr}{(r_2 - r_1)} = \frac{t\ k\ dr}{(r_2 - r_1)\ (ar - br^2)} \ldots\ldots [13]$$

$$\therefore \Sigma\text{ arc} = \frac{t\ k}{(r_2 - r_1)}\int_{r_1}^{r_2}\frac{dr}{ar - br^2}$$

$$\frac{r_2 - r_1}{tk}\Sigma\text{ arc} = \int_{r_1}^{r_2}\frac{dr}{ar - br^2} = \frac{1}{a}\log_e\left|\frac{-2br + a - a}{-2br + a + a}\right|_{r_1}^{r_2}$$

$$= \frac{1}{a}\log_e\left(\frac{br}{br - a}\right)\Bigg|_{r_1}^{r_2}$$

$$= \frac{1}{a}\log_e\frac{r_2}{r_1}\left(\frac{br_1 - a}{br_2 - a}\right) = \frac{1}{a}\log_e\frac{r_2h_1}{r_1h_2}$$

$$\therefore \Sigma\text{ arc} = \frac{tk}{a(r_2 - r_1)}\log_e\frac{r_2h_1}{r_1h_2}$$

$$= \frac{tW\log_e(r_2h_1/r_1h_2)}{2\pi G(r_2 - r_1)\left[h_2 + \left(\frac{h_1 - h_2}{r_2 - r_1}\right)r_2\right]} = \frac{tW\log_e(r_2h_1/r_1h_2)}{2\pi G(h_1r_2 - h_2r_1)} \ldots [14]$$

and $\Sigma\text{ arc} = t\theta$ approximately, where θ = total angle of movement.

$$\therefore \theta = \frac{W}{2\pi G(h_1r_2 - h_2r_1)}\log_e\frac{r_2h_1}{r_1h_2} \ldots\ldots [15]$$

This same problem can be handled in a different manner if it is assumed that the top surface of the rubber is a plane surface as shown in Fig. 10*e*, and that rubber of this shape will behave similarly to that shown in Fig. 10c. Here

$\phi = W/2\pi rhG = k/rh$, where $k = W/2\pi G$. Now $h = a - br$,

where $a = \dfrac{h_1r_2 - h_2r_1}{r_2 - r_1}$ and $b = \dfrac{h_1 - h_2}{r_2 - r_1}$

$$dD/dr = \tan\phi = \tan[k/(ar - br^2)]$$

$$\therefore D = \int_{r_1}^{r_2}\tan\left(\frac{k}{ar - br^2}\right)dr$$

Now ϕ is always less than $\pi/2$.

$$\therefore D = \int_{r_1}^{r_2}\left[\frac{k}{ar - br^2} + \frac{1}{3}\left(\frac{k}{ar - br^2}\right)^3 + \frac{2}{15}\left(\frac{k}{ar - br^2}\right)^5 + \frac{17}{315}\left(\frac{k}{ar - br^2}\right)^7 + \ldots\right]dr$$

The integration of this expression is tedious, and the results are complicated. The first two terms alone yield

$$D = \frac{W}{2\pi G}\frac{(r_2 - r_1)}{(h_1r_2 - h_2r_1)}\log_e\frac{h_1r_2}{h_2r_1} \cdot + 2\left(\frac{W}{2\pi G}\right)^3\frac{(h_1 - h_2)^2(r_2 - r_1)^4}{(h_1r_2 - h_2r_1)^5}\log_e\frac{h_1r_2}{h_2r_1} + \frac{1}{6}\left(\frac{W}{2\pi G}\right)^3\frac{(r_2 - r_1)}{(h_1r_2 - h_2r_1)^2}\left[\frac{h_1r_2 - 2r_1h_1 + r_1h_2}{(h_1r_1)^2} + \frac{h_1r_2 - 2h_2r_2 + h_2r_1}{(h_2r_2)^2}\right] + \left(\frac{W}{2\pi G}\right)^5\frac{(h_1 - h_2)(r_2 - r_1)^2}{(h_1r_2 - h_2r_1)^4}\left[\frac{h_1r_2 - 2r_1h_1 + r_1h_2}{h_1r_1} + \frac{h_1r_2 - 2h_2r_2 + h_2r_1}{h_2r_2}\right] \ldots\ldots [16]$$

For a given set of dimensions, however, this expression reduces to

$$D = c_1(W/G) + c_2(W/G)^3 \ldots\ldots [17]$$

which is quite simply used.

(d) With Height of Rubber Varying Linearly With the Radius, Angles α and β Small, Fig. 10c

Assume AB and XY are straight lines at the start. It is desired to obtain

$$\int_{r = r_1}^{r = r_2} dD$$

$$h = b + ar = h_1 + [(h_2 - h_1)(r - r_1)/(r_2 - r_1)]$$

Let $a = (h_2 - h_1)/(r_2 - r_1)$ and $b = h_1 - [r_1(h_2 - h_1)/(r_2 - r_1)]$

$$\phi = \text{Strain} = W/2\pi rhG = W/[2\pi Gr(ar + b)]$$

Let $W/2\pi G = k$, then $\phi = k/(ar^2 + br)$. Now if β is small and α not too large, $dD = dr\phi$

$$\therefore dD = (kdr)/(ar^2 + br) \ldots\ldots [18]$$

$$\therefore D = k\int_{r_1}^{r_2}\frac{dr}{ar^2 + br} = \frac{k}{b}\log_e\left|\frac{2ar + b - b}{2ar + b + b}\right|_{r_1}^{r_2}$$

$$= \frac{k}{b}\log_e\left[\frac{r_2}{r_1}\left(\frac{ar_1 + b}{ar_2 + b}\right)\right] = \frac{k}{b}\log_e\frac{r_2h_1}{r_1h_2}$$

$$\therefore D = \frac{W}{2\pi G\{h_1 - [(h_2 - h_1)/(r_2 - r_1)]r_1\}}\log_e\frac{r_2h_1}{r_1h_2}$$

$$= \frac{W(r_2 - r_1)}{2\pi G(h_1r_2 - h_2r_1)}\log_e\frac{r_2h_1}{r_1h_2} \ldots\ldots [19]$$

Equation [19] is the one proposed by F. L. Haushalter, mentioned by the author in a previous paper.[2]

Fig. 11 shows agreement of theory and practice for a typical bushing.

[2] "Rubber Mountings," by J. F. Downie Smith, *Journal of Applied Mechanics*, Trans. A.S.M.E., vol. 60, March, 1938, p. A-22.

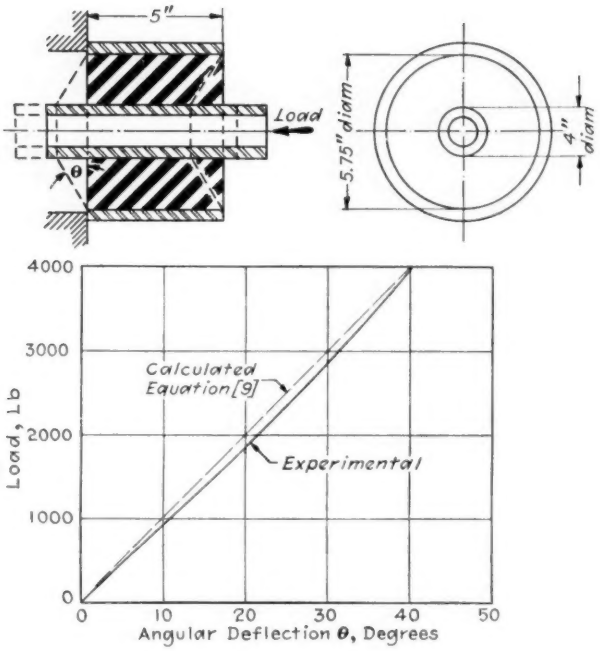


FIG. 11 LOADING CURVES OF CYLINDRICAL SHEAR BUSHING
(Durometer reading, 45.)

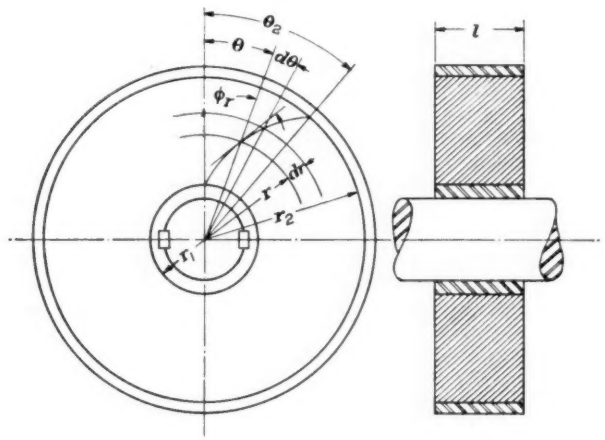


FIG. 12 COAXIAL TUBE OF CONSTANT LENGTH IN TORSION

CASE 3. COAXIAL TUBE OF CONSTANT LENGTH IN TORSION, FIG. 12

$$\text{Torque} = T = 2\pi r^2 lS \quad \ldots\ldots\ldots\ldots\ldots\ldots [20]$$

$$\text{Also, approximately, } r d\theta/dr = \tan \phi_r \quad \ldots\ldots\ldots [21]$$

and $\phi_r = S/G$. $\therefore T = 2\pi l r^2 G \phi_r$

$$\therefore \phi_r r^2 = T/2\pi lG \text{ and } \phi_r = T/2\pi lGr^2 \ldots [22]\ [23]$$

From Equations [21] and [23] $r d\theta/dr = \tan(T/2\pi lGr^2)$

$$\therefore d\theta = (1/r) \tan (T/2\pi lGr^2) dr$$

For a given torque on a given sample, $T/2\pi lG = \text{const} = a$.

$$\therefore \int_0^{\theta_2} d\theta = \int_{r_1}^{r_2} \frac{1}{r} \tan \frac{a}{r^2} dr = \theta_2 \quad \ldots\ldots [24]$$

Let $a/r^2 = t$, or $r = \sqrt{(a/t)}$, then

$$dt = -2ar^{-3}\, dr \text{ and } dr = -a^{1/2} dt/2t^{3/2}$$

$$\therefore \theta_2 = -\int_{r=r_1}^{r=r_2} \left(\frac{t}{a}\right)^{1/2} \tan t \frac{a^{1/2} dt}{2t^{3/2}} = -1/2 \int_{r=r_1}^{r=r_2} \frac{\tan t}{t}\, dt$$

Where $t^2 < \pi^2/4$ the solution of this equation is

$$\theta_2 = -\frac{1}{2}\left[t + \frac{t^3}{9} + \frac{2}{75} t^5 + \frac{17}{2205} t^7 + \ldots \right]_{r=r_1}^{r=r_2}$$

See Dwight's Table of Integrals, No. 481.2

$$\therefore \theta_2 = -\frac{1}{2}\left[\frac{a}{r^2} + \frac{a^3}{9r^6} + \frac{2a^5}{75r^{10}} + \frac{17a^7}{2205r^{14}} + \ldots \right]_{r_1}^{r_2}$$

$$= -\frac{a}{2}\left[\left(\frac{1}{r_2^2} - \frac{1}{r_1^2}\right) + \frac{a^2}{9}\left(\frac{1}{r_2^6} - \frac{1}{r_1^6}\right) + \frac{2a^4}{75}\left(\frac{1}{r_2^{10}} - \frac{1}{r_1^{10}}\right) + \frac{17a^6}{2205}\left(\frac{1}{r_2^{14}} - \frac{1}{r_1^{14}}\right) \ldots \right]$$

$$\therefore \theta_2 = \frac{T}{4\pi lG}\left[\left(\frac{1}{r_1^2} - \frac{1}{r_2^2}\right) + \frac{1}{9}\left(\frac{T}{2\pi lG}\right)^2 \left(\frac{1}{r_1^6} - \frac{1}{r_2^6}\right) + \ldots \right] \quad \ldots\ldots\ldots [25]$$

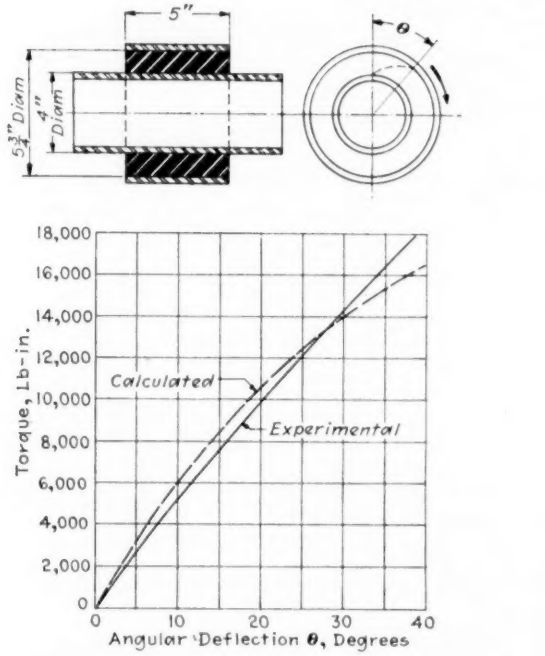


FIG. 13 TORSIONAL LOADING OF COAXIAL TUBE

The solution given in Equation [25] is based on the assumption that $a^2/r^4 < \pi^2/4$, i.e., that $a/r^2 < \pi/2$, that $T/2\pi lGr^2 < \pi/2$, or $\phi_r < \pi/2$ which is ordinarily the case. Therefore the solution is valid.

Fig. 13 shows a typical bushing with both theoretical and experimental curves.

CASE 4. COAXIAL TUBE OF CONSTANT STRESS IN TORSION, FIG. 14

(a) *For Small Deflections*

$$T = (2\pi r \times 2l)Sr = \text{const} = 4\pi r^2 lS \quad \ldots\ldots\ldots [26]$$

$$\therefore r^2 l = T/4\pi S = \text{const} \quad \ldots\ldots\ldots\ldots\ldots [27]$$

which is the equation of a hyperbola.
The stress can be obtained from

$$S = T/4\pi r^2 l = T/4\pi r_1^2 l_1 = T/4\pi r_2^2 l_2 \quad \ldots\ldots [28]$$

For small deflections, $rd\theta/dr = \phi_r$ $\ldots\ldots [29]$

$\phi_r = S/G$

$$\therefore \theta_2 = \int_0^{\theta_2} d\theta = \int_{r_1}^{r_2} \frac{\phi_r}{r}\, dr = \phi_r \log_e \frac{r_2}{r_1} \quad \ldots\ldots [30]$$

$$\therefore \theta_2 \text{ (in radians)} = \frac{S}{G} \log_e \frac{r_2}{r_1} = \frac{T}{4\pi r_2^2 l_2 G} \log_e \frac{r_2}{r_1} \quad \ldots\ldots [31]$$

$$= \frac{2.3026T}{4\pi r_2^2 l_2 G} \log_{10} \frac{r_2}{r_1} = \frac{0.1833T}{r_2^2 l_2 G} \log_{10} \frac{r_2}{r_1} \quad \ldots\ldots [32]$$

$$\theta_2 \text{ (in deg)} = \frac{57.3 \times 0.1833T}{r_2^2 l_2 G} \log_{10} \frac{r_2}{r_1} = \frac{10.5T}{r_2^2 l_2 G} \log_{10} \frac{r_2}{r_1} \quad \ldots [33]$$

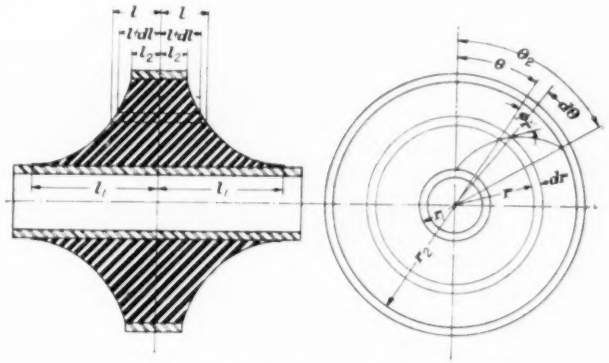


FIG. 14 COAXIAL TUBE OF CONSTANT STRESS

(b) *For Large Deflections, Fig. 14*

$$rd\theta/dr = \tan \phi_r \text{ approximately} \quad \ldots\ldots [34]$$

ϕ_r = strain = S/G = const, $\therefore \tan \phi_r$ = const = a, $\therefore rd\theta/dr = a$

$$\theta_2 = \int_0^{\theta_2} d\theta = \int_{r=r_1}^{r=r_2} \frac{a}{r}\, dr = a \log_e \frac{r_2}{r_1} = \tan \phi_r \times \log_e \frac{r_2}{r_1}$$

$$= \tan S/G \log_e r_2/r_1 \quad \ldots\ldots [35]$$

$$\theta_2 \text{ (in radians)} = \log_e \left(\frac{r_2}{r_1}\right) \tan \left(\frac{T}{4\pi G r_2^2 l_2}\right) \quad \ldots\ldots [36]$$

$$= 2.3026 \log_{10} \left(\frac{r_2}{r_1}\right) \tan \left(\frac{T}{4\pi G r_2^2 l_2}\right) \quad \ldots\ldots [37]$$

$$\theta_2 \text{ (in deg)} = 132 \log_{10} \left(\frac{r_2}{r_1}\right) \tan \left(\frac{T}{4\pi G r_2^2 l_2}\right) \quad \ldots\ldots [38]$$

CASE 5. CIRCULAR DOUGHNUT SANDWICH USED IN ANGULAR SHEAR, FIG. 15

The metal plates are vulcanized to the rubber. Apply torque T to the plates.

(a) *For Small Deflections*

If deflections are small, $\theta l \backsimeq \phi r$ $\ldots\ldots [39]$

Also, $S = G\theta$ and the torque given to the ring at radius $r = dT$, where $dT = (2\pi r dr)rS$

$$\therefore dT = 2\pi r^2 G\theta dr \quad \ldots\ldots [40]$$

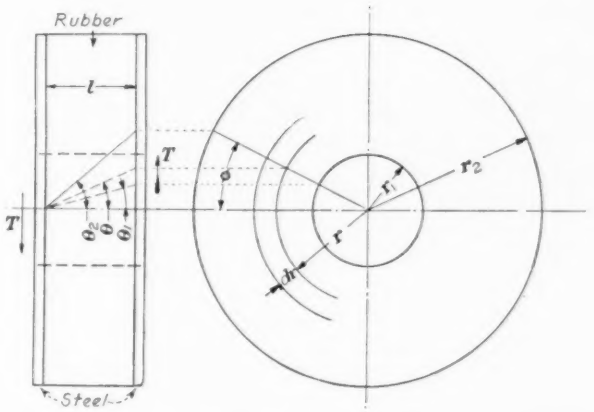


FIG. 15 CIRCULAR DOUGHNUT SANDWICH IN ANGULAR SHEAR

$$\therefore T = 2\pi G \int_{r_1}^{r_2} r^2\theta dr = \frac{2\pi G\phi}{l} \int_{r_1}^{r_2} r^3 dr = \frac{2\pi G\phi}{4l} (r_2^4 - r_1^4)$$

$$= 1.5708 \frac{G\phi}{l} (r_2^4 - r_1^4) \ldots [41]$$

Thus, if ϕ is in radians, $T = 1.5708G\phi(r_2^4 - r_1^4)/l$ $\ldots\ldots [42]$

and, if ϕ is in degrees, $T = 0.0274G\phi(r_2^4 - r_1^4)/l$ $\ldots\ldots [43]$

(b) *For Large Deflections*

In this case $\theta l \neq \phi r$. More correctly, $r\phi/l = \tan \theta$. $S = G\theta$ and $dT = 2\pi r^2 dr S = 2\pi G r^2 \theta dr$.

$$\therefore T = 2\pi G \int_{r_1}^{r_2} r^2\theta dr = 2\pi G \int_{r_1}^{r_2} r^2 \tan^{-1} \frac{r\phi}{l}\, dr \ldots [44]$$

Let $r\phi/l = p$, $\therefore dr = (l/\phi)dp$, and $r^2 = l^2p^2/\phi^2$

$$\therefore T = \frac{2\pi G l^3}{\phi^3} \int_{r=r_1}^{r=r_2} p^2 \tan^{-1} p\, dp$$

$$= \frac{2\pi G l^3}{\phi^3} \left[\frac{p^3}{3} \tan^{-1} p - \frac{p^2}{6} + \frac{1}{6} \log_e (1 + p^2)\right]_{r=r_1}^{r=r_2}$$

$$= \frac{2\pi G l^3}{3\phi^3} \left\{\left(\frac{r_2\phi}{l}\right)^3 \tan^{-1}\left(\frac{r_2\phi}{l}\right) - \left(\frac{r_1\phi}{l}\right)^3 \tan^{-1}\left(\frac{r_1\phi}{l}\right)\right.$$

$$- \tfrac{1}{2}\left[\left(\frac{r_2\phi}{l}\right)^2 - \left(\frac{r_1\phi}{l}\right)^2\right] + \tfrac{1}{2} \log_e \left[1 + \left(\frac{r_2\phi}{l}\right)^2\right] - \tfrac{1}{2} \log_e$$

$$\left.\left[1 + \left(\frac{r_1\phi}{l}\right)^2\right]\right\}$$

$$\therefore T = \frac{2\pi G}{3}\left[r_2^3 \tan^{-1}\left(\frac{r_2\phi}{l}\right) - r_1^3 \tan^{-1}\left(\frac{r_1\phi}{l}\right)\right]$$

$$- \frac{\pi G l}{3\phi} [r_2^2 - r_1^2] + \frac{\pi G l^3}{3\phi^3} \log_e \left[\frac{1 + \left(\frac{r_2\phi}{l}\right)^2}{1 + \left(\frac{r_1\phi}{l}\right)^2}\right] \ldots [45]$$

(c) *First Degree of Approximation*

Let $R = (r_1 + r_2)/2$, $t = r_2 - r_1$, θ = angle at radius R, and S = average stress in rubber. Then $T = 2\pi R^2 tS$ and $S = G\theta$.

$$\therefore T = 2\pi G\theta \{[(r_1 + r_2)^2/4](r_2 - r_1)\}$$

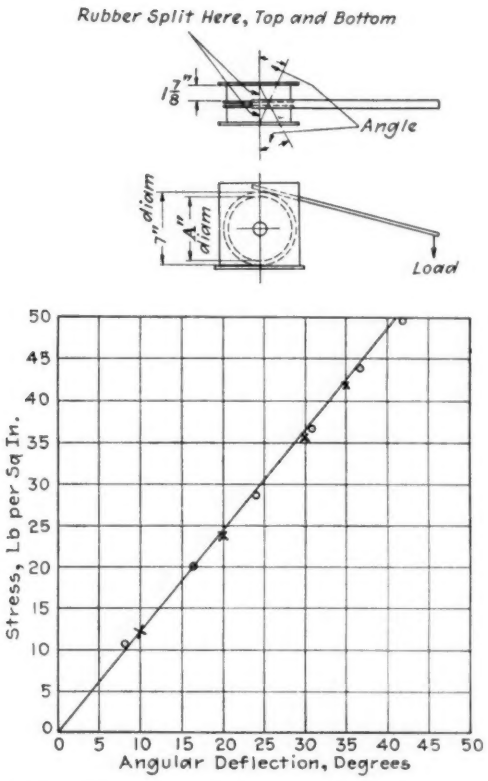


FIG. 17 THEORETICAL LOADING CURVE AND EXPERIMENTAL POINTS

(Rubber: 43 durometer; area (twice cross section) = 25 sq in. for $A = 5\frac{3}{4}$ in. and 45.2 sq in. for $A = 4\frac{1}{2}$ in.
Solid line = theoretical curve for both values of A.
Plotted points are experimental, $A = 4\frac{1}{2}$ in. represented by crosses and $A = 5\frac{3}{4}$ in. represented by circles.)

$$= (\pi G\theta/2)(r_1 + r_2)^2(r_2 - r_1) \quad [46]$$

Also $\tan\theta = d/l$ and $d/R = \phi$, $\therefore \tan\theta = R\phi/l$.
This is definitely applicable where $(r_2 - r_1)$ is small compared with r_1.

In Fig. 16 are shown data for two sizes of rubber doughnut sandwich. The experimental data and the theoretical curves obtained from Equations [43] and [45] are in striking agreement. The remarkably good agreement of experiment with the theoretical Equation [46] (see Fig. 17) makes it appear that this first degree of approximation can be used satisfactorily where $r_2 - r_1$ is small compared with r_1.

CASE 6. DOUGHNUT SANDWICH WITH CONICAL FACES, FIG. 18

Let S = the shear stress in the rubber at radius r
S_1 = the shear stress at the bonded surface = $S\cos\alpha = S\{r_2/[r_2^2 + (l_2/2)^2]^{1/2}\}$
θ = angle of twist of the fiber AB

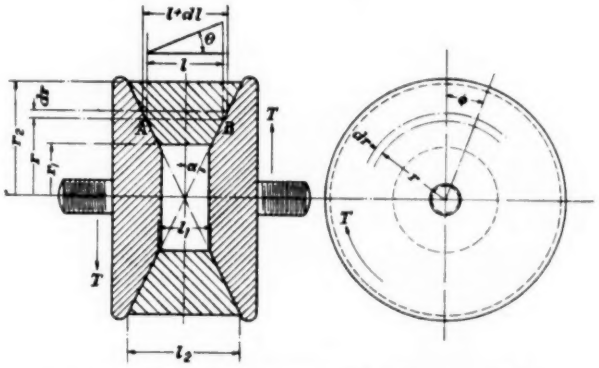


FIG. 18 DOUGHNUT SANDWICH WITH CONICAL FACES

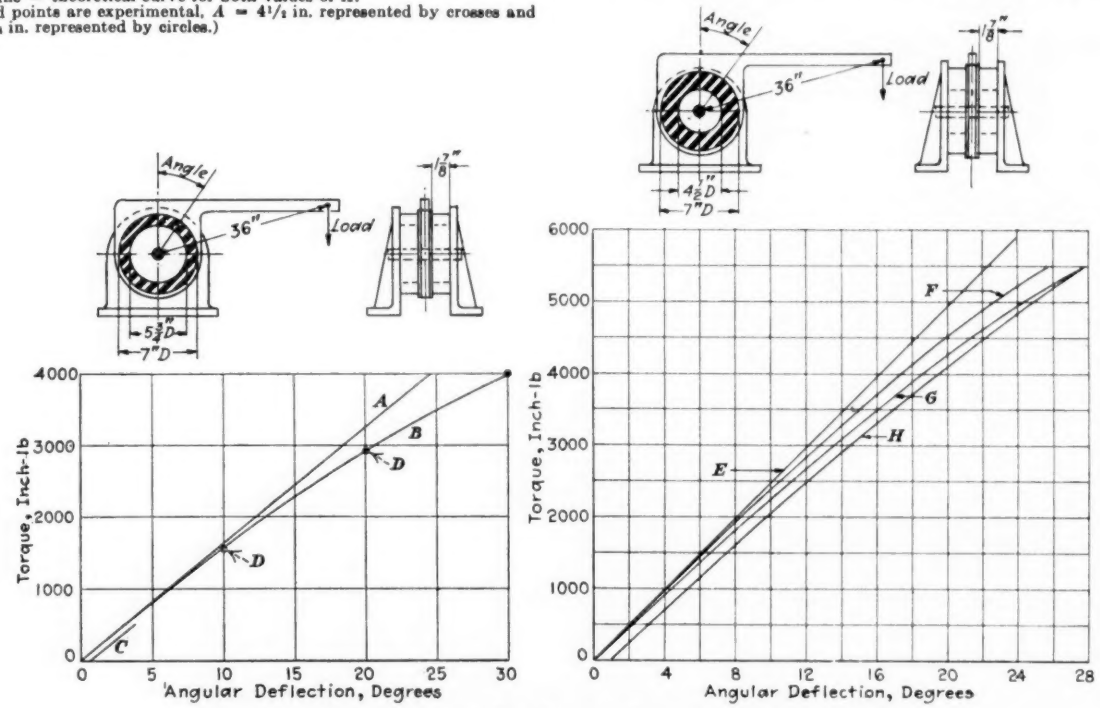


FIG. 16 LOADING CURVES FOR TWO SIZES OF RUBBER DOUGHNUT SANDWICH

(Spring (a). Rubber: area (2 × area of cross section), 25 sq in., 43 durometer.
Curve A, theoretical curve for small deflections.
Curve B, experimental loading curve.
Curve C, experimental unloading curve.
Points D, calculated from theoretical curve for large deflections.)

Spring (b). Rubber: area, 45.16 sq in., 43 durometer.
Curve E, theoretical curve for small deflections, $T = 247.2\phi$.
Curve F, theoretical curve for large deflections.
Curve G, experimental loading.
Curve H, experimental unloading.)

$S = G\theta$, and $\tan\theta = r\phi/l$

But $l/r = l_2/r_2 = l_1/r_1 = \text{const.}$
Therefore, for a definite angle ϕ, both θ and S are constant.
Now $dT = 2\pi r^2 dr S$

$$\therefore\ T = 2\pi S \int_{r_1}^{r_2} r^2 dr = \frac{2\pi S}{3}(r_2^3 - r_1^3) \quad [47]$$

and $S = G\theta = G\tan^{-1}(r_2\phi/l_2)$

$$\therefore\ T = (2/3)\pi G(r_2^3 - r_1^3)\tan^{-1}(r_2\phi/l_2) \quad [48]$$

or
$$\phi = (l_2/r_2)\tan\{3T/[2\pi G(r_2^3 - r_1^3)]\} \quad [49]$$

Notice that Equation [48] is almost the same as the first term of Equation [45].

It is hoped that the preceding calculations and experimental data will be helpful to those who deal with rubber mountings. It is realized, of course, that although in every case plain shear has been assumed for the sake of simplicity, compound stresses are ordinarily encountered. The effect of these stresses on the shear load-deflection data is seen to be small, and thus there is justification for using the simpler calculations.

Acknowledgment is due to F. L. Yerzley of E. I. du Pont de Nemours and Company for helpful criticism after preparation of the paper.

Solution of Rectangular Clamped Plate With Lateral Load by Generalized Energy Method[1]

By GERALD PICKETT,[2] MANHATTAN, KAN.

The author gives formulas by which the energy method may be readily applied for obtaining the moments and deflections for any lateral load on a clamped rectangular plate. He not only gives mathematical computations illustrating the application of the formulas, but also discusses the accuracy of the method compared to others, and points out its limitations.

THE problem of a thin rectangular plate bent with lateral load and fixed at its edges so that neither deflection nor inclination occurs at the edges, is a problem of frequent discussion in recent technical literature. All solutions are in the form of an infinite series. The coefficients of the terms in the series cannot be definitely determined as is the case in Fourier analysis. Approximate solutions using the first few terms in the series involve the solution of as many linear equations as terms taken. Although all methods of solution have these characteristics in common, they differ in details.

The energy method was developed by Timoshenko (1)[3] and Ritz (2). Timoshenko used it for the solution of the simply supported plate, while Ritz used it for the clamped plate. In his solution, Ritz used the normal functions of a doubly clamped bar. These functions are complicated, and the labor involved in obtaining numerical results is almost prohibitive. Shortly after Ritz's death, Knott (3) and Mesnager (4) attempted to obtain numerical values from his method but their results did not agree, as Mesnager's work was incorrect because he did not use the proper functions.

Because the energy method was so laborious, and results from it did not always agree, it lost favor as soon as the methods of Boobnov (5) and Hencky (6) became known.

Generalized Energy Method

In order to obtain correct results with the energy method, it is essential that the proper functions be used. So far as is known, the only correct functions for the deflection of the rectangular clamped plate are those used by Ritz, and the function

$$w = \frac{P}{Dab^3}(x^2 - a^2)^2(y^2 - b^2)^2 \sum_0^\infty{}_m \sum_0^\infty{}_n \alpha_{mn}\left(\frac{x}{a}\right)^m\left(\frac{y}{b}\right)^n \quad \ldots [1]$$

where the plate has dimensions shown in Fig. 1; w is the deflection; P is the total lateral load; D is the flexural rigidity assumed constant; and m and n, as well as r, s, g, and h subsequently used, are integers. In the case of symmetrical loading, only even integers will occur.

The boundary requirements $w = 0$, $\partial w/\partial n = 0$ are satisfied by each term of the series of Equation [1]. The strain energy of bending is given by the integral (7)

$$\int\int \frac{D}{2}\left\{\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right)^2 - 2(1-\nu)\left[\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} - \left(\frac{\partial^2 w}{\partial x dy}\right)^2\right]\right\} dx\,dy \quad \ldots [2]$$

It may be shown by the calculus of variations that the integral of the expression in the brackets, over an area from which there is no flow of energy, vanishes. Since there can be no flow of energy past the boundary of a clamped plate, the expression for the total energy U becomes

$$U = \int\int\left[\frac{D}{2}\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right)^2 - qw\right] dxdy \quad \ldots [3]$$

where q is the intensity of lateral load.

It may be shown also by the calculus of variations that making the total energy a minimum is equivalent to satisfying the differential equations of equilibrium.

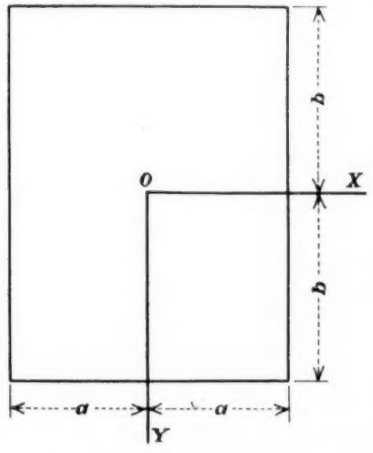


FIG. 1

When Equation [1] is substituted into Equation [3] and the integral minimized by setting

$$\frac{\partial U}{\partial \alpha_{mn}} = 0 \quad \ldots [4]$$

a set of linear equations is obtained from which the matrix of α's may be determined to any desired accuracy. There will be an equation associated with each α; for example, the mn equation associated with α_{mn} is found from Equation [4] and may be written as

$$C_{mn00}\,\alpha_{00} + C_{mn10}\,\alpha_{10} + C_{mn01}\,\alpha_{01} + C_{mn20}\,\alpha_{20} + \ldots\ldots + C_{mnrs}\alpha_{rs} + \ldots\ldots = q_{mn} \quad \ldots [5]$$

---

[1] Including portions of a dissertation submitted to the faculty of the University of Michigan in partial fulfillment of the requirements for the degree of doctor of philosophy.

[2] Associate Professor of Applied Mechanics, Kansas State College.

[3] Numbers in parentheses refer to the Bibliography at the end of the paper.

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Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y., and will be accepted until January 10, 1940, for publication at a later date. Discussion received after the closing date will be returned.

NOTE: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

where C_{mnrs} is a coefficient of α_{rs} in the mn equation and is a function of the integers m, n, r, and s, and the ratio of a to b. Also q_{mn} is a function of m, n, and the loading q. The calculations show that C_{mnrs} may be written as

$$C_{mnrs} = R_{mr}T_{ns} + 2S_{mr}S_{ns}(a^2/b^2) + T_{mr}R_{ns}(a^4/b^4)$$

where

$$R_{gh} = R_{hg} = \frac{384[2g^2h^2 + 2gh(g+h) + g^2 + h^2 - 8gh - 4(g+h) + 3]}{(g+h+5)(g+h+3)(g+h+1)(g+h-1)(g+h-3)}$$

$$S_{gh} = S_{hg} = \frac{128\,[g^2 + h^2 - 4gh - 3(g+h) + 2]}{(g+h+7)(g+h+5)(g+h+3)(g+h+1)(g+h-1)}$$

$$T_{gh} = T_{hg} = \frac{768}{(g+h+9)(g+h+7)(g+h+5)(g+h+3)(g+h+1)}$$

Before the derivation of the foregoing equations, the determination of any coefficient of α represented considerable work and consequently only those necessary for the first few equations had been evaluated. Since the coefficients of the α's are obtained from the strain-energy portion of Equation [3] they are independent of the loading. The effect of loading distribution shows itself in the value of q_{mn} which is obtained from the remaining portion of the integral. In the case of concentrated loads, the q's are obtained without integration. For example, $q_{00} = 1$, and all other q's are zero for a concentrated load at the center. For a uniform load over the entire plate

$$q_{mn} = \frac{64}{(m+5)(m+3)(m+1)(n+5)(n+3)(n+1)}$$

The q's for other distributions are readily obtained as soon as q as a function of x and y is known. As an additional example, suppose there is a uniform load over the middle fourth of the plate, that is, from $-a/2$ to $+a/2$, and $-b/2$ to $+b/2$. We then have

$$q_{mn} = \frac{(9m^2 + 84m + 203)(9n^2 + 84n + 203)}{2^{m+n+8}(m+5)(m+3)(m+1)(n+5)(n+3)(n+1)}$$

It is recommended that the C's be evaluated from a prepared table, as shown by Table 1.

From Table 1 it may be seen, for example, that

$$C_{2044} = C_{4420} = C_{4024} = C_{2440} = \left(\frac{6784}{1155} \times \frac{256}{15015}\right) - \left(2 \times \frac{512}{6435} \times \frac{256}{3465}\frac{a^2}{b^2}\right) + \left(\frac{128}{105} \times \frac{256}{45045}\frac{a^4}{b^4}\right)$$

Using only one parameter α_{00}, for a square plate with uniform load we have

$$C_{0000}\,\alpha_{00} = q_{00}$$

where

$$C_{0000} = \left(\frac{128}{5} \times \frac{256}{315}\right) + \left(2 \times \frac{256}{105} \times \frac{256}{105}\right) + \left(\frac{256}{315} \times \frac{128}{5}\right)$$

and

$$q_{00} = \frac{64}{225}$$

TABLE 1 VALUES OF R, S, AND T FOR A FEW EVEN INTEGERS

g/h	0	2	4	6
		R		
0	$\frac{128}{5}$	$\frac{128}{35}$	$\frac{128}{105}$	$\frac{128}{231}$
2		$\frac{384}{35}$	$\frac{6784}{1155}$	$\frac{10,112}{3003}$
4			$\frac{82,302}{15,015}$	$\frac{12,416}{3003}$
6				$\frac{9088}{2431}$
		S		
0	$-\frac{256}{105}$	0	$+\frac{256}{3465}$	$+\frac{512}{9009}$
2		$-\frac{256}{1155}$	$-\frac{512}{6435}$	$-\frac{256}{9009}$
4			$-\frac{256}{5005}$	$-\frac{1024}{36,465}$
6				$-\frac{13,568}{692,835}$
		T		
0	$\frac{256}{315}$	$\frac{256}{3465}$	$\frac{256}{15,015}$	$\frac{256}{45,045}$
2		$\frac{256}{15,015}$	$\frac{256}{45,045}$	$\frac{256}{109,395}$
4			$\frac{256}{109,395}$	$\frac{256}{230,945}$
6				$\frac{256}{440,895}$

TABLE 2

No. of parameters[10]	Deflection at (0,0)	Deflection at $(\frac{a}{2}, 0)$	Moment at (0,0)	Moment at (a,0)
	(a) Uniform load			
1	.005517 $\frac{Pa^2}{D}$	.002991 $\frac{Pa^2}{D}$	.021267 P(1+μ)	-.04254 P
4	.005058 $\frac{Pa^2}{D}$	.003033 $\frac{Pa^2}{D}$	.01756 P(1+μ)	-.05116 P
9	.0050610 $\frac{Pa^2}{D}$	.003033 $\frac{Pa^2}{D}$	.01751 P(1+μ)	-.05128 P
16	.0050615 $\frac{Pa^2}{D}$	.003033 $\frac{Pa^2}{D}$	.01764 P(1+μ)	-.05127 P
	(b) Uniform load over central fourth of plate			
1	.01337 $\frac{Pa^2}{D}$	.007522 $\frac{Pa^2}{D}$	.05349 P (1+μ)	-.1070 P
4	.01361 $\frac{Pa^2}{D}$	.007506 $\frac{Pa^2}{D}$	.05653 P(1 +μ)	-.1005 P
9	.01362 $\frac{Pa^2}{D}$	.007496 $\frac{Pa^2}{D}$	.05693 P(1+μ)	-.1010 P
16	.01358 $\frac{Pa^2}{D}$	.007540 $\frac{Pa^2}{D}$	.05582 P(1+μ)	-.1075 P
	(c) Concentrated load at center			
1	.01869 $\frac{Pa^2}{D}$	.01051 $\frac{Pa^2}{D}$	.07477 P(1+μ)	-.1495 P
4	.02040 $\frac{Pa^2}{D}$	.01004 $\frac{Pa^2}{D}$	.10195 P(1+μ)	-.08177 P
9	.02135 $\frac{Pa^2}{D}$	.009113 $\frac{Pa^2}{D}$	.1266 P(1+μ)	-.1611 P
16	.02153 $\frac{Pa^2}{D}$	.009785 $\frac{Pa^2}{D}$	.1343 P(1+μ)	-.1363 P

10. The number of parameters will be the same as the number of linear equations in general, but for a square plate, only six equations are necessary for nine parameters: ten equations for sixteen parameters, etc.

from which $\alpha_{00} = 0.0053168$. Using only this one parameter, it is found that it gives a deflection at the center 5.04 per cent too large; a deflection at $(x = a/2, y = 0)$ 1.4 per cent too small; and a moment at the middle of a clamped edge 17.0 per cent too small.

Computed values of deflections and moments of a square plate under three different types of loads are given in Table 2. Examination of this table shows that four parameters are sufficient for

TABLE 3

	Concentrated load	Uniform over entire plate	Uniform over central fourth	α_{00}	α_{20}	α_{22}	α_{40}	α_{42}	α_{44}	
00	-1.000	-0.2044	-0.7154	53.4988	9.7270	0.5404	2.1338	0.3048	0.0525	
20		-0.0406	-0.0512	4.8635	10.9732	0.8729	5.4593	0.6298	0.0953	
22		-0.0058	-0.0367	0.5404	1.7459	0.4722	0.9094	0.3955	0.0794	
40		-0.0135	-0.0072	1.0669	5.4593	0.4547	4.8166	0.5088	0.0887	
42		-0.0019	-0.0005	0.1524	0.6298	0.1977	0.5088	0.2212	0.0530	
44		-0.0003	-0.0001	0.0525	0.1906	0.0794	0.1775	0.1061	0.0309	
00	-19.05	-5.4194	-13.6310	1019.3040	185.3280	10.2960	40.6580	5.8080	1.00	
20		-0.4263	- 0.5374	51.0264	115.1267	9.1585	57.2775	6.6078	1.00	
22		-0.0730	- 0.0461	6.8059	21.9821	5.9454	11.4499	4.9793	1.00	
40		-0.1526	- 0.0807	12.0215	61.5126	5.1232	54.2703	5.7334	1.00	
42		-0.0364	- 0.0096	2.8738	11.8751	3.7282	9.5943	4.1708	1.00	
44		-0.0208	- 0.0023	1.6992	6.1717	2.5713	5.7468	3.4342	1.00	
00	-19.05	-5.3065	-13.6287	1017.6047	179.1562	7.7246	34.9091	2.3737		
20		-0.4054	- 0.5351	49.3271	108.9550	6.5872	51.5307	3.1735		
22		-0.0522	- 0.0438	5.1047	15.8103	3.3740	5.7031	1.5450		
40		-0.1317	- 0.0783	10.3222	55.3409	2.5518	48.5235	2.2992		
42		-0.0156	- 0.0073	1.1745	5.7033	1.1566	3.8475	0.7366		
00	-8.026	-2.2742	- 5.7413	428.6840	75.4727	3.2541	14.7061	1.00		
20		-0.1277	- 0.1686	15.5429	34.3317	2.0756	16.2373	1.00		
22		-0.0337	- 0.0283	3.3038	10.2326	2.1836	3.6911	1.00		
40		-0.0572	- 0.0340	4.4894	24.0694	1.1098	21.1043	1.00		
42		-0.0211	- 0.0099	1.5944	7.7422	1.5704	5.2229	1.00		
00	-8.026	-2.2530	- 5.7313	427.0895	67.7304	1.6836	9.4831			
20		-0.1065	- 0.1586	13.9485	26.5894	0.5051	11.0143			
22		-0.0126	- 0.0184	+ 1.7093	+2.4903	+0.6132	-1.5318			
40		-0.0361	- 0.0241	2.8949	16.3271	-0.4606	15.8813			
00	-0.846	-0.2375	- 0.6043	45.0368	7.1422	0.1775	1.00			
20		-0.0096	- 0.0144	+ 1.2663	2.4140	0.0458	1.00			
22		+0.0082	+ 0.0120	- 1.1158	-1.6257	-0.4003	1.00			
40		-0.0022	- 0.0015	+ 0.1822	+1.0280	-0.0290	1.00			
00	-0.846	-0.2353	- 0.6028	+44.8545	6.1141	0.2065				
20		-0.0074	- 0.0128	+ 1.0841	1.3860	0.0748				
22		+0.0105	+ 0.0135	- 1.2961	-2.6537	-0.3713				
00	-4.097	-1.1392	- 2.9187	217.1661	29.6019	1.00				
20		-0.0988	- 0.1721	14.4907	18.5131	1.00				
22		-0.0202	- 0.0364	3.4961	7.1470	1.00				
00	-4.097	-1.1109	- 2.8823	213.6699	22.4549					
20		-0.0705	- 0.1356	10.9845	11.3661					
00	-0.182	-0.0494	- 0.1283	9.5154	1.000					
20		-0.0062	- 0.0119	0.9664	1.00					
00	-0.182	-0.0432	- 0.1164	8.5490						

SOLUTION

	Concentrated load	Uniform over entire plate	Uniform over central fourth
α_{00}	+0.021346	+ 0.00506103	+ 0.0136184
$\alpha_{02} = \alpha_{20}$	- 0.020629	+ 0.00131830	- 0.0012286
α_{22}	+ 0.072810	+ 0.00117761	- 0.0023844
$\alpha_{04} = \alpha_{40}$	+ 0.019429	+ 0.00006038	+ 0.0002284
$\alpha_{24} = \alpha_{42}$	- 0.090140	+ 0.00089485	+ 0.0003131
α_{44}	+ 0.101736	- 0.00021297	- 0.0095039

13. Calculations were carried to eight significant figures but fewer are shown in the table to conserve space.

an accuracy of three significant figures for uniform load. The values shown using four or more parameters agree with calculations made by other methods.[4] It was pointed out in the author's dissertation[1] that nine parameters were necessary for such accuracy when the ratio of length to breadth was 1.75.

Very good accuracy is also indicated for four parameters when the load is distributed over the central fourth of the plate. In the case of a concentrated load at the center, the values for moments are not as accurate. Timoshenko (9) gives a value of the moment at the middle of the clamped edge of $-0.1257P$.

In addition to the method described in this paper, the author has solved the uniformly loaded clamped square plate by all of the following methods: Energy method using functions proposed by Ritz (2), Hencky method (6), elastic-web method of Marcus (8) power-series integration,[1] and superposition method of Timoshenko (9). Also he has used some of these methods for concentrated loads and for plates not square.

As to which method is the better depends upon the type of loading, the ratio of length to breadth, what is wanted; that is, deflection, moment or shear, and the degree of accuracy desired. The method of Timoshenko is found to be the best for a concentrated load at the center. His method gives very good results in this case because a very rapidly converging series solution is known for the simply supported case.

The advantages of the energy method using the series Equation [1] were discussed in the author's dissertation.[1]

Solving Simulataneous Linear Equations

Since all methods involve the solution of a set of linear equations, and since it may be necessary to solve ten or more such equations to obtain the desired accuracy, the method found to be most satisfactory to the author will be explained.

One does not know at the beginning, how many parameters will be necessary. It will be found most economical of time to start with one or two parameters then continue to take more, as shown in Table 2, until the desired accuracy is indicated.

The author has found that the method illustrated in Table 3, where practically all of the computations are shown for the solution of six equations and six unknowns for three different loading conditions, gives accurate results with a minimum of computations. Every operation is outlined in advance so that two computers working simultaneously would have identical computations unless a mistake had been made. With this method, mistakes are easily detected; therefore, the solutions may be as accurate as the computing machine.

Conclusions

With the general formulas given in this paper available, the energy method may be readily applied for obtaining the moments and deflections for any lateral load on a clamped rectangular plate. It is believed that for fair accuracy, with all except a few types of loading, it has advantages over other known methods. It does not give as good results for concentrated loads as for distributed loads, and is inferior to Timoshenko's method for a concentrated load at the middle.

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[4] The value given by Hencky and others for the moment at $(a, 0)$ is $-0.0513P$, and for the deflection coefficient at (0, 0) is 0.0138 which when reduced to the proper notation becomes 0.005055.

Chain Links Under Cross Forces

By JOS. B. REYNOLDS,[1] BETHLEHEM, PA.

This paper develops theory for stresses and deflections of links with circular neutral axes when freely supported at the ends of one diameter and loaded with two equal loads, perpendicular to the plane of the neutral axis, at the ends of the diameter perpendicular to the first. It also develops theory for deflections of oblong links whose neutral axes consist of semicircles connected by straight sides when supported at the middle points of the end arcs and loaded with equal loads perpendicular to the plane of the neutral axis at the midpoints of the sides. The theory is checked by experimental tests.

IT IS WELL known that a chain will fail under a much smaller load when the links are subjected to a cross pull than when subjected to a direct pull. Cross pulls may arise from many causes. Chains may be pulled around a corner or pole, wound on a cylinder or capstan, or kinked or knotted so as to produce forces across the plane of the neutral axis of a link.

Because of the uncertainty of the occurrence of such situations many engineers avoid as far as feasible the use of chains, replacing them by cables. But cables also have their dangerous characteristics. It is common practice in making anchor chains to put a stud across the middle of an oblong link in order to prevent tangling and kinking of the chain. While this expedient prevents certain dangerous situations it does not eliminate all. For certain purposes the chain, if not the only, is certainly the best device.

It is the purpose of this paper to develop theory for stresses and deflections when links of different shapes are subjected to certain symmetrical loading perpendicular to the plane of the neutral axis of the link and to compare the results of theory with those obtained from tests in an engineering laboratory.[2]

Outline of Tests

Tests were performed on three specimens. No. 1 was a circular link of forged steel $10^{15}/_{16}$ in. OD, accurately made of 2-in. round stock. It will be called the round circular link. This link was first tested for stresses and vertical and angular deflections, then annealed and again tested for deflections.

No. 2 was a circular link of forged steel $10^{15}/_{16}$ in. OD, accurately made of 2-in. square stock. It will be called the square circular link. This link was annealed and tested for vertical and angular deflections.

No. 3 was an oblong link of cast steel assumed to be made up of two semicircular ends connected by equal straight sides. It will be called the oblong link. It was 13.61 in. long and 8.16 in. wide, made of 2.26-in. round stock. It was tested for vertical and angular deflections.

A link whose principal mutually perpendicular axes are AC and BD (Fig. 1) was supported on rollers or spherical bearing blocks at A and C, and two equal vertically downward forces were applied at B and D by a spherical loading block centered in an Olsen testing machine over the center of the link. The setup for the round circular link is shown in Fig. 2.

Stresses at the upper surface of this link were measured by means of an extensometer. In the case of each link tested the vertical deflection was measured under the loaded points and over the supported points by means of Ames gages reading to 0.0001 in. The angular deflection or turning of the stock over the points of support was measured by determining the angular rotation of arms welded to the link in the line of the axis AC over the supports. This rotation measured by Ames gage readings at points at a measured distance apart on the arms was checked by deducing from it a deflection of the neutral axis which was then compared with the deflection determined by direct readings on the gages at A, B, C, and D.

The method of performing these separate tests for stresses, vertical deflection, and angular rotation will now be described and the results compared with theoretical values.

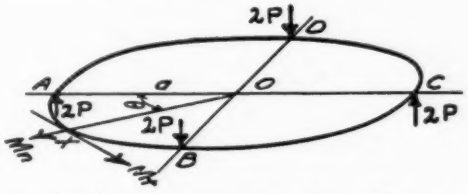


Fig. 1 Circular Neutral Axis of Links Nos. 1 and 2

Notation

- A = point on neutral axis directly over support
- AB = one quadrant of neutral axis
- A_s = area of cross section of stock
- a = radius of neutral axis
- E = Young's modulus
- G = modulus of shear
- I_n = moment of inertia of A_s with respect to the normal to the neutral axis
- I_t = moment of inertia of A_s with respect to the tangent to the neutral axis
- k = angular rotation of stock at supports
- L = total load
- l = length of one half of straight side for oblong links with semicircular ends
- M_o = bending moment at A
- M_n = moment about normal to neutral axis
- M_t = moment about tangent to neutral axis
- O = center of circular neutral axis
- P = $L/4$
- r = radius of circular stock
- S = stress
- X = any point on the arc AB
- z = vertical deflection of point X
- z' = value of z at B
- θ = central angle AOX

Tests for Stresses

On the upper side of the round circular link beginning at the points A and C directly over the supports six small shallow holes

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[2] The specimens required for these tests were generously provided by the Baldt Anchor, Chain, and Forge Corporation of Chester, Pa. The tests were made at the Fritz Engineering Laboratory, Lehigh University, under the direction of Professor Inge Lyse whose suggestions and criticisms have been very helpful to the author.

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Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y., and will be accepted until January 10, 1940, for publication at a later date.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of authors, and not of the Society

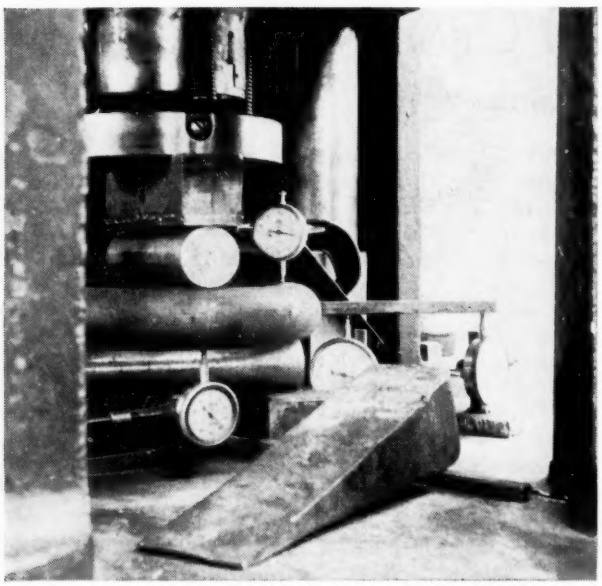


FIG. 2 LOADING ARRANGEMENT OF CIRCULAR LINK

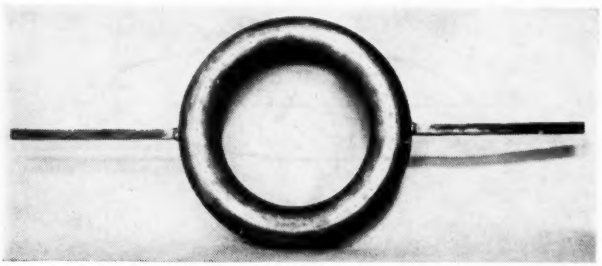


FIG. 3 ROUND CIRCULAR LINK WITH STRAIN-GAGE HOLES AND ANGULAR-ROTATION ARMS

were sunk at distances of about one inch apart in each direction along the circular arc directly above the neutral axis.[3] See Fig. 3. Beginning at the end hole of each series of holes, extensometer measurements were made from each hole to the second following one and the measured strain in each distance was taken as the strain at the midpoint of the arc subtended by the measurement. First readings were taken under a load of 1000 lb.

Conditions for the four quadrants are obviously symmetrical. The results obtained on the four arcs averaged for each of five angular distances from the supports are given in the third column of Table 1. These values represent increases of stresses over the initial stresses. In the fourth column are given corresponding theoretical increases in stresses at these points computed from $SI_n = rM_n$ in which

$$M_n = Pa\,(\sin\theta - \cos\theta) \quad \ldots\ldots\ldots\ldots\ldots [1]$$

The results are depicted graphically in Fig. 4. Because of the limitations of the extensometer, observations are incomplete for $\theta = 0$. Indications are that the elastic limit is reached over the supports under a load of from 20,000 to 30,000 lb. From deflections it was found to occur at about 25,000 lb.[4]

In Fig. 5 are plotted the bending moment along the arc AB as computed from Equation [1] and, also, the moment due to twist given by

$$M_t = Pa\,(\sin\theta + \cos\theta - 1) \quad \ldots\ldots\ldots\ldots [2]$$

[3] The neutral-axis radius is taken as 4.47 in. No consideration is here given to the slight shifting of the neutral axis away from the geometric axis due to curvature. After the stress test this link was annealed to remove initial and set stresses and tested again. Its deflection behavior in the second test nearly duplicated that of the first which is not given for the sake of clarity.

TABLE 1 OBSERVED AND COMPUTED STRESSES[a] IN ROUND CIRCULAR LINK

Angle, θ, from support, deg	Total load L, 10^3 lb	Stress from observed strain, psi	Theoretical stress, psi
0	1	000	000
	10	—11100	—12890
	20	—25400	—27210
	30	—44600	—41530
	35	Out of	—47980
	40	range of	—55950
	1	instrument	
13	1	000	000
	10	— 9110	— 9590
	20	—19100	—20250
	30	—31400	—30910
	35	—48400	—36240
	40	—76600	—41570
	1	—36400	
26	1	000	000
	10	— 5900	— 5890
	20	—12800	—12440
	30	—20300	—18990
	35	—24500	—22265
	40	—29200	—25540
	1	— 4000	
39	1	000	000
	10	— 3100	— 1890
	20	— 4900	— 3990
	30	— 7000	— 6090
	35	— 7500	— 7140
	40	— 8800	— 8190
	1	— 100	
52	1	000	000
	10	+ 3100	+ 2200
	20	+ 5100	+ 4650
	30	+ 7400	+ 7100
	35	+ 8600	+ 8330
	40	+10500	+ 8550
	1	+ 1700	

[a] Stress values represent increases in stress over that at the initial load of 1000 lb.

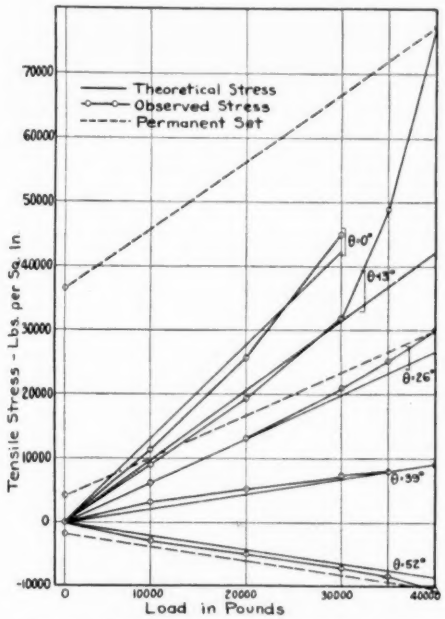


FIG. 4 STRESSES IN ROUND CIRCULAR LINK

[4] On a straight pull the elastic limit for this link was found to be about 17,000 lb, a ratio of 1 to 1.47. The theoretical ratio is about 14 to 3 or 1 to 1.49.

No observations were made to determine directly values of M_t but they are indirectly involved in the deflections upon which careful observations were made. The total stress at any point depends upon M_n and M_t combined with a small stress due to vertical shear.

Tests for Vertical Deflections

The vertical deflection z' of the neutral axis of the links under the applied loads at B and D, Fig. 1, was measured in two ways which may be described by reference to Fig. 6 which shows the setup for the square circular link. First, the gross deflection was measured by gages set up under the specimen at points directly under the loads. From this was subtracted the amount of settling of the specimen determined by gages set up at points directly over the supports. The average values thus found for the two points under the loads were taken as the measured deflection determined by this method. It is called deflection by direct measurement.

Second, from the readings of the two gages farthest right in the figure (one only partially visible), the distance between them, and the distance from the inner gage to the neutral axis, the point of rotation of the arm and downward motion of the neutral axis were determined by simple geometric considerations. The average deflection thus determined by the two arms is called deflection by indirect measurement or rotation of arms.

First readings were taken under an initial load of 1000 lb. In the first run readings were taken at 1000, 5000, 10,000, 15,000, and 20,000 lb. The load was then eased off to 1000 lb and the

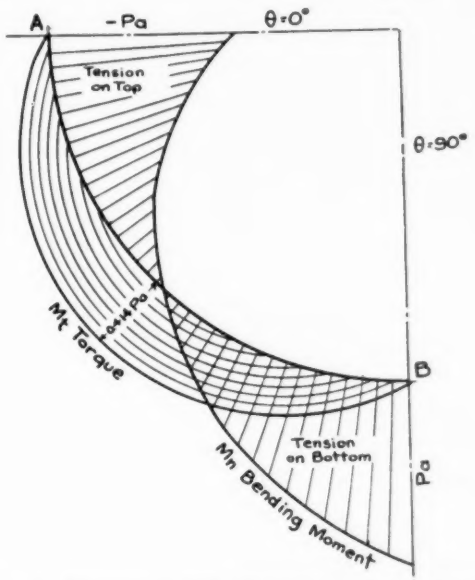


Fig. 5 Moments in One Quadrant of Circular Link

second run made as before to a maximum of 25,000 lb. The maximum was then increased for succeeding runs by steps of 5000 lb until the elastic limit was reached. In the second run, only, in the case of each link the load was eased off from the maximum by steps of 5000 lb until the initial load was again reached. The values of deflections thus obtained are averages of from two to sixteen readings.

The theoretical deflection for the round circular link, as given by Equation [23] in the Appendix with $\theta = \pi/2$, is

$$z' = \frac{Pa^3}{GI_t}(\pi - 3) + \frac{Pa^3}{2EI_n}(\pi - 2) + \frac{P\pi a}{2A_eG}$$

which for $E = 3 \times 10^7$ and $2.6G = E$, a ratio used by Timoshenko,[5] gives

$$z' = 7.68 \times 10^{-7}L \quad [3]$$

Similarly, for the square circular link

$$z' = \frac{42.6Pa^3}{A_e^4G} I_t(\pi - 3) + \frac{Pa^3}{2EI_n}(\pi - 2) + \frac{P\pi a}{2A_eG} = 4.77 \times 10^{-7} L \quad [4]$$

and for the oblong link

$$z' = \frac{Pa^3}{GI_t}(\pi - 3) + \frac{Pa^3}{2EI_n}(\pi - 2) + \frac{Pl(l^2 + 3al + 3a^2)}{3EI_n} + \frac{P}{2A_eG}(\pi a + 2l) = 4.89 \times 10^{-7}L \quad [5][6]$$

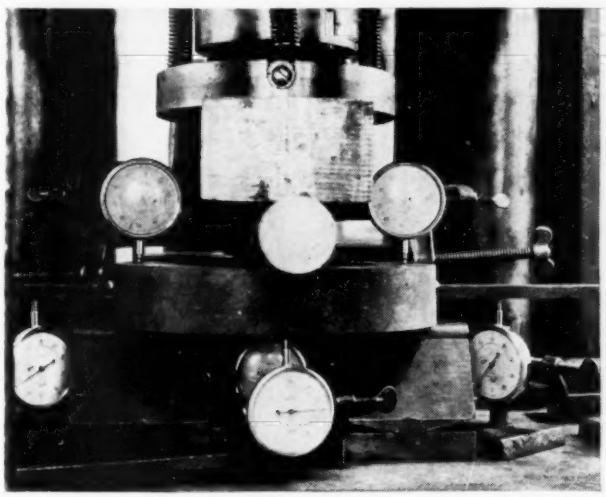


Fig. 6 Loading Arrangement for Square Circular Link

In Table 2 are set forth the observed and theoretical values of the deflection z' at points directly under the loads. These results are exhibited graphically in Figs. 7 and 8.

Tests for Angular Rotation Over Supports

As the link deflects under load the stock turns about the tangent to the neutral axis at points over the supports. The theory gives for the amount k of this turning in radians for the round circular link

$$k = \frac{Pa^2}{4}(\pi - 2)\left(\frac{1}{GI_t} + \frac{1}{EI_n}\right) = 1.39 \times 10^{-7}L \quad [6]$$

for the square circular link

$$k = \frac{Pa^2}{4}(\pi - 2)\left(\frac{42.6I_t}{A_e^4G} + \frac{1}{EI_n}\right) = 0.827 \times 10^{-7}L \quad [7]$$

and for the oblong link

$$k = \frac{Pa^2}{4}(\pi - 2)\left(\frac{1}{GI_t} + \frac{1}{EI_n}\right) + \frac{Pl(2a + l)}{2EI_n} = 1.123 \times 10^{-7}L \quad [8]$$

[5] "Strength of Materials," by S. Timoshenko, part 1, D. Van Nostrand Company, New York, N. Y., 1930, page 59, art. 16.

[6] The link used for this test only approximately meets the slope required by the theory. The values $l = 3$ and $a = 2.68$ in. were judged best from a plain section containing a neutral axis of the link.

TABLE 2 COMPARISON OF OBSERVED AND THEORETICAL DEFLECTIONS AT POINTS UNDER APPLIED LOADS, INCHES

Load, 10^3 lb	Specimen No. 1 Round circular link D	I	T	Specimen No. 2 Square circular link D	I	T	Specimen No. 3 Oblong link D	I	T
1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
5	0.0042	0.0046	0.0030	0.0022	0.0023	0.0019	0.0016	0.0015	0.0020
10	0.0068	0.0072	0.0071	0.0044	0.0043	0.0043	0.0036	0.0033	0.0044
15	0.0103	0.0107	0.0109	0.0064	0.0063	0.0067	0.0047	0.0045	0.0068
20	0.0141	0.0145	0.0146	0.0083	0.0080	0.0090	0.0078	0.0074	0.0093
25	0.0177	0.0180	0.0184	0.0103	0.0099	0.0104	0.0099	0.0096	0.0117
30	0.0224	0.0222	0.0222	0.0122	0.0117	0.0138	0.0115	0.0111	0.0142
35	0.0282	0.0279	0.0261	0.0143	0.0138	0.0162	0.0138	0.0131	0.0166
37.8	0.0366	0.0362	0.0282	....	....	....	....	....	....
40	0.0471	0.0459	0.0299	0.0163	0.0160	0.0186	0.0166	0.0160	0.0191
45	....	....	....	0.0188	0.0183	0.0210	0.0189	0.0181	0.0215
50	....	....	....	0.0216	0.0207	0.0234	0.0210	0.0200	0.0240
55	....	....	....	0.0274	0.0261	0.0258	0.0238	0.0231	0.0264
60	....	....	....	....	....	....	0.0262	0.0254	0.0289
65	....	....	....	....	....	....	0.0286	0.0277	0.0313
70	....	....	....	....	....	....	0.0309	0.0300	0.0338
75	....	....	....	....	....	....	0.0331	0.0323	0.0362
80	....	....	....	....	....	....	0.0355	0.0347	0.0387
90	....	....	....	....	....	....	0.0409	0.0399	0.0436
100	....	....	....	....	....	....	0.0471	0.0458	0.0484
110	....	....	....	....	....	....	0.0555	0.0513	0.0533
120	....	....	....	....	....	....	0.0657	....	0.0582
130	....	....	....	....	....	....	0.0849	....	0.0631

D = direct measurement.
I = indirect measurement or measurement by rotation of arms.
T = theoretical deflection.

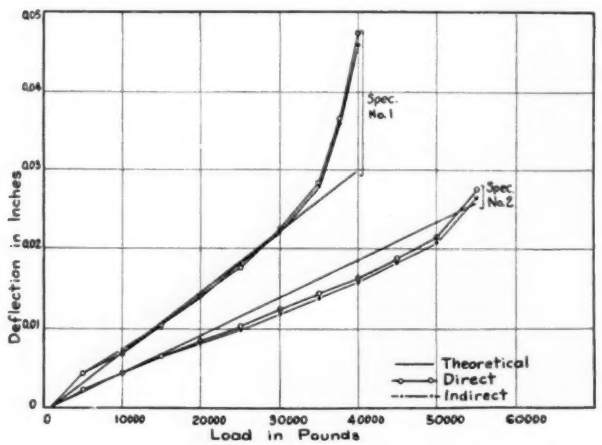


FIG. 7 DEFLECTION OF CIRCULAR LINKS UNDER LOADING POINTS

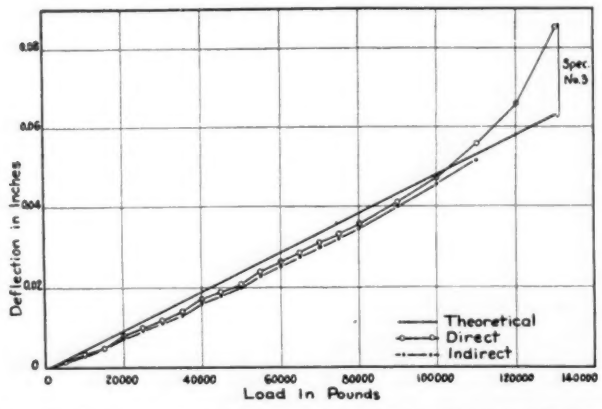


FIG. 8 DEFLECTION OF OBLONG LINK UNDER LOADING POINTS

Values of k were found from observation by dividing the change in elevation of two points on the arms welded to the links, determined by gage readings, by the distance between the points.[7] Each value thus determined is the average of from two to sixteen readings depending upon how often the particular load in question was applied. The results are closely checked by the fact that deflections determined from the values of k, as seen by Figs. 7 and 8 agree closely with those determined by direct measurement.

The observed and theoretical values of k for the different links are set forth in Table 3. This table gives the increases in rota-

TABLE 3 ROTATION OF STOCK OVER SUPPORTS[a]

Load, 10^3 lb	Rotation over supports in radians: Specimen No. 1 R-C link O	T	Specimen No. 2 S-C link O	T	Specimen No. 3 Oblong link O	T
1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
5	0.0005	0.0006	0.0005	0.0003	0.0004	0.0004
10	0.0012	0.0013	0.0009	0.0007	0.0010	0.0010
15	0.0019	0.0020	0.0012	0.0012	0.0014	0.0015
20	0.0026	0.0026	0.0015	0.0017	0.0020	0.0021
25	0.0032	0.0033	0.0018	0.0021	0.0025	0.0026
30	0.0040	0.0040	0.0021	0.0024	0.0031	0.0032
35	0.0050	0.0047	0.0025	0.0028	0.0036	0.0037
37.8	0.0067	0.0051	....	....	....	....
40	0.0084	0.0054	0.0029	0.0032	0.0041	0.0043
45	....	....	0.0033	0.0036	0.0047	0.0048
50	....	....	0.0039	0.0040	0.0053	0.0055
55	....	....	0.0050	0.0045	0.0058	0.0060
60	....	....	....	....	0.0063	0.0066
65	....	....	....	....	0.0068	0.0071
70	....	....	....	....	0.0074	0.0077
75	....	....	....	....	0.0079	0.0082
80	....	....	....	....	0.0085	0.0088
90	....	....	....	....	0.0097	0.0099
100	....	....	....	....	0.0113	0.0111
110	....	....	....	....	0.0131	0.0122
120	....	....	....	....	0.0152	0.0133
130	....	....	....	....	0.0197	0.0144

R-C = round circular, S-C = square circular.
O = observed, T = theoretical.
[a] Increase in rotation after initial load of 1000 lb.

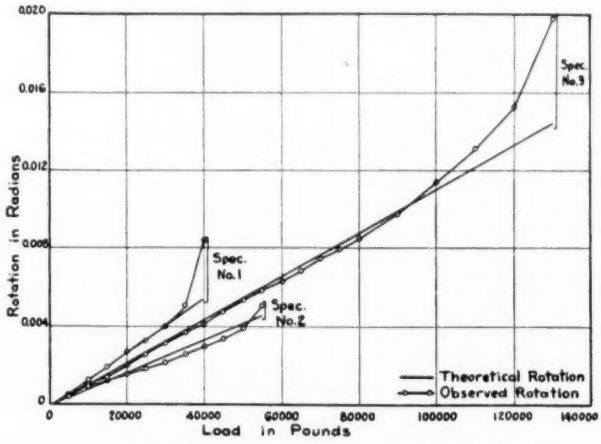


FIG. 9 ROTATION OF LINKS OVER SUPPORTS

[7] This method takes no account of change in slope of the cross section under load. The change must be very small for the loads applied.

tion after the application of the initial load of 1000 lb. These results are depicted graphically in Fig. 9.

Appendix

Theory of Deflection of Circular Ring

The deflection under consideration is caused by a vertical load $4P$, applied in two equal parts at the ends of a diameter of a horizontal circular ring, of geometric axis radius a and circular cross-sectional area A_c, supported by vertical forces at the ends of a diameter perpendicular to the loaded diameter. Let any point X on the neutral axis be located by the central angle θ measured from a support. See Fig. 10. Let M_t and M_n be the moments exerted by the section perpendicular to the neutral axis to the right of X on that to the left about the tangent and normal to the neutral axis, respectively, with positive senses as indicated by arrowheads in the figure.

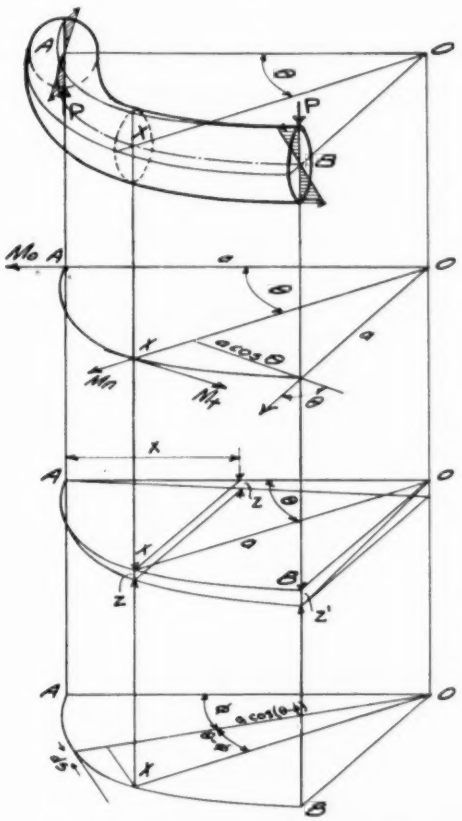


Fig. 10 One Quadrant of Circular Ring

Let z be the vertically downward deflection of the point X. Then four effects contribute to z: The twisting about the tangent; the twisting about the normal or bending; the detrusion due to vertical shear; and a free rotation about the point A. Let the component of z yielded by the twisting about the tangent be represented by z_t, that about the normal by z_n, that due to vertical shear by z_s, and let k be the angle of free rotation. Then

$$z = z_t + z_n + z_s + ka\,(1 - \cos\theta) \quad \ldots\ldots [9]$$

since x in the figure equals $a\,(1 - \cos\theta)$.

Let the stock at a point determined by the central angle ϕ be twisted through an angle dw_t in a length ds of the arc $AX = s$ of the neutral axis. Then the deflection, z_t, at X due to this twist is such that

$$dz_t = -a[1 - \cos(\theta - \phi)]dw_t \quad \ldots\ldots [10]$$

where

$$dw_t = \frac{M_t ds}{GI_t} \quad \ldots\ldots [11]$$

with G being the modulus of shear and I_t the polar moment of inertia of the cross section of the stock with respect to its center. Hence

$$z_t = \frac{-a}{GI_t}\int [1 - \cos(\theta - \phi)]\, M_t ds \quad \ldots\ldots [12]$$

Now

$$M_t = M_0 \cos\phi - Pa\,(1 - \sin\phi) = Pa\,(\sin\phi + \cos\phi - 1) \quad \ldots [13]$$

since by taking moments about A we get $M_0 = Pa$. Replacing ds in Equation [12] by $ad\phi$ and substituting for M_t its value from Equation [13] we have

$$z_t = -\frac{Pa^3}{GI_t}\int_0^\theta [1 - \cos(\theta - \phi)][\sin\phi + \cos\phi - 1]\, d\phi \quad \ldots [14]$$

which evaluates to

$$z_t = \frac{Pa^3}{GI_t}\left[\theta + \cos\theta - 1.5\sin\theta - 1 + \frac{\theta}{2}(\sin\theta + \cos\theta)\right] \quad \ldots [15]$$

To determine the effect due to bending we have

$$dz_n = -a\sin(\theta - \phi)\, dw_n,\ dw_n = \frac{M_n ds}{EI_n} \quad \ldots\ldots [16]$$

in which E is Young's modulus and I_n the moment of inertia of a cross section of the stock with respect to the normal to the neutral axis. Now

$$M_n = M_0 \sin\phi - Pa\cos\phi = Pa\,(\sin\phi - \cos\phi) \quad \ldots [17]$$

and therefore

$$z_n = \frac{Pa^3}{EI_n}\int_0^\theta (\cos\phi - \sin\phi)\sin(\theta - \phi)\, d\phi$$

$$= \frac{Pa^3}{2EI_n}[\theta(\sin\theta + \cos\theta) - \sin\theta] \quad \ldots\ldots [18]$$

Due to vertical shear we have the detrusion

$$z_s = \frac{Pa\theta}{A_c G} \quad \ldots\ldots [19]$$

For the total deflection we have by Equations [15], [19], and [9]

$$z = \frac{Pa^3}{GI_t}\left[\theta + \cos\theta - 1.5\sin\theta - 1 + \frac{\theta}{2}(\sin\theta + \cos\theta)\right] + \frac{Pa\theta}{A_c G} + \frac{Pa^3}{2EI_n}[\theta(\sin\theta + \cos\theta) - \sin\theta] + ka\,(1 - \cos\theta) \quad \ldots [20]$$

By considerations of symmetry we see that k must have such a value that where $\theta = \pi/2$ at B, $\frac{dz}{ds}$ the slope of the neutral axis

shall be the same as at A where $\theta = 0$. From Equation [20]

$$\frac{dz}{ds} = \frac{dz}{ad\theta} = \frac{Pa^2}{GI_t}\left[1 - \frac{\sin\theta}{2} - \cos\theta + \frac{\theta}{2}(\cos\theta - \sin\theta)\right] + \frac{P}{A_cG} + \frac{Pa^2}{2EI_n}[\theta(\cos\theta - \sin\theta) + \sin\theta] + k\sin\theta \ldots [21]$$

Setting Equation [21] equal to $\frac{P}{A_cG}$ for $\theta = \pi/2$, gives

$$k = \frac{Pa^2}{4}(\pi - 2)\left(\frac{1}{GI_t} + \frac{1}{EI_n}\right) \ldots\ldots\ldots [22]$$

which by Equation [20] gives for the total deflection

$$z = \frac{Pa^3}{GI_t}\left[\theta - 1.5\sin\theta + \frac{\theta}{2}(\sin\theta + \cos\theta) - \frac{6-\pi}{4} \times (1 - \cos\theta)\right] + \frac{Pa\theta}{A_cG} + \frac{Pa^3}{2EI_n}\left[\theta(\sin\theta + \cos\theta) - \sin\theta + \frac{(\pi-2)}{2}(1 - \cos\theta)\right] \ldots [23][8]$$

If the cross section of the ring is a square instead of a circle we must replace $\frac{1}{I_t}$ by $\frac{42.6I_t}{A_c^4}$ in Equations [22] and [23].[9] The values z' of z for $\theta = \pi/2$ are obtained from Equation [23] before and after this substitution. Similarly the numerical values for k are found.

Theory for Oblong Link

The neutral axis of the oblong link here considered mathematically consists of two semicircles of radius a connected by straight sides each of length $2l$. The supports are assumed to be at the middle of the semicircular ends and the loads applied at the midpoints of the straight sides though they might be taken vice versa with the same results. Fig. 11 shows one of the symmetrical quarters of the link. Since the force P at B is equivalent to a force P at C combined with a moment of magnitude P, the physical state of the quadrant of the semicircular end is the same as for the circular ring except that the slope at B is different.

Now the moment M_0 at A is, as before, equal to Pa and the moment M_1 at B is equal to $P(a + l)$. Let y be the deflection below C of any point on the straight side at distance x from C; then

$$EI_n\frac{d^2y}{dx^2} = P(l - x) - M_1 = -P(x + a) \ldots\ldots [24]$$

Integration of this equation with the conditions that the slope is P/A_cG at B and the deflection zero at C gives

$$EI_n\frac{dy}{dx} = \frac{P}{2}[(l + a)^2 - (x + a)^2] + \frac{PEI_n}{A_cG} \ldots\ldots [25]$$

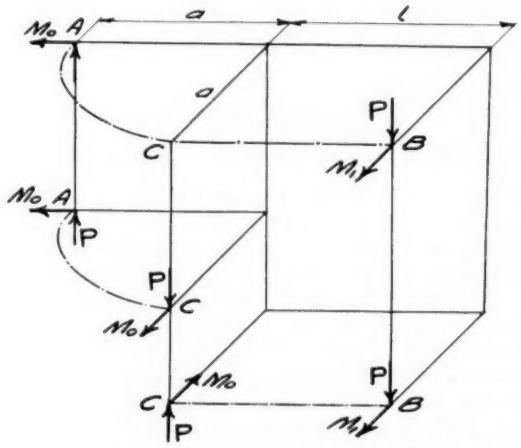


Fig. 11 Neutral Axis of Oblong Ring

and

$$y = \frac{P}{6EI_n}[3(l + a)^2x - (x + a)^3 + a^3] + \frac{Px}{A_cG} \ldots [26]$$

From these we obtain for the slope at C and the deflection at B respectively

$$\frac{dy'}{dx} = \frac{Pl(2a + l)}{2EI_n} + \frac{P}{A_cG} \ldots\ldots [27]$$

and

$$y' = \frac{Pl^2(2l + 3a)}{6EI_n} + \frac{Pl}{A_cG} \ldots\ldots [28]$$

Setting Equation [21], for $\theta = \pi/2$, equal to Equation [27] we find for k, for a circular cross section, the value

$$k = \frac{Pa^2}{4}(\pi - 2)\left(\frac{1}{GI_t} + \frac{1}{EI_n}\right) + \frac{Pl(2a + l)}{2EI_n} \ldots [29]$$

and adding y' from Equation [28] to z from [21] integrated with k from Equation [29] and evaluated for $\theta = \pi/2$, we get for the deflection at B

$$z' = \frac{Pa^3}{GI_t}(\pi - 3) + \frac{Pa^3}{2EI_n}(\pi - 2) + \frac{Pl(l^2 + 3al + 3a^2)}{3EI_n} + \frac{P}{2A_cG}(\pi a + 2l) \ldots [30]$$

for the link of circular cross section.

The deflection at any point on CB at distance x ($x < l$) from C will be given by

$$z = z_c + y \ldots\ldots [31]$$

where z_c is the value of z from Equation [23] for $\theta = \pi/2$ and y is taken from Equation [26].

[8] Since the curve of deflection must be symmetrical with respect to the point where $\theta = \pi/4$ we may apply the following check. Let z_1 be the value of z for $\theta = \pi/4$ and z_2 its value for $\theta = (\pi/2) - \theta$ then we must have $z_1 - z = z_2 - z_1$. This check will be found to be satisfied by Equation [23].

[9] "Strength of Materials," by Arthur Morley, Longmans Green and Co., London, 1911; New York, 1928; art. 112, p. 279.

Vibration of Locomotive Driving Wheels Caused by Unbalance

By O. J. HORGER[1] and C. W. NELSON,[2] CANTON, OHIO

Results of road service tests on steam locomotives are presented to show the influence of unbalance in driving wheels in causing vibrations of the wheel on the rail. These results are accompanied by a correlated theoretical study of conditions under which these vibrations occur. A method is given for eliminating such vibrations based on test results. The effect of unbalance and weight of reciprocating parts is also discussed relative to the track and locomotive.

Introduction

IT HAS been known for some time that main driver wheels of steam locomotives intermittently lifted vertically off the rail in road service under certain conditions of speed and counterbalance. When the speed of the locomotive was such that the inertia force of the overbalance, or what was usually underbalance in the main driver wheel, exceeded the load on the wheel, then lifting of the wheel took place. A frequent result of this action was rail damage commonly termed kinks which were located on the head of the rail at intervals equal to the circumference of the driver wheel. It was therefore a relatively simple matter to determine from the driver-wheel diameter of various locomotives which class of locomotive was doing this damage. Kinks which have been found in rail within the last year on some railroads (1, 2)[3] however, have had a spacing along the rail head which is considerably less than the circumference of any driver wheel and any other locomotive or car wheel in operation. In fact the distance between kinks was found to show considerable variations in spacing in any one stretch of damaged track. Theoretical considerations indicated that this damage could be developed by vibrations of the driver wheels at high speeds although the inertia force at such speeds was considered insufficient to cause lifting. Such high speeds are sometimes attained by trains, and high rotative wheel speeds may occur in fast train service due to occasional slipping of the driver wheels produced by grease on the rail at crossings, frosted and wet rail, and so on. Modern high-speed locomotives require judicious handling and alertness on the part of the operating engineer in order to avert or control such slipping tendencies.

As a means of investigating this problem high-speed moving pictures were taken to determine the action between the driver wheel and rail with various steam locomotives[4] in road service at high rotative wheel speeds.

[1] In charge of Railway Engineering and Research, Timken Roller Bearing Company. Mem. A.S.M.E.

[2] Research Department Engineer, Timken Roller Bearing Company.

[3] Numbers in parentheses refer to the Bibliography.

[4] These tests are outlined in much greater detail in reference (1) and only sufficient information on test method is presented here to provide a basis for the technical discussion.

Presented at the National Meeting of the Applied Mechanics Division of The American Society of Mechanical Engineers, New York, N. Y., June 14–15, 1939.

Discussion of this paper should be addressed to the Secretary, A.S.M.E., 29 West 39th Street, New York, N. Y., and will be accepted until January 10, 1940, for publication at a later date. Discussion received after the closing date will be returned.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors, and not those of the Society.

Tests

Photographic Arrangement. Moving-picture cameras were mounted on both sides of the locomotive in a position near the cylinders as shown in Fig. 1. One camera operating at 100 frames per second was mounted on the left side and two cameras with speeds of 100 and of 400 frames per second, on the right side. All were driven by governor-controlled electric motors and remotely operated from batteries and switches located in a car of the train.

Track Conditions. Tests were made on main-line tangent track having 0.18 per cent ascending grade and 100-lb A.A.R.-A rail laid on fully plated 7 × 8-in. hardwood ties. There was at least 12 in. of chat ballast under the ties and this was spot-tamped before the test. Actual values of the modulus of foundation[5] u were measured from tie to tie over a section of the test track and found to vary from 1060 to 3460 lb per in. per in. with an average value of 1500. These values of u are typical of actual conditions existing on present-day well-maintained track.

Locomotives Tested. While six locomotives were tested only two are discussed here.[4] These two locomotives were of the 4–6–4 type used in passenger service and have counterbalance conditions and other characteristics shown in Table 1. Both were identical except that No. 4003 had lightweight reciprocating and rotating parts[6] with a reciprocating weight of 1026 lb per side, an overbalance of 103 lb,[7] and roller-bearing-equipped axles, while No. 3012 had conventional reciprocating parts, 2109 lb per side, friction bearings, and an overbalance of 315 lb.[7]

[5] The modulus of foundation u is defined as the number of pounds of load per linear inch of rail required to depress the rail one inch vertically. The method of measurement was developed by Dr. A. N. Talbot of the University of Illinois in connection with the Special Committee of the A.A.R. on Stresses in Railroad Track and these measurements were made under his direction.

[6] Reciprocating weight includes piston, piston rod, nut, and so on, crosshead and shoes, and the front-end weight of the main rod which is considered as reciprocating.

[7] Overbalance calculated in accordance with A.A.R. method, "Manual of Standard and Recommended Practice," Section F, Association of American Railroads. This is method 1 shown in Table 1.

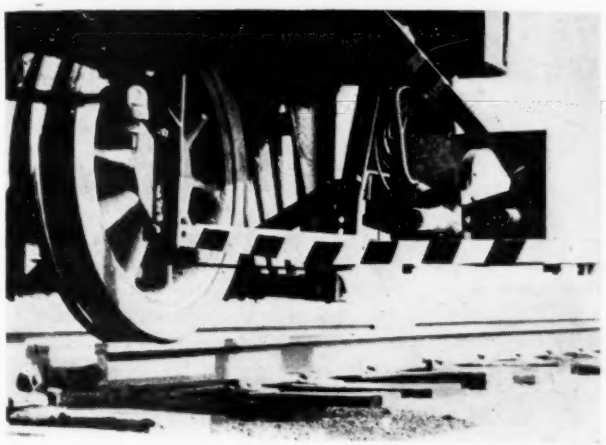


Fig. 1 Camera Arrangement and Location on Extension Platforms in Front of and Below the Locomotive Cylinders

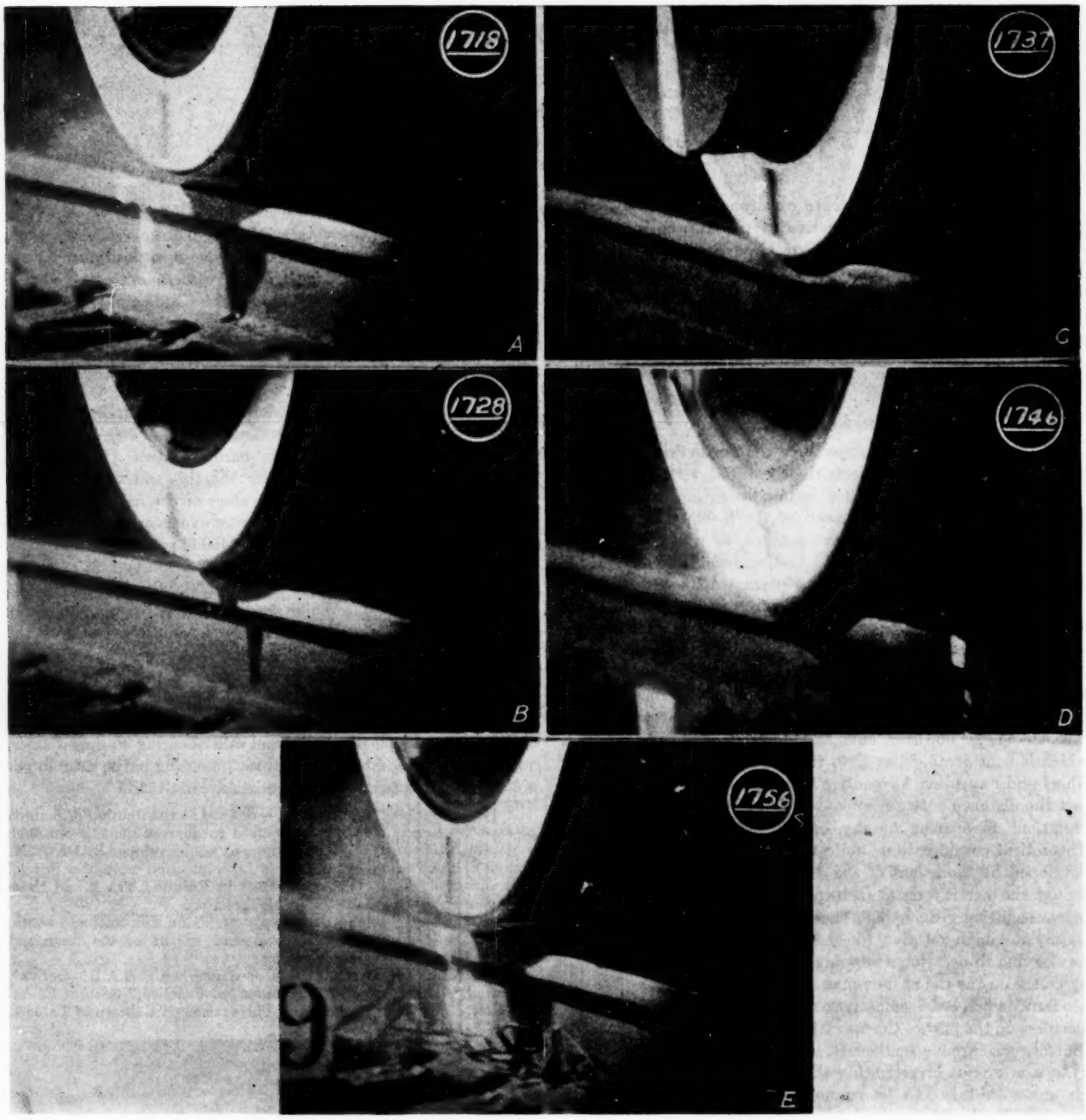


FIG. 2 MAIN DRIVER, LOCOMOTIVE NO. 3012

(Enlargements from 16-mm film taken at 400 frames per sec showing action of wheel on rail for one complete wheel revolution at a slipping speed of 108 mph. Lettered photographs correspond to similarly marked points in Fig. 4.)

Test Procedure. The top of the rail on both sides of the track was greased for a length of from 230 to 504 ft and the locomotive under test, pulling a train of 5 or 6 passenger-equipment cars, entered this greased section at constant speed. Upon entrance of the locomotive into the greased section, the rotative speed of the drivers began to increase. This acceleration (with throttle open) continued even after the locomotive had passed through the greased section due to grease being carried along by the wheels. The locomotive throttle was left open for about 800 ft beyond the greased section to permit slippage of the drivers up to the desired test speed. The motors operating the cameras were started a short distance in advance of entering the greased section. In order to correlate the wheel action with the position of overbalance and the location along the rails, the driver tires were painted with symbols, and markers were placed at frequent intervals along the track so that they were included in the field of view of the cameras.

Results of Tests. These tests definitely indicate forced vibrations of the unsprung weight of the main-axle assembly on the rail, and a condensed statement of the results of tests at several different speeds for each of the two locomotives tested is given in Table 1. Photographs from enlargements of the 16-mm moving pictures for the right side of locomotive No. 3012 at 108 mph slipping speed are shown in Figs. 2 and 3. Fig. 2 shows one

TABLE 1 DATA ON LOCOMOTIVE TESTS

(Both locomotives are 4-6-4 and identical except that No. 3012 is conventionally equipped and No. 4003 is Timken equipped with driver bearings, light-weight reciprocating parts and rods and crank pins. Cylinders, 25 × 28 in.; boiler pressure 250 psi; diameter of drivers, 78 in.; and tractive effort, 47,700 lb)

Locomotive no.	Main-axle weights, lb	Overbalance in main wheels by different methods[a] Method	Lb	Dynamic augment main driver, lb: At diameter speed	At maximum slip speed	Reciprocating weight per side, lb	Lifting speed,[c] mph: Calculated values Due to vibrations[d]	Due to inertia of overbalance	Tests: Slip speed, mph	Train speed, mph	Remarks	Rails requiring replacement
3012	Sprung 47800	1	315[b]	14200	27200	2109	99	122	98	56	Lifted slightly	0
	Unsprung 21980	2	481[b]	21700	41700	2197	86	99				
	Total 69780	3	R 503	22700	43600	2197	85	96	108	66	Lifted 7/8 in.	39
			L 549	24800	47600	2197	82	92				
		4	R 198	9000	17200	2197	115	154				
			L 475	21400	41100	2197	86	99				
4003	Sprung 47550	1	103[b]	4600	12500	1026	129	213	112	72	Intermittent smoke puff	0
	Unsprung 21640	2	189[b]	8500	22800	1080	114	157			Believed just lifting	
	Total 69190	3	R 200	9000	24200	1080	111	153				
			L 237	10700	28800	1080	107	141	123	78	Lifted slightly	0
		4	R 134	6100	16400	1080	122	187				
			L 382	17300	46600	1080	93	111	128	81	Lifted 7/8 in.	0

[a] Method 1 is that now recommended by A.A.R. for cross counterbalance—overbalance given in counterbalance plane.
Method 2 same as 1 except correction is made for proper division of main-rod weight into rotating and reciprocating portions—overbalance given in plane of rail.
Method 3 same as 2 except correction is made for proper balance of eccentric rod and crank—overbalance given in plane of rail.
Method 4 by static method—50 per cent of reciprocating balance distributed equally on all drivers which gives 351 lb nominal overbalance on No. 3012 and 171 lb on No. 4003. Overbalance values given by method 3 were used to determine the actual overbalance present.
[b] Two thirds of the eccentric-crank weight was used for No. 3012 and one half for No. 4003.
[c] Speed at which the main driver just leaves the rail.
[d] Average track modulus of 1500 lb per in. per in. was used here to calculate the lifting speeds.

revolution of the main wheel, with the wheel off the rail during most of the revolution, and Fig. 3 shows all three drivers. Figs. 4 and 5 show graphical records of wheel lift from the rail for both locomotives as measured from the 16-mm film projected on a screen.

Discussion of Tests

Comparison of Two Locomotives. It is evident from Table 1 that (*a*) locomotive No. 4003 went into violent vibrations at a speed which is 20 mph faster than for similar vibration of No. 3012 and (*b*) rail kinking resulted from the 7/8-in. wheel lift of No. 3012 but no rail damage developed with 3/4-in. wheel lift of No. 4003. The higher speed required to produce nearly the same wheel lift on locomotive No. 4003 may be explained by the lower overbalance[7]—103 lb for No. 4003 and 315 lb for No. 3012. The manner in which the amount of overbalance influences the speed at which lifting of the wheel begins will be discussed later. Some consideration will be given also to what happens after the wheel leaves the rail but theoretical investigations have not been carried far enough to state whether the amount of rail damage is determined only by the maximum wheel lift. The difference in rail damage may have been due almost entirely to the from 15 to 20 per cent lower wheel lift for locomotive No. 4003, or the difference in the values of overbalance which produced these wheel lifts may also have contributed to the difference in rail damage.

Method of Calculating Overbalance. There is no unanimous opinion among concerned American engineers as to the proper method of calculating the amount of overbalance in main driver wheels. In view of the importance attached to the value in pounds of the overbalance in causing wheel lifting it becomes necessary to define what is meant. The usual meaning of the term overbalance is used here and refers to the weight added over that required to balance the rotating parts; this added weight is used to balance a portion of the reciprocating parts. There are two general methods of calculating overbalance: (*a*) The static method which balances for forces only and neglects couples produced by the out-of-plane positions of various parts of the rotating axle assembly and (*b*) the cross-counterbalance method which balances for both forces and couples and requires considerably more weight in the balance than the static method. It is not intended to discuss the details of these two methods here as earlier writers have covered this field (3, 4, 5, 6, 7, 8). Neither

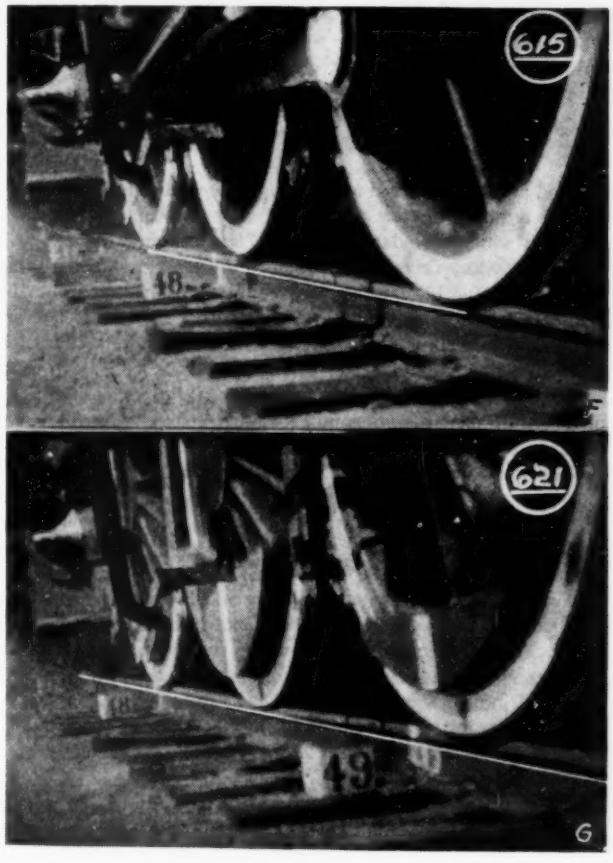


Fig. 3 Three Drivers, Locomotive No. 3012

(Enlargements from 16-mm films taken at 100 frames per sec showing curvature of rail under main wheel at a slipping speed of 108 mph. Letters *F* and *G* correspond to similarly marked points in Fig. 4.)

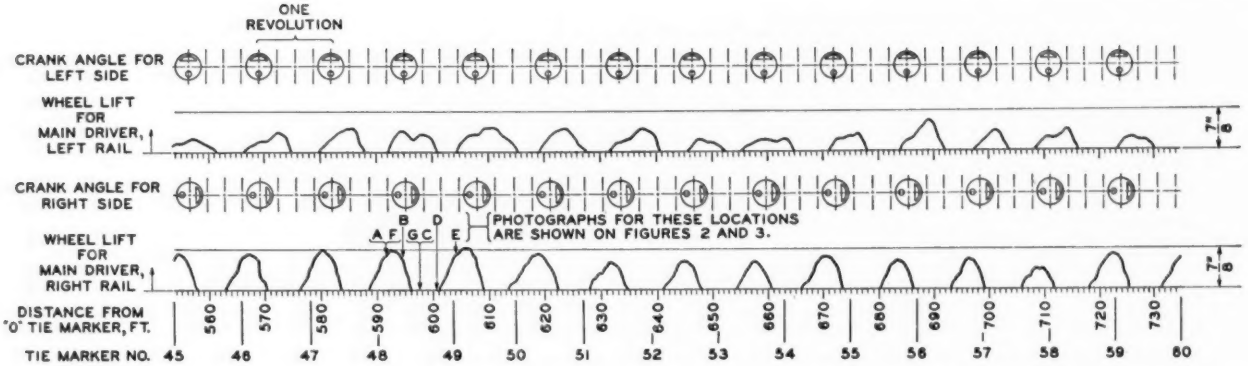


FIG. 4 WHEEL-LIFT CURVES MEASURED FROM MOVING-PICTURE FILM FOR LOCOMOTIVE NO. 3012 AT 108 MPH MAXIMUM SLIPPING SPEED
(Length of greased section, 230 ft, from tie marker 0 to tie marker 19. Train speed, 66 mph. Measurements of lift on left side are only approximate.)

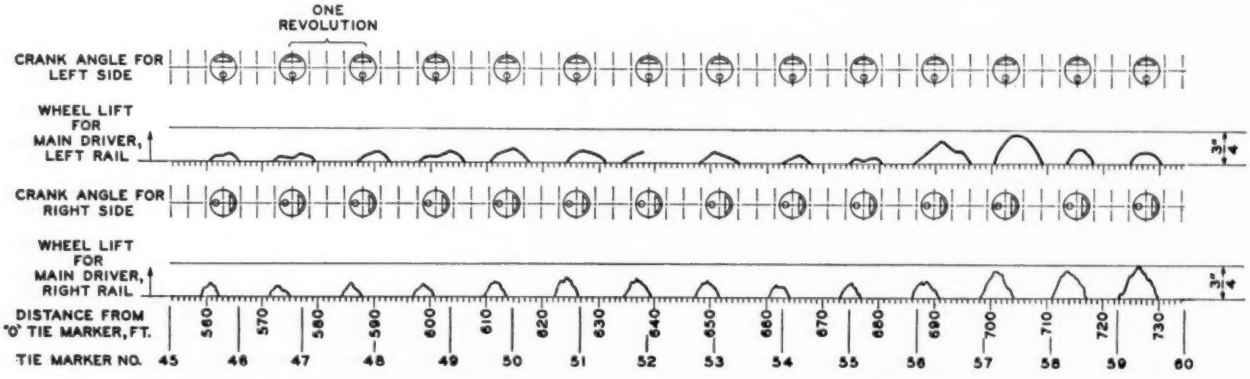


FIG. 5 WHEEL-LIFT CURVES FOR LOCOMOTIVE NO. 4003 AT 128 MPH MAXIMUM SLIPPING SPEED
(Length of greased section, 504 ft, from 274 ft before tie marker 0 to tie marker 19. Train speed, 81 mph. Measurements of lift on left side are only approximate.)

method as generally used is correct. Some defense of this statement is required in view of the thousands of locomotives operating today which are balanced by one of these two methods.

Obviously any driver-axle assembly to be in equilibrium must be balanced for both forces and couples. In this respect the principle of the cross-counterbalance method is correct but assumptions made in carrying out the calculations are often much in error. The static method while basically incorrect is reconcilable; when corrections are made for improper assumptions practiced in usual cross-counterbalance calculations for main drivers, one finds the actual overbalance much higher than when these corrections are omitted, and this actual overbalance may represent a balance condition about as bad as that resulting when the locomotive is statically balanced on the basis of the same improper assumptions.

In order to clarify these discrepancies in balance the two locomotives used in these tests have been calculated for overbalance in accordance with several methods as shown in Table 1. Method 3 is considered a correct statement[8] of the overbalance in these two locomotives as tested. The other methods are given as a matter of information since they are the ones generally followed in practice and with which most engineers are familiar.

Speed at Which Wheel Begins to Lift. The problem of determining the speed at which the driver wheel just begins to leave the rail is treated on the simplified assumption that the locomotive and track structure comprise a system having a single degree of freedom. The vertical motion of the sprung weight of the locomotive is neglected and the problem is reduced to that of determining the motion of the unsprung mass of the axle assembly operating between the driver springs, interposed between the sprung and unsprung masses, and an elastic foundation—the track structure. The overbalance[9] in the wheel produces forced vibrations and if the driving wheel rotates in the region of the natural frequency of this system then the axle assembly will develop severe vibrations on the rail.

Equation 3 and its derivation predicting the speed at which the wheel just begins to leave the rail is given in the Appendix along with remarks relative to the assumptions made. Considering this equation it is apparent that the principal factors influencing the speed at which wheel lifting is initiated are (*a*) modulus of track foundation, (*b*) amount of overbalance, and (*c*) unsprung weight of axle assembly. The relative importance of these factors, for the conditions presented by the locomotives tested and many other locomotives in operation having similar physical characteristics, are shown by curves in Figs. 6 and 7. The influence of various values of modulus of track foundation on the speed at which the wheel just leaves the rail is indicated in Fig. 6 where it is apparent that this speed is little influenced by the track modulus for the larger values of overbalance and that the amount of overbalance has a considerably greater effect on

[8] Footnote in Table 1 partially describes method 3 which includes certain corrections for (*a*) true rotating weight of back ends of main rod and eccentric rod determined from swinging the rods as pendulums per method outlined in reference (5), and (*b*) the eccentric-crank weight was considered acting at its center of gravity instead of assuming an arbitrary allowance acting at crank-pin radius as per methods 1 and 2. It has been assumed without investigation that other rotating parts are properly balanced.

[9] While term overbalance is used throughout this paper the word may be used interchangeably with underbalance, except for the phase of the vibration caused thereby.

this speed. Fig. 7 shows how variations in unsprung weight of the driver-axle assembly influence the speed at which the wheel just lifts for both large and small values of overbalance and spring stiffness of track structure. It is obvious that the value of the unsprung weight has little effect on the speed at which lifting occurs for any stiffness of track shown when the overbalance is large; lower values of overbalance, however, indicate a practical effect on speed for various values of unsprung weight and track stiffness.

After Wheel Begins to Lift. An interesting effect can be observed in Fig. 3 in which the curvature of the rail is shown for two positions of the counterbalance. When the counterbalance is up and the wheel off the rail, the rail is bowed up under the main wheel as shown in (*F*); this condition is emphasized by the straight line drawn tangent to the bottom of the rail head under the front and back driving wheels. Fig. 3 (*G*) illustrates how the rail is bowed down under the main wheel when the counterbalance is down.

Several questions arise as to what happens when the speed is

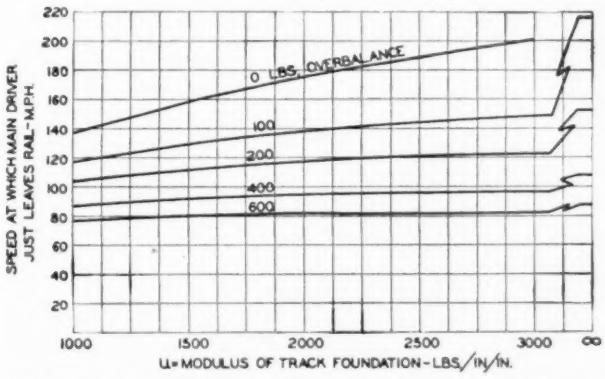


FIG. 6 CALCULATED EFFECT OF AMOUNT OF OVERBALANCE ON SPEED AT WHICH WHEEL JUST LEAVES THE RAIL
(D = 78, R = 14, K = 10,500, S = 24,900, W = 10,900, 100-lb rail.)

increased beyond that required just to cause the wheel to leave the rail such as (*a*) whether the overbalance is in the "up" or "down"[10] position when the wheel lift is about a maximum and (*b*) whether or not higher speeds permit passage through the range of large vibrations and again allow the wheel to remain on the rail without lifting. Sufficient test data do not exist to answer these questions but an attempt has been made to predict theoretically what will happen in the curves of Fig. 8 on the basis of the conditions and assumptions given in the derivation of the formulas presented in the Appendix.

(*a*) Fig. 8 shows, for various speeds and for several values of overbalance, the fraction of a wheel revolution that the wheel is off the rail. It will be noted that each overbalance value is represented by two separate curves, one on the left and one on the right of the single curve for zero overbalance, and that all these curves cross at a common point. In all cases the curve to the left indicates the fraction of the revolution that the wheel is off the rail with the overbalance in the up position, while the right-hand curve indicates that, at higher speeds, the fraction of the revolution that the wheel is off the rail occurs with the overbalance in the down position. The reversal in phase of vibration occurs when the value of this fraction reaches 0.79. It has been reported that actual slipping tests have shown that the balance was in the down position when the wheel lift was a maximum, and Fig. 8 indicates the explanation for this phenomenon. Unfortunately

[10] The "down" position means that the overbalance resultant force is in a direction toward the rail and "up" that the resultant is in a direction away from the rail.

Fig. 8 does not consider the modifying influence that the unknown factors of damping and other conditions mentioned in the Appendix will have on these curves. Oscillator[11] tests could be made to determine these factors so that corrected curves could be obtained representing practical conditions. In such cases it would be expected that the two branches of each curve would be replaced by one single continuous curve lying in the region below the branches of the present curve. The maximum wheel lift,

[11] An oscillator has been designed by the authors which consists of an adjustable out-of-balance-weight device which can be attached to the main crankpin of a locomotive and rotated at various known speeds. The locomotive would be stationary on the track and the out-of-balance rotating weight would produce forced vibrations of the main driver on the rail. Analysis of data from such tests of locomotives on track structures having various spring-stiffness values would permit a determination of more practical curves in Fig. 8. In addition valuable data could be obtained on the effect of other factors influencing stresses in rails produced in service operation of locomotives where wheel lifting is not necessarily involved.

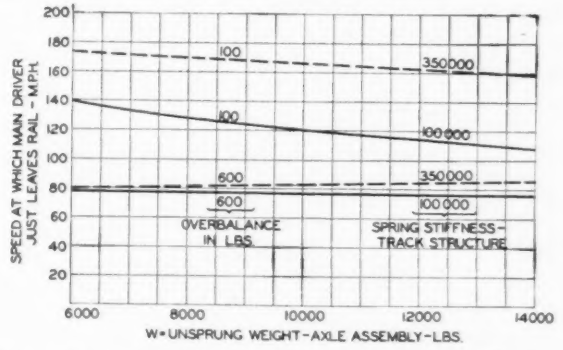


FIG. 7 CALCULATED EFFECT OF AMOUNT OF UNSPRUNG WEIGHT ON SPEED AT WHICH WHEEL JUST LEAVES RAIL
(R = 14, D = 78, S = 23,900, K = 10,500)

Rail weight per yard, lb	Foundation modulus, u K_R = 100,000	K_R = 350,000
90	1120	5960
105	1010	5380
112	930	4910
127	850	4700
131	840	4430
152	740	3920

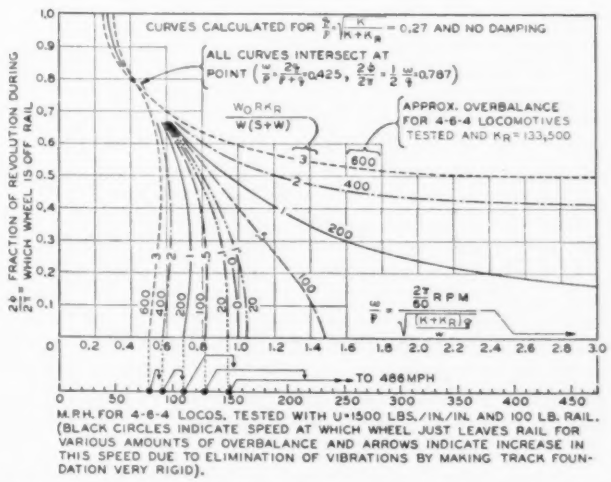


FIG. 8 CALCULATED CURVES FOR VARIOUS AMOUNTS OF OVERBALANCE SHOWING WHEN WHEEL JUST LEAVES RAIL, AND THE FRACTION OF REVOLUTION THAT WHEEL IS OFF RAIL, FOR VARIOUS SPEEDS

(For each overbalance, the curve has two branches except for zero overbalance. For the branch to the left of the curve for zero overbalance, the wheel is off the rail when the overbalance is up. For the other branch, the wheel is off when the overbalance is down. Other branches of the curve are not shown but probably have little practical significance.)

instead of occurring with the overbalance either up or down would lag the disturbing force by an angle which would gradually increase with speed.

(*b*) Fig. 8 indicates that once the wheel has lifted it will not stop lifting except at speeds so high as to be of no practical interest and then only if the overbalance is less than a certain amount. According to Fig. 8 in which friction is neglected, if a locomotive

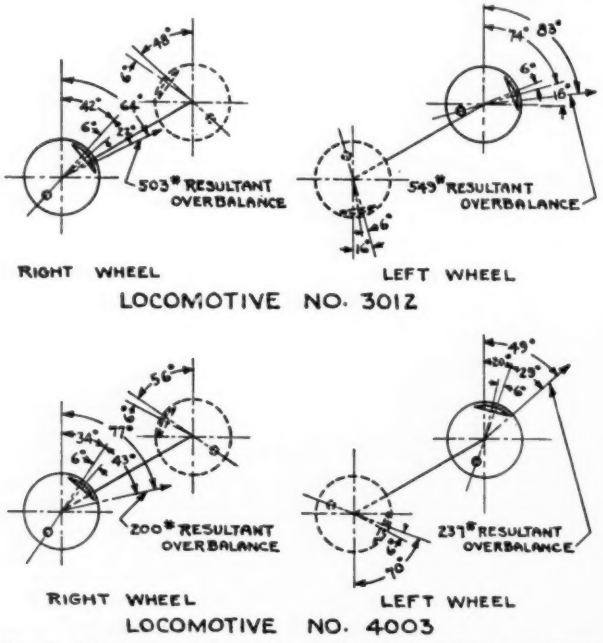


FIG. 9 LOCATION OF OVERBALANCE RESULTANT FORCE IN RIGHT AND LEFT MAIN DRIVERS FOR MAXIMUM WHEEL LIFT OFF RAIL FOR LOCOMOTIVE NO. 3012 AT 108 MPH SLIPPING SPEED AND LOCOMOTIVE NO. 4003 at 128 MPH SLIPPING SPEED

(Each sketch of a pair of wheels indicates by full lines the wheel which is a maximum distance off the rail and dotted lines show relative position of opposite wheel which is not a maximum distance off the rail.)

of the type tested here had about zero overbalance the wheel would start leaving the rail at about 157 mph but would stop lifting after passing through this speed.

The location of the resultant overbalance force acting on each of the right and left main driver wheels for maximum wheel lift off the rail is shown in Fig. 9. It may be noted from the position of the resultant overbalance force that the maximum lift lags behind this force. If higher speeds had been attained than made in these tests it would be expected that the overbalance would lead the maximum lift by an even larger angle than was found in these tests. This phenomenon was discussed in connection with Fig. 8.

Effect on Track and Locomotive. Lifting of the wheel in service does not result in derailment of the locomotive since the other wheels and guiding wheels remain on the rail. The rail was kinked as shown in Fig. 10 by locomotive No. 3012 (overbalance of 315 lb) at 108 mph; at slightly lower speeds the rail was marked on the head as shown in Fig. 11. No rail damage requiring replacement occurred with locomotive No. 4003 in which the overbalance was 103 lb.[7]

The magnitude of the stresses introduced in the rail under such wheel vibrations as shown in Figs. 2 and 3 are not known but even at speeds much lower than that required for the wheel to lift, tremendous variations in rail contact pressure occur during one wheel revolution. This change in pressure is to a large extent due to the resultant of the forces: (*a*) dynamic augment of the overbalance discussed here and, due to angularity of the main rod, vertical components of (*b*) the piston thrust and (*c*) the inertia force on the reciprocating parts.[12] This resultant however is modified by dynamic effects. In the balancing of drivers of locomotives similar to No. 3012 it is sometimes the practice to merely reduce the amount of overbalance conventionally used so as to decrease the force in (*a*) but this procedure does not reduce the resultant rail force in proportion to the change in overbalance or as much as would be anticipated. This is due to the large value of the inertia-force component (*c*) which does not change and the effect it has on the resultant. The desired procedure to reduce the resultant dynamic force on the rail is to decrease the overbalance and at the same time reduce the weight of reciprocating parts as was done with locomotive No. 4003; in this manner both component forces (*a*) and (*c*) are reduced so as to make the resultant dynamic rail force only from a half to a third of the value obtained without reducing the reciprocating weight.

Unbalanced forces have an important effect on locomotive

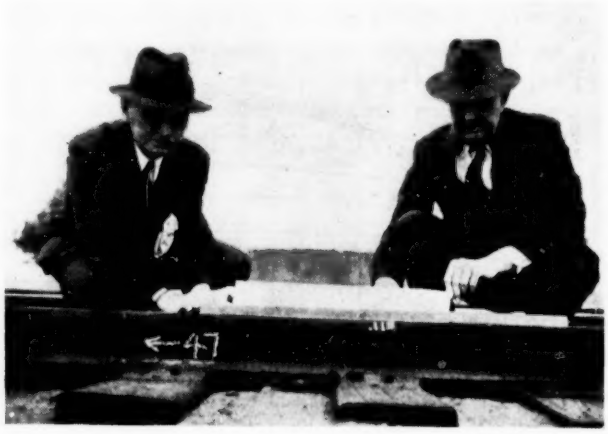


FIG. 10 RAIL KINK PRODUCED BY LOCOMOTIVE NO. 3012 AT 108 MPH



FIG. 11 RAIL MARK PRODUCED BY LOCOMOTIVE NO. 3012 BEFORE REACHING 108 MPH

operation and maintenance, as well as on track structure but the limitations of overbalance and weight of reciprocating parts for various degrees of difficulties and expense involved with locomotive and track are unknown. In this respect lighter reciprocat-

[12] The effects of (*b*) and (*c*) on wheel vibrations have not been considered here because their frequency is twice that of (*a*) and their influence may be small except for low overbalance values. These forces are shown by curves in reference (1) which show the component as well as the composite forces acting on the rail and unbalanced forces tending to shake the locomotive.

ing parts also reduce the unbalanced distributing forces[12] on the locomotive.

Acknowledgment

Considerable credit is due T. V. Buckwalter, vice-president of Timken Roller Bearing Company, under whose direction this photographic study was made. The tests were actually made under the general direction of H. H. Urbach, mechanical assistant to the executive vice-president of the Chicago, Burlington and Quincy Railroad, and F. Zeleny, engineer of tests. Considerable calculating work was done by R. Morgan of the Timken railway engineering department. Valuable discussions were also had with Professor S. Timoshenko and Professor J. Ormondroyd.

Appendix

Figs. 6, 7, and 8 are based on calculations made from theoretical equations for the vertical motion of an unbalanced locomotive driving wheel. In the derivation of these equations, several simplifying assumptions are made. Friction is assumed to be zero and the mass of the rail and of moving parts of the track foundation are neglected and accordingly there can be no energy loss due to impact when the wheel strikes the rail after having lifted from it. It is assumed that the vertical motion of the sprung weight of the locomotive is negligible in the speed range considered. The unsprung weight of an axle, a pair of wheels, and so on, is divided into two equal weights assumed to be concentrated, one in the plane of each rail. Any vertical movement of the driver springs as a whole which may exist due to functioning of the equalizer system is neglected. Vertical forces which act on a main driver due to angularity of the main rod combined with piston thrust due to steam pressure, and inertia forces on the reciprocating parts are neglected. The rail on its foundation is assumed to have a linear load-deflection curve and the effect of rail deflection due to one driver on vibrations of the other drivers is neglected. It is also assumed that the driver tires are perfectly round and that no irregularities are present in the rail surface.

With these and other assumptions, the problem of the vertical motion of a driver wheel due to unbalance reduces to the case of motion of a single mass held between two compressed springs with a sinusoidal disturbing force applied to the mass. For a certain amplitude of vibration, the mass could intermittently lose contact with one of the springs and this would correspond to the lifting of a driver wheel from the rail.

Notation

W = unsprung weight per driver wheel, lb
S = sprung weight per driver wheel, lb
K = driver-spring stiffness, lb per in.
K_R = stiffness of rail on its foundation, assumed to be the same at all points, lb per in.
g = acceleration due to gravity, in. per sec per sec
R = crank radius, in.
D = driver diameter, in.
W_O = unbalanced weight at crank radius, per driver wheel in plane of rail, lb
ω = angular velocity of driver, assumed to be constant, radians per sec
ωt = angular displacement of driver taken as zero when the unbalance is on the vertical center line and above the horizontal center line of the wheel, radians
x_1 = upward displacement of driver from its position of rest for the fraction of a revolution during which the wheel is on the rail, in.
x_2 = upward displacement of driver from its position of rest for the fraction of a revolution during which the wheel is off the rail, in.
ϕ = half the angle during which the wheel is off the rail, radians.

In addition, the following abbreviations will be used

$$p = [(K + K_R)g/W]^{1/2}$$

$$q = (Kg/W)^{1/2}$$

so that $$q/p = [K/(K + K_R)]^{1/2}$$

The differential equation of motion for the wheel when it is on the rail and rotating at a constant speed is

$$\frac{W}{g}\frac{d^2x_1}{dt^2} + (K + K_R)x_1 = \frac{w^2RW_O}{g}\cos \omega t$$

or

$$\frac{d^2x_1}{dt^2} + p^2x_1 = \frac{\omega^2RW_O}{W}\cos \omega t$$

The solution of this equation is (9)

$$x_1 = A \cos pt + B \sin pt + \frac{RW_O}{W}\frac{\omega^2}{(p^2 - \omega^2)}\cos \omega t \quad \ldots [1]$$

where A and B are constants of integration. If the wheel does not leave the rail during any part of a revolution (9) and the vibrations of the wheel have reached a steady state, then A and B are zero so that

$$x_1 = (RW_O/W)\ [\omega^2/(p^2 - \omega^2)]\cos \omega t$$

and

$$(x_1)_{max} = (RW_O/W)\ [\omega^2/(p^2 - \omega^2)]$$

If the wheel just leaves the rail instantaneously once during each revolution, then the value of $(x_1)_{max}$ must equal the rail deflection under the driving wheel due to the static wheel load or

$$(RW_O/W)\ [\omega^2/(p^2 - \omega^2)] = (S + W)/K_R$$

and from this equation, the angular wheel velocity at which the wheel will just leave the rail is

$$\omega = \left\{\frac{(K + K_R)g}{W + [K_R/(S + W)]RW_O}\right\}^{1/2} \quad \ldots\ldots [2]$$

which corresponds to a train speed in miles per hour of

$$\text{mph} = \frac{D}{35.2}\left\{\frac{(K + K_R)g}{W + [K_R/(S + W)]RW_O}\right\}^{1/2} \quad \ldots\ldots [3]$$

The curves of Figs. 6 and 7 were calculated from Equation [3].

For higher speeds, the wheel will be off the rail during a portion of each revolution. The constants A and B in Equation [1] will no longer be zero and, assuming that a steady state of vibrations has been reached, can be determined from the conditions

$x_1 = (S + W)/K_R$ when $\omega t = \phi$ and $dx_1/dt = 0$ when $\omega t = \pi$.

Then

$$x_1 = \frac{\left[\frac{S + W}{K_R} - \frac{RW_O}{W}\frac{\omega^2}{(p^2 - \omega^2)}\cos \phi\right]}{\cos[(p/\omega)\ (\pi - \phi)]}\cos\left[\frac{p}{\omega}(\pi - \omega t)\right] + \frac{RW_O}{W}\frac{\omega^2}{(p^2 - \omega^2)}\cos \omega t \quad .. [4]$$

and this equation is applicable only from $\omega t = \phi$ to $\omega t = 2\pi - \phi$.

When the wheel is off the rail, from $\omega t = -\phi$ to $\omega t = \phi$, the equation of motion is

$$\frac{d^2x_2}{dt^2} + q^2x_2 = \frac{\omega^2 RW_O}{W}\cos\omega t - \frac{g(S+W)}{W}$$

When this equation is integrated and the conditions

$x_2 = (S+W)/K_R$ when $\omega t = \phi$, and $dx_2/dt = 0$ when $\omega t = 0$ are applied to determine the constants of integration, it is found that

$$x_2 = \frac{\left[\frac{S+W}{K_R} + \frac{S+W}{K} + \frac{\omega^2}{(\omega^2 - q^2)}\frac{RW_O}{W}\cos\phi\right]}{\cos(q\phi/\omega)}\cos qt - \frac{S+W}{K} - \frac{RW_O}{W}\frac{\omega^2}{(\omega^2 - q^2)}\cos\omega t \quad [5]$$

One condition remains to be satisfied, namely, that the vertical velocity of the wheel obtained from Equation [4] shall equal that obtained from Equation [5] when $\omega t = \phi$, i.e., $dx_1/dt = dx_2/dt$ when $\omega t = \phi$. To satisfy this condition it is found that

$$\frac{K_R RW_O}{W(S+W)} = \frac{\left(\frac{p^2}{\omega^2} - 1\right)\left\{\tan\left[\frac{p}{\omega}(\pi - \phi)\right] + \frac{p}{q}\tan\frac{q\phi}{\omega}\right\}}{\cos\phi\left\{\tan\left[\frac{p}{\omega}(\pi - \phi)\right] - \frac{q}{p}\left(\frac{p^2 - \omega^2}{\omega^2 - q^2}\right)\tan\frac{q\phi}{\omega}\right\} + \frac{\omega}{p}\left(\frac{p^2 - q^2}{\omega^2 - q^2}\right)\sin\phi} \quad [6]$$

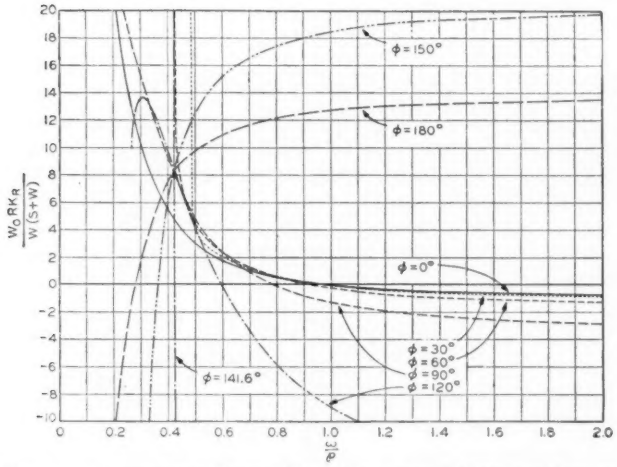


FIG. 12 CALCULATED CURVES FROM EQUATION [6] USED TO OBTAIN FIG. 8

(Only one branch of each curve is shown. Other branches in addition to the branch shown are believed to have little practical significance for the problem of vibration of a locomotive driving wheel.)

and this equation can be solved easily for the value of $K_R RW_O/W(S+W)$ for any assumed values of ϕ and ω/p after assigning a value to the constant $q/p = [K/(K+K_R)]^{1/2}$.

In Fig. 12, values of $K_R RW_O/(S+W)W$ are plotted against ω/p for values of ϕ from 0 to π. These curves are for the case of $q/p = 0.27$ or $K/K_R = 0.0787$ which corresponds to $K = 10{,}500$ lb per in. and $K_R = 133{,}500$ lb per in.; the latter figure is the rail stiffness for 100-lb rail and a track modulus of 1500 lb per in. per in. track modulus. Some of the curves have other branches than the one shown but these additional branches are omitted to simplify the figure and are believed to have little practical significance since they occur at speeds less than that at which the wheel just leaves the rail, except when $K_R RW_O/(S+W)W$ is too large to be of interest. The curves of Fig. 12 were used as a means of obtaining Fig. 8, the significance of which is discussed in the body of the paper. In Fig. 8, two branches of the curves are shown for each value of $K_R RW_O/(S+W)W$ and additional branches are omitted as in Fig. 12. Having Fig. 8, it would be possible to return to Equations [4] and [5] and calculate the maximum wheel lift and the maximum rail depression as well as the value of the rail load corresponding to this depression. However such investigations have not been completed and it may be more important first to obtain curves such as those of Fig. 8 for less idealized conditions, perhaps with some values assumed for friction and the energy loss due to impact when the wheel strikes the rail.

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Discussion

The Determination of the Expansion Forces in Piping by Model Test[1]

EVERS BURTNER.[2] Load-deflection tests of several one-plane expansion bends were made in connection with the undergraduate thesis of William Budd and David Cooper, 1936, Department of Naval Architecture and Marine Engineering, Massachusetts Institute of Technology. The agreement between the calculations and measured deflections was satisfactory. While the loading to represent fixed-end conditions was much simpler than the ingenious one outlined by the author, particular care was taken in the construction of the model bends. They were annealed after bending, being packed in cast-iron chips to avoid formation of scale. It would be interesting to learn whether the Westinghouse Electric & Manufacturing Company has found it advisable to use similar care in constructing more complicated three-dimensional bends.

It is difficult to fix the ends of the tubing without introducing high local stresses at this point. From the appearance of Fig. 6, the writer assumes that this end of the pipe bend is held by the friction or press fit of the pin-supported bushing.

F. M. HILL.[3] The model method for determining forces and moments in three-dimensional pipe runs will unquestionably appeal to the practical-minded engineer because of its slide-rule simplicity and general reliability. The solution of the branch problem is now within easy reach of the designing engineer.

The foundation for work with models was laid by the work already done in the field of mathematics and graphics by such men as Hovgaard, Crocker, McCutchan, Wahl, Karelitz, Marchant, Shipman, Mitchell, Cocks, and others. It is upon their work that the intelligent development and use of the model method stands.

The question arises, to what extent has the author verified his model answers with actual installations or with calculated solutions? This is an important consideration in the minds of inquiring engineers.

The writer is in general agreement with the author's method of translating the forces and moments of the model to those of the actual pipe run. However, his statement: "If the model were exactly to scale and of the same material as the full-size pipe, the forces on the full-size pipe would simply equal the model forces divided by the scale of the model," appears to be misleading or at least incomplete. Should it not be amplified to include moments, temperature, and deflections, since these are essential factors in every problem?

The following revision is suggested: If the model were exactly to scale in all dimensions, of the same material, at the same temperature, and deflected the scale amount, the forces and moments on the full-size pipe would equal the model forces and moments divided by the scale squared and the scale cubed, respectively.

Since departures are generally made in one or more of the factors mentioned, it is quite necessary that the operator understand the fundamental mathematical relationships.

Concerning the experimental error mentioned by the author in the use of his model, the writer's attention has been drawn to Fig. 1, showing a model pipe run set up for testing. It appears that small screws have been used at both ends of the pipe to secure it to the slide base and to the measuring head or frame. Can these screws be depended upon for positive fixation? The sliding head appears to be tilted at an angle that would hardly correspond to the type of pipe run being tested. The bend in the pipe adjacent to the measuring head appears to be considerably flattened in cross section.

The writer has been working for several months with a model-testing and analyzing device, differing considerably from that of the author. Different scales have been used deliberately to obtain exaggerated readings, for the purpose of reducing the relative error in taking measurements, and to make various influences more pronounced for analytical observations. Solid steel die-drawn rods approximately $^1/_4$ in. in diameter, bent to shape at a red heat and further stress-relieved by heat-treating, have given results well within the practical limits of power-plant designing. The device will accommodate tubing as well as solid rods. No practical advantage, however, has been found in the use of tubing over solid rods, while considerable advantage has been found in the use of rods which make it possible to duplicate results by testing several models made in duplicate. The uniformity of cross section of die-drawn rods, together with the uniformity with which they can be bent to shape, makes their use an outstanding feature.

The reliability of the model method in my experience is one of its outstanding qualities. The model-testing and analyzing device, which the writer has developed, is particularly well adapted to demonstrating visually the influence or the effect produced by various changes in loadings, and the relative importance of the various loadings.

From the standpoint of actual practice, several factors enter the problem which determine the degree of accuracy to be sought in any solution. A mathematical solution to five significant figures is obviously not consistent when applied to a pipe run having a wall thickness variation of 12.5 per cent, the allowable mill tolerance, which may or may not result in considerable eccentricity. The cross section of the pipe in a bend may be quite elliptical and eccentric instead of round and concentric as assumed in mathematical formulas.

The time element and the attendant quality of creep in the metal gradually changes the initial stresses toward an equilibrium which may be at considerable variance with answers obtained by any method of solution. The actual pipe run also differs from the run of the problem usually assumed, in that the actual run is stiffened by the installation of valves and restraining and supporting pipe hangers. Ends assumed to be fixed are actually only partially fixed. Finally, the last weld or connection made in the pipe run during construction may introduce stresses and strains of considerable magnitude.

Conservatively speaking, it has been the writer's experience that a day spent with the model-testing and analyzing device on a three-dimensional run will produce more answers of a higher degree of reliability and of greater practical accuracy than one to three weeks spent with calculation methods by the average

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[1] Paper by Harold W. Semar published in the March, 1939, issue of the JOURNAL OF APPLIED MECHANICS, Trans. A.S.M.E., vol. 61, 1939, p. A-21.

[2] Associate Professor, Department of Naval Architecture and Marine Engineering, Massachusetts Institute of Technology, Cambridge, Mass.

[3] Engineer, Commonwealth & Southern Corporation, Jackson, Mich. Mem. A.S.M.E.

engineer. The results, to say the least, have been most satisfactory.

Whatever method is employed to determine stresses and strains in a pipe run, it should be a reliable standard on which rational judgment can be based. In the writer's opinion the model method is the practical engineer's handy tool for this purpose. In the solution of branch problems it is his only workable method at the present time.

WILLIAM HOVGAARD.[4] In view of the complications and elaborate calculations associated with a theoretical determination of the expansion stresses and forces of even an idealized piping system, engineers will undoubtedly welcome a mechanical method such as the one here proposed, provided the results are reliable and sufficiently accurate.

The apparatus described in the paper seems to be simple and robust and the mathematics is simple. The principal source of difficulties in pipe tests lies in obtaining absolute fixity of the ends of the pipe, while at the same time obtaining a record of the reactions. These difficulties seem to be overcome here.

The movable end of the model pipe is clamped in what appears to be a solid block bolted to a tool slide. Thus it can hardly be subject to any angular or linear deflection.

At the fixed end the twofold object of holding the pipe in an angular direction and of measuring the forces is attained ingeniously by employing the same struts or pins, which hold the pipe fixed in direction, as transmitters of the forces, which it is desired to measure. It is not quite clear how R_y is obtained while at the same time the end of the pipe is prevented from moving in its own direction.

It would be interesting to know whether a comparison has been made between the results obtained by a model and those obtained by a purely theoretical calculation.

G. B. KARELITZ.[5] It may be of interest to consider the case of high-pressure steam pipes. For such pipes the ratio

$$\lambda = tR/r^2 = t/r \times R/r$$

is of a definite order of magnitude. R/r is usually about 10 (R equals five diameters of the pipe), while t/R is from $^1/_8$ to $^1/_4$. This gives values of λ from 2 to 2.5. The coefficient of increased flexibility is $k = \dfrac{10 + 12\lambda^2}{11 + 12\lambda^2}$; it varies from $k = 1.1$ for $\lambda = 2$, to $k = 1.11$ for $\lambda = 2.5$. The variation is small. Therefore, if the author's formula gives, say, a scale factor $s = 18.5$, no appreciable error would result in taking a convenient scale factor, such as $s = 20$, close to 18.5. Of course, this does not hold for low-pressure thin-walled pipes.

The writer was invited by Sanderson and Porter to determine the expansion forces and stresses for the high-pressure pipes of an installation comprising three leads from the boilers to a distributing manifold and two leads from the manifold to the turbine throttles. The computation tables[6] referred to by the author were set up by the writer for that purpose. The values obtained for the five pipes will be of interest in comparison with those given in the paper. The pipes were 12.75 in. outside diameter; thickness of wall $t = 1.294$ in.; the bends were of 60 in. radius; temperature of steam 935 F, i.e., close to the conditions obtaining in the paper. The pipes were considered without cold spring, but the motion of the boiler headers and turbine throttle as submitted by the respective manufacturers was taken into account.

The forces and moments are given in Table 1.

TABLE 1 FORCES AND MOMENTS IN HIGH-PRESSURE PIPING SYSTEM

Reaction		Lb P_x	Lb P_y	Lb P_z	Lb-in. M_{xy}	Lb-in. M_{yz}	Lb-in. M_{zx}
Boiler leads	No. 2	1520	900	220	705100	120100	20400
	No. 4	680	1180	370	368900	204000	16600
	No. 6	110	1370	430	39500	213500	30800
Turbine leads	No. 1	110	300	1190	33000	2550	110000
	No. 2	470	470	920	7400	123400	1700

NOTE: The signs, i.e., the directions of the forces and moments, are omitted.

The difference between the first three pipes and the last two is that the first ones are very much longer. The magnitudes of the force components (lateral, axial, and vertical) are of the same order as reported by the author, within 2000 lb. These are within the limits usually allowed by manufacturers of turbines or boilers. However, the moments tending to bend or twist the pipe at the header are much larger in the case of the longer boiler leads.

The boiler-builder's specification called for zero twisting moment at the superheater header, which would mean elaborate clamping of the pipe to take up the twist. Joint action resulted in a corresponding change in the design of the boiler to take care of the twisting and bending moments on the superheater header. To the writer's knowledge there has been no leakage of steam from the superheater header in these three boilers.

When a company is confronted frequently with the computation of end reactions of pipes, it would no doubt be economical to build and operate the instrument suggested by the author. However, if only occasional determinations occur, then it is more economical to obtain the answer by computation. In the discussion of the paper[6] in which the writer's tables were published, it was brought out that a number of concerns and individuals were occupied working out similar schemes for computation. "A similar, although somewhat different method has been developed by H. C. E. Meyer, chief engineer of Gibbs & Cox, Inc.;" "...for a number of years, the Westinghouse Electric & Manufacturing Company has used a method very similar to the one given in this paper;" D. R. Rosenheim of M. W. Kellogg Company states that they "are now able to handle more complex pipe lines, involving three or more points of fixation and similar problems in a reasonable length of time." It is reasonable to assume that these methods may be better than the plates set up by the writer. The publication of these methods would be of service to the art.

J. H. MARCHANT.[7] The author's method is particularly desirable for organizations whose demands justify the cost of such model apparatus and the services of an expert to operate it. While favorably impressed with the model method proposed, the writer remains of the opinion that two different methods should be used for the determination of the expansion forces and moments in pipe lines. In this case the author's method should certainly be included.

The method which is used for the determination of the model deflection constants R_x R_y, etc., is interesting. Here, although it appears at first glance the author has assumed that the deflection constants are valid over the range of the actual displacement of the end of the prototype ($e_a = 2.99$ in.), in reality he merely assumes that they are valid in the range 0 to 0.4 in., which is probably well within the elastic range of the model.

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[5] Professor of Mechanical Engineering, Columbia University, New York, N. Y. Mem. A.S.M.E.

[6] "End Reactions and Stresses in Three-Dimensional Pipe Lines," by G. B. Karelitz and J. H. Marchant, JOURNAL OF APPLIED MECHANICS, Trans. A.S.M.E., vol. 59, 1937, p. A-68.

[7] Associate Professor of Mechanical Engineering, Connecticut State College, Storrs, Conn. Mem. A.S.M.E.

In an effort to check the results, the writer has computed the expansion forces and moments exerted by the actual pipe line the author used, by the method referred to in footnote 6 on page A-186. The elastic equations of this pipe line are as follows:

(1) $-131{,}698\ P_x + 140{,}060\ P_y - 1404\ P_z + 9\ M_{yz} + 6.3\ M_{zz} + 1295\ M_{xy} = 0$

(2) $359{,}615\ P_x - 288\ P_y - 144{,}849\ P_z + 9\ M_{yz} + 1289\ M_{zz} + 6.3\ M_{xy} = 0$

(3) $1692\ P_x - 330{,}099\ P_y + 118{,}137\ P_z + 1232\ M_{yz} + 9\ M_{zz} + 9\ M_{xy} = 0$

(4) $159{,}216{,}945\ P_x - 19{,}586{,}892\ P_y - 46{,}411{,}692\ P_z + 1692\ M_{yz} + 359{,}618\ M_{zz} - 131{,}596\ M_{xy} = 21{,}084 \times 10^6$

(5) $-19{,}084{,}038\ P_x + 135{,}216{,}954\ P_y - 39{,}114{,}902\ P_z - 330{,}102\ M_{yz} - 288\ M_{zz} + 140{,}063\ M_{xy} = -38{,}306 \times 10^6$

(6) $-46{,}411{,}666\ P_x - 39{,}114{,}154\ P_y + 45{,}527{,}440\ P_z + 118{,}139\ M_{yz} - 144{,}847\ M_{zz} - 1404\ M_{xy} = 20{,}440 \times 10^6$

The checks for these equations, as based on Maxwell's reciprocal theorem, are given in Table 2.

TABLE 2 EQUATION CHECKS BASED ON MAXWELL'S RECIPROCAL THEOREM

$P_{x1}:M_{xy4}$	—131698:	—131596	$P_{y3}:M_{yz5}$	—330099:	—330102	$M_{yz1}:M_{xy3}$	9.0 :	9.0
$P_{x2}:M_{xz4}$	359615:	359618	$P_{z1}:M_{xy5}$	—1404:	—1404	$M_{yz2}:M_{xz3}$	9.0 :	9.0
$P_{x3}:M_{yz4}$	1692 :	1692	$P_{z2}:M_{xz5}$	—144849:	—144847	$P_{y4}:P_{z5}$	—19586892:	—19084038
$P_{y1}:M_{xy5}$	140060:	140063	$P_{z3}:M_{yz6}$	118137:	118139	$P_{z4}:P_{x6}$	—46411692:	—46411666
$P_{y2}:M_{xz5}$	—288:	—288	$M_{xz1}:M_{xy2}$	6.3 :	6.3	$P_{x6}:P_{y4}$	—39114902:	—39114154

For purposes of comparison, the results of the writer's calculations are given, together with those of the author, in Table 3.

TABLE 3 FORCES AND MOMENTS EXERTED BY PROTOTYPE PIPE

Reaction	Semar	Marchant	Variation, per cent
P_x	470 lb	495 lb	—5
P_y	—980 lb	—941 lb	—4
P_z	1120 lb	949 lb	+18
M_{xy}	145000 lb-in.	155674 lb-in.	—7
M_{yz}	—318000 lb-in.	—344844 lb-in.	+8
M_{zz}	—9600 lb-in.	—29923 lb-in.	+68

The author's paper has been studied carefully without discovering any error in his model criteria or in his method. True, the scale fidelity between model and prototype is not perfectly maintained, i.e., the small bend in the model of $^7/_8$ in. radius corresponds to 17.5 in. radius rather than the required 18 in. radius in the prototype; and one straight section in the model corresponds to 137.5 in. rather than the required 138 in. in the prototype. However, these departures from geometrical similarity between model and prototype in the author's work are not sufficient to account for the discrepancies between the writer's results and those obtained by him, i.e., 18 per cent and 68 per cent in the cases of P_z and M_{zz}, respectively. Further, λ has obviously not been maintained in the model due to the difficulty of making sharp bends in the model without flattening the pipe. (It is suggested that a low-fusion alloy be used, instead of sand, in making these bends.)

The most logical explanation for the departure in results lies in the method used by the author for the determination of his so-called model deflection constants. When it so happens, as it does in the case of R_{zz}, that the calibration curve has a small slope (—2.1), the determination of this particular model deflection constant is questionable. This possible error in R_{zz} introduces a proportionally large error in M_{zz} and has a smaller effect on P_z. This observation is in agreement with results given in Table 3. The writer's calculations, on the other hand, may be in error, since there are six coefficients without any check within themselves in the equations. However, every precaution has been used in executing them.

M. S. NOYES.[8] This paper leaves one with a feeling of wonder as to why some easier and more direct method of manipulating and measuring the reactions of the model section of piping has not been devised and applied to the author's setup.

The writer has had opportunity to view another piece of apparatus developed for the same purpose which had more accurate, direct, and visual means for obtaining the reactions and observing the behavior of the section of model piping under test. Surprisingly close agreement was obtained with values found by the customary formulas for calculation of moments, forces, and stresses in piping via thermal expansion.

Perusal of the subject paper and observation and discussion of the apparatus just mentioned would seem to indicate that the value of the method can be reaped most directly by the turbine manufacturer who is concerned with the effect of thermal expansion reactions upon his turbine main steam connection. The second and more dubious value lies in the ability of the apparatus to check calculations on single-branch piping sections and to provide the end reactions for making the calculations on multibranch sections of piping. No saving in calculations by the piping designer is involved as in every case the calculations must be made in order to determine the fiber stress in the full-sized piping.

E. O. WATERS.[9] The calculation of piping stresses and deflections is notoriously cumbersome, even though it involves nothing more than patient application of addition, subtraction, multiplication, and division. Designers in the power-plant and process-industry fields should accordingly study with interest any method that promises to lighten the labor of piping computations. The model setup described in the present paper is relatively simple; it requires no equipment beyond that available in the usual machine shop and toolroom; instrument indications are large enough to be read accurately; the procedure has been in use and is, therefore, presumably on a sound practical basis.

The author has been extremely modest as to claims for the value of his method and goes only so far as to emphasize its value for determining end moments and reactions. This is natural since the organization that has been using the method is doubtless more interested in the effect of the pipe on the anchorage than that of the anchorage on the pipe. It is obvious, however, that the method makes it possible to determine the forces and couples at any arbitrary section of the pipe line, so that the pipe stress itself may be calculated.

By way of criticism, the writer believes it worth-while to point out that the scale factors, which are used for extending the model reactions and moments to the full-size values, accurately represent the effect of flattening of the section at bends and the effect of bending actions, but are slightly in error as regards longitudinal extension, which is directly proportional to the first power of the length and inversely proportional to the cross-sectional area instead of the moment of inertia. This error may be eliminated by choosing R_m such that $\frac{R_m}{R_a} = s = \left(\frac{r_m}{r_a}\right)^2 \frac{t_a}{t_m}$, which takes care of the flattening effect and in addition, by making $s = \frac{\rho_m}{\rho_a}$, where ρ = radius of gyration of pipe section. Since for ordinary piping ρ varies approximately as r and is independent of t, this reduces the requirements to

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[9] Associate Professor, Mechanical Engineering, Yale University, New Haven, Conn. Mem. A.S.M.E.

$$\frac{R_m}{R_a} = s, \qquad \frac{r_m}{t_m} = \frac{r_a}{t_a}$$

In most layouts, it is to be expected that direct extensions will be small in comparison with bending displacements, and the ratio of pipe radius to wall thickness in the model will be reasonably close to that in the full-size pipe; hence the error in the author's formula is of the second order and it may be neglected with safety.

As an example, consider a full-size and model pipe of the dimensions given in Fig. 5 of the author's paper, for which $s = 0.0484$, $\frac{r_m}{t_m} = 4.3077$, $\frac{r_a}{t_a} = 4.3588$; the full-size pipe consisting of a 3-ft radius quarter bend, followed by a 24-ft tangent. The pipe is anchored at one end of the bend, and the far end of the tangent is displaced lengthwise without rotation. Then the full-size and model forces at the anchorage are actually in the ratio (E_m assumed the same as E_a):

Axial force, 19.511531
Transverse force, 19.512600
Moment, 403.060854

The author's formula gives

Axial force, 19.506731
Transverse force, 19.506731
Moment, 402.981686

As far as torsion is concerned, the conversion from model to full-scale loads appears to be precise, provided, however, that Poisson's ratio is the same for both, at the temperatures involved.

Author's Closure

The author feels that the method of clamping the ends of the model was not clearly shown in the paper. This was done by extending the model tubing beyond the theoretical point of fixity. A tight-fitting plug was inserted into this extension so that the four clamping screws did not collapse the tubing. The four clamping screws can be seen on the movable end of the model in Fig. 1 on the first page of the paper. The hole in the clamping piece which carried the model end was relieved beyond the first pair of clamping screws so that the tubing was unrestrained beyond that point.

Bends in the tubing have been made by packing the tube with sand and bending the tubing around a die without heating. The low-fusion alloy suggested by Mr. Marchant has been tried and found satisfactory.

It has been pointed out in the discussion that slight departures from the scale criteria in the construction of the model are of little importance, and that the departures of the prototype from the theoretical dimensions due to fabrication and erection tolerances may be considerable. Refinements in the testing procedure and in the construction of the model to approach more closely the result obtained by a calculation of a mathematically precise system are not warranted to obtain a practical solution of a piping problem.

Mr. Marchant's explanation of the difference between his calculated values and the author's test values is correct. With the model-testing method, it is possible to have a large percentage error in the determination of a force or moment which is small in relation to the other forces and moment. In the case of M_{zz} in which the discrepancy is large, the deflection constant R_{zz} is roughly 3 and 6 per cent, respectively, of the other two moment deflection constants and is therefore liable to a considerable percentage error. However, a large percentage error in a small force or moment is of little practical significance.

Analysis of Effect of Wire Curvature on Allowable Stresses in Helical Springs[1]

J. R. Finniecome.[2] The author has referred to Röver and Göhner. The purpose of the writer's contribution is to give a comprehensive survey of the theory, formulas, and tests of the five principal authorities on the design of closely coiled springs.

Röver's Theory.[3] The most striking classical advances in the mathematical theory of the stresses in springs are to be found in Röver's pioneer work, published as early as 1913. It is unnecessary here to emphasize its importance, for it contains a wealth of original information. It would be a mistake to underrate its immense value. Several interesting facts emerge from a close examination of this authoritative analysis. We find that the maximum shear and bending stresses are also a function of the curvature of the spring, the cross section of the wire, the radius of the wire in the case of a circular cross section, and also of the pitch angle.

Wahl's Theory.[4] In 1929 increased interest was taken in Röver's advanced theory by Wahl for he published additional formulas for a pitch angle equal to zero. The latter's formula for the maximum-shear stress is by no means simpler than Röver's for a zero pitch angle.

Honegger's Theory.[5] In the last few years renewed interest and controversy have centered around the stress theory of springs, and it is important to mention here also the mathematical analysis and test of Honegger in 1931. His work is a worthy addition to this literature for his formulas correct for the displacement of the actual center of rotation of the fiber from the geometrical cross section of the wire during the loading and the compression of the spring. Neither Röver nor Wahl has allowed for this correction.

Göhner's Theory[6] *and Bergsträsser's Formulas.*[7] Göhner's lengthy and rather involved mathematical analysis was first published in 1930 and then continued in 1932. It contains a wealth of interesting information and is an extremely valuable study of the subject. It will, however, be found that the formulas when compared with those of the other authors are too complicated. Reference to Tables 1 and 2 shows this very clearly.

Göhner's formulas were simplified mathematically in a very ingenious manner by Bergsträsser (Tables 1 and 2) in 1933 and these are now of real practical value for numerical evaluation. Göhner's formulas are now in general use in Germany, special charts having been produced to reduce the mathematical work. It will be found that the numerical values of Bergsträsser agree closely with Göhner, the discrepancy being very minute.

Comparison of Stress Factors of Five Authors. The formulas

[1] Paper by A. M. Wahl, published in the March, 1939, issue of the Journal of Applied Mechanics, Trans. A.S.M.E., vol. 61, p. A-25.

[2] 4 Rye Bank Road, Seymour Grove, Manchester, England.

[3] "Beanspruchung zylindrischer Schraubenfedern mit Kreisquerschnitt," by A. Röver, *Zeit. V.D.I.*, vol. 57, November, 1913, pp. 1906–1911.

[4] "Stresses in Heavy Closely Coiled Helical Springs," by A. M. Wahl, Trans. A.S.M.E., vol. 51, part 1, 1929, paper APM-51-17.

[5] "Zur Berechnung von Schraubenfedern mit Kreisquerschnitt," by E. Honegger, Third International Congress for Applied Mechanics, Proceedings, vol. 2, 1930, pp. 99–108. Also: "The Calculation of Helical Springs of Round Wire," by E. Honegger, *The Brown Boveri Review*, vol. 18, 1931, pp. 120–125.

[6] "Schubspannungsverteilung im Querschnitt einer Schraubenfedern," by O. Göhner, *Ingenieur Archiv.*, vol. 1, 1930, p. 619. Also: "Die Berechnung zylindrischer Schraubenfedern," by O. Göhner, *Zeit. V.D.I.*, vol. 76, 1932, pp. 269–272.

[7] "Die Berechnung zylindrischer Schraubenfedern," by M. Bergsträsser, *Zeit. V.D.I.*, vol. 77, 1933, p. 198.

TABLE 1 FORMULAS FOR MAXIMUM-SHEAR- AND BENDING-STRESS FACTORS FOR HEAVY CLOSE-COILED SPRINGS OF CIRCULAR CROSS SECTION

$$\tau_{max} = K\frac{16}{\pi d^3}PR \qquad \sigma_{max} = K\sigma\frac{32}{\pi d^3}PR$$

Author	Stress factors
A. Röver[3]	$K = \left[\frac{c}{c-\cos^2\alpha} + \frac{1+\sin^2\alpha}{4c}\right]\cos\alpha$ $K\sigma = \left[\frac{c}{c-\cos^2\alpha} + \frac{\sin^2\alpha}{4c}\right]\sin\alpha$
E. Honegger[5]	$K = \left[\cos\alpha\frac{\sqrt{c^2-1}}{c-\cos^2\alpha}\right]\left[\frac{1}{1+\frac{1}{6c^2}+\frac{1}{16c^4}}\right] + \left[\frac{0.616}{c}\cos\alpha\right]$ $K\sigma = \left[\frac{1}{4}\sin^2\alpha + \left(\frac{1}{1+\frac{1}{4c^2}+\frac{5}{32c^4}}\right)\times\left(\frac{c}{c+\cos^2\alpha}\right)\right]\sin\alpha$
O. Göhner[6]	$K = \cos\alpha\left[\frac{\left(\frac{c}{c-\cos^2\alpha}+\frac{1}{4}\frac{\cos^2\alpha}{c}+\frac{1}{16}\frac{\cos^4\alpha}{c^2}\right)}{\left(c^2-\frac{13}{16}\cos^4\alpha\right)} + \frac{[0.62\cos^2\alpha(c-\cos^2\alpha)+0.53\cos^4\alpha]\tan^2\alpha}{c(c-\cos^2\alpha)}\right]$ $K\sigma = \sin\alpha\left[1+0.87\frac{\cos^2\alpha}{c}+\frac{0.64\cos^4\alpha}{c(c-\cos^2\alpha)}+\frac{1}{4c}\right]$
M. Bergsträsser[7]	$K = \left[\frac{c+0.50+\sin^2\alpha}{c-0.75+1.51\sin^2\alpha}\right]$

NOTE: The notes [3,5,6,7] after authors' names refer to previous footnotes in text.

TABLE 2 FORMULAS FOR MAXIMUM SHEAR STRESS FOR CLOSE-COILED SPRINGS OF CIRCULAR CROSS SECTION FOR $\alpha = 0$

$$\tau_{max} = K_0\frac{16}{\pi d^3}PR$$

Author	Maximum-shear-stress factor K_0 for $\alpha = 0$
A. Röver[3]	$\frac{c}{c-1} + \frac{1}{4c}$
A. M. Wahl[4]	$\frac{4c-1}{4c-4} + \frac{0.615}{c}$
E. Honegger[5]	$\sqrt{\left(\frac{c+1}{c-1}\right)}\times\frac{1}{\left[1+\frac{1}{6c^2}+\frac{1}{16c^4}\right]} + \frac{0.616}{c}$
O. Göhner[6]	$\left[\frac{c}{c-1}+\frac{1}{4c}+\frac{1}{16c^2}\right]\times\left[\frac{c^2-1}{c^2-0.8125}\right]$
M. Bergsträsser[7]	$\frac{c+0.5}{c-0.75}$

τ_{max} = maximum shear stress
K_0 = maximum-shear-stress factor for $\alpha = 0$
d = diameter of wire
P = load
R = mean radius of spring
α = pitch angle of spring
c = $2R/d$

for the maximum-shear- and bending-stress factors by the five authors are briefly summarized in Table 2 for a pitch angle α, and the maximum-shear-stress factors for a pitch angle equal to zero in Table 2, the bending stress being zero for this particular case. We find that Röver's formulas for the stress factor for α equals zero are even simpler than Wahl's. Honegger's formulas give an additional correction factor for the movement of the center of the cross section of the wire due to axial loading. Göhner's formulas are more complicated than any of the others. However, Bergsträsser reduces Göhner's formulas to a simplified form. The difference of both these values for the various values of c is so small that it is only at the third decimal that a mathematical discrepancy is noticed.

The percentage difference in the maximum-shear-stress factors relative to Röver's values are shown in Fig. 1. Here we find that Honegger gives the highest positive value, the maximum being 3.05 per cent for c equals 5.75; Wahl gives a maximum value of 0.8 per cent at c equals 6.0. The values by Göhner and Bergsträsser are below Röver's. The general conclusion to be deduced from this information is that the formulas presented by the five authors are practically identical, the differences being negligible.

It is worthy of note that this analysis entirely clarifies all doubtful points in the calculation of various stresses in springs. There is complete agreement in the various maximum-shear-stress factors published by these authors. Having dealt with the principal facts and formulas developed by Röver, Wahl, Honegger, Göhner, and Bergsträsser, we shall now compare the stress factors in relationship to the simple torsion theory derived from the formulas published by these authors. The relative comparison of Röver, Wahl, Honegger, and Göhner is represented in Fig. 2. This gives a realistic picture of the analysis. All these curves show a shear-stress factor for a pitch angle equal to zero plotted as a function of $\frac{R}{r}$. This shear-stress factor takes into consideration the torsion and the direct stress for a zero pitch angle with a bending stress equal to zero. The curves show distinctly that Röver's, Wahl's, and Göhner's values are very close to each other. We find that the curve based on Wahl's values is between the curves by Röver and Honegger for the values of c above 2.9. For values of c below 2.9, the Wahl curve is between the Göhner and Röver curves, the difference being extremely small.

The Honegger curve shows only a slight deviation from the others. It gives the lowest values for the shear-stress factor for

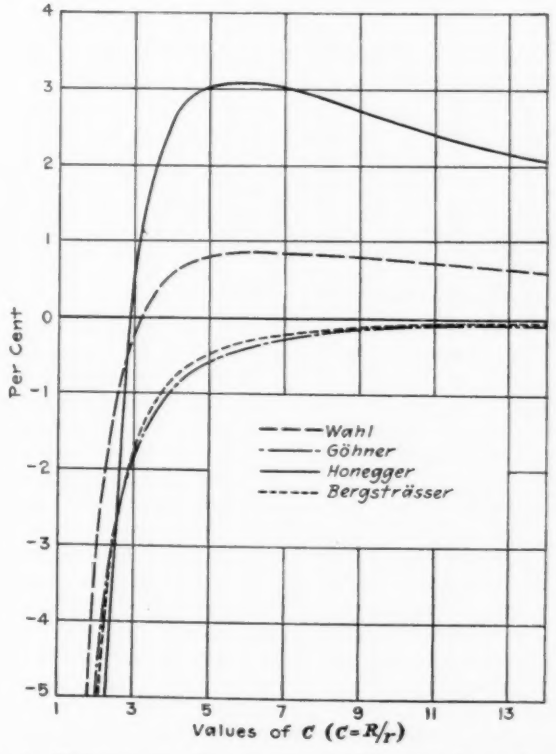


FIG. 1 PERCENTAGE DIFFERENCE IN MAXIMUM-SHEAR-STRESS FACTORS RELATIVE TO RÖVER'S VALUES

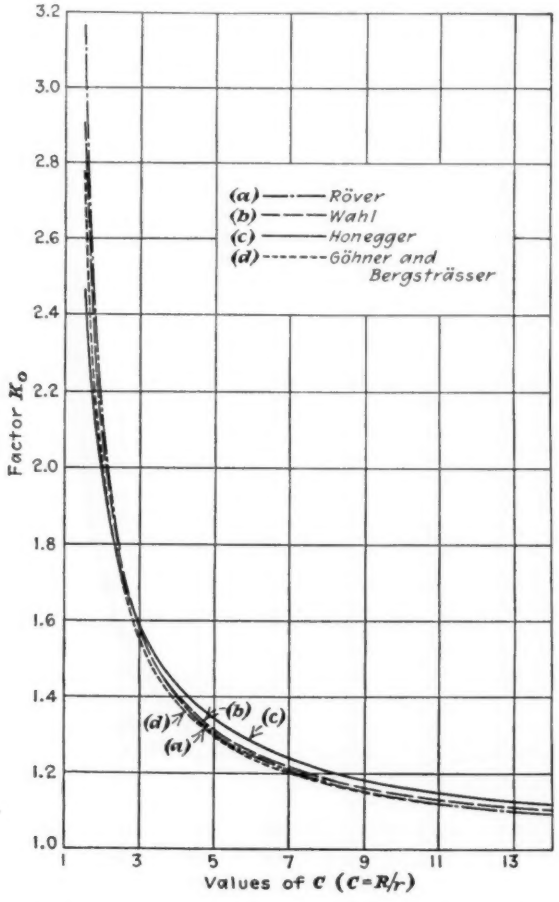


FIG. 2 MAXIMUM-SHEAR-STRESS FACTORS FOR HEAVY CLOSE-COILED HELICAL SPRINGS OF CIRCULAR CROSS SECTION

$$\left(\tau_{max} = K_0 \times \frac{16}{\pi d^3} PR\right)$$

c below 2.9 and the highest for c above 2.9. From an engineer's point of view the values of the five authors are practically equal.

Tests on Stresses in Springs. This intense survey would be incomplete without including some of the major tests carried out by various authorities. Credit is first due to Wahl for having carried out such tests for low values of c. His experiments were made on springs for a value of $c = 3.14$ and 2.98, respectively. He found that the stresses determined on tests agreed closely with his own formulas. Further elaborate tests were made by Honegger to check his own theory and formulas, and these springs had a value of $c = 3$ and 3.9. Also his test values agreed with his formulas. Now for the value of $c = 3$ the calculated stress factor for Wahl and Honegger is 1.580 and 1.592, respectively, the difference being only 0.76 per cent. More than twelve tests were made on a number of springs by Thiersch in 1934, having c values from 3.92 to 14.15. He states that when the test values are compared with the Göhner factors, the variation is +10.9 to —9.4 per cent. In this case the springs had only a wire diameter of 0.06 in. to 0.16 in., while all previous tests were carried out on actual springs.

Twenty-one fatigue tests on springs were made by Zimmerli on a large number of springs for $c = 3.5$ to 12 for a range of stress of 20,000 to 95,000 lb per sq in. He states that these confirm the Wahl stress factor.

As the calculated values of the four authors for the maximum shear stress on springs agree so well, one is not surprised to find that the tests carried out by the various experimenters are also in close agreement.

Conclusions. The object of this comprehensive survey has been to collate published information to enable one to form a definite opinion of the present-day knowledge of the theory and practice of helical springs of circular cross section. A serious attempt has been made to summarize the existing facts, to assist toward a better understanding of the principal theories and formulas, and to simplify the evaluation of the maximum stresses in springs. We obtain the very surprising result that the stress factors relative to the simple torsion theory for the five principal authors agree closely, the differences being very minute. Furthermore, this is confirmed by various authorities who have made tests on actual springs.

WM. HAESSLER.[8] The writer has on various occasions made fairly complete studies of railway-car spring suspensions, chiefly for the purposes of deriving the vibration characteristics.

It is his experience that, for the oscillating stress conditions existing in railway-truck helical springs, the use of the Wahl correction factor will yield a satisfactory spring from the standpoint of life.

It is well to note that the surfaces of helical railway springs are in general quite rough and this rough surface presents the problem of stress concentration in a region where the stresses are at their peak. This condition largely compensates for the overcorrection in using the Wahl factor, which is noticeable when smooth polished surfaces exist.

E. T. P. NEUBAUER.[9] During the discussion of Mr. Wahl's paper a question was brought up regarding the effect on allowable stresses, to which the writer answered: When springs are subjected to fairly wide variations of temperature, the minimum stress or initial stress is important in order to avoid having the spring take a set.

Fig. 3 illustrates this point. The spring subjected to the stresses of curve A took a set while the spring subjected to the stresses of curve B though a higher maximum stress, did not take a set. In both cases the running temperature of the springs was about 300 to 350 F. The maximum stress was applied for only short intervals.

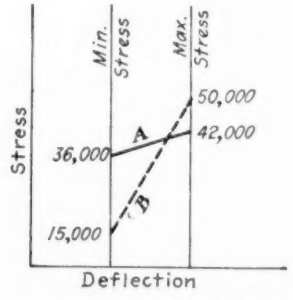


FIG. 3 DIAGRAM OF STRESS APPLICATIONS

AUTHOR'S CLOSURE

The author wishes to thank Mr. Finniecome for his thorough and comprehensive analysis of the results obtained by using spring formulas derived by various authorities. As mentioned by Mr. Finniecome, this analysis shows that the numerical differences between the results, as calculated by using the formulas given by Röver, Göhner, Honegger, Bergsträsser, or the author,

[8] American Car and Foundry Company, New York, N. Y. Mem. A.S.M.E.

[9] York, Pa.

are negligible from a practical standpoint. Since the author's formula is quite widely applied in this country and also agrees closely (within about 1½ per cent for practical springs) with Göhner's more exact equations, which were derived on the basis of the theory of elasticity, it was used in carrying out the analysis given in the present paper. Practically the same results would have been obtained if the formulas given by the other authorities mentioned had been employed.

It should be noted that the primary purpose of the paper was to evaluate in a rational way the effect of wire or bar curvature on the allowable stress in springs under repeated or fatigue loading, and to explain why higher stresses (figured by using the K-factor, Fig. 1 of the paper) are obtained for the springs of smaller index. A discussion of the effects of permanent set under static load and of creep under temperature, although admittedly of much importance to spring designers, is considered to be beyond the scope of the present paper. However, assuming that stress-concentration effects due to curvature may be neglected in figuring stresses due to static loads, then, for a spring subject to a static load, it follows that the stress as obtained by using the K-factor should be divided by K_e (Fig. 1 of the paper) to obtain a comparative stress value for design purposes. It is the author's opinion that, for most spring materials which have some ductility or elongation, this procedure is logical and the resulting value of stress will probably be a closer measure of the tendency of the spring to creep or take a set under static loads than would the peak-stress value obtained by using the K-factor. For extremely brittle materials, this procedure is open to question and should not be used. In the author's opinion there is a definite need for more test data on the creep or settage of springs under constant load at high stress both at normal and elevated temperatures. Although the tests, carried out under the auspices of the Special Research Committee on Mechanical Springs, have constituted a good start in this direction, in addition there exists a definite need for more fatigue-test data on springs of different indexes and materials, and under various loading conditions.

Book Reviews

Mathematics

THE GEOMETRY OF DETERMINENTAL LOCI. By T. G. Room, Professor of Mathematics in the University of Sydney. The University Press, Cambridge. The Macmillan Company, New York, 1939. Cloth, 7¼ × 10½ in., xvi and 483 pp., $10.

REVIEWED BY R. M. THRALL[1]

THIS book is written for the pure mathematician, and it is the reviewer's opinion that it contains very little material of use in applied mathematics, except, perhaps, in problems whose solution requires study of linear equations in which the coefficients are linear functions of several parameters. The language and setting of the treatise are those of projective and algebraic geometry. The loci discussed are also of importance in many fields of algebra, including trilinear form classification, group theory, and linear algebras.

The possible use of this theory in applied mathematics can be seen from the following discussion. The necessary and sufficient condition that the set of q linear homogeneous equations A: $\sum_{j=1}^{p} x_{ij}y_j = 0$, $i = 1, \ldots, q$ shall have a nontrivial solution (i.e. one different from $y_1 = \ldots = y_p = 0$) is that the coefficient matrix (x_{ij}) shall be of rank $p - 1$ or less. In the most elementary case the coefficients x_{ij} are given constants. However, in many cases the x_{ij} are of the form $a_{ij} + xb_{ij}$, where a_{ij} and b_{ij} are constants and x is a parameter. (For instance, vibration problems, where x represents the square of the natural frequencies.) If we set $x = x_1/x_0$ and multiply through by x_0 in each equation we get an equivalent system of equations whose coefficients are linear homogeneous functions of two parameters, x_0 and x_1. If, now, $p = q$ the condition for a nontrivial solution of the equations A is that the determinant $f(x_0, x_1) = |x_{ij}|$ shall vanish, and thus many important properties of the equations A are reflected in the determinental locus $f(x_0, x_1) = 0$.

More generally if $x_{ij} = \sum_{h=0}^{n} a_{hij}x_h$ for any n, p, q the values of $x_0, \ldots, x_n$ for which (x_{ij}) is of rank r constitute a *determinental locus*, V^r, in a projective space of n dimensions. The theory of invariant factors (or elementary divisors) suffices for $n = 1$, so that applications of this work begin with $n = 2$, or with problems whose related equations A involve two or more parameters.

The first chapter explains the notation and gives the projective background and definitions. The next six chapters are concerned with properties of the most general manifolds V^r. The remaining nine chapters are devoted to detailed study of special manifolds V^r (i.e., small values of n, p, q or with specially defined coefficients).

[1] Instructor in Mathematics, University of Michigan, Ann Arbor, Mich.

Modern Developments in Fluid Dynamics

MODERN DEVELOPMENTS IN FLUID DYNAMICS. Vols. I and II. By the Fluid Motion Panel of the Aeronautical Research Committee and others, edited by S. Goldstein. Clarendon Press, Oxford, 1938. 6 × 9¼ in. Vol. I, xxii and 330 pages, 91 figs., 29 plates. Vol. II, xi and 372 pages, 165 figs., 5 plates, 50 s.

REVIEWED BY HUGH L. DRYDEN[2]

WITH secondary title, "An Account of Theory and Experiment Relating to Boundary Layers, Turbulent Motion, and Wakes," these two volumes by a galaxy of distinguished British scientists bring to student, research worker, and practicing engineer a connected account of practically all of the recent developments, both theoretical and experimental, in the aforementioned fields, which have hitherto been available only in scattered papers. As such, it is one of the indispensable books. It is a tribute to the rapid development of fluid mechanics that such a book could be written so shortly after the appearance of the six volumes of Aerodynamic Theory edited by Dr. William F. Durand with so little duplication of subject matter.

The volumes were written at the instigation of the late Sir Horace Lamb and it was hoped that they would appear under his editorship, but Providence decreed otherwise. They are sponsored by the Aeronautical Research Committee of Great Britain and dedicated to Lamb. The actual writing was done

[2] Chief, Mechanics and Sound Division, National Bureau of Standards, Washington, D. C. Mem. A.S.M.E.

by members of the Fluid Motion Panel, L. Bairstow, A. Fage, S. Goldstein, B. M. Jones, A. R. Low, E. Ower, E. F. Relf, L. F. G. Simmons, R. V. Southwell, H. B. Squire, and G. I. Taylor, with the collaboration of Messrs. V. M. Falkner, L. Howarth, L. Rosenhead, and H. C. H. Townend. There is no indication of the authorship of the several sections. So skillfully has the editor, Dr. Goldstein, woven the separate contributions into a continuous fabric, that there are no observable differences in style or quality of treatment.

The subject matter is restricted to rational analysis and experiments of interest in guiding the analysis and checking its validity. After two chapters of an introductory nature giving a general discussion of real and ideal fluids and boundary-layer theory, the equations of viscous fluid flow are developed, followed by the mathematical theory of motion in a boundary layer. The discussion of boundary-layer theory is very complete, including the simple methods of low degree of approximation as well as the relatively more exact methods. The available comparisons with experiment are critically discussed.

A chapter on turbulence, probably written by G. I. Taylor, gives a compact account of the theory of turbulence, including Taylor's important contributions to the theory of isotropic turbulence. This chapter is followed by a brief survey of experimental apparatus and methods of measurement.

Seven chapters deal with general flow phenomena, classified according to boundary conditions. The discussion of pipes, channels, and flat plates together in two chapters aids greatly in bringing out the correspondence in behavior. Laminar flow, transition from laminar to turbulent flow, and turbulent flow are discussed in turn. The next five chapters deal with flow around symmetrical cylinders (drag), flow past asymmetrical cylinders (airfoils, lift), flow past solid bodies of revolution (spheres and airship shapes), boundary-layer control, and wakes.

Two final chapters give an account of heat transfer in laminar and turbulent flow.

Extensive footnotes and summaries at the ends of chapters were inserted at a late stage in the printing to include the very latest work. These added references are to papers appearing in 1937 and early in 1938.

The emphasis is naturally on experiments pertaining to aerodynamics, but everyone interested in fluid mechanics will desire to have these two volumes at hand.

Physics

PROCEDURES IN EXPERIMENTAL PHYSICS. By John Strong, Ph.D., in collaboration with H. Victor Neher, Ph.D., Albert E. Whitford, Ph.D., C. Hawley Cartwright, Ph.D., and Roger Hayward. (Prentice-Hall Physics Series, E. U. Condon, Ph.D., editor.) Prentice-Hall, Inc., New York, 1938. Cloth, 6 × 9¼ in., x and 642 pp., illustrated, $5.

REVIEWED BY FRANCIS BITTER[3]

THIS is an exceedingly good book concerned primarily with the technique of physical experimentation. Although some of the material covered may be found in handbooks and textbooks, most of it is of a sort that is generally acquired by experience only, the sort of thing that may be referred to as "general knowledge" by those familiar with "the art." A feature of the book is the large number of illustrations which help enormously in making the text clear with a minimum of effort on the part of the reader.

The subjects covered include chapters on glass blowing, laboratory optical work, electrometers and electroscopes, Geiger counters, vacuum thermopiles, photoelectric cells and amplifiers, and one on light sources, filters, and optical instruments. These will perhaps be of less interest to engineers and metallurgists than the sections of the book devoted to high vacuum, evaporation and sputtering, the use of fused silica, photography in the laboratory, heat and high temperature, notes on the materials of research, notes on the construction and design of instruments and apparatus, and on molding and casting. Experimenters generally should find it desirable to keep this book on their desks for reference, and to browse in it occasionally for inspiration.

Mathematics

ELEMENTARY MATRICES. By R. A. Frazer, W. J. Duncan, and A. R. Collar. The University Press, Cambridge, The Macmillan Company, New York, 1939. Cloth, 6 × 10 in., xiii and 416 pp., figs., $8.50.

REVIEWED BY LOUIS A. PIPES[4]

THE authors of this book have by means of their investigations for the Aeronautical Research Committee demonstrated the great utility of the application of matrix algebra to classical mechanics. This is the first book that has appeared which develops the subject from its foundations and illustrates the application of matrix algebra to ordinary linear differential equations with constant and variable coefficients and to classical mechanics.

The first four chapters are devoted to the development of matrix algebra and the differential and integral calculus of matrixes without specific applications to problems of dynamics. The nonmathematical reader whose principal interest is mechanics may find the perusal of these chapters somewhat tedious because of the lack of illustrative material to subjects of his special interest. Having developed the necessary theorems, the authors devote chapters 5 and 6 to an excellent detailed exposition of linear ordinary differential equations from the matrix point of view.

Chapter 7 illustrates the use of matrix algebra in the numerical solution of systems of linear differential equations with variable coefficients. The last six chapters contain a very clear exposition of those phases of mechanics of particular importance to the theory of oscillations of aeronautical structures. Powerful iterative methods for the numerical solution of complex oscillatory systems are introduced with actual physical examples such as torsional oscillations of a multicylinder engine, flexural oscillations of a tapered beam, etc. Chapter 11 is devoted to dynamical systems with solid friction and the book is concluded with illustrative applications of friction theory to flutter problems.

This book affords an excellent introduction to the application of matrix algebra to the treatment of linear dynamical systems and although the illustrative examples are chosen from the realm of mechanics, the theory could equally well be applied to electrical systems. The only criticism that might be offered is that the long mathematical preliminary of the first four chapters would have been made more palatable to the nonmathematical reader with strong interests in mechanics by the earlier introduction of illustrative material from this field. However, the procedure followed by the authors is more logical and the reader who masters these chapters is later richly rewarded for his efforts.

[3] Associate Professor of the Physics of Metals, Massachusetts Institute of Technology, Cambridge, Mass.

[4] Faculty Instructor in Electrical Engineering, Harvard University, Cambridge, Mass.

IN TWO SECTIONS—SECTION TWO

Transactions

OF THE

A.S.M.E.

for

FEBRUARY, 1939

TRANSACTIONS

of The American Society of Mechanical Engineers

Published on the tenth of every month, except March, June, September, and December

Vol. 61, No. 2 February, 1939

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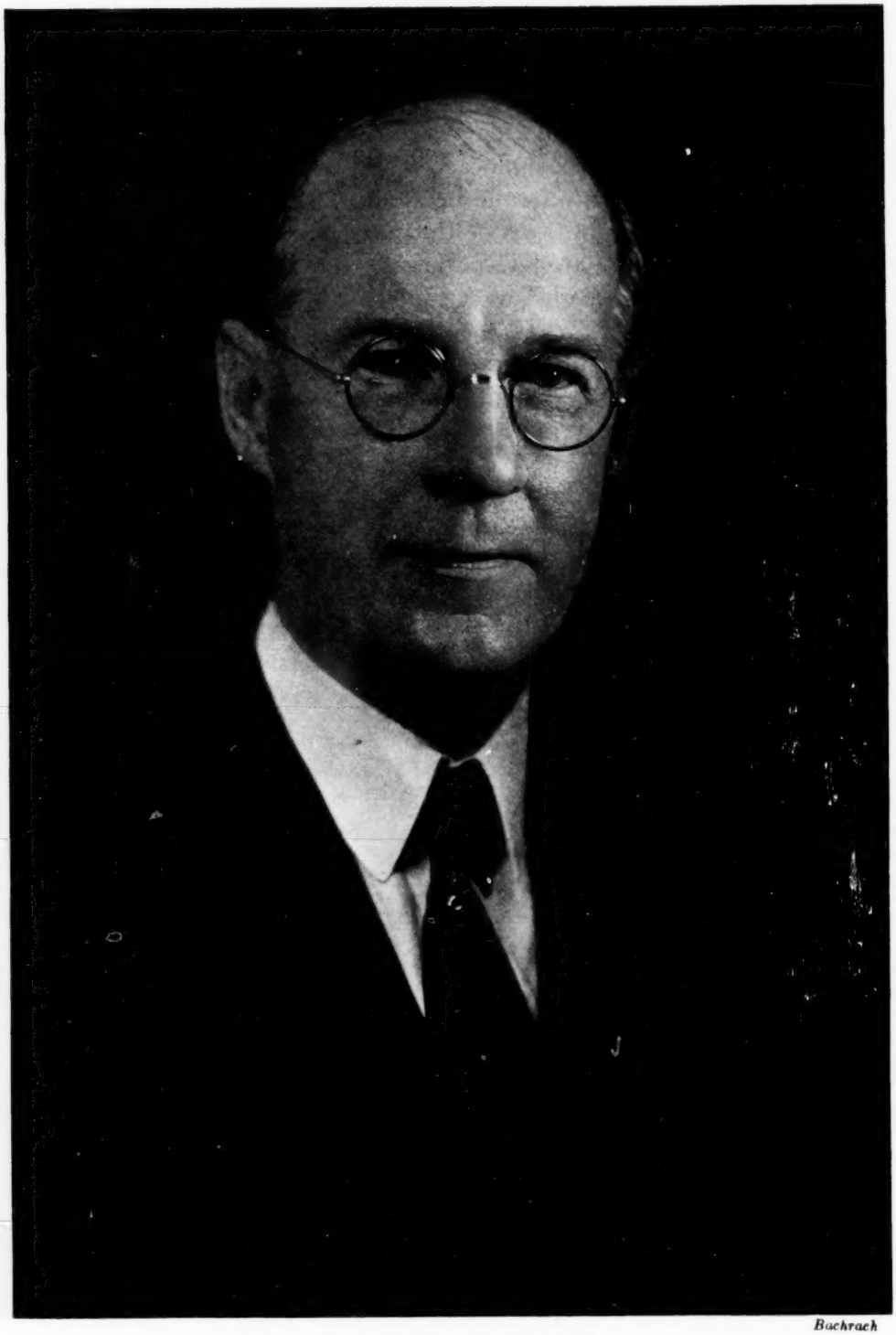
Foreword

THE Transactions of The American Society of Mechanical Engineers include selected technical papers and reports delivered at meetings of the Society, its Professional Divisions, and its Local Sections, the *Journal of Applied Mechanics* (contributions of the Applied Mechanics Division), certain records of the Society of permanent value, and indexes to its publications.

In order to secure the advantages of timeliness and greater usefulness in issuing these Society Records, the material comprising them is divided into a number of parts, each one of which is mailed as a supplement to one of the regular monthly issues of the Transactions. For 1939, the first of these, the present issue, contains the personnel of the Council and committees for the year. Another, to be issued sometime later in the year, will contain memorial notices of deceased members. The indexes to miscellaneous publications, *Mechanical Engineering*, and to the Transactions themselves, must, necessarily, be issued in 1940, and will probably be mailed as a supplement to the January issue of that year.

In binding the 1939 Transactions, all of these parts of the Society Records will be assembled at the back of the volume as has been customary for several years. To aid in locating references in the bound volumes, the page numbers of the sections containing the *Journal of Applied Mechanics* and the Society Records are preceded by the letters A and RI, respectively.

THE COMMITTEE ON PUBLICATIONS



Bachrach

ALEXANDER G. CHRISTIE

PRESIDENT OF THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS
1938–1939

Alexander Graham Christie

ALEXANDER GRAHAM CHRISTIE, president of The American Society of Mechanical Engineers for the year 1938–1939, is professor of mechanical engineering at Johns Hopkins University, Baltimore, Md., but is more widely known throughout the world for his work in the design of power plants. He was born in Manchester, Ont., Canada, in 1880, and was graduated from the School of Practical Science, University of Toronto, in 1901 and received his M.E. degree from the same school in 1913.

Upon his graduation in 1901, Professor Christie started his engineering career as a mechanic in the East Pittsburgh, Pa., plant of the Westinghouse Machine Company, working his way up to the positions of erecting and test engineer and foreman, where he was able to do his part in the early development and construction of steam turbines and gas engines. After being in charge of the Westinghouse turbine and gas-engine exhibits at the World's Fair in St. Louis, Mo., in 1904, he resigned to become an instructor in mechanical engineering at Cornell University. However, he resigned again the following year in order to take charge of the erection, test, and operation of the first steam turbines built by the Allis-Chalmers Company, where he also did work on condensers, steam engines, gas engines, and pumps. In the fall of 1907, he became mechanical engineer in charge of construction and operation of the power plant of Western Canada Cement & Coal Company, Exshaw, Alberta, Canada.

But with his considerable experience in the design and construction of power-plant equipment and machinery, it was not long, 1909 to be exact, before Professor Christie was invited to become assistant professor of steam and gas engineering at the University of Wisconsin and later associate professor. In 1914, he joined the faculty of Johns Hopkins University as associate professor of mechanical engineering, being promoted in 1920 to a full professorship. Since 1916, he has been in charge of the evening engineering and technological school of the University.

In his connections with various engineering and manufacturing firms both here and abroad, he has worked on the design and construction of such power plants as Cahokia, Lake Shore, and Avon Beach in Cleveland, Gould Street in Baltimore, Md., India Basin in San Francisco, Three Rivers in Quebec, Edmonton, Alberta, Can., Salt Creek in Wyoming, and also on power projects in Great Britain, Germany, Sweden, Switzerland, France, Denmark, Perak, Asia, Queensland, Australia, and Argentina, S.A. At the present time, he is consulting engineer to several concerns on designs of a new superimposed power plant and of industrial plants.

Elected to membership in the A.S.M.E. in 1907, Professor Christie served as secretary-treasurer of the Baltimore Section for many years. Since 1918, he has been a member of various committees on Power Test Codes, being today vice-chairman of the Standing Committee on Power Test Codes. He has also given freely of his valuable time to the Society by serving on the Society's Committee on Publications, Nominating Committee, and Professional Conduct Committee; and as manager for the years 1922 to 1925 and as vice-president from 1925 to 1927.

From 1919 to 1922, Professor Christie was chairman of the Joint Committee on Code of Ethics of all the national engineering societies. He is a member of the Engineers' Club of Baltimore as well as that of New York. In the last few years, Professor Christie has been or still is a member of the division of engineering, National Research Council; Prime Movers Committee, Edison Electric Institute; American Advisory Committee, International Electrotechnical Commission; and the World Power Conference.

As an authority on steam turbines and power-plant equipment, Professor Christie has written many articles for technical journals, here and abroad. He is the author of the steam-turbine section of both Sterlings' "Marine Engineer's Handbook" and Kent's "Mechanical Engineer's Handbook."

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W. J. Wohlenberg (1941)
A. L. Kimball (1942)
N. E. Funk (1943)
Junior Advisers: H. G. Oliver, Jr. (1939); G. L. Lingner (1940)

PUBLICATIONS

G. F. Bateman, *Chairman and Representative on the Council* (1939)
C. B. Peck (1940)
Coleman Sellers, 3d (1941)
F. L. Bradley (1942)
C. R. Soderberg (1943)

Advisory Members (1939)

W. L. Dudley
A. N. Goddard
E. J. Kates
L. S. Marks
J. M. Todd

Junior Advisers

A. E. Bliler (1939)
H. B. Fernald, Jr. (1940)

(*Personnel of Special Committee, p.* RI-7)

ADMISSIONS

F. C. Spencer, *Chairman and Representative on the Council* (1939)
R. L. Sackett (1940)
H. E. Molé (1941)
T. M. Knoop (1942)
S. H. Libby (1943)

Advisory Member: L. R. Ford (1939)

PROFESSIONAL DIVISIONS

L. K. Sillcox, *Chairman and Representative on the Council* (1939)
Harte Cooke (1940)
Victor Wichum (1941)
G. B. Karelitz (1942)
W. A. Carter (1943)
Junior Advisers: Elmo Caruthers, Jr. (1939); Sam Schoor (1940)

(*Personnel of Professional Divisions' Executive Committees, p.* RI-10)

LOCAL SECTIONS

D. B. PRENTICE, *Chairman and Representative on the Council* (1939)
A. J. KERR (1940) — J. N. LANDIS (1942)
H. L. EGGLESTON (1941) — J. P. FERRIS (1943)
Junior Advisers { W. F. CARHART (1939); J. R. CAVE, JR. (1940)

(*Personnel of Local Sections' Executive Committees, p.* RI-14)

CONSTITUTION AND BY-LAWS

R. D. BRIZZOLARA, *Chairman and Representative on the Council* (1939)
G. E. HULSE (1940) — A. T. DUPONT (1942)
S. R. BEITLER (1941) — L. H. KENNEY (1943)
Junior Adviser: C. E. LINDENMEYR (1939)

HONORS AND AWARDS

HARTE COOKE, *Chairman and Representative on the Council* (1939)
W. H. CARRIER (1940) — R. V. WRIGHT (1942)
J. W. ROE (1941) — D. C. JACKSON (1943)
Junior Adviser: CORNELIUS KIRBY (1939)

(*Personnel of Special Committee, p.* RI-6)

RELATIONS WITH COLLEGES

F. V. LARKIN, *Chairman and Representative on the Council* (1939)
H. O. CROFT (1940) — A. C. CHICK (1942)
E. W. O'BRIEN (1941) — J. D. CUNNINGHAM (1943)

Advisory Members (1939)

H. E. DEGLER — E. O. EASTWOOD
S. F. DUNCAN — F. H. PROUTY
G. L. SULLIVAN

Junior Adviser: J. I. YELLOTT (1939)

(*Student Branches and Officers, p.* RI-22)

EDUCATION AND TRAINING FOR THE INDUSTRIES

JOHN YOUNGER, *Chairman and Representative on the Council* (1939)
WARNER SEELY (1940) — A. R. STEVENSON, JR. (1942)
J. H. PLOEHN (1941) — M. P. BOWERMAN (1943)

Advisory Members (1939)

E. S. AULT — E. H. NEFF
C. J. FREUND — M. B. RICHARDSON
A. C. HARPER — L. N. ROWLEY, JR.
F. V. LARKIN — S. A. VAULE

LIBRARY

A. R. MUMFORD, *Chairman and Representative on the Council* (1940)
E. F. CHURCH, JR. (1939) — JOHN BLIZARD (1941)
The Secretary, C. E. DAVIES, *Ex-Officio*

POWER TEST CODES

FRANCIS HODGKINSON, *Chairman and Representative on the Council* (1939)
A. G. CHRISTIE, *Vice-Chairman*
JOHN ALLHUSEN, *Junior Observer* (1939)

Term expires 1939
C. H. BERRY
D. S. JACOBUS
FRANCIS HODGKINSON
L. F. MOODY
E. B. RICKETTS

Term expires 1940
A. T. BROWN
P. H. HARDIE
C. F. HIRSHFELD
R. J. S. PIGOTT
M. C. STUART

Term expires 1941
A. G. CHRISTIE
PAUL DISERENS
G. A. ORROK
L. A. QUAYLE
W. M. WHITE

Term expires 1942
W. A. CARTER
HARTE COOKE
E. R. FISH
H. B. OATLEY
W. J. WOHLENBERG

Term expires 1943
LOUIS ELLIOTT
G. A. HORNE
H. B. REYNOLDS
P. W. SWAIN
E. N. TRUMP

(*Personnel of Technical Committees, p.* RI-34)

RESEARCH

N. E. FUNK, *Chairman and Representative on the Council* (1939)
L. W. WALLACE (1940) — W. TRINKS (1942)
E. G. BAILEY (1941) — M. D. HERSEY (1943)

(*Personnel of Special Committees, p.* RI-24)

STANDARDIZATION

O. A. LEUTWILER, *Chairman and Representative on the Council* (1939)
W. C. MUELLER (1940) — J. E. LOVELY (1942)
A. L. BAKER (1941) — J. R. WEAVER (1943)

(*Personnel of Special Committees, p.* RI-26)

SAFETY

D. L. ROYER, *Chairman and Representative on the Council* (1940)
J. B. CHALMERS (1939) — A. W. LUCE (1942)
T. F. HATCH (1941) — A. E. WINDLE (1943)

(*Personnel of Special Committees, p.* RI-32)

PROFESSIONAL CONDUCT

HUGO DIEMER, *Chairman and Representative on the Council* (1939)
B. F. WOOD (1940) — W. H. KENERSON (1942)
E. H. TENNEY (1941) — C. E. WADDELL (1943)

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H. C. BOARDMAN
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JAMES PARTINGTON
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H. LEROY WHITNEY

(*Personnel of Boiler Code Committees, p.* RI-36)

Special Committee of the Board of Honors and Awards

COMMITTEE ON MEDALS

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C. L. BAUSCH
HARTE COOKE
F. M. GUNBY
R. L. SACKETT

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W. H. CARRIER
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H. C. MEYER, JR.
L. W. WALLACE

Terms expire 1941
H. A. EVERETT
H. A. S. HOWARTH
G. A. ORROK
J. W. ROE

Terms expire 1942
ALEXANDER KLEMIN
E. W. O'BRIEN
E. S. PEARCE
R. V. WRIGHT

Terms expire 1943
C. M. ALLEN
R. L. DAUGHERTY
W. A. HANLEY
D. C. JACKSON

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C. H. Berry, Cambridge, Mass., *Alternate*

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A. R. Mumford, New York, N.Y., *Alternate*

III B. F. Rogers, Kingston, Pa., *Secretary*
C. F. Dietz, Syracuse, N.Y., *Alternate*

IV B. R. Van Leer, Raleigh, N.C.
J. B. Jones, Blacksburg, Va., *Alternate*

V F. C. Hockema, Lafayette, Ind.
C. L. Bauer, Springfield, Ohio, *1st Alternate*
S. R. Beitler, Columbus, Ohio, *2nd Alternate*

VI H. A. Smith, Sugar Creek, Mo.
Linn Helander, Manhattan, Kan., *Alternate*

VII D. R. Gray, Spokane, Wash.
Alternate to be appointed

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Providence
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Dayton
Detroit
Erie
Louisville
Ontario
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Pittsburgh
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Kansas City
Mid-Continent
Milwaukee
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Nebraska
Rock River Valley
St. Joseph Valley
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(Dates in parentheses denote expiration of terms)

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C. F. Hirshfeld, Engineers' Council for Professional Development
V. M. Palmer, Registration
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AIMS AND OBJECTIVES

Appointments to be made

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G. L. Knight (1941)

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M. K. Dutton

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Lillian M. Gilbreth R. V. Wright
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G. W. Farny S. H. Libby
W. R. Webster

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C. F. Hirshfeld H. B. Oatley
D. S. Kimball H. L. Whittemore
W. E. Wickenden
Junior Representatives W. F. Carhart, Harold Sizer, W. B. Oakley, Jr.
Chairmen of Committees on Local Sections and Relations With Colleges, *Ex-Officio*

EDUCATION

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H. P. Hammond (1939) V. M. Palmer (1940)
A. R. Stevenson, Jr. (1939) A. D. Bailey (1941)
K. H. Condit (1941)
Representatives of Committees on Local Sections and Relations With Colleges, and Society for the Promotion of Engineering Education, *Ex-Officio*

FREEMAN FUND

Clarke Freeman E. C. Hutchinson G. A. Orrok

MANUAL OF PRACTICE

B. F. Wood, *Chairman* Wynn Meredith
J. A. Willard, *Secretary* J. M. Todd
Alfred Iddles M. X. Wilberding
Junior Adviser, Philip Werner
Chairman of Committee on Professional Conduct, *Ex-Officio*

NATIONAL DEFENSE

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W. C. Dickerman
T. A. Morgan
J. L. Walsh
C. T. Harris (War Dept. Rep.)
A. M. R. Allen (Navy Dept. Rep.)

REGISTRATION

V. M. Palmer, *Chairman*
J. A. Hunter
W. K. Simpson
J. A. McPherson
R. J. Reed

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Being Organized

GEORGE WESTINGHOUSE BUST

D. S. Kimball, *Chairman*
C. E. Davies, *Secretary*
K. T. Compton
S. W. Dudley
A. L. Humphrey
C. N. Lauer
J. H. McGraw
L. A. Osborne
C. F. Scott
R. V. Wright

A.S.M.E. Representatives on Other Activities

See also A.S.M.E. Representatives on Other Research Committees, etc., pages RI-25, 31, 33, 35, *and* 37.

(*Dates in parentheses denote expiration of terms*)

AMERICAN ASSOCIATION FOR THE ADVANCEMENT OF SCIENCE

SECTION M, ENGINEERING

R. F. Gagg
R. L. Sackett

AMERICAN ENGINEERING COUNCIL

(*One year term*)

A. G. Christie, *Chairman*
L. P. Alford
W. L. Batt
R. E. Flanders
E. W. O'Brien

Alternates: W. H. Clapp, Sabin Crocker, A. L. Davis, G. F. McDougall, T. S. McEwan

AMERICAN STANDARDS ASSOCIATION

Walter Samans (1939)
Alfred Iddles (1940)
Alternates: A. L. Baker (1939)
C. B. LePage (1939)

AMERICAN YEAR BOOK CORPORATION

C. E. Davies

BUREAU OF WELDING RESEARCH

James Partington

CENTER FOR SAFETY EDUCATION

J. B. Chalmers

THOMAS ALVA EDISON FOUNDATION

Walter Kidde

THE ENGINEERING FOUNDATION

A. E. White (1939)
K. H. Condit (1939)
W. H. Fulweiler (1940)

RESEARCH PROCEDURE COMMITTEE

W. H. Fulweiler (1939)

ENGINEERING HISTORY

G. A. Orrok
J. W. Roe

ENGINEERING SOCIETIES EMPLOYMENT SERVICE

C. E. Davies, National Board
Hugo Diemer, Chicago Board
S. R. Dows, San Francisco Board
Ernest Hartford, *Chairman*, Metropolitan Board

ENGINEERS' COUNCIL FOR PROFESSIONAL DEVELOPMENT

C. F. Hirshfeld (1939)
A. R. Stevenson, Jr. (1940)
H. T. Woolson (1941)

JOHN FRITZ MEDAL BOARD OF AWARD

R. E. Flanders (1939)
W. L. Batt (1940)
J. H. Herron (1941)
H. N. Davis (1942)

FUEL VALUES

JOINT COMMITTEE WITH THE A.I.M.E.

A. D. Bailey
E. H. Barry
F. M. Gibson
H. Drake Harkins
J. C. Hobbs
A. L. Penniman, Jr.
E. B. Ricketts
E. H. Tenney

GANTT MEDAL BOARD OF AWARD

A. F. Ernst (1939)
W. D. Ennis (1940)
L. C. Morrow (1941)

DANIEL GUGGENHEIM MEDAL FUND, INC.

T. A. Morgan (1939)
W. B. Mayo (1940)
E. A. Sperry, Jr. (1941)

JOSEPH A. HOLMES SAFETY ASSOCIATION

J. F. Barkley

HOOVER MEDAL BOARD OF AWARD

C. N. Lauer (1939)
W. H. Kenerson (1941)
S. F. Voorhees (1943)

INTERNATIONAL ELECTROTECHNICAL COMMISSION

U.S. NATIONAL COMMITTEE

H. N. Davis
Francis Hodgkinson
Paul Diserens
Alternates: C. Harold Berry, E. C. Hutchinson

MARSTON AWARD

G. A. Orrok

NATIONAL BUREAU OF ENGINEERING REGISTRATION

V. M. Palmer

NATIONAL FIRE WASTE COUNCIL

J. A. Neale

NATIONAL MANAGEMENT COUNCIL

L. P. Alford (1940)—C. W. Lytle, *Alternate*
J. A. Piacitelli (1941)—F. L. Eidmann, *Alternate*
L. C. Morrow (1942)—W. H. Kushnick, *Alternate*

NATIONAL RESEARCH COUNCIL

DIVISION OF ENGINEERING AND INDUSTRIAL RESEARCH

W. Trinks (1939) R. C. H. Heck (1940)
C. E. Davies, *Secretary, Ex-Officio*

ALFRED NOBLE PRIZE

A. M. Greene, Jr.

UNITED ENGINEERING TRUSTEES, INC.

D. Robert Yarnall (1939) K. H. Condit (1940)
H. A. Lardner (1942)

ENGINEERING SOCIETIES MONOGRAPHS COMMITTEE

C. B. Peck L. K. Sillcox

VERMILYE MEDAL ADVISORY COMMITTEE

R. A. Wentworth

WASHINGTON AWARD COMMISSION

Bernhard Schroeder (1939) C. B. Nolte (1940)

Professional Divisions

ARTICLE B6A, PAR. 16: The Standing Committee on Professional Divisions shall, under the direction of the Council, have supervision of the Professional Divisions of the Society.

STANDING COMMITTEE

L. K. SILLCOX, *Chairman and Representative on the Council* (1939)
HARTE COOKE (1940) — G. B. KARELITZ (1942)
VICTOR WICHUM (1941) — W. A. CARTER (1943)
Junior Advisers — ELMO CARUTHERS, JR. (1939); SAM SCHOOR (1940)

Aeronautic Division

Organized, 1920

ALEXANDER KLEMIN, *Chairman*

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ALEXANDER KLEMIN, *Chairman*
CHAS. H. DOLAN, *Vice-Chairman*
JEROME LEDERER, *Secretary*
J. M. CLARK
R. M. MOCK
B. M. WOODS

GENERAL COMMITTEE

E. A. SPERRY, *Chairman*

COMMITTEE ON INDUSTRIAL AERODYNAMICS

ALEXANDER KLEMIN, *Secretary*
OLIVER ALLEN
W. H. CARRIER
H. E. DAVISON
H. P. FREAR
W. G. GROVE
O. E. HOVEY
A. L. KIMBALL
R. J. S. PIGOTT
L. K. SILLCOX

REPRESENTATIVES ON OTHER ACTIVITIES

Aircraft Safety and Inspection, JEROME LEDERER
Marking of Obstructions to Air Navigation, J. E. WHITBECK
Daniel Guggenheim Medal Fund, E. E. ALDRIN

Applied Mechanics Division

Organized, 1927

R. EKSERGIAN, *Chairman*

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R. EKSERGIAN, *Chairman*
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H. L. DRYDEN
J. H. KEENAN
B. M. WOODS

Associates

J. C. HUNSAKER
A. L. KIMBALL
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JESSE ORMONDROYD
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C. R. SODERBERG
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JOURNAL OF APPLIED MECHANICS

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J. M. LESSELLS, *Technical Editor*

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Elasticity, STEPHEN TIMOSHENKO
Fluid Mechanics, H. L. DRYDEN
Lubrication, G. B. KARELITZ
Plasticity, A. NÁDAI
Strength of Materials, R. E. PETERSON
Thermodynamics, J. A. GOFF

Fuels Division

Organized, 1920

L. C. BOSLER, *Chairman*

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A. W. THORSON
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H. O. CROFT, *Chairman*

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A.S.M.E. Members — H. O. CROFT; H. F. HEBLEY; R. A. SHERMAN; J. E. TOBEY

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R. A. SHERMAN, *Chairman* — L. C. BOSLER, *Secretary*

COMMITTEE TO COOPERATE WITH CONSUMER'S COUNCIL WITH RELATION TO ACTIVITIES OF NATIONAL BITUMINOUS COAL COMMISSION

K. M. IRWIN, *Chairman* — T. A. MARSH, *Secretary*

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W. G. CHRISTY, *Chairman*

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H. DRAKE HARKINS
J. C. HOBBS
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Graphic Arts Division

Formerly Printing Industries Division. Organized, 1922

B. D. STEVENS, *Chairman*

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Organized, 1938

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Power, G. A. GAFFERT
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Hydraulic Division

Organized, 1926

FORREST NAGLER, *Chairman*

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Honorary Chairman, LORENZO ALLIEVI, Rome, Italy

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Organized, 1927

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J. H. ROMANN
G. T. SNYDER
W. TRINKS
W. R. WEBSTER

Machine Shop Practice Division

Organized, 1921

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ERIK OBERG, *Secretary*
W. A. JOHNSON, *Staff Secretary*
WARNER SEELY
B. G. TANG
J. R. WEAVER

SUBCOMMITTEE ON FOUNDRY PRACTICE

JAMES THOMSON, *Chairman*
R. E. KENNEDY, *Secretary*

SUBCOMMITTEE ON MACHINE DESIGN

E. S. AULT, *Chairman*
A. E. R. DE JONGE
J. MARIN

SUBCOMMITTEE ON LUBRICATION ENGINEERING

W. F. PARISH, *Chairman* C. M. LARSON
H. J. MASSON

SUBCOMMITTEE ON CUTTING METALS

HANS ERNST, *Chairman*

(See also Special Research Committee on Cutting of Metals, page RI-24)

SUBCOMMITTEE ON WELDING

C. W. OBERT, *Chairman*

Management Division

Organized, 1920

W. H. KUSHNICK, *Chairman*

EXECUTIVE COMMITTEE

W. H. KUSHNICK, *Chairman* HAROLD B. BERGEN
L. N. ROWLEY, JR., *Secretary* J. A. JACOBS
J. R. BANGS L. C. MORROW

Associates

C. W. LYTLE F. E. RAYMOND
J. A. PIACITELLI J. R. SHEA

DEPRECIATION STUDIES

P. T. NORTON, JR. H. V. COES

Materials Handling Division

Organized, 1920

R. B. RENNER, *Chairman*

EXECUTIVE COMMITTEE

R. B. RENNER, *Chairman* C. F. DIETZ
F. J. SHEPARD, JR., *Secretary* J. A. JACKSON
G. E. HAGEMANN, *Vice-Chairman* M. C. MAXWELL

Associates

NIXON ELMER F. E. MOORE
R. H. MCLAIN E. D. SMITH
J. B. WEBB

Oil and Gas Power Division

Organized, 1921

L. H. MORRISON, *Chairman*

EXECUTIVE COMMITTEE

L. H. MORRISON, *Chairman* F. G. HECHLER
M. J. REED, *Secretary* C. W. GOOD
CARL BEHN E. J. KATES

Associates

HARTE COOKE ERNEST NIBBS
H. E. DEGLER LEE SCHNEITTER
L. R. FORD C. G. A. ROSEN

SUBCOMMITTEE ON OIL ENGINE POWER COST REPORT FOR 1938

H. C. MAJOR, *Chairman* M. J. REED, *Secretary*

SUBCOMMITTEE ON PAPERS AND PROGRAM

LEE SCHNEITTER, *Chairman* O. H. ALLEN
C. W. GOOD

SUBCOMMITTEE ON EDITING PAPERS SUBMITTED

E. J. KATES ERNEST NIBBS

Petroleum Division

Organized, 1925

Reorganized, 1937

C. J. COBERLY, *Chairman*

EXECUTIVE COMMITTEE

C. J. COBERLY, *Chairman*
J. H. ENGELBRECHT, *Staff Secretary*
T. D. TIFFT, *1st Vice-Chairman*
W. F. HERBERT, *2nd Vice-Chairman*
H. L. EGGLESTON, *3rd Vice-Chairman*
GLENVER MCCONNELL, *4th Vice-Chairman*

GENERAL COMMITTEE

Atlantic	*Mid-Continent*	*Pacific*
E. H. BARLOW	W. H. CARSON	C. J. COBERLY
A. H. MILLER	W. F. HERBERT	H. L. EGGLESTON
V. W. SMITH	GLENVER MCCONNELL	R. J. REED
C. E. SPERRY		L. G. METCALF
T. D. TIFFT		C. C. SCHARPENBERG

Power Division

Organized, 1920

G. A. GAFFERT, *Chairman*

EXECUTIVE COMMITTEE

G. A. GAFFERT, *Chairman* W. A. CARTER
O. F. CAMPBELL, *Secretary* G. C. EATON
E. L. HOPPING

Process Industries Division

Organized, 1934

J. H. SENGSTAKEN, *Chairman*

EXECUTIVE COMMITTEE

J. H. SENGSTAKEN, *Chairman* R. F. O'MARA
T. R. OLIVE, *Secretary* A. F. SPITZGLASS
THEODORE BAUMEISTER, JR. ARNOLD WEISSELBERG
C. E. HARRINGTON VICTOR WICHUM
J. W. HUNTER W. R. WOOLRICH
J. A. MCPHERSON F. L. YERZLEY

COMMITTEE ON INDUSTRIAL INSTRUMENTS AND REGULATORS

EXECUTIVE COMMITTEE

E. S. SMITH, *Chairman* A. F. SPITZGLASS, *Secretary*
E. D. HAIGLER, *Vice-Chairman*
E. S. LEE, *Chairman*, Bibliography Committee
P. G. EXLINE, *Chairman*, Pressure and Temperature Committee
J. J. GREBE, *Chairman*, Program Committee
A. F. SPITZGLASS, *Chairman*, Quantitative Analysis Committee
J. I. YELLOTT, *Chairman*, Terminology Committee
M. J. ZUCROW, *Chairman*, Translation Committee

COMMITTEE ON AIR CONDITIONING

C. F. KAYAN, *Chairman*

COMMITTEE ON DRYING

ARNOLD WEISSELBERG, *Chairman*

COMMITTEE ON FANS AND BLOWERS

THEODORE BAUMEISTER, JR., *Chairman*

COMMITTEE ON FOOD PROCESSING

G. L. MONTGOMERY, *Chairman*

COMMITTEE ON MECHANICAL SEPARATION

R. F. O'Mara, *Chairman*

COMMITTEE ON PAPER AWARDS AND HONORS

C. E. Lucke, *Chairman*

COMMITTEE ON PROGRAM

J. W. Hunter, *Chairman*

COMMITTEE ON PULP AND PAPER

J. A. McPherson, *Chairman*

COMMITTEE ON RUBBER AND PLASTICS

F. L. Yerzley, *Chairman*

COMMITTEE ON SANITATION

William Raisch, *Chairman*

COMMITTEE ON SUGAR

F. M. Gibson, *Chairman*

COMMITTEE ON SULPHUR

B. E. Short, *Chairman*

COMMITTEE ON VEGETABLE OILS

W. R. Woolrich, *Chairman*

Railroad Division

Organized, 1920

C. T. Ripley, *Chairman*

EXECUTIVE COMMITTEE

C. T. Ripley, *Chairman*
M. B. Richardson, *Secretary*
L. H. Fry (1940)
A. I. Lipetz (1941)
D. S. Ellis (1942)
J. R. Jackson (1943)

GENERAL COMMITTEE (RR2)

C. T. Ripley, *Chairman*
J. G. Adair (1939)
F. E. Russell (1939)
R. W. Salisbury (1939)
O. C. Cromwell (1940)
W. E. Dunham (1940)
H. C. Wilcox (1940)
A. Giesl-Gieslingen (1941)
K. F. Nystrom (1941)
L. W. Wallace (1941)
E. G. Young (1941)
R. S. Binkerd (1942)
F. E. Lyford (1942)
F. G. Lister (1943)
John Roberts (1943)
G. A. Young (1943)

PAST-CHAIRMEN (RR3)

James Partington (1923–1924)
C. E. Chambers (1925)
H. B. Oatley (1926–1927)
Wm. Elmer (1928)
R. S. McConnell (1929)
A. F. Stuebing (1930)
Eliot Sumner (1931)
T. C. McBride (1932)
L. K. Sillcox (1933)
C. B. Peck (1934)
C. E. Barba (1935)
Geo. W. Rink (1936)
W. H. Winterrowd (1937)
E. C. Schmidt (1938)

COMMITTEE ON MEETINGS AND PAPERS (RR5)

Lawford H. Fry, *Chairman* Harold C. Wilcox, *Vice-Chairman*

COMMITTEE ON REVIEW AND EDITING (RR5A)

H. B. Oatley
F. E. Lyford
James Partington
W. E. Woodard

COMMITTEE ON SURVEY (RR6)

E. G. Young, *Chairman*
B. S. Cain (Electrical Developments)
A. Giesl-Gieslingen (Foreign Developments)
K. F. Nystrom (Cars)
John Roberts (Canadian Developments)

COMMITTEE ON MEMBERSHIP (RR8)

T. C. McBride, *Chairman*
H. B. Oatley
M. B. Richardson
W. C. Sanders
W. M. Sheehan
L. K. Sillcox
A. G. Trumbull
H. C. Wilcox

Textile Division

Organized, 1921

J. D. Robertson, *Chairman*

EXECUTIVE COMMITTEE

J. D. Robertson, *Chairman*
R. DeVere Hope, *Secretary*
Wendell Brown
H. H. Iler
Albert Palmer
Earle Stall

Associates

A. W. Benoit
Winn Chase
W. L. Conrad
H. D. Learnard

J. J. McElroy

M. A. Golrick, Jr., *Metropolitan Representative*

Wood Industries Division

Organized, 1921

R. H. McCarthy, *Chairman*

EXECUTIVE COMMITTEE

R. H. McCarthy, *Chairman*
P. T. Norton, Jr., *Secretary*
T. D. Perry
A. W. Keuffel
C. B. Norris

Associates

C. L. Babcock
P. H. Bilhuber
H. B. Carpenter
F. P. Cartwright
G. E. French
A. S. Kurkjian
J. S. Mathewson
A. D. Smith, Jr.

MACHINING OF WOOD COMMITTEE

G. E. French, *Chairman*
C. L. Babcock
E. M. Davis
W. Kynoch
R. H. McCarthy
C. M. Thompson

Local Sections

ARTICLE B6A, PAR. 17: The Standing Committee on Local Sections shall, under the direction of the Council, have supervision of the Local Sections of the Society.

STANDING COMMITTEE

D. B. PRENTICE, *Chairman and Representative on the Council* (1939)
A. J. KERR (1940) J. N. LANDIS (1942)
H. L. EGGLESTON (1941) J. P. FERRIS (1943)

Junior Advisers

W. F. CARHART (1939) J. R. CAVE, JR. (1940)

Regional Group Delegates to Annual Conferences

Terms expire October, 1939

P. B. EATON, *Speaker for 1938 Conference*, Group III
C. A. HESCHELES, *Secretary*, Group II
C. P. HOWARD, Group I W. E. MCDOWELL, Group IV
J. M. DRISCOLL, Group II E. R. MCCARTHY, Group V
D. K. HUTCHCRAFT, Group VI

Terms expire October, 1940

S. F. DUNCAN, *Speaker for 1939 Conference*, Group VII
R. A. SPENCE, *Secretary*, Group I
C. A. HESCHELES, Group II N. C. EBAUGH, Group IV
J. T. REA, Group III C. J. FREUND, Group V
R. E. TURNER, Group VI

AKRON-CANTON

Organized: 1920
Territory: Counties of Richland, Ashland, Medina, Summit, Portage, Wayne, Stark, Holmes, Tuscarawas, Carroll, and Coshocton in Ohio
Number of Members: 129

EXECUTIVE COMMITTEE

JAMES FORREST, *Chairman* S. H. HAHN
M. R. BOWERMAN, *Vice-Chairman* G. J. SCHOESSOW
A. D. MACLACHLAN, *Secretary-Treasurer* A. W. SEEKINS
O. BIESHELT A. E. SHETLER
V. R. CAMP O. W. TRAUT
A. W. WALKER

ANTHRACITE-LEHIGH VALLEY

Organized: 1920, as Lehigh Valley; reorganized, 1928, as Anthracite-Lehigh Valley
Territory: Counties of Bradford, Susquehanna, Wayne, Sullivan, Wyoming, Lackawanna, Columbia, Luzerne, Monroe, Pike, Schuylkill, Carbon, Berks, Lehigh, Northampton in Pennsylvania, and Warren in New Jersey
Place of Meeting: One meeting annually at Allentown, Bethlehem, Easton, Hazleton, Pottsville, Reading, Scranton, and Wilkes-Barre
Local Organization: The Engineers' Club of Lehigh Valley
Number of Members: 192

EXECUTIVE COMMITTEE

F. C. PETERS, *Chairman* B. H. JENNINGS
C. W. BELL, *Vice-Chairman* J. B. LAUDIG
P. B. EATON, *Vice-Chairman* R. E. MOYER, JR.
J. T. REA, *Vice-Chairman* W. P. SAUNIER
C. M. MERRICK, 3D, *Secretary* E. A. SCHMIDT
M. C. STUART, *Treasurer* W. T. SOULIS
W. W. HAGERTY S. V. WOOD

ATLANTA

Organized: 1913
Territory: Radius of sixty miles from Atlanta, Ga.
Place of Meeting: Atlanta Athletic Club
Luncheon meeting every Monday at 12:30 p.m. at Atlanta Athletic Club
Number of Members: 83

ATLANTA (*continued*)

EXECUTIVE COMMITTEE

T. E. BELL, *Chairman* E. P. KRAMER
M. F. MERL, *Vice-Chairman* H. M. LANGE
E. A. HARPER, *Secretary-Treasurer* J. M. RITTELMEYER
R. A. TROTTER

BALTIMORE

Organized: 1916
Territory: Radius of thirty miles from Baltimore, Md.
Place of Meeting: Engineers Club of Baltimore
Luncheon meeting every Wednesday at 12:00 noon at Engineers Club
Number of Members: 171

EXECUTIVE COMMITTEE

C. R. CHISHOLM, *Chairman* K. P. HANSON
S. B. SEXTON, 3D, *Secretary-Treasurer* W. P. HILL
L. F. COFFIN G. W. KEEN
A. M. GOMPF P. J. KIEFER
Junior Group Chairman: J. H. BERRYMAN

BIRMINGHAM

Organized: 1915
Territory: Radius of sixty miles from Birmingham, Ala.
Place of Meeting: Tutwiler Hotel
Number of Members: 92

EXECUTIVE COMMITTEE

J. E. GETZEN, *Chairman* J. E. BUELL
R. A. POLGLAZE, *Vice-Chairman* G. R. OZLEY
W. J. MOORE, *Secretary-Treasurer*

BOSTON

Organized: 1909
Territory: Radius of thirty miles from Boston, Mass.
Place of Meeting: Rooms of the Engineering Societies of New England
Local Organization: Engineering Societies of New England
Number of Members: 537

EXECUTIVE COMMITTEE

JAMES HOLT, *Chairman* KERR ATKINSON
H. K. DEAN, *Vice-Chairman* EDGAR MACNAUGHTON
R. A. SPENCE, *Secretary-Treasurer* G. K. SAURWEIN
Junior Group Chairman: F. ROBERT HARTIN
Junior Group Secretary: R. A. SPENCE

BRIDGEPORT

Organized: 1917, as a Branch of Connecticut Section; reorganized as a Section, 1923
Territory: Fairfield County, Conn.
Place of Meeting: Stratfield Hotel
Local Organization: Engineers' Club of Bridgeport
Number of Members: 118

BRIDGEPORT (*continued*)

EXECUTIVE COMMITTEE

A. H. BEEDE, *Chairman*
C. N. HOAGLAND, *Vice-Chairman*
W. H. SNIFFEN, *Secretary*
A. W. HAGAN, *Treasurer*
T. H. BEARD
H. PATTERSON HARRIS
A. H. MOGENSEN
R. O. MOODY
O. J. RICHMOND
D. E. STEER
J. H. VAN YORX, JR.
H. E. WELLS
Junior Group Chairman: A. H. BEEDE
Junior Group Secretary: D. E. STEER

BUFFALO

Organized: 1915
Territory: Radius of thirty miles from Buffalo, N.Y.
Place of Meeting: Hotel Statler
Local Organization: Engineering Society of Buffalo
Number of Members: 175

EXECUTIVE COMMITTEE

PAUL DUBOSCLARD, *Chairman*
E. K. NICHOLSON, *Vice-Chairman*
W. A. MILLER, *Secretary*
C. E. HARRINGTON, *Treasurer*
W. M. KAUFMAN
E. J. WEITZMAN
J. L. YATES

CENTRAL ILLINOIS

Organized: 1937
Territory: All the territory in Central Illinois between the following counties on the northern boundary: Bureau, LaSalle, Knox, Stark, Putnam, Marshall, Livingston, Peoria; counties on the southern boundary: Pike, Scott, Morgan, Sangamon, Macon, Piatt, Douglas, and Edgar
Place of Meeting: Place varies
Local Organization: Indiana Engineering Society

EXECUTIVE COMMITTEE

L. P. WEINER, *Chairman*
C. A. DAVIS, JR., *Vice-Chairman*
R. T. MEES, *Secretary-Treasurer*
Junior Group Chairman: W. L. ACKERMAN
F. L. DAVIS
O. A. LEUTWILER
R. E. LUNEY

CENTRAL INDIANA

Organized: 1916
Territory: Radius of eighty miles from Indianapolis, within Indiana
Place of Meeting: Place varies
Local Organization: Indiana Engineering Society
Number of Members: 127

EXECUTIVE COMMITTEE

C. M. GROSS, *Chairman*
H. H. SKABO, *Vice-Chairman*
R. W. UPDIKE, *Secretary-Treasurer*
J. A. DROGUE
H. L. SOLBERG
J. S. TATMAN

CENTRAL PENNSYLVANIA

Organized: 1921
Territory: Radius of approximately sixty miles from State College, Pa.
Place of Meeting: Pennsylvania State College, State College, Pa.
Number of Members: 72

EXECUTIVE COMMITTEE

F. C. STEWART, *Chairman*
J. O. P. HUMMEL, *Secretary-Treasurer*
A. H. ZERBAN
C. L. ALLEN
R. L. ANTHONY

CHARLOTTE

Organized: As a Branch, 1923; as a Section, 1927
Territory: Radius of seventy-five miles from Charlotte, N.C.
Luncheon Meeting every Monday at 1:00 p.m. at Efirds Department Store
Number of Members: 39

EXECUTIVE COMMITTEE

A. W. BOLLENBACK, *Chairman*
R. P. REECE, *Vice-Chairman*
W. W. LEROY, *Secretary-Treasurer*
M. D. THOMASON
ASA HOSMER
F. R. JACKSON
W. E. MCDOWELL

CHICAGO

Organized: 1913
Territory: Radius of fifty miles from Chicago, Ill.
Headquarters: Mid-West A.S.M.E. Office, Room 1617, 205 West Wacker Drive, Chicago, Ill.
Luncheon Meeting every Tuesday at 12:15 p.m. at Chicago Engineers' Club
Local Organization: Western Society of Engineers
Number of Members: 757

EXECUTIVE COMMITTEE

R. E. TURNER, *Chairman*
L. M. ELLISON, *Vice-Chairman*
F. C. EVERITT, *Vice-Chairman*
H. F. HEBLEY, *Vice-Chairman*
B. D. STEVENS, *Vice-Chairman*
F. B. ORR, *Secretary-Treasurer*
C. C. AUSTIN
A. D. BAILEY
H. M. BLACK
HUGO DIEMER
W. E. DUNHAM
ROBERT ERICKSON
J. A. FOLSE
CHARLES GOLLIN
A. H. JENS
J. S. KOZACKA
F. H. LANE
T. S. MCEWAN
C. W. PARSONS
H. S. PHILBRICK
DANIEL ROESCH
M. W. SHERWOOD
A. F. SPITZGLASS
W. H. WINTERROWD
Junior Group Chairman: CHARLES GOLLIN
Junior Group Secretary: W. R. STEUR

CINCINNATI

Organized: 1912
Territory: Radius of thirty miles from Cincinnati, Ohio
Place of Meeting: Engineers' Club Rooms, Ninth & Race Sts.
Local Organization: Engineers' Club of Cincinnati
Number of Members: 171

EXECUTIVE COMMITTEE

EDWARD MARTIN, *Chairman*
SOL EINSTEIN, *Vice-Chairman*
R. STARR PARKER, *Secretary-Treasurer*
D. S. BROWN
E. H. MITSCH
Junior Group Chairman: R. L. SMITH
T. B. MORRIS
E. H. SCHUBERT
R. L. SMITH
C. E. SOMOGYI
H. P. THOMPSON

CLEVELAND

Organized: 1918
Territory: Counties of Lorain, Cuyahoga Lake, Geauga, and Ashtabula in Ohio
Place of Meeting: Statler Hotel
Local Organization: Cleveland Engineering Society
Luncheon Meeting every Wednesday at 12:30 p.m. at Hotel Statler Cafeteria
Number of Members: 247

EXECUTIVE COMMITTEE

H. A. SCHWARTZ, *Chairman*
G. B. CARSON, *Vice-Chairman*
E. R. MCCARTHY, *Secretary*
F. A. BARNES, *Treasurer*
Junior Group Chairman: A. K. PHELPS
Junior Group Secretary: J. O. GEISSBUHLER
A. E. GIBSON
E. L. LINDSETH
J. M. MAIN
R. R. SLAYMAKER

COLORADO

Organized: 1919
Territory: Entire State of Colorado
Place of Meeting: Parisienne Rotisserie Inn, Denver, Colo.
Local Organization: Colorado Engineering Council (Colorado Society of Engineers)
Number of Members: 83

EXECUTIVE COMMITTEE

A. L. HILL, *Chairman*
R. F. THRONE, *Secretary-Treasurer*
G. E. BEATTY
L. D. CRAIN
J. T. STRATE
J. A. HUNTER
F. H. PROUTY
G. A. RICHTER
S. L. SIMMERING

COLUMBUS

Organized: 1920
Territory: Counties of Union, Delaware, Licking, Madison, Franklin, Fayette, Pickaway, and Ross in Ohio

COLUMBUS (*continued*)

Place of Meeting: Battelle Memorial Institute and The Ohio State University
Local Organization: Engineers' Club of Columbus
Luncheon Meeting third Friday of each month at 12:00 noon at Engineers' Club, Columbus
Number of Members: 75

Executive Committee

W. A. Welcker, Jr., *Chairman*
R. N. Tucker, *Vice-Chairman*
R. C. Cross, *Secretary-Treasurer*
S. R. Beitler
H. N. Booker
H. M. Faust
C. P. Roberts
E. M. Sampson

DAYTON

Organized: 1926
Territory: Counties of Drake, Miami, Champaign, Preble, Montgomery, Greene, and northern part of Butler and Warren in Ohio
Place of Meeting: Engineers' Club of Dayton
Local Organization: Engineers' Club of Dayton
Number of Members: 75

Executive Committee

C. L. Bauer, *Chairman*
B. F. Downey, *Vice-Chairman*
E. L. Hutsel, *Secretary*
B. E. Tate, Jr., *Treasurer*
F. W. Williams
H. M. Gano
P. H. Kemmer
R. W. Martin
A. L. Paschall

DETROIT

Organized: 1916
Territory: Radius of thirty miles from Detroit, Mich.
Place of Meeting: Place varies
Local Organization: Engineering Society of Detroit
Number of Members: 433

Executive Committee

M. W. Benjamin, *Chairman*
B. W. Beyer, Jr., *Secretary-Treasurer*
J. W. Armour
J. A. Carlin
G. C. Daniels
J. J. Grebe
J. F. Jarnagin
T. W. Jeffords
H. E. Keeler
D. E. McGuire
J. P. Schechter
C. J. Freund, *Ex-Officio*
Junior Group Chairman: R. K. Weldy
Junior Group Secretary: O. F. Zehn

EAST TENNESSEE

Organized: 1922
Territory: All counties in Tennessee east of the west boundary of Scott, Morgan, Cumberland, White, Warren, Coffee, Moore, Franklin; Belle County in Kentucky; and Rossville, Dade, Walker, Cattasa, Whitfield, Murray, Gordon, Chattooga in Georgia
Place of Meeting: Place varies
Local Organization: Chattanooga Engineers Club and Knoxville Technical Club
Luncheon Meeting every Monday noon at Chattanooga Engineers Club
Number of Members: 93

Chattanooga Executive Committee

Newell Sanders, *Honorary Chairman*
T. C. Ervin, *Chairman*
H. H. Bailey, *Vice-Chairman*
G. S. Myers, *Secretary-Treasurer*
M. P. Wall

Knoxville Executive Committee

James Ellis, *Chairman*
W. R. Chambers, *Vice-Chairman*
J. M. Kaderly, *Secretary*
D. B. Benscoter
John Callan
D. R. Shearer
J. A. Switzer
Elmer Torok

ERIE

Organized: 1917
Territory: Radius of thirty miles from Erie, Pa.
Place of Meeting: Auditorium of Pennsylvania Telephone Company
Number of Members: 69

ERIE (*continued*)

Executive Committee

C. T. Oergel, *Chairman*
W. L. Hunter, *Secretary*
E. C. Ims, *Treasurer*
G. W. Bach
B. S. Cain
H. E. Goetz
W. L. McClaren
H. G. Mueller
M. S. Reed
R. W. Rogers

FLORIDA

Organized: 1925
Territory: State of Florida
Place of Meeting: Place varies
Local Organization: Florida Engineering Society, Gainesville, Fla.
Number of Members: 68

Executive Committee

Robert Peyinghaus, *Chairman*
N. C. Ebaugh, *First Vice-Chairman*
J. H. Clouse, *Second Vice-Chairman*
P. O. Yeaton, *Secretary-Treasurer*
M. H. C. Brombacher
W. E. Drew
A. B. Hale
John Hunter
A. J. Strong
T. F. Koch, *Student Chairman*
Junior Group Chairman: Robert Peyinghaus

GREEN MOUNTAIN

Organized: 1923
Territory: Entire State of Vermont and neighboring and closely related communities of Claremont and Hanover, N.H.
Place of Meeting: Springfield, Windsor, Vt., and Claremont, N.H.
Local Organization: Vermont Engineering Society
Number of Members: 34

Executive Committee

C. J. Dewell, *Chairman*
Leif Fersing, *Secretary-Treasurer*
M. H. Arms
C. S. Beach
J. B. Johnson
P. M. Klein
H. J. Lockwood
E. L. Sussdorff

GREENVILLE

Organized: As a Branch, 1923; as a Section, 1927
Territory: Radius of sixty miles from Greenville, S.C.
Place of Meeting: Meetings held at Greenville, Clemson College, S.C., Canton, Asheville, and Enka, N.C.
Number of Members: 37

Executive Committee

H. H. Iler, *Chairman*
R. S. Pruitt, *Vice-Chairman*
J. H. Sams, *Secretary-Treasurer*
B. E. Fernow
J. R. Gill
C. E. Hoey
J. A. McPherson
J. E. Waldrep

HARTFORD

Organized: 1917, as Branch of Conn. Section; reorganized, 1923
Territory: Hartford County except that portion served by Meriden and New Britain Sections
Place of Meeting: Hartford Electric Light Company
Number of Members: 119

Executive Committee

G. R. Truedsson, *Chairman*
S. A. Brandenburg, *Vice-Chairman*
R. D. Keller, *Secretary-Treasurer*
P. J. Cubeta
R. F. Dow
William Ferguson
E. P. Herrick
F. O. Hoagland
D. K. Morgan
W. S. Paine
R. S. Shaw
S. Jay Teller
H. B. Van Zelm
J. C. Ward, Jr.
Junior Group Chairman: G. C. Leitch
Junior Group Secretary: J. S. Eyster

INLAND EMPIRE

Organized: 1921
Territory: East of Columbia River in State of Washington, and Counties of Okanogan and Benton, and part of Northern Idaho
Place of Meeting: Davenport Hotel, Spokane
Luncheons Wednesdays at 12:00 noon, Davenport Hotel, Spokane
Local Organization: Associated Engineers of Spokane
Number of Members: 32

INLAND EMPIRE (*continued*)

EXECUTIVE COMMITTEE

E. B. PARKER, *Chairman*
A. R. KARLSTON, *Secretary-Treasurer*
H. F. GAUSS
D. R. GRAY

ITHACA

Organized: 1936
Territory: Radius of thirty miles from Ithaca plus following cities: Binghamton, Corning, Endicott, Geneva, Painted Post
Place of Meeting: Willard Straight Hall, Cornell Campus, Ithaca, N.Y.
Number of Members: 91

EXECUTIVE COMMITTEE

W. M. SAWDON, *Chairman*
E. F. MURPHY, JR., *Vice-Chairman*
F. S. ERDMAN, *Secretary-Treasurer*
CHRISTIAN FYNBOE
L. W. MORROW
F. G. SWITZER
M. P. WHITNEY

KANSAS CITY

Organized: 1921
Territory: Radius of sixty miles from Kansas City, Mo.
Place of Meeting: University Club
Local Organization: Engineers' Club of Kansas City
Number of Members: 127

EXECUTIVE COMMITTEE

W. B. ROLLINS, *Chairman*
H. L. CRAIN, *Vice-Chairman*
R. V. SUTHERLAND, *Secretary*
J. R. STONE, *Treasurer*
H. A. ATWATER
C. E. BECK
C. E. BROWN
E. D. HAY
LINN HELANDER
H. A. SMITH
W. E. WIDAU

Junior Group Chairman: RAYMOND HAHN
Junior Group Secretary: R. F. GLORE

LOS ANGELES

Organized: 1915
Territory: South of southern boundaries of following counties: Monterey, Kings, Tulares, and Inyo, Calif.
Place of Meeting: Barker Bros. Store
Local Organization: Technical Societies of Los Angeles
Luncheon Meetings Thursdays at 12:00 noon at Engineers' Club
Number of Members: 416

EXECUTIVE COMMITTEE

S. F. DUNCAN, *Chairman*
EARL MENDENHALL, *Vice-Chairman*
H. S. HOUGHTON, *Secretary-Treasurer*
W. H. CLAPP
D. S. CLARK
ALADAR HOLLANDER
SCOTT JENSEN
H. M. PERRY
C. H. SMITH
E. M. WAGNER
A. W. WELLS

Junior Group Chairman: EDMOND WAGNER
Junior Group Secretary: A. S. GREEN

LOUISVILLE

Organized: 1922
Territory: Radius of thirty miles from Louisville, Ky.
Place of Meeting: Engineers and Architects Club of Louisville
Local Organization: Engineers and Architects Club
Number of Members: 45

EXECUTIVE COMMITTEE

H. H. FENWICK, *Chairman*
RAY HASSELBERGER, *Vice-Chairman*
L. L. AMIDON, *Secretary*
V. E. FURNAS, JR., *Treasurer*
L. R. JACKSON
M. R. RABUCK
J. H. ROMANN
MELVIN SACK

MEMPHIS

Organized: 1923
Territory: Radius of sixty miles from Memphis, Tenn., and eastern half of Arkansas including all the territory east of a line drawn north and south through the western boundary of the city of Little Rock
Number of Members: 26

MEMPHIS (*continued*)

EXECUTIVE COMMITTEE

M. D. RUST, *Chairman*
W. J. FRANSIOLI, JR., *Vice-Chairman*
R. B. H. SHEPHERD, *Secretary-Treasurer*
T. H. ALLEN
C. M. MCCORD
W. H. ROBERTS

METROPOLITAN

Organized: 1910
Territory: Metropolitan District, New York and New Jersey
Place of Meeting: Engineering Societies Building, New York, N.Y.
Number of Members: 3380

EXECUTIVE COMMITTEE

J. M. DRISCOLL, *Chairman*
O. B. SCHIER, II, *Secretary*
A. F. ERNST, *Treasurer*
T. B. ALLARDICE
E. J. BILLINGS
E. H. FEZANDIE
C. A. HESCHELES
R. T. LIVINGSTON
W. MCC. MCKEE
A. R. MUMFORD
THEODORE BAUMEISTER, JR., *Ex-Officio*

Junior Group Chairman: H. B. FERNALD
Junior Group Secretary: SIDNEY DAVIDSON

MID-CONTINENT

Organized: 1919
Territory: Entire state of Oklahoma; territory in Arkansas not included in Memphis Section; part of Louisiana; and territory in Texas north of the southern boundaries of the counties of Gaines, Dawson, Bordon, Scurry, Fisher, Jones, and Shackelford
Place of Meeting: Tulsa Club
Number of Members: 140

EXECUTIVE COMMITTEE

D. K. HUTCHCRAFT, *Chairman*
G. RAYMOND, *Vice-Chairman*
L. C. PRICE, *Vice-Chairman*
A. G. BLANCHARD, *Vice-Chairman*
HERMAN WIENECKE, *Secretary*
W. L. DUCKER, *Treasurer*
E. E. AMBROSIUS
R. G. AYERS
E. C. BAKER
H. B. BERNARD
W. G. HELTZEL
EMORY KEMLER
A. J. KERR
HENRY SCHAEFER
H. T. SEARS
W. F. STEWART
F. A. STIVERS

Junior Group Chairman: W. F. STEWART

MILWAUKEE

Organized: 1904
Territory: Radius of fifty miles from Milwaukee, Wis.
Place of Meeting: Milwaukee Athletic Club
Local Organization: Engineers' Society of Milwaukee
Number of Members: 204

EXECUTIVE COMMITTEE

R. C. NEWHOUSE, *Chairman*
P. P. FICNERSKI, *Secretary*
W. D. BLISS
HANS DAHLSTRAND
F. H. DORNER
T. F. ESSERKALN
A. H. LUDICKE
ARTHUR SIMON
L. H. STARK

Junior Group Chairman: RUSSELL HAHN
Junior Group Secretary: W. T. SAVELAND, JR.

MINNESOTA

Organized: Minneapolis, 1913; St. Paul, 1913; the two Sections merged, 1934
Territory: Entire State of Minnesota
Place of Meeting: Minnesota Union
Local Organization: Minneapolis Engineers' Club, Minnesota Federation of Architectural and Engineering Societies
Number of Members: 98

EXECUTIVE COMMITTEE

E. B. CURRY, *Chairman*
K. A. POWELL, *Vice-Chairman*
BERTRAM GETSUG, *Secretary-Treasurer*
L. A. BRENEMAN
A. R. FORD
P. S. VAN WYCK

NEBRASKA

Organized: 1922
Territory: State of Nebraska, and Council Bluffs, Iowa

NEBRASKA (*continued*)

Place of Meeting: Lincoln and Omaha
Local Organization: Engineers' Club of Lincoln and Omaha
Luncheon Meeting every Wednesday noon at the Omaha Engineers' Club
Number of Members: 33

EXECUTIVE COMMITTEE

C. F. MOULTON, *Chairman*
H. N. BARNARD, *Vice-Chairman*
G. A. ROGERS, *Secretary-Treasurer*
J. W. BURNETT
OTTO JABELMANN
J. W. WHITE

NEW BRITAIN

Organized: 1921, as a Branch of Connecticut Section; reorganized as a Section, 1923
Territory: New Britain, Plainville, Forestville, Bristol, Kensington, and Berlin, Conn.
Place of Meeting: Auditorium of the State Trade School
Number of Members: 40

EXECUTIVE COMMITTEE

C. C. STEVENS, *Chairman*
W. C. ZINCK, *Vice-Chairman*
H. D. MARTIN, *Secretary-Treasurer*
P. W. BAUER
N. F. HJERPE
J. A. HUTCHINSON
B. S. LEWIS
S. H. STONER

NEW HAVEN

Organized: 1912; reorganized, 1923
Territory: Portions of New Haven and Middlesex Counties, Conn.
Place of Meeting: Mason Laboratory, Yale University
Number of Members: 83

EXECUTIVE COMMITTEE

F. W. KEATOR, *Chairman*
L. H. VON OHLSEN
A. L. BRECKENRIDGE
I. T. HOOK
R. A. NORTH
W. L. TANN
W. F. THOMPSON

NEW ORLEANS

Organized: 1916
Territory: All of Louisiana except the northern part allotted to Mid-Continent Section
Place of Meeting: Room 422, St. Charles Hotel
Local Organization: Louisiana Engineering Society
Number of Members: 95

EXECUTIVE COMMITTEE

K. P. KAMMER, *Chairman*
L. J. LASSALLE, *Vice-Chairman*
G. R. HAMMETT, *Secretary-Treasurer*
L. J. CUCULLU
J. S. HUEY
J. K. MAYER
A. D. REMANJON

Junior Group Chairman: L. J. CUCULLU
Junior Group Secretary: W. S. NELSON

NORTH TEXAS

Organized: 1922
Territory: Radius of one hundred and twenty-five miles from Dallas, in Texas
Place of Meeting: Dallas Power & Light Co. Bldg.
Local Organization: Technical Club of Dallas
Number of Members: 87

EXECUTIVE COMMITTEE

W. B. GREGORY, *Chairman*
N. G. HARDY, *Vice-Chairman*
F. C. JUSTICE, *Secretary-Treasurer*
H. R. PEARSON
H. M. ROBINSON
R. W. SALISBURY
RICHARD WERNER

Junior Group Chairman: L. S. COLE

NORWICH

Organized: 1930
Territory: Counties of Tolland, Windham, and New London in Connecticut, and Westerly District in Rhode Island
Place of Meeting: Arcanum Club, 150 Main St., Norwich
Number of Members: 37

(NORWICH *continued*)

EXECUTIVE COMMITTEE

F. S. ENGLISH, *Chairman*
E. S. DENNISON, *Vice-Chairman*
A. D. ANDRIOLA, *Secretary*
LOVELOCK HOLM
HANS LUEHRS
ERNEST NIBBS
F. E. REED

Junior Group Chairman: F. E. REED

ONTARIO

Organized: 1917
Territory: Province of Ontario, Canada
Place of Meeting: Mining Building, University of Toronto
Number of Members: 160

EXECUTIVE COMMITTEE

W. G. MCINTOSH, *Chairman*
G. E. ELLSWORTH, *Secretary-Treasurer*
T. C. AGNEW
O. H. ANDERSON
W. S. BALL
J. R. BURGESS
S. G. CLARKE
D. F. CORNISH
F. G. EAST
R. B. ELLIOTT
O. W. ELLIS
R. L. RUDE
WILLIAM SANGSTER

OREGON

Organized: 1919
Territory: State of Oregon and that territory in Washington within a radius of thirty miles from Portland, Ore.
Place of Meeting: Portland Hotel, Portland, Ore.
Local Organization: Oregon Society of Engineers
Number of Members: 53

EXECUTIVE COMMITTEE

E. N. BATES, *Chairman*
R. B. ROWAN, *Secretary*
F. L. DAVIS
P. L. HESLOP
LAWRENCE SUPOVÉ

Junior Group Chairman: J. C. OTHUS

PENINSULA

Organized: 1923
Territory: West of the east boundaries of the following counties: Emmet, Charlevoix, Antrim, Kalkaska, Missaukee, Clare, Isabella, Gratiot, Clinton, Eaton, Calhoun, and Branch, Mich.
Place of Meeting: Grand Rapids, Mich.
Local Organization: Engineers' Club of Grand Rapids
Number of Members: 47

EXECUTIVE COMMITTEE

C. A. HAMILTON, *Chairman*
R. E. KLISE, *Secretary-Treasurer*
C. G. LOHMANN
E. E. NORMAN
B. E. PORTER

PHILADELPHIA

Organized: 1912
Territory: Counties of Bucks, Montgomery, Chester, Philadelphia, Delaware, Pa., and the State of Delaware
Place of Meeting: Philadelphia Engineers' Club, 1317 Spruce Street, Philadelphia, Pa.
Local Organization: Philadelphia Engineers' Club
Luncheon Meeting every Tuesday noon at Philadelphia Engineers' Club
Number of Members: 840

EXECUTIVE COMMITTEE

C. C. JONES, *Chairman*
E. L. HOPPING, *Vice-Chairman*
H. E. CORL, *Secretary-Treasurer*
L. P. HYNES
J. S. MOREHOUSE
COLEMAN SELLERS, 3D

Junior Group Chairman: J. H. PETERSON
Junior Group Secretary: J. P. CLARK

PITTSBURGH

Organized: 1920
Territory: Counties bounded by and including Beaver, Butler, Venango, Forest, Jefferson, Indiana, Somerset, Fayette, Greene, and Washington, Pa.
Place of Meeting: Engineers' Society of Western Pennsylvania, William Penn Hotel
Local Organization: Engineers' Society of Western Pennsylvania
Number of Members: 425

PITTSBURGH *(continued)*

EXECUTIVE COMMITTEE

B. M. HERR, *Chairman*
T. E. PURCELL, *Secretary*
K. F. TRESCHOW, *Treasurer*
W. D. CANON
W. M. FRAME
P. G. McVETTY

PLAINFIELD

Organized: 1921
Territory: Plainfield and territory included between Elizabeth, Bound Brook, Metuchen, and Watchung, N.J.
Place of Meeting: Elizabeth Carteret Hotel, Elizabeth, and Plainfield Masonic Temple, Plainfield
Local Organization: Plainfield Engineers Club, Singer Engineering Society
Number of Members: 181

EXECUTIVE COMMITTEE

R. S. BRESCKA, *Chairman*
D. V. WATERS, *Vice-Chairman*
JOSEPH KERR, *Secretary*
C. G. HOLMBERG, JR., *Treasurer*
W. A. APPLEGATE
JOHN HAYDOCK

PROVIDENCE

Organized: 1920
Territory: Radius of thirty miles from Providence, R.I.
Place of Meeting: Providence Engineering Society Building, 195 Angell St., Providence, R.I.
Local Organization: Providence Engineering Society
Number of Members: 157

EXECUTIVE COMMITTEE

F. S. BLACKALL, JR., *Chairman*
H. S. SIZER, *Vice-Chairman*
A. W. CALDER, *Secretary-Treasurer*
Z. R. BLISS
F. A. CHIFFELLE
J. D. ELDERT
E. W. FREEMAN
N. D. MACLEOD
A. W. MOULDER
S. A. VAULE

RALEIGH

Organized: As a Branch, 1923; as a Section, 1927
Territory: Radius of sixty miles from Raleigh, N.C.
Place of Meeting: N. C. State College, Raleigh, N.C.
Local Organization: N.C. Engineering Council, Raleigh Engineers Club
Number of Members: 33

EXECUTIVE COMMITTEE

E. G. HOEFER, *Chairman*
C. E. KERCHNER, *Vice-Chairman*
F. B. TURNER, *Secretary-Treasurer*
R. M. ROTHGEB
H. E. SATTERFIELD
I. W. SUMMERLIN

ROCHESTER

Organized: 1919
Territory: Radius of thirty miles from Rochester, N.Y.
Place of Meeting: Rochester Engineering Society Rooms, Sagamore Hotel
Local Organization: Rochester Engineering Society, Sagamore Hotel
Luncheon Meeting every Tuesday at 12:15 p.m. at Sagamore Hotel
Number of Members: 100

EXECUTIVE COMMITTEE

T. F. HOOKER, *Chairman*
M. D. LEE, *Vice-Chairman*
I. G. McCHESNEY, *Secretary-Treasurer*
R. H. BUSHLEY
K. B. CASTLE
W. T. COWELL

ROCK RIVER VALLEY

Organized: 1926
Territory: Radius of thirty miles from Rockford, Ill., plus members in Madison, Wis.
Local Organization: Rockford Engineering Society
Number of Members: 61

EXECUTIVE COMMITTEE

R. C. GLAZEBROOK, *Chairman*
C. A. JACOBSON, *Vice-Chairman*
W. C. VON FISCHER, *Secretary-Treasurer*
J. H. MANSFIELD
L. H. GEDDES
F. P. GRUTZNER
A. M. JOHNSON

ST. JOSEPH VALLEY

Organized: 1929
Territory: Counties of La Porte, Starke, Pulaski, St. Joseph, Marshall, Fulton, Elkhart, and Kosciusko in Indiana, and Cass and Berrien Counties in Michigan
Place of Meeting: Morningside Hotel, South Bend, Ind.
Local Organization: St. Joseph Valley Engineers' Club
Number of Members: 28

EXECUTIVE COMMITTEE

C. R. ADAMS, *Chairman*
C. C. WILCOX, *Vice-Chairman*
K. W. KNORR, *Secretary-Treasurer*

ST. LOUIS

Organized: 1909
Territory: Radius of thirty miles from St. Louis, Mo.
Place of Meeting: Place varies
Local Organization: Engineers' Club of St. Louis
Number of Members: 203

EXECUTIVE COMMITTEE

A. L. HEINTZE, *Chairman*
D. E. DICKEY, *Vice-Chairman*
G. V. WILLIAMSON, *Secretary-Treasurer*
C. J. COLLEY
A. K. HOWELL
G. L. SHANKS

SAN FRANCISCO

Organized: 1910
Territory: All territory north of the northern boundaries of the counties of San Luis Obispo, Kern, and San Bernardino
Place of meeting: Engineers' Club, 206 Sansome St.
Luncheon Meetings every Thursday at 12:00 noon at the Engineers' Club
Local Organization: San Francisco Engineers' Club
Number of Members: 378

EXECUTIVE COMMITTEE

K. B. ANDERSON, *Chairman*
W. H. ARCHER, *Vice-Chairman*
H. B. LANGILLE, *Secretary-Treasurer*
G. L. SULLIVAN
Junior Group Chairman: EDWIN C. FLOYD
V. F. ESTCOURT
G. H. RAITT
H. J. SMITH

SAVANNAH

Organized: 1923
Territory: Radius of 125 miles from Savannah in Georgia
Place of Meeting: Savannah Hotel
Local Organization: Engineers' Council of Savannah Chamber of Commerce
Number of Members: 20

EXECUTIVE COMMITTEE

L. C. ROESEL, *Chairman*
A. P. KEISKER, *Vice-Chairman*
T. R. JONES, *Secretary-Treasurer*
G. A. MERCER, JR.
A. M. ORMOND
B. J. SAMS

SCHENECTADY

Organized: As a Branch, 1919; as a Section, 1927
Territory: Radius of thirty miles from Schenectady, N.Y.
Place of Meeting: Edison Club Hall
Number of Members: 162

EXECUTIVE COMMITTEE

J. K. SALISBURY, *Chairman*
H. M. OTTO, *Vice-Chairman*
A. L. RUIZ, *Vice-Chairman*
E. E. PARKER, *Secretary*
O. L. WOOD, JR.
R. H. NORRIS, *Treasurer*
J. E. ANDERSON
S. L. JAMESON
A. L. KIMBALL

SOUTH TEXAS

Organized: 1919
Territory: South Texas and the northern part of the State not included in the North Texas Section territory
Place of Meeting: Electric Bldg., Houston, Tex.
Number of Members: 160

SOUTH TEXAS (*continued*)

EXECUTIVE COMMITTEE

D. D. ALTON, *Chairman*
J. J. KING, *Vice-Chairman*
R. M. GARRISON, *Secretary-Treasurer*
J. W. BERETTA
C. W. CRAWFORD
H. E. DEGLER
JOHN DOGGETT, JR.
W. F. HERBERT
F. M. LEVERETT
G. W. LOWTHER
J. H. POUND

SUSQUEHANNA

Organized: 1927
Territory: Counties of Cumberland, Dauphine, Lebanon, Adams, York, and Lancaster
Place of Meeting: Engineering Society of York
Local Organization: Engineering Society of York and Engineers' Society of Pennsylvania, Harrisburg, Pa.
Number of Members: 79

EXECUTIVE COMMITTEE

H. B. MARTIN, *Chairman*
E. D. CLARK
A. B. SNAVELY
O. G. WHEAT

SYRACUSE

Organized: 1920
Territory: Radius of thirty miles from Syracuse, N.Y.
Place of meeting: Ball Room of the Onondaga Hotel
Local Organization: The Technology Club of Syracuse
Number of Members: 93

EXECUTIVE COMMITTEE

M. B. MOYER, *Chairman*
D. V. SHETLAND, *Vice-Chairman*
H. T. AVERY
C. F. DIETZ
F. A. FAILMEZGER
R. C. PAUL

TOLEDO

Organized: 1920
Territory: Radius of thirty miles from Toledo, Ohio
Place of Meeting: University Club, Toledo, Ohio
Local Organization: Affiliated Technical Societies of Toledo
Number of Members: 56

EXECUTIVE COMMITTEE

H. O. KRANICH, *Chairman*
N. R. OHLER, *Secretary-Treasurer*
C. A. ALEXANDER
H. A. BENNETT
S. B. BROOTZKOOS
R. G. N. EVANS
CARL FRAUTSCHI
JOHN GILLETT
L. H. MIDDLETON
D. A. OBERST
WILBUR SCHROEDER
V. E. WALTERS
E. W. WEAVER

TRI-CITIES

Organized: 1920
Territory: Radius of thirty miles from Moline, Ill.
Place of Meeting: Rock Island, Ill., Moline, Ill., and Davenport, Iowa
Luncheon Meeting every Wednesday, Davenport Hotel, 12:00 noon
Number of Members: 65

EXECUTIVE COMMITTEE

R. M. BARNES, *Chairman*
R. A. CROSS, *Vice-Chairman*
C. A. CARLSON, *Secretary-Treasurer*
P. E. ANDERSON
E. G. ERICKSON
J. M. HARTMAN
Junior Group Chairman: C. A. HAND
Junior Group Secretary: W. A. RUMMERY

UTAH

Organized: 1923
Territory: State of Utah
Place of Meeting: University Club, Salt Lake City
Local Organization: Utah Society of Engineers
Number of Members: 35

EXECUTIVE COMMITTEE

W. H. TRASK, JR., *Chairman*
H. D. LANDES, *Vice-Chairman*
W. D. TURPIN, *Secretary-Treasurer*
JULIUS BILLETER
J. D. ROBERTS

VIRGINIA

Organized: 1919
Territory: State of Virginia
Place of Meeting: Richmond, Norfolk, Charlottesville, Roanoke, University, Petersburg
Local Organization: Central Virginia Engineers Club
Number of Members: 129

EXECUTIVE COMMITTEE

G. L. BASCOME, *Chairman*
R. V. TERRY, *Vice-Chairman*
J. B. JONES, *Secretary*
F. S. ROOP, JR., *Treasurer*
E. W. FARLEY
W. H. KNISKERN
C. H. MANNING
G. C. MOLLESON
ARTHUR ROBERTS, JR.

WASHINGTON, D.C.

Organized: 1919
Territory: District of Columbia
Place of Meeting: Auditorium, Potomac Electric Power Co., 10th & E Sts., Washington, D.C.
Number of Members: 207

EXECUTIVE COMMITTEE

A. T. DUPONT, *Chairman*
H. G. THIELSCHER, *Vice-Chairman*
C. E. MILLER, *Secretary-Treasurer*
W. B. ENSINGER
J. FULTON FOX
H. K. KUGEL
P. A. WILLIS

WATERBURY

Organized: 1917, as a Branch; reorganized as a Section, 1923
Territory: Litchfield County and a portion of New Haven County
Place of Meeting: Elton Hotel
Number of Members: 83

EXECUTIVE COMMITTEE

A. J. GERMAN, *Chairman*
H. P. HART, *Vice-Chairman*
W. C. SCHNEIDER, *Secretary-Treasurer*
W. P. DOOLITTLE
L. J. NIEKIRK
R. C. PERRY
Junior Group Chairman: G. N. TYACK
Junior Group Secretary: H. C. ASHLEY

WESTERN MASSACHUSETTS

Organized: 1922
Territory: Includes counties of Berkshire, Franklin, Hampden, and Hampshire
Place of Meeting: Highland Hotel, Springfield, Mass.
Local Organization: Engineering Society of Western Massachusetts
Number of Members: 87

EXECUTIVE COMMITTEE

LESTER CAMPBELL, *Chairman*
J. L. SCHERNER, *Vice-Chairman*
L. G. CARLTON, *Secretary-Treasurer*
A. E. BENSON
A. H. LEWIS
C. E. MAYNARD

WESTERN WASHINGTON

Organized: 1919
Territory: State of Washington west of Columbia River with exception of territory included in 30-mile radius of Portland, Ore.
Place of Meeting: Engineers' Club, Arctic Bldg., Seattle, Wash.
Local Organization: Seattle Engineers' Club
Luncheon Meetings daily at noon at Engineers' Club, Seattle
Number of Members: 84

EXECUTIVE COMMITTEE

B. T. MCMINN, *Chairman*
EDDY FLOODEEN, *Vice-Chairman*
R. E. WALTER, *Secretary-Treasurer*
H. L. ADAMS
L. G. HARTMAN
C. J. YOST, JR.

WEST VIRGINIA

Organized: 1925
Territory: State of West Virginia, South of Parallel 39
Place of Meeting: Charleston, W. Va.
Number of Members: 53

EXECUTIVE COMMITTEE

F. LEROY SCHAEFER, *Chairman*
C. L. JOHNSON, *Vice-Chairman*
H. B. HICKMAN, *Secretary-Treasurer*
B. B. BETTY, JR.
H. L. CARSPECKEN
E. R. HABICHT
F. D. RIPLEY
Junior Group Chairman: H. L. CARSPECKEN
Junior Group Secretary: J. WILLIAMS

WORCESTER

Organized: 1915
Territory: Radius of thirty miles from Worcester, Mass.
Place of Meeting: Sanford Riley Hall, Worcester Poly. Inst.
Local Organization: Worcester Engineering Society
Number of Members: 134

Executive Committee

R. F. Gow, *Chairman*
F. R. Jones, *Vice-Chairman*
R. H. Wood, *Secretary-Treasurer*
E. W. Armstrong
H. P. Crane
R. P. Kolb
Otto Muller
E. H. Smith
W. S. Snow

YOUNGSTOWN

Organized: 1928
Territory: Counties of Trumbull, Mahoning, and Columbiana in Ohio, and Mercer and Lawrence in Pennsylvania
Place of Meeting: Central Y.M.C.A., Youngstown, Ohio
Number of Members: 58

Executive Committee

J. D. Jones, *Chairman*
H. W. Smith, *Vice-Chairman*
C. W. Foard, *Secretary-Treasurer*
F. J. Bowers
C. J. Duby
E. O. Oyen
F. R. Schaeffer
R. J. Wean
J. L. Wick, Jr.

Student Branches

ARTICLE B6A, PAR. 20: The Standing Committee on Relations With Colleges shall, under the direction of the Council, have supervision of the Student Branches of the Society and of such work of the Society as aims to further the education of engineers through the colleges and schools of accepted standing.

STANDING COMMITTEE, RELATIONS WITH COLLEGES

F. V. LARKIN, *Chairman and Representative on the Council* (1939)

H. O. CROFT (1940)
E. W. O'BRIEN (1941)
A. C. CHICK (1942)
J. D. CUNNINGHAM (1943)

Advisory Members (1939)

H. E. DEGLER
S. F. DUNCAN
E. O. EASTWOOD
F. H. PROUTY
G. L. SULLIVAN

Junior Adviser: J. I. YELLOTT (1939)

Communicate with Student Branch through Honorary Chairman

Name and Location	Year Authorized	No. of Members	Chairman	Secretary	Honorary Chairman
Akron, Univ. of, Akron, Ohio	1924	23	D. H. CORNELL	R. E. DAVIES	F. S. GRIFFIN
Alabama Polytechnic Inst., Auburn, Ala.	1920	28	G. M. KEMP	L. O. WESTBROOK, JR.	C. R. HIXON
Alabama, Univ. of, University, Ala.	1931				J. M. GALLALEE
Arizona, Univ. of, Tucson, Ariz.	1937	23	CHARLES NICHOLAS	F. D. AVIS	M. L. THORNBURG
Arkansas, Univ. of, Fayetteville, Ark.	1910	9	J. H. HALL	J. R. DODSON	L. C. PRICE
Armour Inst. of Tech., Chicago, Ill.	1909	142	P. F. HENRIKSEN	J. J. JANICEK	H. L. NACHMAN
British Columbia, Univ. of, Vancouver, B.C., Can.	1938	24	R. A. PHILLIPS	D. A. STEWART	F. W. VERNON
Brown Univ., Providence, R.I.	1923	14	E. D. BROWN	F. L. PARKER	Z. R. BLISS
Bucknell Univ., Lewisburg, Pa.	1916	17	ROBERT LEWIS	CLARENCE WEAVER	G. M. KUNKEL
California Inst. of Tech., Pasadena, Calif.	1914	72	DONALD LAWRIE	CHARLES CARSTARPHEN	D. S. CLARK
California, Univ. of, Berkeley, Calif.	1912	133	J. T. DAWSON	D. T. DOBBINS	E. D. HOWE
Carnegie Inst. of Tech., Pittsburgh, Pa.	1913	52	R. L. MAKEPEACE	M. D. HENDERSON	D. C. SAYLOR
Case School of Applied Science, Cleveland, Ohio	1913	99	R. J. ROCK	W. T. GLOOR	G. B. CARSON
Catholic Univ. of America, Washington, D.C.	1922	28	PAUL O'NEIL	NICHOLAS SARELAS	M. F. WESCHLER
Cincinnati, Univ. of, Cincinnati, Ohio	1909	71	ROBERT HUTZELMAN	ROBERT RUBENDUNST	R. L. SMITH
Clarkson College of Tech., Potsdam, N.Y.	1930	19	V. G. REDMOND	PAUL DEMONG	W. H. PETERS
Clemson A.&M. College, Clemson College, S.C.	1921	37	C. A. DEWEY, JR.	L. A. WILLIAMSON, JR.	J. H. SAMS
Colorado State College of A.&M. Arts, Fort Collins, Colo.	1914	30	JAMES PENNY	RICHARD BICE	J. T. STRATE
Colorado, Univ. of, Boulder, Colo.	1914	38	R. E. SIMMERING	L. G. SHEARER	W. S. BEATTIE
Colorado School of Mines Division, Golden		3	C. R. CRISS	C. R. BLOMBERG	J. C. REED
Columbia Univ., New York, N.Y.	1909	34	W. A. HADLEY	A. E. THOMAS	FRANCIS HODGKINSON
Cooper Union, New York, N.Y.	1920		DAN RUSS	MILTON BREITMAN	E. A. SALMA
Day Division		36			
Evening Division		71			
Cornell Univ., Ithaca, N.Y.	1908	58	R. B. HEYWARD	J. W. SIMONSON	P. H. BLACK
Delaware, Univ. of, Newark, Del.	1929	40	J. D. ROGERS	C. C. BENSON	W. F. LINDELL
Detroit, Univ. of, Detroit, Mich.	1930	61	D. J. LA BELLE	M. A. KILLINGER	F. J. LINSENMEYER
Drexel Inst. of Tech., Philadelphia, Pa.	1920	43	RICHARD BOYAJIAN	T. J. C. MONROE	W. J. STEVENS
Duke Univ., Durham, N.C.	1935	44	R. L. JAMES	W. M. CRAMER	F. J. REED
Florida, Univ. of, Gainesville, Fla.	1926	26	T. F. KOCH	H. H. GRAY	N. C. EBAUGH
George Washington Univ., Washington, D.C.	1924	32	E. J. H. LANE	G. H. WERNER	A. F. JOHNSON
Georgia School of Tech., Atlanta, Ga.	1915	66	H. M. LANGE	T. J. HUGHES	R. S. HOWELL
Idaho, Univ. of, Moscow, Idaho	1925	16	KENNETH ARNETT	GLENN HARDING	H. F. GAUSS
Illinois, Univ. of, Urbana, Ill.	1909	124	J. R. POYSER	J. W. MORRISON	F. H. THOMAS
Iowa State College, Ames, Iowa	1919	73	FRANK SILVER	WILSON DOBSON	H. L. DAASCH
Iowa, State Univ. of, Iowa City, Iowa	1913	22	C. H. PETERSON	T. E. MCVICKER	R. M. BARNES
Johns Hopkins Univ., Baltimore, Md.	1917	38	C. L. WESTENDORF	J. M. WETZLER	K. P. HANSON
Kansas State College, Manhattan, Kan.	1914	64	JOHN PENNINGTON	LEROY CULBERTSON	B. B. BRAINARD
Kansas, Univ. of, Lawrence, Kan.	1909	33	KENNETH WILLEY	CHARLES GODFREY	R. S. TAIT
Kentucky, Univ. of, Lexington, Ky.	1911	30	N. A. WIDES	FRED VANARSDELL	C. C. JETT
Lafayette College, Easton, Pa.	1919	27	C. S. EVANS	W. B. GILMOUR	W. E. REASER
Lehigh Univ., Bethlehem, Pa.	1911	39	DONALD MCKENZIE	R. A. ACKERMAN	T. E. JACKSON
Lewis Inst., Chicago, Ill.	1933	29	C. J. REIMULLER	JOSEPH LEEP	J. S. KOZACKA
Louisiana State Univ., Baton Rouge, La.	1916	20	R. G. WEBB	E. R. PIZZATI	WILLIAM WHIPPLE
Louisville, Univ. of, Louisville, Ky.	1928	25	RALPH DUPPS	BRUCE BARDIN	L. L. AMIDON
Maine, Univ. of, Orono, Maine	1910	35	T. D. VERRILL	C. S. HILL	H. D. WATSON
Marquette Univ., Milwaukee, Wis.	1923	35	RODERICK FOX	RICHARD NELSON	R. J. SMITH
Maryland, Univ. of, College Park, Md.	1937	26	G. E. SEELEY	ROBERT LODGE	J. W. HUCKERT
Massachusetts Inst. of Tech., Cambridge, Mass.	1909	94	SIDNEY SILBER	JOHN MCMULLEN, III	C. L. SVENSON
Michigan College of Min. & Tech., Houghton	1930	40	ERNEST KLEPETKO	LEO SEIDL	A. P. YOUNG
Michigan State College, E. Lansing, Mich.	1917	59	W. R. CLOW	V. C. CARLSON	C. N. RIX
Michigan, Univ of, Ann Arbor, Mich.	1914	109	J. M. STEVENS	T. O. RUETTINGER	E. T. VINCENT

Name and Location	Year Authorized	No. of Members	Chairman	Secretary	Honorary Chairman
Minnesota, Univ. of, Minneapolis, Minn.	1913	94	Robert Wolfe	Frank Powers	J. J. Ryan
Mississippi State College, State College, Miss.	1926	46	B. H. Brown	W. H. Ewart	O. D. M. Varnado
Missouri School of Mines & Metallurgy, Rolla, Mo.	1930	28	E. A. Rhodes	G. E. Machens	A. J. Miles
Missouri, Univ. of, Columbia, Mo.	1909	30	J. A. Thurlo	D. F. Gross	R. L. Scorah
Montana State College, Bozeman, Mont.	1920	32	Tom Murdoch	George Clemow	Eric Therkelsen
Nebraska, Univ. of, Lincoln, Neb.	1909	53	Ellis Smith	Dan Chaney	N. H. Barnard
Nevada, Univ. of, Reno, Nev.	1923	13	Edward Zareh	Ralph Isaac	F. H. Sibley
Newark College of Engineering, Newark, N.J.	1924	71	Richard Foster	John Bermingham	F. D. Carvin
New Hampshire, Univ. of, Durham, N.H.	1926	33	Tenho Kauppinen	J. R. Lovett	E. T. Donovan
New Mexico State College of A.&M. Arts, State College, New Mex.	1938		Gene Gray	Robert Yates	M. T. Lewellen (Temporary)
New Mexico, Univ. of, Albuquerque, New Mex.	1935	22	Robert Burke	William Balmanno	A. D. Ford
New York, College of the City of, New York, N.Y.	1922	41	Philip Scheuble	W. P. Targoff	H. T. Updegrove, Jr.
New York Univ., New York, N.Y.					
Aeronautic Division	1917	26	Julius Raven	B. S. Adams	C. E. Gus *A. F. Spilhaus
Mechanical Division		38	Norman Stone	Fred Schneider	C. E. Gus
New York Univ. Evening School, New York, N.Y.	1933	36	R. J. Sherman	W. G. Heissenbuttel	C. E. Gus
North Carolina State College, Raleigh, N.C.	1920	47	J. B. Lasley	J. T. Power	G. G. Fornes
North Dakota Agricultural College, Fargo, N.D.	1929	17	Kaare Loftheim	Oswald Jorstad	A. W. Anderson
North Dakota, Univ. of, Grand Forks, N.D.	1923	13	Matthew Foreman	Raymond Hitchcock	A. J. Diakoff
Northeastern Univ., Boston, Mass.	1922	76	F. C. Humphreys	I. H. Atwood, Jr.	A. J. Ferretti
Northwestern Univ., Evanston, Ill.	1935	28	Harrison Storms	H. J. Wirth	Edward Obert
Notre Dame, Univ. of, Notre Dame, Ind.	1929	22	C. B. Hayes	F. P. Fransioli	J. A. MacLean
Ohio Northern Univ., Ada, Ohio	1922	8	Paul Adam	Robert Reich	J. A. Needy
Ohio State Univ., Columbus, Ohio	1911	51	G. E. Manker	A. P. Johnson	S. R. Beitler
Oklahoma A.&M. College, Stillwater, Okla.	1921	22	Grant Turrill	Bob Griffin	V. W. Young
Oklahoma, Univ. of, Norman, Okla.	1917	76	Don Walsh	Boyd Christensen	E. E. Ambrosius
Oregon State Agricultural College, Corvallis, Ore.	1909	35	R. C. Borger	Donald Lavey	J. C. Othus
Pennsylvania State College, State College, Pa.	1909	61	K. B. Lawrence	J. P. Rollins	C. L. Allen
Pennsylvania, Univ. of, Philadelphia, Pa.	1925	55	Herman Diehl	George Doll	L. N. Gulick
Pittsburgh, Univ. of, Pittsburgh, Pa.	1917	56	J. G. Rohrman	T. C. Keller	I. A. Gradisar
Polytechnic Institute of Brooklyn, Brooklyn, N.Y.					
Day Division	1909	58	C. H. Hollweg	W. A. Dommers	F. W. Ming
Evening Division		48	Edward Bolsetzian	Rosario Rizzo	F. W. Ming
Pratt Inst., Brooklyn, N.Y.	1923	86	William Morsell	George Woodger	R. B. Dale
Princeton Univ., Princeton, N.J.	1926	31	R. H. Burroughs, Jr.	F. E. Newbold, Jr.	E. P. Culver
Puerto Rico, Univ. of, Mayaguez, P.R.	1923	30	J. M. Santiago Loughran	J. B. Lopez	Guillermo Deyá
Purdue Univ., W. Lafayette, Ind.	1909	210	C. E. Clutter	Robert Aldag	W. J. Cope
Rensselaer Polytechnic Inst., Troy, N.Y.	1910	89	J. M. Fiske	E. E. Henry	G. K. Palsgrove
Rhode Island State College, Kingston, R.I.	1930	30	Raymond Stockard	Alexandra Dobrolet	E. L. Carpenter
Rice Inst., Houston, Tex.	1926	30	J. S. John	S. E. Haggard	R. R. Crookston
Rose Polytechnic Inst., Terre Haute, Ind.	1926	23	Malcolm Steele	R. P. McKee	H. C. Gray
Rutgers Univ., New Brunswick, N.J.	1920	27	Kenneth McAllister	E. C. Gutzwiller	Joseph Marin
Santa Clara, Univ. of, Santa Clara, Calif.	1925	17	Mathew Whitfield	Frank Booth	G. L. Sullivan
South Dakota State College, Brookings, S.D.	1935	17	Clifford Welsh	H. S. Hardy	R. E. Gibbs
Southern California, Univ. of, Los Angeles, Calif.	1929	33	Charles Engle	Louis Bilotta	T. T. Eyre
Southern Methodist, Univ. of, Dallas, Tex.	1933	8	J. D. Wisenbaker	Marvin Moore	R. M. Matson
Stanford Univ., Stanford University, Calif.	1909	18	L. J. Nissen	E. J. Gallagher	A. C. Gullikson
Stevens Inst. of Tech., Hoboken, N.J.	1908	86	Frederick Reines	O. A. Scheller	W. R. Halliday
Swarthmore College, Swarthmore, Pa.	1921	9	L. F. Coffin, Jr.	Vincent Boyer	C. G. Thatcher
Syracuse Univ., Syracuse, N.Y.	1912	25	Robert Guiendon	Robert Beeler	S. T. Hart
Tennessee, Univ. of, Knoxville, Tenn.	1923	25	R. G. Ritter	R. M. Stivers	J. A. Switzer
Texas, A.&M. College of, College Station, Tex.	1921	128	J. W. Clark	B. H. Burns	C. W. Crawford
Texas Technological College, Lubbock, Tex.	1930	44	Henry Meredith	Buster Kirk	H. F. Godeke
Texas, Univ. of, Austin, Tex.	1921	67	Joe Hill, Jr.	H. B. Crockett	C. J. Eckhardt
Toronto, Univ. of, Toronto, Ont., Can.	1933	56	Winnett Boyd	R. N. Boyd	R. C. Wiren
Tufts College, Tufts College, Mass.	1917	24	Donald Bradbury	J. H. Bystrom	Edgar MacNaughton
Tulane Univ. of Louisiana, New Orleans, La.	1933	35	C. W. Nelson	Landry Murphy	J. K. Mayer
Utah, Univ. of, Salt Lake City, Utah	1923	31	Vance McNeilly	Bruce Wiesley	R. D. Baker
U.S. Naval Academy, Post Graduate School, Annapolis, Md.	1925				P. J. Kiefer
Vanderbilt Univ., Nashville, Tenn.	1928	29	Alfred Estes	Richard Cromer	J. E. Boynton
Vermont, Univ. of, Burlington, Vt.	1922	15	Clinton Renfrew	Frank Canary	E. L. Sussdorff
Villanova College, Villanova, Pa.	1925	14	R. E. MacGregor	John Heck	W. J. Barber
Virginia Polytechnic Inst., Blacksburg, Va.	1915	73	C. H. Long	P. D. Dale	J. B. Jones
Virginia, Univ. of, University, Va.	1923	25	H. A. Dinwiddie, Jr.	P. S. Ball, Jr.	H. C. Hesse
Washington, State College of, Pullman, Wash.	1930	19	J. H. Tweet	Delbert Allen	J. G. McGivern
Washington, Univ. of, Seattle, Wash.	1917	53	W. P. Field	H. W. Kuhlman	R. W. Crain
Washington Univ., St. Louis, Mo.	1911	31	R. F. Seifert	J. A. Sporcic	Herbert Kuenzel
West Virginia Univ., Morgantown, W.Va.	1922	29	K. E. Pyle	R. G. Buckley	H. M. Cather
Wisconsin, Univ. of, Madison, Wis.	1909	42	W. E. Hood	Robert Halberg	R. A. Rose
Worcester Polytechnic Inst., Worcester, Mass.	1914	78	J. G. Hollick	F. N. Webster	E. W. Armstrong
Wyoming, Univ. of, Laramie, Wyo.	1925	31	J. C. Webber	Oliver Wormwood	R. S. Sink
Yale Univ., New Haven, Conn.	1910	45	L. P. Doty	J. L. Meriam, Jr.	S. W. Dudley

* Faculty Adviser.

Special Research Committees

ARTICLE B6A, PAR. 24: The Standing Committee on Research shall advise the Council on the research work of the Society.

The first Standing Committee on Research was organized in 1909.

STANDING COMMITTEE

N. E. FUNK, *Chairman and Representative on the Council* (1939)
L. W. WALLACE (1940)
E. G. BAILEY (1941)
W. TRINKS (1942)
M. D. HERSEY (1943)

LUBRICATION

Appointed October, 1915, to investigate the fundamental problems of lubrication, to formulate results of investigations previously made, and to keep in touch with contemporary research in this field

(Reorganized May, 1936)

G. B. KARELITZ, *Chairman*
S. J. NEEDS, *Secretary*
A. L. BEALL
OSCAR BRIDGEMAN
W. E. CAMPBELL
HANS DAHLSTRAND
H. A. EVERETT
A. E. FLOWERS
J. C. GENIESSE
RAYMOND HASKELL
M. D. HERSEY
B. F. HUNTER
G. L. NEELY
B. L. NEWKIRK
A. E. NORTON
E. S. PEARCE
J. F. PELLY
ERNEST WOOLER

FLUID METERS

Appointed 1916 to develop the theory of fluid meters of all kinds and to report on the best methods for their installation and use

(Reorganized July, 1926)

R. J. S. PIGOTT, *Chairman*
J. R. CARLTON, *Secretary*
H. S. BEAN
S. R. BEITLER
R. K. BLANCHARD
B. O. BUCKLAND
LOUIS GESS
A. J. KERR
T. H. KERR
M. P. O'BRIEN
W. S. PARDOE
E. S. SMITH
L. K. SPINK
R. E. SPRENKLE
E. C. M. STAHL
T. R. WEYMOUTH
M. J. ZUCROW

THERMAL PROPERTIES OF STEAM

Appointed in December, 1921, to direct research on the thermal properties of water-vapor and steam from 0 C to the upper limits of temperature and pressure

(Reorganized April, 1929)

G. L. BOURNE, *Chairman*
W. L. ABBOTT, *Vice-Chairman*
H. N. DAVIS
H. C. DICKINSON
ALEX DOW
A. M. GREENE, JR.
R. C. H. HECK
D. S. JACOBUS
MAX JAKOB
J. H. KEENAN
F. G. KEYES
L. S. MARKS
G. A. ORROK
R. J. S. PIGOTT
H. V. RASMUSSEN
E. L. ROBINSON

STRENGTH OF GEAR TEETH

Appointed in December, 1921. Is investigating factors affecting the strength and life of gear teeth

R. E. FLANDERS, *Chairman*
C. H. LOGUE, *Secretary*
C. G. BARTH
EARLE BUCKINGHAM
A. M. GREENE, JR.
C. W. HAM
F. E. MCMULLEN
E. W. MILLER
ERNEST WILDHABER

CUTTING OF METALS

Appointed in September, 1923. Is studying the problems of metal cutting, including tool materials, tool design, lubrication, cooling, and speeds and fuels

M. F. JUDKINS, *Chairman*
L. N. GULICK, *Secretary*
L. P. ALFORD
O. W. BOSTON
R. C. DEALE
A. L. DELEEUW
C. M. THOMPSON, JR.

MECHANICAL SPRINGS

Appointed May, 1924, to determine the status of the mechanical-spring art, to promote and conduct necessary and adequate research, and to develop the art to the point of standardization

J. R. TOWNSEND, *Chairman*
C. T. EDGERTON, *Secretary*
C. E. BARBA
W. G. BROMBACHER
R. W. COOK
W. T. DONKIN
RUPEN EKSERGIAN
G. E. HANSEN
BENJAMIN LIEBOWITZ
DAVID LOFTS
(R. D. BRIZZOLARA, *Alternate*)
D. J. MCADAM, JR.
R. E. PETERSON
J. B. REYNOLDS
J. W. ROCKEFELLER, JR.
B. W. ST. CLAIR
M. F. SAYRE
T. R. WEBER
KEITH WILLIAMS
J. K. WOOD
F. P. ZIMMERLI
O. B. ZIMMERMAN

ELEVATORS

Appointed June, 1924, as a subcommittee of the Sectional Committee on Safety Code for Elevators, to study the function and operation of elevator safeties and buffers and their associated mechanisms and to develop methods of test for the approval of elevator safety devices

D. J. PURINTON, *Chairman*
O. P. CUMMINGS, *Vice-Chairman*
J. A. DICKINSON, *Secretary*
BASSETT JONES
D. L. LINDQUIST
M. G. LLOYD
J. J. MATSON
M. B. MCLAUTHLIN
W. S. PAINE

EFFECT OF TEMPERATURE ON THE PROPERTIES OF METALS

Appointed December, 1924, as a joint research committee of the A.S.T.M. and the A.S.M.E. to encourage the investigation and accumulation of data on the properties of metals used in the mechanic arts at extremely high and low temperatures

N. L. MOCHEL, *Chairman*
H. J. KERR, *Vice-Chairman*
J. W. BOLTON, *Secretary*
R. H. ABORN
W. H. ARMACOST
A. B. BAGSAR
A. D. BAILEY
F. E. BASH
E. S. DIXON
F. B. FOLEY
J. R. FREEMAN, JR.
H. J. FRENCH
H. W. GILLETT
A. J. HERZIG
G. F. JENKS
J. J. KANTER
C. E. MACQUIGG
P. E. MCKINNEY
E. L. ROBINSON
A. E. WHITE

Director, National Bureau of Standards, U.S. Department of Commerce
Representative of Bureau of Engineering, U.S. Navy Department

BOILER FEEDWATER STUDIES

Appointed March, 1925, as a Joint Research Committee of the American Boiler Manufacturers Association, American Railway Engineering Association, American Water Works Association, Edison Electric Institute, the American Society for Testing Materials, and the A.S.M.E. to study methods of analysis and treatment of boiler feedwater for stationary and railroad practice

EXECUTIVE COMMITTEE (Total personnel 41)

C. H. FELLOWS, *Chairman*
R. C. BARDWELL, *Vice-Chairman*
J. B. ROMER, *Secretary*
†A. G. CHRISTIE
R. E. COUGHLIN
B. W. DE GEER
MAX HECHT
†C. F. HIRSHFELD
H. E. JORDAN
W. F. KEENAN, JR.
P. B. PLACE
S. T. POWELL
F. N. SPELLER
M. F. STACK
E. H. TENNEY
†A. E. WHITE

CONDENSER TUBES

Appointed May, 1925, to investigate and report on the causes of failure of tubes used in steam condensers and similar heat interchange apparatus

A. E. WHITE, *Chairman*
BERT HOUGHTON, *Vice-Chairman*
P. A. BANCEL
D. K. CRAMPTON
H. M. CUSHING
R. E. DILLON
O. B. J. FRASER
J. R. FREEMAN, JR.
V. M. FROST
C. F. HARWOOD
C. F. HIRSHFELD
G. C. HOLDER
O. Z. KLOPSCH
H. W. LEITCH
E. F. MILLER
W. B. PRICE
M. F. STACK
H. A. STAPLES
W. R. WEBSTER
D. C. WEEKS

Director, Bureau of Engineering, U.S. Navy Department

BOILER FURNACE REFRACTORIES

Appointed June, 1925, to determine the principal factors governing the failure of refractories in various types of installations, to subject these factors to detailed experimental analysis, to undertake the formulation of suitability tests and, if necessary, to attempt to develop a suitable refractory to meet the needs of severe service

W. A. CARTER, *Chairman*
S. H. BARNUM
G. A. BOLE
M. C. BOOZE
W. H. FULWEILER
C. B. GRADY
R. A. HEINDL
C. F. HIRSHFELD
R. K. HURSH
N. E. LEWIS
J. S. MCDOWELL
(F. A. HARVEY, *Alternate*)
S. J. MCDOWELL
PERCY NICHOLLS
S. M. PHELPS
E. B. POWELL
R. A. SHERMAN
R. B. SOSMAN
L. J. TROSTEL
G. B. WILKES

WORM GEARS

Appointed May, 1927, to investigate certain problems in connection with the action of worm gear drives and to recommend improvements in their design, manufacture, and use

EARLE BUCKINGHAM, *Chairman*
G. H. ACKER
L. R. BUCKENDALE
W. H. HIMES
D. L. LINDQUIST
A. A. ROSS
B. F. WATERMAN

Representative of Bureau of Engineering, U.S. Navy Department

VELOCITY MEASUREMENT OF FLUID FLOW

Appointed October, 1927, to sponsor the development of an absolute method for determining the velocity of the flow of fluids by means of the location of nodal points in wave systems

W. F. DURAND, *Chairman*
T. R. WEYMOUTH

† Official A.S.M.E. representatives serving on this committee.

MEASURES OF MANAGEMENT

Appointed March, 1928, to attempt the reconciliation of certain economic laws affecting production, to develop formulas for management, and to collect and report information on management research

W. E. FREELAND, *Chairman*
F. E. RAYMOND, *Secretary*
J. H. BARBER
T. H. BROWN
R. C. DAVIS
G. E. HAGEMANN

ABSORPTION OF RADIANT HEAT IN BOILER FURNACES

Appointed April, 1928, to make a study of the absorption of radiant heat in boiler furnaces with the purpose of developing recommendations on improved furnace design

W. J. WOHLENBERG, *Chairman*
E. G. BAILEY
R. M. GATES
C. W. GORDON
E. L. LINDSETH
G. A. ORROK
R. J. S. PIGOTT
JOHN VAN BRUNT

STRENGTH OF VESSELS UNDER EXTERNAL PRESSURE

Appointed June, 1929, to develop reliable design data on the strength of cylindrical and spherical surfaces under external pressure, particularly with reference to jacketed vessels

W. D. HALSEY, *Chairman*
F. V. HARTMAN
M. B. HIGGINS
A. W. LIMONT, JR.
H. E. SAUNDERS
E. E. SHANOR
D. B. WESSTROM
F. S. G. WILLIAMS
D. F. WINDENBURG

WIRE ROPE

Appointed April, 1930, to investigate existing rope so that it may be better understood and more effectively used

W. H. FULWEILER, *Chairman*
H. LER. BRINK
D. L. LINDQUIST
G. W. MARTIN
C. A. MCCUNE
A. H. MCDOUGALL
B. V. E. NORDBERG, JR.
W. S. PAINE
W. J. RYAN
GEORGE SIMPSON
L. E. YOUNG

CRITICAL PRESSURE STEAM BOILERS

Appointed June, 1931, to study the characteristics of high-pressure forced-circulation steam-generating units

A. A. POTTER, *Chairman*
W. H. ARMACOST
A. D. BAILEY
E. G. BAILEY
A. G. CHRISTIE
F. S. CLARK
C. H. FELLOWS
H. J. KERR
M. W. LINK
G. A. ORROK
H. L. SOLBERG
P. W. THOMPSON

COTTONSEED PROCESSING

Appointed December, 1932, to study the mechanical problems involved in storing, conditioning, and cooking cottonseed meats

W. R. WOOLRICH, *Chairman*
HOMER BARNES
E. L. CARPENTER
C. E. GARNER
B. J. SAMS
R. B. TAYLOR

ROLLING OF STEEL (PLASTICITY)

Appointed October, 1938, to study plasticity in the particular field of rolling of steel

E. C. BAIN
C. L. EKSERGIAN
C. W. MACGREGOR
A. NÁDAI
MORRIS STONE

A.S.M.E. Representatives on Other Research Committees

See also A.S.M.E. Representatives on Other Activities, page RI-8

AMERICAN COORDINATING COMMITTEE ON CORROSION

American Society for Testing Materials

S. L. KERR
C. H. FELLOWS (*Alternate*)

CORROSION COMMITTEE

American Society of Refrigerating Engineers

(To be appointed)

FATIGUE PHENOMENA OF METALS

American Society for Testing Materials

C. T. EDGERTON

HEAT-TREATMENT OF ROCK DRILL STEELS

Advisory Board of the Bureau of Standards and Bureau of Mines

(To be appointed)

METALLURGICAL RESEARCH

Advisory Committee to the National Bureau of Standards

C. H. BIERBAUM

PROPERTIES OF REFRACTORY MATERIALS

Advisory Committee to the National Bureau of Standards

E. B. POWELL

WATER FOR INDUSTRIAL USES

American Society for Testing Materials

J. H. WALKER

Standardization Technical Committees

ARTICLE B6A, PAR. 23: The Standing Committee on Standardization shall advise the Council on the dimensional standardization work of the Society, including relations with the American Standards Association.

The first Standing Committee on Standardization was organized in April, 1911.

STANDING COMMITTEE

O. A. LEUTWILER, *Chairman and Representative on the Council* (1939)
W. C. MUELLER (1940)
A. L. BAKER (1941)
J. E. LOVELY (1942)
J. R. WEAVER (1943)

STANDARDIZATION AND UNIFICATION OF SCREW THREADS (B-1)

* *Joint sponsorship with the Society of Automotive Engineers. Sectional Committee originally organized in June, 1921. Reorganized in February, 1929*

A.S.M.E. Members (Total personnel, 36)

†R. E. FLANDERS, *Chairman*
EARLE BUCKINGHAM, *Secretary*
E. J. BRYANT
G. S. CASE
T. G. CRAWFORD
†A. M. HOUSER
H. C. E. MEYER
†P. V. MILLER
W. C. MUELLER
R. H. PERRY
O. B. ZIMMERMAN

SUBCOMMITTEE CHAIRMEN

No. 1 on Scope, Arrangement, and Editing of American National Standard, R. E. FLANDERS
No. 2 on Terminology and Thread Specifications, Except Gages, C. W. BETTCHER
No. 3 on Special Threads and Twelve Pitch Series, Except Gages
No. 4 on Acme and Other Similar Threads, Except Gages, EARLE BUCKINGHAM
No. 5 on Screw Thread Gages and Inspection, G. S. CASE
No. 7 on Wood Screws, ARTHUR BOOR

BALL AND ROLLER BEARINGS (B-3)

* *Joint sponsorship with the Society of Automotive Engineers. Sectional Committee organized December, 1920*

A.S.M.E. Members (Total personnel, 14)

†W. P. KENNEDY, *Vice-Chairman*
†D. E. BATESOLE
L. A. CUMMINGS
F. G. HUGHES
†G. E. HULSE
L. F. NENNINGER
ERNEST WOOLER

* *Note:* All of the standards committees for which the Society is sponsor or joint sponsor, or on which it has representation, are organized under the procedure of the American Standards Association.

† Official A.S.M.E. representatives serving on this committee.

PIPE THREADS (B-2)

* *Joint sponsorship with the American Gas Association. Sectional Committee originally organized in 1913. Reorganized May, 1927*

A.S.M.E. Members (Total personnel, 49)

A. S. MILLER, *Chairman*
A. F. BREITENSTEIN
E. J. BRYANT
E. S. CORNELL, JR.
J. J. CROTTY
A. P. DENTON
J. J. HARMAN
†A. M. HOUSER
A. H. JARECKI
P. V. MILLER
F. H. MOREHEAD
W. C. MORRIS
S. F. NEWMAN
E. S. SANDERSON
L. N. SHANNON
W. D. SIZER
J. H. WILLIAMS

SUBCOMMITTEE CHAIRMEN

No. 1 on Editing and Gaging, A. M. HOUSER
No. 2 on Taper Pipe Threads, S. B. TERRY
No. 3 on Straight Pipe Threads, A. S. MILLER
No. 4 on Plumbers' Threads, A. F. BREITENSTEIN
No. 5 on Screw Threads for Rigid Steel Conduit, JAMES BARTON
No. 6 on Special Threads for Thin Tubes, C. C. WINTER

ALLOWANCES AND TOLERANCES FOR CYLINDRICAL PARTS AND LIMIT GAGES (B-4)

**Sole sponsorship. Sectional Committee originally organized in June, 1920. Reorganized in September, 1930*

A.S.M.E. Members (Total personnel, 44)

†J. E. LOVELY, *Chairman*
F. E. BANFIELD, JR.
F. S. BLACKALL, JR.
E. J. BRYANT
†EARLE BUCKINGHAM
†F. H. COLVIN
T. G. CRAWFORD
R. E. W. HARRISON
F. O. HOAGLAND
N. E. JACOBI
H. C. E. MEYER
P. V. MILLER
W. C. MUELLER
†E. C. PECK
R. H. PERRY
W. C. SCHOENFELDT
C. C. STEVENS
O. B. ZIMMERMAN

SUBCOMMITTEE CHAIRMAN

No. 1 on Tolerance Systems, R. E. W. HARRISON

SMALL TOOLS AND MACHINE TOOL ELEMENTS (B-5)

** Joint sponsorship with the National Machine Tool Builders Association and the Society of Automotive Engineers.*
Sectional Committee organized September, 1922

A.S.M.E. Members (Total personnel, 25)

†W. C. Mueller, *Chairman*
F. O. Hoagland, *Vice-Chairman*
J. B. Armitage
F. S. Blackall, Jr.
O. W. Boston
E. J. Bryant
Earle Buckingham
†F. H. Colvin
S. A. Einstein
†H. E. Harris
John Haydock
J. E. Lovely
†E. R. Norris
Erik Oberg

C. W. Spicer

Technical Committee No. 1 on T-slots

A.S.M.E. Members (Total personnel, 9)

†Erik Oberg, *Chairman*
J. B. Armitage
†Harry Cadwallader, Jr.
Herman Casler
S. A. Einstein
†F. O. Hoagland

†E. R. Norris

Technical Committee No. 2 on Tool Posts and Tool Shanks

A.S.M.E. Members (Total personnel, 8)

O. W. Boston, *Chairman*
F. S. Blackall, Jr.
†Granger Davenport
M. E. Lange

Technical Committee No. 3 on Machine Tapers

A.S.M.E. Members (Total personnel, 20)

F. S. Blackall, Jr., *Chairman*
J. B. Armitage
†E. J. Bryant
†Earle Buckingham
F. H. Colvin
J. B. Dillard
(T. F. Githens, *Alternate*)
B. P. Graves
H. E. Harris
†F. O. Hoagland
J. H. Horigan
A. H. Lyon
L. F. Nenninger
C. W. Spicer

SUBGROUP CHAIRMEN

Steep Tapers Series, S. McMullan
Revision of Slow Taper Standard, E. J. Bryant

Technical Committee No. 4 on Spindle Noses and Collets for Machine Tools

A.S.M.E. Members (Total personnel, 26)

J. E. Lovely, *Chairman*
L. F. Nenninger, *Secretary*
J. B. Armitage
B. P. Graves
F. O. Hoagland
A. M. Johnson
M. E. Lange
J. H. Mansfield
H. W. Ruppel
L. D. Spence

SUBGROUP CHAIRMEN

No. 1 on Milling Machines, Small and Medium, J. B. Armitage
No. 2 on Large Milling Machines, F. B. Kampmeier
No. 3 on Grinding Machine Spindles, H. J. Griffing
No. 5 on Drilling Machines and Horizontal Boring Machines, S. McMullan
No. 6 on Turning Machines, Including Automatic Screw Machines, Lathes, Automatic Lathes, Turret Lathes, and Automatic Chucking Machines, J. E. Lovely
No. 7 on Cutting and Hobbing Machines, A. L. Stewart
No. 8 on Correlation of Counter Proposals for Spindle Noses, J. E. Lovely

Technical Committee No. 5 on Milling Cutters

A.S.M.E. Members (Total personnel, 20)

J. B. Armitage
†A. N. Goddard
J. H. Horigan
G. L. Markland, Jr.
E. K. Morgan
†Erik Oberg

E. D. Vancil

SUBGROUP CHAIRMEN

No. 1 on Profile Cutters, E. D. Vancil
No. 2 on Keyways, J. B. Armitage
No. 3 on Nomenclature, A. C. Danekind
No. 4 on Limits, J. H. Horigan
No. 5 on Formed Cutters, H. C. Hungerford
No. 6 on Hobs, G. L. Markland, Jr.
No. 7 on Inserted Tooth Cutters, J. B. Armitage

Technical Committee No. 6 on Designations and Working Ranges of Machine Tools

A.S.M.E. Members (Total personnel, 19)

John Haydock, *Chairman*
Earle Buckingham
T. H. Doan, Jr.
B. P. Graves
J. J. McBride
E. R. Smith

Technical Committee No. 7 on Twist Drill Sizes

A.S.M.E. Members (Total personnel, 7)

W. C. Mueller, *Chairman*
J. H. Horigan

C. W. Spicer

Technical Committee No. 8 on Jig Bushings

A.S.M.E. Members (Total personnel, 8)

J. H. Horigan
H. E. Wells

Technical Committee No. 9 on Punch Press Tools

A.S.M.E. Members (Total personnel, 16)

D. H. Chason
N. W. Dorman
H. E. Harris
D. M. Palmer

H. E. Wells

Technical Committee No. 10 on Forming Tools and Holders

A.S.M.E. Members (Total personnel, 9)

W. C. Mueller, *Chairman*
William Hartman
L. D. Spence
H. E. Wells

Technical Committee No. 11 on Chucks and Chuck Jaws

A.S.M.E. Member (Total personnel, 10)

J. E. Lovely, *Chairman*

SUBGROUP CHAIRMAN

No. 1 on Master Chuck Jaws, J. E. Lovely
No. 2 on Adapters for Air Cylinders, J. E. Lovely

Technical Committee No. 12 on Cut and Ground Thread Taps

(Total personnel, 7)

Technical Committee No. 13 on Splines and Splined Shafts

A.S.M.E. Members (Total personnel, 16)

C. W. Spicer, *Chairman*
J. B. Armitage
R. E. W. Harrison
F. O. Hoagland
J. E. Lovely
B. F. Waterman

SUBGROUP CHAIRMEN

Machine Tool Splines, R. E. W. Harrison
Automotive Splines, W. L. Barth
Involute Splines, C. K. Lenning

Technical Committee No. 17 on Nomenclature for Small Tools and Machine Tool Elements

A.S.M.E. Members (Total personnel, 15)

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F. S. Blackall, Jr.
F. H. Colvin
H. E. Harris

F. O. Hoagland

Ex-Officio Members

A. N. Goddard
B. P. Graves
W. C. Mueller
C. W. Spicer

H. E. Wells

Technical Committee No. 18 on Multiple Spindle Drilling Heads

A.S.M.E. Member (Total personnel, 9)

H. E. Wells

Technical Committee No. 19 on Single Point Cutting Tools

A.S.M.E. Members (Total personnel, 2)

F. H. Colvin, *Chairman*
O. W. Boston, *Secretary*

Technical Committee No. 20 on Reamers

A.S.M.E. Members (Total personnel, 16)

E. J. Bryant
F. H. Colvin
T. F. Githens
J. H. Horigan

H. E. Wells

GEARS (B-6)

* *Joint sponsorship with the American Gear Manufacturers Association. Sectional Committee organized June, 1921*

A.S.M.E. Members (Total personnel, 27)

B. F. Waterman, *Chairman*
†Earle Buckingham, *Vice-Chairman*
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L. H. Fry
C. B. Hamilton, Jr.
D. T. Hamilton
M. R. Hanna
†O. A. Leutwiler
G. L. Markland, Jr.
H. E. Vehslage

Subcommittee Chairmen

No. 1 on Program, B. F. Waterman
No. 2 on Editing Reports, B. F. Waterman
No. 3 on Nomenclature, D. T. Hamilton
No. 4 on Tooth Form (Spur Gears), U. S. Eberhardt
No. 5 on Helical Gears, W. P. Schmitter
No. 6 on Worm Gears, T. R. Rideout
No. 7 on Bevel Gears, F. L. Knowles
No. 8 on Materials, C. B. Hamilton, Jr.
No. 9 on Inspection, J. P. Breuer
No. 10 on Horsepower Rating, Earle Buckingham

PIPE FLANGES AND FITTINGS (B-16)

* *Joint sponsorship with the Heating, Piping, and Air Conditioning Contractors National Association and the Manufacturers Standardization Society of the Valve and Fittings Industry. Sectional Committee organized October, 1921*

A.S.M.E. Members (Total personnel, 50)

†C. P. Bliss, *Chairman*
J. J. Harman, *Secretary*
†L. W. Benoit
A. L. Brown
W. R. Conard
Sabin Crocker
Ferdinand Fink
H. E. Haller
J. S. Hess
†H. A. Hoffer
E. L. Hopping
A. M. Houser
D. S. Jacobus
†C. A. Kelting
J. R. Kruse
(John Blizard, *Alternate*)
M. B. MacNeille
F. H. Morehead
†E. L. Moreland
L. S. Morse
Ludwig Skog
†J. R. Tanner
J. H. Taylor
H. L. Underhill
G. W. Watts
J. H. Williams

Subcommittee Chairmen

No. 1 on Cast Iron Flanges and Flanged Fittings, A. M. Houser
No. 2 on Screwed Fittings, F. H. Morehead
No. 3 on Steel Flanges and Flanged Fittings, C. P. Bliss
No. 4 on Materials and Stresses, A. M. Houser
No. 5 on Face to Face Dimensions of Ferrous Flanged Valves, J. R. Tanner
No. 6 on Malleable Iron or Steel Brass Seat Unions
No. 7 on Rating of Pipe Fittings
No. 8 on Marking of Pipe Fittings
No. 9 on Port Openings, W. W. Hubbard

BOLT, NUT, AND RIVET PROPORTIONS (B-18)

* *Joint sponsorship with the Society of Automotive Engineers. Sectional Committee organized March, 1922*

A.S.M.E. Members (Total personnel, 62)

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F. C. Billings
B. G. Braine
(D. L. Braine, *Alternate*)
Ellwood Burdsall
G. S. Case
T. G. Crawford
H. P. Frear
Herman Koester
S. F. Newman
Martin Slifka
R. J. Whelan
E. M. Whiting
V. R. Willoughby
(J. J. McBride, *Alternate*)
O. B. Zimmerman

Subcommittee Chairmen

No. 1 on Large and Small Rivets, H. N. Wallin
No. 2 on Wrench Head Bolts and Nuts, H. B. Slocum
No. 3 on Slotted Head Proportions
No. 4 on Track Bolts and Nuts
No. 5 on Round Unslotted Head Bolts (Carriage Bolts), M. C. Horine
No. 6 on Plow Bolts, O. B. Zimmerman
No. 7 on Body Dimensions and Materials
No. 8 on Nomenclature, G. S. Case
No. 9 on Socket Head Cap and Set Screws, Herman Koester

SHAFTING (B-17)

* *Sole sponsorship. Organized October, 1918*

A.S.M.E. Members (Total personnel, 13)

†C. M. Chapman, *Chairman*
H. C. E. Meyer
L. C. Morrow
J. M. Shimer
C. W. Spicer
†G. N. Van Derhoef
†L. W. Williams

Subcommittee Chairmen

No. 1 on Shafting Dimensions, L. W. Williams
No. 2 on Stock Key Sizes
No. 3 on Code for Design of Transmission Shafting
No. 4 on Taper Keys, J. E. Bushnell
No. 5 on Woodruff Keys, L. C. Morrow

PLAIN AND LOCK WASHERS (B-27)

* *Joint sponsorship with the Society of Automotive Engineers. Sectional Committee organized August, 1925*

A.S.M.E. Members (Total personnel, 40)

Eugene Caldwell
T. G. Crawford
†B. S. Lewis
C. H. Loutrel
J. J. McBride
H. C. E. Meyer
E. M. Whiting
O. B. Zimmerman

Subcommittee Chairmen

No. 1 on Plain Washers
No. 2 on Lock Washers, C. H. Loutrel

TRANSMISSION CHAINS AND SPROCKETS (B-29)

**Joint sponsorship with the Society of Automotive Engineers and the American Gear Manufacturers Association. Sectional Committee organized September, 1917. Reorganized December, 1926*

A.S.M.E. Members (Total personnel, 15)

W. J. Belcher
†C. H. Jahnke
Joseph Joy
†L. V. Ludy
D. B. Perry
G. A. Young
O. B. Zimmerman

CODE FOR PRESSURE PIPING (B-31)

**Sole sponsorship. Sectional Committee organized March, 1926. Reorganized December, 1937*

A.S.M.E. Members (Total personnel, 101)

E. B. Ricketts, *Chairman*
R. E. Bryant
H. C. Cooper
D. H. Corey
Sabin Crocker
H. D. Edwards
Charles Fitzgerald
E. R. Fish
V. M. Frost
T. W. Greene
H. E. Haller
H. D. Halsey
J. S. Haug
(E. B. Severs, *Alternate*)
H. A. Hoffer
G. G. Hollins
E. L. Hopping
†A. M. Houser
†Alfred Iddles
D. S. Jacobus
T. M. Jasper
C. A. Kelting
G. S. Larsen
M. B. MacNeille
G. W. Martin
H. C. E. Meyer
J. W. Moore
(J. D. Capron, *Alternate*)
F. H. Morehead
(W. W. Crawford, *Alternate*)
(J. J. Harman, *Alternate*)
H. H. Morgan
L. S. Morse
A. W. Moulder
E. W. Norris
C. W. Obert
G. A. Orrok
A. L. Penniman, Jr.
†C. S. Robinson
J. H. Romann
G. W. Saathoff
G. K. Saurwein
Ludwig Skog
H. S. Smith
(H. H. Moss, *Alternate*)
F. N. Speller
J. R. Tanner
J. H. Taylor
J. H. Vance
H. LeR. Whitney
H. L. Whittemore
J. H. Williams
T. F. Wolfe

SUBCOMMITTEE CHAIRMEN

No. 1 on Plan, Scope, and Editing, SABIN CROCKER
No. 2 on Power Piping, ALFRED IDDLES
No. 4 on Gas and Air Piping, J. S. HAUG
No. 5 on Refrigeration Piping, A. H. BAER
No. 6 on Oil Piping, A. D. SANDERSON
No. 7 on Piping Materials and Identification, F. H. MOREHEAD
No. 8 on Fabrication Details, LUDWIG SKOG
No. 9 on Code for District Heating Piping, G. K. SAURWEIN

WIRE AND SHEET METAL GAGING SYSTEMS (B-32)

**Joint sponsorship with the Society of Automotive Engineers. Sectional Committee organized November, 1927*

A.S.M.E. Members (Total personnel, 27)

J. F. HOWE — F. G. WILSON

SUBCOMMITTEE CHAIRMEN

No. 1 on Flat Stocks
No. 2 on Wires and Rods
No. 3 on Tubing, Piping, Conduit and Casing, F. H. NULLMEYER

SCREW THREADS FOR HOSE COUPLINGS (B-33)

**Sole sponsorship. Sectional Committee organized August, 1928*

A.S.M.E. Members (Total personnel, 28)

A. L. BROWN, *Secretary*
†A. F. BREITENSTEIN
J. J. CROTTY
W. L. CURTISS
†W. E. DUNHAM
J. J. HARMAN
(F. C. ERNST, *Alternate*)
A. M. HOUSER
H. C. E. MEYER
J. H. WILLIAMS

SUBCOMMITTEE CHAIRMEN

No. 1 to Draft Recommended Specifications, J. H. HOWLAND
No. 2 on Basic Thread Dimensions, D. R. MILLER

WROUGHT IRON AND WROUGHT STEEL PIPE AND TUBING (B-36)

**Joint sponsorship with the American Society for Testing Materials. Sectional Committee organized April, 1928*

A.S.M.E. Members (Total personnel, 44)

H. H. MORGAN, *Chairman*
SABIN CROCKER, *Secretary*
H. E. ALDRICH
E. L. HOPPING
(A. B. MORGAN, *Alternate*)
†A. M. HOUSER
†D. S. JACOBUS
†(F. S. CLARK, *Alternate*)
J. J. KANTER
H. C. E. MEYER
F. H. MOREHEAD
†H. B. OATLEY
†LUDWIG SKOG
F. N. SPELLER
J. R. TANNER
A. E. WHITE
H. LeR. WHITNEY

SUBCOMMITTEE CHAIRMEN

No. 1 on Plan, Scope, and Editing, H. H. MORGAN
No. 2 on Pipe and Tubing for Low Temperature Service, J. J. SHUMAN
No. 3 on Pipe and Tubing for High Temperature Service, J. R. TANNER
No. 4 on Materials, F. H. MOREHEAD

PRESSURE AND VACUUM GAGES (B-40)

**Sole sponsorship. Sectional Committee organized July, 1930*

A.S.M.E. Members (Total personnel, 45)

M. D. ENGLE, *Chairman*
†A. W. LENDEROTH, *Secretary*
E. J. BRYANT
†J. P. CAVANAUGH
PAUL DISERENS
C. H. GRAESSER
W. F. JONES
R. J. KEHL
†J. C. McCUNE
A. H. MORGAN
H. B. REYNOLDS
W. C. SCHOENFELDT

SUBCOMMITTEE CHAIRMEN

No. 1 on Plan and Scope, M. D. ENGLE
No. 2 on Definitions, C. F. SCHWEP
No. 3 on Gage Sizes and Mounting Dimensions, H. B. REYNOLDS
No. 4 on Accuracy and Test Methods, O. J. HODGE

STOCK SIZES, SHAPES AND LENGTHS FOR HOT AND COLD FINISHED IRON AND STEEL BARS (B-41)

**Sole sponsorship. Sectional Committee organized April, 1929*

A.S.M.E. Members (Total personnel, 27)

F. H. DECHANT
†L. W. WILLIAMS
G. H. WOODROFFE
O. B. ZIMMERMAN

SUBCOMMITTEE CHAIRMEN

No. 1 on Hot Rolled Steel, HENRY WYSOR
No. 2 on Cold Finished Steels, L. E. CREIGHTON
No. 3 on Hot Rolled Iron

SPECIFICATIONS FOR LEATHER BELTING (B-42)

**Sole sponsorship. Sectional Committee organized February, 1931*

A.S.M.E. Members (Total personnel, 27)

H. T. COATES
†R. W. DRAKE
KING HATHAWAY
J. E. RHOADS
G. A. SCHIEREN
†C. O. STREETER
O. B. ZIMMERMAN

SUBCOMMITTEE CHAIRMEN

No. 1 on Standard Specifications, R. C. BOWKER
No. 2 on Recommendations for Selection, Care, and Installation, G. A. SCHIEREN

MACHINE PINS (B-43)

**Joint sponsorship with the Society of Automotive Engineers. Sectional Committee organized, March, 1926*

A.S.M.E. Members (Total personnel, 16)

†E. J. BRYANT
J. J. McBRIDE
H. C. E. MEYER
O. B. ZIMMERMAN

SUBCOMMITTEES

No. 1 on Straight, Taper, and Dowel Pins
No. 2 on Split Pins

FOUNDRY EQUIPMENT AND SUPPLIES (B-45)

**Joint sponsorship with the American Foundrymen's Association. Sectional Committee organized February, 1931*

A.S.M.E. Members (Total personnel, 24)

E. S. CARMAN, *Chairman*
E. W. ELY
F. W. HERENDEEN
A. S. PHELPS
O. B. ZIMMERMAN

SUBCOMMITTEE CHAIRMEN

No. 1, L. M. SHERWIN
No. 2, E. S. CARMAN
No. 3, A. S. PHELPS

CLASSIFICATION AND DESIGNATION OF SURFACE QUALITIES (B-46)

**Joint sponsorship with the Society of Automotive Engineers. Sectional Committee organized May, 1932*

A.S.M.E. Members (Total personnel, 63)

J. R. WEAVER, *Chairman*
E. J. ABBOTT
E. J. BRYANT
JOHN CETRULE
T. G. CRAWFORD
†R. C. DEALE
U. S. EBERHARDT
S. A. EINSTEIN
R. F. GAGG
W. W. GILBERT
J. J. HARMAN
†R. E. W. HARRISON
F. V. HARTMAN
F. O. HOAGLAND
H. J. HOLTZCLAW
R. T. KENT
H. J. KURTZ
A. H. LYON
M. J. REED
F. C. SPENCER
C. C. STEVENS
J. S. TAWRESEY
C. H. WHITAKER
ERNEST WOOLER

SUBCOMMITTEE CHAIRMEN

No. 1 on Surfaces Produced by Tools and Abrasives, JOHN CETRULE
No. 2 on Surfaces Produced by Molds, Dies, Rolls, or Any Other Means of Deforming Materials

CLASSIFICATION AND DESIGNATION OF SURFACE QUALITIES (B-46) *(continued)*

No. 3 on Coated Surfaces, G. B. Hogaboom
No. 4 on Symbols for Indicating Surface Quality on Drawings, T. G. Crawford
No. 5 on Ways, Means, and Apparatus for Measuring Quality of Surface, J. R. Weaver
No. 6 on Finance, R. E. W. Harrison

COMBUSTION SPACE FOR SOLID FUELS (B-50)

**Sole sponsorship. Sectional Committee organized June, 1933*

A.S.M.E. Members (Total personnel, 21)

C. E. Bronson, *Chairman*
W. G. Christy
John Hunter
A. J. Johnson
†V. G. Leach
T. A. Marsh
J. F. McIntire
R. H. Rowland
†John Van Brunt

Subcommittee Chairmen

No. 1 on Purpose and Scope, C. E. Bronson
No. 2 on Combustion and Design, B. M. Guthrie
No. 3 on Warm Air Furnaces, J. H. Manny
No. 4 on Steel Heating Boilers, W. B. Russell
No. 5 on Cast Iron Boilers, J. F. McIntire

SCHEME FOR IDENTIFICATION OF PIPING SYSTEMS (A-13)

**Joint sponsorship with the National Safety Council. Sectional Committee organized June, 1922*

A.S.M.E. Members (Total personnel, 36)

E. E. Ashley
W. L. Bunker
Crosby Field
E. L. Hopping
H. L. Miner
H. S. Smith
Frank Thornton

Subcommittee Chairmen

Identification by Colors
Classification, Crosby Field
Identification Markings Other Than Color, W. S. Morrison

MINIMUM REQUIREMENTS FOR PLUMBING AND STANDARDIZATION OF PLUMBING EQUIPMENT (A-40)

**Joint sponsorship with the American Society of Sanitary Engineering. Sectional Committee organized August, 1928*

A.S.M.E. Members (Total personnel, 46)

J. J. Crotty
J. F. Carney
A. M. Houser
G. W. Martin
(A. H. Morgan, *Alternate*)
†W. R. Webster

Subcommittee Chairmen

No. 1 on Minimum Requirements for Plumbing, W. C. Groeniger
No. 2 on Staple Vitreous China Plumbing Fixtures, E. S. Aitkin
No. 3 on Staple Porcelain (All Clay) Plumbing Equipment, E. S. Aitkin
No. 4 on Enameled Sanitary Ware, A. H. Cline, Jr.
No. 5 on Traps, A. R. McGonegal
No. 6 on Brass Plumbing Products, J. L. Murphy
No. 7 on Brass Fittings for Flared Copper Tubes, F. L. Riggin
No. 8 on Cast Iron Soil Pipe and Fittings, J. J. Crotty
No. 9 on Gasoline, Oil, and Grease Separators, J. J. Crotty
No. 11 on Soldered Fittings for Tubing, A. M. Houser
No. 12 on Minimum Air Gaps in Plumbing Systems, W. K. McAfee

ELECTRIC MOTOR FRAME DIMENSIONS (C-28)

**Joint Sponsorship with the National Electrical Manufacturers Association. Sectional Committee organized November, 1927*

A.S.M.E. Members (Total personnel, 31)

C. A. Adams
S. A. Einstein
E. W. Ely
F. S. English
W. F. Jones
P. G. Rhoads
†A. G. Trumbull

ROLLED THREADS FOR SCREW SHELLS OF ELECTRIC SOCKETS AND LAMP BASES (C-44)

**Joint sponsorship with the National Electrical Manufacturers Association. Sectional Committee organized March, 1929*

A.S.M.E. Members (Total personnel, 17)

†E. J. Bryant
†Earle Buckingham
A. B. Morgan
†E. S. Sanderson

LETTER SYMBOLS AND ABBREVIATIONS FOR SCIENCE AND ENGINEERING (Z-10)

**Joint sponsorship with the American Association for the Advancement of Science, American Institute of Electrical Engineers, American Society of Civil Engineers, and the Society for the Promotion of Engineering Education. Sectional Committee organized January, 1926. Reorganized October, 1935*

A.S.M.E. Members (Total personnel, 37)

†S. A. Moss, *Vice-Chairman*
R. M. Anderson
(E. P. Warner, *Alternate*)
K. H. Condit
†R. J. S. Pigott
(†S. R. Beitler, *Alternate*)
Frank Thornton, Jr

Subcommittee Chairmen

No. 1 on Letter Symbols and Signs for Mathematics, A. A. Bennett
No. 2 on Symbols for Hydraulics, J. C. Stevens
No. 3 on Symbols for Mechanics, R. E. Peterson
No. 4 on Symbols for Structural Analysis, Albert Haertlein
No. 5 on Symbols for Heat and Thermodynamics, S. A. Moss
No. 6 on Symbols for Photometry, E. C. Crittenden
No. 7 on Aeronautical Symbols
No. 8 on Symbols for Electric and Magnetic Quantities, J. F. Meyer
No. 9 on Symbols for Radio, H. M. Turner
No. 10 on Symbols for Physics, H. K. Hughes
No. 11 on Abbreviations for Engineering and Scientific Terms, G. A. Stetson

DRAWINGS AND DRAFTING ROOM PRACTICE (Z-14)

**Joint sponsorship with the Society for the Promotion of Engineering Education. Sectional Committee organized July, 1926*

A.S.M.E. Members (Total personnel, 53)

F. DeR. Furman, *Chairman*
C. W. Keuffel, *Secretary*
T. G. Crawford
H. P. Frear
T. E. French
A. C. Harper
E. R. Hill
A. M. Houser
Alfred Iddles
†Samuel Ketchum
F. R. Laney
H. B. Langille
Rudolph Michel
F. W. Ming
W. C. Mueller
E. B. Neil
J. W. Owens
F. C. Panuska
†E. S. Smith

Subcommittee Chairmen

No. 1 on Specifications for Paper and Cloth
No. 2 on Method of Indicating Dimensions
No. 3 on Lettering, T. G. Crawford
No. 4 on Layout, Douglas Baker
No. 5 on Line Work, Samuel Ketchum
No. 6 on Graphical Symbols on Drawings, T. E. French
Editing Committee, T. E. French

GRAPHIC PRESENTATION (Z-15)

**Sole sponsorship. Sectional Committee organized November, 1926*

A.S.M.E. Members (Total personnel, 31)

†G. E. Hagemann, *Secretary*
C. M. Bigelow
Wallace Clark
H. G. Crockett
T. E. French
†D. B. Porter

Subcommittee Chairmen

No. 1 on Plan and Scope
No. 2 on Terminology
No. 3 on Preferred Practice for Time Series Charts, A. H. Richardson
No. 4 on Engineering and Scientific Graphs, W. A. Shewhart

SPEEDS OF MACHINERY (Z-18)

**Sole sponsorship. Sectional Committee organized May, 1928*

A.S.M.E. Members (Total personnel, 29)

†C. M. Bigelow
J. F. Daggett
†R. C. Deale
Paul Diserens
F. S. English
D. C. Jackson
H. C. E. Meyer
John Reid
P. G. Rhoads
F. C. Spencer
O. B. Zimmerman

Subcommittee Chairmen

No. 1 on Plan and Scope, A. E. Hall
No. 2 on Questionnaire and Canvass to Industry, F. S. English
No. 3—Special Reviewing Committee

GRAPHICAL SYMBOLS AND ABBREVIATIONS FOR USE ON DRAWINGS (Z-32)

**Joint sponsorship with American Institute of Electrical Engineers. Sectional Committee organized April, 1936*

A.S.M.E. Members (Total personnel, 53)

E. E. Ashley
(John James, *Alternate*)
J. M. Barnes
†T. E. French
D. T. Hamilton
G. F. Habach
F. W. Holland
A. M. Houser
(J. J. Harman, *Alternate*)
W. C. Mueller
L. L. Munier
J. W. Owens
†F. C. Panuska
M. J. Reed
T. R. Thomas

Subcommittee Chairmen

No. 1 on Symbols for Use in Mechanical Engineering, T. E. French
No. 2 on Symbols for Use in Electrical Engineering, H. W. Samson

DEVELOPMENT OF APPLICATIONS OF STATISTICS IN ENGINEERING AND MANUFACTURING

Joint sponsorship with the American Society for Testing Materials. Appointed in December, 1929

A.S.M.E. Members (Total personnel, 8)

A. G. Ashcroft
W. H. Fulweiler
†L. K. Sillcox
†J. S. Tawresey

A.S.M.E. Representatives on Miscellaneous Standardization Committees

See also A.S.M.E. Representatives on Other Activities, page RI-8

ACOUSTICAL MEASUREMENTS AND TERMINOLOGY

**Sponsor body: Acoustical Society of America*

E. E. Free
(P. H. Bilhuber, *Alternate*)
R. V. Parsons
(J. S. Parkinson, *Alternate*)
W. B. White

AMERICAN MARINE STANDARDS COMMITTEE

Advisory Committee of Department of Commerce

J. W. Gray (H. W. Brown, *Alternate*)

BUILDING CODE REQUIREMENTS FOR LIGHT AND VENTILATION

**Sponsor bodies: Federal Housing Administration and U.S. Public Health Service*

F. R. Scherer

CLASSIFICATION OF COAL

**Sponsor body: American Society for Testing Materials*

E. G. Bailey

DRAINAGE OF COAL MINES

**Sponsor body: American Mining Congress*

O. M. Pruitt

ELECTRIC WELDING APPARATUS

**Sponsor bodies: American Institute of Electrical Engineers and the National Electrical Manufacturers Association*

R. E. Kinkead

ELECTRICAL DEFINITIONS

**Sponsor body: American Institute of Electrical Engineers*

C. H. Berry

FIRE TESTS OF BUILDING CONSTRUCTION AND MATERIALS

**Sponsor bodies: A.S.A. Fire Protection Group, National Bureau of Standards, and the American Society for Testing Materials*

R. C. Parlett

FOREST FIRE PROTECTION

Committee of National Fire Protection Association

(To be appointed)

GEAR LUBRICANTS

**Sponsor body: American Gear Manufacturers Association*

G. B. Karelitz

LOADING PLATFORMS AT FREIGHT TERMINALS AND WAREHOUSES

**Sponsor body: American Trucking Association*

(To be appointed)

MANHOLE FRAMES AND COVERS

Sponsor bodies: A.S.A. Telephone Group and American Society of Civil Engineers

Anton Hansen
Homer Rupard

MARKING OF OBSTRUCTIONS TO AIR NAVIGATION

Joint committee between aviation interests and combined utility interests

J. E. Whitbeck (Jerome Lederer, *Alternate*)

METHODS OF RATING RIVERS

Sponsor body: U.S. Geological Survey

D. W. Mead

METHODS OF TESTING WOOD

**Sponsor bodies: U.S. Forest Service and the American Society for Testing Materials*

C. M. Bigelow

MISCELLANEOUS OUTSIDE COAL-HANDLING EQUIPMENT

**Sponsor body: American Mining Congress*

(To be appointed)

PETROLEUM PRODUCTS AND LUBRICANTS

**Sponsor body: American Society for Testing Materials*

G. B. Karelitz
(H. J. Masson, *Alternate*)
W. F. Parish
(S. J. Needs, *Alternate*)

PETROLEUM SPECIFICATIONS

Advisory Board of U.S. Bureau of Mines

H. A. S. Howarth

PREFERRED NUMBERS

**Special Committee of A.S.A.*

K. H. Condit

ROTATING ELECTRICAL MACHINERY

*Sponsor bodies: *American Institute of Electrical Engineers and National Electrical Manufacturers Association*

C. A. BOOTH

SPECIFICATIONS FOR CAST IRON PIPE AND SPECIAL CASTINGS

*Sponsor bodies: *American Gas Association, American Society for Testing Materials, American Water Works Association, National Electrical Manufacturers Association*

G. W. BIGGS J. E. GIBSON

SPECIFICATIONS FOR CLEAN BITUMINOUS COAL

*Sponsor body: *American Institute of Mining and Metallurgical Engineers*

R. A. SHERMAN (E. L. LINDSETH, *Alternate*)

SPECIFICATIONS FOR FUEL OILS

*Sponsor body: *American Society for Testing Materials*

W. H. BUTLER H. W. DOW
HARTE COOKE L. H. MORRISON
LEE SCHNEITTER

SPECIFICATIONS FOR MATERIALS FOR USE IN MANUFACTURE OF SPECIAL TRACKWORK

*Sponsor body: *American Transit Association*

W. R. HULBERT

SPECIFICATIONS FOR AND RECOMMENDED PRACTICE IN THE USE OF WIRE ROPE FOR MINES

*Sponsor body: *American Mining Congress*

J. L. HARRINGTON

SPECIFICATIONS FOR SIEVES FOR TESTING PURPOSES

*Sponsor bodies: *American Society for Testing Materials and National Bureau of Standards*

(To be appointed)

SPECIFICATIONS FOR TOOL STEEL

Subcommittee No. XIV of A.S.T.M. Committee A1 on Steel

O. W. BOSTON C. M. INMAN

STANDARDIZATION IN THE FIELD OF PHOTOGRAPHY

*Sponsor body: *Optical Society of America*

J. A. LUCAS (J. F. GUINAN, *Alternate*)

THERMAL INSULATING MATERIALS

*Sponsor body: *American Society for Testing Materials*

R. H. HEILMAN

VOLUME WATER HEATING

*Sponsor body: *American Gas Association*

MARK RESEK

Safety Technical Committees

ARTICLE B6A, PAR. 25: The Standing Committee on Safety shall advise the Council on the activities of the Society having to do with engineering and industrial safety, except the activities of the Boiler Code Committee, for which special provision is made.

The first Standing Committee on Safety was appointed in October, 1921.

D. L. ROYER, *Chairman and Representative on the Council* (1940)
J. B. CHALMERS (1939) A. W. LUCE (1942)
T. F. HATCH (1941) A. E. WINDLE (1943)

SAFETY CODE FOR MECHANICAL POWER-TRANSMISSION APPARATUS (B15)

Joint sponsorship with the International Association of Industrial Accident Boards and Commissions, National Conservation Bureau. Sectional Committee organized February, 1921

A.S.M.E. Members (Total personnel, 31)

P. G. RHOADS, *Secretary* W. S. PAINE
C. C. FOSTER †D. C. WRIGHT
†G. M. NAYLOR (†G. N. VANDERHOEF, *Alternate*)
W. W. NICHOLS

SUBCOMMITTEES

No. 1 on Detail Classification of Belts
No. 2 on Modification of Rule 223 for Cone Pulley Belts
No. 3 on Mechanical Power Control
No. 4 on Use of ASA Code Versus State Codes
No. 5 on Statistics on Place of Occurrence of Accidents

SAFETY CODE FOR ELEVATORS (A17)

Joint Sponsorship with the American Institute of Architects and the National Bureau of Standards. Sectional Committee organized November, 1922

A.S.M.E. Members (Total personnel, 41)

O. P. CUMMINGS, *Vice-Chairman* D. L. LINDQUIST
C. R. CALLAWAY †N. O. LINDSTROM
†D. L. HOLBROOK M. B. MCLAUTHLIN
W. S. PAINE

SUBCOMMITTEE CHAIRMEN

Research, Recommendations, and Interpretations, D. J. PURINTON
Inspectors' Handbook, K. A. COLAHAN
Elevator Systems Safe Against Fire, W. S. PAINE
Interlocks and Hoistway Enclosures, B. JONES

SAFETY CODE ON COMPRESSED AIR MACHINERY AND EQUIPMENT (B19)

Joint sponsorship with the American Society of Safety Engineers—Engineering Section, National Safety Council. Sectional Committee organized May, 1923

A.S.M.E. Members (Total personnel, 24)

D. L. ROYER, *Chairman* W. J. GRAVES
H. D. EDWARDS †H. H. JUDSON

SAFETY CODE FOR CONVEYORS AND CONVEYING MACHINERY (B20)

Joint sponsorship with the National Conservation Bureau. Sectional Committee organized November, 1925

A.S.M.E. Members (Total personnel, 50)

D. L. ROYER, *Chairman* W. S. PAINE
C. T. COLLEY C. G. PFEIFFER
W. J. GRAVES R. B. RENNER
†M. A. KENDALL F. J. SHEPARD, JR.
(†N. W. ELMER, *Alternate*) †WILLIAM STANIAR
P. T. ONDERDONK J. G. WHEATLEY

SUBCOMMITTEE CHAIRMEN

No. 1 on All Types of Chain Conveyors, Belt Conveyors, Belt Elevators Including Steel Belt, and Screw, Track or Scraper Conveyors, C. G. PFEIFFER
No. 2 on Gravity Conveyors and Chutes, Live Roll Conveyors, H. G. DALTON
No. 3 on Cable-Operated and Cable Flight Conveyors and Cableways, R. McA. KEOWN
No. 4 on Air, Steam, or Liquid Conveyors, J. J. McNULTA
No. 5 on Tiering, Piling, and Stacking Conveyors, J. G. WHEATLEY

SAFETY CODE FOR CRANES, DERRICKS, AND HOISTS (B30)

**Joint sponsorship with U.S. Navy Department, Bureau of Yards and Docks. Sectional Committee organized November, 1926*

A.S.M.E. Members (Total personnel, 52)

†LEWIS PRICE — †R. H. WHITE
F. H. SCHWERIN — H. L. WHITTEMORE

SUBCOMMITTEE CHAIRMEN

No. 1 on Overhead and Gantry Cranes, R. H. WHITE
No. 2 on Locomotive and Tractor Cranes, H. H. VERNON
No. 3 on Derricks and Hoists, LEWIS PRICE
No. 4 on Miscellaneous Equipment for Cranes and Hoists, L. W. HOPKINS
No. 5 on Jacks, E. W. CARUTHERS

AIRCRAFT SAFETY AND INSPECTION

A.S.M.E. Committee organized April, 1931

A.S.M.E. Members (Total personnel, 6)

JEROME LEDERER, *Chairman* — ALEXANDER KLEMIN

A.S.M.E. Representatives on Other Safety Committees

See also A.S.M.E. Representatives on Other Activities, page RI-8

SAFETY CODE FOR ABRASIVE WHEELS

**Sponsor bodies: Grinding Wheel Manufacturers Association of United States and Canada, and International Association of Industrial Accident Boards and Commissions*

J. B. CHALMERS

SAFETY CODE FOR AERONAUTICS

**Sponsor body: Society of Automotive Engineers*

(To be appointed)

SAFETY CODE FOR AMUSEMENT PARKS

**Sponsor bodies: National Association of Amusement Parks and National Conservation Bureau*

(To be appointed)

SAFETY CODE FOR CONSTRUCTION WORK

**Sponsor bodies: American Institute of Architects and National Safety Council*

C. H. O'NEIL

COOPERATION WITH OTHER ENGINEERING SOCIETIES

Sponsor body: American Society of Safety Engineers—Engineering Section, National Safety Council

H. L. MINER

A.S.A. SAFETY CODE CORRELATING COMMITTEE

H. H. JUDSON (D. L. ROYER, *Alternate*)

SAFETY CODE FOR EXHAUST SYSTEMS

**Sponsor body: International Association of Industrial Accident Boards and Commissions*

J. C. HARDIGG — T. F. HATCH

SAFETY CODE FOR FLOOR AND WALL OPENINGS, RAILINGS, AND TOE BOARDS

**Sponsor body: National Safety Council*

(To be appointed)

SAFETY CODE FOR FORGING AND HOT METAL STAMPING

**Sponsor bodies: American Drop Forging Institute and National Safety Council*

C. F. PARK

SAFETY CODE ON COLORS FOR IDENTIFICATION OF GAS MASK CANISTERS

**Sponsor body: National Safety Council*

L. C. LICHTY

SAFETY CODE FOR LADDERS

**Sponsor body: American Society of Safety Engineers—Engineering Section, National Safety Council*

H. H. JUDSON

SAFETY CODE FOR LAUNDRY MACHINERY AND OPERATION

**Sponsor bodies: Laundry Owners National Association, International Association of Governmental Labor Officials, and National Association of Mutual Casualty Companies*

E. J. CARROLL

SAFETY CODE FOR LIGHTING FACTORIES, MILLS, AND OTHER WORKS PLACES

**Sponsor body: Illuminating Engineering Society*

H. H. JUDSON

SAFETY CODE FOR LOGGING AND SAWMILL MACHINERY

**Sponsor body: National Bureau of Standards*

(To be appointed)

LOW VOLTAGE ELECTRICAL HAZARD

**Special Committee of the American Society of Safety Engineers—Engineering Section, National Safety Council*

J. P. JACKSON

SAFETY CODE FOR MECHANICAL REFRIGERATION

**Sponsor body: American Society of Refrigerating Engineers*

O. A. ANDERSON — E. W. GALLENKAMP
CROSBY FIELD — W. F. JONES

(A. W. OAKLEY, *Alternate to all A.S.M.E. Representatives*)

SAFETY CODE FOR PAPER AND PULP MILLS

**Sponsor body: National Safety Council*

R. L. WELDON

SAFETY CODE FOR POWER PRESSES, AND FOOT AND HAND PRESSES

**Sponsor body: National Safety Council*

J. B. CHALMERS

SAFETY CODE FOR PREVENTION OF DUST EXPLOSIONS

**Sponsor bodies: National Fire Protection Association and U.S. Department of Agriculture*

J. H. MORROW (R. M. FERRY, *Alternate*)

SAFETY CODE FOR PROTECTION OF HEADS, EYES, AND RESPIRATORY ORGANS OF INDUSTRIAL WORKERS

**Sponsor body: National Bureau of Standards*

T. A. WALSH, JR. (T. F. HATCH, *Alternate*)

SAFETY CODE FOR PROTECTION OF INDUSTRIAL WORKERS IN FOUNDRIES

**Sponsor bodies: American Foundrymen's Association and National Founders Association*

H. M. Lane

SAFETY CODE FOR RUBBER MACHINERY

**Sponsor bodies: National Safety Council and International Association of Industrial Accident Boards and Commissions*

(To be appointed)

SPECIFICATIONS AND METHODS OF TEST FOR SAFETY GLASS

**Sponsor bodies: National Conservation Bureau and National Bureau of Standards*

T. A. Walsh, Jr.

SAFETY CODE FOR TEXTILES

**Sponsor body: National Safety Council*

(To be appointed)

SAFETY CODE FOR VENTILATION

**Sponsor body: American Society of Heating and Ventilating Engineers*

T. F. Hatch

SAFETY CODE FOR WALKWAY SURFACES

**Sponsor bodies: American Institute of Architects and American Society of Safety Engineers—Engineering Section, National Safety Council*

(To be appointed)

SAFETY CODE FOR WINDOW WASHING

**Sponsor body: National Safety Council*

(To be appointed)

SAFETY CODE FOR WORK IN COMPRESSED AIR

**Sponsor body: International Association of Industrial Accident Boards and Commissions*

L. J. Eibsen

Power Test Codes Technical Committees

Article B6A, Par. 27: The Standing Committee on Power Test Codes shall, under the direction of the Council, have supervision of all the activities of the Society in connection with the A.S.M.E. Power Test Codes, including the interpretation of such codes.

The first Standing Committee on Power Test Codes was organized in December, 1918, to revise and extend the Power Test Codes which had been formulated by various technical committees appointed to develop particular codes. This work began in 1886.

STANDING COMMITTEE

Francis Hodgkinson, *Chairman and Representative on the Council* (1939)

A. G. Christie, *Vice-Chairman*

John Allhusen, *Junior Observer* (1939)

Term expires 1939	*Term expires 1940*
C. H. Berry	A. T. Brown
D. S. Jacobus	P. H. Hardie
Francis Hodgkinson	C. F. Hirshfeld
L. F. Moody	R. J. S. Pigott
E. B. Ricketts	M. C. Stuart
Term expires 1941	*Term expires 1942*
A. G. Christie	W. A. Carter
Paul Diserens	Harte Cooke
G. A. Orrok	E. R. Fish
L. A. Quayle	H. B. Oatley
W. M. White	W. J. Wohlenberg
Term expires 1943	
Louis Elliott	H. B. Reynolds
G. A. Horne	P. W. Swain

E. N. Trump

(1) GENERAL INSTRUCTIONS

Appointed December, 1918

W. H. Kavanaugh, *Chairman*
A. M. Greene, Jr.
C. F. Hirshfeld
M. C. Stuart

(2) DEFINITIONS AND VALUES

Appointed December, 1918

Reorganized, 1936

R. J. S. Pigott, *Chairman*
L. J. Briggs
W. F. Davidson
S. L. Kerr
A. L. Kimball
L. S. Marks
F. G. Philo
J. C. Smallwood
P. W. Swain
A. C. Wood

(3) FUELS

Appointed December, 1918

W. J. Wohlenberg, *Chairman*
E. G. Bailey
B. L. Boye
L. P. Breckenridge
H. W. Brooks
S. B. Flagg
D. M. Myers
F. G. Philo
G. S. Pope
E. B. Ricketts
F. M. Rogers
E. X. Schmidt
Nicholas Stahl
E N. Trump

(4) STATIONARY STEAM-GENERATING UNITS

Appointed December, 1918

E. R. Fish, *Chairman*
A. D. Bailey
M. W. Benjamin
B. J. Cross
Martin Frisch
P. H. Hardie
R. M. Hardgrove
Alfred Iddles
E. L. Lindseth
E. L. McDonald
E. B. Powell
R. Shellenberger

R. L. Spencer

(10) CENTRIFUGAL AND TURBO-COMPRESSORS AND BLOWERS

Appointed December, 1918

Reappointed, 1929

A. T. Brown, *Chairman*
E. L. Anderson
C. A. Booth
W. H. Carrier
Thomas Chester
L. E. Day
E. S. Dean
Z. G. Deutsch
S. H. Downs
P. E. Good
J. J. Grob
H. F. Hagen
Paul Hoffman
H. D. Kelsey
A. L. Kimball
L. S. Marks
Arvid Peterson
H. F. Schmidt

M. C. Stuart

(5) RECIPROCATING STEAM ENGINES

Appointed December, 1918

Reappointed, 1931

A. G. Christie, *Chairman*
Harte Cooke
K. S. M. Davidson
Henrik Greger
Thomas Hall
J. A. Hunter
H. G. Mueller
B. V. E. Nordberg
A. V. Saharoff
A. G. Whiting

(6) STEAM TURBINES

Appointed December, 1918

C. H. Berry, *Chairman*
I. E. Moultrop, *Secretary*
O. D. H. Bentley
W. E. Caldwell
C. B. Campbell
A. G. Christie
Hans Dahlstrand
V. M. Frost
A. E. Grunert
Francis Hodgkinson
S. A. Moss
R. O. Muller
T. E. Purcell
C. C. Thomas
G. B. Warren

(7) RECIPROCATING STEAM-DRIVEN DISPLACEMENT PUMPS

Appointed December, 1918

R. D. Hall, *Chairman*
C. H. Anderson
E. H. Brown
J. N. Chester
J. E. Gibson
G. L. Kollberg
M. B. MacNeille
D. W. Mead
L. A. Quayle

(8) CENTRIFUGAL AND ROTARY PUMPS

Appointed December, 1918

Reorganized, 1936

M. B. MacNeille, *Chairman*
H. E. Beckwith
R. L. Daugherty
W. B. Gregory
R. T. Knapp
L. F. Moody
Arvid Peterson
F. H. Rogers
W. C. Rudd
Max Spillman
W. M. White
I. A. Winter

(9) DISPLACEMENT COMPRESSORS AND BLOWERS

Appointed December, 1918

Reorganized, 1935

Paul Diserens, *Chairman*
G. T. Felbeck
C. R. Houghton
J. F. Huvane
R. M. Johnson
J. F. D. Smith

(11) COMPLETE STEAM POWER PLANTS

Appointed December, 1918

Reappointed, 1933

F. M. Van Deventer, *Chairman*
D. S. Wegg, *Secretary*
W. F. Davidson
C. H. Fellows
R. A. Foresman
V. M. Frost
W. W. Johnson
E. W. Norris
H. S. Whiton

(12) CONDENSERS, WATER HEATING, AND COOLING EQUIPMENT

Appointed December, 1918

G. A. Orrok, *Chairman*
P. J. Hardie, *Secretary*
C. H. Baker, Jr.
R. N. Ehrhart
J. F. Grace
D. W. R. Morgan
H. B. Reynolds
P. E. Reynolds

(13) REFRIGERATING SYSTEMS

Appointed December, 1918

(Committee being reorganized)

(14) EVAPORATING APPARATUS

Appointed December, 1918

E. N. Trump, *Chairman*
B. N. Bump
E. A. Newhall
H. L. Parr
L. C. Rogers

(15) STEAM LOCOMOTIVES

Appointed December, 1918

E. C. Schmidt, *Chairman*
W. F. Kiesel, Jr.
H. B. Oatley
G. E. Rhoads
L. K. Sillcox
W. E. Woodard

(16) GAS PRODUCERS

Appointed December, 1918

C. D. Smith
H. F. Smith

(17) INTERNAL-COMBUSTION ENGINES

Appointed December, 1918

Reappointed, 1928

(Committee being reorganized)

(18) HYDRAULIC PRIME MOVERS

Appointed December, 1918

Reappointed, 1931

S. L. Kerr, *Chairman*
C. M. Allen
L. M. Davis
H. L. Doolittle
W. F. Durand
H. N. Eaton
N. R. Gibson
J. P. Growdon
T. H. Hogg
L. J. Hooper
C. W. Hubbard
E. C. Hutchinson
D. J. McCormack
L. F. Moody
W. J. Rheingans
E. B. Strowger
R. V. Terry
W. M. White

(19) INSTRUMENTS AND APPARATUS

Appointed December, 1918

C. F. Hirshfeld, *Chairman*
W. A. Carter, *Secretary*
C. M. Allen
W. C. Andrae
E. G. Bailey
H. S. Bean
L. J. Briggs
J. D. Davis
K. J. De Juhasz
R. E. Dillon
F. M. Farmer
J. B. Grumbein
W. W. Johnson
W. H. Kenerson
E. S. Lee
E. L. Lindseth
Osborn Monnett
S. A. Moss
R. J. S. Pigott
E. B. Ricketts
W. A. Sloan
R. B. Smith
I. M. Stein

(20) SPEED-RESPONSIVE GOVERNORS

Appointed December, 1921

Francis Hodgkinson, *Chairman*
Harte Cooke
F. H. Rogers

(21) DUST SEPARATING APPARATUS

Appointed October, 1934

M. D. Engle, *Chairman*
Ollison Craig, *Secretary*
E. L. Anderson
A. D. Bailey
H. H. Bubar
W. G. Christy
H. O. Croft
J. M. DallaValle
H. C. Dohrmann
Philip Drinker
J. W. Fehnel
H. F. Hagen
P. H. Hardie
C. W. Hedberg
J. H. Leech
d. E. Macomber
H. B. Meller
H. C. Murphy
B. F. Tillson

A.S.M.E. Representatives on Other Committees

See also A.S.M.E. Representatives on Other Activities, page RI-8

DEVELOPMENT OF DEFINITIONS FOR THE NET CALORIFIC VALUE AND GROSS CALORIFIC VALUE OF FUELS

Sponsor body: American Society for Testing Materials

W. J. Wohlenberg

COMMITTEE ON REDEFINING SO-CALLED STANDARD TON OF REFRIGERATION

Sponsor body: American Society of Refrigerating Engineers

G. B. Bright

Boiler Code Committees

ARTICLE B6A, PAR. 26: The Special Committee on Boiler Code shall, under the direction of the Council, have supervision of all the activities of the Society in connection with the A.S.M.E. Codes for Pressure Vessels, including the interpretations of these codes.

SPECIAL COMMITTEE

D. S. JACOBUS, *Chairman*
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M. JURIST, *Acting Secretary*
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A. J. ELY
V. M. FROST
C. E. GORTON
A. M. GREENE, JR.
W. G. HUMPTON
J. O. LEECH
I. E. MOULTROP
C. O. MYERS
H. B. OATLEY
JAMES PARTINGTON
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S. K. VARNES
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W. F. DURAND
T. E. DURBAN
C. L. HUSTON
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H. H. VAUGHAN
H. LEROY WHITNEY

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L. M. BARRINGER, Seattle, Wash.
J. G. BOLLOCK, St. Joseph, Mo.
B. M. BOOK, Pennsylvania
J. C. BRYAN, St. Louis, Mo.
L. M. CAVE, Maryland
JAMES DONOHUE, Indiana
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GERALD GEARON, Chicago, Ill.
J. M. GILLESPIE, Tulsa, Okla.
C. H. GRAM, Oregon
B. GRETZKE, Washington
F. A. HECKINGER, Memphis, Tenn.
H. K. KUGEL, District of Columbia
G. A. LUCK, Massachusetts
R. B. MCCOLLUM, Kansas City, Mo.
C. E. MCGINNIS, Los Angeles, Calif.
H. H. MILLS, Detroit, Mich.
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F. A. PAGE, California
L. C. PEAL, Nashville, Tenn.
E. K. SAWYER, Maine
A. H. SCHLEMAN, Tampa, Florida
J. F. SCOTT, New Jersey
J. N. SEIGER, Evanston, Ill.
C. I. SMITH, Utah
H. B. SMITH, North Carolina
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J. M. HALL
H. B. OATLEY

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OFFICER-IN-CHARGE, Specifications Section, Design Division, Bureau of Engineering, U.S. Navy Department

SPECIAL COMMITTEE ON FEEDWATER

C. W. RICE, *Chairman*

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SPECIAL COMMITTEE ON RULES FOR OPENINGS

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SPECIAL COMMITTEE ON RADIOGRAPHIC EXAMINATION OF WELDED JOINTS

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SPECIAL COMMITTEE ON ALLOYED AND HIGH TENSILE STEELS FOR WELDED PRESSURE VESSELS

C. A. ADAMS, *Chairman*

SPECIAL COMMITTEE TO REVISE SECTION VIII OF THE A.S.M.E. BOILER CODE

E. R. FISH, *Chairman*

SPECIAL COMMITTEE ON RULES FOR CONTAINERS FOR GASES AND LIQUIDS AT SUBZERO TEMPERATURES

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D. S. JACOBUS
T. McLEAN JASPER
JAMES PARTINGTON

A.P.I. Representatives

A. J. ELY
K. V. KING
(P. D. McELFISH, *Alternate*)
R. C. POWELL
WALTER SAMANS
T. D. TIFFT

The Woman's Auxiliary to the A.S.M.E.

The Woman's Auxiliary to the A.S.M.E. was organized on May 10, 1923, and its Constitution and By-Laws was approved by the Council of the A.S.M.E. on October 27, 1924. The objects of the Auxiliary are to render service to all that pertains to the interest of the profession of mechanical engineering; to cooperate with any committees of the A.S.M.E.; and to assist the sons and daughters of the members of the Society or worthy students of mechanical engineering in obtaining scholarships; and to promote any other objects consistent with the aims or objects of the A.S.M.E.

OFFICERS

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First Vice-President, MRS. JOHN H. R. ARMS
Second Vice-President, MRS. E. C. STAHL
Third Vice-President, MRS. RANDALL B. PURDY
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Education, MRS. ROY V. WRIGHT
Ways and Means, MRS. FREDERICK M. GIBSON
Program, MRS. C. B. LEPAGE
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Hospitality, MRS. JAMES T. McQUEENEY
Publicity, MISS BURTIE HAAR
Custodian, MISS BURTIE HAAR

COUNCIL REPRESENTATIVES

R. F. GAGG
J. H. HERRON

Awards

The following paragraphs deal with the medals, awards, scholarships, and loan funds which come within the jurisdiction of the A.S.M.E. Other awards available to Student Members are listed in *Mechanical Engineering*, February, 1938, page 183. The Society also participates with other engineering societies in a number of joint awards. Further details concerning all the awards will be found in a series of articles beginning in the October, 1938, issue of *Mechanical Engineering*.

Honorary Membership, to which persons of acknowledged professional eminence are elected by unanimous vote of Council under the provisions of the By-Laws and Rules. A list of honorary members is given on page RI-41.

Life Membership, which may be conferred by the Council for distinguished service to the Society; or secured by a member by payment for an annuity in accordance with the provisions of the By-Laws.

A.S.M.E. Medal, established by the Society in 1920 to be presented, together with an engraved certificate, for distinguished service in engineering and science. May be awarded for general service in science having possible application in engineering.

Holley Medal, instituted and endowed in 1924 by George I. Rockwood, Past Vice-President of the Society, to be bestowed, together with an engraved certificate, for some great and unique act of genius of engineering nature that has accomplished a great and timely public benefit.

Worcester Reed Warner Medal, provision for which was made in the will of Worcester Reed Warner, Honorary Member of the Society, is a gold medal to be bestowed, together with an engraved certificate, on the author of the most worthy paper received, dealing with progressive ideas in mechanical engineering or efficiency in management.

Melville Medal, established in 1914 by the bequest of Rear-Admiral George W. Melville, Honorary Member and Past-President of the Society, to be presented, together with an engraved certificate, for an original paper or thesis of exceptional merit, presented to the Society for discussion and publication, to encourage excellence in papers. The medal may be presented annually.

Spirit of St. Louis Medal, established by an endowment fund created in 1929 by citizens of St. Louis, Mo., to be awarded for meritorious service in the advancement of aeronautics. This medal will be awarded at the discretion of the Council of the Society at approximately three-year periods upon the recommendation of its Board of Honors and Awards.

Junior Award, annual cash award of $50, estabished in 1914, from a fund created by Henry Hess, Past Vice-President of the Society, to be presented, together with an engraved certificate, for the best paper or thesis submitted by a Junior Member.

Student Awards, two annual cash awards of $25 each, established in 1914, from a fund created by Henry Hess, Past Vice-President of the Society, to be presented, together with engraved certificates, for the best papers or theses submitted by Student Members. The awards for 1932 and subsequent years have been given, one for undergraduate and the other for postgraduate work.

Charles T. Main Award, annual cash award of $150, established in 1919 from a fund created by Charles T. Main, Past-President of the Society, to be awarded, together with an engraved certificate, to a Student Member of the Society, for the best paper within the general subject of the influence of the profession upon public life. The exact subject is assigned by the Board of Honors and Awards, subject to the approval of the Council, and is announced each year through the Honorary Chairman of the Student Branches.

SCHOLARSHIPS AND LOAN FUNDS

Max Toltz: Loan Fund of $15,000 established by Major Max Toltz, former member of the Council of the Society, the income to be used for assistance to Student Members.

John R. Freeman: Fund of $25,000 established in 1926 by John R. Freeman, Past-President of the Society, the income to be used for travel scholarships and research.

Woman's Auxiliary: Scholarship or Fellowship offered by the Woman's Auxiliary to the Society to assist sons and daughters of members or worthy students of mechanical engineering.

RECIPIENTS OF AWARDS

The names of the recipients of the different awards to date are given in the following lists, together with the dates of presentation, and the services or papers for which the awards were made. There were no awards for the years not listed.

A.S.M.E. Medal

1921 Hjalmar Gotfried Carlson, in recognition of the services rendered the Government because of his invention and part in the production of 20,000,000 Mark III drawn steel booster casings used principally as a component of 75-mm high-explosive shells, but also used extensively in gas shells and bombs

1922 Frederick Arthur Halsey, for his paper describing the premium system of wage payments presented before the Society at the Providence Meeting in 1891, as the adoption of the methods there proposed has had a profound effect toward harmonizing the relations of worker and employer

1923 John Ripley Freeman, for his eminent service in engineering and manufacturing by his meritorious work in fire prevention and the preservation of property

1926 R. A. Millikan, in recognition of his contributions to science and engineering

1927 Wilfred Lewis, for his contributions to the design and construction of gear teeth

1928 Julian Kennedy, for his services and contributions to the iron and steel industry

1929 William LeRoy Emmet, for his contributions in the development of the steam turbine, electric propulsion of ships, and other power-generating apparatus

1931 Albert Kingsbury, for his research and development work in the field of lubrication

1933 Ambrose Swasey, for his contributions to the advancement of the engineering profession and for his part in the development of the turret lathe and the astronomical telescope

1934 Willis H. Carrier, in recognition of his research and development work in air-conditioning

1935 Charles T. Main, for distinguished achievements in the textile and other industries, in engineering education, and for eminent service to the engineering profession

1936 Edward Bausch, for meritorious mechanical developments in the field of optics

1937 Edward P. Bullard, for outstanding leadership in the development of station-type machine tools

1938 Stephen J. Pigott, for outstanding leadership in marine propulsion and construction.

Holley Medal

1924 Hjalmar Gotfried Carlson, for his inventions and processes which made possible the timely production of drawn steel booster casings for artillery ammunition, thereby aiding victory in the World War (diploma in recognition of achievements presented in 1921)

1927 Elmer Ambrose Sperry, for achievements and inventions that have advanced the naval arts, including the gyroscope that has freed navigation from the dangers of the fluctuating magnetic compass

1929 Baron Chuzaburo Shiba, for his contributions to knowledge through fundamental research, including the field of aerodynamics, by the development of ultra-rapid kinematographic methods

1934 Irving Langmuir, for his contributions to science and engineering, including the development of gas-filled incandescent lamps, thoriated filament for thermionic emission, atomic hydrogen welding, phase control operation of the thyratron tube, and fundamental research in oil films

1936 Henry Ford, for revolutionary influence through invention and practice on transportation and on mass production methods in manufacturing

1937 Frederick G. Cottrell, for preeminent public service—the invention of electric precipitation—advancement of the science of gas liquefaction—gifts for engineering research

1938 Francis Hodgkinson, for meritorious services in the development of the steam turbine.

Worcester Reed Warner Medal

1933 Dexter S. Kimball, for his contributions to efficiency in management as exemplified by his recently revised "Principles of Industrial Organization" and by his many articles, engineering society papers, and public addresses
1934 Ralph E. Flanders, for his contributions to a better understanding of the relationship of the engineer to economic problems and social trends as exemplified by the many papers which he has presented
1935 Stephen Timoshenko, for his contributions to the theory of the design of elastic structures and the treatment of dynamics of moving machinery
1936 Charles M. Allen, for his early and continued hydraulic laboratory work and for the permanent value of the papers on his development of methods of testing large hydraulic turbine installations
1937 Clarence F. Hirshfeld, for his research and contributions to the theory and practice of heat-power engineering as exemplified by books and papers
1938 Lawford H. Fry, for contributions relating to improved locomotive boiler design and utilization of better materials in railway equipment.

Melville Medal

1927 Leon P. Alford, "Laws of Manufacturing Management"
1929 Joseph W. Roe, "Principles of Jig and Fixture Practice"
1930 Herman Diederichs and William D. Pomeroy, "The Occurrence and Elimination of Surge or Oscillating Pressure in Discharge Lines From Reciprocating Pumps"
1931 Arthur E. Grunert, "Comparative Performance of a Pulverized-Coal-Fired Boiler Using Bin System and Unit System of Firing"
1932 Alexey J. Stepanoff, "Leakage Loss and Axial Thrust in Centrifugal Pumps"
1933 William E. Caldwell, "Characteristics of Large Hell Gate Direct-Fired Boiler Units"
1935 Oscar R. Wikander, "Draft-Gear Action in Long Trains"
1936 H. A. Stevens Howarth, "The Loading and Friction of Thrust and Journal Bearings With Perfect Lubrication"
1937 Alfred J. Büchi, "Supercharging of Internal-Combustion Engines With Blowers Driven by Exhaust-Gas Turbines"
1938 Alphonse I. Lipetz, "Air Resistance of Railroad Equipment."

Spirit of Saint Louis Medal

1929 Daniel Guggenheim, founder of the Guggenheim Fund for the Promotion of Aeronautics
1932 Paul Litchfield, for his work in encouraging and sponsoring airship design and construction in this country
1935 Will Rogers, for his splendid, constructive, and unselfish work in the achievement of aviation, and the building up of public confidence in aviation through his articles in the press, over the radio, and from the speaker's platform
1938 James H. Doolittle, for meritorious service in the advancement of aeronautics.

Junior Award

1915 Ernest O. Hickstein, "Flow of Air Through Thin Plate Orifices"
1916 L. M. McMillan, "The Heat Insulating Properties of Commercial Steam-Pipe Coverings"
1919 E. D. Whalen, "Properties of Airplane Fabrics"
1921 S. Logan Kerr, "Moody Ejector Turbine"
1922 R. H. Heilman, "Heat Losses From Bare and Covered Wrought-Iron Pipe at Temperatures up to 800 Degrees Fahrenheit"
F. L. Kallam, "Preliminary Report on the Investigation of the Thermal Conductivity of Liquids"
1923 S. S. Sanford and S. Crocker, "The Elasticity of Pipe Bends"
1924 R. H. Heilman, "Heat Losses Through Insulating Material"
1925 Gilbert S. Schaller, "An Investigation of Seattle as a Location for a Synthetic Foundry Industry"
1927 William M. Frame, "Stresses Occurring in the Walls of an Elliptical Tank Subjected to Low Internal Pressure"
1928 M. D. Aisenstein, "A New Method of Separating the Hydraulic Losses in a Centrifugal Pump"
1929 Arthur M. Wahl, "Stresses in Heavy, Closely Coiled Helical Springs"
1930 Ed Sinclair Smith, "Quantity-Rate Fluid Meters"
1931 M. K. Drewry, "Radiant-Superheater Developments"
1932 Edmond M. Wagner, "Frictional Resistance of a Cylinder Rotating in a Viscous Fluid Within a Coaxial Cylinder"
1933 Townsend Tinker, "Surface Condenser Design and Operating Characteristics"
1934 John I. Yellott, Jr., "Supersaturated Steam"
1935 Stanley J. Mikina, "Effect of Skewing and Pole Spacing on Magnetic Noise in Electrical Machinery"
1936 Harwood F. Mullikan, Jr., "Evaluation of Effective Radiant Heating Surface and Application of the Stefan-Boltzman Law to Heat Absorption in Boiler Furnaces"
1937 Leslie J. Hooper, "American Hydraulic-Laboratory Practice"
1938 Arthur C. Stern, "Separation and Emission of Cinders and Fly Ash."

Student Award

1916 Boynton M. Green, Stanford University, "Bearing Lubrication"
Howard E. Stevens, Rensselaer Polytechnic Institute, "An Investigation of the Dynamic Pressure on Submerged Flat Plates"
M. Adam, Louisiana State University, "The Adaptability of the Internal Combustion Engine to Sugar Factories and Estates"
1917 H. R. Hammond and C. W. Holmberg, The Pennsylvania State College, "Study of Surface Resistance With Glass as the Transmission Medium"
1919 C. F. Leh and F. G. Hampton, Stanford University, "An Experimental Investigation of Steel Belting"
W. E. Helmick, Stanford University, "An Experimental Investigation of Steel Belting"
1920 Howard G. Allen, Cornell University, "Wire Stitching Through Paper"
1921 Karl H. White, University of Kansas, "Forces in Rotary Motors"
Richard H. Morris and Albert J. R. Houston, University of California, "A Report Upon an Investigation of the Herschel Type of Improved Weir"
1923 Charles F. Olmstead, University of Minnesota, "Oil Burning for Domestic Heating"
H. E. Doolittle, University of California, "The Integrating Gate: A Device for Gaging in Open Channels"
1924 George Stuart Clark, Stanford University, "Two Methods Used for the Determination of the Gasoline Content of Absorption Oils in Absorption Plants"
L. J. Franklin and Charles H. Smith, Stanford University, "The Effect of Inaccuracy of Spacing on the Strength of Gear Teeth"
1925 Harry Pease Cox, Jr., Rensselaer Polytechnic Institute, "A Study of the Effect of End Shape on the Towing Resistance of a Barge Model"
W. S. Montgomery, Jr., and E. Ray Enders, Jr., Pennsylvania State College, "Some Attempts to Measure the Drawing Properties of Metals"
1926 R. E. Peterson, University of Illinois, "An Investigation of Stress Concentration by Means of Plaster of Paris Specimens"
Cecil G. Heard, University of Toronto, "Pressure Distribution Over U.S.A. 27 Aerofoil With Square Wing Tips—Model Tests"
1927 Alfred H. Marshall, Princeton University, "Evaporative Cooling"
Roger Irmin Eby, University of Washington, "Measurement of the Angular Displacement of Flywheels"
1928 Clarence C. Franck, Johns Hopkins University, "Condition Curves and Reheat Factors for Steam Turbines"
1929 Frank Vernon Bistrom, University of Washington, "An Investigation of a Rotary Pump"
William Wallace White, University of Washington, "An Investigation of a Rotary Pump"
1930 Gerard Eden Claussen, Polytechnic Institute of Brooklyn, "High-Temperature Oxidation of Steel"
Harold L. Adams and Richard L. Stith, University of Washington, "A Wind Tunnel for Undergraduate Laboratory Experiments"
1931 Jules Podnossoff, Polytechnic Institute of Brooklyn, "Pressure and Energy Distribution in Multi-Stage Steam Turbines Operating Under Varying Conditions"
1932 H. E. Foster, Jr., University of Tennessee, "Factors Affecting Spray Pond Design" (Undergraduate Award)
William A. Mason, Stanford University, "An Experimental Investigation of the Flame Propagation in Internal-Combustion Engines" (Postgraduate Award)

1933 HUGO V. CORDIANO, Polytechnic Institute of Brooklyn, "Thermal Analysis of Lithium–Magnesium System of Alloys" (Undergraduate Award)
JAMES A. OSTRAND, JR., Princeton University, "Sudden Enlargement in the Open Channel" (Postgraduate Award)

1934 H. REYNOLDS HUDSON, Georgia School of Technology, "Dynamic Balance and Functional Utility Applied to Automotive Design" (Undergraduate Award)

1935 CHARLES P. BACHA, Rutgers University, "The Behavior of Metals Subjected to Combined Stress" (Postgraduate Award)
ROBERT W. BEAL, Oregon State College, "Do Lubricating Oils Wear Out?" (Undergraduate Award)

1936 LEON B. STINSON, Oklahoma Agricultural and Mechanical College, "Polymerized Motor Fuels; Their Economic Significance" (Undergraduate Award)
DEWITT D. BARLOW, JR., Princeton University, "The Critical Speeds of Lateral Vibrations of Shafts With Gyroscopic Effects" (Postgraduate Award)

1937 GINO J. MARINELLI, Rensselaer Polytechnic Institute, "Investigation of the Towing Resistance of a Model Submarine Hull" (Undergraduate Award)

1938 MARSHALL C. LONG, Princeton University, "An Investigation Into the Angular Characteristics of an Adjustable Blade Current Meter" (Postgraduate Award)
DONALD C. MCSORLEY, Michigan State College, "Humidity Insulation" (Undergraduate Award).

CHARLES T. MAIN AWARD

1925 CLEMENT R. BROWN, Catholic University of America. Subject: "The Influence of the Locomotive on the Unity of the United States"

1926 W. C. SAYLOR, Johns Hopkins University. Subject: "The Effect of the Cotton Gin Upon the History of the United States During Its First Seventy Years"

1927 No Award. Subject: "Scientific Management and Its Effect Upon the Industries"

1928 ROBERT M. MEYER, Newark College of Engineering. Subject: "Scientific Management and Its Effect Upon Manufacturing"

1929 No award. Subject: "The Influence of Engineering on Farm Production"

1930 JULES PODNOSSOFF, Polytechnic Institute of Brooklyn. Subject: "The Value of the Safety Movement in the Industries"

1931 ROBERT E. KLISE, University of Michigan. Subject: "Interchangeability—Its Development and Significance in Industry"

1932 MARSHALL ANDERSON, University of Michigan. Subject: "Apprenticeship and Vocational Training"

1933 GEORGE D. WILKINSON, JR., Newark College of Engineering. Subject: "Progress in the Prevention of Smoke and Atmospheric Pollution"

1934 PHILIP P. SELF, Colorado State College, "Air Conditioning—Its Practicability and Relation to Public Welfare"

1935 G. LOWELL WILLIAMS, Lafayette College. Subject: "Coordinated Transportation—An Economic Comparison of Railroad, Bus, Truck, Water, and Air Transportation for Long and Short Haul"

1936 No award. Subject: "Development in the Generation and Distribution of Power and Their Effect Upon the Consumer"

1937 ALLAN P. STERN, Case School of Applied Science. Subject: "The Influence of the Introduction of Labor Saving Machinery Upon Employment in the United States"

1938 EDWARD W. CONNOLLY, University of Detroit. Subject: "Economic Limitations in Engineering Design, With Concrete Examples."

FREEMAN TRAVEL SCHOLARSHIP

1927 HERBERT N. EATON
1928 BLAKE R. VAN LEER
1929 ROBERT T. KNAPP
1931 REGINALD WHITAKER
1932 G. ROSS LORD
1933, 1934 H. J. CASEY
1935, 1936 VICTOR L. STREETER

Honorary Members

HONORARY MEMBERS IN PERPETUITY

ALEXANDER LYMAN HOLLEY, Founder of the Society. Died 1882.

JOHN EDSON SWEET, Founder of the Society. Died 1916.

HENRY ROSSITER WORTHINGTON, Founder of the Society. Died 1880.

DECEASED HONORARY MEMBERS

	ELECTED	DIED
HORATIO ALLEN	1880	1889
SIR WILLIAM ARROL	1905	1913
SIR JOHN AUDLEY FREDERICK ASPINALL	1911	1937
WILLIAM WALLACE ATTERBURY	1925	1935
SIR BENJAMIN BAKER	1886	1907
JOHANN BAUSCHINGER	1884	1893
SIR HENRY BESSEMER	1891	1898
SIR FREDERICK JOSEPH BRAMWELL	1884	1903
JOHN ALFRED BRASHEAR	1908	1920
GUSTAVE CANET	1900	1908
ANDREW CARNEGIE	1907	1919
DANIEL KINNEAR CLARK	1882	1896
RUDOLPH JULIUS EMMANUEL CLAUSIUS	1882	1888
SIR JOHN GOODE	1889	1892
PETER COOPER	1882	1883
CHARLES DE FRÉMINVILLE	1919	1936
CARL GUSTAF PATRICK DE LAVAL	1912	1913
RUDOLPH DIESEL	1912	1913
JAMES DREDGE	1886	1906
VICTOR DWELSHAUVERS-DERY	1886	1913
THOMAS ALVA EDISON	1904	1931
ALEXANDRE GUSTAVE EIFFEL	1889	1923
MARSHAL FERDINAND FOCH	1921	1929
SIR CHARLES DOUGLAS FOX	1900	1921
JOHN RIPLEY FREEMAN	1932	1932
JOHN FRITZ	1900	1913
MAJOR-GENERAL GEORGE WASHINGTON GOETHALS	1917	1928
FRANZ GRASHOF	1884	1893
REAR-ADMIRAL ROBERT STANISLAU GRIFFIN	1920	1933
OTTO HALLAUER	1882	1883
CHARLES HAYNES HASWELL	1905	1907
NATHANAEL GREENE HERRESHOFF	1921	1938
FRIEDRICH GUSTAV HERRMANN	1884	1907
GUSTAV ADOLPH HIRN	1882	1890
JOSEPH HIRSCH	1889	1901
IRA N. HOLLIS	1928	1930
ROBERT WOOLSTON HUNT	1920	1923
BENJAMIN FRANKLIN ISHERWOOD	1894	1915
HENRI LÉAUTÉ	1891	1916
ERASMUS DARWIN LEAVITT	1915	1916
HENRI LE CHATELIER	1927	1936
ANATOLE MALLET	1912	1919
CHARLES H. MANNING	1913	1919
REAR-ADMIRAL GEORGE WALLACE MELVILLE	1910	1912
THE HONORABLE SIR CHARLES ALGERNON PARSONS	1920	1931
CHARLES TALBOT PORTER	1890	1910
AUGUSTE C. E. RATEAU	1919	1930
SIR EDWARD J. REED	1882	1906
FRANZ REULEAUX	1882	1905
CALVIN WINSOR RICE	1931	1934
PALMER C. RICKETTS	1931	1934
HENRI ADOLPHE-EUGENE SCHNEIDER	1882	1898
C. WILLIAM SIEMANS	1882	1883
VISCOUNT EI-ICHI SHIBUSAWA	1929	1931
AMBROSE SWASEY	1916	1937
ELIHU THOMSON	1930	1937
HENRY ROBINSON TOWNE	1921	1924
HENRI TRESCA	1882	1885
WILLIAM CAWTHORNE UNWIN	1898	1933
OSKAR VON MILLER	1912	1934
FRANCIS A. WALKER	1886	1897
WORCESTER REED WARNER	1925	1929
GEORGE WESTINGHOUSE	1897	1914
SIR WILLIAM HENRY WHITE	1900	1913
SIR ALFRED FERNANDEZ YARROW	1914	1932

LIVING HONORARY MEMBERS

	ELECTED
LORENZO ALLIEVI	1937
HUTCHINSON I. CONE	1936
MORTIMER ELWYN COOLEY	1928
ALEX DOW	1936
WILLIAM FREDERICK DURAND	1934
HERBERT CLARK HOOVER	1925
DAVID SCHENCK JACOBUS	1934
MASAWO KAMO	1929
GEORGE A. ORROK	1936
GRANDE UFFICIALE ING. PIO PERRONE	1920
CHARLES M. SCHWAB	1918
WILLIAM H. TSCHAPPAT	1938
SAMUEL MATTHEWS VAUCLAIN	1920
RIGHT HONORABLE LORD WEIR	1920
ORVILLE WRIGHT	1918

Past-Presidents

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ALEXANDER LYMAN HOLLEY, *Chairman of the Preliminary Meeting for Organization of The American Society of Mechanical Engineers* (1882)

1880–1882 ROBERT HENRY THURSTON (1903)
1883 ERASMUS DARWIN LEAVITT (1916)
1884 JOHN EDSON SWEET (1916)
1885 JOSEPHUS FLAVIUS HOLLOWAY (1896)
1886 COLEMAN SELLERS (1907)
1887 GEORGE H. BABCOCK (1893)
1888 HORACE SEE (1909)
1889 HENRY ROBINSON TOWNE (1924)
1890 OBERLIN SMITH (1926)
1891 ROBERT WOOLSTON HUNT (1923)
1892 CHARLES HARDING LORING (1907)
1893–1894 ECKLEY BRINTON COXE (1895)
1895 EDWARD F. C. DAVIS (1895)
1895 CHARLES ETHAN BILLINGS (1920)
1896 JOHN FRITZ (1913)
1897 WORCESTER REED WARNER (1929)
1898 CHARLES WALLACE HUNT (1911)
1899 GEORGE WALLACE MELVILLE (1912)
1900 CHARLES HILL MORGAN (1911)
1901 SAMUEL T. WELLMAN (1919)
1902 EDWIN REYNOLDS (1909)
1903 JAMES MAPES DODGE (1915)
1904 AMBROSE SWASEY (1937)
1905 JOHN RIPLEY FREEMAN (1932)
1906 FREDERICK WINSLOW TAYLOR (1915)
1907 FREDERICK REMSEN HUTTON (1918)
1908 MINARD LAFEVER HOLMAN (1925)
1909 JESSE MERRICK SMITH (1927)
1910 GEORGE WESTINGHOUSE (1914)
1911 EDWARD DANIEL MEIER (1914)
1912 ALEXANDER CROMBIE HUMPHREYS (1927)
1913 WILLIAM FREEMAN MYRICK GOSS (1928)
1914 JAMES HARTNESS (1934)
1915 JOHN ALFRED BRASHEAR (1920)
1916 DAVID SCHENCK JACOBUS
1917 IRA NELSON HOLLIS (1930)
1918 CHARLES THOMAS MAIN
1919 MORTIMER ELWYN COOLEY
1920 FRED J. MILLER
1921 EDWIN S. CARMAN
1922 DEXTER SIMPSON KIMBALL
1923 JOHN LYLE HARRINGTON
1924 FREDERICK ROLLINS LOW (1936)
1925 WILLIAM FREDERICK DURAND
1926 WILLIAM LAMONT ABBOTT
1927 CHARLES M. SCHWAB
1928 ALEX DOW
1929 ELMER AMBROSE SPERRY (1930)
1930 CHARLES PIEZ (1933)
1931 ROY V. WRIGHT
1932 CONRAD N. LAUER
1933 A. A. POTTER
1934 PAUL DOTY (1938)
1935 RALPH E. FLANDERS
1936 WILLIAM L. BATT
1937 JAMES H. HERRON
1938 HARVEY N. DAVIS

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of the

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(Including Indexes to Publications)

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Dresden — Bibliothek der Technischen Hochschule

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Library of the Escuela de Ingenieros Mecanicos y Electricistas

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Oslo — Den Polytekniske Forening

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Moscow.....Supreme Council of National Economy
Tomsk.....Tomsk Polytechnic Institute

Indexes to A.S.M.E. Papers and Publications

THE following pages will serve as a guide to the current publications of the A.S.M.E. during the calendar year 1939, and also to publications developed by the technical committees. The publications of the Society are as follows:

REGULAR SOCIETY PUBLICATIONS, 1939

Mechanical Engineering, monthly (see index on pages RI-53-62)
A.S.M.E. Transactions, monthly (see index on pages RI-63-71)
Mechanical Catalog and Directory, 1940 edition

SPECIAL PUBLICATIONS ISSUED IN 1939

1938 Oil Engine Power Cost Report
1939 Proceedings of Oil and Gas Power Division
Manual on Cutting of Metals

Standards

Indicating Pressure and Vacuum Gages
Terminology and Definitions for Single Point Cutting Tools
Face-to-Face Dimensions of Ferrous Flanged and Welding End Valves
Malleable-Iron Screwed Fittings—150 Lb
Wrought-Iron and Wrought-Steel Pipe
Cast-Iron Pipe Flanges and Flanged Fittings, Class 125
Steel Pipe Flanges and Flanged Fittings for Maximum WSP of 150 to 2500 Lb

Power Test Codes

Displacement Compressors, Vacuum Pumps and Blowers
Instruments and Apparatus, Part 13—Speed Measurements

Research

Bibliography on Machining of Wood

Safety

Compressed-Air Machinery and Equipment

Papers Presented at A.S.M.E. Meetings, 1939

THE complete technical programs of the meetings of the Society and of its Professional Divisions have been published in *Mechanical Engineering* and may be located by consulting the index on pages RI-53-62. A considerable number of papers and reports included in these programs were not published during the year in either Transactions or *Mechanical Engineering*, but were issued in mimeographed or photo-offset form. Complete sets of these are on file for reference purposes at the office of the Society and the Engineering Societies Library, under the title of "Miscellaneous Papers Presented at A.S.M.E. Meetings, 1939." Photostatic copies of any of the papers may be secured from the Library at regular rates. A list of these papers and reports follows:

MISCELLANEOUS PAPERS PRESENTED AT A.S.M.E. MEETINGS IN 1939—NOT PUBLISHED

ANDERSON, A. W., Some Problems in the Burning of Fuel Oil
BERNHARD, R. K., PERRY, T. D., AND STERN, E. G., Super-Pressed Plywood
Bibliography on the Machining of Wood
BLISS, GEORGE F., Worsted System Spinning
BOELTER, L. M. K., Basic Engineering Factors in Agricultural Technology and Illustrations
BOSSE, L. J., Economic Status of Hardwood Dimension Industry
BRANDT, CARL D., Staple Rayon Spinning
COOK, D. D., Notes on Piston Rings and Cylinder Wear
DEGLER, HOWARD E., Interpretation of Smoky Exhaust of Diesel Engines
DODGE, J. M., Factors Which Influence the Design of Piston Rings and Materials Used in Them
DOST, F. J., Making Better Machine-Tool Castings
ELLIS, RAY E., Apprentice Training
FONDA, BAYARD P., Controlled Air Conditions in the Printing Industry
HEINZ, W. B., Performance Characteristics of Dye Jigs
HERBECK, C. J., Burning of Oathulls
KRONERT, JOSEF, Regulation of Power-Plant Variables
LLOYD, WILLIAM, Industrial Firing of Anthracite
MALEEV, V. L., Combustion-Chamber Development and Characteristics, as Applied to American High-Speed, Compression-Ignition Oil Engines
MAXWELL, C. R., AND ROSEN, C. G. A., The Development of the High-Speed Diesel Engine in the United States
MEDLEY, J. W., Development of Dimensional Control and Tolerances for Wood Products
NEFF, ELMER H., Apprenticeship in the Machine-Tool Industry
NEUMANN, G., AND WÜNSCH, G., Regulators
NIXON, STUART, Notes on Piston Rings From the Operator's Viewpoint
OVERBECK, J. E., AND BEITLER, S. R., Methods of Measuring Large Volumes of Natural Gas
PEARSON, HARRY R., Mountain Creek Steam Electric Station an Example of Semi-Outdoor Construction
Report on Oil-Engine Power Cost for 1938
RINCLIFFE, R. G., Modern Coal Purchasing
SCHMIDT, WALTER, Laws of Direct Regulation on a General Basis
SCOTT, DONALD C., Testing the Stiffness of Yarns and Twines
STEIN, THEODORE, Self-Regulation
Technical Memoranda on Automatic Control, 1938
TENOT, N. A., An Experimental Study of Regulation
VEIHMEYER, F. J., Water Regimen of Soils in Place in the Field
VINCENT, E. T., Injection Systems for Oil Engines
VOLZ, F. O., The Use of Electric Welded Construction for Columns and Bases in the Machine-Tool Industry
WEHMHOFF, B. L., Paper Testing
WILLIAMS, PAUL, Lubricating Systems for Diesel Engines
WORKMEN, T. R., Coordination of Fuel Engineering From the Mine to the Consumer

Publications Developed by the Technical Committees

THE Society's technical committees, the first of which was organized many years ago and all of which have been continuously at work on codes, standards, research, and other special reports, have developed a series of publications of permanent value to the membership. The following list is first presented here for record and for ready reference. This list covers the entire group of publications of these committees completed to date which are now available.

To assist the members in securing copies of these publications the sale price is also given. A discount of 10 per cent is allowed to A.S.M.E. members on standards and a 20 per cent discount on all other publications except where otherwise noted.

RESEARCH

Dynamic Loads on Gear Teeth (1932), $1.50
Fluid Meters:
Part 1—Theory and Application (1937), $3.00

Part 2—Description of Meters (1931), $1.75
Part 3—Selection and Installation (1933), $1.50
Report of the AGA-ASME Committee on Orifice Coefficients (1935), $2.75
Tests on Electrical Equipment for Drilling Rotary Drilled Oil Wells (1933), $0.85
Tests on Steam Equipment for Drilling Rotary Drilled Oil Wells (1932), $0.85
Roll-Neck Bearings (1935), $1.50
Bibliography on Cutting of Metals (1866–1930), $1.25
Bibliography on Deterioration of Condensing Equipment (1845–1930), $1.25
Bibliography on Effect of Temperature Upon Properties of Metals (1828–1931), $1.25
Bibliography on Management Literature and Supplement (1903–1935), $2.75
Bibliography on Mechanical Springs (1678–1927), $1.25
Bibliography on Woods of the World (1928), $1.25
Bibliography on Marketing Research (1935), $1.00

POWER TEST CODES

Test Codes for

Atmospheric Water-Cooling Equipment (1930), $0.45
Centrifugal and Rotary Pumps (revision in preparation)
Compressors and Exhausters (1935), $0.95
Displacement Compressors, Vacuum Pumps, and Blowers
Evaporating Apparatus (revision in preparation)
Feedwater Heaters (1927), $0.35
Gas Producers (1928), $0.55
Hydraulic Prime Movers (1938), $0.60
Internal-Combustion Engines (1930), $0.55
Liquid Fuels (1930), $0.35
Reciprocating Steam Engines (1935), $0.65
Reciprocating Steam-Driven Displacement Pumps (1927), $0.65
Refrigerating Systems (1927), $0.55
Solid Fuels (1931), $0.55
Speed-Responsive Governors (1927), $0.45
Stationary Steam-Generating Units (1936), $0.60
Steam Condensing Apparatus (1938), $0.65
Steam Locomotives (1927), $0.55
Steam Turbines (1928), $0.55

Supplementary Codes and Publications

Definitions and Values (1931), $0.40
General Instructions (1929), $0.35
Instruments and Apparatus:
Part 1—General Consideration (1935), $0.35
Part 2—Pressure Measurement; Chapter 1, Barometers, and Chapter 6, Tables, Multipliers, and Standards (revision in preparation)
Part 2—Pressure Measurement; Chapter 2, Static and Total Pressure, Static Holes and Tubes, Impact Tubes, and Chapter 3, Pipes for Pressure Measurements (1936), $0.65
Part 2—Pressure Measurement; Chapter 4, Bourdon, Bellows, Diaphragm, and Deadweight Gages (1938), $0.65
Part 3—Temperature Measurement; Chapter 1, General; Chapter 5, Pyrometric Cones; Chapter 6, Liquid-in-Glass Thermometers; and Chapter 7, Bourdon Tube Thermometers (1931), $0.75
Part 3—Temperature Measurement; Chapter 2, Radiation Pyrometers (1936), $0.55
Part 3—Temperature Measurement; Chapter 8, Optical Pyrometers (1933), $0.35
Part 4—Head Measuring Apparatus (1933), $0.35
Part 6—Electrical Measurements (1934), $1.25
Part 9—Heat of Combustion (1932), $0.40
Part 10—Flue and Exhaust Gas Analyses (1936), $1.35
Part 11—Determination of Quality of Steam (1931), $0.45
Part 12—Measurement of Time (1932), $0.35
Part 13—Speed Measurements (1939), $0.45
Part 14—Linear Measurements (1936), $0.55
Part 15—Measurement of Surface Areas (1937), $0.75
Part 16—Density Determinations (1931), $0.30
Part 17—Determination of the Viscosity of Liquids (1931), $0.45
Part 18—Humidity Determinations (1932), $0.50
Part 20—Smoke-Density Determinations (1936), $0.65
Part 21—Leakage Measurement, Chapter 1, Condenser Leakage Tests (1928), $0.35
Part 21—Leakage Measurement; Chapter 2, Boiler and Piping; Chapter 3, Steam Engine Leakage (1932), $0.35

BOILER CODE

Power Boiler Code (1937) Including Materials Specifications and 1938–1939 Addenda, $2.25
Locomotive Boiler Code (1937) with 1938–1939 Addenda, $0.55
Low-Pressure Heating Boiler Code (1935) with 1936–1938 Addenda, $0.65
Miniature Boiler Code (1937) with 1938–1939 Addenda, $0.50
Suggested Rules for Care of Power Boilers (1935), $0.70
A.S.M.E. Unfired Pressure Vessel Code (1937) with 1938–1939 Addenda, $0.75
A.S.M.E. Boiler Construction Code, Combined Edition (1937) with 1938–1939 Addenda, $5.50
Boiler Code Interpretation Sheets, $2.75 per set with binder. Single sheets, $0.15
Annual Subscription, $2.50

STANDARDS

Machine Shop Practice Standards

Shafting and Stock Keys (B17.1—1934), $0.45
Code for Design of Transmission Shafting (B17c—1927), $0.75
Woodruff Keys, Keyslots, and Cutters (B17f—1930), $0.35
Tolerances, Allowances, and Gages for Metal Fits (B4a—1925), $0.50
Screw Threads for Bolts, Nuts, Machine Screws, and Threaded Parts (B1.1—1935), $0.60
Wrench-Head Bolts and Nuts and Wrench Openings (B18.2—1933), $0.50
Slotted Head Proportions; Machine Screws, Cap Screws, and Wood Screws (B18c—1930), $0.45
Track Bolts and Nuts (B18d—1930), $0.40
Round Unslotted-Head Bolts, Carriage, Step, and Machine Bolts (revision in preparation)
Plow Bolts (B18f—1928), $0.35
Tinners', Coopers', and Belt Rivets (B18g—1928), $0.35
Rotating Air Cylinders and Adapters (B5.5—1932), $0.35
Jig Bushings (B5.6—1935), $0.35
T-Slots, Their Bolts, Nuts, Tongues, and Cutters (B5a—1927), $0.35
Tool Holder Shanks and Tool Post Openings (B5b—1929), $0.25
Milling Cutters (B5c—1930), $0.75
Taps—Cut and Ground Threads (revision in preparation)
Spur Gear Tooth Form (B6.1—1932), $0.45
Shaft Couplings (B49—1932), $0.35
Drawings and Drafting Room Practice (Z14.1—1935), $0.50
Small Rivets (B18a—1927), $0.30
Large Rivets: 1/2 In. Diameter and Larger (B18.4—1937), $0.65
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Fire Hose Coupling Screw Thread (B26—1925), $0.25
Hose Coupling Screw Threads (B33.1—1935), $0.25
Rolled Threads for Screw Shells of Electric Sockets and Lamp Bases (C44—1931), $0.35
Indicating Pressure and Vacuum Gages (B40—1939), $0.40
Terminology and Definitions for Single Point Cutting Tools (B5.13—1939), $0.40

Pipe and Fittings

Wrought-Iron and Wrought-Steel Pipe (B36.10—1939), $0.50
Code for Pressure Piping (no discount) (B31.1—1935), $1.00
Cast-Iron Pipe Flanges and Flanged Fittings for 25 Lb Maximum Saturated Steam Pressure (B16b2—1931), $0.40
Cast-Iron Pipe Flanges and Flanged Fittings for 125 Lb Maximum Saturated Steam Pressure (B16a—1939), $0.60
Cast-Iron Pipe Flanges and Flanged Fittings for 250 Lb Maximum Saturated Steam Pressure (B16b—1928), $0.50
Cast-Iron Pipe Flanges and Flanged Fittings for 800 Lb Maximum Hydraulic Pressure (B16b1—1931), $0.35
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